AN ELECTRON MICROSCOPY STUDY OF PHASE TRANSFORMATIONS
AND ROOM-TEMPERATURE STRENGTHENING MECHANISMS IN A
Co-Cr-Mo-C ALLOY

by

KRISHNA RAJAN

B.A.Sc., University of Toronto
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for the degree of
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Signature of Author

Department of Materials Science
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Accepted by Chairman, Departmental Committee
on Graduate Students

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ABSTRACT

AN ELECTRON MICROSCOPY STUDY OF PHASE TRANSFORMATIONS
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KRISHNA RAJAN

Submitted to the Department of Materials Science and Engineering on May 5, 1978 in partial fulfillment of the requirements for the degree of Doctor of Science.

The primary purpose of this study was to determine, using transmission electron microscopy, the various contributions to the flow stress at room temperature of a cast Co-Cr-Mo-C alloy. These alloys are used as surgical implant materials.

The samples were deformed in tension. The deformation was interrupted at various stages of the stress-strain curve and the dislocation structure was observed. The material was observed in the as cast, solutionized and solutionized and aged conditions. The electron microscopy revealed certain characteristics concerning the deformation substructure of this alloy. The stacking fault energy being low (γ = 0), all perfect dislocations in the matrix were widely dissociated into their respective partials. Thus the structure was characterized by the presence of stacking faults, which were determined to be intrinsic. The perfect dislocations that were observed existed in the confines of the stacking faults. The stacking faults acted as regions of localized slip.

At high degrees of deformation there was evidence of twin formation. Twins provided a major obstacle to slip as noted by the presence of dislocation pileups at twin intersections.

The various possible mechanisms contributing to the overall flow stress were considered. The mechanisms discussed were the following: the intersection of stacking faults, stacking fault intersections as obstacles to slip,
Lomer dislocation locks, dislocation stress field interaction on intersecting planes, solid solution effects, lattice friction and twin-slip interaction. A quantitative estimate of the stress needed to have a dislocation overcome each of the various types of obstacles was made. Of the mechanisms considered it was concluded that stacking fault intersections and twins make the largest contribution to the work hardening rate of the Co-Cr-Mo-C alloy. The stacking fault intersection was modelled as a form of dislocation dipole from which the stress interaction was estimated. In the case of twin-slip interaction, it is suggested that the incorporation of the slip dislocation into the twin is governed by the reaction

\[
\frac{1}{2}[110]_{(111)} \rightarrow \frac{1}{6}[141]_{(115)} + \frac{1}{6}[2\overline{1}2\overline{1}]_{\text{CTB}(111)}
\]

The stress concentration that would arise at a coherent twin boundary (CTB) due to the above dislocation reaction is manifested by the presence of a dislocation pileup at a twin-CTB intersection. The observed dislocation configuration at such a pileup matches well the dislocation distribution calculated from the above reaction. Also, a new mechanism of fcc twinning has been proposed in this thesis. This model suggests an interaction between stacking faults and pre-existing twin nuclei.

A study of the phase transformations focussed on the wrought alloys found that heat treatments in the temperature range 650°C-750°C are capable of producing multiphase structures consisting of the fcc phase, a heavily faulted hcp phase, a recrystallized hcp phase and M\(_{23}\)C\(_6\) carbides. The carbide precipitation occurred in a discontinuous fashion, forming only in the hcp phase and on stacking faults. The nature of the various discontinuous precipitation reaction fronts is discussed.

Finally, the use of computer calculated phase diagrams for the Co-Cr-Mo system was briefly investigated.

Thesis Supervisor: John B. Vander Sande
Title: Associate Professor of Materials Science and Engineering
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I wish to thank Professor John Vander Sande for his guidance, encouragement and patience during my stay at MIT. Also I am indebted to Professor John Wulff for the concern and interest he has shown in me.

My thesis committee members, Professors Nicholas Grant and Robert Rose, provided many valuable comments. Professor John Cahn, Dr. Subash Mahajan of Bell Telephone Laboratories and Dr. Fritz Prinz of the University of Vienna gave an insight into some of the key analytical problems faced in this thesis. The help and encouragement given by Guenther Arndt and Art Gregor has been invaluable.

Finally, I am thankful to all my friends who have made the past few years enjoyable. Many individuals come to my mind, but let me at least mention some of those whom I had the pleasure of working with here at MIT: Bob Allen, Sam Allen, Bob Freed, Ernie Hall, Steve Hansen, Tom Kelly, Buz Kenney, Hank Nusbaum, Greg Olson, Dick Salzbrenner and Bill Sherry.

The financial support of the National Science Foundation is gratefully acknowledged.
I. INTRODUCTION

Investment castings of Co-Cr-Mo-C alloys (commercially known as Vitallium or H.S.21) have had a long history of successful use as surgical implants. Unfortunately, despite the extensive use of these alloys as surgical implant materials, the shortcoming of these alloy systems are evident in the mechanical failure statistics of weight bearing prostheses\(^1\). This provides a disturbing commentary on the extent of knowledge of the metallurgical characteristics of these complex cobalt based alloys. However what is even more disturbing is that many researchers in the field have the attitude as expressed by one person that: "In a sense the essential research on metallic implant materials has been completed"\(^2\). It is interesting to note that this comment was made over ten years ago!

The primary motivation in initiating the present study was the feeling that the "essential research" was far from being complete and there was a need to eliminate the empiricism which has governed the field\(^3-6\). To do this it was felt that a basic understanding of the structure-property relationships in this cobalt based alloy must first be established. Specifically the primary purpose of the study was to determine, using transmission electron
microscopy, the various contributions to the flow stress, at room temperature of a cast Co-Cr-Mo-C alloy.

The material being used for surgical implants falls into the category of superalloys as the chemistry of this alloy is very similar to conventional cobalt based superalloys (Table 1). While the room temperature mechanical behavior of cobalt alloys has not been studied extensively, the high temperature behavior is very well documented\textsuperscript{7-10}. It would be worthwhile to briefly review some of the primary high temperature strengthening mechanisms in cobalt based alloys (Table 2).

The primary strengthening mechanisms in cobalt alloys is precipitation hardening; principally by the presence of carbides in the matrix and grain boundaries\textsuperscript{11-13}. The carbides, especially M\textsubscript{23}C\textsubscript{6}, precipitate at the grain boundaries, pin the grain boundaries preventing grain boundary sliding, or in the case of higher carbon content, the carbide network may support some of the load. The large carbide particles can act as dislocation generators under the influence of a stress and the subsequent dislocation interaction can give rise to an increase in the flow stress of the metal. Regarding carbides, another important mode of strengthening arises from precipitation of fine M\textsubscript{23}C\textsubscript{6} particles on stacking faults\textsuperscript{14}. This provides a rather
**Table 1**

**Cast alloy composition**

<table>
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<tr>
<th>Element</th>
<th>Composition</th>
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<tr>
<td>Cr</td>
<td>28.5%</td>
</tr>
<tr>
<td>Mo</td>
<td>5.7%</td>
</tr>
<tr>
<td>Ni</td>
<td>2.2%</td>
</tr>
<tr>
<td>Fe</td>
<td>0.4%</td>
</tr>
<tr>
<td>C</td>
<td>0.3%</td>
</tr>
<tr>
<td>Si</td>
<td>0.4%</td>
</tr>
<tr>
<td>Mn</td>
<td>0.5%</td>
</tr>
<tr>
<td>Co</td>
<td>Balance</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Comments</td>
</tr>
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<td>---------------------------------------</td>
<td>--------------------------------------------------------------------------</td>
</tr>
<tr>
<td>2. Solid-solution hardening (Ta, W, Mo,Cb)</td>
<td>Effective at temperatures above about 2000°F (1093°C) but not potent below this temperature.</td>
</tr>
<tr>
<td>3. Intermetallics (Co₃W, Co₃Mo, Co₃Ti, Co₃Ta, σ, Laves, μ)</td>
<td>Overaging above ~ 1600°F (870°C); possible deterioration of ductility and oxidation resistance due to σ, μ and Laves. Appreciable hardening can be achieved below 1500°F (815°C).</td>
</tr>
<tr>
<td>4. Grain structure (directional, single, fine)</td>
<td>Directional solidification yields better thermal fatigue and ductility, but higher cost because of special processing. Fine-grained more ductile, possible better fatigue but poorer creep.</td>
</tr>
<tr>
<td>5. Dispersoids (ThO₂, other oxides, carbides)</td>
<td>Expensive, technique very critical, no outstanding alloy yet developed.</td>
</tr>
<tr>
<td>6. Minor elements (B, Zr)</td>
<td>Fairly well explored, effective. Excessive boron must be avoided due to low ductility and incipient melting.</td>
</tr>
<tr>
<td>7. Eutectics (intermetallics)</td>
<td>Requires further investigation, offers promise of high-temperature stability and good strength.</td>
</tr>
<tr>
<td>8. FCC → HCP transformation and dislocation structure</td>
<td>Field requires much more investigation before assessment can be made, question of stability.</td>
</tr>
<tr>
<td>9. Fiber reinforcement</td>
<td>Not enough information developed as yet. Adds to expense so that properties developed must be improved enough for economic justification.</td>
</tr>
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<td>10. Combinations of the above</td>
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Table 2  High temperature strengthening mechanisms in cobalt base superalloys (Ref. 9).
uniform dispersion of interlocking structural effects caused by the hard carbide particles at stacking faults and stacking fault intersections. Dislocation movement is sharply impeded through a structure of this sort, and significant strengthening results. Grant and co-workers¹⁵-²⁰ have studied the carbide precipitation process in detail.

Besides carbides, another form of second phase particles in cobalt based alloys are intermetallic compounds. These include topologically close packed compounds such as the σ, μ and Laves phases. These phases have to be considered as they can cause a significant loss of both strength and ductility at high temperatures. Their deleterious effect is due primarily to the chemical partitioning effect caused by the mass presence of the phase itself, thus depleting the matrix of solute strengthening atoms. The solid solution strengtheners include mainly the refractory elements such as Cr, Mo and W.

These atoms contribute to strengthening a number of ways: they inhibit recovery by binding vacancies; and also they impede dislocation glide through the existence of interstitial complexes around the refractory atoms. The fcc→hcp transformation in cobalt alloys has not been fully utilised in the development of desired mechanical properties. This
is primarily due to the fact that the details of the allotropic transformation have not been well understood; and the role of the fcc→hcp transformation for strengthening in cobalt based alloys has been speculative in nature.

The first detailed examination using transmission electron microscopy of the fcc→hcp transformation in the Co-Cr-Mo-C system was carried out by Vander Sande et al.\textsuperscript{21}. Their work was initiated by earlier findings of property modifications in response to heat treatments and processing variables and the associated microstructural changes\textsuperscript{22-26}. The mechanical failures of cobalt based surgical implant alloys have been attributed to casting porosity\textsuperscript{27-28}. However microradiographic studies have shown that all as-cast specimens contain varying amounts of such porosity and that this porosity can be eliminated by appropriate hot isostatic pressing (HIP) treatments. This results in improved ductility and permits the development of improved strength properties. Based on the microstructural changes observed upon heat treatments, Vander Sande et al. conjectured on the correlation of the microstructure to room temperature mechanical behavior. These workers suggested improved strength properties from the standpoint of slip in the fcc phase being restricted by stacking faults and the formation of the hcp phase. Also it was noted that as
in all precipitation hardening processes, overaging is expected to occur with its associated reduction in the strengthening ability of the precipitates. However Vander Sande et al. did not postulate specific mechanisms contributing to the strength of the material. One of the primary aims of this study was to specify those mechanisms in detail. Due mainly to the availability of the material, the present work focussed on the cast Vitallium rather than the wrought, or cast and HIP material. It should be noted that most of the material in service today is used in the cast condition.
II. EXPERIMENTAL DETAILS

1. Mechanical Testing and Specimen Preparation

The cobalt alloys used in this study were obtained from the Howmedica company. The samples were cast into the form of cylindrical tensile samples. They had a gage length of one inch and a gage diameter of 0.25 inch. The tensile testing was carried out on an Instron testing machine. The stress-strain curve was derived from carrying a test to fracture, from which the total elongation was measured. Using the load-extension curve the stress-strain data was obtained. The testing was done at room temperature with a machine cross head speed of 0.1"/min. Further samples were deformed, and interrupted at various stages of the stress-strain curve; the dislocation structure in these samples were observed by transmission electron microscopy.

In preparing samples for electron microscopy, the deformed tensile samples were first machined to 3 mm diameter rods to suit the dimensions of the electron microscope specimen holder. It was felt that as only the center of the sample will be examined, the effect of the deformation induced by the machining will not be observed. From these rods, discs of thickness .013 inch were cut using a
spark cutter. The use of the spark cutter minimized any damage to the specimen. The samples were finally thinned using conventional jet polishing techniques in a solution of methanol – 15% sulphuric acid, operating at -50°C, and 0.5 A/cm² current density.

2. Electron Microscopy

2.1 Dynamical Theory for Image Contrast

In this section, the wave optical formulation of the dynamical theory of image contrast in electron microscopy will be briefly reviewed; the results of which will be applied to the case of stacking faults and partial dislocations\(^{29-32}\). Using the two beam approximation (Fig. 1), the electrons propagating on a column of crystal (Fig. 2) are described by a wave function given by:

\[
\psi(\mathbf{r}) = \phi_0(z) \exp(2\pi i \mathbf{k} \cdot \mathbf{r}) + \phi_g(z) \exp(2\pi i \mathbf{k}' \cdot \mathbf{r})
\]

(1)

where: \(\mathbf{k}\) = wave vector of incident beam
\(\mathbf{k}'\) = wave vector of scattered beam \(|\mathbf{k}'| = |\mathbf{k}| = \frac{1}{\lambda}\)
\(\mathbf{r}\) = position in crystal
\(\phi_0\) = amplitude of incident beam
\(\phi_g\) = amplitude of scattered beam
Fig. 1  Ewald sphere construction in the reciprocal lattice. (Ref. 30)
Fig. 2  Schematic of column approximations with dynamical theory. (Ref. 30)
also \[ \vec{k} = \vec{k}' + \vec{g}' + \vec{s} \] (2)

where \[ \vec{g}' = \text{reciprocal lattice vector} \]
\[ \vec{s} = \text{deviation from exact Bragg condition} \]

Both \( \phi_o \) and \( \phi_g \) vary with the depth of the crystal. As the wave \( \phi_o \) propagates into the crystal, its amplitude will be depleted by diffraction and the amplitude \( \phi_g \) will increase, i.e. there is a coupling between \( \phi_o \) and \( \phi_g \). This coupling is defined by the differential equations:

\[ \frac{d\phi_o}{dz} = \frac{i\pi}{\xi_o} \phi_o + \frac{i\pi}{\xi_g} \phi_g \exp(2\pi i sz) \] (3)

and \[ \frac{d\phi_g}{dz} = \frac{i\pi}{\xi_o} \phi_g + \frac{i\pi}{\xi_g} \phi_g \exp(-2\pi i sz) \]

where \( \xi_o \) and \( \xi_g \) = extinction distance

Solving these equations, the wave function described in equation (1) than becomes:
\[ \psi(\vec{r}) = \psi^{(1)}[C_o^{(1)} \exp(2\pi i \vec{k}^{(1)} \cdot \vec{r}) + C_g^{(1)} \exp(2\pi i (\vec{k}^{(1)} + \vec{g}) \cdot \vec{r})] + \psi^{(2)}[C_o^{(2)} \exp(2\pi i \vec{k}^{(2)} \cdot \vec{r}) + C_g^{(2)} \exp(2\pi i (\vec{k}^{(2)} + \vec{g}) \cdot \vec{r})] \]

where \( \psi^{(1)} \) and \( \psi^{(2)} \) = arbitrary constants.

