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A MARKOVIAN QUEUEING NETWORK APPROACH TO THE OPTIMAL DESIGN OF HANDLING OPERATIONS AT CONTAINER PORTS.

by

Kasumu Odugbemi Salawu

Doctor of Engineering Science, Columbia University, (1972)

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1978

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A MARKOVIAN QUEUEING NETWORK APPROACH TO THE OPTIMAL DESIGN OF HANDLING OPERATIONS AT CONTAINER PORTS

by

Kasumu Odugbemi Salawu

Submitted to the Department of Civil Engineering on May, 1978 in partial fulfillment of the requirements for the Degree of Master of Science in Civil Engineering

ABSTRACT

This work is an application of Queueing Network Theory to the analysis of container handling operations at container terminals.

A container terminal is characterized as a closed network of tandem, Markovian queues which form as containers flow from, (to), ships, through operations at work stations, to, (from), the port's hinterland. Bulk service facilities, such as the storage, are treated as buffers, not actively involved in the flow of containers through the port. The effects of the variations in handling times, at each work station, on port dwell times and the turnaround times for ships and inland feeder vessels are demonstrated.

In place of the customary, detailed simulation experiments, a brief but versatile PL/I programme, PortQ, iteratively identifies the bottleneck work station and computes many useful performance measures, accurately. Input data requirements are minimal and may be approximate in most instances. The output includes the system's steady-state handling rates and asymptote, along with their associated levels of facilities utilization. These results show a better than 99% correlation factor with those of a detailed, simulation run on an identical system.

PortQ should form a good basis for decision making on the effective assignment of port facilities and capacity planning.

Thesis Supervisor: E. Frankel

Title: Professor of Ocean Engineering
ACKNOWLEDGEMENT

Professor Ernst G. Frankel, my friend, extended numerous professional courtesies to me and provided partial financial support for this work. For these, I am very grateful.

For serving as my academic adviser, I wish to thank Professor Henry S. Marcus. The management of the Moran Public Container Terminal at Mystic Wharf provided useful data for this effort and I also owe some thanks for these.

In addition to typing the entire manuscript, my wife, Norene, tolerated my absences from her and our son, Akikanmi, throughout the year. To her, I am most indebted and I dedicate this work.
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1. A QUEUEING NETWORK APPROACH TO THE ANALYSIS OF CONTAINERPORT OPERATIONS

1.1 THE CONTAINERIZATION TREND

The role of a seaport is to serve as an efficient, reliable and cost competitive link in the intermodal and intramodal transportation systems which facilitate the collection and distribution of cargo between the port's foreland and its hinterland. Where ports have not been recognized and operated as strategic transshipment points, port costs have been known to surpass more than twice the costs of actual movement of the ships that use them. This waste on inefficient port operations derive largely from poor coordination of activities and mismatched modes, thereby causing high turnaround times for both the ships and the inland feeder vessels. In 1975, these time losses accounted for 55% of all port-related costs.¹

Unitization has converted, (through consolidation), bits and pieces of mostly traditional, general break bulk cargo into boxes of standard size containers which are transported and handled much more efficiently. Table 1.1-1 lists the decisive advantages of modern container operations over those of conventional, general cargo liners and ports.²

¹ E.G. Frankel, Lecture: "Port Planning and Development" Course 13.631 Lecture at MIT, February 14, 1978
² Bird, (1971) presents an extensive bibliography on containerization throughout the book. Also see Bruun, (1976), pages 254-255 for more recent references.
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<th>Containerships and Containerports Operations</th>
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<tr>
<td>Throughput of 30,000 to 100,000 dwt a year</td>
<td>Can handle between 500,000 and 1,000,000 dwt a year</td>
</tr>
<tr>
<td>One longshoreman required for every 750 to 1,000 dwt of break bulk cargo per year</td>
<td>Bigger and more economical ships</td>
</tr>
<tr>
<td>Ships in port for 10-15 days which is up to 70% of their total voyage time</td>
<td>Ships in port for 1 to 2 days</td>
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<td>Where 1,980 metres of quay are needed here,</td>
<td>274.3 metres of quay are needed for equivalent berth capacity, hence need fewer berths</td>
</tr>
<tr>
<td>Where 40 cranes are needed here</td>
<td>Only 2 cranes are needed to do equivalent work</td>
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<tr>
<td></td>
<td>Higher overall capital costs but lower handling costs per ton of cargo</td>
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1. The entries in this table were mostly summarized from the National Port Council Bulletin, No. 9, (T1976).
It appears that, every year a higher percentage of containerizable goods — capture ratio, is carried in containers, than in the previous year. This in turn induces more demand for containerized cargo movement. During the decade from 1960 to 1970, worldwide, container movements doubled approximately every 2 to 3 years but that rate of growth was bound to diminish before the end of the next decade.

Nonetheless, it is clear that the advantages in the high level of service of fast, efficient and reliable container handling and delivery, (all of which translate to low-cost services), should not be allowed to be negated by port inefficiencies. Especially on short hauls, (though also on feeder, coastal, medium and long routes), it would therefore be most expedient to keep the turnaround times for all vessels which interface at the port to a minimum.

Table A-1 in Appendix A shows the history of container traffic at our representative port, the Boston-Mystic Moran Terminal. The unexpected decline in traffic in 1977 is directly attributable to the labour union strike which forced the Port to close down between October 1, 1977 and November 30, 1977. However it was estimated that in 1978, a capture ratio of 63% of containerizable cargo moving through the Port of Boston will be containerized and this would be handled in 103,330 full containers, (91% of all 113,510 containers the port will handle in that year).\(^1\)

---

1.2 LITERATURE SURVEY

Different approaches to measuring the performance of containerports were enumerated and compared by Frankel and Tang.\(^1\) Essentially, these were attempts to measure and model the capacity and throughput of a port or some of its links empirically or analytically, using queueing theory or by digital simulation. The subobjectives of these methods include:

a) the maximization of berth occupancy, i.e. the total number of hours ships occupy berth

b) the determination of the optimum number of berths, given ship lengths and berth lengths

c) optimization of berth throughput

d) maximization of ship productivity via the minimization of ship port time and unit port service costs

e) maximization of port labour productivity

f) optimal utilization of storage area

From this grossly abbreviated list of subobjectives, it is clear that multiple criteria should be used in evaluating containerport performances. An attempt is made to incorporate as many of these objectives as feasible and appropriate into the analytic approach used in this work.

---

1. A Comprehensive review of a number of approaches was presented by Frankel and Tang, (1977).
The theoretical background of this work comes from queueing network theory. The thorough work of Jackson\(^1\) laid the general foundation for queueing network theory and in some cases, made definitive statements about occurrences in these networks. In particular, Jackson introduced the idea of dependency of exponentially distributed interarrival and service times on the state of the system of tandem queues interconnected to form a network. Noting the variety of sequencing of service for different customers in these systems, Jackson named them 'Jobshop-like Queueing Systems.'\(^2\)

Next, Gordon and Newell elaborated on Jackson's suggestion that the total number of customers in the system be held fixed by imposing the concepts of closure and cyclicity on Jackson's open network. This way, queueing network theory becomes more readily applicable to a lot of phenomena that involve flows of customers in queueing networks, such behaviour as our containers display at ports.

In this succession of brilliancies, Buzen\(^3\) successfully applied the results of these works to the utilization of the central processing unit in time-shared computer systems. He developed efficient computational algorithms for investigating performance in closed network of queues.\(^4\)

---

2. Ibid, page 135.
Recently, Solberg reported astounding results in applying the works of Jackson, Gordon and Newell, and Buzen to computerized manufacturing systems. By modelling an integrated system of computer controlled, (DNC), metal cutting machine tools and an automatic transport mechanism as a closed network of queues, he obtained results, that agreed to within 3%, as those obtained in a detailed and carefully validated digital simulation of the same system.

Riding this wave of optimism, our work here is offered as an alternative approach to the likes of Creton's simulation which has large data requirements, could be very elaborate and prohibitively expensive and would need extensive statistical validation. Furthermore, whereas this work examines container terminals in particular, it will be seen that even multipurpose ports can be conceptualized to use the model developed here. Any lift-on lift-off container terminal, (wheeled or grounded), serving non-self-sustained containerships may use the model as it is.

1.3 PROBLEM DEFINITION

Among other considerations, which are discussed in the next section, the throughput of a container terminal is a reliable measure of its level of productivity. Though increasing the capacity of a container port does not necessarily attract more traffic, there is a direct variation of throughput with capacity. We shall define port capacity as the amount of cargo traffic that a port can handle in a unit period of time. In operational terms, port congestion occurs when the port is unable to cope with the frequency and volume of cargo traffic intended to pass through it and this in turn causes long waiting and turnaround times for ships and inland feeder vehicles at the port.

It is widely accepted that the capacity of a port is limited by that of its bottleneck path. There is a more readily observable determinant of port capacity, which also unambiguously identifies the bottleneck path and its utilization. This indicator is the length of time containers remain at the port, otherwise referred to as their 'dwell time'.

It has been shown that, for either imports or exports, the longer the shipping service interval, at a port, the longer the dwell time for the containers. It was also reported that though dwell times were chiefly dependent on a port's operating efficiency, consignees tended to use the port as a warehouse more if the quay rents charged were lower. The rate and quality of information flow between the port and its users
along with the nature of the commodity also have an appreciable bearing on dwell time. The sum effect of these influences on dwell time is summarized in the following, causal system shown in Figure 1.2-1.

Whereas our ultimate objective is to minimize the turnaround times for all vessels that interface at a container port, our auxiliary objective would be to minimize the expected dwell time of containers at the terminal. As an indispensable aid to this exercise, let us visualize some Time-Card-carrying containers between a port's entrance and its gates.

1.4 CRITERIA FOR EVALUATION AND UNITS OF MEASURE

Internal congestion plays a very significant role in determining the productive capacity of the port hence our emphases here would be on the nature and causes of queues of containers that form from time to time, as they require the use of the port's resources. The analysis of waiting lines beyond the harbour's entrance or the terminal's gates is beyond the scope of this work.

The central interest is in the availability, performance and utilization of the port's facilities and equipment. The quality of maintenance and repair of the port's facilities has a decisive influence on the effective capacity of these facilities and, of course, on the likelihood of the formation of queues. In this model of tandem queues, blocking is not allowed but queues of containers, of length equal

1. Dally, et.al. (1977).
Figure 1.2-1: The Influence of Container Dwell Time on Vessel's Turnaround Times
to the maximum number permitted in the system at any time, (due to closure), are allowed to form at any service or work station in the system. This model will measure the steady state saturation or congestion flow rate at every node, (station), and in the various links.

Other measurements provided directly by the output of this model include:

a) the level of utilization of all work stations at the port, (identical equipment or facilities e.g. fork lift trucks, may be pooled to form one service station or may each be treated as a separate service station). When the appropriate basis for comparison among single and multiple server stations is established, we identify as the bottleneck station, that which has the highest level of utilization

b) the actual throughput of the port and its asymptotic value, which, of course, is the effective capacity of the bottleneck station

c) the distribution of containers among work stations, (both in queue and in service)

d) the mean time it takes to process a container through the port, (its dwell time), both with congestion and in a free flow

e) other measures that will be better understood when discussed in Chapter 5, after benefiting from the discussions from now till then
Containerports consist of facilities, manpower, equipment processor and functional organizations which work together just as obtains in industrial plants. At the level of resolution of this work, these similarities and the standardization of containers permit us to use the time-tested methods of Industrial and Systems Engineering in analyzing the container polysystem.

The stated objective is the minimization of the total turnaround time of all vessels at the port and how this relates to and derive from measures enumerated above was the subject of 1.3. The next section will be a brief discussion of the model's data requirements.

1.5 DATA REQUIREMENTS

In the world-wide, door-to-door, physical distribution of containers, there are two major systems of handling and storage at containerports, viz. grounded and wheeled. Both have built-in economies of scale and are highly mechanized but there are quite a few, significant differences among their operations.

The wheeled system, such as Sealand's at Castle Island in Boston, has proved to be the more efficient. One container rests on one chassis, which means a high investment in many chassis and in the large acreage on which to park

---

1. The discussion will be simplified by reducing the variety of physical sizes to Twenty-foot Equivalent Units, (TEU's).
these chassis. The grounded system, such as the one in
operation at the Boston Mystic terminal stacks containers,
(could go five-high), one atop the other using straddle
carriers or hoisters. Though the grounded system requires
less land, its facilities require more maintenance; have
lower availability and reliability; are more hazardous and
tiring to operate and, in general, have higher, overall
operating costs.

Unlike the grounded system, Sealand's operations
are heavily computerized which means faster search times and
less container dwell time as the system, by its very nature,
places more efficient, technological constraints on its users.
It would have been best to obtain more readily available data
from Sealand but one cannot physically observe Sealand's
integrated operations without going to New York. To build a
model, this observation stage is very crucial to its concept-
tualization. The Boston Mystic containerport operations
can however be observed in its entirety, in Boston, and
despite data being not readily available there, Mystic was
used in characterizing the containerport.

The craneage/ground system does not otherwise detract
from the powerful yet simple uses to which this model could
be put. Data requirements for this model are minimal and
as will be seen, very large systems can be evaluated with
no severe, computational penalty. Essentially, the inputs
required to use this model are:
a) the mean time it takes to perform a handling or storage operation at the relevant work station and the number of servers at that station

b) the relative frequency with which an operation is performed on the containers. For example, for the quayside cranes, this is 1, as they handle every container that comes in or goes out of the port whereas the inspection operation may be performed on 1 out of every 3 containers.

