PARTIAL VERIFICATION OF A COMPUTER AIDED
ELECTRICAL MACHINE DESIGN PROGRAM

By

Thomas Alan Kush

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Bachelor of Science

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ABSTRACT

Because of the lack of experience and knowledge in the field of high speed electrical machine design a computer aided design program would be a valuable tool for the motor designer. Mr. Ignacio Perez in his work, "Computer Aided Design of High Rotational Speed Electrical Machinery", M.S. Thesis, M.I.T., 1977, has developed such a program. One part of the program is a thermal model of high speed rotating electrical machinery. This program takes as its input the physical dimensions, losses, thermal properties and other parameters describing the machine. As output, the program gives the predicted operating temperatures of various parts of the machine.

The goal of this work is to verify the predicted temperatures. A conventional squirrel cage induction motor was used as the test machine. The physical dimensions required by the program were obtained directly or calculated from information supplied by the manufacturer. The thermal properties of the materials used in the machine were obtained in the literature. Electrical tests were used to determine the losses. Theoretical calculations were used to obtain the bearing losses, flow rate of coolant, and film coefficient of convection from the frame. The data obtained was then used by the program to calculate the operating temperatures of the machine. The temperatures of the test machine were found using thermocouple, semiconductor and resistance thermometry techniques. The agreement between the measured and calculated was in most cases quite good. All discrepancies could be traced either to the inability to make a temperature measurement at the exact location of the calculated temperature or to an inaccurate temperature measurement.

THESIS SUPERVISOR: John G. Kassakian
TITLE: Assistant Professor of Electrical Engineering
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CHAPTER 1
INTRODUCTION

The use of the digital computer to model the thermal characteristics of electrical machinery can provide a valuable aid in the machine design process. Such a model would serve, at least in part, to remove the design of motors and generators from the realm of experience, intuition and rule of thumb methods which are now the basis for a significant part of the design process. The design of electrical machinery is well enough understood that designers of conventional types of motors and generators can undoubtedly manage as they have, without this aid. It is in the design of machines which depart significantly in some way from conventional designs that such a model would prove invaluable. In these unconventional designs there is no extensive experience or well established and tested empirical rules for the designer to draw on. Still it would be nice to be able to predict, prior to actually building a prototype and testing it, the operating temperatures of the machine. A computer implemented thermal model that may serve this purpose for the case of motors with very high rotational speeds has been developed by Mr. Ignacio Perez\textsuperscript{(7)}. The work described here is the experimental verification of this model.

While Mr. Perez's model is intended for high speed machine design, it should be possible to verify the temperature predictions on a conventional squirrel cage induction motor. First it was necessary to determine the input required by the model. This includes the physical dimensions of various parts of the machine, the thermal conductivities of the steels, copper, aluminum and insulations used. Also required are the flow rate of air through the cooling passages of the motor and the electrical and mechanical
losses at load. The physical dimensions are available in the manufacturer's specifications. The thermal conductivities are well known and available in the literature. The determination of the losses and the flow rate of air required experimental procedures. As a starting point the standard I.E.E.E. test procedures were used\(^{(5)}\). These proved inadequate for separating some losses and for measuring flow rate and so some additional tests were developed and used.

The second part of the work was the instrumentation of the test motor to measure the temperatures of the various parts of the machine. Three techniques of thermometry were used. Thermocouples were used to measure the temperature of all parts of the machine. This method was chosen because of the ease with which thermocouples may be placed on even relatively inaccessible parts of the machine and left in place while the machine is running. A semiconductor type digital thermometer was used to measure all air temperatures. The average temperature of the windings was determined by the change in resistance. In all, the temperatures of fourteen locations were measured at three conditions of load.
CHAPTER 2
DESCRIPTION OF THE MODEL

2.1 Introduction

The thermal model program written by Mr. Perez is intended to provide the temperatures at points of interest in a given machine operating under given conditions. Other information including the average temperatures of the elements of the machine and a complete map of heat transfers within the machine is also obtained. This, then, is a check on whether the design in question is within the required limits of temperature rise. This chapter summarizes the thermal model developed in Mr. Perez's thesis (7) with emphasis on the underlying theory and those aspects important to the verification of the model.

The model makes use of the fact that there is an electrical network analogous to the thermal network of the machine. In order to make use of this and keep the model manageable the idealized machine geometry shown in Fig. 2.1, incorporating the important features of a broad range of machine designs was chosen. Next the machine was divided into its basic thermal elements. These basic elements become the nodes of the equivalent circuits. Once the division of nodes was chosen the equivalent thermal resistances between nodes and the equivalent circuit for the conduction and convection modes was determined. Since, unlike the case for conduction and convection, there is direct analogy between heat transfer by radiation and an electrical network, a special thermal circuit was prepared for the radiation mode. This results in two networks, one for the combined conduction and convection modes and one for the radiation mode. The equations describing the conservation of energy at each node of the two thermal circuits were then written and the resulting system of equations was solved for the temperature
1. STATOR EMBEDDED WINDING
2. STATOR END WINDING
3. STATOR TEETH
4. STATOR CORE
5. ROTOR EMBEDDED WINDING
6. ROTOR END WINDING
7. ROTOR TEETH
8. ROTOR CORE
9. FRAME
10. AIR GAP
11. OVERHANG AIR
12. SHAFT
13. BEARINGS
14. END SHIELD
15. AMBIENT AIR
16. STATOR END PLATE
17. ROTOR END

IDEALIZED MACHINE GEOMETRY AND NODE DIVISION

FIG. 2.1
rises.

From the outset the program was not intended to model all types of rotating electric machines and so certain restrictions were assumed. These were:

i) No salient poles

ii) Wound rotor and stator

iii) Enclosed rotor end windings

iv) Four cooling options

a) Totally enclosed. Natural cooling, fins. No air space between frame and stator core.

b) Totally enclosed. External forced cooling. Fins. No air space between frame and stator core.


(i) and (iii) are in keeping with the intent to model high speed machines. The model was later modified to allow for squirrel cage rotors as the test machine was of this type.
2.2 Heat Transfers and the Equivalent Networks

2.2.1 The Conduction Mode

Heat flow by conduction is described by Fourier's Law,

\[ q = -k \nabla \theta \] (2.1)

where \( q \) = heat flow density (watts/m\(^2\))

\( \theta \) = temperature (°K)

which is identical in form to Ohm's Law. It is also true that for steady state conduction

\[ \nabla^2 \theta = 0 \] (2.2)

which is identical to Laplace's equation. This implies that the thermal network of the machine can be treated the same as an equivalent electrical network. In the simplest case of one dimensional heat flow in a homogeneous, isotropic material the two equations above yield

\[ Q = \frac{\Delta \theta}{R_t} \] (2.3)

\[ R_t = \frac{\rho}{kA} \] (2.4)
where \( k \) = length of the element (m)
\[ A = \text{cross sectional area (m}^2\text{)} \]
\[ R_t = \text{thermal resistance of the element (} {}^0\text{K/watt)} \]
\[ \Delta \theta = \text{temperature difference across the length (} {}^0\text{K)} \]
\[ Q = \text{heat flow through the element (watts)} \]

In the more complicated cases of flow through inhomogeneous or anisotropic media or in the case of multi-dimensional heat flow, equations (2.1) and (2.2) must be solved for the particular case. This solution will yield a network of thermal resistances which will be representative of heat flow through the element. When preparing the equivalent thermal circuit the thermal resistances between the nodes must be joined in the same way as the elements of the machine that they represent. Conduction contributes all or part of the heat transfer between these nodes: 3-4, 4-9, 4-16, 7-8, 8-11, 8-12, 8-17, 9-14, 9-16, 10-11, 12-13, 13-14, 14-16.

2.2.2 The Convection Mode

Heat transfer by convection is described by

\[ Q = h \cdot A \cdot \Delta \theta \]  

(2.5)

where \( A \) = surface area of the body in contact with the fluid (m\(^2\))
\[ \Delta \theta = \theta_{\text{surface of body}} - \theta_{\text{bulk of fluid (} {}^0\text{K)}} \]
\[ h = \text{film coefficient of heat transfer (watts/m}^2{}^0\text{K)} \]
\[ Q = \text{heat flow from the body to the fluid (watts)} \]

and an equivalent convective thermal resistance may be defined

\[ R_t = \frac{\Delta \theta}{Q} = \frac{1}{h \cdot A} \]  

(2.6)

Unfortunately this is not as straight forward as the equations would
indicate. The film coefficient, \( h \), is a function of many variables. The properties of the surface of the body, of the flow past the body and of the fluid all influence the value of the film coefficient to varying degrees. This renders the film coefficient, and so the convective thermal resistance, non-linearly dependent on temperature. The determination of the film coefficient for those situations in which convection is an important mode of heat transfer is based on empirical correlations between the dimensionless numbers describing the fluid and its flow across the body. For some of these situations no empirical correlations were found and so others for similar situations were used. The difficulties involved in determining valid film coefficients and their importance to the accuracy of the model presents an obstacle. Convective heat transfer exists between nodes: 4-9, 9-15, 14-15, 3-10, 7-10, 12-11, 17-11, 14-11, 16-11, 2-11.

2.2.3 The Radiation Mode

Radiation does not lend itself to straightforward incorporation into the equivalent thermal circuit. Two aspects of radiative heat transfer are responsible for this. First, the heat transferred by radiation from one surface to another is composed of multiple reflections and absorption and re-radiation from all other surfaces present as well as the direct transfer between the two surfaces being considered. So, the heat transfer between two surfaces is dependent not just on the properties of those surfaces but also on the shape, orientation and radiative properties of all surfaces present. This makes defining a thermal equivalent resistance for radiation between two surfaces impossible. Second, the equation governing heat transfer by radiation is nonlinear in temperature. This necessitated the selection of a method applicable to nonlinear systems for solving the resulting system of equations.
The approach used to arrive at an equivalent thermal circuit for the radiation mode is as follows: The surfaces of the machine that can transfer heat by radiation are grouped into enclosures made up of all those surfaces that can interact with one another. These enclosures are chosen in such a way that all the heat radiated or reflected from any surface of the enclosure falls only on other surfaces of the enclosure. Therefore, no heat is transferred by radiation out of the system of thermal elements comprising the enclosure. Common engineering assumptions which were used regarding radiant heat transfer in such an enclosure are:

i) The enclosure may be divided into a finite number of isothermal surfaces.

ii) The surfaces are grey body emitters, reflecters and absorbers.

iii) The emitted and reflected radiation leaving a surface has diffuse directional distributions.

iv) The magnitude of emitted and reflected radiation is uniform over the surface.

Radiant heat transfer for a surface \( j \) is described by

\[
Q_i = \frac{E_j A_j}{1 - \varepsilon_j} (\sigma \theta_j^4 - B_j) \tag{2.7}
\]

where \( B_j \) = radiosity of the surface \( j \) (watts/m\(^2\))

This is the rate at which radiant energy, both original and reflected, leaves the surface per unit area.

\( \theta_j \) = absolute temperature of the surface \( j \) (\( ^0 \text{K} \))

\( \sigma = \) Stefan-Boltzman constant = \( 5.67 \times 10^{-8} \) (watts/m\(^2 \) \( ^0 \text{K} \)^4)

\( A_j \) = area of the surface \( j \) (m\(^2\))

\( E_j \) = emissivity of the surface \( j \)
If all surfaces of an enclosure are considered the radiant flux balance equations yield:

\[ Q_j = B_j A_j - \sum_{i=1}^{N} B_i F_{j-i} A_j = - \sum_{i=1}^{N} (B_j - B_i) A_j F_{j-1} \quad (2.8) \]

where \( F_{j-i} \) is the shape factor. This is the fraction of radiant heat leaving area \( j \) which strikes area \( i \) directly. It is a function of the shapes and orientations of the surfaces concerned.

