MEASURES OF FUNCTIONAL COUPLING IN DESIGN

by

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Submitted to the Department of Mechanical Engineering on April 15, 1982 in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

ABSTRACT

Minimization of coupling between functional requirements in a design is one of two fundamental principles embodied in a design improvement strategy referred to as an axiomatic approach to design.

A vector field representation of the general design problem has been developed which facilitates understanding the nature of coupling in a design. The characteristics of the vector field representing the design depend on the relationships between functional descriptions and physical descriptions of a design.

Two quantitative measures of coupling are based on the conformation of the vector field representing the design. The first measure, called reangularity, is a measure of the degree to which different design parameters have a similar effect on the set of functional requirements. The second measure, semangularity, is a measure of the degree to which each design parameter affects only one functional requirement of a product.

The measures of coupling are applied to simple passive filter design problems to illustrate the concepts of coupling in design and to demonstrate how measures of coupling are used to select among alternative design configurations.

Thesis Supervisor: Nam P. Suh
Title: Professor of Mechanical Engineering

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To My Parents
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Chapter 1

DESIGN AND DESIGN IMPROVEMENT STRATEGIES

1.1 The Design Process

Design is, in the broadest sense, the process of specifying how articles that are useful to and sought by society are to be achieved. As such, design encompasses all endeavors of society and the efficacy of design, in part, determines the general welfare of society.

This broad definition of design includes activities in all of the engineering disciplines, architecture, urban planning, computer science, management, and the arts. Much of what is said herein relates to all types of design but is intended primarily within the context of engineering design. The topics discussed in later chapters are believed to be applicable in other design disciplines, such as architectural design and design of management systems, but are not discussed in that context here because of the difficulties in quantifying the objectives of designs in those areas.

The engineering design task is to specify the physical components, the topography and the processes required to produce a product which performs a function needed by society. The design objective is to make the specification such that the design, production and utilization of the product consume a minimum of societal resources. Other descriptions of design embody the same basic concept, i.e. a progression from a need to
a means of satisfying the need. Asimow [3] described design as a "progression from the abstract to the concrete." Woodson [49] built on this idea when he described design as a series of transformation processes, including the process of transforming a "nebulous need" to a "specific solution." Yasuhara [50] restated this idea when he described design as a transformation from a functional description of a product to a physical description of a product.

Since design is the process or means for satisfying needs, it is an essential activity of mankind. Accordingly, design activities have been studied a great deal with the purpose of improving the proficiency of designers and the quality of designs. These studies have resulted in a number of strategies for design improvement.

1.2 Approaches to Design Improvement

Design is a complex process. It involves creative, analytical, theoretical and experimental aspects in a complex iterative and recursive structure. The interaction between various aspects of design is widely recognized. Bell [43], Johnson [21], Nann [26], and Asimow [3] have each published a diagramatic description of design showing how they believe problem definition, creativity, analysis and testing interact to produce a final design. Bell uses an analogy to feedback control to point out that accurate analysis within the loop structure of design not only identifies poor design choices but drives the creative process toward solutions that are more likely to be acceptable just as the error signal in a control loop drives the system toward the set-point.

Descriptions of the complete design process, in terms of the complex interactions among creativity, analysis, experimentation, evaluation and
other aspects provide an interesting perspective of the design process, but in practice the individual phases of design have been studied separately. The result is that design studies and methods for improving design are segmented in much the same way that diagrams of the design process are segmented into blocks that describe the individual phases or aspects of design.

One segment of the design process that has received a great deal of attention is that of improving creativity. Osborn [30] and Adams [0] each presented a number of techniques, hints and approaches to enhance the quantity, diversity and quality of design alternatives. Other specialized techniques have also been proposed. These include brainstorming [30], lateral thinking [12], metaphorical techniques [18], synthesis by implication [21], analogs, reversals, visual thinking [25] and a logical building block approach [21] among others. The objective of these techniques is to obtain as many design concepts or possible design configurations as possible. The emphasis is completely on creativity in these techniques. Analysis and discussion of feasibility are usually implicitly or explicitly discouraged during this stage of design with the understanding that all proposed solutions will be analyzed at a later time.

The analysis and optimization of proposed designs has probably received more attention than any other aspect of design. A great number of techniques have been developed to assist the designer in selecting optimum parameter values for a predetermined design configuration. The Method of Optimal Design [22] is one search-based methodology for doing exactly this. Fox [13] and Wilde [45] have also developed techniques for engineering optimization based on search strategies. Search techniques are perhaps the
most important tools available to the optimizing designer primarily because searching a parameter space does not place severe restrictions on the form of the equations that govern system performance or the criterion for selection. Search techniques do have some severe drawbacks however. The most important of these is the computational requirement of searching a parameter space over many dimensions. A second consideration is one of obtaining a global optimal rather than a sub-optimal stationary point or local optimum. A third consideration is the form of the criterion function which is to be optimized. More will be said about criterion functions shortly.

Wilde [46] points out that search can be minimized a great deal and in some cases eliminated altogether by applying other optimization techniques such as monotonicity analysis, geometric programming and partial optimization to simplify an optimization problem. Other techniques, such as linear programming are also useful when the criterion function is of the proper form.

These direct optimization techniques share four serious drawbacks. The first of these drawbacks is that direct optimization is based on finding the extremum value of an of an explicitly stated performance criterion, objective function or performance index. Cost is a very common objective for optimization but to minimize cost requires estimating the life cost of a product including materials, labor, lifetime and depreciation and also requires assigning a dollar value to reliability, performance, convenience and a host of other factors. Establishing rational objective functions that accurately reflect the goals of the design is particularly difficult for multi-attribute designs.

Gall [14] developed a max-ranking criterion specifically to avoid the
need to derive a composite objective function for a control system subjected to stochastic disturbances. Gall's criterion was based on the worst of many criterion functions that each depended on a single factor. This approach raises other questions because forming the objective function is not simply a matter of design goal fidelity but also a matter of computational convenience. Since Gall's method involves a logical condition to select among criterion functions it is likely to introduce sharp discontinuities in the ultimate objective. This may have serious ramifications on the effectiveness and reliability of various search strategies and may render the problem no longer solvable by special purpose techniques. Modern control design is a field in which the true objective function is often modified to be of a standard form, e.g. quadratic, so that the resulting equations can be solved by standard techniques.

A second difficulty of direct optimization with regard to design is that only substantially complete design configurations or components can be optimized. This may not seem to be a liability but one must remember that optimization is often used to select among alternative design configurations by computing the optimum performance criterion for each of the many proposed configurations. The effect is that each alternative design must be substantially detailed before the best of the basic configurations can be selected.

A third difficulty is that the results of the optimization are little more than a criterion for accepting or rejecting the proposed design. The results do not adequately demonstrate the deficiencies of a proposed design in a way that enables the designer to address critical issues and more quickly arrive at an acceptable design. This lack of constructive feedback from the analysis phase to the creative phase of design is characteristic
of the optimization techniques discussed.

A fourth drawback is that optimization is carried out with respect to a fixed design configuration. The implication of a fixed design configuration is that superior solutions that are of a slightly different configuration are not found. To find such solutions the designer must explicitly optimize for all conceivable design configurations. Weinberger [44] addressed this issue by allowing component values of electrical networks to assume values ranging from zero to infinity thereby creating network configurations that were subsets of the master network. He then developed search techniques that are practical for searching an infinite space. The difficulty in applying this technique is in finding a configuration of the design that is a superset of all of the reasonable design alternatives.

Some strategies for design improvement have addressed the design process as a whole rather than the individual aspects of creativity analysis and the like. One group of these can be classified as organizational strategies for design. One of this group is the morphological approach [29, 51], which is a method for structuring the options that are available to the designer to diminish the chance that some options are overlooked. Another is group technology [19], which is a system for classifying parts. The classifications or groups of designs are based on common features that affect production of the items in the group. The similarity of designs in a group technology environment is believed to reduce production and inventory costs.

Decision analysis [37] and value engineering [10] are used in design to structure the design options and evaluate trade-offs among the options. One advantage of applying decision analysis to product design is, as Starr
[39] has shown, the explicit consideration of risk and uncertainty in design.

A number of individuals have proposed guidelines, rules or methodologies for design. These methodologies span the breadth of design activities. Glegg's approach [15, 16, 17] addressed many aspects of design including problem formulation, analysis of relevant data and decision making in design. Datsko's rules [9] on the other hand are oriented toward detailed specification of machine components. Johnson's logical building-block approach [21] is a step by step procedure for creative design based on combining components or devices that are capable of satisfying some requirements of the design.

Each of the individual techniques, approaches and perspectives of design are valuable for improving some phase of the overall design process. However, the individual techniques cannot be combined in such a way that they comprise a comprehensive methodology for design.

A fundamental difficulty is the lack of synergism among the various techniques. The individual processes work well but do not support those with which they interact. A good example is the interaction between creativity and analytical aspects of design.

The object of most of the techniques for enhancing creativity is to increase the number of design concepts or alternatives. A very successful technique may result in an enormous number of design possibilities. If the designer is to choose rationally among the alternatives, each must be designed in detail, analyzed and optimized. Many designs might have to be built as prototypes and tested and then estimates of production cost are needed to select the final configuration. It is obviously a long and expensive process to completely analyze a large number of design
alternatives. Starr [39] describes a relatively simple design for a three
cOMPONENT device. The few options for each of the nine design decisions
result in 5184 possibilities. Starr points out that 11 man-weeks are
required to analyze the options if only five minutes are devoted to each.
The expense and the computational requirements make the exhaustive
evaluation of alternatives impractical. Most often designers must reject a
large number of design alternatives based on their intuition and experience
and concentrate their detailed evaluation efforts on a few of the
possibilities. Most of the poor designs are removed from consideration at
this point, but certainly a few good designs have been discarded with the
bad.

Frequently, the result of the evaluation process is a determination
that all of the alternatives are incapable of satisfying the design
requirements. In such cases the designer seeks new alternatives but he may
find that the analytical results obtained during the previous evaluation
are of little value in directing him toward designs that are more nearly
satisfactory. The failure of the evaluation stage of design to direct or
focus the creative stage results in the process running in an open loop
mode that does not quickly converge on a favorable design alternative.

Good designs are occasionally discarded and the design process may not
converge quickly to an acceptable design because there are no general
principles that enable designers to evaluate design decisions and select
good designs without either extensive analysis or experience.

1.3 An Axiomatic Approach to Design

Wilde [46] states that the key structure in design problems is sensed
and used intuitively by experienced designers. This is often the case and
has resulted in a great number of successful designs accomplished in a short period of time. Design based solely on experience, however, has drawbacks. A designer may have little experience when faced with a large scale interdisciplinary design or when he begins his career. The solution to this problem is training but it would certainly be advantageous if young engineers could be taught basic principles of design to reduce the training required to develop a competent designer.

Although many good designs have resulted from an experienced designers intuition and experience, we must also recognize that many poor designs have also resulted. What is needed is a way of culling from the history of good design and from the collective experience of good designers a set of basic principles of design. The basic principles would enhance a young designer's ability to produce good designs and would assist experienced designers working in new fields. Basic principles of design would enable designers to select among alternative design configurations before detailed design, analysis and optimization are carried out and would be a standard by which the evaluation of unsatisfactory designs helps to focus creative efforts toward superior design configurations.‡

This is the basis of an axiomatic approach to design [40, 43]. Design based on axioms is fundamentally different from all existing techniques. An axiomatic approach is based on the belief that fundamental principles or axioms of good design practice exist. The axioms can be applied at all stages, phases and levels of the design process. As such the axioms can be used to guide and evaluate design decisions.

‡Starr [39] has emphasized this point. "We believe that the use of decision methods encourages creative endeavors. Far-out ideas can be evaluated. The results of evaluation are fed back to the designers. there is more, not less stimulation."
An axiomatic approach has many advantages over conventional design improvement strategies. Since axioms are used as an analytical tool to evaluate decisions, rather than substantially complete designs, an axiomatic approach can greatly simplify the design process by eliminating at an early stage of the process many alternatives that ultimately prove to be unsatisfactory. A set of design axioms is a criterion for evaluating decisions but it also can be thought of as an index that directs the generation of design alternatives. As such axioms can help to focus the creative aspect of design.

The concepts underlying an axiomatic approach to design are introduced in the next chapter.
Chapter 2
AN AXIOMATIC APPROACH TO DESIGN

2.1 Introduction

An axiomatic approach to design is based on the hypothesis that fundamental principles of good design practice exist [40]. The principles are believed to be applicable at all stages and levels of design and are believed to lead to decisions that maximize the productivity of the entire product synthesis operation.

A set of properly verified design axioms, and a methodology for applying the axioms could enable a designer to evaluate individual decisions during the design process. In particular, a design methodology based on axioms would enable the designer to choose among alternative design configurations before the alternatives were detailed, analyzed and optimized. It would thereby eliminate the dependence on exhaustive evaluation of alternatives and the need to construct and search enormous data bases to automate portions of the design process.

A design methodology based on axioms would also simplify the structure of the design process. The axioms focus creativity and allow selection of the best design alternatives, reducing the looping and branching required in the design process.
2.2 Axioms for Design and Manufacturing

A study of many successful designs led Suh, Bell, and Gossard \cite{40} in 1977 to propose a set of hypothetical axioms for design and manufacturing. This set of axioms embodies two basic concepts. The first of these is that each functional requirement of a product should be satisfied independently by some aspect, feature or component within the design. The second basic concept is that good designs are minimally complex. These two concepts are formalized as the working set of design and manufacturing axioms:

Axiom 1: Maintain the independence of functional requirements.

Axiom 2: Minimize information content.

These hypothetical axioms are being studied to determine if they embody fundamental aspects of good design and to develop methods for applying these axioms in design.

2.3 Complexity in Design

The second of the axioms stated above has to do with the overall complexity, or information content, of the design and manufacturing process. As such, a comprehensive measure of complexity must include factors related to all phases of design, manufacturing and product performance. The complexity of the create, analyze, prototype and test phases of the design process must be considered as well as the product specification per se including materials, topography and tolerance. The precision, repeatability and production rates of manufacturing processes contribute to overall complexity as do production volume, lot size and scheduling delays. Many of these factors are very difficult to quantify, indeed a comprehensive measure of complexity in design must ultimately address fundamental questions with enormous impact, e.g. is a cubic solid
more or less complex than a cylindrical solid in a given situation? How does material hardness affect the complexity of manufacturing? How should design complexity be amortized over production volume?

Some preliminary work directed toward the development of a comprehensive measure of complexity has addressed some of these issues. Wilson [47] developed a model of the manufacturing process, analogous to a communication channel, and applied information theoretic [34, 4] results to obtain a measure of complexity for simple manufacturing operations, e.g. positioning a NC machine tool. Kaneshige [23] and Nakazawa [28] applied similar measures to sequential positioning and process planning respectively. Wilson [47] also used fourier descriptors as a basis for characterizing the complexity of plane closed shapes. These preliminary results are encouraging but much work is required before the ideas can be integrated into a complete measure of complexity for even a small portion of the design and manufacturing process.

