COMPUTER-AIDED DESIGN OF HIGH SPEED SYNCHRONOUS MACHINES
by
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Madrid, Spain
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Ignacio José Pérez Arriaga

Submitted to the Department of Electrical Engineering and Computer Science on May 10, 1978, in partial fulfillment of the requirements for the Degree of Master of Science.

ABSTRACT

A wound field synchronous machine has been modeled from electrical, magnetic, and thermal viewpoints. The models are detailed and valid for a wide range of operating conditions, including unusually high and low magnetic saturation, mechanical stresses beyond yield strength, a variety of cooling conditions with the option of operation in a high vacuum environment, and no restriction on the rotational speed. Nonlinear programming is used to select the set of independent design parameters which optimize a prescribed figure of merit. The resultant program may be used to perform either specific designs or parametric studies. A particular application to study the effect of rotational speed on the design of minimum size machines is described.

THESIS SUPERVISOR: John Gabriel Kassakian

TITLE: Assistant Professor of Electrical Engineering
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NOMENCLATURE

(Accounting for the most significant parameters)

$A(I,J)$, coefficient matrix for the linearized equations of the thermal model.

$AL0, AL1, AL2, AL3, AL4$, mean lengths in the magnetic circuit (air gap, stator and rotor cores, stator and rotor teeth)

$AR, AS$, wire cross section of stator (rotor) winding. Insulation is not included.

$BO$, peak value of the air gap magnetic flux density (in time and space)

$B1, B2, B3, B4$, average (space) r.m.s. (time) magnetic flux density at stator and rotor cores, stator and rotor teeth.

$BH0(I), BH1(I)$, coefficients of the magnetization curve polynomial expansion.

$BMIN, BSAT$, min. and max. values of the magnetic flux density in the magnetization curve polynomial expansion.

$CA(I)$, per unit constraint allowances.

$CFINS$, effectiveness coefficient for the fins.

$CFR, CFS$, rotor (stator) conductors number per phase.

$CF1$, air gap friction length.

$CIR$(or $IR$), rotor current intensity (d.c.)

$CIS$ (or $IS$), stator current intensity (phase, r.m.s.)

$CJR$ (or $JR$), rotor current density (d.c.)

$CJS$ (or $JS$), stator current density (r.m.s.)

$CL$ (or $L$), length of the stack of laminations.

$CL1$ (or $L1$) = $CL + WRD$
CLAF, armature-field mutual inductance per phase.
CLAL, leakage inductance per phase.
CLB, bearing losses.
CLCR, CLCS, rotor (stator) winding losses.
CLD, total inductance per phase.
CLER, end ring losses.
CLIRC, CLISC, rotor (stator) core iron losses.
CLIRT, CLIST, rotor (stator) teeth iron losses.
CLMR, CLMS, mean length of a conductor in the rotor (stator) overhang region (only one end).
CLMTR, CLMTS, mean length of rotor (stator) turn.
CLT (or LT), machine total length.
CLWO, CLW1, air gap (overhang) windage losses.
CMECH(I), mechanical properties (Poisson and Young moduli of rotor laminations).
CNR, CNS (or NR, NS), total number of turns in the rotor (stator) winding.
CNTR(I), constraint functions.
CNTRL(I), constraint limits.
CSR, CSS, rotor (stator) conductors numbers per slot.
DCR, DCS, diameter of the rotor (stator) conductor, including insulation.
DEL, power angle $\delta$.
DENS(I), mass densities.
DR, DS, rotor (stator) slot depth.
E(I), emissivities.
EC(I), electrical conductivities at 20°C.

EFF, efficiency.

EP(I), machine electrical parameters.

ES, back emf.

F(I), shape factors.

F, electrical frequency.

FBH(I,J), coefficients of polynomial expansion for the magnetization curve.

FI, power factor angle.

FLR, FLS, stacking factor for the rotor (stator) laminations.

FMAX, FMIN, max. and min. considered values for frequency in the analytical simulation of the magnetization curve.

FR, FS, rotor (stator) insulation factor.

FSL, FSLT, centrifugal force due to one slot.

G, air gap length.

G(I), machine geometrical dimensions.

GR, GS, rotor (stator) wedge depth.

H, width of stator core.

H(I), film coefficient.

HCF, harmonic content factor.

HIR, width of a rotor tooth root.

HR, max. width of the rotor tooth.

INDVIO(I), index accounting for the violated constraints.

Int(.), integer part of the number in brackets.

IR, IS, same as CJR, CJS.

JR, JS, same as CJR, CJS.
kCR, kCS, thermal conductivity (th.c) of the rotor (stator) conductors.
kEI, th.c. of the external layer of insulation of a bunch of wires.
kF, th. c. of the frame.
kI, th. c. of the bulk insulation among the wires of a winding.
kR, kS, rotor (stator) slot factor.
kRR, kSR, th. c. of rotor (stator) core in radial direction.
kRZ, kSZ, th.c. of rotor (stator) core in axial direction.
kSH, th. c. of the shaft.
kXR, kX, kYR, kY, equivalent th. c. of rotor (stator) winding in the circumferential (radial) direction.
L, L1, LT, same as CL, C1L, CLT.
NR, NS, same as CNR, CNS.
OR, OS, rotor (stator) overhang saliency.
P, number of poles.
PR(I), performance requirements.
PS, real output power.
R, rotor external radius.
RBF(I), RPF(I), RWF(I), breadth, pitch and winding factors of the rotor.
REC, rotor conductors electrical conductivity.
RF, frame external radius.
RSH, shaft radius.
S0, S1, S2, S3, S4, magnetic circuit cross sections.
SBF(I), SPF(I), SWF(I), breadth, pitch and winding factors for the stator.
SEC, stator conductors electrical conductivity.
SIGMAX(I), max. allowable mechanical stresses.
SR, SS, total number of rotor (stator) slots.
SRWF(I), overall stator-rotor winding factors.
TA, air gap length between frame and stator core.
TAMB, ambient temperature.
TAVR, TAVS, average temperature of the rotor (stator) winding.
TB, end shield bracket thickness.
TC(I), vector of thermal conductivities.
TF, frame thickness.
TI, temperature of element I.
TXR, TXS, TYR, TYS, insulation thicknesses (Fig. 4.b).
U10, U18, U19, overall heat transfer surface coefficients for the
    air gap, axial duct and radial duct, respectively.
UR, excitation voltage.
US, output r.m.s. voltage (λ-n).
VCUR, VCUS, rotor (stator) conductors volume.
VDUCT, coolant velocity in axial duct.
W, rotational speed.
W1, W2, W0, flow rate of coolant at the air gap, axial duct and total,
    respectively.
WF(I), weighting factors for the constraints.
WRAV, average value of WR and WRI.
WR, WS, rotor (stator) slot width.
WRC, WSC, WRT, WST, weight of the rotor (stator) core and teeth.
WRD, width of the radial duct.
WRI, rotor slot root width.
X(I), unknowns in the thermal model equations.

XD, synchronous reactance (p.u.).

Note: Units in M.k.S. system.
CHAPTER I

INTRODUCTION

One problem confronting anyone wishing to perform energy conversion at higher frequencies is machine definition. How much smaller will the machine be as compared to its 60 Hz counterpart? Will it be sufficiently efficient? How much power can one get out of a machine at a given shaft speed? Which are the most desirable characteristics for the construction materials involved? Parametric studies of the effect of shaft speed on the characteristics of different types of electrical machines have been done by using exclusively analytical means, and therefore by considering only gross effects and introducing strong assumptions, such as to presume geometric similarity for the expected optimum designs at different speeds.

The approach to this problem that would seem to make a significant contribution is to model the machine completely enough so that order of magnitude scalings can be performed, that is, to obtain cross dependences among the main parameters of the machine. Trying to scale the machine with simple approaches does not yield answers that warrant high confidence. The problem then is to make sure that the significant parameters are included and are realistically constrained to provide a useful mechanism for machine prediction.

Operation at high rotational speeds also implies other unconventional operating conditions, such as unusual levels of magnetic saturation, extremely high mechanical stresses, or windage losses becoming so large that a high vacuum environment may be required. The mathematical model for the machine must be valid and reasonably accurate within this wide range of
situations.

In general there is either an infinite number of machine designs or none, which can satisfy a prescribed set of specifications or performance requirements; in other words, that are feasible. In the first case the goal is to precisely select that feasible design which is closest to some combination of desirable features, like minimum material cost, minimum weight or maximum efficiency. In the second case the objective may be to obtain that design which has reasonable properties and is as close as possible to feasibility. In order to be able to perform this choice, the mathematical model must be provided with an objective function which accounts for the preferred characteristics of the machine, feasibility being the most important of all of them.

It now becomes apparent that the required design approach differs widely from conventional design methods. This issue has been discussed by Chalmers\(^2\). The key point in the conventional design approach (Fig. 1.a) is that a good initial guess of the main design parameters may be obtained by analogy with similar existing machines. A good mathematical model is not necessary as far as there is enough information on similar existing machines. Once the first basic design is obtained, experience or heuristic rules are used to introduce minor changes which may improve the design or tailor it to specific requirements. Therefore an explicit formulation of the objective function is not generally required. In this case the computer may serve as an aid to the designer, by enabling more complex and more accurate methods to be used and by disposing reliably and quickly of routine and tedious calculations.
Fig. 1a. Conventional Design Approach:

1. Design Requirements
2. Analogy with similar existing machines: pick up main design parameters
3. Design analysis process
4. Minor changes (heuristically)
5. Does it look good? (hopefully)
6. Yes
   - Design
   - Optimum design (in a specified sense; it may not be feasible)
   - Constitute relations
   - Improve (systematic procedure)
7. No
   - Minor changes (heuristically)
   - Design
   - Constitutive relations
   - Improve (systematic procedure)
8. No
   - Constitute relations
   - Improve (systematic procedure)

Fig. 1b. Non-conventional Design Approach:

1. Design requirements
2. Guess a set of independent design parameters
3. Design
4. Optimum design (in a specified sense; it may not be feasible)
5. Yes
   - Design
   - Optimum design (in a specified sense; it may not be feasible)
6. No
   - Minor changes (heuristically)
   - Design
   - Optimum design (in a specified sense; it may not be feasible)
7. No
   - Design
   - Optimum design (in a specified sense; it may not be feasible)
On the other hand, the design of a highly unusual machine cannot rely on information about similar machines to obtain a reasonable initial guess or to check the results (Fig.1.b). It must rely on a good mathematical model and an adequate choice of an objective function to be optimized. It also requires a strategy to improve the design systematically and to guarantee that the optimum design (in a specified sense) is found. This last part is too tedious to be performed manually and is unthinkable without a computer. During the search, very unusual or even impossible configurations may be encountered, and the mathematical model must be able to accommodate them.

Exploring the capabilities of wound rotor 3-phase synchronous machines operating at high rotational speeds is the specific design problem addressed in this work. Such machines find application in the aircraft industry and have been proposed, and in fact are necessary, for high speed flywheel energy storage systems. Although the synchronous machine has been the vehicle for the work described here, the results of this work are extendable to a much broader class of rotating machinery. Relatively minor modifications to the model details can, for example, produce a program for designing squirrel cage induction machines.

The optimization of the design is approached as a problem in nonlinear programming. The mathematical model is reduced to a set of mostly nonlinear equations in the design parameters, some of which have upper and/or lower bounds. This constrained optimization problem is transformed into an unconstrained one by using a variant, proposed by Menzies\textsuperscript{3}, of the least $p^{th}$ optimization method of Bandler and Charalambous\textsuperscript{4}. This allows
the search to include non-feasible designs. The pattern search method of Hooke and Jeeves\textsuperscript{5} is the adopted optimization technique. The overall mathematical approach is further explained in Chapter II.

The preparation of the mathematical model for the machine represents most of the reported work. The model includes a fair amount of detail, so the outcome of the optimization process provides enough information to allow one to implement the machine (i.e., number and approximate cross section of conductors, resistance of the windings at operating temperatures, leakage reactance, magnetic saturation and effect of the frequency on it, prediction of hot spot temperatures, etc.). The model is described in detail in Chapter III.

A FORTRAN program has been written embodying the machine models and the optimization strategy. Its main inputs are the machine performance requirements and the properties of the selected materials. Description of the program and print-out are given in Chapter IV.

The program is very flexible and may be used to perform specific designs or parametric studies. A particularly significant application has been selected: a parametric study of the effect of the rotational speed in the design and performance of machines intended for minimum size. Results are presented in Chapter V.

The main limitations of the proposed model and suggestions for further improvements are summarized in Chapter VI.
2.1 Machine Definition

The construction and performance of a machine is completely described by the numerical values of a set of quantities, which will be called design parameters (i.e., air gap length, stator winding maximum temperature, rotor slot number, ...). Any specialization of the design parameters will be termed "design requirements", and can be of the following types:

a) Performance requirements: These are the specific requirements which are imposed on the machine (i.e., ambient temperature) or which the machine is intended to meet (i.e., output voltage or power).

b) Constitutive relations: The geometry of the machine and the physical laws governing its behavior impose certain relations among the design parameters. They can be further classified into geometrical, electrical, magnetic, thermal and mechanical relations.

c) Design constraints: Certain combinations of the design parameters yield absurd or impossible designs. Moreover, many possible designs, although in agreement with the constitutive relations, lead to undesirable performance conditions. To prevent a design from being unacceptable certain conditions have to be established. They usually consist of a set of limitations that cannot be exceeded.

The designs satisfying all the requirements will be called "feasible". The hypothetical case of having a unique feasible design for a given set of requirements is extremely unlikely. Some feasible designs, however, should be preferred to others because of a higher efficiency, lower mater-
ial cost, smaller size or weight, etc. Non-feasible designs are compared and evaluated on the basis of a minimum of constraint violations.

A design process will be entirely satisfactory only when it leads to that design which offers more advantages than any other. A function accounting for the preferred characteristics of the machine, thus allowing design comparisons, will be called the objective function. The goal of the design process is to determine the numerical values of all the design parameters for the machine with the optimum objective function. In this work the convention will be used that "optimum" is equivalent to "minimum".

The mathematical model is the formal expression of all the design requirements and the objective function. In summary it consists of $\bar{p}$ design parameters

$$\bar{p} = (p_1, p_2, ..., p_\lambda) ,$$

$k$ equations in the design parameters, corresponding to the performance requirements and the constitutive relations,

$$r_i(\bar{p}) = 0 ; i = 1,2,\ldots,k , \quad (1)$$

$m$ inequalities,

$$c_i(\bar{p}) \leq 0 ; i = 1,2,\ldots,m , \quad (2)$$

and the objective function, $M(\bar{p})$. In general the functions $r_i$, $c_i$ and $M$ are nonlinear algebraic expressions in explicit terms of the design parameters.
The number, \( \lambda \), of design parameters required to specify the construction and performance of the machine within the level of detail in the model proposed in this work is well above 70. However, because of the \( k \) equations, the number of degrees of freedom, \( n=\lambda-k \), is much less. When a usual set of performance requirements is given, the number, \( n \), of independent variables is 11. This number is increased by letting free any of the performance requirements, or decreased by assigning numerical values to certain design parameters from the outset. The numerical values of the \( n \) independent variables completely determine the remaining \( \lambda-n \) design parameters, and therefore the machine design. Not any set of \( n \) design parameters can be used as independent variables, but there is more than one possible set, depending upon the nature of the system of equations.

The problem is restated now in terms of the \( n \) independent variables,

\[
x = (x_1, x_2, \ldots, x_n),
\]

with the \( m \) inequalities becoming

\[
g_i(x) \leq 0, \ i = 1, 2, \ldots, m \tag{3}
\]

and the objective function, \( F(x) \).

The system of equations(1) allows one to solve for the design parameters in terms of \( x \),

\[
p_i = h_i(x) ; \ i = 1, 2, \ldots, \lambda, \tag{4}
\]

The main features of the resultant optimization problem may be summarized as follows:
The functions \( h_i \), \( g_i \), and \( F \) in general are nonlinear algebraic expressions, and not in explicit terms of \( \bar{x} \).

- It is not possible to write down the derivatives of \( F \) with respect to the independent variables in closed form.

- Some of the design parameters are discrete variables (slots and turns numbers).

- It may be nontrivial (and even impossible) to find an initial guess for the independent variables \( \bar{x} \) which yields a feasible design. The higher the rotational speed the more unconventional the design and the lower the probability of a feasible initial guess.

### 2.2 Selection of the Independent Variables

Although there are no rigid constraints on the choice of independent variables, some ground rules may be derived by inspection of the main features of the particular mathematical model and the selected optimization technique, with the goal of assuring good performance of the algorithms to be used and saving computation time. It can be said in general that the independent variables and the program logic must be such that only one design may result from a given set of numerical values for \( \bar{x} \). It is also convenient to choose independent variables with as much significance as possible, so that the design and therefore the objective function are very sensitive to changes in the vector \( \bar{x} \). The more physically meaningful the variables \( \bar{x} \) are, the easier it will be to determine the main design trade-offs from the results of the optimization process. The independent variables should also be selected in such a way that the cal-
clusion of the remaining design parameters and the objective function is not made more complicated than necessary.

Special attention has been paid to this last issue, since the objective function cannot be written in explicit terms of \( \bar{x} \). In order to obtain the remaining \( \lambda \)-n design parameters, \( p \), it is necessary to solve the system of nonlinear equations (1). This in general would require a sophisticated and computationally expensive algorithm, due to the large number of design parameters involved. However, in the present study, a set of physically meaningful independent variables has been found, which allows the calculation of most of the remaining \( \lambda \)-n design parameters without having to resort to such algorithms. This represents an important saving of computation time.

Taking the performance requirements and the vector \( \bar{x} \) as the starting point, a large subsystem of the whole system of equations (1) can be solved immediately for almost all the design parameters in a sequential fashion, much like a conventional design routine. All the design parameters, except for the temperature rises, can be obtained in this way. A system of 13 nonlinear algebraic equations must be solved by the Newton-Raphson method to calculate the temperature rises. The part of the optimization process which concerns the calculation of the design parameters is called the analysis step.

The discreteness of some design parameters is another important issue which is also closely related to the selection of the optimization technique. It is not usually useful to assume that the functions are non-discrete, obtain a solution, and then round the solution to the nearest
discrete values of the variables. Such a solution may have shifted from
the feasible to the nonfeasible region, or the final adjustments might
have to be drastic enough to make the whole optimization an illusion.
Most optimization algorithms require the vector $\bar{x}$ to be continuously
variable, which rules out the choice of slot and turn numbers as inde-
dependent variables, thus translating the problem to the analysis step.
This also creates some difficulties in the logic of the optimization strat-
egy, since every set of numerical values for the vector $\bar{x}$ has to be sub-
ject to certain small adjustments in order to be consistent with the
values of the discrete variables. These adjustments may hamper the con-
vergence properties of the optimization algorithm.

The alternative option, which is used here, consists of incorporating
all the discrete parameters into the independent variables. This simpli-
ifies the analysis step, but restricts the number of applicable optimization
techniques and again may create convergence problems. However, those dif-
ficulties have been solved by using the pattern search algorithm, which
can be easily modified to handle discrete variables, and by checking the
final design for local minima by starting the search at different points.
The design parameters finally selected as the independent variables are:

- Rotor radius, R
- Axial core length, L
- Air gap length, G
- Width of stator core, H
- Number of stator slots per phase and pole, SS/3xP
- Number of rotor slots per pole, SR/P
- Stator slot factor (ratio of slot width to the total of slot plus tooth width), KS
- Rotor slot factor, KR
- Number of turns per stator slot, 2xNS/SS
- Number of turns per rotor slot, 2xNR/SR
- Stator current density, JS
- Stator current intensity, CIS

The last variable is not used when the power factor, voltage, and power outputs are included among the performance requirements.

2.3 Objective Function

The optimization process consists of the search for that design (uniquely defined from the independent variables) which is preferred to all the others, for a given set of performance requirements (always met) and design constraints (met when the design is feasible). The objective function, defined by the designer, is a mathematical expression which measures the "goodness" of a given design. As pointed out before, it must be able to also compare nonfeasible designs for two reasons: firstly because the initial guess for the independent variables may well lead to a nonfeasible design and, secondly, because when there is no feasible design for the given requirements it is of great interest to obtain the design which is closest to feasibility, in order to understand the main trade-offs and capabilities of the machine. This last point is especially important in the design of nonconventional machinery. Since the optimization algorithm proceeds blindly with the only goal being to look for designs with lower values of the objective function, the following features must
be demanded of this function:

a) It must have very high values for absurd designs (i.e., when the values of the independent variables lead to lengths which are negative) and account for all these possibilities, so they are always immediately rejected.

b) Nonfeasible designs must yield higher values for the objective function than feasible ones. Constraint violations must be weighted according to their magnitude and also to the relative importance of the constraint. The objective function should emphasize the effect of the most important constraint violation.

c) It must include a figure of merit function, \( B(\bar{x}) \), which quantifies the preferred characteristics of a design, such as cost, weight, efficiency, etc., without regard for feasibility.

d) Each constraint should be accounted for separately, with its individual constraint limit and weighting factor, so it can be changed for different machine designs or even during an optimization process.

The above requirements mean that the objective function must take care of reducing the initially constrained optimization problem into an unconstrained one. Several methods have been proposed to achieve this purpose\(^7\,^8\), but the one presented by Menzies\(^3\) meets almost completely the aforementioned conditions. This approach is predicated upon the incorporation of the figure of merit function, \( B(\bar{x}) \), into the objective function as an additional constraint, \( g_{m+1}(\bar{x}) \), defined as

\[
g_{m+1}(\bar{x}) = B(\bar{x}) - B_0
\]  

(5)
where $B_0$ is some user defined constant. The following objective function is defined for a system of $m$ constraints:

$$F(\overline{x}) = H(\overline{x}) \left[ \sum_{K} \left( \frac{w_i \cdot g_i(\overline{x})}{H(\overline{x})} \right)^{q} \right]^{\frac{1}{q}}$$  \hspace{3cm} (6)$$

where $w_i \cdot g_i(\overline{x})$ is the weighted $i^{th}$ constraint. $K$ is the index subset of all violated constraints, except when all constraints are met in which case $K$ is the entire index set of $m + 1$ constraints. $H(\overline{x})$ is defined as

$$H(\overline{x}) = \max \left\{ w_i \cdot g_i(\overline{x}) \right\}; \quad i = 1, 2, \ldots, m + 1$$  \hspace{3cm} (7)$$

and $q$ is defined as

- $q = p$ if any constraint is violated,
- $q = -p$ if all constraints are satisfied,

where $p$ is any real number $\geq 1$. The value $p = 3$, suggested in reference 3, has been found to work satisfactorily.

The objective function $F(\overline{x})$ defined in (6) is continuous for all $\overline{x}$ if the constraint functions $g_i(\overline{x})$ and the weighting factors $w_i$ are also continuous. Discontinuities of $w_i$ where the constraint function $g_i(\overline{x})$ becomes zero do not produce discontinuity in $F(\overline{x})$.

If there is any violated constraint the above defined objective function emphasizes the larger weighted constraint violations, so the optimization algorithm will try to reduce these violations first. If all the $m + 1$ constraints are satisfied the objective function considers all of them, and the optimization algorithm will attempt to move the design
away from all the constraint boundaries and into the center of the feasible region.

The selection of the weighting factors \( w_i \) is a very delicate issue. First of all they must weight reasonably the importance of violating each constraint, and here again the designer's criterion influences the final outcome of the design. References 3 and 4 use the same values for each \( w_i \) in the feasible and nonfeasible regions. In the present work, when a constraint is satisfied the inverse of \( w_i \) is used as the new weighting factor, since it is considered that in this way the need for the design to move away from the most important constraint boundaries is emphasized.

For the particular application described in Chapter V, the constant \( B_0 \) has been chosen such that the figure of merit constraint is always violated. Consequently, as seen from (5) and (6), \( F(x) = B(x) \) in the feasible region. Constraints other than \( g_{m+1} \) have been assigned very high weighting factors as compared with \( g_{m+1} \), so they almost completely determine the value of \( F(x) \) outside the feasible region.

Figure 2 shows the relationship between \( F(x) \) and \( B(x) \) in both the feasible and nonfeasible region for this particular case, but assuming for simplicity only two degrees of freedom. In general the feasible region may or may not contain the unconstrained minimum. It is assumed that constraints 1, 2 and 3 have much higher weighting factors than the figure of merit constraint, as pointed out before.

2.4 Optimization Strategy

Many techniques are presently available to solve unconstrained optimization problems\(^7,8\). Most of them have to be eliminated from the outset
Figure 2.a Constraint boundaries and lines of constant figure of merit.

Figure 2.b Lines of constant objective function value.
\( \bar{x} = (x_1, x_2) \); \( m=3 \); \( B(\bar{x}) < B \) everywhere.
due to the presence of discrete independent variables which make things
difficult for methods relying on the use of derivatives or sophisticated
algorithms. Simple direct search methods seem to be the best suited to
this type of application, and among them the pattern search method of
Hooke and Jeeves\(^5\) has proved to be very efficient\(^3,9\).

The pattern search algorithm has been used as described in the ori-
ginal reference\(^5\). Minor modifications had to be included to adapt it to
the case of searching over continuous and discrete variables at the same
time. These changes concern principally the step reduction logic. The
independent variables are not allowed to become negative and the slot fac-
tors KS and KR cannot exceed unity. The overall flowchart is represented
in Figure 3. The independent variables \(\bar{x}\) have not been normalized; suit-
able initial values for the step sizes have been used instead.
Figure 3. Overall Flowchart
CHAPTER III
THE MATHEMATICAL MODEL

3.1 Introduction

This chapter describes the different mathematical models leading to equations (1) and (2). The most significant assumptions are discussed when the corresponding model is presented, thus indicating the range of applicability of the results of this work. Only one type of machine has been considered, a small wound field three-phase synchronous machine. The machine may be cooled in different ways, and even the totally enclosed type operating in a high vacuum has been considered. The design of the bearings and slip rings for machines with high rotational speed are important issues by themselves but have been ignored in the present work. A basic machine topology has been considered from the outset, and it remains unchanged throughout this work (Fig. 4).

The equations have been classified into constitutive relations and design constraints (equality and inequality constraints are the standard terms in nonlinear programming), since they are used in a different way in the optimization, as indicated before. Somewhat arbitrarily they are also divided into five groups, which correspond to five models or points of view in considering the machine: geometry, electrical equivalent circuit, magnetic equivalent circuit, mechanical stresses and cooling.

A flowchart in Appendix D describes how the equations are used in the analysis step. The figure of merit function is not included since it depends on the particular application.

Constraints other than the ones specified in the coming sections may
be required for particular applications of the model (i.e., upper limits for the cost, weight or size of the machine). There is no difficulty in including in the model any constraint function which can be expressed in terms of the design parameters obtained in the analysis step.
3.2 Machine Geometry

Most of the equations concerning the machine geometry come in a straightforward way from the definition of the geometrical parameters. (See Nomenclature and Fig. 4.) The constitutive relations involve the different radii, lengths, depths and widths of the parts, the numbers of slots, turns and conductors, the conductor cross sections and insulation thicknesses, the mean lengths of the turns and the volumes of the machine elements.

Constitutive relations.

- Rotor slot width:
  \[ WR = 2\pi RKR/\mathrm{SR} \]  
  \[ \text{(8)} \]

- Stator slot width:
  \[ WS = 2\pi (R+G)KS/SS \]  
  \[ \text{(9)} \]

- Stator conductor cross section (no insulation):
  Round conductors are assumed. A diameter of \(3 \times 10^{-3} \) m. is about the maximum to be considered for a base conductor (no insulation) in the winding of a small machine\(^{10,11}\). Therefore, the number of conductors per turn (in parallel) is:
  \[ CFS = \mathrm{Int}[0.5+CIS/(CJS \times 7.07 \times 10^{-6})] \]  
  \[ \text{(10)} \]
  \[ AS = CIS/(CFS \times CJS) \]  
  \[ \text{(11)} \]

- Rotor conductor cross section (no insulation):
  \[ CFR = \mathrm{Int}[0.5+CIR/(CJR \times 7.07 \times 10^{-6})] \]  
  \[ \text{(12)} \]
  \[ AR = \mathrm{CIR} / (CFR \times CJR) \]  
  \[ \text{(13)} \]

- Number of conductors per slot of the stator:
CSS = 2.CNS.CFS/SS \hspace{1cm} (14)

- Number of conductors per slot of the rotor:

\[ CSR = 2.CNR.CFR/FR \hspace{1cm} (15) \]

- Diameter of the stator insulated conductor:

The diameter of the bare conductor is:

\[ DBCS = \left( \frac{1}{4.AS/\pi} \right)^{1/2} \hspace{1cm} (16) \]

The following analytical expression approximately reproduces empirical data regarding the relation between the bare and insulated conductor diameters (ref. 11, vol. 1, p. 33):

\[ DCS = 1.092 \times DBCS \hspace{0.5cm}, \text{if } DBCS < 0.65 \times 10^{-3} \text{ m.} \hspace{1cm} (17a) \]

\[ DCS = DBCS + 9 \times 10^{-5} \times (1 - 2 \times e^{-2757 \times DBCS}), \hspace{1cm} \text{if } 0.003 \geq DBCS \geq 0.000065 \text{ m.} \hspace{1cm} (17b) \]

- Diameter of the rotor insulated conductor:

In the same way eqs. (18), (19) and (20) can be written with DCR and DBCR instead of DCS and DBCS.

- Stator slot depth:

The depth corresponding to the wedge can be estimated as (ref. 10, p. 401; ref. 11, II, p. 27):

\[ GS \approx 0.1 \times DS \hspace{1cm} (21) \]

The effect of roughness and slack is to reduce each linear dimension in the slot cross section by a factor of about
0.9¹⁰,¹¹; the slot lining width is assumed to be 0.35 mm.; thus:

\[
DS=1.1x\frac{[CSS.DCS^2/ABS(0.9xWS-2x0.35x10^{-3})+3x0.35x10^{-3}]x \frac{1}{0.9}}{ } ;
\]

\[
DS=1.22x\frac{[CSS.DCS^2/ABS(0.9xWS-0.7x10^{-3})+1.05x10^{-3}]}{ } \tag{22}
\]

For purposes of the thermal model described in section 3.6, the round conductors are approximated by fictitious ones with square cross sections and the following defining dimensions:

\[
WXS = WYS = AS^\frac{1}{2} \tag{23}
\]

\[
UXS = UYS = \frac{DCS}{0.9} - WXS \tag{24}
\]

Fig. 5: Detail of fictitious winding geometry

- Rotor slot depth:

The rotor wedge depth GR will be determined in section 3.5. The final value will be the larger of either the section 3.5
value or the approximate expression 0.1xDR.

In the same way as for the stator slot:

\[ DR = 1.11x[CSR.DCR^2/\text{ABS}(0.9xWR-0.7x10^{-3})+1.05x10^{-3}+GR] \]  \hspace{1cm} (25)

\[ WXR = WYR = AR^2 \]  \hspace{1cm} (26)

\[ UXR = UYR = \frac{DCR}{0.9} - WXR \]  \hspace{1cm} (27)

\[ WR_{ave} = 0.5 \times (WR + WIR) \]

- Length of the mean stator turn:

Assuming circular end turns:

\[ CLMTS = 2x[CL1+\pi x(R+G+.5xDS)x\text{SIN}(\frac{\pi}{P})] \]  \hspace{1cm} (28)

This simple expression agrees reasonably with other more complex empirical equations which have been proposed\textsuperscript{13}.

- Length of the mean rotor turn:

\[ CLMTR = 2x[CL1+\pi x\text{ABS}(R-0.5xDR)x\text{SIN}(\frac{\pi}{P})] \]  \hspace{1cm} (29)

- Volume of the stator conductor:

\[ VCUS = CNS.AS.CFS.CLMTS \]  \hspace{1cm} (30)

- Volume of the rotor conductor:

\[ VCUR = CNR.AR.CFR.CLMTR \]  \hspace{1cm} (31)

**Design constraints**

The geometrical constraints must be carefully selected. Violation of these constraints makes the design impossible, so they must be assigned
very high weighting factors. Any absurd combination of numerical values is detected through the geometrical constraints. They limit the depth of the stator and rotor slots and take care of allowing enough room for conductors and insulation within the slots. These constraints are:

\[
\begin{align*}
DR + RSH - R &< 0 \quad (32) \\
DS - H &< 0 \quad (33) \\
2 \times TXR - WR &< 0 \quad (34) \\
2 \times TXS - WS &< 0 \quad (35) \\
- HIR &< 0 \quad (36)
\end{align*}
\]

In addition, the depth of the rotor slot and the width of the root of the rotor teeth, both relative to the rotor radius, must have an upper and lower bound respectively, because of practicability reasons.

When the geometrical constraints are satisfied, it does not matter if the design stays very close to their boundaries. Therefore weighting factors of zero have been assigned to satisfied geometrical constraints.

3.3 Electrical Equivalent Circuit

The performance of an electrical machine is usually considered in terms of its effect on the system to which it is connected. This does not directly concern the details of the fields within the various elements of the machine, but rather their integrated effects as observed from the terminals of the armature and field windings and the torque produced on the rotor. Thus the machine's characteristics are described as viewed from these terminals in terms of a lumped parameter electrical network and a mechanical torque of electrical origin.
Only steady-state operation of the machine has been considered. Constitutive relations and constraints accounting for transient reactances could be included as desired. The adopted model is entirely conventional and has been presented in many references\textsuperscript{10,11,15,16}.

The elements of the lumped parameter electrical equivalent circuit are obtained from the machine geometry and the performance requirements. The magnetic flux density at the air gap is the key parameter in these calculations, which are closely interrelated with the magnetic circuit model through the effective air gap length. Details of the magnetic equivalent circuit are presented in section 3.4.

The design constraints include a lower bound for the efficiency, an upper bound for the harmonic content and an upper bound for the power angle (which is equivalent to setting an upper bound on the synchronous reactance) in order to maintain stable steady-state operation.

**Constitutive relations**

- Electrical real power output:

\[ PS = 3 \times \text{US.CIS.COS(FI)} \] (37)

Relations associated with the machine phasor diagram (Fig. 6):

The phasor diagram represents peak values for one phase of the stator (\(e_n\) voltages). Resistive voltage drops have been neglected. The equations are written for the fundamental component.
Figure 6: Phasor diagram for the machine electric equivalent circuit.

- Voltage behind synchronous reactance CLD:
\[ \frac{E_{fd}}{\sqrt{Z}} = \left[ \left(\text{CIS.CLD.W}.\frac{P}{2}.\cos(FI)\right)^2 + \left(\text{US} + \text{CIS.CLD.W}.\frac{P}{2}.\sin(FI)\right)^2 \right]^{\frac{1}{2}} \] (38)

- Peak value of the air gap magnetic field:
\[ B_0 = \frac{3.\sqrt{Z}.P}{4.\text{R.CL.CNS.SBF(1)}.\text{SPF(1)}} \left[ \left(\text{CLAL.CIS.COS(FI)}\right)^2 + \left(\frac{2.\text{US}}{P.W} + \text{CLAL.CIS.SIN(FI)}\right)^2 \right]^{\frac{1}{2}} \] (39)

- Excitation magnetomotive force:
\[ \text{CIR} = 2.\sqrt{Z}.ES/(\text{CLAF.W}.P) \]

which with eq. (44) gives:
\[ \text{CNR.CIR} = 3.\sqrt{Z}.\pi.P.\text{GEFF.ES}/(8.\text{CL.R}.\mu_0.\text{CNS.SRWF(1)}.W) \] (40)
The value of the effective air gap length GEFF is determined in section 3.4.

- Rotor current intensity:
  \[ CIR = \frac{(\text{CNR.CIR})}{\text{CNR}} \]  \hspace{1cm} (41)

- Rotor current density:
  \[ CJR = \frac{\text{REC.UR}}{(\text{CNR.CLMTR})} \]  \hspace{1cm} (42)

- Synchronous inductance\(^{15}\):
  \[ CLD = \text{CLAL} + \left(8.\mu_0.\text{CL.R.CNS}^2.\text{SPF}(1)^2.\text{SBF}(1)^2\right)/(3.\pi.\text{GEFF}.P^2) \]  \hspace{1cm} (43)

- Armature-field mutual inductance\(^{15}\):
  \[ CLAF = 16.\text{CL.R.}\mu_0.\text{CNR.CNS.SRWF}(1)/(3.\pi.P^2.G) \]  \hspace{1cm} (44)

- Armature leakage inductance (per phase)\(^{10,17}\):

For windings on a synchronous machine, it is customary to divide fluxes due to currents in these windings into three components: air gap flux, slot leakage flux, and end turn leakage flux. Other types of leakage fluxes, such as zig-zag and differential leakage, have been mentioned\(^ {10,13}\) but they will not be considered here. The second term in eq. (43) accounts for the air gap flux; the armature leakage inductance CLAL takes care of the leakage fluxes. The slot leakage flux will be calculated first\(^ {10}\):

The slot leakage flux crosses the conductors from one tooth to the next. It is in phase with the current for all conductors in a given
phase-group, and depends only on the magnitude of the current, neglecting saturation. It is assumed that the current in the slot-conductors is distributed uniformly over their cross section, implying that skin effect is neglected.

A simplified slot geometry is shown in Fig. 7. More detailed models are available. The leakage flux produced by the current may be considered to have a path straight across the slot and through the iron at the bottom of the slot, and the iron reluctance is assumed to be negligible. Calculations are made for a current intensity of 1 amp. per turn. Ampere's law gives the magnetic field at the slot:

\[ B(x) = \frac{1}{W} \cdot \mu_0 \cdot T \cdot \frac{x}{h}, \text{ which links } \frac{x}{h} \cdot T \text{ turns.} \]

![Diagram of slot leakage idealized geometry]

Fig. 7: Slot leakage idealized geometry

The total leakage flux associated to the slot per unit current is the slot-leakage inductance per slot:
\[ x = h \int_{x=0}^{x=h} T(x)B(X)\,dX = \int_{x=0}^{x=h} T \cdot \frac{1}{h} \cdot \mu_0 \cdot T \cdot \frac{x}{h} \cdot p \cdot dX = \frac{\mu_0 \cdot p \cdot h \cdot T^2}{3 \cdot W} \]

where

\[ T = \frac{2 \cdot CNS}{SS} \quad ; \quad W = WS \quad ; \quad p = CL \quad ; \quad h = DS \]

The same calculation can be repeated for the set of slots corresponding to one single pole for the considered phase. This amount to multiplying \( T \) and \( W \) by the number of involved slots: \( \frac{SS}{(3 \cdot P)} \). This result must be multiplied by the number of poles \( p \) to have the desired slot leakage inductance for one phase:

\[ CLAL_s = P \cdot \frac{\mu_0 \cdot CL \cdot DS}{3 \cdot WS} \cdot \left( \frac{2 \cdot CNS \cdot SS}{SS \cdot 3 \cdot P} \right)^2 = \frac{4 \cdot \mu_0 \cdot CL \cdot DS \cdot CNS^2}{9 \cdot WS \cdot SS} \]  

(45)

In the preceding calculation it has been also assumed that the turns in all the pole pairs are in series and that each slot only holds conductors whose currents belong to the same phase. A correction factor must be included when this is not the case\(^{10}\); this factor is obtained by separate calculation of the contribution of each particular slot\(^{17}\).

The end turn leakage component of the armature inductance is fairly simple in concept but it is extremely difficult to calculate. The end leakage flux involves three-dimensional fields of irregular shapes. The end leakage is related in some way to the length of the end-connectors and their shapes, the type of winding, the spacing between stator and rotor overhangs and the proximity of magnetic housings; it is also affected by many other factors which vary with both design and manufacture. Con-
siderable analytical and experimental work has been done on this problem; references 10, 13, 17 and 19 to 22 mention some of the most important contributions. Most of the proposed equations are quite lengthy\textsuperscript{11,23}; in view of all the uncertainties a complicated formula does not seem to be justified here. Therefore, a very simple expression, presented in ref. 14, will be used:

\[
\text{CLAL}_e = \frac{15.8}{2.\pi} \times 10^{-8} \times \frac{x}{P} \times \left(\frac{\text{CNS}}{3}\right)^2 \times (47 \times \pi - 30 \times 2) \times (R + G + 0.5 \times DS) \times \text{SIN}\left(\frac{\pi}{P}\right);
\]

\[
\text{CLAL}_e = 4.9 \times 10^{-7} \times \frac{\text{CNS}^2}{P} \times (R + G + 0.5 \text{ DS}) \times \text{SIN}\left(\frac{\pi}{P}\right)
\]  \hspace{1cm} (46)

The armature leakage inductance (per phase) includes both slot and end leakage effects:

\[
\text{CLAL} = \text{CLAL}_s + \text{CLAL}_e
\]  \hspace{1cm} (47)

The following concerns the calculation of the harmonic content of the air gap magnetic flux density\textsuperscript{15}.

- Stator breadth factor:

It is assumed that the slots are evenly distributed and all of them have the same number of conductors, therefore the configuration of turns is entirely symmetric. Ref. 15 gives the following expression for the breadth factor corresponding to the kth harmonic:

\[
\text{SBF}(k) = \frac{\sin\left(\frac{n \cdot k \cdot \phi}{2} \cdot \frac{P}{2}\right)}{k \cdot n \cdot \sin\left(\frac{k \cdot \phi}{2} \cdot \frac{P}{2}\right)}
\]  \hspace{1cm} (48)

where
n is the number of slot pairs involved: \( \frac{SS}{3.P} \)

\( \phi \) is the geometrical angle between two neighbor turns: \( \frac{2.\pi}{SS} \)

Thus the above expression becomes:

\[
SBF(k) = \frac{\sin \left( \frac{\pi.k}{6} \right)}{k.\frac{SS}{3.P} \cdot \sin \left( \frac{k.P}{2.SS} \cdot \pi \right)}
\]  \( (49) \)

This only holds for odd \( k \). All the breadth factors corresponding to even \( k \) are zero.

- Stator pitch factor:

If the pitch is shorted by \( \ell \) slots, the pitch factor for the \( k \)th harmonic is:

\[
SPF(k) = \sin \left[ \frac{k.\pi}{2} \cdot \frac{SS}{SS - \frac{P.\ell}{SS}} \right]
\]  \( (50) \)

In the program it has been assumed that:

\[
\ell = \frac{SS}{12}
\]  \( (51) \)

- Rotor breadth factor:

Equation (48) applies, but here

\[
n = \frac{SR}{P}
\]

\[
\phi = \frac{2.\pi}{SR}
\]

so the final expression is:
\[ RBF(k) = \frac{\sin \left( \frac{k \cdot \pi}{2} \right)}{k \cdot SR \cdot \sin \left( \frac{k \cdot p}{2 \cdot SR \cdot \pi} \right)} \] (52)

where again the equation only holds for odd \( k \) and it is zero for even \( k \).

- **Rotor pitch factor:**

  The rotor winding is assumed to be fully pitched; therefore:

  \[ RPF(k) = 1 \] (53)

- **Winding factor:**

  An overall winding factor for the \( k \)th harmonic is defined as:

  \[ SRWF(k) = SBF(k) \cdot SPF(k) \cdot RBF(k) \cdot RPF(k) \] (54)

Only copper and iron losses in the machine are of electrical origin. Stray losses have not been considered. Iron losses will be obtained in section 3.4, while copper losses are calculated here. The increment in resistance due to the skin effect \(^{18}\) is not considered, making the expressions straightforward:

- **Copper losses in the stator winding:**

  \[ CLCS = \frac{1}{SEC} \cdot VCUS \cdot CJS^2 \] (55)

- **Copper losses in the rotor winding:**

  \[ CLCR = \frac{1}{REC} \cdot VCUR \cdot CJR^2 \] (56)

The electrical conductivities depend significantly on the temperature.
This effect is accounted for in the following expressions:\[11\]:

\[
\text{SEC} = \left[EC(1)^{-1} + EC(2) \cdot (TAVS - 20)\right]^{-1}
\] (57)

\[
\text{REC} = \left[EC(3)^{-1} + EC(4) \cdot (TAVR - 20)\right]^{-1}
\] (58)

where

\[
TAVS = \frac{\text{TEMP}(1) + \text{TEMP}(2)}{2}
\] (59)

\[
TAVR = \frac{\text{TEMP}(3) + \text{TEMP}(4)}{2}
\] (60)

and 1, 2, 3, 4 stand for the embedded and end parts of the stator and rotor windings, respectively.

For commercial copper of conductivity 0.965 times the conductivity of the standard value for copper, the following numerical values must be used:\[11\]:

Conductivity at 20°C = \(5.6 \times 10^7 \ (\Omega \cdot \text{m})^{-1}\)

Temperature coefficient: \(68 \times 10^{-12} \ (\Omega \cdot \text{m} \cdot \text{°C})^{-1}\)

- Overall machine efficiency:

The total amount of losses on the machine is:

\[
\text{CLTOT} = \text{CLCS} + \text{CLCR} + \text{CLISC} + \text{CLIST} + \text{CLB} + \text{CLWO} + \text{CLWI}
\] (61)

And the machine efficiency:

\[
\text{EFF} = \frac{\text{PS}}{\text{PS} + \text{CLTOT}}
\] (62)

**Design Constraints**

All the design constraints of electrical origin, except for the first one, may be classified as soft, since they are more like performance
goals than constraints themselves. If the prescribed bounds of these
constraints are violated, this means that the machine design is poor, but
possible. Not too large weighting factors should be assigned to these
constraints.

- Steady state stable operation:

The real electrical power out of a 3-phase synchronous machine can
be expressed as\(^{15}\):

\[
PS = 3 \cdot \frac{\text{US}}{\text{P}} \cdot \frac{\text{ES}}{\text{W.CLD}} \cdot \sin(\text{DEL})
\]

(63)

It is required for stability (in the absence of elaborate controls)
that the power angle \(\text{DEL}\) remains within bounds: \(0 < \text{DEL} < \text{DELMAX}\), where
theoretically \(\text{DELMAX} = \pi/2\). Thus, eq. (63) becomes a constraint on the
maximum electric power delivered by a machine in certain operating condi-
tions:

\[
PS - 6 \cdot \frac{\text{US}}{\text{P}} \cdot \frac{\text{ES}}{\text{W.CLD}} \cdot \sin(\text{DELMAX}) < 0
\]

(64)

The machine should never be operating at the limit \(\text{DELMAX} = \frac{\pi}{2}\). A
more conservative upper bound for \(\text{DEL}\) must be selected in the program.
It must be realized that eq. (64) may be written as an upper limit for the
per unitized synchronous reactance:

\[
X_{d\text{ p.u.}} = \frac{X_d}{X_{\text{base}}} = \frac{\text{CLD.W.PS}}{3 \cdot \text{US}^2} < \frac{\text{ES}}{\text{US}} \cdot \sin(\text{DELMAX})
\]
- Limit on the harmonic content:

So far the basic electric constitutive relations for the machine have been written for the fundamental component. It is desired now to write a constraint to bound the harmonic content of the emf's and magnetic flux densities in the machine. The following index has been proposed to measure the harmonic content:

\[
HCF = \sum_{k=3}^{k'} |SRWF(k)| / |SRWF(1)|
\]

(65)

thus the constraint is:

\[
HCF - HCFMAX < 0
\]

(66)

The value of HCFMAX is again a designer's choice, and it is application dependent.

- Lower limit for the machine efficiency:

The machine efficiency may be explicitly or implicitly included in the figure of merit. However, it is convenient to prescribe a lower bound for it, in order to prevent searching over undesirable designs.

\[
EFFMIN - EFF < 0
\]

(67)

Other design constraints of electrical origin may be considered, depending on the particular application. The present model could be enlarged to include parameters characterizing the transient behavior of the machine.
3.4 Magnetic Circuit

With the magnetic flux density at the air gap being provided by the electrical equivalent circuit model, the magnetic circuit model calculates the required magnetomotive force (expressed in terms of the effective air gap length) and the iron losses. Stray losses are not considered in the model. There is no imposed limit on the level of magnetic saturation, since its effects are accounted for in the losses and MMF values. Because of the broadly different rotational speeds expected, the models for iron losses and magnetization must be accurate over a wide range of frequencies and saturation levels. Sections 3.4.3 and .4 are devoted to finding convenient approximate analytical expressions for the empirically determined magnetization curve and specific iron losses. Certain properties are desirable for magnetic materials to be used in high rotational speed machinery, such as high yield strength, low specific losses and high permeability. Unfortunately, trade-offs among them exist, so it is not a priori obvious which magnetic material is most appropriate for a particular application\textsuperscript{32,33}. These trade-off studies can be performed by feeding the proposed program with the data of the magnetic materials to be considered and by comparing the different optima which are obtained from each of them.

There are no design constraints related to the magnetic circuit.

3.4.1 The equivalent air gap length GEFF.

The equations used in section 3.3 to relate magnetomotive forces to magnetic fluxes in the machine correspond exactly to an ideal machine
with perfectly smooth air gap of length $GEFF$ and iron of infinite permeability. It will be shown in this section that the substitution of the fictitious "effective air gap length" $GEFF$ for the actual machine air gap length, $G$, accounts for the actual geometry of the machine and the iron finite permeability effect. The standard assumptions of magnetic circuit theory will be adopted here (no leakage, and the use of mean lengths and cross sections).

The following equation defines the effective air gap length:

$$\frac{GEFF}{\mu_0 S_0} \cdot FLUX = \sum_{\text{whole magnetic circuit}} H_i \cdot l_i$$  \hspace{1cm}(68)
The magnetic symmetry of the machine allows the consideration of only one fourth of the magnetic circuit corresponding to one pole pair. The following parts will be studied separately: air gap, stator and rotor cores, stator and rotor teeth.

- Air gap:

The presence of the slots (or ducts) increases the gap reluctance by restricting the flux to a degree depending on the width of the opening and the length of the gap. The problem is generally solved by use of the analytical results of Carter\textsuperscript{24,14}:

$$G_{eff \text{ air gap}} = A_{LO} = \xi \cdot \frac{5.6 + WS}{5.6 + WS \cdot (1 - KS)} \cdot \frac{5.6 + WR}{5.6 + WR \cdot (1 - KR)} \quad (69)$$

$$S_0 = CL.R.\frac{\pi}{P}$$

- Stator core:

The estimated mean length and cross section for this part of the magnetic circuit are:

$$AL1 = \frac{2}{3} \times \frac{\pi}{P} \times (R + G + DS + 0.5 \times H) \quad (70)$$

$$S1 = CL.FLS.ABS(H-DS) \quad (71)$$

$$B1 = FLUX/S1 \quad (72)$$

$$H1 = HBF(B1,F) \quad (73)$$

where the approximate analytical expression $HBF(B,FREQ)$ for the magnetiza-
tion curve will be presented in section 3.4.4.

Rotor core:

\[ AL2 \approx (R - DR) \cdot \sin \frac{\pi}{p} \]  \hspace{1cm} (74)

\[ S2 \approx 0.5 \times [(R - RSH - DR) \cdot CL \cdot FLR + (R - DR) \cdot \frac{\pi}{p}] \]

however, to emphasize the effect of the constraint (32), \( S2 \) will be expressed as:

\[ S2 \approx (R - RSH - DR) \cdot CL \cdot FLR \]  \hspace{1cm} (75)

\[ B2 = FLUX/S2 \]  \hspace{1cm} (76)

\[ H2 = HBF(B2, 0) \]  \hspace{1cm} (77)

Stator teeth:

\[ AL3 \approx DS \]  \hspace{1cm} (78)

\[ S3 \approx (R + G + 0.5 \times DS) \times \frac{\pi}{p} \times (1 - kS) \cdot CL \cdot FLS \]  \hspace{1cm} (79)

\[ B3 = FLUX/S3 \]  \hspace{1cm} (80)

\[ H3 = HBF(B3, F) \]  \hspace{1cm} (81)

Rotor teeth:

\[ AL4 \approx DR \]  \hspace{1cm} (82)

\[ S4 \approx (R - 0.5 \times DR) \cdot \frac{\pi}{p} \cdot (1 - kR) \cdot CL \cdot FLR \]  \hspace{1cm} (83)

\[ B4 = FLUX/S4 \]  \hspace{1cm} (84)

\[ H4 = HBF(B4, 0) \]  \hspace{1cm} (85)
Effective air gap length:

Since the distribution of flux in the air gap is sinusoidal:

\[
\text{FLUX} = \int_{0}^{\frac{\pi}{2}} B_0 \cos \alpha \cdot C.L.R. \cdot \frac{d\alpha}{P/2} = \frac{2}{P} R B_0 \cdot C.L
\]  

(86)

\[
G_{\text{EFF}} = A_0 + \frac{\mu_0 \cdot \pi}{2 B_0} \cdot (H_1 A_1 + H_2 A_2 + H_3 A_3 + H_4 A_4)
\]  

(87)

It must be realized that \(B_0\) is the peak value of the distribution of air gap magnetic flux density, under both a space (eq. 86) and time (eq. 39) point of view.

3.4.2 Iron losses

The hysteresis and eddy current losses in the stator and rotor core and teeth will be determined in this section. Although actual empirical data for commercial laminations will be used, the results may not be very accurate due to several additional effects that cannot be taken into account easily: flux not going exactly parallel to the laminations, eddy currents in metallic contacts between laminations, harmonic content of the magnetic flux, etc...

The total weights of the four regions of the iron in which the magnetic flux density has been assumed to be uniform are:

- Weight of the stator core:

\[
W_{\text{SC}} = \pi \cdot F.L.C.L. (H-DS) \cdot (2R+2G+DS+H) \cdot DENS(7)
\]  

(88)
- Weight of the stator teeth:
  \[ \text{WST} = \pi.\text{CL}.\text{FLS}.(1-kS).\text{DS}.(2.R+2.G+DS).\text{DENS}(7) \]  
  \hspace{1cm} (89)  

- Weight of the rotor core:
  \[ \text{WRC} = \pi.\text{CL}.\text{FLR}.[(R-\text{DR})^2 - \text{RSH}^2].\text{DENS}(1) \]  
  \hspace{1cm} (90)  

- Weight of the rotor teeth:
  \[ \text{WRT} = \pi.\text{CL}.\text{FLR}.\text{DR}.(1-kR).(2.R-\text{DR}).\text{DENS}(1) \]  
  \hspace{1cm} (91)  

In section 3.4.3 an expression for the specific total iron losses in terms of the frequency and the magnetic flux density is determined:

\[ \text{SPCL}(B,F) \text{ Watts/kg.} \]  

The iron losses can now be determined:

- Stator core iron losses:
  \[ \text{CLISC} = \text{WSC}.\text{SPCL}(B1,F) \]  
  \hspace{1cm} (92)  

- Stator teeth iron losses:
  \[ \text{CLIST} = \text{WST}.\text{SPCL}(B3,F) \]  
  \hspace{1cm} (93)  

Iron losses in the rotor are zero for a synchronous machine in steady state conditions and where only the fundamental component of the magnetic flux is considered.

\[ \text{CLIRC} = 0 \]  

\[ \text{CLIRT} = 0 \]  

\[ \hspace{1cm} (94) \]  

\[ \hspace{1cm} (95) \]  

3.4.3 Mathematical description of the specific iron losses

The objective now is to find an approximate analytical expression for
the iron losses for laminae of a specified thickness and material, in
terms of the frequency $F$ of the applied magnetic field and of the maximum
instantaneous value $B$ of the average flux density in the cross section of
the laminae.

Theoretical models for the specific iron losses in terms of the fre-
quency and the magnetic flux density can be found in the literature\textsuperscript{26 to 31},
but they do not yield satisfactory agreement with the empirical results
32 to 37, as has been recognized\textsuperscript{27,38,39}. Therefore, from a pragmatic
point of view, an approximate analytical representation of the actual
watt-loss results in terms of the frequency and flux density as provided
by manufacturers, is probably of more value.

The expression

$$P = aB^xF^y$$

where $a$, $x$ and $y$ are constants, has been widely proposed\textsuperscript{16,38}. Unfortunately,
this equation is usually only a crude approximation when broad
ranges of frequency and magnetic flux density must be represented by a
unique expression. Improvements to the above equation are suggested here,
always taking as a target the actual watt-loss empirical curves. The
ranges of interest are:

$$50 \leq F \leq 1500 \text{ Hz}$$

saturation $> B >$ as low as .1 wb/m$^2$ for very high rotational
speeds.

By consistently plotting data from different materials, the following
conclusions have been drawn (Figs. 9 and 10 are representative of the
Figure 9. Specific iron losses empirical curves $p = p(F, B = \text{const})$ approximated by a family of straight lines.

