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CONNECTIVITY MONITORING
IN MOBILE PACKET RADIO NETWORKS

BY

Michael Gene Hluchyj

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Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139
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Michael Gene Hluchyj

B.S., University of Massachusetts at Amherst (1976)

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Submitted to the Department of Electrical Engineering and Computer Science on December 15, 1978, in partial fulfillment of the requirements for the Degrees of Master of Science and Electrical Engineer.

ABSTRACT

Knowledge of connectivity (i.e., what pairs of nodes can communicate directly) in a data communication network is essential for the efficient and reliable operation of the network. In a packet radio network with mobile nodes, the connectivity varies with time and thus must in some way be monitored.

The problem of monitoring connectivity in mobile packet radio networks is considered. Two general methods for monitoring connectivity are developed and compared. It is found that each method has its respective advantages and disadvantages, and thus to choose between them, one must examine both the specific type of packet radio in which one wishes to monitor connectivity and the specific use that is to be made of connectivity information in that network.

Implementations of both monitoring methods in a terminal-oriented mobile packet radio network, where connectivity information is used for updating packet routes, are presented and compared. It is found that a particular implementation is the most flexible and in general uses the least amount of overhead. Its performance is analyzed in detail for a particular network model.

Thesis Supervisor: Cyril Leung
Title: Assistant Professor of Electrical Engineering
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PART I

General Concepts
A packet radio network is a type of data communication network. In a general sense, one may view a data communication network as a finite collection of nodes connected to each other by some form of communication. Attached to some or all of the nodes are resources (e.g., terminals, computers), and the purpose of the network is to transport messages both reliably and efficiently between resources at different nodes. A message is generally transported in the form of one or more packets. Each packet, in addition to message bits, contains binary encoded control information (e.g., source and destination addresses, error control bits). Much of this control information is often located at the beginning of the packet in what is called the packet header.

The main distinguishing feature between packet radio networks and other data communication networks, specifically point-to-point packet switching networks such as the ARPANET [2] or Tymnet [3], is the type of communication used between nodes. Pairs of nodes in a point-to-point packet switching network are connected by separate communication channels (e.g., hardwire channels, microwave links). In contrast, the nodes of a packet radio network are linked together by broadcast radio channel(s). Specifically, the communication section of each node in a packet radio

* See [1] for a discussion on the advantages of transporting data in the form of packets.
network is a radio transceiver, with an omnidirectional antenna* and finite transmission range, accessing one or more common broadcast channels. Although seemingly simple, this one difference between packet radio networks and point-to-point packet switching networks leads to significant differences in the operation of the two types of networks and in the applications for which each is suited.

Due to the broadcast nature of nodes in a packet radio network, a packet may be received by all nodes within range of the transmitting node. Thus, in contrast to a point-to-point network, a channel in a packet radio network is not generally associated with only two nodes. This implies the need for additional information in packet headers informing the receiving nodes for whom a packet is or is not intended. This further implies that if a node receives a packet in error, it may have no way of knowing if the packet was intended for it and consequently, cannot request a retransmission. For this reason positive acknowledgements and time-outs are used for error control in packet radio networks.

Channels in packet radio networks are generally shared by several nodes. Various schemes (see [5], [6], [7] for examples) have been devised which allow nodes to access these common channels. Studies (see above references) have shown that for many applications, schemes which dynamically share channel capacity (e.g., ALOHA) are more efficient than those which assign fixed capacities to nodes (e.g., TDMA). Many of these schemes which dynamically share channel capacity are referred to as

*In certain situations it may be desirable for one or more of the nodes to have directional antennas. See [4] pp. 221-223 for an example.
random access schemes. This is because with each of these schemes, the times at which nodes transmit packets are in some sense chosen at random. Hence, in using a random access scheme, two or more packet transmissions may overlap in time on the same channel, and thus interfere with each other at the receiving node. Consequently, the probability of a node receiving a packet in error in a packet radio network will not only be greater than in a point-to-point packet switching network, but will also be a function of the traffic level and the spatial topology of the nodes.

Packet radio networks are particularly well suited for applications which have one or more of the following properties. 1) The network resources are located in remote areas where hardwire connections are uneconomical, or in hostile locations where hardwire connections may not be feasible and where the capability of rapid deployment is essential. 2) The traffic characteristics of the resources are of a bursty nature (i.e., a high ratio of peak to average data rate), thus making the dynamic allocation of channel capacity a desirable feature. 3) Some or all of the resources are mobile, in which case a radio channel is essential.

1.2 Connectivity Monitoring

In a packet radio network, we say that a communication "link" exists from node i to node j if node j is within the transmission range of node i. Two nodes are said to be connected if such a communication link exists in at least one direction between them. The complete set of links in a network is referred to as the connectivity of the network.

Network control functions often require knowledge of connectivity. For example, directed routing is an efficient routing technique
in which a packet is directed from one node, say i, to another node within its transmission range, say j, by attaching the identity of node j to the packet header. All nodes other than j that receive the packet will ignore it, and only j will accept it and then either forward it on or keep it depending on the packet's final destination. There are various routing schemes which incorporate the use of directed routing. In one scheme, an ordered list of the nodes which are to relay a packet to its final destination is placed in the packet header by the node which originated the packet. In this way, a packet may be directed from node to node along its route with all the routing information contained in the packet header. In another scheme, each node maintains a table which pairs each of the possible destination nodes with a node within its transmission range to which it is to direct packets for that particular destination. In this scheme, each node determines the identity of the node to which it should direct a received packet by examining the packet header, determining the identity of the destination node, and then performing a table look-up. In Chapter 3 we describe a packet radio network in which the routing scheme is in some sense a combination of the above two schemes. In any event, we see that depending on the exact implementation, directed routing requires anywhere from a global knowledge of connectivity where every node knows the entire network connectivity, to a simple local knowledge where each node knows at least one node within its transmission range to which it may direct packets.

In packet radio networks with mobile nodes (e.g., a network for law enforcement may include patrol cars which would constitute mobile nodes), the network connectivity will be a function of an initial position
and the subsequent motions of the nodes. Since updated knowledge of connectivity is necessary for network control functions such as routing, the problem of how a packet radio network with randomly moving nodes monitors its connectivity is one which deserves study.

1.3 Outline of Thesis

The purpose of this thesis is to examine possible methods for monitoring connectivity in mobile packet radio networks. The thesis is divided into two parts. The first part includes, in addition to this introductory chapter, a chapter in which two general methods for monitoring connectivity are developed and compared. Throughout Part I, we keep the discussion as general as possible, making few if any assumptions about the type of packet radio network or the use that is to be made of connectivity information. In the second part of the thesis, however, we consider a specific type of packet radio network and a specific use of connectivity information. We begin Part II with a chapter on the description of this network. Next we discuss how the monitoring methods developed in Part I may be implemented in this network. We find that one monitoring implementation is the most flexible and in general uses the least amount of overhead. In order to gain a better understanding of the trade-offs associated with this implementation, in the fifth chapter we analyze its performance in greater detail. Finally, in the last chapter we make concluding remarks on connectivity monitoring and the results that we have obtained.
CHAPTER 2

Two Methods for Monitoring Connectivity

A method of determining if two nodes in a packet radio network are connected is to simply test the communication channel between them (i.e., try sending a packet from one node to the other). In this chapter, two general methods for monitoring connectivity which employ this simple testing idea are developed and compared.

2.1 Broadcast Method

Consider two nodes, say node i and node j, in a packet radio network. Node j wishes to determine if a communication link exists from node i to itself. One method for doing this is as follows. Node i will transmit a special packet to node j. This special packet will contain the unique identity of node i. If node j successfully receives this packet, after examining the identity, node j will know that it is within the transmission range of node i (i.e., a link exists from i to j). If node j's transmission range is greater than or equal to node i's transmission range, then node j will also know that it will be able to successfully send packets to node i (i.e., a link also exists from j to i). The monitoring extension of this method is to have node i transmit these special packets at various points in time (e.g., periodically). Each time node j receives one of these special packets, it will have updated its knowledge of the connectivity from node i to itself.

Now consider all of the nodes in a packet radio network. One approach to monitoring connectivity is to give each node the responsibility of determining and then monitoring the connectivity between itself and the other nodes in the network. Depending on the network control
functions (and their implementations) which require knowledge of connectivity, this connectivity information may be needed elsewhere in the network (e.g., a packet radio network may be set up so that one node in the network determines the routing of packets for the entire network, and thus would require knowledge of the entire network connectivity). In such cases, after updating its knowledge of the connectivity between itself and the other nodes in the network, each node could transmit this connectivity information to those nodes within its transmission range which require this information and/or are forwarding points to other nodes which require this information.

A method in which each node may monitor the connectivity from the other nodes in the network to itself follows from our discussion of the two node case. Here each node at different points in time will broadcast a special packet containing its identity. Each node within range of the transmitting node that receives one of these special packets will know that a communication link exists from the transmitting node to itself. In the case where every node in the network has the same transmission range, the receiving node will also know a link exists from it to the transmitting node. We call this the broadcast method of connectivity monitoring, because each node simply broadcasts its identity to all nodes within its transmission range.

With the broadcast method of connectivity monitoring, we see that for a packet radio network with \( N \) nodes, only \( N \) transmissions are needed in order for every node to determine the connectivity from the other nodes to itself. Naturally, knowledge of connectivity cannot be perfect. By the very nature of the schemes which are used to access the common
broadcast channel(s) in a packet radio network, packet errors will occur. Thus although a node may be within range of another node transmitting a special packet, it may not successfully receive that packet. So we see communication links may actually exist but not be detected.

To reduce the monitoring overhead associated with the broadcast method, one can take advantage of the connectivity information contained in regular message packets. If at each time a node transmits a packet (whether the node originated the packet or is simply forwarding it on from another node) it attaches to the packet header its identity, all nodes within range of the transmitting node that receive the packet (even those to which it is not intended) can examine the packet header, determine the transmitting node, and update their knowledge of the connectivity from that node. In this way, a node need only broadcast a special packet when it has not transmitted a regular message packet for some length of time. Thus during heavy usage of the network by many nodes, one has a desirable reduction in the overhead needed for connectivity monitoring.

The broadcast method of connectivity monitoring is not, however, without its problems. With the broadcast method, a node which receives special monitoring packets from neighboring nodes is only informed of the connectivity from these nodes to itself. This knowledge of connectivity is in itself rather incomplete, in that without additional transmissions, a node is aware of the neighboring nodes that may transmit

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*Depending on the details of packet routing, this identity may already be in the packet header, and thus need not be added.*
packets to it, but does not in general know to which nodes it may transmit packets. If a node is to use connectivity information for routing packets, it is often necessary that that node be aware of the nodes to which it is connected in both directions. The outbound direction is necessary for sending message packets, and the inbound direction is necessary for receiving the acknowledgments for those packets. In packet radio networks where all nodes have the same transmission range, a test of connectivity between two nodes in one direction may be a sufficient test for both directions. In this situation, the broadcast method can be used to inform each node of its connectivity to and from the other nodes in the network. However, considering that transmission range is a function of transmitter power, such an assumption is not always realistic; possibly making the broadcast method of connectivity monitoring limited in its applications.

Another problem with the broadcast method is that each node must rely on the broadcasts of the other nodes in the network in order to monitor its inbound connectivity. For a mobile packet radio network, the rate at which a node must update its knowledge of connectivity will be a function of the characteristics of node mobility (e.g., velocity, randomness of motion) and the requirements (e.g., maximum average delay in packet delivery) placed upon that node by its associated resources. In general, an increase in node velocity and/or a decrease in the required average delay in packet delivery will require a higher monitoring rate. Suppose there is a large variance in the range of node velocities (e.g., a military packet radio network may include nodes moving by foot, motor vehicle, and aircraft), and/or a variety of network resources
which place different requirements upon the network (e.g., digitized voice communication may require a much smaller delay in packet delivery than say terminal-to-terminal communication). Although each node may have a different required rate of updating its knowledge of connectivity, with the broadcast method, the rate at which all nodes within a geographical area must broadcast special monitoring packets will have to be that needed by the node with the highest required updating rate. This can clearly result in excessive overhead for monitoring connectivity.

In the next section, we shall see that the problems associated with the broadcast method of connectivity monitoring are eliminated with the probing method. However, this is achieved only at the expense of an increase in network overhead.

2.2 Probing Method

Let us first consider just two nodes in a packet radio network. Node j wishes to determine if there exists both a communication link from it to node i and a communication link from node i to itself. Node j can do this by sending node i a **probe packet**, which if received by node i, instructs node i to send a **response packet** back to node j. If received by node j, this response packet informs node j that node i received the probe packet. Thus if after sending a probe packet to node i, node j receives a response, node j will know that it is connected in both directions to node i (i.e., a link exists from j to i and a link exists from i to j). The monitoring extension of this method is to have node j "probe" node i (i.e., send node i a probe packet) at various points in time, with each response packet received by node j
being an indication of a connection in both directions between nodes i and j.

Now consider all of the nodes in a packet radio network. A method in which each node may monitor the connectivity in both directions between it and the other nodes in the network follows from the two node case. Here each time a node wishes to update its knowledge of the connectivity between it and the other nodes in the network, that node will transmit a general probe packet. A general probe packet is simply a probe packet addressed to all nodes in the network. Each node within the transmission range of the probing node (i.e., the node which transmitted the probe packet) that receives the probe packet will respond by sending the probing node a response packet. This response packet will in some way contain the unique identity of the responding node. Thus, each response packet received as a reply to a probe packet, informs the probing node of the existence of a connection in both directions between it and the responding node. We call this the probing method of connectivity monitoring, because each node probes (via probe packets) the nodes within its transmission range in order to update its knowledge of connectivity.

It is important to note that, as with the broadcast method, knowledge of connectivity gained via the probing method cannot in general be perfect. For example, suppose each of two nodes is within the transmission range of the other (i.e., the two nodes are connected in both directions). If a probe packet sent by one node is received in error, or if a probe packet sent by one node is correctly received but the associated response packet sent by the other node is received in error,
then the connection between the two nodes will go undetected by the probing node. Consequently, as with the broadcast method, communication links in a network may actually exist but not be detected when using the probing method of connectivity monitoring.

It is also important to recognize that when a node transmits a general probe packet, some mechanism must be provided to prevent the nodes that receive that probe packet from sending response packets all at the same time on a common broadcast channel. If they did, it is possible that the response packets arriving at the probing node at the same time will destructively interfere with each other, and none of the response packets will be correctly received. To avoid this problem, one could assign to every node in the network a different time delay which each node will wait before responding to a general probe packet. The time delays should be assigned so that no two nodes could respond to a probe packet with response packets which overlapped in time. Alternatively, if the number of nodes in the network is large, one could make the time delay for each node a random variable. Although there still exists the possibility of a conflict of two or more response packets at a probing node, one could, with randomized time delays, reduce the time from when a probe packet is sent until all possible responses are received. This can simply be done by limiting the largest value of time that the time delay random variable associated with each node takes on nonzero probability.

