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AN EMULATOR OF AN ENGINE-CAR SYSTEM
BY AN ENGINE-DYNAMOMETER SYSTEM

by

Wing Hong Lee

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
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AN EMULATOR OF AN ENGINE-CAR SYSTEM
BY AN ENGINE-DYNAMOMETER SYSTEM

by

Wing Hong Lee

Submitted to the Department of Electrical Engineering and
Computer Science on May 15, 1980 in Partial fulfillment of
the requirements for the Degree of Master of Science

ABSTRACT

A linear emulator is designed for an automobile using an engine-dynamometer. Linear quadratic theory is applied to achieve a type-2 servo. Linear simulations and robustness analysis are provided which indicate promising results for eventual implementation. This is achieved by breaking the design into three parts. A linear post-compensator is inserted after the engine making the engine-compensator pair equivalent to the engine-car system in the input-output relationship. Then around each of the engine-compensator and dynamometer, a LQ servo is built using the results developed for type-2 servo and the design is thus complete.

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TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>2</td>
</tr>
<tr>
<td>Acknowledgement</td>
<td>3</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>4</td>
</tr>
<tr>
<td>List of Figures</td>
<td>6</td>
</tr>
<tr>
<td>Chapter I: Introduction</td>
<td>8</td>
</tr>
<tr>
<td>1.1 Motivation</td>
<td>8</td>
</tr>
<tr>
<td>1.2 Defining the Problem Area</td>
<td>9</td>
</tr>
<tr>
<td>1.3 An Overview of the Thesis</td>
<td>13</td>
</tr>
<tr>
<td>Chapter II: Problem Statement</td>
<td>14</td>
</tr>
<tr>
<td>2.1 Preview of Problem Statement</td>
<td>14</td>
</tr>
<tr>
<td>2.2 Modelling of the Internal Combustion Engine</td>
<td>23</td>
</tr>
<tr>
<td>2.3 Modelling of the ECS</td>
<td>26</td>
</tr>
<tr>
<td>2.4 Modelling of the Dynamometer and the EDS</td>
<td>29</td>
</tr>
<tr>
<td>2.5 Incremental Form of the Problem Statement</td>
<td>32</td>
</tr>
<tr>
<td>Chapter III: An LQ Approach</td>
<td>35</td>
</tr>
<tr>
<td>3.1 Decoupling of Design</td>
<td>35</td>
</tr>
<tr>
<td>3.2 LQ Servo</td>
<td>38</td>
</tr>
<tr>
<td>Chapter IV: Numerical Results</td>
<td>48</td>
</tr>
<tr>
<td>4.1 Adding a Compensator</td>
<td>49</td>
</tr>
<tr>
<td>4.2 LQ Controller around the Engine-Compensator</td>
<td>49</td>
</tr>
<tr>
<td>4.3 LQ Controller around the Dynamometer</td>
<td>63</td>
</tr>
<tr>
<td>4.4 Overall Emulator Design</td>
<td>70</td>
</tr>
<tr>
<td>4.5 A Final Note on the Dynamometer State Feedback</td>
<td>72</td>
</tr>
<tr>
<td>Chapter V: Conclusions and Suggestions for Future Research</td>
<td>75</td>
</tr>
<tr>
<td>References</td>
<td>77</td>
</tr>
</tbody>
</table>
Appendices

A. Proof of Corollary of Proposition 3.1 79
B. Type-1 LQ Servo 80
C. Numerical Values of the Matrices $A_E, B_E$, and $C_E$ 82
D. Numerical Values of the matrices $A_D, B_D$, and $C_D$ 84
E. Unit Step and Unit Ramp Responses for type-1 LQ Servo around the Engine-Compensator 85
<table>
<thead>
<tr>
<th>Figure</th>
<th>Figure Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Functional Schema of Test-Cell 1</td>
<td>10</td>
</tr>
<tr>
<td>2.1</td>
<td>Illustrative Diagram for the Control Problem</td>
<td>15</td>
</tr>
<tr>
<td>2.2</td>
<td>Engine-Dynamometer System</td>
<td>17</td>
</tr>
<tr>
<td>2.3</td>
<td>Assumed Structure for ECS</td>
<td>20</td>
</tr>
<tr>
<td>2.4</td>
<td>Assumed Structure for EDS</td>
<td>22</td>
</tr>
<tr>
<td>2.5</td>
<td>Illustrative Diagram for the Problem Statement for Linear Emulator</td>
<td>33</td>
</tr>
<tr>
<td>3.1</td>
<td>3 Stages of Linear Emulator Design</td>
<td>36</td>
</tr>
<tr>
<td>3.2</td>
<td>P-I-I Structure for LQ Type-2 Servo</td>
<td>44</td>
</tr>
<tr>
<td>3.3</td>
<td>P-I-I-D Structure for LQ Type-2 Servo</td>
<td>45</td>
</tr>
<tr>
<td>4.1</td>
<td>Unit Response of Type-2 Servo for Engine-Compensator using (p,q) = (10, 100)</td>
<td>51</td>
</tr>
<tr>
<td>4.2</td>
<td>Unit Response of Type-2 Servo for Engine-Compensator using (p,q) = (10, 10000)</td>
<td>52</td>
</tr>
<tr>
<td>4.3a</td>
<td>$\delta N_c$ Response of Closed Loop LQ Type-2 Servo around Engine-Compensator to series of Ramp and Step Inputs $\delta N_c$</td>
<td>55</td>
</tr>
<tr>
<td>4.3b</td>
<td>Throttle Angle (Incremental) Generated by LQ Type-2 Controller in response to a series of Ramp and Step $\delta N_c$ Inputs</td>
<td>56</td>
</tr>
<tr>
<td>4.4</td>
<td>Structure of Full State Feedback LQ Type-2 Servo around Engine-Compensator</td>
<td>58</td>
</tr>
<tr>
<td>4.5</td>
<td>Bode Plots for LQ Type-2 Servo around Engine-Compensator using Full State Feedback</td>
<td>60</td>
</tr>
<tr>
<td>4.6</td>
<td>LQ Type-2 Servo with Observer around Engine-Compensator</td>
<td>62</td>
</tr>
<tr>
<td>4.7</td>
<td>LQ Type-2 Servo around Dynamometer with Full State Feedback</td>
<td>66</td>
</tr>
</tbody>
</table>
4.8 Bode Plots for LQ Type-2 Servo around Dynamometer using Full State Feedback 67
4.9 LQ Type-2 Servo with Observer around Dynamometer 69
4.10 Simulation of Linear Emulator using a Series of Ramp and Step $\delta N_c$ inputs 71
4.11 Simplified Version of Linear Emulator 74
CHAPTER I

INTRODUCTION

1.1 Motivation

This thesis is a study of designing a linear emulator for a simplified model of an engine-car system, using an engine-dynamometer set.

The long term goal of this study is to develop formal procedures by which, given a drive-train system $S$ composed of 1) an internal combustion engine, 2) an automatic transmission gear box, and 3) an aerodynamic car load driven by the internal combustion engine, one can emulate its behaviour by means of a Ward-Leonard-Motor-Generator set driven by an internal combustion engine with the same characteristics as the one in system $S$. This is part of the effort in improving the engine performance so as to meet the tightening Federal standard in fuel economy and emissions. Also it aims at expanding the capabilities of the various test-cells, which in general consist of engine-dynamometer configurations.

This study focuses on the emulation of a motor vehicle's behaviour by a test-cell consisting of an engine-dynamometer arrangement, and computational facilities. Conventionally the test-cells are used for doing experiments on the engine only and the complete drive-train and load road test simulation are carried out elsewhere on a chassis-dynamometer. Although these experiments are important on their own right, one must not ignore the significance of the correlation between the engine and the drivetrain. This is especially important when one tries to study and to make use of every motor vehicle's characteristic (including the engine's) at the same time in order to meet the stringent Federal regulations. There are two viable alternatives to achieve such goals. The first is to build a test cell around the chassis-dynamometer with a vehicle on it. This is costly to do, not to mention that one has to reconnect all the instrumentations in the test-cell every time a differ-
ent vehicle need to be tested. So we propose the second method: the idea is to incorporate a compensator-controller set into the test-cell engine-dynamometer set so that it can emulate the behaviour of a vehicle on a road. Thus experiments on the engine can be done more accurately and meaningfully. Given this capability, the scope of usefulness of the test-cell can be greatly expanded.

Test-cell 1 in the General Motors Research Laboratories was used for this study. It consists of an Oldsmobile 350 cu.in. internal combustion engine and a General Electric dynamometer of the Ward-Leonard Motor-Generator type. (More precise specifications of the engine and the dynamometer will be given later in Chapter II, in the sections that deal with modeling.) We remark that test-cell 1 has the largest computational and functional capabilities among the test cells in the General Motors Corporation.

Figure 1.1 displays the functional schema of the test-cell 1. With an abuse of language, the internal combustion engine will be referred to simply as the "engine", and the Ward-Leonard-Motor-Generator set as the "dynamometer". Also note that the engine is connected to the generator of the dynamometer through a rigid shaft (no transmission).

In Chapter II we shall present a precise problem definition for this study. Before doing so, we first provide a brief account on defining the scope and area of our problem.

1.2 Defining the Problem Area

Assuming that we are given the dynamics of an engine-car system (ECS) operating about some given set of nominal values of torque, speed, EGR, throttle angle, spark advance angle and air/fuel ratio, we want to emulate its dynamical behaviour with an engine-dynamometer system (EDS) in the sense that the operating characteris-
FIGURE 1.1 FUNCTIONAL SCHEMA OF TEST-CELL 1
tics are made to be "close" to those of the ECS. The precise meaning of the word "close" will become clear in the problem statement in Chapter II.