Equation (4) points out that the wave function splits into two Bloch waves which dynamically interact with each other. At the Bragg reflection position the Bloch waves are expressed as

\[ b^{(1)}(\vec{k}^{(1)}, \vec{r}) = i\sqrt{2} \sin(\pi \vec{g} \cdot \vec{r}) \exp(2\pi i (\vec{k}^{(1)} + 1/2\vec{g}) \cdot \vec{r}) \]
\[ b^{(2)}(\vec{k}^{(2)}, \vec{r}) = \sqrt{2} \cos(\pi \vec{g} \cdot \vec{r}) \exp(2\pi i (\vec{k}^{(2)} + 1/2\vec{g}) \cdot \vec{r}) \] 

These waves, which have both a transmitted and diffracted component are channeled through the crystal (Fig. 3). The Bloch waves of the first type are concentrated between the atoms and therefore well transmitted, while the second is concentrated in the vicinity of the atoms. Thus the
Fig. 3 Standing wave fields at the Bragg reflecting position in a simple cubic lattice. (Ref. 30)
second wave is scattered more by the atoms than the first wave, accounting for the "anomalous absorption effect", that is, for the depletion of the waves in given directions due to large angle inelastic scattering.

In the dynamical theory, the intensity of the incident or transmitted wave in a crystal of thickness \( t \) is given by:

\[
|\phi_0|^2 = 1 - \frac{1}{1+w^2} \sin^2 \frac{\pi (1+w^2)^{1/2} t}{\xi_g} \tag{6}
\]

where \( w = s\xi_g \)

The intensity of the diffracted beam is \( 1 - |\phi_0|^2 \), and this intensity oscillates with a periodicity given by

\[
t = \frac{1}{s_{\text{eff}}} = \frac{\xi_g}{(1+s^2\xi_g^2)^{1/2}} \tag{7}
\]

where \( s_{\text{eff}} = \left[s^2 + (1/\xi_g^2)\right]^{1/2} \)

Thus for a constant thickness, the intensity oscillates as \( s \) varies giving rise to band contours. The anomalous absorption effect noted above gives rise to an asymmetry in the intensity distribution of the band contours as described by the "rocking curve" (Fig. 4). The asymmetry
Fig. 4 Rocking curves for (i) $\xi g/\xi'g = 0.05$ and (ii) $\xi g/\xi'g = 0.1$ in bright field (full line) and dark field (broken line). (Ref. 31)
at the contour corresponds to a region of high intensity outside the reflecting position \((w>0)\), and a region of poor intensity inside the reflecting position. Outside the reflecting position, Bloch wave \((1)\) dominates, while inside the reflecting position, Bloch wave \((2)\) dominates. This indicates that \(\psi^{(2)}\) is scattered more strongly and is attenuated more rapidly while \(\psi^{(1)}\) is well transmitted. It is for this reason, that in any two-beam diffracting condition, the best image is obtained for a slightly positive deviation from the Bragg condition.

Up to now, we have dealt with a perfect crystal. The equations for the wave amplitudes in the imperfect crystal are:

\[
\frac{d\phi}{dz} = \frac{\pi i}{\xi_o} \phi_o + \frac{\pi i}{\xi_g} \phi \exp(2\pi i sz + 2\pi i g \cdot \hat{R}) \tag{8}
\]

\[
\frac{d\phi}{dz} = \frac{\pi i}{\xi_o} \phi_o + \frac{\pi i}{\xi_g} \phi \exp(-2\pi i sz - 2\pi i g \cdot \hat{R})
\]

\(\hat{R}\) describes the displacement in the lattice that is produced by the defect. If we define:
\[ \psi''(z) = \psi_o(z) \exp(-\pi iz/\xi_o) \] (9)

and
\[ \psi''(z) = \psi_g(z) \exp(2\pi isz - \frac{\pi iz}{\xi_o} + 2\pi i \vec{\gamma} \cdot \vec{R}) \]

then,
\[ \frac{d\psi''}{dz} = \frac{\pi i}{\xi_o} \psi''_g \]
\[ \frac{d\psi''}{dz} = \frac{\pi i}{\xi_g} \psi''_0 + (2\pi is + 2\pi i \vec{\gamma} \cdot \frac{d\vec{R}}{dz}) \psi''_g \] (10)

The latter formulation is useful since it points out that the strain which appears as \( \vec{\gamma} \cdot \frac{d\vec{R}}{dz} \) acts like a local rotation of the Bragg planes and changes the effective value of \( s \).

2.2 Image Characteristics of Stacking Faults

A stacking fault may be defined by the displacement vector \( \vec{R} \), where \( \vec{R} \) is the displacement of the crystal below the planar defect with respect to the top of the crystal (Fig. 5). The contrast from a planar defect which is inclined to the foil surface gives rise to a fringe pattern. The origin of the fringe pattern may be understood by recalling the attenuation of the Bloch waves in the crystal. Since the first Bloch wave is concentrated
Fig. 5  Intensity oscillations of periodicity $1/s$ in crystal with the kinematic theory. (Ref. 30)
between the atoms and therefore has a higher average potential energy than the second Bloch wave, it must have a lower kinetic energy, in order to maintain the total energy constant. The boundary condition at the top of the crystal means that the wave vector components parallel to the surface must be identical; consequently \( \mathbf{k}^{(1)} \) and \( \mathbf{k}^{(2)} \) differ only normal to the crystal surface. Due to the difference in wavelength there is a coupling effect between the two waves, resulting in oscillations of total transmitted or diffracted intensity which gives rise to the characteristic fringe pattern of stacking faults (or wedge shaped crystals in which case thickness fringes are produced), (Figs. 6 and 7). The details of the stacking fault fringe pattern are a function of the phase angle \( 2\pi \mathbf{g} \cdot \mathbf{R} \), where \( \mathbf{g} \) is the reciprocal lattice vector, corresponding to the operating reflection plane. If \( 2\pi \mathbf{g} \cdot \mathbf{R} \) is zero or has an integral value of \( 2\pi \), the fault is invisible, since the Bragg reflecting planes above and below the fault are in registry with one another. The contrast is unchanged by the addition of a lattice vector to \( \mathbf{R} \), since the resultant change on the value \( 2\pi \mathbf{g} \cdot \mathbf{R} \) must be an integral number of \( 2\pi \).

Faults in the fcc structure lie on the \{111\} planes and can be formed on different ways: by shearing
Fig. 6  Origin of fringe pattern obtained from stacking faults. (Ref. 30)
Fig. 7  Stacking fault image profile in bright field (full line) and dark field (broken line). (Ref. 29)
parallel to the fault or by expansion or collapse of the lattice in a direction perpendicular to the fault. For a stacking fault in the (111) plane, \( \mathbf{\hat{R}} \) can take the values of \( \pm \frac{1}{3}[111] \). One of these possibilities corresponds to the removal of a layer of atoms followed by a collapse of the two crystals on either side of the fault, and the other corresponds to the insertion of an extra layer of atoms pushing the two crystals apart. These are called intrinsic and extrinsic faults respectively. Faults which form by shear have displacement vectors of the type \( \pm \frac{1}{6}[112] \) but since they differ from \( \pm \frac{1}{3}[111] \) by a lattice translation vector, they can also be classified as either intrinsic or extrinsic.

The phase angle \( \alpha = 2\pi \mathbf{g} \cdot \mathbf{\hat{R}} \) may also take on values of \( \pm \frac{2}{3}n \) where \( n \) is an integer. Again, the principle that the addition of a lattice vector to \( \mathbf{\hat{R}} \) does not change the contrast from stacking faults is applicable. This means that when assessing the magnitude of \( 2\pi \mathbf{g} \cdot \mathbf{\hat{R}} \), only those values which are made as close to zero as possible, by the arbitrary addition or subtraction of \( 2n\pi \), need be considered. Thus the contrast associated with \( 2\pi \mathbf{g} \cdot \mathbf{\hat{R}} = \frac{4}{3}\pi \) is equivalent to that associated with \( 2\pi (\mathbf{g} \cdot \mathbf{\hat{R}}) = \frac{-2}{3}\pi \). For \( \alpha = \pm 2m\pi \) where \( m \) is an integer, the stacking fault is invisible. For \( \alpha = \pm \frac{2}{3}n \), the fault would be invisible for some
\{220\} and \{311\} reflections for example and would always be visible for any \{200\} reflection.

2.3 Contrast Behavior of Partial Dislocations

The application of the invisibility criterion is the method of determining the Burger's vector (displacement vector) of a dislocation. This is achieved by imaging the dislocation under two-beam conditions with successively different diffracting vectors. When $\hat{g} \cdot \hat{b} = 0$ for a perfect screw dislocation, it is invisible. A perfect edge dislocation is invisible if $\hat{g} \cdot (\hat{b} \times \hat{u}) = 0$ and $\hat{g} \cdot \hat{b} = 0$ where $\hat{u}$ is the line direction of the dislocation.

The situation with partial dislocations is slightly more involved. First we should note that, as the Burger's vector of the partial dislocation bounding the stacking fault also describes the displacement vector of the fault, it is to be expected that $\hat{g} \cdot \hat{b}_p$ can take on integer values as well as $\pm 1/3$, $\pm 2/3$, $\pm 4/3$ etc. It was found in this work, as will be discussed later, that the faults present in the deformation substructure were all intrinsic faults formed by a shear displacement of the type $\hat{b}_p = 1/6 \langle 112 \rangle$. Thus our discussion will be limited to Shockley partials.

As in the case of perfect dislocations, if $\hat{g} \cdot \hat{b}_p = 0$ or
an integer the dislocation may be treated as being invisible. However, complications arise for other values of \( \mathbf{g} \cdot \mathbf{b}_p \). Partial dislocations may also be invisible for \( \mathbf{g} \cdot \mathbf{b}_p = \pm 1/3 \). This also holds for small deviations from the Bragg condition. However considerable caution must be exercised when \( \mathbf{g} \cdot \mathbf{b}_p = \pm 2/3 \) since as pointed out by Mahajan and Chin\(^{33}\): i) a weak contrast is observed if the image lies on the perfect crystal side of the partial and ii) a strong contrast is observed when the image lies on the fault side. Another point of caution is that for \( \mathbf{g} \cdot \mathbf{b}_p = -2/3 \), a partial dislocation can be invisible if \( w = \xi g s \) is large\(^{34}\); (Tables 3-5).

2.4 Contrast Analysis

To experimentally set up the two beam condition necessary for contrast analysis, requires the use of Kikuchi bands\(^{30,31,35}\). These bands arise due to the inelastic and incoherent scattering of the incoming electron beam, caused by its interaction with the atoms in the crystal. These electrons can be subsequently rescattered coherently when Bragg's law is satisfied by a suitable set of reflecting planes. The geometry of formation of Kikuchi bands is illustrated in Fig. 8. Cones of radiation are emitted and if the incident waves are symmetrically
Table 3

$\vec{b} \cdot \vec{g}$ values for perfect dislocations on the (111) plane

<table>
<thead>
<tr>
<th>$\vec{b}$</th>
<th>$\frac{1}{2}[1\overline{1}0]$</th>
<th>$\frac{1}{2}[10\overline{1}]$</th>
<th>$\frac{1}{2}[0\overline{1}1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{111}$</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$11\overline{1}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>220</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$\overline{202}$</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$02\overline{2}$</td>
<td>-1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>$22\overline{0}$</td>
<td>-2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\overline{113}$</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>$13\overline{1}$</td>
<td>-1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>$\overline{311}$</td>
<td>-2</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>200</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$1\overline{11}$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$\overline{311}$</td>
<td>-2</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>002</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$1\overline{13}$</td>
<td>1</td>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 4

\( \mathbf{\bar{g}} \cdot \mathbf{\bar{h}} \) values for partial dislocations

on the (111) plane

\[
\begin{array}{|c|c|c|c|}
\hline
\mathbf{\bar{g}} & \frac{1}{6}[11\bar{2}] & \frac{1}{6}[1\bar{2}1] & \frac{1}{6}[\bar{2}11] \\
\hline
\bar{1}1\bar{1} & 1/3 & -2/3 & 1/3 \\
1\bar{1}\bar{1} & 1/3 & 1/3 & 0 \\
1\bar{1}1 & 2/3 & -1/3 & -1/3 \\
220 & 2/3 & -1/3 & -1/3 \\
\bar{2}0\bar{2} & 1/3 & -2/3 & 1/3 \\
02\bar{2} & 1 & -1 & 0 \\
2\bar{2}0 & 0 & 1 & -1 \\
\bar{1}13 & -1 & 0 & 1 \\
13\bar{1} & 1/3 & -1 & 0 \\
\bar{3}1\bar{1} & 0 & -1 & 1 \\
200 & 1/3 & 1/3 & -2/3 \\
\bar{3}3\bar{1} & -1 & 0 & 1 \\
00\bar{2} & -2/3 & 1/3 & 1/3 \\
1\bar{1}3 & -1 & 1 & 0 \\
\hline
\end{array}
\]
Table 5

\( \mathbf{\hat{g}} \cdot \mathbf{\hat{b}} \) values for perfect dislocations on the \((\overline{1}1\overline{1})\) plane

<table>
<thead>
<tr>
<th>( \mathbf{\hat{g}} )</th>
<th>( \frac{1}{2}[101] )</th>
<th>( \frac{1}{2}[011] )</th>
<th>( \frac{1}{2}[1\overline{1}0] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{1}1\overline{1} )</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( 1\overline{1}1 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( 2\overline{2}0 )</td>
<td>1</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>( \overline{1}1\overline{3} )</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>002</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \overline{1}1\overline{3} )</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\( \mathbf{\hat{g}} \cdot \mathbf{\hat{b}} \) values for partial dislocations on the \((\overline{1}1\overline{1})\) plane

<table>
<thead>
<tr>
<th>( \mathbf{\hat{g}} )</th>
<th>( \frac{1}{6}[\overline{2}1\overline{1}] )</th>
<th>( \frac{1}{6}[11\overline{2}] )</th>
<th>( \frac{1}{6}[1\overline{2}\overline{1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{1}1\overline{1} )</td>
<td>2/3</td>
<td>-1/3</td>
<td>-1/3</td>
</tr>
<tr>
<td>( 1\overline{1}1 )</td>
<td>-1/3</td>
<td>-1/3</td>
<td>2/3</td>
</tr>
<tr>
<td>( 2\overline{2}0 )</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \overline{1}1\overline{3} )</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>002</td>
<td>-1/3</td>
<td>2/3</td>
<td>-1/3</td>
</tr>
<tr>
<td>( \overline{1}1\overline{3} )</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 8  Geometry of formation of Kikuchi lines. (Ref. 32)
impinging on the plane AB, cones of equal intensity are scattered, with a semi-vertex angle of 90-θ, to each side and bisecting the reflecting plane AB. If, however, the waves impinge on an inclined reference plane AB, then most of the electrons are initially scattered into the direction $K_1$ and relatively few into the forward direction $K_2$. Thus on the photograph, one then observes a bright line corresponding to $K_1$ near the Bragg spot and a dark line corresponding to $K_2$ near the origin. The intersection of the cones of Kikuchi radiation with the photographic plate produces hyperbolic lines. However they appear as straight lines since θ is small.