Figure 10. Specific iron losses empirical curve $p = p(B, F_0)$ approximated by an analytical curve.
expected shapes and values for common types of electrical steel):

- The family of curves \( \log P = \text{func}(\log F, B = \text{const}) \), (see Fig. 9), can be reasonably represented by a set of straight lines. This depends on the type of material, but it is very accurate for frequencies over 200 Hz. The dotted lines in Fig. 9 are parallel; a better fit can be achieved by slightly varying the slopes. The deviation of the actual curves from being straight lines at low frequencies is likely due to the larger weight of the hysteresis losses at these frequencies.

- The intersections of this family of curves with lines \( f = \log F_0 = \text{const} \), give curves like the one represented in Fig. 10, which are roughly straight lines for nonsaturated iron conditions.

A first approximation to the actual surface \( P = P(B,F) \) can be made by using the dotted lines in Figs. 9 and 10:

Fig. 10 is represented by:

\[
\log P = a + c \cdot \log B
\]

where \( a \) and \( c \) are constants determined from the plot. The curve corresponds to any preselected frequency \( F_0 \). \( F_0 \) can be any frequency in the range \( 50 \leq F \leq 1500 \) Hz.

The family of lines in Fig. 9 is represented by:

\[
\log P = d + e \cdot \log F
\]

where the slope \( e \) is common to all of them and can be determined from the plot; the value of \( d \) is different for each curve of the family and is calculated in terms of \( B \) from the above equations.
\[ d = a + \log(B^c F_0^{-e}) \]
\[ \log P = a + \log \left[ B^c \left( \frac{F}{F_0} \right)^{-e} \right] \]

The constant \( a \) is defined as \( \log P_0 \), and its value is:

\[ a = \log P_0 = \log \left[ P(B = 1 \text{ wb/m}^2, F = F_0) \text{ watts/kg} \right] \]

The wanted equation becomes:

\[ P = P_0 \cdot B^c \cdot \left( \frac{F}{F_0} \right)^e \]  \( (96) \)

where

- \( F_0 \) is the frequency chosen as reference for Fig. 10.
- \( c \) is the slope of the dotted line in Fig. 10.
- \( e \) is the common slope of the dotted lines in Fig. 9.
- \( P_0 = P(B = 1 \text{ wb/m}^2, F = F_0) \text{ watts/kg} \).

This first model can be improved by removing the assumption of uniform slope of the family of lines in Fig. 9. Therefore, the constant \( e \) becomes a function of \( b \). Assuming a linear dependence and a reference \( b_0 = \log B_0 : e(b) = g + h(b - b_0) = g + \log \left( \frac{B}{B_0} \right)^h = g + \log B^h \)

if \( B_0 = 1 \text{ wb/m}^2 \) is taken as reference; \( g \) and \( h \) are constants.

The equation giving the specific losses becomes now:

\[ P = P_0 \cdot B^c \cdot \left( \frac{F}{F_0} \right)^{g + h \cdot \log B} \]  \( (97) \)

This second model can be further improved by removing the simplification of considering the curve in Fig. 10 to be a straight line, which is far from reality for high flux densities. It has been found that the
following analytical expression reproduces reasonably the actual empirical curves for different materials:

\[ p = a + c \cdot b - k \cdot b^r + \lambda \cdot b^s \quad (98) \]

where \( a, c, k, r, \lambda \) and \( s \) are constants. The values of \( a \) and \( c \) were determined above. Reasonable estimates for \( r \) and \( s \) are \( r = 4 \) and \( s = 6 \). The values of \( k \) and \( \lambda \) must be determined by fitting eq. (98) to the empirical curve at a few points. More sophisticated procedures for fitting analytical curves to empirical data are available in the literature.\(^{25}\)

The expression giving the specific losses is:

\[ p = p_o \cdot B^{c-k} \cdot (\log B)^{r-1} + \lambda \cdot (\log B)^{s-1} \cdot (\frac{F}{F_0})^{g+h} \cdot \log B \quad (99) \]

If the fit at low frequencies is still poor, polynomials of second or higher order should be used to simulate the empirical curves in Fig. 9.

3.4.4 Mathematical description of the magnetization curve

In this section the objective is to find analytical expressions to describe the relationship between the magnetic flux density \( B \) in the iron of the machine and the magnetic field intensity \( H \), taking also into account the effect of the frequency \( F \) of operation. In summary: \( H(B,F) \).

In the context of this work, the expression must satisfy the following conditions:

1. If possible, a simple function (or as few functions as possible) should represent the whole range of interest of the variables.

2. The nature of the analytical step in the design requires \( H \) to be obtained from \( B \), so the expression to be determined is \( H = H(B,F) \).
3. The approximation should be simple mathematically, so that:

3.1 The computer and preparation time to determine the analytical expression \( H(B,F) \) for every material should be reasonable, and the method not very sophisticated.

3.2 More important (because of the frequent use) is that the computer time to evaluate \( H \) from \( B \) and \( F \) should be as small as possible.

3.3 The expression can be implemented easily in the overall program.

4. The errors should be reasonable. It must be taken into account that the magnetic circuit approach itself has a limited accuracy.

Many different types of functions have been proposed to reproduce the empirical dc magnetization curve\(^{40}\) to \(^{44}\). It has been found that the simplest proposed analytical expressions\(^{40,41}\) do not give a satisfactory fit over the whole range for the considered magnetic materials, so several of them had to be combined. Numerical interpolation\(^{41}\) has been discarded due to the computer storage requirements and the associated inconvenience of changing the data for different materials. Other more elaborate methods\(^{25,42,43,44}\) do not satisfy the conditions 3.1 and 3.2. Therefore a more suitable method has been devised and used:

Magnetization curves are usually presented by the manufacturers in semi-logarithmic paper (i.e., \( B \) versus \( \log H \))\(^{45,35,36,37}\). The immediate change of variables:

\[
x = B
\]
\[
y = \log H
\]
allows the curve to be approximated by a polynomial \( y(x) \) without any special difficulty. Most of the simplest methods which have been proposed attempt to approximate the smooth shape of the B-H curve with simple analytical expressions, but they run into problems in the long saturation region. On the other hand the curve B-log H has not such a smooth shape, Fig. 11, but the saturation region has been compressed and polynomial interpolation can be used without having numerical problems. For values of

\[ H = H_s + \frac{10^7}{4\pi} \cdot (B - B_s) \quad \text{for} \quad B > B_s \]  

Fig. 11 Empirical magnetization curve log H=f(B).

\( B > B_s \) (complete saturation of the material) the following expression must be used (straight line of slope \( \mu_0 \)):
The polynomial interpolation of the curve in Fig. 11 yields:

\[ \log H = \sum_{n=0}^{N} C_n \cdot B^n \]  \hspace{1cm} (101)

where \( C_n \) are constant coefficients for a given curve.

Therefore for the interval of interest:

\[ \sum_{n=0}^{N} C_n \cdot B^n \]  \hspace{1cm} (102)

\[ H = 10^{\sum_{n=0}^{N} C_n \cdot B^n} \], for \( B_0 \leq B \leq B_s \),

where \( B_0 \approx 0.2 \text{ wb/m}^2 \).

Even with the aforementioned change of variables, a reasonable fit requires polynomials of 8th order, at least. Thus numerical problems appear if an ordinary polynomial fit is intended, and Chebyshev polynomials have been used.

A computer program has been prepared which uses canned subroutines\(^{46}\) to obtain the coefficients of the Chebyshev polynomials expansion for a given set of points \( (B_i, \log H_i) \). This program is trivial and its listing has not been included. Since transformation of Chebyshev expansions into ordinary polynomials normally results in severe loss of accuracy, no attempt is made to return the polynomial expansions, and all the required calculations are performed in terms of Chebyshev polynomials. Thus eq. (101) is only symbolic.

So far the effect of the frequency has not been included. As pointed out in section 3.4.2 the magnetization curves change significantly with the frequency, so the model must account for it. This has been achieved by repeating the aforementioned analysis for several frequencies in the
range of interest. A final expression:

\[ H(B,F) = 10^{\sum_{n=0}^{N} C_n(F)B^n}, \text{ for } B_o \leq B \leq B_s \]  \hspace{1cm} (103)

where the coefficients \( C_n(F) \) are again obtained as a Chebyshev polynomial's least square fit to the corresponding coefficients of the polynomials obtained for different frequencies, in terms of the frequency:

\[ C_n(F) = \sum_{k=0}^{K} C_{n,k} F^k \]  \hspace{1cm} (104)

In this case only three terms of the expansion need be considered to give reasonable accuracy. Again the program CURVEFIT can be used. Improvements in the accuracy of the final expression can be achieved by increasing \( N \) and \( K \) in eqs. (101) and (104). The resultant expression, eq. (102), has been implemented in the subroutine HBF of the main program.

Sometimes the manufacturer does not provide B-H curves for different frequencies, but instead provides apparent losses (volt-amperes) versus B curves, for various frequencies. In this case the curve in Fig. 11 must be previously prepared, point by point, by means of the equation:

\[ H = 0.3185 \times \frac{C.D}{F.B} \]  \hspace{1cm} (105)

where

\begin{align*}
H & \text{ = Peak magnetic intensity (A/m)} \\
B & \text{ = Peak magnetic flux density (wb/m}^2) \\
C & \text{ = RMS volt-amperes per kg at B and F}
\end{align*}
D = Material density (kg/m³)

F = Frequency (Hz)

Specific details for a particular case can be found in Chapter V.
3.5 Mechanical Design and Stress Distribution

The purpose of this section is twofold:

a) To dimension those elements whose function is mainly mechanical, but only if the output of their design affects significantly the overall design of the machine (i.e., it has important electrical, thermal or geometrical consequences). Only two elements: shaft and rotor wedges, have been considered to satisfy these requirements.

b) To obtain constraint equations limiting the value of the stresses at those points where they can result in damage to the machine, when the design of the concerned elements is mostly of a non mechanical character. This happens to the rotor body and the roots of the rotor teeth.

It is therefore clear that a complete or detailed mechanical design of the machine is not intended. Only the mechanical aspects closely interrelated to electrical parameters will be accounted for here.

Since this work deals with high speed rotating machinery, special attention has been paid to the effect of centrifugal forces, which can result in upper limits for the rotational speeds. Calculations for the rotor in the plastic region have been included, although they should likely be ruled out since they result in serious damage of the magnetic properties of the rotor material.

The bearing design has not been included. For very high rotational speeds this deserves a separate study.

Most of the geometrical dimensions will be assumed to be a priori known, as provided by previous steps in the overall design. If some of the constraint equations are not satisfied, corrective measures will be
adopted in the synthesis part of the design.

Following is a list of the mechanical properties of the machine construction materials, which are necessary to calculate the mechanical constitutive relations and design constraints:

- Mass densities $\rho$ of:
  - Stator and rotor iron
  - Stator and rotor conductors
  - Stator and rotor conductors insulation
  - Shaft and frame
  - Rotor wedges

- Yield strengths $\sigma_{\text{max}}$, Poisson's moduli $\mu$ and Young's moduli $E$ of:
  - Rotor iron
  - Rotor wedges

Numerical values for a particular application are given in Chapter V.

When an elastic body is subjected to a system of combined stresses it may fail according to one of a number of limiting conditions. Failure is implied when the material ceases to obey Hooke's law. Several theories have been proposed, each assuming a different criterion for failure$^{48,54}$.

It is usual to apply the maximum shear stress theory to ductile materials, since good approximation is obtained between theory and experimental results, the theoretical results being always on the safe side. This theory assumes that elastic failure occurs when the maximum shear stress for a complex system is equal to the shear stress at the elastic
limit in simple tension. Stated mathematically, failure will occur when (for a two dimensional system):

$$\frac{\sigma_{\text{max}}}{2} = \tau_{\text{max}} = \frac{1}{2} (\sigma_1 - \sigma_2) = \frac{1}{2} \left[ (\sigma_x - \sigma_y)^2 + 4 \cdot \tau^2 \right]^{\frac{1}{2}} \quad (106)$$

where:

- $\sigma_{\text{max}}$ = Elastic limit in simple tension
- $\tau_{\text{max}}$ = Maximum shear stress
- $\sigma_1, \sigma_2$ = Maximum and minimum principal stresses
- $\sigma_x, \sigma_y$ = Stresses corresponding to two perpendicular directions $x$ and $y$.
- $\tau$ = Shear stress for planes $xz, yz$.

The maximum shear stress theory will be applied throughout this section.

**Constitutive relations.**

Some elements of the machine must be designed in order to meet certain performance requirements which are mostly mechanical. Only a few of these elements are such that there is a significant interdependence among their specific design and other aspects of the machine design. The basic design of these elements, shaft and wedges, is presented next.

**3.5.1 Shaft design**

The shaft design consists primarily of the determination of the correct shaft diameter to ensure satisfactory strength, rigidity and critical speed for the system when the shaft is transmitting power under various operating and loading conditions. There is no apparent advantage in
having a shaft radius which is larger than necessary. Therefore the smallest shaft radius which satisfies the three aforementioned conditions will be adopted.

The design criteria that follow are intended for shafts of ductile material and circular cross section $^{49,53,54}$.

**Design of shaft for strength**

The case of a horizontal shaft will be considered first. The rotor is keyed to the shaft and it is assumed to be supported by perfectly inelastic bearings, then the deflections at the supports will be zero. The restraints or moments at the supports may be also assumed to be zero. The shaft is subject to torsion and bending loads.

![Diagram](image)

**Fig. 12  Idealized geometry for shaft design**

The A.S.M.E. Code equation for a solid shaft in these conditions is:

$$
RSH = 0.5 \times \left\{ \frac{16}{\pi S_s} \times \left[ \left(k_b \cdot M_b \right)^2 + \left(k_t \cdot M_t \right)^2 \right] \right\}^{\frac{1}{3}}, \quad (107)
$$

where:
RSH = Shaft outside radius (m)

M_b = Bending moment (nwxm)

M_t = Torsional moment (nwxm)

k_b = Combined shock and fatigue factor applied to bending moment.

For rotating shafts:

k_b = 1.5 for load gradually applied (which is the case)

k_b = 1.5 ÷ 2 for load suddenly applied (minor shock)

k_b = 2 ÷ 3 for load suddenly applied (heavy shock)

k_t = Combined shock and fatigue factor applied to torsional moment.

For rotating shafts:

k_t = 1.0 for load gradually applied

k_t = 1.0 ÷ 1.5 for load suddenly applied (minor shock)

k_t = 1.5 ÷ 3.0 for load suddenly applied (heavy shock, which can be considered the case).

S_s = Allowable stress (maximum shear theory)

The A.S.M.E. Code specifies that for commercial steel shafting:

S_s = 6000 psi = 422 kg/cm² = 41.37 x 10^6 N/m² for shafts with keyway.

Taking S_s = 41.37 x 10^6 N/m², k_b = 1.5 and k_t = 2, the eq. (107) becomes:

\[ RSH \geq 2.49 \times 10^{-3} \times (2.25 \times M_b^2 + 4 \times M_t^2)^{\frac{1}{6}} \]  \hspace{1cm} (108)

The torsional moment acting on the shaft can be determined from:

\[ M_t = \frac{\text{Power rating of the machine}}{2 \times \pi \times \text{(rps)}} \approx \frac{PS}{W} \]  \hspace{1cm} (109)
In order to obtain \( M_b \), the first step must be to draw the bending moment diagram for the loaded shaft, which will give the maximum \( M_b \).
The shaft and rotor weights are the loads per unit length \( W_1 \) and \( W_2 \). The likely unbalanced magnetic pull (roughly proportional to the shaft deflection) is not considered here.

![Diagram of load distribution for the shaft](image)

**Fig. 13** Load distribution for the shaft

\[
W_1 = 9.81 \times \pi \times \rho \times RSH^2 \text{(N/m)} ,
\]

(110)

where \( \rho \) is the mass density of the shaft (kg/m\(^3\))

\[
W_2 = 9.81 \times \pi \times \rho_{ave} \times (R^2 - RSH^2) ,
\]

(111)

where \( \rho_{ave} \) is an average mass density for the whole rotor, without including the shaft. (kg/m\(^3\)).

The expression for the maximum \( M_b \) is given in Timoshenko\(^54\):

\[
M_b = \frac{1}{8} (W_1 \times CLT^2 + 2 \times W_2 \times CL \times CLT - W_2 \times CL^2) ,
\]

(112)

For the case of vertical shaft, as no eccentricity and no unbalanced
magnetic pull are considered here, the only load is the torsional moment. The axial load coming from the weight of the machine is considered to be negligible. The same equation holds:

\[
RSH \geq 3.14 \times 10^{-3} \times M_t^{\frac{3}{2}},
\]

(113)

**Design of shaft regarding critical speeds**

If the machine is going to operate at variable speed, and especially if the speed range to be covered is very large, the operation at critical speeds cannot be easily prevented. Therefore the damping and spring constants of the bearings will have to take care of the vibrations produced at the critical speeds, and the shaft design will not be directly constrained by this effect.

The situation is different if the machine must operate at a prescribed constant speed or within a very narrow range of speeds. In this case the operating speed has to be far enough from any of the critical speeds.

However, the resultant constraint will not be taken into account, since the present work aims to perform a capability study rather than a very specific design. In fact, the values of the critical speeds are very sensitive to parameters of secondary importance in the overall design, such as the radius of the shaft and the distance between bearings; therefore, attempting to avoid the critical speeds would distort the general results of the study, without providing any significant advantage, since a more detailed study of the problem would be required to design entirely a specific machine.

Reference 48 provides a good survey of methods which can be applied
to determine the critical speeds of an electrical machine. The "exact" analytical analysis is the most appropriate, since it can be performed easily and provides all the critical speeds in a single closed form expression. Reference 53 mentions some secondary effects, such as the unbalanced magnetic pull, which can modify the results of conventional critical speeds calculations.

**Design of shaft for rigidity**

a) **Design of shaft for torsional rigidity**

Shaft design for torsional rigidity is based on the permissible angle of twist. No limit will be fixed since a high value of the angle of twist does not introduce any problem in this particular application. Moreover, the remaining constraints that will be fixed will keep this angle of twist within reasonable limits.

b) **Design of shaft for lateral rigidity**

Shaft design for lateral rigidity is based on the permissible lateral deflection. The deflection of a shaft on a rotating machine is an important aspect of its mechanical design. Having first ascertained that the shaft is strong enough to transmit the required power, some consideration must be given to its stiffness. This is particularly important in the case of rotating electrical machines because of the presence of an unbalanced magnetic pull (UMP).

Due to the shaft deflection, the air gap between the stator and rotor will be less at one side than at the other; the UMP will be proportional to the deflection, the magnetic pull being greatest where the air gap is
smallest.

The mechanical stability of the shaft must be such as to restrict the UMP from having a cumulative effect, i.e., for some particular percentage deflection of the air gap the UMP will have a certain value, but if the deflection is increased the UMP will also increase, tending to pull the shaft over still further; the greater the deflection the greater will be the UMP, and if the mechanical stability of the shaft is not sufficient to prevent this cumulative effect taking place, failure may occur. In practice, for rotating electrical machinery, experience indicates that the static deflection of the shaft should not exceed about 20% of the air gap\textsuperscript{48}.

Equation (114) gives the maximum static deflection for the shaft-rotor system, with the load distribution shown in Fig. 13. The UMP is not considered yet. It has been assumed that the rotor only acts on the shaft as a uniformly distributed load, without adding any rigidity to the shaft. This seems to be reasonable, since the rotor is only shrunk on to the shaft by means of the end plates. Anyway it is a conservative assumption. (1). The expression for the maximum static deflection is\textsuperscript{54}:

\[
y_{\text{max}} = \frac{1}{384xEl} \left[ 5.0W_1 CL^4 + W_2.8 CL^3 CL - W_2.4 CL^4 + W_2 CL^4 \right] \quad (114)
\]

where I is the rectangular moment of inertia of the cross section:

\[
I = \frac{1}{4} \cdot \pi \cdot RSH^4 , \quad (115)
\]

(1) If a solid rotor, shrunk completely on the shaft or being one with the shaft, is used, the calculation must be repeated for a shaft of non-constant flexure\textsuperscript{48}. 
Now the UMP is taken into account. It can be shown\textsuperscript{12} that for deflections much smaller than the air gap the UMP is roughly proportional to the deflection. Therefore the force per unit length $F$ of the rotor can be written as:

$$F = k \cdot y,$$

where $k$ is a constant which depends on the geometry and magnetic flux in the machine.

Assuming that the deflection is uniform along the whole rotor length, $y$, and therefore that the UMP is also uniform, the eq. (114) must be modified in the following way to account for the UMP:

$$W_{2_{\text{new}}} = W_{2} + ky$$

$$y_{\text{max}}_{\text{new}} = y_{\text{max}} + \frac{1}{384EI} \cdot (8CLT^3CL - 4CLTCL^3 + CL^4) \cdot ky_{\text{max}}_{\text{new}} = y_{\text{max}} + \frac{C}{EI} \cdot k \cdot y_{\text{max}}_{\text{new}},$$

$$y_{\text{max}}_{\text{new}} = \frac{y_{\text{max}}}{1 - \frac{C}{EI} \cdot k},$$

where $C$ is a constant and to prevent the failure the shaft design must be such that:

$$k < \frac{EI}{C},$$

This analysis is only valid as long as the deflection $y_{\text{max}}$ is small compared with the air gap.

The eq. (118) can be used as a mechanical constraint, not
having any effect on the design unless it is violated.

The constant $k$ is now calculated in terms of the geometry and magnetic conditions of the machine. The calculation is a small variation on the procedure presented in references 33 and 53. Assuming a two pole machine and a sinusoidal azimuthal distribution of the magnetic flux density in the air gap, the mentioned references would yield:

$$\text{UMP} = 4 \cdot \frac{B_0^2}{2 \cdot \mu_0} \cdot R \cdot (\frac{V}{G}) \cdot \int_0^\pi \sin^4 \theta \cdot d\theta = \frac{3 \cdot \pi}{4} \cdot \frac{B_0^2}{\mu_0} \cdot \frac{R \cdot L}{G} \cdot y, \quad (119)$$

Thus:

$$k = \frac{3 \cdot \pi}{4} \cdot \frac{B_0^2}{\mu_0} \cdot \frac{R \cdot L}{G}, \quad (120)$$

And the constraint becomes:

$$\frac{3 \cdot \pi}{4} \cdot \frac{R_0^2}{\mu_0} \cdot \frac{R \cdot L}{G} - \frac{384 \cdot E \cdot I}{8 \cdot C \cdot L^3 \cdot C \cdot L - 4 \cdot C \cdot L \cdot C \cdot L^3 + C \cdot L^4} < 0, \quad (121)$$

As pointed out before, the static deflection of the shaft is also constrained not to exceed 20% of the air gap length:

$$\frac{1}{384E_I} \cdot [5 \cdot w_1 \cdot C \cdot L^4 + 8 \cdot w_2 \cdot C \cdot L^3 \cdot C \cdot L - 4 \cdot w_2 \cdot C \cdot L \cdot C \cdot L^3 + w_2 \cdot C \cdot L^4] - 0.2 \times G < 0, \quad (122)$$

The value adopted in the design for the shaft radius $R_{SH}$ is the smallest value of $R_{SH}$ satisfying the three inequalities 108, 121 and 122.
3.5.2 Wedge design

The theory of rectangular plates can be applied directly. It is assumed that the deflections are small compared to the plate thickness $h$, and also that the supports do not constrain the deformations of the plate in the $y$ direction.

Fig. 14a

Fig. 14b

Fig. 14 Simplified wedge geometry

The load per unit surface $P$ is considered to be uniform throughout the plate, with the value:

$$P = \frac{F_1 + F_2}{CL \cdot WR}$$

(123)

where $F_1$ and $F_2$ are the centrifugal forces corresponding to the material within the slot and to the wedge itself, respectively. Their expressions are calculated in section 3.5.4.

The maximum bending momenta occur at the center of the plate. Since the plate is long and narrow ($CL \gg WR$) and only supported at the long
sides, the only bending momentum to be considered is the one acting on the section $yz$, whose value is:

$$M_{\text{max}} = \beta \cdot P \cdot WR^2,$$

(124)

and $\beta$ is a coefficient dependent on the ratio $CL/WR$. For large values of $CL/WR$ (i.e., bigger than 5):

$$\beta = 1/8 \text{ if the plate is simply supported.}$$

$$\beta = 1/12 \text{ for a built-in plate.}$$

Adopting the most conservative assumption:

$$M_{\text{max}} = \frac{1}{8} \cdot P \cdot WR^2,$$

(125)

The maximum stress will be:

$$\sigma_{\text{max}} = \frac{6}{GR^2} M_{\text{max}} = 0.75 x P x \left[\frac{WR}{GR}\right]^2 = 0.75 x (F_1 + F_2) x \frac{WR}{CL \cdot GR^2},$$

(126)

And the sought value for $GR$:

$$GR = \left[0.75 \cdot \frac{WR}{\sigma_{\text{max}} \cdot CL \cdot (F_1 + F_2)} \right]^\frac{1}{2},$$

(127)

where $\sigma_{\text{max}}$ will be determined from the material to be used and the safety coefficient.

**Design constraints**

The operating conditions of the machine, and especially its rotational speed, yield high mechanical stresses at specific spots. There is nothing
wrong with a design as long as these stresses are below a prescribed limit.

The goal of this section is to determine where these risky spots are located and to calculate their mechanical stresses in terms of the machine geometrical dimensions, material properties, and speed of rotation.

The obtained numerical values will be compared with the maximum allowable stress and if these stress limits happen to be exceeded, corrective measures will be taken in the main design routine.

The elements which will be studied are the following: rotor core, teeth and end plates.

3.5.3 Rotor core

The concerned configuration is shown in Fig. 15. The rotor can be solid (axial length may be comparable with the radius) or laminated (thickness of the laminations is much smaller than the radius). In both cases the variation of radial and tangential stresses in the axial direction can be neglected and the problem becomes independent of z.

If it is assumed that the stress distribution is symmetrical with respect to the axis of rotation (this is not completely true, due to the slots) (1), the stress components do not depend on θ and are functions of r only. From symmetry it follows also that the shearing stress

(1) If the concentration of stresses in the corners of the teeth root is neglected, it will be shown later that the maximum stresses in the rotor core occur well inside the rotor. Therefore it is reasonable to assume symmetry in the azimuthal direction.
\(\tau_{r\theta}\) must vanish.

According to the foregoing assumptions the problem of the determination of the stresses in the rotor core has been reduced to a quite standard one: stress distribution in an axially symmetric flat rotating disk\(^{48,51,54,55}\). The disk has an internal symmetric hole and in general it is symmetrically loaded at both internal and external peripheries with the pressures \(P_o\) and \(P_i\) nW/m\(^2\). The equations giving the stress distribution in the disk have been obtained in appendix B for cases of both nonplastic and plastic design.

![Rotor lamination geometry](image-url)
Stresses in the rotor core under elastic conditions

Since the presence of the rotor slots prevents that part of the rotor between \( r = R \) and \( r = D-DR \) from supporting tensile hoop stresses, the problem becomes one of a rotating disk having outer radius \( R-DR \) and inner radius \( RSH \), being loaded externally by the rotor winding and the rotor teeth (this external pressure is assumed to be evenly distributed). See Fig. 16. The maximum stress in the rotor body can be directly calculated from eqs. B.4, 5 and 6, taking into account the following directions:

- The pressure \( P_0 \) is produced by the inertial force of the teeth and the material within the slots. Assuming that it is uniform, it can be calculated as follows:

\[
P_0 = -\frac{1}{2\pi(R-DR)} \int_{R-DR}^{R} 2\pi R^2 r [r(1-kR)\rho_I + r.kR.\rho_{FLR}] \, dr =
\]
\[ w^2 = -\frac{3}{3.(R-DR)} \cdot \rho_{eq} \cdot [R^3 - (R-DR)^3] \quad , \quad (128) \]

where \( \rho_I \) and \( \rho_S \) are the mass densities of the iron in the laminations and the stuff within the slots respectively. The weighted combined mass density of teeth and slots is \( \rho_{eq} \):

\[ \rho_{eq} = \rho_S \cdot \frac{KR}{FLR} + \rho_I \cdot (1 - KR) \quad , \quad (129) \]

where the stacking factor for the laminations, FLR, makes the slots seem to weight \( FLR^{-1} \) times more than they actually do.

- The rotor core is mounted directly on the shaft, to which it is keyed (but not shrunk on to it), and clamped between end plates secured between a shoulder on the shaft and a shrink-ring. Therefore the internal pressure \( P_i \) can be considered to be zero. This may not be true for the end ring plates, which may be shrunk on to the shaft. In this case it is necessary to estimate the required value of \( P_i \) and to use the complete eq. B.6.

- Due to sudden changes in contours (i.e., in the roots of the teeth or in the keyway), certain concentrations of stress take place. However, such a concentration does not represent a danger, since only local plastic flow will take place; this is generally so when the material is ductile and the stresses are static.

- No radial stresses are transmitted among the neighbor laminations, because of the symmetry. Therefore each lamination can be studied independently and the effect of the insulation in between the laminae will
not be considered.

- It is assumed that the end belts take complete care of the end windings.

With the foregoing assumptions, the eqs. B.4, 5 and 6 can be applied, giving a maximum shear stress at the inner surface of the rotor:

$$\tau_{\text{max}} = \tau(r=\text{RSH}) = 0.5 \left[ \sigma_t(r=\text{RSH}) - \sigma_r(r=\text{RSH}) \right] = 0.5 \sigma_t(r=\text{RSH}) =$$

$$= \rho_{\text{eq}} \frac{w^2 (R-DR)}{3} \cdot \frac{R^3 - (R-DR)^3}{(R-DR)^2 - \text{RSH}^2} +$$

$$+ \frac{\rho_I w^2}{8} \cdot [(3+\mu) \cdot (R-DR)^2 + (1-\mu) \cdot \text{RSH}^2], \quad (130)$$

If the maximum shear stress theory is applied, the following constraint eq. is obtained:

$$\tau_{\text{max}} < 0.5 \sigma_{\text{max}} \quad ; \quad 2 \times \tau_{\text{max}} - \sigma_{\text{max}} < 0 \quad \text{laminations.} \quad \text{laminations} \quad (131)$$

where the value of $\sigma_{\text{max}}$ is determined by the characteristics of the material used for the laminations (where the loss of magnetic properties due to mechanical stresses must be taken into account). An appropriate safety factor can be introduced, by reducing the effective value of $\sigma_{\text{max}}$.

**Stresses in the rotor core under plastic conditions**

The general remarks and assumptions of the preceding section can be
repeated here. The basic equations have been developed in appendix B. If complete yielding of the disk is admissible, the constraint eq. is:

\[
\frac{\rho \text{eq} \cdot w^2}{3.(R-DR)} \cdot [R^3-(R-DR)^3] < \sigma_{\text{max}} \cdot \ln \frac{R-DR}{R SH} - \frac{\rho \text{eq} \cdot w^2}{2} \cdot [(R-DR)^2-RSH^2],
\]

which is more meaningful when written this way:

\[
w = \left[ \frac{\sigma_{\text{max}} \cdot \ln \frac{R-DR}{R SH}}{\frac{R^3-(R-DR)^3}{3.(R-DR)} \cdot \rho \text{eq} + \frac{(R-DR)^2}{2} \cdot \rho \text{eq}} \right]^{\frac{1}{2}} < 0,
\]

(133)

If only partial yielding is allowed, eq. B.13 should be used, which relates the rotational speed and the mass densities to the percentage of material in the plastic region. This eq. has the eqs. (130) and (132) as limit particular cases.

3.5.4 Rotor teeth

In addition to knowing the body stresses, the stresses at the roots of the teeth also require attention. These stresses are roughly obtained by dividing the centrifugal force of the tooth and slot material by the tooth root area.

Figure 17 gives the dimensions of the rotor tooth and slot, the angle of the wedge being taken as 60°, which is usual. The width of the tooth root can be estimated roughly as

\[
\text{HIR} = (R-DR) \cdot \frac{2\pi}{SR} - \text{WR},
\]

(134)
Fig. 17  Rotor teeth idealized geometry

The following approximate expression will be used for the inertial force:

\[ F = M \cdot w^2 \times d \]  

(135)

where \( M \) is the mass of the part, \( w \) is the rotational speed and \( d \) is the distance from the center of rotation to the center of gravity of the body. Three parts will be considered: the tooth, the wedge and the interior of the slot.
- Interior of the slot:

\[ F_1 = \rho_1 \cdot W_{R_{av}} \cdot (DR-GR) \cdot w^2 \cdot (R-.5\ GR-.5\ DR), \quad (136) \]

- Wedge:

\[ F_2 = \rho_2 \cdot WR.\ GR.\ w^2 \cdot (R-.5\ GR), \quad (137) \]

- Tooth:

\[ F_3 = \rho_3 \cdot \frac{1}{2} \cdot DR \cdot (HR+HIR) \cdot w^2 \cdot (R - \frac{DR}{3} \cdot \frac{HR + 2HIR}{HR+HIR}), \quad (138) \]

The combination of forces shown in Fig. 18 gives the following tension stress at the rotor tooth:

\[ \sigma_r \approx \frac{F_{tot}}{HIR} = \frac{1}{HIR} \cdot [F_3 + (F_1 + F_2) \cdot 2 \cdot \cos \left( \frac{\pi}{3} - \frac{\pi}{5R} \right)], \quad (139) \]

The constraint eq. is (maximum shear stress theory):

\[ \sigma_r - \sigma_{\text{max}} < 0, \quad (140) \]

![Figure 18. Diagram of forces at the rotor teeth.](image)
3.6 Temperature rises and cooling system

The thermal model provides the temperature rises at the expected hot spots in the machine. Peripheral results are the temperatures at other significant points and complete maps of the conduction, convection and radiation heat exchanges. Nothing is actually designed with the thermal model, its goal is to check if a given complete design meets the standard requirements on temperature rises in electrical machinery. All the expressions leading in some way to the obtention of the temperatures at the expected hot spots will be considered to be constitutive relations. The inequalities \( T_{\text{spot}_i} - T_{\text{max}_i} \leq 0 \) will be the design constraints.

3.6.1 Overall description of the thermal model

The major steps in the elaboration of the thermal model for the machine are the following:

a) Choice of an idealized machine geometry (Figs. 4, 5, 19)

b) Division of the machine into basic thermal elements (Figs. 19 and 20). These elements are sometimes called nodes, suggesting an analogy (which in general does not hold) with an electrical resistive network.

c) Writing equations expressing the conservation of energy at each node. In steady state conditions:

\[
L_i = Q_i + C_i + R_i, \quad i = 1, ..., N
\]  

(141)

where

- \( N \) is the number of elements,
- \( L_i \) is the rate of heat produced at element \( i \).
$Q_i$, $C_i$ and $R_i$ are the rates of heat flow from element $i$ by conduction, convection and radiation, respectively. In general these flows can be written in terms of the temperatures of the elements, which are not known a priori. Some of the radiative heat exchanges require new unknown parameters called radiosities.

d) Determination of the coefficients for the unknowns in the above equations. (These are the "equivalent thermal resistances" when the electrical analogy holds). Some of the coefficients related to convective heat exchanges are mildly temperature dependent.

e) Solving the resulting system of equations for the temperature rises and the radiosities. Figure 20 shows the considered heat exchanges between the basic elements into which the machine was divided. Many of the branches correspond to more than one node. Radiative heat exchanges in the overhang region do not relate directly to pairs of elements and have not been represented.

The model does not attempt to be valid for all types of rotating electrical machines. The following features have been assumed for the machine:

a) No salient poles.

b) Wound rotor and stator.

c) Enclosed rotor end windings.

d) Axially symmetric cooling system (same temperature distribution at both ends of the machine).

e) Four cooling options:

(1) Mean temperatures of the elements are selected as the unknowns. Maximum or surface temperatures are obtained in terms of the mean temperatures, as needed.
1. STATOR EMBEDDED WINDING.
2. STATOR END WINDING.
3. STATOR TEETH.
4. STATOR CORE.
5. ROTOR EMBEDDED WINDING.
6. ROTOR END WINDING.
7. ROTOR TEETH.
8. ROTOR CORE.
9. FRAME.
10. AIR GAP.
11. OVERHANG AIR.
12. SHAFT.
13. BEARINGS.
14. END SHIELD.
15. AMBIENT AIR.

16. STATOR END PLATE
17. ROTOR END
18. AXIAL DUCT
19. RADIAL DUCT

Figure 19. Thermal model. Machine quarter section showing idealized geometry and node division.
Figure 20. Heat exchange map.

(Note: Radiative heat exchanges in the overhang region do not relate directly pairs of elements and have not been represented.)


With options I and II the choice exists of operating the machine at standard conditions or to simulate operation in a high vacuum (internally).

Some different features may be obtained by introducing minor modifications in the program (i.e., not wound rotor. See section 3.6.8).

Following is a brief presentation of the three modes of heat transfer and the basic assumptions involved.

**Conduction mode**

Fourier's law for the steady flow of heat by conduction can be expressed in differential form by:

\[ q = -k \cdot \nabla \theta \]  \hspace{1cm} (142)

where

- \( q \) = heat flow density (watt/m²)
- \( k \) = thermal conductivity of the medium (watt/m°C)
- \( \theta \) = temperature (°C)
and in steady state conditions also:
\[ \nabla^2 \theta = 0 \quad , \tag{143} \]

Equations (142) and (143) show a complete analogy with Ohm's law and Laplace's equation, which implies that equivalent thermal circuits can be prepared in the same way as electrical circuits.

For one dimensional heat flow in a homogeneous and isotropic medium, the above equations yield the simple expressions

\[ Q = \frac{\Delta \theta}{R_t} \quad , \tag{144} \]

\[ R_t = \frac{1}{k} \cdot \frac{\ell}{A} \quad , \tag{145} \]

where

\( \ell \) and \( A \) = length and cross section of the element.

\( \Delta \theta \) = temperature difference across \( \ell \).

\( R_t \) = thermal resistance of the element

\( Q \) = heat flow through the element.

If heat flow takes place in more than one direction, eqs. (142) and (143) have to be solved with appropriate boundary conditions for the particular case, and a network of thermal resistances will be found that, exactly or more likely approximately, will reproduce the heat flow through the element.

To prepare the thermal equivalent circuit the thermal resistances accounting for the different elements must be joined together, maintaining the same topological relationships in the network as in the original
body.

In this particular application the heat inputs to the different elements of the system are known from the outset, and their temperatures must be determined. Therefore every element whose temperature has to be calculated, should be represented in the equivalent circuit by one node, so that the node temperature coincides with the mean temperature of the element. All the resistances for the equivalent circuit have to be determined taking this into account.

The usually weak temperature dependence of the thermal conductivity will not be taken into account. It could be included without difficulty.

Convection mode

The basic law for the heat flow by convection is:

\[ Q = h \cdot A \cdot \Delta \theta \quad , \tag{146} \]

where

\[ A \quad \text{is the area of the surface of the body in contact with the fluid} \]
\[ \Delta \theta = \text{temperature difference between the surface of the body and the bulk of the fluid, } \Delta \theta = \theta_{\text{surface}} - \theta_{\text{fluid}} \]
\[ Q = \text{heat flow} \]
\[ h = \text{factor of proportionality, called "(film) coefficient of heat transfer".} \]

Heat convection is a very complex process and the simplicity of eq. (146) is delusive. As a matter of fact, the film coefficient, \( h \), defined by that equation, is a function of many variables such as shape,
and dimensions of the surface, kind, direction and velocity of the flow, temperature, density, viscosity, specific heat and thermal conductivity of the fluid.

The value of the coefficient of heat transfer, \( h \), for the different situations in which convection is the type of heat transmission in the electrical machine, has to be determined from empirical correlations between the dimensionless numbers involved in the process. In several cases no empirical correlations are available, so others devised for more or less similar configurations have been used.

One of the main features of \( h \) is its dependence on the temperature of both the body and the fluid. This makes the equation (146) nonlinear in terms of the temperatures. The difficulty in determining heat transfer coefficients which will be applicable to all sizes and types of machines stands in the way of obtaining great accuracy in the calculation of temperature rises.

In order to include the convection processes in the thermal circuit, one can define an equivalent convective thermal resistance:

\[
R = \frac{\Delta \theta}{Q} = \frac{1}{hA},
\]

(147)

where \( R \) is temperature dependent.

In the dimensional analysis of the convection heat transfer processes, the film coefficient \( h \) is always associated with a dimensionless group called \( N_{NU} \), the number of Nusselt, which is defined as

\[
N_{NU} = \frac{h \cdot \ell}{k},
\]

(148)
where \( \ell \) is the characteristic length of the configuration and \( k \) the fluid thermal conductivity. Since \( k \) and \( \ell \) are always known from the outset, \( h \) can be immediately determined once \( N_{NU} \) has been obtained. The thrust of the calculation of the convective thermal resistances, will be therefore directed at obtaining \( N_{NU} \) for the different types of convection that occur in the machine.

The dimensional analysis shows that for the general case of a viscous fluid flowing past a heated surface, there are four dimensionless groups of physical parameters characterizing the process:

The Reynolds number: \( N_{RE} = \frac{V \cdot \rho \cdot \ell}{\mu} \), \( \quad (149) \)

The Grashof number: \( N_{GR} = \frac{g \cdot \ell^3 \cdot \beta \cdot \Delta \theta \cdot \rho^2}{\mu^2} \), \( \quad (150) \)

The Eckert number: \( N_{ER} = \frac{V^2}{C_p \cdot \Delta \theta} \), \( \quad (151) \)

The Prandtl number: \( N_{PR} = \frac{\mu \cdot C_p}{k} \), \( \quad (152) \)

where:

\( V \) is the characteristic velocity of the fluid.

\( \ell \) is the characteristic length of the body whose surface is considered.

\( \rho \) is the fluid density.

\( g \) is the gravitational acceleration.

\( \beta \) is the coefficient of volume expansion of the fluid.

\( \Delta \theta \) is \( \theta_{\text{surface}} - \theta_{\text{fluid}} = \theta_s - \theta_f \).

\( C_p \) is the fluid specific heat.

\( \mu \) is the fluid dynamic viscosity.

\( k \) is the fluid thermal conductivity.
When the fluid is assumed to behave as an ideal gas, which is reasonable for air at about standard pressure and temperature, \( \beta \) is the inverse of the absolute temperature at the bulk of the fluid.

It is customary to evaluate the fluid properties at the mean film temperature \( \theta_m = (\theta_s - \theta_f)/2 \), except for the coefficient of volume expansion. \( \beta \), which is normally evaluated at the temperature of the undisturbed fluid far removed from the surface, namely \( \theta_f \).

Thus, all convection problems for which these are the proper variables of description will be governed by an equation of the form:

\[
\phi \left( N_{RE}, N_{GR}, N_{ER}, N_{PR} \right) = 0
\]

(153)

where \( N_{RE} \) characterizes the flow conditions; \( N_{PR} \) is a measure of the relative magnitudes of momentum and thermal diffusion in the fluid; \( N_{GR} \) results from the inclusion of buoyancy forces in the equations of motion and \( N_{ER} \) represents the rate of dissipation of energy.

It can be shown\(^58\) that the dimensionless heat transfer coefficient \( N_{NU} \) can be expressed in this way:

\[
N_{NU} = f\left( N_{RE}, N_{GR}, N_{ER}, N_{PR} \right)
\]

(154)

In forced convection applications it is usual to neglect any effects of buoyant forces so that:

\[
N_{NU} = f\left( N_{RE}, N_{ER}, N_{PR} \right)
\]

(155)

when viscous dissipation is included, and

\[
N_{NU} = f\left( N_{RE}, N_{PR} \right)
\]

(156)

when viscous dissipation is neglected.
In the case of free convection the main fluid stream is absent so that \( N_{RE} \) becomes no longer significant. In addition the flow is generally so slow that viscous dissipation does not enter. In free convection applications, then, it is expected that:

\[
N_{NU} = f(N_{GR}, N_{PR})
\]  \( \text{(157)} \)

**Radiation mode**

The rate of heat, \( q \), radiated from an area, \( A \), is given by the Stefan-Boltzmann law:

\[
q = \varepsilon \sigma A \theta^4
\]  \( \text{(158)} \)

where

\( \theta \) is the absolute temperature of the surface.
\( \varepsilon \) is a property of the particular emitting surface and of its temperature, and is known as emissivity
\( \sigma \) is the Stefan-Boltzmann constant, independent of both the surface and the temperature. Its numerical value is \( 5.67 \times 10^{-8} \) watts/[m\(^2\)(°K)\(^4\)], in M.K.S. units.

Two features of this law make it difficult to incorporate this mode of heat transfer into an equivalent thermal circuit:

a) It is not possible to establish a bilateral relationship between any pair of bodies without taking into consideration the remaining ones. The net radiant exchange of energy between any pair of surfaces i-j is composed not only of a direct transfer of energy but also of an indirect transfer, the result of multiple reflections, absorption and re-radiation, etc. Thus \( q_{ij} \) will depend upon the geometric shape and orientation of all
surfaces present as well as their emissivities and absorptivities. Therefore there is no way of defining an equivalent thermal resistance for the radiant heat flow between any two surfaces $i, j$.

b) The equation (158) shows a marked non-linearity with respect to the temperatures.

However there is no difficulty in finding an analytical expression for the term $R_i$ in eq. (141) as required. The problem considered here is as follows:

A cavity (1), containing a gas transparent to radiation, is formed by three or more surface regions that can be considered isothermal. The analysis will consider only radiation; the radiation heat flow to or from every node, together with the conduction and convection heat flows has to match the fixed heat input to the node.

The ultimate goal of this analysis will be the obtention of a set of equations relating the temperatures at the nodes with the radiation heat flows to or from them.

Radiation heat transfer computations for engineering systems are commonly carried out under the following assumptions, that here will be accepted:

a) The enclosure can be subdivided into a finite number of isothermal surfaces.

b) The surfaces are gray body emitters, absorbers and reflectors. Therefore $\alpha = \varepsilon$ and $\rho = 1 - \varepsilon$, where $\alpha$, $\varepsilon$ and $\rho$ respectively represent the absorptance, emittance and reflectance. Transmissivity of the surface material is assumed negligible.

c) The emitted radiation and the reflected radiation leaving any
surface in the enclosure both have directional distributions that are diffuse.

d) The magnitude of the radiant energy leaving any surface is uniform over that surface.

The last two assumptions permit the use of the same angle factors for gray surfaces as are used for black surfaces.

In dealing with gray-diffuse enclosures it is convenient to introduce the "radiosity" \( B \), which is the rate at which radiant energy leaves a surface per unit area. This quantity differs from the emissive power, eq. (158), in that the radiosity includes reflected energy as well as the original emission. By definition the radiosity of any gray surface \( j \) is related to the absolute temperature \( \theta_j \) and the net radiative heat loss \( R_j \) as follows\(^92\):

\[
R_j = \frac{\varepsilon_j A_j}{1 - \varepsilon_j} (\sigma \theta_j^4 - B_j) \quad (159)
\]

It is evident that if \( \theta_j \) and \( B_j \) are known, \( R_j \), the total radiative heat loss from surface \( j \), is readily calculable.

Radiant flux balance equations can be written for all the surfaces, giving:

---

(1) An essential first step in these calculations is to envision a complete enclosure made up of all surfaces that can radiatively interact with the surfaces of interest. The purpose of the enclosure concept is to ensure that radiant energy arriving from all directions in space is fully taken into account. One or more of the surfaces of the enclosure may not be material surfaces, for instance the ambient space surrounding the machine. Each of such surfaces may be assigned equivalent radiation properties and an equivalent black body temperature that corresponds to the rate at which radiant energy passes through the surface into the enclosure.
\[ R_j = B_j A_j - \sum_{i=1}^{N} B_i F_{j-i} A_j = (\text{since } \sum_{i=1}^{N} F_{j-i} = 1) \]

\[ = - \sum_{i=1}^{N} (B_i - B_j) A_j F_{j-i}, \quad (160) \]

where \( F_{j-i} \) = shape factor for radiation from surface \( j \) to surface \( i \).

(Fraction of the radiant energy leaving area \( A_j \) that arrives directly at area \( A_i \); none being considered which is transferred by reflection or re-radiation from other surfaces that may be present).

The eqs. (159) and (160) can be rephrased in a form that suggests an analogy with an electric circuit composed of resistances and voltage sources:

\[ \sum_{i=1}^{N} \frac{B_i - B_j}{(A_j F_{j-i})^{-1}} + \frac{\sigma \theta_j^4 - B_j}{(1 - \varepsilon_j)/(\varepsilon_j A_j)} = 0, \quad (161) \]

One may now associate the radiosities \( B \) and the emissive powers \( \sigma \theta^4 \) with potentials, and the quantities \( \frac{1}{A_F} \) and \( \frac{(1 - \varepsilon_j)}{\varepsilon_A} \) with resistances. Then, terms such as

\[ \frac{B_i - B_j}{(A_j F_{j-i})^{-1}}, \quad (162) \]

and

\[ R_j = \frac{\sigma \theta_j^4 - B_j}{(1 - \varepsilon_j)/(\varepsilon_j A_j)}, \quad (163) \]

are clearly identifiable as currents.
Thus, eq. (161) expresses conservation of current at a node. Each term in the summation represents a current flowing from node \( i \) into node \( j \). The term \( R_j \) of eq. (163) is a current flowing into node \( j \) through an external connection that links an externally applied potential \( \sigma \varepsilon_j^4 \) with node \( j \); the resistance of the connection is \( (1-\varepsilon_j)/(\varepsilon_j A_j) \). Therefore \( R_j \) is represented by a current flowing into node \( j \) from an external current source.

In the light of the foregoing interpretation, an equivalent electric circuit exists, in which the radiosities are determinable just by calculating the electric potential of certain nodes. Once the \( B_j \) are known, the surface heat fluxes are computed by evaluating the eq. (159).

In order to avoid confusion in interpretation of the equivalent electrical network, as applied here to radiation, it is helpful to note, first, that it differs significantly from the equivalent electrical networks that have been applied to the conduction and convection modes of heat transfer. For the radiation problem, the electrical resistances in the network do not directly represent physical thermal resistance elements, and the electrical potentials do not directly represent physical temperatures.

The electrical resistance elements represent functions of surface areas combined either with emissivity values, or with shape factors. And the electrical potentials, at the points representing surface regions, represent \( \sigma T^4 \) instead of the absolute temperature, \( T \), itself.

To illustrate the above explanation, the electrical network corresponding to the radiant heat exchange between three surfaces is represented in Figure 21. The sign conventions indicated in the figure correspond exactly
with the signs in the above equations. The expressions \( 1 - \varepsilon_i \) are also the reflectivities \( \rho_i \) for gray surfaces.

\[
\frac{\rho_i}{\varepsilon_i A_i} \rightarrow R_i \rightarrow \sigma \theta_i^4
\]

\[
(A_3, F_{3,1})'; (A_1, F_{1,3})'; (A_1, F_{1,2})'; (A_2, F_{2,1})'; (A_2, F_{2,3})'; (A_3, F_{3,2})';
\]

\[
\frac{\rho_3}{\varepsilon_3 A_3} \rightarrow R_3 \rightarrow \sigma \theta_3^4
\]

\[
\frac{\rho_2}{\varepsilon_2 A_2} \rightarrow R_2 \rightarrow \sigma \theta_2^4
\]

Figure 21. Example of equivalent electrical network for radiant heat exchange among three surfaces.
The above theory in connection with the adopted machine node distribution, yields the particular radiation equivalent electrical network shown in Figure 22. Two main features of this particular equivalent network have to be emphasized:

a) As far as radiation is concerned, four independent cavities must be considered. The relation of the four enclosures and the surfaces forming them is as follows (Figs. 19 and 22):

Enclosure a = 9, 14, 15.
Enclosure b = (3-4), 9 (the contribution of 16 is neglected).
Enclosure c = (3-4), (7-8), since the air gap can be considered as completely enclosed for radiation heat exchange.
Enclosure d = 2, (17-12), 14, 16.

The reason for considering these four independent enclosures, is that the main characteristic of a complete enclosure is that the relation:

\[
\sum_{j=i}^{N} F_{i-j} = 1 , \quad (164)
\]

has to be applicable to any of the surfaces that form the enclosure. This expression is used to obtain the eq. (160).

It happens that in the machine thermal model these are elements, such as 9, having two surfaces such that:

\[ F_{9-15} = 1 ; F_{9-4} \approx 1 ; \]

and therefore

\[
\sum_{i=1}^{N} F_{9-i} > 1 \quad , \text{in contradiction with eq. (164)}. \]
Figure 22. Equivalent electrical network for radiant heat exchange in the machine.
b) As it was pointed out before, one or more of the surfaces of the enclosure may not be material surfaces. This happens in the enclosure a , where it is necessary to assign equivalent radiation properties \( \varepsilon_{15} = 1, \ \rho_{15} = 1 - \varepsilon_{15} = 0 \); \( \frac{\rho_{15}}{\varepsilon_{15} A_{15}} = 0 \) and an equivalent black body temperature \( \theta_{15} \) to the ambient space surrounding the machine.

The contribution of radiation to the total cooling of a conventionally ventilated machine is expected to be small, and the model itself gives values on the order of 5% of the total. However, it has been included in detail since for high rotational speeds the high windage losses may force a switch to other cooling methods, including operation in a vacuum.

3.6.2 Input data required by the model

The calculation of the temperature rises is one part or subroutine of the main design program. Heating calculations only can be achieved once the design parameters of the machine have been completely specified. Therefore it is now convenient to enumerate those design parameters that are required to determine the elements of the equivalent thermal circuit and which are the required inputs to the subroutine THERMO in the final computer program. They can be classified as follows:

Machine dimensions

Most of the required machine dimensions are represented in Figure 4. Following is a list of these dimensions, the definitions of which may be found in the Nomenclature section.
Machine losses

The losses in the different parts of the machine have to be known from the outset in order to perform the thermal analysis. They are the heat inputs $L_i$ into the elements in equation (141). The list is:

CLCS, CLCR, CLISC, CLIRC, CLIST, CLIRT, CLB, CLWO, CLW1.

It must be noticed that copper losses depend on the temperature of the conductors, through its conductivity. Windage losses CLWO and CLW1 may be calculated in the model (Appendix 6.A.2). Non wound induction motors require an additional loss CLER, end ring losses (section 3.6.8).

Thermal properties of the materials

Following is a list of the most important materials taking part in the thermal behavior of the machine. For solid materials only the thermal sensitivity and emissivity are required, and also the maximum operating temperature for the insulating ones (1).

(1) Only typical values will be given for one material of each group. Variations are wide for different types of the same material or different methods of manufacture.

(2) In the plane of the laminations the thermal conductivity $k_x$ will be a function of the silicon content of the steel and of the stacking factor (typical 0.925). Normal to the plane of the lamination the value of the thermal conductivity $k_y$ will depend on: a) individual lamination thick-
<table>
<thead>
<tr>
<th>Material</th>
<th>$\frac{1}{k}$ ($^\circ$C.m/watt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper $^{10}$</td>
<td>0.0026</td>
</tr>
<tr>
<td>Aluminum $^{58}$</td>
<td>0.005</td>
</tr>
<tr>
<td>Sheet steel laminations $^{90}$ (Hot rolled 0.016&quot; thick; flash enameled, 1.83% Si, clamping pressure: 40 lb/in$^2$) $^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>- In the plane of the laminations ($k_x^{-1}$)</td>
<td>0.03</td>
</tr>
<tr>
<td>- Normal to the plane of the laminations ($k_y^{-1}$)</td>
<td>0.58</td>
</tr>
<tr>
<td>(both values $k_x$, $k_y$ include the insulation effect).</td>
<td></td>
</tr>
<tr>
<td>Mica tape (perpendicular to laminations) $^{10}$</td>
<td>$\approx 3$</td>
</tr>
<tr>
<td>Asbestos $^{10}$</td>
<td>$\approx 4$</td>
</tr>
<tr>
<td>Compressed paper $^{10}$</td>
<td>$\approx 8$</td>
</tr>
<tr>
<td>Cotton insulation (untreated) $^{12}$</td>
<td>$\approx 13$</td>
</tr>
<tr>
<td>Cotton insulation (impregnated or varnished) $^{12}$</td>
<td>$\approx 4$</td>
</tr>
<tr>
<td>Empire cloth $^{10}$</td>
<td>$\approx 4$</td>
</tr>
<tr>
<td>Oil $^{10}$</td>
<td>$\approx 8$</td>
</tr>
<tr>
<td>Air (still, 26.7$^\circ$C, 1 atm.) $^{58}$</td>
<td>$\approx 38$</td>
</tr>
</tbody>
</table>

Emissivity values for different materials are available in the literature $^{68,92,12}$. For the materials that form part of electrical machinery it is reasonable to assume an overall value of 0.9. Information (2) cont. ness; b) type and thickness of interlaminar insulation; c) surface finish of lamination (i.e., whether hot or cold reduced); d) pressure under which the laminations are clamped.
on limit temperatures is given in section 3.6.7.