2.4 Functional Independence in Design

The terms "functional requirement" and "independence" have special meanings in the context of the design axioms stated previously. In this context a set of functional requirements (FR) is a complete statement of the design task. Functional requirements must be independent, i.e. the functional requirements must be stated such that any of the functional requirements can be stated without consideration of any other functional requirement. An acceptable set of functional requirements is not necessarily unique, however, the set of FRs must be self consistent and minimal, in the sense that none of the FRs are redundant when considered with the other FRs. It is important to note that there can be a physical
relation between functional requirements without violating the stated conditions. Such a relationship is called a constraint relation. For example, output pressure and temperature of an air stream could be FRs for an air compressor even though pressure and temperature are related through the gas law. Specifying density in addition to the pressure and temperature would not be acceptable since arbitrary density specifications will be either redundant or inconsistent with the first two functional requirements due to the gas law constraint.

With this definition of functional requirements we can say that the designers task is to specify a complete description of a product or process such that constraints on an acceptable solution are not violated and such that all of the functional requirements are accomplished. In this sense design can be considered to be a transformation from a functional description of a product to a physical description of a product. The characteristics of the transformation are the characteristics of the design itself and are therefore the basis for evaluating alternative designs.

One characteristic of the transformation relevant to the present discussion is the effect on the independence of functional requirements. Although functional requirements are specified such that they are independent, a design configuration can render them interdependent, in that satisfying a subset of the FRs makes it difficult or impossible to satisfy the remaining FRs. Let us consider once again the compressor, which takes air at ambient condition \((P_0,T_0)\) and must discharge the air stream at specified pressure \(P\), and temperature, \(T\). If the designer specifies a compression process that is nearly isentropic, he will find that the two functional requirements have become coupled since the output temperature of the process will depend only on the input conditions and the output
pressure. Therefore, satisfying the pressure requirement by isentropic compression will preclude satisfying the temperature requirement. It should be noted that in this case only one degree of freedom of the design (i.e., compression ratio) was available to satisfy two functional requirements and that in the general case the degree of freedom of the design must equal or exceed the number of functional requirements in order to obtain a successful design.

If the compression process was followed by passing the air through a heat exchanger it would again be possible to obtain arbitrary pressure and temperature of the air stream (within the domain of applicability of the gas law). In this case the specified pressure and temperature are attained by adjusting the compression ratio $V_r$, and the heat flux in the heat exchanger, $Q$. These two variables are called design parameters (DP). A design parameter is any free and independent choice made during the design process. Although DPs are independent, they are not necessarily unique. It should be noted that in the case given, varying either DP causes both the temperature and pressure of the air to change. This is called a coupled system in contrast to an uncoupled system in which one and only one FR changes in response to a variation in a single DP. In an uncoupled system there is no interaction between the functional requirements of a design and therefore the FRs are completely independent of each other and the first axiom is satisfied to the greatest extent possible. The essence of the first axiom is that design configurations that couple or render the functional requirements interdependent are inferior to design configurations in which the functional requirements are not coupled. Coupled and uncoupled designs are discussed in greater detail in the following chapters.
The point of view that minimizing interaction improves design is not unique to this study. Simon [38] observed that many designs are nearly decomposable, i.e. that they consist of modules or boxes-within-boxes whose function or performance does not depend on most of the rest of the system. The decomposable structure allows the design of the modules without regard to the details of the final application or installation.

Preiss [31] observed that during the general design phase (as differentiated from detailed design), "decisions are made to decouple potentially interacting subsystems." Preiss believes this to be advantageous because of the dynamic character of design and prototype construction; that the construction of some subsystems begins before other subsystems are designed and that the requirements of the design often change during the design/construction phases. Preiss says that a good designer "defines the problem so that a minimum of work is expended on nonapplicable solutions," and equates this to "[defining] the frames so that the network is hierarchic, with as few interactions across the tree as possible."

Bristol [6] has observed a structure in the design of complex control, which he calls idiomatic control. The use of idioms, or conceptual structures with a standard purpose, appears to minimize the designer's need to consider detailed interactions of subsystems. Bristol believes these structures contribute to design clarity and operational order.

The concept of interaction is very important in the design of multivariable control systems. Mesarovic [27] looked at the nature of interrelated changes in outputs of a system, and the impact on design and synthesis of decouplers and controllers. Decouplers and measures of interaction in multivariable control will be discussed briefly in later
chapters, but it is worth noting now that decouplers in multivariable
control do not change the nature of the interaction, they compensate for
it. Shinsky [35] reports that degraded response or instability can result.
We should also make a distinction as Bristol [7] does between decoupling
control loops and decoupling control objectives. The later is much closer
to the concepts behind maintaining the independence of functional
requirements.

2.5 Examples of Coupled and Uncoupled Designs

Three design examples are presented to illustrate the concepts of
coupled and uncoupled systems. The first two examples, a conventional
refrigerator [41] and a combination bottle opener/can opener [42] have been
developed from other work. These examples are presented in a qualitative
form. The third example, a passive electrical filter network design is
presented in detail and will be referred to throughout the remainder of the
thesis. The quantitative results of the filter case highlight the
importance of coupling in design.

Before we discuss the cases it is worthwhile emphasizing that the
axioms must be satisfied simultaneously to obtain the optimal design, and
that since a general measure of complexity is not yet available, the cases
and examples presented herein are predicated on the assumption that
alternative designs are of approximately equal complexity. This assumption
is normally supported by arguing that tolerances, number of components,
arrangement, etc. of alternative designs are comparable. This restriction
precludes consideration of design alternatives that are grossly different
and usually restricts alternatives to modest variations.
2.5.1 Refrigerator Design

A conventional U.S. residential refrigerator must satisfy many functional requirements. Among these are:

FR₁: Maintain contents at fixed temperature, (e.g. 40°F).

FR₂: Provide access to the contents.

In a conventional refrigerator, FR₁, is satisfied by providing a cooling system and an insulated enclosure. FR₂ is satisfied, in part, by a hinged door. Note however that as the hinged door is opened on a standard refrigerator, the cold air inside the refrigerator falls out and is replaced by the warmer room air, thereby compromising FR₁. The functions are coupled since the means for achieving the second FR interfere with or compromise the means chosen to satisfy the first FR. An alternative design configuration is that of the top-opening chest freezer. Since the door of the chest freezer is the top surface of the enclosure, opening the door does not allow the cold air to fall out of the freezer. The thermal isolation of the chest freezer is compromised to a much lesser extent than that of the conventional refrigerator when the door is opened. Therefore the two FRs listed are less coupled in the chest freezer design than in the conventional refrigerator design. This does not mean that the conventional refrigerator design is poor. There are many other functional requirements for a refrigerator that must be considered. Two additional FRs are likely to include floor area use and convenience of access. Both of these favor the conventional refrigerator design.

2.5.2 Bottle/Can Opener Design

A simple device for opening beverage cans and bottles is pictured in Figure 2.1. The functional requirements for the device are:
Figure 2.1: Combination bottle opener and can opener.
FR$_1$: Open beverage bottles.

FR$_2$: Open beverage cans.

Note that the means for achieving both FRs is contained in the same device. Are the functional requirements coupled? No, they are not, because the capacity for opening cans does not interfere with or compromise opening bottles. The functional requirements are coupled only if a functional requirement of the product is to open bottles and cans simultaneously. The functions have been physically integrated, but not functionally coupled. Physical integration without functional coupling will often be advantageous since the complexity of the product will be reduced in accordance with the second axiom.

2.5.3 Passive Filter Design

A simple instrumentation system is to be designed to obtain a record of the displacement of a mechanical system. Two types of displacement transducers are available. The first is a Pickering precision LVDT model DTM-5. The second is a displacement transducer based on a four active arm strain-gage bridge. The AC excited transducers produce an amplitude modulated displacement signal. The transducer output is passed through a full wave, phase (direction) sensitive demodulator. The signal is to be recorded using a light beam oscillograph equipped with two types of magnetically damped galvanometers. The specifications for the displacement transducers, demodulator and galvanometers are given in Appendix A.

The displacement signal to be recorded has spectral content in the range 0 - 2 hz. The displacement transducers are excited at 60 hz. The demodulated output of the transducers consists of the desired signal, the
rectified carrier near 120 hz. and higher harmonics of the carrier. The
carrier magnitude is 2/3 as great as the signal as shown by a spectral
analysis of the demodulated transducer output, in Appendix B.

The design task is to specify a network to match the demodulated
transducer output to the galvanometer in the light beam oscillograph. The
general configuration is shown in Figure 2.2. The network must suppress
the carrier frequency while passing, undistorted, the displacement signal
of interest. The network must also attenuate the signal so that the
deflection record is properly scaled. The functional requirements can be
stated:

FR₁: Suppress the carrier without distorting the displacement signal.
FR₂: Attenuate the signal to obtain the proper scale.

These functional requirements can be satisfied by using a simple passive
filter. Many filter configurations of different complexity and performance
are possible. For this simple case we will consider only three
configurations of passive, first-order, low-pass filters. Each
configuration includes only a single resistor and a single capacitor and
are considered to be equally complex.

The specifications for the filter characteristics are based on the
degree of carrier suppression and the fidelity of the displacement signal.
Residual carrier and signal distortion both contribute to error in the
recorded signal. The filter cutoff frequency or pole position was selected
to minimize this error. The error analysis and criterion for pole position
are shown in Appendix C. The desired pole position of the filter is 6.94
hz.

The desired full scale magnitude of the deflection record was
specified arbitrarily to be 6", which is roughly full scale on the light
Figure 2.2: General configuration of a displacement recording system.
beam oscillograph. We can now state the functional requirements in a quantitative form. The FRs for a first order, low-pass filter network to match the transducers, demodulator and galvanometers as specified are:

**FR\(_1\):** Obtain filter pole at 6.94 hz.

**FR\(_2\):** Obtain D.C. gain such that full scale deflection results in \(\pm 3^\circ\) light beam deflection.

Consider first the case where the displacement transducer and galvanometer are the LVDT and the model 348 galvanometer respectively.

We might adopt a design strategy to first satisfy FR\(_1\), the filter pole position, without consideration of FR\(_2\). Later we will satisfy FR\(_2\). The filter pole can be set by shunting the transducer and galvanometer with a capacitor as shown in Figure 2.3. The proper capacitance value can be determined either by experimentation or by computation if the galvanometer and transducer impedances are known. A 246 \(\mu\)F capacitor results in the desired pole location.

Having satisfied FR\(_1\) we now set out to satisfy the scale requirement, FR\(_2\), by placing a resistor in the network. We will consider three network configurations as shown in Figure 2.4. The three configurations are obtained by placing a resistor in series with the transducer, in series with the galvanometer, and in the shunt configuration, respectively.

Again, by computation, or by experiment, we find proper resistor values for each of the three configurations to obtain the desired light beam deflection, but we note that in each case the filter pole, which had been previously set, is now changed.

In the first case the filter pole dropped 5\% from 6.94 hz to 6.59 hz. In the second case the pole dropped 95\% to .35 hz. In this case the signal, which has spectral content up to 2 hz is severely distorted and an
Figure 2.3: A simple network to obtain the desired filter cut-off frequency.
Figure 2.4: Three network configurations considered as possible solutions to the design problem posed in Section 2.5.3.
accurate record of the displacement cannot be obtained. In the third case
the pole frequency increases by two orders of magnitude to 792 hz. In this
case the entire carrier is passed with the signal and an unusable record of
the deflection is obtained.

If we use the same design strategy with the strain gage bridge -348
galvo combination, we get similar but less severe results. The results are
summarized in Table 2.1.

It is clear that in all cases, satisfying the second FR affected the
first FR and therefore there is some coupling in each of these
configurations. It is also clear that in some cases the coupling is more
severe than in other cases. The LVDT case, network 1 gave only a 5% pole
frequency change, which is probably acceptable, however networks 2 and 3
clearly are not acceptable. For the strain gage case, all of the networks
resulted in an unacceptable change in pole frequency. Note also that the
greatest changes for the LVDT cases were more extreme than for the strain
gage cases. This might lead one to conclude that the LVDT cases are more
strongly coupled. This is incorrect as we will see in the fourth chapter.
The LVDT cases are actually less strongly coupled but without the resistor,
the LVDT-Galvo case is much "farther" from the desired design point.

As we changed FR2, the scale requirement, we affected the cut-off
frequency, FR1. However, the converse is not true. We are able to change
FR1 without affecting FR2, so if we had specified the resistor value before
specifying the capacitor value we could have attained the desired FRs
without iteration. This special type of coupled design is said to be
quasi-coupled. Quasi-coupled designs are discussed in Section 3.3.
<table>
<thead>
<tr>
<th>Network Configuration</th>
<th>Schematic Diagram</th>
<th>Component Values and Filter Pole Frequency Shift</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Component Values (OHMS)</td>
</tr>
<tr>
<td>LVDT</td>
<td>246</td>
<td>222 K</td>
</tr>
<tr>
<td></td>
<td>246</td>
<td>527</td>
</tr>
<tr>
<td>Strain Gage</td>
<td>246</td>
<td>222 K</td>
</tr>
<tr>
<td></td>
<td>246</td>
<td>527</td>
</tr>
<tr>
<td>LVDT</td>
<td>246</td>
<td>.823</td>
</tr>
<tr>
<td>Strain Gage</td>
<td>245</td>
<td>22.3</td>
</tr>
</tbody>
</table>

Table 2.1: Component Values and Filter Pole Frequency Shift for the Design Procedure Used in Section 2.5.3.
2.6 Benefits from Maintaining Functional Independence

A qualitative description of functional coupling and examples of coupling in design were presented in previous sections. In succeeding chapters, functional coupling will be put on a quantitative basis. Before proceeding let us summarize the benefits to be obtained by choosing designs that are in accordance with the first axiom, i.e. designs in which functional requirements are maintained independent. The most significant benefit obtained by applying this principle is a major reduction in the complexity of the design process. If a design was completely coupled, the designer would need to know in detail how each design parameter effects every functional requirement. In the case of a complicated design, such as an automobile or a power plant, the designer would need to know the consequences of many thousands of interactions. Each of the interactions would have to be considered when any critical design decision was made. Even if all interactions could be predicted, the resulting computational difficulty of simultaneously considering the effects of all interactions would render the problem insolvable. In practice, such interactions are difficult if not impossible to predict precisely and in detail, so the designer adopts a design configuration which minimizes the interaction. When there is no coupling or minimum coupling the designer is able to proceed sequentially with the design of the product. This procedure is similar to Preiss' [31] constraint decoupling frames.

Uncoupled designs are also advantageous because they are more easily adjusted or compensated than coupled designs. In uncoupled designs, a change in a design parameter influences only one functional requirement, making it a simple process to alter or trim that functional requirement.
Preiss [31] believes this feature of design is particularly advantageous because of the normal evolutionary changes prescribed in functional requirements. Uncoupled designs result in modular and hierarchical designs which make interchange of parts and modules among various designs a simple matter. The complexity of standard parts or modules is less than the complexity of a custom design since the complexity of the standard item may be amortized over a greater number of applications. Even in designs with a small number of design variables and predictable interaction among the variables, selecting design configurations with independent functional requirements simplifies the design process. In these cases the design parameters may be optimized sequentially rather than simultaneously, a procedure Wilde [46] calls partial optimization. The result is a simplification of an n\textsuperscript{th} order optimization problem to n, first order optimization problems. Even when there are as few as ten completely coupled design parameters, it may not be possible to identify an acceptable design. Yet, when the same number of design parameters are uncoupled, finding a solution is likely to be a relatively straightforward matter.
Chapter 3

DESIGN REPRESENTATION

3.1 Introduction

In the first two chapters we described products, or design objectives, in terms of the function that they perform and in terms of the physical attributes of the device or process. In this chapter we build on this concept that designs can be described in a functional or a physical domain and that the design process is one of specifying the transformation between the domains. A general representation of design is developed from this perspective.

