
CONE PENETRATION IN CLAYS

by

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ABSTRACT

This thesis investigates the in-situ evaluation of clay properties by means of cone penetrometers and pore pressure probes.

A review of existing theories of cone penetration indicates an almost complete neglect of the fact that continuous deep penetration is a steady-state problem where deformations and strains in the soil can be estimated with far less uncertainty than stresses. A method is developed for estimating strains and strain-rates due to cone penetration. A logical approach to penetration problems combines strain-path of soil elements with appropriate constitutive laws or laboratory testing of soil samples.

For practical reasons, a semi-empirical theory of cone penetration in clays is based on a complete theory of plane-strain steady-state wedge penetration in anisotropic rigid/plastic materials. It predicts the undrained shear strength from measurements of cone resistance and pore pressures.

Extensive penetration testing is conducted in three clay deposits. The continuity of cone penetration records permits a detailed study of soil variability. Results indicate that: (1) Cone resistance $q_c$ measured with electrical cone is repeatable and detects approximately the same inherent soil variability as field vane data. (2) As predicted, $q_c$ increases as the cone angle decreases, and tends to be decreased by tip enlargement depending on soil type and stress-history. (3) Moderate variation in penetration velocity does not change $q_c$ significantly. (4) Pore pressure $u$ during steady penetration is repeatable, varies with soil type, stress-history and the location of the porous stone on the cone or the shaft behind it, and can have values exceeding the initial total vertical stress. (5) The ratio $u/q_c$ is promising with regard to in-situ evaluation of soil type, stratigraphy and possibly stress-history of clays.

Theoretical strength predictions $s_u$(cone) are compared to $s_u$(reference) based on actual embankment performance after strain-rate effects are empirically accounted for. For enlarged cones, $s_u$(cone) is in excellent agreement with $s_u$(reference). For the "standard" cone shape (FUGRO), the theory predicts reasonable upper and lower bounds of the strength, with the possible exception of very sensitive clays.

For immediate practical application, empirical correlations between $q_c$ from standard cones and the strength obtained by the field vane test are presented for the three sites and six additional sites in Scandinavia. These correlations depend on the soil type, stress-history and depth.

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NOTATION

Symbols and abbreviations may be found under the following subject headings:

ANALYSIS OF DEFORMATION AND STRAINS
MEASUREMENTS, PREDICTIONS AND IN-SITU STRESSES
SLIPLINE THEORY
SOIL PARAMETER AND EMPIRICAL FACTORS
STATISTICAL ANALYSIS

ANALYSIS OF DEFORMATION AND STRAINS

\(x,y,z\) Rectilinear coordinates.
\(r,z,\phi\) Circular cylindrical coordinates.
\(R,\theta,\phi\) Spherical coordinates.
\(v_x, v_y, v_z\) Components of the velocity vector.
\(\dot{\epsilon}_{xx}, \dot{\epsilon}_{yy}, \dot{\epsilon}_{xy}\) Components of the strain-rate tensor.

\(a\) Scale factor.
\(B\) Half the base width of a wedge.
\(c_i\) Scalar value of stream function.
\(D_{ij}\) Component of the rate-of-deformation tensor.
\(r_c\) Radius of a penetrating cone.
\(r_0\) Initial \(r\) coordinate of a streamline.
\(t\) Time.
\(\dot{v}\) Eulerian velocity vector.
\(V\) Velocity of the indenter in the initial problem or incident velocity of the conjugate problem.
\(\dot{x}\) Position vector.
\(x_0\) Initial \(x\) coordinate of a streamline.
\(y_0\) Coordinate parallel to the major direction of motion of an undefined element.
\(\gamma\) Half the apex angle of the cavity behind a steady advancing wedge.
\(\delta\) Half the apex angle of a wedge.
Components of the natural strain tensor.

Principal strain components.

Maximum shear strain in an element.

Component of the strain-rate tensor.

An arbitrary constant.

Stream function.

MEASUREMENTS, PREDICTIONS, AND IN-SITU STRESSES

$P_L$: Limit pressure from pressuremeter test or expansion pressure for a long cylindrical cavity.

$p_0$: Initial isotropic stress in the soil.

$P_S$: Expansion pressure for a spherical cavity.

$q_c$: Cone penetration resistance.

$q_u$: Ultimate bearing capacity of footing.

$q_w$: Wedge penetration resistance.

$u$: Pore pressure.

$u_0$: Initial static pore pressure.

$\sigma_h$: Horizontal total stress.

$\sigma_{ho}$: Initial horizontal total stress.

$\sigma_{vo}$, $\sigma_{vo}$: Initial vertical total and effective stresses.

SLIPLINE THEORY AND YIELD CONTOUR

$x, y$: Plane cartesian frame of reference.

$\sigma_x, \sigma_y, \tau_{xy}$: Stress components in frame $xy$.

$1/2(\sigma_y - \sigma_x), \tau_{xy}$: Deviatoric stress components in frame $xy$.

$\alpha, \beta$: Plane rectilinear coordinates inclined at $0$ to $xy$.

$\sigma_\alpha, \sigma_\beta, \tau_{\alpha\beta}$: Stress components in frame $\alpha\beta$.

$1/2(\sigma_\beta - \sigma_\alpha), \tau_{\alpha\beta}$: Deviatoric stress components in frame $\alpha\beta$.

$1, 3$: Subscripts denoting principal directions.

$o, f$: Subscripts denoting initial and failure conditions.
Axes of an elliptic yield contour.

Skempton's pore pressure parameters.

Pore pressure parameter \( A \) at failure.

Half the width of a wedge.

Cohesive strength \( (\phi = 0) \).

Cohesion intercept in Mohr-Coulomb failure envelope based on effective stresses.

\( K_0 \)-consolidated undrained shear test.

\( d = 1 - b^2/a^2 \).

\( f = 1/2(1 - K_0) \).

\( g = 2A_f - 1 \).

Shear modulus.

Angle between \( \sigma_1 \) and the vertical at failure

\( k = 1/2(1 + K_0) \).

Arc length along yield contour.

Wedge resistance factor.

Wedge resistance factor for different friction level.

Origin of planes.

\( 1/2(\sigma_1 + \sigma_3) \).

Pressure acting on the gap behind the wedge.

An arbitrary material point in plastic state.

A point on the yield contour representing stresses at point \( P \).

\( 1/2(\sigma_1 - \sigma_3) \).

\( 1/2(\sigma_1 - \sigma_3) \) at failure.

Ultimate bearing capacity of a strip footing.

Penetration resistance of a wedge (stress).

Velocity of a wedge.

Point representing the wedge on a hodograph.

Slope of the failure envelope on a \( p-q \) diagram.

Half the apex angle of the cavity behind a steady advancing wedge.

Half the apex angle of a wedge.

Normal and shear strain-rate components in frame \( \alpha\beta \).
\( \theta \) Inclination between two rectilinear coordinate systems \( xy \) and \( \alpha \beta \).

\( \lambda \) A scale factor.

\( \mu \) Coefficient of fraction = \( \tau / \sigma_n \).

\( \sigma_1, \sigma_2, \sigma_3 \) Principal stresses.

\( \sigma_n \) Effective normal stress acting on wedge face.

\( \tau \) Shear stress acting on wedge face.

\( \phi \) 2\( \phi \) = inclination of the tangent to yield contour.

\( \psi \) Angle related to the slip-line field for pseudosteady wedge penetration.

\( \omega \) Angle defining the uniform stress zone in front of wedge face = \( 1/2 \cos^{-1}(\tau/c) \).

SOIL PARAMETERS AND EMPIRICAL FACTORS

\( a, b \) Axes of an elliptic yield contour.

\( A, B \) Skempton's pore pressure parameters.

\( c \) Intercept of Mohr Coulomb envelope.

\( c_{\text{CK}} \) \( K_o \)-consolidated undrained shear test.

\( d \) \( 1 - (b/a)^2 \)

\( E \) Young's modulus.

\( G \) Shear modulus.

\( K_o \) Coefficient of earth pressure at rest.

\( K_s \) Anisotropic strength ratio = \( s_u(H)/s_u(V) \).

\( m \) \( (1 - K_s)(1 + K_s) \).

\( OCR \) Overconsolidation ratio = \( \overline{\sigma}_{\text{vm}} / \overline{\sigma}_{\text{vo}}, \overline{\sigma}_{\text{vm}} / \overline{\sigma}_{\text{vc}} \).

\( p, p' \) \( 1/2(\sigma_v + \sigma_h), 1/2(\sigma_v + \sigma_h) \).

\( q \) \( 1/2(\sigma_v - \sigma_h) \).

\( q_f \) \( 0.5(\sigma_1 - \sigma_3)f \).

\( S_t \) Sensitivity = \( s_u \) (undisturbed)/\( s_u \) (remolded).

\( s_u \) Undrained shear strength = \( q_f \) or \( \tau_{ff} = q_f \cos \phi \).

\( s_{u \text{(AVE)}} \) Average \( s_u \) for a combination of failure modes.

\( s_{u \text{(field)}} \) \( s_u \) for embankment design or bearing capacity analyses.

\( s_{u \text{(REF)}} \) Reference \( s_u \) for cone penetration theory.
\( s_u(x) \) from a particular mode of failure or test \( x \).

\( V \): in-situ plane strain compression.

\( H \): in-situ plane strain extension

\( 45^\circ \): in-situ simple shear.

\( FV \): field vane test.

\( PSC: \) \( \overline{CK_u} \) plane strain compression test.

\( PSE: \) \( \overline{CK_u} \) plane strain extension test.

\( DSS: \) \( \overline{CK_u} \) direct simple shear test.

\( TC: \) \( \overline{CK_u} \) triaxial compression test.

\( TE: \) \( \overline{CK_u} \) triaxial extension test.

\( u \) pore water pressure.

\( u_f \) pore water pressure at failure.

\( w_L \) liquid limit.

\( w_n \) natural water content.

\( w_p \) plastic limit.

\( \alpha \) \( \overline{\sigma_u} \) (AVE) = \( \alpha \{ s_u(V) + s_u(H) \} \).

\( \gamma_t \) Total unit weight of soil.

\( \Delta x \) An increment of quantity \( x \).

\( \mu \) Field vane correction factor.

\( \mu_R \) Field vane correction factor for strain-rate effect.

\( \sigma_{vc} \) Vertical consolidation stress.

\( \sigma_{vo} \) Initial vertical stress.

\( \sigma_{vm} \) Maximum past pressure.

\( \phi \) Angle of internal friction.

\( \overline{\phi} \) Effective angle of internal friction.

STATISTICAL ANALYSIS

\( a \) Input parameter in filtering procedure.

\( d \) Depth.

\( i \) Arbitrary layer designation.

\( M \) Median value of data points in a prescribed region.
\( V \) Coefficient of variation (mean/standard deviation).
\( x \) A normalized variable (Eq. (6.3)).

\( \Delta \) Input parameter (depth increment) in filtering procedure.
\( \sigma \) Standard deviation.
\( \sigma_{q_c} \) Standard deviation of \( q_c \).
CHAPTER 1

INTRODUCTION

Prediction of foundation performance for engineering structures represents an essential part of geotechnical engineering (Lambe, 1973). In many cases, the accuracy and reliability of the predictions depend primarily on an adequate knowledge of foundation soil conditions and soil properties. In-situ measurements have recently attracted considerable attention, both in research and practice, as a means of determining soil conditions and properties during site investigation and for design purposes. The stimuli for this renewed interest are:

(1) A concern over the time and the escalating cost required by traditional exploration techniques which rely on boring, sampling, and laboratory tests, and the reliability of the results of such procedures.

(2) The increasingly difficult environments in which engineering structures are founded and where foundation exploration is required, as typified by offshore locations or frozen soils.

(3) Attempts to better define in-situ stresses and soil properties, such as deformability and permeability, which are not readily evaluated by laboratory tests, to complement the enhanced capability of analysis.

(4) The necessity to assess the spatial and inherent variability of soil properties for design and reliability study of more important or complex structures.

The cone penetrometers and the pore pressure probes studied in this thesis represent a new generation of in-situ testing devices which have no
mechanically moving part and are readily amenable to remote control and automated data recording and processing. These instruments, in principle, combine simplicity, consistency, and economy, and are rapidly gaining world-wide acceptance. They are used to define soil stratigraphy and type, assess soil variability, and infer in-situ deformability and strength properties (cone) or permeability properties (pore pressure probe) (e.g., Schmertmann, 1975; Mitchell and Gardner, 1975; Torstensson, 1975; Wissa et al., 1975). For application in medium to soft clays, however, they lack evaluated experience by the profession (Ladd et al., 1977) and a common basis for analysis (Baligh, 1975).

This thesis attempts to establish a better understanding of the penetration mechanisms in clays, through analytical and experimental means.

Chapters 2, 3, and 4 as well as Appendices A and B comprise the analytical part of the thesis. Chapter 2 discusses deformation and strains in cone penetration and reviews existing theories for cone penetration. Chapter 3 discusses possible solution approaches to continuous and deep cone penetration and outlines the strain-path approach which represents a logical approach for solving the cone penetration problem, in view of the complex stress-strain-pore pressure behavior of clays. This approach, however, requires considerable additional effort than is practical. Chapter 4 presents an approximate and more practical theory of cone penetration. Appendix A presents a procedure for estimating deformation, strain, strain-rate, and strain-history in steady penetration problems. Appendix B provides analytical background for plane-strain analysis of isotropic and anisotropic rigid plastic materials.
Chapters 5, 6 and 7 represent the experimental part of the thesis. Important parameters affecting cone penetration are identified and results used to improve interpretation methods, develop new or expanded applications of existing instruments, and suggest improvements of these instruments. Chapter 5 outlines the experimental program which was designed on the basis of extensive soil behavior studies on three soil deposits conducted at M.I.T. in the past 15 years. Chapter 6 presents a new method for studying the variability characteristics of a clay deposit based on in-situ measurements, and illustrates the method on the (standard) cone resistance and the field vane test records. Chapter 7 presents results from extensive cone and pore pressure probe testing which involves several special instruments and more than 6,000 ft of penetration. The experimental results are compared to theoretical predictions and to results reported by other investigators.

Chapter 8 synthesizes the theoretical and experimental work regarding the undrained shear strength of clays, and checks the validity of the proposed cone penetration theory. Chapter 9 presents a purely empirical method of predicting undrained shear strength from cone penetration tests for practical purposes. Chapter 10 presents a summary of the major findings and the conclusions of this research.
CHAPTER 2

THE CONE PENEITRATION PROCESS AND EXISTING THEORIES OF CONE PENEITRATION

2.1 The Cone Penetration Process

Continuous and deep penetration of a rigid indenter in a homogeneous soil mass represents a steady state problem. That is, to an observer moving with the cone (or pile), the deformation pattern, the strain and stress fields in the soils do not change with time. This is different from other important problems in soil mechanics such as the load-displacement of a footing or the expansion of a cavity, where deformations, stresses and strains all change with time.

The steady state condition severely restricts the acceptable modes of deformation (i.e., the velocity fields). Experimental observations by Rourk, (1961), Vesić, (1963), Robinsky and Morrison, (1964), Szechy, (1968), and others indicate that the deformation pattern due to penetration by a rigid indenter is similar in different soils even though the penetration resistance can be drastically different. This suggests that penetration is closer to being a strain-controlled problem than a stress-controlled problem, and that rational solutions of the penetration process should give primary consideration to deformations and strains. Thus we should start by measuring or estimating the strain-history at different locations in the soil mass, and then estimate the corresponding stress field around the cone on the basis of equilibrium and soil properties. Once the stress field is determined, the penetration resistance immediately follows. This approach emphasizes deformations and strains, and, in view of the complex behavior of soils, may fail to
satisfy equilibrium everywhere in the soil mass. It is in contrast to popular approaches in soil mechanics, e.g., the stress path method (Lambe, 1967) or bearing capacity theories presented subsequently that satisfy equilibrium but may lead to strains and deformations entirely different from the ones observed.

Figure 2.1 shows the deformation pattern due to penetration of a flat-end circular pile in bentonite. We note that a hemispherical "dead" zone of clay forms at the pile tip and moves with it.* Below this "dead" zone, the soil is pushed downward and outward. Intense shearing is confined to a zone close to the pile with a thickness roughly equal to its radius. At some distance behind the pile tip, the soil deformation becomes constant (with respect to further penetration), and these soils are left with a permanent distortion. This permanent distortion is very large near the pile but decreases significantly for soils at some distance from the pile centerline.

The strain field in this problem can be computed from the measured displacements in Fig. 2.1 by numerical differentiation. This approach, which was pursued by researchers at Cambridge University, England, especially with regard to the retaining wall problem (see for example Roscoe, 1970), has one major disadvantage: unless deformations are measured very accurately (difficult in clay and non plane-strain problems), the computed strains will be in error because of the highly sensitive process of numerical differentiation. This is particularly true in regions of intense shearing or where irregularities due to testing imperfections are present.

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*A blunt cone whose tip is within this hemispherical zone is likely to cause similar deformation pattern.
An alternative approach is to seek an analytical expression for the velocity field that yields a deformation pattern similar to the one observed, Baligh (1975). With the analytical expression of the velocity, displacements, strains, and strain-history of soil elements can be obtained with any degree of accuracy. The velocity description also provides physical insights into the deformation mechanisms as illustrated subsequently. However, this approach is limited by the difficulty of obtaining adequate analytic expressions for the velocity field. Fortunately, the penetration of blunt cones (or piles) is one case where this approach could be used.

Appendix 1 presents a detailed analysis of deformations, strains, and strain-rate in steady axisymmetric or plane problem in which the velocity of any soil element with respect to the penetrometer can be described as a function of position (Eulerian description). The analysis starts by defining the streamlines along which soil elements move relative to the penetrometer during steady penetration. The displacement and strains in any soil element can then be obtained by integrating the velocity and the strain-rate components (which are related to the spatial derivatives of the velocity) along the appropriate streamline.

Figure 2.2 shows the deformation pattern (thin lines) of a square grid predicted by this technique for a spherically symmetric velocity field given by

\[ v_R = \frac{4}{3} V (\sqrt{c}/R)^2 - V \cos \Theta; \]

\[ v_\Theta = V \sin \Theta; \]

\[ v_\phi = 0, \]  \hspace{1cm} (2.1)
where $V$ is the penetration velocity; $r_c$, the cone (or pile) radius, and $a$, a scale factor. This velocity field applies for undrained soils, and basically combines a uniform field $V$ along the axis of the cone (or pile) with a spherically symmetric field. (In fluid dynamics terminology, it is similar to the flow pattern around a moving point source, see for example, Batchelor, 1970.) The observed deformation pattern of Fig. 2.1 are represented by the heavy lines on the right-hand side of Fig. 2.2; the agreement between the predicted and the observed deformation patterns is very good when $a$ is about 1.35. This implies that the assumed velocity approximates the actual mode of deformation outside a zone with a thickness $0.35 r_c$ around the cone (or pile). In the region close to the pile, the actual deformation pattern is more complicated probably due to the shearing stress along the surface of the cone (or pile).

Since the agreement between the predicted and the observed deformation patterns is good, the velocity field given by Eq. 2.1 can now be used to estimate the strains due to penetration of a blunt cone (or pile). Shearing of the soil around the cone (or pile) is illustrated by the distortion of the initially square elements in Fig. 2.2. The variation in the area of these elements also indicates the magnitude of the circumferential strain which, in this case, is always tensile and represents the minor principal strain, $\varepsilon_3$. The major principal strain, $\varepsilon_1$, in a soil element lies on a vertical (meridian) plane; and is in the vertical direction (along the axis of the cone) before the cone (or pile) reaches the soil element but reclines increasingly toward the horizontal direction as the penetration progresses.

The degree of shearing due to penetration can be described by the
maximum shear strain, $h(\varepsilon_1 - \varepsilon_3)$, and the maximum shear strain on vertical planes, $h(\varepsilon_1 - \varepsilon_2)$. Figure 2.3 shows these shear strains vs. distance to the cone (or pile) tip for two soil elements A and B (shown in Fig. 2.2) which are initially located at $0.7 r_c$ and $2 r_c$ respectively from the cone axis. Figure 2.3 indicates that significant shearing occurs even before the cone (or pile) tip reaches the location of the soil elements. In this stage, $h(\varepsilon_1 - \varepsilon_2)$ is practically equal to $h(\varepsilon_1 - \varepsilon_3)$. As the pile tip reaches the location of the soil elements, the magnitude of the circumferential strain increases rapidly, and $h(\varepsilon_1 - \varepsilon_3)$ becomes very dominant. After the pile tip reaches the location of the soil elements, $h(\varepsilon_1 - \varepsilon_2)$ tends to decrease while $h(\varepsilon_1 - \varepsilon_3)$ asymptotically approaches a limiting value. For soil elements located between $5 r_c$ and $50 r_c$ behind the tip, $h(\varepsilon_1 - \varepsilon_2)$ is approximately half as large as $h(\varepsilon_1 - \varepsilon_3)$.

The rate of shear straining depends on the slope of the curves in Fig. 2.3 (see Appendix A). For a penetration velocity of 2 cm/sec often used in cone penetration tests, the shear rate before the tip reaches the locations of the soil elements is approximately 50% per second for element A close to the cone (or pile) and 5% per second for element B some distance away from the cone (or pile).

### 2.2 Existing Theories of Cone Penetration in Clays

Existing theories for cone penetration and/or point bearing capacity of piles are based on one of two approaches: plane-strain slip-line solutions or expansion of cavities. Both approaches rely on modifications of the rigorous solutions to simplified problems. The simplifications are made on the problem geometry, the stress-strain behavior of the soil, and the mode of deformation. Both approaches cannot or do not predict the deformations
and strains discussed earlier. Table 2.1 summarizes the predicted cone penetration resistance in clays according to these theories which are described below.

(1) The plane-strain bearing capacity solution approach. This approach treats cone penetration as an incipient failure problem and is often based on Prandtl's fundamental solution for a strip footing on the surface of a rigid-plastic half-space, Fig. 2.4, (Terzaghi, 1943; Meyerhof, 1951, etc.). Modifications to Prandtl's solution are made by introducing empirical factors to account for the difference in geometry between the plane strain strip footing and the axisymmetric cone as well as for the effect of embedment. The point resistance \( q_c \) of a cone (or pile) is then written as:

\[
q_c = N_c s_u + \sigma_{vo} ;
\]  

(2.2)

\( \frac{N_c}{N} = \text{(shape factor)(depth factor)} \) (5.14),

where \( N_c \) is the cone resistance factor;

\( s_u \) = undrained shear strength of clay,

and \( \sigma_{vo} \) = initial vertical total stress in the soil.

The shape factor is generally assumed to be 1.2 to 1.3 (Terzaghi, 1943; Skempton, 1951), whereas the depth factor is assumed to be 1.5 to 1.6 for deep foundation (Skempton, 1951; Brinch Hansen, 1961 and 1970). Because the basic solution which these and the following theories rely on applies to incipient failure only, this approach cannot predict deformations or strains.

To account for the effect of varying cone angle, Mitchell and Dorgunoglu (1973) use Meyerhof's solution for incipient (rough) wedge penetra-
tion at the surface (see Appendix 2) instead of Prandtl's solution. Thus for a cone with an apex angle $2\delta$, $q_c$ is given by

$$q_c = N_c s_u + \sigma_{vo}$$

$N_c = \text{(shape factor)} \left(\text{depth factor}\right) (2.57 + 2\delta + \cot\delta)$,

where $\delta$ is in radian and is equal to or less than $\pi/4$.

Meyerhof (1961) presents another approximate solution to point resistance of a cone at depth along this approach. He assumes that the circumferential stress is the minor principal stress and that the slip-line field on a meridional plane in the axisymmetric cone problem is identical to that for the plane-strain wedge penetration. With these assumptions, he obtains the bearing capacity factor numerically, and finds that the bearing capacity of a cone is slightly larger than that of a wedge and is given by:

$$q_c = N_c s_u + \sigma_{vo}$$

$N_c \approx (1.09 \text{ to } 1.15) \left(6.28 + 2\delta + \cot\delta\right)$

(shape factor is introduced here by the Author for simplicity of the presentation.)

(2) The cavity expansion approach. This approach is based on the expansion of a cylindrical or spherical cavity in an infinite medium, starting from zero radius. Because of the relative simplicity of the problems (which are one-dimensional), solutions for more realistic and thus more complex soil properties (e.g., strain-hardening and strain-softening) can be obtained (Bishop et al. 1945; Chadwick et al., 1963; Ladanyi, 1963 and 1967; Baguelin et al., 1972; Palmer, 1972; Prevost and Hoeg, 1975a and 1975b, etc.). For an incompressible material
whose stress-strain behavior can be idealized as elastic-perfectly plastic with a shear modulus $G$, the solution has a very simple form:

$$p_L = p_o + s_u \left( 1 + \frac{\ln G}{s_u} \right) \text{ for a cylindrical cavity;} \tag{2.5}$$

$$p_S = p_o + 1.33 s_u \left( 1 + \frac{\ln G}{s_u} \right) \text{ for a spherical cavity,}$$

where $p_o$ is the isotropic initial stress in the soil (Bishop et al, 1945).

Bishop et al (1945) recognize that the difference between $p_L$ and $p_S$ is not large and propose that, for deep penetration, the resistance $q_c$ of a lubricated cone lies between $p_L$ and $p_S$, and that, for very sharp cones, $q_c$ approaches $p_L$ whereas, in blunt cones, $q_c$ approaches $p_S$. Measurements in metals using $40^\circ$ to $120^\circ$ lubricated cones seem to support this approximation. By relating the geometry of the expanded cavity to that of the cone, deformations and strains can be predicted which are spherically asymmetric for blunt cones. Experimental observations such as that shown in Fig. 2.1 indicate that this is not the case in actual cone penetration.

Gibson (1950) extends the above theory to bearing capacity in clays by making an additional assumption that there is, acting on the surface of the cone or pile tip, a shear stress equal in magnitude to $s_u$. Thus for a cone with an apex angle $2 \delta$, he proposes that $q_c$ is given by:

$$q_c = N_c s_u + \sigma_{vo} ; \tag{2.6}$$

$$N_c = 1.33 \left( 1 + \frac{\ln G}{s_u} \right) + \cot \delta$$

where the term $\cot \delta$ is the friction contribution, and $\sigma_{vo}$ indicates the influence of the initial stresses (assumed isotropic) in the soil.
Vesic (1975 and 1977) proposes the stress field around the cone tip as illustrated by Fig. 2.4. The pressure on a cylindrical surface beneath the cone tip is assumed to equal $p_s$, and the stresses increase towards the cone face in the same manner as in a radial-fan shear zone of the slip-line theory for rigid plastic material in plane strain deformation. (Appendix B). Vesic also suggests that the in-situ octahedral normal stress $\sigma_{oct} = 1/3(\sigma_{vo} + 2 \sigma_{ho})$ be used as $p_o$ in Eq. 2. instead of $\sigma_{vo}$. Thus $q_c$ for a blunt cone ($2 \delta \leq 90^\circ$) is given by:

$$q_c = N_c s_u + \sigma_{oct};$$

$$N_c = 1.33 (1 + \ln G/s_u) + 2.57,$$

where 2.57 is the stress increment in the radial-fan shear zone beneath the cone tip.

The two modifications to the expansion of a spherical cavity presented above have no real theoretical basis, and, in view of the highly approximate nature of Bishop et al's original theory, are unjustified.

Al Awkati (1975) working with Vesic relates $q_c$ to $p_L$ using an empirical factor determined from experiments. For undrained clays, $q_c$ is given by:

$$q_c = N_c s_u + \sigma_{oct};$$

$$N_c = \lambda(1 + \ln G/s_u)$$

Experiments in sands show that $\lambda$ is approximately 1.9 for a 60° cone, and 1.85 for $2 \delta \geq 83^\circ$. This method is purely empirical and provides no basis for estimating deformations and strains.
(3) **Steady Penetration Approach.** To account for the continuous nature of cone penetration, Baligh (1975) bases his approximate solution on the slip-line solution for steady penetration of a rigid wedge in a rigid perfectly plastic material (Baligh, 1972; Baligh and Scott, 1976). The axisymmetric counterpart of this problem is that of a cone with no rigid boundary behind it to constrain the deformation of the material, Fig. 2.5 b. He proposes that the work required to push a conventional cone (Fig. 2.5 a) down a unit distance is the sum of the work required to push the cone of Fig. 2.5 b down the same distance and the work required to keep the cavity open, Fig. 2.5 c. The first component is approximated as 1.2 times the penetration resistance of a wedge with the same apex angle (2 \( \delta \)) as the cone while the second component as the expansion pressure for a cylindrical cavity, \( P_L \) (Eq. 2.5). According to this theory, \( q_c \) is given by:

\[
q_c = N_c s_u + \sigma_{ho} ;
\]

\[
N_c = 1.2(5.71 + 3.33 \delta + \cot \delta) + (1 + \ln c/s_u) .
\]

**2.3 Summary**

Continuous and deep cone penetration represents a steady state deformation problem in which deformation and strains should be given primary consideration. For penetration of blunt cones (or piles), the deformations, strains and strain-rate can be estimated with some limitations using a relatively simple velocity field and the technique described in Appendix A. The analysis indicates that significant shearing occurs in the soil ahead of the cone tip, and that the maximum shear strain,
\( h_1(c_1 - c_3) \), in a soil element occurs in a circumferential direction. The maximum shear strain, \( h_2(c_1 - c_2) \), on vertical (meridian) planes is as large as \( h_1(c_1 - c_3) \) before the cone tip reaches the location of the soil element, but tends to decrease to about 50% of \( h_1(c_1 - c_3) \) afterwards.

All existing theories of cone penetration rely on modifications of rigorous solutions to simplified problems. Most of them assume either an incipient deformation in plane-strain or expansion of a cavity, and neither approach yields meaningful deformations nor strains. Baligh (1975) offers a more rational approach based on steady penetration of a wedge. Chapter 3 discusses various possible approaches to the complex problem of cone penetration. The present work will rely mainly on extensions and modifications of Baligh's (1975) theory on the basis of experimental observations.
<table>
<thead>
<tr>
<th>Type of Approach</th>
<th>Reference</th>
<th>Expression for $N_c$</th>
<th>$N_c$ for $2\delta = 60^\circ$</th>
<th>$P_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Capacity</td>
<td>Terzaghi (1943)</td>
<td>$(\text{shape factor})(\text{depth factor}) \times 5.14$</td>
<td>$9.25$</td>
<td>same</td>
</tr>
<tr>
<td></td>
<td>Meyerhof (1951)</td>
<td>$(\text{shape factor})(\text{depth factor}) \times (2.57 + 2\delta + \cot\delta)$</td>
<td>$9.63$</td>
<td>same</td>
</tr>
<tr>
<td></td>
<td>Mitchell and Dorglunoglu (1973)</td>
<td>$(1.09 \text{ to } 1.15) \times (6.28 + 2\delta + \cot\delta)$</td>
<td>$10.2$</td>
<td>same</td>
</tr>
<tr>
<td></td>
<td>Meyerhof (1961)</td>
<td>$(1 + \ln G/s_u)$</td>
<td>$7.47$</td>
<td>$9.30$</td>
</tr>
<tr>
<td>Cavity Expansion</td>
<td>Bishop et al (1945)</td>
<td>$1.33(1 + \ln G/s_u)$</td>
<td>$9.21$</td>
<td>$11.03$</td>
</tr>
<tr>
<td></td>
<td>Gibson (1950)</td>
<td>$1.33(1 + \ln G/s_u) + \cot\delta$</td>
<td>$10.04$</td>
<td>$11.87$</td>
</tr>
<tr>
<td></td>
<td>Vesic (1975, 1978)</td>
<td>$1.33(1 + \ln G/s_u) + 2.57$</td>
<td>$10.65$</td>
<td>$13.28$</td>
</tr>
<tr>
<td></td>
<td>Al Awkati (1975)</td>
<td>$(\text{correction factor}) \times (1 + \ln G/s_u)$</td>
<td>$11.02$</td>
<td>$11.02$</td>
</tr>
<tr>
<td>Steady Penetration</td>
<td>Baligh (1975)</td>
<td>$1.2(5.71 + 3.33\delta + \cot\delta) + (1 + \ln G/s_u)$</td>
<td>$11.02$</td>
<td>$11.02$</td>
</tr>
</tbody>
</table>

Table 2.1 Summary of Existing Theories of Cone Penetration in Clays
Fig. 2.1 Deformation pattern in bentonite due to cone penetration (from Rourk, 1961).
Fig. 2.2 Predicted and observed deformation patterns due to cone penetration

thin grid = predicted deformation patterns,
heavy lines = observed deformation pattern
(Rourk, 1961)
Fig. 2.3 Strain history on soil elements during cone penetration.
Fig. 2.4 Assumed failure patterns for deep penetration. (From Vesic, 1967; 1977.)
Fig. 2.5 Model for cone penetration mechanism according to Baligh (1975).
CHAPTER 3

THEORY OF STEADY WEDGE PENETRATION

3.1 Why Study Steady Wedge Penetration

Ideally, a theory for cone penetration should determine the stress and displacement fields based on the actual behavior of the soil. But since soil behavior is very complicated, and cone penetration involves large deformations in the soil, simplifications are required. Different approaches to cone penetration can be taken depending on the simplifications made.

1) **Strain-path approach.** The stress path approach (Lambe, 1967) provides a method for systematically simplifying and solving a number of problems in soil mechanics. First the stresses are estimated at selected locations in the soil mass, and laboratory tests conducted on soil samples obtained from these locations wherein the samples are subjected to the same stress paths expected in the problem. Strains obtained from those tests are then integrated to predict displacements. The success of the stress path approach relies on the accuracy in estimating stresses, the number of soil elements investigated, and the ability of laboratory tests to minimize the effects of sample disturbance (in the sampling process) and to duplicate the actual stress paths of the problem. However, even under ideal conditions, the stress path approach fails to satisfy compatibility of displacements (or strains).*

*This simply means that, when deformations are predicted for a very large number of soil elements, the deformed elements will not fit together nicely but will imply voids or overlaps.
In many shallow foundation problems involving surface loading (or unloading), the stress path method proves satisfactory. In penetration problems, and in deep foundation problems as well, the stress pattern is much more difficult to estimate than the pattern of deformation. Research is currently underway at M.I.T. to implement a new approach for these problems: the strain path approach. In this approach, strain paths for selected soil elements are estimated from a deformation pattern (velocity field) chosen on the basis of the kinematic conditions. Laboratory tests are conducted, or alternatively appropriate constitutive laws used, to determine the stresses in these elements when subjected to the estimated strains. These stresses are then combined into a stress field applicable at a certain stage of deformation.*

Chapter 2 presents estimates of strain field and strain history (strain path) in different soil elements due to steady penetration of a blunt cone (or pile). The present work, however, does not pursue the strain path approach in cone penetration further, due to the following problems: (a) strain field and strain paths can be estimated only for blunt cones ($\theta \geq 90^\circ$) (when compared to experimental observations, they also are not applicable very close to the cone); (b) the approach requires extensive sophisticated laboratory tests (e.g., true triaxial) on different soils or, alternatively, the availability of a good stress-strain model for soil behavior and the appropriate parameters for any particular soil; and (c) the procedure to obtain a statically acceptable

* Hill (1963) presents a rigorous theoretical basis for this approach, both for steady and unsteady deformations. For steady deformation problems, the deformation pattern and the strain field do not change with time.
stress field is quite complicated. This approach, however, may be absolutely necessary for obtaining a rational solution to cone penetration for soil exhibiting very complex behaviors, e.g., significant strain-softening.

(2) Numerical Solutions. A second approach uses a numerical technique for solving field problems in solid mechanics, such as the finite element method. However, steady cone penetration at depth presents several major difficulties to this approach: (1) steady cone penetration involves very large displacements and strains (Fig. 2.1); (2) simulation of the steady state condition requires specification of velocity boundary conditions (similar to fluid flow problems) rather than stress or displacement boundary conditions; and (3) the approach requires a good stress-strain model for soil behavior and the appropriate parameters for any particular soil. Of these difficulties, the second is perhaps the most serious, since, if the steady state condition is not satisfied, displacements and strains from the numerical solutions are likely to be inaccurate, and the errors introduced can be large, especially when considering soils with complicated behaviors, e.g., significant strain-softening. Because of these difficulties, this approach is not pursued in the present work, although selected results are described below for illustration.

Baligh (1972) presents finite element method analyses of progressive (hence unsteady) wedge and cone penetration at depth in a bilinear material.** His results show that: (1) the penetration resistance increases

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* In an incompressible material, only the deviatoric stress components can be determined from the estimated strain paths. The hydrostatic component of stress must be determined from the deviatoric stress field. The final (combined) stress field still might not be self-equilibrated.

** Analysis of progressive cone penetration at surface is presented by Chung and Lee (1974).
with the displacement; (2) the resistance on the cone is about 25% to
35% larger than that on the wedge; (3) compressibility of the material
(as reflected by the Poisson's ratio) affects the penetration resistance
but has little influence on the displacement field; (4) the displace-
ment patterns are similar in both cases, but in the case of a cone, the
displacements are generally smaller and decay faster with distance from
the indenter; (5) the region ahead of the cone or the wedge is heavily
distorted, and the displacements are predominantly in the radial direc-
tion. Behind the cone and the wedge, the displacement is mainly in the
direction of penetration.

(3) Ideal Plasticity Solutions. A third approach approximates the
soil behavior by that of a rigid-plastic nonhardening material. Rigorous
mathematical treatment of axisymmetric deformation problems for this
idealized material is only possible after an additional assumption is made
concerning the stress state in the plastic zone. Harr and von Karman
(1909) assume that the circumferential (principal) stress is equal to one
of the principal stresses in the meridian plane during plastic deformation.
This simplification introduces errors of unknown magnitude according to
Hill (1950).

In the case of footings, Shield (1955) presents the complete solution
to the incipient penetration of a rigid circular footing on the surface
of a half-space. His solution indicates that: (1) the plastic zone be-
neath a circular footing is shallower and narrower than beneath a strip
footing (Hill, 1950); (2) the deformation pattern is very similar in
both cases; (3) the contact pressure on the circular footing is non-
uniform and increases from 5.14 c near the edge to 7.2 c near the center
of the footing, where \( c \) is the shear strength of the material; (4) the bearing capacity determined by the average value of the contact pressure is approximately 10\% higher for a circular footing than for a strip footing (for effects of "roughness" see Shield and Eason, 1959).

Ideal plasticity solution for steady cone penetration is made even more complicated by the additional steady state requirement.* Because of this difficulty and the unknown errors involved, this approach is abandoned in favor of the semi-empirical approach presented below which relies exclusively on plane-strain ideal plasticity solutions.