The thermal properties of the air are extensively used in the convection and conduction heat transfer calculations. These properties depend significantly on pressure and temperature (i.e., the thermal conductivity of dry air at atmospheric pressure increases more than 30% when the temperature passes from 30°C to 150°C).

The thermal properties of the air that have to be considered are the following\textsuperscript{58} (numerical values are provided for 1 atm. pressure and 26.7°C):

- Mass density $\rho = 1.1774$ kg/m$^3$
- Dynamic viscosity $\mu = 1.849 \times 10^{-5}$ kg/(m.sec)
- Thermal conductivity $k = 2.629 \times 10^{-2}$ watts/(m°C)
- Specific heat $C_p = 1.008 \times 10^3$ joule/(kg°C)
- Prandtl number $N_{PR} = 0.709$ (dimensionless)

The values of these properties at atmospheric pressure have been plotted against temperature in the interval 0 °C to 250°C, and it has been found that they can be represented by the following expressions, with an accuracy greater than 3%, which makes the approximation acceptable for the model:

\begin{align*}
\rho &= 352.94/(273.16 + T) \text{ kg/m}^3 \\
\mu &= 1.75 \times 10^{-5} + 4.12 \times 10^{-8} \times T \text{ kg/(m.sec)} \\
k &= 2.45 \times 10^{-2} + 7.16 \times 10^{-5} \times T \text{ watt/(m°C)} \\
C_p &= 1006.86 + 0.1694 \times T \text{ joule/(kg°C)} \\
N_{PR} &= 0.716 - 1.44 \times 10^{-4} \times T \text{ (dimensionless)},
\end{align*}

where $T$ must be expressed in °C.
Information on air (and other gases) properties at pressures different from atmospheric can be found in reference 68.

Miscellaneous inputs

Other information required to compute the thermal model equations includes:

- Ambient temperature TAMB.
- Effectiveness coefficient for the fins, if they are used, CFINS.
- Rotational speed W.
- Flow rate of coolant, if any, W0.
- Choice of cooling mode.
- Numerical values for the film coefficients if they are to be prescribed. They may be calculated in the model.
- Control parameters for the computer subroutine, regarding accuracy, initial guesses for the unknowns, peripheral results to be obtained, etc.

3.6.3 Conduction mode related expressions

Every actual process of heat conduction is 3-dimensional, making it difficult to find the exact analytical expression for the temperature distribution and therefore the equivalent system of resistances to be used. Therefore sometimes one-dimensional models will be assumed, and for them the correct analytical expressions will be worked out. When the one-dimensional model is not reasonable, simplified versions of two-dimensional models will be used. No three-dimensional model will be necessary, due to the almost complete azimuthal symmetry of the system.

Some parts of the machine cannot be considered isotropic in their
thermal properties, i.e., conductivity. For instance the iron cores have very different conductivities in the radial and axial directions; this will be included in the model. On the other hand, it would be very inconvenient to consider every wire of the winding separately; thus equivalent transverse (in the directions normal to the main axis of the machine) thermal conductivities will be defined.

Another important point to be emphasized is that the equivalent resistances have to be determined in order to make the node voltages coincide with the mean temperature of the corresponding actual element. The temperature of the hot spots (points of highest temperature of the elements) must be calculated from the mean temperature. This has to be accounted for in order to determine the values of the equivalent system of resistances.

Since different assumptions and simplifications will be required for every specific resistance, the methods used to obtain the equivalent thermal resistances will be presented simultaneously with the particular case.

3.6.3.1 Derivation of the basic models

One-dimensional bulk conduction

At first sight almost none of the conduction heat flow paths in the machine appear to be appropriately modelled considering only one dimensional flow. Nevertheless the figures will show that in many cases the one-dimensional approximation is reasonable. Moreover, this model is the building block to obtain approximate two and three dimensional models.

Figure 23 shows the basic element to be considered. Heat conduction in the x direction and internally distributed heat generation are in-
cluded in the model. The temperatures at the cross sections for $x=0$ and $x=l$ are assumed to be uniform and the lateral surface is perfectly isolated thermally. With all these assumptions the temperature distribution is completely one-dimensional $\theta = \theta(x)$.

![One-dimensional heat conduction diagram](image)

Figure 23. One-dimensional heat conduction.

Therefore it can be written that:

$$\nabla \cdot (-k \nabla \theta) = -k \frac{d^2 \theta}{dx^2} = q$$

(170)

where $q$ (joules/(m$^3$.sec)) is the amount of heat generated internally in the element per unit volume.

Solving the differential equation:

$$\theta = -\frac{q}{2k} x^2 + C_1 \cdot x + C_2$$

Since $\theta(x=0) = \theta_1 = C_2$

$$\theta(x=l) = \theta_2 = -\frac{q}{2k} l^2 + C_1 \cdot l + \theta_1$$

$C_1 = \frac{\theta_2 - \theta_1}{l} + \frac{q \cdot l}{2k}$

Thus:

$$\theta = \frac{q}{2k \cdot l \cdot A} \cdot x \cdot (l-x) + (\theta_2 - \theta_1) \cdot \frac{x}{l} + \theta_1$$

(171)
In order to obtain the heat input $Q_1$ and output $Q_2$ at both ends of the element:

$$\frac{d\theta}{dx} = \frac{q}{2kA} (x - 2x) + \frac{\theta_2 - \theta_1}{l}$$

$$Q_1 = -k.A\left(\frac{d\theta}{dx}\right)_{x=0} = -\frac{Q}{2} + k.A \cdot \frac{\ell_1 - \theta_2}{l}$$

$$Q_2 = -k.A\left(\frac{d\theta}{dx}\right)_{x=l} = \frac{Q}{2} + k.A \cdot \frac{\theta_1 - \theta_2}{l}$$

It can be checked that $Q_2 - Q_1 = Q$.

Now to obtain the hot spot temperature:

$$\frac{d\theta}{dx} = 0 = \frac{q}{2k} (x - 2x_m) + \frac{\theta_2 - \theta_1}{l} ; \quad x_m = \frac{x}{2} + \frac{k}{q.L} \cdot (\theta_2 - \theta_1)$$

$$\theta_{\text{max}} = \frac{q_0}{2k} \left[\frac{k}{2} + \frac{k}{q.L} \cdot (\theta_2 - \theta_1)\right] \cdot \left[\frac{k}{2} - \frac{k}{q.L} \cdot (\theta_2 - \theta_1)\right] +$$

$$+ (\theta_2 - \theta_1) \cdot \frac{1}{l} \cdot \left[\frac{k}{2} + \frac{k}{q.L} \cdot (\theta_2 - \theta_1)\right] + \theta_1 ;$$

$$\theta_{\text{max}} = \frac{Q.L}{8.k.A} + \frac{k.A}{2.Q.L} \cdot (\theta_2 - \theta_1)^2 + \frac{\theta_2 + \theta_1}{2}$$

But the mean temperature of the element is the important magnitude in the equivalent thermal network:

$$\theta_{\text{av}} = \frac{1}{l} \int_{x=0}^{x=l} \theta(x).dx = \frac{\theta_1 + \theta_2}{2} + \frac{Q.L}{12.K.A}$$
The network in Fig. 24 is an attempt to model thermally the element in Fig. 23. The unknown resistances \( R_1, R_2 \) and \( R_3 \) will be determined in order to match the following conditions, for any \( \theta_1, \theta_2 \) and \( Q \):

\[
\theta_{av} = \theta_1 - R_1 Q_1 + R_3 Q = \frac{\theta_1 + \theta_2}{2} + \frac{Q \cdot \ell}{12 \cdot k \cdot A},
\]

(176)

\[
\theta_{av} = \theta_2 + R_2 Q_2 + R_3 Q = \frac{\theta_1 + \theta_2}{2} + \frac{Q \cdot \ell}{12 \cdot k \cdot A},
\]

(177)

The coefficients of \( \theta_1, \theta_2 \) and \( Q \) at both sides of these expressions have to be identical. This is satisfied for the following values of the resistances:

\[
R_2 = R_1 = \frac{\ell}{2 \cdot k \cdot A},
\]

(178)

\[
R_3 = -\frac{\ell}{6 \cdot k \cdot A},
\]

(179)

If a reference resistance, \( R_0 \), is defined as

\[
R_0 = R_{\text{element for only conduction in } x} = \frac{\ell}{k \cdot A},
\]

(180)

the resistances can be rewritten as:

\[
R_2 = R_1 = \frac{R_0}{2},
\]

(181)

\[
R_3 = -\frac{R_0}{6},
\]

(182)

It should be pointed out that a similar equivalent circuit in which the
temperature at the main nodes is $\theta_{\text{max}}$ instead of $\theta_{\text{av}}$, is only possible if "dependent-on-temperatures $\theta_1$ and $\theta_2$ resistances" are used.

![Figure 24. Equivalent network for one-dimensional heat conduction.](image)

The temperature $\theta_{\text{max}}$ can be obtained from the expression (174) once the whole network has been solved and all the heat flows and node temperatures are known. However sometimes it is more convenient to combine eqs. (174) and (175) to obtain:

$$\theta_{\text{max}} = 1.5 \theta_{\text{av}} - .25 (\theta_1 + \theta_2) + \frac{1}{12} \cdot \frac{(\theta_1 - \theta_2)^2}{2 \theta_{\text{av}} - \theta_1 - \theta_2} \quad (183)$$

Figure 25 shows two important particular cases of the network in Fig. 24, for situations in which $\theta_1 = \theta_2 = \theta_0$ (Fig. 25a) and when there is a common element (i.e., a cooling fluid) at both sides (Fig. 25b).
Figure 25. Equivalent symmetric networks for one-dimensional heat conduction.

Two-dimensional bulk conduction

Now it is assumed that there is no heat flow along one of the three main directions. This is the case of the element of Figure 26, where all the heat flow takes place in the directions $x$ and $y$. There is internal heat generation that is assumed to be uniformly distributed, the rate being $q$ joules/(m$^3$.sec).

Figure 26. Two-dimensional heat conduction
The problem is a classical one and the exact analytical solution for the temperature $\theta(x,y)$ in this configuration is in the literature, but the mathematical form is not compact and cannot be handled easily to be modelled with a network of resistances. It has been shown that an approximate model, much more convenient for the equivalent circuit approach, can substitute the exact one with enough accuracy.

The key point in building the approximate model is to consider that the flows of heat in the two main directions are completely independent and that the mean temperature of the element along both directions is the same. This is wrong from a theoretical point of view, but the numerical results are sufficiently accurate for practical purposes.

The model can include the quite common feature of having different thermal conductivities $k_x$ and $k_y$ for heat flow along the two main directions.

The application of this approximate model to a particular case is straightforward once the complete equations for the one-dimensional model are available. Following is one particular application in which there is uniform internal heat generation and net heat flow conduction due to external sources only in the $y$ direction (therefore $\theta_3 = \theta_4 = \theta_0$ and $\theta_1 \neq \theta_2$). It is considered that $k_x \neq k_y$. The equivalent thermal network of the element in Fig. 26 is shown in Fig. 27, where

$$R_{ox} = \frac{1}{k_x} \cdot \frac{L}{w \cdot h}$$  \hspace{1cm} (184)

$$R_{oy} = \frac{1}{k_y} \cdot \frac{w}{l \cdot h}$$  \hspace{1cm} (185)

$$Q = q \cdot w \cdot h \cdot l$$
Figure 27. Equivalent network for two-dimensional heat conduction

The numerical values of $Q_x$ and $Q_y$ are not known from the outset. They result from the solution of the complete network at the same time that node temperatures do, and can be written in terms of the node temperatures and the total internal heat input $Q$ by solving the following system of equations:

\[ Q = Q_x + Q_y \quad \text{,} \tag{186} \]

II) Heat input is at the same average temperature for both directions
(initial assumption implicit in the adopted equivalent network):

\[ \theta_{av_x} = \theta_{av_y} = \theta_0 + \frac{Q_x \cdot l}{12. k_x \cdot h \cdot w} = \frac{\theta_1 + \theta_2}{2} + \frac{Q_y \cdot w}{12. k_y \cdot h \cdot l}, \]  
\hspace{1cm} (187)

Solving for \( Q_x \) and \( Q_y \):

\[ Q_x = \frac{k_x \cdot w^2 \cdot Q + (6 \theta_1 + 6 \theta_2 - 12 \theta_0) \cdot h \cdot l \cdot w \cdot k_x \cdot k_y}{l^2 \cdot k_y + w^2 \cdot k_x}, \]  
\hspace{1cm} (188)

\[ Q_y = Q - Q_x = \frac{k_y \cdot l^2 \cdot Q - (6 \theta_1 + 6 \theta_2 - 12 \theta_0) \cdot h \cdot l \cdot w \cdot k_x \cdot k_y}{l^2 \cdot k_y + w^2 \cdot k_x}, \]  
\hspace{1cm} (189)

The assumption of setting \( \theta_{av_x} = \theta_{av_y} \) yields \( \theta_{max_x} \neq \theta_{max_y} \) in general. In practical situations these values will be only slightly different and it will be reasonable to use the highest value to be on the conservative side.

\[ \theta_{max_x} = \frac{Q_x \cdot l}{8. k_x \cdot w \cdot h} + \theta_0 = (by \ using \ eq.187) = \theta_{av_x} + \frac{1}{2} (\theta_{av_x} - \theta_0), \]  
\hspace{1cm} (190)

\[ \theta_{max_y} = \frac{Q_y \cdot w}{8. k_y \cdot l \cdot h} + \frac{k_y \cdot l \cdot h}{2. w \cdot Q_y} \cdot (\theta_2 - \theta_1)^2 + \frac{\theta_2 + \theta_1}{2}, \]  
\hspace{1cm} (191)

Soderberg\(^{95} \) has improved this model for the particular case of no imposed external heat flow in the element. He assumes a parabolic distribution of the temperature in the element along both \( x \) and \( y \) directions, which results in a known analytical expression \( \theta(x,y) \) and therefore in unique values for the mean and maximum temperatures. This approach will be used later to model the heat flow in the cross section of the winding.
The foregoing procedure can be applied without difficulty to different bidimensional configurations. It can also be generalized to the three-dimensional case by adding new branches to the network in Fig. 27 accounting for the heat flow in the $z$ direction.

From now on it will be assumed that the equivalent networks developed in the preceding sections are an acceptable model for the bulk conduction in the different elements of the machine (the windings constitute an important particular case which is treated separately). This allows an equivalent network accounting for all the heat conduction processes to be built. The point now is that the resultant network is unnecessarily complex. Numerical analysis performed using data of actual machines and the common practice in the thermal modeling of rotating electrical machinery support the following simplifications: 13,94,75,59,74,91:

- The thermal conductivity of the stack of laminations in the transverse direction is roughly 20 times higher than in the axial direction. Moreover, actual machines data show that thermal resistances of the iron laminations in the transverse directions can be neglected with respect to other resistances in series with them in the overall equivalent circuit. Therefore the temperature of every single lamination will be assumed to be uniform, whereas axial changes in temperature will be considered. Consequently nodes 3 and 4 are glued together and likewise nodes 7 and 8.

- The bunch of wires and insulation forming the windings is simulated by means of a homogeneous medium with an effective thermal conductivity (when necessary two different conductivities, for the two main directions,
are considered) in the transverse direction.

- The heat transfer through the wedges is negligible if compared to what flows through the remainder of the slot surface.

Other numerical calculations were made by using realistic data, which showed that some conductive resistances (i.e., in the shaft-bearings region) had a negligible effect in the thermal behavior of the equivalent circuit, so they were eliminated or glued together with others, yielding the working model in Figure 20. Nevertheless it should be noticed that some of the decisions to be made on simplification issues depend heavily on the particular cooling implementation that has been adopted for the machine, and also of the specific construction.

3.6.3.2. Bulk conduction processes

The results of the preceding section will be used here. The application to the particular cases is straightforward.

Stator iron core in the axial direction

\[
R_{(3-4)-16}^{(3-4)} = \frac{1}{12} \cdot \frac{R_{OZ}^{(3-4)}}{24.\pi.kSZ} \cdot \frac{L}{(H-DS).(R+G+.5(H+DS))+DS.(1-kS).(R+G)}
\]

(192)

Rotor iron core in axial direction

The thermal resistances of the laminations and the shaft are in parallel.

\[
R_{(7-8)-17}^{(7-8)} = \frac{1}{12} \cdot \frac{R_{OZ}^{(7-8)}}{12.\pi} \cdot \frac{CL1}{kSH.RSH^2+kRZ.(R^2+DR^2-2.R.DR.kR)}
\]

(193)
Stator iron core in radial direction

As pointed out before this resistance may usually be neglected:

\[ R_4 = \frac{1}{2 \pi L kSR} \cdot \ln \frac{R + G + H}{R + G + DS} \]  \hspace{1cm} (194)

Resistances among the frame, stator end, end shield and bearings

Thermal resistances \( R_a, R_b, \) and \( R_d \) in Figure 28a can be roughly estimated from the characteristics of the external case which encloses the machine:

\[ R_a = \frac{1}{2} \cdot \frac{0.5 \times CL1}{kF \cdot 2 \pi RF \cdot TF} = \frac{CL1}{8 \pi kF RF TF} \]  \hspace{1cm} (195)

\[ R_b = \frac{1}{2} \cdot \frac{0.5 \times (CLT-CL1)}{kF \cdot 2 \pi RF \cdot TB} = \frac{CLT-CL1}{8 \pi kF RF TB} \]  \hspace{1cm} (196)

Fig. 28a

The value of \( R_d \) depends heavily on the actual construction of the end shield. The last expression may provide a reasonable approximation:
\[ R_d = R_b, \quad (197) \]

The value of \( R_c \) varies widely with the actual construction of the machine. It can have either high or low values. A low value improves the cooling of the stator core but at the same time gives a local hot spot in the external case of the machine; in the program

\[ R_c = 0.5 \times (R_a + R_b), \quad (198) \]

which is usually a low value. It can be modified easily.

![Diagram](image)

**Fig. 28b**

**Fig. 28:** Equivalent network for conduction at the machine ends region.

The above configuration of resistances introduces a new node at the junction of \( R_a, R_b \) and \( R_c \). To prevent this a mesh-star transformation is used, giving
\[ R_{14-16} = R_b + R_c + \frac{R_b R_c}{R_a} \]  
(199)

\[ R_{9-16} = R_a + R_c + \frac{R_a R_c}{R_b} \]  
(200)

\[ R_{9-14} = R_a + R_b + \frac{R_a R_b}{R_c} \]  
(201)

**Thermal resistance between the frame and the stator core**

This thermal resistance is the combination of the conductive resistance of the iron laminations of the stator core in radial direction, in series with the parallel combination of the conductive and convective resistances between the neighbor external surfaces of the frame and the stator core. (See Fig. 29.a, where \( R'_{r4} \) comes from eq. (194) and a crude application of Fig. 27. It must be realized that the resistance \( R'_{r4} \) is small if compared with most of the others in the network, but it stands in the main heat transfer path.)

![Fig. 29a](image)

![Fig. 29b](image)

**Figure 29.** Conduction between the frame and the stator core.
The value of \( R_{4-9 \text{ cond.}} \) depends heavily on the actual construction of the machine; it can be made very small by placing plates with good thermal conductivity between the frame and the stator core. But if the machine has a good forced cooling system the resistance \( R_{4-9 \text{ cond.}} \) can be let to be bigger, so the frame has a lower temperature.

The program uses the approximate circuit shown in Fig. 29b, where \( R_{4-9 \text{ cond.}} \) can be assigned any value according with the actual machine characteristics. This approximation allows not to introduce a new unknown temperature, since \( R_{4-9 \text{ conv}} \) is temperature dependent.

**Thermal resistance between the air in the air-gap and in the overhang region.**

With no axial air flow this is a pure conductive resistance. Its value is very high due to the small thermal conductivity of still air, so this resistance will not be included in the thermal circuit.

**Axial thermal resistance of the external part of the shaft**

Accounting for both ends of the machine:

\[
R_{17-13} = \frac{CLT - CL1}{4.\pi.kSH.RSH^2},
\]  
(202)

**Thermal resistance across the bearings**

The information that has been found in the literature \(101,102,100\) does not allow to write down an equation to model the thermal resistance across the bearings. Nevertheless it can be stated that its value will in general be small if compared with neighbor resistances, like the shaft resistance in axial direction, if journal or ball bearings are to be used.
If this is the case an estimated value will be assigned by comparison with data of similar bearings.

In the actual program the adopted value is one fifth of the thermal resistance of the external shaft in the axial direction.

3.6.3.3 The model for the windings

Effective transverse thermal conductivity in the slot region

It is fairly obvious that handling all the wires of some winding separately for modelling purposes is a very time consuming operation. The alternative is to devise some means of finding the properties of a fictitious homogeneous medium with approximately the same thermal conduction properties as the bunch of conductors with their insulations.

The isotropy, with respect to the thermal conductivity, of the equivalent medium depends on the actual arrangement of conductors and insulation within the slots. For instance, the configuration shown in Fig. 30a should be approximated reasonably by an isotropic medium, since

Fig. 30 a

Fig. 30 b

Fig. 30 Slot cross section
the thermal conductivity does not differ greatly for different directions of heat flow across the array of conductors. But obviously this is not the case for the configuration in Fig. 30b, where two different thermal conductivities $k_x$ and $k_y$, where $k_x > k_y$, should be used for the equivalent homogeneous medium.

The approach that has been adopted in order to obtain the apparent thermal conductivity (or conductivities) of the windings in the transverse direction is certainly rough, but it can be applied very easily and it has proved to give satisfactory results $^{75,72,12}$. Numerical results obtained from this procedure have been compared with the ones offered by more elaborate methods $^{68,88,82}$ (but not easily workable in terms of a model to be handled by computer) and again the agreement has been considered satisfactory.

The following initial assumptions have been adopted:

a) All the wires have the same insulation

b) The thickness $t$ of every insulation is uniform around every wire

c) No air gaps are present. Anyway all the interspace will be considered to be filled with the solid insulation

d) The thickness of the peripheral insulation is uniform for every direction (lateral: $a_i$; vertical: $b_i$).

e) There is no significative heat flow through the wedge. This assumption, together with the even azimuthal distribution of slots and teeths and also with the assumed uniform temperature for every lamination, results in no net external heat flow crossing the slot region, which simplifies the model.

f) The exterior surface is at a uniform temperature (which is a
direct consequence of the previous assumption of uniform temperature for each lamination)

g) No heat flow in the axial (Z) direction is assumed. This assumption will be removed later; it is used here to make simpler the presentation of this model which gives parameters per unit of axial length. Therefore the magnitude $Q$, which is used in this section, in a general sense should mean the amount of heat coming out transversely ($x$ and $y$ directions) from the slot per unit time and per unit length; on the other hand $Q$ is equal to the heat generated internally in the slot plus the heat input in the axial (Z) direction minus the heat output in the axial direction, everything per unit time and per unit length.

h) For practical purposes the thermal conductivity of the conductors may be considered to be infinite, since it is much higher than the one for the usual insulating materials. Nevertheless the calculations have been prepared considering two finite conductivities $k_c$ (conductor) and $k_i$ (insulation).

![Diagram of heat flow in the slot](image)

Fig. 31 Heat flow in the slot
The artifice employed to determine the apparent thermal conductivity of the windings is as follows: The wire of the coil is imagined to be replaced by a conductor of rectangular cross section which has the same cross-sectional area as the wire, and an insulation of uniform thickness and of the same cross-sectional area as the actual insulation, including the space between the insulated wires. The ratio of the side lengths of the rectangular cross section of the conductor is further chosen so that the equivalent coil has the same number of windings and the same number of layers as the original coil. An example of this is presented in Figures 32a and b; in this case the fictitious wires have square cross section with the same circular-mil area of copper; the width of these square wires being equal to the diameter over the insulation of the

Fig. 32a

Fig. 32b

Fig. 32 Detailed fictitious geometry of the winding cross section.

original round wires. These equations follow:
\[ A = \frac{\pi}{4} \cdot (d - 2.t)^2 = W^2 \quad , \tag{203} \]

\[ d^2 - A = (W + 2.u)^2 \quad , \tag{204} \]

which is equivalent to

\[ d = 2u + W \quad , \tag{205} \]

In a general case the equivalent cross section is shown in Fig. 32c.

Now it is assumed that for heat flow in the vertical (y) direction the horizontal (xz) planes are isothermals. Likewise for heat flow in the horizontal (x) direction the vertical planes (yz) are considered isothermals. This allows the thermal resistances of one generalized element (Fig. 32c) for heat flow in the two main directions to be calculated in a straightforward manner:

\[ R_x = \frac{1}{k_i} \cdot \frac{2.u}{(W_y + 2.u) \cdot 1} + \frac{W_x}{k_i \cdot 2.u \cdot 1 + k_c \cdot W_y \cdot 1} \quad , \tag{206} \]

\[ R_y = \frac{1}{k_i} \cdot \frac{2.u}{(W_x + 2.u) \cdot 1} + \frac{W_y}{k_i \cdot 2.u \cdot 1 + k_c \cdot W_x \cdot 1} \quad (207) \]
where unit depth (z direction) has been assumed.

The corresponding resistances of an element of the same geometrical dimensions as the fictitious homogeneous medium are:

\[
R_x = \frac{1}{k_x} \cdot \frac{W_x + 2u}{(W_y + 2u) \cdot 1}, \quad (208)
\]

\[
R_y = \frac{1}{k_y} \cdot \frac{W_y + 2u}{(W_x + 2u) \cdot 1}, \quad (209)
\]

Therefore the expressions for the apparent thermal conductivities for heat flow in the x and y directions are:

\[
\frac{1}{k_x} = \frac{1}{k_i} \cdot \frac{2u}{W_x + 2u} + \frac{W_x \cdot (W_y + 2u)}{(2u \cdot k_i + W_y \cdot k_c) \cdot (W_x + 2u)}, \quad (210)
\]

\[
\frac{1}{k_y} = \frac{1}{k_i} \cdot \frac{2u}{W_y + 2u} + \frac{W_y \cdot (W_x + 2u)}{(2u \cdot k_i + W_x \cdot k_c) \cdot (W_y + 2u)}, \quad (211)
\]

And if \( k_c \) is considered to be infinite:

\[
k_x = k_i \cdot (1 + \frac{W_x}{2u}), \quad (212)
\]

\[
k_y = k_i \cdot (1 + \frac{W_y}{2u}), \quad (213)
\]

In many cases \( W_x = W_y \) and therefore the heat conductivity is about the same in both directions, allowing one to write:

\[
k_x = k_y = k_i \cdot (1 + \frac{W_x}{2u}) = \frac{k_i}{1 - \frac{W}{W+2u}} = \frac{k_i}{1 - \sqrt{WSF}}, \quad (214)
\]
where WSF is the winding space factor if the external layer of insulation is not included:

\[
\text{WSF} = \frac{\text{Area of the cross section of the winding (only copper)}}{\text{Area of the cross section of the winding (copper plus insulation of the wires)}} = \frac{W^2}{(W+2u)^2}
\]

(215)

**The model for the transverse heat conduction in the windings**

Since the hottest spots are expected to be in the windings, these should be modelled with special care. One of the main features of the windings is that their axial dimension is much bigger than the transverse ones. It is therefore reasonable to use the approximate expressions of Section 3.6.3.1 to model the transverse heat flow and to use the exact analytical expression for the flow of heat in the axial direction.

The thermal behavior of the different bunches of wires corresponding to each slot is completely equivalent, for rotor and stator separately. It is therefore impractical to handle each of them independently. Hence a fictitious piece of winding has been devised, which summarizes the whole stator or rotor windings (see Fig. 33).

Since the machine is assumed to be double end ventilated, each turn of a bunch of wires can be divided in four parts, as shown in Fig. 33, which are completely equivalent under the thermal point of view. Therefore in each winding (stator or rotor one) there are 2xS elements like these, where S is the total number of slots of the winding. The following overall parameters are defined:

\[
A_{c_1} \quad \text{and} \quad A_{c_2} = 2xS \text{ times the copper cross section of the bunch of wires corresponding to one slot:}
\]
\[ A_c = A \times CS \times 2 \times S \quad , \]  

(216)

where CS is the number of conductors per slot.

\( A_{e_1} \) and \( A_{e_2} \) = 2 \( \times \) S times the area of the external surface of the bunch of wires corresponding to one slot, per unit of length.

\( b_1 \) and \( b_2 \) = length of the embedded and overhang parts, respectively, of one of the 2 \( \times \) S basic elements, where:

\[ b_1 = 0.5 \times CL_1 \quad , \]  

(217)

\[ b_2 = 0.5 \times CL_M \quad , \]  

(218)

\( q_1 \) and \( q_2 \) = 2 \( \times \) S times the total internal heat input per unit length to the parts 1 or 2 of the bunch of wires corresponding to one slot.

Usually:

\[ q_1 = q_2 = \frac{1}{b_1 + b_2} \times \text{copper losses in the whole winding} \quad , \]  

(219)

All these relations come from considering the overall equivalent element as a shunt combination of the 2 \( \times \) S basic elements, under the heat conduction point of view.

Fig. 33 Generalized Winding
In this section the model for the transverse heat conduction in the windings is presented. The goal of this cross section model will be to relate the mean $\theta_{av}$ and maximum $\theta_{max}$ temperature rises of a generic cross section, over the temperature $\theta_a$ of the environment, to the amount of heat $Q$ coming out of the whole embedded winding (overall equivalent model just shown) transversely, per unit of axial length and per unit time (see Fig. 31). $Q$ is a dummy quantity, only used to calculate $R$.

This can be accomplished by determining an overall thermal resistance $R$, per unit of axial length, between the winding (represented as usual by the spot with the mean temperature $\theta_{av}$) and the environment at temperature $\theta_a$. No axial heat flow is considered in this definition. Thus:

$$R_{overall} = \frac{\theta_{av} - \theta_a}{Q} = \frac{1}{2xS} \times R_{for\ the\ bunch} \quad \text{in one slot,} \quad (220)$$

The model for transverse conduction at the embedded part of a winding

Figure 34a shows the cross section to be modelled. For the embedded part of the winding there is no thermal resistance between the exterior surface of the external layer of insulation and the environment (the laminations). Therefore the ambient temperature $\theta_a$ is the laminations temperature in this case. There is no heat flow across the $x$ (because of the wedge) and $y$ (because of the azimuthal symmetry) axis. Therefore the configuration can be studied with a two-dimensional conduction model with uniformly distributed internal heat input and no external heat input.
Figure 34. Transverse conduction in an embedded winding

Figure 34b gives the approximate equivalent network for the transverse heat flow. The values for the resistances are the following:
\[ R_{xc} = \frac{1}{2S} \times \frac{a}{6.\,b\,k_x} = \frac{1}{2S} \times \frac{0.5xW-T_x}{6.(D-G-2xT_y).k_x} \quad , \]  

(221)

\[ R_{yc} = \frac{1}{2S} \times \frac{b}{6.a\,k_y} = \frac{1}{2S} \times \frac{D-G-2.T_y}{6.(0.5xW-T_x).k_y} \quad , \]  

(222)

\[ R_{xi} = \frac{1}{2S} \cdot \frac{1}{2} \cdot \frac{T_x}{b\,k_{ei}} = \frac{1}{2S} \cdot \frac{T_x}{2.(D-G-2.T_y).k_{ei}} \quad , \]  

(223)

\[ R_{yi} = \frac{1}{2S} \cdot \frac{T_y}{2.a\,k_{ei}} = \frac{1}{2S} \cdot \frac{T_y}{2.(0.5xW-T_x).k_{ei}} \quad , \]  

(224)

\[ R_x = R_{xc} + R_{xi} \quad , \]  

(225)

\[ R_y = R_{yc} + R_{yi} \quad , \]  

(226)

where:

- \( 2 \times S \) = Total number of basic elements in the fictitious winding (SS for the stator and SR for the rotor.) (In general most of the variables in these equations can be applied to the stator or rotor of the machine just by adding R or S subscripts.)

- Notice that Q accounts for all the slots filled with the winding. This is the reason for the factor \( \frac{1}{2S} \) in the resistances.

- \( k_{ei} \) is the thermal conductivity of the external layer of insulation around the whole bunch of conductors.

- All the equations are written assuming unit depth.

The circuit of Figure 34b comes directly from the more general case
of Figure 27, as Figure 35 shows. The bunch of conductors can be considered to be one half of the configuration shown in Figure 35a. Its equivalent circuit can be obtained from Figure 27 by setting \( \theta_1 = \theta_2 \) and \( \theta_3 = \theta_4 \), and has been represented in Figure 35b. It can be further simplified into Figure 35c which is equivalent, as far as \( \theta_{av} \) is concerned, to the circuit in Figure 35d, giving the values for \( R_{xc} \) and \( R_{yc} \).

The resistances \( R_{xi} \) and \( R_{yi} \) are just the thermal resistances offered by the external layer of insulation in the directions \( x \) and \( y \):

\[
R_{xi} = \left( \frac{T_x}{b \cdot k_e} \right) \parallel \left( \frac{T_y}{b \cdot k_e} \right)
\]

\[
R_{yi} = \frac{T_y}{k_e x (2a)}
\]

which again must be divided by \( 2xS \).

To obtain the mean and maximum temperatures of the cross section, the procedure shown in section 3.6.3.1 can be applied. However, for this particular case in which there is no externally imposed heat flow, the expressions obtained by Soderberg are applicable and will be used since they provide better accuracy:

\[
\theta_{av} = \theta_a + Q \cdot \frac{1}{\frac{1}{R_x} + \frac{1}{R_y}} \cdot \left(1 - \frac{R_{xc} \times R_{yc}}{5xR_x \times R_y} \right),
\]  
(227)

\[
\theta_{max} = \theta_a + Q \cdot \left(\frac{R_{xi} + 1.5 \times R_{xc}}{R_x} \cdot \frac{R_{yi} + 1.5 \times R_{yc}}{R_y} \right) \times
\]

\[x \left(1 - \frac{R_{xc} \cdot R_{yc}}{8 \times R_x \times R_y} \right),
\]  
(228)
Figure 35. Equivalent networks for embedded windings cross section.
It should be noticed that both $\theta_{av} - \theta_a$ and $\theta_{max} - \theta_a$ depend linearly on $Q$. These equations apply to every transverse section of the winding, and therefore the quantities $Q, \theta_a, \theta_{av}$ and $\theta_{max}$ can change along the axial direction (different transverse sections). However, the overall resistance parameter $R_E$, defined in equation (220), is invariant along the axial direction as far as the geometrical dimensions of the slot do not change:

$$R_E = \frac{\theta_{av} - \theta_a}{Q} = \frac{1}{\frac{1}{R_x} + \frac{1}{R_y}} \cdot \left(1 - \frac{R_{xc} R_{yc}}{5R_x R_y}\right), \quad (229)$$

The model for transverse conduction at the end part of a winding

As in the preceding section, the goal here is to obtain the overall coefficient $R_o$ of transverse thermal resistance between the end winding (represented by a spot with the mean temperature $\theta_{av}$) and the environment at temperature $\theta_a$, for any cross section of the winding:

$$R_o = \frac{\theta_{av} - \theta_a}{Q}, \quad (230)$$

where $Q$ is the heat flowing out of the winding transversely per unit time and per unit of axial length. ($Q$ accounts for the whole end part of the winding.)

The differences with the foregoing section are the following:

a) There are no wedges. Thus, the heat will flow transversely through the whole periphery of the winding. Figure 36 gives the new definition of the geometrical dimensions.

b) The relation between the temperature at the external surface
of the winding and the environment temperature $\theta_a$ depends on the specific implementation of the cooling of the end winding. The simplest case corresponds to a configuration with the winding totally embedded in a solid body at uniform temperature $\theta_a$, since the heat gets out by conduction and the whole external surface has a temperature $\theta_a$ (this can be a reasonable approximation for the rotor end winding if it is totally enclosed by the end ring). Much more complex is the situation for a usual stator end winding, where two modes of heat transfer occur from its external surface: convection and radiation, the point being that the heat transferred by radiation is independent of the temperature of the air in the overhang region. Two approaches are available in order to overcome this difficulty:

1) To handle the radiation process by means of an equivalent convective one. In this way everything can be referred to the ambient temperature $\theta_a$ of the air in the overhang region. But this approach requires a quite sophisticated implementation in the computer program, since the equivalent convective resistance is dependent on the unknown temperatures of the elements "seen" by the stator end winding. This inconvenience can be avoided with the simpler approach that follows:

2) It is assumed that the surface temperature of the winding is uniform around its periphery. By using this temperature (which is a new unknown) both the convection and radiation processes can be expressed easily. This temperature will be $\theta_a$ in the equation (230). So the equations are made easier at the cost of increasing the number of unknowns by one.

The values for the resistances $R_{xc}$, $R_{x_i}$, $R_{yc}$, $R_{yi}$ in the approximate
Figure 36. Transverse conduction in the end part of a winding.

Figure 37. Equivalent networks for end windings cross section.
equivalent network of the Fig. 36b are:

\[
R_{xc} = \frac{1}{2.5} \cdot \frac{a}{6.5 b k_x} = \frac{0.5 \times W - T_x}{2.5 \times 6 \times (D - G - 2x T_y) \times k_x}, \tag{231}
\]

\[
R_{yc} = \frac{1}{2.5} \cdot \frac{b}{24 \times a \times k_y} = \frac{1}{2.5} \cdot \frac{D - G - 2x T_y}{24 \times (0.5 \times W - T_x) \times k_y}, \tag{232}
\]

\[
R_{xi} = \frac{1}{2.5} \cdot \frac{T_x}{2.5 b k_{ei}} = \frac{1}{2.5} \cdot \frac{T_x}{2 \times (D - G - T_y) \times k_{ei}}, \tag{233}
\]

\[
R_{yi} = \frac{1}{2.5} \cdot \frac{T_y}{2.5 a k_{ei}} = \frac{1}{2.5} \cdot \frac{T_y}{2 \times (0.5 \times W - T_x) \times k_{ei}}, \tag{234}
\]

\[
R_x = R_{xc} + R_{xi}, \tag{235}
\]

\[
R_y = R_{yc} + R_{yi}, \tag{236}
\]

The foregoing expressions have been obtained following the same procedure as in the preceding section. It is summarized in Fig. 37, which is self-explanatory. \( R_{xi} \) and \( R_{yi} \) are determined as in the preceding section.

The values for \( \theta_{av}, \theta_{max} \) and \( R_0 \) can be obtained by using the eqs. (227), (228), and (229) with the new values of the resistances as given in eqs. (231) to (236). Thus:

\[
R_0 = \frac{\theta_{av} - \theta_{a}}{\theta_{Q}} = \frac{R_x \cdot R_y}{R_x + R_y} \cdot (1 - \frac{R_{xc} \cdot R_{yc}}{5 \times R_x \times R_y}), \tag{237}
\]

It follows the application of these equations to the particular
case of the rotor end winding. The assumed configuration is characterized by the following features:

- The end winding is completely enclosed by the end ring.
- The thermal resistance between the external surface of the winding and the end ring is negligible. Therefore \( \theta_a = \theta_{\text{external surface}} \) \(^{(238)}\)
- The whole external surface of the winding makes contact with the end ring. This may not be true, if there are insulating pieces of material helping to fix the windings. This depends very much on the actual construction of the machine. Better assumptions can be made tailoring the model to a specific machine.

With these assumptions the model for the end winding is very much like the one for the embedded winding. The value that should be used for \( \theta_a \) is the temperature of the rotor end (different from the temperature of the enclosed air, because of the convective resistance in between). This means

\[
\theta_a = \theta_{(12-17)} ,
\]

\(^{(239)}\)

The following assumption is made in the application of the presented theory to the transverse heat flow in the stator end winding:

- The surface temperature around the periphery is uniform and equal to the reference temperature \( \theta_a \) in the standard equations used in the preceding sections. This temperature will be one of the basic unknowns in the final system of equations to be solved and it will be referred to as \( \theta_{\text{ext}} \). As indicated previously, this assumption is made since the surface coefficients of heat transfer for convection and radiation are
temperature dependent and the calculations would become too involved
(with dubious improvement of the accuracy) if a different temperature for
each face were considered. Therefore:

- All the geometrical and insulating properties of the windings are
uniform along their length.

Therefore, the standard model for the cross section of the end wind-
ings can be used if

\[ \theta_a = \theta_{\text{ext2}} \quad , \]

(240)

The complete model for the windings. Accounting for the axial and
transverse heat flows.

Experience with thermal models for rotating electrical machinery
indicates that the hottest spots might be expected in the windings. The
most outstanding feature of the windings, with regard to the thermal
model, is their length as compared with their transverse dimensions.
Therefore it seems reasonable to use a thermal model accounting specific-
ally for the temperature distribution along the winding length. This has
been done in the past by several authors\textsuperscript{87,82,72,89,59,63}. The equations
will not be derived here, but the results will be offered as presented in
reference 68.

The idea is to treat the heat flow in the embedded and end windings
and the exchange of heat between them and the surrounding elements as a
one-dimensional problem, whose basic features are sketched in Fig. 38.
Fig. 38 Complete model for the winding.

a) Simplified geometry; b) axial temperature distribution ($\theta_{\text{max}_1} > \theta_{\text{max}_2}$)

Following is a list of the symbols used in the subsequent equations:

- $A_e$: Exposed heat-dissipating area of part
- $A_c$: Cross sectional area of part
- $b$: Axial length of part
- $k$: Axial thermal conductivity
- $\theta_a$: Temperature of environment around part
- $\Delta \theta_t$: Temperature rise of part if no axial heat flow could occur,
  
  $\theta_t = \theta_a + \Delta \theta_t$  
  (transverse flow only)

- $\theta_m$: Mean temperature of whole bar or rod
- $\theta_x$: Temperature of any point $x$ of part
- $X$: Distance from exposed end of part
\[ q = \text{Internal heat input to part, per unit of axial length} \]

\[ R = \frac{\Delta \theta t}{q}, \quad (241) \]

overall surface resistance coefficient between the unit of axial length of area \( A \) and the environment (already defined and calculated).

\[ \alpha = \left[ \frac{1}{k \cdot A_c \cdot R} \right]^\frac{1}{2}, \quad (242) \]

\[ \eta_x = \frac{\tanh \alpha \cdot b}{\alpha \cdot b}, \text{ effectiveness factor of part}, \quad (243) \]

The following assumptions are the starting point for the one-dimensional model:

- It has been shown that the transverse internal thermal resistance (conduction) can be modelled by means of the overall resistance coefficient \( R \). The net effect being that no transverse internal thermal resistance is considered.

- Heat flow from the unjoined ends of each part is negligible. If the machine is cooled symmetrically at both ends, \( x_1 = 0 \) and \( x_2 = 0 \) can be made to correspond to the middle of the embedded and end windings, respectively.

- The cross sectional area, overall surface resistance coefficient, internal thermal resistivity, environment temperature, and heat generated per unit volume are uniform throughout each part, but may differ for the two parts.

- The distribution of axial heat flow is assumed uniform over the
cross section of each part, in spite of discontinuities in heat flow
density which this would require at junctions between parts.

Under these assumptions there is a unidimensional heat flow, and
the heat balance in steady state conditions is given by the differential
equation:

\[ q + \frac{\theta_x - \theta_a}{R} - kA_c \frac{d^2 \theta_x}{dx^2} = 0 \]

which means that the heat developed in the unit length of the winding is
equal to the sum of the heat conducted to the neighboring space elements
and the heat transferred through its surface.

The solution to this equation, with the indicated boundary conditions,
gives the local temperature of any cross section of the windings. The
resultant temperature \( \theta_x \) for any cross section is the mean temperature
of the section, since the overall resistance coefficient was designed in
this way. The expressions are:

\[ \theta_{x_1} = \theta_{t_1} - \frac{\theta_{t_1} - \theta_{t_2}}{R_2 \cdot \eta_{x_1} \cdot b_1} \cdot \frac{\cosh \alpha_1 \cdot X_1}{\cosh \alpha_1 \cdot b_1} = \theta_{t_1} - \beta_1 \cdot \frac{\cosh \alpha_1 \cdot X_1}{\cosh \alpha_1 \cdot b_1} \]

\[ (244) \]

for \( 0 \leq X_1 \leq b_1 \), where \( \beta_1 = \frac{\frac{R_2}{R_1} \cdot \eta_{x_1} \cdot b_1}{1 + \frac{R_2}{R_1} \cdot \eta_{x_2} \cdot b_2} \)

\[ (245) \]

\[ \theta_{x_2} = \theta_{t_2} - \frac{\theta_{t_2} - \theta_{t_1}}{R_1 \cdot \eta_{x_2} \cdot b_2} \cdot \frac{\cosh \alpha_2 \cdot X_2}{\cosh \alpha_2 \cdot b_2} = \theta_{t_2} - \beta_2 \cdot \frac{\cosh \alpha_2 \cdot X_2}{\cosh \alpha_2 \cdot b_2} \]

\[ (246) \]
for \( 0 \leq x_2 \leq b_2 \), where

\[
\beta_2 = \frac{\theta_{t2} - \theta_{t1}}{1 + \frac{R_1 \cdot n_{x2} \cdot b_2}{R_2 \cdot n_{x1} \cdot b_1}},
\]

(247)

From these expressions all the required practical information can be obtained.

Fig. 38b shows that the maximum temperature \( \theta_{x_{\text{max}1}} \) of part 1 may be at \( x = 0 \) or \( x = b_1 \), and correspondingly for part 2. Since it is not known a priori whether \( \theta_{x_{\text{max}1}} \) or \( \theta_{x_{\text{max}2}} \) is bigger, the following two temperatures have to be checked out:

\[
\theta_x(X_1 = 0) \quad \text{and} \quad \theta_x(X_2 = 0).
\]

But these temperatures are mean temperatures \( \theta_{av_{\text{max}}} \) in the cross sections. The procedure to obtain the actual \( \theta_{\text{max}} \) for each of these three sections (one of them is the hottest spot of the winding) is the following:

a) Obtain \( Q \) for the corresponding cross section (heat coming out transversally, per unit time and axial length), using the \( \theta_{x_{\text{max}}} \) just obtained here:

\[
Q = \frac{\theta_{x_{\text{max}}} - \theta_a}{R},
\]

(248)

for \( X_1 = 0 \), \( X_2 = 0 \) and \( X_2 = b_2 \).

b) Obtain the actual \( \theta_{\text{max}} \) in the section by using eq. (228) with the \( Q \)'s just calculated:

\[
\theta_{\text{max}} = \theta_a + Q \cdot \frac{(R_{x_i} + 1.5 \ R_{xc}) \cdot (R_{y_i} + 1.5 \ R_{yc})}{R_x + R_y} \cdot (1 - \frac{3}{8} \cdot \frac{R_{xc} \cdot R_{yc}}{R_x \cdot R_y}),
\]

(249)
The heat transferred from part 1 to part 2 is defined as $q_{1\rightarrow 2}$ and is calculated as follows:

$$q_{1\rightarrow 2} = -k_1 \cdot A_{c_1} \cdot \left. \frac{d \theta_{x1}}{dx_1} \right|_{x_1=b_1} = k_1 \cdot A_{c_1} \cdot \alpha_1 \cdot \beta_1 \cdot \tanh \alpha_1 \cdot b_1,$$

$$= k_1 \cdot A_{c_1} \cdot b_1 \cdot \alpha_1 \cdot \eta_{x1} \cdot \beta_1 = \frac{b_1 \cdot \eta_{x1}}{R_1} \cdot \beta_1.$$

(250)

It can be checked easily that

$$q_{1\rightarrow 2} = -q_{2\rightarrow 1} = k_2 \cdot A_{c_2} \cdot \left. \frac{d \theta_{x2}}{dx_2} \right|_{x_2=b_2} = -\frac{b_2 \cdot \eta_{x2}}{R_2} \cdot \beta_2 = \frac{b_1 \cdot \eta_{x1}}{R_1} \cdot \beta_1.$$

(251)

The overall heat transfer equations for the parts 1 and 2 can be written in the form of heat inputs to the environment: ambients "a1" and "a2", as follows:

$$q_{1\rightarrow a_1} = b_1 \cdot q_1 - q_{1\rightarrow 2} = b_1 \cdot (q_1 - \frac{\beta_1 \cdot \eta_{x1}}{R_1}),$$

(252)

$$q_{2\rightarrow a_2} = b_2 \cdot q_2 - q_{2\rightarrow 1} = b_2 \cdot (q_2 - \frac{\beta_2 \cdot \eta_{x2}}{R_2}),$$

(253)

And finally the mean temperatures for the windings are:

$$\theta_{m_1} = \theta_{t_1} - \frac{R_1}{b_1} \cdot \frac{\theta_{t_1} - \theta_{t_2}}{\frac{R_1}{b_1 \cdot \eta_{x1}} + \frac{R_2}{b_2 \cdot \eta_{x2}}},$$

(254)

$$\theta_{m_2} = \theta_{t_2} - \frac{R_2}{b_2} \cdot \frac{\theta_{t_2} - \theta_{t_1}}{\frac{R_1}{b_1 \cdot \eta_{x1}} + \frac{R_2}{b_2 \cdot \eta_{x2}}},$$

(255)
Application to the rotor windings

The objective of this section is twofold:

a) To obtain equations relating the internal heat input of both parts of the winding (end and embedded parts) to the temperatures of the neighboring nodes of the equivalent thermal circuit, as required in the basic equation (141):

\[ f(\text{CLCR}_e, \theta_8, \theta_{17}) = 0 \quad , \quad (256) \]

\[ g(\text{CLCR}_0, \theta_8, \theta_{17}) = 0 \quad , \quad (257) \]

where \( \text{CLCR}_e \) and \( \text{CLCR}_0 \) are the total rotor copper losses for the embedded and overhang parts of the rotor winding. The ambient temperatures to be used are \( \theta_{a1} = \theta_8 \) and \( \theta_{a2} = \theta_{17} \). It should be noticed that here it is assumed that the rotor iron core temperature is uniform over the embedded winding length. This assumption can be removed at the cost of increasing the analytical complexity of the working equations just obtained\(^6^3\).

b) To determine the temperature of the hottest spot of both parts of the winding, in terms of their internal heat inputs \( L_{CR_e} \) and \( L_{CR_0} \) and the temperatures of the neighboring nodes:

\[ \theta_{\text{max}}_{Re} = \theta_{\text{max}}_{Re} (\text{CLCR}_e, \text{CLCR}_0, \theta_8, \theta_{17}) \quad , \quad (258) \]

\[ \theta_{\text{max}}_{Ro} = \theta_{\text{max}}_{Ro} (\text{CLCR}_e, \text{CLCR}_0, \theta_8, \theta_{17}) \quad , \quad (259) \]

The equations obtained in the last section can be applied directly if the following rules are followed:

- The indices 1 and 2 represent the embedded and end part of the
windings, respectively.

- The fictitious winding of Fig. 38a accounts for the whole actual winding:

\[ A_{C_2} = A_{C_1} = 2 \times SR \times AR \times CSR \quad , \]

\[ b_1 = 0.5 \times CL1 \quad , \]

\[ b_2 = 0.5 \times CLMR \quad , \]

\[ q_2 = q_1 = \frac{CLCR}{b_1 + b_2} \quad , \]

- The overall resistance coefficients \( R_1 \) and \( R_2 \) have the values calculated previously. Then eq. (229) gives \( R_1 \) and eq. (237) gives \( R_2 \). Thus:

\[ R_1 = R_x \cdot R_y \left( 1 - \frac{R_{xc} \cdot R_{yc}}{5 \cdot R_x \cdot R_y} \right) \quad , \]

where:

\[ R_{xc_1} = \frac{1}{2 \times SR} \cdot \frac{0.5 \; WR - TXR}{6 \cdot (DR-GR-2.TYR) . k_x} \quad , \]

\[ R_{yc_1} = \frac{1}{2 \times SR} \cdot \frac{DR-GR-2.TYR}{6 \cdot (0.5 \; WR-TXR) . k_y} \quad , \]

\[ R_{x_1} = R_{xc_1} + \frac{1}{2 \times SR} \cdot \frac{TXR}{2 \cdot (DR-GR-2.TYR) . k_{ei}} \quad , \]

\[ R_{y_1} = R_{yc_1} + \frac{1}{2 \times SR} \cdot \frac{TYR}{2 \cdot (0.5 \; WR - TXR) . k_{ei}} \quad , \]

For \( R_2 \) the eq. is also (264) where:
\[ R_{xc2} = R_{xc1} \]  

(269)

\[ R_{yc2} = 0.25 \times R_{yc1} \]  

(270)

\[ R_{x2} = R_{x1} \]  

(271)

\[ R_{y2} = 0.25 \times R_{yc1} + R_{y1} - R_{yc1} = R_{y1} - 0.75 \times R_{yc1} \]  

(272)

- \( k \) is the thermal conductivity of the copper.

- The environment temperatures are:

\[ \theta_{a1} = \theta_8; \quad \theta_{a2} = \theta_{17} \]  

(273)

- Eqs. (241), (242) and (243) are used in the same way:

\[ \theta_t = \theta_a + q \times R \]  

(274)

\[ \alpha = (k \times A_c \times R)^{-0.5} \]  

(275)

\[ \eta_x = \frac{1}{\alpha \times b} \times \tanh (\alpha \times b) \]  

(276)

The temperature of the hottest spot of the winding is the highest of the numerical values that are obtained from eq. (277) with subindexes 1 or 2 (from eqs. (241), (244), (246), (248), and (249)):

\[ \theta_{\text{max}} = \theta_a + \left[ q - \frac{\beta}{R \times \cosh (\alpha \times b)} \right] \left[ \frac{(R_{xi} + 1.5 \times R_{xc})}{R_x} \times \frac{(R_{yi} + 1.5 \times R_{yc})}{R_y} + \left(1 - \frac{3}{8} \times \frac{R_{xc} \times R_{yc}}{R_x \times R_y} \right) \right] \]  

(277)
where:

\[ \beta_i = \frac{\theta_{ti} - \theta_{tj}}{1 + \frac{\eta_{x_i} \cdot b_i}{\eta_{x_j} \cdot b_j}} \]  \quad (278) 

with \( i, j = 1, 2 \) or \( 2, 1 \).

It is now necessary to relate the rotor winding heat input to the temperatures of the surrounding elements. Eqs. (252) and (253) can be used directly:

\[ q_{5+8} = b_1 \cdot \left( q_1 - \frac{\beta_1 \cdot \eta_{x_1}}{R_1} \right) = b_1 \cdot \left( q_1 - \frac{\eta_{x_1}}{R_1} \cdot \frac{\theta_{18} + q_1 R_1 - \theta_{17} - c_2 \cdot R_2}{1 + \frac{\eta_{x_1} \cdot b_1}{\eta_{x_2} \cdot b_2}} \right) \]  \quad (279)

\[ q_{6+17} = b_2 \cdot \left( q_2 - \frac{\beta_2 \cdot \eta_{x_2}}{R_2} \right) - b_2 \cdot \left( q_2 - \frac{\eta_{x_2}}{R_2} \cdot \frac{\theta_{17} + q_2 R_2 - q_1 R_1 - \theta_8}{1 + \frac{\eta_{x_2} \cdot b_2}{\eta_{x_1} \cdot b_1}} \right) \]  \quad (280)

since the embedded and end windings correspond to the elements 5 and 6 in the thermal circuit, respectively.