One of the primary uses of Kikuchi bands is in the experimental determination of the deviation parameter $\hat{s}$. It will be recalled from the "rocking curve" that for maximum contrast, $\hat{s}$ should be slightly positive. The parameter $\hat{s}$ is taken as positive or negative depending on whether the reciprocal lattice point $\hat{g}$ lies inside or outside the Ewald reflection sphere. The magnitude of $\hat{s}$ is determined by the displacement of the Kikuchi line from its corresponding diffraction spot. When the Bragg condition is precisely satisfied, than the bright Kikuchi line will pass exactly through the associated diffraction spot, and the dark line through the directly
transmitted beam. The deviation parameter \( \hat{s} = 0 \) when \( \theta = \theta_B \) and is taken to be positive when the incident angle is greater than \( \theta_B \) and negative when \( \theta < \theta_B \). Thus when \( \hat{s} \) is positive the bright Kikuchi line will be displaced from the diffraction spot in a direction away from the direct beam.

Besides helping to define \( \hat{q} \) and \( \hat{s} \) for contrast analysis, Kikuchi bands are also used to specify the orientation of the crystal as defined by the zone axis, \( \hat{z} \). In the present work, this aspect was very important as it was often necessary to observe the crystal in certain specific orientations. This was true particularly in the case of obtaining information on the crystallography of twinning, as will be discussed later.\(^{36}\) The Kikuchi bands move as the crystal is tilted and only those lines corresponding to planes which make a small angle with \( \hat{z} \) will be seen on the screen. Thus as the crystal is tilted the Kikuchi pattern will change depending on the crystallographic axis about which the specimen is rotated. These various patterns can be represented by Kikuchi maps as shown in Fig. 9. These maps provided the guide for obtaining the desired orientations.
Fig. 9  Kikuchi map for fcc crystal extending over two standard triangles. (Ref. 31)
III. EXPERIMENTAL RESULTS

1. Mechanical Testing

The material was subjected to tensile deformation for three different conditions: as cast, solutionized (15 min. - 1130°C, 2 hrs - 1250°C) and solutionized and aged (200 hrs - 650°C). The mechanical testing data is given in Table 6 and the engineering stress-strain curves are shown in Fig. 10.

The solutionized sample showed the greatest ductility as well as the least strength. The sample that was over-aged (200 hrs) was very brittle. Its strength at the elastic limit was very high but its fracture stress was not that much higher than the as cast material.

2. General Characteristics of Deformation Substructure

2.1 As Cast

Four states of tensile deformation were examined by electron microscopy; the undeformed condition, 3.85% total engineering strain, 7.43% strain and 11% strain (at fracture).
Table 6

Mechanical testing data

<table>
<thead>
<tr>
<th></th>
<th>As Cast</th>
<th>Solutionized 15 min-1130°C</th>
<th>2 hrs-1250°C</th>
<th>Solutionized Aged: 200 hrs-650°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Reduction in Area</td>
<td>9.47%</td>
<td>14.01%</td>
<td>1.47%</td>
<td></td>
</tr>
<tr>
<td>Stress at Elastic Limit</td>
<td>$6.54 \times 10^9$ dynes/cm$^2$</td>
<td>$5.27 \times 10^9$ dynes/cm$^2$</td>
<td>$8.13 \times 10^9$ dynes/cm$^2$</td>
<td></td>
</tr>
<tr>
<td>Total Strain</td>
<td>11%</td>
<td>18.5%</td>
<td>9%</td>
<td></td>
</tr>
<tr>
<td>Stress at Fracture</td>
<td>$1.02 \times 10^{10}$ dynes/cm$^2$</td>
<td>$9.54 \times 10^9$ dynes/cm$^2$</td>
<td>$1.03 \times 10^{10}$ dynes/cm$^2$</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 10a  Stress-strain curve for as cast material.
Fig. 10b  Stress-strain curve for solutionized material (15 min. - 1130°C, 2 hrs. - 1230°C - Air cooled)
Fig. 10c  Stress-strain curve for solutionized and aged material (200 hrs - 650°C)
There were a number of dislocations, probably intrinsic to the as-received state. All the dislocations, however, were dissociated into their respective partials. Thus the structure was characterized by the presence of stacking faults (Fig. 11). A sample trace analysis indicates that these stacking faults exist on all four unique {111} planes of the fcc lattice. On any given plane, the stacking fault width while large was not constant. In many of the stacking faults, the partial dislocations are not parallel, as in an infinite medium, but bound the stacking fault trapezoidally. As discussed in detail by Kroner et al.\textsuperscript{37}, this effect is due to the fact that each of the partials has a different Burgers vector and therefore tends to intersect the surface under a different angle. This leads to constriction at one end and to extension on the other foil surface.

Under even a slight amount of stress, the stacking faults expand to extremely large widths. This gives rise to the long bands of stacking faults as shown in Fig. 12.

The stacking faults formed by deformation were intrinsic faults on {111} planes of the fcc matrix as shown by the contrast experiment of Fig. 13. This contrast experiment as suggested by Gevers et al.\textsuperscript{38} is based on
Fig. 11 A view of the undeformed structure \( \mathbf{z} = [110], \) \( \mathbf{g} = [1\bar{1}1] \)
Fig. 12  Stacking fault ribbons $\hat{z} = [012]$, $\hat{g} = [200]$
Fig. 13  Dark field images illustrating intrinsic nature of stacking faults
the fact that in dark-field, the operating reflection \( \mathcal{g} \) points away from the light fringe, when the \( \mathcal{g} \) vector is centered on the fault, if the fault is intrinsic. The \( \mathcal{g} \) vector points toward the light fringe if the fault is extrinsic. The reflections to be used are \{111\}, \{220\} and \{400\}. The opposite rule applies for \{200\}, \{222\} and \{440\}. This contrast experiment is independent of the inclination of the fault. Thus Fig. 13 proves that the fault is intrinsic. In doing this type of experiment it is important to avoid overlapping stacking faults and to use single faults only.

At 3.85\% strain, the microstructure is characteristic of the early stages of plastic deformation. As noted in Fig. 14 the stacking fault density has increased from the undeformed state. This has resulted in the formation of a number of stacking fault intersection. There is also evidence of overlapping stacking faults. A view of the same area and surrounding region of Fig. 14 using \( g = \{220\} \) thus producing dislocation contrast and making the stacking faults invisible is shown in Fig. 15. Beside delineating the partial dislocations from the stacking faults there appears to be a high dislocation density in the confines of the stacking faults, particularly in the regions marked in Fig. 15. The nature of these dislocations will be discussed in more detail later; however, let is suffice to
Fig. 14  Specimen deformed with 3.85\% strain $\vec{z} = [110]$, $\vec{g} = [002]$
Fig. 15  Same area as in Fig. 14 imaged using dislocation contrast $\hat{g} = [110]$, $\hat{f} = [220]$
point out the localization of the dislocation population in the vicinity of the stacking faults.

The deformation substructure with 7.43% strain was not noticeably different from the sample with 3.85% strain. It consists of a large number of intersecting and overlapping bands of stacking faults (Fig. 16). Again, the stacking faults seem to be regions of intense localized deformation as indicated in Fig. 17. To determine the nature of these deformation induced faults, contrast experiments similar to those described earlier were done. As shown in Fig. 18 the faults are intrinsic.

At fracture the differences in the microstructure from the other deformation states are more appreciable. In Fig. 19 for example, one sees the large number of stacking fault intersections where few of the faults are viewed edge on. As may be noted in Fig. 19, a great deal of fine structure exists within the region of the stacking faults. The most important microstructural feature however is the formation of twins as shown in Fig. 20 and the associated diffraction pattern in Fig. 21. A bright field-dark field pair showing twins in the material is shown in Fig. 22.

2.2 Solutionized

The deformation substructure (8.12% total strain) of the
Fig. 16  Stacking faults in specimen strained 7.43\% \hat{z} = [011] 
\hat{g} = [111]
Fig. 17  Localized deformation in stacking faults $\mathbf{z} = [110]$, $\mathbf{g} = [\bar{1}11]$
Fig. 18  Dark field images illustrating intrinsic nature of stacking faults
Fig. 19  Fine structure within stacking faults $\vec{z} = [110]$, $\vec{g} = [111]$
Fig. 20a  A region of high twin density $\hat{z} = [110], \hat{g} = [00\bar{2}]$
Fig. 20b  Interference contrast micrograph of fractured sample showing possible surface relief effects of twin formation*  Magnification: x1000

*Electrolytic etch: 2% HCl in H₂O; ∼|A
Fig. 21  Diffraction pattern indicating the presence of twins and not hcp.
\[
\hat{Z} = \begin{bmatrix} i \bar{i} & 0 \\ i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

(b)
Fig. 22 Bright field-dark field pair of micrographs showing twins
solutionized material was identical to that of the cast material. Fig. 23 shows the familiar bands of stacking faults which intersect and overlap. As before, the formation of stacking faults is activated on a number of \{111\} planes as indicated in Fig. 24. The stacking faults again seem to be regions of high dislocation activity. Fig. 25 indicates what appears to be a dislocation array in a stacking fault. A similar configuration is seen in Fig. 26 where a dislocation pileup seems to exist near the intersection of some stacking faults, in the deformed material.

2.3 **Solutionized and Aged**

The deformation substructure (6.03% total strain) was strikingly different with this heat treatment. The stacking fault bands existed as clusters as shown in Fig. 27. The diffraction pattern (Fig. 28) shows the existence of a second phase. A closer examination of the structure shows large needle like particles (carbides) as can be seen in Fig 29. These particles which lie in an fcc matrix can act as sources for dislocations which dissociate to form stacking faults (Fig. 30). This could explain the discontinuous nature of the stacking fault distribution. The needle shape precipitates act as
Fig. 23  Intersecting and overlapping stacking faults
\( z = [100], \ \vec{g} = [020] \)
Fig. 24  Stacking fault formation on different \{111\} planes
\( \mathbf{z} = [110], \mathbf{\bar{g}} = [002] \)
Fig. 25  dislocation array in stacking fault $\frac{1}{2} \mathbf{z} = [110]$, $\mathbf{g} = [002]$
Fig. 26  Dislocation pileups at stacking fault intersections
\( \frac{1}{2} = [100], \frac{\mathbf{g}}{\mathbf{g}} = [022] \)
Fig. 27  Stacking fault clusters near precipitates
Fig. 28  Diffraction pattern of overaged sample
Fig. 29  Morphology of carbide particles
Fig. 30  Stacking fault nucleation at precipitates
as stress raisers which could account for the low ductility of the material.

3. **Deformation by Slip**

In this and the following section some of the detail of the nature of the dislocation configurations will be considered. As mentioned earlier, a high dislocation density is associated with certain stacking faults. In particular there are two configurations to be dealt with. The first is the uniform dislocation arrays in the stacking faults. An example of such an array is shown in Fig. 31.

There are three sets of dislocations considered in this case. A Burgers vector analysis was conducted to determine the nature of these dislocations.

Set I consisted of \( \frac{1}{2} [1\overline{1}0] \) dislocations. It might have been expected upon a cursory examination that the dislocations were partials bounding overlapping stacking faults. If the dislocations were of the \( \frac{1}{6} <1\overline{1}2> \) type, they should exhibit weak or no contrast for both \( \frac{1}{2} = [\overline{1}1\overline{1}] \) and \( \frac{1}{2} = [11\overline{1}] \). The dislocations however are noticeably visible for both of these reflections, thereby removing the possibility of them to be of the \( \frac{1}{6} <1\overline{1}2> \) type.

Set II dislocations were also determined to be perfect dislocations, but these had a Burgers vector \( \frac{1}{2} [10\overline{1}] \). It...
may be argued that there is weak contrast for \( \mathbf{g} = [\overline{1}1\overline{1}] \) which may suggest \( \mathbf{b} = \frac{1}{6} [1\overline{2}1] \) for \( \mathbf{g} \cdot \mathbf{b}_p = 2/3 \). If it was, then it should be invisible for \( \mathbf{g} = [1\overline{1}1] \) where \( \mathbf{g} \cdot \mathbf{b}_p = 1/3 \) but in fact it is strongly visible. This also rules out \( \mathbf{b} = \frac{1}{6} [11\overline{2}] \) where \( \mathbf{g} \cdot \mathbf{b}_p = 1/3 \) and \( \mathbf{b} = \frac{1}{6} [\overline{2}11] \) where \( \mathbf{g} \cdot \mathbf{b}_p = 0 \). Therefore Set II dislocations are definitely not of the type \( \frac{1}{6} <1\overline{1}2> \).

Set III dislocations were however partials with Burgers vector \( \frac{1}{6} [11\overline{2}] \) bounding overlapping stacking faults on parallel planes. There is weak contrast for \( \mathbf{g} = [002] \) where \( \mathbf{g} \cdot \mathbf{b} = 2/3 \) when \( \mathbf{b} = \frac{1}{6} [11\overline{2}] \) and \( \mathbf{g} \cdot \mathbf{b} = 1/3 \) for \( \mathbf{b} = \frac{1}{6} [\overline{1}21] \) and \( \mathbf{b} = \frac{1}{6} [\overline{2}11] \). To remove any ambiguity, the dislocation is definitely visible for \( \mathbf{g} = [1\overline{1}3] \). This rules out \( \mathbf{b} = \frac{1}{6} [\overline{2}11] \) as \( \mathbf{g} \cdot \mathbf{b}_p = 0 \). It is also visible for \( \mathbf{g} = [\overline{1}13] \) and this rules out \( \mathbf{b} = \frac{1}{6} [1\overline{2}1] \) where \( \mathbf{g} \cdot \mathbf{b}_p = 0 \). \( \frac{1}{2} <110> \) type dislocations all are visible for \( \mathbf{g} = [2\overline{2}0] \) and \( \mathbf{g} = [\overline{1}13] \). As this is not the case, the Set III dislocations are definitely not perfect.