Up until the time of this writing, no formal record on how much time a handling or storage operation took had been kept at the Boston-Mystic Container Terminal. However in a personal interview with the Terminal Manager, Mr. Frank X. Johnston on Friday, April 28, 1978, Mr. Johnston disclosed that precisely the time-card system used in this work was due to go into effect in August 1978.

The test data used in this work were gleaned from interviews with high level administrators as well as equipment operators at the terminal.

1.6 BEHAVIOURAL CONSIDERATIONS

Port planning should be a dynamic, multi-objective decision making process which takes into account the social, economic, political, cultural and ecological impacts of the port on its environment, for given technology and changing

1. These include Mr. Brian Gafee, the Port Maintenance Manager and Mr. Donald Young, the Control Tower Supervisor, Mr. Dennis Kaye and Customs Inspector Shrull.
user needs. Though it has been stated that this work concerns itself only with what happens to the containers between a port's seaward and landward entrances, it is recognized that the results here would only be an input into the recommended, total systems approach to port planning.

All through this work, it is implicit that the port's environment is able to support the port's performance. For instance, it is assumed that owners and operators of inland feeder vessels and the vehicles themselves show an appreciable level of reliability and that there is reasonable accord among the Port Authority, the ship owners, the shippers and the port workers. Definitely, labour union activities, such as strikes and demands for higher wages have very significant effects on port operations and operating costs but this model does not capture these considerations explicitly. Again the model requires that all containers flowing into the hinterland are returned filled or empty. It is clear that this is not necessarily so in actual operations. (Especially in a developing country, consignees may adapt containers to a thousand and one other uses, to satisfy their needs, without ever returning them to the port!). All told, it is hoped that faster turnaround for all vessels that meet at the port will indirectly pronounce the forward and backward linkage effects on the hinterland economy and even more so, make the port more competitive.
1.7 OVERVIEW OF THE THESIS

In the next chapter, Chapter 2, the conceptual model of the port will be elaborately presented. Against this background, in Chapter 3, a mathematical model will be developed, using relevant results from queueing network theory. This will be followed by the description of the brief but very powerful and easy-to-use computer programme in Chapter 4. This programme was very carefully prepared as it will form the basis for the documentation of a programme package which will be tested at various ports.

Chapter 5 deals with the analysis and interpretation of the results derived from this model. Furthermore it includes discussions on the validation of the model and sensitivity analysis. Concluding remarks are made in Chapter 5 together with suggestions for further research along the line of this work.
2. THE CONCEPTUAL MODEL

2.1 FLOW OF CONTAINERS THROUGH PORT OPERATIONS

A container port is characterized as a multinodal network of queues which form as the free flow of container units, through handling and storage operations at the Port, is obstructed. The containers require inherently random service times at the port's work stations. The sequencing of operations on each container is permitted to vary as required and the causes of random service times are explained later in this chapter.

Figure 2.1-1 is a schematic of the configuration of the port's network of stations, (nodes) and links. Table 2.1-1 lists some of the operations that may be performed on full import containers at the port's work stations. A work station is either a place where a container is physically present for handling and storage or where its movement through the port is being processed. A station may be physically fixed, such as a storage location or may be mobile, such as a pool of fork lift trucks. Note that operations such as administrative processing for receipt or delivery of containers may be performed at places other than where the containers are physically present, in this case, maybe in storage or on an inland feeder truck.

Delays in the movement of containers may occur in any
<table>
<thead>
<tr>
<th>Table 2.1-1 Container Port Operations and Work Stations</th>
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</thead>
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<tr>
<td><strong>Stage:</strong></td>
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**Remarks:**
- Table 2.1-1 provides a detailed breakdown of container port operations and the associated work stations. Each cell in the table represents a specific node within the port's operational structure. The table highlights the processes and stations involved in handling container traffic, from berthing to export and import operations, ensuring smooth and efficient port operations. 

**Notes:**
- The diagram indicates the flow of operations, with arrows pointing from one stage to the next, illustrating the sequential nature of container handling. 
- The table is designed to be easily referenceable, allowing stakeholders to quickly identify specific areas and processes within the port environment.
of the three subsystems, for example,

a) in the harbour/berth subsystem, either when the containers are aboard ships waiting to berth, (deberth), or at the dock while ships are waiting to be unloaded, (loaded)

b) in the transit storage subsystem, while containers in inventory are waiting to be picked up either by an inland feeder vehicle or to be loaded on to ships

c) in the delivery subsystem while paperwork on containers is being processed by the port's administrative unit for exit or entry into the port.

As mentioned earlier, blocking, (the refusal of entry to a station or its queue), does not occur in this sequence of tandem queues. However, we allow, for instance, the frequent unavailability of straddle carriers to affect the effective capacity of the cranes that handle the containers next. In this case, the model either allows a very long queue to build up for the use of the straddle carriers or uses as input, an adjusted service time to which average waiting time per container when equipment is down has been added. We note that the level of maintenance and therefore, reliability of equipment affects the allocation of other equipment at the terminal.

In general, when reference is made to container units as a homogeneous group, the twenty-foot equivalent units,
(TEU's), are assumed. However, this model can handle, as it is, any variety of container types, for example, the twelve different categories which are formed by the cross product of the following sets of containers:

\[ \text{[Import,Export]} \times [20',40'] \times \text{[Full, Partial, Empty]} \].

The one basic requirement that must be satisfied for any category of containers is its flow balance in and out of the port. This model considers three categories, namely:

\[ \text{[Full, Import, TEU]}, \text{[Full, Export, TEU]}, \text{[Empty, Export, TEU]} \].

The reason for mentioning only empty exports is that the Boston-Mystic terminal is an import-oriented terminal where the ratio of full imports to full exports sometimes reaches 2:1. Hence to balance the flow, empty containers must be exported. As necessary, any k-tupled category may be used as input, for instance, the set of what is being carried in the containers or what the container is made of, may be included in the cross product of sets.

2.2 HARBOUR/BERTH SUBSYSTEM

As containerships approach the harbour of a container terminal, we are ready to punch our time-card-carrying containers into the port system. We will continue to punch these cards as we observe the movements of those containers intended for delivery at our port.
2.2.1 HARBOUR ENTRANCE

Tugboats meet the ship at the anchorage area, or channel access, (node 0), so as to guide it until it lies alongside the length of the wharf as needed, that is, at its berth, (node 1). (A node is roughly equivalent to a work station).

Different types and numbers of transfer equipment like tugs are needed for towing and piloting operations, (link(0,1)), even for identical ships, because these operations are highly dependent on weather and sea conditions. Thus, a certain amount of uncertainty is introduced into the time for positioning the ship along the quay where it will be moored, (link (1,2)), and refueled. Links are equivalent to operations and a container may spend time both at a node and at a link.

2.2.2 APRON NODE

Our representative model deals with non-self sustained, lift-on lift off ships that carry no special handling gears, no stowage and no lashing facilities. At the apron, (node 2), shoremounted, quayside container cranes are then used to load or unload the ship, subject to maintaining the equilibrium of the ship.

1. What is actually important here is the average number of containers aboard hence RORO, barge carriers or LASH ships could be used with this model.
Full import containers are discharged from the ship using one or more quayside cranes when available. Just as in industrial plants, time and motion studies, and work sampling have been performed on these operations. Studies have revealed that a single transfer cycle rate for the crane averages about 3 minutes, (both for containers removed from the deck and from the cells), and the double cycle about 4 minutes.\(^1\) Also, the Hook Cycle in a spreader system was found to average 2.5 minutes.\(^2\)

2.3 TRANSIT STORAGE SUBSYSTEM

2.3.1 FULL IMPORT STORAGE

Containers wait under the crane, after they are unloaded, for chassis/trailer or yard tractors. When a container finds one, it is either conveyed, (link(2,3)) to the open full import storage yard, (node 3), or pulled, (link(2,4)), to the open, empty import/export storage yard, (node 4). These containers are then either left on chassis or grounded and stacked.

The duration of this and other movement systems at containerports are directly affected by the skills of the operators of the yard tractors or trailers, space requirements and

---

1. It is assumed that the cranes will be able to handle all sizes and weights of containers that come.
available space, climatic conditions and the volume of work, i.e. the number of containers to be moved. In addition the rectilinear distances a yard tractor or trailer travels to/from storage are not necessarily the same and these motors may have to wait to collect imports from under the cranes or for a highway tractor to free some space in the storage area. These considerations introduce uncertainty into transfer times.

2.3.2 EMPTY IMPORT/EXPORT STORAGE

It is unlikely that the same containerport will import as well as export empty containers because the usual imbalance in full import and full export containers is corrected by the flow of empties in the deficient direction only. The preceding discussion under Full Import Storage Operations also holds here just as the following discussion on stacking, if practised, holds there too.

Empty, (or full), containers are received from tractors, trailers, fork lift trucks etc. For stacking, straddle carriers, toppick equipment, sideloaders, forklift trucks, large cranes extended by cantilever booms, portal cranes or yard loaders may be used. The sequence and height to which stacking takes place depends on the capacity of the stacking machine, the weights of the containers, and the amount and load-bearingness of the available land area. If port operations are wheeled, the containers are just left to rest on chassis, in open storage.
2.3.3 IMPORT CONTAINER FREIGHT STATION

From the Full Import Storage Station a container may or may not be transferred to the Import Container Freight Station, (CFS, node 5). At this CFS, full containers are stripped, sorted or reshuffled into consignments to different consignees. Since not all containers pass through this station, the entry for the relative frequency of performance of operations there will be less than 1.

2.4 DELIVERY SUBSYSTEM

After consignees or their agents have been notified of the arrival of cargo, they send inland feeder vehicles to the port to collect their consignment. About 70% of container traffic is door to door hence these feeder vehicles return through the landward links, to specific addresses of consignees in the hinterland.

2.4.1 INTERCHANGE POINT

Containers may be directed to the multimodal Interchange point, (node 6), after leaving the Full Import Storage Station. Here, containers are placed on rail, truck, waterway or air vehicles for transportation inland. In general, there is a random pattern of arrivals of inland vehicles at the
terminal and this precludes the full pre-planning of the collection of import containers or the delivery of export containers prior to shipment.

2.4.2 ADMINISTRATIVE COMMAND AND CONTROL

In order to provide proper data on containers in the port and in order to efficiently direct their movements and storage in the most adequate areas, a command and control procedure of processing paperwork on containers should be administered. Manual or electronic, (e.g. minicomputers), systems may be instituted to gather and analyse data and documents on the flow and processing of containers. This would reduce the delays in inspection, registration, marking and release of containers after steamship freight and customs duties shall have been paid. Security systems to minimize theft will be improved as would be the rapidity and reliability of information flow to consignees and truckers. It is noteworthy that while all this processing is going on at the administrative office - (node(7)), the containers are resting either in storage or on a vehicle.

Truckers present authorizations on delivery orders from brokers and Trailer Interchange Receipts, (TIR), in duplicate, for the port and steamline's records.
2.4.3 CUSTOM SPOT

On leaving the interchange point, some containers are randomly selected for inspection by Customs at node 8. Since this involves some sort of random sampling, the relative frequency with which this operation is performed or with which this station is visited by every container is less than 1. Inspections are conducted to check fraudulent imports, narcotics, agricultural products from countries with epidemic diseases and to enforce import quota. Exports are very infrequently inspected if they are ammunitions or to ensure that only licenced shippers are involved.

2.4.4 FULL IMPORT EXIT GATE

At the gate, (node 9), some more control operations and weighing of containers take place. The inland feeder vehicle may have to wait in a queue here until every vehicle that arrived at the gate before it has been processed for exit and until all of its papers have been fully processed.

2.4.5 DECONSOLIDATED CARGO EXIT

Cargo extracted from containers at CFS may or may not pass through customs inspection but they exit through node 10 after all their papers have been appropriately processed.
2.5 PROCESSING AND HANDLING EXPORT CONTAINERS

Export containers pass through almost identical stations as import containers, but in reverse order. Minor differences will be noted in the brief description of their processing that follows.

Full export containers from the hinterland may be processed into the containerport through node 12, the full export entrance while empty export containers\(^1\) may enter through the gate for secondary traffic, node 11. Pieces of cargo from different consigners are received through the CFS entrance, node 13.

The inland vehicles that deliver the cargo are relieved of their load by port carriers at interchange points, (nodes 14 and 15), and by yard equipment. It should be noted that each of the three interchange points can be broken down into up to four stations, ---- a station for each of the four modes: waterway, rail, road and air. The cycle time for a port carrier taking an export container from the inland feeder exchange area is, however, significantly less than the cycle time for delivering an import container to a road vehicle because the straddle carriers spend quite some time searching for import containers to hand over.

Empty containers may then move either to the container

\(^1\)This presupposes a discussion of an import-oriented port.
maintenance station, (node 16), if they need repairs, or they may go directly to the Empty Import/Export Storage yard. (Empty containers at node 4, that need repair are also sent to node 16). In some ports, (e.g. Boston's-Mystic Terminal), this maintenance station may not exist, in which case the repair and maintenance of empty containers is subcontracted to outside shops. Where a port has a maintenance shop, we note that only a fraction of the empty containers pass through this station.