If the two expressions (2.6),(2.7) for radiant energy leaving a surface are set equal and rewritten thus:

\[
\frac{(\sigma \theta_j^4 - B_j)}{(1-E_j)/E_j A_j} + \sum_{i=1}^{N} \frac{(B_i - B_j)}{1/A_j F_{j-1}} = 0 \quad (2.9)
\]

and if the radiosities \( B_j \) and emissive powers \( \sigma \theta_j^4 \) are associated with potentials and the terms \( 1/A_j F_{j-1} \) and \( (1-E_j)/E_j A_j \) are associated with resistances, then equation (2.9) may be interpreted as stating Kirchoff's current law at the node \( j \). It should be noted that these resistances do not represent actual thermal resistance but surface areas combined with emissivities or shape factors. Also the potentials of the nodes of the equivalent circuit representing surfaces do not represent absolute temperatures \( \theta \), but rather \( \sigma \theta^4 \).

Four enclosures consisting of the indicated nodes were defined as follows:

i) 9, 14, 15

ii) 3, 4, 9 (16 was neglected)

iii) (3-4), (7,8)

iv) 2, (17-12), 14, 16
2.2.4 The Equivalent Thermal Networks

The node division, as shown in Fig. 2.1, and the heat transfers by conduction and convection (Sections 2.2.1 and 2) give the thermal circuit of Fig. 2.2. The connections with nodes 1, 2, 5, 6 cannot be expressed as a simple resistance. This is because of the combined nature of heat transfer with the windings.

The equivalent circuit for radiation is shown in Fig. 2.3. It should be noted that this is very different from the circuit for conduction and convection.

2.3 Inputs to the Program

The design parameters required as inputs to the program may be classified into four groups.

2.3.1 Physical Dimensions

The following is a list of machine dimensions. Most are shown in the drawing of Fig. 2.4.

OR = Overhang of rotor
OS = Overhang of stator
LT = Total length between bearings
L  = Length of core
RSH = Radius of shaft
R  = Radius of rotor
G  = Gap thickness
GS = Thickness of wedges in stator
DS = Depth of stator slots
RF = Radius of frame
TB = Thickness of end housing
CONDUCTION AND CONVECTION EQUIVALENT THERMAL CIRCUIT

FIG 2.2
RADIATION EQUIVALENT THERMAL CIRCUIT

FIG. 2.3
TA = Thickness of air space between frame and stator core
TF = Thickness of frame
AS = Total conductor cross section in a stator slot
AR = Total conductor cross section in a rotor slot
NS = Number of turns per phase in stator winding
NR = Number of turns in rotor winding
DR = Depth of rotor slots
H = Radial thickness of stator core
KS = Stator slot factor
KR = Rotor slot factor
P = Number of pole pairs
SS = Number of stator slots
SR = Number of rotor slots
TXS = Thickness of stator slot liner insulation in X-direction
TXR = Thickness of rotor slot liner insulation in X-direction
TYS = Thickness of stator slot liner insulation in Y-direction
TYR = Thickness of rotor slot liner insulation in Y-direction
WS = Width of stator slots
WR = Width of rotor slots
WX = Thickness of individual stator conductors in X-direction
WY = Thickness of individual stator conductors in Y-direction
WXR = Thickness of individual rotor conductors in X-direction
WYR = Thickness of individual rotor conductors in Y-direction
U = Thickness of insulation on individual stator conductors
GR = Thickness of wedges in rotor
UR = Thickness of insulation on individual rotor conductors
MACHINE DIMENSIONS

FIG. 2.4
2.3.2 Machine Losses

The losses are required for the program since they are the sources in the thermal equivalent circuit. The losses are:

- $\text{LCS} =$ Copper losses in the stator
- $\text{LCR} =$ Copper losses in the rotor
- $\text{LIS} =$ Iron losses in the stator
- $\text{LIR} =$ Iron losses in the rotor
- $\text{LB} =$ Bearing losses
- $\text{LWO} =$ Windage loss in the air gap
- $\text{LWI} =$ Windage loss in the overhang region

2.3.3 Thermal Properties

The thermal conductivities required as inputs to the model are:

- $\text{KIS} =$ Insulation on individual conductor of stator winding
- $\text{KIR} =$ Insulation on individual conductor of rotor winding
- $\text{KEIS} =$ Slot liner insulation of stator
- $\text{KEIR} =$ Slot liner insulation of rotor
- $\text{KCS} =$ Conductors of stator winding
- $\text{KCR} =$ Conductors of rotor winding
- $\text{KSE} =$ Stator core in the axial direction
- $\text{KSR} =$ Stator core in the radial direction
- $\text{KRE} =$ Rotor core in the axial direction
- $\text{KRR} =$ Rotor core in the radial direction
- $\text{KF} =$ Frame
- $\text{KSH} =$ Shaft
- $\text{KA} =$ Cooling air

It should be noted that the thermal conductivity of air changes significantly
with temperature and so is to be evaluated at the appropriate temperature. Also required are emissivities for the calculations of equivalent resistances of the radiation equivalent network. These values are available but it was considered reasonable to assume a value of 0.9.

2.3.4 Miscellaneous Inputs

Other information required is:

TAMB = Ambient temperature
FINS = Effectiveness coefficient for cooling fins, if any
\( \omega \) = Rotational speed
WO = Flow rate of coolant (see Fig. 2.5)
W1 = Flow rate of coolant through air gap and radial duct
W2 = Flow rate of coolant through axial duct

Choice of cooling mode

Numerical values for film coefficients. These may be calculated in the model instead.

2.4 Calculation of the Thermal Resistances

All of the details of the extensive calculations involved in determining the thermal resistances will not be described here. What follows is a description of the general approaches used. When the details of the method used or the assumptions made have some influence on the provision of inputs for the model or on the verification of the model, those methods or assumptions will be described.

2.4.1 Calculation of Conductive Thermal Resistances

While all heat transfer by conduction is a three-dimensional process it was found that in many cases one or two-dimensional models are quite
ASSUMED COOLANT FLOW

FIG. 2.5
reasonable. Assuming these simpler cases greatly facilitated the calculations. In the case of one-dimensional models the correct analytical expressions were worked out. In those cases where the one-dimensional model proved inadequate, simplified two-dimensional models were used. Because of the nearly complete azimuthal symmetry no three-dimensional models were required.

Some parts or groups of parts of the machine are not homogeneous or are anisotropic. In the case of the cores the thermal conductivities in the axial and radial directions are very different owing to the laminations. The model accounts for this by using two different values of thermal conductivity for the cores. It would have been inconvenient to take this approach for the windings and consider each conductor with its insulation separately. Instead the windings in a slot were considered to be a homogeneous media with equivalent thermal properties.

In order for the equivalent circuit approach to work it is necessary that the node voltages correspond to the mean temperature of the corresponding machine element, the equivalent resistances being calculated while keeping this in mind. The hot spot temperatures are then calculated from the mean temperatures.

Considerable simplification of the equivalent thermal circuit was achieved by noting that the numerical values obtained from actual machines indicate that the following assumptions are valid:

i) The heat transferred through the wedge is small compared to the heat transferred through the slot walls.

ii) The relative values of the thermal conductivity of the cores in the axial and transversal directions is such that each individual lamination may be considered to be isothermal whereas the change in temperature in the axial direction must be accounted for. This
allows nodes 3 and 4 and also nodes 7, 8 and 12 to be considered as one. This leads to the simplified network of Fig. 2.6.

iii) The bunch of conductors and insulation in a slot will be considered as a homogeneous material.

Because of the difficulty presented by accounting for each wire of a winding separately an equivalent homogeneous medium with approximately the same thermal conduction properties will be devised. This fictitious medium may or may not be isotropic depending on the actual construction of the machine. Several assumptions were made. Those that bear on the provision of the input data are:

i) All wires are the same size

ii) All wires have the same insulation

iii) The thickness of all insulation is uniform around every wire

iv) All dead space in the slot is considered to be filled with insulation

v) The thickness of the slot liner insulation is uniform

The actual windings are considered to be replaced by windings as shown in Fig. 2.7, such that the area A is the same area as the actual conductor. \( W_x \) and \( W_y \) are further chosen so that the actual winding and this have the same number
SIMPLIFIED CONDUCTION AND CONVECTION THERMAL CIRCUIT

Fig. 2.6
ACTUAL WINDING GEOMETRY

ASSUMED EQUIVALENT WINDING

FIG. 2.7
of layers and same number of turns per layer. The thickness of insulation \( U \) is chosen such that the cross sectional area of insulation is the same as the cross sectional area of insulation and dead space in the actual slot. The resulting values of \( W_x, W_y \) and \( U \) are then used by the program to calculate the transversal thermal conductivities of the winding.

2.4.2 Calculation of Convective Thermal Resistances

Natural convection occurs only between the exterior of the frame and end shield (nodes 9 and 14) and the ambient air (node 15). This is true only in the case of no forced external cooling. There is also natural convection between the outside surface of the stator core and the inside surface of the frame if there is no imposed coolant flow axially through this passage. If this is the case, the natural convection equivalent resistance is large compared to the conduction equivalent resistance in parallel with it and so may be neglected.

Empirical correlations for heat flow by natural convection from horizontal cylinders and vertical plates were found and applied to the frame and end shield. These correlations allow the calculation of the film coefficient, \( h \), from certain dimensionless numbers which describe the character of the fluid and its flow.

Since a mounted electrical machine in an air conditioned room is far from accurately modeled as a cylinder floating in an infinite medium, the film coefficient so obtained may not be very accurate. However, the sensitivity of the calculated temperature rises to the film coefficient from the frame to ambient air is very high. As an option to the method used by the program to obtain the film coefficient it is possible to assign to the program a value obtained by some external means.
The forced convection process takes place in the overhang region and in the internal ducts, if any. The process as considered for the model was divided into three groups. First, are the non-rotating parts with flow induced over their surfaces by the rotation of other parts. In this group are the stator end windings, the interior of the end shield and the stator end face (nodes 2, 14, 16 respectively). Second, the non-enclosed rotating parts namely the shaft and rotor end (nodes 8 and 12). While these are in fact enclosed by other parts of the machine the distances involved are large enough that the assumption of non-enclosed rotating surfaces is not unreasonable. The third group is the enclosed rotating elements. This includes the rotor core and stator core through the air gap. For all of the machine elements involved in the forced convection process the effects of local geometries and the types of flow involved were considered.

The film coefficient for forced convection depends on the fluid velocity. For the overhang region the accurate determination of the required velocities is not possible and so rough estimates of the actual velocities were determined for use in the program. The value of the film coefficient for the case of free convection from the surface of interest was also obtained and used as a lower bound for the forced convection case.

The cooling scheme which was considered is shown in Fig. 2.5. No cooling ducts in the rotor were considered due to their incompatability with high speed machines due to mechanical stresses. The absence of radial ducts may be accounted for by setting W2 to zero. Axial ducts in the stator are also easily incorporated, so the model is quite general.

The approach used is to apply the theory of heat exchangers to the machine. Relationships between the amount of heat transferred to the coolant, the temperatures of the surfaces in contact with the flow and the
changing temperature of the coolant were obtained. The film coefficients for the surfaces involved must be found for the velocities of the coolant across them.