A cycle test run of a vehicle consists of starting the vehicle at rest and driving the vehicle through a schedule of different vehicle speeds and accelerations, and eventually bringing the vehicle to rest, thus completing a cycle. These cycle test runs are almost always used to check out the overall performance of a finished vehicle product. In particular, the most important cycle test runs are those specified by the Federal Environmental Protection Agency; called the EPA cycles. These cycle test runs are designed by the government to evaluate an automobile in terms of whether it conforms federal regulations.

We emphasize that these cycle test runs are usually made at the final stage of the design of a vehicle, or thereafter. No matter how unsatisfactory the outcome of the test would be, changes in the design would be extremely costly, or in most cases, impossible until the next model year. Therefore, it would be advantageous to the auto-industry if one can reproduce the same experiment in a test-cell before the vehicle is placed on a chassis-dynamometer, or even on the road, for a test run. Then the engine performance during these cycles can be closely examined with the aid of computers and other instrumentation.

Given a cycle or schedule of vehicle speeds, it is possible to specify uniquely the torque and speed outputs of its internal combustion engine. In other words, a map between the vehicle speed, the engine speed, and torque can be stored in a computer, and one can look up their relationship by simply executing a computer routine. This has been done by General Motors Research Laboratories (GMR) staff [18].

In the test-cell, the engine torque is measured by a Himmelstein torque meter,
and the engine speed is represented by the rotating speed of the shaft connecting the engine and the dynamometer; it is measured by a tachometer. Together, the engine torque and speed correspond to a unique vehicle speed.

Given the complex nature of an automobile, and in particular of the engine, it is impossible to reproduce all its dynamic characteristics in a test-cell. As an engineer, one is faced with the problem of choosing a few relevant variables for capturing its essential behavior. The most important variables for this task are the engine torque and engine speed. Together they describe most of the engine operating characteristics and they are also easily measured; thus, we can define the closeness in emulation as a function of the difference between the measurements and the desired levels of the variables. The most significant control variable for these two output variables is the throttle angle position. It is imperative that one should formulate a control problem in terms of these input-output variables.

It is important to note that the engine speed is basically driven by the engine torque via Newton's second law, a momentum equation. In the ECS, one can visualize that a driver steps on the throttle, changes the engine torque, which then changes the engine speed. Thus, any changes in the engine operating condition, e.g. misfiring, should be realized by the engine torque before the engine speed. In other words, changes in the engine speed cannot precede changes in the engine torque. This important relationship is fundamental to the vehicle dynamics and we must realize it while emulating them in a test-cell, i.e. EDS. In this regard, one should first impose the proper cause-and-effect relationship between the engine torque and speed in an EDS, and then generate the control command signal, based on some proper choice of reference signal, to the throttle angle actuator which
directly affects the engine torque. We shall formulate these ideas more precisely in the next chapter.

1.3 An Overview of the Thesis

After one specifies the proper relationship between the engine torque and engine speed, then tracking the engine speed would be equivalent to tracking the vehicle speed, for a given cycle. Thus the engine speed can be used as a reference signal when one wants to simulate a test run in the test cell. Roughly speaking, the design would consist of a compensator taking the engine torque as the input and producing engine speed as the output which is then used to track a reference signal by a controller. This is the subject of Chapter II, where we also discuss the modeling issues, and the precise problem statement. Of utmost importance are the assumptions contained in that chapter.

In Chapter III we propose a solution to this problem using linear quadratic control theory. It is noted that each EPA cycle consists approximately of a series of speed ramps and steps. It is important for the controller to have capability to track this kind of signals. In Chapter III we shall give a derivation of the controller, called a type-2 LQ controller, used in this study. It will be seen that the controller is of a PII (Proportional-Integral-Integral) structure.

In Chapter IV we present some numerical results, including simulations and robustness studies. It will be seen that a successful design has been achieved subject to the simplifying assumptions. Further suggested simplification of the design is also discussed based on these results.

In Chapter V, we draw some conclusions and discuss briefly the open areas for future work.
CHAPTER II

PROBLEM STATEMENT

In this chapter we formally define the notion of an emulator for an engine-car system, by an engine-dynamometer system. First we present a preview of the precise problem statement. Crucial assumptions for this study are listed. Then we describe dynamic models for the engine, drive-train and car load, dynamometer, and hence the engine-car system (ECS) and the engine-dynamometer system (EDS). Finally, we present statement of the control problem that is the subject of this thesis. In Chapter III we shall present a method to solve this control problem, and in Chapter IV, numerical simulations will be presented.

2.1 Preview of Problem Statement

Assuming that we are given the dynamics of an engine-car system (operating about some given set of nominal values of torque, speed, EGR, throttle angle, spark advance angle and air/fuel ratio), we want to emulate its dynamical behaviour with an engine-dynamometer system in the sense that the operating characteristics of the EDS are made to be close to those of the ECS.

The problem is illustrated in the block diagram of Figure 2.1.

In the diagram, the variables are defined as follows.

Let

\[ T_c(t) \] be the engine torque output of ECS
\[ T_d(t) \] be the engine torque output of EDS
\[ N_c(t) \] be the angular engine speed output of ECS
\[ N_d(t) \] be the angular speed output of EDS

and let
FIGURE 2.1 ILLUSTRATIVE DIAGRAM OF THE CONTROL PROBLEM
\( \alpha_c(t) \) be the throttle angle input to ECS
\( \alpha_d(t) \) be the throttle angle input to EDS

In addition to the throttle angle, there is another input signal to the ECS \( T_L(t) \), which represents the torque load in the ECS. \( S(t) \) is the speed setting input to the dynamometer in the EDS.

We shall describe and explain the dynamics involved in the figure in greater detail in the next section when we deal with models of the engine and dynamometer.

As can be inferred from the figure, for a control signal trajectory applied to the throttle actuator of the ECS, an engine torque is produced. This coupled with a given torque load trajectory to the ECS produces an engine angular speed. Our objective is to generate two signals for the EDS: 1) the control signal throttle angle \( \alpha_d(t) \), applied to the throttle actuator of the EDS; 2) the speed setting signal \( S(t) \) which sets the steady state dynamometer speed, such that \( T_d(t) \) and \( N_d(t) \) take values close to \( T_c(t) \) and \( N_c(t) \) of the ECS, respectively. This is shown in Figure 2.1, which contains a finer representation of the EDS. We remark that \( S(t) \) adjusts a variable electromagnetic field control through a gain and a servo. This field control then affects the dynamics of the dynamometer speed \( N_d(t) \) (in generator unit). \( S(t) \) is calibrated so that a unit change in the steady state values of \( S(t) \) corresponds to a unit change in the steady state values of \( N_d(t) \).

We propose to achieve this by designing a compensator (see Figure 2.1) which takes as inputs some or all of \( T_c(t) \), \( T_L(t) \), \( N_c(t) \), \( T_d(t) \) and \( N_d(t) \) and produces as outputs \( \alpha_d(t) \) and \( S(t) \).

From now on we shall impose linear assumptions on the dynamics.* Linear

* It is assumed that the ECS and EDS are both operating about a set of nominal steady state values (of torque, EGR, speed, throttle angle, air/fuel ratio, intake manifold pressure etc.) such that variations in their dynamics are small and can be represented (approximately) by linear models.
Fig. 2.2 ENGINE-DYNAMOMETER SYSTEM
models are given for the engine and dynamometer in sections 2.2-2.4. Then a
precise problem statement is given in terms of these linearized dynamics and
linear compensators in section 2.5. There we shall also discuss the requirements
for closeness between the pairs \( T_c(t), N_c(t) \) and \( T_d(t), N_d(t) \).

In Chapter III we propose to solve this control problem by linear quadratic
theory, and by decoupling the compensator design into two parts, one for the
engine, and the other for the dynamometer.

Linearity is chosen for its simplicity, and linear models for combustion
engines and Ward-Leonard motor-generator sets are readily obtained (i) by direct
frequency measurements or (ii) from other literature studies. Moreover, it paved
our way to apply linear quadratic control theory [14], [15], [16] in the synthesis.
LQ theory is a simple and well-established modern control theory in the time domain.
It is easy to implement since the controller contains only feedback gains on the
state variables. Even small computers or microprocessors have enough capability to
carry out the implementations in many applications since computations for the
synthesis of the control gains are done offline. The only online computations
involve mostly multiplications and additions and sometimes integrations.

We shall see later in Chapter III that the decoupling of the compensators
are strongly motivated by the phasing requirement between the engine torque
and engine speed. We also remark that during an EPA cycle, a car is scheduled to
be driven at certain speeds for certain ranges. One can really view the human
driver as a closed-loop controller, trying to adjust the vehicle speed to that of
the schedule. So in loose terms, what we really want to do is to replace the
car load by a dynamometer load and linear compensators, and the human driver
by an automatic driver. Note that in a sense we are decoupling the design into
two parts: one on the dynamometer, and the other on the "car" — a post-compensated
engine. While it maintains the proper phasing between engine torque and engine speed, this decoupling technique also has many other advantages (e.g. flexibility, adaptability).