The special monitoring packets used in the broadcast method, and the probe and response packets used in the probing method carry approximately the same amount of information. Thus we may assume that they
would be about the same length in bits if implemented in a packet radio network. Therefore, for the purpose of comparing these connectivity monitoring methods, we can use the number of packet transmissions involved in a particular monitoring method as a measure of the network overhead needed to implement that method.

Consider a packet radio network in which each node is connected in both directions to L other nodes. With the probing method of connectivity monitoring, it is clear that each node will require at most \( L+1 \) transmissions* to update its knowledge of the connectivity between it and the other nodes in the network. For an N node network, this comes to a total of at most \( N(L+1) \) transmissions for all of the nodes to update their knowledge of connectivity. This number of transmissions is greater, by a factor of \( L+1 \), than the number of transmissions required by the broadcast method. However, one must keep in mind that the broadcast method only tests the communication channel between two nodes in one direction, whereas the probing method tests both directions. Also, the probing method is more flexible than the broadcast method. With the probing method, each individual node in a packet radio network may send a probe packet at whatever time it deems necessary. Consequently, each node has complete control over the rate at which it can update its knowledge of the connectivity between it and the other nodes in the network. Thus, in contrast to the broadcast method, a large variance in the required updating rates of the nodes in a packet radio network can be

*One transmission of a probe packet, and a transmission of a response packet by each of the L nodes that correctly received the probe packet.
dealt with by the probing method in a very convenient manner. Specifically, each node can simply send probe packets at its own required rate.

Depending on what knowledge of connectivity is needed by the network, it may be that each node is not interested in monitoring the connectivity between it and all of the other nodes in the network. For example, we shall describe a packet radio network in Chapter 3 in which, at any particular time, each node is interested in monitoring the connectivity between it and only one other node in the network. In this case, each node can transmit probe packets addressed only to that node between which it is interested in monitoring connectivity, with only that specific node responding to each received probe packet. Thus we see that at most only $2N$ transmissions are needed for all $N$ nodes in the network to update that knowledge of connectivity which is needed.

At this point it is clear that we cannot recommend the use of one method of monitoring connectivity over another for packet radio networks in general. Both the broadcast and probing methods have their respective advantages and disadvantages. In choosing between them, one must examine both the specific packet radio network in which one wishes to monitor connectivity, and the specific use that is to be made of connectivity information in that network.
PART II

Connectivity Monitoring in a Terminal-Oriented Mobile Packet Radio Network
CHAPTER 3

Network Description

Up to this point we have only examined general ideas in monitoring connectivity. In Part II of this thesis, we narrow our view and examine connectivity monitoring in a specific type of mobile packet radio network. In this chapter we describe this network.

The packet radio network to be described is chosen for two reasons. First, this type of network is one that often arises in a terminal-oriented packet radio network, and is one which has already received considerable attention (see [8] for a discussion and further references). Secondly, we shall see that connectivity information is only used in this network for updating simple packet routes. Consequently, the connectivity monitoring needed in this network has relatively straightforward implementations which are amenable to analysis.

The packet radio network we now describe is one in which all nodes communicate with each other via some random access scheme on one common broadcast channel. For packet transportation, the network uses directed routing in which all traffic flows through a centralized node called a station. The packet routes are configured as a tree structure rooted at the station, where the branch nodes consist of relay devices called repeaters and the end nodes consist of end devices called terminals.* This structure implies that the same repeaters will be traversed by packets going in either direction between a particular device (terminal, repeater) and the station. Figure 3-1 illustrates a layout of

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*"Terminals" may include, for example, TTY-like and CRT terminals, computers, display printers, and unattended sensors.
The dashed lines between nodes indicate the packet routes between the station and the terminals and repeaters.

*Figure 3-1* Packet radio network with tree-structured routes
tree-structured routes for this type of network. We allow the terminals to be mobile, however, for simplicity, we assume that the station and repeaters are stationary. Also, we place no lower limit on the transmitter power associated with each terminal, thus allowing the terminals to take on any size, including hand-held. We will, however, generally assume that the transmitter power of the station and each repeater is greater than that of any terminal. Later we shall see how this assumption enters into the selection of a connectivity monitoring method for this network.

The above network configuration is based primarily on the assumption that we have a terminal-oriented network [9], where the terminals will mainly want to communicate with the station (where say a computer is located) which, in turn, will provide access to other network terminals or to other networks via a gateway. The repeaters are used simply to increase the range over which this can be done.

The terminal-oriented assumption is also the rationale behind using "essentially fixed" routes between each terminal and the station. Since messages will generally be short, the overhead and complexity associated with variable routing outweigh its advantages. Naturally routes cannot be strictly fixed. Link failures due to device breakdowns, and more importantly due to terminal mobility, will necessitate that routes be changed. For efficiency reasons, when a route needs to be changed, the new route, selected from the set of available routes, will in some sense be the best available route (e.g., the minimum hop route, the minimum average delay route) at the time of selection.

Note that, in general, one may want to change routes in this
type of network not only because of changes in connectivity, but also due to changes in traffic flow which result in a significant increase in the delay in packet transportation. However, by choosing the "best" available route after a change in connectivity necessitates that a new route be selected, the network is automatically updating routes in an attempt to improve traffic flow. Whether this updating alone is sufficient, so that additional changes in routing are not needed, of course depends on both the characteristics of terminal mobility (e.g., average velocity and randomness of motion), and the characteristics of traffic flow fluctuations (e.g., the frequency and magnitude of flow changes). For the terminal-oriented network under consideration, we assume that additional changes in routing for improved traffic flow will be relatively infrequent compared to changes needed as the result of terminal mobility.

Having described the structure of the routes in this network, we now explain how a packet is directed along its route. The implementation of directed routing in this network follows from our discussion in section 1.2. Specifically, the first scheme discussed in section 1.2 is used for routing each outbound packet (i.e., a packet traveling from the station to a particular terminal or repeater), and the second scheme discussed in that section is used for routing each inbound packet (i.e., a packet traveling from a terminal or repeater to the station). That is, when sending a packet to a terminal or repeater, the station includes in the packet header an ordered list of the repeaters which are to relay the packet to its destination. Thus each repeater along the route need only examine the packet header in order to determine to whom it should
direct the packet. This form of directed routing is not, however, necessary for inbound packets. In particular, each device does not have to know the entire route to the station. It need only know the identity of the node (either a repeater or the station) to which it is to direct packets which are destined for the station. When a device, not within range of the station, wishes to send a packet to the station, it merely directs that packet to its "relaying repeater" which will then, in turn, forward the packet to the station.

There are of course other schemes for routing packets in a packet radio network (see [10] for examples). However, the scheme described above is of a rather simple nature. In particular, suppose that a terminal has moved out of range of its relaying repeater, and thus must obtain a new route to the station. With tree-structured directed routing, only that terminal and the station need to obtain new routing information. In fact, this new routing information need only consist of the identity of the terminal's new relaying repeater. The reason for this is that this identity is the only information needed by the terminal to send packets to the station; and by knowing the identity of the terminal's new relay repeater, the station can look up the route to that repeater and thus know the route to the terminal.

Although the use of tree-structured routes simplifies the routing of packets in this network, it also, unfortunately, has its drawbacks. In particular, two problems result from having all traffic flow through the station. The first is that the station is a bottleneck to traffic flow, and the second is that if the station fails, packet transportation in the entire network will cease. The first problem may be partially
solved by using directional antennas at the station (see [4] pp. 221-223). Also, the seriousness of the second problem may be reduced by devising back up routing strategies which do not employ the use of the station. Still, it is important to realize that the concept of a packet radio network is a relatively new one, and that there are many problems which need further study.
CHAPTER 4

Monitoring Methods

We are interested in the monitoring of connectivity for the purpose of updating routes in the network described in Chapter 3. Because the station and repeaters are assumed to be stationary, the connectivity between them will not, aside from infrequent repeater/station breakdowns, change with time. Thus, aside from infrequent repeater/station breakdowns and infrequent changes in routing for improved traffic flow, we assume that the route between each repeater and the station remains fixed. The terminals, however, are mobile and changes in connectivity between a terminal and the repeaters and station may necessitate that the route between that terminal and the station be changed. Thus we shall direct our attention to terminal-repeater and terminal-station connectivity monitoring for the purpose of updating the route between each terminal and the station.

For convenience, we refer to the node (either a repeater or the station) to which a device (terminal, repeater) directs packets which are destined for the station, as that device's relaying node. Note that with tree-structured routes, given the route between each repeater and the station, the route between any terminal and the station is completely specified by the identity of the terminal's relaying node. For this reason, we shall often times refer to a terminal obtaining a new route, as that terminal obtaining a new relaying node.

In this chapter we examine how the monitoring methods developed in Chapter 2 may be implemented, in the network we are now considering, for the specific task we have outlined above. We examine first the use of the broadcast method and then the use of the probing method.
4.1 Broadcast Method

Let us assume that the routes in the packet radio network described in Chapter 3 have been initialized. For tree-structured directed routing, this implies that each device knows the identity of its relaying node and the station knows the complete route to every device. With mobile terminals, changes in terminal-repeater and terminal-station connectivity will occur, and thus the route between each terminal and the station will have to be constantly changed.

4.1.1 First Implementation of the Broadcast Method

An implementation of the broadcast method for monitoring terminal-repeater and terminal-station connectivity follows directly from our discussion in section 2.1. Specifically, the repeaters and station broadcast special monitoring packets. Recall that the identity of the node broadcasting a special monitoring packet is contained in that packet. For each special monitoring packet received, a terminal will know that a communication link exists from the broadcasting node (in this case either a repeater or the station) to itself. If the station is given the function of updating routes, then the terminal will, at various points in time (e.g., periodically and/or when the terminal detects an important change in connectivity), send, via a relaying node within its range, its updated knowledge of connectivity to the station. There a decision is made as to whether the terminal should be given a new route to the station. If a new route is to be assigned, the station will send a packet to the terminal which informs that terminal
of its new relaying node. Another option is to give each individual terminal the function of updating its own route to the station. In this situation, it may be desirable to have each repeater include in its special monitoring packet some measure of its ability to relay packets to the station (e.g., number of hops to the station, expected delay to reach the station, number of terminals already using that route to the station). This way, after a terminal concludes a loss of connectivity between it and its relay node (by not having received a special monitoring packet from that node for some period of time), the terminal may select, with some intelligence, a new relay node from its updated list of available relaying nodes. After selecting a new relay node, a terminal must of course inform the station of the identity of this relay node, so that the station will be able to send packets to that terminal.

Unfortunately, the above use of the broadcast method suffers from the problems that we have already discussed in section 2.1. Recall that in Chapter 3 we made the assumption that the transmitter power of the station and each repeater is greater than that of any terminal. Hence the transmission range of the station and each repeater will be greater than that of any terminal. Thus although a terminal can, from the special monitoring packets, monitor its inbound connectivity, it cannot in general be certain of its outbound connectivity. This lack of connectivity information makes it nearly impossible for either the terminal or the station to consistently select usable routes. Another, possible less serious problem is the lack of control given to a terminal in updating its knowledge of connectivity. A terminal must rely on
the broadcasts of the repeaters and station in order to update its knowledge of connectivity. Thus the rate at which the station and all the repeaters must broadcast special monitoring packets will have to be that needed by the terminal with the highest required updating rate. So for a network with a large range of required terminal updating rates, this implementation can clearly result in excessive overhead for monitoring connectivity.

However, if the network is such that the transmission range of each terminal is at least as great as that of the station and any repeater, and if each terminal has the same required rate of updating its knowledge of connectivity, then the problems associated with this first implementation of the broadcast method disappear. In addition, since only the station and repeaters are broadcasting special monitoring packets, very little overhead is generated in the monitoring process. Thus this first implementation of the broadcast method performs very well if the network is uniform in terms of transmission ranges and connectivity updating rates.

4.1.2 Second Implementation of the Broadcast Method

The problems associated with the first implementation of the broadcast method are eliminated in another, somewhat indirect implementation of the broadcast method. In fact, a form of this second implementation is used in an actual packet radio network that is currently being tested by ARPA (see [11] for details). In this second implementation of the broadcast method, for the purpose of monitoring terminal-repeater and terminal-station connectivity, rather than having
the repeaters and station broadcast special monitoring packets to the
 terminals, the terminals broadcast special monitoring packets to the
 repeaters and station. These special monitoring packets are called
 ROP's * and, as with other special monitoring packets, each ROP contains
 the identity of the terminal from which it was broadcasted. When a
 terminal broadcasts an ROP, any repeater with an assigned route to the
 station that receives the ROP will attach its identity to that ROP and
 then relay that ROP on to the station just as if it were a regular
 message packet. Thus when a terminal broadcasts an ROP, a set of
 associated ROP's are generated and relayed to the station. The
 station can then examine this received set of ROP's and update its
 knowledge of the terminal-repeater and terminal-station connectivity
 for the particular terminal which originated the ROP. The station can
 then use this and possibly other information (e.g., statistics on the
 delay in packet transportation for each route available to the terminal)
 to decide whether a new route should be assigned to that terminal, and
 if so, what route should be assigned. If a new route is selected, the
 station must of course send to the terminal a packet which informs that
 terminal of the identity of its new relaying node.

 With this implementation of the broadcast method, we see that
 the problems associated with the first implementation have been elimi-
 nated. Specifically, the channel is now tested from each terminal to
 the repeaters and station. Since the transmission range of the station
 and each repeater is assumed to be greater than that of any terminal

*For the purpose of this discussion, we may assume that "ROP" denotes
"radio-on-packet."
(and this assumption is crucial), a test in the terminal to repeater
(or station) direction is considered a sufficient test for both direc-
tions. We also see that a terminal has complete control over the times
at which it broadcasts ROP's. Thus the implementation is flexible in
that each terminal can broadcast ROP's, and thus initiate connectivity
updates, at its own required rate.

In the next section we discuss an implementation of the probing
method which not only overcomes the problems associated with the first
implementation of the broadcast method, but is also more flexible and in
many cases uses less overhead than the second implementation of the broad-
cast method.

4.2 Probing Method

Aside from the discussion in section 4.2.5, we once again assume
that the routes in the network described in Chapter 3 have been initial-
ized. We still direct our attention to the monitoring of terminal-re-
peater and terminal-station connectivity for the purpose of updating
the route between each terminal and the station. In this section we
examine the use of the probing method to do this monitoring.