(4) **Semi-Empirical Approach.** A fourth approach approximates steady cone penetration by its plane-strain counterpart with the use of an empirical shape factor, as traditionally done in soil mechanics for bearing capacity problems. Similarities between plane-strain and axisymmetric problems are indicated analytically by: (a) slip-line solutions for incipient failure of surface footings; (b) finite element analyses of unsteady wedge and cone penetration at depth (Baligh, 1972); (c) analyses shown in Chapter 2 and later in soil elements around wedge and cone are similar. Experimental results indicating similarities between plane-strain and axisymmetric problems are presented by Thomsen (1955; extrusion), Mulhern (1959; wedge and cone at surface), De Beer (1967; footing), etc.

An approximate theory for cone penetration in clays is presented in

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* Lockett (1963) presents solutions for progressive (hence unsteady) penetration of rigid lubricated cones, with apex angle \( 2\theta > 105^\circ \), at the surface of a half-space (starting from zero penetration). The plane-strain counterpart of this problem is "pseudosteady" wedge penetration presented in Appendix B.
Chapter 4 based on this semi-empirical approach. In the remaining parts of this chapter, we concentrate on the plane-strain counterpart of cone penetration, and estimate both stresses and strains due to steady state wedge penetration. The complete solutions of wedge penetration provide a basis for developing the understanding needed to approach the complex problem of cone penetration, including the effects of cone angle, cone friction, and strength anisotropy of clays.

3.2 Theory of Slip-Line

The mathematical difficulties arising from complex soil behavior require simplifications of the problem. One common simplification neglects soil deformation prior to failure, i.e., it assumes the soil to be rigid when stressed below its "strength." Then we can solve a variety of plane-strain problems with the theory of slip-lines. Appendix B briefly derives a slip-line theory for undrained anisotropic clays and applies the theory to wedge penetration.

Slip-line solutions for wedge penetration can be divided according to the deformation mode into three groups: incipient penetration, progressive (hence unsteady) penetration, and steady penetration. Bearing capacity problems in soil mechanics represent incipient failures where the yield load is determined on the basis of the initial (undeformed) problem geometry. Solutions of incipient deformation problems, however, cease to apply after the onset of motion, and therefore deformations and strains for these problems cannot be estimated. Steady wedge penetration resembles continuous pushing of an instrument into soils at great depth, and is the focus of attention in this chapter.
3.3 A Theory of Steady Wedge Penetration in Clays

A theory for plane-strain steady wedge penetration in rigid plastic material (using slip-lines) was presented by Baligh (1972) and Baligh and Scott (1976). The basic features of this theory are:

1. The wedge is rigid, of infinite length (plane strain), and has an apex angle $2\delta \leq 90^\circ$.

2. The clay is homogeneous, isotropic, massless, rigid perfectly plastic, incompressible, with a shear strength $s_u$ which is independent of the hydrostatic stress ($\phi = 0$) and of strain rate.

3. During steady penetration, the clay forms a "cavity" (zone which no clay flows into) behind the wedge. The cavity has straight surfaces and is subjected to an internal (pressure) stress, $p_b$.

4. The geometry of the slip-line field is determined by the incompressibility requirement as well as the magnitude of the shear stress $\tau$ acting on the wedge faces. Baligh (1972) obtains solutions for the two limiting cases of $\tau = 0$ and $\tau = s_u$. Appendix B extends these solutions for intermediate values of $\tau$, and shows that, for most practical purposes, the solution for $\tau = s_u$ ("rough" wedge) applies approximately.

Figures 3.1 to 3.4 present solutions for stresses, wedge resistance, deformations and strains according to this theory, using for illustration mainly a "rough" 60° wedge.

1. **Stress Field.** Fig. 3.1 shows the slip-line field in the upper left-hand side, and the streamlines, along which soil elements move relative to the wedge, in the upper right-hand side. The streamlines are obtained from the associated velocity field (Appendix B, Section B.6, or Baligh (1972)), using the method presented in Appendix A. The bottom of
Fig. 3.1 shows a stress diagram which describes the stress field in the plastic zone around the wedge. It shows the Mohr circle of stress for a plane next to the wedge face (A), the (radial) planes containing elements C and D, and the triangular region containing element E. The cycloid in the stress diagram is the trace of the origin of planes along a circular arc from the wedge face to the triangular region containing element E. From the stress diagram (or an analytical method presented in Appendix B, Section B.5), the normal and the shear stresses acting on the wedge faces are given by:

\[ p = p_b + (5.71 + 3.33\delta) s_u \quad ; \]

and

\[ \tau = s_u \quad ("rough" \ \text{wedge}). \]

(The expression for \( p \) is a simplified and hence approximate expression suggested by Baligh (1975); see the exact expression in Appendix B). We note that any increment in the cavity pressure, \( p_b \), results in an equal increment in the normal stress on the wedge faces, and hence does not affect \( q_w \).

With the stress field and the streamlines, we can determine the stress changes occurring on a soil element as penetration progresses. The upper right-hand side of Fig. 3.1 shows the rotation of the principal stress increment on a soil element as it moves along its streamline. For the particular streamline shown, the stress system changes from one of plane strain compression to plane-strain extension and back again.

(2) Penetration Resistance. The theory predicts the penetration resistance, \( q_w \), which is the external force per unit area required to push the wedge (or the external work done per unit area per unit displacement)
to be:

\[ q_w + p_b = p + \tau / \tan \delta \]  \hspace{1cm} (3.2)

or

\[ q_w = (p - p_b) + \tau / \tan \delta \]

which we can rewrite as:

\[ q_w = N_w s_u, \]  \hspace{1cm} (3.3)

where the wedge factor, \( N_w \), for a wedge with an apex angle \( 2\delta \), is approximately given by

\[ N_w = 5.71 + 3.33\delta + \cot \delta \]  \hspace{1cm} (3.4)

(see exact expression in Appendix B).

Figure 3.2 shows a plot of \( N_w \) vs. \( 2\delta \) (top curve labeled "Isotropic"; other curves to be discussed subsequently). The wedge resistance factor, \( N_w \), is \( 9.5 \pm 0.3 \) for \( 40^\circ \leq 2\delta \leq 90^\circ \), and increases rapidly with decreasing \( 2\delta \) for \( 2\delta < 40^\circ \). For \( 2\delta = 18^\circ \), \( N_w \) is 12.6.

(3) Deformations and Strains. Deformations and strains due to steady wedge penetration can be obtained by the method presented in Chapter 2 for blunt cones. This procedure and results for wedge penetration are described in detail in Appendix A.

Figure 3.3 shows deformation pattern in the soil around a \( 60^\circ \) wedge. The wedge causes intense deformation in a zone of width \( 2B(1 + 1/\sin \delta) \), where \( 2B \) is the base width of the wedge. The assumption of soil rigidity prior to yielding requires that straining of the soil occurs only within the plastic zone enclosed by the slip-line field shown in Fig. 3.1. This causes a velocity discontinuity across the boundary of the plastic domain and results in sharp kinks in the deformed grid, Fig. 3.3. Baligh (1972) compares this deformation pattern (which he obtains by a graphical method)
with experimental observations in modeling clay and finds that the predicted deformation agrees reasonably well with the observed deformation after some initial penetration. He also finds that the agreement improves as the wedge angle, \( 2\delta \), decreases, i.e., for sharp wedges.

The strain-history due to wedge penetration is illustrated in Fig. 3.4 for the three soil elements A, B, and C, whose locations are shown in Fig. 3.3. The maximum shear strain \( \frac{1}{2} (\varepsilon_1 - \varepsilon_3) \) is plotted vs. the relative vertical distance between the element and the wedge. For a 60° wedge, element C, which is the farthest from the wedge, is subjected to a 15% peak shear strain. This high level of straining, however, decreases significantly for sharper wedges (see Appendix A). Elements A and B, which are closer to the wedge centerline than element C, undergo significant unloading once they move beyond the base of the wedge, but are left with a permanent or "residual" maximum shear strain of 7 to 13%. The rate of shearing is relatively uniform in the soil just in front of the wedge faces. For a 60° wedge (26) with \( 2B = 3.56 \text{ cm} \), \(^*\) and a penetration velocity of 2 cm/sec, the maximum shear strain rate is on the order of 15% per sec (Appendix A, Section A.3).

(4) **Effect of Strength Anisotropy.** Most natural clays exhibit a stress-strain-strength behavior which depends on the applied stress system (e.g., direction of the major principal stress at failure relative to the vertical). This is a result of inherent anisotropic structural arrangement in the clays and/or anisotropic consolidation stresses. Figs. 3.5 a, b, c and d respectively show the stress-strain behaviors, under

\(^*\) Standard cone penetrometers have a cross-sectional area of 10 cm\(^2\) or a diameter of 3.56 cm, and are pushed at a rate of 1 to 2 cm/sec.
different applied stress systems, of normally-consolidated and over-consolidated (OCR = 4) Boston Blue Clay, normally-consolidated Atchafalaya Basin Clay (EABPL Clay), and normally-consolidated Connecticut Valley Varved Clay. These stress-strain behaviors are obtained from laboratory \( \frac{C_k}{U} \) plane-strain undrained shear tests in accordance with the SHANSEP approach (Ladd and Footit, 1974), using three different applied stress systems shown schematically in the figures: plane-strain compression (PSC), plane strain extension (PSE) and direct simple shear (DSS). The horizontal axis shows the maximum shear strain, and the vertical axis, the maximum shear stress (for PSC and PSE) or the applied horizontal shear stress (for DSS). For each clay, we note that: (a) the peak shear stresses under different applied stress systems are different; (b) the shear strains at which peak shear stresses occur are different; (c) the shear moduli (shear stress/shear strain) are different at all stress or strain levels; (d) for Boston Blue Clay and Connecticut Valley Varved Clay, the PSC mode of deformation tends to show significant strain-softening (i.e., stress decreases as strain increases), whereas PSE and DSS do not; and (e) among these clays, the normally-consolidated EABPL clay shows the smallest degree of anisotropy.

To estimate the effects of anisotropic stress-strain-strength behaviors on penetration resistance, Appendix B (Sections B.2 and B.4) introduces the yield contour which defines the shear strength under all plane-strain stress systems. A yield contour general enough to approximate undrained strength anisotropy of most natural clays (within the present limitation of data) has the shape of an ellipse, Fig. 3.6. The parameters \( K_s \) and \( b/a \) controlling the shape of this elliptic yield contour can be determined from strengths under the plane-strain compression, extension and simple
shear modes of failure. Table 3.1 shows the normalized peak shear resistance from laboratory tests on normally consolidated clays, and the corresponding values of $K_s$ and $b/a$. In some cases, however, the peak shear resistance may not necessarily be the appropriate strength to use in the analysis. This will be discussed further in (6).

Appendix B (Section B.7) presents the slip-line solution for steady wedge penetration in clays with elliptic yield contours. The wedge resistance factor $N_w$ relating $q_c$ to strength in a particular mode of failure, say $s_u(V) = s_u$ for vertical loading, is not unique but depends on the shape of the yield contour of the soil, Fig. 3.2.

Appendix B (Section B.8) shows that reasonable approximations can be obtained for bearing capacity and penetration problems in anisotropic clay with the isotropic theory by using a weighted "average" strength defined as:

$$s_u(AVE) = \alpha[s_u(V) + s_u(H)]$$

$$= \alpha(1 + K_s)s_u(V)$$  \hspace{1cm} (3.5)

where $s_u(V)$ and $s_u(H)$ are respectively the in-situ strength for vertical and horizontal loading, and $\alpha$ varies generally between 0.45 and 0.50, depending on the characteristics of the clay anisotropy ($0.5 \leq K_s \leq 1.0; 0.65 \leq b/a \leq 1.0$). An approximate solution (error less than $\pm 15\%$) for wedge resistance is given when the parameters $K_s$ and $b/a$ are uncertain by:

$$q_w = N_w s_u(AVE),$$  \hspace{1cm} (3.6)

where $N_w$ is from the "isotropic" curve in Fig. 3.2.

(5) **Effect of Soil Deformability.** The slip-line theory assumes that the soil remains undeformed or "rigid" when stressed below its strength. Figs. 3.5, however, show that real soils deform before the
peak shear stress is reached, and this, in some problems, may cause a significant difference between the predicted and measured deformation (see Baligh (1972) or Appendix B). A measure of soil deformability prior to yielding is given by the ratio G/c, where G is the shear modulus and c, the shear strength. The higher the ratio G/c, the closer is the clay behavior to the idealized rigid-plastic material. Ladd et al. (1977) present experimental data showing that, for slightly overconsolidated and normally consolidated clays (OCR < 2) in undrained conditions, the ratio G/c (from direct simple shear, i.e. DSS tests) exceeds about 100 when the applied shear stress is below 2/3 c, and is significantly higher at lower stress levels.

To obtain an indication of the effect of material deformability, we look at experimental and analytical studies on progressive (hence unsteady) penetration of wedges and cones at the surface of a half-space presented by Mulhern (1959), Marsh (1964) and Hirst and Howse (1969). Their results indicate that the effect is more significant in blunt wedges and blunt cones than in sharp ones. This is due to a change in the mode of deformation, when the indenter is blunt, from the cutting mode predicted by slip-line theory into one of radial compression. Marsh (1964) suggests that, for a given indenter shape, the penetration resistance increases as G/c increases and approaches the slip-line solution for rigid-plastic material. The apex angle, 2δ, at which the rigid plastic mode applies, depends on the ratio G/c and the problem geometry (i.e., wedge vs. cone). Hirst and Howse (1969) predict that, for a wedge with 2δ < 60°, the rigid-plastic mode applies when G/c exceeds about 80. They also suggest that the effect of soil deformability is less important for a cone than for a wedge with
the same apex angle because the stresses ahead of the cone decay faster with distance.

(6) **Effect of Strain-softening Behavior.** Strictly speaking, Eq. (3.6) applies only to soils which do not show strain-softening behavior. For these soils, it is reasonable to assume that the shear resistance is fully mobilized in the plastic zone, where considerable strains take place. On the other hand, Eq. (3.6) may overestimate the wedge resistance, \( q_w \), in soils which show some strain-softening behavior (e.g., Figs. 3.5a and c). If the strain-softening phenomenon is very pronounced, rational solutions of wedge (or cone) resistance should be obtained with the strain path approach discussed earlier.

For foundation bearing capacity or stability problems, it is not unreasonable to make an ad hoc assumption on the variation of the average strength with a "representative" strain level when the stress-strain behavior of the soil is available from laboratory tests. If strain softening occurs in one or more deformation modes (see Figs. 3.5a and c), the average "strength" at any strain level is always smaller than the average peak shear resistance. Ladd (1975) attributes this reduction in the average strength to the effects of strain compatibility. This concept is not considered in the present work, but may be useful for penetration problems in sensitive clays.

3.4 **Summary**

Rigorous solutions for steady state cone penetration in clay is difficult to obtain because the problem is axisymmetric and involves large deformations and strains, and because soil behavior is very complicated.
Different approaches to the problem are discussed. The most rigorous approach relies on the fact that the deformation pattern (velocity field) for penetration problems can be predicted with far less uncertainty than the stress pattern. In this strain path approach, strain history is estimated for selected soil elements, and the resulting stresses determined from laboratory tests or appropriate constitutive laws. The present work estimates strain fields and strain paths for wedge and (blunt) cone penetration. The procedure for obtaining a stress field from these results are, however, quite complicated. The present work pursues, therefore, a semi-empirical approach where a plane-strain slip-line solution of wedge penetration, which is simple to obtain, provides the basis for understanding the complex problems of cone penetration.

Baligh (1972) and Baligh and Scott (1976) present a complete ideal-plasticity solution of steady state wedge penetration applicable for isotropic clays in undrained conditions. From this theory, we can predict the wedge resistance, the stress-increment, deformation and strain fields around the wedge. The wedge resistance is relatively constant for apex angles, $2\theta$, between 40° and 90° and approximately equal to $9.5 \pm 0.3$ times the isotropic undrained shear strength of the clay, but increases rapidly for sharper wedges. Appendix A provides a method for estimating deformations, strains and strain paths from any given steady velocity field. The deformation around a penetrating wedge is intense, especially for blunt wedges ($2\theta > 40^\circ$). The maximum shear strain in a soil element increases as it is approached by the wedge, but, depending on its location, may remain constant or decrease afterwards.

Most natural clays show anisotropic stress-strain-strength behavior.
Appendix B extends the ideal-plasticity solution described above to a special class of anisotropic clays with an elliptic yield contour and thus allows a rational estimate of the effects of strength anisotropy on wedge resistance. Other factors investigated which have only small effects on wedge penetration in clays are friction at the wedge-soil interface and soil deformability before yielding. However, because the theory neglects the actual stress-strain behavior of the soil, it cannot directly account for complex soil behavior, e.g., significant strain-softening and the effects of strain-rate. Table 3.2 summarizes the probable range of errors due to simplifications made in this rigid/plastic analyses (rough wedge).
<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>Index Properties</th>
<th>$s_u/\overline{\sigma}_{vc}$ (1)</th>
<th>$K_s$ (5)</th>
<th>b/a = (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_p$ (%)</td>
<td>P.I. (%)</td>
<td>L.I.</td>
<td>PSC</td>
</tr>
<tr>
<td>Portsmouth Clay</td>
<td>35</td>
<td>15</td>
<td>1.8</td>
<td>0.350</td>
</tr>
<tr>
<td>Haney Sensitive Clay</td>
<td>44</td>
<td>18</td>
<td>0.75</td>
<td>0.296</td>
</tr>
<tr>
<td>Boston Blue Clay</td>
<td>41</td>
<td>21</td>
<td>0.81</td>
<td>0.340</td>
</tr>
<tr>
<td>AGS CH Clay</td>
<td>71</td>
<td>40</td>
<td>----</td>
<td>0.370</td>
</tr>
<tr>
<td>San Francisco Bay Mud</td>
<td>88</td>
<td>45</td>
<td>1.04</td>
<td>0.370</td>
</tr>
<tr>
<td>Connecticut Valley Varved Clay</td>
<td>35-65</td>
<td>12-39</td>
<td>1.00</td>
<td>0.280</td>
</tr>
</tbody>
</table>

* From $\overline{CK_oU}$ plane strain tests with $\overline{\sigma}_{vc} > (1.5 \text{ to } 2) \times \overline{\sigma}_{vm}$

(1) $s_u = q_f = 0.5(\sigma_1 - \sigma_3)_f$ except for DSS where $s_u = (\tau_h)_{max}$

(2) L.I. = $\frac{w_n - w_p}{w_k - w_p}$

(3) plane strain compression  

(*) direct simple shear  

(5) plane strain extension

** Assuming $s_u(DSS) = s_u(45^\circ)$

Table 3.1 Normalized undrained shear strength in different modes of failure
(data from Ladd et al., 1977; table courtesy of A. S. Azzouz).
<table>
<thead>
<tr>
<th>Factor</th>
<th>Variation of Influence</th>
<th>Probable range of $q_e^{(actual)} - q_e^{(predicted)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Anisotropy</td>
<td>Lean clays $\uparrow$</td>
<td>$\pm 15%$</td>
</tr>
<tr>
<td>2. Soil deformability prior to yielding</td>
<td>Blunt wedges $\uparrow$</td>
<td>- 0 to 10%</td>
</tr>
<tr>
<td></td>
<td>Cones $\uparrow$ (relative to wedges)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>OC clays $\uparrow$ $(G/s_u)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fat clays $\uparrow$</td>
<td></td>
</tr>
<tr>
<td>3. Strain softening</td>
<td>&quot;Soft&quot; clays $\uparrow$</td>
<td>- 5 to 20%</td>
</tr>
<tr>
<td></td>
<td>Lean clays $\uparrow$</td>
<td></td>
</tr>
<tr>
<td>4. Strain rate</td>
<td>Fat clays $\uparrow$</td>
<td>$\pm (30 \div 15)%$</td>
</tr>
<tr>
<td>5. Friction at wedge (or cone) - soil interface</td>
<td>&quot;Soft&quot; clays $\uparrow$</td>
<td>- 0 to 20%</td>
</tr>
<tr>
<td></td>
<td>sensitive clays $\uparrow$</td>
<td></td>
</tr>
<tr>
<td>6. Anisotropic initial stresses</td>
<td>&quot;Soft&quot; clays $\uparrow$</td>
<td>Difficult to estimate</td>
</tr>
<tr>
<td></td>
<td>lean clays $\uparrow$</td>
<td></td>
</tr>
<tr>
<td>7. Applicability of $\phi = 0$ plasticity theory analysis to clays</td>
<td>Increases with $\delta$</td>
<td>- 0 to 20%</td>
</tr>
</tbody>
</table>

Table 3.2  Probable errors in wedge penetration theory (estimated for clays considered in this research, Chapter 5)
Fig. 3.1 Slip-line solution for steady wedge penetration.
Fig. 3.2 Wedge resistance factors for steady penetration in anisotropic clays in terms of $q_u(V) = q_u$ (PSC or plane-strain compression).
Fig. 3.3 Predicted deformation pattern around a 60° wedge (strain-history on soil elements A, B, and C is shown in Fig. 3.4).
Fig. 3.4 Strain-history of soil elements A, B, and C shown in Fig. 3.3.
Fig. 3.5a Normalized stress-strain relations from $\psi_{ck}$ tests on normally consolidated Boston Blue Clay.
(from Azzouz and Baligh, 1978)
Fig. 3.5b Normalized stress-strain relations from CKU tests on Boston Blue Clay (OCR = 4) (from Azzouz and Baligh, 1978).
Fig. 3.5c Normalized stress-strain relations from $C_{KU}$ tests on normally consolidated Connecticut Valley Varved Clay (from Ladd, 1975).
Fig. 3.5d Normalized stress-strain relations from CKU tests on normally consolidated EABPL Clays
(from Fuleihan and Ladd, 1976)
ELLiptic Yield Contour  Davis and Christian (1972)

\[ a = \frac{1}{2} \left[ s_u(V) + s_u(H) \right] \quad ; \quad m = \frac{(1 - K_s)}{(1 + K_s)} \]

\[ b = \frac{s_u(45^\circ)}{\sqrt{s_u(V)s_u(H)}} \quad ; \quad K_s = \frac{s_u(H)}{s_u(V)} \]

Fig. 3.6 Elliptic Yield Contour
CHAPTER 4

A THEORY OF CONE PENETRATION IN CLAYS

4.1 Introduction

The continuous penetration of cones into deep soil layers represents a steady state problem. Chapter 2 estimates deformations and strains around a penetrating cone (or pile) in clays and emphasizes the difference between steady state penetration and the more common bearing capacity theories based on incipient failure modes. Chapter 3 discusses the difficulties encountered in obtaining rigorous solutions for the steady state cone penetration. This chapter presents an approximate semi-empirical theory of cone penetration in clays based on solutions for penetration of a wedge.

4.2 Cone Resistance

The cone resistance, \( q_c \), is the external force per unit area required to push the cone, or, alternatively, the external work done per unit area per unit displacement during steady state penetration.

1. Idealized Cone. Chapter 3 (Section 3.2) presents an ideal-plasticity theory for steady state wedge penetration in clays. According to the theory, the clay forms a "cavity" behind the wedge (Figs. 3.1 and 3.2) which is subjected to an isotropic state of stress, \( p_b \). The wedge resistance \( q_w \), which is the external force per unit area required to push the wedge, depends on the wedge angle and the undrained shear strength of the clay, \( s_u \), but is independent of \( p_b \). The axisymmetric counterpart of the plane strain wedge problem is that of a cone pushed by means of a
shaft with a much smaller diameter, Fig. 4.1a. Based on a comparison of
plane and axisymmetric deformation problems (Baligh, 1972) and the empir-
icial shape factors often used in soil mechanics (Skempton, 1951; Brinch
Hansen, 1970; Vesic, 1973), Baligh (1975) approximates $q_c$ by

$$q_c = 1.2 q_w.$$ (4.1)

Therefore, $q_c$ is independent of the stress in the cavity, $p_b$, and, in iso-
tropic clays, $q_c$ is given by:

$$q_c = N_c \cdot s_u,$$ (4.2)

where

$N_c$ is the cone resistance factor plotted in Fig. 4.2 (solid line),

and

$2\delta$ is the apex angle in radians.

Laboratory tests of "undisturbed" samples indicate that most clays
exhibit an anisotropic undrained strength behavior, i.e., $s_u$ varies with
the modes of failure (see Ladd et al. (1977) for summary of test results).
Actual bearing capacity or stability problems as well as the cone penetra-
tion problem involve a combination of different failure modes, and hence
their solutions may not be uniquely related to $s_u$ determined (in situ or
by laboratory test) from any one mode of failure, say, plane-strain com-
pression.

Based on the results of laboratory plane-strain $K_o$-consolidated un-
drained shear ($\overline{C K U}$) tests, Chapter 3 and Appendix B describe undrained
strength anisotropy of clay with an elliptic yield contour and show that
for plane-strain bearing capacity and wedge penetration problems, the
isotropic theories provide reasonable approximation to more rigorous solu-
tions, provided that a weighted "average" strength, \( s_u(AVE) \), is used. This average strength reflects the variation in the shearing resistance the clay can mobilize in different modes of deformation, and can be approximately related to \( s_u \) for simple modes of failure by:

\[
s_u(AVE) = \alpha [s_u(V) + s_u(H)]. \tag{4.3}
\]

or

\[
s_u(AVE) = \alpha (1 + K_s) s_u(V),
\]

where \( s_u(V) \) and \( s_u(H) \) denote the in-situ shear strength under plane-strain vertical and horizontal loading (i.e., compression and extension) respectively; \( K_s = s_u(H)/s_u(V) \), and \( \alpha \) depends on the characteristics of the clay anisotropy. Using the peak strength from laboratory \( K_o \)-consolidated undrained shear tests, Appendix B (Section B.8) shows that \( \alpha \) generally varies between 0.45 to 0.50 for typical "medium" to "soft" layered and nonlayered clay and can be reasonably approximated by 0.47 for nonlayered clays.*

Based on the similarity between wedge and cone penetration and Eq. (4.2), we write the cone resistance in anisotropic clays as:

\[
q_c = N_c \cdot s_u(AVE) \tag{4.4}
\]

Henceforth, we will denote \( s_u(AVE) \) for anisotropic clays plainly by \( s_u \) when no confusion might arise.

Eq. (4.3) may be used to predict the strength for the plane-strain compression or extension modes of failure (PSC or PSE) from \( q_c \). This, however, requires evaluation of the parameter \( K_s = s_u(PSE)/s_u(PSC) \). Fig. 4.3a

---

* Eq. (4.3) gives the average strength when the slip-line field is a circle (or 1/2 or 1/4 of a circle), and is based on the yield contour of the soil (Appendix B). Azzouz and Baligh (1978) report that a similar defined "average" strength and a similar range of \( \alpha \) applies for plane-strain circular arc analysis of anisotropic clays.
(from Ladd et al., 1977) shows correlations between $K_s$ determined from laboratory tests and the plasticity index (PI) and the overconsolidation ratio (OCR) of the clay. Ladd et al. (1977) also present data from six normally consolidated clays, indicating that the plane strain $s_u$ (PSC) is typically $9 \pm 5\%$ greater than $s_u$ determined from the more common $K_o$-consolidated triaxial compression test.

(2) **Enlarged Cone.** Realistically, a shaft with a finite diameter $d$ is required to push a cone with base diameter $D$ into the soil, Fig. 4.1b. Clay deformation due to this "enlarged" cone ($D/d > 1$) is believed to be similar to the idealized case presented earlier ($d = 0$). The penetration resistance for enlarged cones can thus be obtained from Eq. (4.2), provided that a correction is made to account for the presence of the shaft behind the cone, i.e.,

$$q_c + p_b(1 - d^2/D^2) = N_c \cdot s_u + p_b$$

or

$$q_c = N_c \cdot s_u + (d/D)^2 p_b \quad (4.5)$$

where $N_c$ is given by Eq. (4.2) and Fig. 4.2a (solid line).

Thus, the point resistance of "enlarged" cones depends on stress $p_b$ within the cavity behind the cone which cannot, at present, be determined from the theory. For $D/d = 2$ used in field tests (to be described in Chapter 5), the correction due to the shaft equals $2 s_u$ when $p_b = 8 s_u$ and equals $s_u$ when $p_b = 4 s_u$. 
(3) **Regular (Unenlarged) Cone.** Recent developments in cone penetrometers lead to the adoption of the Fugro design (de Ruiter, 1971 and Chapter 5) as the "standard" geometry for future electric penetrometers. The Fugro cone has a 60° tip (26) and a straight cylindrical shaft behind the cone tip having the same 10 cm² cross-sectional area as the base of the cone tip, i.e., D/d = 1. This geometry is shown schematically in Fig. 4.1c and will henceforth be referred to as the "regular" or "unenlarged" cone.

The shaft behind an unenlarged cone imposes a rigid constraint on soil deformation and is most likely subjected to non-uniform stresses. Denoting the average normal stress on this shaft by \( p'_b \), and assuming that the stress increment in the soil between behind the cone and the cone face is the same as in an enlarged cone, we can write

\[
q_c = N_c \cdot s_u + p'_b, \tag{4.6}
\]

where \( N_c \) is given by Eq. (4.2) and plotted in Fig. 4.2a (solid line).

Baligh (1975) presents an approximate theory for penetration of unenlarged cones in isotropic clay (see Chapter 2). His theory is equivalent to assuming that \( p'_b \) equals the pressure required to expand an infinitely long cylindrical cavity in the clay mass. If the clay is approximated as an elastic perfectly plastic material, this expansion pressure is given by:

\[
p'_b = \sigma_{ho} + (1 + \ln G/s_u)s_u, \tag{4.7}
\]

where \( \sigma_{ho} = \) initial horizontal total stress in the soil;
\( G = \) undrained shear modulus of the clay.

Hence, for \( D/d = 1 \),

\[
q_c = [N_c + (1 + \ln G/s_u)] \cdot s_u + \sigma_{ho}. \tag{4.8}
\]
This approach is believed to give an upper bound for $q_c$. Fig. 4.2a shows the estimated range of $q_c / s_u$ for unenlarged cones. The lower bound is provided by the idealized cone with very large $D/d$, whereas the upper-bound, by Eq. (4.8). Thus the possible range of $q_c$ is given by:

$$N_c \cdot s_u \leq q_c \leq N_c \cdot s_u + (1 + \ln G/s_u) \cdot s_u + \sigma_{ho}.$$  (4.9)

Fig. 4.2b shows the influence of soil parameters on the predicted range of $q_c$. In typical "medium" to "soft" onshore clay deposits, the range of $q_c$ for a 60° unenlarged cone is approximately given by:

$$11 \: s_u \leq q_c \leq (22 \text{ to } 28) \: s_u$$  (4.10)

($\gamma_t / \gamma_w = 1.6 \text{ to } 1.9$; $K_o = 0.55 \pm 0.15$; $G/s_u = 150$; $s_u / \sigma_{vo} = 0.20 \text{ to } 0.33$).

Fig. 4.3b shows empirical correlations for the coefficient of earth pressure at rest, $K_o$, required for evaluating $\sigma_{ho}$.

Based on earlier discussion, Eqs. (4.8), (4.9) and (4.10) approximately apply for anisotropic clays as well, where a weighted "average" strength (Eq. (4.3)) is used. Baligh et al. (1977) gives an approximate expression for $q_c$ lying roughly in the middle of its lower and upper bounds:

$$q_c \approx \sigma_{ho} + N_c \cdot s_u,$$  (4.11)

where $N_c$ is given by Eq. (4.2) and plotted in Fig. 4.2a (solid line), which is also shown in Figs. 4.2a and 4.2b. This expression has no theoretical basis, but provides a convenient means for estimating the influence of apex angle on $q_c$. An alternative expression following the traditional bearing capacity equation, i.e.,

$$q_c \approx \sigma_{vo} + N_c \cdot s_u,$$  (4.12)

where $N_c$ is given by Eq. (4.2) and plotted in Fig. 4.2a (solid line), does not follow directly from the analytical expression for the upper bound (Eq. (4.8)), but generally represents less than 15% change from Eq. (4.11).
4.3 Pore Pressure During Steady Penetration

During steady cone penetration, a pore pressure field develops around the cone; its magnitude and extent depend on the stress-strain-strength properties and consolidation characteristics of the soil, and the penetration velocity. Fig. 4.4a shows two profiles (one partial) of steady penetration pore pressure in a deposit consisting of sand and phosphatic clay whose fine content is shown versus depth in Fig. 4.4b. * Generally, u is close to in-situ static pore pressure when the fine content is less than 15% but increases significantly when the fine content exceeds 15% (this does not necessarily apply to other soils).

When penetration stops, this pore pressure will dissipate. The rate of dissipation of pore pressure depends on the consolidation or permeability characteristics of the soil (see Wissa et al., 1974). The present work concentrates on pore pressure during steady penetration which may influence \( q_c \).

For steady state penetration in clays at a velocity of 1 to 2 cm/sec normally used in penetration testing, the undrained condition approximately applies near the penetrometer. In this case, \( u \) can be estimated from the total stress field around an advancing wedge (Chapter 3). Fig. 4.5 shows schematically the development of stresses and pore pressure in front of an enlarged cone. On the cone face, the hydrostatic component of stress is given by

\[
p = p_b + 1.2(5.71 + 3.33\delta) s_u \tag{4.13}
\]

* Courtesy of Dr. R.T. Martin (M.I.T.) and the Florida Phosphatic Clays Research Project.
For a non-softening soil with isotropic properties, the effective stress at yielding can be estimated as

$$\bar{p} = s_u / \sin \bar{\phi}$$  \hspace{1cm} (4.14)

where $\bar{\phi}$ is the effective friction angle of the soil. With this additional assumption, we can predict the pore pressure on the face of an enlarged cone as:

$$u_{\text{front}} = p - \bar{p} ;$$

Since $p_b = u_{\text{back}}$ cannot be determined from the theory, we can only estimate the pore pressure increment between the front and the back of the cone:

$$u_{\text{front}} - u_{\text{back}} = (p - p_b) - s_u / \sin \bar{\phi} ;$$  \hspace{1cm} (4.15)

$$= [1.2(5.71 + 3.33\delta) - 1 / \sin \bar{\phi}] \cdot s_u .$$

For a $18^\circ$ enlarged cone ($2\delta = 18^\circ$, $D/d > 1$) and $\bar{\phi} = 32^\circ$, the pore pressure increment between the front and the back of the cone is approximately equal to $5.6 \ s_u$.

The pore pressure behind an unenlarged cone, $p_b'$, is of particular interest since it resembles pore pressure generated around piles during the installation process which controls pile "setup." A number of investigators have attempted to measure or predict this pore pressure (e.g., Bjerrum and Johannessen (1961); Lo and Stermac (1965); Koizumi and Ito (1967), Butterfield and Bannerjee (1970), Vesci (1973), Esrig et al. (1977), Desai (1978)). Existing analytical methods are based exclusively on plane-strain expansion of a cylindrical cavity in an infinite soil mass. For an isotropic elastic perfectly plastic material, the pore pressure at the cavity wall in the undrained state can be obtained from Eqs. (4.6) and (4.14) as
\[ u(\text{cavity expansion}) = \sigma_{ho} + \left( 1 + \ln \frac{G}{s_u} \right) s_u - \frac{s_u}{\sin \overline{\phi}}; \]

\[ = \sigma_{ho} + \left( 1 + \ln \frac{G}{s_u} - \frac{1}{\sin \overline{\phi}} \right) s_u. \quad (4.16) \]

For \( G/s_u = 60 \) to 150 and \( \overline{\phi} = 28^\circ \) to 36\(^\circ\), the \((u(\text{cavity expansion}) - \sigma_{ho})/s_u\) ratio is approximately 3 to 4.
a. IDEALIZATION

\[ q_c + p_b = N_c \cdot s_u + p_b \]
\[ N_c = 1.2[5.71 + 3.33\delta + 1/\tan \delta] \]

b. CONE WITH ENLARGED TIP

\[ (D/d > 1) \]
\[ q_c = N_c \cdot s_u + (d/D)^2 \cdot p_b \]

c. UNENLARGED CONE

\[ (D/d = 1) \]
\[ N_c \cdot s_u \leq q_c \leq N_c \cdot s_u + \]
\[ \left[ \sigma_{ho} + (1 + \ln G/s_u) \left( s_u \right) \right] \]

Fig. 4.1 Predictions of cone resistance
Fig. 4.2a Theoretical cone penetration resistance in clays.
(Upper bound computed for a total unit weight of
1.8 T/m³, K = 0.55, G/sₜ = 150, sₛᵢ/σₒ = 0.25, and a
hydostatic condition with water table at surface.)
Fig. 4.2b  Influences of soil parameters on predictions of cone resistance $q_c$.
(for a hydrostatic condition with ground water table at the surface)
Fig. 4.3a Empirical correlations for the anisotropic strength ratio $K_s$ (from Ladd et al., 1977)
Fig. 4.3b Empirical correlations for $K_o$ in normally consolidated clay deposit (from Ladd et al., 1977). For overconsolidated clays, $K_o$ is approximately given by

$$K_o (OCR > 1) = K_o (OCR = 1) \cdot (OCR)^m$$

where $m = 0.35$ to $0.40$ (unloading condition only).
Fig. 4.4 Effects of soil type on pore pressure measured during cone penetration. (Data courtesy of Dr. R. T. Martin and the Florida Phosphatic Clay Research Project.)
Fig. 4.4  Effects of soil type on pore pressure measured during cone penetration. (Data courtesy of Dr. R.T. Martin and the Florida Phosphatic Clay Research Project.)
\[ u_{\text{front}} = p - s_u / \sin \phi \]
\[ = p_b + [1.2 (5.71 + 3.33 \delta) - 1 / \sin \phi] s_u \]

Fig. 4.5 Schematic development of stress and pore pressure in front of an advancing cone.
CHAPTER 5

FIELD TESTING PROGRAM

5.1 Objectives and Approaches

The primary objective of the field testing program is to check the validity of cone penetration theory presented in Chapter 4 and hence provide a better understanding of the cone penetration process. Thus we measure cone resistance, for different cone sizes and shapes and at different penetration rates, and pore pressures at various locations on the cone, by means of pore pressure probes, to provide an indication of the stress field around the cone. Measurements are also repeated to check reproducibility and soil variability.

Penetration tests were conducted in three clay deposits: Boston Blue Clay, Atchafalaya Basin Clay, and Connecticut Valley Varved Clay. These clays are post glacial clays having different depositional environments and characteristics and thus represent a wide spectrum of cohesive soils. M.I.T. studied the engineering properties of these deposits extensively in the last decade using various laboratory and field tests.

This chapter describes the equipment used in this study, the clay deposits tested, the testing program conducted at each site, and finally the similarities and differences among the deposits that may influence the cone penetration process.
5.2 Test Equipment

(1) The Cone Penetrometers. Most of the penetrometers and the supporting equipment (signal recorder and pushing apparatus) used in this study are provided by Fugro, Inc. The Fugro cone, Fig. 5.1, uses electronic strain gages to measure the tip resistance and the friction on the friction sleeve. De Ruiter (1971) provides a thorough description of the apparatus. The "load cell" of this penetrometer is contained in a straight cylindrical shaft, 10 cm² in cross-section, behind the cone tip. The standard cone tip has the same 10 cm² base area and an apex angle (2δ) of 60°. Table 5.1 summarizes the size and shape of the additional cone tips especially constructed for this research program.