A representation of design is useful for examining some of the relationships between design parameters and the degree to which a particular design satisfies the functional requirements. In this regard a design representation assists the designer in determining the degree of coupling that exists in a design or to find and eliminate designs that cause the functions to become interdependent.

3.2 The Function Vector, the Design Vector and a Representation of Design

If the requirements of a design are specified as a set of n functional requirements then the performance of the design can be written as a n-dimensional function vector.

\[ \overline{F}^T = [FR_1, FR_2, FR_3, \ldots FR_n] \]  (3.1)
The n-dimensional function vector can be represented in an n-dimensional space, called the function space. At certain points or within regions of this space all functional requirements are satisfied. If the function vector endpoint of a particular design lies at such a point, the design is acceptable. At other points in the space, one or more of the functional requirements is not satisfied and a design that corresponds to a function vector at such a point is not acceptable. Figure 3.1 shows a 3-dimensional function space and the region for an acceptable design.

Similarly, the physical attributes of a particular design configuration or set of design configurations can be described as a set of n design parameters, where as before, design parameters are free and independent choices made during the design process. The set of n design parameters completely characterize the physical design. The n design parameters can also be written as a vector, as Fox described [13]. The design vector, \( \vec{D} \) is

\[
\vec{D}^T = [D_{P_1}, D_{P_2}, D_{P_3}, \ldots D_{P_n}]
\]

(3.2)

The design vector can be represented in an n-dimensional space, called the design space. The design space can also be thought of as the option space or as the physical space, to emphasize the form of the design that is represented. The design space may be divided into regions if parameter constraints (e.g., standard sizes) have been imposed on acceptable solutions.

If we assume that the performance of a product or process is a function of the full set of design parameters alone, then there is a unique point in the function space corresponding to every point in the design space. Since each point in the function space can be represented as a
Figure 3.1: The region for an acceptable design shown in a 3-D function space.

\[
\begin{align*}
FR_1 &\leq \overline{FR}_1 \\
FR_2 &= \overline{FR}_2 \pm \Delta FR_2 \\
FR_3 &= \overline{FR}_3 \pm \Delta FR_3
\end{align*}
\]
vector, then a unique function vector is associated with every point in the
design space and we have obtained a vector field representation of the
design. The vector field can be expressed in a mathematical form if the
functional requirements can be expressed in terms of the design parameters.

If we assume

\[ FR_1 = f_1(DP_1, DP_2, DP_3 \ldots DP_m) \]
\[ FR_2 = f_2(DP_1, DP_2, DP_3 \ldots DP_m) \]
\[ FR_n = f_n(DP_1, DP_2, DP_3 \ldots DP_m) \]  \hspace{1cm} (3.3)

and the function vector can be expressed in terms of unit function vectors

\[ \bar{F}^T = [FR_1, FR_2 \ldots FR_n] \]  \hspace{1cm} (3.4)

\[ \bar{F} = FR_1 \bar{u}_1 + FR_2 \bar{u}_2 + FR_3 \bar{u}_3 \ldots FR_n \bar{u}_n \]

then the vector field is given by specifying the function vector at each
point in the design space. If \( \bar{f} \) is a vector function of the design vector
\( \bar{D} \), then

\[ \bar{F} = \bar{f}(\bar{D}) \]  \hspace{1cm} (3.5)

and

\[ \bar{F} = f_1(DP_1, DP_2 \ldots DP_m)\bar{u}_1 + \]

\[ f_2(DP_1, DP_2 \ldots DP_m)\bar{u}_2 + \]

\[ \ldots \ldots \ldots \ldots \ldots + \]  \hspace{1cm} (3.6)

\[ f_n(DP_1, DP_2 \ldots DP_m)\bar{u}_n \]

where the \( f_j \) are scalar functions of the design parameters and each \( \bar{u}_j \) is a
unit vector in the FR\(_j\) direction.
The essence of the vector field representation of design is the mapping or transformation of the function space into the design space.

As an example of such a transformation consider the passive filter design problem posed in Section 2.5.3. For network configuration #3, the strain gage displacement transducer and type 348 galvanometer, we have the network shown if Figure 3.2. The acceptable ranges of the functional requirements, filter cutoff frequency and DC deflection, are indicated in the function space shown in Figure 3.3a. The range of design parameters, C and R3, corresponding to the allowable function range is shown in Figure 3.3b. The relative scale of the figures is not meaningful but note how the shape of the region of acceptable designs changed as the function space was mapped into the design space.

The vector field representation of design is based on the transformation or mapping of the function space into the design space. However, the design space can also be mapped into the functional space to obtain a similar representation. This representation is often more useful than the first one presented, but it has two severe disadvantages. First, the design vector field in the function space is not normally unique. A unique representation can often be obtained when the acceptable domain of DPs is restricted or when only a single design configuration is considered. A second disadvantage is that design equations, when available are normally given in the form of Equation 3.3 and must be solved simultaneously to obtain design vectors in the function space. The severe nonlinearity of design equations makes formal solution extremely difficult or even impossible to obtain for an arbitrary design. In spite of these liabilities, the design vectors will often be shown mapped into the function space in this thesis. The reader should keep in mind that this
Figure 3.2: A configuration of passive filter matching a displacement transducer and a galvanometer.
Figure 3.3a: The region for an acceptable design. a) Shown in the function space. b) Shown in the design space.
Figure 3.3b: The region for an acceptable design. a) Shown in the function space. b) Shown in the design space.
representation is not always possible and is presented only to clarify some concept. The results presented in this thesis do not require that such a representation can be obtained.

3.3 Linear Design Examples

Some insight into the nature of functional coupling can be obtained by examining the relationship between functional requirements and design parameters.

If the n FRs are known linear functions of n DPs, Equation 3.6 can be written in a matrix form with the function vector related to the design vector through a coupling matrix, \( \overline{C} \).

\[
\overline{F} = \begin{bmatrix}
FR_1 \\
FR_2 \\
\vdots \\
FR_n
\end{bmatrix} = \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1n} \\
c_{21} & \cdots \\
\cdots \\
c_{n1} & \cdots & c_{nn}
\end{bmatrix} \begin{bmatrix}
DP_1 \\
DP_2 \\
\vdots \\
DP_n
\end{bmatrix}
\]  

(3.7)

In linear cases, and when the determinant of the coupling matrix is nonzero, the design vector can be given in terms of the coupling matrix.

\[
\overline{D} = \overline{C}^{-1}\overline{F}
\]  

(3.8)

Equation 3.8 can be used to plot the design vector field in the function space.

We will examine four design examples where the two functional requirements are linear functions of the two design parameters. The examples are intended to illustrate the characteristics of different

\*\* The significance of the term, coupling matrix will become apparent in Chapter 4.
transformations and not the character of real design problems.

Equation 3.9 is the matrix form of a set of linear coupled design
equations.

\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} =
\begin{bmatrix}
0.87 & 0.20 \\
-0.50 & 0.98
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix}
\] (3.9)

Isograms of the functional requirements\(^*\) are plotted in the design
space in Figure 3.4. Note that the function isograms in the design space
are not orthogonal to each other nor are they parallel to the coordinate
axis of the space as they would be if plotted in the function space. The
function space was rotated and distorted as it was mapped into the design
space.

Isograms of FR\(_1\) intersect lines of constant DP\(_1\) at angle \(\beta_1\) and FR\(_2\)
isograms intersect lines of constant DP\(_2\) at angle \(\beta_2\) as shown in Figure
3.4. In this case the angles are the arctangents of ratios of elements in
the design matrix.

\[
\begin{align*}
\beta_1 &= \tan^{-1}(c_{12}/c_{11}) \\
\beta_2 &= \tan^{-1}(c_{21}/c_{22})
\end{align*}
\] (3.10)

If one of the functional requirements is to be adjusted without
changing the other FR, (e.g., from point A to B or from C to D in Figure
3.4) changes in both DP\(_1\) and DP\(_2\) are required. In this linear case it is a
simple matter to compute the required design parameter changes but in the
general case of a nonlinear, coupled design it is likely to be very
difficult to determine the proper values for the DPs. A search for the new

\(^*\) Isograms of functional requirements are plotted rather than the function
vectors to simplify the diagram and aid comprehension. The function
isograms provide sufficient information to construct the function vectors
at each isogram intersection.
Figure 3.4: Function isograms plotted in the design space for the coupled design described by Equation 3.9.
design point either computationally or experimentally would be iterative and might not be convergent on the desired design point due to discontinuities or local extremum points on the function response surfaces. This is an important point because in general, relationships between FRs and DPs can only be roughly estimated, at best.

As mentioned previously, design parameter isograms can be plotted in the function space for nonsingular, linear designs. DF isograms are plotted in the function space in Figure 3.5 for the design described by Equation 3.9. The design isograms intersect the function coordinates at angles \( \alpha_1 \) and \( \alpha_2 \), which are negative in Figure 3.5. The alphas are also arctangents of ratios of design matrix elements.

\[
\begin{align*}
\alpha_1 &= \tan^{-1}(-c_{12}/c_{22}) \\
\alpha_2 &= \tan^{-1}(c_{21}/c_{11})
\end{align*}
\]  

(3.11)

The effect of a change in a single design parameter is easily discerned from this figure. \( \text{DP}_1 \) varies as we move along an isogram of \( \text{DP}_2 \) (e.g., from point A to point B in Figure 3.5). In this case \( \text{FR}_2 \) changes by .58 units for each unit change in \( \text{FR}_1 \).

Equation 3.12 is the matrix form of an uncoupled design.

\[
\begin{bmatrix}
\text{FR}_1 \\
\text{FR}_2
\end{bmatrix} =
\begin{bmatrix}
c_{11} & 0 \\
0 & c_{22}
\end{bmatrix}
\begin{bmatrix}
\text{DP}_1 \\
\text{DP}_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\text{DP}_1 \\
\text{DP}_2
\end{bmatrix}
\]  

(3.12)

The design space and function space for this design are shown in Figures 3.6 and 3.7 respectively. Note that the isograms are orthogonal to each other and parallel to the coordinates of the space making the alphas and betas equal to zero. Neither space was rotated or distorted as it was mapped into the other domain and only diagonal elements of the coupling.
Figure 3.5: Design parameter isograms plotted in the function space for the coupled design described by Equation 3.9.
Figure 3.6: Function isograms plotted in the design space for the uncoupled design described by Equation 3.12.
Figure 3.7: Design parameter isograms plotted in the function space for the uncoupled design described by Equation 3.12.
matrix are non-zero. These are characteristics of uncoupled designs.

A change in a single functional requirement (e.g., from A to B in Figure 3.6) can be obtained by changing a single DP, and conversely a change in any design parameter (e.g., from A to B in Figure 3.7) affects one and only one functional requirement. This form of functional dependency is very important even when the exact relationship is not known. Monotonicity analysis and single parameter search greatly expedite finding the proper magnitudes of design parameters in this case, and are not subject to the extreme difficulties that often complicate multi-parameter techniques.

Two special cases of coupled designs deserve mention. The first of these is described by Equation 3.13 and represented in Figures 3.8 and 3.9.

\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} = 
\begin{bmatrix}
c_{11} & c_{12} \\
-c_{12} & c_{11}
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} = 
\begin{bmatrix}
.8 & .6 \\
-.6 & .8
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix}
\]  
(3.13)

In this case function isograms in the design space are mutually orthogonal as are design parameter isograms in the function space. The alphas are equal in this case and the betas have the same magnitude but opposite sign of the alphas. When design parameter isograms are mutually orthogonal the two design parameters have a maximally different effect on the set of functional requirements and therefore smaller changes in the DPs are required to make changes in the functional requirements. This point is discussed in much greater detail in the next chapter.

Another interesting design is described by Equation 3.14.
Figure 3.8: Function isograms plotted in the design space for the design described by Equation 3.13. Note that the isograms are orthogonal.
Figure 3.9: Design parameter isograms plotted in the function space for the design described by Equation 3.13. Note that the isograms are orthogonal.
\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} \\
0 & c_{22}
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} =
\begin{bmatrix}
1 & .6 \\
0 & .8
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix}
\]  

(3.14)

In this case one set of function isograms is parallel to one of the coordinates of the design space and one set of design parameter isograms is parallel to a function space coordinate as shown in Figures 3.10 and 3.11. Note that FR₁ can be changed, without changing FR₂ (e.g., from A to B in Figure 3.10) by adjusting only DP₁ but a change in only FR₂ (e.g., from C to D in Figure 3.10) requires a change in both DP₁ and DP₂. If DP₂ is adjusted before DP₁, the new design point can be located using a sequence of one parameter adjustments or searches, which greatly simplifies the compensation process. This type of design is called quasi-coupled and is characterized by a triangular coupling matrix.

3.4 Passive Filter Design Problem

The passive filter design problem which was introduced in Chapter 2 is useful for demonstrating some of the concepts presented in this chapter. We will discuss network configurations 2 and 3 as shown in Figure 3.12 for both the LVDT and strain-gage transducer driving type 348 galvanometer. Figure 3.13 shows function isograms plotted in the design space for the LVDT - network 2 case. Note that the isograms are nearly aligned with the coordinates of the design space indicating that this is a nearly uncoupled design configuration. Function isograms for the strain gage - network 2 case are plotted in Figure 3.14. The isograms of filter cut-off frequency are not quite aligned with the R₂ coordinate in the design space. The

* The logarithmic scaling used in Figures 3.14 - 3.17 is not conformal and therefore the angles between isograms and coordinates in the figures should not be used as an estimate of the degree of coupling.
Figure 3.10: Function isograms plotted in the design space for the quasi-coupled design described by Equation 3.14.
Figure 3.11: Design parameter isograms plotted in the function space for the quasi-coupled design described by Equation 3.14.
Figure 3.12: Network configurations 2 and 3.
Figure 3.13: Function iso-grams plotted in the design space for the LVDT - Network 2 - 348 galvanometer case.
Figure 3.14: Function isograms plotted in the design space for the strain gage - Network 2 - 348 galvanometer case.
design parameter isograms in the function space, Figure 3.15, show that a change in the capacitance affects only the filter cutoff frequency but a change in the resistance, R₂, affects both deflection and cut-off frequency. This is a quasi-coupled system, as described earlier.

Design isograms for cases using network configuration 3 are shown in Figures 3.16 and 3.17. Note that both of these configurations are quasi-coupled but they are more strongly coupled than the previous cases. Note also that the functional requirements in the LVDT-Network 3 case are extremely responsive to changes in the design parameters. In this case R₃ should be set at .823 ohms. If instead R₃ is equal to 1.00 ohms, the deflection will be 22% too great at ±3.64 inches and the filter pole position will be 22% low at 5.7 hz. The 22% changes in functional requirements that result from a .18 ohm change in resistance are not desirable. The effects of sensitivity and responsivity are discussed briefly in the next chapter.