4.2.1 An Implementation of the Probing Method

The implementation of the probing method which we now describe
follows from our general description of probing in section 2.2. With
the use of probe packets, each terminal assumes the responsibility for
both initiating and determining an update in its knowledge of the
connectivity between it and the repeaters and station. Specifically,
whenever a terminal deems it necessary, that terminal will transmit a general probe packet. When the station or any repeater with an assigned route to the station receives a general probe packet, it will respond by sending the probing terminal a response packet.* As before, the response packet informs the terminal that the responding node (either a repeater or the station) received the terminal's probe packet. Thus for each general probe packet transmitted, a terminal will obtain, from the received response packets, an update in its knowledge of the two directional connectivity between it and the repeaters and station. This connectivity information can then be transported to the station via the terminal's assigned route (if it is still usable) or via a route (i.e., a relaying node) selected from the terminal's updated knowledge of connectivity. As with the other implementations, the station can then make a decision as to whether a new route should be assigned. If a new route is selected, the station must then inform the terminal of this change.

At this point, let us compare this implementation of the probing method with the two implementations of the broadcast method discussed in section 4.1. To begin with, the two problems associated with the first implementation of the broadcast method have been eliminated with this implementation of the probing method. That is, this use of the probing method 1) tests the channel between a terminal and repeater (or station) in both directions, and 2) gives complete control to each

*As mentioned in section 2.2, some mechanism must be provided to avoid the possibility of having two or more response packets arrive at the probing terminal at the same time.
terminal of the rate at which it may update its knowledge of connectivity. As for the second implementation of the broadcast method, assuming repeater/station transmission range is greater than terminal transmission range, the end result of using either the second implementation of the broadcast method or the above implementation of the probing method is the same. That is, whether a terminal broadcasts an ROP or transmits a general probe packet, the end result is that the station receives an update in its knowledge of the connectivity between that terminal and the repeaters and station. Although the end result is the same, the overhead used by each of the two methods may be different. To illustrate this point, let us determine the overhead, measured by the expected number of transmitted overhead packets \(E[P]\), associated with each implementation in order for a terminal to update its route to the station. Suppose that the terminal we are considering is connected in both directions to \(L\) relaying nodes. For simplicity, we assume that the route between the terminal and the station via each of these relaying nodes consists of \(H\) hops (i.e., there are \(H-1\) repeaters along each route). Figure 4-1 illustrates the transmission of overhead packets for each of the two implementations. Ignoring the possibility of channel errors and the transmission of acknowledgements, the overhead associated with each implementation is given by

\[
E[P | \text{broadcast}] = 1 + L(H-1) + H \cdot Pr[\text{new route assigned}] \tag{4.1}
\]

and

\[
E[P | \text{probing}] = 1 + L + H + H \cdot Pr[\text{new route assigned}]
\]
I. As a new route assigned ROP's relayed to the station
ROP broadcast
1. ROP broadcast
2. ROP's relayed to the station
3. new route assigned

(a) Broadcast implementation (via ROP's)

(b) Probing implementation

Figure 4-1 Illustration of the overhead packets associated with updating the route between a terminal and the station
From this analysis we see that the probing implementation is favored in the case where the terminal is far (i.e., several hops) from the station and surrounded by many repeaters. Whereas the broadcast implementation is favored in the case where the terminal is close (i.e., very few hops) to the station and surrounded by few repeaters. We can, however, take advantage of the properties of the network we are now considering in order to better refine the use of the probing method and thus reduce its associated overhead.

4.2.2 A Revised Implementation of the Probing Method

Before assigning to the station the responsibility for updating packet routes, one must be certain that either this is the only way or at least a reasonably efficient way for routes to be updated. Suppose that the route between a terminal and the station is changed only after that terminal experiences a loss of connectivity between it and its relaying node. In this case, we can revise the implementation of the probing method given in the previous section, so that less responsibility in updating routes is given to the station and more responsibility is given to the individual terminal, with the end result being a reduction in the associated overhead. Specifically, we give each terminal the responsibility for determining the loss of connectivity between it and its relaying node. A terminal may do this by transmitting a probe packet, at certain points in time, which is addressed only to its relaying node, with only that relaying node responding to each such received probe packet. Only when a terminal concludes that a loss of connectivity between it and its relaying node has occurred (by not having
received a response to one or more probe packets), will it then transmit a general probe packet, and thus update its knowledge of the connectivity between it and the repeaters and station. Two of the options available to the terminal at this point are 1) send this connectivity information to the station where a new route will be selected and sent to the terminal, or 2) use this connectivity information, and possibly other information sent in the received response packets, to select its own route to the station, and then inform the station of this new route. Suppose that the procedure used in selecting a new route for a terminal is of the nature, "choose the route with the minimum ______", where the blank, for example, is filled in by "number of hops to the station," or "average delay to the station."

If the station and each repeater could maintain the required measure of its ability to relay packets to the station, then this information could be included in the response to each received general probe packet. Thus a terminal could, just as well as the station, choose its own route to the station and in so doing, not only reduce the overhead associated with obtaining a new route to the station, but also reduce the processing that is usually performed at the station.

Let us suppose that the procedure used for updating routes is such that one may use the above revised implementation of the probing method where each terminal is able to select its own route to the station. We now determine the overhead associated with this implementation of the probing method, and compare it with that associated with the ROP implementation of the broadcast method. Once again we determine the expected number of transmitted overhead packets
associated with a particular terminal when it updates its route to the station. We assume the terminal is connected in both directions to \( L \) relaying nodes, where the route via each of these relaying nodes consists of \( H \) hops. Figure 4-2 illustrates the transmission of overhead packets associated with the revised implementation of the probing method. Again ignoring the possibility of channel errors and the transmission of acknowledgements, the overhead associated with the revised probing implementation is given by

\[
E[P | \text{revised probing}] = 2 \cdot P_r \{\text{new route not selected}\} \\
+ (1+1+L+H) \cdot P_r \{\text{new route selected}\} \\
= 2 + (L+H) \cdot P_r \{\text{new route selected}\} \quad (4.2)
\]

Comparing equations (4.1) and (4.2), we see that for the case \( P_r \{\text{new route selected}\} \approx 0 \), only when the terminal is within range of the station with no surrounding repeaters (i.e., \( L=1, H=0 \)) will the broadcast implementation use less overhead than the probing implementation. The case \( P_r \{\text{new route selected}\} \approx 1 \) is the worst case situation for using the probing implementation, but even in this unlikely situation, the probing implementation is favored when \( 2L < 1+LH \) (e.g., when \( L=2, H=2 \)). Furthermore, one must keep in mind that when using the ROP implementation of the broadcast method, not only does it generally involve a greater number of transmitted overhead packets, but even worse, most of these packets are being sent to the station which, without the use of directional antennas, is already a bottleneck to traffic flow. Also, by giving the station less responsibility in monitoring connectivity and updating routes, one can reduce the
Figure 4-2 Illustration of the overhead packets associated with the revised probing implementation
processing capability needed at the station to perform these functions. Of course this implies the need for added processing capability at the terminals. However, a trend in technology has been toward decreasing the cost and increasing the capability of small processors. In an effort to improve network reliability and efficiency, a trend in the design of data communication networks has been to distribute the control of the network among its various nodes. The above implementation of the probing method is a step in this direction.

4.2.3 Probing via Message Packets

What makes the revised probing implementation even more attractive is its ability to easily incorporate the connectivity information obtained when regular message packets are transmitted, in order to further reduce its associated overhead. With this implementation of the probing method, each individual terminal is given the function of determining the loss of connectivity between it and its relaying node, and does this by probing its relaying node at certain points in time (e.g., periodically). This probing, however, is essentially performed each time the terminal sends a regular message packet to the station. Each packet a terminal wishes to send to the station is first sent to the terminal's relaying node. For each such packet received, the relaying node sends back to the terminal an acknowledgement which informs the terminal that the relaying node successfully received the packet. Thus, for the purpose of monitoring the connectivity between a terminal and its relaying node, a regular message packet acts as the probe packet and the acknowledgement for that packet acts as the response packet. In this way, a terminal
need only send a probe packet to its relaying node when it has not sent a regular message packet for some length of time. Thus an increase in the rate at which a terminal sends packets to the station will generally result in a desirable decrease in the overhead needed for that terminal to monitor the connectivity between it and its relaying node.

4.2.4 Choosing the Probing Times - Periodic Probing and Probing with Position Information

Given that the revised probing implementation is to be used, one must determine the times at which a terminal should probe its relaying node. A long period of an undetected loss of connectivity between a terminal and its relaying node is undesirable in that it can result in a significant increase in the delay experienced by packets traveling from the station to a terminal. That is, as mentioned in the previous section, a packet being sent from a terminal to the station acts as a probe packet. Thus the added transportation delay for that packet, due to a loss of connectivity between the terminal and its relaying node, is only the delay associated with the terminal selecting a new relaying node. However, when a terminal suffers a loss of connectivity between it and its relay node, that terminal is effectively cut off from receiving packets sent by the station. This loss of connectivity is only discovered when the terminal attempts sending a packet (e.g., a probe or message packet) to its relaying node. Thus, the added delay experienced by an outbound packet, due to a loss of connectivity between the destination terminal and its relaying node, can be significant. Therefore, it is desirable for a terminal to learn of a loss of connectivity between it and its relaying node soon after the loss
actually occurs.

One approach is to have each terminal periodically probe its relaying node. The probing rate for a particular terminal will depend on the mobility characteristics of that terminal, the network topology, and the requirements (e.g., maximum duration of an undetected loss of connectivity) the terminal is to satisfy. Of course a terminal will not need to transmit probe packets in a strictly periodic manner. As discussed in section 4.2.3, each time a terminal sends a message packet to the station, it is effectively probing its relaying node. Thus, in this situation, periodic probing implies that a terminal will probe its relaying node at time \( t \) only if the last transmission of either a probe or message packet was at time \( t - T_p \), where \( T_p \) is the probing period. By decreasing the probing period, a terminal can decrease the time of an undetected loss of connectivity. However, this decrease in probing period will generally result in an undesirable increase in the network overhead.

A terminal could improve upon periodic probing if it could keep track of its position relative to that of its relaying node. With periodic probing, the probing period is the same regardless of where the terminal is located relative to its relaying node. Ideally, the probing period should be smaller at those locations where the terminal is more likely to suffer a loss of connectivity between it and its relaying node. Such a location may, for example, be in the outermost region of the relaying node's transmission range.

If a terminal could continually monitor its position relative to that of its relaying node (see [12] for a discussion of possible
methods), then it could use this information to vary the rate at which it probes its relaying node. In fact, if position and connectivity are highly correlated, then with the station and repeaters fixed in position, a terminal would not even have to probe its relaying node. The terminal could just select a new relaying node when, by the use of position information, it anticipates a possible loss of connectivity between it and its current relaying node. However, one must keep in mind that to enable a terminal to continually monitor its position will generally involve the use of hardware and radio spectrum other than that which is provided in the basic packet radio network.

An alternative is to have each terminal obtain position information only at each effective probing of its relaying node, and doing so using only the radio channel already provided. For example, the distance between two radio transceivers (e.g., a terminal and its relaying node) can be estimated by measuring the radio wave propagation delay from one radio transceiver to the other. Thus a terminal can estimate the distance to its relay node by taking the difference in the time at which it transmits a probe(message) packet and the time at which it receives the associated response(acknowledgement) packet, and then subtracting off the processing time (which may be included in the response(acknowledgement) packet) at the relaying node. With the additional use of directional antennas at the relaying node, angle can be estimated at the relaying node and then sent back to the terminal in the response(acknowledgement) packet (see [13] for related discussion). With this information, the terminal can estimate its position relative to its relaying node, and then use this information to decide when it would
be best to send the next probe packet. Furthermore, with this position information, a terminal can anticipate a possible loss of connectivity between it and its relaying node, and thus initiate rerouting before the loss occurs.

It would be reasonable to say that probing with position information is more efficient (i.e., less overhead for the same duration of an undetected loss of connectivity) than periodic probing. However, it is also more complex in that it requires the capability of determining position. In Chapter 5 we make use of analytical comparisons of the two probing schemes to determine under what conditions the increased efficiency of probing with position information might outweigh its complexity.

4.2.5 Using the Probing Implementation in a more General Network

In our discussion, in this chapter, of using connectivity monitoring for updating packet routes, we assumed that the packet routes in the network had been previously initialized and that the station and all repeaters are stationary. These assumptions, however, were only made to simplify both the discussion of the monitoring implementations developed in this chapter and the analysis performed in the next chapter. In this section we show that the implementation of the probing method discussed in section 4.2.2 can be generalized so as to incorporate both the initialization of packet routes and the mobility of the repeaters and station.

If we allow the repeaters and station to be mobile, then changes in connectivity between a repeater and the network's other repeaters
and station may necessitate that the route between that repeater and the station be changed. Just as would a terminal, this repeater can use probe and message packets to monitor the connectivity between it and its relay node (either another repeater or the station). When a repeater concludes a loss of connectivity between it and its relaying node, that repeater could then transmit a general probe packet and, from the received response packets, update its knowledge of the connectivity between it and the network's other repeaters and station. Just as in the case of a terminal, the repeater could send this connectivity information to the station where a new route will be selected, or if the routing procedure allows it, the repeater could select its own route to the station. Thus we see that for the purpose of updating routes, each repeater could function just as though it were a terminal. Of course when a repeater does conclude a loss of connectivity between it and its relaying node, until it obtains a new route to the station, that repeater will not respond to any received probe packet nor acknowledge any received message packet. In this way, a terminal or repeater to which this repeater is acting as a relaying node will not receive a response nor an acknowledgement after sending a probe or message packet, respectively. Thus that terminal or repeater will assume a loss of connectivity between it and this repeater, and will then initiate obtaining a new route to the station. The idea here is that for the purpose of routing, each device, terminal or repeater, only concerns itself with having a usable route to the station. We see that the process of updating routes, even with the station and each repeater mobile, is relatively simple, because, with the use of tree-structured routing, a route to the station
is specified by the identity of a relaying node, and to change a route, a device simply changes its relaying node.