As noted earlier, all the dislocations present in the matrix were widely dissociated into their respective partials. However, as just pointed out perfect dislocations do exist, seemingly in the confines of stacking faults. It is suggested here that these perfect dislocations lie on the stacking fault plane. In fact the only type of dislocations that could lie on a stacking fault
Fig. 31a  Schematic of dislocation array in a stacking fault
Fig. 31b \( \hat{z} = [110] \) \( \hat{g} = [002] \)
Fig. 31c  \( \hat{z} = [110] \)  \( \hat{g} = [\overline{111}] \)
Fig. 31d  \( \mathbf{z} = [110] \)  \( \mathbf{g} = [1\bar{1}3] \)
Fig. 31e  $\mathbf{z} = [110]  \quad \mathbf{g} = [\bar{1}11]$
Fig. 31f \( \hat{z} = [110] \) \( g = [1\overline{1}3] \)
Fig. 31g  $\tilde{z} = [110]  \quad \tilde{g} = [\bar{2}20]$
would be perfect dislocations since a partial dislocation on a stacking fault would provide a displacement in the lattice on the fault plane resulting in an atom sitting on top of another. This of course is an energetically unfavorable situation; thus the perfect dislocations that are present lie directly on stacking faults.

Besides the straight dislocation array in the stacking faults, there are some stacking faults exhibiting dislocation tangles. A Burgers vector analysis was done on these type of dislocations as well (Fig. 32). Again it was found that these tangles consist of perfect dislocations in the stacking faults.

The point to make from these results is that the stacking faults act as regions of localized slip. This demonstrates one important mode of deformation in this material.

4. Twinning and Twin-Slip Interaction

As pointed out earlier, at high degrees of deformation there was clear evidence of twin formation. Thus the production of twins is another important mode of deformation. Twins in a fcc metal may be characterized as a set of intrinsic stacking faults which overlap on parallel but adjacent \{111\} planes.
Fig. 32a  Dislocation tangles in stacking faults \( \mathbf{z} = [110] \)
\( \mathbf{f} = [\bar{2}20] \)
Fig. 32b  Same area as Fig. 32a  \( \mathbf{z} = [110] \)  \( \mathbf{g} = [\overline{1}11] \)
Fig. 33 shows how thin twins may impede the passage of partial dislocations. The displacement of the stacking fault fringes at the intersection with the twins (which are viewed edge on) is indicative of a barrier to the movement of the partial through these thin twins.

A similar type of obstruction occurs at the intersection of two twins. Fig. 34 shows an area of twin intersection and the corresponding diffraction pattern. The matrix has a $[\bar{1}14]$ orientation relative to the electron beam and the extra spots are characteristic of a $<110>$ orientation which would be obtained from a $<111>$ type twinning. It can be shown that twinning on $(\bar{1}11)$, $(\bar{1}1\bar{1})$, $(11\bar{1})$ and $(111)$ planes transforms the $[\bar{1}14]$ direction to the $[\bar{1}10]$, $[\bar{1}1\bar{3}]$, $[\bar{5} \bar{1} \bar{4}]$ and $[11 \bar{5} \bar{4}]$ directions respectively. Of these, the $[\bar{1}10]$ is the only orientation that is consistent with the observed diffraction pattern and thus the faults are produced by a twinning shear on the $(\bar{1}11)$ plane. The dark field micrograph shows one set of the twins much more clearly. It should be noted that at some twin-twin intersections, one twin prevents another from crossing it in a continuous fashion.

As has already been established, slip occurs on stacking faults. Thus slip dislocations may also exist within twins formed by a layer of stacking faults formed on adjacent
Fig. 33  Twins as obstacles to partial dislocations
Fig. 34a  Twin intersections imaged in dark field using (002) twin reflection

Fig. 34b  Associated dark field diffraction pattern
MATRIX and TWIN

x TWIN

$\mathbf{2} = [114]$
planes. The intersection of two such twins forms an obstacle to the motion of these slip dislocations as evidenced by the dislocation pileup in Fig. 35.

If we take the Set I dislocations to lie on the (111) plane, then \( b_I = \frac{1}{2} [10\bar{1}] \). Set II dislocations would then lie on the (\( \bar{1}11 \)) plane and then \( b_{II} = \frac{1}{2} [101] \). The dislocations are perfect slip dislocations and the periodic array of these dislocation would suggest a pile up at the twin intersection.

The above results have shown that the presence of twins provides an obstacle to deformation. A more detailed analysis is necessary to understand how the presence of twins contributes to work hardening. What will be examined now is specifically the interaction observed between slip dislocations and a thin twin boundary as well as between slip and twinning dislocations.

Fig. 36 shows an example of both twin-twin and twin-slip interaction. Let us first consider some general features of the region. Faults A and B which are viewed almost edge on lie on the (11\( \bar{2} \)) and (1\( \bar{1} \)1) planes respectively. These indices are defined by the \( \vec{g} \) vectors when the matrix is viewed in the [110] direction. Thus each of the faults and thin twins C and D lie on one of the (111) or (\( \bar{1}11 \)) planes. A Burgers vector analysis
Fig. 35  Dislocation pileup of perfect dislocations

(a) \( \mathbf{g} = [110], \mathbf{\bar{g}} = [111] \), (b) \( \mathbf{g} = [002] \)
Fig. 36a  Schematic of twin-twin and twin-slip interaction
Fig. 36b  $\hat{z} = [110], \hat{g} = [2\bar{2}0]$
Fig. 36c \( \hat{g} = [\overline{1}1\overline{3}] \)

Fig. 36d \( \hat{g} = [1\overline{1}1] \)
Fig. 36e \( \hat{g} = [00\bar{2}] \)

Fig. 36f \( \hat{g} = [1\bar{1}\bar{1}] \)
Fig. 36g  \( \hat{g} = [1\overline{1}3] \)
of the dislocations provided consistent and sensible results only when C was assigned to be on the (111) plane and D on the (111) plane. The dislocations marked Set I on the (111) plane and Set II on the (111) plane were found to perfect dislocations with \( \mathbf{b}_I = \frac{1}{2} [\overline{1}10] (111) \) and \( \mathbf{b}_{111} = \frac{1}{2} [10\overline{1}] (111) \).

Fig. 37 gives a closer look at Set III dislocations on the (111) plane. Dislocations 1-9 (Group 1) are visible for all the reflections except for \( \mathbf{g} = [\overline{1}13] \) and \( \mathbf{g} = [002] \). This is consistent for \( \mathbf{b}_{\text{group 1}} = \frac{1}{2} [\overline{1}10] \).

The Group 2 dislocations are visible for \( \mathbf{g} = [\overline{2}20] \) but invisible for \( \mathbf{g} = [\overline{1}13] \) which is consistent only for \( \mathbf{b} = \frac{1}{6} [\overline{2}1\overline{1}] \) and \( \mathbf{b} = \frac{1}{2} [1\overline{1}0] \). However it is also invisible for \( \mathbf{g} = [1\overline{1}1] \) which agrees with \( \mathbf{g}.\mathbf{b} = 1/3 \) and not \( \mathbf{g}.\mathbf{b}_T = 1 \). Thus \( \mathbf{b}_{\text{Group 2}} = \frac{1}{6} [\overline{2}1\overline{1}] \). This would indicate a region of overlapping stacking faults and thus possibly a thin twin with the perfect dislocations lying within this twin. The final group of dislocations of interest lying among the Set III dislocations of marked Group 3. These were visible for all the reflections used except for \( \mathbf{g} = [\overline{1}1\overline{1}] \) which would mean \( \mathbf{b}_{\text{Group 3}} = \frac{1}{2} [011] \). These dislocations lie near the intersection of the regions C and D.

Finally let us consider the Set IV dislocations on the
Fig. 37a  A detailed view of Set III dislocations $\mathbf{g} = [\overline{2}20]$
Fig. 37b  Schematic of Set III dislocations
(111) plane (Fig. 38). Here there are a number of distinct groups of dislocations to deal with.

Dislocations 1-7 and 9 (Group 1) were determined to have a Burgers vector $\mathbf{b}_{\text{Group 1}} = \frac{1}{2} \langle 10\overline{1} \rangle$. There could be some question for $\mathbf{g} = \langle 11\overline{1} \rangle$ where we could have strong contrast for $\mathbf{g} \cdot \mathbf{b}_P = 2/3$ when $\mathbf{b}_P = \frac{1}{6} \langle 121 \rangle$. If $\mathbf{b} = \frac{1}{6} \langle 121 \rangle$ then the dislocations should be invisible or have weak contrast for $\mathbf{g} = \langle 11\overline{1} \rangle$ and $\mathbf{g} = [002]$ since $\mathbf{g} \cdot \mathbf{b} = 0$ and $1/3$ respectively. However they are definitely visible, thus $\mathbf{b} \neq \frac{1}{6} \langle 121 \rangle$. Also they are visible for $\mathbf{g} = \langle 11\overline{1} \rangle$ ruling out $\mathbf{b} = \frac{1}{6} \langle 11\overline{2} \rangle$ and $\mathbf{b} = \frac{1}{6} \langle 211 \rangle$ for which $\mathbf{g} \cdot \mathbf{b} = 1/3$ and 0 respectively.

Dislocations 6D, 7D and 9D forming Group 2 seem to be connecting or junction type dislocations branching from the perfect dislocations. They were invisible for $\mathbf{g} = \langle 11\overline{1} \rangle$ and $\mathbf{g} = [11\overline{1}]$ thus ruling out $\mathbf{b} = \frac{1}{6} \langle 110 \rangle$. They were visible for $\mathbf{g} = [11\overline{3}]$ eliminating $\mathbf{b} = \frac{1}{6} \langle 211 \rangle$ and visible for $\mathbf{g} = [2\overline{2}0]$ ruling out $\mathbf{b} = \frac{1}{6} \langle 11\overline{2} \rangle$ as $\mathbf{g} \cdot \mathbf{b} = 0$ for both of these cases. Thus $\mathbf{b}_{\text{Group 2}} = \frac{1}{6} \langle 121 \rangle$.

Dislocations 8, 10, 11 and 13 which constitute Group 3 were found to have a Burgers vector of $\mathbf{b}_{\text{Group 3}} = \frac{1}{6} \langle 211 \rangle$. They were invisible for both $\mathbf{g} = \langle 11\overline{1} \rangle$ and $\mathbf{g} = [11\overline{1}]$ thus eliminating the possibility of being the type $1/2\langle 110 \rangle$. The Group 3 dislocations were visible for both $\mathbf{g} = [2\overline{2}0]$ and
Fig. 38a  A detailed view of Set IV dislocations $\mathbf{q} = [220]$
GROUP 1  1 - 7,9
GROUP 2  6A,7A,9A
GROUP 3  8,10,11,13

Fig. 38b  Schematic of Set IV dislocations
\( \mathbf{\hat{g}} = [\bar{1}13] \) which is consistent only for \( \mathbf{b}^* = \frac{1}{6} [\bar{2}11] \) where 
\( \mathbf{g} \cdot \mathbf{b}^*_p = -1 \) and 1 respectively.

So far, the dislocations identified among Set IV have the Burgers vectors: 
\( \mathbf{b}^*_\text{Group 1} = \frac{1}{2} [10\bar{1}] \), 
\( \mathbf{b}^*_\text{Group 2} = \frac{1}{6} [\bar{2}11] \) and 
\( \mathbf{b}^*_\text{Group 3} = \frac{1}{6} [1\bar{2}1] \). There is however another group of dislocations which appear to form a closed dislocation network and with the reflections used could not satisfactorily indexed. These dislocations (Group 4) could be the product of a dislocation reaction between any two of the above mentioned dislocations:

\[
\frac{1}{2}[10\bar{1}] + \frac{1}{6}[\bar{2}11] \rightarrow \frac{1}{6}[11\bar{2}] \tag{11}
\]

\[
\frac{1}{2}[10\bar{1}] + \frac{1}{6}[1\bar{2}1] \rightarrow \frac{1}{3}[2\bar{1}1] \tag{12}
\]

\[
\frac{1}{6}[\bar{2}11] + \frac{1}{6}[1\bar{2}1] \rightarrow \frac{1}{6}[\bar{1}2] \tag{13}
\]

\[
\frac{1}{2}[10\bar{1}] + \frac{1}{6}[2\bar{1}1] \rightarrow \frac{1}{6}[5\bar{1}4] \tag{14}
\]

\[
\frac{1}{2}[10\bar{1}] + \frac{1}{6}[\bar{1}2\bar{1}] \rightarrow \frac{1}{3}[11\bar{2}] \tag{15}
\]

\[
\frac{1}{6}[\bar{2}11] + \frac{1}{6}[\bar{1}2\bar{1}] \rightarrow \frac{1}{2}[\bar{1}0] \tag{16}
\]
None of the resultant dislocations in the above reactions matched the observed Burgers vectors mentioned earlier. As all the dislocations in the network were visible for \( \mathbf{\hat{g}} = [2\overline{2}0] \) this clearly eliminated \( \frac{1}{6}[1\overline{1}2], \frac{1}{6}[\overline{1}12] \) and \( \frac{1}{2}[\overline{1}10] \) as products of any dislocation reaction occurring in this case since \( \mathbf{\hat{g}} \cdot \mathbf{\hat{b}} \) for these dislocations is zero. Next we consider reactions (12) and (14).

\[
\begin{align*}
\frac{1}{2}[10\overline{1}] + \frac{1}{6}[1\overline{2}1] &\rightarrow \frac{1}{3}[2\overline{1}1] & (12) \\
\frac{a^2}{2} + \frac{a^2}{6} &\rightarrow \frac{2}{3}a^2 \\
\frac{1}{2}[10\overline{1}] + \frac{1}{6}[2\overline{1}1] &\rightarrow \frac{1}{6}[5\overline{1}4] & (14) \\
\frac{a^2}{2} + \frac{a^2}{6} &\rightarrow \frac{7}{6}a^2
\end{align*}
\]

Since the square of the Burgers vector is proportional to the elastic energies of the dislocations, it can be seen that reaction (14) is clearly unfavorable. Reaction (12) however shows no change in the elastic energies between product and reactant dislocations, and thus no driving force exists for the reaction to occur either way. A Burgers vector analysis shows that for \( \mathbf{\hat{g}} = [2\overline{2}0], [\overline{1}13] \) and \([1\overline{1}3], \mathbf{\hat{g}} \cdot \mathbf{\hat{b}} \) produces integer values \((2, -2, \text{ and } 0 \text{ respectively})\) and thus the visibility criterion is unambiguous (Table 7). The micrographs indicate that some
Table 7

\[ g^* \cdot b^* \text{ values for } b^* = \frac{1}{3}[211] \]

<table>
<thead>
<tr>
<th>( g^* )</th>
<th>( g^* \cdot b^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>-4/3</td>
</tr>
<tr>
<td>111</td>
<td>2/3</td>
</tr>
<tr>
<td>002</td>
<td>-2/3</td>
</tr>
<tr>
<td>220</td>
<td>2</td>
</tr>
<tr>
<td>113</td>
<td>-2</td>
</tr>
<tr>
<td>113</td>
<td>0</td>
</tr>
</tbody>
</table>
of the dislocations in the network seem to follow the values of \( \frac{2}{3} \) for the above three reflections. The point is that it is possible that \( \frac{1}{3} [2\Pi] \) dislocations are in fact present.