**Application to the stator winding**

This section is merely a repetition of the preceding one, but now with the stator winding. Therefore only the new list of eqs. to be used will be given here:

\[ A_{C_2} = A_{C_1} = 2 \times SS \times AS \times CSS \]  \quad (281)

\[ b_1 = 0.5 \times CL1 \]

\[ b_2 = 0.5 \times CLMS \]  \quad (282)
\[ q_1 = q_2 = \frac{CLCS}{b_1 + b_2} \tag{283} \]

\[ R_1 \text{ is given by eq. (229):} \]
\[ R_1 = \frac{R_x \cdot R_y}{R_x + R_y} \cdot (1 - \frac{R_{xc} \cdot R_{yc}}{5 \cdot R_x \cdot R_y}) \tag{284} \]

where:
\[ R_{xc1} = \frac{1}{2 \times SS} \cdot \frac{a}{6 \cdot b \cdot k_x} \tag{285} \]
\[ R_{yc1} = \frac{1}{2 \times SS} \cdot \frac{b}{6 \cdot a \cdot k_y} \tag{286} \]
\[ R_{x1} = R_{xc1} + \frac{1}{2 \times SS} \cdot \frac{TXS}{2 \cdot b \cdot k_e} \tag{287} \]
\[ R_{y1} = R_{yc1} + \frac{1}{2 \times SS} \cdot \frac{TYS}{2 \cdot a \cdot k_e} \tag{288} \]
\[ a = 0.5 \text{ WS} - \text{TXS} \tag{289} \]
\[ b = \text{DS} - \text{GS} = 2 \cdot \text{TYS} \tag{290} \]

\[ R_2 \text{ is given by eq. (237):} \]
\[ R_2 = \frac{R_x \cdot R_y}{R_x + R_y} \cdot (1 - \frac{R_{xc} \cdot R_{yc}}{5 \cdot R_x \cdot R_y}) \tag{291} \]

where:
\[ R_{xc2} = R_{xc1} \tag{292} \]
\[ R_{yc2} = 0.25 \times R_{yc1} \tag{293} \]
\[ R_{x_2} = R_{x_1} \]  
\[ R_{y_2} = R_{y_1} - 0.75 \times R_{yc_1} \]  
\[ \theta_{a_1} = \theta_4 \]  
\[ \theta_{a_2} = \theta_{ext_2} \]  
\[ \theta_t = \theta_a + q.R \]  
\[ \alpha = (k.A_c.R)^{-0.5} \]  
\[ \eta_x = \frac{1}{\alpha.b} \cdot \tanh(\alpha.b) \]  

The temperature of the hottest spot of the winding is the largest value of eq. (301) when evaluated with subindices 1 or 2:

\[ \theta_{max} = \theta_a + [q - \frac{\beta}{R.\cosh(\alpha.b)}] \cdot \left[ \frac{(R_{xi} + 1.5 \times R_{xc} \times R_{yi} + 1.5 \times R_{yc})}{R_x + R_y} \right] \cdot (1 - \frac{3}{8} \cdot \frac{R_{xc} \times R_{yc}}{R_x \times R_y}) \]  

where:

\[ \beta_i = \frac{\theta_{ti} - \theta_{tj}}{\frac{R_{ji \cdot n_{xi \cdot b_i}}}{1 + \frac{R_{ij \cdot n_{xj \cdot b_j}}}{R_{ij \cdot n_{xj \cdot b_j}}}} \]  

(302)
with \( i, j = 1,2 \) or \( 2,1 \).

The following equations relate the stator winding heat input to the rest of the system. Equation (252) gives:

\[
Q_{1 \rightarrow 4} = b_1 \cdot (q_1 - \frac{\beta \cdot \eta x_1}{R_1}) = b_1 \cdot (q_1 - \frac{\eta x_1}{R_1}) \cdot \frac{\theta_4 + q_1 \cdot R_1 - \theta_{ext2} - q_2 \cdot R_2}{1 + \frac{R_2 \cdot \eta x_1 \cdot b_1}{R_1 \cdot \eta x_2 \cdot b_2}} = f(\theta_4, \theta_{ext2}, \text{CLCS}) , \tag{303}
\]

which in the final system of eqs. can be considered as a heat input to element 4, and the node 1 can be completely ignored.

In the same way:

\[
Q_{2 \rightarrow ext2} = b_2 \cdot (q_2 - \frac{\eta x_2}{R_2}) \cdot \frac{\theta_{ext2} + q_2 \cdot R_2 - \theta_4 - q_1 \cdot R_1}{1 + \frac{R_1 \cdot \eta x_2 \cdot b_2}{R_2 \cdot \eta x_1 \cdot b_1}} = f(\theta_4, \theta_{ext2}, \text{CLCS}) \tag{304}
\]

3.6.4 Convection mode related expressions

3.6.4.1 Natural convection processes

Heat transfer in the machine by means of natural convection occurs only between the external part of the machine (frame and end shield) and the ambient when it is assumed that there are no forced means of external cooling.
There is also a natural convection process between the stator core and the frame, if no forced axial flow is considered; however, it is expected to make a small contribution to the cooling of well designed machines, as pointed out before.

**Thermal convective resistance between the frame and the ambient**

Much work has been done on the dissipation of heat by free convection from wires, tubes and flat plates, and some results obtained from those configurations will be used here. There are some available results obtained specifically from electric machinery, but they do not allow the consideration of the frame and the end shield separately. Therefore, more general equations will be used instead.

For free convection in air at normal atmospheric conditions, the general equation

\[ h = \frac{k}{\lambda} \cdot N_{Nu} = \frac{k}{\lambda} \cdot C \cdot (N_{GR} \cdot N_{PR})^m \quad , \]

will be used, as applied to horizontal cylinders, where the characteristic length \( \lambda \) is equal to the exterior diameter of the frame:

\[ \lambda = 2 \times RF \quad , \]

which gives:

\[ h_{9-15} = \frac{k_{15-9}}{2 \times RF} \cdot C \cdot (N_{GR_{15-9}} \cdot N_{PR_{15}})^m \quad , \]

The mean film temperature \( \theta_m = 0.5 \times (\theta_9 + \theta_{15}) \) must be used for the evaluation of the fluid properties. This is indicated with a subscript (15-9). Therefore:
\[
N_{GR}^{15-9} = \frac{9.8 \times 8 \times RF^3 \times (\frac{157}{\theta_9 - \theta_{15}})}{273 + \theta_{15}} \cdot \frac{(\rho)}{\mu_{15-9}}^2,
\]

\[
N_{PR}^{15-9} = (\frac{\mu \cdot C}{k})^{15-9},
\]

where \( C \) and \( m \) are determined as indicated below for the expected range of \( N_{PR} \cdot N_{GR} \) (which indicates the flow conditions):

- For \( 10 < (N_{GR} \cdot N_{PR})_{9-15} < 2 \times 10^9 \) (laminar flow): \( C = 0.53, \ m = 0.25 \):

\[
h^{9-15 \text{ laminar}} = \frac{0.265}{RF} \cdot [k \cdot (N_{GR} \cdot N_{PR})^{0.25}]_{9-15} \text{ watts/(m}^2 \cdot \text{C) , (310)}
\]

- For \( 5 \times 10^9 < (N_{GR} \cdot N_{PR})_{9-15} < 10^{12} \) (turbulent flow): \( C=0.104; \ m=\frac{1}{3} \):

\[
h^{9-15 \text{ turbulent}} = \frac{0.052}{RF} \cdot [k \cdot (N_{GR} \cdot N_{PR})^{\frac{1}{3}}]_{9-15} \text{ watts/(m}^2 \cdot \text{C) , (311)}
\]

In the transition region the eq. (311) is usually used as an approximation. In the program this equation is extrapolated down to

\[
N_{GR} \cdot N_{PR} = 3.07 \times 10^8
\]

so there are no jumps in the value of \( h \).

A mounted electrical machine cannot be perfectly modelled by a cylinder floating in an infinite medium. The expected accuracy of the above equations is therefore not very high. Unfortunately, temperature rises in the machine are very sensitive to the value of \( h_{9-15} \), especially in naturally cooled machines. The program contains the option of assigning a value to \( h_{9-15} \) externally.
The active surface in the convection from the frame to the ambient is:

\[ A_{9-15} = 2 \cdot \pi \cdot CL \cdot RF \]  \hspace{1cm} (312)

Thus the convective resistance is:

\[ R_{9-15} = \frac{1}{h_{9-15} \cdot A_{9-15}} \]  \hspace{1cm} (313)

Thermal convective resistance between the end shield and the ambient

This resistance consists of two parts in parallel, namely, the cylindrical part and the two vertical plates. The procedure is exactly the one followed in the preceding section, since the same correlations can be used for horizontal cylinders and vertical uniform-temperature plane surfaces.\(^{68}\)

Therefore the convective resistance \( R_{14-15} \) is:

\[ R_{14-15} = \frac{1}{(h_{14-15} \times A_{14-15})_{\text{plate}} + (h_{14-15} \times A_{14-15})_{\text{cylinder}}} = \frac{1}{h_{14-15} \times A_{14-15}_{\text{total}}} \]  \hspace{1cm} (314)

where \( h_{14-15} \) is obtained by using the expressions in the last section.

3.6.4.2 Forced convection processes

The forced convection processes taking place in the machine can be classified into three groups. The first concerns the non rotating elements, but with an impinging fluid due to the movement of the rotating parts; in this category are the resistances \( R_{16-11}, R_{14-11} \) and \( R_{2-11} \). The second
includes the forced convection from the non enclosed (non tightly enclosed) rotating elements, described by the resistances $h_{8-11}$ and $R_{12-11}$. The third group is for the enclosed rotating elements, such as the rotor iron related to the stator iron through the air gap.

In section 3.6.1 it was pointed out how $N_{Nu}$ depends on $N_{PR}$ and therefore on the fluid velocity in the forced convection processes. The determination of the required fluid velocities in the overhang region is a very involved problem in itself and only rough estimates of the actual velocities will be obtained.

If there is any doubt that the always present free convection is negligible, the corresponding film coefficient must be obtained. Then whichever $h$ is the highest, it should be used as a crude first conservative approximation. Sometimes more precise information is available\textsuperscript{98}.

a) Non rotating elements

**Thermal convective resistance for the stator end windings**

The external surface of the stator end winding is assumed to have a uniform temperature $\theta_{ext\ 2}$, as indicated before. The temperature of the surrounding air will be also considered to be uniform: $\theta_{11}$. The approximate effective area for convection can be calculated as (accounting for both ends):\[A_{2-11} = SS \times CLMS \times 2 \times (WS + DS - GS)\] (315)

The correlations to be used in the calculation of the film coefficient $h_{2-11}$ depend strongly on the characteristics of the flow of the coolant in the overhang region, which can be estimated only very roughly.
The movement of the rotating parts produces an overall movement of the bulk of the fluid in the overhang region (even without a fan), due to the fluid viscosity. The effect of the shaft is to produce a circular movement of the fluid around its axis of rotation. This effect is also produced by the end of the rotor, but this disk also produces a pump effect, which circulates a mass of flow such as indicated in Fig. 39.

![Diagram of coolant flow in the overhang region](image)

Fig. 39 Coolant flow in the overhang region

This mass flow has been evaluated for certain configurations of disks and enclosures, including baffle ring if any; but further research in this subject is needed to establish valid correlations for conditions of practical applications, like this one. Therefore no simple satisfactory approach is available to obtain the velocity in the bulk of the fluid outside the boundary layer for the elements of the overhang region. Following is the adopted approach, which is different for the natural and forced cooling situations.
Natural cooling:

It is reasonable to assume that the film coefficient $h_{2-11}$ lies in between these two extremes: pure free convection and the value corresponding to the most ventilated surfaces (those closest to the rotor, for which the correlations for enclosed cylinders can be reasonably used).

The correlation for free convection is given in eq. (305), where:

- The characteristic length $\ell_{2-11}$, which is also used for $N_{GR}$ and $N_{PR}$ is the equivalent diameter:

$$\ell_{2-11} = \frac{2}{\sqrt{\pi}} \left[ WS \cdot (DS - GS) \right]^2 ,$$

(316)

- The mean film temperature

$$\theta_m = 0.5 \times (\theta_{2 \text{ext}} + \theta_{11}) ,$$

(317)

is advised for the evaluation of the fluid properties.

- For $10 < (N_{GR} \cdot N_{PR})_{2-11} < 3.07 \times 10^8$, use $C = 0.53$ and $m = 0.25$

- For $3.07 \times 10^8 < (N_{GR} \cdot N_{PR})_{2-11} < 10^{12}$, use $C = 0.104$ and $m = \frac{1}{3}$.

The value for $h_{2-11}$ in the air gap enclosed between the stator windings and the rotor end can be assumed to be equal to the value for the completely enclosed part of the air gap, which will be determined later. The value for the film coefficient is twice the value given for $U$ in the mentioned expression.

The adopted value for the surface heat transfer coefficient $h_{2-11}$ is a weighted average of the values obtained from these two extreme assumptions.
b) Forced cooling

It will be assumed that a volume flow rate of \( W_2 \) m\(^3\)/sec. is forced to pass through the stator end windings (both ends. See section 3.6.4.3). The configuration will be studied like a bundle or bank of tubes with a fluid flowing across, normal to the axis of the tubes. The cross section and length of the tubes will be the same as the cross section and length of the end windings. The equation of Colburn\(^{60}\), is suggested by Chapman\(^{58}\) and may be used to obtain an idea of the expected film coefficient for this case:

\[
h_{2-11} = \left( \frac{kN_{NU}}{\ell} \right)_{2-11} = 0.33 \times \left( \frac{k}{\ell} \times \frac{0.6}{N_{RE}} \times \frac{1}{N_{PR}} \right)_{2-11},
\]

(318)

where \( \ell_{2-11} \) and the mean film temperature are the same as in section A, and the Reynolds number is based on the mean velocity \( V_{2-11} \) in the minimum free cross section for fluid flow, which can be estimated as:

\[
V_{2-11} = \frac{0.5 \times W_2}{2 \times \pi \times (R + G + GS) \times OS \times (1 - kS)}.
\]

(319)

\[
N_{RE} = \left( \frac{V_{2-11} \rho}{\mu} \right)_{2-11},
\]

(320)

\[
N_{PR} = \left( \frac{\mu \cdot C_p}{k} \right)_{2-11},
\]

(321)

The relative importance of free convection must always be checked. Even if there is forced cooling it must be used the biggest value of \( h_{2-11} \) obtained from either natural or forced cooling.

The convective resistance is:
\[ R_{2-11} = \left( \frac{1}{h \cdot A} \right)_{2-11} \]  

**Thermal convective resistance for the end shield**

The motion of the rotating parts of the machine makes the air flow tangentially to the end shield. Since the radii of curvature involved are quite large, this process of forced convection can be modelled as flow past a flat plate, which is a standard case \(^{58}\).

The equations to be applied are different according to the flow regime. If the flow in the film is laminar (which requires \(N_{RE} < 4 \times 10^5\)) the expression for the local film coefficient is:

\[ N_{NU_x} = 0.332 \times \left( \frac{1}{N_{RE_x}} \right)^{\frac{1}{2}} \times \left( \frac{1}{N_{PR}} \right)^{\frac{1}{3}}, \quad \text{for } N_{RE} < 4 \times 10^5 \]  

laminar

and for turbulent flow, from Colburn \(^{60}\):  

\[ N_{NU_x} = 0.0292 \times N_{RE_x}^{0.8} \times \left( \frac{1}{N_{PR}} \right)^{\frac{1}{3}}, \quad \text{for } N_{RE} > 4 \times 10^5 \]  

turbulent

If the surface temperature, fluid velocity and temperature are assumed to be uniform, these local expressions give the following average equations (for the cylindrical part of the end shield):

\[ h_{14-11} = \left( \frac{k}{\lambda} \cdot N_{NU} \right)_{14-11} = 0.664 \times \left( \frac{k}{\lambda} \times \left( \frac{0.5}{N_{RE}} \times N_{PR} \right) \right)^{\frac{1}{3}}, \quad \text{for } N_{RE,14-11} < 4 \times 10^5 \]  

(325)
\[ h_{14-11}^{\text{turbulent}} = \left( \frac{k}{\epsilon} \cdot N_{\text{NU}}^{\text{turbulent}} \right)_{14-11} = 0.036 \times \left( \frac{k}{\epsilon} \times \frac{0.8}{N_{\text{RE}}^{\text{turbulent}}} \times N_{\text{PR}}^{\frac{1}{3}} \right)_{14-11}, \]

for \( N_{\text{RE}} > 4 \times 10^5 \),

(326)

where

- All the fluid properties are evaluated at the mean film temperature \( \theta_m = \frac{1}{2} (\theta_{14} + \theta_{11}) \),
- The velocity in the undisturbed stream outside the boundary layer should be used in \( N_{\text{RE}} \). A rough and simple estimate for this velocity is just the average velocity between the velocity of the fluid at the shaft: \( \omega \times R_{\text{SH}} \) and at the end shield: zero. This value will be maintained even if the cooling is forced since usually there are baffles addressing the flow towards the end windings and plates.

\[ V_{14-11} \approx 0.5 \times \omega \times R_{\text{SH}} \]

(327)

- The characteristic length is: \( \ell_{14-11} = 2 \times \pi \times RF \).
- In the program the transition from the laminar to the turbulent regime occurs at \( N_{\text{RE}} = 1.658 \times 10^4 \) to avoid jumps in \( h_{14-11} \).

It will be assumed now that the obtained value of \( h \) is also valid for the vertical end plates of the end shield. This can be justified by indicating that the model uses only one temperature to describe the end shield and also considers a unique bulk velocity of the fluid. With this assumption the convective resistance becomes:
\[ R_{14-11} = \frac{1}{h_{14-11} \cdot A_{14-11}} \quad , \quad(328) \]

where

\[ A_{14-11} = 2 \times \pi \times RF \times (LT - CL1) + 2 \times \pi \times (RF^2 - RSH^2) \quad , \quad(329) \]

Finally it is necessary to check the relative importance of free convection. The expressions for horizontal and vertical plates will be used, since they are the closest available equations to the actual considered system. Since the coefficients for horizontal and vertical plates are very similar\(^5\), a unique set of averaged coefficients will be used:

- For \(10^4 < (N_{GR} \cdot N_{PR})_{14-11} < 5.94 \times 10^8\):

\[ h_{14-11}^{laminar} = \frac{0.28}{RF} \cdot \left[k \cdot (N_{GR} \cdot N_{PR})_{14-11}^{0.25}\right] \text{ watts/(m}^2\cdot\text{C)} \quad , \quad(330) \]

- For \(5.94 \times 10^8 < (N_{GR} \cdot N_{PR})_{14-11} < 10^{12}\):

\[ h_{14-11}^{turbulent} = \frac{0.052}{RF} \cdot \left[k \cdot (N_{GR} \cdot N_{PR})_{14-11}^3\right] \text{ watts/(m}^2\cdot\text{C)} \quad , \quad(331) \]

**Thermal convective resistance for the stator end plate**

The effective area of the stator end plate surface is smaller than most of the other surfaces that have been considered; otherwise the velocity of the neighboring air is difficult to estimate. Since the conditions for the convection process are reasonably similar to those in the region between the stator and the frame, which can be estimated more accurately,
it will be assumed that

\[ h_{16-11} \approx h_{3-18} \]  \hspace{1cm} (332)

The effective area for convection is (both ends):

\[ A_{16-11} = 2 \times \pi \times [(R + G + H)^2 - (R + G + DS)^2] + \]

\[ + 2 \times 2 \times \pi \times (R + G) \times (1 - kS) \times DS = \]

\[ = 2 \times \pi \times [(H - DS) \cdot (H + DS + 2R + 2G) + \]

\[ + 2 \cdot DS \cdot (R + G) \cdot (1 - kS)] \]  \hspace{1cm} (333)

And the convective thermal resistance:

\[ R_{16-11} = \left( \frac{1}{h \cdot A} \right)_{16-11} \]  \hspace{1cm} (334)

**Thermal convective resistance in the annular duct between the stator and the frame.**

The values for this resistance and the methods of calculation are entirely different for natural and forced cooled machines.

\textit{Natural cooling} \hspace{1cm} (80)

In this case the region between the stator and the frame must be treated as an enclosed space. The first point is to decide whether natural convection or just conduction control the process. Typically
for Grashof numbers (based on clearance and the difference in temperature between both surfaces) below $2 \times 10^3$, natural convection is suppressed and conduction dominates, yielding:

$$R_{3-9} = \frac{1}{2 \times \pi \times CL \times k_{18}} \cdot \ln \frac{RF - TF}{RF - TF - TA}$$

(335)

For $N_{GR} > 2 \times 10^3$ natural convection dominates. There are no correlations available for the particular case of a horizontal annular air space. But it happens that the correlations for horizontal and also vertical enclosed air spaces are almost identical, and therefore any of them can be used, based on the clearance $TA$. The adopted expression is:

$$h_{3-18} \approx h_{9-18} = 2 \times \frac{k_{18}}{TA} \times C \times \left[ N_{GR} \cdot N_{PR} \right]^{n}_{18},$$

(336)

where

- For $2.496 \times 10^5 > N_{GR} > 2 \times 10^3$ : $C = 0.20$, $n = 0.25$.

- For $10^7 > N_{GR} > 2.496 \times 10^5$ : $C = 0.071$, $n = 1/3$.

The value $2.496 \times 10^5$, boundary between both sets of values, has been slightly modified to prevent jumps in the expression which could hamper the convergence of the resultant system of equations.

The resultant thermal resistances are:

$$R_{3-18} = \frac{1}{2 \times \pi \times CL \times (RF - TF - TA) \times h_{3-18}}$$

(337)
\[ R_{9-18} = \frac{1}{2 \times \pi \times CL \times (RF - TF) \times h_{9-18}} \]  

(338)

It can be checked easily that the series combination of these two resistances yields a value very close to eq. (335) for air at atmospheric pressure and in the expected temperature range. Therefore the expression which gives the lowest value for \( R_{3-9} \) will always be used, so jumps cannot occur and the results will be accurate enough.

**Forced cooling.**

In this situation the coolant is forced to flow through the annulus in the axial direction and the Reynolds number becomes the key parameter. The most reliable results at the present time seem to be simple modifications of the correlations for flow in a cylindrical tube\(^{58,68}\). Here the approximation suggested by Chapman\(^{58}\) will be used:

An "equivalent diameter" is defined as four times the cross sectional flow area divided by the wetted perimeter. The equivalent diameter, \( D_{\text{eq}} \), is the diameter of a hypothetical circular pipe which has the same pressure gradient along its axis as the noncircular ducts if the mean flow velocities are the same.

\[ D_{\text{eq}} = 4 \times \frac{\frac{\pi}{4} \times (D_{\text{ex}}^2 - D_{\text{in}}^2)}{\pi \times (D_{\text{ex}} + D_{\text{in}})} = D_{\text{ex}} - D_{\text{in}} = 2 \times TA \]  

(339)

For forced convection inside the annular space the Nusselt and Reynolds numbers are based on this equivalent diameter:

\[ N_\text{NU} = \frac{h \times D_{\text{eq}}}{k} \]  

(340)
\[ N_{RE} = \frac{V_m \cdot D_{eq} \cdot \rho}{\mu} , \] (341)

The film coefficient \( h \) at either surface of the annulus can be evaluated by using the following expression suggested by McAdams:

\[ h_{3-18} \approx h_{9-18} = \frac{k_{18}}{2 \times \Delta A} \times 0.023 \times 0.8 \times 0.4 \times N_{RE} \times N_{PR} , \] (342)

where

\[ V_m \] is the average flow velocity:

\[ V_{m18} = \frac{0.5 \times W_2}{2 \times \frac{n}{n} \times (RF - TF) \times TA} , \] (343)

and the fluid properties must be determined at the bulk fluid temperature \( \theta_{18} \).

The effective areas for convection are:

\[ A_{3-18} = 2 \times \pi \times CL \times (RF - TF - TA) , \] (344)

\[ A_{9-18} = 2 \times \pi \times CL \times (RF - TF) , \] (345)

and the corresponding resistances:

\[ R_{3-18} = \frac{1}{(h.A)_{3-18}} \]

\[ R_{9-18} = \frac{1}{(h.A)_{9-18}} \]
Thermal convective resistance in the radial duct

Since there are no available correlations for this specific geometry, a crude modification of the equations for flow in a cylindrical tube will be used, like in the preceding section. Here it will be assumed that the duct has uniform cross section, and the one corresponding to the mean radius \( R + G + 0.5 H \) will be used. The equivalent diameter is:

\[
D_{eq} = 4 \times \frac{2 \times \pi \times (R + G + 0.5H) \times \text{WRD}}{2 \times 2 \times \pi \times (R + G + 0.5H)} = 2 \times \text{WRD} \quad , \quad (346)
\]

The expressions are the same ones used in the preceding section, where:

\[
V_{m_{19}} = \frac{W_1}{2 \times \pi \times (R + G + 0.5H) \times \text{WRD}} \quad , \quad (347)
\]

and the fluid properties must be determined at the bulk fluid temperature \( \theta_{19} \).

The effective area for convection is:

\[
A_{3-19} = 2 \times \pi \times [(H - DS) \cdot (H + DS + 2R + 2G) + \\
\quad + 2 \cdot DS \cdot (R + G) \cdot (1 - kS)] \quad , \quad (348)
\]

and the convective thermal resistance:

\[
R_{3-19} = \frac{1}{(hA)_{3-19}}
\]
Enclosed rotating elements.

Convective heat transfer in the air gap.

The air gap is the only region of the machine where convective heat transfer for actually enclosed rotating elements takes place.

Gazley's classical paper \(^{67}\) and its precedents \(^{81,93}\) are the basic references concerning the convective heat transfer specifically for the air gap of smooth rotor electric machinery. Nevertheless a proprietary reference from G.E. \(^{85}\), as summarized in reference 68, is more complete and updated and therefore will be used.

All the correlations that will be presented are based on tests or analysis for surfaces where any roughness is too small to have appreciable effect on the heat transfer. Unfortunately heat transfer correlations for the effect of surface roughness seem to be lacking, with the one exception of Gazley's tests of enclosed rotating cylinders with axial grooves. His results show that the presence of slots may cause counteracting effects on the heat transfer rate. The presence of slots decreases the exposed cylindrical surface area and thus decreases the heat transfer area; the presence of insulation in the slots also reduces the effectiveness of the slot surface. On the other hand, the slots presumably disturb the flow in the gap and thereby increase the convective heat transfer rate across the gap. Gazley's conclusions are that both with or without axial flow the slots appear to have no appreciable effect on the rotor-stator heat transfer characteristics. It should be indicated that his heat-transfer coefficients are based on the nominal mean cylindrical area so that the two mentioned counteracting effects apparently compensate each other.
The machine air gap is modelled as an inner smooth rotating cylinder and an also smooth stationary coaxial one. Only the equation for zero axial flow (no forced cooling in the machine) will be presented in this section, although there are available equations including the possibility of non-zero axial flow (see Appendix C).

Since high velocities are expected, the effect of viscous dissipation may be significant. Therefore the effect of kinetic energy recovery and windage on the surface temperature rise must be included. The reason for this temperature excess is the conversion into heat of some of the kinetic energy (relative to the surface) of the gas molecules from the main gas stream which penetrate the boundary layer, and therefore become slowed down. The remainder of this kinetic energy is transferred back into the gas by the conduction and convection in the boundary layer.

The working equations for this configuration have been obtained in Appendix C. If the condition of zero axial flow is added, which in steady state conditions means:

$$\frac{d\theta_m}{dz} = 0$$

the equation C.17 becomes:

$$q_i + q_o + q_{tw} = 0$$

(349)

And subtracting eq. C.14 from eq. C.13:

$$\theta_i - \theta_o = \frac{q_i'' - q_o''}{2 \times U} = q_i \cdot \frac{1}{2 \times U \times 2 \times \pi \times R \times CL} - q_o \times \frac{1}{2 \times U \times 2 \times \pi \times (R+G) \times CL}$$

(350)
where the temperature rise due to the kinetic energy recovery does not appear, since it is roughly equal for both surfaces and here only the difference in temperature between the rotor and stator surfaces is relevant.

Fig. 40 Convective heat flow in the air gap

The situation is easily modelled as Fig. 40 indicates, where:

\[
R_{7-10} = \frac{1}{4 \times \pi \times U \times C_L \times R} \quad ,
\]

(351)

\[
R_{3-10} = \frac{1}{4 \times \pi \times U \times C_L \times (R + g)} \quad ,
\]

(352)

The values for \( U \) and \( q_{tw} \) are obtained by means of the equations provided in Appendix C.
The final working equations, which give the amount of heat delivered to the air gap by the two cylindrical surfaces in terms of their temperatures, are obtained by substituting eq. (349) into eq. (350), giving:

\[ q_0 = 2 \times \pi \times R \times CL \times U \times (\theta_0 - \theta_i) - \frac{q_{tw}}{2}, \quad (353) \]

\[ q_i = 2 \times \pi \times R \times CL \times U \times (\theta_i - \theta_0) - \frac{q_{tw}}{2}, \quad (354) \]

Non enclosed rotating elements

Two heat transfer processes in the machine have been classified in the category of forced convection from non enclosed rotating elements. The first one is the convection from the rotating shaft, which has been modeled as a rotating cylinder without enclosure. The second one is the convection from the rotor end plates, which will be considered as disks rotating without enclosure.

Both elements are certainly enclosed by the remaining parts of the machine; nevertheless the distances to the enclosures are large enough so the most convenient assumption is to consider them to be non enclosed elements.

Thermal convective resistance for the shaft.

For the expected range of the parameters the following correlation should be used to obtain \( N_{NU} \):

\[ N_{NU} = 0.1075 \times N_{PR} \times [0.5 \times N_{RE} + N_{GR}]^{0.35} \times 2 \quad \text{0.35} \quad 0.35, \quad (355) \]
where

\[ N_{RE} = \frac{\rho \times \omega \times RSH \times (2 \times RSH)}{\mu} = \frac{2 \times \rho \times \omega \times RSH^2}{\mu} \quad , \quad (356) \]

\[ N_{GR} = \frac{9.8 \times (2 \times RSH)^3 \times \beta \times \Delta t \times \rho^2}{\mu^2} = \]

\[ = \frac{78.4 \times RSH^3 \times (\theta_{12} - \theta_{11})}{273 + 0.5 (\theta_{12} + \theta_{11})} \times \frac{\rho^2}{\mu^{12-11}} \quad , \quad (357) \]

since the present correlations need \( \beta \) to be evaluated at the film temperature.

The Prandtl number must be evaluated also at the film temperature,

\[ N_{PR}^{12-11} \quad . \]

\[ h_{11-12} = \frac{k}{2 \times RSH} \times N_{NU}^{11-12} \quad , \quad (358) \]

These equations hold as far as the free convection process, also accounted for in eq. (355) is unimportant, which is measured by the condition:

\[ y = \left( N_{RE} \times N_{GR} \right)^{-0.5} \quad < 0.2 \quad , \quad (359) \]

If this condition is not satisfied the correlation for a stationary cylinder, described previously, should be used.

The experimental verification of eq. (355) by Dropkin\textsuperscript{62} covers the following ranges:
\[ 10^3 < N_{RE} < 5 \times 10^5 \] ,  

\[ 5 \times 10^6 < 0.5 \times N_{RE}^2 + N_{GR} < 10^{11} \] ,  

The total active area of the shaft for convection is:

\[ A_{11-12} = 2 \times \pi \times RSH \times (LT - CL1 - 2 \times OR) \] ,  

And the equivalent thermal resistance:

\[ R_{11-12} = \left( \frac{1}{h \cdot A} \right)_{11-12} \] ,  

**Thermal convective resistance for the rotor end plates.**

The two ends of the rotor will be modelled as completely flat plates of outer radius \( R \) and inner radius \( RSH \).

The correlation to be used for this model gives the local \( h \) at any radius \( r \), and assumes a uniform surface temperature:

\[ h(r) = C_1 \times \frac{k}{r} \times N_{RE}^m \times \left( \frac{NPR}{0.70} \right)^a \] ,  

where

\[ N_{RE} = \frac{\rho \times \omega \times r^2}{\mu} \] ,  

For high rotational speeds a well developed turbulent regime is expected and the following values for the constants \( C_1 \) and \( m \) \(^{84}\) and \( a \) \(^{76,61}\) are recommended:
\[ C_1 = 0.019 \quad , \quad m = 0.80 \quad , \quad a = 0.60 \quad , \quad \text{in the range } 0.6 < N_{PR} < 4 \quad . \]

The average value for \( h \) will be calculated as if the disk were complete, which yields a value:

\[
h_{av} = \frac{20}{26} \times h(R) = 0.018 \times \left[ k \times \left( \frac{\rho \omega}{\mu} \right)^{0.8} \times (R \times N_{PR})^{0.6} \right]_{17-11} \quad ,
\]

(366)

The area of convection is:

\[
A_{17-11} = 2 \times \pi \times (R^2 - RSH^2) \quad ,
\]

(367)

And the equivalent thermal resistance is:

\[
R_{17-11} = \left( \frac{1}{h \times A} \right)_{17-11} \quad ,
\]

(368)

The checking of the actual flow regime will be done using the following criterion for turbulence of the boundary layer\(^97\):

\[
N_{RE} > 50,000 \quad ,
\]

(369)

This is the recommended value for "very rough" surfaces. This fits well in this configuration, which usually departs considerably from a flat plate. If the actual machine construction suggests that the rotor ends be modeled as "smooth" surfaces, the following condition will be used instead\(^97\):

\[
N_{RE} > 310,000 \quad ,
\]

(370)
If the flow regime is found to be laminar, the values:

\[ C_1 = 0.35, \ m = 0.50 \ \text{and} \ a = 0.44 \]

are recommended if no wide variation of \( N_{PR} \) is expected. For more accurate evaluation of the effect of variation of \( N_{PR} \) on \( h \) reference 69 should be consulted. There are no data available for the transition regime, although the value of \( h \) will be presumably intermediate between both extremes, turbulent and laminar. For smooth disks the transition begins at \( N_{RE} \approx 180,000 \). No data are available for rough surfaces.

In many electrical machines, and especially if they have high rotational speed, the effective length of the rotor is bigger than the stator length, because of the end rings enclosing and protecting the rotor windings.

This additional rotor length \( 2 \times OR \) is usually small compared with the remainder rotor length \( \ell \), and a separate thermal study for its film coefficient would be very involved due to the presence of the stator windings. Therefore it will be assumed that the film coefficient for this surface is the same as the one for the enclosed rotor surface.

\[ h_{\text{rotor end}} = 2 \times U_{\text{air gap}}, \quad (371) \]

\[ h_{\text{cylindrical part}} \]

### 3.6.4.3 Modifications and new features for the forced cooling case

So far it has been assumed that the cooling of the machine is natural, which means that no external means are used to ventilate the
machine. This constraint will be removed now, and it will be considered that somehow (usually by means of a fan) a cooling gas is blown over certain critical parts of the machine, in order to increase the coefficient of heat transfer between the part and the coolant, and also to remove the heated air.

The configuration to be considered is shown in Fig. 41. It has been assumed that the cooling is symmetrical (fans at both ends of the machine) and no ventilating ducts in the rotor (which would yield high mechanical stresses in a high speed rotating machine) have been included. Despite these limitations the adopted configuration is quite general and allows the most important features of a forced cooling machine to be modeled. If the stator had internal axial ducts, these could be modeled
just by increasing the effective convection area between the stator and
the frame; this is very reasonable since the stator core presents very
low thermal resistance in the radial direction.

In this section the equations required to model this configuration
have been obtained. This requires that the amounts of heat delivered to
the coolant be related to the temperatures of the different surfaces in
contact with it and to the varying temperature of the coolant itself.
This is a particular application of the theory of heat exchangers. The
film coefficients of some surfaces must be changed according to the
velocity of the coolant passing by (this has been indicated in the pre-
ceding sections, when it was appropriate).

No study has been attempted to relate the flow rate of coolant wo
to the power dissipated in the fan or to the fan design or selection.
The same holds for the analysis of the distribution of wo into w1 and
w2 for the two available paths. Volume I of reference 68 includes
abundant material on these topics. The results are very dependent on
the particular construction of the machine. Therefore values for wo, w1
and w2 must be supplied to the program. If only wo is supplied, w1 and
w2 are obtained by means of the ratio between the duct cross sections,
which is not reliable at all.

The theory of heat exchangers applied to the ventilating ducts.

The process of heat exchange in a ventilating duct can be treated as
a particular case of heat exchangers. The duct can be modelled as
Figure 42 shows. In general the duct is bounded by two different sur-
faces, A and B, with different temperatures and film coefficients. The
Fig. 42 Ventilating duct as a heat exchanger

following assumptions are made:

a) The film coefficients $h_A$ and $h_B$ are constant throughout the duct. They will be considered to be equal, since this is suggested by the sections where film coefficients were determined.

b) The specific heat and mass density of the fluid are constant.

c) No heat is exchanged with anything but the mentioned two surfaces.

d) The temperatures of the two walls will be considered to remain constant along the duct, and equal to their actual average temperatures. This assumption is reasonable since the change of temperature for the walls is much smaller than for the fluid along the duct; moreover, the heat conduction along the walls has been accounted for separately.

e) At any cross section in the duct the fluid may be characterized by a single temperature.

f) The geometry of the configurations in which the model is to be used suggests the assumption that the areas of the two surfaces are equal.
The heat exchanged in an incremental length $dx$ of the duct can be expressed in the following two ways:

$$
d_q = h \frac{A}{x} \cdot dx \cdot (\theta_{av_A} + \theta_{av_B} - 2x\theta_f) = \frac{2 \cdot h \cdot A}{x} \cdot (\theta_S - \theta_f) \cdot dx = \frac{2 \cdot h \cdot A}{x} \cdot \Delta \theta \cdot dx,
$$

$$
d_q = W \cdot C_p \cdot \rho \cdot d\theta_f = - W \cdot C_p \cdot \rho \cdot d\Delta \theta,
$$

where

$$
\theta_S = \frac{1}{2} (\theta_{av_A} + \theta_{av_B}),
$$

$$
\Delta \theta = \theta_S - \theta_f,
$$

This yields the following differential equation in $\Delta \theta$:

$$
\frac{d\Delta \theta}{dx} = - \frac{2 \cdot h \cdot A}{x \cdot W \cdot C_p \cdot \rho} \cdot \Delta \theta,
$$

whose solution is:

$$
\Delta \theta = \Delta \theta_{in} \cdot e^{- \frac{2 \cdot h \cdot A}{x \cdot W \cdot C_p \cdot \rho} \cdot x}.
$$

The total amount of heat delivered to the coolant is:

$$
q_{total} = W \cdot C_p \cdot \rho \cdot (\theta_{f \ out} - \theta_{f \ in}) = W \cdot C_p \cdot \rho \cdot (\Delta \theta_{in} - \Delta \theta_{out}) =
$$

$$
= W \cdot C_p \cdot \rho \cdot \Delta \theta_{in} \cdot (1 - e^{-\frac{2 \cdot h \cdot A}{W \cdot C_p \cdot \rho}});
$$

$$
q_{total} = W \cdot C_p \cdot \rho \cdot \left(\frac{\theta_{av_A} + \theta_{av_B}}{2} - \theta_f \cdot \text{in} \right) \cdot (1 - e^{-\frac{2 \cdot h \cdot A}{W \cdot C_p \cdot \rho}}),
$$

(378)
This heat can be assumed to be exchanged with a fluid at a constant temperature $\theta_f^{\text{eq}}$, which can be obtained in the following way:

\[
q_{\text{total}} = W.C_p \cdot \rho \left( \frac{\theta_{\text{av}}_A + \theta_{\text{av}}_B}{2} - \theta_f^{\text{in}} \right) \cdot (1 - e^{-\frac{2hA}{W.C_p \cdot \rho}}) = \\
= h.2.A \left( \frac{\theta_{\text{av}}_A + \theta_{\text{av}}_B}{2} - \theta_f^{\text{eq}} \right);
\]

\[
\theta_f^{\text{eq}} = \frac{\theta_{\text{av}}_A + \theta_{\text{av}}_B}{2} - \frac{W.C_p \cdot \rho}{h.2.A} \cdot \left( \frac{\theta_{\text{av}}_A + \theta_{\text{av}}_B}{2} - \theta_f^{\text{in}} \right) \cdot (1 - e^{-\frac{2hA}{W.C_p \cdot \rho}})
\]

(379)

Now follows the calculation of the amount delivered by each surface:

\[
dq_{A+F} = h.\frac{A}{\ell} \cdot (\theta_{\text{av}}_A - \theta_f) \cdot dx;
\]

\[
q_{A+F} = h.A.\theta_{\text{av}}_A - \frac{hA}{\ell} \cdot \left[ \theta_{\text{av}}_A \cdot \frac{\ell}{2} + \frac{\theta_{\text{av}}_A + \theta_{\text{av}}_B}{2} - \theta_f^{\text{in}} \right] \cdot e^{-\frac{2hA}{\ell . W.C_p \cdot \rho}} \cdot \frac{\chi}{x} \cdot dx =
\]

\[
= h.A.\frac{\theta_{\text{av}}_A - \theta_{\text{av}}_B}{2} + \frac{W.C_p \cdot \rho}{2} \cdot \left[ \frac{\theta_{\text{av}}_A + \theta_{\text{av}}_B}{2} - \theta_f^{\text{in}} \right] \cdot (1 - e^{-\frac{2hA}{W.C_p \cdot \rho}}), (380)
\]

In the same way:

\[
q_{B+F} = h.A.\frac{\theta_{\text{av}}_A - \theta_{\text{av}}_B}{2} + \frac{W.C_p \cdot \rho}{2} \cdot \left[ \frac{\theta_{\text{av}}_A + \theta_{\text{av}}_B}{2} - \theta_f^{\text{in}} \right] \cdot (1 - e^{-\frac{2hA}{W.C_p \cdot \rho}}), (381)
\]

The exhaust temperature $\theta_f^{\text{out}}$ is obtained from eq. (377):
\[ \theta_{S} - \theta_{f, out} = (\theta_{S} - \theta_{f, in}) \cdot e^{- \frac{2hA}{W \cdot C_{p} \cdot \rho}} \cdot \frac{\theta_{av_{A}} + \theta_{av_{B}}}{2} \cdot \left( \frac{\theta_{av_{A}} + \theta_{av_{B}}}{2} - \theta_{f, in} \right) \cdot e^{- \frac{2hA}{W \cdot C_{p} \cdot \rho}} \]

(382)

Heat exchange in the ventilating duct between the stator and the frame.

The equations of the preceding section will be applied to one of the two sides of the machine. Therefore:

\[ W = 0.5 \times W_{2} \]

\[ A = \frac{1}{4} (A318 + A918) \]

\[ h = h_{18} \]

\[ \theta_{fin} = \theta_{11} ; \quad \theta_{av_{A}} = \theta_{3} ; \quad \theta_{av_{B}} = \theta_{9} \]

The properties of the coolant, \( C_{p} \) and \( \rho \), are calculated at the temperature:

\[ \theta = \frac{1}{2} (\theta_{18, in} + \theta_{18, out}) = \frac{1}{2} (\theta_{11} + \theta_{18, out}) \]

The amounts of heat delivered by the stator and the frame (whole machine) are:

\[ Q_{3-18} = \frac{1}{4} \cdot h_{18} \cdot (A318 + A918) \cdot (\theta_{3} - \theta_{9}) + \frac{1}{2} \times W_{2} \times C_{p} \times \frac{\theta_{3} + \theta_{9}}{2} - \theta_{11} \]

\[ \cdot (1 - e^{-C}) = \frac{1}{4} \cdot [h_{18} \cdot (A318 + A918) + W_{2} \times C_{p} \times \theta_{3} \times (1 - e^{-C})] \cdot \theta_{3} + \]

\[ + \frac{1}{4} \times [-h_{18} \cdot (A318 + A918) + W_{2} \times C_{p} \times \theta_{9} \times (1 - e^{-C})] \cdot \theta_{9} - \]

\[ - \frac{1}{2} \times W_{2} \times C_{p} \times \theta_{11} \times (1 - e^{-C}) \cdot \theta_{11} \]

(383)
\[ Q_{9-18} = \frac{1}{4} h_{18} \cdot (A318 + A918) \cdot (\theta_9 - \theta_3) + \frac{1}{2} W2 C_p \cdot \rho \cdot \left( \frac{\theta_3 + \theta_9}{2} - 0_{11} \right) \cdot (1 - e^{-c}) = \]

\[ = \frac{1}{4} \left[ h_{18} \cdot (A318 + A918) + W2 x C_p \cdot \rho \cdot (1 - e^{-c}) \right] \cdot \theta_9 + \]

\[ + \frac{1}{4} \left[ - h_{18} \cdot (A318 + A918) + W2 C_p \cdot \rho \cdot (1 - e^{-c}) \right] \cdot \theta_3 - \frac{1}{2} W2 C_p \cdot \rho \cdot (1 - e^{-c}) \cdot 0_{11} \]

(384)

where

\[ C = \frac{h_{18} \cdot (A318 + A918)}{W2 \times C_p \times \rho} , \]  

(385)

The exhaust temperature is:

\[ \theta_{18_{out}} = \frac{\theta_3 + \theta_9}{2} - \left( \frac{\theta_3 + \theta_9}{2} - 0_{11} \right) \cdot e^{-c} , \]  

(386)

Heat exchange in the air gap.

There are some differences with respect to the approach in the last section, since there are significant internal losses in the duct. The starting points are the eqs. C.13, 14 and 17:

\[ \theta_i - \theta_f = 0.22 \times \frac{\omega^2 R^2}{C_p} + \frac{q_i'' - 0.2 \times q''_0}{2.4 U} , \]  

(387)

\[ \theta_i - \theta_f = 0.22 \times \frac{\omega^2 R^2}{C_p} + \frac{q_o'' - 0.2 q''_1}{2.4 U} , \]  

(388)

\[ \frac{d\theta_m}{dx} = \frac{2 \pi R}{C_p W \rho} \left[ q_i'' + q_o'' + \frac{C_f \omega^3 R^3 \rho}{2} \right] , \]  

(389)

where \( \theta_i \) and \( \theta_o \) have the same role as \( \theta_{av_B} \) and \( \theta_{av_A} \) in the preced-
ing section. In order to obtain a differential equation for \( Q_m \), the first and second of the equations above are added and the result obtained for \( q_i'' + q_o'' \) is substituted in the third equation:

\[
q_i'' + q_o'' = -1.32 \times \frac{\omega^2 R^2 U}{C_p} + 3U(\theta_i + \theta_o - 2\theta_f), \quad (390)
\]

\[
\frac{d\theta_f}{dx} = \frac{\pi Rc \omega^3 R^3}{C_p W} - 2.64 \times \frac{\pi R^2 U}{C_p W \rho} + 12 \pi U R x \frac{\theta_i + \theta_o}{2} - \theta_f, \quad (391)
\]

\[
\frac{d\Delta \theta}{dx} = a - b \Delta \theta, \quad (391)
\]

where:

\[
a = \frac{2.64 \times \pi \times \omega^2 \times R^3 \times U - \pi \times C_f \times \omega^3 \times R^4 	imes C_p \times \rho}{C_p^2 \times W \times \rho}, \quad (392)
\]

\[
b = \frac{12 \times \pi \times U \times R}{C_p \times W \times \rho}, \quad (393)
\]

\[
\Delta \theta = \theta_s - \theta_f, \quad (394)
\]

\[
\theta_s = \frac{1}{2}(\theta_i + \theta_o), \quad (395)
\]

Solving the differential equation with the boundary condition

\[\Delta \theta(X = 0) = \theta_s - \theta_f \text { in } = \theta_s - \theta_{11}; \]

\[\Delta \theta(X) = \frac{\theta_i + \theta_o}{2} - \theta_f(X) = \Delta \theta_{\text{in}} \cdot e^{-bx} + \frac{a}{b} \cdot (1 - e^{-bx}), \quad (396)\]
The heat exchanges can be calculated now. The total amount of heat received by the fluid in the duct (internally and externally) is:

\[ q_{\text{total}} = W \cdot C_p \cdot \rho \cdot (\theta_f \text{ out} - \theta_f \text{ in}) = W \cdot C_p \cdot \rho \cdot (\Delta \theta \text{ in} - \Delta \theta \text{ out}) = \]

\[ = W \cdot C_p \cdot \rho \cdot [\Delta \theta \text{ in} \cdot (1-e^{-b \Delta \theta}) - \frac{a}{b} \cdot (1-e^{-b \Delta \theta})] = Q_i + Q_o + Q_{tw} , \quad (397) \]

It must be realized that subtracting eqs. (387) and (388) yields:

\[ q_i'' - q_o'' = 2 \cdot U \cdot (\theta_i - \theta_o) ; \quad Q_i - Q_o = 4 \cdot \pi \cdot R \cdot U \cdot (\theta_i - \theta_o) , \quad (398) \]

Therefore:

\[ q_{\text{total}} = 2 \times Q_i + 4 \cdot \pi \cdot R \cdot U \cdot (\theta_3 - \theta_7) + \pi \cdot R \cdot U \cdot C_f \cdot \omega^3 \cdot R^3 \cdot \rho \]

\[ Q_i = -\frac{1}{2} \cdot \pi \cdot R \cdot U \cdot C_f \cdot \omega^3 \cdot R^3 \cdot \rho + 2 \cdot \pi \cdot R \cdot U \cdot (\theta_i - \theta_o) + \]

\[ + \frac{W \cdot C_p \cdot \rho}{2} \cdot (\Delta \theta \text{ in} - \frac{a}{b}) \cdot (1-e^{-b \Delta \theta}) , \quad (399) \]

\[ Q_o = Q_i - 4 \pi R \cdot U \cdot (\theta_i - \theta_o) = -\frac{1}{2} \cdot \pi \cdot R \cdot U \cdot C_f \cdot \omega^3 \cdot R^3 \cdot \rho + \]

\[ + 2 \cdot \pi \cdot R \cdot U \cdot (\theta_0 - \theta_i) + \frac{W \cdot C \cdot \ell}{2} \cdot (\Delta \theta \text{ in} - \frac{a}{b}) \cdot (1-e^{-b \Delta \theta}) , \quad (400) \]

Applying these eqs. to the particular case of the air gap, and obtaining values for the heat exchanges in the whole machine, although the eqs. are valid for one duct:

\[ W = .5 \times W_l \]

\[ \ell = .5 \times CL \]
\[ U = U_{10} \]

\[ \theta_i = \theta_f^3; \theta_o = \theta_3; \theta_{f \text{ in}} = \theta_{11} \]

The fluid properties are calculated at the temperature:

\[ \theta = \frac{1}{2} ( \theta_{10 \text{ in}} + \theta_{10 \text{ out}} ) = \frac{1}{2} ( \theta_{11} + \theta_{10 \text{ out}} ) \]

The amounts of heat delivered by the stator and the rotor are (for the whole machine):

\[ Q_{3-10} = - \frac{Q_{tw}}{2} + 2.\pi.R.C.L.U_{10} \times \left( \theta_3 - \theta_7 \right) + \frac{W_1 C_p x \rho}{2} \cdot \left( \frac{\theta_3 + \theta_7}{2} - \theta_{11} - \frac{a}{b} \right) \cdot (1 - e^{-bCL/2}) \]

\[ + \left[ 2.\pi.R.C.L.U_{10} + \frac{1}{4} W_1 C_p x \rho x (1 - e^{-bCL/2}) \right] \cdot \theta_3 + \]

\[ + \left[ -2.\pi.R.C.L.U_{10} + \frac{1}{4} W_1 C_p x \rho x (1 - e^{-bCL/2}) \right] \cdot \theta_7 - \]

\[ - \frac{1}{2} W_1 C_p \rho (1 - e^{-bCL/2}) \cdot \theta_{11} \] \hspace{1cm} \text{(401)}

\[ Q_{7-10} = - \frac{Q_{tw}}{2} - \frac{W_1 C_p \rho a}{2 \cdot b} \cdot (1 - e^{-bCL/2}) + \left[ -2.\pi.R.C.L.U_{10} + \frac{1}{4} W_1 C_p \rho (1 - e^{-bCL/2}) \right] \]

\[ \cdot \theta_3 + \left[ 2.\pi.R.C.L.U_{10} + \frac{1}{4} W_1 C_p \rho (1 - e^{-bCL/2}) \right] \theta_7 - \]

\[ - \frac{1}{2} W_1 C_p \rho (1 - e^{-bCL/2}) \cdot \theta_{11} \] \hspace{1cm} \text{(402)}
\[ Q_{tw} = \pi R CL C_f \omega^3 R^3 \rho , \quad (403) \]

The exhaust temperature of the coolant (at the end of the air gap and before entering the radial duct) is:

\[ \theta_{10_{out}} = \frac{1}{2}(\theta_3 + \theta_7) - \left( \frac{\theta_3 + \theta_7}{2} - \theta_1 \right) e^{-b CL/2} - \frac{a}{b}(1-e^{-b CL/2}) , (404) \]

**Heat exchange in the radial duct**

The radial duct will be modeled as a constant cross section duct. Its geometrical characteristics will be (accounting for both sides):

- length = \( l = H \)
- Cross section: \( AC19 = WRD \times 2 \times \pi \times (R+G+0.5H) \), \( (405) \)
- Effective surface for convection: \( A_{3-19} \) (see eq. 348)

The equations for heat exchangers can be applied, where:

\[ W = W1 \]

\[ A = 0.5 \times A_{3-19} \]

\[ h = h_{19} \]

\[ \theta_{fin} = \theta_{10_{out}} \]

\[ \theta_{av_A} = \theta_{av_B} = \theta_{wall} \]

and the equation giving the amount of heat delivered to the coolant in the radial duct is (both sides):

\[ Q_{3-19} = W1 C_p \rho \left( \theta_{wall} - \theta_{10_{out}} \right) \cdot (1-e^{-U_{319} W1 C_p \rho}) , \quad (406) \]
The temperature $\theta_{\text{wall}}$ is unknown and will be determined by looking at the conduction process in the bulk of one of the halves of the stator core, in axial direction. The equivalent network in Fig. 43a comes from the theory of one dimensional heat conduction (section 3.6.3.1), where:

$$ R_0 = \frac{0.5 \times CL}{0.5 \times A_{3-19} \times k_z} = \frac{CL}{A_{3-19} \cdot k_z} \text{, stator iron} $$

Fig. 43 Equivalent network for heat exchange in the radial duct.
This network is equivalent to the one shown in Fig. 43b, from which the following equation can be written:

\[
\frac{1}{2} \cdot Q_{3-19} = \frac{6}{R_0} \cdot (\theta_{av} - \theta_{wall}) + \frac{2}{R_0} \cdot (\theta_{wall} - \theta_{16}) ;
\]

\[
Q_{3-19} = \frac{12}{R_0} \cdot \theta_3 - \frac{4}{R_0} \cdot \theta_{16} - \frac{8}{R_0} \cdot \theta_{wall} ,
\]

(408)

Eliminating now \(\theta_{wall}\) between the equations (406) and (408) and solving for \(Q_{3-19}\) :

\[
Q_{3-19} = \frac{8 \times C_{19}}{8+C_{19}xR_0} \cdot [1.5 \times \theta_3 - 0.5 \times \theta_{16} \times \theta_{10_{out}}]
\]

(409)

where

\[
C_{19} = W_1C_p \cdot \rho \cdot (1- \frac{U_{319}}{C_p \cdot \rho})
\]

(410)

\[
\theta_{wall} = -\frac{R_0}{8} \cdot Q_{3-19} + 1.5 \theta_3 - 0.5 \theta_{16}
\]

(411)

The exhaust temperature is:

\[
\theta_{19_{out}} = \theta_{wall} - \left(\theta_{wall} - \theta_{10_{out}}\right) \cdot e^{-\frac{U_{319}}{W_1C_p \cdot \rho}}
\]

(412)

and the fluid properties will be calculated at the temperature:

\[
\theta = \frac{1}{2} \left(\theta_{10_{out}} + \theta_{19_{out}}\right)
\]

(413)

From eq. (183) and Figs. 24 and 43a the maximum stator iron temperature in these conditions may now be calculated:
\[
\theta_{\text{max}} = 1.5 \theta_3 - 0.25 (\theta_{16} + \theta_{\text{wall}}) + \frac{1}{12} \cdot \frac{(\theta_{16} - \theta_{\text{wall}})^2}{\theta_3 - \theta_{16} - \theta_{\text{wall}}}, \quad (414)
\]

3.6.5 Radiation mode related expressions.

From the analysis of the radiation heat transfer in section 3.6.1 (summarized in Fig. 22) it results that the following parameters must be calculated in order to determine the network completely:

a) The surface areas \( A_i \)

b) The shape factors \( F_{ij} \).

3.6.5.1 Surface areas.

The calculation of the areas \( A_i \) is straightforward from the idealized sketches of the machine in Figs. 4 and 44. The parameters \( \lambda_i \) can be easily written in terms of more basic geometric dimensions. It is necessary to distinguish between the four independent enclosures: a, b, c and d. Although the sketches only represent one quarter of the machine, the calculated values and the corresponding elements of the network are for the whole machine.