The penetrometer is pushed into the soils by a hydraulic system. During penetration, the electronic signal from the cone is transmitted to the surface by a cable strung through the pushing rods, and is recorded continuously on graph paper as a function of the penetrated depth (or time). This signal is adjusted electronically to produce a graph of cone tip resistance and sleeve friction at selected scales; the actual voltage of the signal from the load cell is not normally recorded.

(2) The Pore Pressure Probes. The pore pressure probe was developed at M.I.T. by Wissa et al. (1975). The original design is conical in shape (2δ = 18°) and measures pore pressure at the cone tip, Fig. 5.2a. Three pore pressure probes of different shapes were designed and constructed for this research program following the same
design concepts. Figures 5.2b and c show pictures of two of these probes, and Table 5.1 summarizes the measuring capability available. Each probe consists essentially of a porous stone made of zinted steel connected hydraulically to an electro-mechanical pressure transducer. The pushing and the signal-recording apparatus are similar to those used for the cone with the exception that the voltage output of the pressure transducer is always recorded.

5.3 Boston Blue Clay

Table 5.2 presents the sources of information on geology and engineering properties of Boston Blue Clay on which the following summary is based.

(1) Geology. The Boston Blue Clay was formed during the wane of the late Pleistocene ice age (about 14,000 years ago) under a marine environment in the Boston Basin, probably not very far from the ice margin. The clay deposit overlaid a glacial till which covered the bedrock, and had a typical thickness in excess of 50 to 125 ft depending on the topography of the till. It included numerous lenses of fine sands, isolated sand pockets and occasional stones or pebbles. Subsequent to clay deposition, movements of the earth crust and of the sea level resulted in emergence of the clay above the sea, followed by extensive weathering, desiccation, and erosion of the upper part of the deposit. This was in turn followed by at least two periods of submergence and deposition, of lesser significance, in which outwash sand, and peat and silt were deposited above the clay.
(2) **Soil Conditions at the Test Site.** The test site in Boston Blue Clay is located in Saugus, Ma, at Station 246, 160 to 200 ft offset to the east from the centerline on the unfinished Interstate 95 embankment. This and the adjoining section of the embankment has been studied extensively by M.I.T. in the last 15 years; see for example, D'Appolonia and Lambe (1970), on excess pore pressure during construction: M.I.T. (1975), on planned embankment failure; Ladd et al (1978), on pressuremeter tests and measurement of lateral earth pressure.

Fig. 5.3 shows the soil conditions at the site; the clay is about 130 ft thick and underlies 20 ft of peat and sand. The top 50 ft of the clay is heavily precompressed, probably due to desiccation, whereas the bottom 80 ft is only slightly overconsolidated. Fig. 5.4 shows field vane measurements at this and nearby locations; the "smoothed" and averaged profile of four field vane tests* is shown in Fig. 5.5 (smoothing procedure to be described in Chapter 6); Fig. 5.5 also shows the undrained shear strength profiles for different failure modes based on the SHANSEP approach, computed using the stress history of the clay, Fig. 5.3, and the normalized undrained strength parameters determined from laboratory tests†, Fig. 5.6. The "average" strength, denoted by curve AVE, reflects the shearing resistance the soil can mobilize in a combination of different failure modes and should govern cone resistance or bearing capacity problems in nonsoftening soils (Chapters 3 and 4) provided that the SHANSEP laboratory testing procedure accurately simulates the in situ soil

*Two preconstruction tests and two tests relatively unaffected by construction of the embankment fill.

†Selected stress-strain curves under different applied stress systems are shown in Fig. 3.4.
behavior and the effect of strain rate is not large. The average strength is greater than the average field vane strength throughout the profile; the difference being about 20% in the heavily over-consolidated region and 40% in the slightly-overconsolidated region.

Table 5.3a summarizes the different penetration tests performed at this site and Fig. 5.7 shows the layout of the test locations. The record of individual tests is presented in Appendix C.

5.4 EABPL Clays

Table 5.2 presents the sources of information on geology and engineering properties of the East Atchafalaya Basin Protective Levee (EABPL) Clay on which the following summary is based.

(1) Geology. The EABPL clays are backswamp deposits formed by floodwater in the Atchafalaya Basin of the Mississippi River Valley following the retreat of the last Pleistocene ice age when the sea level was considerably lower than the present level. As the sea level rose, a thick bed of sand and gravel was deposited in the valley followed by a layer of clay. The clay was deposited by floodwater in shallow ponded areas left behind natural levees, often in regular layers over large extent. The earliest backswamp deposits in the Atchafalaya Basin were laid down about 15,000 years ago, but sedimentation continued to the present day. The thickness of the clay at the test site is over 120 ft.
(2) **Soil conditions at the test site.** The test site is located at station 1381 + 50 ft, 150 to 200 ft offset from the centerline to landside, along the East Atchafalaya Basin Protective Levee (EABPL), about 35 miles south of Baton Rouge, La. Fig. 5.8 presents the soil conditions at this site which indicates a zone of very high water content approximately between El. -10 and -25 ft (varying slightly among different test borings) containing peat, wood and plant roots. The bottom of this zone marks the transition of the depositional environment from one of swamp to one of shallow lake, perhaps associated with the formation of the Atchafalaya River about 500 to 1000 years ago. For this reasearch program, attention is concentrated in the lower, more uniform clay. Fig. 5.9 shows the field vane strength which varies between 0.2 and 0.8 kg/cm² (TSF), and tends generally to increase with depth, but with a significant scatter. Fig. 5.10 presents the profile of $s_u$ (DSS) according to the SHANSEP approach together with the "smoothed" average of the field vane strength (smoothing procedure to be described in Chapter 6); the SHANSEP strength is computed based on the stress history of the clay, Fig. 5.8, and the normalized undrained strength parameter shown in Fig. 5.11. Because of the relatively isotropic behavior of this clay, $s_u$ (DSS) approximately applies for all modes of failure and should govern the cone penetration resistance, provided that the effect of strain rate is not large. Between the depths of 40 and 120 ft, $s_u$ (DSS) is
approximately 20 to 40% smaller than \( s_u \) (field vane).

Table 5.3b summarizes the different penetration tests performed at this site, and Fig. 5.12 shows the layout of the test locations. The record of individual tests is presented in Appendix D.

### 5.5 The Connecticut Valley Varved Clay

Table 5.2 presents the sources of information on the geology and engineering properties of this clay deposit on which the following summary is based.

1. **Geology.** The Connecticut Valley Varved Clay (CVVC) is a glacial lake deposit formed during the retreat of the late-Pleistocene ice sheet from New England, approximately between 13,000 to 10,700 years ago. The soil contains alternate layers of different composition (high and low clay contents) as a result of the annual depositional sequences, and overlie a till layer which has a widely variable character ranging from fine silt to coarse gravel. The varved clay deposits had an original thickness considerably in excess of 100 ft, but, in some locations, were extensively eroded after the glacial lake drained (about 10,700 years ago). Present extent of CVVC consists of a band, five to ten miles wide, along most of the Connecticut River in Western Massachusetts, with a thickness ranging from 50 to over 150 ft. The surface is generally covered with a layer of sand, typically 10 to 20 ft thick. The upper portion of the clays may also have been desiccated.
(2) Soil conditions at the test site. The test site is adjacent to the University of Massachusetts campus in Amherst, MA, about two miles east of the Connecticut River. The soil conditions at this site are presented in Fig. 5.13. The clay is highly precompressed above a depth of 35 ft, probably due to desiccation; below this depth, the degree of precompression is moderate, being about 0.6 kg/cm² (this could be caused by a net erosion of 35 ft of overburden). The (Geonor) field vane strength, Fig. 5.14, below the desiccated crust is almost constant at about 0.40 ± 0.05 kg/cm² with a sensitivity of 5 to 7. Fig. 5.14 also shows the SHANSEP strength profiles for the plane-strain compression (PSC) and the direct simple shear (DSS) modes of failure computed from stress history of the clay (Fig. 5.13) and the normalized undrained strength parameters shown in Fig. 5.15. The plane-strain extension strength, $s_u^{(PSE)}$, not shown here, is only about 10% lower than $s_u^{(PSC)}$ for this varved material. If the effect of strain rate is small, the cone penetration resistance should be governed by the average strength denoted by curve AVE in Fig. 5.14 which is approximately equal to $s_u^{(field vane)}$ in the bottom 20 ft of the clay, but is approximately 15% lower than $s_u^{(field vane)}$ in the top 30 ft.

Table 5.3c summarizes the different penetration tests performed at this site; the record of individual tests is presented in Appendix E.

5.6 Similarities and Differences Among the Three Clay Deposits

In this section, we look at the similarities and differences among
the three clay deposits tested, regarding several important soil properties that, according to the theory presented in Chapter 4, can influence cone penetration, namely: the strength of the clay (including strength anisotrophy), its stiffness, and the in situ state of stress.

The Boston Blue Clay and CVVC deposits show similar patterns of overconsolidation, Figs. 5.16a and c; both are heavily overconsolidated in the upper part and are slightly overconsolidated (OCR ≤ 1.5) in the lower part. The field vane strength is relatively uniform in both deposits, being about 0.45 ± 0.1 kg/cm² in Boston Blue Clay and 0.40 ± 0.05 kg/cm² in the Connecticut Valley Varved Clay (Figs. 5.5 and 5.14).

The Atchafalaya Basin Clay (EABPL) exhibits a more complex stress history than the other two sites, with alternate zones of overconsolidated and normally consolidated clays. The general trend of the field vane strength increases with depth (from about 0.2 to 0.7 kg/cm²) with a larger scatter than in the other two sites, perhaps due to more varied depositional conditions in a backswamp environment than in a lacustrine or marine environment. In addition, while the undrained shear strength of EABPL clay varies little with the mode of failure (e.g., vertical vs. inclined loading), the Boston Blue Clay and the Connecticut Valley Varved Clay possess pronounced strength anisotropy as shown by the strength profiles in Figs. 5.5 and 5.14.
Indication of the in situ state of stress in these deposits is given by the ratio $\sigma_{vo}/s_u$ (field vane), where $\sigma_{vo}$ is the initial vertical total stress. The value of this ratio, plotted in Figs. 5.16a, b, and c is inversely related to the OCR of the clays. In the slightly overconsolidated regions, $\sigma_{vo}/s_u$ (field vane) varies between 7 and 12 in all three deposits, whereas it is less than 5 in heavily-overconsolidated regions.

Absolute comparison of stiffness (e.g., Young's modulus or shear modulus) among these clays is difficult since clays exhibit a highly nonlinear stress-strain behavior as shown by the stress-strain curves in Fig. 3.4. Fig. 5.17 shows the ratio of Young's modulus, $E$, to $s_u$, measured in $K_0$-consolidated undrained direct simple shear (DSS) tests for an applied shear stress ($\tau_h$) equal to $1/3 s_u$. The ratio $E/s_u$ is only 200–250 for EABPL clay at all OCR's, whereas it is $800 \pm 100$ for normally consolidated and slightly overconsolidated Boston Blue Clay and Connecticut Valley Varved Clay and decreases significantly with increasing OCR. From this figure and the SHANSEP DSS strength profiles (Figs. 5.5, 5.10), $E$ is estimated for a DSS mode of deformation with $\tau_h = 1/3 s_u$ as shown in Figs. 5.18a to c. In all three deposits, this $E$ generally increases with depth. The EABPL clay has the lowest value of $E$, only about $100 \pm 50$ kg/cm², whereas $E$ in the Boston Blue Clay and Connecticut Valley Varved Clay deposits are generally 4 and 2, times, respectively, higher.
<table>
<thead>
<tr>
<th></th>
<th>D/d</th>
<th>Cone Angle $2\delta$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical Cone</td>
<td>1.0</td>
<td>60°</td>
<td>Fugro Apparatus with specially made tips; d = 3.56 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30°</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30°</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18°</td>
<td></td>
</tr>
<tr>
<td>Mechanical Cone</td>
<td>not applicable</td>
<td>60°</td>
<td>Begeman type (with friction mantle)</td>
</tr>
<tr>
<td>Pore pressure Probes (porous stone at tip)</td>
<td>1.0</td>
<td>60°</td>
<td>Wissa et al (1975)'s design, d = 3.81 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>18°</td>
<td></td>
</tr>
<tr>
<td>Pore Pressure Probe (porous stone on cone face)</td>
<td>1.0</td>
<td>18°</td>
<td>Stone at mid-cone</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>18°</td>
<td>Stone at $\frac{1}{4}$ x cone from tip</td>
</tr>
<tr>
<td>Pore Pressure Probe (porous stone behind cone)</td>
<td>1.0</td>
<td>60°</td>
<td>Distance between porous stone and tip can be varied</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18°</td>
<td></td>
</tr>
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Table 5.1 Summary of Equipment Characteristics
<table>
<thead>
<tr>
<th>Type of Deposit Information</th>
<th>Boston Blue Clay</th>
<th>EABPL Clay</th>
<th>Connecticut Valley Varved Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of Deposit</strong></td>
<td>Marine illitic CL clay, medium sensitivity</td>
<td>Highly plastic deltaic clay, low sensitivity</td>
<td>Glacial lake clay, varved, medium sensitivity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Krinitzsky and Smith (1969)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foott and Ladd (1973)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ladd et al. (1971a)</td>
<td></td>
<td>Ladd (1975)</td>
</tr>
<tr>
<td></td>
<td>Ladd and Edgers (1972)</td>
<td></td>
<td>Sambhandharaska (1977)</td>
</tr>
<tr>
<td></td>
<td>Lambe et al (1972)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Marr (1974)</td>
<td>USCE (1968)</td>
<td></td>
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Table 5.2 Sources of Information on Clay Deposits Tested
<table>
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<tr>
<th>Test No.</th>
<th>Cone Angle</th>
<th>Diameter Ratio D/d</th>
<th>Rate Effect</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60°</td>
<td>1</td>
<td>---</td>
<td>(St 261, 160 ft West)</td>
</tr>
<tr>
<td>2</td>
<td>60°</td>
<td>1</td>
<td>---</td>
<td>(St 263, 160 ft West)</td>
</tr>
<tr>
<td>3</td>
<td>60°</td>
<td>1</td>
<td>---</td>
<td>Cone</td>
</tr>
<tr>
<td>4</td>
<td>60°</td>
<td>1</td>
<td>---</td>
<td>(11 tests)</td>
</tr>
<tr>
<td>6</td>
<td>30°</td>
<td>1</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>60°</td>
<td>2</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>60°</td>
<td>1</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>30°</td>
<td>2</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>18°</td>
<td>2</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>60°</td>
<td>N.A.</td>
<td>---</td>
<td>Begemann Mechanical Cone</td>
</tr>
<tr>
<td>5</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td>stone at tip</td>
</tr>
<tr>
<td>8</td>
<td>60°</td>
<td>1</td>
<td>---</td>
<td>stone at tip (Feb.'77)</td>
</tr>
<tr>
<td>21*</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td>Pore Pressure Probe</td>
</tr>
<tr>
<td>22*</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td>stone at tip</td>
</tr>
<tr>
<td>23*</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td>stone at mid-cone</td>
</tr>
<tr>
<td>24*</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td>stone just behind cone</td>
</tr>
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<td>25*</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td>stone 3.2 d behind cone</td>
</tr>
<tr>
<td>26*</td>
<td>18°</td>
<td>1</td>
<td>---</td>
<td>stone 4.7 d behind cone</td>
</tr>
<tr>
<td>31*</td>
<td>18°</td>
<td>1</td>
<td>yes</td>
<td>stone at mid-cone (St 263, 160 ft West) (June, '77)</td>
</tr>
<tr>
<td>32*</td>
<td>18°</td>
<td>1.9</td>
<td>yes</td>
<td>stone 11 d behind cone</td>
</tr>
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<td>33*</td>
<td>18°</td>
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<td>stone 11 d behind cone</td>
</tr>
<tr>
<td>34*</td>
<td>18°</td>
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<td>35*</td>
<td>60°</td>
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<td>stone at tip</td>
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<td>36*</td>
<td>60°</td>
<td>1</td>
<td>yes</td>
<td>stone 0.84 d behind cone</td>
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<td>60°</td>
<td>1.9</td>
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<td>stone 3.2 d behind cone (July, '77)</td>
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Note: *Pushed through predrilled hole filled with water

Table 5.3a Penetration Test Programs at Saugus, MA
<table>
<thead>
<tr>
<th>EABPL</th>
<th>Test No.</th>
<th>Cone Angle</th>
<th>Diameter Ratio, D/d</th>
<th>Rate Effect</th>
<th>Remarks</th>
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<td>St 1381 +50</td>
<td>1</td>
<td>60</td>
<td>1</td>
<td>--</td>
<td>---</td>
</tr>
<tr>
<td>180 ft Landside</td>
<td>2</td>
<td>60</td>
<td>1</td>
<td>--</td>
<td>zero shift</td>
</tr>
<tr>
<td>Cone</td>
<td>3</td>
<td>18</td>
<td>1</td>
<td>--</td>
<td>zero shift</td>
</tr>
<tr>
<td>(13 tests)</td>
<td>4</td>
<td>18</td>
<td>2</td>
<td>yes</td>
<td>---</td>
</tr>
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<td>yes</td>
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<tr>
<td></td>
<td>12</td>
<td>18</td>
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<tr>
<td></td>
<td>13</td>
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<td>1</td>
<td>yes</td>
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</tr>
<tr>
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<td>17*</td>
<td>18</td>
<td>2</td>
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<td>wet hole</td>
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<tr>
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<td>18*</td>
<td>60</td>
<td>2</td>
<td>--</td>
<td>wet hole</td>
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<tr>
<td></td>
<td>21</td>
<td>60</td>
<td>1</td>
<td>--</td>
<td>ST 1385 + 50</td>
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<td>Pore Pressure Probe</td>
<td>6</td>
<td>60</td>
<td>1</td>
<td>yes</td>
<td>stone at tip</td>
</tr>
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<td></td>
<td>9</td>
<td>18</td>
<td>1.9</td>
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<td>stone close to tip</td>
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<td>18</td>
<td>1</td>
<td>yes</td>
<td>stone at mid cone</td>
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<td></td>
<td>14</td>
<td>18*</td>
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<td>yes</td>
<td>stone lid behind cone</td>
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<tr>
<td></td>
<td>15</td>
<td>60*</td>
<td>1.9</td>
<td>--</td>
<td>stone 1.6 d behind cone</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>60*</td>
<td>1.9</td>
<td>--</td>
<td>stone 1.6 d behind cone</td>
</tr>
<tr>
<td>St 1385 +50</td>
<td>23</td>
<td>18*</td>
<td>1</td>
<td>--</td>
<td>stone at tip</td>
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<td></td>
<td>24</td>
<td>18*</td>
<td>1.9</td>
<td>--</td>
<td>stone 1.6 d behind cone</td>
</tr>
</tbody>
</table>

Note: There are 4 additional standard cone tests conducted by Fugro Gulf, Inc. in 1976 in the same general area.

* Push through predrilled hole fill with water

Table 5.3b  Penetration Test Program at EABPL, LA, January, 1978
<table>
<thead>
<tr>
<th>AMHERST, MA</th>
<th>Test No.</th>
<th>Cone Angle 26</th>
<th>Diameter Ratio, D/d</th>
<th>Rate Effect</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>Cone</td>
<td>1</td>
<td>60°</td>
<td>1</td>
<td>--</td>
<td>performed at 200 ft from main test site</td>
</tr>
<tr>
<td>(6 tests)</td>
<td>2</td>
<td>60°</td>
<td>1</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>60°</td>
<td>1</td>
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<td></td>
<td>4</td>
<td>30°</td>
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<td>7</td>
<td>18°</td>
<td>2</td>
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<td></td>
<td>8</td>
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<tr>
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<td>18°</td>
<td>1</td>
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<td>stone at tip</td>
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<tr>
<td>(2 tests)</td>
<td>6</td>
<td>60°</td>
<td>1</td>
<td></td>
<td>stone at tip</td>
</tr>
</tbody>
</table>

Table 5.3c Penetration test program at Amherst, Massachusetts, 1977
Fig. 5.1 Diagram of the Fugro electrical cone with friction sleeve (from Sanglerat (1972))
Fig. 5.2 The Pore Pressure Probe

(a) Diagram of original design (Wissa et al., 1975)
(b) 18° pore pressure probe with porous stone at tip.

Fig. 5.2 The pore pressure probe.
(b) 18° pore pressure probe with porous stone at tip.

Fig. 5.2 The pore pressure probe.
(c) Pore pressure probe with porous stone behind the conical tip.

Fig. 5.2 The pore pressure probe.
(c) Pore pressure probe with porous stone behind the conical tip.

Fig. 5.2 The pore pressure probe.
Fig. 5.3 Soil conditions at the Saugus, MA, test site (I95 embankment St. 246, 200 ft offset from centerline to the east; data from unpublished MIT reports; drawing courtesy of S. M. Lacasse).
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Fig. 5.15 SHANSEP undrained strength parameters and $K_o$ for Connecticut Valley Varved Clay (from Ladd and Poot, 1977)
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CHAPTER 6

STATISTICAL ANALYSIS OF CONTINUOUS PENETRATION RECORDS

6.1 Introduction

Chapters 6 and 7 present the results of the field testing program described in Chapter 5. In this chapter, the characteristics of the measurements are described, and the variability of the standard cone resistance \( (2\delta = 60^\circ, \ D/d = 1) \) analyzed. A procedure for filtering penetration data is introduced in order to reduce the influence of local soil variability on the data and allow easy comparison of test results. This procedure will be used extensively in Chapter 7 where we compare measurements from different cones and pore pressure probes.

6.2 Continuous Penetration Records

The electrical cone and the pore pressure probe provide essentially continuous data with depth, and hence can describe stratification and variability of a soil deposit more clearly than discrete measurements obtained by other means such as the field vane test or laboratory testing of soil samples. Figs. 6.1a and b shows profiles of \( q_c \) measured with the standard cone \( (2\delta = 60^\circ, \ D/d = 1) \) in the relatively uniform Boston Blue Clay and the more variable EABPL Clay (see site description in Chapter 5). These plots are made from data points discretized at depth intervals of 3 to 5 cm
(constant in each test) with a computer-controlled cathode-ray tube plotting device. For all clay deposits considered, these intervals reproduce the original continuous $q_c$ record obtained from the strip chart recorder with sufficient accuracy.

During cone and pore pressure probe tests, penetration stops every 1 m (or 5 ft depending on rod length) for about one minute or less in order to add a new pushing rod. When penetration starts again, the cone resistance, $q_c$, and the pore pressure, $u$, goes through a transient period before reaching a steady state. This transient behavior is very brief in the case of $q_c$ and can be neglected for all practical purposes. On the other hand, the pore pressure, $u$, typically does not reach a steady state until after about 4 to 15 cm of penetration (at a penetration rate of 1 to 2 cm/sec) depending on the characteristics of the soil and the probe*, Fig. 6.2. A profile of pore pressure $u$ during steady penetration is obtained by deleting the transient records of $u$; an example of a profile is shown in Fig. 6.3 for a 60°, $D/d = 1$, probe with the porous stone at the cone tip.

Comparing Figs. 6.3 and 6.1b, we note the strong correlation between the high values ('peaks') of $q_c$ and the low values of $u$ ('troughs'). e.g., at depths 22 ft, 42 ft, 80 to 90 ft, 106 ft, etc. These peaks and troughs probably indicate the presence of sand

*If the response of the probe is slow due to improper deairing, the transient period will increase significantly.
lenses, or perhaps highly over-consolidated clay. The implications of the ratio \( u/q_c \) will be discussed further in Chapter 7.

### 6.3 Repeatability of Measurements and Data Filtering

Soil exploration data obtained from field and laboratory tests always include some variability, which, in many cases, seriously affects major design decisions. These variations are due to:

1. actual variability of the deposit;
2. variability caused by the measuring device, e.g., strain-gage, pressure transducer, readout equipment, etc.;
3. variability in the test procedures, e.g., soil disturbance, rate of testing, etc.

In situ measurements obtained from the cone penetrometer and the pore pressure probe involve very simple test procedures that minimize human error and soil disturbance. Measurement variability in these tests is therefore due mainly to the equipment and the soils. Figs. 6.4a, 6.5a and 6.6a show the variability of cone penetration resistance \( q_c \), in three clay deposits: the relatively uniform Boston Blue Clay and Connecticut Valley Varved Clay, and the more variable EABFL Clay (Chapter 5). In each case, the (discretized) \( q_c \) measurements from two or three standard cone tests at close proximity (within a 40 ft radius of each other) are plotted together using different symbols. These figures indicate that \( q_c \) measurements generally form a well-defined band with a number of anomalies with small thickness which often do not appear in all tests. Many of these anomalies are less than 1 to 2 ft thick, and are probably due
to local clay variability, inhomogeneities (e.g. sand lenses, stones, sea shells or pieces of wood) or equipment malfunction.

Inhomogeneities in soft clay deposits generally have a higher shear strength than the clay mass, and thus do not have as much influence on the mass behavior as in granular deposits (or rock mass) where inhomogeneities with low strength might control the mass behavior. A study of the basic strength properties of clays therefore requires elimination of these anomalies. The most common method of data smoothing is simply to eliminate "bad" points from the analysis on the basis of "engineering judgement". However, this procedure suffers from lack of repeatability, especially when different investigations are involved, and may lead to controversy and disagreement. For a field exploratory program where large amounts of data are obtained, a more systematic method of data smoothing is therefore essential.

Data smoothing can be accomplished systematically simply by averaging all the data points that fall within a moving "window". This procedure may, however, be strongly influenced by a few extreme values in the data, and, because the anomalies are not excluded, cannot estimate the variability in the properties, e.g., the undrained shear strength, of the clay mass. A systematic method of filtering or eliminating the "bad" points from the data is therefore developed for this research program as described below.
The computerized data smoothing procedure used herein emphasizes the importance of the median (rather than the mean) because of its insensitivity to anomalies in the data (Turkey, 1970). We first divide the soil into sublayers of thickness $\Delta$, where $\Delta$ is an input parameter, and assume that the quantity of interest ($q_c, u$, etc.) is stationary within each sublayer in order to compute its statistical parameters from data points, obtained from one or more tests, within this layer. Anomalies contained within an arbitrary sublayer $i$ are filtered as follows:

1. Compute the standard deviation $\sigma$ of data points contained in each of the three consecutive sublayers $i-1$, $i$, and $i+1$.

2. Find the median $M$ of data points in all the three sublayers.

3. Select a "representative" standard deviation $S$ defined as:

$$S = \text{Minimum} \left\{ \frac{1}{2}[\sigma(i-1) + \sigma(i)], \frac{1}{2}[\sigma(i) + \sigma(i+1)], \right\}
\frac{1}{2}[\sigma(i-1) + \sigma(i+1)] \right\} \quad (6.1)$$

4. Remove all data points in layer $i$ outside the range

$$M \pm aS, \quad (6.2)$$

where $a > 0$ is a second input parameter.

This filtering process basically depends on the two parameters

*The computer software for this smoothing procedure is included as several subroutines in a software package that processes discretized data and makes plots of $q_c$, $u$, etc.
Δ and a. Small values of sublayer thickness, Δ, are necessary to satisfy the assumption of stationarity, and hence avoid errors due to actual variation with depth in the quantity of interest. On the other hand, reliable statistics require each sublayer to contain a sufficient number of data points (5 to 10 as a minimum). Also, Δ should not be less than the thickness of the anomalies to be filtered from the data. The parameter a influences the degree of filtering; small values of a (a < 1) retain only data points which are close to the median of the data, whereas large values of a (a > 3) filters only extreme values of the data. The procedure is not based on a rigorous statistical analysis but provides reasonable results which, given Δ and a, can be exactly duplicated.

Using Δ = 2 ft and a = 2, Figs. 6.4b, 6.5b and 6.6b show the filtered q_c measurements in the three clay deposits presented in Figs. 6.4a, 6.5a and 6.6a, respectively. By comparing unfiltered and filtered q_c data at the three sites, we note that, except for a few isolated regions, the filter performs a reasonable smoothing of anomalies and gives results which are probably similar to ones obtained on the basis of engineering judgement. A major advantage of this procedure is its ability to assess soil variability after filtering. Figs. 6.4c, 6.5c and 6.6c show the mean and the band of ± 2σ, where σ is the standard deviation, of the filtered data.

Assuming a normal distribution about the mean, this band should contain 95% of the data points. The reader can easily verify the
performance of the proposed filtering procedure by comparing this band to one drawn based on his judgement to include roughly 15% of the data points. Compare especially the widely variable measurements between the depths of 80 and 95 ft in EABPL clay, and the mean ± 2σ band, computed with and without filtering, Figs. 6.5a, 6.5b, 6.5c and 6.7a.

The influence of the parameters Δ and a on the filtering process is illustrated for the EABPL clay in Figs. 6.7a to d by comparing mean and standard deviation of: unfiltered data with Δ = 2 ft; filtered data with Δ = 2 ft and a = 1.5; filtered data with Δ = 2 ft and a = 2.5; and filtered data with Δ = 6 ft and a = 2.0. Analysis of these figures and other results not shown here indicates that:

(1) By increasing Δ from 2 to 6 ft, and a from 1.5 to 2.5, the mean values of q_c in different sublayers are not significantly affected. The filtered and unfiltered means are generally not very different; however, the filtered mean can be as much as 15% lower than the unfiltered mean (remembering that anomalies in q_c tend to be on the high side) and shows a smoother profile in regions where the variation in q_c is large, e.g., between the depths of 80 to 95 ft in EABPL clay.

(2) The standard deviation σ of the filtered data is much more uniform (with depth) than that computed from the unfiltered data, and should be more representative of the variability of the clay mass
(this will be discussed further in section 6.4). $\sigma$ is also more sensitive to the parameters $\Delta$ and $a$ than the mean, and tends to increase with increasing values of $\Delta$ and $a$. The fact that $\sigma$ increases with $\Delta$, even in a relatively uniform region, indicates nonstationarity of $q_c$. With $\Delta = 2$ ft, the effects on $\sigma$ of varying $a$ between 1.5 and 2.5 is less than 20%.

(3) Reliability study of designs based on the filtered $q_c$ data requires the probability distribution function of the data, in addition to the mean and the standard deviation. Each sublayer of thickness $\Delta$ generally contains 10 to 25 data points (depending on total number of tests and discretization frequency), which are insufficient for a reliable assessment of data distribution. To overcome this problem we study the distribution of a normalized random variable $x$ defined as:

$$x = \frac{q_c - \text{mean } q_c}{\sigma q_c},$$

(6.3)

where $\text{mean } q_c$ and $\sigma q_c$ are computed for each sublayer of thickness $\Delta$. If the computed distribution of $x$ is approximately the same for all sections of the profile (each containing several sublayers of thickness $\Delta$), then we can say that the distribution $x$ is independent of depth and write $q_c$ as a probabilistic function of depth, $d$, with a distribution related to that of $x$ by:
\[ q_c(d) = \text{mean}_{q_c}(d) + \sigma_{q_c}(d) \cdot x \]  

(6.4)

Figs. 6.8a and 6.8b show the cumulative probability distribution function of \( x \) (curve '1'), computed from 3 standard cone tests for two 46 ft layers of Boston Blue Clay: a heavily to moderately overconsolidated layer (depth = 26 to 72 ft), and a slightly overconsolidated layer (depth = 72 to 118 ft). Also shown for comparison are the theoretical normal probability distribution function (curve '2') and the Kolmogorov-Smirnov 95% confidence band (curves '3' and '4') around the sample probability distribution function (Kendall and Stuart, 1961; IMSL, 1975, subroutine USPC). These figures indicate that \( x \) has approximately the same distribution in both layers, that the distribution is close to a normal distribution (with zero mean and \( \sigma = 1 \)), and hence that Eq. (6.4) is valid.

6.4 Soil vs. Equipment Variability

The variability in the filtered \( q_c \) records can be due to soil variability and/or equipment errors. In order to have an indication of the relative importance of these two sources of measurement variability, we compare the variability of \( q_c \) from different soil deposits, and compare the variabilities of \( q_c \) and field vane measurements.

In these comparisons, our measure of variability is the coefficient of variation, \( V \), defined as:
\[ V = \frac{\text{standard deviation}}{\text{mean}}. \quad (6.5) \]

For cone resistance measurements, we use the difference between \(q_c\) and the horizontal total stress, \(\sigma_{ho}\), as a measure of the undrained shear strength of clay consistent with the theory presented in Chapter 4, thus the measure of variability of the cone penetration test is

\[ V(q_c - \sigma_{ho}). \quad (6.6) \]

However, results obtained on the basis of \(V(q_c - \sigma_{vo})\), where \(\sigma_{vo}\) is the vertical total stress, show identical patterns as \(V(q_c - \sigma_{ho})\), but generally, \(V(q_c - \sigma_{vo})\) is 5 to 15% higher than \(V(q_c - \sigma_{ho})\).

Figs. 6.9a, b and c show \(V(q_c - \sigma_{ho})\) for the three clay deposits, computed with two filters from 3 standard cone tests in the case of Boston Blue Clay and EABPL Clay, and 2 standard cone tests in the case of Connecticut Valley Varved Clay (Figs. 6.4, 6.5 and 6.6). For Boston Blue Clay and EABPL Clay, \(V(s_u\ Field\ Vane)\), computed with filters from 4 tests located within the same 200 ft radius area as the cone tests (Figs. 5.4 and 5.9), is also shown for comparison. We note from these figures that:

1. A filter with large sublayer thickness, \(\Delta\), produces a smoother \(V\) profile than a filter with small \(\Delta\), and tends to increase \(V\), especially when \(V\) is small (this can be attributed to the increased sample size and/or the errors due to the stationary assumption).
(2) In Boston Blue Clay, $V(\text{cone})$ decreases with depth, from about 0.20 to about 0.05. In EABPL Clay, two layers can be distinguished: one between depths of 45 and 75 ft, and one below 75 ft depth. In both layers, $V(\text{cone})$ decreases with depth, from more than 0.30 to about 0.12 and 0.10 respectively. In Connecticut Valley Varved Clay, $V(\text{cone})$ is relatively uniform and varies between 0.03 to 0.08, except at the top of the layer where it increases significantly.

(3) $V(\text{field vane})$ has approximately the same magnitude and shows the same pattern as $V(\text{cone})$. This is somewhat surprising since presumably the field vane test is more strongly affected by soil disturbance than the cone test, and might be expected to show more scatter than the cone penetration test. Possible explanations are: (1) the variability in $q_c$ is increased due to the smaller size of the cone relative to the field vane; (2) the variability detected both by the cone and the field vane is due mostly to the soil variability rather than random equipment (and test procedure) errors, or (3) uncertainties in the interpretation of cone resistance and/or field vane data which will be discussed in Chapter 8.

(4) In Boston Blue Clay and EABPL Clay, $V(\text{cone})$ and $V(\text{field vane})$ show a good correlation with the overconsolidation ratio, OCR, of the soils (Figs. 5.16). Low values of $V$ occur in clays with low OCR ($V \approx 0.05$ in Boston Blue Clay, and $V \approx 0.10$ in EABPL Clay), while in the more heavily overconsolidated regions, $V$ increases by 2 to 4 times. This significant variation of $V$ with stress history pro-
vides another indication that equipment errors are not the main source of measurement variability.* In Connecticut Valley Varved Clay where OCR varies almost as much as in Boston Blue Clay, \( V \) (cone) is however very small throughout (less than 0.08). This suggests that high OCR is not a direct cause of soil variability. Geology (Chapter 5) indicates that the highly overconsolidated Boston Blue Clay and the overconsolidated EABPL Clay have both been strongly affected by desiccation, whereas overconsolidation in Connecticut Valley Varved Clay is primarily due to removal of overburden. Since the effects of desiccation is likely to be non-uniform, desiccated clays can be expected to show high variability.** The variation in the \( V \) profiles of Fig. 6.9, therefore, indicates different degrees of desiccation rather than OCR.

(5) The relatively large values of \( V \) computed for the EABPL Clay relative to the Boston Blue clay indicates quantitatively that the EABPL Clay is less uniform, and furthermore that the point variability of the estimated \( s_u \) is roughly twice as high in the EABPL Clay as in the Boston Blue Clay.

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* This by no means rules out the possibility that the measurements contain systematic equipment errors.

** Ladd et al. (1971b) presents variability in the "apparent" OCR of Bangkok Clay due perhaps to different degrees of desiccation.
6.5 Summary

The continuous measurements of cone resistance, $q_c$, and pore pressure, $u$, during cone penetration allow better identification of soil stratigraphy and variability than discrete field measurements, e.g., field vane tests or laboratory tests on selected samples. In a marine illitic clay, a plastic deltaic clay and a glacial lake varved clay, $q_c$ is repeatable within a reasonable margin of uncertainty ($\pm 5$ to 15%). Typical $q_c$ profiles include small scale anomalies, probably due to soil inhomogeneity, which must be discarded when the variability of the clay mass is investigated. A computerized filtering procedure to eliminate these anomalies is developed and applied to records of $q_c$ at the three test sites. The filtered data are approximately normally distributed about the mean and have a standard deviation which varies with depth. An analysis of soil variability based on the coefficient of variation ($= \text{standard deviation/mean}$) shows that both the cone resistance and the field vane data detect the same soil variability which depends on the soil type and shows a significant increase in desiccated regions.
Fig. 6.1a Typical cone resistance profile in Boston Blue Clay. (* test number)
Fig. 6.1b Typical cone resistance profile in EABPL Clay.
Fig. 6.2 Details of pore pressure measurements for an 18°, D/d = 1.0, conical probe with porous stone at tip.
Fig. 6.3 A profile of steady penetration pore pressure in EABPL Clay (60°, D/d = 1, conical probe with porous stone at tip).
Fig. 6.4 Variability of $q_c$ in Boston Blue Clay.
(a) Measurements from 3 standard cone tests.
Fig. 6.4 Variability of $q_c$ in Boston Blue Clay.
(b) Filtered $q_c$ data ($\Delta = 2$ft, $a = 2.0$).
Fig. 6.4 Variability of $q_c$ in Boston Blue Clay.

(c) Statistics of filtered $q_c$ data ($\Delta = 2\text{ft}$, $a = 2.0$).
Fig. 6.5 Variability of $q_c$ in EABPL Clays.
(a) Measurements from 3 standard cone tests.
Fig. 6.5 Variability of $q_c$ in EABPL Clays.
(b) Filtered $q_c$ data.
Fig. 6.5 Variability of $q_c$ in EABPL Clays.
(c) Statistics of filtered $q_c$ data ($\Delta = 2\text{ft}$, $a = 2.0$).
Fig. 6.6 Variability of $q_c$ in Connecticut Valley Varved Clays.
(a) Measurements from 2 standard cone tests.
Fig. 6.6 Variability of $q_c$ in Connecticut Valley Varved Clays. 
(b) Filtered $q_c$ data ($\Delta = 2\text{ft}, \ a = 2.0$).
Fig. 6.6 Variability of $q_c$ in Connecticut Valley Varved Clays.