2.4.3 Calculation of Radiation Thermal Resistances

The calculation of the radiation thermal resistance of Fig. 2.3 requires the evaluation of the areas $A_i$ and shape factors $F_{i-j}$ of Eqn. (2.9). To facilitate these calculations for enclosure iv a new idealized geometry for the overhang region was chosen. The areas and shape factors for each enclosure were then calculated. The calculation of the areas was straightforward. The shape factors, however, were not so easily determined except for the most elementary geometries.

The actual calculation of the shape factors relies heavily on the information on shape factors for coaxial cylinders found in the literature. Extreme accuracy in determining the shape factors is not warranted because of the inaccuracies of other parts of the model and the inaccuracy introduced by the further simplification of the geometry. Because of the similarity of some of the terms of the shape factor equations it was necessary to use double precision arithmetic in the program. For these reasons it was considered desirable to use some less accurate equations that are not so likely to give rise to problems of computation.

2.5 The System of Equations and Method of Solution

Analysis of the resulting networks for conduction and convection and for radiation allows the heat exchanges between the modelled machine elements to be written in terms of known coefficients (the equivalent thermal resistances of the two networks) and the following thirteen unknowns:
Nine average temperatures

\[ X(1) = \theta_{3-4} \] (stator core and teeth)
\[ X(2) = \theta_{7-8} \] (rotor core and teeth)
\[ X(3) = \theta_9 \] (frame)
\[ X(4) = \theta_{11} \] (overhang air at entrance to the axial duct)
\[ X(5) = \theta_{12-17} \] (shaft and rotor end face)
\[ X(6) = \theta_{13} \] (inner bearing race)
\[ X(7) = \theta_{14} \] (end shield)
\[ X(8) = \theta_{16} \] (stator end plate)
\[ X(9) = \theta_{\text{ext 2}} \] (exterior of the stator end winding)

Four radiosities of the elements in the overhang region

\[ X(10) = B_{\text{ext 2}} \]
\[ X(11) = B_{12-17} \]
\[ X(12) = B_{14d} \]
\[ X(13) = B_{16} \]

For any element the total internal heat produced (the losses occurring in the element) can be equated with the heat transferred to those elements which are in thermal contact with it

\[ L(I) = \sum_{j=1}^{9} [Q(I,J) + C(I,J) + R(I,J)] \]  \hspace{1cm} (2.10)

where \( L(I) \) are the losses in the element \( I \) and \( Q(I,J) \) stands for conduction, \( C(I,J) \) stands for convection and \( R(I,J) \) stands for radiation all from element \( I \) to element \( J \). Heat transfer with those elements not included in the list of nine is accounted for by the \( I=J \) terms.
It was shown that the heat exchanges may be written as:

\[ Q(I,J) = k_1 \cdot [X(I) - X(J)] \], \quad Q(I,I) = k_2 \cdot X(I) + k_3 \]  \hspace{1cm} (2.11)

\[ C(I,J) = k_4 \cdot [X(I) - X(J)] \], \quad C(I,I) = k_5 \cdot X(I) + k_6 \]  \hspace{1cm} (2.12)

\[ R(I) = \sum_{j=1}^{9} R(I,J) = \sum_{j=1}^{9} k(I) \cdot X_{ABS}(I) + \sum_{j=1}^{13} k(I) \cdot X(I) \]  \hspace{1cm} (2.13)

where the \( k_1, 2, 3, 4, 5, 6, I \) have been determined.

Substituting Eqs. (2.11), (2.12), (2.13) into Eqn. (2.10) results in nine nonlinear equations of the form:

\[ \sum_{j=1}^{9} B(I,J) \cdot X_{ABS}(J)^4 + \sum_{j=1}^{13} D(I,J) \cdot X(J) + E(I) = 0 \] for \( I = 1, \ldots, 9 \)  \hspace{1cm} (2.14)

The additional four equations result from applying Eqn. (2.9) to the four nodes of the radiation network with unknown radiosities:

\[ \sum_{j=1}^{9} B(I,J) \cdot X_{ABS}(J)^4 + \sum_{j=10}^{13} D(I,J) \cdot X(J) = 0 \] for \( I = 10, \ldots, 13 \)  \hspace{1cm} (2.15)

where \( B(I,J), D(I,J) \) and \( E(I) \) are coefficients.

These equations are solved, using the Newton Raphson method, for the nine average temperatures. From these average temperatures the temperatures at the expected hot spots are calculated. These are:

- \( TEMP(1) \) = Mid-point of the embedded stator winding
- \( TEMP(2) \) = Mid-point of the stator end winding
- \( TEMP(3) \) = Mid-point of the embedded rotor winding
- \( TEMP(4) \) = Mid-point of the rotor end winding
- \( TEMP(5) \) = Hot spot of the stator iron
- \( TEMP(6) \) = Hot spot of the rotor iron
2.6 Modification of the Model to Coincide with the Test Machine

In order to adapt the model to the squirrel cage machine used in the tests the following assumptions and modifications were adopted.

- In the absence of any way of separating the end ring loss and bar loss components of the rotor copper losses it was assumed that the bar losses and end ring losses were 2/3 and 1/3 of the rotor copper losses.

- Because there is no thermal insulation between the rotor bars and rotor core it was assumed that all heat produced in the bars is transferred to the core with no temperature difference.

- The thermal resistance of the rotor core in the axial direction was made equal to the axial resistance of the iron in parallel with the resistance of the bars.

- The temperatures of the rotor bars and rotor body are equal.

- The temperatures of the end rings and rotor ends are equal.

- The presence of the cooling fins (fan blades) on the rotor ends was accounted for by making the effective surface area equal to 2.25 times the actual surface area of the rotor end. This is in keeping with the increase in effective surface area usually obtained with cooling fins.

- A value for the thermal resistance due to conduction out through the shaft was chosen which made the temperature of the shaft obtained in the model equal the temperature obtained experimentally. The value used is quite reasonable.

- Values for the thermal resistances due to conduction from the stator end and stator core to the frame based on the actual machine dimensions and the thermal conductivity of steel were used.
3.1 Physical Dimensions

The required physical dimensions of the machine are defined in Section 2.3. Most of these were obtained directly from the drawings and production specifications provided by the manufacturer. In some cases, however, it was necessary to calculate the required parameters from this documentation. Also, because of the assumptions and simplifications made in the model, some interpretation of the actual machine geometry was required in order to conform to the model.

For those cases where the information was obtained directly from the manufacturer's drawings, the values are given without explanation. For those cases where the required information was not directly available, the calculations follow. In both cases the parameters appear on the summary page at the end of this chapter.

- AS: Conductor cross section of stator winding. From the winding data provided by the manufacturer the following information was obtained:

36 coils, 36 slots, 11 turns/coil, each turn consists of four individual conductors one #17 and three #16.5 AWG

With 36 coils and 36 slots there must be part of two coils in each slot since the coils must go down through one slot and back through another. This implies that there are conductors from 22 turns in each slot. There are, therefore, 22 #17 conductors and 66 #16.5 conductors in each slot. The cross sectional area of these conductors is:

\[ #17 = 0.001609 \text{ sq. in.} \]

\[ #16.5 = 0.001819 \text{ sq. in.} \]
Therefore, the total cross sectional area of conductor in each stator slot, \( AS \), is:

\[
AS = 22 \cdot 0.001609 \text{ sq. in.} + 66 \cdot 0.001819 \text{ sq. in.} = 0.155 \text{ sq. in.}
\]

- **AR**: Conductor cross section of rotor winding. From the rotor disk specifications, the total slot area is given as 6.22 sq. in. Since the rotor bars are die cast into the assembled rotor core they completely fill the slots, of which there are 48. So, the conductor area per rotor slot, \( AR \), is:

\[
AR = \frac{6.22 \text{ sq. in.}}{48} = 0.130 \text{ sq. in.}
\]

- **NS**: Number of conductors per phase in the stator winding. From the manufacturer's specifications the following was obtained

\[
NS = \frac{36 \text{ coils} \cdot 11 \text{ turns/coil} \cdot 4 \text{ conductors/turn}}{3 \text{ phases}} = 528 \text{ conductors/phase}
\]

- **KS**: Stator slot factor. This is the ratio of the total circumferential length of the stator slots to the total stator circumference. This is usually taken at the body of the slot, rather than at the actual opening of the slot, which may be quite narrow or not exist at all as in the case of the rotor slots of this machine. Figure 3.1 shows the geometry of the stator core in the vicinity of the slot.

\[
\text{circumference at neck} = 2\pi \cdot \text{rad.} = 2 \cdot 3.14 \cdot (2.25 \text{ in.} + 0.0525 \text{ in.}) \approx 20.75 \text{ in.}
\]

\[
\text{total slot length at neck} = 36 \text{ slots} \cdot 0.321 \text{ in.} = 11.56 \text{ in.}
\]

so, \( KS = \frac{\text{Total Slot Length}}{\text{Circumference}} = \frac{11.56}{20.75} = 0.557 \)
- KR: Rotor slot factor. Calculated in the same way as KS. The geometry is shown in Fig. 3.2.

\[
\text{circumference} = 2\pi \cdot \text{rad} = 2 \cdot 3.14 \cdot (3.216 \text{ in.} - 0.031 \text{ in.}) = 20.01 \text{ in.}
\]

\[
\text{total slot length} = 48 \text{ slots} \cdot 0.157 \text{ in.} = 7.54 \text{ in.}
\]

\[
KR = \frac{\text{Total Slot Length}}{\text{circumference}} = \frac{7.54 \text{ in.}}{20.01 \text{ in.}} = 0.377
\]

- WX, WY: Dimensions of equivalent rectangular conductors. In accordance with the assumptions detailed in Section 2.4.1.2 regarding the simplified structure, shown in Fig. 2.7, used to model the windings, a fictitious winding composed of the same number of conductors of rectangular cross section with the same cross sectional area will be found. Since the windings of the test motor are not wound in layers it will not be necessary to preserve the arrangement of conductors in the slot. Because of this it is reasonable to take the simplest approach and assume square conductors. The following are known: 22 turns/slot, 4 conductors/turn, the total cross sectional area of conductors in the slot, \( AS = 0.155 \text{ sq. in.} \). So there are 88 conductors per slot.

\[
\text{Average area of the conductors} = \frac{0.155 \text{ sq.in.}}{88} = 0.00176 \text{ sq.in.}
\]

\[
WX = WY = (\text{average area})^{1/2} = 0.042 \text{ in.}
\]

- U: Thickness of insulation on each conductor. This also conforms to the assumed geometry of the windings presented in Section 2.4.1 and shown in Fig. 2.7. It was assumed that all of the cross section of the slot not occupied by conductors is occupied by the insulating material and the insulation is of uniform thickness.
STATOR SLOTS
FIG. 3.1

ALL DIMENSIONS IN INCHES

ROTOR SLOTS
FIG. 3.2
Total area of stator slot = 0.410 sq.in.
Total area of conductors, AS = 0.155 sq.in.
so, Total area of insulation = 0.410 sq.in - 0.155 sq.in. = 0.255 sq.in.

With 88 conductors there must be \( \frac{0.255 \text{ sq.in.}}{88} = 0.0029\) sq. in. of insulation on each conductor. From Fig. 2.7 it can be seen that the area of insulation around each conductor is given by:

\[
(2U + W)^2 - W^2
\]

\[
(2U + 0.042 \text{ in.})^2 - (0.042 \text{ in.})^2 = 0.0029 \text{ sq.in.}
\]

Solving for the thickness of insulation, \( U = 0.013 \) in.