The initialization of packet routes follows from the above discussion of updating routes with the repeaters and station mobile. When the network is first started (or restarted), no device has a route to the station. Thus each device begins transmitting general probe packets. Since only those relaying nodes with routes to the station will respond to received general probe packets, initially only the station will be responding to received general probe packets. Thus each device with a connection in both directions between it and the station will eventually receive a response to a transmitted general probe packet, and will subsequently obtain a route to the station which involves sending its packets directly to the station. After obtaining this route, each of these devices which is a repeater will then begin responding to general probe packets transmitted by other devices to which it is connected in both directions. Each of these other devices will eventually receive a response to one of its general probe packets, and will subsequently obtain a two-hop route to the station. This process then continues on for devices which are three hops and further from the station, until the packet routes for the entire network have been initialized.
Analysis of the Probing Implementation*

In Chapter 4 we found that the implementation of the probing method developed in section 4.2.2 is more flexible and in general uses less overhead than the implementations of the broadcast method developed in section 4.1. However, we have left some questions concerning this use of the probing method unanswered. For example, we have not yet determined how the characteristics of terminal mobility affect the performance of this implementation, nor have we determined how much better is probing with position information than periodic probing. In this chapter we examine these and other aspects associated with the use of the probing implementation.

In the analysis in this chapter, we generally consider just one terminal as it moves within the region covered by a packet radio network. For reasons of clarity, and without loss of generality, we assume that this terminal always sends (receives) packets to (from) the station via at least one repeater. In this way, the terminal's relaying node will in fact always be a repeater.

5.1 Performance Criterion

In this section we establish the criterion that will be used to evaluate the performance of the probing implementation. To motivate the choice of criterion, let us consider a terminal as it moves within the region covered by a packet radio network. For

*In this chapter, the words "probing implementation" implicitly refer to the revised probing implementation (discussed in section 4.2.2) where each terminal selects its own relaying node.
simplicity, we ignore the possibility of channel errors. Figure 5-1 illustrates a path taken by the terminal. Each location marked by $X$ denotes a point along the path where the terminal probed its relaying repeater and received a response. Each location marked by $H$ denotes a point where the terminal probed its relaying repeater, but did not receive a response due to a loss of connectivity between it and that repeater. So at each of these locations, the terminal also transmitted a general probe packet and, from the received response packets, selected a new relaying repeater. The terminal's $i^{th}$ ($i=1,2,...$) selected relaying repeater is denoted as $R_i$. The transmission range of the terminal and all repeaters are assumed equal. Thus, only after the terminal exits the transmission range of its current relaying repeater, does it then experience a loss of connectivity between it and that repeater. The portions of the path marked by a solid line indicate that the terminal is connected to its current relaying repeater, and the portions marked by a broken line indicate that the terminal is not connected to its current relaying repeater. As mentioned in section 4.2.4, it is desirable that a terminal learn of a loss of connectivity between it and its relaying repeater soon after the loss occurs. Thus it is desirable to reduce that fraction of the path marked by broken lines in Figure 5-1. However, we note that to do so would require that the terminal probe its relaying repeater more often. This illustrates that there is a fundamental trade-off between the fraction of time a

*At least the terminal would have to probe its relaying repeater more often as it approaches the outermost region of its relaying repeater's transmission range.*
Figure 5-1 Illustration of the probing implementation

- The path of the mobile terminal is shown.
- The "boundary" of R2's transmission range is indicated.
- Probing of R2 with a response received.
- Probing of R2 with no response received.

R1, R2, R3, R4
terminal has a usable route to the station and the associated probing rate (and thus the network overhead). It follows that a reasonable method for evaluating the performance of the probing implementation is to examine how well the probing implementation is able to trade off these quantities. Thus, we use as the performance criterion, the fraction of time a terminal has a usable route to the station for a given associated average transmission rate of overhead packets. The first quantity will be referred to as the fraction of time connected (FTC)*, and the second quantity will be referred to as the average transmission rate (ATR). Being more precise about these quantities, we define

\[ T_i = \text{the duration of time between the terminal's (i-1)th and i}^{\text{th}} \text{ probing} \]

\[ TC_i = \text{the total time that the terminal is connected to its current relaying repeater between probing i-1 and i} \]

\[ P_i = \text{the number of transmitted overhead packets associated with the routing update at the time of the } i^{\text{th}} \text{ probing} \]

With these definitions, we may now express FTC and ATR as

\[
FTC = \lim_{M \to \infty} \frac{\sum_{i=1}^{M} TC_i}{\sum_{i=1}^{M} T_i}
\]

(5.1)

* The word "connected" refers to the terminal being connected to its current relaying repeater and thus, in a sense, to the network itself.
and

\[
\text{ATR} = \lim_{M \to \infty} \frac{\sum_{i=1}^{M} P_i}{M} = \frac{\sum_{i=1}^{M} T_i}{M}
\]  

(5.2)

if the limits exist.

5.2 Network Model

In order to evaluate the performance of the probing implementation, we must formulate both a model of terminal mobility and a model of network topology. Our objective in the analysis is not so much to obtain specific quantitative results, but rather to obtain qualitative results which indicate trends associated with using the probing implementation. Thus our mobility and topology models will have to be realistic enough to correctly indicate trends, and yet simple enough to allow the use of analytical techniques.

In the analysis, we consider just one terminal. We model this terminal's mobility as a constrained random walk characterized by a homogeneous, discrete-time, discrete-state Markov process. Specifically, the terminal is constrained to move along the path shown in Figure 5-2. The path is considered to extend in each direction for an infinite distance. At intervals of \( s \) units of distance along the path are locations identified by consecutive integers below the path. These locations correspond to the states of the Markov process. The terminal's movement between these states is defined as follows. If the terminal enters state \( i \) at time \( t \), at time \( t+T_s \) it will move to state \( i+1 \) with
boundary of $R_1$'s transmission range

states identified relative to each relaying repeater

states identified relative to the path

path upon which the terminal is constrained to move

Figure 5-2 Illustration of the mobility and topology models
probability \( p \), move to state \( i-1 \) with probability \( q \), or remain in state \( i \) with probability \( 1-p-q \); where \( T_s \) is the state transition time.

It is desirable to characterize the terminal's motion in terms of velocity and randomness. To do so, we first define \( d(t) \) as the total distance the terminal has moved up to time \( t \), where \( d(0) = 0 \). It follows that the expected value of \( d(t) \) is given by

\[
E[d(t)] = (p+q)\frac{s}{T_s} t \quad t=nT_s, \; n=0,1,2,... \tag{5.3}
\]

Thus we may interpret \( (p+q)\frac{s}{T_s} \) as the terminal's average velocity. As for the randomness of motion, we first define the random variable \( z \) as the change in the terminal's position after any particular transition. As such, we note that

\[
z = \begin{cases} 
  s & \text{with probability } p \\
  0 & \text{"} \quad l-p-q \\
  -s & \text{"} \quad q
\end{cases}
\]

It follows that the variance of \( z \) is given by

\[
\text{var}[z] = s^2[(p+q) - (p-q)^2] \tag{5.4}
\]

We interpret \( \text{var}[z] \) as a measure of the terminal's randomness of motion. For given values of \( s \) and \( T_s \), we note from (5.3) that the terminal's average velocity may be fixed by fixing the value of \( p+q \). Furthermore, for a fixed average velocity, we note from (5.4) that the terminal's randomness of motion may be varied by varying the value of \( p-q \). In section 5.5, we vary the values of \( p \) and \( q \) in this manner in order to examine how changes in the characteristics of terminal mobility affect
the performance of the probing implementation.

As for the network topology, shown in Figure 5-2 are repeaters which are equally spaced along the path. Each of these repeaters may act as the terminal's relaying repeater during one or more segments of time. The transmission range of each of these repeaters is shown to encompass N consecutive states along the path. The states relative to each repeater are identified by the integers 1 through N. For simplicity, we assume that the terminal and the repeaters along the path each have the same transmission range.* Also, we assume that the repeaters are spaced so that each state along the path is within the transmission range of at least one repeater. Not shown in Figure 5-2, but nevertheless present, are other network repeaters and the station.

In the derivation of FTC and ATR in section 5.4, it is necessary to distinguish between a state relative to the path and a state relative to the terminal's current relaying repeater. To do so, we use the notation $s(n)=j$ to denote the event that immediately after the $n$th transition (i.e., at time $nT_g$), the terminal is in state $j$ ($j$ an integer) relative to the path. Also, where appropriate, we use the notation $s_r(n)=j$ to denote the event that immediately after the $n$th transition, the terminal is in state $j$ ($j=1,2,\ldots,N$) relative to its current relaying repeater.

---

*In the actual analysis, it is only necessary that the repeaters along the path have the same transmission range.
5.3 Probing and the Selection of a New Relaying Repeater

In the analysis of the probing implementation, we assume that the terminal's knowledge of the connectivity between it and its relaying repeater is only updated each time the terminal sends its relaying repeater a probe packet. That is, we do not directly incorporate in the analysis the terminal's effective probing of its relaying repeater each time it sends a message packet to the station. Also, aside from the discussion in section 5.6, we ignore the possibility of packet errors. In particular, if the terminal and a repeater are each within the transmission range of the other, then a probe packet sent from the terminal to the repeater and the associated response packet sent from the repeater back to the terminal will each be received correctly. These assumptions are made to simplify the mathematical analysis. We shall, however, comment on the expected changes in the obtained results when each of these assumptions is removed.

With the model of terminal mobility described in section 5.2, we note that the connectivity between the terminal and its relaying repeater does not change from the end of one transition time until the beginning of the next. Thus the best times for the terminal to probe its relaying repeater are immediately after transitions. However, the question is, after which particular transitions should the terminal probe its relaying repeater? As mentioned in section 4.2.4, we examine two schemes for choosing these probing times. The first scheme is periodic probing in which the terminal probes its relaying repeater immediately after every \( n^{th} \) \((n=1,2,\ldots)\) transition and consequently, \( nT_s \) is the probing period.
The second scheme is probing with position information. With this scheme, at each probing while within the transmission range of its relaying repeater, the terminal learns of its position relative to that repeater. If at a probing the terminal is out of the transmission range of its relaying repeater, then through the process of obtaining a new relaying repeater, the terminal learns of its position relative to that new relaying repeater. In either case, the terminal uses this position information to determine when (i.e., after how many more transitions) to send the next probe packet to its relaying node.

Before proceeding with the derivation of FTC and ATR, there is one other aspect associated with the probing implementation that must be mentioned. In the discussion in section 4.2.2, we stated that when a terminal selects a new relaying repeater, it bases its choice on the relative relaying abilities of the available repeaters. For the model described in section 5.2, we assume that a repeater's position along the path is uncorrelated with its ability to relay packets to the station. Thus, choosing a new relaying repeater on the basis of relaying ability will, as far as the analysis is concerned, correspond to choosing a repeater at random from the set of available repeaters. However, it is also desirable to examine the performance of the probing implementation when the choice of a new relaying repeater is based on other criteria. In section 5.5, besides examining the use of a random choice of a new relaying repeater, we also examine the use of basing the choice of a new relaying repeater on the terminal's position relative to each of the available relaying repeaters. The motivation for doing this comes from equation (4.2). We note from (4.2) that the largest amount of overhead is used in the updating process when a terminal must select a new
relaying repeater. Thus it is desirable to maximize the time between needed changes in the terminal's relaying repeater. One method for doing this is to have the terminal base its choice of a new relaying repeater on its position relative to each of the available relaying repeaters. In section 5.5 we discuss and evaluate two uses of position information for choosing a new relaying repeater.

5.4 Derivation of FTC and ATR

To incorporate in the performance analysis both the use of periodic probing and probing with position information, we associate with the $i$th ($i=1,2,...,N$) state relative to each repeater the waiting time $\tau_i$ ($\tau_i$ a positive integer). Furthermore, we define the associated waiting time vector $\mathbf{T} = (\tau_1, \tau_2, ..., \tau_N)$. The significance of $\mathbf{T}$ is as follows. If at a probing the terminal is in state $i$ relative to its current relaying repeater, then the terminal will wait $\tau_i$ transition times before again probing its relaying repeater. If at a probing the terminal is out of the transmission range of its current relaying repeater, but is in state $i$ relative to its newly selected relaying repeater, then the terminal will wait $\tau_i$ transition times before probing its new relaying repeater.*

For periodic probing, it is clear that $\tau_1 = \tau_2 = ... = \tau_N = k$, where $k$ takes on values 1, 2, 3, ... for probing periods $T_s, 2T_s, 3T_s, ...$, respectively. For probing with position information, $\mathbf{T}$ should be selected so as to optimize the performance of the probing implementation. Later (in section 5.5) we shall discuss both the selection of $\mathbf{T}$ and what is meant by the optimal performance of the probing implementation.

* We assume that the time delay from when the terminal detects a loss of connectivity between it and its relaying repeater until it selects a new relaying repeater is very small compared to the transition time $T_s$. 
For now, let us assume that a value for $T$ and values for $p$ and $q$ have been selected along with a strategy for choosing a new relaying repeater. On the route to deriving expressions for FTC and ATR, we define

$$\psi_{ij} = \Pr[s_{r}(T_i+m) = j \mid s_{r}(m) = i] \quad i,j=1,2,\ldots,N; \; m=0,1,2,\ldots$$

where state $j$ is taken to be relative to the terminal's current or newly selected relaying repeater if, immediately after transition $T_i+m$, the terminal is, respectively, within range or out of range of its current relaying repeater. In words, $\psi_{ij}$ is the probability that the next probing will take place while the terminal is in state $j$ relative to its relaying repeater, given that the last probing took place while the terminal was in state $i$ relative to its relaying repeater at that time.