(c) Statistics of filtered $q_c$ data ($\Delta = 2$ft, $a = 2.0$)
Fig. 6.7 Effects of filtering on the mean and standard deviation of $q_c$.
(a) No filtering.
Fig. 6.7 Effects of filtering on the mean and standard deviation of $q_c$.
(b) Filtered data with $\Delta = 2\text{ft}$, $a = 1.5$. 
Fig. 6.7 Effects of filtering on the mean and standard deviation of \( q_c \).

(c) Filtered data with \( \Delta = 2\) ft, \( a = 2.5 \).
Fig. 6.7 Effects of filtering on the mean and standard deviation of $q_c$.

(d) Filtered data with $\Delta = 6\text{ft}$, $a = 2.0$. 

$\sigma_{vo}$

$\pm 2\sigma$ band

$60^\circ$ $D/d = 1.0$

$EABPL, LA$
Fig. 6.8 Cumulative probability density function of $x$, its Kolmogorov-Smirnov band, and the cumulative normal probability density function (mean = 0, $\sigma = 1$). Circles denote where normal pdf falls on the confidence limit.
Fig. 6.8 (cont'd.).
**Fig. 6.9a** Coefficients of variation of cone resistance and the field vane strength, Boston Blue Clay.
Fig. 6.9b Coefficients of variation of cone resistance and field vane measurements in EABPL Clay.
Fig. 6.9c Coefficients of variation of cone resistance in Connecticut Valley Varved Clay.
CHAPTER 7

TEST RESULTS

7.1 Introduction

This chapter presents measurements obtained by means of cone penetrometers and pore pressure probes with different sizes and shapes. The testing program is outlined in Chapter 5 and consists of more than 6,000 ft of penetration. The data presentation is divided into three parts: cone resistance $q_c$, pore pressure $u$ measured along a cone during steady penetration, and the comparison between $q_c$ and $u$. In each part, we consider the effects of cone size and shape and penetration rate. In the case of $u$, we also determine the effect of the location of the porous stone. Results are compared with the theoretical predictions in Chapter 4 and with similar results reported in the literature.

Available equipment (Chapter 5) allow one type of measurement per test, i.e., $q_c$ or $u$. In order to show the trend in the data and allow easy comparison of test results, measurements are filtered and averaged by the procedure described in Chapter 6 ($\Delta = 2$ ft; $a = 2.0$ unless otherwise noted). When more than one test of the same characteristics (same $2\delta$ and $D/d$) is performed, the average of these tests is presented.

7.2 Cone Penetration Resistance

(1) Effect of Cone Angle. Figs. 7.1 show $q_c$ profiles in Boston Blue Clay and EABPL Clay from "enlarged" cones ($D/d = 2$) with three different apex angles ($2\delta$): $60^\circ$, $30^\circ$ and $18^\circ$. As predicted by the theory in Chapter 4, $q_c$ in both deposits increases as $2\delta$ decreases. In Boston Blue Clay,
Fig. 7.1a, \( q_c(2\delta = 18^\circ) \) is about 30\% greater than \( q_c(2\delta = 60^\circ) \) throughout the profile as predicted for the idealized enlarged ones with very large \( D/d \) (Eq. (4.2)). For EABPL Clay, Fig. 7.1b, the difference between \( q_c(2\delta = 18^\circ) \) and \( q_c(2\delta = 60^\circ) \) is only 10 to 20\% between depths of 40 and 80 ft, but increases to about 30\% below depths of 90 ft. Measurements of enlarged cone resistance in Connecticut Valley Varved Clay (CVVC) were not conducted.

Figs. 7.2 show the effect of cone angle on \( q_c \) for unenlarged or "regular" cones (\( D/d = 1 \)). As for enlarged cones, \( q_c \) increases as \( 2\delta \) decreases. In Boston Blue Clay, the difference between \( q_c(2\delta = 18^\circ) \) and \( q_c(2\delta = 60^\circ) \) is generally more pronounced than for enlarged cones, whereas in EABPL and Connecticut Valley Varved Clays, it is generally less pronounced (less than 20\%).

Muromachi (1974) measures similar effects on cone angles on \( q_c \) in a re-sedimented clay. The cone used is a hand penetrometer similar in shape to that shown in Fig. 7.3a, with a \( D/d = 3.2 \) and a (cone tip) base area of 6.45 cm\(^2\). He finds that \( q_c \) remains relatively constant for \( 2\delta \geq 30^\circ \), and that \( q_c(2\delta = 15^\circ) \) is about 25\% higher than \( q_c(2\delta = 60^\circ) \).

(2) **Effect of Cone Shape.** Figs. 7.3 show various cone types and shapes in common use. The force required to push the mechanical cones in Figs. 7.3a, b and c is measured at the surface by means of oil pressure gage, proving ring, etc., whereas electro-mechanical transducers are used for the electric cones in Figs. 7.3d and e. In order to study the effect of tip shape (enlargement) on cone resistance, tests were conducted by means of Fugro equipment using cones with different tip enlargement ratios (\( D/d \)), and the results compared with the mechanical cone (Fig. 7.3c).
Figs. 7.4 show the effect of tip enlargement on $q_c$ ($D/d = 2$ vs. $D/d = 1$) in Boston Blue Clay and EABPL Clay. Above a depth of 70 ft, both regular and enlarged cones have approximately the same penetration resistance in the two clay deposits. On the other hand, below a depth of 70 ft in both deposits, the cone resistance of regular cones $q_c$ ($D/d = 1$) generally exceeds the cone resistance of enlarged cones (with some exceptions in EABPL Clay). This difference in $q_c$ tends to increase with depth and is in general about 50 to 80% of the initial vertical total stress, $\sigma_{vo}$ (or about 20% of $q_c$ for $D/d = 1$). Figs. 5.16 show profiles of two parameters: $\sigma_{vo}/s_u$ and OCR, that may influence cone resistance. In view of these results, tip enlargement appears to affect cone resistance when the ratio $\sigma_{vo}/s_u$ (field vane) is high (more than 7 or 8), or when OCR is close to unity (i.e., normally consolidated clays).

Thomas (1965) determines the effect of tip enlargement on $q_c$ in London Clay for mantle cones, Fig. 7.3b. The base area of his "regular" cone tip equals 10 cm$^2$ and his "enlarged" tip is 20 cm$^2$. He finds that the effect of tip enlargement is not noticeable at a depth less than 15 to 20 ft, but becomes pronounced at greater depth where $q_c$ (regular tip) exceeds $q_c$ (enlarged tip) by about 15 to 25%. At his test site, however, the clay is much stiffer below a depth of 25 ft than the clay above. This suggests that OCR alone is not sufficient to explain the variation in the effect of tip enlargement on $q_c$.

Joustra (1974) compares $q_c$ measurements from Fugro and Delft electrical cones (Figs. 7.3d and c). The Fugro cone has an unenlarged tip, whereas the Delft is slightly enlarged ($D/d = 1.3$). The soil at the test site consists basically of 25 ft of soft clay overlying dense sand. Results
indicate that \( q_C \) (Fugro) equals or only slightly exceeds \( q_C \) (Delft) in the clay, but exceeds \( q_C \) (Delft) by 30%, on the average, in the deeper sand. This provides another indication that a tip enlargement tends to reduce \( q_C \) when the overburden stress is high (compared with the strength of the soil). At shallow depth, enlargement of the tip has little or no effect on \( q_C \), probably because the hole left behind the cone tip has little tendency to close.

(3) **Electrical Cone vs. Begemann Cone.** The Begemann mechanical cone in Fig. 7.4c has a large mantle area behind the tip, on which some friction acts and hence can overestimate the actual cone resistance, \( q_C \). Fig. 7.5 compares \( q_C \) for Fugro cones with 60° enlarged and unenlarged tip with \( q_C \) for a Begemann mechanical cone in Boston Blue Clay. \( q_C \) (Begemann cone) exceeds \( q_C \) (electrical cones) by about 20% above a depth of 70 ft., where \( q_C (D/d = 1) \approx q_C (D/d = 2) \), but is approximately equal to \( q_C (D/d = 2) \) below that depth. However, the profile for \( q_C \) (Begemann) differs substantially from that of \( q_C (D/d = 1) \) or \( q_C (D/d = 2) \). This difference is most likely due to friction on the mantle behind the tip and the weight of the pushing rods* in the Begemann cone. The dashed line in Fig. 7.5 shows \( q_C \) (Begemann) "corrected" for the weight of the rods. The "corrected" \( q_C \) (Begemann) profile agrees reasonably well with \( q_C (D/d = 2) \) but consistently exceeds it by 40 to 75%. This is probably due to the friction on the mantle behind the tip.

* The steel inner pushing rods have a diameter of 1.3 cm and a cross-sectional area of 1.7 cm². The contribution of their weight to \( q_C \) (Begemann) is approximately equal to 70% of \( \sigma_{vo} \) and should be added to the cone resistance measured by a load cell at the surface (Schmertmann, 1975).
Other investigators report similar effects of friction on the mantle behind the tip. De Beer et al. (1974) observe that, in stiff clay, \( q_c \) (mantle cones) exceeds \( q_c \) (mechanical cone without mantle, Fig. 7.4a) by about 40%. Brand (1974) reports that, in soft clay, \( q_c \) (mantle cones) exceeds \( q_c \) from a hand penetrometer (similar in shape but about 2/3 the size of cone in Fig. 7.4a) by about 25%.

(4) Effects of Penetration Rate. Penetration rate (velocity) affects the cone penetration process by influencing the rate of straining in the soil around the cone. Figs. 7.6a and b show changes in \( q_c \) due to changes in penetration rate (from a standard rate of 2 cm/sec) in Boston Blue Clay and EABPL Clay.* These results are obtained by increasing or decreasing the penetration rate by 5 to 10 times for a distance of 0.5 to 2 m.

The results in Figs. 7.6a and b show no noticeable difference in the effect of penetration rate between "regular" and "enlarged" cones. In both cone types, \( q_c \) increases or decreases by about 10% when the penetration velocity increases or decreases tenfold. The scatter is, however, significant. This is approximately the same effect of strain rate determined by laboratory undrained shear strength tests on the same clays (Ladd et al., 1977). Similar effects of penetration velocity on \( q_c \) are reported by Thomas (1965) and Muromachi (1974).

* The \( q_c \) ratios are computed from average values of \( q_c \) over a depth increment of 0.5 to 2 m in relatively uniform soil. Tests performed in the more variable regions of the profile are excluded. The range of the \( q_c \) (fast) and \( q_c \) (slow) from all tests is presented with the results of the standard-rate tests in Appendices C, D and E.
(5) Access to Water. For enlarged cones \((D/d = 2)\), the theory predicts that a cavity forms behind the cone and that the pressure in this cavity affects cone resistance. One possible method of checking this prediction is to compare \(q_c\) when this cavity is filled with water to \(q_c\) when it is only partially filled with water or is filled with air by controlling the access to water. Penetration tests using enlarged cones where the cone is pushed through a predrilled hole (about 2 ft deep) filled with water are therefore compared with penetration tests conducted from the surface. Figs. 7.7 compare \(q_c\) (wet), where water is supplied, with \(q_c\) (dry), where the cone is pushed from the surface. For a cone angle \(2\delta = 60^\circ\), Fig. 7.7a indicates that \(q_c\) from "wet" and "dry" tests are essentially identical, except at a depth of 85 ft, where the \(q_c\) (dry) is very high, probably due to the presence of a sand lens. For a cone angle \(2\delta = 18^\circ\), Fig. 7.7b shows that \(q_c\) (wet) is 10% to 20% higher than \(q_c\) (dry) along most of the deposit.* Since this difference can result entirely from soil and/or test variability (see variability of identical tests in Fig. 6.5a), no clear effect of access to water can be reached from the limited tests performed.

(6) Scale Effects. With the same cone geometry, a change in the size of the cone can affect the cone resistance in two ways. First, it influences strain-rate in the soil around the cone. Appendix A shows that, for a fixed penetration velocity, the strain-rate is inversely proportional to the diameter of the cone. Results of strain-rate effects

* If the cavity in the "dry" test is not filled with water, the theory predicts the \(q_c\) difference to be 25% of the pore pressure measured behind the cone tip, which is approximately the same as the left-most curve in Fig. 7.11a.
presented earlier indicate that this component of scale effect is relatively unimportant. The second scale effect results from the soil inhomogeneities. Larger cones cause deformations, strains and stresses in a larger volume of soil than a smaller cone. Inhomogeneities in the soil due to varves, fissures or other impurities having sizes of the same order as the cone can have a significant effect on $q_c$.

The present testing program did not investigate the scale effects. Muromachi (1974) varies the base area of the cone from less than 2 cm$^2$ to 20 cm$^2$ while keeping the shape* constant. He finds that $q_c$ in remolded clay decreases about 15% due to a tenfold increase in diameter. De Beer et al. (1974) varies the cone diameter from 3.6 cm to 25 cm, using the cone shape shown in Fig. 7.3a. Penetration tests in stiff clay indicate that the increase in cone diameter generally causes a 0 to 30% reduction in $q_c$.

In conclusion, scale effects can affect $q_c$. Their importance is probably on the same order as scale effects on laboratory testing of soil samples. However, in uniform clays, no significant scale effects are expected due to doubling the diameter of the cone (as in our tests on enlarged cones).

(7) **Friction Sleeve Measurements.** Because of the analogy between cone penetrometers and piles, many investigators have revised methods to measure the frictional resistance of soil during cone penetration in addition to cone resistance $q_c$. The relation between the force per unit area, $f_s$, exerted on the friction sleeve behind the cone and $q_c$ are widely correlated with soil types. Experience with the Fugro electrical cone is some-

* 60° cone. Geometry unclear from his paper.
what limited. For mechanical cones, the two widely quoted correlations are:

**Begemann (1965)**

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>( f_s/q_c (%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse to fine sand</td>
<td>1-1.5</td>
</tr>
<tr>
<td>Silty sand</td>
<td>1.5-2.5</td>
</tr>
<tr>
<td>Clayey sand</td>
<td>2.5-3.5</td>
</tr>
<tr>
<td>Loam</td>
<td>3.5-4</td>
</tr>
<tr>
<td>Clay</td>
<td>4-6</td>
</tr>
</tbody>
</table>

**Sanglerat (1972)**

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>( f_s/q_c (%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sands</td>
<td>1-2</td>
</tr>
<tr>
<td>Clays, silts, sands</td>
<td>1-4</td>
</tr>
<tr>
<td>Clays</td>
<td>4-8</td>
</tr>
<tr>
<td>Clays and peat</td>
<td>3-10</td>
</tr>
</tbody>
</table>

Measurements of \( f_s \) made with the Fugro electrical cone at the Saugus and Amherst, MA, test sites do not give intelligible results (\( f_s \) is often negative at large depth) probably due to the small value of the actual frictional resistance compared to equipment uncertainties. In the Atchafalaya Basin Clay, the \( f_s/q_c \) ratio varies generally between 2 and 7%, with \( f_s \) tending to increase with \( q_c \). This makes the \( f_s/q_c \) ratio less sensitive to changes in soil characteristics, especially in clays, than the ratio of pore pressure to cone penetration, which will be discussed in the next section.

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* Data including cone tests at Bayou Sorral, LA, performed by Fugro Gulf, Inc., for Ardaman and Assoc., Inc., in 1977, courtesy of Ardaman and Assoc.

+ Test loading of the load cells (Fig. 5.1) with no friction sleeve attached indicates a friction reading under a purely axial load. This suggests that there is an unwarranted interaction between the load cells. The magnitude of this interaction cannot be determined with the limited data available.
7.3 Pore Pressures During Steady Cone Penetration

Pore pressures developed during steady state cone penetration are essential in the understanding of the cone penetration process. This section presents measurements of pore pressures at different locations on four cones with different geometries. The data in this section are from the Saugus, Massachusetts, test site, where extensive pore pressure measurements were obtained* and where the clay deposit is relatively uniform, thus allowing an easy comparison of test results.

Figs. 7.8 shows the measured pore pressures, $u$, during the steady penetration of 18° enlarged and unenlarged cones. Each curve represents the results obtained from an individual test conducted within a close distance of one another. For the unenlarged cone, Fig. 7.8a indicates that the $u$ profiles are somewhat variable above a depth of 40 ft, probably because of significant soil variability. Below 40 ft, all $u$ profiles are approximately parallel and show identical pattern with depth. The largest pore pressure $u$ is measured at mid-height of the cone (curve "2") and decreases behind the tip. Torstensson (1975) observes a similar trend in a normally consolidated clay among pore pressures measured at the tip of and behind a 60° cone during steady penetration, using a different probe design.

Pore pressures measured at the tip of the cone (curve "1") are about 1 kg/cm$^2$ lower than at the middle of the cone (curve "2")**. On the

* In addition to steady penetration, pore pressures during driving, after penetration stops, during and after monotonic or cyclic axial or rotational motion, were measured and will be reported elsewhere.
** Pore pressure at the tip is measured with probes similar in design to Fig. 5.2a. The porous stone in these probes protrudes slightly in front of the conical tip of the instrument. Figure 5.2b shows a photograph of the 18° conical probe.
cylindrical shaft behind the conical tip, \( u \) decreases for a distance of 4\( d \) to 5\( d \) (\( d \) is the shaft diameter = 3.8 cm for the pore pressure probes), and then appears to remain constant, at least to a distance of 11\( d \) behind the tip.

Pore pressure variation along the cone and the shaft behind it can result from one of two reasons:

1. a change in the total stress along the cone and shaft;
2. pore pressure dissipation, i.e., soil consolidation.

In order to assess the effect of consolidation on the pore pressure difference between points 3 and 6 on the shaft behind the cone, we first estimate the time required for a soil particle to travel the distance 11\( d \) between the two points. For a penetration velocity of 1 to 2 cm/sec used in the tests, this time is about 20 to 40 sec. During such a time period, records of pore pressure decay after penetration is stopped (for a measurement behind the tip) indicate that a negligible amount of consolidation takes place. This, in addition to the very close values of \( u \) at points 5 and 6, show that the measured pore pressures in Fig. 7.8a reflect the variation in the total stresses along the cone and shaft. The magnitude and variation of these total stresses, in turn, affects cone resistance.

Fig. 7.8b shows the steady state pore pressures \( u \) along an 18° enlarged cone (\( D/d = 1.9 \)). As in the case of an unenlarged cone, we note that \( u \) at cone tip (curve "1") is greater than \( u \) behind the cone (curves "2" and "3"). Pore pressure at the tip is also essentially the same as \( u \) (tip) for an unenlarged cone, Fig. 7.8a. Furthermore, \( u \) behind the
enlarged cone is uniform along the shaft at least up to a distance 11d. This supports the theoretical prediction of a cavity behind enlarged cones. Comparing this cavity pressure to the pore pressures far behind an 18° unenlarged cone (curves "5" and "6" in Fig. 7.8a), we note that they are essentially identical below a 70-ft depth, but at shallower depths u behind the enlarged cone is about 0.5 to 1 kg/cm² smaller than u behind the unenlarged cone.

Figs. 7.9 show the steady penetration pore pressures around enlarged and unenlarged 60° cones. As in the case of 18° cones, Fig. 7.9a indicates that u at the tip of an unenlarged cone is higher than u on the shaft behind that tip. Fig. 7.9b shows that u is uniform behind an enlarged cone. u at a distance of 3.2d behind the tip of the unenlarged 60° cone is consistently about 1 kg/cm² larger than u behind the enlarged 60° cone, and is essentially identical to u at the same distance behind an 18° unenlarged cone (curve "4" in Fig. 7.8a). Since u is not measured very far behind the 60° unenlarged cone, no conclusion can be drawn regarding the effect of cone angle on this pore pressure. For the enlarged cones, however, u behind the 60° cone is consistently about 0.5 to 1.0 kg/cm² larger than u behind the 18° cone.

The pore pressure far behind an unenlarged cone is of some practical interest, since it resembles the pore pressure induced around a pile during installation. From the data presented above, we note that the pore pressure is very large in magnitude (> σ_vo). Koizumi and Ito (1967) present measurements of normal stresses and pore pressures on and around a jacked pile in silty clay which show that the total stress and the pore pressure on the pile shaft are essentially equal, and that their magnitude is as much as 2 to 3 times σ_vo.
7.4 Comparison Between Steady State Pore Pressures And Cone Resistance

This section presents pore pressures, \( u \), measured during steady penetration at the tip of 60° and 18° unenlarged cones and far behind an 18° unenlarged cone, and compares them to \( q_c \) in three clay deposits.

(1) **Boston Blue Clay.** Fig. 7.10a shows \( q_c \), \( u \) (tip) and \( u \) (far behind) for an 18° unenlarged cone \((2\delta = 18^\circ, D/d = 1)\). Clearly \( q_c \) exceeds \( u \) throughout the profile. Above a depth of 60 ft \((OCR > 2)\), both \( u \) (tip) and \( u \) (far behind) are small compared to \( q_c \). Below a 60-ft depth \((OCR = 1.2 \text{ to } 2)\), \( u \) (tip) is about 70% of \( q_c \), whereas \( u \) (far behind) is about 55% of \( q_c \).

Fig. 7.10b shows \( q_c \) and \( u \) (tip) for a 60° unenlarged \((2\delta = 60^\circ, D/d = 1)\) cone which exhibits trends similar to the 18° cone in Fig. 7.10a. However, below a depth of 60 ft, \( u \) (tip) exceeds \( q_c \) by about 10%. This is contrary to theoretical predictions in Chapter 4 and is believed to reflect inaccuracies in \( q_c \) measurements.*

Fig. 7.10c shows the smoothed profiles of the \( u/q_c \) ratio obtained from filtered \( u \) and \( q_c \) data where anomalies due to small scale inhomogeneities in the soil (e.g., sand or silt lenses) have been eliminated. All three \( u/q_c \) profiles in Fig. 7.10c increase with depth to about 70 ft and thereafter remain constant. Curve 1, corresponding to a 60° cone where \( u \) is measured at the tip, shows the highest value of \( u/q_c \). Values of \( u/q_c > 1 \)

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* \( q_c \) measurements for unenlarged cones can underestimate the actual cone resistance by as much as 15% due to pore pressures \( u \) acting behind the tip (location 3 in Fig. 7.8a). This error is, however, reduced when \( u \) at this location is small compared to \( q_c \).
attained by this curve below a depth of 60 ft are believed to be caused by inaccuracies in \( q_c \) measurements as discussed earlier.

(2) Atchafalaya Basin Clay. Fig. 7.11a shows \( q_c \), \( u \) (mid-cone) and \( u \) (far behind) for an 18° unenlarged cone (\( u \) (tip) is not available at the main test site; for location of mid-cone, see Fig. 7.8, "2"). Generally, steady penetration pore pressures are smaller in this plastic (high PI) clay deposit than in the less plastic (low PI) Boston Blue Clay. \( u \) (far behind) is less than \( \sigma_{vo} \) except between depths of 40 to 53 ft, 70 to 82 ft, and below 95 ft, where it is approximately equal to \( \sigma_{vo} \). These locations correspond to regions of normally consolidated clay (Fig. 5.16b). \( u \) (tip) shows a variation with depth identical to that of \( u \) (far behind), and the difference in their magnitudes is almost uniform in the normally consolidated regions, as was observed in the Boston Blue Clay below the desiccated crust. Both pore pressures tend to decrease when \( q_c \) increases. This important feature will be amplified by considering the \( u/q_c \) ratio.

Fig. 7.11b shows \( q_c \) and \( u \) (tip) for a 60° unenlarged cone. In this case, \( u \) (tip) exceeds \( q_c \) at a depth of about 50 ft, and is almost equal to \( q_c \) at a depth of 70 ft. Again, these locations of high pore pressure correspond to normally consolidated regions in the deposit.

Fig. 7.11c shows the smoothed \( u/q_c \) profiles which clearly show three distinct strata. It is perhaps worth mentioning that these three curves, showing identical patterns, are obtained from five different instruments and seven soundings. This consistent feature of the \( u/q_c \) ratio indicates its great potential in soil profiling and soil identification. Additional research is needed in this area, however.
(3) Connecticut Valley Varved Clay. Fig. 7.12a shows \( q_c \) and \( u \) (tip) for an 18° unenlarged cone. In this deposit, \( u \) (tip) is relatively close to \( q_c \) except in the desiccated crust near the surface and in the sandy material below a depth of 60 ft. Fig. 7.12b shows \( q_c \) and \( u \) (tip) for a 60° unenlarged cone which shows the same pattern as the 18° cone, except that the difference between \( u \) and \( q_c \) becomes very small. Fig. 7.12c shows that the smooth \( u/q_c \) profiles are relatively constant and equal to 0.8 to 0.9 between depths of 30 to 60 ft, where OCR varies between 1.3 and 2.

7.5 Comparison with Other In-Situ Measurements

(1) Lateral earth pressure cell. This instrument is installed in the ground to measure the "in-situ" lateral stress (and pore pressure) Fig. 7.13a shows two M.I.T. earth pressure cells designed and built for a parallel research project (Ladd et al., 1978). They consist basically of a flat plate with a "stress cell" and a porous element in the middle. Both the stress cell and the porous element are connected hydraulically with an electro-mechanical pressure transducer. The plate has a wedge-shaped tip and is connected to the pushing rods at the top. The assembly is pushed from the bottom of a predrilled hole (filled with drilling mud) for a distance of 4 to 6 ft. The electronic signals from the transducers are monitored continuously during penetration and periodically for several weeks afterwards.

Fig. 7.13b shows the stresses and pore pressure measured during steady penetration at the Saugus, Massachusetts, test site.* We note that the pore

* Courtesy of Prof. C.C. Ladd, Dr. S. M. Lacasse and Mr. J.T. Germaine of M.I.T.
pressure is almost equal to the total stress and that both of them are on
the same order as, or higher than, the pore pressure measured behind
enlarged cones during steady penetration. These stress and pore pressures
cannot be determined from the steady wedge penetraion theory presented in
Chapter 3 and Appendix B. Their large magnitude suggests that strains
have a significant influence on the pore pressure and hence the stresses.
Rational solution for this case should therefore follow the strain path
approach outlined in Section 3.1, using the strain history of soil elements
provided by Appendix A.

(2) **Pressuremeter.** The pressuremeter is an in-situ testing device
consisting basically of an expandable cylindrical membrane. The membrane is
expanded, during the test, while the volume and the pressure inside are
monitored. The pressure-volume curve can analytically be related to the
undrained stress-strain properties of the clay via solutions for the plane-
strain expansion of a long vertical cylindrical cavity in clay mass. (Baguel-
lin et al., 1972; Ladanyi (1972); Palmer (1972); Prevost and Hoeg (1975a
and b)). However, there is still, at present, a great deal of uncertainty
about interpretation of the test results for use in stability analysis of
clays (Ladd et al., 1977).

Fig. 7.14 shows the limit pressure $p_L$ (solid squares) at the
Saugus, Massachusetts, test site. These results are obtained with the
self-boring pressuremeter (PAPSOR; Baguelin et al., 1972).* Also shown in

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* $p_L$ is obtained from the pressure-volume curve by "eyeballing" by Mr.
J. T. Germaine.
Fig. 7.14 are $q_c$ from a Fugro cone (60°, D/d = 1) and $u$ measured at the
tip of and at a distance of 3.2d behind a 60° unenlarged cone (see discus-
sion of $u$ vs $q_c$ in Section 7.4). The limit pressure $p_L$ is essentially
identical to $q_c$. This is quite surprising, since cone penetration and
the pressuremeter test involve quite different deformation mechanisms.
Similar results were observed by M. Jamiołkowski in Italy (private commu-
ication, 1977).

7.6 Summary

This chapter presents extensive cone penetration measurements ($q_c$ and
$u$) in three clay deposits conducted to identify important parameters affect-
ing cone penetration. The results show the following:

(1) $q_c$ increases as the apex angle $2\delta$ decreases (sharper cone). The
effect of $2\delta$ on $q_c$ is of the same order as the theoretical prediction (Chap-
ter 4).

(2) Tip enlargement tends to reduce $q_c$; its effect, however, depends
on the soil type. In "soft" clays (OCR ≈ 1), doubling the cone tip diame-
ter reduces $q_c$ by 0.5 to 1 times $\sigma_{vo}$ (relative to "unenlarged" cone). In a
"stiff" desiccated region, the effect of tip enlargement becomes very small
or nonexistent.

(3) Mantle cone can significantly overestimate $q_c$, especially in
"stiff" desiccated clay, due to friction acting on the sleeve. However,
it may underestimate $q_c$ in deep "soft" clay (OCR ≈ 1), due to the weight
of the pushing rods.

(4) Moderate variation (say, by a factor 2) from the standard penen-
tration velocity of 1 to 2 cm/sec is unlikely to cause any noticeable change
in $q_c$. The limited tests performed do not allow extrapolation to very slow shearing rate comparable to field failures developing over days or weeks.

(5) For penetration at 1 to 2 cm/sec in homogeneous "medium" to "soft" clays, pore pressure $u$ induced by cone penetration around the cone does not have time to dissipate. The variation of $u$ along a cone thus provides an indication of the stress field. Measurement indicates that $u$ is largest at the middle of the cone and decreases along the shaft. $u$ at the tip is slightly smaller than $u$ at the middle of the cone. For unenlarged cones (tip diameter $D = $ shaft diameter $d$), $u$ decreases behind the cone for a distance of 4$d$ to 5$d$ and then remains constant at least to a distance of 11$d$. For enlarged cones, $u$ is uniform behind the cone.

(6) Measurements from 18° cones indicate that $u$ (tip) is not significantly affected by tip enlargement. In "soft" clays, $u$ at a distance greater than 4$d$ to 5$d$ behind the cone is essentially identical to $u$ behind an enlarged cone. In Boston Blue Clay where extensive pore pressure measurements are available, $u$ at the tip of unenlarged cones and $u$ behind enlarged cones increases with increasing apex angle.

(7) $u$ tends to decrease when $q_c$ increases. In "soft" clay (OCR $\approx 1$) regions, $u$ (tip) varies between 0.6 and 1.1 times $q_c$. Values of $u > q_c$ are believed to indicate inaccuracies in $q_c$ measurement. The ratio $u / q_c$, using $u$ at tip, mid-cone, or behind the cone, provides an excellent indication of the variation in stress history and soil type.

(8) Pore pressure behind enlarged cones is of the same order as or lower than pore pressure and total stress behind enlarged and unenlarged wedges.
(9) In Boston Blue Clay, \( q_c \) from standard Fugro cones \( (2\delta = 60^\circ, \quad D/d = 1) \) is essentially identical to the limit pressure, \( p_L \), from the self-boring pressuremeter (PAFSOR).
Fig. 7.1a Effect of cone angle on $q_c$ from "enlarged" cones, Boston Blue Clay.
Fig. 7.1b Effect of cone angle on cone resistance from "enlarged" cones, EABPL Clay.
Fig. 7.2a Effect of cone angle on cone resistance of "regular" cones, Boston Blue Clay.
Fig. 7.2b  Effect of cone angle on cone resistance of "regular" cones, EABPL Clay.
Fig. 7.2c Effect of cone angle on cone resistance from "regular" cones, Connecticut Valley Varved Clay.
Fig. 7.3a
Original Dutch Cone

Fig. 7.3b
Mantle Cone

Fig. 7.3c
Adhesion Jacket Cone (Begemann)
Fig. 7.3d
Fugro Electric Cone

friction mantle (Ø 35.6)

Fig. 7.3e
Delft Electric Cone

friction mantle

300
200
Ø 28

Dimensions in millimeters
1 in = 25.4 mm.

(Drawings adapted from Heijnen, 1974.)
Fig. 7.4a Effects of tip enlargement on resistance, Boston Blue Clay.
SAUGUS, MA (1)

\[ 30° \ D/d = 1.0 \ [6] \]
\[ 30° \ D/d = 2.0 \ [11] \]

Fig. 7.4a (cont'd.).
SAUGUS, MA (1)

$\sigma_{vo}$

Depth, ft

$\sigma_{c}, \text{kg/cm}^2$

Fig. 7.4a (cont'd.).
Fig. 7.4b  Effects of tip enlargement on cone resistance, EABPL Clay.
Fig. 7.4b (cont'd.).
Fig. 7.5 Cone resistance of Fugro electric cone vs. Begemann mechanical cone, Boston Blue Clay.
Fig. 7.6a Effect of penetration rate on cone resistance, Boston Blue Clay.
Fig. 7.6b  Effect of penetration rate on cone resistance, EABPL Clays.
Fig. 7.7 Effect of access to water on $q_c$.
(a) 60° enlarged cones.
Fig. 7.7 Effect of access to water on $q_c$.

(b) $18^\circ$ enlarged cones.
Fig. 7.8a Pore pressures measured along an 18° unenlarged cone during steady penetration.
Fig. 7.8b Pore pressures measured along an 18° enlarged cone during steady penetration.
Fig. 7.9a Pore pressures measured along a 60° unenlarged cone during steady penetration
Fig. 7.9b  Pore pressures measured along a 60° enlarged cone during steady penetration.
Fig. 7.10a Cone resistance and pore pressures for an 18° unenlarged cone (2δ = 18°, D/d = 1) in Boston Blue Clay
Fig. 7.10b  Cone resistance and pore pressures for a 60° unenlarged cone (2δ = 60°, D/d = 1) in Boston Blue Clay
Fig. 7.10 c  Pore pressure to cone resistance ratios in Boston Blue Clay.
Fig. 7.11a Cone resistance and pore pressure for an 13° unenlarged cone (2θ = 18°, D/d = 1) in EABPL Clay
Fig. 7.11b Cone resistance and pore pressure for a 60° unenlarged cone ($2\delta = 60°$, D/d = 1) in EABPL Clay
Fig. 7.11c Pore pressure to cone resistance ratios in EABPL clay
Fig. 7.12a Cone resistance and pore pressure for an 18° unenlarged cone ($2\delta = 18^\circ$, D/d = 1) in Connecticut Valley Varved Clay
Fig. 7.12b  Cone resistance and pore pressure for a 60° unenlarged cone (2δ = 60°, D/d = 1) in Connecticut Valley Varved clay
Fig. 7.12c  Pore pressure to cone resistance ratios in Connecticut Valley Varved Clay.
1 = pressure cell
2 = porous element

Dimensions in millimeters; 1 in = 25.4 mm.

Fig. 7.13a  Diagram of M.I.T. lateral earth pressure cell (courtesy of S.M. Lacasse)
Fig. 7.13b Comparison between pore pressure behind cones and pore pressure and stress behind wedges. (Data courtesy of J.T. Germaine and S.M. Lacasse, M.I.T.)
Fig. 7.14 Comparison between penetration measurements, $q_c$ and $u$, and limit pressure, $p_L$, from pressuremeter test in Boston Blue Clay.
CHAPTER 8

EVALUATION OF THEORETICAL PREDICTIONS

8.1 The "Field" Strength

The semi-empirical theory of cone penetration presented in Chapter 4 provides a rational basis for predicting the undrained shear strength, \( s_u \), of clays based on cone resistance \( q_c \). Table 8.1 summarizes theoretical predictions of \( s_u \) for enlarged and unenlarged cones as well as predictions of \( s_u \) based on pore pressure measurements during steady penetration in front of and behind enlarged cones.

Evaluation of the theoretical predictions is complicated by the difficulty in defining the undrained shear strength of clays. The ultimate objective of an in-situ test, such as the cone penetration test, is to determine the "field" strength, \( s_u \) (field), to use in design. Due to the complicated behavior of soils, the value of \( s_u \) (field) varies with the problem at hand, e.g., embankment stability, footing bearing capacity, pile resistance, etc., and can best be evaluated by full-scale tests. This chapter concentrates on undrained bearing capacity or stability problems.* The method for estimating \( s_u \) (field) for these problems depends on the type of information available:

(1) Actual embankment performance. In the course of several earlier research programs, M.I.T. analyzed the performance of embankments built on

* Appendix F, Section F.1, discusses the validity of using the same strength for these two types of analyses.
the three clay deposits considered in this research thoroughly on the basis of extensive laboratory and field test. Table 8.2 summarizes the values of $s_u$ (field) for the three test sites recommended by these analyses. Actual embankment performance analyzed in light of laboratory studies of soil behavior to be discussed in (3) provides the best estimate of $s_u$ (field) to use in the design of similar problems, and will subsequently be used in the evaluation of the theoretical predictions.

(2) **The field vane test.** When the evaluated performance of actual embankments is not available, a fairly reliable indication of $s_u$ (field) for bearing capacity and stability analyses is provided by the field vane test.* Bjerrum (1972) reviews 16 well-documented embankment failures on cohesive foundations and plots the factors of safety computed from circular arc analysis ($\phi = 0$) using the field vane "strength," $s_u$ (FV), versus the plasticity index (PI) of the soil. From the best fit line through these data points, he proposes a correction factor, $\mu$, Fig. 8.1a, to be applied to $s_u$ (FV) in the design of embankments. Subsequently, Bjerrum (1973) evaluates failures of footings and unsupported excavations and concludes that the same correction factor is also applicable for these problems.

---

* The field vane consists basically of four metal blades welded to a small circular shaft. The blades are often rectangular with a height $h$ and a width (diameter) $d = h/2$. This "vane" is pushed into the soil (usually from the bottom of a predrilled hole) and rotated at a constant rate (0.1° per sec according to ASTM D-2573). Traditional interpretation of the test assumes that the maximum shear strength, $s_u$, is simultaneously mobilized along the surface of revolution described by the vane (a closed end right cylinder) when the maximum torque, $T_{\text{max}}$, is measured, i.e.,

$$s_u (\text{FV}) = \frac{T_{\text{max}}}{\pi d^2/6} \quad \text{for } h/d = 2 \quad \text{(8.5)}$$
Table 8.3 presents a comparison between the recommended $s_u(\text{field})$ from Table 8.2 and the field vane (FV) data. We note that the $s_u(\text{field})/s_u(\text{FV})$ ratio varies between 0.6 and 1.1 and is generally within 15% of the empirical correction factor $\mu$.

(3) **Laboratory Tests.** Ladd and Fooott (1974) propose a design procedure for evaluating the stability of soft clays based on laboratory testing: the SHANSEP approach (**Stress History And Normalized Soil Engineering Properties**). This approach applies to clays exhibiting normalized behavior and requires a careful evaluation of the stress history of the clay. By using different laboratory equipment and applying different stress paths, this approach predicts the undrained shear strength of clays for different stress histories and different modes of failure, e.g., plane-strain compression, extension, simple shear, etc., and thus contributes significantly to the fundamental understanding of soil behavior. Chapter 5 presents SHANSEP strength profiles for the three clay deposits considered in this research.