3.2 Losses

The losses in the machine are the sources of heat which produce the temperature rises to be calculated with the computer model. The model, therefore, requires as inputs the losses, both electrical and mechanical in nature. Further, the losses must be sufficiently localized, i.e., attributed to some particular part of the machine. The required losses are given in Section 2.3.2 and another listing, with their values, is given at the end of this chapter.

With one exception, the determination of the losses is based on data obtained from tests on the motor. The methods used are based on the procedures set forth in the I.E.E.E. publication on testing induction motors(5). These procedures alone were not entirely satisfactory, however, as they did not provide for the required separation of losses or were not sufficiently accurate. In particular, they provided no means of separating the stator iron losses, LIS, from the rotor iron losses, LIR. Nor do they take into account the effects of the iron losses and the friction and windage losses.
in calculating the rotor copper losses, LCR. To overcome these two problems the calculation of LCR was modified and an additional test, the synchronous speed test, described in Section 3.2.1.2 was performed to allow LIS and LIR to be separated. The details of all of the electrical tests performed and the data obtained from them appears in Section 3.2.1.

It was not possible, using exclusively experimental means, to separate the total mechanical losses into the bearing losses, LB, and the windage losses, LW. The electrical tests provided data from which the sum of LB and LW could be found, Section 3.2.2.2. Unfortunately, no method was described for separating them since their individual values are normally of no interest to motor designers. In order to separate the two mechanical losses a theoretical method of calculating the bearing losses was found. This is described in Section 3.2.2.1. From the value obtained for LB and the total mechanical losses, LW could, of course, be found.

3.2.1 Electrical Tests

The data obtained from the tests described here will be used in Section 3.2.2 and Section 3.2.3 to calculate the losses. For all of the tests, power was supplied to the motor through the circuit shown in Fig. 3.3. The variac allowed the voltage applied to the machine to be varied and also prevented excessive starting currents from damaging the instruments. The current drawn by the machine was in all cases considered to be the average of the currents indicated by the three ammeters. The power consumed by the machine is the algebraic sum of the two powers indicated by the watt meters.

Slip was determined using a stobe light. The light was triggered with line voltage and so was flashing at 3600 flashes per second, twice synchronous speed. The apparent rotational speed (taking into account the
stroboscopic effect of two markers) of the shaft seen under the stobe light is, therefore, the slip speed. The slip, $S$, is then given by:

$$ S = \frac{\text{Slip Speed (rpm)}}{1800 \text{ rpm}} $$

Since the apparent rotational speeds were on the order of one revolution per second, it was practical to measure them using a stop watch to find the time required for, say, 100 apparent revolutions, and calculating the apparent revolutions per minute from this.

Winding resistance was determined using the circuit of Fig. 3.4(4). This was done by observing the voltage and current and using Ohm's Law to calculate the resistance. This method was used because it eliminates the effect of lead resistances and poor connections which may be significant when compared to the winding resistances. Another advantage of this method over others which were considered is the speed with which a measurement may be taken. It was always possible to obtain the voltage and current in well under 60 seconds after removing power from the winding. This speed was important when using resistance measurements to determine the winding temperature since the measurement must be made before the winding begins to cool appreciably.

The error introduced by the current drawn by the voltmeter was neglected since the resistance of the voltmeter, about 300 $\Omega$, is much greater than resistance of the winding, about 0.20 $\Omega$. The error resulting from this is less than the limit of accuracy in reading the meters.

3.2.1.1 No Load Test

The no load power and current data obtained in this test will be used in Section 3.2.2.2 to calculate the friction and windage losses and in
SUPPLY CIRCUIT

FIG. 3.3

CIRCUIT USED TO DETERMINE WINDING RESISTANCE

FIG. 3.4
Section 3.4 to calculate the flow rate of coolant and film coefficient.

For this test the motor was run with no load and the voltage applied was varied from 240 V down to 20 V. The current (the average of the ammeter reading in each line), the power consumed as indicated by the wattmeters, and the slip were determined for various voltages. Curves were plotted using the voltage as the abscissa and the average current and the power as ordinates. Another curve plotting power against the square of the voltage was also made (see Section 3.2.2.2). Data and curves obtained appear in Figs. 3.5, 6, 7.

The winding resistance was determined by the method already described. This measurement was made after the machine had reached its steady state no load operating temperature at rated voltage. Two hours were allowed for the temperatures to stabilize. The machine was then shut down and the winding resistance determined as quickly as possible. It was possible to obtain this measurement in under 60 sec. This is the time given in Ref. 4 as that in which no significant change in resistance due to the motor cooling down should occur.

3.2.1.2 Synchronous Speed Test

The synchronous speed test provides data from which the stator iron losses may be calculated. For this test a dc machine was used to turn the test machine at exactly synchronous speed as determined by the stobe tach triggered on line voltage. Maintaining synchronous speed for more than a few minutes was difficult because the speed tended to drift slightly. Despite this it was not too difficult to make the required measurements while there was no apparent rotation of the shaft, this condition indicating exactly synchronous speed. By triggering the strobe at line frequency
<table>
<thead>
<tr>
<th>Voltage</th>
<th>Current</th>
<th>Power</th>
<th>Slip (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>240</td>
<td>13.7</td>
<td>560</td>
<td>0.031</td>
</tr>
<tr>
<td>230</td>
<td>12.7</td>
<td>500</td>
<td>0.033</td>
</tr>
<tr>
<td>220</td>
<td>11.8</td>
<td>440</td>
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<tr>
<td>210</td>
<td>11.5</td>
<td>420</td>
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<td>200</td>
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<td>0.035</td>
</tr>
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<td>180</td>
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<td>300</td>
<td>-</td>
</tr>
<tr>
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<tr>
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<td>-</td>
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<td>0.256</td>
</tr>
<tr>
<td>30</td>
<td>-</td>
<td>55</td>
<td>0.417</td>
</tr>
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</table>

Winding Resistance  0.190 Ω

Fig. 3.5 No Load Test Data
FIG. 3.6. NO LOAD CURRENT AND POWER
both it and the motor are synchronized with the same source. This made obtaining exact synchronous speed possible. This is an important consideration since even small amounts of slip, either positive or negative, would result in power being dissipated in the rotor, precisely what this test was intended to prevent.

The measurements made were the current in each line and the total power input at 240, 230 and 220 volts. Also the winding resistance was obtained in the same manner as has been previously described. The data obtained from the synchronous speed test appears in Fig. 3.8.

3.2.1.3 Test Under Load

These tests were made running the machine at 3/4 and full load. The load was a dc machine wired as a generator which was in turn loaded with a bank of resistors. Both tests were carried out at rated voltage. The loading was determined by the average current drawn by the machine, 36 amps for full load and approximately 27 amps for 3/4 load. The total power input, the slip and the winding resistance were determined as has already been described. The results are listed in Fig. 3.9.

3.2.2 Mechanical Losses

3.2.2.1 Calculation of the Bearing Losses

The following is a description of the theoretical method found in Ref. 6 for calculating the frictional torque of ball or roller bearing. It should be noted that this method is not strictly applicable to the bearings used in the test machine. The equations presented were derived for oil bath lubricated bearings. Those used in the machine are lubricated with grease. However, the equations are approximately valid for grease lubrication if the viscosity of the grease's base oil is used in the equations.
Voltage | Current | Power
--- | --- | ---
240 V | 13.7 a | 360 W
230 V | 12.9 a | 280 W
220 V | 11.8 a | 240 W

Winding Resistance 0.190 Ω

Fig. 3.8 Synchronous Speed Test Data

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Full Load</th>
<th>3/4 Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
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<td>230 V</td>
</tr>
<tr>
<td>Current</td>
<td>36 a</td>
<td>26.4 a</td>
</tr>
<tr>
<td>Power</td>
<td>12600 W</td>
<td>11400 W</td>
</tr>
<tr>
<td>Slip</td>
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<td>1.50%</td>
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<tr>
<td>RPM</td>
<td>1756</td>
<td>1773</td>
</tr>
<tr>
<td>Winding Resistance</td>
<td>0.209 Ω</td>
<td>0.201 Ω</td>
</tr>
</tbody>
</table>

Fig. 3.9 Load Test Data
Another possible source of error is introduced by the uncertainty regarding the actual temperature of the lubricant, which is required in order to find the viscosity of the base oil. The temperatures of both the inner and outer bearing races were determined (see Section 4.3) and the temperature of the grease was considered to be something in between. However, this uncertainty is not serious since it was determined that the value obtained for \( LB \) varies by only ten percent as a result of assuming a temperature equal to that of one of the bearing races rather than the intermediate value and, as was said, the method is not too accurate to begin with, so extreme accuracy in determining the temperature is not justified.

It was first necessary to find the dynamic viscosity of the lubricant at its operating temperature. The information supplied by the manufacturer was; a mineral oil viscosity of 600 Saybolt universal seconds (SUS) at 100\(^\circ\)F (38\(^\circ\)C) and a mineral oil viscous index of 90. The viscous index is an indicator of the degree to which the viscosity is effected by temperature. From the viscosity at 100\(^\circ\)F and the viscous index it is possible to find the viscosity at 210\(^\circ\)F (99\(^\circ\)C) from tabulated standards. This viscosity was found to be 67 SUS at 210\(^\circ\)F. To find the viscosity at any temperature it is only necessary to interpolate between these two values. Since, in this range, viscosity in Saybolt seconds is very nearly linearly dependent on temperature, the following relation was obtained.

\[
\text{SUS} = - 8.74 \ (T) + 332 \quad (3.1)
\]

where \( T = \) temperature \( (^\circ\text{C}) \)

The temperature of the inner bearing race was found to be 58\(^\circ\)C and that of the outer bearing was 40\(^\circ\)C. An intermediate value of 50\(^\circ\)C was chosen for
this calculation. Substituting this into Eq. (3.1) gives a viscosity of 495 SUS. The kinematic viscosity, \( \nu \), in Stokes is related to the viscosity in Saybolt seconds by the equation \(^{(2)}\)

\[
\nu = 2.2 \text{ (SUS)} - \frac{180}{(\text{SUS})}
\]

(3.2)

hence

\[
\nu = 108 \text{ cSt}
\]

and

\[
\mu = \rho \nu
\]

(3.3)

where \( \mu \) = dynamic viscosity, cP
\( \rho \) = mass density, gm/cm\(^3\)

(the mass density of common mineral oils is approximately 0.9 gm/cm\(^3\))

and so

\[
\mu = 0.9 \text{ gm/cm}^3 \quad 108 \text{ cSt} = 98 \text{ cP} = 9.9 \times 10^{-9} \text{ kgf-sec/mm}^2
\]

Although the two bearings in the machine are not identical it is assumed, as in the model, that the temperatures of both are the same and so the value for \( \mu \) will be used for both.