Empty containers from nodes 4 and 16 are sent to the Container freight station, (node 17), where by stuffing them with pieces of cargo from node 13, consignments from different consigners, but intended for delivery at the same port, are consolidated into container loads. When filled, these containers and those from node 15 are stored in the open Full Export storage yard until they are loaded on a ship. 1 Occasionally, the full export containers are delivered directly to under the quayside cranes from where, in fact, the delivering vehicle may immediately pick up an import container. However, these vehicles usually have to queue to leave the export container for storage under the crane.

The full export containers are next loaded on to the ship at the apron. In general, loading times for export containers are higher than unloading times for imports because

---

1. However, when there is competition for storage space, imports get priority over exports.
a higher degree of organization is needed to get the container to the crane, to locate the spreader accurately on the export container and then place it within a carefully designated cell on the ship.

The ship, loaded with exports, deberths and is again guided by tugboats until it can gain access to deep waters and turning basins.

2.6 CLOSURE OF THE PORT QUEUEING NETWORK

Especially for computational reasons, it is more advantageous to consider the container terminal as a closed network of queues than as an open one.\(^1\) We wish then, to conceptualize a container port as a closed network of queues with a fixed number, N, of containers whirling through it. An auxiliary approach to imagining this situation is to first consider creating an additional station at the port which acts as a generator as well as a receiver of containers in maintaining this perpetual motion.

It is important to note that any work station through which all containers moving in and out of the port pass, (for example, the station of cranes used to load and unload ships at the apron), will be treated in the exact same manner, mathematically, as the grand marshalling station

\(^1\) See Buzen, (1973).
suggested above. No supposition is hereby made that our station of cranes be designated as the bottleneck station.

The grand marshalling station, station M, say, precedes and succeeds every operation at every one of the other M-1 stations as shown in Figure 2.6-1. Station M may be thought of as a ubiquitous transporter which

a) generates and delivers a container to a station j with probability $p_{Mj}$, in an infinitesimally small amount of time or rather instantaneously.

At station j, $s_j$ servers perform some operation i on the container in a finite period of time $t_{ij} > 0$. For example, $t_{ij} = 3$ minutes for the unloading operation, i, by cranes at station j, in a single cycle. At the end of every port handling or storage operation, the container is certainly returned to the 'transporter', again in an infinitesimal amount of time and with probability of 1!

The grand marshalling station then

a) regenerates a new entry into the system with probability $p_{MM} = 1$, in a feedback motion. Thus a new 'arrival' to the system is instantaneously dispatched to another work station $j'$ with probability $p_{Mj'}$. 
Figure 2.6-1: Closed Port Queueing Network
3. THE MATHEMATICAL MODEL

3.1 THE STOCHASTIC PROCESS

The queueing network model examined in this work represents a discrete, state-space movement of containers, through a port, in continuous time. Essentially, this movement is treated as a continuous-time Markov Chain, with or without statistical dependence among the indexing parameters or the random variables defined for them.

3.1.1 THE INPUT SOURCE

Suppose \( p_{ij} \) is the probability of transition of a container from node \( i \), where it has just been handled or stored to where it would be handled or stored next, node \( j \). For now, a container may enter or leave the port through any node \( i \), with probability

\[
1 - \sum_{j=1}^{M} p_{ij}
\]

Assume that arrivals into the network are generated by a Poisson Source. The arrival rate of containers to the \( i^{th} \) station is

\[
\lambda_i = \gamma_i + \sum_{j=1}^{M} \lambda_j p_{ji}
\]

Let \( \gamma = \{ \gamma_1, \gamma_2, \ldots, \gamma_N \} \), the vector of external rates of arrival into the system,

\[
P = [p_{ij}]
\]

and
\[ \lambda = \{ \lambda_1, \lambda_2, \ldots, \lambda_n \} \]

Then the above equation becomes

\[ \lambda = \gamma + \lambda P \]

which is independent of exponential assumptions and of parameters \( \mu_j \) (the mean service rate at node \( j \)), and \( \delta_j \) (the number of servers at station \( j \)), but depends only on \( \gamma \) and \( P \).

Since these are tandem queues, the interdeparture time from station \( i \) generates the interarrival time at the next station \( j \). The mean arrival rate may depend almost arbitrarily upon the total number of containers at the port.

At containerports, it is not uncommon that containers destined for a later port of call are unloaded at the current port of call, so as to gain access to those intended for the current port. Meanwhile the originally misstowed containers are held in storage under the cranes until they are reloaded again, as if for export.

In queueing networks with this sort of feedback, Jackson, in his famous theorem, stated that though the arrival process at various nodes will not, in general, be Poisson, the system would behave as if the arrival input were Poisson!\(^1\)

This result is of great relief to us here, in that it permits us to make progress despite a generally non-Poisson arrival rate of containers at containerports.

Table 3.1.1-1 is the daily log of import and export

\[ \text{Table 3.1.1-1} \]

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1. Jackson, (1957). In this work, though, containers may enter or leave the system only from the ports gates or its anchorage.
TABLE 3.1.1-1: LOG OF IMPORT/EXPORT OF CONTAINERS

AT THE BOSTON-MYSTIC TERMINAL
(January 1, 1978 - May 5, 1978)

<table>
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<th>DATE</th>
<th>IMPORT (FULL)</th>
<th>EXPORT (FULL)</th>
<th>EXPORT (EMPTIES)</th>
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<td>20' 40'</td>
<td>20' 40'</td>
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<tr>
<td>1/1, 2/78</td>
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<td>24 29</td>
<td>21</td>
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<td>25</td>
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<td>2 32</td>
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<td>11 36</td>
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<td>63</td>
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<tr>
<td>3/31</td>
<td>29 76</td>
<td>21 16</td>
<td>8 57</td>
</tr>
<tr>
<td>4/1</td>
<td>60 37</td>
<td>14 12</td>
<td></td>
</tr>
<tr>
<td>4/4</td>
<td>58 82</td>
<td>13 25</td>
<td></td>
</tr>
<tr>
<td>4/5</td>
<td>103 6</td>
<td>11 12</td>
<td>2</td>
</tr>
<tr>
<td>4/5</td>
<td>3 17</td>
<td>2 3</td>
<td></td>
</tr>
<tr>
<td>4/6</td>
<td>45 92</td>
<td>46 77</td>
<td>3 13</td>
</tr>
<tr>
<td>4/5</td>
<td>9 59</td>
<td>3 15</td>
<td>2</td>
</tr>
<tr>
<td>4/7</td>
<td>29 31</td>
<td>6 48</td>
<td></td>
</tr>
<tr>
<td>4/7</td>
<td>15 41</td>
<td>6 22</td>
<td>23</td>
</tr>
<tr>
<td>4/10</td>
<td>33 59</td>
<td>61 34</td>
<td>98 78</td>
</tr>
<tr>
<td>4/10</td>
<td>20 50</td>
<td>2 57</td>
<td></td>
</tr>
<tr>
<td>4/11</td>
<td>36 83</td>
<td>30 61</td>
<td>21 46</td>
</tr>
<tr>
<td>4/12</td>
<td>37 54</td>
<td>16 21</td>
<td>3</td>
</tr>
<tr>
<td>4/13</td>
<td>53 9</td>
<td>27 13</td>
<td>37</td>
</tr>
<tr>
<td>DATE</td>
<td>IMPORT (FULL)</td>
<td>EXPORT (FULL)</td>
<td>EXPORT (EMPTY)</td>
</tr>
<tr>
<td>------------</td>
<td>---------------</td>
<td>---------------</td>
<td>----------------</td>
</tr>
<tr>
<td>4/14/78</td>
<td>46</td>
<td>33</td>
<td>15</td>
</tr>
<tr>
<td>4/18</td>
<td>37 74</td>
<td>11 24</td>
<td>16 36</td>
</tr>
<tr>
<td>4/18-19</td>
<td>37 79</td>
<td>26 49</td>
<td>1 23</td>
</tr>
<tr>
<td>4/20</td>
<td>15 20</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4/24</td>
<td>36 84</td>
<td>1 46</td>
<td></td>
</tr>
<tr>
<td>4/24</td>
<td>25 17</td>
<td>103</td>
<td>5</td>
</tr>
<tr>
<td>4/25</td>
<td>81 124</td>
<td>8 62</td>
<td>48 55</td>
</tr>
<tr>
<td>4/25-26</td>
<td>41 78</td>
<td>31 54</td>
<td>2 31</td>
</tr>
<tr>
<td>4/26-27</td>
<td>30 73</td>
<td>21 44</td>
<td>10</td>
</tr>
<tr>
<td>4/27</td>
<td>12 54</td>
<td>9 44</td>
<td>24</td>
</tr>
<tr>
<td>4/28</td>
<td>61 2</td>
<td>15 8</td>
<td>10</td>
</tr>
<tr>
<td>4/30</td>
<td>79 54</td>
<td>30 36</td>
<td>10 65</td>
</tr>
<tr>
<td>5/1</td>
<td>12</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>74 4</td>
<td>12 8</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>32 54</td>
<td>27 44</td>
<td>4 19</td>
</tr>
<tr>
<td>5/3</td>
<td>8 168</td>
<td>2 27</td>
<td>24</td>
</tr>
<tr>
<td>5/4</td>
<td>95 120</td>
<td>17 76</td>
<td>88 24</td>
</tr>
<tr>
<td>5/5</td>
<td>61 79</td>
<td>26 76</td>
<td>5 22</td>
</tr>
</tbody>
</table>
of containers at the Boston-Mystic terminal for the first 125 days of 1978. The figures were extensively analysed separately and in contingency tables in attempts to determine their goodness of fit with the Poisson Distribution using $\chi^2$ as the test statistic. In particular, import and export figures for the month of April, (reasonably far away from the strike which lasted from October 1, 1977 to November 30, 1977, so that steady-state could be assumed), were converted into TEU's and summed up to give the daily number of TEU's handled at the port, --- see Table 3.1.1-2. (Where loading or unloading operations extended over several days, the number of TEU's was shared equally among the days in question).

The average number of TEU's handled per day, (including days when no import or export arrived at or left the port) was $\bar{n} = 244.63$ and the sample standard deviation was was $s = 255.576$, giving a variance of $s^2 = 65319.1$, a far cry from $\bar{n} = 244.63$ whereas for a Poisson process, the mean and variance should be approximately equal.

3.1.2 SERVICE TIMES AND NETWORK TOPOLOGY

In chapter 2, the randomness of service times at each station, $j$, was pervasive. It is assumed that there are $s_j$ compatible, exponential servers, each with mean service time $= 1/\mu_j$. This mean is permitted to depend almost arbitrarily
TABLE 3.1.1-2 Container Traffic at Moran in April 1978.

<table>
<thead>
<tr>
<th>DATE</th>
<th>#TEU's</th>
<th>DATE</th>
<th>#TEU's</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/1/78</td>
<td>172</td>
<td>4/16</td>
<td></td>
</tr>
<tr>
<td>4/2</td>
<td></td>
<td>4/17</td>
<td></td>
</tr>
<tr>
<td>4/3</td>
<td></td>
<td>4/18</td>
<td>515</td>
</tr>
<tr>
<td>4/4</td>
<td>285</td>
<td>4/19</td>
<td>183</td>
</tr>
<tr>
<td>4/5</td>
<td>361</td>
<td>4/20</td>
<td>65</td>
</tr>
<tr>
<td>4/6</td>
<td>458</td>
<td>4/21</td>
<td></td>
</tr>
<tr>
<td>4/7</td>
<td>386</td>
<td>4/22</td>
<td></td>
</tr>
<tr>
<td>4/8</td>
<td></td>
<td>4/23</td>
<td></td>
</tr>
<tr>
<td>4/9</td>
<td></td>
<td>4/24</td>
<td>572</td>
</tr>
<tr>
<td>4/10</td>
<td>770</td>
<td>4/25</td>
<td>819</td>
</tr>
<tr>
<td>4/11</td>
<td>467</td>
<td>4/26</td>
<td>343</td>
</tr>
<tr>
<td>4/12</td>
<td>676</td>
<td>4/27</td>
<td>383</td>
</tr>
<tr>
<td>4/13</td>
<td>161</td>
<td>4/28</td>
<td>106</td>
</tr>
<tr>
<td>4/14</td>
<td>188</td>
<td>4/29</td>
<td></td>
</tr>
<tr>
<td>4/15</td>
<td></td>
<td>4/30</td>
<td>429</td>
</tr>
</tbody>
</table>
upon the total number of containers at that node.

What then is the topological structure of the network and how are the permissible transitions between two nodes described? Though containers of the same type may or may not proceed through the network in exactly the same sequence, (especially because some containers skip some stations altogether), any two consecutive nodes constitute tandem queues in the sense that containers that have been served at node $i$ and immediately enter node $j$ for service have as their interarrival times to node $j$, their inter-departure times from node $i$. For our purposes, these times are exponentially distributed with exactly the same parameter.

This result is derived from Burke's theorem which states that the steady-state output of a stable, stationary $M/M/m$ queue with a Poisson input parameter $\lambda$ and service-time parameter $1/\mu$ for each of the $m$ channels is in fact a Poisson process at the same rate $\lambda$. Not only is another Poisson process generated for departure but also, this output process is completely independent of other processes in the system.\(^1\) Hence, many multiple-server nodes (each with exponential pdf), connected together in a feedforward and, for that matter, feedback\(^2\) network may be analysed through node-by-node decomposition.