3.5 The Algebraic Form of Functional Dependence

The discussion of uncoupled, coupled and quasi-coupled designs was based, in part, on the form of the matrix relating functional requirements to design parameters. The coupling matrix of uncoupled designs is diagonal, the matrix of quasi-coupled designs is triangular and all elements of coupled design matrices can be nonzero. In the field of multivariable control, the form of the matrix relating controlled variables to manipulated variables is also very important. If the matrix is diagonal, each controlled variable responds only to changes in one of the manipulated variables. This is called a decoupled controller.
Figure 3.15: Design parameter isograms plotted in the function space for the strain gage - Network 2 - 348 galvanometer case.
Figure 3.16: Design parameter isograms plotted in the function space for the LVDT - Network 3 - 348 galvanometer case.
Figure 3.17: Design parameter isograms plotted in the function space for the strain gage - Network 3 - 348 galvanometer case.
Methods for obtaining a "diagonal" or "triangular" control structure are important to the control engineer. Rosenbrock [33], Leninger [24] and Hung [20] have developed methods for obtaining and analyzing transfer matrices with a prescribed structure. However, the methods and concepts in multivariable control are different than those presented in this thesis. In control system design, the engineer is usually faced with controlling a plant that cannot be modified. If a plant is coupled, the only way of obtaining a decoupled structure is to compensate for the known interactions. This is accomplished by building decouplers, but the use of decouplers also has disadvantages. Bristol [7] and Shinsky [35] have pointed out that this type of control structure can lead to degraded response and instabilities. In multivariable control, partial decoupling, characterized by a triangular matrix is often preferable to a fully decoupled controller.

Selecting designs on the basis of functional independence is fundamentally different than compensating for an inherently coupled plant. Multiple interacting loops in a decoupled control structure give rise to instabilities whereas the basis for measures of functional coupling is the absence of interaction.
Chapter 4

GENERAL MEASURES OF COUPLING

4.1 Introduction

In the second chapter we discussed the basic concepts of coupling and independence in design and examined alternative designs for a passive filter. A vector field representation of design was presented in the third chapter. We found that for simple designs the characteristics of the representation, such as the conformation of function isograms, are related to the degree of coupling in design.

The degree of coupling in a design is essential information to a designer who wishes to apply the axioms stated in Chapter 2. Almost all designs are coupled to some small extent and often in redesign or design modification problems the alternative designs may be significantly coupled since options are severely constrained. In such cases the designer needs to know the degree to which alternative configurations are coupled in order to select the design which is minimally coupled. A measure of coupling is particularly valuable to a designer if the measure draws attention to the design deficiency and thereby directs the design process toward designs alternatives that are less coupled.

The value of measures of coupling or interaction have been recognized for some time in the multivariable control discipline. Mesarovic [27] stated that in multivariable control, "A system is interrelated if changes
in the outputs of the system influence other outputs." Mesarovic devised a measure of interaction based on the ratios of changes of outputs and used the measure as a criterion for selecting control configurations. Bristol [5] proposed that a relative gain array (RGA) could be used to characterize the degree of interaction in a multivariable system. The elements of the RGA are the ratios of control loop gains with all other control loops open to control gains with all other loops closed and are therefore a direct measure of the interaction of control loops. Shinsky [36] uses the RGA as the basis for a procedure for configuring the control of a multivariable system. The RGA and the procedure for using it are primarily tools applicable to steady state analysis of systems. Davison [11] proposed an interaction index that includes dynamic effects but is limited to linear systems. Interaction tests, such as the diagonal dominance test of Rosenbrock [33] have also been useful in the control field but are not directly applicable to general design problems.

Interaction in multivariable control and the multivariable control problem are similar but not the same as coupling in design and design problems. To minimize coupling in multivariable control we select a control configuration to minimize interaction between designated inputs and outputs of a system. To minimize coupling in design we select a design configuration such that each of the resulting design parameters affects only one functional requirement of the product. Measures of coupling in design are the subject of this chapter.

As mentioned previously the coupling matrix representing the transformation of the function space into the design space shows how functions are interrelated in a design. Ratios of the elements of the coupling matrix are related to the conformation of function isograms in the
design space and therefore to the degree of coupling in the design, but the coupling matrix as presented in the previous chapter is only applicable to designs in which the functional requirements are linearly related to the design parameters. Furthermore, simple and meaningful measures of coupling based on the coupling matrix are needed to evaluate designs.

Two general measures of functional coupling are presented in this chapter. The measures are used conjointly to evaluate design alternatives and are applicable to complex, nonlinear design problems as well as the simple type of designs presented previously. Alternative measures of coupling and the impact of scaling on measures of coupling are also discussed.

4.2 The Coupling Matrix

The coupling matrix, used previously with linear designs, described the transformation from the function space to the design space over the entire domain of design parameters. Since designs are not generally linear, it is not usually possible to describe the relationships between functional requirements and design parameters using a constant matrix, however it is usually meaningful to discuss the response of functional requirement to changes in design parameters at a point in the design space.

When the functional requirements are continuous functions of a set of continuous design parameters, the response of one FR to a change in one of the DPs is simply the partial derivative of that FR with respect to the DP. The response of the function vector to an arbitrary infinitesimal change in the design vector can be described at a point in the design space by a matrix of partial derivatives. The matrix is called the coupling matrix,
\( \overline{C} \), and has elements, \( c_{ij} \), defined by:

\[
c_{ij} = \frac{3(\text{FR}_i)}{3(\text{DP}_j)}
\]

(4.1)

The coupling matrix has \( n \) rows and \( m \) columns where \( n \) and \( m \) are the number of FRs and DPs respectively. In most designs the number of design parameters is much greater than the number of functional requirements however in this section and the two that follow we will consider only designs for which \( n = m \). The selection of design parameters is discussed in Section 4.7.

Note that in linear designs the coupling matrix, as defined here, is constant over the design space and is identical to the matrices used to describe linear designs in the previous chapter. The coupling matrices for non-linear designs vary throughout the design space.

As an example consider once again network configuration 2 for the passive filter design (Figure 3.12). At the specified design point, i.e. filter cut-off frequency, \( \text{FR}_1 \), equal to 6.94 hz., beam deflection, \( \text{FR}_2 \), equal to \( \pm 3 \) inches, \( \text{DP}_1 = C = 228 \mu \text{F} \) and \( \text{DP}_2 = R_2 = 527 \text{ ohms} \), the coupling matrix is:

\[
\overline{C} = \begin{bmatrix}
-1.91 & -0.11 \\
\mu \text{F} & \text{ohm}
\end{bmatrix}
\]

(4.2)

In section 2.5.3 we experimented with the same network. We first set \( C = 425 \mu \text{F} \) and \( R_2 = 0 \) to obtain the proper cut-off frequency. At this point in the design space the coupling matrix is:
The differences between the coupling matrix at different design points indicate how the response of the functional requirements to changes in design parameters varies between points in the design space. Note that only the relative magnitudes of the elements of the coupling matrices in Equations 4.2 and 4.3 are significant. The magnitudes are sensitive to the scale of the FRs and DPs. Scale is discussed in section 4.5.

In most cases the functional requirements are not given as analytical functions of the design parameters and therefore Equation 4.1 cannot be used directly to obtain the elements of the coupling matrix. In such cases the coupling matrix may be approximated experimentally or computationally by finite differences. In other cases the design parameters might be constrained to take one of a set of discrete values, e.g. when standard parts are to be used or when an integer value is required. When the scale of discretization is fine, the elements of the coupling matrix may still be approximated by finite differences. When the scale of discretization of a particular DP is course, other approximation techniques may be useful, but in general that DP is not well suited for controlling the function of the product. Optimization of designs constrained by discrete choices are discussed by Fox [13] and Wilde [46].

* The scale of discretization of a DP is considered to be fine when the smallest change in the DP results in changes in the functional requirements that are small compared to the tolerances on the FRs.
4.3 Reangularity: A Measure of Coupling

We have mentioned a number of times before that coupling in design is related to the coupling matrix and to the conformation of function isograms when plotted in the design space. We are now prepared to discuss in detail how the coupling matrix and representations of design are relevant to the design process.

First consider the design given by Equations 4.4 below and a very similar design given by Equation 4.5. These design equations will be referred to as Examples 4.1 and 4.2, respectively.

Example 4.1

\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} =
\begin{bmatrix}
.8 & -.6 \\
+.6 & .8
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix}
\]

(4.4)

Example 4.2

\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} =
\begin{bmatrix}
.8 & .6 \\
.6 & .8
\end{bmatrix}
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix}
\]

(4.5)

The only difference between these two cases is the sign of \(c_{12}\). In Example 4.2 the angles \(\alpha_1\) and \(\beta_1\) are negatives of those angles in Example 4.1, as shown in Figures 4.1 and 4.2. The magnitudes of the angles, \(\alpha_1\), \(\alpha_2\), \(\beta_1\) and \(\beta_2\) are 36.9\(^\circ\) for both cases.

In these simple linear cases we can solve for the DPs in terms of the FRs and the elements of the coupling matrix.

\[
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} = \frac{1}{c_{11}c_{22} - c_{12}c_{21}}
\begin{bmatrix}
c_{22} & -c_{12} \\
-c_{21} & c_{11}
\end{bmatrix}
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix}
\]

(4.6)
Figure 4.1: Function isograms plotted in the design space for Example 4.1.
Figure 4.2: Function isograms plotted in the design space for Example 4.2.
Then Equations 4.4 and 4.5 can be written as Equations 4.7 and 4.8.

\[
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} = \begin{bmatrix}
.8 & +.6 \\
-.6 & .8
\end{bmatrix} \begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} \tag{4.7}
\]

\[
\begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix} = 3.57 \begin{bmatrix}
.8 & -.6 \\
-.6 & .8
\end{bmatrix} \begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} \tag{4.8}
\]

In Example 4.1, if FR_1 is required to be 32 units and FR_2 must be 31 units, DP_1 must equal 25 and DP_2 must equal 20. If we now would like to increase FR_1 by 10% without changing FR_2 we find that DP_1 must be increased by 5.8% and DP_2 must be decreased by 34.3%. To obtain a 10% increase in the original value of FR_1 in Example 4.2 requires a 36.6% increase in DP_1 and the same 34.3% decrease in DP_2. The required change in DP_1 for this case is much greater than the design parameter change required in Example 4.1 for the same change in functional requirement. These examples are summarized in Table 4.1.

This mathematical result at first appears anomalous since the coupling matrices show that the functional requirements in the two examples are equally responsive to changes in design parameters. We will "walk through" these two examples to better understand the differences between the two.

In both examples, to obtain a unit change in FR_1 we increase DP_1 by 1/.8 = 1.25 as shown in Figures 4.3 and 4.4. As DP_1 is changed FR_2 increases by .6 x 1.25 = .75. If we had desired FR_2 to remain unchanged, DP_2 must be decreased by .75/.8 = .94. When DP_2 is decreased however, we notice that design Examples 4.1 and 4.2 behave differently.

In Example 4.1, FR_2 returns to the original value as desired and FR_1 increases by .56 augmenting the desired increase in FR_1. In Example 4.2,
### Table 4.1
Comparison of Similar Linear Design Examples, 4.1 and 4.2

<table>
<thead>
<tr>
<th></th>
<th><strong>EXAMPLE 4.1</strong></th>
<th><strong>EXAMPLE 4.2</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design Equations</strong></td>
<td>$\begin{bmatrix} FR1 \ FR2 \end{bmatrix} = \begin{bmatrix} .8 &amp; -.6 \ .6 &amp; .8 \end{bmatrix} \begin{bmatrix} DP1 \ DP2 \end{bmatrix}$</td>
<td>$\begin{bmatrix} FR1 \ FR2 \end{bmatrix} = \begin{bmatrix} .8 &amp; .6 \ .6 &amp; .8 \end{bmatrix} \begin{bmatrix} DP1 \ DP2 \end{bmatrix}$</td>
</tr>
<tr>
<td><strong>Initial Function Vector</strong></td>
<td>$[32, 31]$</td>
<td>$[32, 31]$</td>
</tr>
<tr>
<td>[FR1, FR2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initial Design Vector</strong></td>
<td>$[44.2, 5.60]$</td>
<td>$[25, 20]$</td>
</tr>
<tr>
<td>[DP1, DP2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>New Function Vector</strong></td>
<td>$[35.2, 31.0]$</td>
<td>$[35.2, 31.0]$</td>
</tr>
<tr>
<td><strong>New Design Vector</strong></td>
<td>$[46.8, 3.68]$</td>
<td>$[34.1, 13.1]$</td>
</tr>
<tr>
<td><strong>Design Vector Change</strong></td>
<td>$[+2.56, -1.92]$</td>
<td>$[+9.14, -6.86]$</td>
</tr>
<tr>
<td><strong>% Change of Design Vector</strong></td>
<td>$[+5.8, -34.3]$</td>
<td>$[+36.6, -34.3]$</td>
</tr>
</tbody>
</table>
Figure 4.3: Function isograms plotted in the design space for Example 4.1 showing the relationships between changes in functional requirements and changes in design parameters.
Figure 4.4: Function isograms plotted in the design space for Example 4.2 showing the relationships between changes in functional requirements and changes in design parameters.
FR₂ also returns to the original value but FR₁ decreases by .56 nullifying most of the change obtained in FR₁. The net result is that for a unit change in DP₁ and compensating change in DP₂, we obtained a change in FR₁ that was 3.6 times greater in Example 4.1 than in Example 4.2. This result occurs because in Example 4.2 the two DPs have a similar effect on the set of FRs and therefore a change in the second DP that compensates for undesired effects from a change in DP₁ also negates most of the desired effects. When two or more design parameters have nearly the same effect on the set of functional requirements, larger excursions of the DPs are required to adjust the functions of the design.

The first measure of coupling, called reangularity, reflects the degree to which different design parameters have the same effect on the set of functional requirements.

The basis for this measure can be explained using the coupling matrix and design representation. The changes required in DPs to accommodate changes in FRs depend on the coupling matrix. From Equation 4.6 we have

\[ \Delta D P_1 = \frac{1}{c_{11}c_{22} - c_{12}c_{21}} c_{22}(\Delta FR_1) - c_{12}(\Delta FR_2) \]  
\[ \Delta D P_2 = \frac{1}{c_{11}c_{22} - c_{12}c_{21}} -c_{21}(\Delta FR_1) + c_{11}(\Delta FR_2) \]  

Minimal changes of the DPs are required when the quantity \((c_{11}c_{22} - c_{12}c_{21})\) is a maximum, as shown in Appendix D. If the columns of the coupling matrix have been normalized, this quantity is identically equal to \(\cos(\alpha₁ - \alpha₂)\), which is equal to the sine of the angle between DP isograms.

⁺ Scale and normalization of FRs, DPs and the coupling matrix are discussed in Section 4.5.
when plotted in the function space. This quantity is maximized, hence DP excursions are minimized, when DP isograms are mutually orthogonal.