It must be kept in mind that the \( \frac{1}{3} [2\Pi] \) dislocation is produced by the reaction of a \( \frac{1}{2} <110> \) type dislocation with a \( \frac{1}{6} <112> \) type dislocation and that the perfect dislocation must lie on a stacking fault. It is unlikely that the perfect dislocation will combine with a partial dislocation which bounds a stacking fault the \( 1/2 <110> \) dislocation lies on. It is therefore visualized that the \( \frac{1}{2} [10\Pi] \) dislocation must react with the \( \frac{1}{6} [1\Sigma] \) dislocation lying on a plane above or below it, i.e. the \( \frac{1}{2} [10\Pi] \) and \( \frac{1}{6} [1\Sigma] \) dislocations are not strictly co-planar (Fig. 39).

Dislocations on different planes cannot produce reactions on the conventional sense. However, as suggested by Mahajan et al.\(^{39}\), the separation of dislocations is sufficiently close (< 100 Å) for their elastic strain fields to interact and the displacements resulting from such an interaction would be crystallographically similar (although smaller in magnitude) to those resulting from an actual dislocation reaction. Then it is feasible that the electron beam will interact with these displacements
Fig. 39  Schematic of dislocation network formation
to produce an apparent dislocation image, which could account for the network appearance observed in the micrographs.
IV DISCUSSION

1. Introduction

Based on the observations made through electron microscopy, we wish now to consider the possible mechanisms which contribute to the macroscopic flow stress of the material. The following discussion is intended to provide, in as analytical a treatment as possible, the relative importance of the various strengthening mechanisms.

The optical microscopy indicated the presence of second phase particles which were both too large and too widely spaced to have any noticeable effect on the strength of the material (Fig. 40). Besides carbides which are usually considered a primary strengthening feature of cobalt alloys, intermetallic compounds may also be present. In particular, the sigma phase can have an important effect. The improved ductility of the solutionized sample is due to the solutionizing of embrittling again at the grain boundaries. Second phase particles also act as sources of dislocations and therefore are nucleation sites for stacking faults. The role of second phase strengthening based on dispersion effects is not emphasized in the present study. Based on the work of Grant and
Optical micrographs of cast sample: a) x50, b) x100.
Fig. 40c  Optical micrograph of solutionized sample* x100
co-workers\textsuperscript{15-20}, one would not expect a large presence of carbides or sigma phase anyway.

When dealing with room temperature strengthening mechanisms, one normally attempts to achieve as fine a grain size is possible. However, as in the case of most commercially available surgical implant alloys, the grain size is in fact quite large (\(\sim 500 \ \mu \text{m}\)). Thus strengthening due to grain size effects would be minor.

2. The Role of Stacking Faults in Strengthening

The primary characteristic of the deformation substructure is the presence of wide stacking faults. To analyse the contribution these stacking faults make to the work hardening behavior of this alloy, one must consider the mechanism by which they impede dislocation motion. As perfect dislocations are all widely dissociated in this material, the only matrix dislocations intersecting stacking faults are partials. In other words, we must examine the energetics of the intersection of two stacking faults.

As was found experimentally, under seemingly very low stresses, short segments of stacking faults would extend into very long ribbons or bands. With faults on
different planes activated, these bands may intersect. What we want to ask ourselves is, whether the stress needed for two stacking faults to successfully intersect ($\tau_{SF\ intersection}$) is comparable to the stress needed to produce the long stacking fault bands ($\tau_{SF\ formation}$). If the latter ($\tau_{SF\ formation}$) is much less than the former ($\tau_{SF\ int.}$), then, it is argued here that this indicates that stacking faults provide a major obstacle to the propagation of other stacking faults. Hence this would define the role of stacking faults in strengthening.

In a fcc lattice the mutual repulsive force between Shockley partials extends a dislocation to a width that is inversely proportional to the stacking fault energy. Therefore we must first calculate the stresses necessary to create the very wide extensions observed in this alloy.

Neglecting Peierls-Nabarro friction, the stress needed to develop a stacking fault ribbon is given by Friedel\(^40\) as:

$$\tau_{SF\ form} = \frac{\gamma}{|b_p^\dagger|} \quad (17)$$

where $\gamma = $ stacking fault energy

$b_p^\dagger = $ Burgers vector for partial dislocation = $\frac{1}{6}<112>$
Thus for a low stacking fault energy, say \( \gamma \approx 2 \frac{\text{ergs}}{\text{cm}^2} \)

\[ \tau_{\text{SF \, form}} = 10^8 \text{ dynes/cm}^2 \]

Let us now expand the problem to that of the intersection of two of these stacking faults.

Marcinkowski and Miller\(^{41}\) have suggested one approach. Dislocations which are on one \(\{111\}\) plane may intersect a stacking fault on another \(\{111\}\) plane and cut through it in the \(\langle 110 \rangle\) direction. The Burgers' vectors of the cutting dislocations are such that they displace the lower portion of the stacking fault with respect to the upper part (Fig. 41). Since after a dislocation has passed through a stacking fault, a number of the atoms in the fault are brought very close to one another these workers conclude that the stress to drive a dislocation through the fault would be on the order of the theoretical shear stress given by

\[ \tau_{\text{SF int.}} = \frac{|\mathbf{b}|}{a} \frac{\mu}{2\pi} \quad (18) \]

where \( \mu = \text{shear modulus} \)

\( a = \text{lattice parameter} \)

To achieve these stresses then requires a stress concentration such as that from a dislocation pile-up.
Fig. 41 Intersection of dislocations with stacking faults (Ref. 41)
Saarinen and Miekk-oja\textsuperscript{42} have a slightly more refined model although their conclusions are the same. They consider the intersection of a dislocation with a stacking fault with width $d$. The dislocation is parallel to the line of intersection. According to these workers the shear stress $\tau$, acting on it can perform the work:

$$W = \tau b^2 d \sqrt{3}/2$$

This intersection will lead to a jog in the stacking fault if

$$W > U_{\text{jog}}$$

where $U_{\text{jog}}$ is the jog energy which in this case reduces to the constriction energy of a dislocation given by Friedel\textsuperscript{43} as:

$$U_{\text{jog}} = \frac{\mu b^2 d}{30}$$

This approximation of the jog energy is based on the fact that the jog energy is the sum of the line energy of the jog itself plus the constriction energy. If the stacking fault width is much larger than the length to of the jog, the energy of jog formation is then reduced to that of the constriction.
The limiting condition is given by:

$$W = U_{jog} \tag{22}$$

Thus the minimum shear stress needed for intersection to occur is given as:

$$\tau_{SF \text{ int.}} = \frac{\mu}{25} \tag{23}$$

Saarinen and Miekk-oja do however point out that metals of high stacking fault density are deformed at much lower stresses. They account for the discrepancy by suggesting various possibilities:

1) The intersection of stacking faults may be a thermally activated process;

2) The dislocation may not be parallel to the line of intersection; and

3) The dislocation is pressed at the stacking fault by a pileup of other dislocations behind it.

In both of the models suggested by Marcinkowski & Miller and Saarinen&Miekk-oja, the dislocation intersecting the stacking fault is taken to be a perfect dislocation. What we wish to consider is the intersection of a partial dislocation with a stacking fault, as this is what describes the intersection of two stacking faults. Before considering the details of this process, let us present an outline of
the argument to be formulated. The intersection of two stacking faults results in the creation of a defect at the intersection. Associated with this defect is an energy which would provide a measure of the energy needed to produce this defect. This in turn gives an indication of the extent to which a stacking fault is an obstacle to the motion of an intersecting partial dislocation.

Ashbee has suggested from geometric arguments (Fig. 42) that the intersection of two stacking faults results in the formation of a volume dilatation along the line of intersection. He has shown that this dilatation is described by an increase in volume of $a^3/18$ per unit cell, which is equivalent to having two atoms in nine missing. Thus the energy to produce this intersection is expected to be approximately $2/9 E_v (\sim 0.2eV)$ per atomic plane, where $E_v$ is the energy of formation of a vacancy. Thus the 0.2 eV may be considered as a measure of the energy needed to create such a defect (i.e. dislocation dipole). The stress necessary for two stacking faults to intersect each other may be viewed as the stress needed to produce a defect with a self energy of 0.2eV. The energies involved here are of the order of core energies which is what one should expect for such short range interactions. The energy associated
Fig. 42  (110) projection of stacking fault intersection producing a volume dilation (Ref. 44)
with the interaction may be given as

\[ U = b \int \tau dx \]  \hspace{1cm} (24)

where \( b \) = magnitude of Burger's vector of a partial dislocation, \( 1/6 <112> \)

\( \tau \) = stress needed to form the stacking fault intersection

Thus

\[ \tau = \frac{1}{b} \frac{dU}{dx} \approx \frac{1}{b} \frac{\Delta U}{b} \]  \hspace{1cm} (25)

\( \Delta U \approx 0.2 \text{eV} \)

\[ = 3.2 \times 10^{-12} \text{erg} \]

\( \therefore \) \( \tau_{\text{SF int.}} \approx 5.0 \times 10^3 \text{ dynes/cm}^2 \)

Recall that

\[ \tau_{\text{SF form.}} = \frac{\gamma}{|\vec{b}|} \approx 10^8 \text{ dynes/cm}^2 \]

Therefore

\[ \tau_{\text{SF int.}} \ll \tau_{\text{SF form.}} \]  \hspace{1cm} (26)
The point to be made from the above discussion is that, while stacking faults do act as obstacles to the growth of other stacking faults, they are not a strong barrier. The high number of successful stacking fault intersections observed attests to this hypothesis.

Once this intersection is produced, this can prove to be a formidable obstacle to any further dislocation motion. To treat this problem more analytically, it is worthwhile to consider the stacking fault intersection as a dislocation dipole. Since it has been shown in this work that stacking faults act as regions for localized slip, the nature of the interaction between slip dislocations and the dipole existing at this intersection of stacking faults becomes very important to consider. Dipoles have a stress field which tends to zero with \(1/r^2\) where \(r\) is the distance from the dipole. Thus it is necessary to consider the interaction between a dislocation and a dipole only at very small distances (\(\sim 5b\) or less). At this stage one is considering the interaction of dislocation cores. Thus the dislocation intersecting the dipole experiences the interaction force between the dipole dislocations themselves. In order to pass through this dipole, the intersecting dislocation must overcome the attractive force between the dipole
dislocations. The maximum attractive force per unit length between the dipole dislocations when separated by a distance \( y \) is given by Olson and Cohen\textsuperscript{45} as:

\[
F = \frac{-\mu b^2}{8\pi(1-v)y}
\]  

(27)

Following the treatment by Ashbee, the row of vacancies at the stacking fault intersection is actually a partial dislocation dipole. Thus for our purpose, \( b \) in the above equation refers to a partial dislocation in the fcc lattice. The dipole separation \( y \) may also be estimated by using Ashbee's geometric argument. The volume dilation produced by the intersection is \( a^3/18 \), then \( d \) may be taken as \( \left( \frac{a^3}{18} \right)^{1/3} \). The interactive force is thus approximately 260 dynes/cm. The corresponding stress needed to separate the dipole dislocations given by:

\[
\tau \approx \frac{F}{|\vec{b}|} \approx 2 \times 10^{10} \text{ dynes/cm}^2
\]

This would be indicative of the stress needed for a perfect dislocation to pass through a stacking fault intersection (i.e. a dipole). The point of these numerical calculations is simply to emphasize that stacking fault intersections when treated as dislocation dipoles can prove to be a major obstacle for the motion of slip dis-
locations.

3. **Dislocation Interaction on Intersecting Planes**

It has been shown in this work that stacking faults act as glide planes. The dislocations lying on the stacking fault planes are perfect dislocations. In trying to ascertain the various contributions to the flow stress of the alloy, one possibility to consider is the interaction between these perfect dislocations. Naturally, the dislocations of the same sign lying on the same plane will interact with each other repulsively and if they reach an obstacle, a dislocation pileup will occur. This particular aspect will be analysed in detail later. At the present time, another situation that must be considered is the interaction of dislocations on intersecting stacking fault planes. As has been shown, at the early stages of deformation, stacking fault formation is activated on all the \{111\} planes. Thus intersecting paths for the perfect dislocations are readily available.

When two dislocations meet at the intersection of the \{111\} planes, various possible reactions may occur\textsuperscript{46}. The favorable reactions are those which lead to a lowering of the total energy. With two intersecting perfect dislocations, one such reaction is that described by Lomer:
\[ \frac{a}{2} [01\overline{1}]_{(111)} + \frac{a}{2} [101]_{(11\overline{1})} + \frac{a}{2} [110] \] (28)

The resultant dislocation is of pure edge character and is sessile on all \{111\} planes. Thus it is pinned at the intersection and would serve as a barrier to other dislocations passing down the \{111\} and \{11\overline{1}\} planes.