---

1. Burke, (1956). The $M/M/m$ system is the only such FCFS system with this property.
3.1.3 QUEUE DISCIPLINE

It will be quite realistic to assume a first-come-first-served priority discipline for the various queues that form at work stations, at the port. Though ships might be admitted in orders other than this, our model is unaffected as it picks up the containers only after the decision to pilot a ship to the apron has been made.

Export containers may, of course, not follow this discipline and Muntz and Wong have shown that the FCFS assumption may be unnecessary. Furthermore, they developed computationally efficient algorithms for handling non-FCFS priority disciplines.¹

The leading assumptions for the model have been stated. We must digress a little now to examine the form in which raw data will be supplied to this model and how the model will convert them into useful performance measures.

3.2 AVERAGE PROCESSING TIMES AND PROBABILITY OF STATION VISITS

The work loads at various stations are functions of both the frequency and duration of use of those stations. From Figure 2.6-1, we recall that \( p_{mj} \) is the probability that the central station directs a container to station j for hand-

¹ Muntz and Wong, (Honolulu, 1974).
ling or storage while \( p_{MM} \) is the probability that the container has completed all the required handling, storage and processing operation. As our system is memoryless, these probabilities are independent of previous routing decisions.

Let the average time spent in performing operation \( i \) at station \( j = t_{ij} \). This average is taken over all container visits to the station, \( t_{ij} = 0 \) if the performance of operation \( i \) does not take place at station \( j \).

Table 3.2-1 gives the general form of our input table for each container type that passes through the containerport.

The total time spent by a container at each station \( j \) is given by

\[
T_j = \sum_i t_{ij} f_j \quad \text{for} \quad j = 1, 2, \ldots, M
\]

where \( f_j \) is the selection or sampling frequency for station \( j \).

Similarly, \( V_j \), the total number of visits to each station is given by

\[
V_j = \sum_i v_{ij}
\]

where

\[
v_{ij} = \begin{cases} 
0 & \text{for} \quad t_{ij} = 0 \\
1 & \text{for} \quad t_{ij} > 0 \quad \text{for all containers} \\
\frac{f_j}{\langle f_j \rangle} & \text{for} \quad t_{ij} > 0 \quad \text{for only a fraction of all containers}
\end{cases}
\]

1. See explanation of sampling inspection at custom spots in Section 2.4.3
TABLE 3.2-1

Operation Times at Containerport Work Stations

<table>
<thead>
<tr>
<th>Station</th>
<th>Operation</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>j</th>
<th>...</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t_{ij}</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th># Servers</th>
<th>S_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection Frequency</td>
<td>f_j</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>T_j</th>
<th>Σ t_{ij}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V_j</td>
<td>Σ v_{ij}</td>
</tr>
</tbody>
</table>
Let \( V = \sum_j V_j \) then \( \phi_{m_j} \) the frequency of visits by containers to station \( j \), \( = \frac{V_j}{V} \) and the mean processing time at station \( j \), \( \bar{T}_j = \frac{T_j}{V_j} \) for \( j = 1, \ldots, M \)

3.2.1 MULTIPLE CONTAINER TYPES

Suppose the containerport handles a variety of container types e.g. full import 20's, full import 40's, full export 20's, full export 40's, empty export 20's and empty export 40's. One could assume that these various categories of containers, (which may require different amounts, frequencies and sequences of operations), come in unequal numbers but that there is a balance of flow between all imports and all exports. Along with every Table, specify \( n_k \), the number of containers of type \( k \) then,

\[
\phi(k) = \frac{n_k}{\sum_{k=1}^{c} n_k}
\]

A slight modification of the previous notation may now be suggested i.e.

let \( V_j(k) \) be the total number of visits of container type \( k \) to station \( j \) hence for each container type, the total number of operations will be

\[
V(k) = \sum_j V_j(k)
\]
The total number of operations at the port will be

\[ V = \sum_{k} V(k) \]

\[ \phi_{Mj} = \frac{\sum_{k} \phi(k) V_j(k)}{\sum_{k} \phi(k) V(k)} \]

\[ \sum_{k} \phi(k) V_j(k) = \text{average number of operations at station } j \text{ per container while } \]

\[ \sum_{k} \phi(k) V(k) = \text{average number of operations per container that passes through the port} \]

Similarly,

\[ \bar{T}_j = \frac{\sum_{k} \phi(k) T_j(k)}{\sum_{k} \phi(k) V_j(k)} \]

From these values, we can obtain the relative utilizations, \( X_j \), for each station \( j \) thus:

\[ X_j = \frac{\phi_{Mj} \bar{T}_j}{\bar{T}_M} \text{ ; M is the central station} \]

It will be observed in the next chapter that any consistent multiple of \( X_j \) will produce the same results but to control overflow and underflow in the programme, the magnitudes of these numbers should be manipulated.
3.3 STATION RELATIVE UTILIZATION

We shall now derive the same relative utilizations, $X_j$'s, from queueing network theory. After that shall have been done, the model would have a sound theoretical basis as well as empirical meaning. From that point on, the model relies almost exclusively on the use of these $X_j$'s to develop all of its output.

3.3.1 OPEN AND CLOSED QUEUEING NETWORKS

With $N_t$ containers present in a queueing network of $M$ handling or storage stations, we define the state variable to be the vector

$$\mathbf{n} = \{n_1, n_2, \ldots, n_M\}$$

where $n_j$ is the number of containers at the $j$th station, (those in queue plus those in service).

Suppose we associate a unique equilibrium probability $\hat{p}(n_1, n_2, \ldots, n_M)$ with this state. For this probability distribution to exist, our discrete—state Markov process must be irreducible. Let the marginal distribution of finding $n_j$ containers at station $j$ be given by $\hat{p}_j(n_j)$.

---

1. For an open network, $N_t$ varies with time $t$ but by definition, $N_t$ would be a constant in a closed system.
Jackson proved, in another famous theorem,\textsuperscript{1} that
\[
\phi(n_1, n_2, \ldots, n_M) = \phi_1(n_1) \phi_2(n_2) \ldots \phi_M(n_M)
\]
An ingenious modification of open networks of queues
was introduced by Gordon and Newell through the idea of
closure.\textsuperscript{2} Essentially, they analysed a closed Markovian net-
work, which in our sense translates to a constant number of
containers, $N$, whirling around a network into which no more
containers are admitted nor from which any are permitted to
depart.

If $\phi_{ij}$\textsuperscript{3} is the probability that a container in station
i goes to station j next for processing, then
\[
\sum_{j=1}^{M} \phi_{ij} = 1 \quad \text{for all } i
\]
and furthermore,
\[
\sum_{j=1}^{M} n_j = N
\]
Clearly, this introduces dependency among the elements
of the state vector $n = \{n_1, n_2, \ldots, n_M\}$. The number of ways
in which $N$ containers can be distributed among $M$ stations is
given by the binomial coefficient,
\[
\binom{N + M - 1}{M - 1}
\]
which is also the number of distinguishable states.

\textsuperscript{1} Jackson, (1963). In general, it is not unusual to have
similar product forms of solution for Markovian queues in equi-
librium.
\textsuperscript{2} Gordon and Newell, (1967).
\textsuperscript{3} $\phi_{ij}$ is the transition among stations or nodes,
not among states!
As Kleinrock\(^1\) suggested, the behavior of the equilibrium distribution of customers in this closed network could be described thus:

\[ p(n_1, n_2, \ldots, n_M) \left( \sum_{j=1}^{M} \phi_i(n_i) / \lambda_i \right) \]

is equal to the flow of probability into that state from neighbouring states,

\[ \sum_{i=1}^{M} \sum_{j=1}^{M} \phi_i(n_i) / \lambda_i [ \phi_j(n_j, n_1, n_2, \ldots, n_{j-1}, \ldots, n_{i+1}, \ldots n_M) ] \]

where the discrete unit step-function\(^2\)

\[ \phi_k = \begin{cases} 1 & \text{for } k = 0, 1, 2, \ldots \\ 0 & \text{for } k < 0 \end{cases} \]

and the number of customers in service at station \(i\) when \(n_i\) containers are present at that station

\[ \phi_i(n_i) = \begin{cases} n_i & n_i \leq m_i \\ m_i & n_i > m_i \end{cases} \]

The local balance equations are of the form

\[ \mu_i X_i = \sum_{j=1}^{M} \mu_j X_j \phi_i \]

\(X_i, (>0)\), is the relative utilization of station \(i\). The solution for this \(X_i\)'s can be determined only to within a multiplicative constant because all the equations

---

2. It is necessary to include this step function so as to ensure that when a station is empty, its service rate is zero.
3. The special nature of these networks simplifies the extraction of this set of equations from the global-balance or flow-conservation equations which are obtained by balancing flow in and out of states in the general birth-death processes.
are homogeneous and one of them is redundant. We are sure a unique solution exists because we are dealing with an irreducible and ergodic Markov process.

3.3.2 TRUE STATION UTILIZATIONS

The equation describing the flow of probability in and out of a state then reduces to

\[ \Phi(n_1, n_2, \ldots, n_M) = \frac{1}{G_j(N)} \prod_{j=1}^{M} \frac{X_j n_j^{\beta_j(n_j)}}{\beta_j(n_j)} \]

where

\[ \beta_j(n_j) = \begin{cases} n_j \text{!} & \text{for } n_j \leq S_j \\ S_j \cdot S_j^{n_j-S_j} & \text{for } n_j > S_j \end{cases} \]

\( (S_j=\text{number of servers at station } j) \) and the normalization constant,

\[ G_j(N) = \sum_{n \in A} \prod_{j=1}^{M} \frac{X_j n_j^{\beta_j(n_j)}}{\beta_j(n_j)} \]

the summation being over all state vectors \( n = \{n_1, n_2, \ldots, n_M\} \) that lie in \( A \).

3.3.3 DISTRIBUTION OF CONTAINERS AT THE PORT

It may be desirable to determine the probability that there are exactly \( k \) containers at the \( i \)th workstation. This probability may be expressed as

1. We again encounter the product form
\[ \mathcal{P}(n_i = k) = \sum_{n \in A} \mathcal{P}(n_1, n_2, \ldots, n_M) \]

An auxiliary approach is to consider
\[
\mathcal{P}(n_i \geq k) = \sum_{n \in A} \mathcal{P}(n_1, n_2, \ldots, n_M) \quad \text{and} \quad n_i \geq k
\]
\[
= \sum_{n \in A} \frac{1}{G(N)} \prod_{j=1}^{M} (X_j)^{n_j}
\]
\[
= \left( X_i \right)^k \frac{1}{G(N)} \sum_{n \in A'} \prod_{j=1}^{M} (X_j)^{n_j}
\]

where \( A' = \{(n_1, n_2, \ldots, n_M) \mid \sum_{j=1}^{M} n_j = N - k \quad \text{and} \quad n_j \geq 0 \forall j \} \)

Clearly, \( \mathcal{P}(n_i \geq k) \) reduces to \( (X_i)^k \frac{G(N-k)}{G(N)} \)

Find a similar expression for
\[
\mathcal{P}(n_i \geq k+1)
\]
and then
\[
\mathcal{P}(n_i = k) = \mathcal{P}(n_i \geq k) - \mathcal{P}(n_i \geq k+1),
\]
\[
\mathcal{P}(n_i = k) = \frac{(X_i)^k}{G(N)} \left[ G(N-k) - X_i (G(N-k-1)) \right]
\]

where \( G(N) = 0 \quad \text{for} \quad n < 0. \)

We note that for \( k=1 \), one obtains the probability that the \( i^{th} \) station is not idle as exactly one container is there to keep it active. Furthermore, \( E[n_i] \), the expected number of customers present at the \( i \)th facility, is given by \(^1\)
\[
E[n_i] = \sum_{k=1}^{N} (X_i)^k \frac{G(N-k)}{G(N)}
\]

Once the \( G(n) \)'s have been determined, numerous performance measures can be derived from them.

4. THE COMPUTER MODEL

4.1. PORTQ

The computer programme for this model, Port Q, was written in PL/1 on the interactive, time sharing, Conversational Monitor System, (CMS). The Virtual Machine Facility/370, (VM/370) was the system control programme for this work, done on the IBM 370/168. The PL/1 optimizer did the compilation.

The programme was structured, top-down hence making it easy to follow in sequential order. The programme segments, their flowcharts and computational requirements will be discussed in that order. Mention will be made of the overall core storage and central processing units requests by the programme.

Some data developed from the operations at the Boston-Mystic Container Terminal, shown in Table 4.1-1 will be used to test the programme. A fuller version of this table is Table A-2 in the Appendix. That table considers many bulk service stations like the ship, the storage, the container freight stations etc. Because queueing network theory has not yet analyzed bulk services and containers spend, by comparison, an excessive amount of time at these stations, they were eliminated from the analysis, as explained further in the next section.
Table 4.1-1: Sample Data for Computer Models

<table>
<thead>
<tr>
<th></th>
<th>(1) Container Cranes</th>
<th>(2) Trans-Trainers</th>
<th>(3) Forklifts</th>
<th>(4) Yard Hostlers</th>
<th>(5) Customs</th>
<th>(6) Interchange Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Unloading</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Quay To Storage</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Quay To Backreach</td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Inspection</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>(5) Delivery To Truck</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Storage To Quay</td>
<td>9.5</td>
<td>12</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Inspection</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>(4) Delivery From Truck</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>(1) Loading</td>
<td>3.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Storage To Quay</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Delivery From Quay</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>NUMBER OF SERVERS</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>12</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>SAMPLING-FREQUENCY</td>
<td>1</td>
<td>.75</td>
<td>.25</td>
<td>.85</td>
<td>.15</td>
<td>1</td>
</tr>
</tbody>
</table>
4.2 BULK SERVICES

Clearly, the tugboat that pilots the containership to/from the apron and the storage yards that hold containers in inventory may be thought of as M/M/1 systems which provide service to bulk shipments of containers of varying sizes and numbers.