Design parameters have the same relative effect on functional requirements when isograms (or surfaces) of different design parameters are parallel to each other. Design parameters have a maximally different effect on the set of functional requirements when isograms of the DPs are mutually orthogonal, hence the name reangularity.*

Based on these concepts we define reangularity, R.

\[
R = \prod_{i=1,n-1}^{j=1,n} \left( 1 - \frac{\left( \sum_{k=1,n} C_{ki} C_{kj} \right)^2}{\left( \sum_{k=1,n} C_{ki}^2 \right) \left( \sum_{k=1,n} C_{kj}^2 \right)} \right)^{1/2} \tag{4.11}
\]

Although this measure of coupling looks quite formidable at first glance, it is really very simple. The summations in the denominator of the expression are the squared magnitudes of columns in the coupling matrix, and are therefore normalizing factors. If the columns have already been normalized, the denominator summations are unity and have no effect on the measure. The normalization is included to make the measure insensitive to changes in scale of any of the design parameters. Scale is discussed in more detail in Section 4.5.

The summation in the numerator is the dot product of the \(i^{th}\) and \(j^{th}\) columns of the coupling matrix. Since the denominator has the effect of normalizing the columns, this summation can be thought of as the cosine of the angle between vectors in the \(i^{th}\) and \(j^{th}\) columns of the coupling matrix.

* The word reangularity is derived from three Latin roots, [reg- + angulus + -ity], meaning literally right angle quality. A design in which there is a high degree of reangularity is said to be reangulous.
matrix. Since this term is squared and subtracted from one, the argument of the product operator is the square of the sine of the same angle. The product operates on all possible pairs of columns in the matrix, so reangularity is the absolute value of the product of the sines of the angles between all pairs of DP isograms in the function space. The maximum value of reangularity, unity, is obtained when the isograms are mutually orthogonal. The minimum and least desirable value for reangularity is zero and occurs when one or more pairs of isograms are parallel. Contours of reangularity are shown in Figure 4.5 as functions of $\alpha_1$ and $\alpha_2$. The contours are straight lines, since in the two dimensional case, $R = \cos(\alpha_1 - \alpha_2)$.

4.4 Semangularity: A Measure of Coupling

Let us compare Example 4.1 with the uncoupled case.

\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} = \begin{bmatrix}
.8 & -.6 \\
.6 & .8
\end{bmatrix} \begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix}
\]  \hspace{1cm} (4.12)

\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
DP_1 \\
DP_2
\end{bmatrix}
\]  \hspace{1cm} (4.13)

If we compute the reangularity, $R$, we find that for both of these cases, $R$ is equal to unity which is the maximum possible value. But the later case is certainly preferable to the former, so some other factor must be considered as part of a measure of coupling in design. That factor is the degree to which each design parameter affects one and only one functional requirement of the design. In Example 4.1 above, a change in either of the DPs results in a change in both FRs. In the uncoupled case, changing
Figure 4.5: Contours of reangularity for 2-D designs as a function of alpha one and alpha two.
either DP causes a change in only the corresponding FR.

This statement brings us to the first issue to be addressed, and that is establishing the correct correspondence between DPs and FRs. The previous measure, reangularity, depended only on the relative conformation of all DP isogram pairs. There was no need to associate a functional requirement with a particular design parameter. Since the second measure, which we will call semangularity, is a measure of correspondence between FRs and DPs, it is necessary to associate each functional requirement with a particular design parameter. We will address this issue in more detail in Section 4.7, but for the time being we assume that the order of the design parameters has been selected such that DP\(_1\) predominately affects FR\(_1\), DP\(_2\) predominately affects FR\(_2\) and so on. The measure of semangularity itself will determine if the most favorable order of design parameters has been selected. The section on design parameter selection and the case studies in Chapter 5 provide some insight into this question.

The extent to which DPs affect FRs other than the one they were intended to affect, is determined by the magnitude of off-diagonal elements relative to diagonal elements in the coupling matrix.

In the linear, two-dimensional case, we have

\[
\Delta FR_1 = c_{11}(\Delta DP_1) + c_{12}(\Delta DP_2) \tag{4.14}
\]

\[
\Delta FR_2 = c_{21}(\Delta DP_1) + c_{22}(\Delta DP_2) \tag{4.15}
\]

If \(c_{21}\) is nonzero, then a change in DP\(_1\) affects both functional requirements. Similarly, if \(c_{12}\) is nonzero then a change in DP\(_2\) affects both FRs. A measure of coupling, called semangularity, is defined such that the measure has a maximum value of unity when all off-diagonal elements of the coupling matrix are zero. The measure is zero when there
is no correlation between a DP and the FR it was intended to affect, i.e. when one of the diagonal elements of the coupling matrix is zero. Semangularity is defined as:

\[
S = \prod_{j=1, n} \left( \frac{|C_{ij}|}{\left( \sum_{k=1, n} C_{kj}^2 \right)^{1/2}} \right) \quad (4.16)
\]

Once again the denominator in Equation 4.16 has no effect if the columns of the coupling matrix have been normalized by the euclidian magnitude. In this case, semangularity is simply the absolute value of the product of diagonal elements in the coupling matrix. The product of diagonal elements is meaningful because of the normalization. When all off-diagonal elements are zero, the diagonals are unity. When the semangularity is unity, DP isograms (or surfaces) are parallel to coordinates in the function space and FR isograms (or surfaces) are parallel to the coordinates of the design space, hence the name semangularity.

Contours of semangularity are shown as solid lines in Figure 4.6 as a function of \( \alpha_1 \) and \( \alpha_2 \). Dashed lines are contours of reangularity. Both measures are equal to unity when \( \alpha_1 = \alpha_2 = 0 \). This case corresponds to the uncoupled design given by Equation 4.13. Designs parameters have not been associated with the proper functional requirements if the design is in the shaded region of the figure.

4.5 Design Parameter and Functional Requirement Scale

The measures, reangularity and semangularity, were defined such that

*The word semangularity is derived from three Latin roots, \([\text{sem-} + \text{angulus} + \text{ity}]\), meaning literally same angle quality. A design in which there is a high degree of semangularity is said to be semangulous.*
Figure 4.6: Contours of semangularity (solid) and reangularity (dashed) as a function of alpha one and alpha two. Design parameters have not been selected properly if the design is represented in the shaded region of the figure.
the measures are insensitive to the scale of the design parameters. This is as it should be for a number of reasons.

The first reason is that a change in a metric of one of the DPs, e.g. from inches to centimeters, should not influence the measure of coupling since the design is unchanged. The second and more fundamental reason is that coupling in design depends on the interaction of functional requirements and therefore depends only on the relative changes of the individual FRs. Coupling does not depend on sensitivity or responsivity of the FRs to the DPs.

The effects of sensitivity and responsivity of a design are included within the context of an axiomatic approach to design by considering the tolerances necessary on design parameters to maintain the functional requirements within tolerance. As such the issues of sensitivity and responsivity fall within the domain of the complexity or information context axiom, and are not covered here.

Although the measures of coupling are insensitive to the scale of the design parameters, they are not insensitive to functional requirement scale. Consider Example 4.1.

\[
\begin{bmatrix}
\text{FR}_1 \\
\text{FR}_2
\end{bmatrix} =
\begin{bmatrix}
.8 & -.6 \\
.6 & .8
\end{bmatrix}
\begin{bmatrix}
\text{DP}_1 \\
\text{DP}_2
\end{bmatrix}
\]

(4.17)

In this case \( R = 1 \) and \( S = .64 \). If the scale of \( \text{FR}_1 \) is doubled the design equations become

\[
\begin{bmatrix}
\text{FR}_1 \\
\text{FR}_2
\end{bmatrix} =
\begin{bmatrix}
.4 & -.3 \\
.6 & .8
\end{bmatrix}
\begin{bmatrix}
\text{DP}_1 \\
\text{DP}_2
\end{bmatrix}
\]

(4.18)

The coupling measures become, \( R = .81 \) and \( S = .52 \).
The dependence of the measures on the FR scale is essential, since we are interested in relative changes in functional requirements. Meaningful scales must ultimately be based on the significance of a change in one of the FRs relative to the significance of changes in other FRs. The level of significance for each FR depends on the specified tolerance of that functional requirement. Often tolerances on the functional requirements are not explicitly stated and therefore must be inferred from other information. In the absence of other information it may be reasonable to assume the tolerance on a functional requirement is in proportion to the magnitude. The measures of coupling are not sensitive to an absolute scale or tolerance on the functional requirements, only to the relative scale. Therefore, when FR tolerances are not specified it is permissible to arbitrarily establish the scale for one of the FRs and then determine tolerances for the other FRs such that the maximum or expected error in each of the FRs is equally significant. Since the measures are not sensitive to an absolute scale, a proportionate change in scale of all of the FRs does not affect the measures of coupling. Such a change of scale is identical to a change in responsivity of the FRs to all of the DPs.

4.6 The Degree of Coupling of Passive Filter Networks

The passive filter designs introduced in Chapter 2 can be used to demonstrate how measures of coupling in a design can be computed. The functional requirements of the filter, cut-off frequency and DC gain, are non-linear functions of the design parameters, capacitance and resistance. After suitable scales for the FRs have been adopted, the coupling matrix is computed as a function of the design parameters by determining the rate of change of each FR with respect to each DP.
Tolerances were not specified for either of the FRs. Since no other information regarding scale or tolerance is available, we adopt the intrinsic scale, i.e. the tolerance on each FR is assumed to be a fixed fraction of the magnitude of the FR. Then the elements of the coupling matrix are:

\[ c_{11} = \frac{\partial W}{\partial C} \]  
\[ c_{12} = \frac{\partial W}{\partial R} \]  
\[ c_{12} = \frac{\partial D}{\partial C} \]  
\[ c_{22} = \frac{\partial D}{\partial R} \]  

where  
\( W \) \equiv filter cut-off frequency \quad (\text{Hz})  
\( D \) \equiv DC light beam deflection \quad (\text{in.})  
\( C \) \equiv capacitance \quad (\mu\text{F.})  
\( R \) \equiv resistance \quad (\text{ohms})

The terms of the coupling matrix are given as functions of the design parameters in Table 4.2 for each of the network configurations. The table also includes a simplified form of the coupling matrix, obtained by a change in DP scale.

The coupling matrix can be obtained by evaluating the equations in Table 4.2 at the design point. The evaluation involves determining the values of the DPs required to obtain the desired function then substituting the DP values into the equations in Table 4.2. The reangularity and semangularity can then be computed using Equations 4.11 and 4.16. The measures of coupling and component values are tabulated in Table 4.3 for
Table 4.2
Coupling Matrices for Networks 1, 2 and 3

<table>
<thead>
<tr>
<th>Network</th>
<th>Coupling Matrix</th>
<th>Scaled Matrix</th>
</tr>
</thead>
</table>
| 1       | \[
\begin{bmatrix}
-1 \\
R_s + R_1 + R_g \\
0
\end{bmatrix}
\frac{-R_g}{(R_s + R_1 + R_g)(R_s + R_1)}
\] | \[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\frac{R_g}{(R_s + R_1)}
\] |
| 2       | \[
\begin{bmatrix}
-1 \\
R_s + R_2 + R_g \\
0
\end{bmatrix}
\frac{-R_s}{(R_s + R_2 + R_g)(R_2 + R_g)}
\] | \[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\frac{R_s}{(R_g + R_2)}
\] |
| 3       | \[
\begin{bmatrix}
-1 \\
R_3(R_3 + R_g) + R_s \\
0
\end{bmatrix}
\frac{-R_g R_s}{R_3(R_3 + R_g) + R_g + R_3 R_g}
\] | \[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
-1
\] |

Rs = Transducer Source Impedance
Rg = Galvanometer Impedance
Table 4.3

Measures of Coupling and Component Values for

Twelve Passive Filter Case Studies

<table>
<thead>
<tr>
<th>Network Config.</th>
<th>LVDT 345</th>
<th>LVDT 348</th>
<th>Strain Gage 345</th>
<th>Strain Gage 348</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure*</td>
<td>1.000</td>
<td>1.000</td>
<td>0.8304</td>
<td>0.9887</td>
</tr>
<tr>
<td>R (ohms)</td>
<td>60818</td>
<td>222 K</td>
<td>5.21</td>
<td>527.2</td>
</tr>
<tr>
<td>C (mf)</td>
<td>273.5</td>
<td>234.2</td>
<td>456.3</td>
<td>269.6</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure*</td>
<td>0.9995</td>
<td>1.000</td>
<td>0.5966</td>
<td>0.9821</td>
</tr>
<tr>
<td>R (ohms)</td>
<td>60818</td>
<td>222 K</td>
<td>5.21</td>
<td>527.2</td>
</tr>
<tr>
<td>C (mf)</td>
<td>12.71</td>
<td>12.44</td>
<td>448.4</td>
<td>227.9</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure*</td>
<td>0.7071</td>
<td>0.7071</td>
<td>0.7071</td>
<td>0.7071</td>
</tr>
<tr>
<td>R (ohms)</td>
<td>2.57</td>
<td>0.823</td>
<td>1937</td>
<td>22.31</td>
</tr>
<tr>
<td>C (mf)</td>
<td>9215</td>
<td>28135</td>
<td>476.1</td>
<td>1454</td>
</tr>
</tbody>
</table>

* The measures, reangularity, semangularity and the geometric mean, have equal magnitude because network configurations 1, 2 and 3 result in quasi-coupled designs.
the twelve cases involving three network configurations, two displacement transducers and two galvanometers. Note that in all of the cases the reangularity is equal to the semangularity. This is identically true for all second order, quasi-coupled designs. An examination of Table 4.3 allows the designer to select the network configuration which is minimally coupled. In many cases the designer will be able to identify the minimally coupled design without carrying out the detailed computations.

The reangularity and semangularity of the design vary throughout the design space and the physical space and therefore the network configuration that is minimally coupled is likely to change if the functional requirements are changed. The coupling of the passive filters depend only on FR$_2$, the beam deflection specification. The reangularity (or semangularity, since the measures are equal to each other) of the three network configurations are plotted as a function of required beam deflection in Figure 4.7 for one of the cases. Note that Network 1 is less coupled than the other configurations. Network 2 is preferable to Network 3 over most of the allowable deflection range, but when the deflection is large, the converse is true.

4.7 Selecting Design Parameters

The measures of coupling depend on how n of the m design parameters can be used to control the n functional requirements. To properly evaluate a design configuration it is necessary to find the set and sequence of design parameters that maximize the measures of coupling. The number of permutations of the design parameters for a design is
Figure 4.7: Reangularity and semangularity for three network configurations matching the strain gage displacement transducer and the type 348 galvanometer.
\[ \frac{m!}{(m-n)!} \]  

(4.23)

For simple designs the number of permutations is small. For example, when five DPs are available to control 3 FRs there are sixty possible permutations of the DPs. In such cases the best sequence of DPs can be quickly identified by exhaustion. For complex designs, the number of possible DP sequences is enormous. When there are 100 DPs and 75 FRs there are \(10^{132}\) possible DP sequences, which is too great to be evaluated by exhaustion. It is important then to consider expedient methods for determining the optimal DP selection.