**Some basic assumptions and modeling:**

Some assumptions are in order. Partly this is due to engineering simplification and partly this is due to the preliminary nature of this research. These assumptions are summarized below. They are all physical approximations, enabling us to use and justifying the use of linearized time-invariant dynamic systems in the control system design.

It is assumed that the throttle actuator dynamics are included in the engine dynamics and that the sensor dynamics are so fast that they can be ignored. We remark that these are good assumptions since the actuator dynamics is responsible for part of the time-delay in the dynamics between the throttle angle and the engine torque. Furthermore, the sensors consist mainly of electrical components, hence their dynamics are much faster than the dynamics of the engine. The rigid shaft assumption for the ECS and EDS is acceptable since the angles between the three components of the shafts are small. Hence the effects of the universal joints can be ignored. The nominal steady state requirement is met in many applications.

(i) Assumptions on ECS:

The structure in Figure 2.3 is assumed for the ECS. It is assumed that the engine drives the dynamic load via a rigid nondeformable shaft; that the engine torque and speed are accurately measured and the sensors involve no dynamics; that the engine is operating about a set of nominal steady state values (of torque, speed, EGR, throttle angle, spark angle, indicated intake manifold pressure,
Fig. 2.3  ASSUMED STRUCTURE OF ECS
(See also Fig. 2.4.)
air/fuel ratio); that there are no stochastic effects, that is, no unknown disturbances; that the system is operating in a constant environment, typically characterized by atmospheric conditions and altitude, road conditions, etc. The torque load is assumed to be an algebraic function of the vehicle speed (typically a polynomial of order not greater than 3)[18]. Also the vehicle speed is an algebraic function (linear) of the engine torque and speed. This means that the engine torque and speed can be represented by an input-output relationship.

The above assumptions justify the use of a linearized time-invariant deterministic model to represent the ECS. We remark that these assumptions imply that the dynamic effects induced on the ECS due to transmission gear shifting are ignored, or rather, that there is no gear shift.

(ii) Assumptions on EDS

The structure in Figure 2.4 is assumed for the EDS. It is assumed that the engine drives the dynamometer load via a rigid non-deformable shaft; that the speed setting control of the dynamometer sets the steady state engine speed; the engine torque and speed are accurately measured, by a Himmelstein torque meter and a tachometer, respectively, and no dynamics are involved in the sensor measurements; that the system is operating about a set of nominal values (of torque, speed, EGR, throttle angle, spark angle, indicated intake manifold pressure, air/fuel ratio).

Note that in the test-cell, i.e. the EDS, the engine is started by first running the dynamometer so that the generator in the motor-generator set starts and drives the engine to a certain desired speed, say, the nominal speed; then their roles are reversed, i.e. the engine then drives the generator unit, hence,
Fig. 2.4 Assumed Structure for EDS
(See also fig. 2.2 & 2.3)
the dynamometer set. It is about the latter operating phase that this research is centered around.

(iii) Overall Assumptions:

Note that in subsections (i) and (ii) above, both shafts are assumed to be rigid and non-deformable. In reality, they are each composed of three sections connected by universal joints and rubber couplings. It is known that universal joints introduce variations in the angular velocity of the driven shaft at a frequency that is twice the rotating frequency of the driving shaft (see [1]), but we choose to ignore their effects initially, with the understanding that a filter on the measurements might eventually have to be incorporated. Otherwise a time-varying model would have to be used.

The two sets of nominal operating conditions are assumed to be the same for both the ECS and EDS. This has to be the case for the design to make sense.

Last, but definitely not least important, it is assumed that the engines in the ECS and EDS have the same operating characteristics, that is, the same dynamical representation. The engine torque output depends only on the engine variables and is independent of the load in both cases. This permits and partly motivates us to decouple the design into a compensator for the engine, and another one for the dynamometer.

2.2 Modeling of the Internal Combustion Engine

In the last two years, researchers at General Motors Research Laboratories have reported a few realistic models for the 350 cu.in. Oldsmobile internal combustion engine used in Test cell 1. Cassidy [2] reported a 16 states, 4 controls models which describe the main characteristics of the engine with a drivetrain attached on it. The model was based partly on physical considerations and partly on
frequency domain measurements. Also available are engine transfer functions at
600 r.p.m. and 2000 r.p.m., based on frequency measurements by Fruechte and
Kade [3]. A more recent dynamic model has been developed by Lewis [4] and Lewis
and Cassidy [5] who developed an alternate linear state model to [2]. This model
is particularly suitable for studying variations around a certain nominal constant
speed.

It was decided that a reduced version of the dynamic model reported in [4],
[5] was adequate for the purpose of this study. It represents an engine model
which takes the throttle angle as input and the engine torque thus produced as
the output, around 1200 r.p.m. (which corresponds approximately to 30 m.p.h.).
In transfer function form, it is given as follows.

Assuming that the engine-car system operates about an angular engine speed
of 1200 r.p.m., there is correspondingly a set of nominal values for the engine
torque, engine speed, EGR, throttle angle, spark advance angle and air/fuel ratio.
Let

\[ \alpha(t) \text{ be the throttle angle input to the engine (in\%, 100\% is full throttle)} \]
\[ T(t) \text{ be the engine torque output of the engine (in N.m.)} \]

and let \( \alpha, T \) denote the nominal values of the throttle angle, and engine torque,
respectively. Then define the incremental variables,

\[ \delta T(t) = T(t) - T \quad (2.1) \]
\[ \delta \alpha(t) = \alpha(t) - \alpha \quad (2.2) \]
The reduced version of the linearized time invariant dynamics of the engine can then be represented by the transfer function (assuming that EGR, spark advance angle, air/fuel ratio are fixed during the operation).

\[
G_e(s) = \frac{\delta T(s)}{\delta \alpha(s)} = \frac{K_e e^{-s\tau}}{s + p_e}
\]  

(2.3)

where

\[
\tau = 0.035 \text{ sec.} \quad (2.4)
\]

\[
p_e = 23.2 \quad (2.5)
\]

\[
K_e = p_e \times 4.83 \quad (2.6)
\]

Equations (2.3) says that the engine basically is a first order system with a time-delay. The pole at 23.2 is due to the dynamics of manifold pressure.

To translate the transfer function, eq. (2.3), into a linearized lumped state representation, a Pade approximation is used. It is decided to use the second order Pade approximation

\[
e^{-s\tau} \approx \frac{12 - 6\tau s + \tau^2 s^2}{12 + 6\tau s + \tau^2 s^2}
\]  

(2.7)

because of favorable reports in simulation studies [4], [5].

After some arrangement, we arrived at the following linearized version
of the dynamics of the engine

\[
\begin{bmatrix}
\dot{x}_3(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t)
\end{bmatrix} =
\begin{bmatrix}
-P_e & K_e & 0 \\
0 & 0 & 1 \\
0 & -12/\tau^2 & 6/\tau
\end{bmatrix}
\begin{bmatrix}
x_3(t) \\
x_4(t) \\
x_5(t)
\end{bmatrix} +
\begin{bmatrix}
K_e \\
-12/\tau \\
72/\tau^2
\end{bmatrix}
\delta\alpha(t) \quad (2.8)
\]

\[
\delta T(t) =
\begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_3(t) \\
x_4(t) \\
x_5(t)
\end{bmatrix} \quad (2.9)
\]

Equations (2.8) and (2.9) will represent the dynamics of the engine in the remainder of this thesis.

2.3 Modeling of the ECS

Assuming the same set of nominal values in section 2.2, let \( N(t) \) be the angular engine speed output of the engine (in r.p.m) and \( N \) its nominal value; define the incremental (angular) engine speed by

\[
\delta N(t) = N(t) - N \quad (2.10)
\]

The engine-car system is assumed to be a cascade of the engine and the drivetrain. Assuming that there is no gear shift, and that the dynamic load resistances are known, one can then interpret the ECS shown in Figure 2.3 in terms of linearized
dynamics. The incremental throttle angle input to the engine produces an incremental engine torque which then produces an incremental engine speed. The modeling of the engine has been discussed in section 2.2. The modeling of the rest of the ECS is given by the transfer function [6],

\[
G_c(s) = \frac{\delta N(s)}{\delta T(s)} = \frac{K_c(s + z_c)}{(s + p_{c1})(s + p_{c2})}
\]

(2.11)

where

\[
K_c = 4.439 \\
z_c = 2\pi(0.2) \\
p_{c1} = 2\pi(0.0051) \\
p_{c2} = 2\pi(1.7)
\]

where the poles correspond roughly to the drag/inertia and the slippage factor in the transmission. In linear state representation, we can write

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-p_{c1}p_{c2} & -(p_{c1} + p_{c2})
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} +
\begin{bmatrix}
K_c \\
K_c(z_c - p_{c1} - p_{c2})
\end{bmatrix} \delta T(t)
\]

(2.12)

\[
\delta N(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\
x_2(t) \end{bmatrix}
\]

(2.13)
thus a full state representation (for the ECS) would be

\[
\dot{x}_E(t) = A_E x_E(t) + B_E \delta \alpha(t) \quad (2.14)
\]

\[
\delta N(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} x_E(t) \quad (2.15)
\]

\[
C_E
\]

where

\[
x_E(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ \vdots \\ \vdots \\ x_5(t) \end{bmatrix} \quad (2.16)
\]

\[
A_E = \begin{bmatrix} 0 & 1 & K_c & 0 & 0 \\ -p_{c1}p_{c2} & -(p_{c1}+p_{c2}) & K_c(z_c-p_{c1}-p_{c2}) & 0 & 0 \\ 0 & 0 & -p_e & K_e & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -12/\tau^2 & -6/\tau \end{bmatrix} \quad (2.17)
\]

and
by combining eqs. (2.8), (2.9), (2.12) and (2.13)

2.4 Modeling of the Dynamometer and the EDS

It is assumed that the two systems (ECS and EDS) have the same nominal trajectories (see sections 2.2 and 2.3) and the engines used in the two systems have the same models. Therefore it suffices to give the dynamics of the Ward-Leonard motor-generator in order to describe the dynamics of the EDS, given the engine model described in section 2.2.