We may express $\psi_{ij}$ in terms of the n-step transition probability $\phi_{ij}(n) \triangleq \Pr[s(n) = j \mid s(0) = i]^*$ of the Markov process which characterizes the terminal's motion along the path. In doing so, we obtain

$$\psi_{ij} = \phi_{ij}(T_i) + \sum_{k>N}^{k<i} \phi_{ik}(T_i) \cdot \Pr \left\{ \begin{array}{l} \text{state } k \text{ relative to the path is the same location as state } j \text{ relative to the newly selected relaying repeater} \\ \text{state } k \text{ relative to the path} \end{array} \right\}$$ (5.5)

The first term on the right-hand side of (5.5) is the probability that, immediately after transition $T_i+m$, the terminal is in state $j$ relative to its current relaying repeater, given $s_{r}(m) = i$. The second term is the probability that, immediately after transition $T_i+m$, the terminal

*Equation (A.1) defines a recursive method for calculating $\phi_{ij}(n)$ for any finite state Markov chain. See Appendix C for details on how one may compute $\phi_{ij}(T_i)$ for the Markov chain under consideration.
is outside of the range of its current relaying repeater and is in state \( j \) relative to its newly selected relaying repeater, given \( s_j(m) = i \). The second probability term within the summation in (5.3) is of course a function of the strategy used for selecting a new relaying repeater. We note that the associated \( N \times N \) matrix \( \Psi \) with elements \( \Psi_{ij} \) is stochastic, and thus may be viewed as representing the transition probabilities of a discrete-time Markov chain. Furthermore, we note that \( p+q < 1 \) is a sufficient condition for this Markov process to be ergodic. Assuming this condition to hold, from the discussion in Appendix A, we can determine the steady-state probabilities

\[
\tilde{\pi}_i = \lim_{m \to \infty} \tilde{\pi}_i[m] \quad i=1,2,\ldots,N
\]

where

\[
\tilde{\pi}_i[m] = \Pr \{ \text{the } m\text{th probing is made while the terminal is in state } i \text{ relative to its current or newly selected relaying repeater} \}
\]

by solving the set of equations given by

\[
\tilde{\pi}_i = \sum_{k=1}^{N} \tilde{\pi}_i \psi_{ik} \quad i=1,2,\ldots,N
\]

\[
\sum_{i=1}^{N} \tilde{\pi}_i = 1
\]  

(5.6)

*With \( p+q < 1 \), the process is an irreducible, aperiodic, finite Markov chain and is thus ergodic (see [14] section 15.7). If \( p+q = 1 \), then there are certain cases (e.g., \( \tau_i = 1 \) for all \( i \) and relaying repeaters spaced such that there is no overlap of their transmission ranges) in which the Markov chain defined by \( \Psi \) is periodic and thus not ergodic.
We now express (5.1) and (5.2) in the forms given by

\[
FTC = \lim_{M \to \infty} \frac{\frac{1}{M} \sum_{i=1}^{M} TC_i}{\frac{1}{M} \sum_{i=1}^{M} T_i} = \lim_{M \to \infty} \frac{\frac{1}{M} \sum_{i=1}^{M} TC_i}{\frac{1}{M} \sum_{i=1}^{M} T_i} \tag{5.7}
\]

and

\[
ATR = \lim_{M \to \infty} \frac{\frac{1}{M} \sum_{i=1}^{M} P_i}{\frac{1}{M} \sum_{i=1}^{M} T_i} = \lim_{M \to \infty} \frac{\frac{1}{M} \sum_{i=1}^{M} P_i}{\frac{1}{M} \sum_{i=1}^{M} T_i} \tag{5.8}
\]

Since the Markov chain defined by \( \Psi \) is ergodic, from (5.7) and (5.8) we obtain, respectively,

\[
FTC = \frac{E[TC]}{E[T]} \tag{5.9}
\]

and

\[
ATR = \frac{E[P]}{E[T]} \tag{5.10}
\]

where, under steady-state conditions, \( E[T] \) is the expected time between two consecutive probings, \( E[TC] \) is the expected time between two consecutive probings that the terminal is connected to its current relaying repeater, and \( E[P] \) is the expected number of overhead packet transmissions associated with a probing and the resulting possible change in the route to the station. For notational convenience, we normalize \( T_s = 1 \). From the definition of expectation, we may write
Defining the conditional expectation

\[ E[TC^i] = E[TC \mid \text{the first of the two consecutive probings is made while in state i relative to the relaying repeater}] \]

we may write

\[ E[TC^i] = \sum_{k=1}^{N} E[\text{time the terminal occupies state } k, \text{ starting from when a probing is made while in state i until just before the next probing}] \]

\[ = \sum_{k=1}^{N} \sum_{n=0}^{T_{i-1}} \phi_{ik}(n) \]  

Thus from (5.12), we obtain

\[ E[TC] = \sum_{i=1}^{N} \sum_{k=1}^{N} \bar{\pi}_i \sum_{n=0}^{T_{i-1}} \phi_{ik}(n) \]

We define \( \lambda \) as the expected number of additional overhead packets transmitted when, at a probing, the terminal must obtain a new route to the station. It follows that

\[ E[P] = 2 + \lambda \cdot Pr[\text{new relaying repeater selected}] \]

\[ = 2 + \lambda \sum_{i=1}^{N} \bar{\pi}_i \left[ 1 - \sum_{k=1}^{N} \phi_{ik}(T_i) \right] \]

Thus substituting (5.11) and (5.13) into (5.9) yields
Likewise, substituting (5.11) and (5.14) into (5.10) yields

\[
ATR = \frac{2 + \sum_{i=1}^{N} \frac{\pi_i}{n} \left[ 1 - \sum_{k=1}^{n} \phi_{ik}(\tau_i) \right]}{\sum_{i=1}^{N} \frac{\pi_i}{n} \tau_i} \quad (5.16)
\]

Equations (5.15) and (5.16) are the expressions we had set out to derive in this section.

5.5 Performance Results

In this section we evaluate and compare the performances associated with periodic probing and probing with position information, along with those associated with the random and position based selections of a new relaying repeater. This is done as the values of \( p \) and \( q \) are varied in order to model changes in the terminal's average velocity and randomness of motion. Initially we select both a value for the number of states within the transmission range of each repeater along the path, and a value for the spacing between the repeaters along the path. Later we examine the affects on the obtained results when each of these two parameters is changed.

As illustrated in Figure 5-3, \( N \) is chosen to be 11 and a repeater is located every 5 states along the path. In the analysis,
boundary of $R_1$'s transmission range

path upon which the terminal is constrained to move

states identified relative to the path

Figure 5-3 Network model for $N = 11$ and repeater spacing = 5s
we examine average velocities corresponding to \( p+q = 0.8 \) and \( 0.4 \). To give some physical significance to these values, if we choose \( s = 1 \) km, then each repeater's transmission range will be approximately 11 km and a repeater will be spaced every 5 km along the path. Moreover, if we choose \( T_s = 30 \) seconds, then for \( p+q = 0.8 \) and \( 0.4 \), the terminal's average velocity will be, respectively, 96 and 48 km/hour.

We examine three methods for selecting a new relaying repeater. With the first method, the terminal selects at random its new relaying repeater from the set of available repeaters. This corresponds to the situation where position information is not available and/or where the choice of a new relaying repeater is based only on the relative relaying abilities of the available repeaters. With the second method, the terminal selects the nearest repeater as its relaying repeater. This corresponds to the situation where the terminal is aware of its distance to each of the available repeaters and uses only this information, in what seems to be a reasonable way, to select a new relaying repeater. Finally, with the third method, the terminal selects the repeater for which the expected time to first exit that repeater's transmission range \( (E[TE]) \) is a maximum. With this last method, the terminal bases its choice both on its position relative to each of the available repeaters and on its mobility characteristics (i.e., the type of motion and the values of \( p \) and \( q \)). Later we comment on whether this use of position information is optimal.

* See Appendix C for an expression that may be used to compute the conditional expectation \( E[TE_i] = E[TE|s_r(0)=i] \) for \( i=1,2,\ldots,N \).
5.5.1 Periodic Probing

We first examine periodic probing. Let us, for the present time, assume that the route between the station and each repeater along the path consists of two hops. It follows that the expected number of additional overhead packets transmitted when, at a probing, the terminal must select a new relaying repeater is given by

\[ E[\text{number of hops along the route between the terminal and the station}] + E[\text{number of repeaters within range of the terminal at the time of selection}] = 3 + 2 + \Pr(\text{terminal is within range of 3 repeaters at the (5.17) time of selection}) \]

The probability term in (5.17) may be computed via a straightforward summation on \( \bar{\tau}_{ij} (\tau_i) \). Using (5.17) along with the results of section 5.4, values for FTC and ATR were computed for \( p=0.8, q=0; p=0.6, q=0.2; \) and \( p=0.4, q=0.4 \) and are plotted in Figures 5-4, 5-5, and 5-6, respectively. For each of the three methods for selecting a new relaying repeater (i.e., random, nearest, and max \( E[TE] \)), the probing period is increased and the corresponding points (ATR, FTC) are plotted. The consecutive points for each selection method are connected by straight line segments in order to indicate values of FTC and ATR that can be achieved if one were to time-share between the points which define each line segment. Time-sharing in this case means that for fraction \( \theta \) \( (0 \leq \theta \leq 1) \) of the time the value of \( \bar{T} \) associated with one point is used, and for fraction \( 1-\theta \) the value of \( \bar{T} \) associated with the other point is used. The proof of this time-sharing result is given in Appendix B.
Figure 5-4 FTC vs. ATR for periodic probing with $p=0.8$, $q=0$
Figure 5-5  FTC vs. ATR for periodic probing with p=0.6, q=0.2
Figure 5-6 FTC vs. ATR for periodic probing with p=0.4, q=0.4
Note that the values of $p$ and $q$ corresponding to Figures 5-4 through 5-6 represent the same average velocity (i.e., $p+q=0.8$), but where the randomness of motion varies from nearly deterministic to highly random. From Figures 5-4 through 5-6, observe that for a given value of FTC, as the terminal’s motion becomes more random (i.e., as $p-q$ decreases), the associated value of ATR decreases. This is reasonable because a more random motion generally implies a longer period of time before the terminal exits its relaying repeater’s transmission range, and thus a smaller value of ATR for a given value of FTC. Also note that for $p=0.4$, $q=0.4$, the performance results for the nearest and max $E[TE]$ selection methods are identical. This, as can be seen from equation (C.1), is always the case for $p=q$. Observe, however, that for $p=0.6$, $q=0.2$, and for FTC $>0.716$ when $p=0.8$, $q=0$, selecting at random a new relaying repeater has a better performance than selecting the nearest repeater. In fact, we shall see this sort of behavior throughout the performance results in this section. That is, as the terminal’s motion becomes less random, choosing the nearest repeater becomes the least desirable selection method. Also note from Figures 5.4 through 5.6 that for a given value of FTC, although the absolute difference in the values of ATR corresponding to the random and max $E[TE]$ selection methods decreases for increasing randomness of motion, the relative difference remains approximately the same. Finally, for $p=0.8$, $q=0$, observe the rather surprising behavior of the max $E[TE]$ selection method for probing periods $10T_s$ through $15T_s$. In particular, note that the terminal can obtain a higher value of FTC at a lower value of ATR by probing with period $13T_s$ as opposed to $10T_s$. This behavior, which is not
exhibited elsewhere in the performance results, is thought to be due to
the terminal having a higher probability of being in a "good" state (i.e.,
a state for which max E[TE] is large) at the time at which it selects a
new relaying repeater, when the probing period is 13Ts rather than 10Ts.

Still considering periodic probing, we now examine the affects on
the performance when the terminal's average velocity is changed. Specifi-
cally, Figures 5-7 through 5-9 indicate the changes in performance for,
respectively, Figures 5-4 through 5-6 when the terminal's average velocity
is reduced by one half (i.e., p+q=0.4), but where the conditional
probability (conditioned on moving to another state) of moving left or
right along the path remains the same. As expected, the lower average
velocity results in an increase in the time between needed changes in the
terminal's relaying repeater, and thus a decrease in ATR for a given value
of FTC. The percent decrease in ATR is not, however, the same for all
values of FTC. For example, in the case being examined, with a decrease
in the average velocity by one half, for FTC = 1.0, 0.9, and 0.7, the
percent decrease in ATR is, respectively, on the order of 10, 45, and 50
percent. However, we observe that for a given value of FTC < 1.0, the
probing period (τ·Ts) associated with the low velocity case is approxi-
mately twice that associated with the high velocity case. This is
reasonable since one would expect that in order to remain at the same
value of FTC, a particular change in velocity would require a proportional
change in the probing rate. Finally, note that both the high and low
velocity cases represent the same basic qualitative differences between
the performances associated with the three methods for selecting a new
relaying repeater.
Figure 5-7 FTC vs. ATR for periodic probing with $p=0.4$, $q=0$
Figure 5-9 FTC vs. ATR for periodic probing with $p=0.3$, $q=0.1$
Figure 5-9 FTC vs. ATR for periodic probing with p=0.2, q=0.2
5.5.2 Probing with Position Information

Up to this point we have not stated how the waiting time vector \( T \) should be chosen when using probing with position information. Having gained insight from the performance results of periodic probing, we are now in a better position to define the optimal values of \( T \). Note that for given values of \( p \) and \( q \) and a method for selecting a new relaying repeater, each possible choice of \( T \) (where \( T_i \) is a positive integer, \( i=1,2,...,N \)) is mapped via equations (5.15) and (5.16) on to a point on the FTC vs. ATR graph. We define the set of optimal points on FTC vs. ATR (corresponding to the optimal values of \( T \)), as that subset of possible points which lie on the left boundary of the smallest convex region containing all points. That is, if the consecutive points, in the optimal set, corresponding to decreasing values of FTC are connected by straight line segments, the resulting curve will be convex with all nonoptimal points lying to the right of it. For the network model being considered, the optimal selection of \( T \) corresponding to FTC = 1.0 is for \( p \) and \( q > 0 \),

\[
T_i = \begin{cases} 
  i & \text{for } i=1,2,\ldots,\left\lfloor \frac{N}{2} \right\rfloor \\
  N+1-i & \text{for } i=\left\lfloor \frac{N}{2} \right\rfloor+1,\ldots,N
\end{cases}
\]

and for \( p \) or \( q = 0 \),

\[
T_i = \begin{cases} 
  i & \text{for } i=1,2,\ldots,N \text{ and } p=0 \\
  N+1-i & \text{for } i=1,2,\ldots,N \text{ and } q=0
\end{cases}
\]

Because these are the largest possible values of the \( T_i \)'s for which FTC = 1.0, it follows that the selection is optimal. Unfortunately,
equations (5.15) and (5.16) are of sufficient complexity that the task of determining the optimal values of T for FTC < 1.0 seems to be one of extreme difficulty. For this reason, we turn to a heuristic approach for selecting the values of T to be used in evaluating the performance of probing with position information. Before describing this heuristic approach, it is appropriate to mention that the max E[TE] method for selecting a new relaying repeater is not in general an optimal use of position information. Determining the optimal selection of a new relay repeater seems to be on the same order of difficulty as determining the optimal values of T for FTC < 1.0. Thus the max E[TE] method was chosen for its good heuristic qualities.