The problem is not as formidable as it first seems to be because many of the elements of the coupling matrix are likely to be imperceptibly different from zero, and furthermore the coupling matrix may fall into a block decoupled structure as shown in Equation 4.24, in which the X's indicate a nonzero value.

\[
\begin{bmatrix}
\text{FR}_1 \\
\text{FR}_2 \\
\text{FR}_3 \\
\text{FR}_4 \\
\text{FR}_5 \\
\text{FR}_6 \\
\text{FR}_7 \\
\text{FR}_8 \\
\text{FR}_9 \\
\end{bmatrix} = \begin{bmatrix}
X & X & X \\
X & X & X \\
X & X & X \\
X & X & X \\
X & X & X \\
X & X & X \\
X & X & X \\
X & X & X \\
X & X & X \\
\end{bmatrix} = \begin{bmatrix}
\text{DP}_1 \\
\text{DP}_2 \\
\text{DP}_3 \\
\text{DP}_4 \\
\text{DP}_5 \\
\text{DP}_6 \\
\text{DP}_7 \\
\text{DP}_8 \\
\text{DP}_9 \\
\text{DP}_{10} \\
\text{DP}_{11} \\
\text{DP}_{12} \\
\end{bmatrix}
\]

(4.24)
Because of the decoupled structure this, 9 x 12 design problem which in general has 80 million DP sequences can be solved as three subproblems involving a total of 90 DP sequences. Designs problems often fall into a partially decoupled form because of the division of products into subassemblies and parts.

It appears that the optimal sequence of DPs can be identified in most cases even if all elements of the coupling matrix are nonzero. This can be accomplished by establishing a good lower bound on the values for the coupling measure and then utilizing a branch and bound search strategy [48]. Consider how we might find the DP sequence which results in the maximum value for semangularity. First we normalize the columns of the coupling matrix and take the absolute value of all elements. We then find a lower bound on the measure by selecting the maximum value of the first row of the coupling matrix and then we cross out that column of the matrix. Similarly we find the maximum remaining value in the second row and cross out that column. We continue this process until the maximum remaining element in each row has been identified. The product of these elements is a lower bound on semangularity. Optionally, a better lower bound can be obtained by examining column interchanges of the resulting matrix, which requires approximately m^2 multiplications. The second stage is to carry out a branch and bound search with a good bound already established. In such a search only sequences of DPs which could possibly be better than the sequence already identified, are explored. If the bound is a good one, the search is very efficient.

A similar strategy can be used to find the set of DPs which maximize reangularity. In this case we first create the m x m symmetric matrix of
sines of angles between column vectors in the coupling matrix. This operation requires approximately \( m^2n/2 \) multiplications. We then establish a lower bound on reangularity by taking a product of terms and again use a branch and bound search to find the optimum.

4.8 Alternative Measures of Coupling

The measures of coupling defined in this chapter reflect two aspects of coupling in designs. The measures are convenient to apply and they appear to be useful metrics but they are not unique.

A precursor to the measure of semangularity was called alignment [32]. Alignment, like semangularity, is a measure of correspondence between DPs and FRs but alignment was normalized by reangularity. Alignment has some desirable characteristics, most notably that the alignment can achieve the most favorable value even when reangularity is poor. This is a desirable characteristic because the designer is immediately aware that the deficiency in the design lies in the fact that at least two of the DPs have a similar effect on the set of FRs. The normalization of the alignment measure results in some undesirable characteristics of the measure as well. These include the need to use logical conditions to compute the measure, the fact that the measure has very little meaning unless considered with reangularity and most importantly that it is difficult to acquire a "feel" for the meaning of the measure. As a result, alignment was abandoned in favor of semangularity.

It is not inconceivable, perhaps likely, that the measures presented here will be modified to reflect the insight obtained by applying these measures to more complex cases. A very desirable modification would be to combine reangularity and semangularity into a single measure that reflects
the relative importance of the individual measures. For the cases presented in the next chapter the measures reangularity and semangularity are tabulated with the geometric mean of these two measures to demonstrate a simple way that a single measure can be developed. The geometric mean has desirable characteristics as a measure of coupling at the extremes of reangularity and semangularity. However, the characteristics of the geometric mean have not been studied in enough detail to determine if it is an acceptable criterion for selecting among design alternatives. The geometric mean of reangularity and semangularity is plotted as a function of $\alpha_1$ and $\alpha_2$ in Figure 4.8.
Figure 4.8: Contours of the geometric mean of reangularity and semangularity as a function of alpha one and alpha two.
Chapter 5

DESIGN OF A PASSIVE ELECTRICAL NETWORK WITH
THREE FUNCTIONAL REQUIREMENTS

5.1 Overview

In Chapters 2 through 4 we repeatedly discussed the design of an electrical network with two functional requirements. The three configurations considered in those chapters consisted of a single capacitor and a single resistor chosen to satisfy the filter cut-off frequency requirement and the DC gain or light beam deflection requirement. All of the configurations were capable of satisfying the functional requirements for the four combinations of displacement transducers and galvanometers. However, Configuration 1, consisting of a shunt capacitor and a resistor in series with the transducer, was found to exhibit less coupling than the other configurations.

In this chapter we will discuss a slightly more complicated design problem and more complex network configurations based on the passive network design already presented. The design must satisfy three functional requirements, rather than two, as in the previous example. Seven network configurations having up to five components are evaluated using the measures of coupling presented in the previous chapter.
5.2 The Design Problem

In Section 2.5.3 we discussed the design of an electrical network to match a displacement transducer to a recording galvanometer. The specifications for the transducers and galvanometers are given in Appendix A. The functional requirements for a first order, low pass filter were specified as:

FR₁: Obtain filter pole at 6.94 hz.
FR₂: Obtain DC gain such that full scale deflection results in +3" light beam deflection.

The FRs were established without considering the dynamic response of the galvanometer, i.e. the galvanometer response was assumed to be proportional to the instantaneous current. This is not true of an electro-mechanical device such as a galvanometer, which consists of a lightly damped spring-mass system. If external damping is not added or if too much damping is added, the galvanometer will exhibit poor transient response and will have a limited flat response frequency range. The manufacturer recommends that the damping of galvanometers to be 64% of critical damping. This is a third functional requirement for the passive electrical network.

The ramifications of not considering damping are significant in the examples already presented. Network configurations one, two and three, designed to match the LVDT transducer to the type 348 galvanometer, result in damping ratios of .13, .13 and 1.26 respectively. The low damping ratios are particularly disadvantageous because the undamped natural frequency of the type 348 galvanometer is 100 hz. and fundamental carrier frequency is 120 hz. The result is a tripling of the residual carrier recorded by the galvanometer.
When galvanometer dynamics are considered, the complete set of FRs can be written as:

\( \text{FR}_1: \) Obtain filter pole at 6.94 hz.

\( \text{FR}_2: \) Obtain DC gain such that full deflection results in \( +3'' \) light beam deflection.

\( \text{FR}_3: \) Obtain 64% critical damping of the galvanometer.

The proper damping level can be obtained by designing the network such that the driving impedance of the galvanometer is as specified by the manufacturer. The driving impedances for the CEC type 345 and 348 galvanometers should be 180 and 120 ohms respectively.

Since a tolerance on damping is not specified, we once again adopt the intrinsic scale, i.e. a tolerance proportional to the absolute magnitude of the damping ratio, \( H \). If the damping requirement is specified in terms of the driving impedance rather than damping ratio the tolerance on the impedance must be selected to correspond to the allowable tolerance on the damping ratio.

5.3 Alternative Network Configurations

Seven configurations of low pass filter networks are proposed as means for satisfying the functional requirements listed at the end of the previous section. The networks, identified by the numbers four through ten, are shown in Figure 5.1. The networks four through nine are simplifications of Network 10 obtained by allowing components in Network 10 to assume extreme values. Figure 5.2 shows Network 10 coupled to a displacement transducer and galvanometer. The table included as part of
Figure 5.1: Seven configurations of low-pass filter networks.
<table>
<thead>
<tr>
<th>Network</th>
<th>FR Vector</th>
<th>Active Components</th>
<th>Null Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>[W,D,H]</td>
<td>C,R3,R1</td>
<td>R2,R4</td>
</tr>
<tr>
<td>5</td>
<td>[W,D,H]</td>
<td>C,R3,R2</td>
<td>R1,R4</td>
</tr>
<tr>
<td>6</td>
<td>[W,D,H]</td>
<td>C,R2,R4</td>
<td>R1,R3</td>
</tr>
<tr>
<td>7</td>
<td>[W,D,H]</td>
<td>C,R4,R3,R2</td>
<td>R1</td>
</tr>
<tr>
<td>8</td>
<td>[W,D,H]</td>
<td>C,R3,R2,R1</td>
<td>R4</td>
</tr>
<tr>
<td>9</td>
<td>[W,D,H]</td>
<td>C,R4,R2,R1</td>
<td>R3</td>
</tr>
<tr>
<td>10</td>
<td>[W,D,H]</td>
<td>C,R4,R3,R2,R1</td>
<td>none</td>
</tr>
</tbody>
</table>

**Figure 5.2:** Master filter network. The table lists active and null components and the possible design parameters for each sub-network.
Figure 5.2 lists the active and null components for each network as well as the design parameters and function vector for each configuration.

Networks 4-6 consist of three components, a capacitor and two resistors. The component values are the design parameters for the configuration. In these cases there are only as many design parameters as functional requirements and there is a unique combination of design parameters for each configuration such that the FRs are satisfied.

Networks 7-9 have four components and four design parameters. Since there are only three FRs, one of the DPs can be selected arbitrarily (over a limited range) to minimize the coupling in the design. The configuration with the minimum coupling can be selected after the design parameters have been chosen to minimize coupling in the individual configurations.

Network 10 has five components and five design parameters. It is the most complex of the networks being considered. Two parameters can be varied arbitrarily to minimize the coupling in the design.

In the sections that follow we will examine the benefits of the different configurations with respect to coupling in design. We will limit our discussion to the case of the LVDT driving the type 348 galvanometer. The details of this case and the others are included in Appendix E.

5.4 Filter Configurations with Three Design Parameters

The filter cut-off frequency, light beam deflection and galvanometer damping ratio can be easily computed in terms of the network design parameters, LVDT characteristics and galvanometer characteristics for network configurations four through six. The equations describing the relationships between FRs and DPs can be differentiated and then evaluated at the desired design point to obtain the coupling matrix. The normalized
coupling matrices for Networks four through six are listed in Table 5.1. The FRs are shown as a column vector to the left of the coupling matrix in the figure and the appropriate DPs are shown as a row vector above the coupling matrix to indicate the correspondence between FRs, DPs and elements of the coupling matrix. Note that Networks 4 and 6 are quasi-coupled and Network 5 is nearly quasi-coupled. Measures of coupling can be computed from the coupling matrices using Equations 4.11 and 4.16, with the caveat that the correct or best correspondence between DPs and FRs must be identified. The measures of coupling for the three network configurations matching the LVDT with the type 348 galvanometer are tabulated in Table 5.2.

Network 5 is the most strongly coupled of the three. A change in $R_3$ has significant effects on both cut-off frequency, $W$, and gain, $D$, since $R_3$ directly shunts the capacitor. Similarly, $R_2$ affects both gain and galvanometer damping, $H$, since it is in series with the galvanometer. $R_2$ does not affect the cut-off frequency to a large extent because $R_2$ is much greater than $R_3$ and therefore does not affect the apparent impedance across the capacitor. The capacitor value affects only the cut-off frequency since the other requirements are of a steady state nature.

There is less coupling in Network 4 because $R_1$ is very large compared to $R_3$ and therefore only affects gain. $R_3$ however has significant effects on all of the FRs since $R_3$ directly shunts both the capacitor and the galvanometer.

Network 6 is the least coupled of the three. $R_2$ is very large and effectively decouples the cut-off frequency requirement from the other FRs. $R_4$ strongly affects both gain and damping since it shunts the galvanometer. This coupling between the gain requirement and the damping requirement is
Table 5.1

Coupling Matrices for Networks Four, Five and Six

| Network | \[
<table>
<thead>
<tr>
<th>W ]</th>
<th>C</th>
<th>R1</th>
<th>R3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-1.0</td>
<td>0.0</td>
<td>-.582</td>
</tr>
<tr>
<td>D</td>
<td>0.0</td>
<td>-1.0</td>
<td>.582</td>
</tr>
<tr>
<td>H</td>
<td>0.0</td>
<td>0.0</td>
<td>-.568</td>
</tr>
<tr>
<td>\</td>
<td>C</td>
<td>425 mf</td>
<td></td>
</tr>
<tr>
<td>\</td>
<td>R1</td>
<td>121.2 Kohm</td>
<td></td>
</tr>
<tr>
<td>\</td>
<td>R3</td>
<td>120.1 ohms</td>
<td></td>
</tr>
</tbody>
</table>

| Network | \[
<table>
<thead>
<tr>
<th>W ]</th>
<th>C</th>
<th>R3</th>
<th>R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.0</td>
<td>-.707</td>
<td>-.007</td>
</tr>
<tr>
<td>D</td>
<td>0.0</td>
<td>.707</td>
<td>-.782</td>
</tr>
<tr>
<td>H</td>
<td>0.0</td>
<td>.005</td>
<td>-.623</td>
</tr>
<tr>
<td>\</td>
<td>C</td>
<td>12754 mf</td>
<td></td>
</tr>
<tr>
<td>\</td>
<td>R3</td>
<td>1.82 ohms</td>
<td></td>
</tr>
<tr>
<td>\</td>
<td>R2</td>
<td>118.2 ohms</td>
<td></td>
</tr>
</tbody>
</table>

| Network | \[
<table>
<thead>
<tr>
<th>W ]</th>
<th>C</th>
<th>R2</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-1.0</td>
<td>-.015</td>
<td>0.0</td>
</tr>
<tr>
<td>D</td>
<td>0.0</td>
<td>-1.0</td>
<td>.716</td>
</tr>
<tr>
<td>H</td>
<td>0.0</td>
<td>0.0</td>
<td>-.698</td>
</tr>
<tr>
<td>\</td>
<td>C</td>
<td>12.5 mf</td>
<td></td>
</tr>
<tr>
<td>\</td>
<td>R2</td>
<td>121.1 Kohms</td>
<td></td>
</tr>
<tr>
<td>\</td>
<td>R4</td>
<td>120.1 ohms</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2
Measures of Coupling for Networks 4, 5 and 6
Matching the LVDT to Type 348 Galvanometer.

<table>
<thead>
<tr>
<th></th>
<th>Network 4</th>
<th>Network 5</th>
<th>Network 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reangularity</td>
<td>.662</td>
<td>.593</td>
<td>.699</td>
</tr>
<tr>
<td>Semangularity</td>
<td>.568</td>
<td>.441</td>
<td>.698</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td>.613</td>
<td>.511</td>
<td>.698</td>
</tr>
</tbody>
</table>
the primary source of coupling in all of the networks.

5.5 Filter Configurations with Four Design Parameters

Network configurations seven, eight and nine can be obtained by adding a single resistor to one or more of the networks described in the previous section as shown in Figure 5.3. Four design parameters are used to completely specify the design for each of these configurations. If one of the design parameters is specified arbitrarily, the other three are uniquely determined if the FRs are satisfied. Since one of the DPs can be specified arbitrarily, each of these configurations represent a continuum of possible solutions rather than a single solution. As the parameter varies so do the other component values, the coupling matrix and the measures of coupling.