Laboratory testing offers the advantage of having well-defined and controllable boundary conditions, making data interpretation relatively simple. Application of the data to actual in-situ loading conditions, however, may still present problems. These complications arise from the anisotropic, highly nonlinear, and rate-dependent stress-strain-strength behavior of soils and the simplifications necessary for analyzing an actual problem. Important factors that should be considered are: (a) strength anisotropy, (b) strain-softening behavior, (c) strain-rate
effects and (d) the uncertainty in the inclination of the failure plane with respect to the applied stress system (Ladd, 1971 and 1975).

Appendix F (Section F.1) uses the Saugus, MA, test site to illustrate the procedure for predicting $s_u^{(field)}$ from laboratory testing in accordance with the SHANSEP approach. The predicted strength is in good agreement with that estimated from actual embankment performance, Method (1). This procedure is therefore used to estimate $s_u^{(field)}$ at large depth for which Method (1) is not readily applicable.

8.2 The Reference Strength

The correction factor for the field vane test empirically accounts for:

(1) uncertainties in the interpretation of the test (Eq. 8.5) and possible effects of progressive failure both in the test and in field performance;

(2) the difference in failure modes between the field vane test and embankment stability or bearing capacity (this anisotropy effect is pronounced in lean clays), and

(3) the difference in strain rate between the field vane test and actual failures. *

* Laboratory triaxial compression tests indicate that $s_u$ often decreases by $10 \pm 5\%$ for a 10-fold decrease in strain-rate (Bjerrum, 1971; Berre and Bjerrum, 1973).
Based on some evidence of strength anisotropy in soft clays, Bjerrum (1973) isolates the anisotropy effect from the empirical correction factor $\mu$ and proposed another empirical correction factor $\mu_R$ "to be applied to the result of a shear test with a duration of a few minutes in order to obtain the strength which can be mobilized over a period of some weeks or several months."

In cone penetration, $\mu_R$ provides a reasonable basis for estimating from $s_u$ (field) a reference strength, $s_u$ (REF), for evaluating the theoretical predictions, i.e.,

$$s_u (\text{REF}) = \frac{s_u (\text{field})}{\mu_R} ;$$

(8.6)

= strength to be compared with $s_u$ (cone),

and

$$s_u (\text{cone}) = \text{strength predicted from cone resistance } q_c$$

(8.7)

with the theoretical expressions in Table 8.1.

Table 8.2 presents the value of $s_u$ (REF) for the three clay deposits studied.

In the following case studies, $s_u$ (cone) will be compared to $s_u$ (REF). However, $s_u$ (FV) or the SHANSEP strengths will also be shown to provide an indication of the uncertainties involved in strength evaluation.

8.3 Case Studies

Table 8.4 summarizes the cone penetration tests used in predicting $s_u$ (cone) using the formulas in Table 8.1. Detailed measurements obtained in these tests are presented in Chapter 7 and their variability studied in Chapter 6.

(1) Boston Blue Clay. Fig. 8.2 shows $s_u$ (cone) predicted both from 18° and 60° enlarged cones ($D/d = 2$) using Eq. (8.1) in Table 8.1. The
results show that the two cone shapes give essentially identical strengths and hence that the theory (Chapter 4) accounts for the effect of the cone angle appropriately. Furthermore, the predicted strength, \( s_u (\text{cone}) \), shows a similar pattern and value as the reference strength, \( s_u (\text{REF}) \), except at large depths (below 80 ft) where \( s_u (\text{REF}) \) is underestimated by less than 20%.

Figure 8.3 shows the strength, \( s_u (\text{cone}) \), predicted from 60°, 30° and 18° unenlarged cones \((D/d = 1)\) using the "best estimate" cone interpretation method given by Eq. (8.2). The initial horizontal total stress, \( \sigma_{ho} \), is evaluated with the SHANSEP approach using the stress history and \( K_o \) presented in Chapter 5. The predicted strengths, \( s_u (\text{cone}) \), from these cones show a larger scatter than in the case of enlarged cones (Fig. 8.2). This scatter, however, follows no consistent trend. In the "stiff" or heavily overconsolidated clay (about 65-ft depth), the 18° cone yields the highest values of \( s_u (\text{cone}) \), whereas in the slightly overconsolidated clay or "medium" clay below 65-ft depth, the 30° cone gives the highest values of \( s_u (\text{cone}) \).

Generally, \( s_u (\text{cone}) \) predicted from these unenlarged cones tends to be lower than \( s_u (\text{REF}) \) in the slightly overconsolidated clay (above 65-ft depth), but equals or exceeds \( s_u (\text{REF}) \) in the slightly overconsolidated clay (below 65-ft depth). The average of \( s_u (\text{cone}) \) from all three cone angles underestimates \( s_u (\text{REF}) \) by about 20% in the stiff clay, but overestimates it by 10% in the medium clay. These results indicate that Eq. (8.2) does not give as satisfactory predictions of the undrained shear strength as Eq. (8.1) for enlarged cones in this clay. The alternative prediction technique using the upper and lower bounds of strength given by Eq. (8.3) in Table 8.1 will be presented subsequently in (4).

Figure 8.4 shows all predicted strengths presented earlier from enlarged and unenlarged cones. These strengths form a \( \pm (20 - 30\%) \) band
about their average, which is approximately 20% lower than \( s_u^{(\text{REF})} \) in the heavily overconsolidated clay, but is approximately equal to \( s_u^{(\text{REF})} \) in the slightly overconsolidated region. Figure 8.4 also shows the strength predicted from pore pressure increments around an 18° enlarged cone according to Eq. (8.4) in Table 8.1. This predicted strength is surprisingly close to the unenlarged cones below a depth of 60 ft, but is low above 60 ft*.

Also shown in Fig. 8.4 are the SHANSEP strength profiles from plane strain compression (PSC) and direct simple shear (DSS) tests. The DSS strength is about 10% lower than \( s_u^{(\text{field})} \) whereas the PSC strength is about 50% higher. This difference in strength is an indication of the level of difficulty involved in strength prediction.

(2) Atchafalaya Basin Clay. Fig. 8.5 shows \( s_u^{(\text{cone})} \) predicted from 18°, 30°, and 60° enlarged cones (D/d = 2) using Eq. (8.1)** First, we note that the three cone shapes give almost identical values of the undrained shear strength which shows a pattern similar to both (uncorrected) field vane strength, \( s_u^{(\text{FV})} \), and the reference strength, \( s_u^{(\text{REF})} \), based on SHANSEP \( s_u^{(\text{DSS})} \) from a direct simple shear test.

Fig. 8.6 shows \( s_u \) predicted from 18° and 60° unenlarged cones (D/d = 1)

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* \( u_{\text{front}} \) is estimated by pore pressure measurements obtained at the tip of an 18° enlarged cone (see Chapter 7). Pore pressure measurements at the tip of a 60° enlarged cone are not available. However, if they are assumed to be equal to measurements at the tip of a 60° unenlarged cone (Fig. 7.9d), the predicted strength lies between 0.4 and 0.6 kg/cm² throughout the profile.

** Measurements of pore pressures \( u \) behind 60° and 18° enlarged cones were conducted at a location 400 ft from the main test site. The results indicate no noticeable effects of cone angle and show a pattern and value very similar to \( u \) measured far behind an 18° unenlarged cone (test conducted at the main test site). In view of the soil variability at this site, the average of these three tests was used to compute \( p_b \) in Eq. (8.1) for the strength prediction.
using Eq.(8.2).* We note that the two cones predict \( s_u \) (cone) with a similar pattern, but the strength from the 18° cone is about 15 to 20% lower than that of the 60° cone, which, in turn, is about 10 to 20% lower than \( s_u \) (REF). The scatter in the predicted strength with these unenlarged cones is larger than in enlarged cones, as was also observed in Boston Blue Clay. The relative value of \( s_u \) from the enlarged and unenlarged cones, are, however, different in the two deposits. In Boston Blue Clay, \( s_u (D/d = 1) \) tends to be higher than \( s_u (D/d = 2) \) except in the highly desiccated crust, whereas the \( s_u (D/d = 1) \) tends to be lower than \( s_u (D/d = 2) \) in EABPL Clay.

(3) **Connecticut Valley Varved Clay.** In this deposit, measurements are available from unenlarged cones only. Fig. 8.7 shows the predicted \( s_u \) from 60°, 30°, and 18° unenlarged cones \((D/d = 1)\) using Eq. (8.2). The three cones show very similar strength profiles which agree with the reference strength, \( s_u \) (REF), except for a weaker region detected by the cones and the field vane at slightly different depths. This discrepancy is probably due to variability of the deposit. (Note that the SHANSEP strength profiles, Fig. 5.14, which are based on a simplified stress history and soil profile, do not show this weaker region.)

(4) **Fugro Cones.** Recent developments in cone penetrometer testing lead to the adoption of the Fugro cone geometry \((2\delta = 60°, D/d = 1)\) as the "standard" shape for electrical cones. Unfortunately, unenlarged cones \((D/d = 1)\), are harder to interpret analytically than enlarged cones \((D/d > 1)\),

* Due to variability of the stress history in this deposit (Fig. 5.16c), \( \sigma_{ho} \) is estimated as 0.87 \( c_{vo} \) throughout the profile.
Chapter 4. Eq. (8.3) represents the lower and upper bounds of the undrained shear strength predicted on the basis of the cone penetration theory in Chapter 4. The "best estimate" given by Eq. (8.2) lies approximately in the middle of the upper and lower bounds (see Fig. 4.2a).

For the Fugro cone, the upper and lower bounds of $s_u$ are given for the three case studies which also represent a wide spectrum of cohesive soils by:

$$ \frac{q_c}{(23 \text{ to } 25)} \leq s_u \leq \frac{q_c}{11} \tag{8.8} $$

while Eq. (8.2) corresponds approximately to a $q_c/s_u$ ratio of 17 to 20.

Table 8.5 presents the representative soil parameters used in the prediction of the lower bound of $s_u$. The influence of the soil parameters on the prediction is shown in Fig. 4.2b.

Figs. 8.8a, 8.8b and 8.8c show the mean and the $\pm 2\sigma$ band of the $q_c/s_u$ (REF) ratio for Boston Blue Clay, Atchafalaya Basin Clay and Connecticut Valley Varved Clay respectively. We note from these figures that:

1. In Boston Blue Clay, $q_c/s_u$ (REF) increases steadily with depth from 10 to 16 in the heavily overconsolidated clay (top 40 ft). In the slightly overconsolidated region (bottom 60 ft), $q_c/s_u$ (REF) increases only slightly with depth from 16 to 20. Thus the $q_c/s_u$ ratio agrees with the predicted bounds of 11 to 24.5 (Table 8.5). Eq. (8.2), corresponding to a $q_c/s_u$ ratio of 17 to 20, underestimates $s_u$ (REF) in the heavily overconsolidated clay and gives a good estimate of $s_u$ (AVE) in the slightly overconsolidated clay.
2. In the Atchafalaya Basin Clay (below 44-ft depth) and Connecticut Valley Varved Clay (above 50-ft depth), the value of the $q_c/s_u(\text{REF})$ ratio is fairly constant and lies between 13 and 18. This should be compared to the value of 17 predicted by Eq. (8.2) and the predicted bounds of 11 to 23 (Table 8.5).

3. Excluding the desiccated crust in Boston Blue Clay, the range of the $q_c/s_u(\text{REF})$ ratio in "medium" to "soft" clays from all three sites is thus 13 to 20.

(5) Other Case Studies. To check applicability of the predicted bounds, (Eqs. (8.3) and (8.6) and Fig. 4.2b) in other deposits, we apply it to results of standard Fugro cone tests ($2\delta = 60^\circ$, $D/d = 1$) from medium to soft Scandinavian clays performed by NGI-FUGRO and reported by Lunne et al. (1976). The field strengths, $s_u(\text{field})$, for these deposits are estimated from the field vane test using the empirical correction factor $\mu$ presented by Bjerrum (1972), Fig. 8.1a. Thus the reference strength, $s_u(\text{REF})$, for evaluating the theoretical predictions is given by:

$$s_u(REF) = s_u(\text{field})/\mu_R;$$
$$\quad = \mu s_u(FV)/\mu_R;$$
$$\quad = s_u(FV)/(\mu_R/\mu),$$  \hspace{1cm} (8.9)

where $\mu_R/\mu$ is approximately given by (see Fig. 8.1b):

$$\mu_R/\mu = 0.82 + 0.0013 \text{ PI}(%), \quad 10 \leq \text{PI} \leq 100.$$  \hspace{1cm} (8.10)

Table 8.6 shows that the $q_c/s_u(\text{REF})$ ratios from these clay deposits, * from which we note that:

* These results exclude regions in the profile in which $s_u(FV)$ varies significantly.
1. Generally $q_c/s_u$ (REF) increases with depth, the increment being on the order of 5 to 10.

2. Except for the lower stratum at site 3, where the clay has very low PI (4 to 5) and high sensitivity (25 to 125), the range of $q_c/s_u$ (REF) is 11 to 25, which is the same as the range predicted by Eq. (8.8) for typical "medium" to "soft" clays.

3. The lowest values of $q_c/s_u$ occur in a desiccated crust (site 6).

4. Excluding the highly sensitive soil and the slightly higher than the range of the $q_c/s_u$ (REF) is 16 to 25, which is slightly lower than the range of 13 to 20 obtained from "medium" to "soft" clays at the M.I.T. test sites.

8.4 Summary and Discussion

The semi-empirical cone penetration theory presented in Chapter 4 provides a rational basis for the interpretation of cone penetration measurements. Its adequacy can be checked by comparing the results from cones of different geometries, and by comparing the predicted strength to the "field" strength which can best be estimated from full-scale tests. To account for the difference in strain rate between cone penetration and actual failures developing over days or weeks, the "field" strength is empirically adjusted to provide a reference strength for evaluation of the theoretical predictions. In all three deposits studied, this reference strength is within $\pm$30 % of the uncorrected field vane strength.

The comparison indicates that the accuracy of the theory depends on the cone shape. Enlarged cones (see diagram in Table 8.1), supplemented by pore pressure measurements behind the cones, predict undrained shear strength in perfect agreement with the reference strength in a marine
illitic clay (low PI, medium sensitivity) and a plastic deltaic clay (high PI, low sensitivity). Fig. 8.9 presents the procedure and requirements for strength prediction using enlarged cones.

For unenlarged cones (see diagram in Table 8.1) including the "standard" cone (60° tip), the theory predicts the upper and lower bounds of the undrained shear strength on the basis of cone resistance $q_c$. Eq. (8.2) represents a "best estimate" expression lying approximately in the middle of the bounds. It reasonably well predicts the strength of "soft" clays but with a significantly larger scatter than in predictions based on enlarged cones, and may distort the shape of the strength profile depending on the variation of soil type and stress history with depth.

On the other hand, the theory predicts the upper and lower bounds of $s_u$ (Eq. 8.3) from standard cone (FUGRO) with reasonable reliability. The observed values of the $q_c$-reference strength ratio vary between 10 and 20 as compared to the predicted range of 11 to 25. Several case studies of cone penetration tests in Scandinavian clays performed by NGI-FUGRO (Lunne et al., 1976) are also analyzed in light of these results. The "field" strength for these cases is based on the field vane tests with Bjerrum's correction factor. The analysis indicates that the predicted upper and lower bounds of $s_u$ are similarly applicable in these clays, with the possible exception of very sensitive soils. Fig. 8.9 presents the procedure and requirement for predicting the upper and lower bounds of the undrained shear strength.

These results suggest that the Fugro cone design may not be the ideal cone shape from an analytical point of view. The rigid shaft behind the
cone tip constrains the soil deformation and complicates the interpretation of cone resistance. A more accurate interpretation technique for this cone geometry requires additional measurements of pore pressures and/or more sophisticated methods of analysis following the strain-path approach outlined in Chapter 3. Chapter 9 will present empirical correlations for immediate practical applications of cone resistance in design.
<table>
<thead>
<tr>
<th>Type of Measurements</th>
<th>$s_u$ (cone) given by</th>
<th>Special Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_c$ from enlarged cones ($D/d &gt; 1$)</td>
<td>$[q_c - (d/D)^2 p_b]/N_c$</td>
<td>Measurement of pore pressure behind the cone, $p_b$.</td>
</tr>
<tr>
<td>$q_c$ from unenlarged cones ($D/d = 1$)</td>
<td>Best Estimate:</td>
<td>Evaluation of the initial horizontal total stress, $\sigma_{ho}$. For most soft clays, $\sigma_{ho} = (0.80 \pm 0.08) \sigma_{vo}$</td>
</tr>
<tr>
<td></td>
<td>$[q_c - \sigma_{ho} - (1 + \ln G/s_u)]/N_c \leq s_u \leq q_c/N_c$</td>
<td>(8.2)</td>
</tr>
<tr>
<td>Pore pressure increment from enlarged cones</td>
<td>$[u_{front} - u_{back}]$</td>
<td>Measurements of pore pressure at the front of and behind the enlarged cone.</td>
</tr>
<tr>
<td></td>
<td>$N_c - 1/\tan \delta - 1/\sin \phi$</td>
<td>(8.3)</td>
</tr>
</tbody>
</table>

| $N_c$ = cone factor;                       |                                        |                                                           |
| $= 1.2 (5.71 + 3.33\delta + 1/\tan \delta)$; |                                        |                                                           |
| $= 11.0, 12.4, 15.1$ for $2\delta = 60^\circ, 30^\circ, 18^\circ$ respectively; |                                        |                                                           |
| $\sigma_{ho}$ = initial horizontal total stress; |                                        |                                                           |
| $\phi$ = effective friction angle of the soil; |                                        |                                                           |
| $G$ = undrained shear modulus             |                                        |                                                           |

Table 8.1 - Predictions of undrained shear strength from different measurements.
<table>
<thead>
<tr>
<th>(1) Deposit</th>
<th>(2) $s_u$ (field)*</th>
<th>(3) Correction Factor for Rate Effect† $\mu_R$ (Fig. 8.1b)</th>
<th>(4) Reference Strength $s_u$ (REF) $= (2)/(3)$</th>
<th>(5) Relevant Figures</th>
<th>(6) References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston Blue Clay</td>
<td>0.9 $s_u$ (FV)</td>
<td>0.82</td>
<td>1.1 $s_u$ (FV)</td>
<td>Fig. 5.4, Fig. 5.5</td>
<td>Azzouz and Baligh (1978), Appendix F</td>
</tr>
<tr>
<td>25 to 65 ft depth</td>
<td>1.1 $s_u$ (FV)</td>
<td>0.82</td>
<td>1.35 $s_u$ (FV)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>65 to 125 ft depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atchafalaya Basin Clay</td>
<td>$s_u$ (DSS)</td>
<td>0.65</td>
<td>1.5 $s_u$ (DSS)</td>
<td>Fig. 5.9, Fig. 5.10</td>
<td>Fuleihan and Ladd (1976)</td>
</tr>
<tr>
<td>Connecticut Valley Varved Clay</td>
<td>0.85 $s_u$ (FV)</td>
<td>0.80</td>
<td>$\sim s_u$ (FV)</td>
<td>Fig. 5.14</td>
<td>Ladd (1975)</td>
</tr>
</tbody>
</table>

* Based on evaluation of embankment performance
† Recommended by Bjerrum (1973) for clays as a function of plasticity index.

Table 8.2 Strengths recalculated from actual embankment performances and correction for the effects of strain-rate ($FV$ = field vane test; $DSS$ = direct simple shear test).
<table>
<thead>
<tr>
<th>Deposit</th>
<th>(2) Depth (ft)</th>
<th>(3) $\frac{s_u \text{ (field)}}{s_u \text{ (PV)}}$</th>
<th>(4) Bjerrum's $\mu$ (Fig. 8.1a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston Blue Clay (PI = 20 – 24%)</td>
<td>25 - 65</td>
<td>0.9*</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>65 - 125</td>
<td>1.1*</td>
<td>1.0</td>
</tr>
<tr>
<td>Atchafalaya Basin Clay (PI = 60 – 80%)</td>
<td>40 - 120</td>
<td>0.6 – 0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Connecticut Valley Varved Clay (PI ≈ 30%, bulk)</td>
<td>15 - 50</td>
<td>0.85</td>
<td>0.9</td>
</tr>
</tbody>
</table>

*See appendix F, Section F.2, for prediction methods.

Table 8.3 Comparison between the recommended field strength, $s_u \text{ (field)}$, and the strengths obtained from field vane data.
<table>
<thead>
<tr>
<th>Type of Measurements</th>
<th>Cone Type</th>
<th>Number of Tests Used in $s_u$ Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 $\delta$</td>
<td>Boston Blue Clay</td>
</tr>
<tr>
<td>$q_c$</td>
<td>18°</td>
<td>2</td>
</tr>
<tr>
<td>$q_c$</td>
<td>30°</td>
<td>2</td>
</tr>
<tr>
<td>$q_c$</td>
<td>60°</td>
<td>2</td>
</tr>
<tr>
<td>$q_c$</td>
<td>18°</td>
<td>1</td>
</tr>
<tr>
<td>$q_c$</td>
<td>30°</td>
<td>1</td>
</tr>
<tr>
<td>$q_c$</td>
<td>60°</td>
<td>1</td>
</tr>
<tr>
<td>$u$ (tip)</td>
<td>18°</td>
<td>2</td>
</tr>
<tr>
<td>$u$ (behind cone)</td>
<td>18°</td>
<td>2</td>
</tr>
<tr>
<td>$u$ (behind cone)</td>
<td>60°</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 8.4 - Summary of number of tests used in strength prediction
<table>
<thead>
<tr>
<th>Soil Parameters and Predictions</th>
<th>Boston Blue Clay</th>
<th>Atchafalaya Basin Clay</th>
<th>Connecticut Valley Varved Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heavily Overconsolidated</td>
<td>Slightly Overconsolidated</td>
<td></td>
</tr>
<tr>
<td>$\gamma_t / \gamma_w$</td>
<td>1.85</td>
<td>1.85</td>
<td>1.65</td>
</tr>
<tr>
<td>OCR</td>
<td>2.5</td>
<td>1.3</td>
<td>1</td>
</tr>
<tr>
<td>$K_o$</td>
<td>0.77</td>
<td>0.55</td>
<td>0.57</td>
</tr>
<tr>
<td>$s_u / \sigma_{vo}$</td>
<td>0.3</td>
<td>0.23</td>
<td>0.35</td>
</tr>
<tr>
<td>Upper and lower bounds of $s_u$</td>
<td>$q_c/11$</td>
<td>$q_c/11$</td>
<td>$q_c/11$</td>
</tr>
<tr>
<td>$q_c/23.5$</td>
<td>$q_c/24.5$</td>
<td>$q_c/23.1$</td>
<td></td>
</tr>
<tr>
<td>Eq. (8.2)</td>
<td>$q_c/17.5$</td>
<td>$q_c/20.0$</td>
<td>$q_c/17.0$</td>
</tr>
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Table 8.5 - Predictions of the $q_c / s_u$ ratios for the three case studies (see influence of soil parameters in Fig. 4.2b).
<table>
<thead>
<tr>
<th>Test Site</th>
<th>Depth m</th>
<th>PI %</th>
<th>$S_t$</th>
<th>$N_c$ (FV)</th>
<th>$\frac{q_c}{s_u}$ (FV)</th>
<th>$\frac{q_c}{s_u}$ (REF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sunland</td>
<td>3 to 9</td>
<td>25</td>
<td>10 to 15</td>
<td>17 to 18</td>
<td>22 to 26</td>
<td>18.5 to 22</td>
</tr>
<tr>
<td></td>
<td>11 to 14</td>
<td>8</td>
<td>1 to 2</td>
<td>18 to 19</td>
<td>24 to 26.5</td>
<td>20 to 22</td>
</tr>
<tr>
<td></td>
<td>15 to 21</td>
<td>10</td>
<td>1 to 4</td>
<td>15 to 18</td>
<td>26 to 30</td>
<td>22 to 25</td>
</tr>
<tr>
<td>2. Danviks</td>
<td>4 to 11</td>
<td>30</td>
<td>8</td>
<td>13.5 to 17</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>11 to 15</td>
<td>18</td>
<td>3</td>
<td>17 to 18</td>
<td>28</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>20 to 29</td>
<td>10</td>
<td>3</td>
<td>13 to 16</td>
<td>28 to 30</td>
<td>23 to 25</td>
</tr>
<tr>
<td>3. E. Borresens</td>
<td>6 to 10</td>
<td>18</td>
<td>15 to 20</td>
<td>17 to 19</td>
<td>22 to 27</td>
<td>18.5 to 23</td>
</tr>
<tr>
<td></td>
<td>15 to 27</td>
<td>4 to 5</td>
<td>25 to 100$^+$</td>
<td>20 to 26</td>
<td>45 to 48</td>
<td>37 to 40$^{(1)}$</td>
</tr>
<tr>
<td>4. Onsøy</td>
<td>2 to 10</td>
<td>28</td>
<td>5 to 10</td>
<td>17 to 19</td>
<td>22 to 29</td>
<td>19 to 25</td>
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<td>10 to 26</td>
<td>36</td>
<td>4 to 7</td>
<td>12 to 17</td>
<td>22 to 27</td>
<td>19 to 23</td>
</tr>
<tr>
<td>5. Göteborg</td>
<td>3 to 10</td>
<td>40 to 60</td>
<td>13 to 24</td>
<td>13 to 15</td>
<td>18 to 21</td>
<td>16 to 19</td>
</tr>
<tr>
<td></td>
<td>10 to 28</td>
<td>40 to 60</td>
<td>11 to 19</td>
<td>12 to 14</td>
<td>20 to 22</td>
<td>18 to 20</td>
</tr>
<tr>
<td>6. Ska-Edeby</td>
<td>2 to 5</td>
<td>40</td>
<td>8</td>
<td>8 to 9</td>
<td>13 to 19</td>
<td>11 to 16.5$^{(2)}$</td>
</tr>
<tr>
<td></td>
<td>5 to 12</td>
<td>40</td>
<td>10</td>
<td></td>
<td>19 to 23</td>
<td></td>
</tr>
</tbody>
</table>

(1) Validity of $\mu$ and $\mu_R$ doubtful at PI < 10%

(2) Desiccated crust

Table 8.6 - The $q_c/s_u$ ratio for standard cone tests ($2\delta = 60^\circ$, $D/d = 1$), Scandinavian test sites.
Fig. 8.1a Empirical correction factor derived from embankment failures for the field vane test (from Ladd, 1975).
Empirical correlation established from embankment failures (Bjerrum, 1972)

Estimated effect of anisotropy

Estimated time effect, \( \mu_R \)

Fig. 8.1b Components of the field vane correction factor according to Bjerrum (1973).
Fig. 8.2 Prediction of undrained shear strength from enlarged cones with Eq. (8.1), Boston Blue Clay (REF = reference strength).
Fig. 8.3 Prediction of undrained shear strength from unenlarged cones with Eq. (8.2), Boston Blue Clay. (REF = reference strength)
Fig. 8.4 Prediction of undrained shear strength from all cone shapes and from pore pressure increment around an 18° enlarged cone, Eq. (8.4).
Fig. 8.5 Prediction of undrained shear strength from enlarged cones with Eq. (8.1), Atchafalaya Basin Clay. (REF = reference strength, FV = field vane strength.)
Fig. 8.6 Prediction of undrained shear strength from unenlarged cones with Eq. (8.2), Atchafalaya Basin Clay. (REF = reference strength, FV = field vane strength.)
Fig. 8.7 Prediction of undrained shear strength from unenlarged cones with Eq. (8.2), Connecticut Valley Varved Clay (REF = reference strength).
Fig. 8.8a. The cone resistance-reference strength ratio at the Saugus, MA, test site (Fugro cone).
Fig. 8.8b The cone resistance-reference strength ratio at the EABPL, LA, test site (Fugro cone).
Fig. 8.8c The cone resistance-reference strength ratio at the Amherst, MA, test site (Fugro cone).
PROCEDURES AND REQUIREMENTS FOR STRENGTH PREDICTION

ENLARGED CONES

1. Measure cone resistance \( q_c \) and pore pressure \( p_b \) behind cone.
2. Compute \( s_u \) (cone) from \( q_c \) with theory (Eq. (8.1) in Table 8.1)
3. Estimate effects of strain-rate by \( \mu_R \), Fig. 8.1b.
4. Compute \( s_u \) (field) = \( \mu_R s_u \) (cone) for bearing capacity and stability analyses.

UNENLARGED CONES (INCLUDING STANDARD CONE SHAPE)

1. Measure cone resistance \( q_c \).
2. Estimate the maximum and minimum values of the \( q_c/s_u \) ratio using Figs. 4.2a and b or Eq. (8.3).
3. Estimate effects of strain-rate by \( \mu_R \), Fig. 8.1b.
4. Compute \( s_u \) (field) = \( \mu_R s_u \) (cone) for bearing capacity and stability analyses.

Fig. 8.9 Prediction of undrained shear strength using the proposed theory
CHAPTER 9

DESIGN CORRELATIONS FOR THE FUGRO CONE

9.1 Introduction

A semi-empirical theory of cone penetration is presented in Chapter 4 and evaluated in Chapter 8. For the Fugro cone design (Chapter 5), the theory predicts reasonable upper and lower bounds for the undrained shear strength of clays. This chapter provides empirical correlations between cone resistance, \( q_c \), and the undrained shear strength obtained by the field vane test, \( s_u \). These correlations, together with the evaluated experience of \( s_u \) (Bjerrum, 1972 and 1973; Ladd et al., 1977) provide a good framework for the practical use of \( q_c \) in design.

9.2 Correlations with the Field Vane Test

Following the traditional bearing capacity equation in soil mechanics, an empirical cone factor \( N_c \) can be defined as:

\[
q_c = N_c(FV)s_u(FV) + \sigma_{vo},
\]

or

\[
N_c(FV) = \frac{q_c - \sigma_{vo}}{s_u(FV)} = (q_c - \sigma_{vo}) \left( \frac{T_{max}}{7\pi d^3/6} \right),
\]

where

\[
q_c = \text{cone resistance;}
\]

\[
\sigma_{vo} = \text{initial vertical total stress},
\]

and where \( s_u \) and \( T_{max} \) are the "strength" and the maximum torque measured in the field vane test respectively (see Chapter 8, Section 8.1, or Ladd et al., 1977). Eqs. (9.1) and (9.2) essentially relate the force
required for continuous cone penetration for electric cone penetrometers considered herein) to the maximum torque measured during vane rotation, and the initial vertical total stress in the soil.

Figs. 9.1a, 9.1b and 9.1c show the mean and the \( \pm 2\sigma \) band of \( N_c^{(FV)} \) \( (\sigma = \text{standard deviation}) \) computed from Eq. (9.1) for tests in Boston Blue Clay, Atchafalaya Basin Clay, and Connecticut Valley Varved Clay, respectively. The geology and soil conditions at these sites are presented in Chapter 5. In computing the factor \( N_c^{(FV)} \), we use \( q_c \) data obtained from two to three Fugro cone tests (see Table 8.3) performed within a close proximity (40-ft radius). On the other hand, \( s_u^{(FV)} \) are the average of existing field vane measurements conducted within a 200-ft radius of the cone tests* (data were presented in Chapter 5). The computed \( N_c^{(FV)} \) data are filtered by the procedure presented in Chapter 6 to reduce the influence of local soil inhomogeneities. The variability of \( N_c^{(FV)} \) shown in Fig. 9.1 thus reflects the variability of the cone tests but not the scatter of the field vane data. (Chapter 6 studies the variability of both tests individually.) The three case studies and some results reported by others are discussed below.

(1) **Boston Blue Clay.** \( (q_c \) and \( s_u^{(FV)} \) profiles are shown in Figs. 6.4 and 5.5 respectively). Fig. 9.1a shows that the average value at \( N_c^{(FV)} \) decreases from 12 (or higher) in the stiff crust to about 7.5 at a depth of 35 ft. Below this depth, the average \( N_c^{(FV)} \) increases almost linearly

*The field vane tests in the Boston Blue Clay and Connecticut Valley Varved Clay were performed by M.I.T., using the ASTM Standard with the Geonor equipment; in the Atchafalaya Basin Clay, they were performed by the U.S. Corps of Engineers.
to 15 at a depth of 120 ft. Furthermore, the point variability of \( N_c(FV) \) i.e., its standard deviation \( \sigma \), is quite uniform throughout the deposit, though it tends to decrease slightly with depth. The \( \pm 2\sigma \) band which includes approximately 95% of the (filtered) data points (Chapter 6, Section 6.3) corresponds to an uncertainty of \( N_c(FV) \) of \( \pm 2 \) in the top 40 ft (heavily overconsolidated) and \( \pm 1.5 \) in the bottom 60 ft (slightly overconsolidated). Finally, we notice in Fig. 9.1a that below a depth of 60 ft where OCR \( \leq 2 \), the average value of \( N_c(FV) \) is approximately given by 12+3. This uncertainty in \( N_c(FV) \) is due to its dependence on depth in addition to its point variability caused by changes in the soil and possible equipment errors.

(2) **EABPL Clay.** \( q_c \) and \( s_u(FV) \) profiles are shown in Figs. 6.5 and 5.10 respectively.) The soil profile at this test site is rather complex and the profile of \( N_c(FV) \) shows (at least) three distinct strata, Fig. 9.1b. In each stratum, \( N_c(FV) \) decreases to a minimum but then increases with depth. The point variability of \( N_c(FV) \) is relatively uniform, i.e., \( 2\sigma = \pm 2.5 \), except for the more variable regions above 44-ft depth (where the soil is highly organic) and between 78 and 92 ft (which contains numerous silt lenses; USCE, 1964). The range of \( N_c(FV) \) for this clay, which has an OCR \( \leq 1.5 \), is 9+4. Again, this range of uncertainty is significantly higher than the point variability of \( N_c(FV) \).

(3) **Connecticut Valley Varved Clay.** \( q_c \) and \( s_u(FV) \) profiles are shown in Figs. 6.6 and 5.14 respectively). In this deposit, \( N_c(FV) \) decreases from a high value (> 15) in the crust to a very uniform value of 10 to 12 in most of the deposit. A dip in the value of \( N_c(FV) \) near
depth = 45 ft can probably be attributed to soil variability between the locations of the cone and the field vane tests. (The cone tests detect slightly weaker soil at depths of about 46 ft, whereas the field vane test, conducted 200 ft away, detects it at about 52-ft depth.) The point variability of $N_c(FV)$ is relatively small, i.e., $2\sigma = \pm (1$ to $1.5)$. The average value of $N_c(FV)$ is $11 \pm 2$ (excluding the region between 42 and 52 ft). Thus the variation of $N_c(FV)$ with depth in this case is of the same order as its point variability.

4) Comparison of $N_c(FV)$ in Different Clay Deposits. Fig. 9.2 shows the average field vane strength, $s_u(FV)$, at the three M.I.T. test sites (curves B, C, L), and also at six Scandinavian sites (curves 1 to 6) where cone resistance data were obtained by NGI-FUGRO (Lunne et al., 1976). Table 9.1 provides some soil information for these Scandinavian sites; more detailed soil descriptions are given by Lunne et al. (1976).

Fig. 9.3a presents the average $N_c(FV)$ profiles obtained from these nine sites. We note from Fig. 9.3a that:

1. Excluding site No. 3, where $s_u(FV)$ below a depth of 15 m is unusually low (probably because of the high sensitivity of the clay, Table 9.1 and Fig. 9.2), the values of $N_c(FV)$ are between 5 and 21 for the remaining eight sites.

2. $N_c(FV)$ profiles obtained by M.I.T. are either constant with depth or tend to increase with depth, whereas $N_c(FV)$ profiles obtained by NGI-FUGRO tend to decrease with depth.

3. Comparing the average value of $N_c(FV)$ in each deposit, we note that the values obtained by M.I.T. are generally lower than those obtained by NGI-FUGRO.
The different trends and values in \( N_c \) (FV) shown in Fig.9.3a can be attributed to one or more of the following reasons:

1. Errors or distortions in \( s_u \) (FV), \( q_c \), or both.

2. The inability of Eq. (9.1) to account for the dependency of \( q_c \) on depth.

3. Differences in the clay properties among these sites, e.g., plasticity index and/or sensitivity as well as the stress history and/or strength. For example, curves 1 and 4 showing \( N_c \) (FV) decreasing with depth correspond to deposits where sensitivity decreases with depth. Curve L with the lowest value of \( N_c \) (FV) corresponds to the highest value of plasticity index. Curve 3 below 12 m with very high value of \( N_c \) (FV) corresponds to a very sensitive clay.

Lunne et al. (1976) do not consider the dependence of \( N_c \) (FV) with depth, but indicate a relationship between \( N_c \) (FV) and PI for "medium" to "very soft" clays. Their data are shown in Table 9.1 and are replotted in Fig. 9.4a with the uncertainty bands indicated by Lunne et al. (1976). Also shown for comparison in Fig. 9.4a are the results from the M.I.T. test sites, excluding the "stiff" or heavily overconsolidated clays and the more variable regions in the profiles, Table 9.2 and Figs. 9.1. The data in Fig. 9.4a indicate a decreasing trend in \( N_c \) (FV) with increasing PI. At any value of PI, the scatter in the values of \( N_c \) (FV) is approximately ±5.

(5) Application of Bjerrum's Correction Factor. Case studies of embankment and footing failures indicate that \( s_u \) (FV) is not always
appropriate strength to use in bearing capacity or stability analysis.*

A better measure of the in-situ strength is obtained by correcting $s_u(FV)$ by an empirical correction factor, Fig. 8.1, based on actual failures (Bjerrum, 1972 and 1973, Ladd et al., 1977). Lunne et al. (1976) use Bjerrum's empirical correction factor, $\mu$, to compute another empirical cone factor, $N'_c(FV)$ defined by:

$$N'_c(FV) = \frac{q_c - \sigma_{vo}}{\mu s_u(FV)}.$$  (9.3)

Fig. 9.3b shows profiles of the average values of $N'_c(FV)$ for the same deposits described in Fig. 9.3a (data tabulated in Tables 9.1 and 9.2).

Comparing Fig. 9.3b to Fig. 9.3a, we note that the correction factor slightly reduces the scatter in the empirical cone factor at any given depth. However, each $N'_c(FV)$ curve shows almost as much dependence on depth as $N_c(FV)$.

Fig. 9.4b shows a plot of $N'_c(FV)$ vs. PI for "medium" to "very soft" clays at these sites. Comparing this figure to Fig. 9.4a, we note that $N_c(FV)$ does not vary with the plasticity index, PI, as much as $N'_c(FV)$. The range of $N'_c(FV)$ for these "medium" to "very soft" clays is 8 to 20 for PI greater than 10%. At any PI, the uncertainty in $N'_c(FV)$ is the same as in $N_c(FV)$, and is about $\pm 5$. For a mean value of $N_c(FV)$ of 15, this uncertainty range is $\pm 33\%$, which is significantly larger than the scatter of the data associated with Bjerrum's correction factor for the field vane test ($\pm 20\%$, Fig. 8.1).