It is possible to write down dynamical equations for the Ward-Leonard motor-generator set from basic principles in physics. We elect, however, to use the model presented [3] since it is sufficient for our purpose.

For practical purposes, it is assumed that the shaft between the engine and the dynamometer is rigid and nondeforming, thus the engine speed is equivalent to the dynamometer speed. Technically this speed is affected by both the torque developed in the shaft by the engine and the speed setting control for the dynamometer. However, the speed setting is much more dominant than the torque, whose effects can thus be neglected in the dynamometer.

Let \( S(t) \) be the speed setting (in r.p.m.) of the dynamometer and \( S \) be its corresponding nominal value; define the incremental speed setting by
\[ \delta S(t) = S(t) - S \]  
(2.19)

The dynamics of the dynamometer are then represented by a fifth order system [3]

\[ \frac{\delta N_d(s)}{\delta S(s)} = \frac{K_d}{(s + p_d) (s^2 + 2\zeta_1 \omega_1 s + \omega_1^2) (s^2 + 2\zeta_2 \omega_2 s + \omega_2^2)} \]  
(2.20)

where

\[ p_d = 2\pi (2.39) \]
\[ \zeta_1 = 0.277 \]
\[ \omega_1 = 2\pi (1.98) \]
\[ \zeta_2 = 0.362 \]
\[ \omega_2 = 2\pi (8.63) \]
\[ K_d = p_d \omega_1 \omega_2 \]

and \( N_d(t) \) is the dynamometer speed (again in r.p.m.). The incremental dynamometer speed is taken as the difference between \( N_d(t) \) and the nominal value \( N \) of the engine speed.

\[ \delta N_d(t) = N_d(t) - N \]  
(2.22)

Note that in steady state, one should have
\[ \delta N_{d_{ss}} = \delta S_{s_{ss}} \]  

(2.23)

the incremental dynamometer speed equal to the incremental speed setting. One can, of course, represent this transfer function by a linear time-invariant state equations of the form

\[ \dot{X}_D(t) = A_D X_D(t) + B_D \delta S(t) \]  

(2.24)

\[ \delta N_D(t) = C_D X_D(t) \]  

(2.25)

where matrices $A_D$, $B_D$ and $C_D$ are the following:

\[
A_D = \begin{bmatrix}
-p_d & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\omega_1^2 & -\omega_1^2 & -2\zeta_1\omega_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & \omega_2^2 & 0 & -\omega_2^2 & -2\zeta_2\omega_2
\end{bmatrix}
\]  

(2.26)

\[
B_D = \begin{bmatrix}
p_d \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  

(2.27)
\[ C_D = [0 \ 0 \ 0 \ 1 \ 0] \]  \hspace{1cm} (2.28)

The numerical values of the matrices \( A_D \), \( B_D \) and \( C_D \) are given in Appendix D.

2.5 Incremental Form of the Problem Statement

Suppose that we are given the incremental trajectory of the engine speed of an automobile, denoted by \( \delta N_c(t) \), operating about a set of nominal values of engine torque, engine speed, EGR, throttle angle, spark advance angle and air/fuel ratio, our task is to emulate the automobile in the test-cell about this same set of operating conditions.

In the test-cell, the engine is started by first activating the dynamometer, then through the torque generated by the generator in the dynamometer, and appropriate calibrations, the engine can be driven by the shaft to a steady state corresponding to the same set of operating conditions of the automobile. Then the roles of the engine and dynamometer are reversed; the engine drives the dynamometer as if it is a load. It is about this nominal trajectory that this problem is defined.

Given the linearized models and the notations in section 2.2 - 2.4 and \( \delta N_c(t) \) described above, we are required to find a linear compensator (plus controllers) (Figure 2.5), which takes \( \delta N_c(t) \), \( \delta N_d(t) \), and \( \delta T(t) \) as inputs and produces as outputs \( \delta a(t) \) and \( \delta S(t) \) such that \( \delta N_d(t) \) is made to be close to \( \delta N_c(t) \), i.e. \( \delta N_d(t) \) tracks \( \delta N_c(t) \). In particular, one should have zero steady-state error, i.e.

\[ \delta N_d^{ss} - \delta N_c^{ss} = 0 \]  \hspace{1cm} (2.29)
Fig. 2.5 ILLUSTRATIVE DIAGRAM FOR THE PROBLEM STATEMENT FOR LINEAR EMULATOR
Note that, as we have remarked in section 2.1, this tracking of the scheduled incremental engine speed is equivalent to tracking the incremental vehicle speed of the car, and thus its characteristics are emulated. Moreover, the definition of closeness is part of the problem. In the LQ formulation, this is guaranteed by a quadratic penalty term on the error.
CHAPTER III

AN LQ APPROACH

Part of the purpose of this thesis is to investigate the applicability of LQ theory [14], [15], [16] to this emulation problem. In this chapter we show how one can apply the LQ formulation to construct a linear emulator for the problem, which has been described in detail in Chapter II. First of all, we utilize the assumptions and models in Chapter II to describe a decoupled design. Then LQ theory is applied to each of the two parts in the decoupled design. In each case, a type-2 servo is proposed. Numerical results are given in Chapter IV.

3.1 Decoupling of Design

It is desirable to divide the complete compensator design into three parts, as shown in Figure 3.1.

First a linear compensator with transfer function \( H(s) = G_c(s) \) (eq.[2.11]) is inserted after the engine. It takes the incremental engine torque \( \delta T(t) \) as the input and produces an incremental compensator generated engine speed \( \delta N_g(t) \). Then a LQ controller is used to track this \( \delta N_g(t) \) with respect to the reference signal \( \delta N_c(t) \). Next another LQ controller is used to track the incremental dynamometer speed \( \delta N_d(t) \) with respect to the speed \( \delta N_g(t) \). We discuss the reasons for these procedures in the following.

(i) Insertion of the compensator \( H(s) \): It was proposed in section 2.3 that an engine-car system can be modeled as a series combination of an engine (transfer function) and a car load (transfer function) \( G_c(s) \), where the incremental engine torque serves as the output of the engine and as the input to the car load. Moreover, by our assumptions on the ECS and EDS, the engines in both systems have the same transfer function. Therefore, by post-compensating the engine in the EDS by a compensator \( H(s) = G_c(s) \), which uses the incremental engine torque \( \delta T(t) \) from
the EDS as the input (and produces as output an incremental compensator generated engine speed \( \delta N_g(t) \)), it is seen that the engine-compensator series combination has the same transfer function as the ECS, i.e., it exhibits the same input-output relationship from the incremental throttle angle \( \delta \alpha(t) \) to \( \delta N_g(t) \) as the input-output relationship from the incremental throttle angle to the incremental engine speed in the ECS.

(ii) **A sequence of two servos:** We propose to incorporate two servos in a sequence. The first one is built around the engine-compensator such that the incremental engine speed of the ECS, \( \delta N_c(t) \), is the reference input to this servo, which causes the incremental compensator generated engine speed \( \delta N_g(t) \) to track the input \( \delta N_c(t) \). Then a second servo is used to follow \( \delta N_g(t) \) by the incremental engine speed of the EDS, \( \delta N_d(t) \), which is the actual engine speed in the test cell. In this way, we can infer that the two servos together cause \( \delta N_d(t) \) to track \( \delta N_c(t) \). Hence, in the vicinity of the given nominal conditions, the engine characteristics in performance are properly reproduced (see section 2.1).

Apart from the fact that this approach allows us to work with smaller and simpler and more tractable design tasks, it has an important implication. Note that we are essentially decoupling the design so that the engine and the dynamometer are treated as two different entities interconnected by a linear compensator. The compensator generated engine speed is due to the engine torque directly. Hence for any changes in the throttle angle, or more generally in the engine (e.g. misfiring), the engine torque should realize a change before the compensator generated speed. Moreover, this compensator generated speed serves as the reference input to a servo around the dynamometer, to be tracked by the dynamometer speed, which
is the actual engine speed in the EDS. Therefore the engine torque should precede
the engine speed when there is any flutuation in the engine.

We remark further that when the LQ theory derived in the next section is
applied to each of the two controllers, the dynamometer speed will track the given
reference engine speed (if it is a type-2 signal) without steady state error. This
is true since controller 1 will cause $\delta N_g(t)$ to track $\delta N_c(t)$ without steady state
error, and controller 2 will cause $\delta N_d(t)$ to track $\delta N_g(t)$ without steady state
error. Together the error $\delta N_c(t) - \delta N_d(t)$ goes to zero in steady state.

In this application, type-2 (LQ) servos* are used for both controllers 1 and
2. In reality, one is more flexible in his choice for controllers. For example,
if one wants to test how a given trajectory of the throttle angle will affect the
engine performance in an automobile, one can merely replace controller 1 by an
automatic driver which simply generates the given trajectory of throttle angle
as scheduled. As another application, one can actually use a human driver as
controller 1. Moreover, at the end of Chapter IV, we shall discuss how one can
eliminate controller 2. This flexibility resulted from the decoupling is certainly
an advantage in establishing the choice of this design, apart from the phasing
requirement between the engine torque and speed.