First consider Network 7 which is obtained from Network 5 by adding \( R_4 \) or from Network 6 by adding \( R_3 \). \( R_4 \) is used as a parameter which determines the values of \( R_2, R_3 \) and \( C \) required to achieve the desired functional requirements. The coupling matrix and measures of coupling can then be computed. Reangularity, semangularity and the geometric mean of the two measures are plotted as a function of \( R_4 \) in Figure 5.4. The reangularity is greatest when \( R_4 \) is large but semangularity is greatest when \( R_4 \) is small, so it is immediately obvious that the relative importance of these measures must be considered. If the geometric mean is used as a single measure of coupling the minimum coupling occurs when \( R_4 \) is equal to 367 ohms, \( R_2 = 176 \) ohms, \( R_3 = 2.7 \) ohms and \( C = 8603 \) \( \mu \)F. The coupling matrix at this design point is:
Figure 5.3: Relationships between Networks 4 through 10. The vectors indicate the component values that can be varied in each configuration.
Figure 5.4: Reangularity, semangularity and the geometric mean as a function of R4 for the LVDT - Network 7 - 348 galvanometer case.
\[ \begin{bmatrix} \mathbf{Q} & R_2 & R_4 & R_3 \\ \mathbf{W} & -1 & -0.10 & -0.003 & -0.707 \\ \mathbf{D} & 0 & -0.921 & 0.716 & 0.707 \\ \mathbf{H} & 0 & -0.390 & -0.698 & -0.003 \end{bmatrix} \] (5.1)

The dotted lines partitioning the design vector and the coupling matrix indicate those DPs which control the functional requirements and the corresponding portion of the coupling matrix. If \([\mathbf{Q}, R_2, R_4]\) is the design vector, the cut-off frequency requirement is decoupled from the other FRs and we have a two degree of freedom coupled design. This reduced form allows us to examine the coupling between gain and damping and the tradeoff between reangularity and semangularity in the design.

Figure 5.5 shows \(\alpha_1\) and \(\alpha_2\) plotted as a function of \(R_4\) where the angle between an isogram of gain, \(D\), and an isogram of \(R_2\) is \(\alpha_1\) and \(\alpha_2\) is the angle between an isogram of damping and an isogram of \(R_4\), at the design point in the function space. This figure shows that as \(R_4\) increases so does \(\alpha_2\) but with little change in \(\alpha_1\). As \(\alpha_2\) becomes more nearly equal to \(\alpha_1\), reangularity improves because the vectors are more nearly orthogonal. Similarly, as \(\alpha_2\) increases, semangularity decreases since the correspondence between DPs and FRs diminishes. When \(R_4\) is a minimum we have a quasi-coupled design. As \(R_4\) increases we have a block decoupled structure, similar to Equation 4.24, in which semangularity has been compromised to improve reangularity. Network 7 is significantly less coupled than Network 5 and modestly less coupled than Network 6 at a cost of increasing complexity by adding an additional component.
Figure 5.5: DP isogram angles as a function of R4 for the LVDT - Network 7 - 348 galvanometer case. Alpha 1 is the angle between isograms of gain and R2. Alpha 2 is the angle between isograms of damping and R4.
Network 8 can be obtained from Networks 4 or 5 by adding a single resistor. One of the components is again used as a parameter which determines the remaining component values, the coupling matrix and measures of coupling. The measures of coupling for Network 8 are plotted as functions of $R_1$ in Figure 5.6. The maximum values or semangularity and the geometric mean are obtained when $R_1 = 0$. In this case the coupling matrix and measures of coupling are:

$$
\begin{bmatrix}
Q & R_1 & R_2 & R_3 \\
W & -1 & -0.001 & -0.007 & -0.707 \\
D & 0 & -1.000 & -0.782 & 0.707 \\
H & 0 & 0 & -0.623 & -0.005 \\
\end{bmatrix}
$$

(5.2)

Note that since $R_1 = 0$, all of the component values are identical to those in Network 5 but the measures of coupling are greatly improved. The reason for this apparent anomaly is that even though $R_1 = 0$, we are allowing $R_1$ to vary. In Network 5, $R_3$ affected both cut-off frequency and gain whereas in Network 8, $R_1$ affects only gain. The reangularity can be made greater if $R_1$ is as large as possible and $R_2$ becomes zero. In this case Network 8 degenerates to Network 4 and the reangularity is .662. The reangularity is greatest in the region labelled B in Figure 5.6 but in this case the design vector that maximizes reangularity does not maximize semangularity or the geometric mean.

Network configuration 9 is as strongly coupled as Network 6, from which it is derived, and therefore does not justify the added complexity of
Figure 5.6: Reangularity, semangularity and the geometric mean as a function of R1 for the LVDT - Network 8 - 348 galvanometer case. Regions A, B, C and D are governed by different design vectors.
an additional component. Therefore Network 7 is the least coupled of the networks that consist of four components.

5.6 A Filter Configuration with Five Design Parameters

Network configuration 10 consists of five discrete components, the values of which are the design parameters. Two of the DPs can be used as free parameters to uniquely identify a design that satisfies the functional requirements. Figures 5.7, 5.8 and 5.9 show how reangularity, semangularity and the geometric mean vary throughout the parameter space. Note that the maximum for all three of the measures is obtain along the line where $R_1=0$. In this case, Network 10 degenerates to Network 7 with the identical design vectors and measures of coupling as Network 7. Network 7 is preferable to Network 10 since it is less complex and coupled to the same degree.
Figure 5.7: Contours of reangularity as a function of two design parameters for the LVDT - Network 10 - 348 galvanometer case. The maximum value is .974. The minimum value is .701.
Figure 5.8: Contours of semangularity as a function of two design parameters for the LVDT - Network 10 - 348 galvanometer case. The maximum value is .698. The minimum value is .694.
Figure 5.9: Contours of geometric mean as a function of two design parameters for the LVDT - Network 10 - 348 galvanometer case. The maximum value is .770. The minimum value is .696.
Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

An axiomatic approach to design [40] is an alternative to conventional design improvement strategies. The axiomatic approach to design embodies two fundamental principles. The first of these is that designs should be configured to maintain the independence of functional requirements. The second is that designs should be minimally complex. An understanding of the nature of coupling and independence in design, measures of coupling and means for applying these concepts are necessary to utilize the axiomatic approach to design.

Toward that end, a representation of the general design problem has been developed based on the relationships between functional descriptions and physical descriptions of a design. A function vector specifies the performance of a product or process in a function space. Similarly a design vector consisting of all the parameters needed to specify a design can be represented in a design or option space. The relationships between design parameters and the functional performance of a product can be used to map one space into the other to obtain a vector field representation of the design. The characteristics of the vector field are related to the degree of coupling in a design and form the basis for measures of coupling.

A geometric interpretation of the vector field is useful for
understanding the relationships between design parameters and functional requirements in a design. The angles between isograms of design parameter values when plotted in the function space determine the extent to which changes in different design parameters have the same effect on the set of functional requirements. When two or more design parameters have a similar effect on the set of FRs, larger excursions of the DPs are required to obtain a prescribed change in one of the FRs. A measure of coupling called reangularity is based on this concept.

The angles between DP isograms and the coordinates of the function space determine the extent to which a design parameter change affects only one functional requirement of the product. This concept is embodied in a second measure of coupling called semangularity.

Reangularity and semangularity can be used as measures of coupling in designs with any number of functional requirements. They are insensitive to changes in design parameter scale. However, the scales of the functional requirements are critical since coupling depends on relative changes in functional requirements.

The measures of coupling have been applied to the design of simple first order low pass filter networks. The measures of coupling and the coupling matrix are useful for identifying the features of a design that cause it to be coupled and they enable the designer to minimize coupling or to select the least coupled among equally complex networks.

The validity and usefulness of the measures must be ascertained by applying the measures to more complicated design problems. The principle of minimizing coupling in a design is to be used in conjunction with the principle of minimum complexity. It is, therefore, necessary to test and perhaps refine the principle and measures of coupling by comparing equally
complex design configurations. More general measures of complexity can then be investigated to determine the validity and utility of the axiomatic approach as a comprehensive design improvement strategy.
REFERENCES


Appendix A
DISPLACEMENT TRANSDUCER, GALVANOMETER AND DEMODULATOR SPECIFICATIONS

Two displacement transducers are considered in the case studies in Chapters 2 through 5. The first is a precision LVDT, model DTM-5, manufactured by Pickering and Co., Inc., Plainview, NY. The characteristics of the LVDT are given in Table A.1.

The second displacement transducer is built from a four active arm strain gage bridge. The strain gage bridge characteristics are listed in Table A.2.

The signals from the displacement transducers are operated on by full wave-phase sensitive demodulators. The configurations for the demodulators for the LVDT and strain gage displacement transducers are shown in Figure A.1. The secondary windings of the LVDT do not have a common reference so the outputs can be rectified and differenced directly. The two sides of the strain gage bridge do have a common reference so the demodulator uses the bridge excitation as a reference signal to detect the phase or direction of the deflection.

The demodulator output drives the network which is to be designed. The output of the passive network is fed directly to a light beam galvanometer to obtain a record of the deflection.

The galvanometers are standard magnetically damped galvanometers manufactured by the Consolidated Electrodynamics Division of Bell and Howell. The galvanometer specifications are listed in Table A.3. Both magnetic and fluid damping contribute to the total damping of the galvanometers. The fluid damping factor is constant for each type of
galvanometer and the magnetic damping depends on the driving point impedance. The total damping is

\[ H_T = H_m + H_v \] 

(A.1)

where \( H_v \) is the viscous damping factor and \( H_m \) is the magnetic damping.

where \( R_{\text{nom}} \) is the recommended source impedance

\[ H_m = 0.64 \frac{R_{\text{nom}} + R_g}{R_d + R_g} \] 

(A.2)

\( R_d \) is the actual source impedance

\( R_g \) is the galvanometer resistance

Equations A.1 and A.2 are used to determine the effect of source impedance on galvanometer damping.
Table A.1
Pickering LVDT Model DTM-5 Specifications

**Device Characteristics**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Linear Range</td>
<td>+ .500</td>
<td>inches</td>
</tr>
<tr>
<td>Linearity</td>
<td>+ .50</td>
<td>percent</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>1.5</td>
<td>volt/volt/inch</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>1860</td>
<td>ohms</td>
</tr>
<tr>
<td>Nominal Input Voltage</td>
<td>6 volts @ 60 Hz.</td>
<td></td>
</tr>
<tr>
<td>Output Voltage (min)</td>
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<td>volts</td>
</tr>
<tr>
<td>Primary Resistance</td>
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<td>ohms</td>
</tr>
<tr>
<td>Secondary Resistance</td>
<td>240</td>
<td>ohms</td>
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</table>

**In Situ Performance**

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<tr>
<th>Performance</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
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<tr>
<td>Full Scale Output</td>
<td>+ 4.5</td>
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</tr>
<tr>
<td>Full Scale Deflection</td>
<td>+ 0.5</td>
<td>inches</td>
</tr>
<tr>
<td>Input Voltage</td>
<td>6 volts @ 60 Hz.</td>
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</tr>
</tbody>
</table>
Table A.2

Strain Gage Bridge Displacement Transducer Characteristics

**Device Characteristics**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge Configuration</td>
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<tr>
<td>Strain Gage Manufacturer</td>
<td>Micro-Measurements</td>
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<tr>
<td>Gage Type</td>
<td>Resistance</td>
</tr>
<tr>
<td>Model Number</td>
<td>EA-06-187BB-120</td>
</tr>
<tr>
<td>Gage Resistance</td>
<td>120.0 ohms + .15%</td>
</tr>
<tr>
<td>Gage Factor</td>
<td>2.03 + .5%</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>120 ohms</td>
</tr>
</tbody>
</table>

**In Situ Performance**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Displacement Strain</td>
<td>.00116</td>
</tr>
<tr>
<td>Bridge Excitation</td>
<td>10 volts @ 60 hz.</td>
</tr>
<tr>
<td>Full Scale Bridge Output</td>
<td>.015 volts (after demod)</td>
</tr>
</tbody>
</table>
Table A.3
Galvanometer Characteristics

<table>
<thead>
<tr>
<th></th>
<th>CEC 7-345</th>
<th>CEC 7-348</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Damping Resistance (ohms)</td>
<td>180</td>
<td>120</td>
</tr>
<tr>
<td>Undamped Natural Frequency (hz.)</td>
<td>335</td>
<td>100</td>
</tr>
<tr>
<td>Flat Frequency Range [5%] (h)</td>
<td>0-200</td>
<td>0-60</td>
</tr>
<tr>
<td>Terminal Resistance (ohms)</td>
<td>84</td>
<td>98</td>
</tr>
<tr>
<td>Sensitivity (micro-amp/inch)</td>
<td>23.9</td>
<td>6.71</td>
</tr>
<tr>
<td>Sensitivity (inches/micro-amp)</td>
<td>0.0418</td>
<td>0.149</td>
</tr>
<tr>
<td>Maximum Safe Current (ma)</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Viscous Damping</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Nominal Magnetic Damping</td>
<td>0.50</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Figure A.1: Demodulator configurations for the LVDT and strain gage displacement transducers.
Appendix B

SPECTRAL ANALYSIS OF THE DEMODULATOR OUTPUTS

The output of the displacement transducer is an amplitude modulated displacement signal. If the displacement is a simple sinusoid, as shown in Figure B.1, then the transducer output is as shown in Figure B.2. The transducer output is passed through a full wave, phase sensitive demodulator. The demodulator output for the sinusoidal displacement is shown in Figure B.3. The demodulator output drives the networks being considered in the case studies. The networks filter and attenuate the signal which then drive the galvanometer to make a permanent record of the deflection. A simulated deflection record is shown in Figure B.4. Note that a significant ripple occurs on the deflection record as a result of incomplete filtering of the carrier. The carrier frequency and filter cut-off frequency were selected to demonstrate how poor filter selection affects the accuracy of the record. In general, the optimum filter parameters depend on the magnitude of the signal relative to the noise or carrier as shown in Appendix C.

The relative magnitudes of signal and carrier can be determined by carrying out a spectral analysis of the demodulator output when the displacement is a simple sinusoid. If the displacement, \( D \), is

\[
D = A_s \sin(\omega_s t)
\]  

(B.1)

where the subscript \( s \) denotes signal, then the transducer output, \( X \), is the amplitude modulated sine wave

\[
X = \{A_s \sin(\omega_s t)\}\{A_c \sin(\omega_c t)\}
\]  

(B.2)

where the subscript, \( c \), denotes carrier.
The demodulator output, $Y$, is

$$Y = |A_g \sin(\omega_g t)|A_c \sin(\omega_c t)|$$

$Y$ can be expanded into Fourier components by expanding the portion within the absolute value, carrying out the multiplication and then using a trigonometric identity to replace the $(\sin)(\cos)$ terms. The result is:

$$Y = \frac{2A_cA_g}{\pi} \left\{ \sin(\omega_g t) \right. - \frac{1}{3}\sin(2\omega_c+\omega_g)t + \frac{1}{3}\sin(2\omega_c-\omega_g)t \right. \right.$$

$$\left. - \frac{1}{15}\sin(4\omega_c+\omega_g)t + \frac{1}{15}\sin(4\omega_c-\omega_g)t \right\} \quad (B.3)$$

If we presume that the low pass filter will adequately attenuate frequency components above $3\omega_c$ we can truncate the series to the first three terms. The second and third terms are at slightly different frequencies near $2\omega_c$, so they will superpose at the beat frequency. When the two sidebands are $180^\circ$ out of phase the carrier magnitude is $2/3$ and the signal magnitude is $1$, therefore, the worst case signal to carrier or signal to noise ratio is 1.5.
Figure B.1: A 3 Hz. sinusoidal displacement to be recorded.
Figure B.2: The displacement signal shown in Figure B.1 amplitude modulated at 40 Hz.
Figure B.3: Full wave, phase sensitive demodulation of the signal shown in Figure B.2.
Figure B.4: The displacement record obtained by passing the signal shown in Figure B.3 through a low-pass filter with a single pole at 10 Hz. A comparison with Figure B.1 shows the errors introduced by non-ideal filtering.
Appendix C

CRITERION FOR FILTER POLE PLACEMENT

The pole locations of low pass filters determine the degree to which a carrier frequency or high frequency noise is suppressed and the fidelity with which the signal is passed, so filter pole selection is a trade-off between pass band fidelity and reject band suppression. Filter pole positions can be selected in a rational manner by examining the errors introduced because of non-ideal filtering.