Define:

\[
\lambda_9 = \text{RF} - \text{TF} - \text{TA} \quad (415)
\]

\[
\lambda_{10} = \text{R} + \text{G} \quad (416)
\]

\[
\lambda_{11} = \text{RF} - \text{TF} \quad (417)
\]

Enclosure a:

\[
A_{9a} = 2.\pi.L.\text{RF} \quad (418)
\]

\[
A_{14a} = 2.\pi.[\text{RF}.(LT-L)+\text{RF}^2-\text{RSH}^2] \quad (419)
\]
Enclosure b:
\[ A_{9b} = 2 \pi L \ell_1 \]  \hspace{1cm} (420)
\[ A_{(3-4)b} = 2 \pi L \ell_9 \]  \hspace{1cm} (421)

Enclosure c:
\[ A_{(3-4)c} = 2 \pi L \ell_{10} \]  \hspace{1cm} (422)
\[ A_{(7-8)} = 2 \pi L R \]  \hspace{1cm} (423)

Enclosure d:
\[ A_2 = A_{B1} + A_{B2} + A_{B3} + A_{B4} = 2 \pi \left[ 2 \left( \ell_1 + \ell_2 \right) \left( \ell_6 + \ell_7 \right) + \ell_2^2 - \ell_6^2 \right] \]  \hspace{1cm} (424)
\[ A_{(12-17)} = A_{C1} + A_{C2} + A_{C3} + A_{C4} = 4 \pi \left[ \ell_1 \ell_5 + \ell_2 + \ell_3 \right] . \ell_4 + 5 \pi \left( \ell_5^2 - \ell_4^2 \right) \]  \hspace{1cm} (425)
\[ A_{14d} = A_{A1} + A_{A2} = 4 \pi \left[ \ell_8 . \left( \ell_1 + \ell_2 + \ell_3 \right) + 5 \pi \left( \ell_8^2 - \ell_4^2 \right) \right] \]  \hspace{1cm} (426)
\[ A_{16} = 2 \pi \left( \ell_8^2 - \ell_7^2 \right) \]  \hspace{1cm} (427)

In the calculation of the shape factors the areas must be the actual ones for only one enclosure (i.e., only one end of the machine). But the final results remain unchanged if all the areas are doubled.

Fig. 44 Idealized geometry for enclosure d. (\( \ell_i \) represents a length; AI, BI, CI and D are surfaces).
3.6.5.2 Shape factors

The definition of the shape factor is

$$ F_{1-2} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cdot \cos \theta_2}{\pi r^2} \cdot dA_2 \cdot dA_1 , $$

where

- \( r \) is the distance between \( dA_1 \) and \( dA_2 \)
- \( A_1 \) and \( A_2 \) are the two surfaces, and \( dA_1 \) and \( dA_2 \) are elements of them.
- \( \theta_1 \) and \( \theta_2 \) are the angles that the normal directions to \( dA_1 \) and \( dA_2 \) make with the straight line joining them.

The two integrals in eq. (428) are double integrals, each taken over a surface. For even the simplest geometrical shapes the evaluation of the fourfold integral becomes quite involved. Nevertheless there are references giving the results of the integral in eq. 428 for many elemental geometries 96, 92, 68, 58, 70.

More complex shapes often may be reduced to simpler cases by the application of "shape factor algebra", i.e., the use of some of the shape factor properties. Some of them follow:

a) $$ \sum_{j=1}^{N} F_{i-j} = 1 , $$

where all the \( N \) surfaces in a complete enclosure have been accounted for.

b) $$ A_i \cdot F_{i-j} = A_j \cdot F_{j-i} , $$

c) If radiant energy leaves a surface \( A_i \) and arrives at a surface \( A_j \) that is subdivided into parts \( A_j' \) and \( A_j'' \) (\( A_j = A_j' + A_j'' \)), then:
\[ F_{A_i-A_j} = F_{A_i-A_j'} + F_{A_i-A_j''}, \quad (431) \]

and also
\[ A_j' \cdot F_{A_j'-A_i} + A_j'' \cdot F_{A_j''-A_i} = A_j \cdot F_{A_j-A_i}, \quad (432) \]

Now with these tools and the shape factors available in the literature, it is possible to obtain all the \( F_{j-i} \)'s that are required for the network shown in Fig. 22. Again a simplified geometry, Figs. 4 and 44, has to be used.

It is interesting to realize that, because of eq. (430), only one of the two shape factors \( F_{i-j} \), \( F_{j-i} \) has to be calculated.

Since most of the geometries involved in the shape factor calculations are related coaxial cylinders, it is convenient now to give some standard shape factors that will be extensively used and that can be found in the literature.\textsuperscript{96}

For the configuration in Figs. 45

Define:
\[ x = \frac{b}{a} ; \quad y = \frac{c}{a} ; \quad A = x^2 + y^2 - 1 ; \quad B = y^2 - x^2 + 1, \quad (433) \]

The shape factors are:
\[ F_{outer - inner \ cylinder} = f_1(x,y) = \frac{1}{x} - \frac{1}{\pi x}. \]

\[ \left[ \cos^{-1} \frac{B}{A} - \frac{1}{2y} x \left\{ \left( (A+2)^{1/2} - 4 x^2 \right) \right\} \cos^{-1} \frac{1}{x.A+B} + \sin^{-1} \frac{1}{x} - \frac{\pi A}{2} \right], \quad (434) \]
\[
F_{\text{outer - outer cylinder}} = f_2(x, y) = 1 - \frac{1}{x} + \frac{2}{\pi x} \cdot \tan^{-1}\left(\frac{2}{y(x^2-1)}\right) - \frac{y}{2 \cdot \pi x} \left\{ \frac{1}{y} \cdot \left(4 x^2 + y^2\right) \sin^{-1}\left[\frac{\frac{4.(x^2-1)+y^2}{x^2}}{y^2+4.(x^2-1)}\right] - \frac{1}{2} \right\} 
\]

\[
F_{\text{outer - base cylinder (shadowed area)}} = f_3(x, y) = 0.5 \times [1 - f_1(x, y) - f_2(x, y)] \tag{436}
\]

It is obvious that

\[
F_{\text{inner - inner cylinder}} = 0 \tag{437}
\]

and now using eq. (429):

\[
F_{\text{inner - base cylinder (shadowed area)}} = f_4(x, y) = 0.5 \times [1 - 0 - F_{\text{inner - outer cylinder}}] \tag{438}
\]

\[f_4(x, y) = 0.5 \times [1 - x \cdot f_1(x, y)] \tag{438}\]

![Figure 45. Coaxial cylinders of equal length](image.png)
It is also necessary to determine the shape factor $F_{1-4}$ between the two coaxial cylinders numbered 1 and 4 in Fig. 46. Other auxiliary surfaces have also been numbered.

![Coaxial cylinders diagram]

Fig. 46 Coaxial cylinders of different lengths

The following relation is obvious:

$$F_{1-4} = F_{1-7} - F_{1-5} \quad ,$$  \hspace{2cm} (439)

where:

$$F_{1-7} = (\text{eq. 438}) = f_4 \left(x, \frac{y}{z}\right) \quad ,$$  \hspace{2cm} (440)

and it is defined:

$$z = \frac{C}{h} \quad ,$$  \hspace{2cm} (441)

Eq. (432) gives:

$$F_{1-5} = \frac{A_1 + A_2}{A_1} \cdot F_{(1-2)-5} - \frac{A_2}{A_1} \cdot F_{2-5} = z \cdot f_4(x,y) - (z-1) \cdot f_4(x,y,\frac{y}{z}) \quad ,$$  \hspace{2cm} (442)
and finally:

\[ F_{1-4} = f_5(x,y,z) = f_4(x, \frac{y}{z}) - z \cdot f_4(x,y) + (z-1) \cdot f_4(x,\frac{y}{z}) \] (443)

The equations which have been just described must be handled with extreme care under a computational point of view\textsuperscript{96}. Therefore the following ground rules will be followed in the calculation of the shape factors:

a) Use of double precision in the FORTRAN IV program to prevent cancellations of very similar terms.

b) Try to use expressions where all the terms are positive.

c) Keep the calculation of each shape factor and the subsequent assumptions as independent as possible of previously obtained results.

Following is the calculation of the required shape factors:

\[ F_{(4-3)-(7-8)} = F_{37} = f_1(1 + \frac{G}{R}, \frac{L}{R}) \] (444)

\[ F_{9-(4-3)} = F_{93} = f_1(1 + \frac{T \cdot A}{\lambda g}, \frac{L}{\lambda g}) \] (445)

\[ F_{9-15} = F_{915} = 1 \] (446)

\[ F_{14-15} = F_{1415} = 1 \] (447)

Calculation of \( F_{2-16} \):

From eq. (432):

\[ F_{216} = F_{B-D} = \frac{1}{A_2} \times [A_{(B2-B3-B4)} \times F_{(B2-B3-B4)-D} + A_{B1} \times F_{B1-D}] = \]

\[ = \frac{A_{B1}}{A_2} \times f_4 \left( \frac{\lambda_8}{\lambda_7}, \frac{\lambda_1 + \lambda_2}{\lambda_7} \right) \] (448)
where

$$A_{B1} = 4 \times \pi \times \ell_7 \times (\ell_2 + \ell_1)$$  \hspace{1cm} (449)

Calculation of $F_{16-14}$:

From eq. (429):

$$F_{1614} = F_{D-A} \approx 1 - F_{16-2} = 1 - \frac{A_2}{A_{16}} \times F_{216}$$ \hspace{1cm} (450)

Calculation of $F_{2(12-17)}$:

$$F_{212} = F_{B-C} = \frac{1}{A_B} \times \{ A_{B1} x F_{B1-C} + A_{B2} x F_{B2-C} + A_{B3} x F_{B3-C} + A_{B4} x F_{B4-C} \} =$$

$$= \frac{1}{A_B} \times \{ A_{B3} x F_{B3-(C1-C2-C3)} + A_{B4} x F_{B4-C1} \} =$$

$$= \frac{1}{A_B} \times \left\{ A_{B3} \left[ f_1 \left( \frac{\ell_6}{\ell_4}, \frac{\ell_2}{\ell_4} \right) + f_3 \left( \frac{\ell_6}{\ell_4}, \frac{\ell_2}{\ell_4} \right) \right] + A_{B4} \left[ f_1 \left( \frac{\ell_6}{\ell_5}, \frac{\ell_1}{\ell_5} \right) \right] \right\} =$$

$$= 4 \times \frac{x \pi}{A_2} \times \left\{ \ell_2 \times \ell_6 \left[ f_1 \left( \frac{\ell_6}{\ell_4}, \frac{\ell_2}{\ell_4} \right) + f_3 \left( \frac{\ell_6}{\ell_4}, \frac{\ell_2}{\ell_4} \right) \right] + \ell_3 x \ell_4 \left[ f_5 \left( \frac{\ell_6}{\ell_4}, \frac{\ell_2}{\ell_4}, \frac{\ell_3}{\ell_3} \right) + \ell_1 x \ell_6 \left[ f_1 \left( \frac{\ell_6}{\ell_5}, \frac{\ell_1}{\ell_5} \right) \right] \right\} \hspace{1cm} (451)$$

Calculation of $F_{2-14}$:

From eq. (429):

$$F_{214} = F_{B-A} = 1 - F_{B-B} - F_{B-C} - F_{B-D}$$ \hspace{1cm} (452)
\[ F_{B-B} = \frac{1}{A_B} \times [A_{B3} \times F_{B3-B} + A_{B4} \times F_{B4-B}] = (\text{small air gap}) = \]

\[ \approx \frac{4 \pi x \ell_6}{A_2} \times \left[ \ell_2 \times f_2 \left( \frac{\ell_6}{4}, \frac{\ell_2}{4} \right) + \ell_1 \times f_2 \left( \frac{\ell_6}{5}, \frac{\ell_1}{5} \right) \right] , \quad (453) \]

\[ F_{214} = 1 - \frac{4 \pi x \ell_6}{A_2} \times \left[ \ell_2 \times f_2 \left( \frac{\ell_6}{4}, \frac{\ell_2}{4} \right) + \ell_1 \times f_2 \left( \frac{\ell_6}{5}, \frac{\ell_1}{5} \right) \right] - F_{212} - F_{216} , \quad (454) \]

**Calculation of** \( F_{12-14} \):

From eq. (429):

\[ F_{1214} - F_{C-A} = 1 - F_{C-C} - F_{C-B} - F_{C-D} \]

\[ F_{C-C} = \frac{A_{C2}}{A_C} \times F_{C2-C} + \frac{A_{C3} + A_{C4}}{A_C} \times F_{C3-C4} - C^2 = \]

\[ = \frac{A_{C2}}{A_C} \times F_{C2-C} + \frac{A_{C3} + A_{C4}}{A_C} \times F_{C3-C4} - C^2 = \frac{A_{C3} + A_{C4}}{A_C} \times F_{C3-C4} - C^2 = \]

\[ = \frac{8 \pi x \ell_4}{A_{12}} \times (\ell_2 + \ell_3) \times f_4 \left( \frac{\ell_5, \ell_2 + \ell_3}{\ell_4} \right) \]

\[ F_{1214} = 1 - \frac{8 \pi x \ell_4}{A_{12}} \times (\ell_2 + \ell_3) \times f_4 \left( \frac{\ell_5, \ell_2 + \ell_3}{\ell_4} \right) - \frac{A_2}{A_{12}} \times F_{212} , \quad (455) \]

**3.6.6 The resultant system of equations. Solving for the temperature rises.**

**The unknowns.**

In the preceding sections it has been shown that all the heat exchanges between the different modeled elements (nodes) of the machine, can be written in terms of known coefficients and the following 13 unknown
quantities:

- 9 average temperatures (in °C; or $X_{\text{abs}}(I)$ in °K):
  
  \[
  \begin{align*}
  X(1) &= \theta_{3-4} \\
  X(2) &= \theta_{7-8} \\
  X(3) &= \theta_9 \\
  X(4) &= \theta_{11} \\
  X(5) &= \theta_{12-17} \\
  X(6) &= \theta_{13} \\
  X(7) &= \theta_{14} \\
  X(8) &= \theta_{16} \\
  X(9) &= \theta_{\text{ext}2} \quad \text{(temperature of the external surface of stator end winding)}
  \end{align*}
  \]

- 4 radiosities for the elements involved in the overhang region

\[
\begin{align*}
X(10) &= B_{\text{ext}2} \\
X(11) &= B_{12-17} \\
X(12) &= B_{14d} \\
X(13) &= B_{16}
\end{align*}
\]

The equations.

As indicated in section 3.6.2 the internal heat inputs to the different elements of the machine (all types of losses) are assumed to be known from the outset. Actually most of them are provided by the electrical and magnetic models.

It is therefore possible to write an equation for each one of the nine elements whose average temperature is unknown, by expressing the
equality between its internal heat input (if any) \( L(I) \) and the heat transferred to its neighboring elements by the three modes: conduction, convection and radiation:

\[
L(I) = \sum_{J=1}^{9} \left[ Q(I,J) + C(I,J) + R(I,J) \right],
\]

(456)

where \( Q \) stands for conduction, \( C \) for convection and \( R \) for radiation.

The terms with \( J=1 \) include heat exchanges with elements not included in the given list of nine, like the ambient whose temperature \( \theta_{15} \) is prescribed.

It has been shown that the conductive and convective exchanges can always be written as:

\[
Q(I,J) = k_1 \times [X(I) - X(J)] ; Q(I,I) = k_2 \cdot X(I) + k_3,
\]

(457)

\[
C(I,J) = k_4 \times [X(I) - X(J)] ; C(I,I) = k_5 \cdot X(I) + k_6,
\]

(458)

where \( k_1, k_2 \) and \( k_3 \) are slightly temperature dependent and can be considered to be constants. But \( k_4, k_5 \) and \( k_6 \) are usually clearly temperature dependent coefficients.

And for the radiative exchanges (from eq. 159)

\[
R(I) = \sum_{J=1}^{9} R(I,J) = (in \ general) = \sum_{J=1}^{9} k(I) \cdot X(I)^4 + \sum_{J=1}^{13} k(I) \cdot X(I)_{abs},
\]

(459)

where \( k(I) \) are constant coefficients.

The substitution of eqs. (457), (458), and (459) into eq. (456) gives 9 nonlinear algebraic equations of the type:
\[
\sum_{J=1}^{9} B(I,J) \cdot X(J)^4 + \sum_{J=1}^{13} D(I,J) \cdot X(J) + E(I) = 0 \quad I = 1,2,\ldots 9 ,
\]

(460)

where \(B(I,J)\), \(D(I,J)\) and \(E(I)\) are coefficients.

The remaining 4 equations are obtained from a direct application of the eq. (161) to the four nodes with unknown radiosities, giving expressions of the form:

\[
\sum_{J=1}^{9} B(I,J) \cdot X(J)^4 + \sum_{J=10}^{13} D(I,J) \cdot X(J) = 0 \quad I = 10,\ldots 13 ,
\]

(461)

In more explicit terms the 13 equations are:

(462)

\[
CLISC + CLIST = Q(3,1)+Q(3,9)+Q(3,16)+C(3,10)+C(3,18)+C(3,19)+R(3,7)+R(3,9)
\]

(463)

\[
CLIRC + CLIRT = Q(7,5)+Q(7,12)+C(7,10)+R(7,3)
\]

\[
O = Q(9,3)+Q(9,16)+Q(9,14)+C(9,15)+C(9,18)+R(9,3)+R(9,15)
\]

(464)

\[
CLW1 = C(11,2)+C(11,12)+C(11,14)+C(11,15)+C(11,16)
\]

(465)

\[
O = Q(12,6)+Q(12,7)+Q(12,13)+C(12,11)+R(12)
\]

(466)

\[
CLB = Q(13,12)+Q(13,14)+Q(13,15)
\]

(467)

\[
O = Q(14,9)+Q(14,13)+Q(14,16)+C(14,11)+C(14,15)+R(14,15)+R(14d)
\]

(468)

\[
O = Q(16,3)+Q(16,9)+Q(16,14)+C(16,11)+R(16)
\]

(469)

\[
O = Q(2,2\text{ EXT}) + C(2\text{ EXT},11) + R(2\text{ EXT})
\]

(470)

\[
(B_2^{\text{ext}} \cdot \sigma - \sigma_2^{\text{ext}})^{\frac{4}{1-\varepsilon_2}} + (A.F)_2^{12} \cdot (B_2^{\text{ext}} - B_{12}) + (A.F)_2^{14} \cdot (B_2^{\text{ext}} - B_{14}) + (A.F)_2^{16} \cdot (B_2^{\text{ext}} - B_{16}) = 0 ,
\]

(471)
\( \left( B_{12} - \sigma \cdot \theta_{12}^4 \right) \frac{e_{12} \cdot A_{12}}{1 - e_{12}} + (A.F)_{12-2} \cdot (B_{12} - B_{2\text{ext}}) + (A.F)_{12-14} \cdot (B_{12} - B_{14}) = 0 \),

\( \left( B_{14} - \sigma \cdot \theta_{14}^4 \right) \frac{e_{14} \cdot A_{14}}{1 - e_{14}} + (A.F)_{14-2} \cdot (B_{14} - B_{2\text{ext}}) + (A.F)_{14-12} \cdot (B_{14} - B_{12}) +

\( + (A.F)_{14-16} \cdot (B_{14} - B_{16}) = 0 \),

\( \left( B_{16} - \sigma \cdot \theta_{16}^4 \right) \frac{e_{16} \cdot A_{16}}{1 - e_{16}} + (A.F)_{16-2} \cdot (B_{16} - B_{2\text{ext}}) + (A.F)_{16-14} \cdot (B_{16} - B_{14}) = 0 \).

Sections 3.6.3, 4 and 5 provide the elements to write the final numerical version of these equations, as they are presented in the computer program printout of the subroutine THERMO (see Appendix D).

**Solving for the unknowns.**

The preceding section has provided a system of nonlinear algebraic equations in the 13 unknowns. There are only terms in the first and fourth power of the unknowns, but it should not be forgotten that in addition many of the coefficients are temperature dependent.

All of the equations can be written in this format:

\[ \sum_{J=1}^{13} B(I,J) \cdot X_{\text{abs}}(J)^4 + \sum_{J=1}^{13} D(I,J) \cdot X(J) + E(I) = 0 \), \quad I=1, \ldots 13 \]  

Different approaches have been suggested to solve this problem\(^{68,70}\). Most of them are devised for the particular case in which radiation is either overwhelmingly important or almost negligible. The approach to be followed here is a "brute force" Newton-Raphson technique in which the
coefficients will be assumed to be constant during each iteration, but they are updated after each iteration.

Therefore the approximated linearized Newton-Raphson equation is (from eq. 475):

\[
\sum_{J=1}^{13} 4B(I,J)X_{\text{abs}}^3(J) \Delta X(J) + \sum_{J=1}^{13} D(I,J) \Delta X(J) + F(I) = 0, \quad I = 1, \ldots, 13
\]  

(476)

where

\[
F(I) = \sum_{J=1}^{13} B(I,J)X_{\text{abs}}^4(J) + \sum_{J=1}^{13} D(I,J)X(J) + E(I)
\]  

(477)

but now the true unknowns are the increments \( \Delta X(J) \) and the \( X(J) \)'s are the best values obtained for the unknowns so far. Therefore the set of linear Newton-Raphson equations in \( \Delta X(J) \) can be written as follows:

\[
\sum_{J=1}^{13} A(I,J) \Delta X(J) + A(I,14) = 0, \quad I = 1, \ldots, 13
\]  

(478)

where the coefficients \( A(I,J) \) are constants during the process of solving the system of linear equations for \( \Delta X(J) \), but they are updated each time that the unknowns \( X(J) \) are.

The subroutine THERMO may be used in two different modes:

1) As part of the proposed integral design of a machine, the subroutine THERMO provides the thermal design constraints. In other words, for a given proposed design, THERMO checks if its temperature rises are below the prescribed limits.
II) THERMO may be used alone to perform more detailed studies on the cooling system of a particular design or actual machine. It receives the aforementioned data and provides:

a) Temperatures at the expected hot spots in the machine:
   - Middle points of the embedded and end stator windings, TEMP(1) and TEMP(2).
   - Middle points of the embedded and end rotor windings, TEMP(3) and TEMP(4).
   - Hot spot of stator iron, TEMP(5).
   - Hot spot of rotor iron, TEMP(6).

b) Mean temperatures of the main parts of the machine:
   - Stator iron, X(1)
   - Rotor iron, X(2)
   - Frame, X(3)
   - Overhang air, X(4)
   - Rotor ends, X(5)
   - Bearings, X(6)
   - End shield, X(7)
   - Stator ends, X(8)
   - External surface of stator end winding, X(9)

c) All possible heat exchanges among the above mentioned parts of the machine, separately for the 3 modes of heat transfer (see Fig. 20).

d) Other temperatures of interest
   - Exhaust temperature of coolant (if any) after passing the air
gap, TEMP(7), the axial duct (if any), TEMP(9), and the radial duct (if any), TEMP(10).

- Temperature of the radial duct wall (if any), TEMP(11).

Therefore THERMO may also be used as a tool to analyze the behavior of the cooling system of an actual machine or a proposed design. For instance, parametric studies can be easily performed, showing the sensitivity of the temperature rises and heat exchanges to the variation of a particular dimension, loss, flow rate or material thermal property. Relative to the other models, the thermal model is the most voluminous. It will be seen later that this is justified, since the thermal constraints are in general the most active limitations in the machine design.

3.6.7 Design constraints. Standards for limits of temperature rises in electrical machinery.

Following is the list of the expected hot spots in the machine, with the equations which have been used to obtain their temperatures:

- Stator winding hot spot; eq. (301) gives this temperature, which is the higher of either TEMP(1) or TEMP(2).

- Rotor winding hot spot; same with eq. (277), TEMP(3) and TEMP(4).

- Stator core hot spot; eq. (414) gives this temperature TEMP(5) when radial duct exists. Otherwise eq. (183) yields:

  \[ TEMP(5) = 1.5 \times X(1) - .5 \times X(8) \quad (479) \]

- Rotor core hot spot; again eq. (183) gives:

  \[ TEMP(6) = 1.5 \times X(2) - .5 \times X(5) \quad (480) \]
Hot spot temperatures in electrical machines must be limited according to the insulation class$^{103, 11, 10}$. Assuming an ambient temperature of 40°C, the maximum allowable hot spot temperatures in the machine are:

For Class A insulation: 105°C
   " E " 120°C
   " B " 130°C
   " F " 155°C
   " H " 180°C

Therefore the thermal design constraints will have the form:

$$\text{TEMP}(I) < \text{TMAX}, \ I = 1 \ or \ 2, \ 3 \ or \ 4, \ 5 \ and \ 6,$$

where TMAX will be chosen from the above table according to the desired type of insulation.

3.6.8 Experimental checking

The thermal model was checked against measurements made on a 15 hp squirrel cage induction motor. The motor, a Gould high efficiency design designated "E-Plus", was supplied by the manufacturer with complete documentation, including detailed drawings and materials properties. The availability of this information was important since the proposed thermal model relies on a very complete knowledge of these machine characteristics. The other important source of information required by the thermal model is the set of heat inputs to the different nodes. These were provided by detailed measurements yielding complete separation of the losses up to the level required by the thermal model. The flow rate of coolant and the film coefficient for the exterior surface of the machine were also deter-
mined experimentally. All the significant temperatures provided by the model were checked by placing thermocouples at the corresponding spots of the machine. The table shows the correspondence between the thermal model predictions and the measured temperatures.

<table>
<thead>
<tr>
<th>Location</th>
<th>Measured Temperature:°C</th>
<th>Temperature Predicted:°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator core</td>
<td>67 (external surface)</td>
<td>71.7 (avg)</td>
</tr>
<tr>
<td>Rotor core</td>
<td>94 (max)</td>
<td>94.9 (max)</td>
</tr>
<tr>
<td>Frame</td>
<td>47</td>
<td>49.0</td>
</tr>
<tr>
<td>Air at the axial duct entrance</td>
<td>37/49 (two locations)</td>
<td>44.9 (avg)</td>
</tr>
<tr>
<td>Rotor end</td>
<td>83</td>
<td>90.8</td>
</tr>
<tr>
<td>Bearing</td>
<td>58</td>
<td>57.9</td>
</tr>
<tr>
<td>End shield</td>
<td>47/40 (max and min)</td>
<td>40.6 (avg)</td>
</tr>
<tr>
<td>Stator end</td>
<td>62</td>
<td>65.0</td>
</tr>
<tr>
<td>Stator end windings</td>
<td>52</td>
<td>64.7</td>
</tr>
<tr>
<td>external surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stator windings</td>
<td>65 (avg)</td>
<td>71.6/73.1 (min/max) along middle line.</td>
</tr>
<tr>
<td>Exhaust air</td>
<td>51</td>
<td>50.4</td>
</tr>
<tr>
<td>Ambient</td>
<td>28</td>
<td>28 (assigned)</td>
</tr>
</tbody>
</table>

The significant discrepancies in the temperatures of the stator end windings surface and the rotor end, are due to the excessive ventilation of the thermocouples measuring these two temperatures, therefore giving values averaging the temperatures of the coolant and the solid part. Com-
plete details on the experimental checking of the model may be found in reference 6.

The thermal model was specifically addressed to wound rotor synchronous machines with totally enclosed rotor end windings. Therefore the model had to be slightly modified in order to be applicable to a squirrel cage induction motor. A list of the modifications follows:

- Add the end ring losses to node 17.
- Suppress insulation in rotor windings. Rotor bar losses go directly into the rotor core.
- Rotor bars axial thermal resistance is placed in parallel with rotor core axial thermal resistance.
- Increase the effective convective surface of the rotor end because of the fins.
- Enter a finite thermal resistance from the bearings to the ambient. This unknown resistance had to be adjusted until a reasonable bearing temperature was obtained.
- Enter values for the thermal resistances $R_{4-9}$ and $R_c$ (fig. 28a) in accordance with the specific construction of the machine.

The matching of the experimental and calculated values was considered to be very satisfactory; it should be noticed that the proposed thermal model does not "realize" if a machine is conventional or not.
CHAPTER IV

THE COMPUTER PROGRAM

A FORTRAN program has been written gathering together what has been described in the preceding sections. Given a set of performance requirements and material properties, this program designs the machine that is optimum in a prescribed sense. This prescribed sense is determined by the designer through the definition of the figure of merit, and the values and weighting factors for the constraint limits. The program has been used in Time Sharing Option, allowing the designer to interact with the optimization process at certain key points.

Detailed information about the program is presented in Appendices A (computer printouts) and D (flowcharts). What follows is a brief description of the functions performed by the major blocks (or subroutines) in which the complete program has been divided. Figures 3 and 47 show the overall philosophy.

Fig. 47 Computer program outline
The program consists of a MAIN program which assigns default values to the performance requirements, material properties, control variables, constraint limits, weighting factors, initial guesses, required accuracy levels, step sizes and other optional parameters for the optimization algorithm, all of which may be changed from the terminal without requiring a recompilation of the program. The program starts by a call to the subroutine HOOKE. When control is returned from HOOKE all the available information on the best design up to this moment is printed.

The subroutines HOOKE and LOCAL contain the pattern search algorithm. LOCAL only takes care of the exploratory moves; both call the subroutine OBJECT to calculate the objective function. Whenever a step size reduction is required, or a prescribed number of OBJECT calls has been exceeded, the program gives control to the terminal. The designer may then simply let the process continue or introduce modifications by changing the step sizes, entering a new guess for \( \bar{x} \), increasing the number of allowed calls to OBJECT, skipping the search over any of the variables, declaring any variable to be discrete or continuous, or by stopping the search.

The subroutine OBJECT performs a complete design evaluation, and calculation of the corresponding objective function, for a given set of independent variables \( \bar{x} \) provided by HOOKE or LOCAL. OBJECT first goes through the analysis step, then calls THERMO to calculate the temperature rises at the expected hot spots, obtains the constraint functions and finally determines the value of the objective function. Certain calculations in OBJECT and especially in THERMO may be performed with variable accuracy; for instance, re-evaluation of the shape factors and film coefficients is not done at every call to THERMO, and the level of accura-
The accuracy demanded of the Newton-Raphson solution of the thermal model equations may be changed. Appropriately defined control variables take care of all this automatically.

The complete program has over 2300 FORTRAN sentences and takes 9 sec. of central processor time to compile on an IBM 360, Model 75. To perform one design evaluation with its objective function calculation takes 0.025 seconds on the average. A complete optimization process, starting from a poor initial guess, may require 30 base points, 1000 function evaluations and about 25 seconds to run. The program fits in a 250K region.

The comment lines in the program printouts (Appendix A) explain specific details of the performance of each subroutine, which cannot be discussed here. Three options are available to print relevant intermediate information during the optimization process; some intermediate information is always required since the control is sometimes returned to the terminal; in the debugging process or when tracing back the design trade-offs it is convenient to know more about the intermediate steps of the optimization.
AN APPLICATION: DESIGN FOR MINIMUM SIZE

5.1. STATEMENT

The performance of the program is now illustrated with a significant application: a parametric study of the effect of the rotational speed on the design of machines intended for minimum size. The machines are wound field 3-phase synchronous, as has been assumed throughout this work. Two cooling options will be considered:

a) Open double end ventilated machine. No axial or radial ducts. External natural cooling with no fins. In order to have a better comparison basis for the designs, the flow rate of coolant will be assigned a value of .07 m³/sec for any design, which is a reasonable figure for a 20 kW machine with an efficiency of .85, the lower design limit for the efficiency. Thus machines with better efficiencies will be hyperventilated.

b) Totally enclosed machine operating in a vacuum. External forced cooling and fins. A value of 150 W/°C-m² has been assigned to the film coefficient for the exterior surface.

The performance requirements are summarized as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical Power Output</td>
<td>20 kW</td>
</tr>
<tr>
<td>Power Factor</td>
<td>1</td>
</tr>
<tr>
<td>Terminal Voltage</td>
<td>220 V(λ-λ)</td>
</tr>
<tr>
<td>Number of Poles</td>
<td>2</td>
</tr>
<tr>
<td>Excitation Voltage</td>
<td>110 V</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>40°C</td>
</tr>
<tr>
<td>Flow Rate of Coolant</td>
<td>.07 m³/s</td>
</tr>
<tr>
<td>(cooling option a)</td>
<td></td>
</tr>
<tr>
<td>Rotation Speed</td>
<td>3600, 20000, 40000, 60000 rpm</td>
</tr>
</tbody>
</table>

As pointed out in section 3.4 the choice of a suitable magnetic material is a delicate issue, since it involves tradeoffs among different properties of the considered materials. The most systematic and reliable procedure for making a decision for a particular application is to use the present computer program; once the figure of merit function is prescribed, the whole design program must be run for each of the several considered materials, the one which yields the lowest figure of merit being finally selected. Such tradeoff studies, although possible, will not be performed here, and a particular magnetic material has been selected from the outset: Venadium Permendur$^{36}$.

Venadium Permendur consists nominally of 49 percent iron, 49 percent cobalt and 2 percent vanadium. It is a commercially produced alloy, and it provides the high saturation induction typical of certain compositions of the iron-cobalt system. Moreover, its mechanical properties (yield and tensile strength) are better than the properties of the more conventional low silicon grade alloys. This gives to Venadium Permendur a weight-saving advantage over other materials in the design of light, high speed generators.

To realize its optimum magnetic properties Venadium Permendur must be annealed in the temperature range of $1300^\circ$F to $1600^\circ$F. With increased anneal temperature below $1625^\circ$F, magnetic properties improve (core losses diminish significantly) and mechanical strength decreases; on the other hand the dc magnetization curve is not appreciably changed. The foregoing properties suggest the following modes of utilization of the material:

Since the stator is not subject to important mechanical stresses,
the only concern are the magnetic properties. Therefore thin laminations will be used: 0.006" (to reduce core losses, although the cost increases), annealed at 1550°F. On the other hand, for the rotor the main concern is mechanical strength; the rotor works mostly under stationary magnetic fields, therefore non annealed (cold rolled) material will be used; however, in order to prevent high losses due to the effect of harmonics and stray fields a laminated rotor will be used: 0.014" of thickness; this does not harm its mechanical performance, as is shown in section 3.5.3.

For this particular application, the mechanical design constraints will be set in such a way that the magnetic material of the rotor will be always under elastic conditions, so magnetic properties will not be seriously damaged by the action of mechanical stress. The effect of mechanical stresses when the material is under plastic conditions will not be considered here.

The following approximate analytical expressions have been obtained from the data presented in ref. 36, by using the procedures explained in section 3.4.3 and 4.

a) Iron losses:

Material = Vanadium-Permendur, thickness: .006", annealed at 1550°F. Ref. 36 provides values for \( F = 60, 400, 800 \) and 1600 Hz, in the range \( 1 \leq B \leq 2 \text{ wb/m}^2 \).

The resultant expression will be applicable for the whole range of interest of \( B \).

By using the second model presented in section 3.4.3 and taking \( F_0 = 800 \text{ Hz}, B_0 = 1 \text{ wb/m}^2 \), the following expression results:
\[ P = 28.22 \times B^{1.6456} \times \left( \frac{F}{800} \right)^{1.2052 + .3697 \times \log B} \]

This expression simulates the empirical family of curves with errors within \( \pm \, 5\% \) for \( F = 400 \) and \( 800 \) Hz, \( \pm \, 10\% \) for \( 1600 \) Hz and \( \pm \, 15\% \) for \( 60 \) Hz. These results are considered to be satisfactory, given the frequency range of interest and the difficulty in obtaining a good estimate of the iron losses, as pointed out before.

b) Magnetization curve:

The magnetization curve for the rotor does not include frequency dependence. The material is cold rolled Vanadium Permendur, thickness \( = .014" \). The resultant expression is:

\[ y = \sum_{I=1}^{9} BHO(I) \cdot P_{I-1}(B) \text{, for } B \leq 2.4 \text{ wb/m}^2 \]

\[ H = 79.58 \times 10^7 \text{ A/m} \]

where \( P_n(x) \) is the Chebyshev polynomial of order \( n \), in the variable \( x \). And the vector of coefficients \( BHO(9) \) is (for increasing values of \( I \)):

\[ (.708747, 1.06096, .629306, .371509, .170543, .0805209, .0182731, .00959134, -.0284367) \]

It has been determined that 6 digits give enough accuracy in the calculations.

\[ H = 8.352 \times 10^4 + 7.958 \times 10^6 \times (B-2.4) \text{ A/m, for } B \geq 2.4 \text{ wb/m}^2 \]

The expression for the stator magnetization curve must include the frequency dependence. The material is Vanadium Permendur, of thickness
= .006" and annealed at 1550°F. The expression is:

\[ y = \sum_{i=1}^{g} P_{i-1}(B) \times \sum_{k=1}^{3} FBH(I,k) \times P_{k-1}(F), \text{ for } B \leq 2.4 \text{ wb/m}^2 \text{ and } 60 \leq F \leq 1600 \text{ Hz.} \]

\[ H = 79.58 \times 10^y \ A/m. \]

The coefficient matrix FBH (9,3) is:

\[
\begin{pmatrix}
0.665909 & 0.0755612 & -0.0910742 \\
1.021717 & -0.0436253 & -0.00872816 \\
0.627468 & -0.0317297 & 0.00754077 \\
0.425449 & -0.0227534 & 0.00739603 \\
0.244077 & 0.00203121 & 0.0014826 \\
0.106227 & 0.00913556 & -0.00149603 \\
-0.000428736 & 0.00799372 & 0.00112546 \\
-0.00997758 & 0.00529107 & -0.00169356 \\
-0.0528178 & -0.00103540 & -0.00202001
\end{pmatrix}
\]

\[ H = 8.475 \times 10^4 + 7.958 \times 10^6 \times (B-2.4) \ A/m \text{ for } B \geq 2.4 \text{ wb/m}^2. \]

The above expressions reproduce the empirical family of curves with errors ranging between less than 5% in the region of greatest interest in B and F, and less than 15% in the far saturated region. It must be realized that, in the region of interest for the magnetization curve, obtaining H for a given value of B, \( H_0 = H(B_0) \) is much more inaccurate than the inverse operation \( B_0 = H^{-1}(H_0) \).
The program requires numerical values for the following mechanical properties of the materials (section 3.5):

**Rotor iron:**
\[ \rho = 8,200 \text{ kg/m}^3 \]
\[ \sigma_{\text{max}} = 1.275 \times 10^9 \text{ N/m}^2 \]
\[ \mu \approx 0.3 \]
\[ E \approx 1.96 \times 10^{11} \text{ N/m}^2 \]

**Rotor wedges:**
\[ \rho = 7,650 \text{ kg/m}^3 \]
\[ \sigma_{\text{max}} = 1.96 \times 10^8 \text{ N/m}^2 \]
\[ \mu \approx 0.3 \]
\[ E \approx 1.96 \times 10^{11} \text{ N/m}^2 \]

**Conductors (copper):**
\[ \rho = 8,900 \text{ kg/m}^3 \]

**Insulation (average):**
\[ \rho = 1,200 \text{ kg/m}^3 \]

An upper limit of 45° has been set for the power angle, which is equivalent to an upper limit of 1 p.u. for the synchronous reactance. A hot spot temperature limit of 130°C, corresponding to a Class B insulation system, is also specified.

The assumptions relative to the values of the resistances \( R_{4-9}, \)
\( R_{\text{bearings}} \) and \( R_c \) (fig. 28.a) and also from the frame to the ambient through the mountings and external shaft were discussed in the corresponding sections in Chapter III.
The Figure of Merit Function

As pointed out before the figure of merit function is the size, which has been defined as

\[
\text{SIZE} = \pi \times \text{length of the stack of laminations} \times (\text{external radius})^2 = \\
= \pi \cdot L \cdot RF^2
\]

This figure of merit is directly sensitive to variations of only four independent variables: \( R, L, G, \) and \( H \). This fact raises some interesting questions, since apparently the optimal design does not depend on the values of the remaining independent variables.

Actually this would be true only if at the optimum point \( \hat{x} \) there were no active constraints; here this is clearly not the case, since one or more of the specified constraints must limit the machine size, and the constraint functions (in particular the thermal ones) are in general very complex functions of all the independent variables. Therefore, with the objective function defined in equation (6), the actual hypersurfaces \( F(\hat{x}) = \text{const.} \) in the unconstrained problem will depend on all the independent variables \( \hat{x} \). Fig. 48 illustrates this fact for a simple case with only two variables involved; it is assumed that weighting factors have been chosen so that the figure of merit constraint is violated everywhere.
Fig. 48 The objective function and the figure of merit. a) Constrained problem, \( B(\bar{x}) = B(x_1) \); b) Unconstrained problem, \( F(\bar{x}) = F(x_1, x_2) \).
Therefore any well designed optimization strategy must be able to find the optimum $\hat{x}'$. However it is a fact that $F(\hat{x})$ will only be sensitive to all the independent variables variations for points located at the steep slopes where constraints other than $g_{m+1}$ are active. And this might introduce numerical problems related to the small step sizes required there.

It has been decided to add a term related to the machine efficiency to the figure of merit function, so it finally becomes:

$$B(\hat{x}) = WF \times [\text{SIZE} + \varepsilon \times (1 - \text{EFFICIENCY})]$$

where WF is a weighting factor and $\varepsilon$ is a constant.

Choosing a value of $\varepsilon$ such that both terms in the figure of merit function are comparable in magnitude would solve the problem mentioned above, since the efficiency $\eta$ depends on all the independent variables $\hat{x}$. However $\varepsilon$ has been chosen to be small, so the effect of the efficiency term is secondary and the optimization looks primarily for minimum size. The efficiency term helps in the early stages of the optimization by addressing the "undecided" independent variables, so the designed machines tend to be more efficient; this is considered to be convenient, since thermal constraints are the most likely ones to limit the machine. However this term can also create other types of problems, as will be shown later.

The value of $B_0$ is set so that the figure of merit constraint, (5), is always violated. This has the effect of making the objective function equal to the figure of merit inside the feasible region, as shown in (6). Therefore movement within the feasible region is always in a direction to
minimize (make better) the figure of merit.

**Constraint Functions and Weighting Factors**

The table below shows the per-unitized constraint functions CA(I) and their assigned limits CNTRL(I) and weighting factors WF(I). Very high weighting factors have been assigned to the geometrical constraints which detect impossible designs (1,2,3,4); violations of 10% (or CA= .1) are translated into values of $10^7$ for WF.CA. High values have been given to the geometrical constraints accounting for practicability (5,15), so 10% violations yield WF.CA = $10^8$. Mechanical and thermal constraints (6 to 11) have intermediate values, with WF.CA = $10^6$ for CA = 0.1. Finally, the constraints for efficiency, synchronous reactance and harmonic content (12,13,14) have been assigned the smallest weighting factors, so for CA = .1 is WF.CA = $10^3$. The initial value of WF.CA for the figure of merit constraint is about 100. Thus, the weighting factors have been chosen so that outside the feasible region, the violated constraints dominate the search almost independently of the figure of merit.
<table>
<thead>
<tr>
<th></th>
<th>CA(I) = CNTR(I)/CNTRL(I)-1</th>
<th>CNTRL(I)</th>
<th>WF(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(DR-RSH)/R-1</td>
<td>-</td>
<td>10^{15}</td>
</tr>
<tr>
<td>2</td>
<td>DS/H-1</td>
<td>-</td>
<td>10^{15}</td>
</tr>
<tr>
<td>3</td>
<td>2.TXR/WR-1</td>
<td>-</td>
<td>10^{15}</td>
</tr>
<tr>
<td>4</td>
<td>2.TXS/WS-1</td>
<td>-</td>
<td>10^{15}</td>
</tr>
<tr>
<td>5</td>
<td>(R/HIR)/(R/HIR)_{MAX}-1</td>
<td>(R/HIR/{MAX} = 57.3</td>
<td>10^{8}</td>
</tr>
<tr>
<td>6</td>
<td>\sigma_{\text{ROTOR}}/\sigma_{\text{MAX}}-1</td>
<td>\sigma_{\text{MAX}} = 1.275 \times 10^{9}</td>
<td>10^{6}</td>
</tr>
<tr>
<td>7</td>
<td>\sigma_{\text{TEETH}}/\sigma_{\text{MAX}}-1</td>
<td>\sigma_{\text{MAX}} = 1.275 \times 10^{9}</td>
<td>10^{6}</td>
</tr>
<tr>
<td>8</td>
<td>\theta_{\text{MAX STAT.WNDG}}/\theta_{\text{MAX}}-1</td>
<td>\theta_{\text{MAX}} = 130</td>
<td>10^{6}</td>
</tr>
<tr>
<td>9</td>
<td>\theta_{\text{MAX ROT.WNDG}}/\theta_{\text{MAX}}-1</td>
<td>\theta_{\text{MAX}} = 130</td>
<td>10^{6}</td>
</tr>
<tr>
<td>10</td>
<td>\theta_{\text{MAX STAT.CORE}}/\theta_{\text{MAX}}-1</td>
<td>\theta_{\text{MAX}} = 130</td>
<td>10^{6}</td>
</tr>
<tr>
<td>11</td>
<td>\theta_{\text{MAX ROT.CORE}}/\theta_{\text{MAX}}-1</td>
<td>\theta_{\text{MAX}} = 130</td>
<td>10^{6}</td>
</tr>
<tr>
<td>12</td>
<td>(-n + 2.\eta_{\text{MIN}})/\eta_{\text{MIN}}-1</td>
<td>\eta_{\text{MIN}} = .85</td>
<td>10^{4}</td>
</tr>
<tr>
<td>13</td>
<td>HCF/HCF_{\text{MAX}}-1</td>
<td>HCF_{\text{MAX}} = .1</td>
<td>10^{4}</td>
</tr>
<tr>
<td>14</td>
<td>\sin\delta/\sin\delta_{\text{MAX}}-1</td>
<td>\sin\delta_{\text{MAX}} = .7071</td>
<td>10^{4}</td>
</tr>
<tr>
<td>15</td>
<td>(DR/R)/(DR/R)_{\text{MAX}}-1</td>
<td>(DR/R)_{\text{MAX}} = .5</td>
<td>10^{8}</td>
</tr>
<tr>
<td>16</td>
<td>WF.[\pi.\text{CL}.RF^{2} + \epsilon.(1-\eta)]-1</td>
<td>-</td>
<td>WF=7,500; \epsilon%=0.001</td>
</tr>
</tbody>
</table>

Initial step sizes of about 1% the initial value of the corresponding independent variable have been chosen. For discrete variables the step sizes are equal to the closest non zero integer.
5.2 DISCUSSION OF RESULTS

Optimum designs have been outlined for $\omega = 3.6, 20, 40$ and $60$ krpm for machines with cooling option a, and for $40$ and $60$ krpm for machines with cooling option b. Figures 49a and b represent per unitized values for the size, efficiency and the constraints which happened to be active. Figure 50 shows the main geometrical features of the optimum designs for the open ventilated type. The efficiency is fictitiously high since stray losses and the fan power to blow the coolant have not been included. The table below gives the most significant numerical values for each design.

Certain conclusions can be drawn from the results of this design example. For low rotational speed (3600rpm), the machine tries to decrease its size by reducing the cross section of the magnetic circuit, causing an increase in the level of magnetic saturation. This process increases the MMF requirement and is eventually limited by the available rotor slot real estate. This problem is relieved by more armature turns and a shorter air-gap, causing the reactance limit to be reached. Increasing $\omega$ to 20 krpm for the same output voltage relieves the magnetic saturation problem in the rotor. The limit for the rotor slot depth is then set by the practicability constraint relating slot depth to rotor radius, which remains at its limit for all higher rotational speeds. Again, smaller air gap requires less room for rotor conductors, which means smaller size, and the reactance constraint reaches its limit. So far the thermal constraints are still not active.

Soon after 20 krpm, the windage losses (which are roughly proportional to the air gap region area, and the cube of the rotor linear speed at the air gap, and decrease for larger air gap lengths) become high enough so that the temperature rise at the rotor core reaches it limit.
<table>
<thead>
<tr>
<th>W, k rpm</th>
<th>3.6</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>40</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(\bar{x})$</td>
<td>57.3</td>
<td>23.2</td>
<td>12.6</td>
<td>$1.77 \times 10^5$</td>
<td>31.3</td>
<td>$6.3 \times 10^4$</td>
</tr>
<tr>
<td>SIZE/SIZE (W=3.6)</td>
<td>1</td>
<td>0.41</td>
<td>0.22</td>
<td>0.71</td>
<td>0.55</td>
<td>0.82</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.960</td>
<td>0.934</td>
<td>0.906</td>
<td>0.874</td>
<td>0.976</td>
<td>0.981</td>
</tr>
<tr>
<td>$X_d$</td>
<td>1.0</td>
<td>1.0</td>
<td>0.71</td>
<td>1.59</td>
<td>1.0</td>
<td>0.93</td>
</tr>
<tr>
<td>$\theta_{hotspot} / \theta_{max}$</td>
<td>0.52 (R.C.)</td>
<td>0.68 (R.C.)</td>
<td>1 (R.C.)</td>
<td>1.28 (R.C.)</td>
<td>1 (R.W.)</td>
<td>1.07 (R.C. &amp; W.)</td>
</tr>
<tr>
<td>$\sigma_{highest} / \sigma_{max}$</td>
<td>0.09 (R.C.)</td>
<td>0.22 (R.C.)</td>
<td>0.56 (R.C.)</td>
<td>0.99 (R.T.)</td>
<td>0.86 (R.C.)</td>
<td>0.92 (R.C.)</td>
</tr>
<tr>
<td>(R/HIR)/57.3</td>
<td>0.41</td>
<td>0.42</td>
<td>0.60</td>
<td>0.67</td>
<td>0.37</td>
<td>1.0</td>
</tr>
<tr>
<td>(DR/R)/.5</td>
<td>0.67</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.84</td>
</tr>
<tr>
<td>R</td>
<td>0.086</td>
<td>0.080</td>
<td>0.064</td>
<td>0.051</td>
<td>0.080</td>
<td>0.060</td>
</tr>
<tr>
<td>CL</td>
<td>0.105</td>
<td>0.061</td>
<td>0.048</td>
<td>0.134</td>
<td>0.071</td>
<td>0.106</td>
</tr>
<tr>
<td>G</td>
<td>0.0002</td>
<td>0.0016</td>
<td>0.0026</td>
<td>0.0026</td>
<td>0.0008</td>
<td>0.0010</td>
</tr>
<tr>
<td>II</td>
<td>0.052</td>
<td>0.034</td>
<td>0.030</td>
<td>0.050</td>
<td>0.046</td>
<td>0.063</td>
</tr>
<tr>
<td>SS/(3.P)</td>
<td>6</td>
<td>12</td>
<td>15</td>
<td>12</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>SR/P</td>
<td>18</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>20</td>
<td>41</td>
</tr>
<tr>
<td>kS</td>
<td>0.73</td>
<td>0.75</td>
<td>0.75</td>
<td>0.47</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>kR</td>
<td>0.64</td>
<td>0.36</td>
<td>0.55</td>
<td>0.66</td>
<td>0.39</td>
<td>0.60</td>
</tr>
<tr>
<td>2.CNS/SS</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2.CNR/SR</td>
<td>103</td>
<td>55</td>
<td>61</td>
<td>67</td>
<td>123</td>
<td>50</td>
</tr>
<tr>
<td>CJS</td>
<td>$3.94 \times 10^6$</td>
<td>$2.41 \times 10^6$</td>
<td>$3.73 \times 10^6$</td>
<td>$1.67 \times 10^6$</td>
<td>$2.24 \times 10^6$</td>
<td>$2.06 \times 10^6$</td>
</tr>
<tr>
<td>GEFF</td>
<td>$2 \times 6.7 \times 10^{-3}$</td>
<td>$5.75 \times 10^{-3}$</td>
<td>$3.72 \times 10^{-3}$</td>
<td>$3.55 \times 10^{-3}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$2.41 \times 10^{-3}$</td>
</tr>
<tr>
<td>DR</td>
<td>.029</td>
<td>.040</td>
<td>.032</td>
<td>.025</td>
<td>.039</td>
<td>.025</td>
</tr>
<tr>
<td>DS</td>
<td>.013</td>
<td>.019</td>
<td>.011</td>
<td>.042</td>
<td>.010</td>
<td>.011</td>
</tr>
<tr>
<td>RF</td>
<td>.153</td>
<td>.129</td>
<td>.107</td>
<td>.114</td>
<td>.138</td>
<td>.138</td>
</tr>
<tr>
<td>RSH</td>
<td>.018</td>
<td>.007</td>
<td>.005</td>
<td>.005</td>
<td>.008</td>
<td>.008</td>
</tr>
<tr>
<td>Cooling option</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>b</td>
</tr>
</tbody>
</table>
At 40 krpm the problem is solved by increasing the air gap length (so the reactance is below its limit) and decreasing the rotor radius, so windage losses do not increase much. A small rotor radius is now possible since low magnetic flux densities are enough to maintain the output voltage.

The design for 60 krpm is not feasible. The main problem is the high rotor temperature rise, caused by the large windage losses. This is the reason for the significant increase in the length of the machine. A larger air gap area requires smaller temperature rises of the rotor and stator surfaces above the mean fluid temperature at the air gap, in order to remove the heat associated with windage. The rotor radius is kept small to minimize the increase in windage losses and because the mechanical stresses at the rotor core do not allow a larger radius. The machine length increases the reactance, which is also above its limit.

Minor trade-offs are difficult to trace back. Such trade-offs include the effect on the air-gap reluctance vs. the effect on machine size of having either deep or shallow slots, the effect on reluctance vs. the effect on the conductor-insulating ratio of increasing the slot number while maintaining the slot factor, etc.

Designs for 40 and 60 krpm are also presented for the cooling option b. They are bigger than their counterparts cooled as option a, but two facts must be emphasized. First, the design for 60 krpm is almost feasible, showing that option b cooled machines can operate at higher rotational speeds than option a cooled machines. Second, thermal constraints again limit the machine, and much may be done to improve the present design of machines operating in a high vacuum. For instance, specially designed fins could be added at the rotor end to increase the
heat evacuated at this point, possibly yielding smaller size. No fins have been considered here. The program reported in this work might be used to devise the most suitable modifications in the design of these machines.
Figure 49. Normalized parameters of optimized design
Figure 49. Normalized parameters of optimized design
Rotor slots = 48; slot factor = .36
Stator slots = 72; slot factor = .73
Rotor slots = 48; slot factor = .55
Stator slots = 90; slot factor = .75

Fig. 50.c) 40,000 rpm. Main geometrical features of optimized designs.
Fig. 50.d) 60,000 rpm. Main geometrical features of optimized designs.