3.2 LQ Servo

The controllers 1 and 2 in Figure 3.1 are type 2 LQ servos. In this section
we present the derivations of such a controller. It is well known how one can
derive a type-1 servo from LQ theory [7], [8], [9], [17]. The type-2 servo given
here is an extension of these results, and the generalization to general type-$\ell$
system is straightforward. So is the discrete time version. This servo consists

* A type 2 servo will track a ramp command input with zero steady-state error.
of two integrators and feedback gains on the state variables.

Type-2 servos are used since typically a complete EPA cycle (of vehicle speed, and hence engine speed) consists of a series of ramps and steps. It will be demonstrated in Chapter IV that a type-1 system is not adequate in the sense that too large a steady state error might be obtained when a steep ramp input is applied. We remark also that this type-2 servo controller derived from LQ theory gives almost as good responses to step input as those given by LQ type-1 controller, even though an additional integrator is used. As in the case for type-1 system, there are two configurations in which this controller can be implemented. One is a P-I-I controller, and the other, P-I-I-D controller. Of course when full state feedback is available, these two configuration are identical. However, they are somewhat different when estimation of the states is required, since the feedback gains are different in the two configurations. In either case, implementation is not a problem. In Chapter IV, we shall show the results obtained by using the P-I-I structure of the controller.

Proposition 3.1 Suppose that we are given a linear time-invariant dynamical system in continuous time

\[ \dot{x}(t) = Ax(t) + Bu(t) \]  
\[ y(t) = Cx(t) \]

where \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \), \( y(t) \in \mathbb{R}^m \), and \( A, B, C \) are \( n \times n \), \( n \times m \), \( m \times n \), real constant matrices respectively. Assume also that \([A,B]\) is completely controllable and \([A,C]\) completely observable, and that the \((n + m) \times (n + m)\) matrix

\[
\begin{bmatrix}
    A & B \\
    C & D
\end{bmatrix}
\]

is invertible. Let
\[
S = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^{-1}
\] (3.3)

where \(S_{11}, S_{12}, S_{21}, S_{22}\) are dimensions \(n \times n, n \times m, m \times n, m \times m\) respectively.

Define a cost functional

\[
J(u) = \int_0^\infty \left[ (\dot{y}(t) - \dot{y}_o)^T P (\dot{y}(t) - \dot{y}_o) + (y(t) - y_o(t))^T Q (y(t) - y_o(t))
+ \ddot{u}(t) R \ddot{u}(t) \right] dt
\]

where \(y_o(.)\) is an arbitrary ramp input, i.e., \(\dot{y}_o\) is an arbitrary constant and \(P, Q\) are \(n \times n\) positive semi-definite matrices, and \(R\) is a \(m \times m\) positive definite matrix. Then there exists a control time function such that the performance index \(J\) is minimized. The inputs are tracked with zero steady state error for all ramp and step inputs and the closed loop system is stable. The second time derivative of the control \(u(t)\) is given by

\[
\ddot{u}(t) = -G_1(x(t) - x_o(t)) - G_2(u(t) - u_o(t)) - G_3(\dot{u}(t) - \dot{u}_o(t))
\] (3.5)

where \(x_o(t)\) is the state trajectory corresponding to the given input, and \(u_o(t)\) is the corresponding control.

\[
\dot{x}_o(t) = Ax_o(t) + Bu_o(t)
\] (3.6)

\[
y_o(t) =Cx_o(t)
\] (3.7)
The gain matrices $G_1$, $G_2$ and $G_3$ are found from the positive definite solution of the algebraic matrix equation.

$$\dot{\dot{K}} A + \dot{K}' \dot{\dot{K}} - \dot{K} B R^{-1} \dot{B}' \dot{K} + \dot{Q} = 0$$

(3.8)

where

$$K = \begin{bmatrix} A & B & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}$$

(3.9)

$$I^* = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$$

(3.10)

$$Q^* = \begin{bmatrix} A' C' P C A + C' Q C & A' C' P C B & 0 \\ B' C' P C A & B' C' P C B & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3.11)

and

$$[ G_1, G_2, G_3 ] = R^{-1} \dot{B}' \dot{K}$$

(3.12)

**Corollary:** The control time function can be written as
\[ u(t) = L \int_0^t \int_0^\sigma (y_\sigma(\tau) - y(\tau)) \, d\tau \, d\sigma + M \int_0^t (y_\sigma(\tau) - y(\tau)) \, d\tau + N x(t) + f(t) \]  

(3.13)

where \( L, M, \) and \( N \) are constant matrices of dimensions \( m \times m, m \times m \) and \( n \times n \) respectively, and \( f \) depends on the initial conditions \( x(0) - x_0(0), u(0) - u_0(0), \) and \( \dot{u}(0), \)

\[
f(t) = (F + tH) \times \begin{bmatrix} x(0) - x_0(0) \\ u(0) - u_0(0) \\ \dot{u}(0) \end{bmatrix} \]  

(3.13a)

where \( F, H \) are some constant \( m \times (n + 2m) \) matrices.

The matrices \( L, M, \) and \( N \) are given by

\[
L = \begin{pmatrix} G_1 S_{12} + G_2 S_{22} \end{pmatrix} \\
M = \begin{pmatrix} G_1 S_{11} + G_2 S_{21} \\ S_{12} + G_3 S_{22} \end{pmatrix} \\
N = -\begin{pmatrix} G_1 S_{11} + G_2 S_{22} \\ S_{11} + G_3 S_{21} \end{pmatrix} 
\]

We shall conclude this chapter by giving the proof of this proposition. Before we do so, we give a few remarks concerning the theorem.

Remarks:

(i) The proposition minimizes \( J \) over an infinite horizon and imposes no restriction on \( u(0) \) and \( \dot{u}(0) \), or the initialization of the integrators. Zero steady
state error will always be achieved. Generally speaking, one wants to impose continuity of \( u(t) \) and \( \dot{u}(t) \) even at time 0 to reduce the possibility of rate saturation of the actuator in actual implementation.

(ii) The main proposition gives a P-I-I structure of the controller, and the corollary, a P-I-I-D structure. They are shown in Figure 3.2 and 3.3 respectively.

(iii) Note that both structures require feedback of the states, but with different gains. Therefore, when estimates of the states are used, they generally produce different results in the two controllers. In either case, the implementation would require 2 integrators in the forward loop and some constant feedback and/or feedforward gains.

(iv) The invertibility and controllability and observability assumptions guarantee the uniqueness and existence of \( x_0(t) \) and \( u_0(t) \) in the proposition (eqs. (3.6), (3.7)).

(v) As shown in Figure 3.1, we discussed in Section 3.1 that we need to design two servo controllers 1 and 2. This is achieved by applying Proposition 3.1 to each of these two designs. The P-I-I structure is used. Details of such are given in Chapter IV. Since full state feedback is not available, a suboptimal scheme using an observer for estimating the states is used. Good results are obtained.

(vi) One can incorporate in the cost functional quadratic terms involving \( u(t) \) and \( \dot{u}(t) \), but this is usually not necessary in studying the tradeoffs.

(vii) It is straightforward to extend the results to include more general type-L inputs, and/or discrete time systems.
Fig. 3.2 P-I-I STRUCTURE FOR LQ TYPE-2 SERVO
Fig. 3.3 P-I-I-D STRUCTURE FOR LQ TYPE-2 SERVO
Proof of Proposition 3.1 Let \( \dot{y}_o(t) = a \), constant vector

Then

\[
\begin{bmatrix}
\dot{x}_o(t) \\
\dot{u}_o(t)
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{x}_o(t) \\
\dot{y}_o(t)
\end{bmatrix}
\]  

(3.14)

In particular

\[
x_o = S_{11} \dot{x}_o + S_{12} y_o
\]  

(3.15)

Differentiating, and noting that \( \dot{x}_o \) must be a constant by the controllability and observability assumptions, we get

\[
\dot{x}_o = S_{12} a
\]  

(3.16)

Therefore

\[
\begin{bmatrix}
x_o \\
u_o
\end{bmatrix} =
\begin{bmatrix}
S_{11}S_{12}a + S_{12} y_o \\
S_{21}S_{12}a + S_{22} y_o
\end{bmatrix}
\]  

(3.17)

Thus \( x_o, u_o \) are determined.

Now eqs. (3.1), (3.2) and (3.4) can be written as
\[
\frac{d}{dt} \begin{bmatrix} x - x_o \\ u - u_o \\ \hat{u} - \hat{u}_o \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x - x_o \\ u - u_o \\ \hat{u} - \hat{u}_o \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \dot{u} 
\]
\[ (3.18) \]

\[
\begin{bmatrix} y - y_o \\ \dot{y} - \dot{y}_o \end{bmatrix} = \begin{bmatrix} C & 0 & 0 \\ CA & CB & 0 \end{bmatrix} \begin{bmatrix} x - x_o \\ u - u_o \\ \hat{u} - \hat{u}_o \end{bmatrix} 
\]
\[ (3.19) \]

\[
J = \int_0^\infty \begin{bmatrix} x - x_o \\ u - u_o \\ \hat{u} - \hat{u}_o \end{bmatrix}^T \begin{bmatrix} x - x_o \\ u - u_o \\ \hat{u} - \hat{u}_o \end{bmatrix} + \dot{u}^T R \ddot{u} \, dt 
\]
\[ (3.20) \]

where \( \tilde{Q} \) is given by eq. (3.11). Then LQ optimal regulator theory gives the optimal acceleration of \( u \) as

\[
\ddot{u} = -G_1 (x - x_o) - G_2 (u - u_o) - G_3 (\hat{u} - \hat{u}_o) 
\]

where \( G_1, G_2, G_3 \) are determined by eqs. (3.8) - (3.12). And the proposition is proved.