The heuristic approach we use for selecting values for T is as follows. For various values of the variable γ, where 0 ≤ γ < 1, we determine for each i (i=1,2,...,N) the minimum value of Ti such that

$$\Pr\{s(T_i) > N \text{ or } s(T_i) < 1 \mid s(0) = i\} = \sum_{k>N} \sum_{k<1} \frac{1}{T_i} \gamma$$

That is, for each state i relative to the terminal's current relaying repeater, we assign the waiting time to be the smallest value of Ti for which the probability that the terminal has exited its current relaying repeater's transmission range, Ti, transition times after probing while in state i, is greater than γ. Figures 5-10 through 5-12 illustrate the performance of this use of position information, and Tables 5-1 through 5-3 give for each selected value of γ, the corresponding value of Ti. Comparing Figures 5-10 through 5-12 with Figures 5-4 through 5-6 (i.e., the periodic probing counterparts), we see that
Figure 5-10 FTC vs. ATR for probing with position information with $p=0.8$, $q=0$
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**Table 5-1** The value of $\tau$ corresponding to each point plotted in Figure 5-10 where $p=0.8$, $q=0$
Figure 5-11 FTC vs. ATR for probing with position information with $p=0.6$, $q=0.2$
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Table 5-2  The value of $T$ corresponding to each point plotted in Figure 5-11 where $p=0.6$, $q=0.2$
Figure 5-12 FTC vs. ATR for probing with position information with p=0.4, q=0.4
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**Table 5-3**  The value of $\tau$ corresponding to each point plotted in Figure 5-12 where $p=0.4$, $q=0.4$
the same basic qualitative differences between the performances of the three methods for selecting a new relaying repeater are present in both probing schemes. Observe, however, that for values of FTC in the vicinity of 1.0, the value of ATR associated with probing with position information is less than one half of the corresponding value associated with periodic probing. The two values of ATR do, however, converge as FTC decreases, and are in fact generally very close for FTC < 0.8.

In order to get an indication of how close the performance curves generated by the heuristic method for choosing the values for T are to the optimal curves, the values of T corresponding to the points plotted in Figure 5-12 were perturbated and then used to obtain new values of FTC and ATR. It was found that for those perturbations which resulted in an improvement, the improvement was very slight and in many cases would not have been shown to be an improvement if a smaller increment had been chosen for \( \gamma \). Thus, although it is not strictly proven, the perturbation analysis seems to indicate that the performance of the heuristic method for choosing values for T is not significantly different from that of the optimal.

It is worth mentioning that another heuristic approach for generating values for T was also examined. This alternate heuristic is as follows. For various values of the variable \( \delta \), where \( \delta \geq 0 \), we determine for each i (i=1,2,...,N) the minimum value of \( T_i \) such that

\[
T_i - E[TC^i] > \delta
\]

That is, for each state i relative to the terminal's current relaying
repeater, we assign the waiting time to be the smallest value of $\tau_i$ for which the expected time the terminal is outside of its current relaying repeater's transmission range, during the $\tau_i$ transition times after probing while in state $i$, is greater than $\delta$. The performance of this heuristic was determined for the same parameters as represented in Figures 5-10 through 5-12. The corresponding performance curves generated by each heuristic were found to lie very close to each other, however, more often than not, the curve generated by the second heuristic was to the right of the corresponding curve generated by the first heuristic. For this reason, the results of the second heuristic are not presented here.

At this point, one might comment that a reduction of ATR by at most a little more than one half of that corresponding to periodic probing is not sufficient to warrant the complexity and expense associated with being able to determine position at each probing. However, before jumping to this conclusion, one must take into consideration the fact that the obtained performance results are based on a discrete model of terminal motion. That is, the terminal is assumed to move only at discrete instances of time corresponding to the transition times of a Markov process. Because of this, to achieve FTC = 1.0 with periodic probing, the terminal need only probe its relaying repeater immediately after each transition time. In an actual mobile packet radio network, the motion of each terminal is continuous. In this situation, in order to achieve FTC = 1.0 with periodic probing, a terminal would have to probe its relaying repeater nearly continuously, implying an extremely large value of ATR. Moreover, considering the finite capacity of the
broadcast channel, this value of ATR may not even be allowable. However, in using probing with position information in a situation where a terminal's motion is continuous, as long as the terminal's velocity is bounded and the transmission range of its relaying repeater is well defined, \( FTC = 1.0 \) is achievable at values of ATR not significantly different from those associated with the discrete model of motion. The reason for this is twofold. First, by knowing its position at the last probing and by keeping track of its velocity since that probing, a terminal can send the next probe packet just before there is a nonzero probability that it will have exited its current relaying repeater's transmission range. Secondly, with position information, the terminal is able to anticipate a possible loss of connectivity between it and its current relaying repeater and thus select a new relaying repeater before the loss can occur. Hence, \( FTC = 1.0 \) is clearly achievable without a significant increase in the value of ATR associated with the discrete model. In the performance analysis, a discrete model of terminal motion was chosen in order to facilitate the derivation of FTC and ATR. In fact, except in the vicinity of \( FTC = 1.0 \), the performance curves obtained by using the discrete model of motion may be considered to be a good approximation to those which would be obtained with a similar, but continuous model of motion. Thus we see that the main advantage of using probing with position information is when the terminal's motion is continuous in nature and when the required value of FTC is close to 1.0. In this situation, the value of ATR associated with periodic probing can be reduced significantly by switching to probing with position information.
5.5.3 Sensitivity Analysis

The performance results obtained thus far have been based on the assumption that the route between the terminal and the station consists of three hops. Let us now examine the changes in performance as the number of hops between the terminal and the station is increased. Although the same behavior exists for either probing scheme and each of the three methods for selecting a new relaying repeater, Figure 5-13 illustrates the changes in performance for periodic probing (p=0.6, q=0.2) with the random selection of a new relaying repeater. Note that the essential characteristics of the performance curve are not changed, only shifted to higher values of ATR as the number of hops between the terminal and station is increased.

Suppose we maintain the same repeater transmission range and spacing (e.g., 11 and 5 km, respectively), and we require that the terminal's average velocity remain the same. However, we wish to increase the number of states $N$ within the transmission range of each repeater along the path. We can do this by making the changes $s' = \theta s$ and $T_s' = \theta T_s$, where $0 < \theta < 1$. It follows from equation (5.4) that the variance in the terminal's position is proportional to $s^2/T_s$. Thus by increasing $N$ in this manner, we are decreasing the terminal's randomness of motion and consequently, we expect to see the same sort of change in performance as when we increased the value of $p-q$. Moreover, this increase in $N$ makes for a better approximation to a continuous form of terminal motion. Thus, as mentioned above, the advantages of using probing with position information for $FTC = 1.0$ will become more evident.
Figure 5-13 FTC vs. ATR for periodic probing ($p=0.6$, $q=0.2$) with the random selection of a new relaying repeater
Now suppose we maintain the same values for $s$, $T_s$, and $N$, but increase the spacing between the repeaters along the path. Figure 5-14 represents the performance for the same parameters as in Figure 5-5, except that the spacing between the repeaters along the path has been increased from 5s to 8s. Comparing Figures 5-5 and 5-14, we note that by increasing the distance between repeaters, we decrease the difference between the performances of the three methods for selecting a new relaying repeater. This is certainly reasonable, because by increasing the spacing between repeaters, we are decreasing the expected number of repeaters from which the terminal can choose when selecting a new relaying repeater. This is not to say that it is necessarily desirable to decrease the spacing between repeaters. To do so would certainly increase the desirability of the max $E[TE]$ method for selecting a new relaying repeater, not to mention the increase in the network's reliability. However, if the repeaters share a common broadcast channel, a decrease in the spacing between repeaters will result in an undesirable increase in the interference of packet transmissions.

We now comment on the expected changes in the performance results if we were to incorporate the terminal's effective probing of its relaying repeater each time it sends a message packet to the station. Note that in this situation, the overhead associated with selecting a new relaying repeater and informing the station of this choice will not change. However, the overhead associated with detecting a loss of connectivity between the terminal and its relaying repeater will decrease. To see how this decrease affects the performance results, let us suppose that the terminal's motion is of a continuous nature, and
Figure 5-14 FTC vs. ATR for periodic probing with $p=0.6$, $q=0.2$, and with a repeater located every 8 states along the path.
that it is using periodic probing with probing period $T_p$. Also, let us assume that initially the terminal is sending no messages to the station. We may view this initial situation as corresponding to some point on FTC vs. ATR. Recall that when using the form of periodic probing which incorporates the effective probing via message packet transmissions, the terminal sends a probe packet to its relaying repeater at time $t$, only if the last transmission of either a probe or message packet was at time $t-T_p$. Thus as the terminal increases its rate of message packet transmissions, we expect this point on FTC vs. ATR to first move horizontally to the left, and then at some value of ATR we expect it to begin moving vertically toward FTC = 1.0. The initial horizontal movement of the point (i.e., ATR decreasing with FTC remaining the same) is due to the actual probing rate decreasing, but the overall (i.e., actual + effective) probing rate remaining the same. However, when the rate of message packet transmissions equals the required probing rate, any additional increase in the rate of message packet transmissions will be an effective increase in the overall probing rate with no change in the overhead associated with probing. Thus the performance point will move upward. Since a continuous form of terminal motion is assumed, with periodic probing, FTC = 1.0 is approached with an increasing rate of message packet transmissions, but never reached. The same sort of result as stated above for periodic probing is expected for probing with position information. However, if position information is gained with each message packet transmitted, FTC = 1.0 is eventually reached for some finite rate of message packet transmissions. This is because, as we previously stated, with position information, the terminal is able to
anticipate a possible loss of connectivity between it and its relaying repeater and thus select a new relay repeater before the loss can occur.

Finally, we comment on the expected changes in the performance results if we allow the possibility of packets being received in error. We expect two basic changes in the performance curves. First, the number of transmitted overhead packets associated with informing the station of a change in the terminal's relaying repeater will increase (due to necessary retransmissions) with increasing error probability. Thus the performance curves will move to the right in the same fashion as illustrated in Figure 5-13. Secondly, when the terminal sends out a general probe packet, it will not necessarily receive responses from all of the repeaters within its transmission range. Consequently, its choice of a new relaying repeater will be based on incomplete information. As such, we expect the difference in the performance curves associated with the three selection methods to decrease. As a final comment, note that when a terminal probes its relaying repeater, it may not receive a response due to either the probe or response packet being received in error. Thus in this situation, the terminal may conclude that a loss of connectivity between it and its relaying repeater has occurred, when in fact there is no loss. In the next section we examine the desirability of having the terminal probe its relaying repeater one or more additional times before concluding that a lack of a response is due to a loss of connectivity.
5.6 Probing and Packet Errors

If a terminal receives a response to a probe packet that it sent to its relaying repeater, then the terminal will know that a connection exists in both directions between it and that repeater. If the terminal does not receive a response, then the terminal will know that either there has been a loss of connectivity between it and its relaying repeater, or the probe or response packet was received in error and thus ignored. If the amount of overhead associated with obtaining a new route to the station is large and/or if the error probability on the channel is sufficiently high, then at a routing update, it may be desirable to have the terminal probe its relaying repeater one or more additional times before concluding that the lack of a response is due to a loss of connectivity. This section is in some sense a departure from sections 5.1 through 5.5 in that we do not concern ourselves with the performance of the probing implementation in terms of FTC and ATR. Rather, we now assume the possibility of packet errors and consider the question, at a routing update, what is the maximum number of times a terminal should probe its relaying repeater without receiving a response, before concluding that a loss of connectivity between it and that repeater has occurred?

The situation where a terminal will, at any particular routing update, send up to \( k \) \((k=0,1,2,\ldots)\) probe packets before concluding that a loss of connectivity has occurred, will be referred to as probing policy \( k \). Also, we define the optimal probing policy as that selection of \( k \) which minimizes the expected number of transmitted overhead packets
(E[P]) associated with a routing update. To determine this optimal value of \( k \) (\( k_{\text{opt}} \)), we begin by deriving an expression for the conditional expectation \( E[P_k] \), which is the expected number of transmitted overhead packets associated with a routing update, given probing policy \( k \) is adopted. We first define the following events:

- \( c \) - at the time of a routing update, the terminal is connected in both directions to its relaying repeater
- \( r \) - given event \( c \), the terminal receives a response to a probe packet sent to its relaying repeater

We also define the following probabilities:

\[
\begin{align*}
\varepsilon &= \Pr\{\text{a packet sent over the broadcast channel is received in error}\}^* \\
\alpha &= \Pr\{\bar{r}\}^{**}
\end{align*}
\]

Noting that with probability \( \varepsilon \) a probe packet will be received in error, and with probability \( (1-\varepsilon)\varepsilon \) the probe packet will be received correctly but the response packet will be received in error, it follows that

\[
\alpha = \varepsilon + (1-\varepsilon)\varepsilon \quad (5.18)
\]

Finally, we define the following expectations:

\[
\beta = E[\text{number of transmitted overhead packets associated with a probing } | \bar{r}] \\
\lambda = E[\text{number of transmitted overhead packets associated with selecting a new relaying repeater and informing the station of this choice}]
\]

*We assume that packets transmitted over the broadcast channel are received with or without error independently of each other.

**The notation "\( \bar{r} \)" means "not \( r \)."
It follows that

$$\beta = (\varepsilon + 2(1-\varepsilon)\varepsilon)/\alpha = 1 + (1-\varepsilon)\varepsilon/\alpha \quad (5.19)$$

Later we shall derive an expression for $\lambda$. For now, note that if the terminal is not connected in both directions to its relaying repeater, then with probing policy $k$, the terminal will, at a routing update, send a total of $k$ probe packets to its relaying repeater without receiving a response. In addition, an average of $\lambda$ overhead packets will be transmitted in order for the terminal to obtain a new route. Thus we may write

$$E[P_k \mid c] = k + \lambda \quad (5.20)$$

If the terminal is connected in both directions to its relaying repeater, then for $k = 0$, it follows that $E[P_0 \mid c] = \lambda$. For $k > 0$, with probability $(1-\alpha)\alpha^{i-1}$, the terminal will receive a response to the $i^{th}$ ($i=1, 2, \ldots, k$) probe packet sent to its relaying repeater. With probability $\alpha^k$, the terminal will not receive a response after sending $k$ probe packets to its relaying repeater. Thus, it follows that

$$E[P_k \mid c] = \begin{cases} 
\lambda & k=0 \\
\sum_{i=0}^{k-1} (2 + i\beta)(1-\alpha)^i + (k\beta + \lambda)\alpha^k & k=1, 2, \ldots
\end{cases} \quad (5.21)$$

Noting that

$$\frac{k-1}{\sum_{i=0}^{k-1} \alpha^i} = \frac{1 - \alpha^k}{1 - \alpha}$$
and

\[ \sum_{i=0}^{k-1} ia^i = \frac{\alpha(1-a^k)}{(1-a)^2} - \frac{ka^k}{1-a} \]

we may rewrite (5.21) as

\[ E[P_k | c] = 2 + \frac{\beta a}{1-\alpha} + (\lambda - 2 - \frac{\beta a}{1-\alpha})a^k \quad k=0,1,2,... \quad (5.22) \]

Thus from (5.20) and (5.22), we obtain

\[ E[P_k] = Pr(\bar{c})(k + \lambda) + Pr(c) \left[ 2 + \frac{\beta a}{1-\alpha} + (\lambda - 2 - \frac{\beta a}{1-\alpha})a^k \right] \quad k=0,1,2,... (5.23) \]

We now determine \( k_{opt} \) (i.e., the value of \( k \) which minimizes \( E[P_k] \)). For the case where \( \lambda < 2 + \beta a/(1-\alpha) \), we see by inspection of (5.23) that \( E[P_k] \) is minimized when \( k = 0 \). For the case where \( \lambda > 2 + 3\alpha/(1-\alpha) \), it is easily shown that \( E[P_k] \) is a convex function of \( k \). Solving for \( k_0 \) (\( k_0 \in \text{reals} \) in the equation

\[ E[P_{k_0}] = E[P_{k_0+1}] \]

yields

\[ k_0 = \frac{\ln(Pr(\bar{c})/Pr(c)(\lambda-2)(1-\alpha) - \beta a))}{\ln \alpha} \quad (5.24) \]

It follows that

\[ k_{opt} = \begin{cases} \lceil k_0 \rceil \text{ and } \lfloor k_0 \rfloor + 1 & \text{for } \lfloor k_0 \rfloor > 0 \\ 0 & \text{for } \lfloor k_0 \rfloor < 0 \end{cases} \]
Note from (5.24) that as \( \lambda \) and/or \( \Pr(c) \) increases, the value of \( k \) which minimizes \( E[P_k] \) is nondecreasing. This is certainly an intuitively pleasing behavior.