Theoretically, these sort of systems are analogous to Erlangian arrival systems, of the Er/M/1 type. Consequently, the distribution of customers in the system follows a geometric pattern.

However, these sort of systems should not be of practical interest to us here as we are more interested in the dynamics of the port workstations that provide non-bulk services. All port operations constitute a stationary process in that whatever goes into the system must sooner or later come out of it. We recognize the necessity for these sort of operations but we will eliminate them from our considerations of containers flowing through the port. Rather, we consider these stations to be buffers.¹

4.3 PROGRAMME FLOW

4.3.1 INPUT OPERATIONS

The flowchart in Figure 4.3.1-1 describes how and which data are fed into Port Q. For purposes of verification, an honest out-

¹. In the sense used by Frankel and Tang, (1977).
Figure 4.3.1-1: Flowchart for Input Data Verification

Start

Get data on
Stations,
Central Station
Type & number
Circulating

for each Station,
read in number
of servers &
Sampling freq.

for each type
of containers,
read in number
and operations

Read in Processing
Time for type
at each Station

Write out
input data
for checking

A
put of the input of Table 4.1-1 is requested and received
in the form shown in Table A-5 in Appendix A.
The programme segment is shown in Figure B-1 in Appendix B.

4.3.2 DETERMINATION OF AVERAGE PROCESSING TIMES AND THE
PROBABILITIES OF STATION VISITS

Figure 4.3.2-1 gives the flowchart and the programme
segment is given in Figure B-2. The outputs are
shown in two columns of Table 4.3.2-1.

4.3.3 DETERMINATION OF STATION RELATIVE UTILIZATIONS

As explained in Section 3.3.1, the station relative
utilizations, $X_j$'s may be solved for to within multiplica-
tive constants. Hence, here, we set $X_M$ the relative utiliza-
tion of the central station, $M$, equal to some convenient
value and then solve for every other $X_j$, in terms of $X_M$.

It may happen that large $X_j$'s applied consecutively
and raised to higher and higher powers may cause an over-
flow of digits in the computer\(^1\) or if the $X_j$'s are too low,
an underflow of digits might occur. Hence in this model,

---

1. When the storage was first thought of as a station,
its relative utilization was in an unacceptable 200:1 ratio
to that of the container crane station. It is preferable to
have no such ratio higher than 10:1 and then reduce them
all to between 1 and 10.
Figure 4.3.2-1:
Flowchart for Calculating Mean Processing Times and Probabilities of Station Visits

A

Initialize Total Processing Time and # Station visits at zero

Yes

Have Calculations for all stations been done?

Get new station

Does the next operation at this station have positive processing time?

No

No

Increase Total Processing time by this time x Sampling frequency. Increase number of visits by Sampling frequency

Yes

Yes

Have all operations been checked at this station?

Calculate what fraction of all containers is each container type. Calculate total number of station visits by each container type.

Weight with the respective fraction of each container type and calculate the average # operations per container at each station at the port. Calculate the average processing time per container

For each station, derive the probability of a visit by a container and the average processing time per operation at each station.
### Table 4.3.2-1  Derived Station Characteristics

<table>
<thead>
<tr>
<th>Station Fraction</th>
<th>Number of Servers (MS)</th>
<th>Average Processing Time (TM)</th>
<th>Frequency of Visits (PM)</th>
<th>Relative Utilization (UM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3:0.250</td>
<td>4</td>
<td>6.590</td>
<td>0.06290</td>
<td>0.41451</td>
</tr>
<tr>
<td>6:1.000</td>
<td>12</td>
<td>7.090</td>
<td>0.25180</td>
<td>1.78526</td>
</tr>
<tr>
<td>5:0.150</td>
<td>6</td>
<td>16.620</td>
<td>0.03020</td>
<td>0.50192</td>
</tr>
<tr>
<td>2:0.750</td>
<td>2</td>
<td>5.140</td>
<td>0.18890</td>
<td>0.97094</td>
</tr>
<tr>
<td>4:0.850</td>
<td>12</td>
<td>3.090</td>
<td>0.21410</td>
<td>0.66156</td>
</tr>
<tr>
<td>1:1.000</td>
<td>2</td>
<td>3.120</td>
<td>1.00000</td>
<td>3.12000</td>
</tr>
</tbody>
</table>
the \( x_j \)'s have been sorted and applied by alternatively selecting the unapplied and currently smallest \( x_j \) and next selecting the then largest one, and so on. Though this approach does affect the intermediate values of \( g(n,m) \), the final column, \( g(n,m) \) remains unaltered! See Table 4.3.3-1.

The rightmost column of Table 4.3.2-1 shows these relative utilizations. The flowchart is Figure 4.3.3-1 while the programme segment is in Figure B-3 in the appendix.

<table>
<thead>
<tr>
<th>Table 4.3.3-1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>THE RIGHTMOST TWO COLUMNS OF THE MATRIX G</strong></td>
</tr>
<tr>
<td>1.000</td>
</tr>
<tr>
<td>4.331</td>
</tr>
<tr>
<td>9.384</td>
</tr>
<tr>
<td>13.630</td>
</tr>
<tr>
<td>15.010</td>
</tr>
<tr>
<td>13.441</td>
</tr>
<tr>
<td>10.258</td>
</tr>
<tr>
<td>6.906</td>
</tr>
<tr>
<td>4.216</td>
</tr>
<tr>
<td>2.388</td>
</tr>
<tr>
<td>1.278</td>
</tr>
</tbody>
</table>
Figure 4.3.3-1:
Sorting Station Relative Utilizations for the Calculation of the G(N)'s

1. Determine the relative utilization of each station. The bottleneck station has the highest ratio of this to # servers.

2. Sort stations into single and multiple server categories.

3. Post the central station to position M.

4. Giving priority to single servers, rank the station with the smallest relative utilization first, then pick the station with the highest relative utilization next, and so on, alternately.

5. Iteratively calculate the G(N)'s starting with the single server stations if any.
4.3.4 COMPUTATION OF $G(N)$

4.3.4.1 SINGLE SERVERS

Expressions for $G(N)$, were derived from products of relative utilizations of the stations and $N$, the number of circulating containers, in the last chapter. For the single server, this involves the summation of

$$\binom{M+N-1}{N}$$

terms, each of which is a product of $M$ powers of the relative utilizations. However, Buzen proposed a simple, iterative algorithm which computes the entire set of values $G(1), G(2), \ldots, G(N)$, using a total of $N \cdot M$ additions.\(^1\)

Suppose the vector, \(\{X_1, X_2, \ldots, X_M\}\), is given. Then define an auxiliary function

$$g(n, m) = \sum_{n \in S(n, m)} \prod_{i=1}^{m} (X_i)^{n_i}$$

where \(S(n, m) = \{(n_1, n_2, \ldots, n_m) \mid \sum_{i=1}^{m} n_i = n \text{ and } n_i \geq 0 \forall i\}\)

$$G(n, M) = G(n) \text{ for } n = 0, 1, \ldots, N.$$ 

for $m > 1$ and $n > 0$

$$g(n, m) = \sum_{n \in S(n, m)} \prod_{i=1}^{m} (X_i)^{n_i} + \sum_{n \in S(n, m)} \prod_{i=1}^{m} (X_i)^{n_i}$$

where $n_m = 0$.

The initializing conditions for this algorithm are

$$g(0, m) = 1 \text{ for } m = 1, 2, \ldots, M$$

and

$$g(n, 1) = (X_i)^{n} \text{ for } n = 0, 1, \ldots, N$$

\(^1\)Buzen, (1973) p.528
The iteration used in the programme makes it unnecessary to store more than \(N\) values at any given time. The process is however more complex for the multiple server cases.

4.3.4.2 MULTIPLE SERVERS

For these cases,

\[
G(n, m) = \sum_{n_j=0}^{n} \frac{\binom{n}{n_j} X_{n_j}^{n_j}}{\beta_j(n_j)} G(n-n_j, m-1)
\]

where \(\beta_j(n_j)\) is as defined in section 3.3.1

\[
G(n, 1) = \frac{X_1^n}{\beta_1(n)} \text{ for } n = 0, 1, \ldots, N
\]

\[
G(0, m) = 1 \text{ for } m = 1, 2, \ldots, M
\]

The computation of \(G(N) = g(N, M)\), by the iterative method used in the programme, will require \(MN(N+1)/2\) additions, \(MN(N+1)/2\) divisions and \(MN(N+1)\) multiplications for a total of \(2MN(N+1)\) arithmetic operations. Thus the computation of \(G(N)\) for every multiple-server case requires \(N+1\) as many operations as are required in the single-server case.

The flowchart for the calculation of the \(G(N)\)'s is shown in Figure 4.3.3-1 and the programme segment is in Fig B-4. Table 4.3.3-1 shows the rightmost two columns of the \(G\) matrix for our, (all), multiple-server stations.

4.3.5 DETERMINATION OF STATION PERFORMANCE MEASURES

4.3.5.1 TRUE STATION UTILIZATIONS

The most important performance measure of a work station is its utilization, which is purposely defined here as the average number of busy servers at the station per unit
time, in the long run.

The utilization of the central station, for a given number, \( n=1,2,...,N \) containers in the system would be given by

\[
(X_M) \cdot \left[ \frac{g(n-1,M)}{g(n,M)} \right]
\]

while the utilization for any station, with \( N \) containers moving through the port per hour =

\[
X_j \cdot \left[ \frac{g(N-1,M)}{g(N,M)} \right]
\]

Figure 4.3.5-1 is the flowchart for the calculation of all station performance measures in this section. Table 4.3.5-1 is the output and Figure B-5 gives the programme segment.

4.3.5.2 THE DISTRIBUTION OF CONTAINERS IN THE SYSTEM

The marginal probability distribution for the number of parts at the central station is given by

\[
P(n_M = k) = \frac{X_M^k \cdot \beta_M(k)}{\beta_M(n_M)}
\]

Our results here are shown in Table 4.3.5-1.

For any other station, if the station is a single server, the equivalent expression would be

\[
P(n_j = k) = \frac{X_j^k \cdot \beta_{j}(N-k,M)}{\beta_{j}(N,M)} \left[ G_j(N-k,M) - X_j \cdot G_j(N-k-1,M) \right]
\]

To obtain this marginal distribution for any other multiple-server station \( j \), however, one must permute the indices of all \( M \) stations in such a way as to make the current Station \( j \), station \( M \).
Figure 4.3.5-1: Flowchart for the Calculation of Station Performance Measures

1. Calculate the true Station Utilizations i.e. the average number of busy servers at the station at any one time.

2. For all single servers and the central station, find the idleness, 
   # containers at station and # containers in queue = # at station less utilization.
Table 4.3.5-1

<table>
<thead>
<tr>
<th>STATION</th>
<th>UTILIZATION</th>
<th>IDLENESS</th>
<th>CONTAINERS AT STATION</th>
<th>CONTAINERS IN QUEUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.2652</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
</tr>
<tr>
<td>6</td>
<td>1.1423</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
</tr>
<tr>
<td>5</td>
<td>0.3212</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
</tr>
<tr>
<td>2</td>
<td>0.6213</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
</tr>
<tr>
<td>4</td>
<td>0.4233</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
<td>PERMUTE</td>
</tr>
<tr>
<td>1</td>
<td>1.9964</td>
<td>0.0005</td>
<td>7.16</td>
<td>5.17</td>
</tr>
</tbody>
</table>
This is why the word 'permute' appears in the right-most three columns for all multiple-server stations except the central station, (whether or not it is a multiple-server), which is listed last. Without altering the relative utilizations, Xj's, one must recompute the G matrix and then apply the relevant expressions developed earlier to obtain the marginal distribution for the multiple-server station of interest.

4.3.6 DETERMINATION OF SYSTEM PERFORMANCE MEASURES

4.3.6.1 SYSTEM HANDLING RATE

The most important performance measure of the system that this model provides is the steady-state average number of containers, \( Q^* \), that pass through the port per day. This handling rate in \( t \) minutes, for \( n \) containers circulating per hour is given by

\[
Q^* = \frac{g(n-1,M)}{g(n,M)} (\phi_{mm} = 1)(t) / \bar{T}_M
\]

where \( \bar{T}_M \) is the mean processing time at station M.