* Appendix F, Section F.1, discusses the validity of using the same strength for these two types of analyses.
9.3 Summary and Discussion

Empirical correlations between cone resistance, $q_c$, and corrected and uncorrected $s_u(FV)$ are presented for a marine illitic clay (low PI, medium sensitivity), a plastic deltaic clay (high PI, low sensitivity), and a glacial lake varved clay (medium PI and sensitivity). The empirical cone factor $N_c(FV) = (q_c - \sigma_{vo})/s_u(FV)$ varies between 5 and 15 depending on stress history, sensitivity, depth, and possibly other factors. This range of $N_c(FV)$ is lower than that presented by Lunne et al. (1976) for "medium" to "very soft" Scandinavian clays. Bjerrum (1972)’s empirical correction factor, $\mu$, for $s_u(FV)$ slightly reduces the variation of $N_c(FV)$ with soil type (PI) but cannot account for the dependence of $N_c(FV)$ on depth, stress history, or sensitivity. The range of $N'_c(FV) = (q_c - \sigma_{vo})/\mu s_u(FV)$ for the three deposits tested by M.I.T. and the six sites tested by Lunne et al. (1976) is 9 to 20 for medium to very soft clays with PI greater than 10. This large range of uncertainty in $N'_c(FV)$ poses a serious limitation on the use of cone resistance for design purposes.

The complications associated with the empirical interpretation of $q_c$ through the "corrected" $s_u(FV)$ arise due to the highly approximate nature of Eqs. (9.1) and (9.2), the uncertainty in the interpretation of the field vane test, and the complicated and different behavior of various soils. The uncertainty in this empirical correlation can be significantly reduced by using local experience and noting the dependence of $N'_c(FV)$ on depth. It can perhaps be further reduced if measurements of pore pressure during penetration are also available. Chapters 7 and 8 show that this pore pressure depends on the soil type and stress history and can influence the
penetration resistance. Additional study on correlation between $N'_c(FV)$ and pore pressure is, however, required for this approach.

For unfamiliar clay deposits, Chapter 8 provides a method for estimating the upper and lower bounds of the undrained shear strength based on the cone penetration theory presented in Chapter 4.
<table>
<thead>
<tr>
<th>SITE</th>
<th>DEPTH m</th>
<th>PLASTICITY INDEX P.I.,%</th>
<th>SENSITIVITY $S_t$</th>
<th>$N_c$ (FV)</th>
<th>$N_c$ (FV)* corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sundland (Drammen Clay)</td>
<td>4-9</td>
<td>23-30</td>
<td>6-14</td>
<td>17-18</td>
<td>17-19</td>
</tr>
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<td></td>
<td>9-14</td>
<td>7-10</td>
<td>1-2</td>
<td>18-20.5</td>
<td>16.5-18</td>
</tr>
<tr>
<td></td>
<td>16-22</td>
<td>9-12</td>
<td>2-3</td>
<td>15.5-16.5</td>
<td>14-15</td>
</tr>
<tr>
<td>2. Dansviks Gate (Drammen)</td>
<td>3-10</td>
<td>20-35</td>
<td>6-9</td>
<td>13.5-17</td>
<td>15-16</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>10-14</td>
<td>2-4</td>
<td>14-18</td>
<td>13-18</td>
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<td>10</td>
<td>3-4</td>
<td>13-16</td>
<td>12-14.5</td>
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<tr>
<td>3. Børresen Gate (Drammen)</td>
<td>6-12</td>
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<td>13-22</td>
<td>17-20</td>
<td>16-19</td>
</tr>
<tr>
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<td>8</td>
<td>5-14</td>
<td>25-26</td>
<td>22.5-23.5</td>
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<td></td>
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<td>4-5</td>
<td>40-130</td>
<td>20-26</td>
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<tr>
<td>4. Onsøy</td>
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<td>25-30</td>
<td>5-10</td>
<td>17-19</td>
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<td>5. Göteborg</td>
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<td>13-17</td>
<td>12-14</td>
<td>14-16</td>
</tr>
<tr>
<td>6. Ska-Edeby</td>
<td>2-4</td>
<td>40</td>
<td>6-9</td>
<td>8-9</td>
<td>10</td>
</tr>
<tr>
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<td>4-12</td>
<td>35-50</td>
<td>10-12</td>
<td>10.5-12.5</td>
<td>12.5-14.5</td>
</tr>
</tbody>
</table>

* corrected in each 1 - m section using applicable PI, exclude extreme values

Table 9.1 Tabulation of correlations between cone resistance and the field vane strengths at NGI-FUGRO sites (data from Lunne et al. 1976).
<table>
<thead>
<tr>
<th>Test Site</th>
<th>Depth (ft)</th>
<th>PI (%)</th>
<th>$S_t^{(1)}$</th>
<th>$N_c(FV)$</th>
<th>$N'_c(FV)^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saugus, MA (Boston Blue Clay)</td>
<td>65 - 125</td>
<td>≈ 24</td>
<td>6</td>
<td>9 - 15</td>
<td>9 - 15</td>
</tr>
<tr>
<td>EABPL, LA (Atchafalaya Basin Clay)</td>
<td>50 - 120</td>
<td>60 - 80</td>
<td>1?</td>
<td>5.5 - 12.5</td>
<td>9 - 17</td>
</tr>
<tr>
<td>Amherst, MA (Connecticut Valley Varved Clay)</td>
<td>15 - 65 (excluding) 42 - 52 (bulk)</td>
<td>≈ 30</td>
<td>4 - 7</td>
<td>9 - 13</td>
<td>10 - 14</td>
</tr>
</tbody>
</table>

(1) from field vane data
(2) computed from the range of $N_c(FV)$

Table 9.2 Tabulation of correlations between cone resistance and the field vane strengths at M.I.T test sites
Fig. 9.1a Empirical Cone Factor, $N_c (FV) = [q_c - \sigma_{vo}] / s_u (FV)$,
at the Saugus, MA, test site
Fig. 9.1b  Empirical Cone Factor, $N_c(FV) = \left[ q_c - \sigma_{vo} \right] / s_u(FV)$, at EABPL, LA
Fig. 9.1c Empirical Cone Factor, $N_c(FV) = \frac{q_c - \sigma_{vo}}{s_u(FV)}$, at Amherst, MA
Fig. 9.2 Average field vane strength profiles at M.I.T. and NGI-FUGRO test sites (see Tables 9.1 and 9.2 for site identification).
Fig. 9.3a  Empirical cone factor $N_c(FV)$ vs. depth from all case studies.
Fig. 9.3b  Empirical cone factor $N'_c(FV)$ vs. depth from all case studies.
Fig. 9.4 Empirical cone factors $N_c$ (FV) and $N'_c$(FV) from correlations with field vane and "corrected" field vane strengths.
CHAPTER 10

SUMMARY AND CONCLUSIONS

(1) In-situ soil testing is gaining importance in site investigation and in the determination of the necessary soil properties for foundation engineering design. This results from (a) a growing concern over the escalating cost and the reliability of traditional exploration techniques based on boring, sampling and laboratory testing; (b) the increasingly difficult and unfamiliar environments in which engineering structures are founded; and (c) the necessity to assess soil conditions, in-situ properties, and their variability in more detail to complement the development in analytical capability of the geotechnical profession.

(2) The electric cone penetrometer and the pore pressure probe represent a new generation of in-situ testing devices combining wide applicability with simplicity, consistency and economy. Both have no mechanically moving parts and are readily amenable to remote control and automated data recording and processing, and are thus ideal instruments for difficult locations with no easy access, e.g., in deep water. They provide continuous measurements, allowing better identification of soil stratigraphy and variability than discrete field measurements, e.g., the field vane test, or laboratory tests on selected samples. For application in medium to soft clays, however, they lack a solid evaluated experience by the profession and a well-defined common basis for analysis, especially in the U.S. This thesis attempts to establish a better understanding of the cone penetration mechanism through analytical and experimental means in order to pro-
vide a more rational interpretation of cone penetration test results.

Analytic Work

(3) Continuous penetration of a rigid indentor such as a cone or a wedge in deep soil strata represents a steady state problem where deformation and strains should be given primary consideration. Chapter 2 and Appendix A present and illustrate a method for estimating deformation, strain, strain-rate and strain-history in the soil during steady state penetration.

During penetration of blunt cones (or piles), the deformations and strains can be estimated, with some limitations, using a relatively simple velocity field. The analysis indicates that significant shearing occurs in the soil ahead of the cone tip, and that the maximum shear strain, $1/2(\varepsilon_1 - \varepsilon_3)$, in a soil element occurs in a circumferential direction. The maximum shear strain, $1/2(\varepsilon_1 - \varepsilon_3)$, on vertical (meridian) planes is as large as $1/2(\varepsilon_1 - \varepsilon_2)$ ahead of the cone, but decreases to about 50% of $1/2(\varepsilon_1 - \varepsilon_3)$ behind the cone.

(4) Rigorous solutions for steady state cone penetration in clay are difficult to obtain because the problem is axisymmetric and involves very large deformations and strains, and because soil behavior is very complicated. All existing theories of cone penetration rely either on rigorous solutions to simplified problems or on simple incomplete solutions. Most of the existing theories are based on incipient plane-strain deformation modes or on expansion of cavities; neither of these approaches yields
meaningful deformations nor strains. Baligh (1975) offers a more rational approach based on steady penetration of a wedge.

(5) Chapter 3 discusses different approaches to the cone penetration problem and concludes that the strain-path approach is the most promising. The strain path approach relies on the fact that the deformation pattern (velocity field) for penetration problems can be predicted with far less uncertainty than the stress pattern (Baligh, 1975). In this approach, strain history is estimated for selected soil elements, and the resulting stresses determined from laboratory tests or appropriate constitutive laws. Appendix A estimates strain fields and strain paths for wedge and (blunt) cone penetration. The procedure for obtaining an associated self-equilibrating stress field is, however, quite complicated, and requires considerable additional effort. This thesis does not pursue this approach further, but provides a basis for more rigorous future research which may have applications in the analyses of pile driving and bearing capacity.

(6) Other possible solution approaches are numerical method and axisymmetric ideal plasticity solutions. Both approaches have difficulties and complications. This thesis pursues a semi-empirical approach where a plane-strain slip-line solution of wedge penetration, which is relatively simple to obtain, is used to provide the basis for understanding the complex problems of cone penetration.

(7) Baligh (1972) and Baligh and Scott (1976) present a complete ideal plasticity solution of steady state wedge penetration in isotropic clays under undrained conditions. The theory predicts the wedge resistance,
the stress-increment, deformation and strain fields around the wedge. The wedge resistance is relatively constant for apex angles, $2\delta$, between $40^\circ$ and $90^\circ$ and approximately equal to $9.5 \pm 0.3$ times the isotropic undrained shear strength of the clay, but increases rapidly for sharper wedges. The deformation around a penetrating wedge is intense, especially for blunt wedges ($2\delta > 40^\circ$). The maximum shear strain in a soil element increases as it is approached by the wedge, and, depending on its location, may remain constant or decrease afterwards.

(8) Natural clays generally exhibit anisotropic stress-strain strength behavior, i.e., the shearing resistance varies with the applied stress system. Results of laboratory $K_0$-consolidated undrained shear tests indicate that the undrained strength anisotropy can be approximately described by an elliptic yield contour.

Appendix B extends the ideal-plasticity solution obtained by Baligh (1972) to anisotropic clays. The wedge resistance in anisotropic clay is approximately given by $N_{wu}(\text{AVE})$, where $s_u^{\text{(AVE)}}$ is a weighted "average" strength reflecting the shearing resistance of the clay in a combination of failure modes; $N_w$ is the wedge resistance factor for isotropic clay. Other factors investigated which proved to have small effect on wedge penetration are the friction at the wedge-soil interface and the soil deformability before yielding. However, because the theory neglects the actual stress-strain behavior of the soil, it cannot account for complex soil behavior, e.g., significant strain-softening.

(9) Based on a comparison of plane and axisymmetric deformation problems and the empirical shape factors often used in soil mechanics, the
wedge penetration theory was used to estimate penetration resistance, stress and deformation fields during cone penetration in clays.

(10) For enlarged cones with a significant reduction in diameter immediately behind the tip, the cone resistance $q_c$ is given by

$$q_c = N_c s_u$$

where $N_c$ is the cone resistance factor = 11.0, 12.4 and 15.1 for apex angle $2\delta = 60^\circ$, 30$^\circ$ and 18$^\circ$ respectively, and $s_u$ denotes the average strength of the clay.

(11) For unenlarged cones of the FUGRO type, which are pushed forward by a cylindrical shaft having the same cross-sectional area as the cone, the theory predicts lower and upper bounds for $q_c$

$$N_c s_u \leq q_c \leq N_c s_u + [\sigma_{ho} + (1 + \ln G/s_u) s_u]$$

where $N_c$ is the same as for enlarged cones; $\sigma_{ho}$ is the initial horizontal total stress in the soil, and $G$ is the undrained shear modulus.

Experimental Work

(12) Extensive penetration testing was conducted in three clay deposits representing a wide spectrum of cohesive soils. The testing program identifies important parameters controlling cone penetration and evaluates the validity of the proposed cone penetration theories. The clay deposits studied are: Boston Blue Clay (marine illitic clay, low plasticity index and medium sensitivity), Atchafalaya Basin Clay, (back-
swamp clay, high plasticity index and low sensitivity), and Connecticut Valley Varved clay (glacial lake varved clay, medium plasticity index and sensitivity). In the last decade, M.I.T. had extensively studied both the engineering properties of these clays by laboratory and field tests, and the performance of embankments constructed on these clays.

(13) In the three clay deposits studied, the cone resistance \( q_c \), measured with standard Fugro equipment, is repeatable within a reasonable margin of uncertainty (+5 to 15%). Typical \( q_c \) profiles include small-scale anomalies, probably due to soil inhomogeneity, which must be discarded when the variability of the clay mass is investigated. A computerized filtering procedure to eliminate these anomalies in a consistent manner is developed and applied to records of \( q_c \) at the three test sites. The procedure emphasizes the importance of the median (rather than the mean) because of its insensitivity to anomalies in the data. Though it is not based on a rigorous statistical analysis, this procedure provides reasonable results which can be exactly duplicated.

The filtered data are approximately normally distributed about the mean and have a standard deviation which varies with depth. An analysis of soil variability based on the coefficient of variation (= standard deviation/mean) shows that both the cone resistance and the field vane detect approximately the same soil variability which depends on the soil type and shows a significant increase in desiccated regions.

(14) Factors investigated that influence cone resistance \( q_c \) (measured with electrical cones) are:
a. **Apex Angle.** $q_c$ increases as the apex angle $2\theta$ decreases (sharper cone). The effect of $2\theta$ on $q_c$ agrees with theoretical predictions (Chapter 4).

b. **Tip Shape.** As predicted by the theory, tip enlargement reduces $q_c$. This reduction, however, depends on the soil type. In "soft" clays (OCR $\approx 1$), doubling the cone tip diameter reduces $q_c$ by 0.5 to 1 times $\sigma_{vo}$ (relative to "unenlarged" cones). In a "stiff" desiccated region, the effect of tip enlargement becomes negligible.

c. **Penetration Velocity.** Moderate variation (say, by a factor of 2) from the standard penetration velocity of 1 to 2 cm/sec causes no noticeable change in $q_c$. The limited data obtained in this research on penetration rate do not allow extrapolation to very slow shearing rates encountered in field failures developing over days or weeks.

(15) Pore pressures measured during cone penetration by means of special pore pressure probes varies with the location on the cone, i.e., location of the porous element. During cone penetration at the standard rate of 1 to 2 cm/sec in homogeneous "medium" to "soft" clays, the pore pressure $u$ along the cone does not have time to dissipate. The variation of $u$ along a cone thus provides an indication of the variation of the stress field. Measurements indicate that $u$ is largest at the middle of the cone and decreases along the shaft behind the cone. Furthermore, $u$ at the tip is slightly smaller than $u$ at the middle of the cone. For unenlarged cones (tip diameter $D = \text{shaft diameter } d$), $u$ decreases behind the cone for a distance of 4$d$ to 5$d$ and then remains constant at least to a distance of 11$d$. For enlarged cones ($D > d$), $u$ is uniform behind the cone.
(16) Measurements from 18° cones indicate that \( u \) (tip) is not significantly affected by tip enlargement. In "soft" clays, \( u \) at a distance greater than \( 4d \) to \( 5d \) behind the cone is very close to \( u \) behind an enlarged cone. In Boston Blue Clay, where extensive pore pressure measurements are available, \( u \) at the tip of unenlarged cones and \( u \) behind enlarged cones increases with increasing apex angle.

(17) In all three clay deposits tested, steady penetration pore pressure \( u \) tends to decrease when cone resistance \( q_c \) increases. In "soft" clay (OCR \( \approx 1 \)) regions, \( u \) (tip) varies between 0.6 and 1.1 times \( q_c \). Values of \( u > q_c \) are believed to indicate inaccuracies in \( q_c \) measurements. The ratio \( u/q_c \), using \( u \) at tip, mid-cone or behind the cone, provides an excellent indication of the variation in stress history and soil type.

Thus a new instrument capable of measuring both \( q_c \) and \( u \) simultaneously will have great potential in soil exploration. This new instrument will be very sensitive to inhomogeneities or variation in soil properties and will be more valuable for soil identification than the existing friction ratio method (see Sanglerat, 1972, or Begemann, 1965). In addition, this new instrument will be capable of providing information on the in-situ static pore pressure and permeability or consolidation properties of one soil.

**Prediction of Undrained Shear Strength**

(18) The proposed semi-empirical cone penetration theory provides a rational basis for the prediction of undrained shear strength from cone penetration measurements. Its adequacy can be checked by comparing the strength predicted from cones of different geometries, and by comparing
the predicted strength to the "field" strength estimated from actual embankment performance. To account for the difference in strain rate between cone penetration and actual failures developing over days or weeks, the "field" strength is empirically adjusted to provide a reference strength for evaluation of the theoretical predictions. In all three deposits studied, this reference strength is close (within ±30%) to the strength obtained by the field vane test (uncorrected).

(19) The comparison indicates that the accuracy of the theory depends on the cone shape. **Enlarged cones** (with diameter reduction behind cone tip), supplemented by pore pressure measurements behind the cones) predict undrained shear strength in perfect agreement with the reference strength in a marine illitic clay (low PI, medium sensitivity) and a plastic deltaic clay (high PI, low sensitivity).

(20) For **unenlarged cones** (no diameter reduction) including the "standard" cone (60° tip), the theory predicts the upper and lower bounds of the undrained shear strength on the basis of cone resistance $q_c$. A "best estimate" expression lying approximately in the middle of the bounds predicts the strength with a significantly larger scatter than in predictions based on enlarged cones, and can distort the shape of the strength profile depending on the variation of soil type and stress history with depth.

On the other hand, the theory predicts the upper and lower bounds of the undrained shear strength from standard cone (FUGRO) with reasonable reliability. The observed values of the $q_c$/reference strength ratio vary between 10 and 20 as compared to the predicted range of 11 to 25. Several
case studies of cone penetration tests in Scandinavian clays performed by NGI-FUGRO (Lunne et al., 1976) are also analyzed using the "field" strength based on field vane tests with Bjerrum's correction factor. The analysis indicates that the predicted upper and lower bounds of the undrained shear strength are similarly applicable in these clays, with the possible exception of very sensitive soils.

(21) These results suggest that the Fugro cone geometry is not the ideal cone shape from an analytical point of view. The rigid shaft behind the cone tip constrains the soil deformation and complicates the interpretation of cone resistance. A more accurate interpretation technique for this cone geometry requires additional measurements of pore pressures and/or more sophisticated methods of analyses. For immediate practical application of cone resistance in design, empirical correlations based on local experiences are necessary.

(22) Empirical correlations between cone resistance, $q_c$, and corrected and uncorrected field vane strengths $[s_u(FV)\ 	ext{and} \ \mu s_u(FV)]$ are presented for a marine illitic clay (low PI, medium sensitivity), a plastic deltaic clay (high PI, low sensitivity), and a glacial lake varved clay (medium PI and sensitivity). The empirical cone factor $N_c(FV) = [q_c - \sigma_v] / s_u(FV)$ varies between 5 and 15 depending on stress history, sensitivity, depth, and possibly other factors. This range of $N_c(FV)$ is lower than that presented by Lunne et al. (1976) for "medium" to "very soft" Scandinavian clays.

Bjerrum's empirical correction factor, $\mu$, slightly reduces the variation of $N_c(FV)$ with soil type (PI) but cannot account for the dependence
of $N'_c(FV)$ on depth, stress history, or sensitivity. The range of another empirical cone factor, $N'_c(FV) = \frac{q_c - \sigma_{vo}}{\mu s_u(FV)}$ for the three sites and the six sites tested by Lunne et al. (1976) is 9 to 20 for medium to very soft clays with PI greater than 10. The uncertainty in this empirical correlation can be significantly reduced by using local experience and noting the dependence of $N'_c(FV)$ on depth. It can perhaps be further reduced if measurements of pore pressure during penetration are also available. Field experiments indicate that this pore pressure depends on the soil type and stress history and can perhaps influence the penetration resistance. Additional correlations between $N'_c(FV)$ and pore pressure is, however, required for practical application of this approach.
DEFORMATION AND STRAINS IN STEADY MOTION

A.1 Introduction

Continuous and deep penetration of a rigid indenter in a homogeneous soil mass represents a steady state problem. To an observer moving with the indenter, the deformation pattern, the strain and incremental stress fields in the soils do not change with time. This is different from other important problems in soil mechanics such as the load-displacement of a footing or the expansion of a cavity, where deformations, stresses and strains all change with time.

The steady state condition severely restricts the acceptable modes of deformation (i.e., the velocity fields). Experimental observations generally indicate that the deformation pattern due to penetration by a rigid indenter is similar in different soils even though the penetration resistance can be drastically different. This suggests that penetration is closer to being a strain-controlled problem than a stress-controlled problem, and that rational solutions of the penetration process should give primary consideration to deformations and strains. In this Appendix, we develop a procedure with which deformation, strain, and strain-rate may be numerically obtained from any given steady velocity field, and apply the procedure to wedge and cone penetration.
A.2 An Eulerian Formulation

(a) **The conjugate problem.** When gravity can be neglected, the problem of a rigid indenter moving in a rigid-plastic material with a constant velocity \(-V\) (downward) is statically identical to the problem in which the material is moving with a velocity \(+V\) (upward) with respect to a fixed indenter. In the first (initial) problem, the coordinate axes are fixed in space. In the second problem they are fixed to the indenter. The second problem is referred to as the **conjugate problem** of the first. It allows us to follow a material element through the penetration process, and provides a basis for this formulation. The kinematics of a conjugate problem are such that its velocity field is equal to the velocity field of the initial problem superimposed on a uniform velocity field of magnitude \(+V\).

(b) **Description of the flow field.** Motions of a group of particles can be described by two different methods: (1) The Lagrangian method specifies the velocity of a material point as it travels through the flow field. And (2) the Eulerian method specifies the instantaneous spatial distribution of velocity (referring to a coordinate system which may be at rest or moving at a uniform speed). In steady motion, the Eulerian velocity vector \(\vec{v}\) is independent of time:

\[
\vec{v} = \vec{v}(\vec{r}),
\]  

(A.2.1)
where \( \mathbf{x} \) is the position vector. The velocity fields associated with the ideal-plasticity theory of steady wedge penetration (Chapter 3 and Appendix B) are of this type.

(c) **Streamline.** A streamline has a tangent which is everywhere parallel to the velocity vector \( \mathbf{v} \). In steady motion, the streamlines do not vary with time, and are also the paths along which material points follow while they travel through the flow field.

(d) **Incompressibility.** For an incompressible material in steady motion, mass-conservation requires that the velocity vector has a zero divergence:

\[
\nabla \cdot \mathbf{v} = 0.
\]

(A.2.2)

(e) **Plane or axisymmetric steady flow of an incompressible material.** In these types of motion, we can choose a coordinate system such that the motion depends only on two coordinates (\( x \) and \( y \) in plane flow; \( r \) and \( z \) in axisymmetric flow). Furthermore, the incompressibility requirement, Eq. (A.2.2), makes it possible to define a scalar **stream function**, \( \psi \), from which the velocity components can be obtained by differentiation (for differential formulas in different coordinate systems, see Malvern (1969)). Thus

\[
\psi = \psi (x,y) \quad \text{in plan flow};
\]

\[
\psi = \psi (r,z) \quad \text{in axisymmetric flow};
\]

(A.2.3)
and the derivatives are:

\[ \frac{\partial \psi}{\partial y} = v_x \quad \text{and} \quad \frac{\partial \psi}{\partial x} = -v_y \quad \text{in plane flow}; \]

\[ \frac{\partial \psi}{\partial z} = r v_r \quad \text{and} \quad \frac{\partial \psi}{\partial r} = -rv_z \quad \text{in axisymmetric flow}, \]

where \( v_x, v_y, v_r \) and \( v_z \) are the velocity components.

A distinct streamline is given by:

\[ \psi = c_1 \]  \hspace{1cm} (A.2.4)

where \( c_1 \) is a scalar constant related to the volume flux within the region bound by this streamline (see Hildebrand, 1962; Batchelor, 1970). With some mathematical manipulation, it is often possible to find an explicit expression of the streamline in the form:

\[ x = x(y, c_1) \quad \text{for plane flow}; \]

\[ r = r(z, c_1) \quad \text{for axisymmetric flow}. \]  \hspace{1cm} (A.2.5)

where \( y \) and \( z \) are the major direction of motion.
(f) **Deformation grid.** In steady motion, time \( t \) has no real influence besides denoting the progress of deformation, and thus can be replaced by other more convenient variable with the same function. One such variable is the coordinate \( y_o \), parallel to the major direction of motion, of an *undeformed* material element.

On a streamline, \( \psi = c_1 \), a material element moves from \( y_1 \) to \( y_2 \), in the major direction of the motion, while an undeformed element moves from \( y_1 \) to \( y_1 + \Delta y \), where \( \Delta y \) is given by:

\[
\Delta y = \int_{y_1}^{y_2} \frac{dy_o}{dy} \, dy = \int_{y_1}^{y_2} \frac{dy_o}{dt} \frac{dt}{dy} \, dy = \int_{y_1}^{y_2} \frac{V}{v_y} \, dy,
\]

(A.2.6)

where the integral is taken along the appropriate streamline \( \psi = c_1 \), and \( V \) is the velocity (in the \( y_o \)-direction) of an undeformed element.* By joining points on different streamlines corresponding to

\[
\Delta y = n\lambda, \ n = 1, 2, 3, \ldots
\]

(A.2.7)

where \( \lambda \) is an arbitrary constant, we obtain the deformed shapes of straight lines initially perpendicular to the major direction of motions. These lines together with the streamlines provide the deformed shape of an initially square (or rectangular) grid.

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*\( V \) is equal to the velocity of the indenter in the initial problem.
(g) **Strains.** In an Eulerian velocity field, the strain-rate tensor \([\varepsilon]\) is identical to the rate-of-deformation tensor \([D]\) (also called stretching or velocity-strain tensor) given in a cartesian coordinate system by:

\[
\dot{\varepsilon}_{ij} = D_{ij} = \frac{1}{2} (v_{i,j} + v_{j,i}), \quad (A.2.8)
\]

or

\[
d\varepsilon_{ij} = D_{ij} \, dt. \quad (A.2.9)
\]

The left hand side of Eq. A.2.9 represents the **natural strain increments** which can be integrated along a streamline for components of the **natural strain tensor** \([\varepsilon]\):

\[
\varepsilon_{ij} = \int_{t_1}^{t_2} d\varepsilon_{ij} = \int_{y_1}^{y_2} \frac{D_{ij}}{v} \, dy, \quad (A.2.10)
\]

where integral is again taken along an appropriate streamline, and \(t_1, t_2\) correspond to \(y_1, y_2\).
A.3 Steady Wedge Penetration

In this section, we consider steady penetration of a "rough" wedge in an isotropic rigid plastic medium (Chapter 3). The associated velocity field (see Appendix B, Fig. B.6.4) was presented by Baligh (1972) who also obtained the deformation pattern experimentally.

(a) **Basic equations.** The problem geometry and the appropriate coordinate system are as shown in Fig. A.1. The velocity components are

\[
\begin{align*}
\dot{v}_x &= -V \sin \delta \cos (\pi/4 - \gamma) \\
\dot{v}_y &= V \sin \delta \sin (\pi/4 - \gamma) + V \\
\dot{v}_x &= -V \sin \delta y[(x - \sin \delta)^2 + y^2]^{-1/2} \\
\dot{v}_y &= V \sin \delta (x - \sin \delta) [ (x - \sin \delta)^2 + y^2]^{-1/2} + V \\
\dot{v}_x &= 0 \\
\dot{v}_y &= V
\end{align*}
\]

in plastic region I

in plastic region II

outside regions I and II

(A.3.1)
The streamlines are parallel to the $y$-axis outside the plastic regions, and are parallel to the cavity face in region I. In region II, they are found by integration of the velocity components to be:

$$x(y, x_o) = \sin \delta + \frac{x_o - \sin \delta \sqrt{x_o^2 + y^2 \cos^2 \delta}}{\cos^2 \delta}$$

(A.3.2)

where $x_o$ is the $x$-coordinate of the streamline in the undeformed region.

The strain-rate components are zero everywhere except in region II which have (compression positive):

$$\dot{\varepsilon}_{xx} = -\dot{\varepsilon}_{yy} = -V (\sin \delta) y (x - \sin \delta) [y^2 + (x - \sin \delta)^2]^{-3/2};$$

$$\dot{\varepsilon}_{xy} = -1/2 V (\sin \delta) [y^2 - (x - \sin \delta)^2] [y^2 + (x - \sin \delta)^2]^{-3/2}.$$ 

(A.3.3)

In incompressible plane deformation, the state of strain is best represented by the maximum shear strain:

$$\varepsilon_{\text{max}} = \sqrt{\varepsilon_{xx}^2 + \varepsilon_{xy}^2}$$

(A.3.4)

which is also the radius of the Mohr circle of strain.
(b) **Predicted deformation.** The deformation pattern of an initially square grid due to steady wedge penetration is shown in Figs. A.2 a, b, c and d for apex angles of 20°, 30°, 45° and 60°.*

From these figures and Fig. A.1, we note that

1. For a base width $2B$, the region in which the material retains permanent deformation has a width $2B (1 + 1/\sin \delta)$ which decreases as the apex angle $2\delta$ increases.

2. The permanent displacement of the material can be seen at the top of each figure. Table A.1 presents the maximum magnitude of this permanent displacement in terms of $B$. Between apex angles of 20° and 60°, the maximum permanent displacement is on the order of $B$ to $1.3B$ and decreases as $2\delta$ increases.

3. The intensity of deformation is indicated by the curvature of the deformed grids, and increases as $2\delta$ increases.

4. The material is undeformed in front of the wedge and outside the width $2B(1 + 1/\sin \delta)$ as required by the rigid plastic assumption. A deformed horizontal line slopes toward the wedge near the wedge but away from the wedge near the edges of the plastic zone.

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*In order to include the entire plastic zone, it is necessary to choose the scale such that the wedges have different base-width but the same face-length.
(c) **Comparison with experimental observations.** Baligh (1972) presents a detailed comparison of the predicted deformation pattern, (which he obtains graphically) and the actual deformation pattern in a modeling clay. The comparison is complicated because the clay shows some compressibility and has a time-dependent behavior. He finds that:

1. The cavity face behind the wedge is not straight but somewhat curved. Moreover, there is a definite contact between the material and the back of the wedge. These two behaviors, he believes, are mostly the effects of creep.

2. There is a substantial compression of the material in front of the wedge, which increases as $2\delta$ increases. Also a deformed horizontal line never slopes away from the wedge, and contains no sharp kink. These aspects are the major discrepancies between the prediction and experimental observations.

3. The rigid-plastic theory predicts the plastic components of the deformation whereas real clays deform elastically and plastically at the same time. Baligh (1972) therefore compares the predicted deformation to the deformation observed after some initial penetration, and finds that the agreement between the prediction and the observation improves significantly from the previous comparison. A deformed horizontal line is observed to slope away from the wedge at some distance from it but is always smooth.
Hirst and Howse (1969) present similar discussion related to pseudosteady penetration of a rigid wedge at the surface (Appendix B, Fig. B.6.3). They suggest that the rigid-plastic mode of deformation applies only when two conditions are met. First, the apex angle of the wedge must be sufficiently acute. This follows from Mulhern (1959) who finds that the mode of deformation in metals changes to one of radial compression when the apex angle exceeds about 60°. Second, the ratio of the shear modulus to shear strength, G/c, must be sufficiently high. This is because materials with low G/c deviates more from the rigid-plastic assumption than materials with high G/c.

Based on an analysis of the average stress in the plastic region, Hirst and Howse (1969) presents a plot which postulates the applicable mode of deformation for any combination of 2δ and G/c. According to this analysis, the rigid-plastic mode of deformation applies for wedges with $2\delta \leq 60^\circ$ when the G/c ratio is 130 or greater. Experimental data presented by Ladd et al. (1977) indicates that, for a number of normally consolidated and slightly overconsolidated clays, the G/c ratio exceeds about 100 when the applied shear stress is less than 2/3 the shear strength, and is much higher at lower stress levels.
On the basis of these discussions, we conclude that, in normally consolidated and slightly overconsolidated clays, the rigid-plastic theory can reasonably well predict the deformation, and hence the strains, caused by steady penetration of a rigid wedge with $2\delta \leq 60^\circ$. The prediction does not apply, however, at the edges of, or outside, the plastic zone which has a width $2B(1 + 1/\sin \delta)$.

(d) **Strains and Strain Rate.** The straining behavior of the material around a steady moving rigid wedge is obtained by a numerical procedure using the method formulated in Section A.2. The results are presented in Figs. A.3 a, b, c and d. For each wedge, the maximum plastic shear strain, $\varepsilon_{max}$ (Eq. (A.3.4)) is plotted versus the coordinate in the direction of motion for three elements which are originally at a distance roughly equal to 0.2, 0.5, and 0.8 times $B(1 + 1/\sin \delta)$ from the axis of the wedge. We note from these figures that:

1. The maximum plastic shear strain $\varepsilon_{max}$ increases as the apex angle $2\delta$ increases. For a representative strain, we take the strain occurring on element B which is in the middle of the plastic zone. The peak value of $\varepsilon_{max}$ in this element increases from 14.5% to 33% as $2\delta$ increases from $20^\circ$ to $60^\circ$ whereas the residual value of $\varepsilon_{max}$ remains roughly 6-8%, Table A.1.
(2) The straining behavior is approximately monotonous only on element C farthest from the wedge. Elements A and B closer to the wedge undergo significant unloading as they move past the wedge.

(3) The rate of straining is related to the slope of the $\varepsilon_{\text{max}} \text{ vs. } y$ curves shown in Figs. A.3 and is given by

$$\dot{\varepsilon}_{\text{max}} = \frac{d \varepsilon_{\text{max}}}{d(y/B)} \cdot \frac{d(y/B)}{dt};$$

$$\varepsilon_{\text{max}} \approx \frac{V}{B} \frac{d \varepsilon_{\text{max}}}{d(y/B)}, \quad (A.3.5)$$

where $V$ is the penetration velocity; $B$ is half the base width of the wedge, and where $d \varepsilon_{\text{max}} / d(y/B)$ is obtained from the slope of the curves in Figs. A.3 (and is independent of the wedge dimension). Eq. (A.3.5) says that, at a fixed penetration velocity, the strain rate decreases as the size of the wedge increases.

In the early phrase of deformation (when a material element moves pass half the wedge), $d \varepsilon_{\text{max}} / d(y/B)$ is surprisingly uniform for all three material elements. With a penetration velocity $V$ of 2 cm/sec and a base width $2B$ of 3.56 cm, the strain rate $\dot{\varepsilon}_{\text{max}}$ is found to increase from 2% per sec to 14% per sec as the apex angle $2\delta$ increases from 20° to 60°. This range of straining rate is about $3 \times 10^3$ times faster than the normal rate of 1 to 5% per hour for laboratory consolidated-undrained shear tests.
A.4 Penetration of a Blunt Cone

In the preceding section, we rely on the velocity field from the rigid-plastic theory for estimates of displacements and strains around a wedge. For the axisymmetric cone penetration problem, rigorous solutions are difficult to obtain even for this highly idealized material. In this case we rely mainly on experimental observations. Model tests by Rourk (1961), Vesic (1963), Robinsky and Morrison (1964), Szechy (1968), Tavenas et al (1973), Muromachi (1974) and others indicate that the deformation pattern around blunt cones or piles at depth are similar in different soils even though the penetration resistance can be drastically different. From an observed deformation pattern, strains can be obtained by numerical differentiation. Such procedure, however, requires very extensive and accurate measurements of the displacements, and is susceptible to errors from the numerical differentiation process and test imperfections. An alternative approach is to seek an anlytical expression for the velocity field that yields a deformation pattern similar to one observed, Bajich (1975). With an anlytical expression of the velocity, displacements, strains, and strain-history of soil elements can be obtained with high degree of accuracy. This approach is, however, limited by the difficulty of obtaining adequate analytic expressions for the velocity field. Fortunately, the penetration of blunt cones (or piles) is one case where this approach can be used.
(a) **Selection of velocity field.** Figure A.4 shows the deformation pattern (solid lines) of an initially square grid predicted for an axisymmetric velocity field given, in spherical coordinates \((R, \Theta, \phi)\), by:

\[
\begin{align*}
    v_R &= \frac{1}{4}V \left( \frac{a_r}{r_c} \right)^2 - V \cos \Theta; \\
    v_\Theta &= V \sin \Theta; \\
    v_\phi &= 0,
\end{align*}
\]

where \(V\) is the penetration velocity; \(r_c\), the cone (or pile) radius, and \(a\), a scale factor (prediction method presented subsequently). This velocity field has zero divergence (incompressible) and zero curl (irrotational). In fluid dynamics terminology, it is similar to the flow pattern around a moving point source (see for example, Batchelor, 1970). An observed deformation pattern (Chapter 2, Fig. 2.1) is represented by the heavy lines on the right-hand side of Fig. A.4; the agreement between the predicted and the observed deformation patterns is very good when \(a\) is about 1.35. This implies that the assumed velocity field approximates the actual mode of deformation outside a zone with a thickness \(0.35 r_c\) around the cone (or pile). The less satisfactory agreement in the
region close to the pile is probably due to the shearing stress along the cone (or pile) surface.