To estimate the nature of the obstacles provided by the Lomer lock, the force experienced by a \(a/2[01\overline{1}]_{(111)}\) dislocation due to the \(a/2[110]\) Lomer dislocation must be calculated. The interaction energy (per unit length) between parallel dislocations with arbitrary Burgers' vectors, while ignoring end effects, is given as \(^{47}\):

\[
W = \frac{-\mu (\hat{b}_1 \cdot \xi) (\hat{b}_2 \cdot \xi)}{2\pi} \ln \frac{R}{R_0}
\]

\[
- \frac{\mu}{2\pi (1-\nu)} [(\hat{b}_1 x \xi) \cdot (\hat{b}_2 x \xi)] \ln \frac{R}{R_0}
\]

\[
- \frac{\mu}{2\pi (1-\nu) R^2} [(\hat{b}_1 \cdot \xi) \cdot \hat{R}] [(\hat{b}_2 \cdot \xi) \cdot \hat{R}]
\]

(29)

where: \(\mu = \) shear modulus

\(\hat{b}_1, \hat{b}_2 = \) Burger's vector of dislocations

\(\xi = \) dislocation line direction
\[ R = \text{distance between dislocation} \]
\[ \nu = \text{Poisson's ratio}. \]

The first term of the above equation refers to the screw component of the dislocations, while the latter two terms refer to the edge components. From the interaction energy, the interaction force may be derived. The radial component of the force is:

\[
F_R = -\frac{\partial W}{\partial R} = \frac{\mu}{2\pi R^2} (\mathbf{\hat{b}}_1 \cdot \mathbf{\hat{r}}) (\mathbf{\hat{b}}_2 \cdot \mathbf{\hat{r}}) + \frac{\mu}{2\pi (1-\nu) R} \left[ (\mathbf{\hat{b}}_1 \times \mathbf{\hat{r}}) \cdot (\mathbf{\hat{b}}_2 \times \mathbf{\hat{r}}) \right]
\]

(30)

The tangential component of the force is:

\[
F_\theta = -\frac{1}{R} \frac{\partial W}{\partial \theta}
\]

\[
= \frac{\mu}{2\pi (1-\nu) R^3} \left\{ (\mathbf{\hat{b}}_1 \times \mathbf{\hat{r}}) [ (\mathbf{\hat{b}}_2 \times \mathbf{\hat{r}}) \cdot \mathbf{\hat{r}} ] + (\mathbf{\hat{b}}_2 \times \mathbf{\hat{r}}) [ (\mathbf{\hat{b}}_1 \times \mathbf{\hat{r}}) \cdot \mathbf{\hat{r}} ] \right\}
\]

(31)

In this particular case, there is no tangential component to consider, (Fig. 43), and since the Lomer dislocation has only an edge component, the repulsive force experienced by the incoming dislocation on the (111) plane is given by

\[
F_R = \frac{\mu}{2\pi (1-\nu) R} \left[ (\mathbf{\hat{b}}_1 \times \mathbf{\hat{r}}) \cdot (\mathbf{\hat{b}}_2 \times \mathbf{\hat{r}}) \right]
\]

(32)
Fig. 43  Dislocation interaction with Lomer lock
where:

\[
\begin{align*}
\mathbf{b}_1 &= \frac{a}{2} [110]_{\text{Lomer}}; \quad a_{\text{Co}} = 3.5 \times 10^{-8} \text{ cm} \\
\mathbf{b}_2 &= \frac{a}{2} (01\overline{1}) \langle 111 \rangle \\
\mathbf{c} &= [001] \\
\mu &= 3 \times 10^{11} \text{ dynes/cm}^2 \\
|\mathbf{\hat{R}}| &\approx \frac{|\mathbf{b}_1|}{2<110>} = 2.5 \times 10^{-8} \text{ cm} \\
\gamma &\approx 0.3
\end{align*}
\]

Therefore

\[
F_R = 585 \text{ dynes/cm}.
\]

The stress which must be applied to overcome such a repulsive interaction is:

\[
\tau \approx \frac{F_R}{|\mathbf{b}_1|/2<110>} = 2.3 \times 10^{10} \text{ dynes/cm}^2
\]

While the Lomer lock can certainly be a major contributor to the work hardening of this alloy, a Lomer lock, however, is not always produced at every intersection of \{111\} planes. However, before they meet at the intersection, the dislocations always do experience the stress fields of dislocations lying on other planes. What is presented in the following is the calculation of the interactive force between two perfect dislocations, each lying an intersecting \{111\} planes. For the sake of analysis, one dislocation is treated as being fixed, while the
other moves (Fig. 44). The dislocations are considered as being parallel since this give rise to the maximum interaction. For the purpose of this calculation, the vectors described in the previous equations were defined in a cartesian coordinate system relative to the direction defined by the Thomson tetrahedron (Fig. 45). This was done since the four sides of the tetrahedron define the four unique \{111\} planes in an fcc unit cell and the edge defines the \langle 110 \rangle directions of the Burgers' vectors \( \mathbf{q} \) the perfect dislocations. In this case both the radial and tangential components of the force must be calculated. The resultant force in the direction of the moving dislocation is given by

\[
F_x = F_R \cos \theta - F_\theta \sin \theta \tag{33}
\]

From the geometry illustrated in Fig. 45

\[
\hat{\mathbf{R}} = R [y_0 \cot \theta \quad y_0 \quad 0] \]

\[
\hat{\mathbf{b}_1} = a [\cos 30^\circ \quad 0 \quad \cos 60^\circ] \]

\[
\hat{\mathbf{b}_2} = 0.85a [\cos 60^\circ \quad \cos 35.28^\circ \quad \cos 60^\circ] \]

\[
|\hat{\mathbf{b}_1}| = |\hat{\mathbf{b}_2}| = a
\]

\[
\hat{\mathbf{r}} = [001]
\]

This results in
Fig. 44 Dislocations on intersecting planes
Fig. 45 Schematic of coordinate system in terms of the Thomson tetrahedron
\[ F_R = \frac{\mu a^2}{2\pi R} [0.213 + \frac{0.368}{1-\nu}] \]

and

\[ F_\theta = \frac{.867\nu a^2}{2\pi(1-\nu)R} [0.85\cot\theta - 0.694\cot^2\theta + 0.694] \]  \hspace{1cm} (34)

With \( Y_o \approx 0.5\mu m = 5 \times 10^{-5} \text{ cm} \), \( F_\theta \ll F_R \). Thus we can approximate \( F_x \) as \( F_x \approx F_R \cos \theta \). The variation in the interactive force as a function of the distance \( R \) is shown in Fig. 46. As can be seen the maximum repulsive force experienced by the dislocation is only 0.434 dyne/cm. One should in fact consider an array of dislocations and not just one on each of the \{111\} planes. To account for this we may assign a Burger's vector for this array \( \vec{B} \) where \( \vec{B} = n\vec{b} \), \( n \) being the number of dislocations in the array. This has the effect of increasing the interactive force by a factor of \( n^2 \). Thus for example, if \( n = 10 \), \( F_x \) in our calculations then becomes \( \approx 43 \text{ dynes/cm} \). Therefore for a single dislocation to overcome a repulsive force of 43 dynes/cm, a stress of \( \approx 17 \times 10^8 \text{ dynes/cm}^2 \) needs to be applied. While this is less than the stress needed to bypass a Lomer lock, this type of interaction does occur more frequently than that between slip and Lomer dislocations.
Fig. 46  Variation of interactive force with distance
4. **Solid Solution Strengthening Effects**

Given the high alloy content, solute elements are definitely to be expected to contribute to the flow stress of the metal. To provide some quantitative estimate of this contribution, the various types of dislocation interactions with solute atoms may be examined: elastic, electrical and chemical. Any analytical description of the latter two effects is very difficult, and only phenomenological explanations based on experimental data can be discussed^{48,49}. Consequently, we shall dwell here primarily on the elastic interactions. It should be noted that even in this case, solute-dislocation interaction models must be treated with a great deal of caution as there exist many inadequacies in trying to relate them to macroscopic properties.

For the sake of analysis, we shall approximate our alloy system by the Co-Cr binary. Elastic interactions may be classified into size and modulus differences between the two types of atoms. In this case only the size differences will be considered quantitatively due to the limited availability of data. One approach to study the effect of size misfit on solute-dislocation interaction is that suggested by Argon^{50}.

The stress field of an edge dislocation produces a
hydrostatic pressure $P$:

$$p = -\frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$  \hspace{1cm} (35)$$

but for an edge dislocation: (Fig. 47)

$$\sigma_{xx} = \frac{\mu b}{2\pi (1-\nu)} \frac{y(3x^2 + y^2)}{(x^2 + y^2)^2}$$

$$\sigma_{yy} = \frac{-\mu b}{2\pi (1-\nu)} \frac{y(x^2 - y^2)}{(x^2 + y^2)^2}$$  \hspace{1cm} (36)$$

$$\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{yy})$$

$$p = \frac{1+\nu}{1-\nu} \cdot \frac{1}{3\pi} \cdot \frac{\mu b y}{x^2 + y^2}$$  \hspace{1cm} (37)$$

The introduction of a rigid sphere into a somewhat smaller spherical hole in an infinite elastic body produces no dilatation anywhere outside the sphere; it only displaces the boundaries of any closed surface enclosing the misfitting sphere by an amount that would increase its volume by the misfit volume of the rigid sphere.

The volume change produced by the misfitting sphere is:

$$\Delta V = 4\pi a^3 \varepsilon = \frac{\pi \varepsilon b^3}{2}$$  \hspace{1cm} (38)$$
Fig. 47 Interaction of edge dislocation with substitutional solute atom
where \( \varepsilon = \) size misfit parameter

The misfit parameter has been approximated a number of ways. One such estimate is the following as described by Kovács and Zsoldos\(^{51}\).

According to observations in solid solutions, at least to a limited concentration, the average volume \( V \) relating to one atom, is a linear function of the concentration \( c \) of the solute atoms:

\[
V(c) = (1-c)V_o + cv^* \tag{39}
\]

where \( V_o \) is the atomic volume in the case of the pure material and \( v^* \) is a constant characteristic of the solute atoms. This volume change is caused by the size effect of the solute atoms. If the number of solute atoms is \( N_s \), and the total number of atoms is \( N \), \( c = N_o/N \) and the volume change produced by one solute atom is given by the equation:

\[
V' = \frac{N(V-V_o)}{N_s} = c(V^*-V_o)\frac{N}{N_s} = v^*-V_o \tag{40}
\]

From this, the relative volume change per atom, the volume size factor is:

\[
\eta_V = \frac{\Delta V'}{V_o} = \frac{V^*-V_o}{V_o} = \frac{1}{V} \frac{\partial V}{\partial c} \tag{41}
\]
Fleischer\textsuperscript{52} has suggested an approximation for $\eta_v$ as:

$$\epsilon = \frac{1}{a_0} \frac{da}{dc} \quad (42)$$

where $a = \text{the lattice parameter}$

It should be noted that $\Delta V/V_o$ denotes the relative difference of the "free" volume of the solute atom as related to the cavity, whereas $\Delta V'/V_o$ refers to the deformed volume of the solute atom (i.e. its volume in the matrix). Eshelby\textsuperscript{53} has shown that the insertion of a sphere into an elastic medium produces an increase in the lattice parameter $a$, which is given by the real difference in radius multiplied by $\xi$ where:

$$\xi = \frac{3(1-\nu)}{l+\nu} \quad (43)$$

$$\therefore \quad \epsilon = \frac{1}{a_0} \frac{da}{dc}/\xi \quad (44)$$

Thus returning to the expressions for $P$ and $\Delta V$ (equation (35) and (38) respectively), the energy of interaction originating from the size effect of an edge dislocation and a small, spherical defect (e.g. solute atom) is:

$$E = P\Delta V = \frac{\theta}{6} \left( \frac{6b}{\mu} \frac{a \epsilon_y}{x^2 + y^2} \right) \quad (45)$$
where: \[ \beta = (1-\nu)/(1-\nu) \]

Therefore the dislocation will resist the introduction of any misfitting sphere by its hydrostatic pressure by means of raising the energy of the crystal. The force between the dislocations and the solute atom is:

\[
F = -\frac{\partial E}{\partial x} = \beta \frac{\mu \epsilon b^4 x \sqrt{3}}{3(x^2 + y^2)^2} \tag{46}
\]

At low temperatures the solute cannot move because the required diffusion of vacant lattice sites is too slow. As the dislocation glides on its slip plane, it will experience a varying interaction which is strongest at \( x = y/\sqrt{3} \). This maximum force opposing the motion of the dislocation is:

\[
F_{\text{Max}} = \frac{\beta \mu \epsilon b^4 \sqrt{3}}{y^2} \frac{1}{16} \tag{47}
\]

If \( Y \) is approximately equal to the mean distance of the solute atoms randomly distributed in the lattice, the force on the dislocation per unit length is:

\[
f = \frac{F_{\text{Max}}}{Y} = \frac{\sqrt{3}}{16} \beta \mu \epsilon b (\frac{b}{Y})^3 \equiv \frac{\sqrt{3}}{16} \beta \mu \epsilon bc \tag{48}
\]
where: \( c \) = concentration of solute in atom fraction

The motion of a dislocation through a unit cube will produce a shear strain \( b \), which makes an externally applied shear stress \( \tau \) to do work on the crystal described by \( \tau b \). This work must appear on the slip plane as the displacement of the dislocation a distance unity against an opposing force of the magnitude given by \( f \). This force can be overcome by an external shear stress, which defines the shear strength of the solution hardened crystal:

\[
\tau = \frac{f}{b} = \frac{\sqrt{3}}{16} \beta \mu c
\]  

(49)

Let us now apply the above relation to the Ni-Cr system. As we are considering only size effects, it is reasonable to approximate the Co-Cr system by the Ni-Cr system for which room temperature solid solution strengthening data is available. Using the data of Pelloux and Grant\(^ {54} \), we wish to estimate the size misfit parameter \( \varepsilon \).

\[
\varepsilon = \frac{1}{\xi a_o} \frac{\text{d}a}{\text{d}c}
\]
but \[ \frac{da}{dc} = \frac{d\tau}{da} \] (50)

Pelloux and Grant took \( d\tau/dc \) and \( d\tau/da \) to be constant up to approximately 30 a/o Cr.

For Ni-Cr alloys at room temperature (Fig. 48):

\[ \frac{d\tau}{dc} = 5.0 \times 10^9 \text{ dynes/cm}^2 - \text{atomic fraction} \]

and

\[ \frac{d\tau}{da} = 4.0 \times 10^{18} \text{ dynes/cm}^2 - \text{cm} \]

Therefore:

\[ \frac{da}{dc} = \frac{d\tau}{dc} \]

\[ = 1.2 \times 10^{-9} \text{ cm/atom fraction} \]

For pure nickel

\[ a_\circ = 3.52 \times 10^{-8} \text{ cm} \]

\[ \xi = \frac{3(1-\nu)}{1+\nu} = 1.62 \text{ for } \nu = .3 \]
Fig. 48  Solid solution strengthening effects in some nickel alloys (Ref. 54)
Thus

\[ \tau = \frac{\sqrt{3}}{16} \mu \varepsilon \sigma \]

\[ = 5 \times 10^8 \text{ dynes/cm} \]

where:

\[ \mu_{\text{Ni}} = 1.6 \times 10^{11} \text{ dynes/cm}^2 \]

\[ c = .3 \]

5. A Nucleation Model for Twinning

As pointed out earlier, one of the primary modes of deformation is by slip occurring on stacking faults. Also, the presence of microtwins constituted another major feature of the deformation substructure. These twins form an important obstacle to slip. Thus it is necessary to examine the nature of twin-slip interaction. Before this is done however, a mechanism to nucleate and grow the twins must be formulated. Deformation twinning in fcc crystals takes place by shear in \( \langle 112 \rangle \) directions on \( \{111\} \) planes. Thus it would be expected that there is a relationship between the presence of stacking faults and the formation of twins. Recall that for an fcc structure the stacking sequence is

...ABCABCABC...
for an intrinsic stacking fault

\[ \ldots \text{ABCA/CABC} \ldots \]

and for a fcc twin

\[ \ldots \text{ACBACBACB} \ldots \]

Thus the formation of intrinsic stacking faults on every plane produces a twin. The problem of establishing a nucleation model comes in suggesting a process which will allow for the generation of a partial dislocation of just the right sign on each successive \{111\} plane or the transfer of one partial from one plane to the next. Various models have been proposed for the nucleation of twins in fcc metals, some of which will now be reviewed.