The first column of Table 4.3.6-1 shows the number of containers circulating in the system per hour. The next column gives the corresponding, handling rates per day. Figure 4.3.6-1 is the flowchart for all calculations in this section and Figure B-6 is the corresponding programme segment. The handling rate asymptote given for the system is the maximum
Table 4.3.6-1:

Container Terminal System Performance Measures

<table>
<thead>
<tr>
<th># CONTAINERS</th>
<th>HANDLING OR STORAGE RATE PER DAY</th>
<th>AVERAGE HANDLING TIME PER CONTAINER</th>
<th>UTILIZATION OF CENTRAL STATION</th>
<th>RELATIVE EFFICIENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64.4295</td>
<td>7.45</td>
<td>0.419</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>128.8045</td>
<td>7.45</td>
<td>0.837</td>
<td>0.9996</td>
</tr>
<tr>
<td>3</td>
<td>186.1725</td>
<td>7.73</td>
<td>1.210</td>
<td>0.9632</td>
</tr>
<tr>
<td>4</td>
<td>232.2486</td>
<td>8.27</td>
<td>1.510</td>
<td>0.9012</td>
</tr>
<tr>
<td>5</td>
<td>265.3198</td>
<td>9.05</td>
<td>1.725</td>
<td>0.8236</td>
</tr>
<tr>
<td>6</td>
<td>286.2808</td>
<td>10.06</td>
<td>1.861</td>
<td>0.7406</td>
</tr>
<tr>
<td>7</td>
<td>297.9419</td>
<td>11.28</td>
<td>1.937</td>
<td>0.6606</td>
</tr>
<tr>
<td>8</td>
<td>303.6560</td>
<td>12.65</td>
<td>1.974</td>
<td>0.5891</td>
</tr>
<tr>
<td>9</td>
<td>306.1526</td>
<td>14.11</td>
<td>1.990</td>
<td>0.5280</td>
</tr>
<tr>
<td>10</td>
<td>307.1423</td>
<td>15.63</td>
<td>1.996</td>
<td>0.4767</td>
</tr>
</tbody>
</table>

FREE FLOW HANDLING OR STORAGE TIME = 7.45 MINUTES

HANDLING OR STORAGE RATE ASYMPTOTE = 307.6924 CONTAINERS PER DAY

STATION NUMBER 1 IS THE BOTTLENECK STATION

R; T=0.45/1.64  20:47:13
Figure 4.3.6-1: Flowchart for Calculating System Performance Measures

1. For number of containers in the system = 1, 2, ..., N, calculate the corresponding system performance measures such as the utilization of the central station (the true utilization) X \[ G(N-1) / G(N) \] to obtain the system utilization factor.

2. Calculate the daily average handling rate using the system utilization. From this, obtain the average handling time. System Efficiency = Free Flow handling time / Average Handling time.

3. Calculate the handling rate asymptote and specify the bottleneck station.

Stop
throughput allowed by the bottleneck station which is here identified as our central station, the container crane station.

4.3.6.2 AVERAGE HANDLING TIME PER CONTAINER

The average time a container spends being handled at the stations considered at the port, T, is simply related to Q through Little's formula which states that for any steady-state process,

\[ T^* = \frac{\text{n}t}{Q^*} \]

The middle column in Table 4.3.6-1 gives the output. The value corresponding to only one container whirling around the system is the free flow handling time as this one container would never have to wait in a queue before being served!

4.3.6.3 UTILIZATION OF CENTRAL STATION

This is given by the expression \((\text{Number of Servers}) \times (\text{Handling Rate Per Day})/ \text{Handling Rate Asymptote.}\)

4.3.6.4 THE EFFICIENCY RATIO

This is simply the ratio of the free flow time to the corresponding Average handling time per container.
5. ANALYSIS AND SUMMARY OF FINDINGS

5.1 MODEL VALIDATION

The detailed and carefully prepared, GPSS programme in Appendix C, was used to simulate the operations at Moran, using the same data as in Port Q. Table 5.1-1 and 5.1-2 below gives the results of the simulation study.

Table 5.1-3 compares these results with those of Port Q. Suppose $X_j$ is Port Q's result and $Y_j$ is the corresponding result from the simulation. We note that

$$Y_j = 0.96524X_j + 0.005627$$

and the correlation coefficient between $X$ and $Y$ is 0.9969, meaning that we are within one percent of the simulation results!

Furthermore, it was estimated that the likely capacity of the Moran Terminal when expanded to its practical limit is estimated at 100,000 TEU's annually.\(^1\) Port Q gives us an asymptotic handling rate of 307.69\(^2\) containers per day, in 365 days this would be $= 11230685$. Reasons for this 12% overestimation are given in Section 5.3.\(^3\)

3. Machine breakdowns, vacations, strikes etc were not considered.
### Table 5.1-2: Flow Characteristics Given by the Simulation Run

<table>
<thead>
<tr>
<th>TIME/TRANS</th>
<th>UTILIZATION</th>
<th>CONTENTS</th>
<th>CAPACITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>424.901</td>
<td>1766</td>
<td>0.699</td>
<td>1922</td>
</tr>
<tr>
<td>499.909</td>
<td>222</td>
<td>0.955</td>
<td>448.432</td>
</tr>
<tr>
<td>180.769</td>
<td>1521</td>
<td>0.935</td>
<td>493.452</td>
</tr>
<tr>
<td>128.577</td>
<td>1282</td>
<td>0.931</td>
<td>561.462</td>
</tr>
<tr>
<td>180.352</td>
<td>1762</td>
<td>0.931</td>
<td>603.252</td>
</tr>
</tbody>
</table>

Queue Maximum Average = 2.5 Stations

Average

Average

Average

Average

Average

Average

Average

Average

Average

Average

Average

Average

Average

Average

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Average

Average

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Average

Average

Average

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Average

Averag
Table 5.1-3 Comparison of Analytical Model and Simulation Results

<table>
<thead>
<tr>
<th>Utilization of Station Number:</th>
<th>Analytical Results</th>
<th>Simulation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative Utilization</td>
<td>Normalized Contents</td>
</tr>
<tr>
<td>1. Cranes (2)</td>
<td>0.781*</td>
<td>0.153</td>
</tr>
<tr>
<td>2. Straddle Carriers(2)</td>
<td>0.971</td>
<td>0.190</td>
</tr>
<tr>
<td>3. Fork Lifts (4)</td>
<td>0.415</td>
<td>0.081</td>
</tr>
<tr>
<td>4. Yard Hustlers(12)</td>
<td>0.662</td>
<td>0.129</td>
</tr>
<tr>
<td>5. Custom Inspectors(6)</td>
<td>0.502</td>
<td>0.098</td>
</tr>
<tr>
<td>6. Trucks at Interchange (12)</td>
<td>1.785</td>
<td>0.349</td>
</tr>
</tbody>
</table>

* 0.781, the true relative utilization of station 1 comes, (as others do), from the product of its mean processing time of 3.12 minutes and it probability of visit, 0.2503.
5.2 SENSITIVITY ANALYSIS

By varying the levels of input into Port Q, one would be able to identify the most critical elements in the system, (such as the bottleneck node), and concentrate on improving the performance of the system by manipulating parameters dealing with those critical points.

Here, we first increase the number of containers whirling around the closed network and see what happens. Table 5.2-1 shows that once the asymptotic level of handling is reached, (here at about 12), the system would not be able to support more traffic.

We note that the last but one column of the matrix G reaches zero and stays there, (Table 5.2-2).

5.2.1 THE BOTTLENECK STATION

At any time, a queueing network is as sensitive as its bottleneck station hence it will be useful to vary the characteristics of this bottleneck station so as to gain some insight into the behaviour of the system.

Our bottleneck station here is the station of container cranes. At the time of this writing, the Hitachi crane is down. Table 5.2.1-1 shows the corresponding daily handling rates with 1,3,4 and 10 cranes working at this station. The respective asymptotic handling rates are 153;461;615 and 988. There is a linear relationship among the number of cranes working and the
Table 5.2-1
Number of Containers in System and System Asymptotic Behaviour

<table>
<thead>
<tr>
<th># CONTAINERS</th>
<th>HANDLING OR STORAGE RATE PER DAY</th>
<th>AVERAGE HANDLING TIME PER CONTAINER</th>
<th>UTILIZATION OF CENTRAL STATION</th>
<th>RELATIVE EFFICIENCY</th>
</tr>
</thead>
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<td>1</td>
<td>64.4209</td>
<td>7.45</td>
<td>0.419</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>128.7754</td>
<td>7.45</td>
<td>0.837</td>
<td>0.9995</td>
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<tr>
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<td>186.1434</td>
<td>7.74</td>
<td>1.210</td>
<td>0.9632</td>
</tr>
<tr>
<td>4</td>
<td>232.2147</td>
<td>8.27</td>
<td>1.509</td>
<td>0.9012</td>
</tr>
<tr>
<td>5</td>
<td>265.2913</td>
<td>9.05</td>
<td>1.724</td>
<td>0.8236</td>
</tr>
<tr>
<td>6</td>
<td>286.2603</td>
<td>10.06</td>
<td>1.861</td>
<td>0.7406</td>
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<tr>
<td>7</td>
<td>297.9309</td>
<td>11.28</td>
<td>1.937</td>
<td>0.6607</td>
</tr>
<tr>
<td>8</td>
<td>303.6501</td>
<td>12.65</td>
<td>1.974</td>
<td>0.5892</td>
</tr>
<tr>
<td>9</td>
<td>306.1504</td>
<td>14.11</td>
<td>1.990</td>
<td>0.5280</td>
</tr>
<tr>
<td>10</td>
<td>307.1414</td>
<td>15.63</td>
<td>1.996</td>
<td>0.4768</td>
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<td>307.5056</td>
<td>17.17</td>
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<td>12</td>
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<td>18.72</td>
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<td>0.3979</td>
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<tr>
<td>13</td>
<td>307.6726</td>
<td>20.28</td>
<td>2.000</td>
<td>0.3674</td>
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<tr>
<td>14</td>
<td>307.6863</td>
<td>21.84</td>
<td>2.000</td>
<td>0.3412</td>
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<tr>
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<td>307.6902</td>
<td>23.40</td>
<td>2.000</td>
<td>0.3184</td>
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<tr>
<td>16</td>
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<td>2.000</td>
<td>0.2985</td>
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<td>17</td>
<td>307.6921</td>
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<td>28.08</td>
<td>2.000</td>
<td>0.2653</td>
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<td>307.6921</td>
<td>31.20</td>
<td>2.000</td>
<td>0.2388</td>
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FREE FLOW HANDLING OR STORAGE TIME = 7.45 MINUTES
HANDLING OR STORAGE RATE ASYMPTOTE = 307.6924 CONTAINERS PER DAY
STATION NUMBER 1 IS THE BOTTLENECK STATION
R; T=0.54/2.28 14:41:06
Variations of the G Matrix with Number of Containers

Table 5.2-2

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<th>THE RIGHTMOST TWO COLUMNS OF THE MATRIX G</th>
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<td>4.331</td>
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<td>9.384</td>
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</table>

1.000 7.450 27.763 71.580 147.938 267.640 448.745 722.951 1142.794 1791.723 2800.092 4370.789 6819.781 10639.527 16597.988 25893.011 40393.167 63013.367 98300.812 153349.312 239224.812 373190.562 582177.125 908196.312 1416786.000 2210186.000 3447890.000 5378708.000 8390784.000 13089623.000 20419808.000
Table 5.2.1-1

Daily Handling Rates with Various Number of Cranes

<table>
<thead>
<tr>
<th># CONTAINERS</th>
<th>1 CRANE</th>
<th>3 CRANES</th>
<th>4 CRANES</th>
<th>10 CRANES</th>
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<td>HANDLING OR STORAGE RATE PER DAY</td>
<td>HANDLING OR STORAGE RATE PER DAY</td>
<td>HANDLING OR STORAGE RATE PER DAY</td>
<td>HANDLING OR STORAGE RATE PER DAY</td>
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<td>64.4295</td>
<td>64.4295</td>
<td>64.4295</td>
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<td>109.6024</td>
<td>128.8045</td>
<td>128.8045</td>
<td>128.8045</td>
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<td>3</td>
<td>135.6796</td>
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<td>192.9967</td>
<td>192.9967</td>
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<tr>
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<td>147.6905</td>
<td>254.1757</td>
<td>256.7671</td>
<td>256.7671</td>
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<tr>
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<td>152.0995</td>
<td>309.2590</td>
<td>318.8931</td>
<td>319.9072</td>
</tr>
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<td>355.7800</td>
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<td>392.3821</td>
<td>430.7153</td>
<td>443.3352</td>
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<td>153.8281</td>
<td>419.0483</td>
<td>476.9106</td>
<td>503.0625</td>
</tr>
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<td>153.8429</td>
<td>436.9807</td>
<td>515.1738</td>
<td>561.0432</td>
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<td>10</td>
<td>153.8455</td>
<td>448.1226</td>
<td>545.3364</td>
<td>616.3896</td>
</tr>
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</table>
asymptotic handling rates, as long as the crane station remains the bottleneck station. When 10 cranes were working, Station 2 became the bottleneck station. In fact one is assured that when

\[ \frac{3.12}{X} > \frac{0.97094}{2}, \]

i.e \[ X > 6.427 \Rightarrow X > 7 \]

(where \( X = \) number of cranes at station 1, which has a relative utilization of 3.12; and 0.97094 is the relative utilization of station 2, which has two transittainers), then station 2 becomes the bottleneck station.