Rotor slots = 48; slot factor = .64
Stator slots = 72; slot factor = .47
5.3 IDENTIFIED AREAS OF TROUBLE

Many small problems had to be overcome to completely debug and fix up the computer program. Most of them do not deserve to be mentioned here; however, there are a few of these problems which concern important issues either in the optimization strategy or in the numerical techniques which have been adopted. They will be stated now:

a) The system of eqs. (475) for the unknowns in the thermal model is solved by means of iterations over the linearized system (478). An acceleration factor ALPHA (see flowchart of the subroutine THERMO in appendix D) is used to speed up the process. The value of ALPHA must be $0 \leq \text{ALPHA} \leq 1$, and the experience with this program showed that ALPHA:.6 achieved the fastest convergence towards the solution. However, undamped oscillations about the true solution have occurred for specific numerical values; the oscillations disappear by setting ALPHA = 0; they are believed to occur due to coupling between the acceleration scheme and the temperature dependence of some of the coefficients in equations (475).

b) The subroutine SOLVER, which gives the solution of the linear system of equations (478), has sometimes returned values which are totally off. This seems to be related to the weak coupling of some of the unknowns (like the bearing temperatures) to the rest of the system, making the system matrix approach singularity conditions. Therefore the subroutine SOLVER should be replaced by another equivalent subroutine without these numerical problems.

c) It has been pointed out that in designs for minimum size at low rotational speed, there is a major trade-off between the level of magnetic saturation and the MMF requirements. The rotor slot depth DR is a
key parameter in these calculations, since it determines the cross-
section of the magnetic circuit and the available room for the rotor
windings. Unfortunately DR must be calculated by the trial and error
method: an initial guess of DR is required to obtain the cross-section
of the magnetic circuit, and this value must be checked against the MMF
requirements of the rotor slots. The process easily tends to be numerically
unstable due to the small slope of the magnetization curve in the
saturation region; it must be realized that the "numerical loop" has
positive feedback: the larger the value of DR the small the cross-
section and the higher the magnetic saturation leading to higher MMF,
which requires more room in the slots and therefore a larger value of DR.

d) The logic of the adopted optimization strategy demands uniqueness in
the unilateral correspondence between the point \( \bar{x} \) and the value
of the objective function \( F(\bar{x}) \). In other words, the value of \( \bar{x} \) must
determine the whole design completely. The point is that the initial
guesses which are required for some parameters (DR, RSH, GR, CL1, TEMP)
in the analysis step (see flowcharts of subroutines OBJECT and Hooke in
appendix D) may create some problems if these guesses depend on something
different from \( \bar{x} \) (i.e., if the guesses depend on values obtained in pre-
vious designs). In the final version of the program the guesses for the
winding temperatures TEMP(I), I = 1,2,3,4, are the values corresponding
to the last base point; these temperatures are required to evaluate the
electrical conductivities and it was decided that a call to THERMO just
to obtain these temperatures was too time-consuming. The effect of this
policy is that \( F = F(\bar{x}, \text{previous designs}) \); this fact, together with the
numerical problems mentioned in part (c), has given some trouble at cer-
tain stages of the optimization process when the points $\bar{x}$ to be compared are very close. One method of eliminating this problem is to update the guesses only after step size reductions, where the optimization process is restarted again.

e) A universal problem in nonlinear programming is the existence of local minima. The best approach to the problem seems to be to start the optimization process for a single design from several initial points $\bar{x}_0$ and to compare the final results which are obtained. This approach has revealed the existence of at least one local minima in the function $F(\bar{x})$ of concern. Its existence seems to be closely related to the addition of the efficiency term to the figure of merit function: the effect of this term is to move the design (by means of the "undecided" variables) towards regions of higher efficiency, where a local minimum exists. Only an extensive study, which would require many computer runs, could describe reliably these features of the objective function $F(\bar{x})$. 
CHAPTER VI

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

A set of models for the electrical, mechanical, thermal, magnetic, and geometrical aspects of a small wound field synchronous machine have been developed. These models have been employed in a computer program which performs a pattern search for optimum machine designs. The thermal model, being the most important, was verified experimentally. A study was performed to determine the size constraints vs. speed for a 20kVA machine whose figure of merit was size. Both conventionally cooled and vacuum designs were considered.

Several important results of this work are:

1. A new and demonstrably accurate model for heat transfer in electrical machinery has been developed.

2. New models have been derived for the losses and magnetization curves, as function of both frequency and flux density, for magnetic materials.

3. A set of independent variables has been found which reduces significantly the computational effort in the analysis step of the program.

4. The application of the program to the minimum size design has provided some insights on the main trade-offs and expected capabilities of high rotational speed synchronous machines.

5. Capability studies using criteria other than size may be performed by simply changing the performance requirements and redefining the figure of merit.

For example, the question of how much power can be handled by a
machine at a certain rotational speed can be answered with this program. Or a study on the most economical range of rotational speeds for a machine running coupled to a flywheel energy storage system.

Certain modeling issues still remain to be investigated. Stray losses have not been considered and could become significant in certain applications. Slip rings have been ignored; the bearings and the fan, with their corresponding losses, have not been modeled explicitly; configurations for the rotor end winding other than the proposed here could be tried; all these parts of the machine may require innovative designs for high rotational speeds and are themselves topics of separate research work. Although the thermal model has so far worked satisfactorily, the topics of bearing thermal resistance, conduction of heat to the ambient through shaft and mountings, and the magnitude and distribution of flow rate of coolant require a more detailed study. From the results of the program it has been learned that the assumption of uniform iron temperature in the analysis of the embedded part of the windings is quite crude. It can be removed, at the cost of increasing the analytical complexity of the model.63
APPENDIX A. COMPUTER PRINTOUT

C THE 'MAIN' PROGRAM OF DESIGN:
C -ASSIGNS VALUES TO THE FOLLOWING PARAMETERS (IN BLOCKDATA):
C -DEFAULT GUESSES FOR THE INDEPENDENT VARIABLES P(I)
C -DEFAULT GUESSES FOR THE UNKNOWNS IN THERMO X(I)
C -# OF INDEPENDENT VARIABLES NSTAGE
C -MAX. ALLOWED # OF OBJ EVALUATIONS MKOBJ (DEFAULT)
C -MAX. ALLOWED # OF STEP SIZE REDUCTIONS MKSTEP (DEFAULT)
C -MAX. ALLOWED # OF ITERATIONS IN THERMO NTOP (DEFAULT)
C -PATTERN SEARCH MOVE FACTOR ALPHA (DEFAULT)
C -STEP SIZE REDUCTION FACTOR BETA (DEFAULT)
C -CONVERGENCE CRITERION PARAMETER IN THERMO XINC (DEFAULT)
C -CONTROL VARIABLES IC(I) (DEFAULT IN GENERAL):
  IC(1)=0, WRITE MINIMUM AMOUNT OF INFORMATION; 1,MEDIUM;
  2,MAXIMUM.
C -IC(2)=14 (THERMO), DETERMINES VENTILATION SYSTEM
C -IC(3)=1, ALL FLM COEFFICIENTS ARE READ FROM TERMINAL
C -IC(4)=1, OVERHANG WINDAGE & BEARING LOSSES ARE READ
C FROM TERMINAL
C -IC(5)=1, JUST AFTER A STEP SIZE REDUCTION
C -IC(6)=1, OPTIMIZ. PROCESS ENDED; OBTAIN FINAL RESULTS
C -IC(7) IS THE VARIABLE SEARCHED OVER
C -IC(8)=1, CONTROL IS NOT GIVEN TO TERMINAL (NOT IMPLEMENTED)
C -IC(9)=1, IF OBJ=1, SO OBJ IS BEING CALCULATED FOR THE
C FIRST TIME
C -IC(10)=1, MACHINE OPERATES IN A HIGH VACUUM
C -THERMAL CONDUCTIVITIES TC(I)
C -ELECTR. CONDUCT. & TEMP. CORRECTION FACTORS EC(I)
C -LOSSSES CL(I)
C -GUESSES FOR TEMPERATURES OF INTEREST TEMP(I)
C -LIMITS FOR THE CONSTRAINTS CNTRL(I) (DEFAULT)
C -CONSTRAINTS WEIGHTING FACTORS WP(I) (DEFAULT)
C -MAX. ALLOWABLE MECH. STRESSES SIGMA(I)
C -MASS DENSITIES DENS(I)
C -POISSON & YOUNG MODULUS CMACH(I)
C -P FOR THE PTH-OPTIMIZATION METHOD EXPP (DEFAULT)
C -PINS EFFECTIVENESS COEFFICIENT CFINS
-FILM COEFFICIENT FOR EXT. FORCED VENTILATION NEXT
-ESTIMATED VELOCITY OF COOLANT IN VENT. DUCTS VDUCT
-COEFF. FOR THE IRON LOSSES EXPRESSION CLOSS(I)
-COEFF. FOR THE MAGNETIZATION CURVE PBH(9,3)
-BOUNDS FOR B & F IN MAGN. CURVE EXPRESSION BSAT, BMN,FMAX,PMN
-PERFORMANCE REQUIREMENTS PR(I):
  -PR(1), ROTATIONAL SPEED W (RAD/SEC)
  -PR(2), REAL POWER PS (WATTS)
  -PR(3), OUTPUT VOLTAGE US (L-N, RMS, VOLTS)
  -PR(4), POWER ANGLE PI (RAD) (+ IF CURRENT LEADS)
  -PR(5), VOLUME RATE FLOW OF COOLANT WQ (M3/SEC)
  -PR(6), AMBIENT TEMPERATURE TAM, (CELSIUS)
  -PR(7), NUMBER OF POLES P
  -PR(8), EXCITATION VOLTAGE UR (VOLTS DC)
-ASKS FOR PARAMETER VALUES FROM TERMINAL IF DEFAULT VALUES ARE NOT SELECTED
-CALLS THE OPTIMIZATION SUBROUTINE Hooke
-PRINTS VALUES FOR THE RESULTANT DESIGN

COMMON
FIRST/X,TC,NTOP,XINC,H,ACCELE,CPINS,HEXT
THIRD/EPS,ALPHA,BETA,PSTEP,MOBJ,NSTAGE
FOURTH/EXP,CA,CNTRL,WF,SIGMAX,DENS,EP,CMECH,VDJCT
CLOSS,BH,PBH,BSAT,MIN,FMAX,PMN,BH0
FIFTH/KSTEP,KLOCAL,KPTTRN,OBJBST,OBJBAS,OBJMIN,OBJHD
IDISC,ISKP
SIXTH/IC,PR,G,CL,TEMP
SEVEN/P,OBJB,OBJ,EC

DIMENSION X(13),TC(11),H(10),
EPS(12),
CA(25),CNTRL(25),WF(25),SIGMAX(3),DENS(10),EP(25),
CMECH(2),CLOSS(5),BH(9),PBH(9,3),BH0(9),
IDISC(12),ISKP(12),
IC(10),PR(10),G(75),CL(10),TEMP(11),
P(12),EC(4),
IDEF(7)
C P,EPS,IDISC,ISKIP,BASEO,BASEN HAVE DIMENSION 12 BUT ONLY
ICONT=0
CONTINUE
IC(5)=0
IC(6)=0
IC(9)=1
IC(8)=0
WRITE(6,10)
10 FORMAT(5X,'TYPE 1 TO CHANGE DEFAULT DATA VALUES')
READ(5,20) I1
20 FORMAT(I1)
IF(I1.EQ.0) GO TO 30
WRITE(6,40)
40 FORMAT(5X,'TYPE 1 TO MODIFY: ',/5X,'1-IC; 2-NSTAGE, MKSTEP,'),
C 'MKOBJ, NTOP; 3-X; 4-ALPHA, BETA, XINC, EXP, ACCEL; '/
C 5X,'5-CNTRL OR WF; 6-PR; 7-P')
READ(5,20) IDEF
C MODIFYING DEFAULT VALUES
IF(IDEF(1).EQ.0) GO TO 50
WRITE(6,60)
60 FORMAT(5X,'ENTER 10 IC; FORMAT I1; ONLY 1,2,3,4,8,10 MATTER')
READ(5,20) IC
50 CONTINUE
C
IF(IDEF(2).EQ.0) GO TO 70
WRITE(6,80)
80 FORMAT(5X,'ENTER NSTAGE, MKSTEP, MKOBJ, NTOP; FORMAT I4')
READ(5,90) NSTAGE, MKSTEP, MKOBJ, NMAX
90 FORMAT(I4)
70 CONTINUE
C
IF(IDEF(3).EQ.0) GO TO 110
WRITE(6,110)
110 FORMAT(5X,'ENTER 13 X(I); FORMAT L12.6')
READ(5,120) X
120 FORMAT(E12.6)
100 CONTINUE
C
   IF (IDEF(4).EQ.0) GO TO 130
   WRITE (6, 140)
140 FORMAT(5X,'ENTER ALPHA, BETA, XINC, EXPP, ACCELE; FORMAT E12.6')
   READ (5, 120) ALPHA, BETA, XINC, EXPP, ACCELE
130 CONTINUE
C
   IF (IDEF(5).EQ.0) GO TO 150
180 WRITE (6, 160)
160 FORMAT(5X,'ENTER INDEX, FORMAT I4, CNTRL & WF FORMAT E12.6')
   READ (5, 90) INDEX
   READ (5, 120) CNTRL(INDEX), WF(INDEX)
   WRITE (6, 170)
170 FORMAT(5X,'TYPE 1 TO ENTER NEW CNTRL & WF')
   READ (5, 20) INDEX
   IF (INDEX.EQ.1) GO TO 180
150 CONTINUE
C
   IF (IDEF(6).EQ.0) GO TO 190
   WRITE (6, 200)
200 FORMAT(5X,'ENTER 10 PR; FORMAT E12.6')
   READ (5, 120) PR
190 CONTINUE
C
   IF (IDEF(7).EQ.0) GO TO 195
   WRITE (6, 197)
197 FORMAT(5X,'ENTER 12 P; FORMAT E12.6')
   READ (5, 120) P
195 CONTINUE
C
   MODIFYING DEFAULT VALUES DEPENDING ON IC
   IF (IC(3).EQ.0) GO TO 210
   WRITE (6, 220)
220 FORMAT(5X,'ENTER 10 H; FORMAT E12.6')
   READ (5, 120) H
210 CONTINUE
C
IF(IC(4).EQ.0) GO TO 230
WRITE(6,240)
240 FORMAT(5X,'ENTER OVERHANG WINDAGE & BEARING LOSSES; FORMAT E12.6')
READ(5,120) CL(9),CL(7)
230 CONTINUE
30 CONTINUE
C
IF(ICONT.EQ.1) GO TO 350
DO 235 I=1,75
235 G(I)=0.
DO 236 I=1,25
CA(I)=0.
236 EP(I)=0.
350 CONTINUE
C STARTING THE OPTIMIZATION PROCESS
CALL Hooke
C WRITING PARAMETER VALUES OF THE RESULTANT DESIGN
WRITE(6,255) OBJMIN
255 FORMAT(5X,'OBJMIN=',E12.6)
WRITE(6,250) P
250 FORMAT(5X,'P='/(5X,5(E12.6,3X)))
WRITE(6,260) CA
260 FORMAT(5X,'CA='/(5X,5(E12.6,3X)))
WRITE(6,270) G
270 FORMAT(5X,'G='/(5X,5(E12.6,3X)))
WRITE(6,280) EP
280 FORMAT(5X,'EP='/(5X,5(E12.6,3X)))
WRITE(6,290) TEMP
290 FORMAT(5X,'TEMP='/(5X,5(E12.6,3X)))
WRITE(6,300) H
300 FORMAT(5X,'H='/(5X,5(E12.6,3X)))
WRITE(6,310) CL
310 FORMAT(5X,'CL='/(5X,5(E12.6,3X)))
WRITE(6,320) DENS
320 FORMAT (5X, 'DENS='/(5X, 5(E12.6, 3X)))
WHITE (6, 400)
400 FORMAT (5X, 'ENTER 0 TO END; ENTER 1 TO START AGAIN')
READ (5, 20) ICONT
IF (ICONT.EQ.1) GO TO 410
STOP
END

C**********************************************************************
BLOCKDATA
C THIS SUBPROGRAM ASSIGNED VALUES TO MOST OF THE A PRIORI KNOWN
C PARAMETERS.
C NOTE: VARIABLES IN COMMON CANNOT BE GIVEN IN USUAL DATA.
COMMON
C /FIRST/X, TC, NTOP, XINC, H, ACCELE, CFINS, HEXT
C /THIRD/EPS, ALPH, ETA, MKSTEP, KOB, NSTAGE
C /FOURTH/EXPP, CA, CNTRL, WF, SIGMAX, DENS, EP, CMMECH, VDUCT
C , CLOSS, BH, FBH, BSAT, BMIN, FMAX, FMIN, BHO
C /SIXTH/IC, PR, G, CL, TEMP
C /SEVENTH/P, KOB, OBJ, EC
DIMENSION X(13), TC(11), H(10),
C EPS(12),
C CA(25), CNTRL(25), WF(25), SIGMAX(3), DENS(10), EP(25),
C CMMECH(2), CLOSS(5), BH(9), FBH(9, 3), BHO(9),
C IC(10), PR(10), G(75), CL(10), TEMP(11),
C P(12), EC(4)
DATA X/9*100., 4*2000./
C 1-KL, 2-KEI, 3-KCS, 4-KCR, 5-KSZ,
C 6-KSR, 7-KRZ, 8-KRR, 9-KP, 10-KSH, 11-KA
DATA TC/.25, .12, 400., 400., 2.,
C 30., 2., 30., 40., 50., .25/
DATA NTOP/10, XINC/.001, ACCELE/.6/, CFINS/2.25/, HEXT/150./
DATA EPS/.001, 001, 0005, 0001, 1., 1., 005, 005, 1., 1., 1.44, 1./
DATA ALPH/1.25, ETA/5., MKSTEP/25, KOB/200., NSTAGE/11/
DATA EXPP/3, VDUCT/25., BSAT/2.4, BMIN/2., FMAX/1600., FMIN/60./
DATA CNTRL/1., 4*0., 57.3, 2*0., 2*1.275E9, 4*130., .85,
C .1, 7071., 5, 7*0./
DATA WF/7500, 4*1.E15, 1.E8, 8*1.E6, 3*1.E4, 1.E8, 6*0, 0.001/
DATA SIGMAX/2*1.275E9, 1.96E8/
DATA DENS/8200, 2*8900, 2*1200.7650, 8200.2*0.7650. /
DATA CMCH/.3, 1.96E11/
DATA CLOSS/28.22, 1.6456, 800, 1.2052, 3697/
DATA BH0/708747, 1.06096, 629306, 371509,
C  .170543,  .0805209,  .0182731,  .00959134,  .0284367/
DATA PEB/
C  .665909,  1.021717,  .627468,  .425449,  .24.4077,
C  .106227,  -.000428736,  -.00997758,  -.0528.78,  .0755612,
C  -.0436253,  -.0317297,  -.0227534,  .00203812,  -.00913556,
C  .00799372,  .00529107,  -.001035,  -.00910742,  -.000872816,
C  .00754077,  .0073963,  .00114826,  .000149563,  .000112546,
C  -.00169356,  -.00202001/
DATA IC/0.3.6*0.1.0,
DATA PIB/377.20000, 127.0.0.07.40, 2.100.2*0.0/ 
DATA CL/6*0.50.3*0.0/ 
DATA TEMP/11*100.0/ 
DATA P/1.1.12.003.0.75.3.1.12.0.6.4.6.5.0.200000.52.0/
DATA EC/5.6E7, 6.8E-11, 5.6E7, 6.8E-11/
END
C*****************************************************************************
C SUBROUTINE HOOKE:
C -TAKES THE INITIAL GUESST TO THE OPTIMUM FOLLOWING A PATTERN SEARCH
C -ALLOWS TO RESTART THE SEARCH AT ANY POINT
C -ALLOWS TO REINITIALIZE THE STEP SIZES
C -ALLOWS TO SKIP THE SEARCH OVER ANY SELECTED VARIABLE
C -ALLOWS TO DECLARE VARIABLES TO BE DISCRETE
C -ALLOWS TO STOP THE SEARCH
C -GIVES CONTROL TO THE TERMINAL:
C -WHEN A STEP SIZE REDUCTION IS NEEDED
C -WHEN KOBJ OR KSTEP BECOME EQUAL TO PRESCRIBED VALUES
C -CALLS OBJECT TO CALCULATE THE OBJECTIVE FUNCTION
C -RETURNS TO DESIGN WHEN SEARCH IS TERMINATED
C -CALLS LOCAL FOR EXPLORATORY MOVE AROUND A GIVEN HEAD
SUBROUTINE HOOKE

COMMON
C /THIRD/EPS,ALPHA,BETA,MKSTEP,MKOBJ,NSTAGE
C /FIFTH/KSTEP,KLOCAL,KPTTRN,OBJBST,OBJBAS,OBJMIN,OBJHD
C ,IDISC,ISKIP
C /SIXTH/IC,PR,G,CL,TEMP
C /SEVEN/P,KOBJ,OBJ,EC
C /EIGHTH/TEMPBA,CABST,TEMPBS
    DIMENSION
    C EPS(12),
    C IDISC(12),ISKIP(12),
    C IC(10),PR(10),G(75),CL(10),TEMP(11),
    C P(12),EC(4)
    C ,BASEO(12),BASEN(12)
    C ,TEMPBA(4),CABST(25),TEMPBS(4)
C INITIALIZING
    DO 1000 I=1,NSTAGE
    ISKIP(I)=0
1000 IDISC(I)=0
    IDISC(5)=1
    IDISC(6)=1
    IDISC(9)=1
    IDISC(10)=1
    KSTEP=0
    KOBJ=0
    KLOCAL=0
    KPTTRN=0
    OBJBST=0.
    DO 347 J=1,4
    TEMPBA(J)=TEMP(J)
C EVALUATING OBJECTIVE FUNCTION AT INITIAL BASE POINT
    C CONTINUE
    CALL OBJECT
    OBJBAS=OBJ
    OBJHD=OBJ
C BASE POINT TO START WITH
    DO 20 I=1,12
20 BASEN(I)=P(I)
C GIVE CONTROL TO TERMINAL BEFORE STARTING THE SEARCH
C -TO SKIP THE SEARCH OVER SELECTED VARIABLES
C -TO DECLARE VARIABLES TO BE DISCRETE
C -TO ENTER A NEW GUESS P (OVERRIDING BLOCKDATA)
C -TO ENTER NEW STEP SIZES EPS (OVERRIDING BLOCKDATA)
   IF(KOBJ.EQ.1) GO TO 260
C SENTENCES # 1,2,3 FOLLOW HOOKE'S PAPER NOTATION
   1 CONTINUE
      IF(KOBJ.GE.MKOBJ) GO TO 90
C EXPLORATORY MOVE
C RETURNING TO PREVIOUS BASE AFTER A FAILURE OR
C JUST INITIATING THE PROCESS
   DO 25 I=1,NSTAGE
   25 P(I)=BASEN(I)
      OBJHD=OBJBAS
      CALL LOCAL
C CHECKING SUCCESS OF EXPLORATORY MOVE
      IF((OBJBST-OBJBAS).LT.0.) GO TO 2
C FAILURE IN EXPLORATORY MOVE AROUND BASE POINT WHEN:
C A) THIS IS THE INITIAL BASE POINT
C B) THE PREVIOUS PATTERN PLUS EXPLORATION MOVES FAILED
C SO: REDUCE THE STEP SIZE
   GO TO 3
2 CONTINUE
   DO 26 I=1,NSTAGE
   26 OBJBST=OBJBAS
      BASEO(I)=BASEN(I)
      IF(KOBJ.GE.MKOBJ) GO TO 90
C PATTERN MOVE
   WRITE (6,500)
      500 FORMAT('5X,'PATTERN MOVE:')
   DO 40 I=1,NSTAGE
      IF(ISKIP(I).EQ.1) GO TO 40
P(I)=P(I)*(ALPHA+1.)-ALPHA*BASEO(I)
IF(IDISC(I).EQ.0) GO TO 32
P(I)=AINT(P(I)+.5)
IF(P(I).LE.0) P(I)=1.
GO TO 40
32 IF(P(I).LE.0) P(I)=.5*BASEO(I)
IF(P(7).GE.1.) P(7)=.95
IF(P(8).GE.1.) P(8)=.95
40 CONTINUE
KPTTRN=KPTTRN+1
WRITE(6,520) P
520 FORMAT(5X,'P=/(5X,5(E12.6,3X))
CALL OBJECT
WRITE(6,530) OBJ
530 FORMAT(5X,'OBJ=',E12.6)
OBJHD=OBJ
CALL LOCAL
C CHECKING SUCCESS OF PATTERN PLUS EXPLORATION MOVE
C IF SUCCESS: NEW BASE AND PATTERN MOVE
   IF((OBJBST-OBJBAS).GE.0.) GO TO 345
   DO 346 J=1,4
   346 TEMPBA(J)=TEMPBS(J)
   CONTINUE
   IF((OBJBST-OBJBAS).LT.0.) GO TO 2
C FAILURE: EXPLORATORY MOVE AROUND LAST BASE
C IF THIS FAILS TOO: REDUCE STEP SIZE
   GO TO 1
3 CONTINUE
C A STEP SIZE REDUCTION IS REQUIRED
C CHECKING # OF STEP SIZE REDUCTIONS & FUNCTION EVALUATIONS
C IF(KSTEP.GE.MKSTEP) GO TO 90
   IF(KOBJ.GE.MKOBJ) GO TO 90
C CONTROL IS GIVEN TO TERMINAL
260 WRITE(6,250)
250 FORMAT(5X,'TYPE 1 FOR MAKING CHANGES/',
C 5X,'NO CHANGE IF ISHORT,KDISC,NWSTEP,IGUESS,ISTLP=O')
READ (5,60) ICHANG
IF (ICHANG .EQ. 0) GO TO 240
C TO SKIP SEARCH OVER SELECTED VARIABLES
WRITE (6,120)
120 FORMAT (5X,'TYPE 1 TO MODIFY THE VARIABLES OVER WHICH '
C 'THE SEARCH IS SKIPPED'/5X,'?')
READ (5,60) ISHORT
IF (ISHORT .EQ. 0) GO TO 130
WRITE (6,140)
140 FORMAT (5X,'TYPE 12 I1 FORMAT VALUES:'/5X,'?')
C 5X,'0 TO SEARCH OVER VARIABLE I; 1 TO SKIP IT'/5X,'?')
READ (5,60) ISKIP
130 CONTINUE
C
C TO ENTER NEW STEP SIZES EPS(I)
WRITE (6,320)
320 FORMAT (5X,'TYPE 1 TO ENTER NEW STEP SIZES EPS(I)'/5X,'?')
READ (5,60) NWSTEP
IF (NWSTEP .EQ. 0) GO TO 330
WRITE (6,220)
READ (5,230) EPS
WRITE (6,230) EPS
330 CONTINUE
C
C TO TREAT DISCRETE VARIABLES EITHER AS DISCRETE OR CONTINUOUS
WRITE (6,150)
150 FORMAT (5X,'TYPE 1 TO MODIFY THE ASSUMED DISCRETE VARIABLES')
READ (5,60) KDISC
IF (KDISC .EQ. 0) GO TO 160
C INDEXING THE DISCRETE VARIABLES
WRITE (6,180)
180 FORMAT (5X,'TYPE 12 I1 FORMAT VALUES:'/5X,'?')
C 5X,'0 FOR CONTINUOUS VARIABLE; 1 FOR DISCRETE'/5X,'?')
READ (5,60) IDISC
DO 190 I=1,NSTAGE
IF (IDISC(I) .EQ. 0) GO TO 190

00003370 LINE0361
00003380 LINE0362
00003390 LINE0363
00003400 LINE0364
00003410 LINE0365
00003420 LINE0366
00003430 LINE0367
00003440 LINE0368
00003450 LINE0369
00003460 LINE0370
00003470 LINE0371
00003480 LINE0372
00003490 LINE0373
00003500 LINE0374
00003510 LINE0375
00003520 LINE0376
00003530 LINE0377
00003540 LINE0378
00003550 LINE0379
00003560 LINE0380
00003570 LINE0381
00003580 LINE0382
00003590 LINE0383
00003600 LINE0384
00003610 LINE0385
00003620 LINE0386
00003630 LINE0387
00003640 LINE0388
00003650 LINE0389
00003660 LINE0390
00003670 LINE0391
00003680 LINE0392
00003690 LINE0393
00003700 LINE0394
00003710 LINE0395
00003720 LINE0396
C ROUNDING OFF PREVIOUS CONTINUOUS VALUES
C BUT PREVENTING EPS(I) FROM BECOMING 0
   P(I)=AINT(PI+.5)
   EPS(I)=AINT(EPS(I)+.5)
   IF(EPS(I)=EQ.0) EPS(I)=1.
   190 CONTINUE
C CURRENT BASE POINT MAY HAVE BEEN MODIFIED
C SO PROCESS MUST BE REINITIALIZED
   GO TO 110
   160 CONTINUE
C
C TO ENTER A NEW GUESS FOR P(I)
   WRITE(6,200)
   200 FORMAT(5X,'TYPE 1 TO ENTER A NEW GUESS FOR P(I)'/5X,'?')
   READ(5,60) IGUESS
   IF(IGUESS.EQ.0) GO TO 210
   WRITE(6,220)
   220 FORMAT(5X,'ENTER 12 VALUES FORMAT E12.6'/5X,'?')
   READ(5,230) P
   230 FORMAT(E12.6)
   WRITE(6,230) P
   GO TO 110
   210 CONTINUE
C STEP SIZE REDUCTION
   WRITE(6,50)
   50 FORMAT(5X,'TYPE 0 FOR STEP SIZE REDUCTION OR '/'
   C 5X,'CONTINUATION IF KOBJ=1 '/
   C 5X,'TYPE 1 FOR TERMINATION OF THE OPTIMIZATION PROCESS')
   READ(5,60) ISTEP
   60 FORMAT(I1)
   240 IF(ISTEP.EQ.1) GO TO 90
   IF(KOBJ.EQ.1) GO TO 1
   DO 70 I=1,NSTAGE
   IF(ISKIP(I).EQ.1) GO TO 70
   EPS(I)=BETA*EPS(I)
   IF(IDISC(I).EQ.0) GO TO 70
   70 Continue
EPS(I) = AINT(EPS(I) + .5)
IF(EPS(I) .EQ. 0.) EPS(I) = 1.
70 CONTINUE
KSTEP = KSTEP + 1
IC(5) = 1
WRITE(6, 350) BASEN
350 FORMAT(5X, 'STEP SIZE REDUCTION: '/
C 5X, 'DESIGN FOR P=BASEN; BASEN = '/(5X, 5 (E12.6, 3X))
DO 360 I = 1, NSTAGE
360 P(I) = BASEN(I)
CALL OBJRCT
GO TO 1
90 WRITE(6, 80) KGLOBAL, KPTTRN, KOBJ, KSTEP
80 FORMAT(5X, 'END OF THE OPTIMIZATION: '/
C 5X, 'KGLOBAL = ', I4/5X, 'KPTTRN = ', I4/
C 5X, 'KOBJ = ', I4/5X, 'KSTEP = ', I4)
WRITE(6, 310) EPS
310 FORMAT(5X, 'EPS = '/(5X, 5 (E12.6, 3X))
C CONTROLLING TERMINATION FROM THE TERMINAL
WRITE(6, 270)
270 FORMAT(5X, 'TYPE 0 FOR TERMINATION'/
C 5X, 'TYPE 1 TO ENTER NEW MKSTEP, MKOBJ'/5X, '?')
READ(5, 60) IEND
IF(IEND .EQ. 0) GO TO 280
WRITE(6, 290)
290 FORMAT(5X, 'TYPE NEW MKSTEP, MKOBJ; FORMAT I4'/
READ(5, 300) MKSTEP, MKOBJ
300 FORMAT(I4)
GO TO 1
280 CONTINUE
OBJMN = OBJBAS
DO 100 I = 1, NSTAGE
100 P(I) = BASEN(I)
IF(KOBJ .EQ. 1) GO TO 340
C CALLING OBJECT FOR THE LAST TIME
IC(5) = 1
CALL OBJECT
340 CONTINUE RETURN END

SUBROUTINE LOCAL
C SUBROUTINE LOCAL PERFORMS EXPLORATORY MOVES ABOUT P
C OBJBST IS THE BEST VALUE PRODUCED BY THE SET OF EXPL. MOVES
C OBJHD IS THE VALUE AT THE HEAD OF EXPLORATION P (I0
COMMON
C /THIRD/EPS,ALPHA,BETA,MSKSTEP,MKOBJ,NSTAGE
C /FOURTH/EXPP,CA,CNTRL,WF,SIGMAX,DENS,EP,CMECH
C ,VDUCT,COSS,BH,FBH,BSAT,BMIN,PMAK,PMIN,BHO
C /FIFTH/KSTEP,KLOCAL,KPTTRN,OBJBST,OBJBAS,OBJMIN,OBJHD,
C IDISC,ISKIP
C /SIXTH/IC,PR,G,CL,TEMP
C /SEVEN/P,KOBJ,OBJ,EC
C /EIGHTH/TEMPBA,CABST,TEMPBS
DIMENSION
C EPS(12),
C CA(25),CNTRL(25),WF(25),SIGMAX(3),DENS(10),EP(25),
C CMECH(2),COSS(5),BH(9),FBH(9,3),BHO(9),
C IDISC(12),ISKIP(12),
C IC(10),PR(10),G(75),CL(10),TEMP(11),
C P(12),EC(4)
C ,CABST(25),TEMPBA(4),TEMPBS(4)
WRITE(6,600)
600 FORMAT(/5X,'EXPLORATORY MOVE:')
KLOCAL=KLOCAL+1
OBJBST=OBJHD
C SEARCHING ABOUT P
DO 100 I=1,NSTAGE
POLD=P(I)
IC(7)=I
IF(ISKIP(I).EQ.1) GO TO 100
P(I)=P(I)+EPS(I)
IF (P(7) .GE. 1.) P(7) = .95
IF (P(8) .GE. 1.) P(8) = .95
CALL OBJECT

C CHECKING SUCCESS OF LOCAL MOVE IN +1 DIRECTION
C IF SUCCESSFUL: CHANGE P(I) & OBJBST & GO TO SEARCH OVER I+1
    IF ((OBJ-OBJBST) .LT. 0.) GO TO 200
C MOVE FAILED; TRY -1 DIRECTION
    P(I) = P(I) - 2.*EPS(I)
    IF (P(I) .GT. 0.) GO TO 10
    IF (IDISC(I).EQ.1) P(I) = 1.
    IF (IDISC(I).EQ.0) P(I) = .5*(P(I)+EPS(I))
10 CALL OBJECT
    IF ((OBJ-OBJBST) .LT. 0.) GO TO 200
C BOTH DIRECTIONS FAILED; RETURN TO INITIAL POINT
    P(I) = POLD
    GO TO 100
200 OBJBST = OBJ
    DO 20 J = 1,25
20    CABST(J) = CA(J)
    DO 40 K = 1,4
40   TEMPBS(K) = TEMP(K)
100 CONTINUE
    WRITE(6,400) OBJBST,EP(24),EP(16),P
400 FORMAT (5X,'END OF EXPLORATORY MOVE:'/'
C  5X, 'OBJBST=','E12.6,3X,'SIZE=','E12.6,3X,'EPF=','E12.6/
C  5X, 'P=','/(5X,5(E12.6,3X))')
    WRITE(6,300) CABST
300 FORMAT (5X,'CABST=','/(5X,5(E12.6,3X))')
    IF (IC(1).NE.2) GO TO 800
    WRITE(6,700) EP,G
700 FORMAT (5X,'EP,G=','/(5X,5(E12.6,3X))')
800 CONTINUE
    DO 30 I = 1,25
30   CA(I) = CABST(I)
    RETURN
END
C SUBROUTINE OBJECT:
C INCLUDES THE DESIGN ANALYSIS STEP
C - CALCULATES THE CONSTRAINT FUNCTION VALUES (INCLUDING THE
C FIGURE OF MERIT CONSTRAINT)
C - CALCULATES THE OBJECTIVE FUNCTION ACCORDING TO THE PTH
C OPTIMIZATION METHOD
C - CALLS THERMO TO OBTAIN THE STEADY STATE TEMP. RISES FOR
C THE RESULTANT MACHINE
C - SKIPS CALCULATIONS &/OR MODIFIES THE ACCURACY ACCORDING
C TO INDICATIONS RECEIVED FROM HOOKE
C - CALLS HBF TO OBTAIN THE MAGNETIC FIELD INTENSITY H FOR
C GIVEN FLUX DENSITY B & FREQUENCY F
C
SUBROUTINE OBJECT
COMMON
C /FOURTH/EXPP,CA,CNTRL,WF,SIGMAX,DENS,EP,CMECH,VDUCT
C ,CROSS,BH,PBH,BSAT,BNIN,P2MAX,P2MIN,BH0
C /SIXTH/IC,PR,G,CL,TEMP
C /SEVEN/PI,KOBJ,OBJ,EC
C /EIGHTH/TEMPBA,CA,BST,TEMPBS

DIMENSION
C CA(25),CNTRL(25),WF(25),SIGMAX(3),DENs(10),EP(25),
C CMECH(2),CROSS(5),BH(9),PBH(9,3),BH0(9),
C IC(10),PR(10),G(75),CL(10),TEMP(11),
C P(12),EC(4),
C C(3),SBF(9),SPF(9),SWF(9),RBF(9),RPF(9),BWF(9)
C ,CNTR(25)
C ,IVIO(25),TEMPBA(4),CAAST(25),TEMPBS(4)

FUNCTIONS DEFINITION:
C IRON SPECIFIC LOSSES FOR GIVEN FLUX DENSITY B & FREQUENCY F
C THE LOSSES ARE GIVEN PER UNIT WEIGHT (WATTS/KG)

SPCL(X,Y) = CROSS1*X**CROSS2
C *(Y/CROSS3)**(CROSS4+CROSS5+ALOG10(X))
CROSS1 = CROSS(1)
CROSS2 = CROSS(2)
CLOSE3=CLOSE(3)
CLOSE4=CLOSE(4)
CLOSE5=CLOSE(5)
PI=2.*ARCSIN(1.)
CMO=4.*PI*1.E-7
KOBJ=KOBJ+1
IC(9)=0
IF(KOBJ.EQ.1) IC(9)=1
C CONTROL IS GIVEN TO TERMINAL AFTER STEP SIZE REDUCTION
C TO UPDATE ANY CONSTRAINT LIMIT OR WEIGHTING FACTOR
IF(IC(5).EQ.0) GO TO 200
260 WRITE(6,210)
210 FORMAT(5X,'ENTER 1 TO MODIFY ANY CNTRL(I) &/OR WF(I)')
READ(5,220) IMOD
220 FORMAT(1X)
IF(IMOD.EQ.0) GO TO 200
WRITE(6,230)
230 FORMAT(5X,'ENTER INDEX, I2, & CNTRL(INDEX) & WF(INDEX),E12.6')
READ(5,240) INDEX
240 FORMAT(1X)
READ(5,250) CNTRL(INDEX),WF(INDEX)
250 FORMAT(E12.6)
GO TO 260
200 CONTINUE
F=PR(1)*PR(7)/(4.*PI)
C SLOTS # SS,SR
G(23)=3.*PR(7)*P(5)
G(24)=PR(7)*P(6)
C BREATHE & PITCH FACTORS
AUX1=PI/6.
AUX2=PI/2.
AUX4=PR(7)*PI/(2.*G(23))
C IT IS ASSUMED STATOR WNDG. IS SHORT PITCHED BY SS/12 SLOTS
PITCH=AINT(G(23)/12.+.5)
IF(PITCH.EQ.0.) PITCH=1.
AUX5=PI*ABS(G(23)-PR(7)*PITCH)/(2.*G(23))
HCF=0.
AUX7=PI*PR(7)/(2.*G(24))
DO I=1,9,2
C STATOR BREADTH FACTORS SBF
SBF(I)=SIN(I*AUX1)/(I*P(5)*SIN(I*AUX4))
C STATOR PITCH FACTOR SPF
SPF(I)=SIN(I*AUX5)
C STATOR WINDING FACTOR SWF
SWF(I)=SBF(I)*SPF(I)
C ROTOR BREADTH FACTOR RBF
RBF(I)=SIN(I*AUX2)/(I*P(6)*SIN(I*AUX7))
C ROTOR PITCH FACTOR (FULL PITCH ASSUMED)
RPF(I)=1.
C ROTOR WINDING FACTOR
RWF(I)=RBF(I)*RPF(I)
C TOTALS OF HARMONIC CONTENT
IF(I.EQ.1) GO TO 1
HCF=HCF+ABS(SWF(I)*RWF(I))
1 CONTINUE
HCF=HCF/ABS(SWF(1)*RWF(1))
XD=2./((BSAT-BMIN)
X0=-(BSAT+BMIN)/(BSAT-BMIN)
C GUESSES FOR DR,GR,HSH,CL1
C LOW ESTIMATES ON PURPOSE TO PREVENT CNRIR & DR FROM BLOWING
C UP DUE TO MAGNETIC SATURATION.
C GUESSES DO NOT HAVE MEMORY, SO P(I) COMPLETELY DETERMINES
C THE DESIGN.
G(31)=P(2)
G(18)=.2*P(1)
G(35)=.05*G(18)
G(5)=0.
C COEFF. OF MAGNETIZATION CURVE FOR FREQ.=P
IF(KOBJ.NE.1) GO TO 5
ZD=2./(FMAX-PMIN)
Z0=-(FMAX+PMIN)/(FMAX-PMIN)
FN=ZD*P+Z0
DO 1020 I=1,9
DO 1025 J=1,3
1025 C(J)=PBH(I,J)
1020 CALL CNPS(BH(I),PN,C,3)
5 CONTINUE

C ROTOR SLOT WIDTH WR
G(54)=2.*PI*(P(1)*P(8)/G(24)
G(53)=2.*PI*(P(1)-G(18))*P(8)/G(24)
G(30)=-.5*(G(53)+G(54))

C STATOR SLOT WIDTH WS
G(29)=2.*PI*(P(1)+P(3))*P(7)/G(23)

C STATOR CURRENT (IF NOT AN INDEPENDENT VARIABLE) IS
IF(NSTAGE.EQ.12) GO TO 10
EP(10)=PR(2)/(3.*PR(3)*COS(PR(4)))
P(12)=EP(10)
GO TO 11
10 PR(2)=3.*PR(3)*P(NSTAGE)*COS(PR(4))
EP(10)=P(NSTAGE)
11 CONTINUE

C # OF CONDUCTORS PER PHASE (STATOR) CFS
G(43)=AINT((EP(10)/P(11))/7.07E-6+.5)
IF(G(43).EQ.0.) G(43)=1.

C # OF CONDUCTORS PER SLOT (STATOR) CSS
G(39)=G(43)*P(9)

C # OF STATOR TURNS NS
G(16)=P(9)*G(23)/2.

C COPPER CROSS SECTION AREA OF A PHYSICAL CONDUCTOR AS
G(14)=(EP(10)/P(11))/G(43)

C BARE CONDUCTOR DIAMETER (STATOR) DBCS
DBCS=SQRT(4.*G(14)/PI)

C INSULATED CONDUCTOR DIAMETER (STATOR) DTOT
IF(DBCS.LE.0.65E-3) GO TO 20
DCS=9.*5*(1.-2.*EXP(-2757.*DBCS))+DBCS
GO TO 21
20 DCS=1.092*DBCS

C THICKNESS OF INSULATION LAYERS (ESTIMATED)
21 G(25) = .35E-3
   IF ((2.*G(25)) .GE. G(29)) GO TO 500
   G(26) = .35E-3
   IF ((2.*G(26)) .GE. G(30)) GO TO 500
   G(27) = 1.5*.35E-3
   G(28) = 1.5*.35E-3
C STATOR SLOT DEPTH DS
   G(9) = 1.22*(G(39)*DCS**2/ABS(-.9*G(29)-2.*G(25))+3.*G(25))
   IF (G(9) .GE. P(4)) GO TO 500
C STATOR WEDGE THICKNESS
   G(8) = .1*G(9)
C FICTITIOUS DIMENSIONS FOR THERMO
   G(32) = SQRT(G(14))
   G(33) = G(32)
   G(34) = ABS(DCS/- .9 - G(32))
C LEAKAGE INDUCTANCE CLAL
   CLALE = 4.9E-7*G(16)**2*(P(1)+P(3)+.5*G(9))*SIN(PI/PR(7))/PR(7)
   CLALS = 4.*CM0*P(2)*G(9)*G(16)**2/(9.*G(29)*G(23))
   CLAL = CLALE+CLALS
   EP(6) = CLAL
C MAGNETIC CIRCUIT
C PEAK MAGNETIC FLUX DENSITY AT THE AIR GAP B0
   EP(1) = SQRT((EP(6)*EP(10)*COS(PR(4)))**2*
   C (2.*PR(3)/PR(1)*PR(7)) +
   C EP(6)*EP(10)*SIN(PR(4)))**2)*
   C 1.061*PR(7)/(P(1)*P(2)*G(16)*ABS(SWF(1)))
   B0 = EP(1)
C AIR GAP (CARTER)
   AL0 = P(3)*(.5*P(3)+G(29))*(.5*P(3)+G(30))/
   C ((.5*P(3)+G(29)*ABS(1.-P(7)))*
   C (.5*P(3)+G(30)*ABS(1.-P(8)))
   S0 = P(2)*P(1)*PI/PR(7)
C STATOR CORE
   AL1 = .66*PI*(P(1)+P(3)+G(9)+.5*P(4))/PR(7)
C STACKING FACTOR IS ASSIGNED A VALUE
   G(45) = .94
S1 = P(2) * G(45) * ABS(P(4) - G(9))
FUX = 2. * P(1) * E(1) * P(2) / PR(7)
G(22) = PR(7)
EP(2) = FLUX/S1
CALL HBF(H1, EP(2), BH, XD, X0, 0)
CMMF1 = H1 * AL1

C STATOR TEETH
AL3 = G(9)
S3 = (P(1) + P(3) + 0.5 * G(9)) * PI
C * ABS(1 - P(7)) * P(2) * G(45) / PR(7)
EP(4) = FLUX/S3
CALL HBF(H3, EP(4), BH, XD, X0, 0)
CMMF3 = H3 * AL3
INDSH = 0
30 INDDR = 0
32 CONTINUE
C Rotor Core
C Rotor Stacking Factor is assigned a Value
G(46) = 0.94
AL2 = ABS(P(1) - G(18)) * SIN(PI/PR(7))
S2 = ABS(P(1) - G(18)) * P(2) * G(46)
EP(3) = FLUX/S2
CALL HBF(H2, EP(3), BH, XD, X0, 1)
CMMF2 = H2 * AL2

C Rotor Teeth
AL4 = G(18)
S4 = ABS(P(1) - 0.5 * G(18)) * PI
C * ABS(1 - P(8)) * P(2) * G(46) / PR(7)
EP(5) = FLUX/S4
CALL HBF(H4, EP(5), BH, XD, X0, 1)
CMMF4 = H4 * AL4

C EFFECTIVE AIR GAP LENGTH
EP(8) = AL0 + (CM0 * PI / (2 * EP(1))) * (CMMF1 + CMMF2 + CMMF3 + CMMF4)

C SYNCHRONOUS INDUCTION CLD
EP(14) = 0.49 * CM0 * P(1) * P(2)
C *(G(16) *SWF(1)/PR(7)) **2/EP(8)
C ACCOUNTING FOR THE LEAKAGE INDUCTANCE CLAL
C BACK EMF
EP(9)=SQRT((EP(10)*EP(7)*PR(1)*.5*PR(7)*COS(PR(4)))*2+
C (PR(3)*EP(10)*EP(7)*PR(1)*.5*PR(7)*
C SIN(PR(4)))*2)
C ROTOR AMPERE TURNS
CNRIR=1.666*PR(7)*EP(8)*EP(9)
C /(P(1) *P(2) *CM0*G(16) *PR(1) *ABS(SWF(1) *RWF(1)))
C # OF ROTOR TURNS NR
G(17)=P(10)*G(24)/2.
C ROTOR CURRENT INTENSITY
EP(11)=CNRIR/G(17)
C AVERAGE LENGTH OF ROTOR TURN CLMTR
DSLOT=2.*ABS(P(1)-.5*G(18))
CLMTR=.5*PI*DSLOT*SIN(PI/PR(7))
CLMTR=2.*(G(31)*CLMTR)
C ELECTRICAL CONDUCTIVITY OF ROTOR CONDUCTORS REC
TAVR=.5*(TEMPBA(3)+TEMPBA(4))
REC=1./((1./EC(3)+EC(4))*(TAVR-20.))
C ROTOR CURRENT DENSITY JR
EP(13)=REC*PR(8)/(G(17)*CLMTR)
C # OF CONDUCTORS PER PHASE (ROTOR) CFR
G(44)=AINT((EP(11)/EP(13))/.07E-6+.5)
IF(G(44).EQ.0) G(44)=1.
C # OF CONDUCTORS PER SLOT (ROTOR) CSH
G(40)=G(44)*P(10)
C COPPER CROSS SECTION AREA OF A PHYSICAL CONDUCTOR AR
G(15)=(EP(11)/EP(13))/G(44)
C ROTOR SLOT DEPTH (SEE SAME STEPS IN STATOR)
DBCR=SQRT(4.*G(15)/PI)
IF(DBCR.LE.0.65E-3) GO TO 50
DCR=9.E-5*(1.-2.*EXP(-2757.*DBCR)) + DBCR
GO TO 51
50 DCR=1.092*DBCR
51 CONTINUE
C DR & GR ARE INTERDEPENDENT
  DR = (2.*G(28) + G(35) + G(40) * DCR**2) / C
  ABS (.9*G(30) - 2.*G(26))) / .9
  G(53) = 2.*PI*(P(7)-DR) * P(8) / G(24)
  G(30) = .5* (G(53) + G(54))
  IF (DR.GE. (.95*P(1) - G(5))) G(18) = DR
  IF (DR.GE. (.95*P(1) - G(5))) GO TO 500
C FICTITIOUS DIMENSIONS FOR THERMO
  G(36) = SQRT(G(15))
  G(37) = G(36)
  G(38) = ABS(DCR/ .9 - G(36))
C OVERALL DENSITY OF THE STUFF WITHIN THE SLOT
  DENS(8) = (G(35) * G(30) * DENS(6) +
  C * DENS(5) * ABS (ABS(DR - G(35)) * G(30) - G(40) * G(15)) +
  C * DENS(3) * G(40) * G(15)) / (G(30) * DR)
C CENTRIFUGAL FORCE SUPPORTED BY THE WEDGE
  FSLOT = DENS(8) * G(30) * DR*PR(1) ** 2*ABS(P(1) - .5*DR)
C ROTOR WEDGE WIDTH GR
  G(35) = SQRT(.75*G(30) * FSLOT / (P(2) * SIGMAX(3)))
C CHECKING THE PREVIOUS GUESS FOR DR
  INDDR = INDDR + 1
  IF (INDDR .GE. 25) GO TO 60
  ERRDR = ABS ((DR - G(18)) / DR)
  G(18) = DR
  IF (ERRDR .GE. 0.01) GO TO 32
  GO TO 61
60 G(18) = DR
  WHITE (6, 62) ERRDR
  62 FORMAT (5X, 'INDDR IS GE 25; ERRDR=', E12.6)
61 CONTINUE
C CALCULATION OF MORE GEOMETRICAL DIMENSIONS
  G(1) = ABS (P(1) - .5*G(18)) * SIN (PI/PR(7))
  G(2) = (P(1) + P(3) + .5*G(9)) * SIN (PI/PR(7))
C WIDTH OF THE EXTERNAL AXIAL DUCT
  IF (IC(2) .NE. 2) GO TO 70
C A VALUE WAS ASSIGNED TO VDUCT
C ASSUMING W2=W0/2.

   G(12) = .5*PR(5)/(VDUCT*2.*PI*(P(1)+P(3)+P(4)))
GO TO 71

70 G(12)=0.

71 CONTINUE

   G(4)=P(2)
   G(10)=(P(1)+P(3)+P(4)+G(12))/9
   G(11)=.05*G(10)
   G(13)=1.1*G(10)
   G(3)=P(2)+2.*G(10)
   WRD=0.

   IF(IC(2).EQ.2) WRD=2.*P(3)
   G(31)=P(2)+WRD

C SHAFT RADIUS

   IF(G(5).LE.0.) G(5)=.05*P(1)
   DENS(9)=DENS(8)*P(8)/G(46)+DENS(1)*ABS(1.-P(8))
   AUXW1=30.32*DENS(10)*G(5)**2
   BROOT=ABS(P(1)-G(18))
   AUXW2=30.82*(DENS(9)*ABS(P(1)**2-BROOT**2)

C +DENS(1)*ABS(BROOT**2-G(5)**2))
   BENDMO=.125*ABS(AUXW1*G(3)**2+AUXW2*G(31)*(2.*G(3)-G(31)))
   TORSMO=PR(2)/PR(1)
   IF(INDRSH.GE.10) GO TO 640
   RSH=2.49E-3*(2.25*BENDMO**2+4.*TORSMO**2)**(1./6.)
   INDRSH=INDRSH+1
   IF(RSH.GE.(-.95*P(1)-G(18))) G(5)=RSH
   IF(RSH.GE.(-.95*P(1)-G(18))) GO TO 500
   IF(INDRSH.EQ.1) G(5)=RSH
   IF(INDRSH.EQ.1) GO TO 30

C CHECKING PREVIOUS GUESS FOR RSH
C CALCULATIONS ARE ONLY REPEATED IF GROSS ERROR IS DETECTED

   ERRRSH=ABS((RSH-G(5))/RSH)
   IF(INDRSH.GE.3) GO TO 80
   G(5)=RSH
   IF(ERRRSH.GE.0.1) GO TO 30
GO TO 81
80 G(5)=RSH
   WRITE(6,82) ERRRSH
82 FORMAT(5X,'INDRSH IS GE 3; ERRRSH=',E12.6)
81 CONTINUE
640 CONTINUE

C CALCULATION OF THE REMAINING GEOMETRICAL DIMENSIONS
G (6)=P (1)
G (7)=P (3)
G (19)=P (4)
G (20)=P (7)
G (21)=P (8)
G (41)=PI*G (2)
CLMTS=2.*(G(31)+G(41))
G (42)=CLMR
EP (12)=P (11)

C COPPER LOSSES
C STATOR (UPDATING FIRST THE ELECTRICAL CONDUCTIVITY)
     TAVS=.5*(TEMPBA(1)+TEMPBA(2))
     SEC=1.*(1./EC(1)+EC(2)*(TAVS-20.))
     VCUS=G(16)*G(14)*G(43)*CLMTS
     CL(1)=VCUS*P(11)**2/SEC

C ROTOR
VCUR=G(17)*G(15)*G(44)*CLMT
CL(2)=VCUR*EP(13)**2/REC

C WEIGHT OF COPPER
G(47)=VCUS*DENS(2)
G(48)=VCUR*DENS(3)

C WINDINGS RESISTANCE
     EP (17)=CLMTS*G (16)/(SEC*G (14)*G (43))
     EP (18)=CLMT*G (17)/(REC*G (15)*G (44))

C WEIGHT OF IRON
AUX1=PI*G(45)*P(2)*DENS(7)
G(49)=AUX1*(G(19)-G(9))*(2.*P(1)+2.*P(3)+G(9)+G(19))
G(51)=AUX1*ABS(1.-P(7))*G(9)*(2.*P(1)+2.*P(3)+G(9))
AUX2=PI*P(2)*G(46)*DENS(1)
G (50) = AUX2 * ABS((P(1) - G(18)) ** 2 - G(5) ** 2)
G (52) = AUX2 * G(18) * ABS(1 - P(8)) * ABS(2 - P(1) - G(18))

C IRON LOSSES (SYNCHR. MACH.: NO ROTOR LOSSES)
CL(3) = G(49) * SPCL(E(2), F)
CL(4) = 0.
CL(5) = G(51) * SPCL(E(4), F)
CL(6) = 0.

C BEARING LOSSES
IF(IC(4) = EQ. 1) GO TO 84
CL(7) = .0025 * PR(2)
84 CONTINUE

C CALCULATION OF THE CONSTRAINT FUNCTIONS
C CNTRF = (ACTUAL VALUE) - (UPPER LIMIT VALUE) = CNTR-CNTRL < 0.
C BUT THEY ARE HANDLED AS P.U. CONSTRAINT ALLOWANCES:
C CA = (ACTUAL VALUE)/(UPPER LIMIT VALUE) - 1 < 0.; CNTR/CNTRL-1 < 0
C THE CONSTRAINT IS SATISFIED WHEN CA IS NEGATIVE

C INITIALIZING
IGEOM = 0
GO TO 700
500 IGEOM = 1
700 CONTINUE

C DO 85 I = 1, 25
85 CNTR(I) = 0.