In order to find the frictional torques of the bearings it is also necessary to know the loadings of the bearings. The loading may be divided into two components, a radial component, \( F_r \), and an axial or thrust component, \( F_a \). For this application there is no thrust on the motor shaft so \( F_a = 0 \). \( F_r \) was determined by weighing the rotor with each bearing resting on a scale. The values obtained were \( F_{rA} = 13.2 \text{ kgf} \) and \( F_{rB} = 15.7 \text{ kgf} \). Where the subscripts A and B refer to the two bearings.
It is now possible to calculate the frictional torque of the bearings. This torque has been divided into two parts. The first, $\tau_0$, is the running resistance or speed dependent part. The second, $\tau_1$, is the loading resistance which depends on bearing load but is independent of speed.

$$\tau_0 = f_0 \, p \, d_m^3 \left( \frac{u \omega}{p} \right)^{2/3}$$

(3.4)

where $\tau_0 = \text{torque (kg-mm)}$

$f_0 = \text{a constant depending on bearing type. For single row deep groove ball bearings with no filling slot, } f_0 = 1.5$

$p = \text{atmospheric pressure} = 1.0 \times 10^{-2} \text{ kgf/mm}^2$

$d_m = \text{pitch circle diameter of the rolling elements (see Fig. 3.10)}$

$d_{mA} = 54 \text{ mm, } d_{mB} = 74 \text{ mm}$

$\omega = \text{angular velocity of rotation} = 184 \text{ sec}^{-1}$

Substituting the given values into the equation gives

$$\tau_{OA} = 1.5(1.0 \times 10^{-2} \text{ kgf/mm}^2)(54 \text{ mm})^3 \left( \frac{9.9 \times 10^{-9} \text{ kgf-sec/mm}^2 \text{ 184 sec}^{-1}}{1.0 \times 10^{-2} \text{ kgf/mm}^2} \right)^{2/3}$$

$= 7.6 \text{ kgf-mm}$

and similarly

$$\tau_{OB} = 19.5 \text{ kgf-mm}$$

The calculation of $\tau_1$ requires two preliminary results which will now be obtained. They are, $P_0$, the static equivalent bearing load and $C$, the basic static load rating.

$$P_0 = X_0 \, F_r + Y_0 \, F_a \geq F_r$$

(3.5)

where $X_0$ and $Y_0$ are factors dependent on bearing type and are equal to 0.6
BEARING DIMENSIONS AND FORCES

FIG. 3.10
and 0.5 respectively. With the values of $F_r$ and $F_a$ substituted into Eq. (3.5), $P_0$ is seen to take the value of its lower limit, $F_r$, so,

$$P_{0A} = 13.2 \text{ kgf} \quad P_{0B} = 15.7 \text{ kgf}$$

$$C_0 = 0.2 \ k_0 \ n \ cos \alpha \ d_w^2 \quad (3.6)$$

where $k_0$ = a parameter dependent on bearing type = 6.2 kgf/mm$^2$

$n$ = number of rolling elements, $n_A = 9 \quad n_B = 8$

$d_w$ = ball diameter, $d_{WA} = 10 \text{ mm}, \ d_{WB} = 13 \text{ mm}$

so substituting these values

$$C_{0A} = 0.2 \ (6.2 \text{ kgf/mm}^2)(9)(\cos 0)(10 \text{ mm})^2 = 1100 \text{ kgf}$$

and

$$C_{0B} = 1700 \text{ kgf}$$

Now the calculation of

$$\tau_1 = f_1 \ g_1 \ P_0 \ d_m \quad (3.7)$$

where $f_1$ is a factor depending on bearing design and relative load.

For radial groove ball bearings,

$$f_1 = 0.0009 \left(\frac{P_0}{C_0}\right)^{0.55}, \quad f_{1A} = 7.9 \times 10^{-5}, \quad f_{1B} = 6.8 \times 10^{-5} \quad (3.8)$$

$g_1$ is a factor depending on direction of load. For this type of bearing in this application

$$g_1 = \frac{F_r}{P_0} = 1$$
so

\[ \tau_{1A} = 7.9 \times 10^{-5} (1) (13.2 \text{ kgf}) (54 \text{ mm}) = 0.056 \text{ kgf-mm} \]

and

\[ \tau_{1B} = 0.079 \text{ kgf-mm} \]

The total frictional torque of a bearing is simply the sum of \( \tau_0 \) and \( \tau_1 \) so,

\[ \tau_A = \tau_{OA} + \tau_{1B} = 7.6 \text{ kgf-mm} + 0.056 \text{ kgf-mm} = 7.66 \text{ kgf-mm} = 0.075 \text{ N-m} \]

and

\[ \tau_B = \tau_{OB} + \tau_{1B} = 19.5 \text{ kgf-mm} + 0.079 \text{ kgf-mm} = 19.6 \text{ kgf-mm} = 0.192 \text{ N-m} \]

The power dissipated in a body rotating at angular velocity, \( \omega \), against a torque, \( \tau \), is given by the product of \( \tau \) and \( \omega \) so

\[ L_{BA} = \omega \tau_A = 184 \text{ sec}^{-1} \times 0.075 \text{ N-m} = 14 \text{ W} \]

and

\[ L_{BB} = \omega \tau_B = 184 \text{ sec}^{-1} \times 0.192 \text{ N-m} = 35 \text{ W} \]

so

\[ L_B = L_{BA} + L_{BB} = 49 \text{ W} \]

3.2.2.2 Calculation of the Windage Losses

The approach that was used to calculate the windage losses was to obtain the total mechanical losses, \( L_B + L_W \), using the standard I.E.E.E. procedure, and since \( L_B \) is now known, \( L_W \) may be easily found.
The I.E.E.E. procedure makes use of the fact that the mechanical losses are speed dependent while all other losses depend on the voltage and current. With constant speed, if the voltage could be reduced to zero all electrical losses would be zero and only the speed dependent mechanical losses would remain. Obviously this cannot be achieved in practice. It can, however, be approached. At no load the speed may be assumed to be independent of voltage. This is in fact not true but even at as low as 20V the slip is 1.04% which corresponds to a speed of 1781 rpm. A drop of only about one percent from the speed at 230V.

In view of this, the no load test data (sec. 3.2.1.1) may be taken as representing the desired conditions of constant speed with a voltage approaching zero. Since the voltage cannot go to zero it was necessary to extrapolate the data to zero volts. If this is done with the power curve, where this curve reaches the zero volts line will be the total power lost to mechanical losses. Since power is proportional to the voltage squared, it was easier and more accurate to plot power against the square of the voltage. The method of least squares was used to obtain the best linear fit to the no load power versus voltage squared data, Fig. 3.7. From this, the Y-intercept, equal to the sum of the mechanical losses, was found to be 55 watts. Hence

\[ LW = \text{Total Mechanical Losses} - LB = 55W - 49W = 6W \]

The value obtained for the total mechanical losses seemed to be unusually low. In order to check this, the dc machine was used to drive the test machine at operating speed. The power input to the dc machine at no load and when loaded by the test machine were compared in order to find the power required to turn the test machine, this being the sum of the friction
and windage losses. The value obtained verified the previous results.

3.2.3 Calculation of the Electrical Losses

With the data obtained from the electrical tests described in Section 3.2.1 and knowing the mechanical losses, it is now possible to calculate all other losses in the machine. The following is a description of the calculations and the theory of the assumptions on which some of them are based.

- LCS: The Stator Copper Losses

This is simply the $I^2R$ losses in the stator conductor. Since the resistance measured is the terminal to terminal resistance of a star connected winding it must be divided by two in order to obtain the resistance per phase. The $I^2R$ losses per phase must then be multiplied by three, the number of phases, in order to obtain the total copper losses in the stator. This, then, results in the factor of $3/2$ in the equation for LCS.

From the full load current and winding resistance:

\[ LCS = \frac{3}{2} (26.4a)^2 (0.201 \, \Omega) = 210 \, \text{watts at 3/4 load} \quad (3.8a) \]

\[ LCS = \frac{3}{2} (12.8a)^2 (0.190 \, \Omega) = 46.7 \, \text{watts at no load} \quad (3.8b) \]

- LI: The Total Iron Losses

This is not one of the required inputs to the model but is necessary in order to calculate LCR and LIR as discussed below. In order to calculate LI from the information now available it is necessary to first realize that even at no load currents the core saturates and hence the iron losses at no load have already attained their full load value. Since at no load the measured stator input power (MSI) is the total losses in the machine, it is true that:
The power transferred to the rotor is dependent on the changing flux caused by the relative motion of the rotor and the magneto motive force traveling wave induced by the stator current. This relative motion is characterized by the slip and for zero slip, i.e., synchronous speed, there is no power transferred. Because the slip is so low at no load there is very little power transferred to the rotor and so LCR is small and may be ignored for this calculation without introducing serious error. With this in mind, Eq. (3.9) may be written as:

\[ LI = MSI - LCS - (LB + LW) \]  \hspace{1cm} (3.10)

where all values are taken at no load and rated voltage. LCS was calculated in Eq. (3.8b) and LB + LW was found in Section 3.2.2.2 and, as was pointed out, is essentially independent of load. The total losses, given by the no load MSI, was found in the no load test. Substituting these values into Eq. (3.10) gives

\[ LI = 500 - 46.7 - 55 = 398W \]  \hspace{1cm} (3.10a)

This value is valid for all loads for the reason given above.

- **LCR**: The Rotor Copper Losses

Unlike the situation at no load, the slip at full load and 3/4 load is large enough that the power transferred to the rotor is significant. The method used for the determination of LCR uses a modification of the I.E.E.E. procedures. The I.E.E.E. method neglects the effects of iron losses and mechanical losses. The method was modified to take these into account. The equation was
\[
LCR = (MSI - LCS - LI - (LB + LW)) \times S \quad (3.11)
\]

Substituting the full and 3/4 load values obtained for LCS, LI and LB + LW and the values for MSI and S obtained in the tests at load (Section 3.2.1.3)

\[
LCR = (12,600 - 406 - 398 - 55) \times 0.0242 = 284W \text{ at full load} \quad (3.11a)
\]

\[
LCR = (11,400 - 210 - 398 - 55) \times 0.0150 = 161W \text{ at 3/4 load} \quad (3.11b)
\]

- LIS: The Stator Iron Losses

The results of the synchronous speed test are used here to find the stator iron losses. Because the rotor is being turned at synchronous speed it is matching the speed of the magneto motive force traveling wave induced by the stator current. Hence, there is no changing flux seen by the rotor and so no current flows and no power is dissipated in the rotor. Also, since the rotor is being driven by the dc machine, that machine is supplying the power dissipated by friction and windage. The only losses remaining now that LCR, LIR, and LB + LW have been eliminated are LCS and LIS. The sum of these two losses is the synchronous speed MSI at rated voltage. LCS under these conditions is easily calculated from the current and winding resistance as was done previously. The value of LCS is:

\[
LCS = \frac{3}{2} I^2 R = \frac{3}{2} (12.9a)^2 (0.190 \Omega) = 47.4W \quad (3.12)
\]

and using this and the synchronous speed MSI:

\[
LIS = MSI - LCS = 280 - 47.4 = 233W \quad (3.13)
\]

This value is valid for all loads since the iron saturates even at this low current.
-LIR: The Rotor Iron Losses

This is calculated simply by subtracting the value just obtained for LIS from the value for LI, the total iron losses, found earlier.

\[
LIR = LI - LIS = 398 - 233 = 165W \quad (3.14)
\]

Again, this is the value for all loads.

3.2.4 Total Losses and Efficiency

The total losses may now be found simply by adding the individual contributions calculated in the two previous sections.

\[
L = LCS + LCR + LI + (LB + LW) \quad (3.15)
\]

at full load

\[
L = 406 + 284 + 398 + 55 = 1143W \quad (3.15a)
\]

and at 3/4 load

\[
L = 210 + 161 + 398 + 55 = 824W \quad (3.15b)
\]

The efficiency may be calculated as a check of the values obtained for the total losses in Eq. (3.15a). Using the equation

\[
EFF. = \frac{MSI - L}{MSI} \times 100 = \frac{12,600W - 1143W}{12,600W} \times 100 = 91\% \quad (3.16)
\]

This value agrees reasonably well with the designed efficiency given by the manufacturer.