5.1 Venables:

Venables\(^{55}\) has extended the concept of the pole mechanism suggested by Cottrell and Bilby\(^{56}\), to fcc crystals. The basis of this mechanism is described by the location reaction

\[ \text{AC} + \text{A}_\alpha + \alpha \text{C} \quad (51) \]

using the notation of the Thomson tetrahedron (Fig. 49). \(\text{A}_\alpha\) which is a Frank partial is the "pole" which leaves the plane and \(\alpha \text{C}\) rotates around this pole. Venables has proposed that having revolved once around the pole, the twinning dislocation (Shockley partial, \(\alpha \text{C}\)) meets \(\text{A}_\alpha\) and
Fig. 49  The Thomson tetrahedron (Ref. 46)
recombines to form AC again (Fig. 50). This perfect dislocation can redissociate at the next atomic plane, producing a second fault. Thus a twin of finite thickness can be built up from a single stacking fault. As pointed out by Mahajan and Williams\textsuperscript{57} however, in the Cottrell and Bilby approach a strongly pinned pole dislocation is a prerequisite; and it is unlikely that the $\frac{1}{2}\langle 110 \rangle$ (AC) dislocation, being glissile, will be anchored strongly enough to prevent it from moving under the stress causing the sweeping dislocation to move.

5.2 Cohen and Weertman

These workers\textsuperscript{58} have suggested that a perfect dislocation stress dissociates into a Frank partial and a Shockley partial. The glide of the Shockley partials on the appropriate \{111\} planes could then lead to the formation of a (probably imperfect) twin.

5.3 Hirth

Hirth\textsuperscript{59} has proposed a pole mechanism based on the following dissociation reaction:

$$\frac{1}{2}\langle 110 \rangle \rightarrow \frac{1}{6}\langle 41\overline{1} \rangle + \frac{1}{6}\langle 121 \rangle$$

(51)

However, the $\frac{1}{6} \langle 41\overline{1} \rangle$ dislocation being a slip dislocation
Fig. 50  Cottrell and Bilby's twinning mechanism and its extension (Ref. 55)
in the twin may not be anchored strongly to act as a pole dislocation.

5.4 Mahajan and Chin

Unlike the pole mechanisms utilised in the above mentioned models Mahajan and Chin\textsuperscript{60} have used a totally different approach. They have proposed a governing dislocation reaction as:

\[ \frac{1}{2}\langle 110 \rangle + \frac{1}{2}\langle 101 \rangle + 3 \times \frac{1}{6}\langle 211 \rangle \]  

(52)

In other words three layer twins form when two coplanar 1/2\langle 110 \rangle dislocations of different Burgers vectors zip together to form three 1/6\langle 112 \rangle twinning partials (Fig. 51). The basis of this model is the formation of fault pairs by considering the interaction of partials on adjacent and coplanar stacking faults. A macroscopic twin could form when these three layer twins distributed within a slip band, grow into each other. This nucleation model is similar to one suggested by Sleeswyk\textsuperscript{61} for the nucleation of twins in bcc crystals; where three layer twins evolve from the dissociation of 1/2 \langle 111 \rangle screw dislocations into three 1/6\langle 111 \rangle twinning partials.

While Mahajan and Chin have supported their model by electron microscopy evidence found in a Co-9 wt % Fe
Fig. 51  Fault configurations during growth of a three layer twin (Ref. 60)
alloy, it is very questionable whether this model can be applied to the Co-Cr-Mo-C alloy under study here. The interaction of partials requires that they are close enough to react. However with the long separation of partials present in this system, this is clearly not possible. Another possibility is that fault dislocations on different\{111\}planes may react through cross-slip. This requires that the dislocation must constrict. However as shown in the undeformed material, every dislocation is dissociated and the partial separation in all cases is very large. Consequently the possibility of constrictions is very unlikely.

5.5 Olson and Cohen

These workers\textsuperscript{62} have considered in detail a mechanism of martensitic nucleation which may also be readily extended to consider twin nucleation. An important aspect of this mechanism is the formation of a martensitic embryo by a faulting process derived from a group of already existing dislocations which happen to be appropriately spaced. The fault energy per unit area associated with such an embryo, \( n \) atomic planes in thickness can be expressed as:

\[
\gamma = n \rho_A (\Delta G^{\text{Chemical}} + E^{\text{Strain}}) + 2\sigma(n) \quad (53)
\]
where $\rho_A$ is the density of atoms in a close packed plane in moles per unit area, $G_{\text{chemical}}$ and $E_{\text{strain}}$ are the parent-product chemical free energy difference and coherency strain energy respectively, expressed as molar quantities, and $\sigma(n)$ is the particle/matrix interfacial energy. The critical condition for the spontaneous formation of an embryo from a defect was taken as $\gamma = 0$. The above equation was then used to define the size of the existing defects necessary to account for embryo formation under the observed conditions for martensite nucleation.

As pointed out by Olson and Cohen, mechanical twinning can be viewed as a special class of martensitic transformation where the parent and product phases have the same structure but differ in orientation. Twinning requires the passage of a Shockley partial dislocation on every close packed plane rather than on every second plane as is the case of the fcc $\rightarrow$ hcp transformation. Thus the defect necessary for the nucleation of an fcc twin involves a denser array of dislocations than for the nucleation of hcp from fcc.

Given the existence of a fault embryo ($n = 4$ or $5$) of the appropriate spacing, Olson and Cohen have suggested that the embryo may thicken by a pole mechanism. Once conditions are such that the fault embryo can extend to
large distances in the fault plane, there is a high probability that it will intersect "forest dislocations" inclined to the fault plane. If such a forest dislocation is strongly pinned and has a Burger's vector with a screw component normal to the fault plane one interplanar spacing in magnitude, it can act as a pole for the spiraling of the transformational dislocation onto consecutive planes.

When a single transformational dislocation interesects a lattice pole dislocation, a jog is formed in the transformational dislocation which leaves behind a partial dislocation dipole. The partial dislocations of this dipole are bound together by a strong attractive interaction. However, if the entire array of transformational dislocations at the leading front of an embryo, of the critical thickness required for spontaneous embryo formation, intersects a pole dislocation, a dipole is produced on each plane. The dipoles will intersect to produce a single transformational dislocation dipole separated by the thickness of the embryo and connected to the pole dislocation. The dipole interaction is now much weaker and in response to the force provided by the negative stacking fault energy, the transformational dislocations can then spiral around the pole dislocation. (Fig. 52).
Fig. 52  Embryo thickening by pole mechanism (Ref. 45)
5.6 Stacking Fault Mechanism

What makes the Olson and Cohen model initially appealing for applying it to the present study is its criterion used for embryo formation in martensitic nucleation. Olson and Cohen suggest that the critical condition for the spontaneous formation of an embryo is taken by setting the fault energy to equal zero. This condition is applicable to the alloy under study here since as has been shown earlier, the fault energy is so low (i.e. \( \gamma = 0 \)) that no perfect dislocation exists by itself in the lattice and that they are totally unstable, thus dissociating widely into their respective partials. This evidence lends some physical support to the concept of a twin embryo as the probability of the formation of a twin nucleus increases as the fault energy decreases.

Given the presence of a twin nucleus, the problem is now to explain the growth of this embryo. Olson and Cohen have used a pole mechanism. However the presence of such a pole dislocation, which would be sessile on the plane of the transformation dislocation, was not evident. For example, Frank partials, which would have a screw component normal to the fault plane, were never found. However there are faults inclined to the fault plane containing the embryo. What we are suggesting is
that stacking faults may provide the pole dislocations needed for the embryo to grow. Thus when the twin embryo intersects a stacking fault lying on a different plane, it effectively encounters two pole dislocations which are the two partials bounding the stacking fault.

In the limit, if a single transformational dislocation, \( \frac{1}{6}\langle 112 \rangle \), intersects these two pole dislocations, this is equivalent to the intersection of two stacking faults. As Ashbee\(^{44}\) and Heidenreich and Shockley\(^{64}\) have pointed out such an intersection produces a volume dilatation in the lattice which Ashbee describes as a "partial dislocation doublet". This is analogous to the dipole formation suggested by Olson and Cohen in their model.

If a twin embryo intersects the stacking fault instead, a "partial doublet" is formed on each plane and the resulting doublet or dipole interaction is now much weaker for the same reasons as suggested in the Olson and Cohen model. This would mean that the transformational dislocations can spiral around the stacking fault, moving on to consecutive close packed planes with each revolution (Fig. 53). The idea of dislocations wrapping themselves around a stacking fault may seem more acceptable if we keep in mind that what is essentially operating is a type of "double pole" mechanism where for the
Fig. 53  Twin growth by "stacking fault mechanism"
embryo growth to occur, the transformational dislocations must intersect two partial dislocations. Olson and Cohen have pointed out that in the fcc → hcp transformation where embryo thickening requires the passage of a $1/6<112>$ transformational dislocation onto every second close packed plane, the dipole would not begin to spiral until a second inclined lattice dislocation is intersected. In our case both the dislocations happen to be partials.

The next point to consider is how applicable it is to use this modified version of the Olson and Cohen model. The key to the growth of the embryo is knowing whether the driving force for the transformation is great enough to overcome the dipole attraction given the thickness required for embryo formation. We shall return to discussing this parameter in a moment; however first what is more useful for our purposes is to understand how this nucleation mechanism contributes to the work hardening mechanism of this alloy. The stress necessary for twin nucleation from such a defect is:

$$\tau^* = \frac{2\sigma}{n\rho_A V_M s}$$  \hspace{1cm} (54)
where: \( \sigma = \) coherent twin boundary energy for fcc cobalt \( ^{65} \quad \sigma = 1.3 \text{ergs/cm}^2 \)

\( n = \) number of dislocations in embryo = 4

\( \rho_A = \) number of moles/unit area of close packed plane = \( 3 \times 10^{-9} \) moles/cm\(^2\)

\( V_m = \) molar volume = 6.6 cm\(^3\)/mole

\( s = \) twinning shear = \( \frac{\text{twinning displacement}}{\text{distance between twinning planes}} \)

\[ = \frac{b}{6<112> \left/ d_{(111)} \right.} = 0.707 \]

Therefore the critical resolved shear stress necessary for twin nucleation is

\[ \tau^* = 4.5 \times 10^8 \text{ dynes/cm}^2 \]

Recall that the .2% yield stress for this alloy is \( 8 \times 10^9 \) dynes/cm\(^2\). The shear stress is approximately one half of this, \( \tau_{.2\%} = 4 \times 10^9 \) dynes/cm\(^2\). Thus \( \tau_{.2\%} \approx \tau^* \).

In other words, the available stress at yield is high enough to nucleate twins by this embryo mechanism. The difference between \( \tau_{.2\%} \) and \( \tau^* \) may in part be attributed to the lattice friction stress.

As pointed out earlier, to make the pole mechanism operate, a force must be applied which is great enough to overcome the attractive force of the dipole dislocations. The attractive force is given by
\[ F = \frac{-\mu b^2}{8\pi(1-\nu)nd} \quad (55) \]

\[ \hat{b} = \frac{1}{6}\langle112\rangle, \quad d = d_{\{111\}} \]

\[ F \approx 43 \text{ dynes/cm} \]

The force available is given by \( F = \tau \cdot 2\% \cdot b \approx 57 \) dynes. Thus there is enough force to overcome the dipole interaction, thereby allowing for the dipoles to spiral around the dissociated pole dislocation.

6. **Twin-Slip and Twin-Twin Interaction**

6.1 **Behavior in Other Fcc Metals**

Before considering the details of twin-slip interaction in our alloy let us review some of the findings made in other alloy systems.

6.1.1 **Co-Fe Alloys**

Mahajan and Chin\(^{33,65}\) examined the crystallographic features of twin-slip and twin-twin interactions in fcc Co – 8% and 9.5% Fe single crystals deformed in plane strain compression, by optical and electron microscopy. The results of their metallography showed that: (i) during
the twin-slip interaction, the strain associated with a slip band can be accommodated by slip in the existing twin; (ii) the twin interactions are generally accomplished by the occurrence of secondary twinning in the crossed twin; and (iii) the existing slip bands, as a result of their interaction with the twins, are displaced from their original position.

In considering twin-slip interactions, the choice of the glide plane in a twin is restricted by the geometrical condition that the line of intersection of the slip planes on the parent crystal and twin must lie on the boundary. The restriction must be satisfied so that dislocations can transfer from their slip plane in the parent crystal to that in the twin without undergoing reorientation at the boundary. From the twinning matrix (which algebraically defines the relation between the parent crystal and twin), it may be shown that \{115\}_{\text{Matrix}} is parallel to \{111\}_{\text{Twin}}. For slip on the (111) plane, the slip dislocations \(1/2[0\overline{1}1]\) and \(1/2[\overline{1}01]\) could be incorporated into the twin without dissociation. However in the case of \(1/2[\overline{1}01]\) dislocations, dissociation at the (\(\overline{1}11\)) coherent twin boundary (CTB) may occur according to the following reactions:
\[ \frac{1}{2}[\bar{1}01] (111) + \frac{1}{6}[\bar{1}14] (\bar{1}11) \parallel (111) T + \frac{1}{6}[2\bar{1}11] CTB (\bar{1}11) \]  

(56)

\[ \frac{1}{2}[\bar{1}01] (111) + \frac{1}{6}[\bar{1}41] (\bar{1}11) + 2 \times \frac{1}{6}[\bar{1}21] (\bar{1}11) \]  

(57)

\[ \frac{1}{2}[\bar{1}01] (111) + \frac{1}{6}[411] (\bar{1}51) + \frac{1}{6}[1\bar{1}2] (\bar{1}11) \]  

or

(1\bar{1}5)  

(58)

The decomposition reactions are energetically unfavorable, but they could occur when a sufficient number of the \[ \frac{1}{2}[\bar{1}01] \] dislocations have piled up against the twin to relieve the stress concentration that may exist at the head of the pile up. Reaction (58) is unlikely to occur since the slip planes in the parent and twin do not intersect along a line which lies in the coherent twin boundary. Mahajan and Chin suggest that reaction (57) is the most probable on the basis of a strain compatibility argument. Similarly for slip on the (1\bar{1}1) plane into the (\bar{1}11) twin the governing reactions are:

\[ \frac{1}{2}[\bar{1}0\bar{1}] (1\bar{1}1) + \frac{1}{2}[\bar{1}01] (\bar{1}51) \]  

(59)
and

\[ \frac{1}{2} [0\overline{1}1] (\overline{1}1\overline{1}) + \frac{1}{6} [4\overline{1}1] \overline{1} (151) \parallel (\overline{1}1\overline{1}) T + 2x \frac{1}{6} [2\overline{1}1] \text{CTB} \]  

(60)

The analysis of twin-twin intersections follows a similar pattern, but now they are accomplished by secondary twinning. For secondary twinning on the (1\overline{1}5) \parallel (\overline{1}1\overline{1}) planes for example, the \(\frac{1}{6} [\overline{1}1\overline{2}]\) dislocation gliding on the (\overline{1}1\overline{1}) planes must undergo dissociation at the (\overline{1}1\overline{1}) coherent twin boundary. Mahajan and Chin envisaged three \(\frac{1}{6} [\overline{1}1\overline{2}]\) partials when piled up against the intersected twin, could combine to form a \(\frac{1}{6} [\overline{1}1\overline{2}]\) dislocation which could subsequently dissociate as follows:

\[ 3x \frac{1}{6} [\overline{1}1\overline{2}] (\overline{1}1\overline{1}) + 3x \frac{1}{18} [5\overline{5}2] (1\overline{1}5) + 2x \frac{1}{6} [1\overline{1}2] \text{CTB} \]  

(61)

or

\[ 3x \frac{1}{6} [\overline{1}1\overline{2}] (\overline{1}1\overline{1}) + 3x \frac{1}{18} [721] (1\overline{1}5) + 3x \frac{1}{2} [\overline{1}0\overline{1}] (\overline{1}1\overline{1}) T + \frac{1}{6} [\overline{1}1\overline{2}] (\overline{1}1\overline{1}) \]  

(62)

\[ 3x \frac{1}{6} [\overline{1}1\overline{2}] (\overline{1}1\overline{1}) + 3x \frac{1}{18} [\overline{27}1] (1\overline{1}5) + 3x \frac{1}{2} [0\overline{1}\overline{1}] (\overline{1}1\overline{1}) T + \frac{1}{6} [\overline{1}1\overline{2}] (\overline{1}1\overline{1}) \]  

(63)

The glide of either of the three twinning dislocations, \(\frac{1}{18} [5\overline{5}2] \equiv \frac{1}{6} [\overline{1}1\overline{2}] T, \frac{1}{18} [721] \equiv \frac{1}{6} [\overline{1}21] T\) and \(\frac{1}{18} [\overline{27}1] \equiv \frac{1}{6} [211] T\) on the (1\overline{1}5) planes produces a three layer twins. These
dislocations before leaving the twin could appropriately
dissociate into three $\frac{1}{6} [\overline{1}12]$ dislocations in the matrix
plus $\frac{1}{6} [\overline{1}12]$ dislocations left in the $[\overline{1}1\overline{1}]$ boundary.
The above reactions are energetically unfavorable but
they could occur when a sufficient number of the $\frac{1}{6} [\overline{1}12]$ dislocations has piled up against the twin to relieve the
stress concentration that may exist at the head of the pile up.