Before one can actually use the above procedure for determining \( k_{opt} \), one must determine a value for \( \lambda \). We may write

\[
\lambda = \lambda_s + \lambda_i
\]

(5.25)

where

\[
\lambda_s = E[\text{number of transmitted overhead packets associated with selecting a new relaying repeater}]
\]

and

\[
\lambda_i = E[\text{number of transmitted overhead packets associated with informing the station of a change in relaying repeater}]
\]

Recall that a terminal bases its choice of a new relaying repeater on the responses that it receives after transmitting a general probe packet. We assume that only in the case where a terminal does not receive at least one response to a general probe packet will it then transmit another general probe packet. Note that if the terminal is connected in both directions to \( L \) (\( L=1,2,... \)) available repeaters, then the probability that exactly \( i \) (\( i=0,1,...,L \)) of these repeaters will correctly receive the terminal's transmitted general probe packet is given by \( \binom{L}{i} \epsilon^{L-i}(1-\epsilon)^i \) (i.e., \( i \) successes in \( L \) trials). Furthermore, given that \( i \) repeaters correctly receive the terminal's general probe packet, with probability \( \epsilon^i \), the terminal will not receive any of the \( i \) corresponding response
packets. Thus it follows that

\[ \lambda_s = 1 + \sum_{i=0}^{L} (i + \lambda_s \epsilon^i) \left( \frac{L}{i} \right) e^{L-i} (1-\epsilon)^i \quad L=1,2,\ldots \]  \hspace{1cm} (5.26)

Rearranging the terms in (5.26), we obtain

\[ \lambda_s \left[ 1 - \epsilon^{L} \sum_{i=0}^{L} \left( \frac{L}{i} \right)(1-\epsilon)^i \right] = 1 + \sum_{i=0}^{L} i \left( \frac{L}{i} \right) e^{L-i} (1-\epsilon)^i \]  \hspace{1cm} (5.27)

Noting that

\[ \sum_{i=0}^{L} \left( \frac{L}{i} \right)(1-\epsilon)^i = (2-\epsilon)^L \]

and

\[ \sum_{i=0}^{L} i \left( \frac{L}{i} \right) e^{L-i} (1-\epsilon)^i = L(1-\epsilon) \]

we may rewrite (5.27) as

\[ \lambda_s [1 - \epsilon^{L} (2-\epsilon)^L] = 1 + L(1-\epsilon) \]  \hspace{1cm} (5.28)

Noting that \( \alpha = \epsilon(2-\epsilon) \), and solving for \( \lambda_s \) in (5.28), we obtain

\[ \lambda_s = \frac{1 + L(1-\epsilon)}{1-\alpha^L} \quad L=1,2,\ldots \]  \hspace{1cm} (5.29)

Figure 5-15 is a plot of \( \lambda_s \) vs. \( \epsilon \) for various values of \( L \). As for \( \lambda_i \), if the terminal's newly selected route to the station involves \( H \) hops, then the expected number of transmitted overhead packets associated with in-
Figure 5-15 $\lambda_s$ vs. $\varepsilon$
forming the station of this new route is simply given by

\[
\lambda_i = \frac{H}{1-\varepsilon} \tag{5.30}
\]

Figure 5-16 is a graph of \( \lambda_i \) vs. \( \varepsilon \) for various values of \( H \). Finally, substituting (5.29) and (5.30) into (5.25), we obtain

\[
\lambda = \frac{1 + L(1-\varepsilon)}{1-\alpha L} + \frac{H}{1-\varepsilon} \tag{5.31}
\]

Thus for given values of \( \varepsilon \), \( \Pr(c) \), \( L \), and \( H \), using (5.31) and the procedure for determining the optimal probing policy, we can compute \( k_{opt} \). Figures 5-17, 5-18, and 5-19 are graphs of \( k_{opt} \) vs. \( \varepsilon \) for various values of, respectively, \( \Pr(c) \), \( H \), and \( L \). Note that in each case, as \( \varepsilon \) increases, \( k_{opt} \) increases to a peak and then falls rapidly to zero. This implies that for sufficiently large \( \varepsilon \), the terminal should, at a routing update, select a new relay repeater without even probing its current relaying repeater.
Figure 5-16 $\lambda_i$ vs. $\varepsilon$
Figure 5-17 $k_{opt}$ vs. $\varepsilon$ with $H = 3$ and $L = 3$
Figure 5-18 $k_{opt}$ vs. $\varepsilon$ with $Pr(c) = 0.75$ and $L = 3$
Figure 5-19a  $k_{opt}$ vs. $\varepsilon$ with $Pr(c) = 0.75$ and $H = 3$
Figure 5-19b $k_{opt}$ vs. $\varepsilon$ with $Pr(c) = 0.75$ and $H = 3$
CHAPTER 6

Summary and Conclusions

This thesis has been concerned with the problem of monitoring connectivity in mobile packet radio networks. The initial approach to this problem was to make as few assumptions as possible about the type of mobile packet radio network and the use that the network is to make of connectivity information. Considering just two nodes, we presented two monitoring methods, the broadcast method and the probing method. These two methods were generalized so that each node in a packet radio network could monitor the connectivity between it and the other nodes in the network. It was found that in this situation, there is little overhead associated with the broadcast method. However, unlike the probing method, the broadcast method does not test the connection between two nodes in both directions, and is not efficient when there is a large range in the required connectivity updating rates of the nodes in the network. Thus we concluded that there are trade-offs associated with each method and that to choose between them, one must examine both the specific type of packet radio network in which one wishes to monitor connectivity, and the specific use that is to be made of connectivity information.

In the second part of this thesis, we examined connectivity monitoring in a terminal-oriented mobile packet radio network. This network uses a tree-structured form of routing in which all traffic flows through a centralized node called a station. Each of the other nodes in the network is classified as either a repeater or a terminal. We allowed the terminals to be mobile, however, for simplicity, we required the station and repeaters to be stationary. We examined how
the broadcast and probing methods may be implemented in this network for the purpose of updating the route between each terminal and the station.

Two implementations of the broadcast method were first presented. The first implementation, a direct application of the broadcast method, consists of the repeaters and station broadcasting special monitoring packets to the terminals. Unfortunately, we found that it suffered from the two problems that are generally associated with the broadcast method. Thus, although possibly suitable for a network in which the repeaters and station have the same transmission range as the terminals, we considered it an unsuitable choice for the network under consideration. The second implementation, a form of which is currently in use in an actual packet radio network, is a somewhat indirect application of the broadcast method which does not suffer the problems associated with the first implementation. The second implementation consists of each terminal broadcasting special monitoring packets called ROP's. Any repeater with an assigned route to the station which receives an ROP attaches its identity to that ROP and then forwards it to the station. The station can then update its knowledge of the terminal-repeater and terminal-station connectivity for the particular terminal which originated the ROP, and then if needed, assign that terminal a new route to the station.

Next we presented an implementation of the probing method. This implementation consists of each terminal updating, via probe packets, the two-directional connectivity between it and the repeaters and station. This connectivity information is then forwarded to the
station where a decision is made as to whether the terminal should be assigned a new route. We found that although the end result of using either this probing implementation or the ROP broadcast implementation is the same, the overhead associated with each may be different. In particular, we found that if a terminal is close to the station and surrounded by few repeaters, the ROP broadcast implementation is favored, whereas if the terminal is far from the station and surrounded by many repeaters, the probing implementation is favored. However, in the situation where 1) a terminal's route is only changed when necessitated by a change in connectivity, and 2) the terminal is able to select its own route to the station, we found that by revising the probing implementation, the associated overhead can be reduced significantly, and thus make the probing implementation a clear choice over the ROP broadcast implementation.

In addition, we found that this revised implementation can easily make use of transmitted message packets in order to further reduce its associated overhead, and that it can be generalized for the situation where the repeaters and station are mobile.

The revised probing implementation was next analyzed in greater detail. We chose as our measure of performance, the trade-off between FTC and ATR. After formulating a network model, we derived expressions for FTC and ATR. These were then used to evaluate the performance of the revised probing implementation for variations in a terminal's average velocity and randomness of motion, for two methods for choosing the probing times (i.e., periodic probing and probing with position information), and for three methods for selecting a new relaying repeater
(i.e., random, nearest, and max $E[TE]$). The form of the results for variations in a terminal's average velocity and randomness of motion is illustrated in Figure 6-1(a). The comparison of the performances of periodic probing and probing with position information is illustrated in Figure 6-1(b). Here we found that the complexity and expense associated with determining position may well be worth it if the required value of FTC is in the vicinity of 1.0. As for the methods for choosing a new relaying repeater, two points are of importance. First, we found that basing the choice of a new relaying repeater on complete position information (i.e., the max $E[TE]$ method) had a somewhat better performance than choosing a repeater at random. However, the improvement is not necessarily significant enough to warrant basing the choice of a new relaying repeater on position rather than on ability to relay packets to the station. Secondly, we found that one must be careful about using incomplete position information (e.g., distance information only) when choosing a new relaying repeater, because in some situations, the performance is actually worse than that associated with choosing a repeater at random. Finally, we examined the problem of probing in the presence of transmission errors. Specifically, we determined the optimal number of times a terminal should attempt probing its relaying repeater before concluding that a loss of connectivity between it and that repeater has occurred.

Concerning the network model used in the performance analysis, it is important to recognize that, conceptually, we could have just as easily used a two-dimensional Markov model of terminal mobility. The only requirements that the model must satisfy are 1) the positions of
Figure 6-1  Form of the results for (a) variations in a terminal's average velocity and randomness of motion, and (b) the use of periodic probing and probing with position information.
the states and the transition probabilities between the states within
the transmission range of each repeater are the same for every repeater,
2) the position of each repeater relative to each of its neighboring
repeaters is the same for every repeater, and 3) the transition pro-
babilities from the states in each repeater to the states in each of its
neighboring repeaters are the same for every repeater. These spatial
stationarity requirements are necessary in order to have steady-state
probabilities. However, note that the number of states associated
with a two-dimensional model of mobility will generally be the square
of the number of states associated with a similar one-dimensional model.
Thus, one can expect a substantial increase in the computation time
associated with a performance analysis which employs the use of a two-
dimensional model.

Finally, in closing, we would like to reemphasize that the
choice of a monitoring method is highly dependent on both the specific
type of packet radio network that is being considered, and the specific
use that is to be made of connectivity information in that network.
Although we examined in detail only one specific type of network with
one particular use of connectivity information, it is believed that
many of the ideas presented here will carry over to other types of
packet radio networks and other uses of connectivity information. In
particular, distributing among the nodes the function of updating the
network's knowledge of connectivity, and using the connectivity informa-
tion gained via the transmission and reception of, for example, message
packets are two important concepts. In addition, it is believed that
the results in Appendix A on Markov processes with observations could
be useful in the performance analysis of other monitoring implementations.
APPENDIX A

Markov Processes with Observations*

We consider an arbitrary, finite state, discrete-time, homogeneous Markov process.** For notational convenience, we assume that the Markov process has \( N \) states which are identified by the integers 1 through \( N \), and that state transitions occur at integer times. Also, we use the notation \( s(n) = j \) to denote the event that the process is in state \( j \) at time \( n \). The Markov process is completely characterized by its state transition probability matrix \( P \) (where element \( p_{ij} \) \( \Delta \) \( \Pr\{s(n)=j|s(n-1)=i\} \) for \( i,j=1,2,\ldots,N \) and \( n=1,2,\ldots \)) and its initial state probability vector \( \pi(0) \) (where component \( \pi_i(0) \) \( \Delta \) \( \Pr\{s(n)=i\} \) for \( i=1,2,\ldots,N \) and \( n=0,1,2,\ldots \)). Associated with the process is an "observer" and a waiting time vector \( \tau \). The function of the observer is to observe the state of the Markov process. Observations are made at discrete instances of time. The time at which each successive observation is made is determined by \( \tau \) and the state of the process at the most recent observation. Specifically, \( \tau_i \), the \( i^{th} \) \( (i=1,2,\ldots,N) \) component of \( \tau \), is a positive integer associated with state \( i \). If state \( i \) is observed at time \( m \), the observer will wait until time \( m+\tau_i \) before again observing the state of the process. If at time \( m+\tau_i \) state \( j \) is observed, the observer will wait until time \( m+\tau_i+\tau_j \) before making the next observation. This continues for all subsequent observations.

* As related to the derivation of FTC and ATR in section 5.4, observations and probes are equivalent. The word "observation" is used here to lend some generality to the results that are derived.