The proof of the corollary is given in Appendix A.

In Appendix B, the corresponding results for type-1 system is presented for reference purposes.
CHAPTER IV

NUMERICAL RESULTS

In this chapter, we apply the ideas and results in Chapter III to the linear models we described in Chapter II, to design linear emulator for the ECS using the EDS. We do this by dividing the design task in several steps.

Step 1: A compensator \( H(s) = G_c(s) \) is connected in series with the engine, as discussed in Section 3.1. Thus the transfer function of the engine-compensator would look like the transfer function of the ECS, i.e. an automobile. This compensator consists of a zero and 2 poles, and can be properly implemented by the computation facilities, by generating an output trajectory for \( \dot{\omega}_g(t) \) (compensator generated engine speed) when an incremental throttle angle \( \delta \alpha(t) \) is applied at the engine.

Step 2: \( \dot{\omega}_g(t) \) tracks \( \dot{\omega}_c(t) \) by using a LQ type-2 servo as derived in proposition 3.1. Use of an observer to reconstruct the engine states that cannot be measured is investigated.

Step 3: \( \dot{\omega}_d(t) \) then tracks \( \dot{\omega}_g(t) \) by using a second LQ type-2 servo (by applying proposition 3.1 again). Use of an observer to reconstruct unavailable states is investigated.

Step 4: Combine all the above steps resulting into a linear emulator for the ECS.

Step 5: Discuss on the basis of numerical results what improvements can be made on this design.

We shall evaluate these procedures by simulations and robustness analysis as appropriate. Favorable results are obtained.
4.1 Adding a Compensator

A compensator $H(s)$ is connected to the engine that uses the incremental engine torque as the input and produces an output, compensator generated engine speed, $\delta N_g(t)$.

$$H(s) = G_c(s)$$

(4.1)

where $G_c(s)$ is given by eq. (2.11)

This compensator corresponds to the car-load in the ECS. Thus the engine-compensator is equivalent to the ECS in the input-output characteristics, and converts to the same linear state space representation (eqs.(2.14) - (2.18)).

$$\dot{x}_E(t) = A_E x_E(t) + B_E \delta \alpha(t)$$

(4.2a)

$$\delta N_g(t) = \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array} \right] x_E(t)$$

(4.2b)

Numerical values for the matrices $A_E$ and $B_E$ are given in Appendix C.

4.2 LQ Controller around the Engine-Compensator

Our next step is to design a servo controller which can track ramp and step inputs around the engine-compensator, so that $\delta N_c(t)$ can be tracked by $\delta N_g(t)$ using the incremental throttle angle $\delta \alpha(t)$ as the control.

First we evaluate the performance of full state feedback design by applying proposition 3.1 directly. Then since the only available states and measurements are the two states associated with the compensator, which are readily
obtained from the computer, and the engine torque from the Himmelstein torque meter, an observer scheme is required to estimate the two states associated with the time delay. (Recall that a second order Padé approximation has been used in the linearization). We shall evaluate the design again after an observer is incorporated into it.

A quick check shows that \([A_E, B_E]\) is controllable and \([A_E, C_E]\) is observable. Also

\[
\begin{bmatrix}
A_E & B_E \\
C_E & 0
\end{bmatrix}
\]

is invertible. Therefore all assumptions required in the derivations of proposition 3.1 are satisfied.

We define a cost functional

\[
J_E = \int_0^\infty \left( (\delta N_c(t) - \delta N_q(t))^2 p + (\delta N_c(t) - \delta N_q(t))^2 q + \delta u^2(t) \right) \, dt
\]

(4.3)

and apply proposition 3.1 using a few pairs of \(p, q > 0\) of varying magnitudes. Selected on the basis of simulations we are led to believe that one of these 2 pairs of values should be used:

\[
(p, q) = (10,100) \text{ or } (10,10000)
\]

(4.4)

The step responses for these 2 pairs of \((p, q)\) are included (Figures 4.1 and 4.2). Note that \((p, q) = (10,100)\) results in a slower but much smoother response with virtually no overshoot. The pair \((p, q) = (10,10000)\) results in a much faster response, but with an overshoot of about 8%. It also requires larger gains and faster control.
Fig. 4.1 UNIT STEP RESPONSE OF TYPE-2 LQ SERVO FOR ENGINE-COMPENSATOR USING \((p,q) = (10,100)\)
Fig. 4.2 UNIT STEP RESPONSE OF TYPE-2 LQ SERVO FOR ENGINE-COMPENSATOR USING \((p, q)=(10, 10000)\)
On further analysis, it is found that the control system design resulting from the pair

\[(p, q) = (10, 100)\]  \hspace{1cm} (4.5)

is better when combined with the dynamometer (discussed in Section 4.4), since a fast control loop around the engine would induce a significant overshoot in the subsequent loop around the dynamometer. Indeed the unit response settles in about 1.25 seconds which seems comparable to what a human driver can do.

Let us look at a few things about this type-2 servo, the structure of which is given in Figure 4.4. The open loop poles are

REAL PART = -0.320E-01  IMAG PART = 0.0  
REAL PART = -0.107E+02  IMAG PART = 0.0  
REAL PART = -0.232E+02  IMAG PART = 0.0  
REAL PART = -0.857E+02  IMAG PART = 0.495E+02  
REAL PART = -0.857E+02  IMAG PART = -0.495E+02

The first two poles are the compensator poles. The third one is the engine pole while the last two correspond to the two poles associated with the time delay.

The closed loop poles are

REAL PART = -0.857E+02  IMAG PART = 0.495E+02  
REAL PART = -0.857E+02  IMAG PART = -0.495E+02  
REAL PART = -0.235E+02  IMAG PART = 0.0
REAL PART = -0.125E+01 IMAG PART = 0.0
REAL PART = -0.395E+01 IMAG PART = 0.0
REAL PART = -0.698E+01 IMAG PART = 0.231E+01
REAL PART = -0.698E+01 IMAG PART = -0.231E+01

We note that a pair of complex poles are added due to the two integrators. The two poles (which are fast) related to the time delay have not moved at all, while the other engine pole moved only a little. The two compensator poles show the most significant movements.

Next we look at the gains

\[ G_{1E} = [ 9.8479E+00 \quad 1.333E+00 \quad -1.5356E-01 \quad -2.7101E-01 \quad -1.3839E-03 ] \]
\[ G_{2E} = 3.754E+01 \]
\[ G_{3E} = 8.6657E+00 \]

To see the effects of a series of ramp and step inputs, the closed loop system was simulated as shown in Figure 4.3. In Figure 4.3a, we show the \( \delta N_g \) responses against variations of \( \delta N_c \). Observe how well the trajectory \( \delta N_g(t) \) follows the input signal trajectory \( \delta N_c(t) \). For all practical purposes, especially when one considers the duration of an EPA cycle, which usually requires the change be accomplished within 5-10 seconds, the overshoot and transient are not significant. Figure 4.3b shows the incremental throttle angle generated by the controller during this schedule. Note that the requirement for \( \delta \alpha(t) \) is rather small. Since the system is assumed to perform close to the nominal
Fig. 4.3a $\delta N_g$ RESPONSE OF CLOSED LOOP LQ TYPE-2 SERVO AROUND ENGINE-COMPENSATOR TO SERIES OF RAMP AND STEP INPUTS $\delta N_c$
Fig. 4.3b THROTTLE ANGLE (Incremental) GENERATED BY LQ TYPE-2 CONTROLLER IN RESPONSE TO A SERIES OF RAMP AND STEP $N_c$ INPUTS
conditions, large $\Delta N_c(t)$, hence $\Delta \alpha(t)$ is not expected. We remark however that there seems to be some sharp corners in this plot of $\Delta \alpha(t)$ due to sudden changes in $\Delta N_c(t)$, so we make a check on the control rate to make sure that the rate saturation does not occur. It is found that (by analyzing the simulations in a 0.01 second time intervals), the maximum rate during this cycle is $-2.7\%$/sec. (100% corresponds to full throttle position), which occurs at about $t = 18.95$ seconds. Reference [10] suggested that generally this is an acceptable amount. Therefore, we conclude that this servo exhibits no control rate saturation.

Figure 4.4 shows the structure of this LQ servo around the engine-compensator. To evaluate its robustness, we break the loop (as indicated in Figure 4.4) at the incremental throttle angle (control). The Bode plots are given in Figure 4.5. It is seen that the servo has a gain margin of 13 db and a phase margin of about 70°, and hence is rather robust.

We remarked earlier that the two states associated with the time delay need to be estimated using the engine torque as a measurement. A three state observer was used in conjunction with the engine model described in eqs. (2.8), (2.9). The observer gains are chosen such that the closed loop filter poles are somewhat shifted to the left (as compared with the open loop poles), in the usual ad hoc fashion [16].