Consider the simple case of a low pass filter with prescribed pass band and noise at a known frequency and magnitude. The maximum error possible is

\[ E_{\text{max}} = E_{\text{pb}} + 2A_nR_r \]  

(C.1)

where \( E_{\text{pb}} \) is the maximum pass band error

\( A_n \) is the noise amplitude

\( R_r \) is the rejection ratio of the noise frequency

If we divide by the signal magnitude to obtain the error ratio, we obtain

\[ E_{\text{rat}} = 1 - G_{\text{bp}} + \frac{2R_r}{R_{\text{sn}}} \]

where \( G_{\text{bp}} \) is the minimum pass band gain

\( R_{\text{sn}} \) is the signal to noise ratio.

The error ratio can be used as a criterion for pole placement.

The pole position for a first order low pass filter can be determined from Figure C.1 for a pass band ranging from 0 to 2 hz and the carrier at 120 hz. as in the case studies presented in Chapters 2 through 5. The optimal pole position of 6.94 hz. corresponds to the minimum error ratio for the case of signal to noise ratio equal to 1.5.
A contour map indicating the effect of pass band distortion and residual noise on the error ratio is given in Figure C.2.
Figure C.1: Signal error ratio as a function of filter pole frequency and S/N ratio for a pass band between 0-2 hz. and the carrier at 120 hz.
Figure C.2: Contour map of the signal error ratio as a function of pass band gain, S/N ratio and filter rejection ratio.
Appendix D

A CONDITION FOR MINIMIZING DP CHANGES

Equations 4.9 and 4.10 are repeated below.

\[ \Delta D P_1 = \frac{1}{c_{11}c_{22} - c_{12}c_{21}} \{ c_{22}(\Delta F R_1) - c_{12}(\Delta F R_2) \} \]  \hspace{1cm} (D.1)

\[ \Delta D P_2 = \frac{1}{c_{11}c_{22} - c_{12}c_{21}} \{ -c_{21}(\Delta F R_1) + c_{11}(\Delta F R_2) \} \]  \hspace{1cm} (D.2)

The changes required in the DPs depend on the relative magnitudes of the FR changes. We can assume without loss of generality that the Euclidean magnitude of the change in the function vector is unity. In this case, the changes in functional requirements can be expressed in terms of an imaginary angle, \( \gamma \).

\[ \Delta F R_1 = \cos(\gamma) \]  \hspace{1cm} (D.3)

\[ \Delta F R_2 = \sin(\gamma) \]  \hspace{1cm} (D.4)

Furthermore, we assume that the coupling matrix has been normalized. Using Equation 3.11 we can write Equations D.1 and D.2 as:

\[ \Delta D P_1 = \frac{1}{\cos(a_1)\cos(a_2) + \sin(a_1)\sin(a_2)} \{ \cos(a_1)\cos(\gamma) + \sin(a_1)\sin(\gamma) \} \]  \hspace{1cm} (D.5)

\[ \Delta D P_2 = \frac{1}{\cos(a_1)\cos(a_2) + \sin(a_1)\sin(a_2)} \{ -\sin(a_2)\cos(\gamma) + \cos(a_2)\sin(\gamma) \} \]  \hspace{1cm} (D.6)

These expressions can be simplified.

\[ \Delta D P_1 = \frac{\cos(\gamma - a_1)}{\cos(a_2 - a_1)} \]  \hspace{1cm} (D.7)
\[
\Delta P_2 = \frac{\sin(\gamma - \alpha_2)}{\cos(\alpha_2 - \alpha_1)}
\] (D.8)

Consider the case where \(\alpha_1\) is fixed and we would like to select \(\alpha_2\) so as to minimize the expected change required of \(P_2\). If \(\gamma\) is distributed evenly over the range \(0 - 2\pi\) then the expected value of the numerator in Equation D.8 is \(2/\pi\) for any \(\alpha_2\). Therefore, \(\Delta P_2\) is minimized when the denominator of Equation D.8 is maximized. This occurs when \(\alpha_1 = \alpha_2\) which corresponds to the case when isograms of design parameters in the function space are mutually orthogonal and when the quantity \((c_{11}c_{22} - c_{12}c_{21})\) is a maximum.

The result is identical when we seek to minimize change in \(P_1\).
Appendix E

PASSIVE FILTER CASE STUDIES

The detailed results of the passive filter design problem with three functional requirements are presented in this appendix. The three functional requirements of a first order passive filter are:

FR₁: W Obtain filter pole at 6.94 hz.

FR₂: D Obtain DC gain such that full scale deflection results in +3" light beam deflection.

FR₃: H Obtain 64% critical damping of the galvanometer.

The tolerances on the FRs are assumed to be proportional to the magnitudes of the FRs.

The seven networks proposed as solutions to the design problem are shown in Figure E.1. Networks 4-6 consist of three components, the values of which are the design parameters. Networks 7-9 consist of four components and have four design parameters. Network 10 consists of five components and is described by five design parameters. The relationships between the different network configurations are depicted in Figure E.2. Networks 4-9 are sub-networks of Network 10 obtained by setting one or more of the resistor values to zero or infinity. Figure E.3 shows the master network with a table of design parameters and null components for each of the networks.

The reangularity, semangularity, geometric mean, component values and design vector for Networks 4-6 are listed in Table E.1 for three combinations of displacement transducers and galvanometers. It is not possible to satisfy all three functional requirements when the strain gage drives the type 345 galvanometer.
The value of one of the four components in Networks 7-9 is chosen as an arbitrary parameter which determines the remaining component values. The measures of coupling are plotted in Figures E.4 through E.12 as a function of the parameter for each of the cases under consideration. The arrays of variables beneath the diagram in each figure indicate the design vector which maximizes the particular measure of coupling in the different domains of the design. The design vector which maximizes one of the measures of coupling may not maximize the other measures as can be seen in Figures E.7, E.8, E.9 and E.11.

The results for cases involving Network 10 are summarized in Table E.2. Figures E.13-E.21 are contour plots of reangularity, semangularity and geometric mean for each of the cases being considered. In all cases the measures of coupling are maximized when $R_1 = 0$. This is a degenerate case which is identical to Network 7.

Figures E.22 through E.24 summarize the case studies for the three combinations of displacement transducers and galvanometers.
Table E.1

Measures of Coupling, Component Values and Design

Vectors for Nine Passive Filter Case Studies

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<tr>
<th>Network</th>
<th>Reangularity</th>
<th>LVDT 345</th>
<th>Semangularity</th>
<th>LVDT 348</th>
<th>Geometric Mean</th>
<th>Strain Gage 348</th>
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<td>4</td>
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<td>[C,R1,R3]</td>
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<tr>
<td></td>
<td>Design Vector</td>
<td></td>
<td>[C,R3,R2]</td>
<td>LVDT 348</td>
<td>[C,R3,R2]</td>
<td>[C,R3,R2]</td>
</tr>
<tr>
<td>6</td>
<td>.859</td>
<td>.699</td>
<td>.858</td>
<td>.698</td>
<td>.858</td>
<td>.783</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>40.9</td>
<td>.783</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>180.8</td>
<td></td>
<td>120.1</td>
<td>.716</td>
</tr>
</tbody>
</table>
Table E.2

Measures of Coupling for Three Passive Filter Cases Involving Network 10

<table>
<thead>
<tr>
<th></th>
<th>LVDT 345</th>
<th>LVDT 348</th>
<th>Strain Gage 348</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reangularity</td>
<td>.991 (.840)</td>
<td>.974 (.701)</td>
<td>.958 (.707)</td>
</tr>
<tr>
<td>Semangularity</td>
<td>.858 (.689)</td>
<td>.698 (.694)</td>
<td>.654 (.506)</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td>.893 (.710)</td>
<td>.770 (.696)</td>
<td>.752 (.588)</td>
</tr>
</tbody>
</table>

Note 1: The minimum value of coupling in the parameter range inspected is shown in parenthesis.

Note 2: All cases of maximum coupling measure degenerate to Network 7.
Figure E.1: Seven configurations of low-pass filter networks.
Figure E.2: Relationships between Networks 4 through 10. The vectors indicate the component values that can be varied in each configuration.
<table>
<thead>
<tr>
<th>Network</th>
<th>FR Vector</th>
<th>Active Components</th>
<th>Null Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>[W,D,H]</td>
<td>C,R3,R1</td>
<td>R2,R4</td>
</tr>
<tr>
<td>5</td>
<td>[W,D,H]</td>
<td>C,R3,R2</td>
<td>R1,R4</td>
</tr>
<tr>
<td>6</td>
<td>[W,D,H]</td>
<td>C,R2,R4</td>
<td>R1,R3</td>
</tr>
<tr>
<td>7</td>
<td>[W,D,H]</td>
<td>C,R4,R3,R2</td>
<td>R1</td>
</tr>
<tr>
<td>8</td>
<td>[W,D,H]</td>
<td>C,R3,R2,R1</td>
<td>R4</td>
</tr>
<tr>
<td>9</td>
<td>[W,D,H]</td>
<td>C,R4,R2,R1</td>
<td>R3</td>
</tr>
<tr>
<td>10</td>
<td>[W,D,H]</td>
<td>C,R4,R3,R2,R1</td>
<td>none</td>
</tr>
</tbody>
</table>

Figure E.3: Master filter network. The table lists active and null components and the possible design parameters for each sub-network.
Figure E.4: Reangularity, semangularity and the geometric mean as a function of $R_4$ for the LVDT - Network 7 - 345 galvanometer case. The design vector, $[C,R_2,R_4]$, corresponds to the function vector $[W,G,H]$. 
Figure E.5: Reangularity, semangularity and the geometric mean as a function of R4 for the LVDT - Network 7 - 348 galvanometer case. The design vector, [C,R2,R4], corresponds to the function vector [W,G,H].
Figure E.6: Reangularity, semangularity and the geometric mean as a function of R4 for the Strain Gage - Network 7 - 348 galvanometer case. The design vector, [C,R2,R4], corresponds to the function vector [W,G,H].
Figure E.7: Reangularity, semangularity and the geometric mean as a function of R1 for the LVDT - Network 8 - 345 galvanometer case. Regions A, B, C and D are governed by different design vectors as shown in the table.
Figure E.8: Reangularity, semangularity and the geometric mean as a function of R1 for the LVDT - Network 8 - 348 galvanometer case. Regions A, B, C and D are governed by different design vectors as shown in the table.
Figure E.9: Reangularity, semangularity and the geometric mean as a function of $R_1$ for the Strain Gage - Network B - 348 galvanometer case. Regions A, B, C and D are governed by different design vectors as shown in the table.
Figure E.10: Reangularity, semangularity and the geometric mean as a function of R1 for the LVDT - Network 9 - 345 galvanometer case. Regions A and B are governed by different design vectors as shown in the table.
Figure E.11: Reangularity, semangularity and the geometric mean as a function of R1 for the LVDT - Network 9 - 348 galvanometer case. Regions A, B and C are governed by different design vectors as shown in the table.
Figure E.12: Reangularity, semangularity and the geometric mean as a function of R1 for the Strain Gage - Network 9 - 348 galvanometer case. Regions A and B are governed by different design vectors as shown in the table.
Figure E.13: Contours of reangularity as a function of two design parameters for the LVDT - Network 10 - 345 galvanometer case. The maximum value is .991. The minimum value is .840.
Figure E.14: Contours of semangularity as a function of two design parameters for the LVDT - Network 10 - 345 galvanometer case. The maximum value is .8584. The minimum value is .689.
Figure E.15: Contours of the geometric mean as a function of two design parameters for the LVDT - Network 10 - 345 galvanometer case. The maximum value is .893. The minimum value is .710.
Figure E.16: Contours of reangularity as a function of two design parameters for the LVDT - Network 10 - 348 galvanometer case. The maximum value is .974. The minimum value is .701.
Figure E.17: Contours of semangularity as a function of two design parameters for the LVDT - Network 10 - 348 galvanometer case. The maximum value is .698. The minimum value is .694.
Figure E.18: Contours of the geometric mean as a function of two design parameters for the LVDT - Network 10 - 348 galvanometer case. The maximum value is .770. The minimum value is .696.
Figure E.19: Contours of reangularity as a function of two design parameters for the Strain Gage - Network 10 - 348 galvanometer case. The maximum value is .958. The minimum value is .707.
Figure E.20: Contours of semangularity as a function of two design parameters for the Strain Gage - Network 10 - 348 galvanometer case. The maximum value is .654. The minimum value is .506.
Figure E.21: Contours of the geometric mean as a function of two design parameters for the Strain Gage - Network 10 - 348 galvanometer case. The maximum value is .752. The minimum value is .588.
Figure E.22: Coupling measures and design vectors at the optimum design point for each network configuration for the case of the LVDT driving the type 345 galvanometer.
Figure E.23: Coupling measures and design vectors at the optimum design point for each network configuration for the case of the LVDT driving the type 348 galvanometer.
Figure E.24: Coupling measures and design vectors at the optimum design point for each network configuration for the case of the strain gage transducer driving the type 348 galvanometer.

† The appropriate design vector is different from the one shown.
BIOGRAPHICAL NOTE

James R. Rinderle was born in Erie, Pennsylvania on June 9, 1954. He was raised in Euclid, Ohio and attended St. Joseph High School in Cleveland where he participated in interscholastic sports and was co-editor of the yearbook.

James did his undergraduate work at M.I.T. where he was active in his fraternity, Phi Kappa Sigma, and in intramural sports. He received the Bachelor of Science Degree in Mechanical Engineering in June 1976.

James entered graduate school at M.I.T. in the fall of 1976. He was a teaching assistant in "Mechanical Behavior of Materials" and wrote a thesis on precision injection molding under the sponsorship of the MIT-Industry Polymer Processing Program. He was awarded the Master of Science Degree in Mechanical Engineering in 1979.

James has been working on his doctoral thesis in the Laboratory for Manufacturing and Productivity at M.I.T. since that time. Upon completion of his degree he will join the faculty of the Carnegie-Mellon University as Assistant Professor of Mechanical Engineering.