3.3 Thermal Properties

The thermal conductivities provided as inputs to the model, the materials used and the values\(^{(3)}\) appear below.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Material</th>
<th>Thermal Conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>KIS</td>
<td>'Formvar' polyvinyl formal</td>
<td>0.00393 w/in⁻⁰°C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00155 w/cm⁻⁰°C</td>
</tr>
<tr>
<td>KIR</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>KEIS</td>
<td>Nomex polyamide</td>
<td>0.00619</td>
</tr>
<tr>
<td>KEIR</td>
<td>None</td>
<td>0.00244</td>
</tr>
<tr>
<td>KCS</td>
<td>Copper</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.94</td>
</tr>
<tr>
<td>KCR</td>
<td>Aluminum</td>
<td>6.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.40</td>
</tr>
<tr>
<td>KSZ, KRZ</td>
<td>Magnetic sheet steel laminations</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0197</td>
</tr>
<tr>
<td>KSR, KRR</td>
<td>Magnetic sheet steel laminations</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.500</td>
</tr>
</tbody>
</table>

KIR and KEIR do not apply to squirrel cage induction motors.

There was no information regarding KSZ and KRZ, the axial thermal conductivity of the stator and rotor cores for the specific construction used. This value depends on the thickness of the laminations and inter-laminar insulation and on the clamping pressure used when assembling the core. The value given is typical of the type of sheet used. The error introduced in the final temperatures predicted by the model should not be very large. This is because the thermal conductances KRZ and KSZ are in parallel with the much higher thermal conductances of the shaft and rotor bars of the rotor and the frame and windings of the stator.

The thermal conductivity of air, KA, is determined in the model for the temperature in the area of interest. This was necessary on account of the importance of this value in the model's calculations and its relatively strong dependence on temperature.

A typical value for the thermal conductivity of steel was used for KF and KSH.
3.4 Miscellaneous Inputs

Of the inputs listed in Section 2.3.4, those applicable to this particular case are the ambient temperature, the rotational speed, the flow rate of coolant and the film coefficient for convection from the exterior of the machine.

The ambient temperature, $T_{AMB}$, was simply taken with a thermometer a few feet from the machine. This measurement was taken along with the machine temperatures while the machine was running at load. The values obtained were influenced by the load since the air in the room was heated by the machine and load resistors. The values ranged from $24^\circ$ at no load to $28^\circ$ at full load and they may be found with the temperature data of the next chapter.

The rotational speed was determined from the slip as measured with the strobe and stop watch (see Section 3.2.1). The rotational speed, $\omega$, is given by:

$$\omega = 2\pi \cdot 30 \text{ rps} \ (1 - S) \quad (3.17)$$

Substituting the values of $S$ at various loads:

$$\omega = 2\pi \cdot 30 \text{ rps} \ (1 - 0.0242) = 184 \text{ sec}^{-1} \quad (3.17a)$$

at 3/4 load

$$\omega = 2\pi \cdot 30 \text{ rps} \ (1 - 0.0150) = 186 \text{ sec}^{-1} \quad (3.17b)$$

The flow rate of coolant, $W_0$, and the film coefficient for the frame, $h_F$, are not independent values. $W_0$ and $h_F$ are each terms in equations describing two of the three ways in which heat may be transferred out of the machine. These heat transfer modes are the heat exchange to the coolant and
convection, and are described by Eqs. (3.18) and (3.19). The third mode of heat transfer out of the machine is radiation which, for this particular case, is described by Eq. (3.20).

\[ Q = C_p \cdot \rho \cdot W_0 \cdot (T_{OUT} - T_{IN}) \]  \hspace{1cm} (3.18)

\[ Q = h_F \cdot A_F \cdot (T_F - T_{AMB}) \]  \hspace{1cm} (3.19)

\[ Q = \sigma \cdot A_F \cdot (\theta_F^4 - \theta_{AMB}^4) \]  \hspace{1cm} (3.20)

where

- \( Q \) = heat flow (W)
- \( C_p \) = specific heat of air (J/kg °C)
- \( \rho \) = mass density of air (kg/m³)
- \( W_0 \) = flow rate (m³/sec)
- \( h_F \) = film coefficient (W/m² °C)
- \( \sigma \) = Stefan-Boltzman constant = 5.67 x 10⁻⁸ W/m² °K⁴
- \( A_F \) = surface area of frame (m²)
- \( T_{OUT} \) = output temperature of coolant (°C)
- \( T_{IN} \) = input temperature of coolant (°C)
- \( T_F \) = average temperature of the frame (°C)
- \( T_{AMB} \) = ambient temperature (°C)
- \( \theta_F \) = average temperature of the frame (°K)
- \( \theta_{AMB} \) = ambient temperature (°K)

If, as was assumed in the model, these are the only modes of heat transfer out of the machine, then the sum of the heat flow due to each of these modes must equal the total losses, \( L \), in the machine. Equations (3.18), (3.19) and (3.20) may be combined as Eq. (3.21) giving the relation between \( W_0 \) and \( h_F \) in terms of values which may be found.
The terms of this equation may be found as follows. All temperatures were found by the temperature measurements to be described in Chapter 4. The values obtained were

<table>
<thead>
<tr>
<th></th>
<th>Full Load</th>
<th>3/4 Load</th>
<th>No Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{OUT}$</td>
<td>51°C</td>
<td>44°C</td>
<td>30°C</td>
</tr>
<tr>
<td>$T_{IN}$</td>
<td>26°C</td>
<td>28°C</td>
<td>24°C</td>
</tr>
<tr>
<td>$T_F$</td>
<td>47°C</td>
<td>41°C</td>
<td>40°C</td>
</tr>
<tr>
<td>$T_{AMB}$</td>
<td>28°C</td>
<td>28°C</td>
<td>24°C</td>
</tr>
<tr>
<td>$\theta_F$</td>
<td>320°C</td>
<td>314°C</td>
<td>313°C</td>
</tr>
<tr>
<td>$\theta_{AMB}$</td>
<td>301°C</td>
<td>301°C</td>
<td>297°C</td>
</tr>
</tbody>
</table>

- $C_p$ is temperature dependent and the value corresponding to the mean value of $T_{OUT}$ and $T_{IN}$ should be used. The values obtained were

$$C_p = 1014 \text{ joules/kg°C at full load}$$
$$C_p = 1013 \text{ joules/kg°C at 3/4 load}$$
$$C_p = 1011 \text{ joules/kg°C at no load}$$

- $\rho$ is temperature dependent but may be considered constant in this range of temperatures

$$\rho = 1.1774 \text{ kg/m}^3$$

- $A_F$ may be easily determined from the machine dimensions

$$A_F = 2\pi(RF)(LT) + 2\cdot2\pi(RF)^2$$

$$A_F = 0.579 \text{ m}^2$$
- L was found in Section 3.2.4 for full load and 3/4 load and in the no load test for that condition.

\[ L = 1143 \text{ watts at full load} \]
\[ L = 824 \text{ watts at 3/4 load} \]
\[ L = 500 \text{ watts at no load} \]

Substituting these values into Eq. (3.21) and simplifying yields the following three linear equations in \( h_F \) and \( W_0 \).

From the full load values:

\[ h(11.001) + W_0(27,459) = 1068 \quad (3.23) \]

From the 3/4 load values:

\[ h(7.527) + W_0(19,083) = 774 \quad (3.24) \]

and from the no load values:

\[ h(9.264) + W_0(7,142) = 440 \quad (3.25) \]

From this point two courses of action are available. The first is to solve pairs of these equations for \( h_F \) and \( W_0 \). This may be done since neither \( h_F \) or \( W_0 \) is temperature dependent to a significant degree and the speed, which controls \( W_0 \), is essentially constant. The alternative is to use the theoretical methods of the program to obtain a value for \( h_F \). This value may then be used in conjunction with the equation for the load of interest to find the corresponding value for \( W_0 \).

In solving for \( h_F \) and \( W_0 \), three possible pairs of Eqs. (3.23), (3.24) and (3.25) may be chosen. It happens, however, that Eqs. (3.23) and (3.24) define two nearly parallel lines and hence are extremely sensitive to errors in determining the temperatures. The values obtained by solving (3.23) and (3.25) or (3.24) and (3.25) are quite reasonable and are in the
range predicted by theory. It is these two values which may be used as input to the model. The values obtained are:

From Eqs. (3.23) and (3.25) \[ h_F = 25.37 \text{ W m}^{-2} \text{C} \quad W_O = 0.0287 \text{ m}^3/\text{sec} \]

From Eqs. (3.24) and (3.25) \[ h_F = 23.35 \text{ W m}^{-2} \text{C} \quad W_O = 0.0314 \text{ m}^3/\text{sec} \]

Performing an error analysis to determine the sensitivity of the last two sets of values to errors in measuring the temperatures showed that these values are fairly sensitive to any errors. It was determined that the worst case is an error in \( T_{OUT} \) or \( T_{IN} \). The values obtained by introducing a 1°C change in the value of \( T_{OUT} \) for full load and solving Eqs. (3.23) and (3.25) are

\[ h_F = 30.35 \text{ W m}^{-2} \text{C} \quad W_O = 0.0267 \text{ m}^3/\text{sec} \]

A change of about 20% in \( h_F \). If a similar calculation is made changing the full load \( T_F \) (which is probably the least accurate of the temperatures) the values obtained are:

\[ h_F = 27.90 \text{ W m}^{-2} \text{C} \quad W_O = 0.0277 \text{ m}^3/\text{sec} \]

Because of the sensitivity to errors in the temperatures and the fact that the temperatures are accurate to only ± 1°C, the alternative of obtaining \( h_F \) by theoretical means may be more desirable. Both methods were tried. Because of the construction of the machine \( W_2 = W_O \) and \( W_1 = 0 \).

Two additional inputs were supplied in order to tailor the model to the test machine. The first is the dimensions of the mounting pads which connect the stator core to the frame. This information allowed a more accurate value for the thermal resistance between nodes 4 and 9 of the equivalent network to be determined. The values obtained were a thickness from stator core to frame of 0.83 in. and a total cross section area for all
12 pads of 36.38 in. The second input was a rough estimate of the mean length of the coolant path through the machine. This made a more accurate calculation of the heat transferred to the coolant possible. The length was found to be about 8 in.