** See [15] for a more detailed discussion of this type of Markov process.
We would like to determine, probabilistically, at which times and in which states observations are made. We start by defining the indicator random variable

\[ x(n) = \begin{cases} 1 & \text{if an observation is made at time } n \\ 0 & \text{if an observation is not made at time } n \end{cases} \quad n=0,1,2,... \]

We now define two conditional probabilities that are of interest.

\[ \phi_{ij}(n) = \Pr\{s(n) = j | s(0) = i\} \quad i,j=1,2,...,N; \quad n=0,1,2,... \]

\[ \rho_{j}(n) = \Pr\{s(n) = j, x(n) = 1 | x(0) = 1\} \quad j=1,2,...,N; \quad n=0,1,2,... \]

The first conditional probability, \( \phi_{ij}(n) \), is well known in the study of Markov processes and is often called the n-step transition probability. It may be evaluated recursively by using a simple form of the Chapman-Kolmogorov equation given by

\[ \phi_{ij}(n) = \begin{cases} \sum_k \phi_{ik}(n-1)p_{kj} & n=1,2,... \\ \delta_{ij} & n=0 \end{cases} \quad (A.1) \]

Note that \( \phi_{ij}(n) \) is independent of the observation process. The second conditional probability, \( \rho_{j}(n) \), is dependent on the observation process and in words, is the probability that at time \( n \) the Markov process will be in state \( j \) and an observation will be made, given that at time \( 0 \) an observation was made. An equation similar to (A.1) may be used to eval-

*All summations without labeled indices implicitly run from 1 to N (i.e., \( \sum_k \) is used to denote \( \sum_{k=1}^{N} \)), and \( \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \).
u ate $\rho_j(n)$ recursively. This equation and its proof are as follows:

$$
\rho_j(n) = \begin{cases} 
\sum_k \rho_k(n-\tau_k) \phi_{kj}(\tau_k) & n=1,2,\ldots \\
\pi_j(0) & n=0 \\
0 & n=-1,-2,\ldots
\end{cases}
$$

(A.2)

Proof of Equation (A.2):

We assume that the first observation is made at time 0, thus it follows that $\rho_j(n)=0$ for $n<0$. Also, from the definition of $\rho_j(n)$ and $\pi_j(n)$, it is clear that $\rho_j(0)=\pi_j(0)$. We begin the proof for $n>0$ by noting that, given $x(n)=1$ and $s(n)=j$, the set of paired events $\{s(n-\tau_k)=k, x(n-\tau_k)=1\}$ $k=1,2,\ldots,N$ forms a mutually exclusive and collectively exhaustive set. Thus, from the definition of $\rho_j(n)$, we may write

$$
\rho_j(n) = \sum_k \Pr\{s(n)=j, x(n)=1, s(n-\tau_k)=k, x(n-\tau_k)=1|x(0)=1\}
$$

$$
= \sum_k \Pr\{s(n)=j, x(n)=1|s(n-\tau_k)=k, x(n-\tau_k)=1, x(0)=1\} \cdot \Pr\{s(n-\tau_k)=k, x(n-\tau_k)=1|x(0)=1\} 
$$

(A.3)

The first term within the summation of (A.3) may be rewritten as

$$
\Pr\{s(n)=j, x(n)=1|s(n-\tau_k)=k, x(n-\tau_k)=1, x(0)=1\}
$$

$$
= \Pr\{x(n)=1|s(n)=j, s(n-\tau_k)=k, x(n-\tau_k)=1, x(0)=1\}
$$

$$
\cdot \Pr\{s(n)=j|s(n-\tau_k)=k, x(n-\tau_k)=1, x(0)=1\}
$$

$$
= 1 \cdot \phi_{kj}(\tau_k) 
$$

(A.4)

Furthermore, the second term within the summation of (A.3) is by definition $\rho_j(n-\tau_k)$. Using this fact and substituting (A.4) into (A.3), we obtain the desired result.
\[ \rho_j(n) = \sum_k \rho_k(n-\tau_k) \phi_{kj}(\tau_k) \]

Q.E.D.

We now use the above result to investigate the steady-state behavior of a Markov process with observations. We begin by defining

\[ \bar{\pi}_j(n) = \Pr\{s(n) = j | x(n) = 1, x(0) = l\} \quad j = 1, 2, \ldots, N; \ n = 0, 1, 2, \ldots \]

From the definition of conditional probability, as long as

\[ \Pr\{x(n) = 1 | x(0) = l\} \neq 0, \]

we may write

\[ \bar{\pi}_j(n) = \frac{\Pr\{s(n) = j, x(n) = 1 | x(0) = l\}}{\Pr\{x(n) = 1 | x(0) = l\}} \]

\[ = \frac{\rho_j(n)}{\sum_i \rho_i(n)} \quad \text{for } \sum_i \rho_i(n) \neq 0 \quad (A.5) \]

Summing, with respect to \( j \), both sides of (A.2), we obtain

\[ \sum_j \rho_j(n) = \sum_k \rho_k(n-\tau_k) \phi_{kj}(\tau_k) \]

\[ = \sum_k \rho_k(n-\tau_k) \sum_j \phi_{kj}(\tau_k) \]

\[ = \sum_k \rho_k(n-\tau_k) \quad (A.6) \]

Substituting (A.2) and (A.6) into (A.5), we obtain

\[ \bar{\pi}_j(n) = \sum_k \frac{\rho_k(n-\tau_k)}{\sum_i \rho_i(n-\tau_i)} \phi_{kj}(\tau_k) \]

\[ = \sum_k \frac{\bar{\pi}_k(n-\tau_k)}{\rho_k(n-\tau_k)} \phi_{kj}(\tau_k) \quad \text{for } \sum_i \rho_i(n) \neq 0 \quad (A.7) \]

Suppose that at time \( n \) the \( m \)th observation is made. We can say that for
some \( k \) \( (k=1,2,\ldots,N) \), at time \( n-z_k \) the \((m-1)\text{th}\) observation was made while in state \( k \). Using the notation \( \pi_j[m] \) to denote the conditional probability that, given \( x(0)=l \), the process is in state \( j \) at the time of the \( m \text{th} \) observation, we may rewrite (A.7) as

\[
\pi_j[m] = \begin{cases} 
\sum_k \pi_k[m-1] \phi_k(\tau_k) & m=1,2,\ldots \\
\pi_j(0) & m=0 
\end{cases} \tag{A.8}
\]

Note that (A.8) is of the same structure as the familiar (see [15] section 1.3) equation

\[
\pi_j(n) = \begin{cases} 
\sum_k \pi_k(n-1) P_{kj} & n=1,2,\ldots \\
\pi_j(0) & n=0 
\end{cases}
\]

Recall from basic Markov theory that if the stochastic matrix \( P \) represents an ergodic Markov process (e.g., an irreducible aperiodic, finite Markov chain), then the limiting probabilities

\[
\pi_j = \lim_{n \to \infty} \pi_j(n)
\]

always exist and are independent of the initial state probability distribution. Furthermore, these limiting probabilities are uniquely determined by the following equations:

\[
\pi_j = \sum_i \pi_i P_{ij} \quad \text{and} \quad 1 = \sum_i \pi_i
\]
Thus, if we interpret the stochastic matrix $\Phi$, with elements $\Phi_{kj}(T_k)$, as representing the transition probabilities of a Markov process, and if $\Phi$ is such that the associated Markov process is ergodic, then the limiting probabilities

$$\tilde{\pi}_j = \lim_{m \to \infty} \tilde{\pi}_j[m]$$

always exist, are independent of $\tilde{\pi}_j[0]$, and may be uniquely determined by

$$\tilde{\pi}_j = \sum_i \tilde{\pi}_i \Phi_{ij}(T_i)$$

$$1 = \sum_i \tilde{\pi}_i$$ (A.9)

Equation (A.9), in a slightly different form, is used in the derivation of FTC and ATR in section 5.4.
APPENDIX B

Time-Sharing on FTC vs. ATR

Given any two points, say \( Q_1 \) and \( Q_2 \), on FTC vs. ATR corresponding to two different sets of parameters (e.g., two different values of \( T \) with all other parameters the same), we show that with time-sharing, any point lying on the straight line segment connecting \( Q_1 \) and \( Q_2 \) is achievable. By time-sharing we mean that for fraction \( \theta \) \((0 \leq \theta \leq 1)\) of the time the parameter set associated with one of the two points is used, and for fraction \( 1-\theta \) the parameter set associated with the other point is used.

Let us assume that for fraction \( \theta \) of the time the parameter set associated with \( Q_2 \) is used, and for fraction \( 1-\theta \) the parameter set associated with \( Q_1 \) is used. For notational convenience, we define

\[
\begin{align*}
a_i &= \text{E}[TC] \text{ for the parameter set associated with } Q_i \ (i=1,2) \\
b_i &= \text{E}[P] \\
c_i &= \text{E}[T] \text{ for the parameter set associated with } Q_i \ (i=1,2)
\end{align*}
\]

Using these definitions and equations (5.7) and (5.8), we may write

\[
\begin{align*}
\text{FTC}(\theta) &= \frac{\theta a_1 + (1-\theta) a_2}{\theta c_1 + (1-\theta) c_2} \\
\text{ATR}(\theta) &= \frac{\theta b_1 + (1-\theta) b_2}{\theta c_1 + (1-\theta) c_2}
\end{align*}
\]

Let us define point \( Q \) as that point corresponding to FTC(\( \theta \)) and ATR(\( \theta \)). To prove that point \( Q \) is on the line segment connecting points \( Q_1 \) and \( Q_2 \), it is sufficient to show that the slope of the line connecting \( Q_1 \) and \( Q \) is independent of \( \theta \) for \( 0 \leq \theta \leq 1 \). Following this line of proof, we write
slope(Q₁,Q₂) = \frac{FTC(0) - FTC(1)}{ATR(0) - ATR(1)} \tag{B.3}

Substituting (B.1) and (B.2) into (B.3), we obtain

\[
slope(Q₁,Q₂) = \left[ \frac{\theta a₁ + (1-\theta)a₂}{\theta c₁ + (1-\theta)c₂} - \frac{a₁}{c₁} \right] \div \left[ \frac{\theta b₁ + (1-\theta)b₂}{\theta c₁ + (1-\theta)c₂} - \frac{b₁}{c₁} \right]
\]

\[
= \frac{c₁[\theta a₁ + (1-\theta)a₂] - a₁[\theta c₁ + (1-\theta)c₂]}{c₁[\theta b₁ + (1-\theta)b₂] - b₁[\theta c₁ + (1-\theta)c₂]}
\]

\[
= \frac{(1-\theta)a₂c₁ - (1-\theta)a₁c₂}{(1-\theta)b₂c₁ - (1-\theta)b₁c₂}
\]

\[
= \frac{a₂c₁ - a₁c₂}{b₂c₁ - b₁c₂}
\]

Q.E.D.

Thus we have shown that, with the use of time-sharing, any point on the line segment connecting Q₁ and Q₂ can be achieved by an appropriate choice of \( \theta \).
APPENDIX C

Some Computational Details

Computing $\phi_{ij}(\tau_i)$

The computation of FTC and ATR as defined by, respectively, equations (5.15) and (5.16) incorporate (directly, and indirectly through the computation of $\Pi$) the quantity $\phi_{ij}(\tau_i)$ for $i=1,2,...,N$ and $j = \{\text{all integers}\}$. Although the recursive method for computing $\phi_{ij}(n)$, as defined by equation (A.1), is only valid for a Markov process with a finite number of states, by limiting the value of $\tau$, we may view the infinite state Markov process which defines the terminal's motion along the path as having a finite number of states. We define

$$\tau_{\text{max}} = \max_i \tau_i$$

As such, we note that for $i=1,2,...,N$

$$\phi_{ij}(\tau_i) = 0 \quad \text{for} \quad j > \tau_{\text{max}} + N \quad \text{or} \quad j \leq -\tau_{\text{max}}$$

Thus for our purposes, we need only consider the Markov process to have $N + 2\tau_{\text{max}}$ states, and hence we may use equation (A.1) to compute $\phi_{ij}(\tau_i)$.

There are, however, more efficient means for computing FTC and ATR other than first computing $\phi_{ij}(\tau_i)$ in this manner and then making a direct substitution into equations (5.5), (5.15), and (5.16). To see this, let us consider the network model illustrated in Figure 5-3 where $N = 11$ and a repeater is located every 5 states along the path. Let us suppose the restriction $\tau_{\text{max}} \leq 14$ has been imposed. Note from Figure 5-3 that state 0 and state 25 both represent the same position relative
to the repeaters whose transmission ranges encompass each. The same is true for states -1 and 24, -2 and 23, all the way up to and including states -13 and 12. After examining the methods for selecting a new relaying repeater (see section 5.5), one realizes that in so far as choosing a new relaying repeater is related to the computation of $\psi_{ij}$ in equation (5.5), these paired states are equivalent. That is, the term within the summation on the right hand side of (5.5) is the same for $k$ equal to either of the paired states. Thus the logical thing to do is to combine each of these paired states, but doing so in such a way that the values of $\phi_{ij}(T_i)$ (i,j=1,2,...,N) used directly in equations (5.15) and (5.16) are unaffected. We may do this by modifying the current Markov chain so that it is of the form illustrated in Figure C-1. The transition probabilities for moving one state in the clockwise direction, moving one state in the counterclockwise direction, and remaining in the current state are, respectively, $p$, $q$, and $1-p-q$. Note that as long as $T_{\text{max}} \leq 14$, the values of $\phi_{ij}(T_i)$ (i,j=1,2,...,N) used directly in equations (5.15) and (5.16) are the same for either the modified or unmodified Markov chain. This type of modification of the Markov chain can of course be made for any $T_{\text{max}}$, $N$, and repeater spacing. One must only make certain that the paired states are indeed equivalent and that $N_s$, the total number of states in the modified Markov chain, is greater than or equal to $N+T_{\text{max}}$. As a final note, in computing the n-step transition probability $\phi_{ij}(n)$ for a Markov chain of the form illustrated in Figure C-1, recognize that

$$\phi_{ij}(n) = \phi_{((i+k)\mod N_s)+1, ((j+k)\mod N_s)+1}(n) \quad i,j=1,2,...,N_s \quad k,n=0,1,2,...$$
nonpositive states of the unmodified Markov chain paired with the equivalent positive states to form the modified Markov chain.

Figure C-1  Modified Markov chain for $\tau_{\text{max}} \leq 14$, $N = 11$, and repeater spacing = 5s.
Thus to determine \( \phi_{ij}(n) \) for \( i,j=1,2,...,N_s \), one need only actually compute \( \phi_{ij}(n) \) for one value of \( i \) and \( j=1,2,...,N_s \).

**Computing \( E[TE_i] \)**

In section 5.5, one method discussed for choosing a new relaying repeater consisted of the terminal selecting the repeater for which the expected time to first exit that repeater’s transmission range is a maximum. In order to use this method, one must compute the conditional expectation \( E[TE_i] \), which is the expected time for the terminal to first exit a repeater’s transmission range, given that the terminal began while in state \( i \) (\( i=1,2,...,N \)) relative to that repeater. The following is an expression that may be used to compute \( E[TE_i] \):

\[
E[TE_i] = \begin{cases} 
\frac{1}{p-q} \left[ (N+1) \frac{1}{1 - (q/p)^{N+1}} - i \right] & p \neq q, \ i=1,2,...,N \\
\frac{1}{2p} i(N+1-i) & p = q, \ i=1,2,...,N 
\end{cases}
\]  

(C.1)

The derivation of this result may be found, with obvious modifications, in [15], pages 460-462.
REFERENCES