C GEOMETRICAL CONSTRAINTS
C DR + HSH - R < 0
C CNTRL (2) = .95 * P(1)
C CNTRL (2) = G(18) + G(5)
C DS - H < 0
C CNTRL (3) = P(4)
C CNTRL (3) = G(9)
C 2*TXR - WR < 0
C CNTRL (4) = G(30)
CNTR(4)=2.*G(26)

C 2*TXS-WS<0
    CNTRL(5)=G(29)
    CNTR(5)=2.*G(25)
    IF(IGEOM.EQ.1) GO TO 600
C ALL GEOMETRICAL CONSTRAINTS ARE SATISFIED.
C MECHANICAL CONSTRAINTS
C ROTOR TEETH PRACTICALITY (LIMIT TO HIR/R)
    HIR=2.*PI*(P(1)-G(18))*ABS(1.-P(8))/G(24)
    CNTR(6)=P(1)/HIR
C LIMIT FOR THE SHAFT STATIC DEFORMATION: Y-YLIMIT<0
C AN ARRANGEMENT IS MADE SO CONSTRAINTS 7 & 8 CANNOT BE VIOLATED
    AUXI=3.32E-3/(CMECH(2)*G(5)**4)
    CNTR(7)=AUXI*(5.*AUXW1*G(3)**4+8.*AUXW2*G(3)**3*G(4)
    C  -4.*AUXW2*G(4)**4)
    CNTR(7)=-2.*P(3)
    RSH1=0.
    IF(CNTR(7).LE.CNTRL(7)) GO TO 610
    RSH1=1.05*(0.0166*CNTR(7)/(AUXI*CMECH(2)*P(3)))**.25
    610 CONTINUE
C LIMIT FOR THE UNBALANCED MAGNETIC PULL
    CNTR(8)=2.36*EP(1)**2*P(1)*P(2)/(C00*P(3))
    CNTR(8)=(1./AUXI)/ABS(8.*G(3)**3*G(4)-4.*G(3)**3*G(4)**4)
    RSH2=0.
    IF(CNTR(8).LE.CNTRL(8)) GO TO 620
    RSH2=1.05*(3.32E-3*CNTR(8))/(AUXI*CMECH(2)*CNTRL(8)))**.25
    620 CONTINUE
    IF((RSH1.EQ.0.) AND (RSH2.EQ.0.)) GO TO 630
    G(5)=RSH1
    IF(RSH2.GT.RSH1) G(5)=RSH2
    INDRESH=10
    GO TO 30
    630 CONTINUE
C STRESS IN THE ROTOR BODY: 2*MASSHEAR-SIGMAX(TENSION)<0
    CNTR(9)=.66*DENS(9)*PR(1)**2*ABS(P(1)-G(18))
    C  *(P(1)**3-ABS(P(1)-G(18))**3)/
C  (ABS (P(1)-G(18)) **2-G(5)**2) +
C  *0.25*DENS(1)*PR(1) **2*
C  ((3.*CMECH(1))*ABS (P(1)-G(18)) **2*
C  *ABS((1.-CMECH(1) )G(5)**2)
C STRESS IN THE TEETH ROOTS: STRESS-SIGMAX (TENSION) < 0
   HR=ABS(2.*PI*P(1)/G(24)-G(30))
C CENTRIFUGAL FORCE
   FTOOTH=-.5*DENS(1)*G(18)*(HR+HIR)*PR(1) ** 2
C  *ABS((P(1)-G(18))*(HR+2.*HIR)/(3.*(HR+HIR)))
C STRESS
   CNTR(10)=(FTooth+FSLGT*2.*COS (PI/3.-PI/G (24)))/HIR
C THERMAL CONSTRAINTS
   IC(9)=0
   IF ((KOBJ.EQ.1).OR. (IC(8).EQ.0)) IC(9)=1
   CALL THERMO
C TMAX STATOR WNDG-TLIMIT STATOR WNDG<0
   CNTR(11)=TMAX(2)
   IF(TEMP(1).GE.TEMP(2)) CNTR(11)=TEMP(1)
C TMAX ROTOR WNDG-TLIMIT ROTOR WNDG<0
   CNTR(12)=TMAX(4)
   IF(TEMP(3).GE.TEMP(4)) CNTR(12)=TEMP(3)
C TMAX STATOR CORE-TLIMIT STATOR CORE<0
   CNTR(13)=TMAX(5)
C TMAX ROTOR CORE-TLIMIT ROTOR CORE<0
   CNTR(14)=TMAX(6)
C ELECTRICAL CONSTRAINTS
C LOWER LIMIT FOR THE EFFICIENCY: -EFF+EFFMIN<0
C (-EFF+2*EFFMIN)/EFFMIN-1<0
C TOTAL LOSSES ARE CALCULATED FIRST
   CLOT=0.
   DO 90 I=1,10
     CLOT=CLOT+CL(I)
  90   CLOT=CLOT+CL(I)
C EFFICIENCY
   EP(16)=PR(2)/(PR(2)+CLTOT)
C CONSTRAINT FUNCTION
   CNTR(15)=-EP(16)+2.*CNTRL(15)
C UPPER LIMIT TO HARMONIC CONTENT
C HCF-HCFMAX<0
   CNTB(16)=HCF
C STABLE OPERATION: SIN(DEL)-SIN(DELMAX)<0
   CNTB(17)=PR(2)*PR(1)*EP(7)*G(22)/(6.*PR(3)*EP(9))
C UPPER LIMIT TO DR/R (END WINDING PRACTICALITY PROBLEMS)
   CNTB(18)=G(18)/P(1)
600 CONTINUE
00009425 LINE1017
C COST CONSTRAINT (FIGURE OF MERIT): MIN. VOLUME & MAX. EFFIC.
   CNTB(1)=WF(1)*(PI*G(4)*G(10)*G(10)*ABS(1.-EP(16))*WF(25))
C PER UNIT CONSTRAINT ALLOWANCES CA
   DO 100 I=1,25
   CA(I)=0.
   IF(WF(I) .EQ. 0.) GO TO 100
   IF(CNTB(I) .EQ. 0.) GO TO 100
   IF(CNTBL(I) .EQ. 0.) GO TO 100
   CA(I)=CNTRB(I)/CNTRBL(I)-1.
100 CONTINUE
00009460 LINE1021
C DETERMINE IF THERE ARE VIOLATED CONSTRAINTS
C DETERMINA THE VALUE OF THE MOST VIOLATED ONE (OR CLOSEST TO BE) CAWFMX
C WEIGHTING FACTORS WF ARE ACCOUNTED FOR; CHANGE TO 1/WF IF CONSTRAINT
C IS SATISFIED (ONLY USED IF ALL CONSTRAINTS ARE SATISFIED)
C EXCEPT FOR I=2,3,4,5,6, IN WHICH CASE WF=0.
   CAWFMX=CA(1)
   IVIO(1)=0
   IF(CA(1).GT.0.) IVIO(1)=1
   CNTR(1)=CA(1)
   DO 110 I=2,25
   IVIO(I)=0
   IF((CA(I) .EQ. 0.) .OR. (WF(I) .EQ. 0.)) CNTR(I)=0.
   IF(WF(I) .EQ. 0.) GO TO 110
110 CONTINUE
00009550 LINE1032
C CNTR IS USED AGAIN TO SAVE STORAGE
   IF(CA(I)) 120,110,130
120 CNTRB(I)=CA(I)/WF(I)
   IF((I.GE.2) .AND. (I.LE.6)) CNTRB(I)=0.
00009605 LINE1044
GO TO 125
130 IVIO(I)=1
   CNTR(I)=CA(I)*WP(I)
125 IF(CNTR(I).GE.CAWFMX) CAWFMX=CNTR(I)
110 CONTINUE

C MENZIES' APPROACH:
C IF VIOLATED CONSTRAINTS: OBJ ONLY ACCOUNTS FOR THEM
C IF NO VIOLATED CONSTRAINTS: ALL CONSTRAINTS ARE INCLUDED
C EXPONENT IN PTTH-OPTIMIZATION METHOD
   EXPQ=SIGN(EXPP,CAWFMX)
C OBJECTIVE FUNCTION
   OBJ=0.
   DO 140 I=1,25
      IF(CAWFMX) 150,160,170
   140 IF(CNTR(I).LE.0.) GO TO 140
   150 IF(CNTR(I).EQ.0.) GO TO 140
      OBJ=OBJ*(CNTR(I)/CAWFMX)**EXPQ
   140 CONTINUE
      OBJ=CAWFMX*OBJ**((1./EXPQ)
   GO TO 180
   160 OBJ=0.
   WRITE(6,190)
   190 FORMAT(5X,'CAWFMX=0.'
   180 CONTINUE
C STORING VOLUME TO BE WRITTEN
   EP(24)=PI*G(4)*G(10)*G(10)
C PER UNIT SYNCR. & LEAKAGE REACTANCES.
   DUMMY=PR(1)*PR(2)/(3.*PR(3)**2)
   EP(19)=EP(7)*DUMMY
   EP(20)=EP(6)*DUMMY
   IWR=1
   IF(IC(7).EQ.0) IC(7)=1
   IF((IC(5).NE.1).AND.(IC(9).NE.1)) IWR=0
   IF((IC(1).EQ.0).AND.(IWR.EQ.0)) GO TO 320
   WRITE(6,310) IC(7),P(IC(7)),OBJ,EP(24),EP(16)
310 FORMAT (5X, I2, 4 (3X, E12.6))
   WRITE (6, 370) IVIO
370 FORMAT (5X, 'IVIO=', 25 (1X, I1))
320 CONTINUE
   IF ((IC(1) NE. 2) AND (IWR.EQ.0)) GO TO 350
   WRITE (6, 330) CA
330 FORMAT (5X, 'CA='/(5X,5(E12.6,3X)))
   WRITE (6, 340) EP
   WRITE (6, 360) G, DENS, CNTR
340 FORMAT (5X, 'EP='/(5X,5(E12.6,3X)))
360 FORMAT (5X, 'G, DENS, CNTR='/(5X,5(E12.6,3X)))
350 CONTINUE
C RESETTING THE CONTROLLERS
   IC (5) = 0
   IC (6) = 0
   RETURN
   END
C**********************************************************************
   SUBROUTINE HBF (H,B,BH,XD,X0,IND)
C THE SUBROUTINE HBF SIMULATES THE MAGNETIZATION CURVE FOR A
C PRESCRIBED FREQUENCY. IT GIVES H FOR A GIVEN B.
   DIMENSION BH(9)
   BN=XD*B*X0
   IF (BN.GT.1.) GO TO 30
   CALL CNPS (Y,BN,BH,9)
   H=79.58*10.**Y
   GO TO 10
   30 H=8.475E4+7.958E6*(B-2.4)
   10 RETURN
   END
C**********************************************************************
   SUBROUTINE THERMO :
C -RECEIVES DATA OF A COMPLETE MACHINE DESIGN FROM OBJECT
C -RECEIVES DATA ON THERMAL PROPERTIES OF MATERIALS (BLOCKDATA) 00010190 LINE1117
C -CALCULATES THE FILM COEFF. IF THEY ARE NOT GIVEN EXTERNALLY 00010200 LINE1118
C -CALCULATES WINDAGE LOSSES IF THEY ARE NOT GIVEN EXTERNALLY 00010210 LINE1119
C -CALCULATES THE TEMPERATURES AT THE EXPECTED HOT SPOTS 00010220 LINE1120
C -CALCULATES THE HEAT EXCHANGES AMONG THE MAIN PARTS OF THE MACHINE 00010230 LINE1121
C SEPARATELY FOR CONDUCTION, CONVECTION & RADIATION. 00010240 LINE1122
C 00010250 LINE1123
C IF I0=1 WRITE INTERMEDIATE INFORMATION. IF I0=0 DO NOT . 00010260 LINE1124
C IF I1=1 THE MACHINE OPERATES IN A HIGH VACUUM. 00010270 LINE1125
C IF I2=1 VALUES FOR H(I) ARE ASSIGNED EXTERNALLY 00010280 LINE1126
C IF I3=1 WINDAGE LOSSES CL(8) & CL(9) ARE ASSIGNED EXTERNALLY. 00010290 LINE1127
C I4 GIVES THE VENTILATION SYSTEM:
C I4=0 : ENCLOSED MACHINE. PURE NATURAL COOLING. FINS. TA=0. 00010300 LINE1128
C I4=1 : ENCLOSED MACHINE. EXTERNAL FORCED COOLING. FINS. TA=0. 00010310 LINE1129
C I4=2 : OPEN VENTILATED MACHINE. INTERNAL FAN. W0,W1,W2. 00010320 LINE1130
C NO FINS. TA NOT 0. EXTERNAL NATURAL COOLING. 00010330 LINE1131
C I4=3 : OPEN VENTILATED MACHINE. INTERNAL FAN. W0. NO FINS. 00010340 LINE1132
C TA=0 . EXTERNAL NATURAL COOLING. 00010350 LINE1133
C 00010360 LINE1134
SUBROUTINE THERMO
IMPLICIT REAL(K)
COMMON
C /FIRST/X,TC,NTOP,XINC,H,ACCELE,CPINS,HEXT
C /SECOND/I0,I1,I2,I3,I4,I5,ILAST,K,A,T10,T18,T19,D,S
C /SIXTH/IC,PR,G,CL,TEMP
DIMENSION
C X(13),TC(11),H(10),
C K(13,16),A(13,14),D(20),S(15),
C IC(10),PR(10),G(75),CL(10),TEMP(11),
C X1(13),X2(13)
IC(8)=IC(8)+1
TAMB=PR(6)
W=PR(1)
W0=PR(5)
N=1
ALPHA=ACCELE
I4=IC(2)
I1=IC(10)
I2=1
I3=IC(4)
ICOOL=2
IF((IC(2).EQ.3).AND.((IC(10).EQ.0)) ICOOL=0
IF((IC(2).EQ.1).AND.((IC(10).EQ.1)) ICOOL=1
ISHAPE=1
IF((ICOOL.EQ.0).OR.(ICOOL.EQ.1)) GO TO 1500
WRITE(6,1510)
1510 FORMAT(5X,'NONVALID COOLING OPTION')
1500 CONTINUE
IBEST=0
I0=IC(1)
IF((IC(5).EQ.1).OR.((IC(6).EQ.1).OR.((IC(9).EQ.1)) IBEST=1
IF(IBEST.EQ.1) I0=1
NMAX=NTOP
IF(IBEST.EQ.1) NMAX=100
IF((ICOOL.EQ.0).AND.((IBEST.EQ.1)) ISHAPE=0
IF((ICOOL.EQ.1) ISHAPE=0
IF((ICOOL.EQ.0).AND.((IC(3).EQ.0).AND.((
C (IBEST.EQ.1).OR.((IC(7).EQ.3)) ) I2=0
IF(I0.EQ.0) GO TO 135
WRITE(6,190)
190 FORMAT(5X,'THERMO')
135 CONTINUE
C
C FIRST GUESSES FOR THE FLUID TEMPERATURE AT THE AIR GAP T10 ,
C EXTERNAL AXIAL DUCT T18 , AND RADIAL DUCT T19 .
C THEY ARE UPDATED IN SUBROUTINE CONVEC .
C
T10=.5*(X(1)+X(2))
T18=.5*(X(1)+X(3))
T19=T10
DO 1 I=1,13
DO 1 J=1,16

00010540 LINE1153
00010550 LINE1154
00010560 LINE1155
00010570 LINE1156
00010580 LINE1157
00010590 LINE1158
00010600 LINE1159
00010610 LINE1160
00010620 LINE1161
00010630 LINE1162
00010640 LINE1163
00010650 LINE1164
00010660 LINE1165
00010670 LINE1166
00010680 LINE1167
00010690 LINE1168
00010700 LINE1169
00010710 LINE1170
00010720 LINE1171
00010730 LINE1172
00010740 LINE1173
00010750 LINE1174
00010760 LINE1175
00010770 LINE1176
00010780 LINE1177
00010790 LINE1178
00010800 LINE1179
00010810 LINE1180
00010820 LINE1181
00010830 LINE1182
00010840 LINE1183
00010850 LINE1184
00010860 LINE1185
00010870 LINE1186
00010880 LINE1187
00010890 LINE1188
1 K(I,J)=0.
DO 8 I=1,13
DO 8 J=1,14
8 A(I,J)=0.
CALL GEOMET
CALL RADIAT(G,10,TAMB,K,D,S,ISHAPE)

C

C THE INDEX ILAST CHANGES TO 1 WHEN THE SYSTEM OF EQUATIONS
C IS SOLVED ACCURATELY ENOUGH, SO THE HOT SPOT TEMPERATURES
C MAY BE CALCULATED.
   ILAST=0

C

C CALL CONDUC

C

C STORING X(I) PRIOR TO ACCELERATION FOR THE N ITERATION
1000 DO 1001 I=1,13
      1001 X2(I)=X(I)
C THERE IS NO ACCELERATION IN THE FIRST ITERATION
   IF(N.EQ.1) GO TO 1004
C ACCELERATION SCHEME FOR THE N ITERATION
C X1(I) IS X(I) PRIOR TO ACCELERATION FOR THE N-1 ITERATION
1002 DO 1003 I=1,13
      1003 X(I)=X(I)+ALPHA*(X(I)-X1(I))
C STORING X(I) PRIOR TO ACCELERATION FOR THE N ITERATION
1004 DO 1005 I=1,13
      1005 X1(I)=X2(I)
C
C CALL CONVEC
CALL COEFFI

C

C
C ISING=0
C SOLVER SOLVES A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS.
C A(I,J) IS THE COEFFICIENT MATRIX
C L IS THE NUMBER OF EQUATIONS
C ISING IS AN ERROR INDICATOR (=1) IF SINGULAR MATRIX
C IROW AND ICOL ARE THE DIMENSIONS OF A(IROW,ICOL)
C SOLVER PUTS ANSWERS IN A(N,14) N=1,13
C SIGN OF THE INHOMOGENEOUS TERMS : AS WRITTEN AT THE R.H.S.
C CALL SOLVER(A,13,ISING,13,14)
C IF(ISING.EQ.0) GO TO 1010
C WRITE(6,170) ISING

170 FORMAT(//,10X,'SINGULAR',1110,///)
1010 CONTINUE
C
C UPDATING THE UNKNOWNS
C DO 1011 I=1,13
1011 X(I)=X(I)+A(I,14)
C
C CORRECTING THE EVENTUAL NEGATIVE ROOTS: OLD VALUE / 2
C DO 1020 I=1,13
1020 IF(X(I).LE.0.) X(I)=(X(I)-A(I,14))/2.
C
N=N+1
ALPHA=ACCELE/SQRT(FLOAT(N))
C COMBINED TEST FOR CONVERGENCE AND
C MAXIMUM EXPECTED NUMBER OF ITERATIONS
C DO 1025 I=1,13
1025 IF(AABS(A(I,14))-XINC*X(I)) 1025,1025,1025
1025 CONTINUE
C GO TO 1030
1026 IF(N.LE.NMAX) GO TO 1000
C
1030 CONTINUE
C
C
IF(IBEST.EQ.0) GO TO 132
IF((IC(5).EQ.1).AND.(IC(1).EQ.0)) GO TO 132
6 CALL HEATEX(X)
WRITE(6,120) N
120 FORMAT(5X,'ITERATIONS #:',I3/)
WRITE(6,125)
125 FORMAT(5X,'LAST INCREMENTS:')
WRITE(6,130) (A(I,14),I=1,13)
130 FORMAT(5X,E9.3)/
132 CONTINUE
C CALCULATION OF THE TEMPERATURES TEMP
ILAST=1
CALL CONDUC
TEMP(5) = 1.5*X(1) - .25*(X(8)*TEMP(11))
C + (X(8) - TEMP(11))**2/(24.*X(1) - 12.*X(8) - 12.*TEMP(11))
TEMP(6) = X(2) + .5*(X(2) - X(5))
TEMP(8) = X(4)
IF(I4.EQ.2) GO TO 340
330 TEMP(7) = 0.
TEMP(9) = 0.
TEMP(10) = 0.
TEMP(11) = 0.
TEMP(5) = X(1) + .5*(X(1) - X(8))
340 CONTINUE
IF(IBEST.EQ.0) GO TO 2000
IF((IC(5).EQ.1).AND.(IC(1).EQ.0)) GO TO 2000
WRITE(6,140) X
140 FORMAT(5X,'VALUES FOR THE UNKNOWN:/',5X,'I3=+',E12.6/)
C 5X,'T7=',E12.6/5X,'T9=',E12.6/5X,'T11=',E12.6/5X,'T13=',E12.6/5X,'T14=',E12.6/5X,'T16=',E12.6/5X,'T2E=',E12.6/5X,'T2EX=',E12.6/5X,'T2X=',E12.6/5X,'T2=',E12.6/5X,
C 5X,'B12=',E12.6/5X,'B14=',E12.6/5X,'B16=',E12.6/5X)
WRITE(6,160) TEMP
160 FORMAT(5X,'TEMPERATURES'/)
C 5X,'TMAX & TMIN STATOR WNDG=',E12.6,5X,E12.6/
C 5X, 'TMAX & TMIN ROTOR WNDG=' , E12.6 , '3X', E12.6/
C 5X, 'TMAX STATOR CORE=' , E12.6/
C 5X, 'TMAX ROTOR CORE=' , E12.6/
C 5X, 'EXHAUST TEMP COOLANT AIR GAP=' , E12.6/
C 5X, 'TEMP COOLANT OVERHANG=' , E12.6/
C 5X, 'EXHAUST TEMP COOLANT AXIAL DUCT=' , E12.6/
C 5X, 'EXHAUST TEMP COOLANT RADIAL DUCT=' , E12.6/
C 5X, 'TEMP RADIAL DUCT WALL=' , E12.6//)
WRITE (6, 110) H
2000 CONTINUE
RETURN
END
C
C**************************************************************************
C
C**************************************************************************
C
C SUBROUTINE RADIAT(G,IO,TAMB,K,D,S,ISHAPE)
IMPLICIT REAL*8 (A,F,X,Y,Z), REAL(K)
C COMMON CANNOT BE USED WITHOUT A CHANGE IN NOTATION
C SINCE MATRICES A AND X BECOME DOUBLE PRECISION
DIMENSION G(75), K(13,16), D(20), F(9), E(9), S(15)
C EMISIVITIES
C E2,E3C,E3B,E7,E9B,E9A,E12,E14D,E16
DATA E/9*0.9/
C FUNCTIONS DEFINITION
PA(X,Y)=X**2+Y**2-1.
FB(X,Y)=Y**2-X**2+1.
C
\[ P_1(x, y) = \]
\[
\frac{1.0}{x - \text{DARCos}(P_B(x, y) / P_A(x, y)) - \left( (\text{DARCos}(P_B(x, y) / (x \times P_A(x, y))) + \right) \text{DARSin}(1.0 / x) - \text{P_A}(x, y) \times \text{DARSin}(1.0)}{(2.0)} \]

\[ P_2(x, y) = \]
\[
\frac{1.0 - 1.0 / (x \times \text{DARSin}(1.0))}{/ (x \times \text{DARSin}(1.0) \times 2.0)} \]

\[ P_3(x, y) = \]
\[
\frac{1.0}{-5.0 \times (1.0 - P_1(x, y) - P_2(x, y))} \]

\[ P_4(x, y) = \]
\[
\frac{1.0}{-5.0 \times (1.0 - x \times P_1(x, y))} \]

\[ P_5(x, y, z) = \]
\[
P_4(x, y, z) - z \times \text{P_4}(x, y) \times (1.0 - z) \times \text{P_4}(x, y, y - z) \]

\[ \text{RR}(E, S) = \text{SIGMA} \times E \times S / (1.0 - E) \]

\[ \text{IF}(\text{ISHAPE} \cdot \text{EQ} \cdot 1) \text{ GO TO 50} \]

C

C CALCULATION OF THE SHAPE FACTORS

C SHAKE FACTOR P37
1 x1=1.0 + d(9) / d(5)
  y1=d(10) / d(5)
  \text{P}(1) = \text{P}_1(x, y)

C SHAKE FACTOR P93
1 x1=1.0 + d(11) / d(12)
  y1=d(10) / d(12)
  \text{P}(2) = \text{P}_1(x, y)
C SHAPE FACTOR P915
  F(3) = 1.

C SHAPE FACTOR P1415
  F(4) = 1.

C SHAPE FACTOR P216
  AB1 = 8. * DARSIN(1. D0) * D(7) * (D(1) + D(2))
  X1 = D(8) / D(7)
  Y1 = (D(1) + D(2)) / D(7)
  F(5) = F4( X1, Y1) * AB1 / S(1)

C SHAPE FACTOR P1614
  F(6) = 1. - F(5) * S(1) / S(10)

C SHAPE FACTOR P212
  X1 = D(6) / D(5)
  Y1 = D(1) / D(5)
  X2 = D(6) / D(4)
  Y2 = D(2) / D(4)
  Y3 = (D(2) + D(3)) / D(4)
  Z3 = (D(2) + D(3)) / D(3)
  AUX1 = 8. * DARSIN(1. D0) / S(1)
  AUX2 = AUX1 * D(2) * D(6) * (F1(X2, Y2) + F3(X2, Y2))
  AUX3 = AUX1 * D(3) * D(4) * F5(X2, Y3, Z3)
  AUX4 = AUX1 * D(1) * D(6) * F1(X1, Y1)
  F(7) = AUX2 + AUX3 + AUX4

C SHAPE FACTOR P214
  F(8) =
    C 1. - (D(2) * F2(X2, Y2) +
    C D(1) * F2(X1, Y1)) * 8. * DARSIN(1. D0) *
    C D(6) / S(1) - F(7) - F(5)

C SHAPE FACTOR P1214
  X5 = D(5) / D(4)
  F(9) =
    C 1. - F4(X5, Y3) * (D(2) + D(3)) *
    C D(4) * 16. * DARSIN(1. D0) / S(5) -
    C F(7) * S(1) / S(5)
  DO 300 I = 1, 9
  300 G(60+I) = F(I) * 1.
50 CONTINUE
DO 400 I=1,9
400 F(I)=G(60+I)*1.
SIGMA=5.67E-8
TK=273.16
C CALCULATION OF THE COEFFICIENTS K(I,J)
C ASSOCIATED TO THE RADIATIVE HEAT EXCHANGES
K(1,14)=1./(1./RR(E(2),S(2)) +
C 1./(SIGMA*S(2)*F(1)) +
C 1./RR(E(4),S(4)))
K(1,15)=1./(1./RR(E(3),S(3)) +
C 1./(SIGMA*S(7)*F(2)) +
C 1./RR(E(5),S(7)))
K(2,8)=K(1,14)
K(3,6)=K(1,15)
K(3,7)=SIGMA*E(6)*S(6)
K(3,8)=K(3,7)*(TAMB+TK)**4
K(5,6)=-S(1)*F(7)
K(5,8)=-S(5)*F(9)
K(5,7)=-K(5,6)-K(5,8)
K(7,5)=SIGMA*E(6)*S(8)
K(7,6)=K(7,5)*(TAMB+TK)**4
K(7,7)=RR(E(8),S(9))
K(7,8)=-K(7,7)/SIGMA
K(8,5)=-S(1)*F(5)
K(8,6)=-S(10)*F(6)
K(8,4)=-K(8,5)-K(8,6)
K(9,4)=RR(E(1),S(1))
K(9,5)=-K(9,4)/SIGMA
K(10,1)=-K(9,4)
K(10,3)=K(5,6)
K(10,4)=-S(1)*F(8)
K(10,5)=K(8,5)
K(10,2)=K(9,4)/SIGMA-K(10,3)-K(10,4)-K(10,5)
K(11,1)=-RR(E(7),S(5))
K(11,2)=K(5,6)
K(11, 4) = K(5, 8)
K(11, 3) = -K(11, 1) / SIGMA - K(11, 2) - K(11, 4)
K(12, 16) = -K(7, 7)
K(12, 3) = K(10, 4)
K(12, 2) = K(11, 4)
K(12, 4) = K(8, 6)
K(12, 3) = -K(7, 8) - K(12, 1) - K(12, 2) - K(12, 4)
K(13, 16) = -RR(E(9), S(10))
K(13, 1) = K(8, 5)
K(13, 2) = K(8, 6)
K(13, 3) = -K(13, 16) / SIGMA - K(13, 1) - K(13, 2)

RETURN
END

SUBROUTINE GEOMETR
IMPLICIT REAL(K)
COMMON /SECOND/I0, I1, I2, I3, I4, I5, ILAST, K, A, T10, T18, T19, D, S
/ SEXT/N, IC, PR, G, CL, TEMP

DIMENSION K(13, 16), A(13, 14), D(20), S(15)
IC(10), PA(10), G(75), CL(10), TEMP(11)

C GEOMETRICAL DIMENSIONS WHICH ARE FREQUENTLY USED
D(1) = G(1)
D(2) = ABS(G(2) - G(1))
D(3) = ABS(.5 * (G(3) - G(31)) - G(2))
D(4) = G(5)
D(5) = G(6)
D(6) = G(6) + G(7) + G(8)
D(7) = G(6) + G(7) + G(9)
D(8) = ABS(G(10) - G(11))
D(9) = G(7)
C SURFACE AREAS WHICH ARE FREQUENTLY USED
C AR2, AR3C, AR3B, AR7, AR12, AR9A, AR9B,
C AR14A, AR14D, AR16, A3Z

C
PI=2.*ARSIN(1.)
S(1)=2.*PI*(2.* (D(1) + D(2)) *
C (D(6) + D(7)) + D(7)**2 - D(6)**2)
S(2)=2.*D(10)*PI*(D(5) + D(9))
S(3)=2.*PI*D(10)*D(12)
S(4)=2.*PI*D(10)*D(5)
S(5)=4.*PI*D(1)*D(5) + D(4)* (D(2) + D(3)) *
C .5* (D(3)**2- D(4)**2)
S(6)=2.*PI*G(31)*G(10)
S(7)=2.*PI*D(10)* (G(10) - G(13))
S(8)=2.*PI*(G(10)*(G(10) + G(3) - G(4)) - G(5)**2)
S(9)=4.*PI*D(8)*(D(1) + D(2) + D(3)) *
C .5* (D(8)**2- D(4)**2)
S(10)=2.*PI*(D(8)**2- D(7)**2)
S(11)=2.*PI*( (G(19) - G(9))* (G(19) + G(9))
C +2.*D(13) + 2.*G(9)*D(13) *(1.-G(20)))
DO 4 I=12, 15
4 S(I)=0.
DO 2 I=1, 15
2 S(I)=ABS(S(I))
RETURN
END
C
C
C******************************************************************************
SUBROUTINE HEATEX
IMPLICIT REAL(K)
COMMON
C /FIRST/X,TC,NTOP,XINC,H,ACCEL,CFINS,HEXT
C /SECOND/I0,I1,I2,I3,I4,I5,I6ST,K,A,T10,T18,T19,D,S
C /SIXTH/IC,PR,G,CL,TEMP
C X(13),TC(11),H(10)
C K(13,16),A(13,14),D(20),S(15)
C IC(10),PR(10),G(75),CL(10),TEMP(11)
C Q(13),C(12),K(8)
TAMB=PR(6)
W=PR(1)
W0=PR(5)
DO 1 I=1,13
Q(I)=0.
1 C(I)=0.
DO 3 I=1,8
3 R(I)=0.
TK=273.16
Q(1)=K(1,16)+K(1,1)*X(1)+K(1,2)*X(9)
Q(2)=K(1,3)*X(1)-X(3)
Q(3)=K(1,4)*(X(1)-X(8))
C(1)=K(1,5)+K(1,6)*X(1)+K(1,7)*X(2)+K(1,8)*X(4)
C(2)=K(1,9)*X(1)+K(1,10)*X(3)+K(1,11)*X(4)
C(3)=K(1,12)*X(1)+K(1,13)*X(8)+K(2,10)
R(1)=K(1,14)*((X(1)+TK)**4-(X(2)+TK)**4)
R(2)=K(1,15)*((X(1)+TK)**4-(X(3)+TK)**4)
Q(4)=K(2,16)+K(2,1)*X(2)+K(2,2)*X(5)
Q(5)=K(2,3)*(X(2)-X(5))
C(4)=K(2,4)+K(2,5)*X(1)+K(2,6)*X(2)+K(2,7)*X(4)
Q(6)=K(3,1)*X(3)-X(8))
C(5)=K(3,2)*X(3)-K(3,9)
C(6)=K(3,3)*X(1)+K(3,4)*X(3)+K(3,5)*X(4)
R (3) = K (3, 7) * (X (3) + TK) **4 - K (3, 8)
C (7) = K (4, 16) *(X (4) - X (9))
C (8) = K (4, 1) *(X (4) - X (5))
C (9) = K (4, 2) *(X (4) - X (7))
C (10) = K (4, 3) *X (4) - K (4, 5)
C (11) = K (4, 4) * (X (4) - X (8))
Q (7) = K (5, 16) + K (5, 1) *X (2) + K (5, 2) *X (5)
Q (8) = K (5, 4) *(X (5) - X (6))
R (4) = K (5, 6) *X (10) + K (5, 7) *X (11) + K (5, 8) *X (12)
Q (9) = K (6, 1) *(X (6) - X (7))
Q (10) = K (6, 2) *X (6) - K (6, 3)
Q (11) = K (7, 1) *(X (7) - X (8))
Q (12) = K (7, 3) *X (7) - K (7, 4)
R (5) = K (7, 5) *(X (7) + TK) **4 - K (7, 6)
R (6) = K (7, 7) *(X (7) + TK) **4 + K (7, 8) *X (12)
R (7) = K (8, 4) *X (13) + K (8, 5) *X (10) + K (8, 6) *X (12)
R (12) = K (9, 16) + K (9, 1) *X (9) + K (9, 2) *X (1)
R (8) = K (9, 4) *(X (9) + TK) **4 + K (9, 5) *X (10)
Q (13) = K (3, 10) *(X (3) - X (7))

WRITE (6, 160) CL

WRITE (6, 130) Q
130 FORMAT (5X, 'Q3T1=', E9.3, '5X, 'Q3T9=', E9.3, '5X)

WRITE (6, 140) C
C 5X, 'C14T15=', E9.3, '5X

00014530 LINE 1549
00014540 LINE 1550
00014550 LINE 1551
00014560 LINE 1552
00014570 LINE 1553
00014580 LINE 1554
00014590 LINE 1555
00014600 LINE 1556
00014610 LINE 1557
00014620 LINE 1558
00014630 LINE 1559
00014640 LINE 1560
00014650 LINE 1561
00014660 LINE 1562
00014670 LINE 1563
00014680 LINE 1564
00014690 LINE 1565
00014700 LINE 1566
00014710 LINE 1567
00014720 LINE 1568
00014730 LINE 1569
00014740 LINE 1570
00014750 LINE 1571
00014760 LINE 1572
00014770 LINE 1573
00014780 LINE 1574
00014790 LINE 1575
00014800 LINE 1576
00014810 LINE 1577
00014820 LINE 1578
00014830 LINE 1579
00014840 LINE 1580
00014850 LINE 1581
00014860 LINE 1582
00014870 LINE 1583
00014880 LINE 1584
WHITE(6,150) R
150 FORMAT (5X,'R3T7=',E9.3/5X,'R3TY=',E9.3/
C 5X,'R14D=',E9.3/5X,'R16=',E9.3/5X,'R2=',E9.3///)
RETURN
END

C
C *****************************************************
C
C SUBROUTINE CONDUC
C IMPLICIT REAL(K)
C COMMON
C /FIRST/X,TC,NTOP,XINC,H,ACCEL,CFINS,HEXT
C /SECOND/I0,I1,I2,I3,I4,I5,ILAST,K,A,T10,T18,T19,D,S
C /SIXTH/IC,PR,G,CL,TEMP
C DIMENSION
C X(13),TC(11),D(10)
C K(13,16),A(13,14),D(20),S(15)
C IC(10),PR(10),G(75),CL(10),TEMP(11)
C
C TAMBR=PR(6)
C W=PR(1)
C W0=PR(5)
C ASSIGN VALUE TO THE CONDUCTIVE RESISTANCE RC4T9
C RC4T9=0.
C
C PI=2.*ARSIN(1.)
C AC1=2.*G(23)*G(14)*G(39)
C AC2=AC1
C B1=.5*G(31)
C B2=.5*G(41)
C Q1=CL(1)/(B1+B2)
C Q2=Q1
AS=-.5*G (29)-G (25)
B=G (9)-G (8)-2.5*G (27)
KX=TC (1)*((1.+5*G (32)/G (34))
KY=TC (1)*((1.+5*G (33)/G (34))
RXC1=AS/(12.*G (23)*B*KX)
RYC1=B/(12.*G (23)*AS*KY)
RXI=G (25)/(4.*G (23)*TC (2)*B)
RYI=G (27)/(4.*G (23)*TC (2)*AS)
RX1=RXC1+RXI
RY1=RYC1+RYI
R1=RX1*RY1*(1.-RXC1*RYC1)/
(C-(5.*RX1*RY1)/(RX1+RY1)
RXC2=RXC1
RYC2= .25*RYC1
RX2=RX1
RY2=RY1-.75*RYC1
R2=RX2*RY2*(1.-RXC2*RYC2)/
C-(5.*RX2*RY2)/(RX2+RY2)
ALPHA1=1./(SQRT(TC (3)*AC1*R1))
ETAX1=TANH(ALPHA1*B1)/(ALPHA1*B1)
ALPHA2=1./(SQRT(TC (3)*AC2*R2))
ETAX2=TANH(ALPHA2*B2)/(ALPHA2*B2)
AUX1=B1*ETAX1/R1
AUX2=B2*ETAX2/R2
IF(ILAST) 10,10,20
20 BETA1=(X (1)+Q1*R1-X (9)-Q2*R2)/(1.+AUX1/AUX2)
BETA2=(X (9)+Q2*R2-X (1)-Q1*R1)/(1.+AUX2/AUX1)
AUX5=(1.-3.*RXC1*RYC1/(8.*RX 1*RY1))/(RX1+RY1)
AUX6=(1.-3.*RXC2*RYC2/(8.*RX 2*RY2))/(RX2+RY2)
AUX7=AUX5*(RXI+1.5*RXC1)*(RYI+1.5*RYC1)
AUX8=AUX6*(RXI+1.5*RXC2)*(RYI+1.5*RYC2)
TEMP (1)=X (1)+(Q1-BETA1/(R1*COSH(ALPHA1*B1)))*AUX7
TEMP (2)=X (9)+(Q2-BETA2/(R2*COSH(ALPHA2*B2)))*AUX8
10 CONTINUE
K (1,1)=AUX1/(1.+AUX1/AUX2)
K (1,2)=-K (1,1)
K (1, 16) = -B1*Q 1 + K (1, 1) *(Q 1*R 1 - Q 2*R 2)

K (1, 3) = 2.*PI*G (31) * TC (6) / (ALOG ((D (13) + G (19)) / D (7)))
K (1, 3) = 1. / (RC4*T9 + 1./K (1, 3))
C .5*S(11) IS THE STATOR CORE CROSS SECTION
K (1, 4) = 6.*TC (5) * S(11) / G (4)

AC1R = 2.*G (24) * G (15) * G (40)
AC2R = AC1R
B1R = .5*G (31)
B2R = .5*G (42)
Q1R = CL (2) / (B1R + B2R)
Q2R = Q1R
AR = .5*G (30) - G (26)
B R = G (18) - G (35) - 2.*G (28)
KXR = TC (1) * (1.*G (36) * .5/G (38))
KYR = TC (1) * (1.*G (37) * .5/G (38))
RXC1R = AR / (12.*G (24) * BR*KXR)
RYC1R = BR / (12.*G (24) * AR*KYR)
RXR = G (26) / (4.*G (24) * BR*TC (2))
RYR = G (28) / (4.*G (24) * AR*TC (2))
RX1R = RXC1R + RXR
RY1R = RYC1R + RYR
R1R = RX1R*RY1R*(1.-RXC1R*RYC1R/

C (5.*RX1R*RY1R)) / (RX1R + RY1R)
RXC2R = RXC1R
RYC2R = .25*RYC1R
RX2R = RX1R
RY2R = RY1R - .75*RYC1R
K2R = RX2R*RY2R*(1. - RXC2R*RYC2R/

C (5.*RX2R*RY2R)) / (RX2R + RY2R)
ALPH1R = 1. / (SQRT (TC (4) *AC1R*R 1k))
ETA1R = TANH (ALPH1R*B1R) / (ALPH1R*B1R)
ALPH2R = 1. / (SQRT (TC (4) *AC2R*R 2R))
ETA2R = TANH (ALPH2R*B2R) / (ALPH2R*B2R)
\[
\begin{align*}
\text{AUX3} &= B1R*\text{ETAX1B}/R1R \\
\text{AUX4} &= B2R*\text{ETAX2B}/R2R \\
\text{IF (ILAST)} &= 40, 40, 30 \\
30 &\text{ BETAR1} = (X(2) + R1R*Q1R - X(5) - R2R*Q2R) / (1 + AUX3/AUX4) \\
&\text{ BETAR2} = (X(5) + R2R*Q2R - X(2) - R1R*Q1R) / (1 + AUX4/AUX3) \\
&\text{ AUX5} = (1 - 3.8*RXC1R*RYC1R/(8*RX1R*RY1R)) / (RX1R*RY1R) \\
&\text{ AUX10} = (1 - 3.8*RXC2R*RYC2R/(8*RX2R*RY2R)) / (RX2R*RY2R) \\
&\text{ AUX11} = \text{AUX9}*(RXIR+1.5*RXC1R) - (RYIR+1.5*RXC1R) \\
&\text{ AUX12} = \text{AUX10}*(RXIR+1.5*RXC2R) - (RYIR+1.5*RXC2R) \\
\text{TEMP (3)} &= X(2) + (Q1R - BETAR1)/(R1R*COSH(ALPH1R*B1R)) \times \text{AUX11} \\
\text{TEMP (4)} &= X(5) + (Q2R - BETAR2)/(R2R*COSH(ALPH2R*B2R)) \times \text{AUX12} \\
40 &\text{ CONTINUE} \\
K(2,1) &= \text{AUX3}/(1+\text{AUX3}/\text{AUX4}) \\
K(2,2) &= -K(2,1) \\
K(2,16) &= -B1R*Q1R+K(2,1)*(Q1R*R1R-Q2R*R2R) \\
C &= K(2,1) = 12.8*\text{PI}*(TC(10)\times G(5)\times 2+ \\
C &= TC(7)\times (G(6)\times 2+G(18)*2-2*G(6)\times G(18)\times G(21)))/G(31) \\
K(3,16) &= K(1,3) \\
\text{C RIBS EFFECT SHOULD BE ACCOUNTED FOR} \\
R*A/G(31) &= (8.8*\text{PI}+TC(9)\times G(10)\times G(13)) \\
K(5,1) &= -\text{AUX4}/(1+\text{AUX4}/\text{AUX3}) \\
K(5,2) &= -K(5,1) \\
K(5,16) &= -B2R*Q2R+K(5,2)*(Q2R*R2R-Q1R*R1R) \\
K(5,3) &= K(2,3) \\
K(5,4) &= 4.8*\text{PI}+TC(10)\times G(5)\times 2/(G(3)-G(31)) \\
K(6,16) &= K(5,4) \\
\text{RBEAR} &= 2/K(5,4) \\
K(6,1) &= 1.8/(RBEAR+G(3)-G(31))/(8.8*\text{PI}+TC(9)\times G(10)\times G(11)) \\
\text{C ASSUME 1% OF RATED POWER CAN GO TO AMBIENT THROUGH BEARINGS} \\
\text{C IP: TAMB=40, TBEAR=130.} \\
K(6,2) &= 0.1*PR(2)/(130-40) \\
K(6,3) &= K(6,2)*\text{TAMB} \\
K(7,16) &= K(6,1) \\
\text{RB} &= (G(3)-G(31))/(8.8*\text{PI}+TC(9)\times G(10)\times G(11)) \\
\text{RC} &= 0.5*(\text{RA}+\text{RB}) \\
00015970 &\text{ LINE1693} \\
00015980 &\text{ LINE1694} \\
00015990 &\text{ LINE1695} \\
00016000 &\text{ LINE1696} \\
00016010 &\text{ LINE1697} \\
00016020 &\text{ LINE1698} \\
00016030 &\text{ LINE1699} \\
00016040 &\text{ LINE1700} \\
00016050 &\text{ LINE1701} \\
00016060 &\text{ LINE1702} \\
00016070 &\text{ LINE1703} \\
00016080 &\text{ LINE1704} \\
00016090 &\text{ LINE1705} \\
00016100 &\text{ LINE1706} \\
00016110 &\text{ LINE1707} \\
00016120 &\text{ LINE1708} \\
00016130 &\text{ LINE1709} \\
00016140 &\text{ LINE1710} \\
00016150 &\text{ LINE1711} \\
00016160 &\text{ LINE1712} \\
00016170 &\text{ LINE1713} \\
00016180 &\text{ LINE1714} \\
00016190 &\text{ LINE1715} \\
00016200 &\text{ LINE1716} \\
00016210 &\text{ LINE1717} \\
00016220 &\text{ LINE1718} \\
00016230 &\text{ LINE1719} \\
00016240 &\text{ LINE1720} \\
00016250 &\text{ LINE1721} \\
00016260 &\text{ LINE1722} \\
00016262 &\text{ LINE1723} \\
00016264 &\text{ LINE1724} \\
00016270 &\text{ LINE1725} \\
00016280 &\text{ LINE1726} \\
00016290 &\text{ LINE1727} \\
00016300 &\text{ LINE1728}
RABC = RA*RB + RA*RC + RB*BC
K(3,1) = RB/RABC
K(7,1) = RA/RABC
K(3,10) = RC/RABC
K(7,9) = K(3,10)
K(8,16) = K(1,4)
K(8,1) = K(3,1)
K(8,2) = K(7,1)
K(9,1) = AUX2/(1.0 + AUX2/AUX1)
K(9,2) = -K(9,1)
K(9,16) = -B2*Q2*K(9,1) * (Q2*B2 - Q1*R1)
RETURN
END

C *******************************************************************************
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************

SUBROUTINE CONVEC
IMPLICIT REAL(K)

COMMON
C /FIRST/X, TC, NTOP, XINC, H, ACCEL, CPINS, HEXT
C /SECOND/I0, I1, I2, I3, I4, I5, I6, K, A, T10, T18, T19, D, S
C /SIXTH/IC, PR, G, CL, TEMP

DIMENSION
C X(13), TC(11), H(10)
C ,K(13,16), A(13,14), D(20), S(15)
C ,IC(10), PR(10), G(75), CL(10), TEMP(11)

C

C FUNCTIONS DEFINITION
C CP, RO, MU, K, NPR, FOR AIR AT ATMOSPHERIC PRESSURE
C IN THE TEMPERATURE RANGE 0 TO 400 DEGREES CELSIUS
C
F1(T) = 1006.86 + 1.694*T
F2(T) = 352.94/(273.16 + T)
\[ F3(T) = 1.75E-5 + 4.12E-8 \times T \]
\[ F4(T) = 2.45E-2 + 7.16E-5 \times T \]
\[ F5(T) = 4.12E-8 \times T \]

\[ \text{C} \]
\[ \text{TAMB}=\text{PR}(6) \]
\[ \text{W}=\text{PR}(1) \]
\[ \text{WO}=\text{PR}(5) \]
\[ \text{T10}=.5* (X(1) + X(2)) \]
\[ \text{IF}(\text{I4.EQ.2}) \text{ T10}=.5* (X(4) + \text{TEMP}(7)) \]
\[ \text{T18}=.5* (X(4) + \text{TEMP}(9)) \]
\[ \text{T19}=.5* (\text{TEMP}(7) + \text{TEMP}(10)) \]
\[ \text{IF}(\text{I4.NE.1}) \text{ GO TO 400} \]

\[ \text{C} \text{ MACHINE RUNNING IN A VACUUM. ONLY H915 & H1415 ARE NOT ZERO.} \]
\[ \text{C} \text{ TOTALLY ENCLOSED MACH. PINS. EXT. FORCED VENT.: H915, H1415=HEXT} \]
\[ \text{HEXT9}=\text{H}(1) \]
\[ \text{HEXT14}=\text{H}(7) \]
\[ \text{DO} 450 \text{ I}=1,10 \]
\[ 450 \text{ H}(1)=0. \]
\[ \text{K}(3,2)=\text{HEXT9}\times S(6)\times \text{CFINS} \]
\[ \text{K}(3,9)=\text{K}(3,2)\times \text{TAMB} \]
\[ \text{K}(7,3)=\text{HEXT14}\times S(8)\times \text{CFINS} \]
\[ \text{K}(7,4)=\text{K}(7,3)\times \text{TAMB} \]
\[ \text{CL}(8)=0. \]
\[ \text{CL}(9)=0. \]
\[ \text{X}(4)=0. \]
\[ \text{H}(1)=\text{HEXT9} \]
\[ \text{H}(7)=\text{HEXT14} \]
\[ \text{GO TO 30} \]

\[ 400 \text{ CONTINUE} \]
\[ \text{TK}=273.16 \]
\[ \text{PI}=2. \times \text{ARCSIN}(1.) \]

\[ \text{C} \text{ ALGORITHM TO PREVENT TOO SMALL TEMPERATURE INCREMENTS} \]
\[ \text{C} \text{ IN THE FIRST ITERATIONS, WHICH CAN YIELD TO OVERFLOW.} \]
\[ 520 \text{ CONTINUE} \]
\[ \text{DO} 500 \text{ I}=1,8 \]
\[ \text{L}=\text{I}+1 \]
DO 500 J=I,9
   IF(ABS(X(I)-X(J))-.001) 510,510,530
510  X(I)=X(I)+.01*I
   GO TO 520
530  IF(ABS(X(I)-TAMB)-.001) 510,510,500
500  CONTINUE

C

C COOLANT FLOW DISTRIBUTION (TO BE IMPROVED)
   IF(I4.EQ.2) GO TO 50
   W1=0.
   W2=0.
   IF(I4.LE.1) W0=0.
   GO TO 51
50  W1=W0*G(7)/(G(7)+G(12))
    W2=W0-W1
51  CONTINUE

C CONVECTION IN THE AIR GAP
   TF=T10
   CNTA=P2(TF)*W*G(6)**.5*G(7)**1.5/F3(TF)
   IF(CNTA-.63.) 311,311,312
311  GR=G(7)/G(6)
   IF(CNTA-.41.) 313,313,314
313  CF=2.*GR**.5*(1.+GR)**2/(CNTA*(1.+5.*GR))
   GO TO 310
314  CF=2.*(CNTA/GR)**.5
   GO TO 310
312  CONTINUE
   CALL FACTI(G(6),G(7),F3(TF),P2(TF),W,CF,I0)
310  IF(CNTA-.100.) 300,300,301
300  IF(CNTA-.41.) 302,302,303
302  CNU=1.
   GO TO 15
303  CNU=.106*CNTA**.63*F5(TF)**.27
   GO TO 15
301  CNU=.193*CNTA**.5*F5(TF)**.27
C NNUB=.44*CF*(G(6)/G(7))**.5*CNTA*F5(TP)**(1./3.)
IF(CNNU-CNNUB) 14, 14, 13
13 CNNU=CNNUB
   GO TO 15
14 CNNU=CNNUB
15 U10=F4(TP)*CNNU/G(7)
304 H10=2.*U10
C SKIP CALCULATION OF H(I) IF THEY ARE PRESCRIBED
C THE ABOVE CALC. IS NOT SKIPPED (CP IS NEEDED FOR CL(8)).
   IF(I2.EQ.1) GO TO 710
C
C CONVECTION 3-18 AND 9-18
C U18 AND U19 ARE NOT FILM COEFFICIENTS.
C THEY INCLUDE THE AREA OF THE SURFACES.
   IF(I4.EQ.2) GO TO 24
C NO AXIAL DUCTS
   H18=0.
   H319=0.
   CNRE18=0.
   CNRE19=0.
   GO TO 25
C FORCED COOLING THROUGH AXIAL DUCTS
C NATURAL CONVECTION AND CONDUCTION ARE NOT CHECKED
24 VM18=W2/(4.*PI*D(14)*G(12))
   CNRE18=VM18*.5*F2(T18)/F3(T18)
   H18=.0115*F4(T18)*CNRE18**.8*F5(T18)**.4/G(12)
C
C CONVECTION 3-19
   WRD=G(31)-G(4)*.001
   VM19=W1/(2.*PI*(D(13)+.5*G(19)))*WRD)
   CNRE19=VM19*.5*WRD*F2(T19)/F3(T19)
   H319=.0115*F4(T19)*CNRE19**.8*F5(T19)**.4/WRD
25 DUMMY=0.
C CONVECTION 9-15
   IF(I4.GE.2) GO TO 600
   IF(I4.LT.1) H915=HEXT
IF(I4.EQ.1) GO TO 3

C PURE NATURAL CONVECTION
TF= .5* (X(3) + TAMB)
CNGR=78.4*G(10)**3*ABS(X(3)-TAMB)
C*(F2(TF)/F3(TF)**2/(TK+TAMB)
AUX1=CNGR+F5(TF)
IF(AUX1-3.07E8) 1,1,2
1 H915= .265*F4(TF)*AUX1**.25/G(10)
GO TO 3
2 H915= .052*F4(TF)*AUX1**(1./3.)/G(10)
GO TO 3

C ALTERNATIVE EXPRESSION FOR NATURAL CONVECTION (TAKEN FROM
C ACTUAL MACHINES BEHAVIOR)
600 H915=4.22*ABS(X(3)-TAMB)**.25/SQRT(2.*G(10))
3 CONTINUE
IF(I4.NE.0) AUX1=0.
C CONVECTION 11-EXT2
TF= .5* (X(4) + X(9))
C VALUES FOR NATURAL COOLING
CHL=2.*ABS(G(29) *(G(9)-G(8))/PI)**.5
CNGR2=9.8*CHL**3*ABS(X(9)-X(4))*(F2(TF)/F3(TF)**2
C / (TK+X(4))
AUX2EX=CNGR2+F5(TF)
IF(AUX2EX-3.07E8) 255,255,256
255 COEFc=.53
COEFN=.25
GO TO 257
256 COEFc=.104
COEFN=1./3.
257 HNAT=F4(TF)*COEFc*AUX2EX**COEFN/CHL
C VALUES FOR ROTATION INDUCED COOLING
HR=H10
258 H2EX11=(HR+2.*HNAT)/3.
IF(I4.LE.1) GO TO 261
262 VM211=W0/(4.*PI*D(6)*G(2)*ABS(1.-G(20))})
CNRE2=VM211*CHL*F2(TF)/F3(TF)
HFOR= .33\*F4(TF)*CNRE2**.6*F5(TF)**(1./3.) /CHL  
IF (HFOR=H2EX11) 261, 261, 264  
264 H2EX11=HFOR  
261 CONTINUE  

C

C CONVECTION 12-11

TF= .5* (X(4) + X(5))  
RO=F2(TF)  
CMU=F3(TF)  
CK=F4(TF)  
CNPR=F5(TF)  
CNRE=2. * RO*W*G(5) **2/CMU  
CNGR=78.4*G(5) **3*ABS (X(5) - X(4))  
C

C THE SURFACE OF THE STATOR END IS CONSIDERED TO BE ROUGH AND

C THE FLOW REGIME TO BE TURBULENT (NRE>50,000)

H1711=.018*CK*(RO*W/CMU) **.8*(G(6)*CNPR)**.6  
270 CONTINUE

C

C CONVECTION 14-11

V1411=.5*W*G(5)  
TF= .5* (X(4) + X(7))  
CNRE=2. * PI*F2(TF) *G(10)*V1411/F3(TF)  
IF (CNRE<4.5) 4, 4, 5  
4 H1411=.664*F4(TF)*CNRE**.5*F5(TF)**.33/(2.*PI*G(10))  
GO TO 290  
5 H1411=.036*F4(TF)*CNRE**.8*F5(TF)**.33/(2.*PI*G(10))  
290 CONTINUE  

C

C CHECKING IMPORTANCE OF NATURAL CONVECTION

CNGR14=78.4*(PI*G(10)) **3*ABS (X(7) - X(4)) *  
C

AUX14=CNGR14*F5(TF)  
IF (AUX14<5.94E8) 275, 275, 275  
000 18  110  LINE 1909  
000 18  120  LINE 1910  
000 18  130  LINE 1911  
000 18  140  LINE 1912  
000 18  150  LINE 1913  
000 18  160  LINE 1914  
000 18  170  LINE 1915  
000 18  180  LINE 1916  
000 18  190  LINE 1917  
000 18  200  LINE 1918  
000 18  210  LINE 1919  
000 18  220  LINE 1920  
000 18  230  LINE 1921  
000 18  240  LINE 1922  
000 18  250  LINE 1923  
000 18  260  LINE 1924  
000 18  270  LINE 1925  
000 18  280  LINE 1926  
000 18  290  LINE 1927  
000 18  300  LINE 1928  
000 18  310  LINE 1929  
000 18  320  LINE 1930  
000 18  330  LINE 1931  
000 18  340  LINE 1932  
000 18  350  LINE 1933  
000 18  360  LINE 1934  
000 18  370  LINE 1935  
000 18  380  LINE 1936  
000 18  390  LINE 1937  
000 18  400  LINE 1938  
000 18  410  LINE 1939  
000 18  420  LINE 1940  
000 18  430  LINE 1941  
000 18  440  LINE 1942  
000 18  450  LINE 1943  
000 18  460  LINE 1944
276 HNAT 14 = .052 * F4 (TF) * AUX 14 ** (1. / 3.) / G(10)
    GO TO 277
275 HNAT 14 = .28 * F4 (TF) * AUX 14 ** .25 / G(10)
277 IF (HNAT 14 - H1411) 278, 278, 279
279 H1411 = HNAT 14
278 CONTINUE
280 CONTINUE