### 3.5 Summary of Input Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OR</td>
<td>0.65 in.</td>
</tr>
<tr>
<td>OS</td>
<td>2.50 in.</td>
</tr>
<tr>
<td>LT</td>
<td>16.60 in.</td>
</tr>
<tr>
<td>L</td>
<td>5.25 in.</td>
</tr>
<tr>
<td>RSH</td>
<td>1.18 in.</td>
</tr>
<tr>
<td>R</td>
<td>3.23 in.</td>
</tr>
<tr>
<td>G</td>
<td>0.051 in.</td>
</tr>
<tr>
<td>GS</td>
<td>0.053 in.</td>
</tr>
<tr>
<td>DS</td>
<td>0.98 in.</td>
</tr>
<tr>
<td>RF</td>
<td>6.25 in.</td>
</tr>
<tr>
<td>TB</td>
<td>0.24 in.</td>
</tr>
<tr>
<td>TA</td>
<td>0.83 in.</td>
</tr>
<tr>
<td>TF</td>
<td>0.65 in.</td>
</tr>
<tr>
<td>AS</td>
<td>0.155 sq.in.</td>
</tr>
<tr>
<td>AR</td>
<td>0.130 sq.in.</td>
</tr>
<tr>
<td>NS</td>
<td>528 turns/phase</td>
</tr>
<tr>
<td>NR</td>
<td>48 bars</td>
</tr>
<tr>
<td>DR</td>
<td>1.13 in.</td>
</tr>
<tr>
<td>H</td>
<td>3.63 in.</td>
</tr>
<tr>
<td>KS</td>
<td>0.557</td>
</tr>
<tr>
<td>KR</td>
<td>0.377</td>
</tr>
<tr>
<td>P</td>
<td>2 pole pairs</td>
</tr>
<tr>
<td>SS</td>
<td>36 slots</td>
</tr>
<tr>
<td>SR</td>
<td>48 slots</td>
</tr>
<tr>
<td>TXS</td>
<td>0.015 in.</td>
</tr>
<tr>
<td>TXR</td>
<td>N/A</td>
</tr>
<tr>
<td>TYS</td>
<td>0.015 in.</td>
</tr>
<tr>
<td>TYR</td>
<td>N/A</td>
</tr>
<tr>
<td>WS</td>
<td>0.42 in.</td>
</tr>
<tr>
<td>WR</td>
<td>0.16 in.</td>
</tr>
<tr>
<td>WX</td>
<td>0.042 in.</td>
</tr>
<tr>
<td>WY</td>
<td>0.042 in.</td>
</tr>
<tr>
<td>U</td>
<td>0.013 in.</td>
</tr>
<tr>
<td>GR</td>
<td>0.010 in.</td>
</tr>
<tr>
<td>WXR</td>
<td>N/A</td>
</tr>
<tr>
<td>WYR</td>
<td>N/A</td>
</tr>
<tr>
<td>UR</td>
<td>N/A</td>
</tr>
</tbody>
</table>

- **KIS** = 0.155 watts/m°C
- **KIR** = N/A
- **KEIS** = 0.244 watts/m°C
- **KEIR** = N/A
- **KCS** = 394 watts/m°C
- **KCR** = 240 watts/m°C
- **KSZ, KRZ** = 1.97 watts/m°C
- **KSR, KRR** = 50 watts/m°C
- **WO** = 0.0287 m³/sec
- **W1** = 0
- **W2** = 0.0287 m³/sec
- **TAMB** = 28°C
- **ω** = 184 sec⁻¹
- **h_f** = 25.37 watts/m²°C
- **LCS** = 406 watts
- **LCR** = 284 watts
- **LIS** = 233 watts
- **LIR** = 165 watts
- **LB** = 49 watts
- **LW** = 6 watts
CHAPTER 4
TEMPERATURE MEASUREMENTS

The second part of this work is the measurement of the actual operating
temperatures of the various parts of the machine. Three methods of thermom-
etry were used and are described below. Some problems were encountered in
obtaining the temperatures at the points or bodies which were specified by
the model. In those cases where it proved too difficult to obtain the tem-
perature of the desired point the temperature of some nearby point in more
or less good thermal contact with the point of interest was taken. These
instances will be discussed in Section 4.2

4.1 Temperature Measurement Techniques

Digital thermometer, thermocouple and resistance techniques were used
to measure the temperatures of the machine. Because the expected accuracy
of the model's temperature predictions was in the range of ±5°C to ±10°C it
was not considered necessary to use particularly refined procedures in apply-
ing these three methods of thermometry.

4.1.1 Digital Thermometer

The digital thermometer used was of the semiconductor type which uses
the temperature dependence of the characteristics of a transistor's base-
 emitter junction to define the temperature.

This instrument was used to find the temperature of the reference junc-
tion of the thermocouples and to measure the air temperatures in the machine.
The thermometer, of course, indicates the temperature of its probe. The ac-
curacy was checked in ice water and boiling water (the thermal contact be-
tween the water and the probe may be assumed to be so good that the probe
is in fact at the temperature of the water) and was found to be better than 
±0.5°C. The degree to which the temperature of the probe reflects the tem-
perature of the air is open to question. Heat conducted away through the
body of the probe or lost by radiation to the surrounding surfaces if they
are colder than the probe would cause a low reading. Alternatively, if the
surrounding surfaces are hotter than the probe the heat gained by radiation
may cause a high reading. These effects are reduced to some extent by the
velocity of the flow past the probe. The net error should not be too large
and the accuracy of the temperatures is undoubtedly adequate for this work.

4.1.2 Thermocouples

Thermocouples were used for many of the temperature measurements because
of the ease with which they can be placed in even relatively inaccessible
places, such as the rotor or embedded stator windings. Also, they do not
interfere with the operation of the machine and may be left in place while
the machine is running.

There are three fundamental limitations to thermocouple thermometry\(^{(1)}\). The first of these limitations is that the temperature that is actually being
measured is the temperature of the bimetallic junction. This temperature
may or may not be the same as that of the point of interest. This problem
is very likely the greatest source of error for the measurements discussed
here. Unfortunately, it is also the most difficult source of error to
quantify. All that can really be done is to attempt to minimize the error.
This generally involves making good thermal contact with the point of in-
terest. For this work it was considered satisfactory to hold the thermo-
couples in place with fiberglass tape or epoxy and covered with a patch of
asbestos felt. Those thermocouples which were placed on the frame and end
bell were held in place beneath the heads of screws set into the frame and end bell. This undoubtedly produces a better thermal contact but this technique could not be used elsewhere since the cores and windings cannot be drilled and tapped. The other two limitations involved concern the accuracy with which the temperature of the junction itself may be determined.

Thermocouple thermometry is based on measurement of the differential potential between two bimetallic junctions created by their temperature difference. The occurrence of this potential is known as the Seebeck effect. Clearly any error in measuring the potential difference will lead to a corresponding error in the temperature. Also, since the difference in temperature is all that can really be determined it is necessary to know the temperature of one junction in order to find the temperature of the other. The result of this is that any error in determining the temperature of the reference junction will cause an equal error in the temperature found for the measuring junction. The usual practice when great accuracy is required is to immerse the reference junction in ice water slush. This was not deemed necessary and the temperature of the reference junction was simply measured with the digital thermometer as discussed in Section 4.1.

The temperature of the reference junction was determined to an accuracy of ±0.5°C. The potential was measured with a digital voltmeter to an accuracy of ±0.01 mv which corresponds to about ±0.25°C. Therefore, it seems reasonable to say that the temperature of the junction was measured to ±1°C. The actual accuracy obtained in measuring the temperature of the point of interest is probably not this good due to the difference in temperature that may exist between the junction and the point of interest.

The average temperature of the frame and end bells was obtained in a single measurement by connecting the several thermocouples attached to the
frame and end bell in parallel. Because all of the thermocouple extensions were the same length each had the same resistance. So the potential difference of the parallel combination was the average of the potential differences of each one separately and therefore was indicative of the average temperature $(T)$. 

4.1.3 Resistance

The average temperature of the stator winding was found by observing the change in resistance occurring when the winding temperature changed from some known temperature to the operating temperature. The temperature-resistance relation for copper is given by

$$T_H = \frac{R_H}{R_C} (234.5^\circ C + T_C) - 234.5^\circ C \quad (4.1)$$

where

- $T_H =$ hot temperature $(^\circ C)$
- $T_C =$ cold temperature $(^\circ C)$
- $R_H =$ hot resistance $(\Omega)$
- $R_C =$ cold resistance $(\Omega)$

$R_H$ and $R_C$ were measured as described in Section 3.2.1. $T_C$ was found by allowing the machine several hours to come to ambient temperature throughout. At this time the corresponding resistance, $R_C$, was measured. $R_H$ was measured as soon as possible after shutdown. Using the values of these three parameters, $T_H$ could then be calculated.

The resistances were calculated from voltage and current measurements which were readable to $\pm 0.005V$ and $\pm 0.025A$ respectively. This corresponds to an accuracy of about $\pm 0.002\Omega$ in the range of interest. This and the accuracy of the measurement of $T_C (\pm 0.5^\circ C)$ lead to a possible error of
±7°C for \( T_H \). There was no convenient way to obtain a more accurate measurement of temperature and since the model does not predict the average temperature of the winding this was considered adequate.

4.2 The Locations Selected for Measurement

Three air temperatures were measured within the machine. These were: the overhang air at a point near the center of the lower quadrant of the overhang region, the axial duct at a point about 1/3 of the way from the overhang region to the circumferential duct, the exhaust air near the center of the exhaust port. These locations are shown in Fig. 4.1 as positions 1, 2 and 3 respectively. Locations 1 and 2 were selected in order to check the model's prediction for \( X_4 = \theta_{11} \), the temperature of the air in the overhang region. The temperature given by the model is for the air as it is about to pass from the overhang region into the axial duct and so the value predicted should fall between the temperatures measured at positions 1 and 2. The temperature of the exhaust air, taken at position 3, was required for the calculation of the film coefficient for the frame and the flow rate of coolant as was done in Section 3.4.

The average temperature of the frame and end shield was required in the calculation of the film coefficient for the frame and the flow rate of coolant. The average temperature was obtained by placing eight thermocouples on the frame and end bell and connecting them in parallel. The average temperature was then found from the potential of the parallel combination. The locations of these thermocouples is indicated by positions 4 in Fig. 4.1. The average temperature obtained for the frame and end bell will also provide a check of the model predictions \( X_3 = \theta_9 \), the temperature of the frame and \( X_7 = \theta_{14} \), the temperature of the end bell.
TEMPERATURE MEASUREMENT LOCATIONS

FIG. 4.1 a
TEMPERATURE MEASUREMENT LOCATIONS

FIG. 4.1b
Thermocouple positions 6, 8, 10 and 11 were chosen to correspond with the temperatures predicted for $X_8 = \theta_{16}$, the stator core end face, $X_9 = \theta_{\text{ext} 2}$, the exterior of the stator end winding, $X_5 = \theta_{12-17}$, the shaft and rotor end, and $X_6 = \theta_{13}$, the temperature of the inner bearing race and shaft end. These four thermocouples were placed in precisely the locations of the model's predictions so there should be good agreement between the experimental and theoretical temperatures. Drilling into the stator core and placing a thermocouple on the axial center line of a bundle of conductors in the stator winding was impossible. Therefore, in order to be able to provide a rough check on the model's predictions for $X_1 = \theta_{3-4}$, the temperature of the stator core and teeth, and for the temperatures calculated for the center line of the winding, thermocouples were placed at positions 5 and 7. The thermocouple at position 7 was slid under the slot liner from the end so that it is between the bundle of conductor and the slot liner. Because of the locations good agreement was not expected between the temperatures measured at locations 5 and 7 and the temperatures for the stator core and winding center line. The thermocouple at position 9 should provide a good check of the prediction for $X_2 = \theta_{7-8}$, the rotor core. Because of the high thermal conductivity of the rotor core and the absence of anything that might influence the temperature of the thermocouple, there should be good agreement between the experimental and theoretical temperatures. The final thermocouple location is at position 12 on the outer bearing race. This temperature and the temperature obtained at position 11 were required in order to find the viscosity of the bearing lubricant as in Section 3.2.2.1.

The average temperature of the windings was used in conjunction with the temperatures of locations 7 and 8 to check the prediction of the winding temperatures along the winding's axial center line. The maximum temperatures
of the windings are expected along the center line, the predicted temperature should be a good deal higher than the average and surface temperatures obtained experimentally.

4.3 Measurement Results

The table below shows the temperatures measured at each location for the specified load.