Table 5.2.1-2a and 5.2.1-2b show the system's performances when the handling times at the crane station are halved and doubled respectively. Again, there is a linear relationship between crane cycle time and the daily handling rate. The queueing network performance is predictable and stable, as long as the bottleneck node does not change.
Table 5.2.1-2a

System Performance with Crane Cycle Times Halved

CONTAINER TERMINAL SYSTEM PERFORMANCE MEASURES
AS A FUNCTION OF THE NUMBER OF CONTAINERS
PRESENT AT THE PORT —— TIME IN MINUTES

<table>
<thead>
<tr>
<th># CONTAINERS</th>
<th>HANDLING OR STORAGE RATE PER DAY</th>
<th>AVERAGE HANDLING TIME PER CONTAINER</th>
<th>UTILIZATION OF CENTRAL STATION</th>
<th>RELATIVE EFFICIENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>81.4940</td>
<td>5.89</td>
<td>0.265</td>
<td>1.0000</td>
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<td>2</td>
<td>162.8853</td>
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<td>0.529</td>
<td>0.9994</td>
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<tr>
<td>3</td>
<td>241.5726</td>
<td>5.96</td>
<td>0.785</td>
<td>0.9881</td>
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<td>4</td>
<td>315.0664</td>
<td>6.09</td>
<td>1.024</td>
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<td>5</td>
<td>381.3662</td>
<td>6.29</td>
<td>1.239</td>
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<td>574.4863</td>
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</tbody>
</table>

FREE FLOW HANDLING OR STORAGE TIME = 5.89 MINUTES

HANDLING OR STORAGE RATE ASYMPTOTE = 615.3845 CONTAINERS PER DAY

STATION NUMBER 1 IS THE BOTTLENECK STATION

Table 5.2.1-2b

System Performance with Crane Cycle Times Doubled

CONTAINER TERMINAL SYSTEM PERFORMANCE MEASURES
AS A FUNCTION OF THE NUMBER OF CONTAINERS
PRESENT AT THE PORT —— TIME IN MINUTES

<table>
<thead>
<tr>
<th># CONTAINERS</th>
<th>HANDLING OR STORAGE RATE PER DAY</th>
<th>AVERAGE HANDLING TIME PER CONTAINER</th>
<th>UTILIZATION OF CENTRAL STATION</th>
<th>RELATIVE EFFICIENCY</th>
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<td>0.3388</td>
</tr>
</tbody>
</table>

FREE FLOW HANDLING OR STORAGE TIME = 10.57 MINUTES

HANDLING OR STORAGE RATE ASYMPTOTE = 153.8461 CONTAINERS PER DAY

STATION NUMBER 1 IS THE BOTTLENECK STATION
5.3 POSSIBLE EXTENSIONS OF MODEL

Owing to the assumption of some idealized circumstances, this model could be extended to include more and more practical containerport phenomena. The following directions for improvement are indicated:

i) Frankel and Tang stated that ports attempt to assign cranes to the longest hatch time on a priority basis.\(^1\) To reflect this practice, queue disciplines other than first-come-first served may be investigated along the lines suggested by Koontz and Muntz \(^2\) and by Kleinrock.\(^3\)

ii) The problem of equipment unavailability is a very real one, especially at Moran \(^4\) where frequent equipment breakdowns and downtimes affect the effective assignment of equipment to operations. A model may be built which will consider this occurrence explicitly. In Port Q, without this consideration, the throughput is overstated and either a scaling factor, derived from the availability\(^5\) of the machine should be applied or processing times at individual workstations may be adjusted to include the

---

2. Muntz and Wong, (Hawaii, 1974).
5. The availability of a piece of equipment is defined here as the ratio of its mean time between failures to the sum of this mean time between failures and the mean time to repair. From renewal theory, no assumptions about the distribution of these mean times are required for this definition to hold.
average waiting time when equipment is down.

iii) This model is not a dynamic model in that it does not anticipate trends in the volume of containers through the port, for example. Furthermore, dynamic considerations such as behavioural factors like operational work stoppages, labour conditions, management, information and reliable communication and co-ordination are not considered here in this steady-state operation. This model could be given these capabilities or used in a heuristic and adaptive manner to observe the influence of trends.

iv) Blocking was not allowed in this model hence stations succeeding the bottleneck station were not forced into idleness, and the productivities of those stations were not immediately depressed. Buzen's dependency factor could be used to model this blocking phenomenon. In another improvement, the bulk service stations which were essentially considered as infinite-server stations could benefit from the use of these dependency factors to reflect their limiting, designed capacity or effective capacity.

v) Finally, should our assumptions about the exponential distribution of times in this model become too bothersome, Kendall's method of Imbedded Markov Chains may be used to decompose them into a family of structured, exponential processes.

5.4 CONCLUSIONS

The effective assignment and utilization of Port facilities is the chief determinant of a port's internal operating efficiency. Port Q, our simple but versatile and flexible model provides us with these utilizations very easily. It can handle very large systems without any difficulty especially as the number of work stations at any port would hardly exceed 50 and the daily rate of handling of containers would hardly exceed 3000 in the foreseeable future. In fact, with proper definition, Port Q may be used to investigate the effective assignment of even multi-purpose port's facilities.

It can be seen that smaller versions of Port Q may be adapted to hand-held, programmable calculators. The size of the system these calculators can handle, (in terms of number of containers circulating around the system), being limited entirely by the number of available storage registers they have. As data about port operations and stations are entered and processed sequentially, even these pocket calculators will be capable of handling an unlimited number of work stations. Without doubt then, minicomputers can be programmed to use the full-scale model described here.

The output of Port-Q reveals which stations are most heavily utilized. This presents opportunities for pooling stations with compatible service or servers if they show a wide disparity in their levels of utilization.
The input requirements of Port-Q are minimal and one should only concentrate on gathering accurate data about stations with the highest ratio of utilization to the number of servers, to improve the accuracy of the results. A further testimony to the stability of this model is that it is not terribly sensitive to any single input parameter value. Even when many of the inputs are slightly off, results will still be fairly accurate. The work of Williams and Bhandiwad provides the theoretical justification for the stability of this model.

Bulk service stations or buffers, like the storage, are not, now, amenable to the methods of queueing network theory. It may therefore be suggested, that Port Q be used to investigate only the utilization of non-bulk service, mobile port equipment.

Finally, that the results of this analytical model came within 1% of those obtained from a detailed, time-consuming and expensive simulation study, is very pleasing.


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<tr>
<td>1975</td>
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<td>1976</td>
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<tr>
<td>1977</td>
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</tr>
</tbody>
</table>

Table A-1: Container Traffic at the Moran Dock, 1971-1977

Appendix A
Table A-2: Expanded Operations and Station Data

<table>
<thead>
<tr>
<th>STATIONS</th>
<th>OPERATIONS</th>
<th>(1) Harbour Entrance</th>
<th>(2) Berth</th>
<th>(3) Container Cranes</th>
<th>(4) Transittainers</th>
<th>(5) Fork Lifts</th>
<th>(6) Yard Hustlers</th>
<th>(7) Storage</th>
<th>(8) Interchange</th>
<th>(9) CFS(Striping)</th>
<th>(10) CFS(Stuffing)</th>
<th>(11) Customs</th>
<th>(12) Clerk's Office</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10. Paper Processing</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 76 TEU's</td>
<td>8. Inspection</td>
<td>11</td>
<td>9. Paper Processing</td>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 37 TEU's</td>
<td>7. Paper Processing</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># Served per time</td>
<td>250</td>
<td>250</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>12</td>
<td>4000</td>
<td>12</td>
<td>15</td>
<td>15</td>
<td>6</td>
<td>24</td>
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<tr>
<td>Sampling Frequency</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>.75</td>
<td>.25</td>
<td>.85</td>
<td>.95</td>
<td>1</td>
<td>.2</td>
<td>.3</td>
<td>.15</td>
<td>1</td>
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</table>
Table A-3: Computer Reproduction of Input Values

THERE ARE 6 STATIONS AT THIS CONTAINER TERMINAL
STATION NUMBER 1 IS THE CENTRAL MARSHALLING STATION
AVERAGELY, 10 CONTAINERS ARE PRESENT IN 3 DIFFERENT TYPES

5 TYPE 1 CONTAINERS GO THROUGH 5 OPERATIONS EACH.
ALL OTHER HANDLING AND STORAGE TIMES NOT LISTED BELOW ARE ZERO.

<table>
<thead>
<tr>
<th>OPERATION</th>
<th>DURATION IN MINUTES</th>
<th>STATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4.00</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>4</td>
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<tr>
<td>4</td>
<td>20.00</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>10.00</td>
<td>6</td>
</tr>
</tbody>
</table>

3 TYPE 2 CONTAINERS GO THROUGH 4 OPERATIONS EACH.
ALL OTHER HANDLING AND STORAGE TIMES NOT LISTED BELOW ARE ZERO.

<table>
<thead>
<tr>
<th>OPERATION</th>
<th>DURATION IN MINUTES</th>
<th>STATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.25</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>9.50</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>12.00</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4.00</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>11.00</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5.00</td>
<td>6</td>
</tr>
</tbody>
</table>

2 TYPE 3 CONTAINERS GO THROUGH 3 OPERATIONS EACH.
ALL OTHER HANDLING AND STORAGE TIMES NOT LISTED BELOW ARE ZERO.

<table>
<thead>
<tr>
<th>OPERATION</th>
<th>DURATION IN MINUTES</th>
<th>STATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.25</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4.00</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5.00</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>6</td>
</tr>
</tbody>
</table>
PORT_Q:PROC OPTIONS(MAIN);
DCL CONTAIN FILE STREAM INPUT;
OPEN FILE(CONTAIN) INPUT;
GET FILE(CONTAIN) LIST(M,MCENTRE,MT,N);
PUT SKIP(2) EDIT('THERE ARE','M',' STATIONS AT THIS CONTAINER TERMINAL','STATION NUMBER','MCENTRE', 'IS THE CENTRAL MARSHALLING STATION','AVERAGELY','N',' CONTAINERS ARE PRESENT IN','MT', 'DIFFERENT TYPES')
(SKIP(2),A,F(3),A,SKIP(2),A,F(3),A,SKIP(2),A,F(5),A,F(2),A);
/**
*/
/***/
M IS THE NUMBER OF HANDLING OR STORAGE STATIONS AT THE TERMINAL
MCENTRE IS THE 'CENTRAL MARSHALLING STATION'
MT IS THE NUMBER OF CONTAINER TYPES CONSIDERED IN THE MODEL
N IS THE AVERAGE NUMBER OF CONTAINERS PRESENT AT ANY ONE TIME
/*/ COMMON:BEGIN;
DCL MS(M) DEC FIXED(4), /* NUMBER OF SERVERS AT EACH STATION */
TM(M) DEC FIXED(10,2), /* AVERAGE PROCESSING TIME IN MINUTES */
FM(M) DEC FIXED(5,4), /* PROBABILITY OF PROCESSING AT STATION */
FM(M) DEC FIXED(5,4); /* FRACTION OF CONTAINERS THAT VISIT STATION*/
GET FILE(CONTAIN) LIST(MS,FM);
TIMETBL:BEGIN;
DCL CN(MT) DEC FIXED(10,4), /* AVERAGE NUMBER OF CONTAINER TYPE */
MO(MT) DEC FIXED(4), /* # OF OPERATIONS FOR CONTAINER TYPE */
SV(MT) DEC FIXED(8,3), /* TOTAL # OF VISITS BY CONTAINER TYPE*/
SM(MT,N) DEC FIXED(10,2), /* PROCESSING TIME AT STATION */
VM(MT,N) DEC FIXED(8,3); /* VISITS TO STATION BY CONTAINER TYPE */
/**
*/
/***/ LET THE INDEX I_ BE ASSOCIATED ONLY WITH CONTAINERS AND
LET THE INDEX J_ BE ASSOCIATED ONLY WITH THE STATIONS.
/*/ D10:DO K=1 TO MT;
GET FILE(CONTAIN) LIST(CN(K),MO(K));
COLUMNS:BEGIN;
DCL TIME(MO(K),M) DEC FIXED(10,2);
/* PROCESSING TIME FOR CONTAINER TYPE AT STATION */
GET FILE(CONTAIN) LIST(TIME);
PUT SKIP(2) EDIT('CN(K)', 'TYPE', 'K', ' CONTAINERS GO THROUGH', 'MO(K)', ' OPERATIONS EACH', 'ALL OTHER HANDLING AND STORAGE TIMES NOT LISTED BELOW ARE ZERO')
(SKIP(2),F(5),A,F(3),A,F(4),A,SKIP(2),A);
PUT SKIP(2) EDIT('OPERATION __ DURATION IN MINUTES _ STATION')
(SKIP(2),X(15),A);
D19:DO L=1 TO MO(K); D29:DO J=1 TO M;
IF TIME(L,J)=0.0 THEN
PUT SKIP EDIT(L,TIME(L,J),J)(SKIP,X(17),F(3),X(13),F(7,2),X(12),F(3));
END D29; END D19;
Table B-2

Programme Segment to Calculate Average Processing Times and the Probabilities of Station Visits

```
D20: DO J=1 TO M;
  VM(K,J), SM(K,J)=0.0;
D30: DO L=1 TO MQ(K);
  IF TIME(L,J)>0.0 THEN D40: DO;
  SM(K,J)=SM(K,J)+TIME(L,J)*FM(J);
  VM(K,J)=VM(K,J)+FM(J);
END D40; END D30; END D20;
END COLSUM; END D10;
XN=N;
D50: DO K=1 TO MT;
  CN(K)=CN(K)/XN;
  SV(K)=0.0;
D60: DO J=1 TO M;
  SV(K)=SV(K)+VM(K,J);
END D60; END D50;
D70: DO J=1 TO M;
  P1=P2+T1=0.0;
D80: DO K=1 TO MT;
  P1=P1+CN(K)*VM(K,J);
  P2=P2+CN(K)*SV(K);
  T1=T1+CN(K)*SM(K,J);
END D80;
IF P1=0.0; P2=0.0 THEN D90: DO;
  PUT SKIP(2) EDIT('STATION NUMBER', J, '
  IS NEVER USED. KINDLY ELIMINATE IT AND RESTART.')(X(5), A, F(4), A);
STOP;
END D90;
PM(J)=P1/P2;
TM(J)=T1/P1;
END D70;
END TIMETBL;
```
Figure B-3