C CONVECTION 16-11
    H1611 = H1411
C CONVECTION 14-15 (LIKE 9-15)
    IF (I4. GE. 2) GO TO 610
    IF (I4. EQ. 1) H1415 = NEXT
    IF (I4. EQ. 1) GO TO 12
    TF = .5 * (TAMB * X (7))
    CGN = 7.4 * G (10) ** 3 * ABS (X (7) - TAMB)
    C = *(F2 (TF) / F3 (TF)) ** 2 / (TK + TAMB)
    AUX3 = CGN * F5 (TF)
    IF (AUX3 = 3.07E8) 10, 10, 11
10  H1415 = .265 * F4 (TF) * AUX3 ** .25 / G (10)
    GO TO 12
11  H1415 = .052 * F4 (TF) * AUX3 ** (1. / 3.) / G (10)
    GO TO 12
C ALTERNATIVE EXPRESSION
610 H1415 = 4.22 * ABS (X (7) - TAMB) ** .25 / SQRT (2. * G (10))
12 CONTINUE
    IF (I4. NE. 0) AUX3 = 0.
C STORING THE JUST OBTAINED FILM COEFFICIENTS IN H(I)
    H (1) = H915
    H (2) = H2EX11
    H (3) = H1211
    H (4) = H1711
    H (5) = H1411
    H (6) = H1611
    H (7) = H1415
    H (8) = H18
    H (9) = H319
\[
\begin{align*}
H(10) &= H10 \\
710 \text{ CONTINUE} \\
\text{C CALCULATION OF THE COEFFICIENTS K(I,J) RELATED TO CONVECTION} \\
\text{C WINDAGE LOSSES IN THE AIR GAP} \\
\text{C CL(8) IS CALCULATED. IF TOTAL WINDAGE LOSSES ARE ESTIMATED} \\
\text{C THE REMAINDER IS ASSIGNED TO THE OVERHANG (NOT IMPLEMENTED)} \\
H10 &= H(10) \\
U10 &= 0.5 * H(10) \\
\text{IF (I3.EQ.1) QTW = CL(8)} \\
\text{IF (I3.EQ.1) GO TO 420} \\
QTW &= CP * PI * G(6)^*4 * (G(4) + 2. * G(1)) * F2(TF) * W^**3 \\
CL(8) &= QTW \\
420 \text{ CONTINUE} \\
\text{C AIR GAP} \\
AUX &= PI * G(6) * G(4) \\
TF &= T10 \\
U10 &= 0.5 * H(10) \\
\text{IF (I4.EQ.2) GO TO 21} \\
20 \text{ K(1,5) = -0.5 * QTW} \\
K(1,6) &= 2. * AUX * U10 \\
K(1,7) &= -K(1,6) \\
K(1,8) &= 0. \\
T19 &= T10 \\
\text{C T19 HAS NO MEANING IF NO RADIAL DUCTS} \\
\text{GO TO 22} \\
21 \text{ CP = F1(TF)} \\
R0 &= F2(TF) \\
A1 &= (2.64 * PI * W^**2 * G(6) * 3 * U10 - PI * CP * W^**3 \\
* G(6) * 4 * CP * R0) / (.5 * CP * 2 * W1 * R0) \\
B &= 12. * PI * U10 * G(6) / (.5 * CP * W1 * R0) \\
\text{C THIS EXPRESSION & THE ONE FOR THE RADIAL DUCT MAY UNDERFLOW} \\
\text{C FOR CERTAIN VALUES OF THE PARAMETERS. SAVEGARDS SHOULD BE} \\
\text{C DESIGNED.} \\
Y1 &= W1 * CP * R0 * (1. - \exp(-.5 * B * G(4))) \\
K (1,5) &= -0.5 * QTW - 0.5 * A1 * Y1 / Y \\
Y2 &= 2. * PI * G(6) * G(4) * U10 \\
\end{align*}
\]
K(1,6) = Y2 + .25*Y1
K(1,7) = -Y2 + .25*Y1
K(1,8) = -.5*Y1
Y3 = -5*(X(1) + X(2))
Y4 = EXP (-5*B*G(4))
TEMP(7) = Y3 - (Y3 - X(4)) * Y4 - A1* (1. - Y4) / B

CONTINUE

C AXIAL AND RADIAL DUCTS
C AXIAL DUCT LENGTH MAY BE LONGER THAN CL/2
IF (I4 .EQ. 2) GO TO 26
K(1,9) = 0.
K(1,10) = 0.
K(1,11) = 0.
K(1,12) = 0.
K(1,13) = 0.
K(2,10) = 0.
GO TO 720

26 A318 = 2.*PI*G(4)*D(12)
A918 = 2.*PI*G(4)*D(14)
H18 = H(8)
U18 = H18 + 5*(A318 + A918)
CP = F1(T18)
RO = F2(T18)
Y1 = 2.*U18
Y2 = Y1 / (w*CP*RO)
Y3 = w*CP*RO* (1. - EXP (-Y2))
K(1,9) = .25* (Y1 + Y3)
K(1,10) = .25* (-Y1 + Y3)
K(1,11) = -.5*Y3
Y4 = 5*(X(1) + X(3))
TEMP(9) = Y4 - (Y4 - X(4)) * EXP (-Y2)

C
H319 = H(9)
U19 = H319*S(11)
C
RO = F2(T19)
CP=F1(T19)
Y1=U19/(W1*CP*RO)
Y2=W1*CP*3.0*(1.-EXP(-Y1))
R03=G(4)/{TC(5)*S(11)}
Y3=8.*Y2/(8. + Y2*R03)
K(1,12)=1.5*Y3
K(1,13)=-.5*Y3

C NON CONSISTENT NOTATION. K(2,10) SHOULD BE K(1,17)
K(2,10)=Y3*TEMP(7)
C3T19=K(1,12)*X(1)*K(1,13)*X(8)+K(2,10)
TEMP(11)=-R03*C3T19/8. +1.5*X(1)-.5*X(8)
TEMP(11)=ABS(TEMP(11))
TEMP(10)=TEMP(11)-(TEMP(11)-TEMP(7))*EXP(-Y1)

720 CONTINUE
C CONVECTION FOR THE MOTOR SIDE OF THE AIR GAP
K(2,4)=K(1,5)
K(2,5)=K(1,6)-4.*PI*G(6)*G(4)*U10
K(2,6)=K(1,7)*4.*PI*G(6)*G(4)*U10
K(2,7)=K(1,8)
C
C CONVECTION 9-18
K(3,3)=K(1,10)
K(3,4)=K(1,9)
K(3,5)=K(1,11)
C
C CONVECTION FRAME-AMBIENT
C ONLY FINS IF ENCLOSED MACHINE
H915=H(1)
K(3,2)=H915*S(6)
IF(I4.LE.1) K(3,2)=H915*S(6)*CFINS
K(3,9)=K(3,2)*TAMB
C CONVECTION 11-EXT2
H2EX11=H(2)
A11EX2=2.*G(23)*G(41)*ABS(G(29)+G(9)-G(8))
K(4,16)=H2EX11*A11EX2
C CONVECTION 12-11
A1211=2.*PI*G(5)*ABS(G(3)-G(31))-2.*G(1))
A1711=2.*PI*ABS(G(6)**2-G(5)**2)
H1711=H(4)
H1211=H(3)
U1211=U10
K(4,1)=8.*PI*G(6)*G(1)*U1211+H1211*A1211+H1711*A1711
C CONVECTION 14-11
H1411=H(5)
K(4,2)=H1411*S(9)
C COOLANT INFLOW
K(4,3)=F1(-.5*(X(4)+TAMB))*W0*F2(-.5*(X(4)+TAMB))
K(4,5)=K(4,3)*TAMB
C CONVECTION 16-11
H1611=H(6)
K(4,4)=H1611*S(11)
C OTHERS
K(5,5)=K(4,1)
K(7,2)=K(4,2)
C CONVECTION END SHIELD-AMBIENT
C ONLY PINS IF ENCLOSED MACHINE
H1415=H(7)
K(7,3)=H1415*S(8)
IF(I4.LE.1) K(7,3)=H1415*S(8)*CPINS
K(7,4)=K(7,3)*TAMB
C OTHERS
K(8,3)=K(4,4)
K(9,3)=K(4,16)
C WINDAGE LOSSES IN THE OVERHANG REGION
C ROTOR ENDS CONTRIBUTION
IF(I3.EQ.1) GO TO 730
TF=.5*(X(4)+X(5))
RO=F2(TF)
CMU=F3(TF)
CNR17=W*G (6) **2*RO/CMU
IF (CNR17-5.553E4) 800,800,810
800 CM17=1.935/CNR17**.5
GO TO 840
810 IF (CNR17-1.E6) 820,820,830
820 CM17=.073/CNR17**.2
GO TO 840
830 CM17=-.491/ABS(ALOG10(CNR17))**2.58
840 CL(9)=CM17*RO*W**3*G (6)**5
C SHAFT CONTRIBUTION
CNR12=RO*W*G (5)**2/CMU
IF (CNR12-30.) 850,850,860
850 CF12=4./CNR12
GO TO 890
860 DUMMY=1.
IF (CNR12-2000.) 870,870,880
870 CF12=10.**(-.0762-.5483*ALOG10(CNR12))
GO TO 890
880 CF12=10.**(-1.1072-.236*ALOG10(CNR12))
890 CL(9)=CL(9)+CF12*PI*G (5)**4*ABS(G(3)-G(31)-2.*G(1))*
C RO*W**3
730 CONTINUE
30 CONTINUE
RETURN
C END
C
C**************************************************************************
C
C SUBROUTINE FRICTI (U,G,CMU,RO,W,CF,IO)
IMPLICIT REAL(K)
CP1 = .476*(CMU/(RO*W))**.5/(G**.25*R**.75)
ESTCP2 = CF1

C THIS ESTIMATE CAN BE IMPROVED WITH EXPERIENCE
CP2 = ESTCP2
N = 1
C1 = 1.18*(R+G)/(R+ .5*G)
C2 = -2.*ALOG(2.83*CMU*((R+G)/
C    (RO*W*G*R**2)) + 17.16
10 CONTINUE
FUNC = ALOG(CF2) - C1/CF2**.5 + C2
DFUNC = 1./CF2**.5*C1/CF2**1.5
CP2NEW = CF2 - FUNC/DFUNC

C CORRECTING THE EVENTUAL NEGATIVE ROOT
IF(CP2NEW.LE.0.) CP2NEW = CP2/2.
IF(N-20) 1, 2, 2
1 IF(ABS(CP2NEW-CF2) - 1.E-6) 3, 4, 4
4 CP2 = CP2NEW
   N = N + 1
   GO TO 10
2 WRITE (6, 100)
100 FORMAT (19H WARNING IN FRIC TI)
3 IF(DF1-CF2) 5, 5, 6
5 CF = CF2
20 GO TO 7
6 CF = CF1
7 RETURN
END

C
C ************************************************************
C
C SUBROUTINE COEFFI
IMPLICIT REAL(K)
COMMON
C /FIRST/X,TC,NTOP,XINC,H,ACCEL,CFINS,HEXT
C /SECOND/I0,I1,I2,I3,I4,I5,ILAST,K,A,T10,T18,T19,D,S
C /SIXTH/IC,PR,G,CL,TEMP
DIMENSION
C X(13),TC(11),H(10)
C K(13,16),A(13,14),D(20),S(15)
C IC(10),PR(10),G(75),CL(10),TEMP(11)

C TAMB=PR(6)
W=PR(1)
W0=PR(5)

C CALCULATION OF THE COEFFICIENTS A(I,J) FOR THE
C NEWTON-RAPHSON EQUATIONS

TK=273.16
C11=K(1,14)+K(1,15)
D11=K(1,1)+K(1,3)*K(1,4)+K(1,6)+K(1,9)+K(1,12)
A(1,1)=4.*C11*(X(1)+TK)**3+D11
A(1,2)=4.*K(1,14)*(X(2)+TK)**3+K(1,7)
D13=-K(1,13)+K(1,10)
A(1,3)=-4.*K(1,15)*(X(3)+TK)**3+D13
D14=K(1,8)+K(1,11)
A(1,4)=D14
D18=-K(1,4)+K(1,13)
A(1,8)=D18
A(1,9)=K(1,2)
E1=K(1,16)+K(1,5)-CL(3)-CL(5)+K(2,10)
F1=C11*(X(1)+TK)**4-K(1,14)*(X(2)+TK)**4
C -K(1,15)*(X(3)+TK)**4+D11*X(1)+K(1,7)*X(2)+
C D13*X(3)+D14*X(4)+D16*X(5)+K(1,2)*X(9)+E1
A(1,14)=-F1

C A(2,1)=-4.*K(2,8)*(X(1)+TK)**3+K(2,5)
D22=K(2,1)+K(2,3)+K(2,6)
A(2,2)=4.*K(2,8)*(X(2)+TK)**3+D22
A(2,4)=K(2,7)
D25=K(2,2)-K(2,3)
E2 = K(2, 16) + K(2, 4) - CL(4) - CL(6)
F2 = -K(2, 8) * (x(1) + TK) ** 4 + K(2, 8) * (x(2) + TK) ** 4
C = +K(2, 5) * x(1) + D22 * x(2) + K(2, 7) * x(4) + D25 * x(5) + E2
A(2, 14) = -F2

C
D31 = -K(3, 16) + K(3, 3)
A(3, 1) = -4 * K(3, 4) * (x(1) + TK) ** 3 + D31
C33 = K(3, 6) + K(3, 7)
D33 = K(3, 16) + K(3, 1) + K(3, 2) + K(3, 4) + K(3, 10)
A(3, 3) = 4 * C33 * (x(3) + TK) ** 3 + D33
A(3, 4) = K(3, 5)
A(3, 7) = -K(3, 10)
A(3, 8) = -K(3, 1)
F3 = -K(3, 6) * (x(1) + TK) ** 4 + C 33 * (x(3) + TK) ** 4
C = +D31 * x(1) + D33 * x(3) + K(3, 5) * x(4) - K(3, 1) * x(8) + E3
C = -K(3, 10) * x(7)
A(3, 14) = -F3

C
D44 = K(4, 16) + K(4, 1) + K(4, 2) + K(4, 3) + K(4, 4)
CIF I1 = 1 ALL THE COEFFICIENTS A(4, 1) ARE 0. SO THE MATRIX A(I, J) C BECOMES SINGULAR. TRICK: FORCING A(4, 4) = 1 MAKES INCR X(4) = 0.
C & PREVENTS A(I, J) FROM BEING SINGULAR.
A(4, 4) = D44
A(4, 5) = -K(4, 1)
A(4, 7) = -K(4, 2)
A(4, 8) = -K(4, 3)
A(4, 9) = -K(4, 16)
A4 = -K(4, 5) - CL(9)
F4 = D44 * x(4) - K(4, 1) * x(5) - K(4, 2) * x(7)
C = -K(4, 4) * x(8) - K(4, 16) * x(9) + E4
A(4, 14) = -F4
IF(I1, EQ, 1) A(4, 4) = 1.

C
D52 = K(5, 1) - K(5, 3)
D55 = K(5, 2) + K(5, 3) + K(5, 4) + K(5, 5)
C
A (6, 5) = K (6, 16)
D66 = K (6, 16) + K (6, 1) + K (6, 2)
E6 = -K (6, 3) - CL (7)
A (6, 6) = D66
A (6, 7) = K (6, 1)
P6 = -K (6, 16) * X (5) + D66 * X (6) - K (6, 1) * X (7) + E6
A (6, 14) = -P6

C
C77 = K (7, 5) + K (7, 7)
D77 = K (7, 16) + K (7, 1) + K (7, 2) + K (7, 3) + K (7, 9)
E7 = -K (7, 4) - K (7, 6)
A (7, 3) = -K (7, 9)
A (7, 4) = -K (7, 2)
A (7, 6) = -K (7, 16)
A (7, 7) = 4 * C77 * (X (7) + TK) **3 + D77
A (7, 8) = -K (7, 1)
A (7, 12) = K (7, 8)
F7 = C77 * (X (7) + TK) **4 - K (7, 2) * X (4) - K (7, 16) * X (6)
C + D77 * X (7) - K (7, 1) * X (8) + K (7, 8) * X (12) + E7
C - K (7, 9) * X (3)
A (7, 14) = -P7

C
D88 = K (8, 16) + K (8, 1) + K (8, 2) + K (8, 3)
A (8, 1) = -K (8, 16)
A (8, 3) = -K (8, 1)
A (8, 4) = -K (8, 3)
A (8, 7) = -K (8, 2)
A (8, 8) = D88
A (8, 10) = K (8, 5)
A (8, 12) = K (8, 6)
A (8, 13) = K (8, 4)
F8 = -K (8, 16) * X (1) - K (8, 1) * X (3) - K (8, 3) * X (4) - K (8, 2) * X (7)
C + D88 * X (8) + K (8, 5) * X (10) + K (8, 6) * X (12) + K (8, 4) * X (13)
A (8, 14) = -F8

C
D99 = K (9, 1) * K (9, 3)
A (9, 1) = K (9, 2)
A (9, 4) = -K (9, 3)
A (9, 9) = 4. * K (9, 4) * (X (9) + TK) ** 3 + D99
A (9, 10) = K (9, 5)
F9 = K (9, 4) * (X (9) + TK) ** 4 + K (9, 2) * X (1) - K (9, 3) * X (4)
C + D99 * X (9) + K (9, 5) * X (10) + K (9, 16)
A (9, 14) = -F9

C
A (10, 9) = 4. * K (10, 1) * (X (9) + TK) ** 3
A (10, 10) = K (10, 2)
A (10, 11) = K (10, 3)
A (10, 12) = K (10, 4)
A (10, 13) = K (10, 5)
F10 = K (10, 1) * (X (9) + TK) ** 4 + K (10, 2) * X (10)
C + K (10, 3) * X (11) + K (10, 4) * X (12) + K (10, 5) * X (13)
A (10, 14) = -F10

C
A (11, 5) = 4. * K (11, 1) * (X (5) + TK) ** 3
A (11, 10) = K (11, 2)
A (11, 11) = K (11, 3)
A (11, 12) = K (11, 4)
F11 = K (11, 1) * (X (5) + TK) ** 4 + K (11, 2) * X (10)
C + K (11, 3) * X (11) + K (11, 4) * X (12)
A (11, 14) = -F11
A (12, 7) = 4. * K(12, 16) * (X(7) + TK) ** 3
A (12, 10) = K(12, 1)
A (12, 11) = K(12, 2)
A (12, 12) = K(12, 3)
A (12, 13) = K(12, 4)
P12 = K(12, 16) * (X(7) + TK) ** 4 + K(12, 1) * X(10)
C = + K(12, 2) * X(11) + K(12, 3) * X(12) + K(12, 4) * X(13)
A (12, 14) = -P12

A (13, 8) = 4. * K(13, 16) * (X(8) + TK) ** 3
A (13, 10) = K(13, 1)
A (13, 12) = K(13, 2)
A (13, 13) = K(13, 3)
P13 = K(13, 16) * (X(8) + TK) ** 4 + K(13, 1) * X(10)
C = + K(13, 2) * X(12) + K(13, 3) * X(13)
A (13, 14) = -P13
RETURN
END

*********************************************************************************************

SUBROUTINE SOLVER (A, L, ISING, IROW, ICOL)
DIMENSION A (IROW, ICOL)
ISING = 0
M = L
N = M + 1
I1 = 1
I3 = 11
SUM = ABS (A(I1, I1))
DO 3 I = I1, M
   IF (SUM - ABS (A(I, I1))) 2, 3, 3
2 I3 = I
   SUM = ABS (A(I, I1))
3 CONTINUE
IF (I3-I1) = 4, 6, 4
4 DO 5 J=1,N
   SUM=-A(I1,J)
   A(I1,J)=A(I3,J)
   5 A(I3,J)=SUM
6 I3=I1+1
   IF (A(I1,I1)) = 7, 15, 7
7 DO 8 I=I3,M
8 A(I,I1)=A(I,I1)/A(I1,I1)
9 J2=I1-1
   I3=I1+1
   IF (J2) = 10, 12, 10
10 DO 11 J=13,N
    DO 11 J=13,N
   11 A(I1,J)=A(I1,J)-A(I1,I)*A(I,J)
      IF (I1-M) = 12, 14, 12
12 J2=I1
    I1=I1+1
    DO 13 I=I1,M
    DO 13 J=1,J2
13 A(I1,I1)=A(I1,I1)-A(I,J)*A(J,I1)
      IF (I1-M) = 1, 9, 1
14 IF (A(M,M)) = 16, 15, 16
15 GoTo 19
16 DO 18 I=1,M
    J2=M-I
    I3=J2+1
    A(I3,N)=A(I3,N)/A(I3,I3)
    IF (J2) = 17, 19, 17
17 DO 18 J=1,J2
18 A(J,N)=A(J,N)-A(I3,N)*A(J,I3)
19 RETURN
END
APPENDIX B

CALCULATION OF THE STRESS DISTRIBUTION IN ROTATING CIRCULAR DISKS

Elastic Conditions

The configuration of concern is shown in fig. B.1, and the basic assumptions are presented in section 3.5.3. The resultant problem has been studied extensively\textsuperscript{48,51,54,55} and here only the main steps will be described and the final working eqs. will be given.

![Figure B.1 Rotor core infinitesimal element](image)

The eq. of equilibrium in radial direction for an element of the disk (fig. B.1) in polar coordinates is:

\[
r \cdot \frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} + \sigma_r - \sigma_\theta + R = 0,
\]

which because of symmetry conditions reduces to:

\[
r \cdot \frac{\partial \sigma_r}{\partial r} + \sigma_r - \sigma_\theta + R = 0,
\]

where \(\sigma_r(r)\) and \(\sigma_\theta(\theta)\) are unknowns and \(R\) is the body force in the radial direction:
\[ R = \rho \cdot \omega^2 \cdot r^2, \quad (B.3) \]

where \( \rho \) is the mass density.

By means of the strain-stress relations, the eq. (B.2) can be transformed into a differential equation with the radial shift \( u \) as the only unknown. Solving for \( u \) and writing \( \sigma_r \) and \( \sigma_\theta \) in terms of \( u \), the following expressions for the stresses are obtained:\(^{51}\):

\[ \sigma_r(r) = \frac{-p_i \cdot a^2 \cdot (b^2 - r^2) - p_0 \cdot b^2 \cdot (r^2 - a^2)}{r^2 \cdot (b^2 - a^2)} + \rho \cdot \omega^2 \cdot \frac{3+\mu}{8} \cdot \left( \frac{a^2 + b^2 - \frac{b^2 \cdot a^2}{r^2} - r^2}{r^2} \right), \quad (B.4) \]

\[ \sigma_t(r) = \frac{p_i \cdot a^2 \cdot (b^2 + r^2) - p_0 \cdot b^2 \cdot (a^2 + r^2)}{r^2 \cdot (b^2 - a^2)} + \rho \cdot \omega^2 \cdot \frac{3+\mu}{8} \cdot \left( \frac{a^2 + b^2 - \frac{a^2 \cdot b^2}{r^2} - 1+3\mu}{3+\mu} \cdot r^2 \right), \quad (B.5) \]

and the maximum shear stress:

\[ \tau(r) = 0.5(\sigma_t - \sigma_r) = \frac{p_i \cdot a^2 \cdot b^2 - p_0 \cdot a^2 \cdot b^2}{r^2 \cdot (b^2 - a^2)} + \rho \cdot \omega^2 \cdot \frac{3+\mu}{8} \cdot \left[ \frac{a^2 \cdot b^2}{r^2} \cdot (3+\mu) + (1-\mu) \cdot r^2 \right], \quad (B.6) \]

**Plastic conditions**

So far it has been assumed that the material obeys Hooke's law and is homogeneous. Such assumptions, however, are no longer valid once the yield point has been reached. The study of the stress distribution after yielding has occurred comes under the heading of the theory of plasticity\(^{48,54}\). Exact analysis in the theory of plasticity are very complex;
here a simple solution to the concerned configuration will be presented, where standard basic assumptions will be used.

To exceed the yield point stress and work in the plastic range is sometimes admissible, as will be shown later. This has the obvious advantage of allowing greater loads to be applied, which means higher rotational speed. However this usually implies permanent damage to the magnetic properties of the material, and a compromise must be reached.

The mechanical effect of stressing a component beyond its yield point stress is to cause partial or complete plastic penetration. The resulting stress distribution for both cases will be studied next. When the applied load is sufficient to cause yielding, some permanent set remains in the material when load is removed; however, the material will withstand subsequent loads equal to, or almost equal to, the load initially applied to cause yielding, without suffering any further permanent deformation.

The effect of stressing beyond the elastic limit and then removing the applied load not only causes some permanent set, but also gives rise to residual stresses in the component. This is because that region which has suffered some permanent set prevents the elastic region from returning to its initial condition on removal of the applied load, and so some stresses are locked in. This effect presents the following two advantages:

a) If the load that stresses a component causing plastic penetration is removed and then applied again, the new stress distribution will be more favorable for the component and likely the yield point will not be reached again.
b) It is possible to overstress a component to increase its yield point by imposing residual stresses. It enables the component to be stressed to a higher value (that before prestressing would lead to failure) while still behaving elastically. For instance, the rotor could be subjected to a high over speed test, such that the whole rotor yields under centrifugal stresses. The rotor should likely be machined down to the required dimensions, although this could damage the magnetic properties. If the test speed is higher than the maximum speed attainable in practice, it is reasonably certain that the rotor will not suffer any permanent deformation in service.

Figure B.2  Strain-stress idealized characteristics for different types of materials.

a. Perfectly elastic body
b. Rigid body, perfectly plastic
c. Elastic body, perfectly plastic
In section 3.5.3 it was determined under which conditions the maximum shear stress in the rotor laminations is reached. The internal pressure $P_i$ was considered to be zero, which will be also assumed for the rest of this section. As the load (rotational speed) is increased still further, more material will yield and plastic penetration will occur. Assuming an elastic and ideally plastic material (see fig. B.2), the maximum stress can never exceed the yield point stress, so that, assuming the shear stress theory of failure, for any point in the plastic region:

$$\sigma_{\text{max}} = \sigma_t - \sigma_r,$$

and substituting this expression in eq. B.2 the differential equation for points in the plastic region is obtained:

$$\frac{\sigma_r}{dr} = \frac{\sigma_{\text{max}}}{r} - \rho \cdot \omega^2 \cdot r,$$

which when integrated and the boundary conditions of zero internal pressure applied, gives the radial stress at any point in the plastic region:
\[ \sigma_r = \sigma_{\text{max}} \cdot \ln \frac{r}{a} - \frac{\rho \cdot \omega^2}{2} \cdot (r^2 - a^2) \]  

(B.9)

If the whole disk is working in the plastic region, this eq. holds for \( r = b \), resulting in the condition for complete yielding of the disk:

\[ -p_o = \sigma_{\text{max}} \cdot \ln \frac{b}{a} - \frac{\rho \cdot \omega^2}{2} \cdot (b^2 - a^2) \]  

(B.10)

with the following distribution of tangential and radial stresses:

\[ \sigma_r(r) = \sigma_{\text{max}} \cdot \ln \frac{r}{a} - \frac{\rho \cdot \omega^2}{2} \cdot (r^2 - a^2) \]  

(B.11)

\[ \sigma_t(r) = \sigma_r(r) + \sigma_{\text{max}} \]  

(B.12)

which has been represented in fig. B.3.b as compared to the elastic case, eqs. B.4 and 5 with \( P_i = 0 \), in fig. B.3.a.

Fig. B.3. Rotor core stress distribution for totally elastic (a) and plastic (b) conditions.
Eq. B.6 with $\tau(r=a) = 0.5\sigma_{\text{max}}$ gives the conditions that just cause yielding at the bore, whereas eq. B.10 holds for complete yielding of the disk. In between these two limiting conditions of the fully elastic and fully plastic state, the disk will be in a state of elasto-plasticity, i.e. the yield or plastic penetration will have spread to some radius $c$ between $a$ and $b$. Now follows the calculation of the stress distribution corresponding to this elasto-plastic state.

The radial pressure $\sigma_r(r=c)$ will be called $P_c$. Eqs. B.4 and 5 hold for the elastic region $c \leq r \leq b$, if $P_c$ is used instead of $P_1$. Eqs. B.7 and 9 must be applied in the plastic region $a \leq r \leq c$. The continuity of the stresses $\sigma_t$ and $\sigma_r$ provides equations to calculate $c$ and $P_c$ for a given value of $\omega$. The following equation relates the main parameters for the elasto-plastic state:

$$\omega^2 \cdot \left\{ \frac{\rho}{2}(c^2-a^2) - \frac{P_0}{\omega^2} + \left( \frac{c^2}{2b^2} + 0.5 \right) \cdot \frac{\rho}{4} \cdot \left[ b^2(3+\mu) = c^2(1-\mu) \right] \right\} =$$

$$= \sigma_{\text{max}} \cdot \left[ \ln \frac{c}{a} + (0.5 - \frac{c^2}{2b^2}) \right], \quad \text{(B.13)}$$
APPENDIX C

CONVECTIVE HEAT TRANSFER FOR AN ENCLOSED ROTATING CYLINDRICAL ROTOR,
INCLUDING THE EFFECT OF KINETIC ENERGY RECOVERY
AND WINDAGE ON SURFACE TEMPERATURE RISE

Norris' report\textsuperscript{85}, as presented in the Data Books of G.E.\textsuperscript{68} is the
main source used to write this appendix, especially the sections
G.511.3, 4, and 7 of reference 68.

Relation of new symbols used in this appendix

\( \theta_i \) and \( \theta_o \) = temperatures of the rotor and stator surfaces, respectively, at the location considered. (\(^{\circ}\text{C}\))

\( \theta_m \) = mean value (mixed-mean) for the temperature of the fluid
at the streamwise location considered. (\(^{\circ}\text{C}\))

\( \omega \) = rotational speed of the rotor. (rad/sec)

\( r_i, r_o \) = radii of the rotor and stator surfaces of the enclosure,
respectively: R and R+G, (m.)

\( V_m \) = mean velocity (averaged over the velocity profile) of the
fluid. (m/sec)

\( C_p \) = specific heat of the fluid at constant pressure
(joule/\(^{\circ}\text{C} \times \text{kg}\))

\( q_i, q_o \) = total heat transferred from the rotor and stator surfaces,
respectively; positive when heat flows into the fluid
(watts).

\( A_i, A_o \) = heat transfer surface areas, for rotor and stator respec-
tively (m\(^2\)).
\( q_i", q_o" = \frac{q_i}{A_i}, \frac{q_o}{A_o} \), respectively.

\( F_i, F_o \) = adiabatic - depression factors for rotor and stator, respectively.

\( \eta_{r,i}, \eta_{r,o} \) = recovery factors for rotor and stator, respectively.

\( h_{i,i}, h_{o,o} \) = heat transfer coefficients for rotor and stator surfaces, respectively, when the other surface is adiabatic (insulated). \((\text{watts/(m}^2.\text{°C)})\).

\( U \) = overall stator-rotor heat transfer coefficient with no axial fluid flow conditions. \((\text{watts/(m}^2.\text{°C)})\).

\( V_z \) = fluid velocity in the axial direction. \((\text{m/sec})\).

\( Q \) = volumetric flow rate of the fluid in axial direction \((\text{m}^3/\text{sec})\).

\( W \) = weight flow rate of the fluid in axial direction \((\text{Kg/sec})\).

\( V_m \) = mean total velocity of the fluid (averaged over the velocity profile), \((\text{m/sec})\).

\( V_{rel,i}, V_{rel,o} \) = velocity of the rotor and stator surfaces relative to the fluid velocity, \((\text{m/sec})\).

\( q_{tw}'' \) = total (rotor and stator) windage losses per unit of surface area of the rotor, \((\text{watts/m}^2)\).

\( C_f \) = friction factor (dimensionless).

\( N_TA \) = Taylor number (dimensionless).
Working equations

The equations that follow account for the effects of convective heat transfer in the gap between a stationary cylinder (stator) and a coaxial rotating cylinder (rotor). The surfaces of both cylinders are smooth enough so no effects of their roughnesses are included. The equations account for the possibility of axial flow. Equations will only be given for the expected ranges of the parameters in the particular application of this work; therefore it will be assumed that:

- The gap is smaller than the radius:
  \[ g \ll R \], \text{(C.1)}

- The azimuthal velocity of the fluid is much bigger than its axial velocity:
  \[ W.R \gg V_z \], \text{(C.2)}

which is reasonable since high rotational speeds are expected, whereas the axial speed comes from the eventual utilization of forced cooling.

- Only turbulent regimes for the flow of the gas will be considered, due to the expected high rotational speeds. When necessary the status of the flow regime will be checked in order to use the right equations.

- Temperature rise due to the effect of kinetic energy recovery and windage must be included when high rotational speeds exist (say above 30 m/sec) and therefore the effect of viscous dissipation may be significant. Here these effects are included in the equations from the outset.
A remark should be made on the practical significance of the presence of appreciable axial flow and its repercussions on the equations. The function of the axial flow in electrical machinery is to absorb the heat generated in the rotor and stator, thereby keeping their temperature rises within the desired limits. Accordingly, the temperature rises of primary interest here usually are $\theta_i - \theta_m$ and $\theta_o - \theta_m$, the rises of the rotor and stator temperatures $\theta_i$ and $\theta_o$ above the mean fluid temperature $\theta_m$ (at some particular local axial location or for the axial average).

This case differs in this respect from the case of zero axial flow, since in the latter, the temperature rise of the rotor above the stator is, in general, the temperature rise of practical interest, and the fluid temperature requires no consideration except as the basis for properties.

Accordingly, when axial flow exists the heat transfer process of interest is usually not the rotor-to-stator one (given by the overall coefficient $U$), but the heat transfer from the rotor and stator to the fluid (which is described by means of the heat transfer coefficients $h_{i,i}$ and $h_{o,o}$ and the adiabatic-depression factors $F_i$ and $F_o$).
The general equations for the temperature rises are the following:

\[
\theta_i - \theta_m = \frac{\eta_{r,i} \times V_{rel,i}^2}{2 \times c_p} + \frac{q_i''}{h_{i,i}} - F_i \times \frac{q_o''}{h_{o,o}}, \quad (C.3)
\]

\[
\theta_o - \theta_m = \frac{\eta_{r,o} \times V_{rel,o}^2}{2 \times c_p} + \frac{q_o''}{h_{o,o}} - F_o \times \frac{q_i''}{h_{i,i}}, \quad (C.4)
\]

where the first term on the R.H.S. in both expressions is the temperature rise due to the kinetic energy recovery, the recovery factors \(\eta_{r,i}\) and \(\eta_{r,o}\) being the parameters that give the magnitude of this temperature rise in terms of the fluid flow conditions.

For turbulent flow in the considered configuration, the value of 0.88 for both \(\eta_{r,i}\) and \(\eta_{r,o}\) is recommended, which is close to the value of \((\mu_c p/k)^{1/3}\) suggested by theory. The recovery factor \(\eta_r\) is dimensionless.

The definitions of the "adiabatic-depression factors" \(F_i\) and \(F_o\) are:

\[
F_i = \frac{t_{m,o} - t_{i,o}}{t_{o,o} - t_{m,o}}, \quad (C.5)
\]

giving the depression of the adiabatic temperature of the rotor surface (now insulated) below the mixed mean fluid temperature, as a fraction of the temperature rise of the stator surface above the mixed mean fluid temperature. (The second subscript in the pair is used to indicate which surface is the sole source of heat, the other surface being adiabatic.)

In the same way:

\[
F_o = \frac{t_{m,i} - t_{o,i}}{t_{i,i} - t_{m,i}} \quad (C.6)
\]
Fig. C.2. Air gap fluid temperature rise.

The coefficients $F_i$ and $F_0$ are used to determine $h_{i,i}$ and $h_{o,o}$ from the geometric dimensions and the overall heat transfer coefficient for the gap, $U$.

The values of $F_i$ and $F_0$ are expected to range, with air or other gas in turbulent flow conditions, from 0 to around 0.2, the lower the value the more turbulent the flow. Unfortunately there is a lack of data in situations where the effect of rotation is not negligible, and the closest correlations are for stationary annular ducts, which give values for $F_i$ and $F_0$ of about 0.15 for $g \ll R$ and well developed turbulent flow. A conservative assumption is to consider $F_i, F_0 = 0.2$, which gives the highest expected value for the transfer coefficients $h_{i,i}$ and $h_{o,o}$. This is the assumption that will be adopted here.

The following expression, obtained for stationary annular ducts, gives the relation among the heat transfer coefficients $h_{i,i}, h_{o,o}$ and $U$: 
\[ h_{i,i} = \frac{1}{2} \frac{r_i + r_0}{r_i} \cdot U \cdot \left[ 1 + \frac{r_i}{r_0} \times \frac{h_{i,i}}{h_{o,o}} \times (1 + F_i) + F_i \right], \quad (C.7) \]

Correlations with stationary annular ducts show that \( h_{i,i} \approx h_{o,o} \) for \( \frac{r_0 - r_i}{r_i} = \frac{G}{R} \leq 0.3 \), which is widely surpassed in electrical machines. Therefore this equation becomes:

\[ h_{o,o} \approx h_{i,i} \approx 2 \times (1 + F) \times U \quad , \quad (C.8) \]

where the overall heat transfer coefficient \( U \) will be determined later.

The following equations concerning the velocities of the surfaces and the fluid are immediate:

\[ V_z = \frac{Q}{\pi x (r_0^2 - r_i^2)} = \frac{Q}{\pi x G \times (2R+G)} = \frac{W}{\pi x G \times (2R+G) \times \rho} \quad , \quad (C.9) \]

\[ V_m = \left[ V_z^2 + \left( \frac{\omega \cdot r_i}{2} \right)^2 \right]^{1/2} \quad , \quad (C.10) \]

\[ V_{rel, o} = V_m \quad , \quad (C.11) \]

\[ V_{rel, i} = \left| \omega \times r_i - V_m \right| = \omega \times r_i - V_m \approx \frac{\omega \times r_i}{2} \quad , \quad (C.12) \]

since usually \( V_z/(\omega \cdot r_i) \) is small enough to make this vector difference approximately equal to the scalar difference of its components.
After all this manipulation the initial eqs. C.3 and 4 become:

\[ \theta_i - \theta_m = 0.22 \cdot \frac{\omega x R^2}{C_p} + \frac{q_i'' - 0.2 x q_o''}{2.4 \times U}, \quad (C.13) \]

\[ \theta_o - \theta_m = 0.22 \cdot \frac{\omega x R^2}{C_p} + \frac{q_o'' - 0.2 x q_i''}{2.4 \times U}, \quad (C.14) \]

where

\[ q_i'' = \frac{q_i}{2 \times \pi x R x \ell}, \quad (C.15) \]

\[ q_o'' = \frac{q_o}{2 \times \pi x (R+G) x \ell}, \quad (C.16) \]

If there is axial fluid flow and with a stationary regime of flow and temperature, the fluid temperature must change in the axial direction according to the following straightforward equation:

\[ \frac{d\theta_m}{dz} = \frac{2 \times \pi x R}{C_p x W_{,\rho}} [q_i'' + \frac{R+G}{R} x q_o'' + q_{tw}''], \quad (C.17) \]

where

\[ q_{tw}'' = \frac{q_{tw}}{2 \times \pi x R x \ell} = \frac{1}{2 \times \pi x R x \ell} x C_f x \pi x R^4 x \ell x \rho x \omega^2 = \frac{C_f x (\omega x R)^3 x \rho}{2}, \quad (C.18) \]

and the friction factor \( C_f \) will be determined later.

The fluid temperature (as basis for evaluation of fluid properties) will be:

\[ \theta_f = \frac{\theta_i + \theta_o + 2 \theta_m}{4}. \]

which can be simplified for the particular case of no axial flow to:
\[ \theta_f = \frac{\theta_i + \theta_o}{2} \]

Calculation of the overall rotor-stator surface heat transfer coefficient \( U \).

The overall heat transfer coefficient \( U \) is defined as follows under conditions of no axial flow (which implies \( \frac{d\text{tm}}{dx} = 0 \) in practice, and \( q_i = -q_o = q \)):

\[ U = \frac{1}{2 \times \pi \times L \times r_m} \times \frac{\dot{q}}{\theta_i - \theta_o}, \quad \text{(C.19)} \]

where \( r_m \) is the logarithmic mean radius:

\[ r_m = \frac{r_o - r_i}{\ln \frac{r_o}{r_i}} \approx \frac{r_o + r_i}{2} = R + \frac{G}{2}, \quad \text{(C.20)} \]

where \( G \) is the air gap length.

The value of \( U \) can be obtained directly from the Nusselt number \( N_{NU} \). There are correlations available to obtain \( N_{NU} \), the different equations being dependent on the conditions of the flow, which are measured by the dimensionless Taylor number \( N_{TA} \):

\[ U = \frac{k \times N_{NU}}{G}, \quad \text{(C.21)} \]

\[ N_{TA} = \frac{\rho \times \omega \times R \times G}{\mu} \times \left( \frac{G}{R} \right)^{0.5}, \quad \text{(C.22)} \]

The Nusselt number \( N_{NU} \) must be evaluated from whichever of the following alternative formulas is pertinent to the range of \( N_{TA} \) values.
which covers the value found in eq. C.22 above:

- For \( N_{TA} < 41 \):
  \[
  N_{NU} = 1.00
  \]  
  \( \quad (C.23) \)

- For \( 41 < N_{TA} < 100 \) if \( N_{PR} \approx 0.7 \) (air in standard conditions)

and

- For \( N_{NU} > 1.00 \) if \( N_{PR} \neq 0.7 \):
  \[
  N_{NU} = 0.106 \times N_{TA}^{0.63} \times N_{PR}^{0.27}
  \]  
  \( \quad (C.24) \)

- For \( N_{TA} > 100 \) use the one of the two following expressions which gives the highest value for \( N_{NU} \):
  \[
  N_{NU} = 0.193 \times N_{TA}^{0.5} \times N_{PR}^{0.27}
  \]  
  \( \quad (C.25) \)

which is expected for values of \( N_{TA} \) closer to 100.

  \[
  N_{NU} = 0.44 \times C_f \times \left( \frac{R}{g} \right)^{0.5} \times N_{TA} \times N_{PR}^{\frac{1}{3}}
  \]  
  \( \quad (C.26) \)

which is expected for the turbulence dominant regime \( (N_{TA} >> 100) \).

The expressions to evaluate the friction factor \( C_f \) are also dependent on the value of \( N_{TA} \):

- For \( 0 < N_{TA} < 41 \) (laminar):
  \[
  C_f = 2 \times \left( \frac{G}{R} \right)^{0.5} \times \frac{1}{N_{TA}} \times \frac{(1 + \frac{G}{R})^2}{1 + 0.5 \times \frac{G}{R}}
  \]  
  \( \quad (C.27) \)

- For \( 41 < N_{TA} < 63 \) (transition):
  \[
  C_f = 2 \times N_{TA}^{0.5} \times \left( \frac{R}{G} \right)^{0.5}
  \]  
  \( \quad (C.28) \)
- For \( N_{TA} > 63 \) use the one of the two following expressions (the second is an implicit equation) which gives the highest value for \( C_f \). The first one is expected for lower values of \( N_{TA} \) and the second one for the turbulence dominant regime:

\[
C_f = 0.476 \times \left( \frac{G}{R} \right)^{0.5} \times \frac{1}{\sqrt[0.5]{N_{TA}}} ,
\]

\[
\ln C_f - 1.18 \times \frac{R+G}{R+0.5G} \times C_f^{-0.5} = 2 \times \ln[2.83 \times \frac{\mu x(R+G)}{\rho x w G x R}] - 17.16 ,
\]

(C.29)

(C.30)

The values of \( U \) and \( C_f \) as calculated in this section are applicable when axial flow exists, as long as \( V_z \ll \omega \times R \).

**Windage losses.**

Following is an estimation of the windage losses in the machine. The expressions presented in reference 68 for drag of rotating surfaces have been used. Three sources of windage losses have been considered in the machine.

a) The air gap region.

b) The rotor end.

c) The shaft.

The losses originated because of the eventual forced cooling (fan and pressure drops in ventilating ducts) have not been considered here.

**Windage losses in the air gap.**

The configuration is modeled as a pair of smooth, coaxial cylinders, the inner cylinder having a rotational speed \( \omega \). The resultant windage
is given by the following expression:

\[ P_w = C_f \cdot \pi \cdot R^4 \cdot (CL + 2 \cdot OR) \cdot \rho \cdot \omega^3, \quad (C.31) \]

where \( C_f \) is the friction factor, which was determined previously.

This relation excludes the contribution of the ends, which are considered separately and modeled as unenclosed flat rotating disks.

**Windage losses in the rotor ends.**

It has been assumed that the rotor windings are enclosed in some type of end bell in order to diminish the windage and prevent damage due to centrifugal forces. It is therefore reasonable to model the rotor ends as flat rotating disks.

The expression for the windage losses originating at both ends of the machine is therefore:

\[ P_w = C_m \cdot \rho \cdot \omega^3 \cdot R^5, \quad (C.32) \]

where the coefficient \( C_m \) depends on the flow regime. The following equations are used to represent the available information, and prevent discontinuities in the analytical expression:

- For \( N_{RE} < 5.553 \times 10^4 \) (laminar regime):

  \[ C_{m_l} = 1.935 \times N_{RE}^{-0.5}, \quad (C.33) \]

- For \( 10^6 > N_{RE} > 5.553 \times 10^4 \) (turbulent regime):
\( C_{m1} = 0.073 \times N_{RE}^{-0.2} \), \hspace{1cm} (C.34)

- For \( N_{RE} > 10^6 \) (turbulent regime):

\[
C_{m1} = 0.491 \times \left[\log_{10} N_{RE}\right]^{-2.58} , \hspace{1cm} (C.35)
\]

The transition regime has been ignored, so the laminar and turbulent regimes have been joined without discontinuity.

The expression for the Reynolds number for a disk is:

\[
N_{RE} = \frac{\omega x R^2 \times \rho}{\mu} , \hspace{1cm} (C.36)
\]

Windage losses originating in the shaft.

The shaft is modeled as an unenclosed smooth cylinder. The expression for the windage power loss is (both sides):

\[
P_w = C_f \cdot \pi \cdot R SH^4 \cdot (LT - CL1 - 2.0R) \cdot \rho \cdot \omega^3 , \hspace{1cm} \text{shaft} \hspace{1cm} (C.37)
\]

where the friction coefficient \( C_f \) depends on the flow conditions. The experimental results presented in reference 12 can be roughly represented by the following analytical expression:

- For \( N_{RE} < 30 \):  
  \[
  C_f = \frac{4}{N_{RE}} , \hspace{1cm} (C.38)
  \]

- For \( 30 < N_{RE} < 2000 \):
\[ C_f = 10^{-(0.0762 + 0.5483 \log_{10} N_{RE})} \quad (C.39) \]

- For \( 2000 < N_{RE} < 10^6 \):

\[ C_f = 10^{-(1.1072 + 0.2360 \log_{10} N_{RE})} \quad (C.40) \]

where

\[ N_{RE} = \frac{\rho \cdot \omega \cdot RSH^2}{\mu} \quad (C.41) \]

Values of \( N_{RE} \) bigger than \( 10^6 \) cannot be reasonably expected.
D.1 Flowchart of MAIN program

START 'MAIN' PROGRAM

INITIALIZE INPUT DATA WITH DEFAULT VALUES
CALL SUBPROGRAM 'BLOCKDATA'

VARIABLES: TC, CFINS, SIGMAX, DENS, CMECH, VDUCT, CLOSS, FBH, BHO, BSAT, BMIN, FMAX, FMIN, EC, NSTAGE, X, NTOP, XINC, ACCELE, HEXT, EPS, ALPHA, BETA, MKSTEP, MKOBJ, EXPP, CNTRL, WF, IC, PR, CL, TEMP, P.

MUST ANY DEFAULT VALUE BE CHANGED?

YES

READ FROM TERMINAL (OPTIONAL)
IC, NSTAGE, MKSTEP, MKOBJ, NTOP, X, ALPHA, BETA, XINC, EXPP, ACCELE, CNTRL, WF, PR, P, H, CL.

CLEAR MATRICES G, CA, EP

IS THIS THE FIRST CASE?

YES

OPTIMIZATION PROCESS
CALL SUBROUTINE 'HOOKE'

RESET CONTROL PARAMETERS

NO
WRITE OPTIMAL DESIGN PARAMETERS
OBIMIN , P, CA, G, EP, TEMP, H, CL, DENS

MORE CASES?

YES

NO

STOP
START SUBROUTINE 'HOOKE'

DATA (FROM COMMON)
EPS, ALPHA, BETA, MKSTEP, IC, P, MKOBJ, NSTAGE

INITIALIZE CONTROL PARAMETERS
ISkip, KSTEP, KOBJ, KLOCAL, KPTTRN, OJBST = 0 ;
IDISC

CALCULATE OBJECTIVE FUNCTION
AT INITIAL BASE POINT P.
CALL SUBROUTINE 'OBJECT'
OBJ
KOBJ: KOBJ + 1
OBJBAS, OBJHD = OBJ
BASEN = P

IS KOBJ = 1 ?

READ FROM TERMINAL (OPTIONAL)
• VARIABLES NOT TO SEARCH OVER, SKIP
• NEW STEP SIZES, EPS
• VARIABLES TREATED AS DISCRETE, IDISC

ANY CHANGE IN IDISC?

ROUND-OFF P, EPS

READ FROM TERMINAL (OPTIONAL)
• NEW GUESS P

a b c d
2

- Set new base point
- Actualize guess for windings temperature

- Objbas: Objbst; Based: Basen; Basen: P
- Pattern move
- Check P(I) are within bounds
- Kptrn = Kptrn + 1
- Objhd = Obj

Exploratory move
Call subroutine 'local'

Is present function value below that at base point?

3

- Is Kstep > Mkstep or Kobj > Mkobj?

Write control parameters
Klocal, Kptrn, Kobj, Kstep

Read from terminal
Iend

Iend?

Read from terminal
Mkstep, Mkobj
NOTE: Fully detailed analysis step and intermediate printing are performed only when KOBJ=IC(9)=1 (first analysis step) and/or IC(5)=1 (just after a step size reduction) and/or IC(6)=1 (optimization has been stopped).

NOTE: The numbers 1, 2 and 3 in the flowchart above correspond to the same numbers in the flowchart of reference 5.
D.3 Flowchart of subroutine LOCAL

START SUBROUTINE 'LOCAL'

SELECT NEW COORDINATE

INCREASE COORDINATE

CHECK IF P(I) IS WITHIN BOUNDS

YES

IS THE MOVE A SUCCESS?

NO

DECREASE COORDINATE

CHECK IF P(I) IS WITHIN BOUNDS

YES

IS THE MOVE A SUCCESS?

NO

RESET NEW COORDINATE AND NEW FUNCTION VALUE

RESET COORDINATE

ANY COORDINATE LEFT?

YES

NO

KLOCAL = KLOCAL + 1

RETURN
D.4 Flowchart of subroutine OBJECT

START SUBROUTINE 'OBJECT'

IS
IC(5) = 1
?

OPTIONAL MODIFICATION
OF CNTRL & WF FROM
THE TERMINAL

KOBJ = KOBJ + 1

DATA (FROM COMMON)

- PERFORMANCE REQUIREMENTS, PR
- MATERIAL PROPERTIES, DENS, SIGMAX,
  CMECH, FBH, BHO, BSAT, BMIN, FMAX, FMIN
- OTHER VARIABLES: YDUCT, FLS, FLR

CALCULATE
F, SS, SR, SWF, RWF, HCF

ASSIGN PRESCRIBED GUESSES TO:
DR, GR, RSH, CL1

IS
KOBJ = 1
?

CALCULATE COEFFICIENTS OF
MAGNETIZATION CURVE
XD, XO, ZD, Z0, FN, BH
CALCULATE
WR, WS, CIS, CFS, CSS, CNS, AS, DBCS, DCS, TXS, TXR, TYS, TYR, DS, GS, WXS, WYS, CLAL, BO, SO, AL0, S1, AL1, S3, AL3, FLUX, BI, B3

CALCULATE
S2, AL2, B2, SH, ALH, BH

M.M.F. CALCULATIONS
CALL SUBROUTINE 'HBF'
HI, H2, H3, H4, GEFF

CALCULATE
CNRIR, CNR, C1R, CLMR, CLMTR, REC, CJR, CFR, CSR, AR, DBCR, DCR, HIR, DR

IS DR > R - RSH ?
YES

IS DR ACCURATE ENOUGH ?
NO

CALCULATE
WXR, WYR, UR, DENS(8), FSLOT, GR

IMPROVE GUESS

CALCULATE
OR, OS, TA, RF, TA, TF, CLT, WRD, CL1, DENS(9), RSH

b c d e
IS RSH > R-DR?

IS RSH ACCURATE ENOUGH?

CALCULATE
CLMS, CLMTS, SEC, WCUS, WCUR, CLCS, CLCR, WSC, WST, WRC, WRT, CLSC, CLIST, CLIRC, CLIRT, CLB

CALCULATE
GEOMETRICAL CONSTRAINT FUNCTIONS

IS ANY GEOMETRICAL CONSTRAINT VIOLATED?

CALCULATE
MECHANICAL CONSTRAINT FUNCTIONS

MUST RSH BE MODIFIED?

CALCULATE
THERMAL CONSTRAINT FUNCTIONS

CALL SUBROUTINE 'THERMO'
CALCULATE ELECTRICAL CONSTRAINT FUNCTIONS

CALCULATE FIGURE OF MERIT CONSTRAINT FUNCTION

CALCULATE PER UNIT CONSTRAINT ALLOWANCES

CALCULATE OBJECTIVE FUNCTION (MENZIES)

WRITE INTERMEDIATE INFORMATION

RESET THE CONTROL PARAMETERS IC(5), IC(6), IC(7) = 0

RETURN
D.5 Flowchart of subroutine THERMO

START SUBROUTINE 'THERMO'

DATA (FROM COMMON)
- GEOMETRICAL DIMENSIONS, G
- LOSSES, CL
- DESIRED COOLING MODE & OTHER CONTROL PARAMETERS NTOP, ACCELE, XINC, IC
- ESTIMATES FOR THE UNKNOWNS, X
- MATERIALS & COOLANT PROPERTIES TC, E, TAMB, W, CFINS, HEXT, W0, W1, W2, H (OPTIONAL)

CALCULATE
T10, T15, T19 (FROM X INITIAL GUESS)

CLEAR MATRICES
K(I,J) = 0 ; A(I,J) = 0

N = 1

CALCULATE AUXILIARY GEOMETRICAL QUANTITIES
CALL SUBROUTINE 'GEOMET'

CALCULATE RADIATION RELATED COEFFICIENTS K(I,J)
CALL SUBROUTINE 'RADIAT'

CALCULATE CONDUCTION RELATED COEFFICIENTS K(I,J)
CALL SUBROUTINE 'CONDUC'
STORE BEFORE ACCELERATION SCHEME: $X_2(I) = X(I)$

IS $N = 1$?

ACCELERATION SCHEME:

$x(I) = x(I) + \alpha [x(I) - x_1(I)]$

STORE VALUE PREVIOUS TO THE ACCELERATION SCHEME:

$x_1(I) = x_2(I)$

UPDATE (OR FIRST CALCULATION IF N=1) THE CONVECTION RELATED COEFF. $K(I,J)$

CALL SUBROUTINE 'CONVEC(FRICTI)'

CALCULATE COEFFICIENTS $A(I,J)$ OF THE LINEARIZED NEWTON-RAPHSON EQUATIONS (FROM $X(I)$ AND $K(I,J)$)

CALL SUBROUTINE 'COEFFI'

ISING = 0

SOLVE THE LINEAR SYSTEM OF NEWTON-RAPHSON EQUATIONS FOR $DX(I)$

CALL SUBROUTINE 'SOLVER'
b

IS SING = 1 ?

WRITE

MATRIX A(I,J) IS SINGULAR

UPDATE Unknowns X(I)

X(I) = X(I) + DX(I)

ANY

X_i(I) < 0 ?

YES

X_i(I) = 0.5 * (X_i(I) - DX_i(I))

N = N + 1

NO

ANY |DX(I)| > .01 * X(I) ?

YES

IS N > NMAX ?

NO

WRITE

INTERMEDIATE INFORMATION

c

d
HEAT EXCHANGES WANTED?

CALCULATE HEAT EXCHANGES
$Q(i,j), C(i,j), R(i,j)$
CALL SUBROUTINE 'HEATEX'

WRITE
$Q, C, R$

CALCULATE TEMPERATURES OF INTEREST
CALL SUBROUTINE 'CONDUC'

RETURN
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