<table>
<thead>
<tr>
<th>Location</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full Load</td>
</tr>
<tr>
<td>1. Overhang air</td>
<td>37</td>
</tr>
<tr>
<td>2. Axial duct air</td>
<td>49</td>
</tr>
<tr>
<td>3. Exhaust air</td>
<td>51</td>
</tr>
<tr>
<td>4. Frame and end seal</td>
<td>47</td>
</tr>
<tr>
<td>5. Stator core</td>
<td>67</td>
</tr>
<tr>
<td>6. Stator end</td>
<td>62</td>
</tr>
<tr>
<td>7. Stator embedded winding</td>
<td>61</td>
</tr>
<tr>
<td>8. Stator end winding</td>
<td>52</td>
</tr>
<tr>
<td>9. Rotor core</td>
<td>94</td>
</tr>
<tr>
<td>10. Rotor end and shaft</td>
<td>83</td>
</tr>
<tr>
<td>11. Inner bearing race</td>
<td>58</td>
</tr>
<tr>
<td>12. Outer bearing race</td>
<td>40</td>
</tr>
<tr>
<td>Winding (by resistance)</td>
<td>65</td>
</tr>
<tr>
<td>Ambient</td>
<td>28</td>
</tr>
</tbody>
</table>
CHAPTER 5

THE RESULTS

The agreement between the temperatures predicted by the model and those measured turned out to be much better than expected. In only two cases were there any great discrepancies. This chapter contains a general discussion of the possible sources of differences between the predicted and measured temperatures. Following this is a detailed comparison of the theoretical and experimental values.

5.1 Sources of Discrepancy

There are three sources of disagreement between the predicted and measured temperatures. The first of these is the possibility of error in the input values to the model. Of these inputs, the most likely to be in error are the losses. Everything else was obtained in a more or less straightforward way from the manufacturer's drawings or published data. It should be noted that the effects of an error in the losses would be quite wide spread, affecting a number of temperatures. On the other hand, an error in, say, a thermal conductivity would have more localized effects. The second possible source of disagreement is the model itself. This would be difficult to determine except by eliminating all other possibilities because there is no characteristic error that can be associated with the model in general. The third source of discrepancies are errors in the temperature measurements. These would, of course, be localized. The best indication of any errors in the temperatures would be any inconsistencies between adjacent temperatures. In the comparison of the predicted and measured temperatures these sources of error will be considered and any major discrepancies will
be examined to determine their cause.

5.2 Comparison of Predicted and Measured Values

The solution of the system of equations described in Section 2.5 for the unknowns $X_1$ through $X_9$ which correspond to the temperatures of the machine elements was carried out. The solutions obtained and the most nearly corresponding measured temperatures are tabulated below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Predicted Temperatures</th>
<th>Measured Temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = \theta_{3-4}$</td>
<td>Average 73°C, Maximum 76°C</td>
<td>Position 5, Value 67°C</td>
</tr>
<tr>
<td>$X_2 = \theta_{7-8}$</td>
<td>Average 94°C, Maximum 96°C</td>
<td>Position 9, Value 94°C</td>
</tr>
<tr>
<td>$X_3 = \theta_9$</td>
<td>49°C</td>
<td>Position 4, Value 47°C</td>
</tr>
<tr>
<td>$X_4 = \theta_{11}$</td>
<td>46°C</td>
<td>Position 1, Value 37°C</td>
</tr>
<tr>
<td>$X_5 = \theta_{12-17}$</td>
<td>92°C</td>
<td>Position 10, Value 83°C</td>
</tr>
<tr>
<td>$X_6 = \theta_{13}$</td>
<td>58°C</td>
<td>Position 11, Value 58°C</td>
</tr>
<tr>
<td>$X_7 = \theta_{14}$</td>
<td>39°C</td>
<td>Position 4, Value 47°C</td>
</tr>
<tr>
<td>$X_8 = \theta_{16}$</td>
<td>66°C</td>
<td>Position 6, Value 62°C</td>
</tr>
<tr>
<td>$X_9 = \theta_{\text{ext} 2}$</td>
<td>66°C</td>
<td>Position 8, Value 52°C</td>
</tr>
</tbody>
</table>

- $X_1 = \theta_{3-4}$ = Average 73°C, Maximum 76°C. The temperature at position 5 was 67°C. Because position 5 is on the outer surface of the core in an axial cooling duct, the temperature here would be expected to be at or near the minimum temperature of the core. With this in mind the average and maximum temperatures predicted by the model appear to be entirely reasonable.

- $X_2 = \theta_{7-8}$ = Average 94°C, Maximum 96°C. The temperature measured at the center rotor core in the keyway, position 9, was 94°C. The agreement
is excellent.

- $X_3 = \theta_9 = 49^\circ C$, $X_7 = \theta_{14} = 39^\circ C$. The measured temperature corresponding to these two predicted temperatures is the average temperature of the frame and end bells taken at positions 4. This was found to be $47^\circ C$. While it is difficult to say how well this average temperature verifies the predictions for $\theta_9$ and $\theta_{14}$, they do not appear to be greatly in error.

- $X_4 = \theta_{11} = 46^\circ C$. This is the predicted temperature of the air on entering the axial duct. The air temperatures measured in the overhang region, position 1, and in the axial duct, position 2, were $37^\circ C$ and $49^\circ C$ respectively. The predicted value for $\theta_{11}$ falls between the two measured temperatures, just as it should.

- $X_5 = \theta_{12-17} = 92^\circ C$. The temperature measure at position 10 was $83^\circ C$. This is a particularly large discrepancy in view of the fact that this is one of the places where the ideal position for the thermocouple was possible. Upon investigating the three possible sources of this discrepancy the following conclusions were reached. The five input parameters influencing this temperature, namely LCR, LIR, KRZ, KCR and KSH must not be greatly in error since they would also influence $\theta_{7-8}$, where the agreement was excellent. It was then observed that $\theta_{12-17}$ seemed low when compared with $\theta_{7-8}$. To investigate this apparent inconsistency the simple yet reasonably accurate model for conduction (7) shown in Fig. 5.1 was used to find the temperature gradient that should be expected between positions 9 and 10. The calculations follow.

The one-dimensional thermal resistances of the shaft, rotor core, and rotor bars are found with the equation

$$R_0 = \frac{L}{K \cdot A}$$

(5.1)
IDEALIZED ROTOR GEOMETRY AND
EQUIVALENT THERMAL NETWORK

FIG. 5.1
where \( \lambda \) = length of the body (cm)

\[ K = \text{thermal conductivity (W/cm}^\circ\text{C)} \]

\[ A = \text{cross sectional area (cm}^2\text{)} \]

The length, \( \lambda \), is the length of the rotor core, \( L = 13.34 \text{ cm} \).

so,

\[ K_{SH} = 0.50 \text{ W/cm}^\circ\text{C} , \ A_{SH} = \pi R_{SH}^2 = 28.2 \text{ cm}^2 \Rightarrow R_{SH} = 0.372 \text{ }^\circ\text{C/W} \]

\[ K_{CR} = 2.40 \text{ W/cm}^\circ\text{C} , \ A_{BARS} = 40.13 \text{ cm}^2 \Rightarrow R_{BARS} = 0.134 \text{ }^\circ\text{C/W} \]

\[ K_{RZ} = 0.0197 \text{ W/cm}^\circ\text{C} , \ A_{CORE} = \pi R^2 - A_{SH} - A_{BARS} = 143.11 \text{ cm}^2 \Rightarrow R_{CORE} = 4.73 \text{ }^\circ\text{C/W} \]

Since, in the network of Fig. 5.1, \( R_{BARS} \) is dominant in the parallel combination, if \( R_{SH} \) and \( R_{CORE} \) are neglected an approximation representing an upper limit of the temperature difference will be obtained. Also, since it is not accurately known how much of \( L_{CR} \) is due to the bars as opposed to the end rings, \( L_{CR} \) will be assumed to be entirely in the bars. This assumption will lead to a calculated temperature gradient which is larger than should actually exist. So, with these two approximations, an upper bound for the temperature gradient, \( \Delta T \), is given by

\[ \Delta T = RQ = \frac{R_{BARS}}{12} \cdot (L_{CR} + L_{IR}) \quad (5.2) \]

\[ \Delta T = \frac{0.134 \text{ }^\circ\text{C/W}}{12} \cdot (284W + 165W) = 5.01^\circ\text{C} \quad (5.2a) \]

The temperatures obtained at positions 9 and 10 would indicate that \( \Delta T = 11^\circ\text{C} \). It therefore seems reasonable to conclude that the temperature of position 10 is in error. This may be due to the close proximity of the thermocouple to the fan and the resulting high air flow past the thermocouple. This could, perhaps, have been remedied by providing better
insulation from the air flow and using some type of thermal compound to improve the thermal contact between the thermocouple and the core.

- \( x_6 = \theta_{13} = 58^\circ C \). The thermal resistance and heat loss of the shaft to the outside was not known so values which made the predicted temperature agree with the measured temperature at position 11 were used. This then provides no check since the agreement was forced. It should be noted that the thermal resistance and heat loss introduced to obtain \( \theta_{13} = 58^\circ C \) were quite reasonable.

- \( x_8 = \theta_{16} = 66^\circ C \). The temperature measured at position 6 was 62\(^\circ\)C. While this was one of the places where better than usual agreement would have been expected this discrepancy is still small. Even if there is in fact a four degree difference between the actual and predicted values, the model is sufficiently accurate and the error is on the conservative side.

- \( x_9 = \theta_{\text{ext 2}} = 66^\circ C \). The temperature measured at position 8 on the stator end winding was 52\(^\circ\)C. This was one of those instances where the thermocouple was placed in the position for which the model predicts a temperature. As was the case for the temperature at position 10 it seems that the high flow rate of air caused the temperature of the thermocouple to be much lower than the temperature at the point of interest. This was confirmed by performing calculations similar to those made for \( \theta_{12-17} \). Again, the use of a thermal compound and better insulation would have improved this measurement.

- Winding Temperature. The model calculates the maximum temperature of each cross section \( d\lambda \) (see Fig. 5.2) and gives as output the highest and lowest of these values. They were found to be: maximum 74\(^\circ\)C, minimum 73\(^\circ\)C. Since the temperature at position 8 has been determined to be in error, only the temperature of position 7 and the average temperature of the
WINDING

MAXIMUM TEMPERATURE OF ELEMENT \( dl \)

SECTION

FIG. 5.2
winding as obtained by resistance can provide a check on the predicted maximums. The temperature of position 7 was 61°C and the average temperature was 65°C. Because position 7 is on the surface of the bundle of conductors and its contact with the conductors is of unknown quality and because the average temperature is so inaccurate (Section 4.1.3) they provide only a rough check. However, it seems that the winding temperatures predicted by the model cannot be grossly in error.

5.3 Conclusion

Based on the results obtained in this work it seems that Mr. Perez has been successful. The only major discrepancies between the measured and the predicted temperatures can be attributed to two causes. First, it was not always possible to make the temperature measurement at the locations of the model's temperature predictions. The other discrepancies were found to be due to erroneous temperature measurements caused by a combination of poor thermal contact with the point of interest and cooling of the thermocouple by the air flow. Mr. Perez's thermal model appears to accurately model the test machine.

As was pointed out, the model was intended for high speed machines. The differences between high speed machines and the conventional machine used in these tests lie in increased windage losses and film coefficients of convection. Should the model prove inadequate for the higher values, discrepancies would be expected in the predicted temperatures for the air gap and overhang regions and, depending on the method of cooling, those surfaces in the coolant path. There is, however, no reason to suppose that the model should not work as well with high speed machines as it did with the test machine.
REFERENCES