Specifically, the observer equation is

$$\dot{x}_e(t) = A_{ee} x_e(t) + B_e \dot{\alpha}(t) + H_e (T(t) - C x_e(t))$$  \hspace{1cm} (4.6)
Fig. 4.4 STRUCTURE OF FULL STATE FEEDBACK LQ TYPE-2 SERVO AROUND ENGINE-COMPENSATOR
Fig. 4.5 BODE PLOTS FOR LQ TYPE-2 SERVO AROUND ENGINE-COMPENSATOR USING FULL STATE FEEDBACK
where

\[ H_e' = \begin{bmatrix} 2 & -5 \end{bmatrix} \]

and \( A_e, B_e, C_e, x_e \) are given in eqs. (2.8) and (2.9), which describe the engine subsystem of the engine-compensator. \( \hat{x}_e(t) \) is the estimate of \( x_e(t) \), generated at time \( t \). Recall that \( x_e \) consists of the last three states of \( x_E \).

The complete servo with the observer is shown in Figure 4.6. It is simulated for transient and steady state responses. It is seen that the observer introduces negligible transient, even at the instant at which the observer is started. This is because the observer poles correspond roughly to the engine poles which are relatively faster than the dominant pole of the closed loop system. Bode plots are also analyzed by breaking the control loop at the incremental throttle angle. Similar Bode plots (not shown) to those for the full state feedback case, Figure 4.5, are obtained. In particular, they indicate a gain margin of 14 db and a phase margin of roughly 75°.

At this point one might want to compare the above results with those had a type-1 servo design been used. For this purpose, we define a cost functional

\[ J_1 = \int_0^\infty \left( (\delta N_c(t) - \delta N_g(t))^2 q_1 + \delta \dot{\alpha}^2(t) \right) dt \]  \hspace{1cm} (4.7)

and apply the type-1 LQ servo given in Appendix B and simulate it with step inputs for a few \( q_1 \)'s of varying magnitudes. Results for \( q_1 = 1 \) and 100 were obtained. Simulations for these \( q_1 \)'s with unit step input are given in Appendix E.
Fig. 4.6 LQ TYPE-2 SERVO WITH OBSERVER AROUND ENGINE-COMPENSATOR
It is seen that the type-2 LQ design (Figure 4.1 and 4.2) gives comparable results to the type-1 controller as far as unit step inputs are concerned, which is certainly encouraging. Moreover, for ramp responses, a steady state error is noted in the case of the type-1 controller. This steady state error is not desirable in the emulator design. (However it is interesting to note that the type-1 servo roughly correspond to a human driver when he is asked to track a schedule of speed (ramps), since human beings are, roughly speaking, type-1 servos.)

4.3 LQ Controller around the Dynamometer

As in the previous step, first we evaluate the full state design performance; then an observer is incorporated since the only available measurement is the dynamometer speed measurement, obtained from the tachometer connected to the engine-dynamometer shaft. Same types of evaluations are carried out, i.e. time simulations and robustness analysis.

As noted in section 2.4, the dynamometer dynamics can be written in linear state space form, as follows

\[ \dot{x}_D(t) = A_D x_D(t) + B_D \delta S(t) \]  
(4.8)

\[ \delta N_d(t) = C_D x_D(t) \]  
(4.9)

Numerical values for the matrices $A_D$, $B_D$, $C_D$ are given in Appendix D. They form a completely controllable and observable system. Invertibility of

\[
\begin{bmatrix}
A_D & B_D \\
C_D & 0
\end{bmatrix}
\]

is also satisfied.
We define a cost functional

$$J_D = \left[\int_0^\infty (\delta_{g} \dot{N}(t) - \delta_{d} \dot{N}(t))^2 + (\delta_{g} N(t) - \delta_{d} N(t))^2 + \ddot{S}^2(t)\right] dt$$

Assuming that $\delta_{g} N(t)$ is either a ramp or step input, we apply proposition 3.1 to find the control gains. Again through simulations, we selected the weights

$$r = 10$$
$$s = 10000$$

for subsequent analysis. The structure of this servo is shown in Figure 4.7.

Let us take a look at the poles and the gains: The open loop poles are:

- REAL PART = -0.150E+02, IMAG PART = 0.0
- REAL PART = -0.345E+01, IMAG PART = 0.120E+02
- REAL PART = -0.345E+01, IMAG PART = -0.120E+02
- REAL PART = -0.196E+02, IMAG PART = 0.505E+02
- REAL PART = -0.196E+02, IMAG PART = -0.505E+02

Note that these dynamometer poles are faster than the engine-compensator poles.

The closed loop poles are:

- REAL PART = -0.196E+02, IMAG PART = 0.505E+02
- REAL PART = -0.196E+02, IMAG PART = -0.505E+02
REAL PART = -0.368E+01 IMAG PART = 0.130E+02
REAL PART = -0.368E+01 IMAG PART = -0.130E+02
REAL PART = -0.152E+02 IMAG PART = 0.0
REAL PART = -0.728E+01 IMAG PART = 0.552E+01
REAL PART = -0.728E+01 IMAG PART = -0.552E+01

The dynamometer poles are moved by an extremely small amount, as compared to the open loop poles. Again two poles are introduced due to the addition of the two integrators. The structure of the closed loop system is shown in Figure 4.7, where the control gains are given by

\[
G_{1D} = \begin{bmatrix} 1.1948E+01 & -2.7643E+01 & -5.4174E-01 & 9.6184E-02 & 1.4683E-03 \end{bmatrix} \\
G_{2D} = 1.1560E+02 \\
G_{3D} = 1.5205E+01
\]

The control loop was broken at \( \delta S(t) \), as indicated in Figure 4.7, to study the stability margins. Bode plots for both magnitude and phase are given in Figure 4.8. The system exhibits huge stability margins. Indeed it is stable for all phase shifts. The loop gain is extremely small even at low frequencies. We remark here that this is because the dynamometer itself is a very fast and stable system, thus the controller finds that it is not necessary to wrap additional tight loops around the dynamometer. We shall return to this point in the last section of this chapter.

Now let us evaluate an estimation scheme for the dynamometer states. Available is the measurement of dynamometer speed from the tachometer. It is used in a
Fig. 4.7 LQ TYPE-2 SERVO AROUND DYNAMOMETER USING FULL STATE FEEDBACK
fifth order observer which provides estimates for all five states associated with the dynamometer (eqs. (4.8), (4.9)). The differential equation associated with the observer is

$$\dot{\hat{x}}_D(t) = A_D\hat{x}_D(t) + B_D\delta(t) + H_D(\delta N_d(t) - C_D\hat{x}_D(t))$$  \hspace{1cm} \text{(4.11)}$$

where $\hat{x}_D(t)$ denotes the estimate of $x_D(t)$, generated at time $t$, which are then used in the feedback. The overall compensator with both the controller and the observer incorporated is shown in Figure 4.9. Again, as in the case for the engine, in an ad hoc fashion, the observer gains are picked to be

$$H_D = \begin{bmatrix} 5. \\ 5. \\ 25. \\ 10. \\ 500. \end{bmatrix}$$

with the closed loop observer poles placed at

- REAL PART = -0.420E+01 IMAG PART = 0.142E+02
- REAL PART = -0.420E+01 IMAG PART = -0.142E+02
- REAL PART = -9.173E+02 IMAG PART = 0.0
- REAL PART = -0.228E+02 IMAG PART = 0.559E+02
- REAL PART = -0.228E+02 IMAG PART = -0.559E+02
Fig. 4.9 LQ Type-2 Servo with Observer Around Dynamometer

Feed forward term from $\delta N(t)$ and $\delta N(t)$

Break loop here to study stability margins
Note that the observer is somewhat faster than the (open loop) dynamometer itself.

Linear simulations were carried out to study the effects of the observer in the closed loop system. It is seen that in steady state, it presents not a single bit of problem. There is a small transient, however, at the instant when the observer is turned on, which lasts for about 0.5 sec.

When we break the control loop at $\delta S(t)$, as indicated in the Figure 4.9, to study the stability margins, it is observed that the system exhibits the same behaviour as the full state feedback (see Figure 4.8). We shall return to this point in section 4.5.

4.4. Overall Emulator Design

Our last step is to connect the two controllers in the previous steps together to see how it behaves.

First we note that robustness is not an issue here since these two controllers, which are in sequence, have been determined to be robust.

A complete design (with the observers) is obtained by simply connecting the two controllers in Figures 4.6 and 4.9. We evaluate this design by simulating the system using a series of ramp and step $\delta N_c(t)$ inputs. In the simulation, it is assumed that the observers have been switched on for a while and any transient (which are generally small, as we discussed previously) has died out. In Figure 4.10 we show response trajectories for $\delta N_g(t)$, $\delta N_d(t)$ and $\delta S(t)$. $\delta \alpha(t)$ is omitted since its behaviour is the same as that shown in Figure 4.3. Observe that even though $\delta N_g$ is not exactly a sequence of ramps and steps, $\delta N_d$ tracks it and hence $\delta N_c$ without steady state error. This demonstrates that the linear emulator design is a good design. Observe also that $\delta S(t)$, the input to the dynamometer, which
Fig. 4.10 SIMULATION OF LINEAR EMULATOR USING A SERIES OF RAMP AND STEP & NC INPUTS

N_c(t)  N_g(t)  N_d(t)  S(t)

TIME (sec.)

SPEED (rpm)
serves as the speed setting for $\delta N_d(t)$, follows $\delta N_g(t)$ extremely closely. $\delta N_d$ lags behind a little bit - but a very insignificant amount since the dynamometer is so fast.