(b) **Basic equations.** Since the velocity field given by Eq. (A.4.1) predicts a deformation pattern which agrees reasonably well with experimental observation. It can now be used to estimate strains around a cone (or pile).* Fig. A.5 shows the appropriate coordinate systems for this problem.

First we obtain the stream function by using properties of its derivatives:

\[
\frac{\partial \psi}{\partial \Theta} = R^2 \sin \Theta \, v_R \quad \text{and} \quad \frac{\partial \psi}{\partial R} = -R \sin \Theta \, v_R \quad (A.4.2)
\]

(for differential formulas in cylindrical and spherical coordinates, see Malvern, 1969), and is found to be:

\[
\psi(R, \Theta) = -\frac{1}{2} \, V \, R^2 \sin^2 \Theta - \frac{1}{4} \, V(\ar_c)^2 \cos \Theta. \quad (A.4.3)
\]

Far in front of the cone, the streamlines run parallel to the cone-axis and have

\[
R \sin \Theta = \text{constant} = r_o, \quad (A.4.4)
\]

*The scale factor \(a\) may have to be adjusted to match observed deformation pattern in different soils. Further research in this area will be very useful.*
where \( r_o \) denotes the perpendicular distance between a streamline and the axis \( \Theta = 0 \). Combining Eqs. (A.4.3) and (A.4.4) we obtain an explicit expression for the streamlines:

\[
R(\Theta, r_o) = \left[ r_o^2 + 1/2 \left( a_r c \right)^2 (1 - \cos \Theta) \right]^{1/2} (\sin \Theta)^{-1}
\]

(A.4.5)

The innermost streamline which is defined by \( R(\Theta, 0) \) has a radius:

\[
r = R \sin \Theta = a_r c \sqrt{1/2 (1 - \cos \Theta)}
\]

(A.4.6)

from the axis of the cone (or pile), which approaches \( a_r c \) at some distance behind the tip.

The components of the strain-rate tensor [\( \varepsilon \)] is spherical symmetric (compression positive):

\[
\dot{\varepsilon}_{RR} = D_{RR} = -\frac{\partial v_R}{\partial R} = 1/2 V (a_r c)^2 R^{-3};
\]

\[
\dot{\varepsilon}_{\Theta \Theta} = D_{\Theta \Theta} = -(\frac{1}{R} \frac{\partial v_\Theta}{\partial \Theta} + \frac{v_R}{R}) = -1/4 V (a_r c)^2 R^{-3};
\]

\[
\dot{\varepsilon}_{\phi \phi} = D_{\phi \phi} = -(\frac{1}{R \sin \Theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_\Theta}{R} \cot \Theta + \frac{v_R}{R}) = -1/4(a_r c)^2 V R^{-3}.
\]

(A.4.7)
The $\dot{\varepsilon}_{RR}$ and $\dot{\varepsilon}_{\Theta\Theta}$ components cannot be directly integrated for strains since they refer to a rotating coordinate system. We thus transform them to a cylindrical coordinate system by:

$$
\dot{\varepsilon}_{rr} = 1/2 (\dot{\varepsilon}_{RR} + \dot{\varepsilon}_{\Theta\Theta}) - 1/2 (\dot{\varepsilon}_{RR} - \dot{\varepsilon}_{\Theta\Theta}) \cos 2\theta;
$$

$$
\dot{\varepsilon}_{zz} = 1/2 (\dot{\varepsilon}_{RR} + \dot{\varepsilon}_{\Theta\Theta}) + 1/2 (\dot{\varepsilon}_{RR} - \dot{\varepsilon}_{\Theta\Theta}) \cos 2\theta;
$$

$$
\dot{\varepsilon}_{rz} = 1/2 (\dot{\varepsilon}_{RR} - \dot{\varepsilon}_{\Theta\Theta}) \sin 2\theta,
$$

(A.4.8)

which are then integrated numerically along an appropriate streamline for $\varepsilon_{rr}$, $\varepsilon_{zz}$ and $\varepsilon_{rz}$ following Eq. (A.2.10) in the form:

$$
\varepsilon_{rr} (\theta) = \int_0^\theta \dot{\varepsilon}_{rr} \frac{R}{v_\theta} d\theta
$$

(A.4.9)

The circumferential strain $\varepsilon_{\phi\phi}$ is given analytically by:

$$
\varepsilon_{\phi\phi}(\theta, r_o) = -1/2 \ln \left[ \frac{r_o^2 + 1/2(ar_c)^2 (1 - \cos \theta)}{r_o^2} \right],
$$

(A.4.10)

which says that very far behind the cone tip, the circumferential strain is:
\[ \varepsilon_{\phi\phi} (\pi, r_o) = -1/2 \ln [1 + (ar_c / r_o)^2] \quad (A.4.11) \]

identical to that resulting from a plane-strain expansion of a cylindrical cavity from zero radius to radius of \( ar_c \) (Bishop et al, 1945).

(c) **Strains and strain-rate.** Shearing of the soil around the cone (or pile) is illustrated by the distortion of the initially square elements in Fig. A.4. The variation in the area of these elements also indicates the magnitude of the circumferential strain which, in this case, is always tensile and represents the minor principal strain, \( \varepsilon_3 \). The major principal strain, \( \varepsilon_1 \), in a soil element lies on a vertical (meridian) plane; and is in the vertical direction (along the axis of the cone) before the cone (or pile) reaches the soil element but reclines increasingly toward the horizontal direction as the penetration progresses.

The degree of shearing due to penetration can be described by the maximum shear strain, \( 1/2 (\varepsilon_1 - \varepsilon_3) \), and the maximum shear strain on vertical planes, \( 1/2 (\varepsilon_1 - \varepsilon_2) \). Fig. A.6 shows these shear strains vs. distance to the cone (or pile) tip for two soil elements A and B (shown in Fig. A.4) which are initially located at \( 0.7 r_c \) and \( 2 r_c \) respectively from the cone axis. Fig. A.6 indicates that significant shearing occurs even before the cone (or pile) tip reaches the location of the soil elements. In this stage,
1/2 (ε₁ - ε₂) is practically equal to 1/2 (ε₁ - ε₃) As the pile tip reaches the location of the soil elements, the magnitude of the circumferential strain increases rapidly, and 1/2 (ε₁ - ε₃) becomes very dominant. After the pile tip reaches the location of the soil elements, 1/2 (ε₁ - ε₂) tends to decrease while 1/2 (ε₁ - ε₃) asymptotically approaches a limiting value. For soil elements located between 5 r_c and 50 r_c behind the tip, 1/2 (ε₁ - ε₂) is approximately half as large as 1/2 (ε₁ - ε₃).

The rate of shearing can be obtained from the slope of the curves in Fig. A.6 using Eq. (A.3.5) (with B replaced by r_c). For a cone diameter (2r_c) of 3.56 cm and a penetration velocity of 2 cm/sec often used in cone penetration tests, the shear rate before the tip reaches the locations of the soil elements is approximately 50% per second for element A close to the cone (or pile) and 5% per second for element B some distance away from the cone (or pile). These rates of straining are of the same order as the rates predicted for blunt wedges, Table A.1.
<table>
<thead>
<tr>
<th>Apex Angle 2δ</th>
<th>20°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Permanent Displacement</td>
<td>1.3B</td>
<td>1.2B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>(1) Peak $\varepsilon_{\text{max}}$</td>
<td>14.5%</td>
<td>18.4%</td>
<td>17.3%</td>
<td>33.0%</td>
</tr>
<tr>
<td>(1) Residual $\varepsilon_{\text{max}}$</td>
<td>6.3%</td>
<td>8.0%</td>
<td>8%</td>
<td>7.2%</td>
</tr>
<tr>
<td>(2) Strain Rate $\dot{\varepsilon}_{\text{max}}$ %/sec</td>
<td>2.0</td>
<td>4.1</td>
<td>9.2</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Note (1) Use element B as the representative element

(2) With a penetration velocity of 2 cm/sec, and a base width 2B of 3.56 cm.

Table A.1 Straining Characteristics of Different Wedges
Fig. A.1 Problem geometry and coordinate system for steady state wedge penetration.
Fig. A.2 Predicted deformation pattern of an initially square grid due to penetration of a 20° wedge
Fig. A.2 Predicted deformation pattern of an initially square grid due to penetration of a 30° wedge.
Fig. A.2 Predicted deformation pattern of an initially square grid due to penetration of a 45° wedge
Fig. A.2 Predicted deformation pattern of an initially square grid due to penetration of a 60° wedge
Fig. A.3 Strain-history of soil elements A, B and C as shown in Figs. A.1
Fig. A.3 Strain history of soil elements (cont.) A, B and C as shown in Figs. A.1
Fig. A.4 Predicted and observed deformation patterns due to cone penetration

thin grid = predicted deformation patterns,
heavy lines = observed deformation pattern
(Rourk, 1961)
Fig. A.5 Problem geometry and coordinate system for steady penetration of a blunt cone
Fig. A.6 Strain history of soil elements A and B shown in Fig. A.4
APPENDIX B

SLIP LINE SOLUTIONS FOR WEDGE PENETRATION IN ANISOTROPIC CLAYS

B.1 Introduction

In many soil mechanics problems, the mathematical difficulties arising from complex soil behavior require simplifications to be made. One common simplification consists of neglecting soil deformation prior to failure, i.e., assuming the soil to be rigid when stressed below its "strength." Then a variety of plane-strain problems can be solved using the theory of slip-lines. In this appendix, we look at the causes and models for undrained strength anisotropy of clays. Yield contour is introduced to describe undrained strength anisotropy in plane-strain deformations, complying with the theory of ideal plasticity. The basic equations of slip-line theory for anisotropic materials are briefly described, and then specialized to elliptic yield contours. Existing solutions for various modes of wedge penetration in isotropic material are presented. The solution for steady-state penetration, which resembles continuous pushing of instruments in soils, is extended to anisotropic materials so that the effects of strength anisotropy can be determined.
B.2 Yield Contours

For plane strain deformations, let $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ represent the stress components in a right-handed orthogonal cartesian frame of reference $xy$. Rice (1973) shows that, for all homogeneous, massless, rigid-perfectly plastic, and incompressible materials which comply with the principle of maximum plastic work in plasticity theory (Hill, 1950), the yield criterion can be expressed solely in terms of the deviatoric stress components $1/2(\sigma_y - \sigma_x)$ and $\tau_{xy}$ as

$$f[1/2(\sigma_y - \sigma_x), \tau_{xy}] = 0,$$

(B.2.1)

provided that yielding does not explicitly depend on the mean in-plane stress, $1/2(\sigma_y + \sigma_x)$.

On a $\tau_{xy}$ vs. $1/2(\sigma_y - \sigma_x)$ diagram, Eq. (B.2.1) describes a unique "yield contour," of arbitrary shape, which must enclose and be always concave to the origin (Fig. B.2.1). The convexity of the yield contour ensures that a unique set of stresses corresponds to a prescribed set of strain increments (Hill, 1950).

The shape of the yield contour is independent of the frame of reference. By using the stress transformation law, it can be shown that the yield contour referring the frame $\alpha\beta$ inclined at an angle $\theta$ to frame $xy$ ($\theta$ positive counterclockwise) is given by a $2\theta$ rotation of the yield contour referring to frame $xy$ (Fig. B.2.1). This provides the basis for two methods of estimating strength anisotropy: (1) by testing samples with a fixed orientation under different stress systems, or (2) by testing samples with varying orientation under a fixed stress system.
Fig. B.2.1 Yield contour on a deviactoric stress diagram
B.3 Undrained Strength Anisotropy of Clays

Stress-strain-strength anisotropy of clay results from its structure (fabric), composition, and stress history. Structural anisotropy occurs during clay deposition due to the platelike shape of the clay particles, and to the (one-dimensional) gravity loading. X-ray diffraction shows that clay particles tend to align themselves perpendicular to the direction of gravity loading (e.g., Martin and Ladd, 1975). Compositional anisotropy develops due to heterogeneity on a larger scale as a result of fissures, periodic variation in the soil composition, etc. A prime example of compositional anisotropy is varved clay, which consists of alternate layers of clay and silt an inch thick or less (e.g., Ladd and Wissa, 1970). When the scale of interest is at least several varve thicknesses, varved clays behave as homogeneous anisotropic materials. Structural and compositional anisotropy are inherent properties of the material and are usually called inherent anisotropy (Casagrande and Carillo, 1944; Ladd et al., 1977).

Inherent anisotropy influences the undrained shear strength $s_u$ by causing a dependence of the basic strength parameters: $C$, $\phi$ and $A_f$ (see Lambe and Whitman, 1969) on orientation. But even a clay with isotropic structure and homogeneous composition can still exhibit anisotropic behavior if previously subjected to anisotropic consolidation stresses. Normally consolidated clay deposits are typically subjected to a horizontal effective stress which is about one half of the vertical effective stress (Brooker and Ireland, 1965; Kenney, 1967); they are therefore highly influenced by this stress-induced anisotropy.
Strength anisotropy of a clay, with constant $\bar{c}$, $\bar{\phi}$ and $A_f$, which is consolidated under $K_o$ condition, can be shown based on Brinch-Hansen and Gibson (1948). This work preceded the introduction of pore pressure parameters $A$ and $B$ by Skempton (1954). Simpler derivation leading to the same results is given below for the case of $\bar{c} = 0$.

Consider a set of stress increments applied to a $K_o$-consolidated sample (Fig. B.3.1). The $\alpha$ and $\beta$ directions are the minor and major principal directions of the stress increments to cause yielding. The undrained shear strength $q_f$ is given by:

$$q_f = \bar{p}_f \sin \bar{\phi} ;$$  \hspace{1cm} (B.3.1)

where

$$\bar{p}_f = \bar{p}_o + (\Delta p)_f - (\Delta u)_f ;$$

$$\bar{p}_o = \frac{1}{2}(1 + K_o)\bar{\sigma}_{vo} ;$$

$(\Delta p)_f$ = mean total stress increment;

$(\Delta u)_f$ = pore pressure increment;

$$p = \frac{1}{2}(\sigma_1 + \sigma_3) ;$$

$$q = \frac{1}{2}(\sigma_1 - \sigma_3) ;$$

$\sigma_1$, $\sigma_3$ = major and minor principal stress respectively,

and where subscripts $o$ and $f$ denote the initial and the final conditions. Using the pore pressure parameters, the pore pressure increment $(\Delta u)_f$ is given by

$$(\Delta u)_f = (\Delta \sigma)_3 + A_f[(\Delta \sigma)_1 - (\Delta \sigma)_3] ;$$

$$= (\Delta \sigma)_\alpha + A_f[(\Delta \sigma)_\beta - (\Delta \sigma)_\alpha] . \hspace{1cm} (B.3.2)$$

(for saturated clays, $B = 1$).

From the Mohr circle of stresses in Fig. B.3.1 and Eqs. (B.3.1) and
(B.3.2) it can be shown that

\[
\left( \frac{(\Delta \sigma)_{\beta} - (\Delta \sigma)_{\alpha}}{2} \right)^2 = q_f^2 - 2 q_f f \frac{\sigma_{\varphi}}{\sigma_{\varphi}} \cos 2i + \left( f \sigma_{\varphi} \right)^2 ; \quad (B.3.3)
\]

\[
q_f = \sin \phi \left[ k \frac{\sigma_{\varphi}}{\sigma_{\varphi}} - g \left( \frac{(\Delta \sigma)_{\beta} - (\Delta \sigma)_{\alpha}}{2} \right) \right] \quad (B.3.4)
\]

where

\[
f = 1/2 \left( 1 - K_o \right)
\]

\[
g = 2A_f - 1
\]

\[
k = 1/2 \left( 1 + K_o \right)
\]

\[
i = \text{angle between } \sigma_1 \text{ and the vertical at failure.}
\]

Combining Eqs. (B.3.3) and (B.3.4), we get

\[
q_f = \frac{g \sin \phi \sqrt{k \sin^2 \phi - 2f k \sin \phi \cos 2i - f^2 \sin^2 \phi + f^2 \sin^2 \phi \sin^2 2i + f^2 - k \sin \phi + g f \sin^2 \phi \cos 2i}}{\sigma_{\varphi}}
\]

\[
= \frac{g \sin \phi \sqrt{k \sin^2 \phi - 2f k \sin \phi \cos 2i - f^2 \sin^2 \phi + f^2 - k \sin \phi + g f \sin^2 \phi \cos 2i}}{g^2 \sin^2 \phi - 1}
\]

\[
(B.3.5)
\]

The predicted yield contour for the case of \( \overline{c} = 0 \) treated herein and for \( \overline{\phi} = 38^\circ, K_o = 0.5 \) and \( A_f = 1 \) is shown in Fig. B.3.2; its shape is close to an ellipse with center on the 1/2(\( \sigma_y - \sigma_x \)) axis.
Fig. B.3.1 Stress history induced anisotropy.
Fig. B.3.2 A yield contour for undrained shear of an isotropic soil under anisotropic consolidation stresses

\((c = 0, \phi = 38^\circ, K_o = 0.5, A_f = 1)\)
B.4 Mathematical Models for Undrained Strength Anisotropy

Most existing strength anisotropy models for undrained clays attempt to describe the combined effect of both inherent and stress-induced anisotropies. In this section, these models are described using the yield contour in order to simplify the comparison. Their original expressions can be found in the references noted, or in Davis and Christian (1972). We also consider only the plane-strain condition, and use as basic strength parameters the shear strengths in the vertical, horizontal, and diagonal directions: $s_u(V)$, $s_u(H)$, and $s_u(45^\circ)$, except when noted, from $K_o$-consolidated undrained shear tests. The strength parameters for six normally consolidated clays are shown in Table B.4.1. Additional data are presented by Ladd et al. (1977).

(1) **Two-degree of Freedom Models.** Hill (1948, 1950) suggests a yield contour which is an ellipse with center at the origin. When applied to soils, this model requires $s_u(H)$ to be equal to $s_u(V)$. Soft nonlayered clays are, however, highly influenced by stress-induced anisotropy, and have as a result $s_u(H)$ considerably smaller than $s_u(V)$. Casagrande and Carillo (1944) suggest varying the strength gradually between $s_u(V)$ and $s_u(H)$ by a trigonometric expression, Table B.4.2. Ranganatham and Matthai (1967) use the same expression to describe strength measured by the direct shear test.

(2) **Three-degree of Freedom Models.** Numerous measurements including those in Table B.4.1 indicate that two-parameter models cannot accurately describe undrained strength anisotropy for all soils. Bishop (1966) suggests another trigonometric expression for the yield contour with three
parameters, Table B.4.2; the corresponding yield contours are shown in Fig. B.4.1 for a layered and a nonlayered clay, both normally consolidated, using the strengths from Table B.4.1. It is clear that Bishop's model does not always lead to convex yield contours and therefore violates an important requirement of plasticity theory.

Davis and Christian (1972) generalize Hill's model by allowing the center of the ellipse to be shifted along the \( \frac{1}{2}(\sigma_y - \sigma_x) \) axis, Fig. B.4.1. They also specify the y-direction to be the vertical direction in the physical plane. This model predicts strength anisotropy similar to Bishop's model, but always leads to convex yield contours and thus can be applied to slip-line theory presented subsequently. The shape and location of the center of the elliptic yield contour are given in Table B.4.1. The factor \( b/a \) indicates the shape of the ellipse. When \( b/a \) equals unity, the yield contour is a circle corresponding to a clay with no inherent anisotropy. Varved clays have a small value of \( b/a \), due to a high degree of inherent anisotropy. The factor \( K_s = s_u (H)/s_u (V) \) describes the location of the center of the ellipse which is at the origin when \( K_s = 1 \) and is shifted to the right as \( K_s \) decreases.

(3) **Four-degree of Freedom Model.** Livneh and Greenstein (1974), propose a four-parameter model. However, because of insufficient experimental data to define all four parameters, they arbitrarily specify one parameter (an exponent). This effectively reduces the model to a three-parameter one, Table B.4.2, which is also very similar to Bishop's. As with Bishop's model, the yield contour is not always convex, and the model is therefore unacceptable in terms of plasticity theory.
<table>
<thead>
<tr>
<th>Type of Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portsmouth Clay</td>
</tr>
<tr>
<td>Haney Sensitive Clay</td>
</tr>
<tr>
<td>Boston Blue Clay</td>
</tr>
<tr>
<td>AGS CH Clay</td>
</tr>
<tr>
<td>San Francisco Bay Mud</td>
</tr>
<tr>
<td>Connecticut Valley Varved Clay</td>
</tr>
<tr>
<td>** Index Properties **</td>
</tr>
<tr>
<td>$W_f$ (%)</td>
</tr>
<tr>
<td>P.I. (%)</td>
</tr>
<tr>
<td>L.I.</td>
</tr>
<tr>
<td>(2)</td>
</tr>
<tr>
<td>(3)</td>
</tr>
<tr>
<td>(4)(5)</td>
</tr>
<tr>
<td>$s_u/\bar{\sigma}_{vc}$</td>
</tr>
<tr>
<td>PSC</td>
</tr>
<tr>
<td>**(4)</td>
</tr>
<tr>
<td>DSS</td>
</tr>
<tr>
<td>PSE</td>
</tr>
<tr>
<td>$K_s$ (5)</td>
</tr>
<tr>
<td>$b/a = \frac{(4)}{(5)}$</td>
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</table>

<table>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>15</td>
<td>1.8</td>
<td>0.350</td>
<td>0.200</td>
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<td>Haney Sensitive Clay</td>
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<td>18</td>
<td>0.75</td>
<td>0.296</td>
<td>---</td>
</tr>
<tr>
<td>Boston Blue Clay</td>
<td>41</td>
<td>21</td>
<td>0.81</td>
<td>0.340</td>
<td>0.200</td>
</tr>
<tr>
<td>AGS CH Clay</td>
<td>71</td>
<td>40</td>
<td>---</td>
<td>0.370</td>
<td>0.250</td>
</tr>
<tr>
<td>San Francisco Bay Mud</td>
<td>88</td>
<td>45</td>
<td>1.04</td>
<td>0.370</td>
<td>0.250</td>
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<tr>
<td>Connecticut Valley Varved Clay</td>
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<td>12-39</td>
<td>1.00</td>
<td>0.280</td>
<td>0.165</td>
</tr>
</tbody>
</table>

*From CK_U plane strain tests with $\bar{\sigma}_{vc} > (1.5 \text{ to } 2) \times \bar{\sigma}_{vm}$

(1) $s_u = q_f = 0.5(\sigma_1 - \sigma_3)_{f}$ except for DSS where $s_u = (\tau_h)_{max}$

(2) $\text{L.I.} = \frac{w_n - w_p}{w_f - w_p}$

(3) Plane strain compression  

(4) Direct simple shear  

(5) Plane strain extension

** Assuming $s_u (\text{DSS}) = s_u (45^\circ)$

Table B.4.1 Normalized undrained shear strength in different modes of failure* (data from Ladd et al, 1977; table courtesy of A. S. Azzouz).
<table>
<thead>
<tr>
<th>Reference</th>
<th>Yield Criterion</th>
<th>Parameters</th>
<th>Convexity of Yield Contours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Casagrande and Carillo (1944)</td>
<td>$1/2(\sigma_\beta - \sigma_\alpha) = s_u(V)[1-(1-K_s) \sin^2 \theta]$</td>
<td>$\tau_{\alpha\beta} = 0$</td>
<td>Always</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tau_{\alpha\beta} = \tau_f(0)[1-n \sin^2 \theta]$</td>
<td></td>
</tr>
<tr>
<td>Ranganatham and Matthai (1967)</td>
<td>$1/2(\sigma_\beta - \sigma_\alpha) = 0$</td>
<td>$n=1-\frac{\tau_f(90^\circ)}{\tau_f(0^\circ)}$</td>
<td>Always</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tau_f(\theta) = \tau_{\alpha\beta}$ to cause yielding at $\theta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>with $1/2(\sigma_\beta - \sigma_\alpha) = 0$</td>
<td></td>
</tr>
<tr>
<td>Hill (1948 and 1950)</td>
<td>$\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \frac{a^2}{b^2} \tau_{xy} = a^2$</td>
<td>$a = s_u(V) = s_u(H)$; $b = s_u(45^\circ)$ when $x$ is horizontal direction.</td>
<td>Always</td>
</tr>
<tr>
<td>Bishop (1966)</td>
<td>$1/2(\sigma_\beta - \sigma_\alpha) = s_u(V)[1-b_1 \sin^2 \theta][1-b_2 \sin^2 2\theta]$</td>
<td>$b_1 = 1 + K_s \frac{s_u(45^\circ)}{s_u(V)}$</td>
<td>Not Always</td>
</tr>
<tr>
<td>Davis and Christian (1972)</td>
<td>$[1/2(\sigma_y - \sigma_x-ma)]^2 + \frac{a^2}{b^2} \tau_{xy} = a^2$</td>
<td>$b/a = s_u(45^\circ)/\sqrt{s_u(V)s_u(H)}$</td>
<td>Always</td>
</tr>
<tr>
<td>Livneh and Greenstein (1974)</td>
<td>$1/2(\sigma_\beta - \sigma_\alpha) = 1-n_1 \sin^2 \theta - n_2 \sin^2 2\theta$</td>
<td>$n_1 = (1-K_s)$ $\frac{1+K_s}{2} - \frac{s_u(45^\circ)}{s_u(V)}$</td>
<td>Not Always</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n_2 = \frac{s_u(45^\circ)}{s_u(V)}$</td>
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</tbody>
</table>

Table B.4.2 - Anisotropic Yield Criteria ($\phi = 0$)
Fig. B.4.1 Strength anisotropy models.
B.5 Theory of Slip-Lines (following Rice, 1973)

(1) **Stress Field.** Let $\alpha \beta$ be an orthogonal curvilinear coordinate system inclined at an angle $\theta$ to the cartesian orthogonal frame of reference $xy$. A material point $P$ is in a plastic state when its stress state corresponds to $P^*$ on the yield contour, Fig. B.5.1. The deviatoric stress increments in the neighborhood of $P^*$ are

$$
d 1/2(\sigma_\beta - \sigma_\alpha) = -dl \cos 2(\theta - \phi) ;$$  \hspace{1cm} (B.5.1)

$$
d^{\tau} \alpha \beta = dl \sin 2(\theta - \phi),$$

where $l$ is the arc length along the yield contour measured from any fixed point, and $2\phi$, the inclination of the tangent to the contour at $P^*$:

$$
\tan 2\phi = \frac{d^{\tau} xy}{d 1/2(\sigma_y - \sigma_x)}. \hspace{1cm} (B.5.2)
$$

In the absence of body forces, the equilibrium conditions in the $\alpha \beta$ system can be written in terms of the mean in-plane normal stress $p$, and the deviatoric stresses, $1/2(\sigma_\beta - \sigma_\alpha)$ and $\tau_{\alpha \beta}$, as

$$
\frac{\partial p}{\partial \alpha} - \frac{\partial}{\partial \alpha} 1/2(\sigma_\beta - \sigma_\alpha) + \frac{\partial \tau_{\alpha \beta}}{\partial \beta} = 0 ;
$$

$$
\frac{\partial p}{\partial \beta} + \frac{\partial}{\partial \beta} 1/2(\sigma_\beta - \sigma_\alpha) + \frac{\partial \tau_{\alpha \beta}}{\partial \alpha} = 0
$$

which, when combined with Eq. (B.5.1) become

$$
\frac{\partial p}{\partial \alpha} + \cos 2(\theta - \phi) \frac{dl}{d\alpha} + \sin 2(\theta - \phi) \frac{dl}{d\beta} = 0 ;
$$

$$
\frac{\partial p}{\partial \beta} - \cos 2(\theta - \phi) \frac{dl}{d\alpha} + \sin 2(\theta - \phi) \frac{dl}{d\beta} = 0,
$$

applicable for a homogeneous material with arbitrary anisotropy.
If the $\alpha \beta$ system is chosen such that

$$20 = 2\phi,$$

(B.5.5)

Eq. (B.5.4) will reduce simply to

$$\frac{\partial}{\partial \alpha} (p + \ell) = 0;$$

$$\frac{\partial}{\partial \beta} (p - \ell) = 0,$$

(B.5.6)

applicable at any arbitrary material point $P$ in the plastic domain.

Taking the $\alpha \beta$ system as a moving curvilinear coordinate system, we can cover the entire plastic zone with two orthogonal families of "slip lines" along which one of Eq. (B.5.6) holds. The $\alpha$-lines are herein defined as ones which, at a material point $P$, inclines at an angle $\phi$ to the $x$-axis, where $2\phi$ is the inclination of the yield contour at the corresponding stress state $P^*$ (Eq. (B.5.2)). The stress field in the plastic zone can be described by:

$$p + \ell = \text{constant along an } \alpha\text{-line};$$

$$p - \ell = \text{constant along a } \beta\text{-line},$$

(B.5.7)

together with the appropriate boundary conditions.

For an isotropic material, the yield contour is a circle and the arc length $\ell$ is $2c\phi$, where $c$ is the (unique) shear strength of the material. Eq. (B.5.7) thus reduces to:

$$p + 2c\phi = \text{constant along an } \alpha\text{-line};$$

$$p - 2c\phi = \text{constant along a } \beta\text{-line},$$

(B.5.8)

which are known as the Hencky equations.
(2) **Velocity and Strain Rate Field.** Incompressibility requires that, at any point,

\[ \varepsilon_\alpha + \varepsilon_\beta = 0 \quad , \]  

(B.5.9)

where \( \varepsilon_\alpha \) and \( \varepsilon_\beta \) are the normal components of the strain rate tensor in the \( \alpha \beta \) system.

Furthermore, the normality rule requires that the deviatoric strain rate vector \( (\gamma_{\alpha\beta}, \varepsilon_\beta - \varepsilon_\alpha) \) at a point \( P \) which is in a plastic state be parallel to the outward normal to the yield contour at the corresponding stress state \( P^* \), i.e.,

\[ \varepsilon_\beta - \varepsilon_\alpha = \lambda \cos \left( \frac{\pi}{2} + 2\phi - 2\theta \right) \quad ; \]  

(B.5.10)

\[ \gamma_{\alpha\beta} = \lambda \sin \left( \frac{\pi}{2} + 2\phi - 2\theta \right) \quad , \]

where \( \lambda \) is a scale factor. Along the \( \alpha \) and \( \beta \) slip-lines, \( 2\phi = 2\theta \) (Eq. (B.5.5)), and the first of Eq. (B.5.10) reduces to

\[ \varepsilon_\beta - \varepsilon_\alpha = 0 \quad , \]  

(B.5.11)

which together with Eq. (B.5.9) indicates that

\[ \varepsilon_\alpha = \varepsilon_\beta = 0 \quad . \]  

(B.5.12)

Thus the slip-lines are the directions of zero extension rate, and hence of maximum shear strain rate.*

---

*In mathematical terminology, the slip lines are the real characteristics both of velocities and stresses, the basic equations of which are hyperbolic (Hill, 1950, Chapter 6 and Appendix III).
The velocity vector of every material point in the physical plane can be mapped onto a hodograph. Because of the zero extension rate, the velocity increment vector between two neighboring material points must be normal to the line element joining the two points. Hence, corresponding elements of a slip line and its image in the hodograph are orthogonal. For identical slip-line fields (described subsequently), the velocity field is independent of material anisotropy.

(c) **Elliptic Yield Contour.** Section B.4 shows that elliptic yield contours with center along the $\frac{1}{2}(\sigma_y - \sigma_x)$ axis adequately describe the undrained strength anisotropy of most natural clays. We now specialize the slip-line theory to this particular class of materials.

**Arc-Length.** The arc length \( \ell \) of an elliptic yield contour can be related to the \( \alpha \) slip-line inclination \( \phi \) by an elliptic integral. Starting from the top vertex of the contour, Fig. B.5.1, and moving clockwise, \( \ell \) is given by:

\[
\ell(\phi, b/a) = E(d, \eta),
\]

where

\[
d = 1 - (b/a)^2;
\]

\[
\eta = \tan^{-1}(-a/b \tan 2\phi);
\]

\( E(d, \eta) \) = the elliptic integral of the second kind defined as

\[
\int_0^{\eta} \sqrt{1 - d \sin^2 \theta} \, d\theta
\]

(Milne-Thomson, 1964; Belyakov et al., 1965).

**Slip-line Geometry.** Straight-line (uniform stress) and centered-fan regions are admissible slip-line fields for anisotropic materials satisfying incompressibility and the normality rule (Rice, 1973).
The inclination $\phi$ of an $\alpha$ slip-line at a material point $P$ is related to the stress state at that point by Eqs. (B.5.2) and (B.5.5):

$$\tan 2\phi = -\frac{b^2}{a^2} \frac{1/2(\sigma_y - \sigma_x) - ma}{\tau_{xy}}.$$  \hspace{1cm} (B.5.14)

Geometrically, Eq. (B.5.14) relates the inclination of an elliptic yield contour at a point $P^*$ and its position with respect to the origin, Fig. B.5.1. If the position vector from the origin to $P^*$ has a length $q_f$ and an inclination $2\theta$, then

$$q_f = \sqrt{\frac{2}{\tau_{xy}} + 1/4(\sigma_y - \sigma_x)^2};$$ \hspace{1cm} (B.5.15)

$$\tan 2\theta = \frac{\tau_{xy}}{1/2(\sigma_y - \sigma_x)},$$

where $q_f$ is the maximum possible shear stress or "strength" in a compression test with $q_1$ acting on the $\theta$-plane. For the yield contours of interest, we can write:

$$q_f = q_f(\phi, b/a, K_s);$$  

$$\theta = (\phi, b/a, K_s).$$  \hspace{1cm} (B.5.16)

When the shear stress $\tau_{xy}$ vanishes, Eq. (B.5.14) shows that the $\alpha$ slip-line inclination $\phi = \pm \pi/4$. Thus slip-lines intersect a horizontal plane at $\pm 45^\circ$ if the shear stress on this plane vanishes. Furthermore, Eq. (B.5.15) indicates that these slip-lines are also the directions of maximum shear stress. Therefore, the slip-line field for some simple problems, e.g., the incipient penetration of a smooth flat punch, is independent of material anisotropy if the problem has an axis of symmetry which coincides with the axis of material anisotropy.
Fig. B.5.1 Elliptic yield contour and relation to slip-line theory.
B.6 Different Modes of Wedge Penetration in Isotropic Materials

Consider a rigid wedge with an apex angle $2\delta$ penetrating into an isotropic rigid perfectly-plastic material with a shear strength $c$. The yield contour (section B.2) of this material is a circle with radius $c$; for isotropic clays in undrained conditions, $c$ is equal to the undrained shear strength $s_u$. The friction between the wedge and the soil, $\tau$, can vary between 0 and $c$. The limiting cases of $\tau = 0$ and $\tau = c$ will be referred to as the "smooth" and "rough" wedge respectively.

Solutions of wedge penetration can be divided according to the mode of deformation into three groups: incipient penetration, pseudo-steady penetration, and steady penetration. Solutions for different penetration modes may have similar slip-line fields and hence similar wedge resistances, but may imply very different deformations and strains. Deformation and strain cannot be estimated for incipient wedge penetration, since the associated velocity field applies only at the instant of motion initiation. They are also difficult to estimate in unsteady wedge penetration (including pseudosteady penetration), because the pattern of motion changes with time. For steady wedge penetration, where the pattern of motion remains constant with time, deformation and strain can be estimated by the method described in Appendix A. These solutions and factors affecting wedge penetration are described below.

(a) **Incipient penetration.** Bearing capacity problems in soil mechanics represent incipient failures where the yield load is determined on the basis of the initial (undeformed) problem geometry. Solutions of incipient deformation problems do not consider the progress of deformation and
yielding, and hence do not apply after the initiation of motion.

Figs. B.6.1 a and b show the plane-strain solutions for incipient penetration of a rigid wedge at the surface of, or in a deep vertical cut in a half-space. These solutions require that the wedges be already below the surface, or below the vertical cut before the initiation of motion. Solutions for the limiting cases of "rough" and "smooth" wedges are obtained by Meyerhof (1961).* Each solution consists of the slip-line field, the associated velocity field, and the wedge resistance factor, \( N_w \), (defined for all solutions presented herein) by:

\[
q_w = N_w c + p_b ,
\]

(B.6.1)

where \( q_w \) = force per unit area required for wedge penetration, and \( p_b \) = normal stress acting on the surface at the boundary of the plastic zone, Fig. B.6.1.

For surface penetration, \( p_b \) vanishes when no surcharge is applied to the surface of the half-space. Fig. B.6.2a shows \( N_w \) vs. the apex angle \( 2\delta \) for the limiting cases of "rough" and "smooth" wedge.

In the plastic domain, zone I (ACE) is a zone of uniform state of stress adjacent to the boundary of the plastic domain. Zone III (ABD), adjacent to the wedge face (AB), also has a uniform state of stress; its geometry (angle \( \omega \)) is determined by friction \( \tau \) on the wedge face, i.e.,

\[
\tau = c \cos 2\omega
\]

or

\[
\omega = \frac{1}{2} \cos^{-1} \left( \frac{\tau}{c} \right)
\]

(B.6.2)

* His solution for deep incipient penetration of a "rough" wedge, however, assumes existence of a shear stress \( \tau = c \) on surface AC, resulting in a slightly different solution than given herein.
The slip lines intersect the wedge face (AB) in zone III at an angle \( \omega \) or \( \omega + \pi/2 \). Zones I and III are connected by a radial fan shear zone (section B.5). The state of stress at a point in the plastic domain can be determined by drawing the Mohr circle of stress representing that point, as will be shown subsequently for the case of steady wedge penetration.

(b) **Pseudosteady deformation.** Problems in which deformation changes with time (unsteady) are difficult to solve, because the size and shape of the plastic zone also change with time. Johnson and Kudo (1964) obtain upperbound solutions for shallow penetration of a rigid wedge (with rigid vertical boundaries behind) by assuming different shape and velocities for the deforming region, and equating the rate of energy dissipation to the rate of work done by the wedge. The predicted profiles of the indented surface, however, differ substantially from those actually observed.

In some unsteady problems, a geometrical similarity is maintained during the deformation. Such problems are called pseudosteady and have simpler solutions. Fig. B.6.3 presents the pseudosteady solution for surface wedge penetration (Hill et al., 1947; Grunzweig et al., 1954; Haddow, 1967). Initially, the surface is flat (dashed line). As the wedge penetrates into the material, the surface on both sides of the wedge is displaced upward. Hill et al. (1947) assume that the displaced free surface is straight, as indicated by AC, in order to obtain a geometrically similar configuration at every stage of penetration. Fig. B.6.2b presents the wedge resistance factor \( N_w \) for the limiting cases of "rough" and "smooth" wedges, which are similar to that for incipient surface penetration (Fig. B.6.2a).
(c) **Steady deformation.** Steady state wedge penetration requires that the velocity (relative to the wedge) of points around the wedge remain constant with time. To satisfy this requirement, some simplifications have to be made. Baligh (1972) obtains the solutions for the limiting cases of "rough" and "smooth" wedges by assuming that a cavity (where no material flows in) exists behind the wedge, and, furthermore, that the boundaries of this cavity are straight (Fig. B.6.4). The geometry of this cavity depends on angle $\gamma$, given by

$$\sin \gamma = \frac{\sin \delta}{\sqrt{2} \cos \omega} \quad (B.6.3)$$

For the limiting cases of "rough" and "smooth" wedge, $\gamma$ reduces to $\sin^{-1}[(\sin\delta)/\sqrt{2}]$ and $\delta$ respectively. Fig. B.6.2b shows the wedge resistance factor, $N_w$, for these two limiting cases; the effects of wedge friction will be discussed in (f). For sharp wedges ($2\delta \leq 20^\circ$), $N_w$ is similar to that for incipient deep penetration (Fig. B.6.2a), whereas for blunter wedges, $N_w$ for the two cases differ, especially for "smooth" wedges. Fig. B.6.4.c describes the stress field in the plastic zone using a graphical method described by Prager (1959) and Baligh (1972).