Programme Segment to Sort Stations

```
NETWORK:BEGIN;
DCL UM(M) DEC FIXED(10.5), /* RELATIVE UTILIZATION OF STATION */
    IN(M) DEC FIXED(4), /* ORDERING INDEX */
    G(O,N) DEC FIXED (15.5),
    H(O,N) DEC FIXED(15.5);
BTLNECK=0.0;
FM(MCENTRE)=1.0;
D105:DO J=1 TO M;
    UM(J)=FM(J)*TH(J);
    IF UM(J)/MS(J)>BTLNECK THEN D105:DO;
BTLNECK=UM(J)/MS(J);
    NECK=J;
END D105; END D15;
MC=MCENTRE;
JSV1,JSVS=0;
D25:DO J=1 TO M;
    IF J=MC THEN
        IF MS(J)=1 THEN D35:DO;
            JSV1=JSV1+1;
            IN(JSV1)=J;
        END D35;
    ELSE D45:DO;
        JSVS=JSVS+1;
        IN(M-JSVS)=J;
        END D45;
    END D25;
    IN(M)=MC;
    IF JSV1>1 THEN
        D55:DO J=1 TO JSV1-1;
        INDEX=((-1)**J);
        CALL MINMAX(INDEX,UM,IN,J,JSV1);
    END D55;
    IF JSVS>1 THEN
        D65:DO J=JSVS+1 TO M-2;
        INDEX=((-1)**J)
        CALL MINMAX(INDEX,UM,IN,J,M-1);
    END D65;
    MINMAX:PROC(INDEX,R,IN,J,JS);
    DCL R(*) DEC FIXED(10.5),
    IN(*) DEC FIXED(4);
    D75:DO NEW=J+1 TO JS;
    IF R(IN(J))*INDEX<R(IN(NEW))*INDEX THEN D85:DO;
        KTEMP=IN(NEW);
        IN(NEW)=IN(J);
        IN(J)=KTEMP;
    END D85;
    END D75;
END MINMAX;
PUT SKIP(2) EDIT( 'STATION: ' NUMBER AVERAGE FREQUENCY RELATIVE') (A);
PUT SKIP EDIT( 'OF SERVERS PROCESSING OF VISITS UTILIZATION') (A);
PUT SKIP EDIT( 'FRACTION (MS) TIME (TH) (FM) (UM)') (A);
PUT EDIT(IN(J)), ' ', FM(IN(J)), MS(IN(J)), TH(IN(J)), PM(IN(J)), UM(IN(J)) DO J=1 TO M)
(SKIP(2), F(4), A, F(5), X(8), F(3), X(7), F(10), X(4), F(7), X(2), F
(10), 5));
```
Figure B-4
Programme Segment to Calculate G(N)

G,H=0.0;
G(0),H(0)=1.0;
GMATRIX:DO JJ=1 TO M;
J=IN(JJ);
IF MS(J)=1 THEN D95:DO;
R=UM(J);
SINGLE:DO I=1 TO N;
G(I)=G(I)+R*G(I-1);
END SINGLE; END D95;
ELSE D95:DO;
TEMP=1.0;
MULTI:DO II=1 TO N;
I=N-II+1;
D13:DO K=1 TO I;
SERVERS=MS(J);
CONT=I-K+1;
IF SERVERS>CONT THEN SERVERS=CONT;
TEMP=(TEMP*UM(J))/SERVERS+G(K);
END D13;
H(I)=G(I);
G(I)=TEMP;
TEMP=1.0;
END MULTI; END D95;
END GMATRIX;
PUT SKIP(2) EDIT(’THE RIGHTMOST TWO COLUMNS OF THE MATRIX G’)
(X(15)**A);
PUT EDIT((H(I),G(I)) DO I=0 TO N))
(SKIP, X(9),(2) (F(15,3), X(14)));
Figure B-5

Programme Segment to Calculate Station Performance Measures

```
FORM:FORMAT(SKIP(2),X(10),F(3),X(8),F(6,4),X(5),F(6,4),
X(4),F(7,2),X(5),F(7,2));
PUT SKIP(2) EDIT('STATION PERFORMANCE MEASURES')(COL(22),A);
PUT EDIT('__________________________________________')(COL(22),A);
PUT SKIP(2) EDIT('CONTAINERS CONTAINERS')(COL(42),A);
PUT SKIP EDIT('STATION UTILIZATION IDLENESS AT STATION IN QUEUE')
(COL(10),A);
XNORM=UM(MC)*G(N-1)/G(N);
PERFORM:DO JJ=1 TO M;
J=IN(JJ);
UTIL=UM(J)*XNORM/UM(MC);
IF MS(J)=1 THEN D23:DO;
XIDLE=1.0-UTIL;
PRESENT=1.0;
D33:DO  I=1 TO N;
PRESENT=PRESENT*UM(J)+G(I);
END D33;
PRESENT=PRESENT/G(N) -1.;
QUEUE=PRESENT -UTIL;
PUT EDIT(J,UTIL,XIDLE,PRESENT,QUEUE)(R(FORM));
END D23;
ELSE IF J=MC THEN
PUT EDIT(J,UTIL,'PERMUTE PERMUTE PERMUTE')
(SKIP(2),X(10),F(3),X(8),F(6,4),COL(33),A);
ELSE D43:DO;
A=H(N);
B=G(N);
XIDLE=A/B;
TEMP=N;
SERVERS=MS(MC);
D53:DO  K=1 TO N;
CONT=N-K+1;
IF SERVERS>CONT THEN SERVERS=CONT;
TEMP=(TEMP*UM(MC)/SERVERS)+(N-K)*H(K);
END D53;
PRESENT=TEMP/G(N);
QUEUE=PRESENT-UTIL;
PUT EDIT(MC,UTIL,XIDLE,PRESENT,QUEUE)(R(FORM));
END D43;
END PERFORM;
```
Programme Segment to Calculate System's Performance Measures

PUT SKIP(2) EDIT('--------------------------------------------------------------------')(A);
PUT SKIP(2) EDIT('CONTAINER TERMINAL SYSTEM PERFORMANCE MEASURES:')(A);
PUT SKIP EDIT(  AS A FUNCTION OF THE NUMBER OF CONTAINERS:')(A);
PUT SKIP EDIT(  PRESENT AT THE PORT --- TIME IN MINUTES:')(A);
PUT SKIP(2) EDIT('--------------------------------------------------------------------')(A);
PUT SKIP(2) EDIT('HANDLING AVERAGE UTILIZATION')(A);
PUT SKIP EDIT('OR STORAGE HANDLING TIME OF CENTRAL RELATIVE')(A);
PUT SKIP EDIT('CONTAINERS RATE PER DAY PER CONTAINER STATION EFFICIENCY')(A);
PUT SKIP EDIT('--------------------------------------------------------------------')(A);

D63: DO I=1 TO N;
   X0=G(I-1); X1=G(I);
   XXX=I;
   XUTIL=X0*UM(MC)/X1;
   RATE=XUTIL*480.*PM(MC)/TM(MC);
   AVET=XXX*TM(MC)/(XUTIL*PM(MC));
   EFFI=XUTIL*G(1)/(XXX*UM(MC));
   PUT SKIP EDIT(I,RATE,AVET,XUTIL,EFFI)(X(2),F(5),X(8),F(9,4),X(7),F(7,2),X(7),F(5,3),X(7),F(6,4));
END D63;
ASYM=PM(MC)*480.*UM(MC)/(TM(MC)*BTLNECK);
PUT SKIP(2) EDIT('FREE FLOW HANDLING OR STORAGE TIME:')(A,F(8,2),A);
PUT SKIP(2) EDIT('HANDLING OR STORAGE RATE ASYMPTOTE:')(A,F(9,4),A);
PUT SKIP(2) EDIT('CONTAINERS PER DAY:')(A,F(9,4),A);
PUT SKIP(2) EDIT('STATION NUMBER',NECK, 'IS THE BOTTLENECK STATION')
   (X(13),A,F(4),A);
END NETWORK;
END COMMON;
CLOSE FILE(CONTAIN);
END PORT_0;
<table>
<thead>
<tr>
<th>BLOCK NUMBER</th>
<th>*ioc</th>
<th>OPERATION</th>
<th>A, B, C, D, E, F, G</th>
<th>COMMENTS</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>EXPON FUNCTION RN1, C24</td>
<td>THIS IS THE EXPONENTIAL DISTRIBUTION</td>
<td></td>
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<tr>
<td></td>
<td>1</td>
<td>GENERATE</td>
<td>.360, FNSEX Pon</td>
<td>ONE ARRIVES EVERY SIX MINUTES</td>
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<td>2</td>
<td>TRANSFER</td>
<td>.5, XPORT</td>
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<td></td>
<td>3</td>
<td>QUEUE</td>
<td>CRANE</td>
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<tr>
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<td>4</td>
<td>ENTER</td>
<td>CRANE</td>
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<tr>
<td></td>
<td>5</td>
<td>DEPART</td>
<td>CRANE</td>
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<tr>
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<td>6</td>
<td>ADVANCE</td>
<td>180, FNSEX Pon</td>
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</tr>
<tr>
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<td>7</td>
<td>LEAVE</td>
<td>CRANE</td>
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<td>8</td>
<td>TRANSFER</td>
<td>.25, LIFT1</td>
<td>75% SAMPLING</td>
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<td>9</td>
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<td>STRAD</td>
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<td>STRAD</td>
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<td>DEPART</td>
<td>STRAD</td>
<td></td>
</tr>
<tr>
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<td>12</td>
<td>ADVANCE</td>
<td>180, FNSEX Pon</td>
<td></td>
</tr>
<tr>
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<td>STRAD</td>
<td></td>
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<td>.75, YARD1</td>
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<tr>
<td></td>
<td>15</td>
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<td>17</td>
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<td>FORKL</td>
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<td>240, FNSEX Pon</td>
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<td>FORKL</td>
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</tr>
<tr>
<td></td>
<td>20</td>
<td>YARD1 TRANSFER</td>
<td>.15, CUST1</td>
<td></td>
</tr>
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<td></td>
<td>21</td>
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<td>HUSTL</td>
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</tr>
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<td>23</td>
<td>DEPART</td>
<td>HUSTL</td>
<td></td>
</tr>
<tr>
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<td>24</td>
<td>ADVANCE</td>
<td>180, FNSEX Pon</td>
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<td>LEAVE</td>
<td>HUSTL</td>
<td></td>
</tr>
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<td>26</td>
<td>CUST1 TRANSFER</td>
<td>.85, CHNG1</td>
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<td>27</td>
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</tr>
<tr>
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<td>DEPART</td>
<td>CUSTH</td>
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<td>ADVANCE</td>
<td>1200, FNSEX Pon</td>
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49 LEAVE CUSTM
50 YARD2 TRANSFER .15,,LIFT2
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52 ENTER HUSTL
53 DEPART HUSTL
54 ADVANCE 240, FN$EXPO
55 LEAVE HUSTL
56 LIFT2 TRANSFER .75,,TRAN2
57 QUEUE FORKL
58 ENTER FORKL
59 DEPART FORKL
60 ADVANCE 720, FN$EXPO
61 LEAVE FORKL
62 TRAN2 TRANSFER .25,,CRAN2
63 QUEUE STRAD
64 ENTER STRAD
65 DEPART STRAD
66 ADVANCE 570, FN$EXPO
67 LEAVE STRAD
68 CRAN2 QUEUE CRANE
69 ENTER CRANE
70 DEPART CRANE
71 ADVANCE 195, FN$EXPO
72 LEAVE CRANE
73 TRANSFER ,OUT
74 EMPTY QUEUE INTER
75 ENTER INTER
76 DEPART INTER
77 ADVANCE 180, FN$EXPO
78 LEAVE INTER
79 TRANSFER .15,,LIFT3
80 QUEUE HUSTL
81 ENTER HUSTL
82 DEPART HUSTL
83 ADVANCE 120, FN$EXPO
84 LEAVE HUSTL
85 LIFT3 TRANSFER .75,,TRAN3
86 QUEUE FORKI
87 ENTER FORKL
88 DEPART FORKL
89 ADVANCE 300, FN$EXPO
90 LEAVE FORKL
91 TRAN3 TRANSFER .25,,CRAN3
92 QUEUE STRAD
93 ENTER STRAD
94 DEPART STRAD
95 ADVANCE 240, FN$EXPO
96 LEAVE STRAD
97 CRAN3 QUEUE CRANE
98 ENTER CRANE
99 DEPART CRANE
100 ADVANCE 195, FN$EXPO
101 LEAVE CRANE
102 OUT TABNIA LATE FLOW
103 TERMINATE FLOW TABLE M1,0,60,100
CRANE STORAGE ?
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**104**
- **GENERATE** 8640000
- **THREE MONTHS TO REACH STEADY STATE**

**105**
- **TERMINATE** 1
- **START** 1, NP
- **RESET**

**106**
- **GENERATE** 31536000
- **ONE YEAR OF OBSERVATION**

**107**
- **TERMINATE** 1
- **START** 2
- **END**