A step change generally has a settling time of about 1 second and an overshoot occurs when the input changes from a ramp to a step. These are typical behaviours of type-2 servos. Nevertheless, we think that these amounts are tolerable for the problem at hand.

Overall speaking, this is a successful design for a linear emulator for an automobile.

4.5 A Final Note on the Dynamometer State Feedback

We observe in the Bode plots (Figure 4.8) that the dynamometer LQ servo is insensitive to phase shifts in the state feedback gains. This indicates that the controller does not really utilize the information contained in the dynamometer states since the dynamometer is itself (open loop) very fast and stable. Furthermore by examining the responses in Figure 4.10, we see that $\delta S$ follows $\delta N_g$ almost perfectly. This means that what determines $\delta S$ is basically $\delta N_g$ alone, and that the state feedback is not important.

This is interesting because it means that we can break all the dynamometer state feedback loops, i.e. zero out the gains, and hence throw away the dynamometer observer. Computations are reduced and the implementation is simplified.

Suppose we do this; then the closed loop poles are $-15$, $-3.45 \pm j12.$, $-19.6 \pm j50.5$, $7.6 \pm j.76$. Observe that these are all very close to the full state feedback closed loop poles. Indeed the first five poles are the dynamometer open loop poles. The last two are introduced by the integrators, and are close
to the closed loop poles \(-7.28 + j5.2\) for the full state feedback case. Thus one should use this suboptimal implementation because one is not incurring any loss in performance.

We remark that the gain \(G_{2d}\) on the error signal \(\delta N_g - \delta N_d\) is relatively higher than the rest of the gains, indicating that this term is more important in the control, i.e., it explains why the simulations behave the way they are. Moreover this large gain of \(G_{2d}\) is responsible for the small loop gain we saw in the Bode plots (Figure 4.8). Thus the behaviour in the robustness analysis are explainable.

On deeper analysis, we find that further simplification is possible, and indeed desirable. Such a simplified design is depicted in Fig.4.11. In this design, the controller around the dynamometer is completely eliminated, and we set the incremental speed setting of the dynamometer (directly) equal to the incremental compensator generated engine speed. This is easily achieved in the implementations since the compensator is simulated by the computation facilities in the test cell and the signal \(\delta N_g\) that it generates can be connected directly to the speed setting, which is basically a gain times a variable electromagnetic field, through some electrical connections. In steady state, we know that \(\delta N_d\) follows \(\delta S\). By referring to Figure 4.10 again, we see that \(\delta N_d\) follows \(\delta S\) extremely well along ramps as well. Thus one would expect that this much simpler version of the design would achieve no worse than what we see in figure 4.10. Moreover this design has the merit that it has wider bandwidth than the original one, since the two integrators are not used here. We conclude that this is the version that should be implemented.
CHAPTER V

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

A successful linear emulator design for an engine-car system using an engine-dynamometer system has been presented. It is expected that this would expand the capabilities of the test-cells.

This emulator can be used to study incremental changes about a nominal trajectory, e.g. a certain range of EPA cycle. More work is needed if one desires to cover the whole EPA cycle since the entire operating range generally calls for several linear models. This is a noteworthy direction to be investigated in the future.

We showed how one can apply LQ theory to derive type-2 servo. In particular, details are given for type-2 servo which we applied successively to the engine and the dynamometer. Incidentally this decoupling simplified the complexity of the design.

A compensator is added to make the engine-compensator look like the ECS in the input-output configuration. Tracking is then performed around the engine-compensator, and then the dynamometer by following an incremental trajectory of the engine speed from an automobile which is usually stored in and available from a computer.

All in all, this seems to be a simple but effective approach to do engine experiments in the test-cells.

Also noted at the end of Chapter IV, suboptimal designs are possible by ignoring some state feedback loops as indicated from a study of robustness and simulations. Indeed one can eliminate the servo around the dynamometer
completely. This simplifies the design a great deal and actually provides better bandwidth.

In the future it would also be beneficial to study the effect of universal joints in the shaft and its nonrigidity. A notch filter might be used to eliminate the frequencies introduced by the U-joints which are known to introduce frequencies twice the rotating speed. Also it would increase our understanding further if one can propose a physical model for the dynamometer, in contrast to the frequency measured model we used in this thesis. Also stochastic effects should eventually be considered, as are other effects due to nonlinearities.
References


[10] Personal Communication with J.B. Lewis


[18] Personal Communication with J.F. Cassidy.
APPENDIX A

PROOF OF COROLLARY OF PROPOSITION 3.1

We know from Proposition 3.1 that

\[ \ddot{u}(t) = -G_1 (\dot{x} - \dot{x}_0) - G_2 (\dot{u} - \dot{u}_0) - G_3 (\ddot{u} - \ddot{u}_0) \]  

(A.1)

Since

\[
\begin{bmatrix}
\dot{x} - \dot{x}_0 \\
\dot{u} - \dot{u}_0
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{x} - \dot{x}_0 \\
\dot{y} - \dot{y}_0
\end{bmatrix}
\]  

(A.2)

integrating eq. (A.1), we obtain

\[ \ddot{u}(t) - \ddot{u}(0) = - (G_1 S_{11} + G_2 S_{21}) (x - x_0 - x(0) + x_0(0)) \]
\[- (G_1 S_{12} + G_2 S_{22}) \int_0^t (y - y_0) \, dt \]
\[- G_3 (u - u_0 - u(0) + u_0(0)) \]  

(A.3)

Thus applying eq. (A.2) again,

\[ \ddot{u}(t) = - [(G_1 S_{11} + G_2 S_{22}) S_{11} + G_3 S_{21}] (\dot{x} - \dot{x}_0) \]
\[- [(G_1 S_{11} + G_2 S_{21}) S_{12} + G_3 S_{22}] (\dot{y} - \dot{y}_0) \]
\[- (G_1 S_{12} + G_2 S_{22}) \int_0^t (y - y_0) \, dt \]
\[ + H(x'(0) - x'_0(0), u'(0) - u'_0(0), \ddot{u}_0(0))' \]  

(A.4)
where $H$ is some $m \times (n+2m)$ constant matrix. Integrate eq. (A.4) once more and the corollary is proved.
APPENDIX B

TYPE-1 LQ SERVO

If in Proposition 3.1, J (eq. (3.4)) is changed to

\[ J = \int_0^\infty [(y(t) - y_0)'Q(y(t) - y_0) + \dot{u}'(t)R\dot{u}(t)] \, dt \]  \hfill (B.1)

where \( y_0 \) is an arbitrary constant vector. Then

\[ u(t) = L \int_0^t (y_0 - y(s))ds + N(x(t) - x(0)) + u(0) \]  \hfill (B.2)

minimizes \( J \) and produces \( y(t) \) which tracks step input of size \( y_0 \) with no steady state error, where

\[ L = \hat{G}_1 S_{12} + \hat{G}_2 S_{22} \] \hfill (B.3)

\[ N = -(\hat{G}_1 S_{11} + \hat{G}_2 S_{21}) \] \hfill (B.4)

and the matrix \( \hat{G} = [\hat{G}_1 \quad \hat{G}_2] \) is found from the positive definite solution of the algebraic matrix equation

\[ \hat{R}A + \hat{R}B - \hat{R}BR^{-1}\hat{R} + \hat{G} = 0 \]  \hfill (B.5)

where
\[ \hat{A} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \quad \text{(B.6)} \]

\[ \hat{B} = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad \text{(B.7)} \]

\[ \hat{C} = \begin{bmatrix} C^T C & 0 \\ 0 & 0 \end{bmatrix} \quad \text{(B.8)} \]

and

\[ \hat{G} = R^{-1} B^T K \quad \text{(B.9)} \]

(See [7], for example)
APPENDIX C

NUMERICAL VALUES OF THE MATRICES $A_E$, $B_E$, AND $C_E$

\[
A_E = \begin{bmatrix}
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
\end{bmatrix}
\]

\[
B_E = \begin{bmatrix}
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
    & & & & & & \\
\end{bmatrix}
\]

\[
C_E = \begin{bmatrix}
    1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix}
\]
NUMERICAL VALUES OF THE MATRICES $A_D$, $B_D$, AND $C_D$

$$A_D = \begin{bmatrix}
-1.5017E+01 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0000E+00 & 0.0 & 0.0 \\
1.5477E+02 & -1.5477E+02 & -6.8921E+00 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0000E+00 & 0.0 \\
0.0 & 2.9402E+03 & 0.0 & -2.9402E+03 & -3.9258E+01
\end{bmatrix}$$

$$B_D = \begin{bmatrix}
1.5017E+01 \\
0.0 \\
0.0 \\
0.0 \\
0.0
\end{bmatrix}$$

$$C_D = \begin{bmatrix}
0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0
\end{bmatrix}$$
APPENDIX E

UNIT STEP AND UNIT RAMP RESPONSES FOR TYPE-1 LQ SERVO AROUND THE ENGINE-COMPENSATOR USING

\[ J = \int_0^\infty [(\delta N_c(t) - \delta N_g(t))^2 + \delta \alpha^2(t)] \, dt \]

IN APPENDIX B

Input - $\delta N_c(t)$
Output - $\delta N_g(t)$
Control - $\delta \alpha(t)$
Fig. E-2 UNIT STEP RESPONSE OF TYPE-1 LQ SERVO FOR ENGINE-COMPENSATOR USING q=100
Fig. E-3 UNIT RAMP RESPONSE OF TYPE-1 LQ SERVO FOR ENGINE-COMPENSATOR USING q=100