(d) **Range of validity of the solutions.** The solutions for the smooth wedge ($\tau = 0$) is valid for apex angle, $2\delta$, between $0^\circ$ and $180^\circ$. For a wedge with some friction, the solution is somewhat restricted by the requirement that no yielding is possible outside the plastic region. By this requirement, the slip-lines cannot intersect at the tip of the wedge at an angle sharper than $90^\circ$ (Hill, 1954), and as a result the apex angle of the wedge is limited to
\[ 2\delta \leq \pi/2 + \cos^{-1} \frac{\tau}{c}, \quad \text{(B.6.4)} \]

or

\[ 2\delta \leq \pi/2 + 2\omega. \]

Experiments on blunt wedges indicate that a dead zone of material forms in front of the penetrating wedge (Johnson, 1970; Baligh, 1972). If this dead zone is assumed to be wedge-shaped with an apex angle of 90°, the solution will be identical to that of a 90° rough wedge (\( \tau = c \)). The shearing stress on the wedge faces is then given as

\[ \tau = c \cdot \cos 2(\delta = \pi/4) \quad \text{(B.6.5)} \]

(e) **Principle of minimum wedge resistance (energy dissipation).**

Baligh (1972) suggests, on the basis of the concept of minimum energy dissipation, that a dead zone of (penetrated) material develops in front of a blunt wedge if its presence reduces the wedge resistance, \( q_w \). Thus if the predicted \( N_w \) of a blunt wedge (wedge friction = \( \tau \)) exceeds \( N_w \) of a sharper wedge (with \( \tau = c \)), the dead zone will develop in front of the blunt wedge with the shape of the sharper wedge, and will reduce the value of \( N_w \) to that of the sharper wedge. This principle specifies the maximum value of \( N_w \) for blunt wedges with \( 2\delta \geq 67.2^\circ \) to be 9.19 in steady penetration.

(f) **Effects of wedge friction and apex angle.** The wedge-soil interface friction and the wedge apex angle affect the normal and shear stresses on the wedge, and hence influence wedge penetration resistance (or \( N_w \)). The contribution to \( q_w \) from the normal stress increases linearly (or almost linearly) with the apex angle, \( 2\delta \), whereas the contribution from the friction is proportional to \( 1/\tan \delta \). For blunt wedges (\( 2\delta > 50^\circ \)), normal stress predominates, and the influence of \( \tau/c \) on \( N_w \) is small; for sharp wedges (\( 2\delta > 50^\circ \)) friction
predominates, and the influence of \( \tau/c \) on \( N_w \) is large. For wedges with intermediate degree of wedge friction \((0 < \tau/c < 1)\), \( N_w \) increases rapidly with decreasing apex angle, as shown in Fig. B.6.5 for steady wedge penetration. We note that each curve has a minimum value of \( N_w \) at a critical value of \( 2\alpha \), which is less than 67.2°. This critical wedge apex angle decreases as the degree of wedge friction, \( \tau/c \), decreases. The curve for smooth wedges, which approximates the contribution to \( N_w \) from the normal stress for all wedges, has a minimum of \( 2\alpha = 0° \).

To estimate the appropriate range of \( \tau/c \), it is necessary to relate it to the coefficient of friction which, for clays whose strength is determined by the effective stress, should be in terms of the effective stresses. Fig. B.6.6a shows the total stress state of soil elements adjacent to a steady penetrating wedge in relation to the effective strength envelope of a clay. The Mohr circle on the left represent the effective stress state in the plastic domain (provided that the clay is nonstrain-softening). The coefficient of friction, \( \mu \), is given by

\[
\mu = \frac{\tau}{\bar{\sigma}_n}
\]

\[
= \cos 2\omega/(\sin 2\omega + \cot \bar{\alpha})
\]

where

\( \tau = \) friction on wedge face = \( c \cos 2\omega \)

\( \bar{\sigma}_n = \) effective normal stress on wedge face

\( = c(\sin 2\omega + \cot \bar{\alpha}) \)

\( \bar{\alpha} = \) angle defining the effective strength envelop in

a p-q diagram (see Lambe and Whitman, 1969).

Fig. B.6.6b shows \( \omega \) as a function of \( \mu \) and \( \bar{\alpha} \). For a range of \( \bar{\alpha} = 20° \) to 32° and \( \mu = 0.3 \) to 0.4, \( \omega \) is between 0° and 23.5°, corresponding to \( \tau/c = 0.68 \) to 1. Thus the "rough" wedge solution approximately applies for practical purposes.
(g) **Effects of depth below surface.** The wedge assistance factor $N_w$ for deep penetration always exceeds that for surface penetration, Table B.6.1, as a result of the greater volume of soil resisting the motion. The effect of depth on bearing capacity of foundations has received considerable attention in soil mechanics, where it is customary to estimate deep penetration resistance from surface penetration resistance through empirical depth factors, Table B.6.2. Fig. B.6.7 shows the ratio of $N_w$ (incipient deep penetration)/$N_w$ (incipient surface penetration) in comparison with empirical depth factors for large depth. It is clear from this illustration that no single depth factor can account for the predicted effect of depth for all apex angles or degrees of wedge friction ($\tau/c$).
<table>
<thead>
<tr>
<th>Deformation Mode</th>
<th>$q_w = N_w \cdot c + p_b$</th>
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<tr>
<td></td>
<td>$2 \delta = 20^\circ$</td>
</tr>
<tr>
<td></td>
<td>$\tau = 0$</td>
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<tr>
<td>(a) <strong>Incipient</strong></td>
<td></td>
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<tr>
<td><strong>Penetration</strong></td>
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<tr>
<td>Shallow</td>
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<tr>
<td>Deep</td>
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<tr>
<td>(b) <strong>Pseudosteady</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Penetration</strong></td>
<td></td>
</tr>
<tr>
<td>(Shallow)</td>
<td>2.09</td>
</tr>
<tr>
<td>(Deep)</td>
<td>5.84</td>
</tr>
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</table>

Table B.6.1 Wedge resistance factor, $N_w$, for $20^\circ$ and $60^\circ$ wedges
<table>
<thead>
<tr>
<th>Source</th>
<th>Depth Factor (1 &lt;&lt; D/B)</th>
<th>At great depth in cohesive soils</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Skempton (1951)</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>2. Brinch-Hansen (1961) recommended by Mitchell and Durgunoglu (1973)</td>
<td>$1.0 + \frac{0.35}{B/D + 0.6/(1 + 7 \tan^4 \phi)}$</td>
<td>1.58</td>
</tr>
<tr>
<td>3. Brinch-Hansen (1970) recommended by Vesic (1973)</td>
<td>$1.0 + 0.4 \tan^{-1} \frac{D}{B}$</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Note: $D =$ depth, $B =$ wedge width, $\phi =$ friction angle

Table B.6.2 Empirical Depth Factor for Bearing Capacity of Deep Foundations
\[ N_w = 1 + \frac{\pi}{2} + 2(\beta - \omega) + \sin 2\omega + \cos 2\omega / \tan \beta \]

**Fig. B.6.1(a) INCIPIENT SURFACE WEDGE PENETRATION**
\[ N_W = 1 + \frac{3 \pi}{2} + 2(\delta - \omega) + \frac{\sin \omega \cos \omega}{\tan \beta} \]

Fig. B.6.1(b)  INCIPIENT DEEP WEDGE PENETRATION
Fig. B.6.2(a) Wedge resistance factor for shallow and deep incipient penetration.
Fig. B.6.2(b) Wedge resistance factor for pseudosteady (shallow) and steady (deep) penetration.
Slip-Line Field

\[ N_w = 1 + 2\psi + \sin 2\omega + \cos 2\omega / \tan \beta \] (28 \leq 90\degree);

(see Grunzweig et al (1954) and Haddow (1967) for \(\psi\))

Fig. B.6.3 PROGRESSIVE SURFACE PENETRATION (pseudo-steady)
\[ N_w = 1 + 3\pi/2 + 2(3+\gamma-\omega) + \sin 2\omega + \cos 2\omega / \tan \beta \]

Fig. B.6.4 DEEP STEADY WEDGE PENETRATION
Fig. B.6.5 Effects of wedge-soil interface friction on wedge resistance factor, $N_w$ for steady penetration.
Fig. B.6.6(a) Stress state in soil next to wedge-faces.

Fig. B.6.6(b) Relationships among $\omega$, coefficient of friction $\mu$, and effective strength envelope $\alpha$.

Coefficient of friction, $\mu = \frac{\tau}{\sigma_n}$
Fig. B.6.7 Effects of depth on wedge resistance factor $N_w$. 

$q_w = N_w \cdot c + p_b$

$q_w =$ Wedge Resistance

**Recommended Depth Factors**

1. Skempton (1951)
B.7 Steady Wege Penetration in Anisotropic Materials

In anisotropic materials, the magnitude of the shear stress acting on a plane parallel to a slip-line varies with the inclination of the slip-line. This significantly complicates solutions for wedge penetration when the shear stress acting on the wedge faces, \( \tau \), is specified (i.e., in terms related to the yield contour). The solutions are much simpler, however, when the wedge faces represent slip-line boundaries of the plastic domain. This requires slippage to take place, in the material immediately adjacent to the wedge faces, in directions parallel to the wedge faces, as in the case of the "rough" wedge in isotropic material (\( \tau = c \)).

Analysis in the preceding section indicates that the "rough" wedge solution applies approximately for steady penetration in isotropic clays, for wide ranges of clay and wedge-clay interface properties considered. In this section, we consider steady penetration of a "rough" wedge in a material with an elliptic yield contour (Section B.4; Davis and Christian, 1972) in order to determine the effects of undrained strength anisotropy on penetration resistance.

Except for the geometry of the cavity behind the wedge and of zone I (ACE), the slip-line field illustrated in Fig. B.7.1 is identical to that for the steady penetration of a "rough" wedge in isotropic clay (Fig. 3.1). The geometry of the cavity behind the wedge is governed by Eq. (B.5.14) in addition to the incompressibility requirement:

\[
\gamma = \gamma(\delta, b/a, K_s);
\]

\[
\sin \delta = \sin \gamma / \cos (\gamma - \phi);
\]  

(B.7.1)
\[ \tan 2\phi = - \frac{b^2 \frac{1}{2}(\sigma_y - \sigma_x) - ma}{a^2 \tau_{xy}}. \]

Numerical solutions for \( \gamma \) and the slip-line inclination \( \phi \) are shown in Table B.7.1, together with the inclination of \( \sigma_1 \), at the wedge faces.

The states of stresses in the uniform stress region I and at the wedge face are shown in Fig. B.7.2; the normal and shear stresses on the wedge face are:

\[
\sigma_n = p_b + q_f(\gamma) + \Delta \lambda - q_f(\theta) \cos 2(\theta + \delta); \tag{B.7.2}
\]

\[ \tau = q_f(\theta) \sin 2(\theta + \delta), \]

where

\( p_b = \) isotropic stress in the cavity behind the wedge

\( q_f(\gamma), q_f(\theta) = \) strengths in compression tests when \( \sigma_1 \) acts on \( \gamma \) and \( \theta \)-plane respectively;

\( \theta = \) inclination of \( \sigma_1 \) at the wedge faces;

\( \Delta \lambda = \) increment of arc length along the yield contour from the stress state of region I to the stress state at the wedge face.

The wedge penetration resistance is

\[ q_w = p_b + \sigma_n + \tau / \tan \gamma, \tag{B.7.3} \]

which can be obtained in terms of the vertical strength \( s_u(V) \) and is plotted in Fig. B.7.3. It is clear that the relation between \( q_w \) and \( s_u(V) \) depends on the characteristics of the material anisotropy (\( K_s \) and \( b/a \)), and that undrained strength anisotropy, for a given \( s_u(V) \) tends to reduce penetration resistance (\( K_o < 1, b/a < 1 \)). For a 60° wedge, the penetration factor is 6.9 for a typical normally consolidated nonlayered clay (\( K_s = 0.5, b/a = 0.8 \)), 7.6 for a typical normally consolidated layered clay (\( K_s = 0.9, b/a = 0.65 \)), and 8.5 for a typical highly plastic clay (\( K_s = 0.8, b/a = 1.0 \)), approximately.
<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\gamma$</th>
<th>$\phi$</th>
<th>$\theta$</th>
<th>$\gamma$</th>
<th>$\phi$</th>
<th>$\theta$</th>
<th>$\gamma$</th>
<th>$\phi$</th>
<th>$\theta$</th>
<th>$\gamma$</th>
<th>$\phi$</th>
<th>$\theta$</th>
<th>$\gamma$</th>
<th>$\phi$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°</td>
<td>8.58</td>
<td>-22.18</td>
<td>17.31</td>
<td>7.95</td>
<td>-29.32</td>
<td>22.13</td>
<td>7.35</td>
<td>-35.23</td>
<td>27.15</td>
<td>8.09</td>
<td>-27.76</td>
<td>24.64</td>
<td>7.52</td>
<td>-33.62</td>
<td>30.19</td>
</tr>
<tr>
<td>60°</td>
<td>25.90</td>
<td>-3.22</td>
<td>5.10</td>
<td>25.03</td>
<td>-7.18</td>
<td>7.57</td>
<td>23.20</td>
<td>-14.82</td>
<td>11.32</td>
<td>24.32</td>
<td>-10.20</td>
<td>6.86</td>
<td>22.91</td>
<td>-15.95</td>
<td>10.14</td>
</tr>
<tr>
<td>70°</td>
<td>29.95</td>
<td>0.81</td>
<td>3.27</td>
<td>28.87</td>
<td>-3.81</td>
<td>4.91</td>
<td>27.21</td>
<td>-9.93</td>
<td>7.92</td>
<td>27.10</td>
<td>-8.12</td>
<td>4.38</td>
<td>25.33</td>
<td>-13.02</td>
<td>6.56</td>
</tr>
<tr>
<td>80°</td>
<td>32.96</td>
<td>0.80</td>
<td>1.60</td>
<td>32.45</td>
<td>-0.94</td>
<td>2.41</td>
<td>31.15</td>
<td>-5.27</td>
<td>3.75</td>
<td>30.77</td>
<td>-5.41</td>
<td>2.13</td>
<td>29.48</td>
<td>-10.54</td>
<td>3.22</td>
</tr>
<tr>
<td>90°</td>
<td>36.02</td>
<td>2.32</td>
<td>0.0</td>
<td>35.77</td>
<td>1.53</td>
<td>0.0</td>
<td>34.95</td>
<td>-0.93</td>
<td>0.0</td>
<td>33.53</td>
<td>-5.07</td>
<td>0.0</td>
<td>32.37</td>
<td>-8.40</td>
<td>0.0</td>
</tr>
</tbody>
</table>

* $\gamma$ = inclination of the cavity face to the vertical.
* $\phi$ = inclination of the $\alpha$ slip-line at the cavity face.
* $\theta$ = inclination of the plane on which $\sigma_1$ acts at the wedge face.

Table B.7.1 Geometry of the slip-line field for steady wedge penetration in anisotropic clays.
Fig. B.7.1  DEEP STEADY WEDGE PENETRATION IN ANISOTROPIC CLAY
Fig. B.7.2 Mohr circles of stress for region I and wedge face AB.
Fig. B.7.3 Wedge resistance in terms of $s_u$ (V)
B.8 Approximation Based on the Average Strength

Slip-line solutions for clays with anisotropic strength tend to be very complicated (e.g., Fig. B.7.3) and may require soil parameters that are not readily available. For practical purposes, a more convenient strength measure reflecting the shearing resistance the clay can mobilize in a combination of different failure modes (i.e., applied stress systems) is the "average" strength defined as:

$$s_u(\text{AVE}) = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} q_f(\phi, b/a, K_s) d\phi,$$  \hspace{1cm} (B.8.1)

where $2\phi$ is the inclination of a point on the yield contour, Fig. B.5.1. Conceptually, $s_u(\text{AVE})$ is the average value of the strength when the slip-line field is a circle (or 1/2 or 1/4 of a circle). Fig. B.8.1 shows that, for the typical values of $0.5 < K_s < 1$ and $0.65 < b/a < 1$ (low $b/a$ tends to associate with high $K_s$), $s_u(\text{AVE})$ is approximately given by:

$$s_u(\text{AVE}) = (0.90 \text{ to } 1.00)a;$$

or

$$s_u(\text{AVE}) = (0.45 \text{ to } 0.50)(1 + K_s)s_u(V).$$  \hspace{1cm} (B.8.2)

For slightly OC and NC nonlayered and insensitive clays, $K_s$ (determined by laboratory tests, Table B.4.1) varies between 0.4 and 0.7. Available data for a varved clay indicates that $K_s$ in layered soils is close to unity.

Approximations using $s_u(\text{AVE})$ are illustrated below for steady wedge penetration and incipient failure of a strip rigid footing.

(1) **Steady wedge penetration:** The wedge resistance factor, $N_w$, in terms of $s_u(\text{AVE})$ is shown in Table B.8.1. For the typical anisotropy parameters of $0.5 \leq K_s \leq 1$ and $0.65 \leq b/a \leq 1$, $N_w$ (in terms of $s_u(\text{AVE})$) falls in a small band around $N_w$ given by the isotropic theory, Fig. B.8.2. Thus, we
can write

\[ q_w \approx p_b + N_w s_u(\text{AVE}) \]  \hspace{1cm} (B.8.3)

where \( N_w \) is the wedge penetration factor for an isotropic material ("rough wedge"), with an error of less than 15% (error is smaller than 10% for \( 2\delta \geq 30^\circ \)).

(2) **Incipient failure of a strip footing:** Davis and Christian (1972) present the slip-line solution for incipient failure of a strip rigid footing on soils with an elliptic yield contour. The ultimate bearing capacity is given by:

\[ q_u = 4 \left[ 1 + E(d) \right] \left[ s_u(V) + s_u(H) \right] \]  \hspace{1cm} (B.8.4)

where \( E(d) \) is the complete elliptic integral of the second kind defined as

\[ \int_{\frac{\pi}{2}}^{\pi/2} \frac{1}{\sqrt{1 - d \sin^2 \theta}} \, d\theta \]

(see Milne-Thomson (1964), Belyakov et al. (1965)),

and

\[ d = 1 - (b/a)^2 \]

An approximation of the exact solution using the average strength is given by

\[ q_u^* = 5.14 \, s_u(\text{AVE}) \]  \hspace{1cm} (B.8.5)

where the bearing capacity factor of 5.14 is given by the isotropic theory (Prandtl, 1920; Hill, 1950), and \( s_u(\text{AVE}) \) by Eq.(B.8.1). Table B.8.2 shows the \( q_u/q_u^* \) ratio which indicates the error due to this approximation. For the expected range of the strength anisotropy parameters \( b/a \) and \( K_s \), the error (1- \( q_u/q_u^* \)) due to this approach is less than 10%.
<table>
<thead>
<tr>
<th>δ</th>
<th>b/a = 0.65</th>
<th>0.80</th>
<th>1.0</th>
<th>b/a = 0.65</th>
<th>0.80</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>14.62</td>
<td>16.31</td>
<td>17.65</td>
<td>14.11</td>
<td>15.94</td>
<td>17.44</td>
</tr>
<tr>
<td>10</td>
<td>10.97</td>
<td>11.83</td>
<td>12.38</td>
<td>10.37</td>
<td>11.32</td>
<td>11.977</td>
</tr>
<tr>
<td>15</td>
<td>9.96</td>
<td>10.55</td>
<td>10.82</td>
<td>9.33</td>
<td>9.98</td>
<td>10.335</td>
</tr>
<tr>
<td>20</td>
<td>9.58</td>
<td>10.05</td>
<td>10.18</td>
<td>8.93</td>
<td>9.45</td>
<td>9.646</td>
</tr>
<tr>
<td>35</td>
<td>9.36</td>
<td>9.76</td>
<td>9.80</td>
<td>8.73</td>
<td>9.15</td>
<td>9.196</td>
</tr>
</tbody>
</table>

Table B.8.1 Wedge resistance factor, $N_w$, in terms of the average strength, $s_u$ (AVE)

<table>
<thead>
<tr>
<th>b/a</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_u/q_u^*$</td>
<td>0.92</td>
<td>0.94</td>
<td>0.97</td>
<td>1.00</td>
<td>1.03</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table B.8.2 Errors due to approximation based on the average strength, bearing capacity, of a strip footing.
Fig. B.8.1 The average strength, $s_u (AVE)$, in terms of $a = 0.5[s_u (V) + s_u (H)]$
Fig. B.8.2 Wedge resistance in terms of the "average" strength, $s_u$ (AVE)
APPENDIX C

Records of Field Tests in
Boston Blue Clay
(Saugus, MA)
SAUGUS, MA (1)

60° D/d = 1.0 (2)
Note: Results of pore pressure probe tests no. 21 through 26 are not digitized.
SAUGUS, MA (1)

60° D/d = 1.0 (3)
SAUGUS, MA (1)
18° D/d=1.0 [7]
\( q_c, k \Omega/cm^2 \)

DEPTH, ft

SAUGUS, MA (1)

- 30\(^\circ\) D/d=2.0
- F FAST PUSHING RATE
- S SLOW PUSHING RATE

\( \sigma_{w} \)
SAUGUS, MA (1)

- \(18^\circ \frac{D}{d}=2.0 \) [12]
- F FAST PUSHING RATE
- S SLOW PUSHING RATE
SAUGUS, MA (1)
○ BEGEMANN CONE
SAUGUS, MA (1)

18° D/d=1.0 [5]
$u$, k$\text{g/cm}^2$

DEPTH, ft

SAUGUS, MA (1)

$18^\circ$ D/$d=1.0$ [31]
SAUGUS, MA (1)

18° D/d = 1.9 (32)
F FAST PUSHING RATE
SAUGUS, MA (1)

- $18^\circ \ D/d=1.9$ [34]
- F FAST PUSHING RATE
- S SLOW PUSHING RATE
SAUGUS, MA (1)

-- 60° 1/v/d = 1.9 1351
F FAST PUSHING RATE
S SLOW PUSHING RATE
SAUGUS, MA (1)

F FAST PUSHING RATE
S SLOW PUSHING RATE
APPENDIX D

Records of Field Tests in Atchafalaya Basin Clay

(EABPL, LA)
EABPL, LA

- $60^\circ$ D/d = 1.0 [B]
- F FAST PUSHING RATE
- S SLOW PUSHING RATE
EABPL, LA

- 30° D/d = 1.0 [10]
- F: FAST PUSHING RATE
- S: SLOW PUSHING RATE
\( q_c \cdot k \varphi / cm^2 \)

DEPTH, ft

EABPL, LA

\( 18^\circ \ D/d = 1.0 \) [12]
$q_c$, $kg/cm^2$

DEPTH, ft

EABPL, LA

- $60^\circ$, $D/d=1.0$ [13]
- F FAST PUSHING RATE
- S SLOW PUSHING RATE
$u, \, k\, \rho/\, cm^2$

DEPTH, ft

EABPL, LA

18° D/D=1.0 0119
F FAST PUSHING RATE
S SLOW PUSHING RATE
$u$, $k \rho / cm^2$

DEPTH, ft

EABPt, LA

$18^\circ \theta / d = 1.9$ [24]
APPENDIX E

Records of Field Tests in
Connecticut Valley Varved Clay
(Amherst, MA)
In-situ stresses, \( \sigma / \text{cm}^2 \)

DEPTH, ft

\( u \), \( \sigma_{vo} \)

AMHERST, MA

0 5 10 15 20

0 10 20 30 40 50 60 70 73
AMHERST, MA

$60^\circ \ D/d = 1.0 [1]$
AMHERST, MA
30° D/d = 1.0 (4)
F FAST PUSHING RATE
S SLOW PUSHING RATE
F.1 Strengths for Undrained Bearing Capacity and Stability Analyses

The shear strength of soils is governed by effective stresses in accordance with the Mohr-Coulomb yield criterion or variations thereof. However, the use of effective stresses in the analysis of short-term stability of saturated clays requires the difficult task of measuring or estimating the pore water pressure at failure. For convenience and simplicity, these problems can be analyzed on the basis of total stresses \((\phi = 0)\), Skempton (1948). This is due to the fact that, when no drainage takes place, the strength of clays is independent of the confining stress [see, for example, Lambe and Whitman (1969)].

Total and effective stress analyses imply different inclinations of the failure plane with respect to the principal stress directions. The total stress analysis requires that the angle \(\theta\) between the failure plane and the major principal stress be 45\(^\circ\), whereas the effective stress analysis requires \(\theta = 45 + \phi/2\), where \(\phi\) is the effective friction angle of the clay (with no cohesion intercept, \(c = 0\)).* When related to the principal stresses, the shearing stress acting on the failure plane (at failure) according to the two approaches is therefore different. The total stress approach requires this shear stress to be \(q_f = 1/2 (\sigma_1 - \sigma_3)\), whereas the effective stress approach requires a shear stress \(\tau_{ff} = q_f \cos \phi\). For typical values of \(\phi = 30^\circ\), \(\tau_{ff}\) is about 15\% smaller than \(q_f\).

*Lo (1965) reports laboratory test results on about 650 samples of two clay types, indicating that \(\theta\) is about 34\(^\circ\). These results contradict the predictions of the total stress analysis, but also imply a relatively low effective friction angle \((\phi = 22^\circ)\) from the effective stress analysis.
Ladd (1971) recommends the use of $s_u = q_f$ in undrained bearing capacity analyses, but $s_u = r_{ff}$ in "total stress" circular arc stability analyses. Bjerrum (1973) presents case studies of embankment and footing failures which imply no noticeable difference in the empirical correction factors to be applied to field vane data for these two types of problems. The two recommendations by Ladd and Bjerrum, however, do not necessarily contradict, since conventional two-dimensional stability analyses neglecting end effects generally underestimate the factor of safety by about 10% (Azzouz and Baligh, 1978).

**F.2 Design Strength for the Saugus, Massachusetts, Test Site**

The Saugus, Massachusetts, test site and the adjoining section of an unfinished highway embankment is one of the most extensively instrumented and studied clay sites. In addition, the foundation clay, the Boston Blue Clay, has been the subject of extensive research at M.I.T. for decades. However, this massive volume of information available (see Table 5.2 for example) do not readily or simply lead to the appropriate strength to use in embankment stability and/or bearing capacity analyses. This results from the immensely complicated soil behavior coupled with the highly simplified methods of analysis available at present.

The following paragraphs illustrate four different approaches for predicting the "field" strength, $s_u (field)$:
1. Kinner and Ladd (1970, 1973) studied the load deformation behavior of model footings on resedimented Boston Blue Clay under carefully controlled laboratory conditions at OCR values of 1, 2 and 4. The results are summarized below:

<table>
<thead>
<tr>
<th>OCR</th>
<th>$\frac{s_u(AVE)}{\sigma_{vc}}$</th>
<th>$\frac{s_u(MFT)}{\sigma_{vc}}$</th>
<th>$\frac{s_u(AVE)}{s_u(MFT)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.26</td>
<td>0.96</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>0.47</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>0.76</td>
<td>0.81</td>
<td>0.94</td>
</tr>
</tbody>
</table>

where $s_u(AVE)$ is the "average" strength approach presented in Appendix B (Section B.8) based on results of laboratory $K_o$ consolidated-undrained plane strain shear tests and $s_u(MFT)$ is the $s_u$ back calculated from the model strip footing tests interpreted using the Davis and Christian (1971) anisotropic bearing capacity equation. These data suggest excellent agreement between SHANSEP predicted $s_u$ values and measured bearing capacity results. However, the agreement is considered fortuitous since two aspects of soil behavior now considered to be potentially important were ignored. These are the effects of strain compatibility and strain rate. Thus the above data will be re-interpreted considering these two factors.

Analysis of laboratory $CK_oU$ plane strain data on Boston Blue Clay indicates that the $s_u(AVE)$ should be decreased by about 10% to account for strain compatibility (Ladd, 1975; Azzouz and Baligh, 1978). That is, the $s_u$ that can be mobilized along an actual failure surface, assuming
uniform strain along this surface, is 10% less than the average of the peak strengths due to the strain-softening behavior shown in Figs. 3.5a and b. Thus, $s_u$ (AVE) should be multiplied by 0.9 to account for strain compatibility. The model footing tests were carried to failure in about 15 seconds, in order to minimize consolidation and creep effects during loading, whereas the laboratory $Ck_U$ tests had typical times to failure on the order of 1 to 2 hours. If one assumes a 5 to 10% reduction in $s_u$ per log cycle of time to failure, the measured bearing capacities would then be decreased by:

$$(5 \text{ to } 10\%) \times \frac{\log (1.5 \times 3600)}{(15)} = 13 \text{ to } 26\%$$

Thus, multiplication of $s_u$ (MFT) by 0.8 to 0.9 should account for strain rate effects.

The results presented by Kinner and Ladd (1970, 1973) are now adjusted in accordance with the above estimated effects of strain compatibility and strain rate:

<table>
<thead>
<tr>
<th>OCR</th>
<th>$s_u$ (AVE)$	imes(0.90)$</th>
<th>$s_u$ (MFT)$	imes(0.8-0.9)$</th>
<th>Adjusted $s_u$ (AVE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.225</td>
<td>0.21 - 0.235</td>
<td>0.96 - 1.07</td>
</tr>
<tr>
<td>1</td>
<td>0.395</td>
<td>0.375 - 0.425</td>
<td>0.93 - 1.05</td>
</tr>
<tr>
<td>4</td>
<td>0.685</td>
<td>0.65 - 0.73</td>
<td>0.94 - 1.05</td>
</tr>
</tbody>
</table>

Hence after adjusting the laboratory shear tests for strain compatibility and the model footing tests for strain rate, the resulting values of undrained strength agree within about ± 5%. This suggests that use of $s_u$ (field) = 0.9 $s_u$ (AVE) would be appropriate for bearing
capacity problems at the Saugus, Mass. test site assuming:

(1) In situ Boston Blue Clay behaves similarly to resemented samples of the same soil (CK\textsubscript{o} U triaxial compression and extension test data obtained at M.I.T. on both types of samples show good agreement in \( s_u / \sigma_{vc} \) values).

(2) Negligible strain rate effects between CK\textsubscript{o} U laboratory tests and the in situ condition (note that decreases in the field \( s_u \) due to longer times to failure may be partly compensated by partial consolidation).

(3) That the in-situ history is sufficiently well defined to provide a reliable estimate of the OCR profile required to compute \( s_u \) (AVE) from previously established \( s_u / \sigma_{vc} \) in OCR relationships (e.g. Figs. 5.3 and 5.16a).

2. Extensive field vane test data exist at the test site and since the PI of Boston Blue Clay is about 20%, Bjerrum’s (1972) empirical correlation would suggest that the measured FV strengths are appropriate for evaluating the stability of embankments via circular arc analyses. Bjerrum (1973) later concluded that the same empirical correlation also applies to bearing capacity analyses using plasticity theory. The FV \( s_u \) values have been normalized with respect to \( \sigma_{vo} \) and correlated versus the stress history shown in Fig. 5.3 (Lacasse et al, 1978). These data are compared below to \( s_u (\text{field}) = 0.9 \times s_u \) (AVE) established from Method 1 as appropriate for bearing capacity analyses.
<table>
<thead>
<tr>
<th>OCR</th>
<th>$\frac{s_u}{\bar{\sigma}_{vc}}$ (FV)</th>
<th>$\frac{0.9 \times s_u}{\bar{\sigma}_{vc}}$ (AVE)</th>
<th>$\frac{0.9 \times s_u}{s_u}$ (FV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.165</td>
<td>0.225</td>
<td>1.36</td>
</tr>
<tr>
<td>2</td>
<td>0.32</td>
<td>0.395</td>
<td>1.23</td>
</tr>
<tr>
<td>4</td>
<td>0.625</td>
<td>0.685</td>
<td>1.10</td>
</tr>
</tbody>
</table>

The above comparison shows that Method 1 yields strengths $23 \pm 13\%$ larger than those recommended by Bjerrum. This apparent discrepancy will be discussed shortly.

3. According to Ladd (1971, 1975), $s_u$ (field) for circular arc stability analyses should be based on $\tau_{ff} = q_f \cos \phi$ rather than $q_f$. For Boston Blue Clay, $\cos \phi = 0.83$ and hence $s_u$ (field) for circular arc analysis based on SHANSEP becomes equal to $0.9 \times s_u$ (AVE) $\times 0.83 = 0.745 \times s_u$ (AVE). Comparing this to $s_u$ (FV), as shown below:

<table>
<thead>
<tr>
<th>OCR</th>
<th>$\frac{s_u}{\bar{\sigma}_{vo}}$ (FV)</th>
<th>$\frac{s_u}{\bar{\sigma}_{vc}}$ (SHANSEP)</th>
<th>$\frac{s_u}{s_u}$ (SHANSEP) (FV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.165</td>
<td>0.185</td>
<td>1.12</td>
</tr>
<tr>
<td>2</td>
<td>0.32</td>
<td>0.33</td>
<td>1.03</td>
</tr>
<tr>
<td>4</td>
<td>0.625</td>
<td>0.57</td>
<td>0.91</td>
</tr>
</tbody>
</table>

We note that: Method 3 gives field strengths about $10\%$ larger than $s_u$ (FV) for normally consolidated Boston Blue Clay and that the implied FV correction factor decreases with increasing OCR of the clay. Note also that Ladd would use $s_u = \tau_{ff}$ for circular arc analyses and $s_u = q_f$ for bearing capacity analyses whereas Bjerrum (1973) implies that the
same strength applies to both types of analyses.

4. Azzouz and Baligh (1978) analyzed the planned embankment failure close to the test site (M.I.T. 197) with \( s_u \) (FV) and \( s_u \) (SHANSEP) using both 2 and 3-D methods of analysis. It should be noted that the critical failure surface extended down to about EL. 70 ft and thus primarily involved the upper "stiff" clay which had an average OCR of about 3 to 4. First looking at the result of the 2-D analyses, shown below

<table>
<thead>
<tr>
<th>Method</th>
<th>F.S.</th>
<th>Implied Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 2 ( s_u ) (FV)</td>
<td>0.91 ± 0.02</td>
<td>1.10 ± 0.02</td>
</tr>
<tr>
<td>No. 3 ( s_u ) (SHANSEP)</td>
<td>0.82</td>
<td>1.22</td>
</tr>
</tbody>
</table>

we see that Methods 2 and 3 both underpredicted the in situ strength. However, the failure involved substantial end effects compared to most embankment failures (Azzouz and Baligh, 1978); thus the results of the 3-D analyses should give a better indication of the actual in situ \( s_u \). This analyses showed

<table>
<thead>
<tr>
<th>Method</th>
<th>F.S.</th>
<th>Implied Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 2 ( s_u ) (FV)</td>
<td>1.13 ± 0.04</td>
<td>0.885 ± 0.03</td>
</tr>
<tr>
<td>No. 3 ( s_u ) (SHANSEP)</td>
<td>1.03 ± 0.03</td>
<td>0.97 ± 0.03</td>
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which would lead to the following conclusions regarding \( s_u \) (field) for circular arc stability analysis in "stiff" Boston Blue Clay

\[
\text{s}_u \text{ (field)} \approx 0.9 \text{s}_u \text{ (FV)} \approx 1.0 \text{s}_u \text{ (SHANSEP)}
\]
Note that the ratio of the implied correction factors of
0.885/0.97 = 0.91 is also equal to $s_u(\text{SHANSEP})/s_u(\text{FV}) = 0.91$ at an
OCR = 4. Further, the results of the model footing tests suggest
that laboratory $\text{CK}_o\ U$ test data give a good indication of the
changes in $s_u$ with OCR. Thus one might logically conclude that the in-
situ $s_u$ of Boston Blue Clay appropriate for circular arc stability
analyses should also vary with stress testing as predicted by
$s_u(\text{SHANSEP})/\bar{\sigma}_{vc}$ vs. OCR. This conclusion, if correct, has two
important implications:

(1) That the FV correction factor for Boston Blue Clay
varies significantly with OCR, e.g. from about 1.1 for normally
consolidated clay to about 0.9 for heavily overconsolidated clay.

(2) That the correct $s_u$ to use in $\phi = 0$ bearing capacity
analyses is different from that for circular arc stability analyses,
i.e., $s_u = q_f$ vs. $s_u = \tau_{ff} = q_f \cos \bar{\phi}$. (Note that both values should
be reduced by about 10% for strain compatibility.)
Figure F.1 presents in summary the normalized strength data though to apply to Boston Blue Clay at the Saugus test site based on the above discussion. It assumes that:

\[ s_u (\text{field-bearing capacity analyses}) = 0.9 \ s_u (\text{AVE}) \]

where

\[ s_u (\text{AVE}) = 0.47 \ [q_f (\text{PSC}) + q_f (\text{PSE})] \]

and

\[ s_u (\text{field - circular arc analyses}) = s_u (\text{SHANSEP}) \]

where

\[ s_u (\text{SHANSEP}) = (0.47)(0.9) \ [\tau_f (\text{PSC}) + \tau_f (\text{PSE})] \]

Also shown are \( s_u (FV)/\sigma_{vo} \) and the corresponding correction factor for circular arc stability analyses.
Fig. F.1 Recommended undrained strength parameters for Boston Blue Clay and the corresponding field vane correction factor, $\mu$. 

\[ \frac{s_u}{\bar{\sigma}_{vc}} \]
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BIOGRAPHICAL NOTE

The author was born in Bangkok, Thailand, in 1951. He received his early education in Bangkok before coming to the United States in 1969. He entered the Massachusetts Institute of Technology in 1970 and graduated with a Bachelor of Science degree in Civil Engineering in September 1973 majoring in Engineering Mechanics. He then entered the graduate program in Geotechnical Engineering at M.I.T. and received a Master of Science degree in February 1976. During his graduate studies at M.I.T. between 1973 and 1978 he worked at various times as a research assistant, teaching assistant and laboratory technician under Professors M.M. Baligh, C.C. Ladd, T.W. Lambe, and J.K. Mitchell, and became a member of Chi Epsilon and Sigma Xi.