It is clear that the dislocation reaction suggested
for the incorporation of slip dislocations with twins
and twin-twin intersection, are energetically unfavorable.
This incorporation can occur only when the applied stress
is locally magnified as in the presence of a dislocation pileup. Mahajan and Chin consequently suggest that the
rate of damage accumulation in a crystal, deforming si-
multaneously by slip and twinning, is likely to be higher
than that in a crystal deforming exclusively by slip.
Therefore, in such a crystal the work hardening rate
should be higher.

6.1.2 Copper

Other possible dislocation reactions may occur for
twin-twin intersections in fcc metals, as suggested by
Mahajan et al.$^{39}$ in the case of copper. These workers
showed through electron microscopy that when a thin twin intersects a coherent twin boundary it is possible that the \( \frac{1}{6}[\overline{1}12] \) partial dislocation bounding the fault may dissociate at the boundary according to a reaction of the following type:

\[
\frac{1}{6}[\overline{1}12] \quad (111) + \frac{1}{6}[\overline{1}21] \quad (111) + \frac{1}{6}[\overline{1}21] \quad \text{CTB} + \frac{1}{6}[11\overline{4}] \quad \overline{151}
\]

As in the case of the Co-Fe alloys, this reaction is also energetically unfavorable but can proceed when the stress concentration existing at the head of the pileup of twinning dislocations is of the suitable magnitude to move the resulting dislocations away from the reaction zone. This reaction provides a mechanism for distributing the twinning strain and relaxing the stress concentration at the twin CTB intersection.

6.1.3 Hadfield Manganese Steel

Raghavan et al.\(^7\) have provided some electron microscopy evidence of twins as obstacles to slip in Hadfield manganese steel. They point out that in order to pass a dislocation through a twinned crystal, interface dislo-
cations must in general be generated. This generation costs energy which in turn raises the applied stress for dislocation motion. The finer the distribution of twins the greater will be the density of interface dislocations and thus the applied stress for dislocation motion.

6.1.4 Co-Ni-Cr-Mo Alloys

Remy\textsuperscript{68} provided more conclusive experimental evidence of interface dislocation generation using electron microscopy and concluded in the case of a Co-30Ni-10Cr-5Mo alloy, that twin-slip interaction produces a reaction as follows:

\[
\frac{1}{2}[110]_{(\overline{1}11)} + \frac{1}{2}[10\overline{1}]_T + \frac{1}{6}[2\overline{1}1]_{CTB} \tag{65}
\]

He points out that this observed reaction is consistent with an incorporation reaction which minimizes the dislocation self energy, as compared to the other possible reaction

\[
\frac{1}{2}[110]_{(\overline{1}11)} + \frac{1}{2}[110]_T + 2\times \frac{1}{6}[11\overline{2}]_{CTB} \tag{66}
\]
Rémy suggests that this indicates (differing slightly from Mahajan and co-workers) that for twin-slip interaction in fcc crystals the operating accommodation mechanism satisfies an energetic criterion provided that the resulting dislocation may slip away.

6.2 Geometric Analysis of Twin-Slip Interactions

Friedel\textsuperscript{69} has pointed out that slip across an epitaxial layer produces, on the two crystal planes in contact, two steps of different heights or inclinations. The matching is thus made more difficult and internal stresses are developed. These stresses can be ascribed to an imperfect dislocation $\mathbf{b}_e$ along the step which describes the differences between the incoming dislocation $\mathbf{b}_1$ and the outgoing one $\mathbf{b}_2$, (Fig. 54). Thus

$$\mathbf{b}_1 = \mathbf{b}_2 + \mathbf{b}_e$$

(67)

As more dislocations $\mathbf{b}_1$ cross from one crystal to the other along a given slip plane, each leaves behind a dislocation $\mathbf{b}_e$. These dislocations, as well as the epitaxial dislocations, will soon make it difficult for new dislocations to cross. The same kind of imperfect dislocations make it
Fig. 54  Slip across an epitaxial layer (Ref. 69)
difficult for a slip band to cut across a twin boundary.

Thus as pointed out by Mahajan\textsuperscript{70}, when a dislocation with Burgers vector $\mathbf{b}_m$ passes into a twin oriented crystal it may dissociate into a dislocation within the twin $\mathbf{b}_T$ and an interface dislocation $\mathbf{b}_{\text{INT}}$. Therefore,

$$\mathbf{b}_M = \mathbf{b}_T + \mathbf{b}_{\text{INT}} \quad (68)$$

In order to pass a dislocation through a twinned crystal, interface dislocations must in general be generated. This generation requires energy and this in turn must raise the applied shear stress for dislocation motion. It can, therefore, be inferred that the finer the distribution of twins, the greater will be the density of interface dislocations and thus higher the shear stress required for dislocation motion.

Remy\textsuperscript{68} has provided a good summary of the geometrical analysis of the twin-slip interaction in fcc metals and the following discussion is based on his treatment on the subject. As pointed out in the above discussion, during the incorporation of the slip dislocation, the Burger's vector is conserved. Also the dislocations which form at the coherent twin boundary must slip away from the intersection giving rise to either thinning or thickening of the obstacle twin. Another geometric constraint is that the re-
laxation plane in the twin must contain the intersection axis of the incident slip or twinning plane and of the coherent twin boundary. Since in fcc crystals, both the slip and twinning planes are \{111\} planes, the intersection of the incident slip plane with the obstacle twin boundary is a \(<1\bar{1}0>\) direction. Therefore the only relaxation plane in the twin is the \(\{111\}_T\) mirror plane of the incident matrix slip plane. Let us consider for example the case for an obstacle twin with a (111) composition plane and a (\(\bar{1}\bar{1}1\)) incident slip plane.

For the (111) plane, the possible Burger's vectors are \(\pm \frac{1}{2}[01\bar{1}], \pm \frac{1}{2}[101]\) and \(\pm \frac{1}{2}[110]\). The first one is parallel to the slip plane-twin boundary intersection while the other dislocations make a 60° angle with their intersection. The simplest form of incorporation is described by:

\[
\frac{1}{2}[01\bar{1}] + \frac{1}{2}[0\bar{1}1]_T
\] (69)

which occurs by cross-slip thus leaving no interface dislocation. The other dislocations can only be incorporated in the twin by means of a dislocation reaction at the boundary since \(\frac{1}{2}[110]=\frac{1}{6}[114]_T\) and \(\frac{1}{2}[101]=\frac{1}{6}[141]_T\) are not slip vectors in the twin. These crystallographic
relationships results from the utilisation of the twinning matrix on the matrix vectors (which will be derived in the next section). Each dislocation must dissociate into one slip dislocation in the twin and a partial dislocation which describes the step, one atomic plane high, left at the interface.

6.3 Twinning Matrix for the fcc Lattice

In order to explicitly define the Burger's vectors of the dislocations in a twin relative to the parent crystal as discussed in the previous sections, we must understand the algebraic relationship between the twin crystal and the parent crystal. The parent crystal is defined by an orthonormal coordinate system or basis \((\hat{x}_1, \hat{x}_2, \hat{x}_3)\). What we wish to do is to define a matrix which describes what happens to vectors in the fcc lattice when a twinning shear occurs on a \(\{111\}\) plane in a \(<112>\) direction. The parent crystal can be described algebraically by an orthogonal matrix\(^7\). Let us first review some properties of orthogonal matrices:

i) For an orthogonal matrix \(C\), \(C\) is non-singular (i.e. \(\det C \neq 0\))

ii) \(C^T = C^{-1}\) (transpose matrix and inverse matrix are equal)
iii) $C$ is defined by an $n \times n$ square matrix
iv) $\det C = \pm 1$

v) The magnitude of vectors is unchanged due to orthogonal transformation

vi) Angular relationships between vectors are preserved.

Consider now another orthogonal matrix with its own set of orthonormal vectors defining the basis of the twin crystal's coordinate system $(\hat{x}_1', \hat{x}_2', \hat{x}_3')$. The new coordinate system is defined as follows:

i) the unit shear direction = $\hat{x}_1'$

ii) the unit vector parallel to shear plane normal = $\hat{x}_2'$

iii) the direction normal to $\hat{x}_1'$ and $\hat{x}_2'$ = $\hat{x}_1' \times \hat{x}_2' = \hat{x}_3'$

The relationship between the two sets of axes is shown in Fig. 55. With respect to the fcc basis the unit vectors are:

\[
\hat{x}_1' = [100]
\]

\[
\hat{x}_2' = [010]
\]

\[
\hat{x}_3' = [001]
\]

and
Fig. 55  Relationship between parent axes and new axes in an fcc structure (Ref. 67)
\[ \hat{x}_1' = \frac{1}{\sqrt{6}}[112] \]
\[ \hat{x}_2' = \frac{1}{\sqrt{3}}[111] \]
\[ \hat{x}_3' = \frac{1}{\sqrt{2}}[1110] \]

The coordinate transformation between the two orthonormal bases is described by an orthogonal matrix \([A]\). Therefore \([\hat{x}'] = [A] [\hat{x}']\). The components of the transformation matrix \(a_{ij}\), represent the cosine of the angle between the \(\hat{x}_i\) and \(\hat{x}_j\) axes. Therefore \(a_{ij} = \cos^{-1}(\hat{x}_i \cdot \hat{x}_j)\).

Therefore:
\[
[A] = \begin{bmatrix}
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0
\end{bmatrix}
\]

(70)

It can be checked that

\[ a_{ik}a_{jk} = 1 \quad (l=j) \]

and

\[ a_{ik}a_{jk} = 0 \quad (l \neq j) \]
as in the case for an orthogonal matrix.

Up to this point we still have not introduced the concept of a twin relation between the two sets of crystal axes. This can be accomplished by a $180^\circ$ rotation about the $x_2^*$ axis thus producing a crystal with a twin relationship to the parent structure. This rotation is described by a matrix:

$$[\bar{T}] = \begin{bmatrix}
\bar{1} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \bar{1}
\end{bmatrix}$$ \hspace{1cm} (71)

Consequently, the matrix which expresses the twinning process relative to the parent fcc coordinate system is given by:

$$[T] = [A][\bar{T}][A^T]$$ \hspace{1cm} (72)

Thus:

$$[T] = \frac{1}{3} \begin{bmatrix}
\bar{1} & 2 & 2 \\
2 & \bar{1} & 2 \\
2 & 2 & 1
\end{bmatrix}$$ \hspace{1cm} (73)

The above derivation was for the specific case for twinning on the $(111)$ plane. Similarly, transformation matrices may be formulated on the other $(111)$ planes. Matrices for twinning on the $(\bar{1}11)$, $(1\bar{1}1)$, and $(11\bar{1})$
planes are found to be respectively\textsuperscript{72}:

\[
\frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}, \quad \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \quad \text{and} \quad \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}
\]

Therefore any vector in the twin can be defined with respect to the parent crystal by the following relationship

\[
\vec{b}_T = [T]\vec{b}
\]  \hspace{1cm} (74)

6.4 \textbf{Slip Accommodation in a Twin}

In the electron microscopy observations described earlier, there were a number of examples shown of twin-twin and twin-slip interactions. Consider the situation as shown in Fig. 56 where two thin twins or stacking faults are intersecting each other. An important feature of these micrographs is the presence of dislocation pile-ups at the intersections of the twins. The nature of these pileups will be considered in detail later, but such a dislocation configuration does indicate the presence of an obstacle to slip. As was analyzed earlier these dislocations were found to be of the $1/2 <110>$ type. On the basis of the arguments presented in the previous
Fig. 56  Dislocation pileup at coherent twin boundary
\( \vec{z} = [110], \vec{g} = [220] \)
sections the interaction of the $1/2\langle110\rangle$ dislocation would produce an interface dislocation at the twin boundary and a dislocation within the twin. This process costs energy and thus increases the applied stress needed for dislocation motion. The incorporation of the slip dislocation into the twin may be governed by the following reaction

$$\frac{1}{2}[110]_{(111)} + \frac{1}{6}[141]_{(115)} + \frac{1}{6}[2\overline{1}\overline{1}]_{\text{CTB}(111)}$$ (75)

The $\frac{1}{6}[141]$ dislocation could glide away on a $(\overline{115})$ plane in the twin leaving the $\frac{1}{6}[2\overline{1}\overline{1}]$ dislocation at the interface (Fig. 57). It should be noted that these indices are with respect to the parent crystal reference system and can be transformed to the twin reference system by operating on them with the transformation matrix for (111) twinning. Therefore:

$$\frac{1}{6}[141] = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & \overline{1} & 2 \\ 2 & 2 & \overline{1} \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = \frac{1}{2}[101]_{(\overline{111})T}$$ (76)

We must also define the Burger's vector of the interface dislocation in terms of the twin crystal; therefore:
\[ \frac{1}{6}[141] = \frac{1}{2}[101] \]

Fig. 57 Twin-slip interaction
\[
\frac{1}{6}[\overline{2}11] \equiv \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \cdot \frac{1}{6} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{6}[\overline{2}11]_{\text{CTB}}
\] (77)

The dissociation reaction is energetically unfavorable and physically this implies that the \( \frac{1}{2}[101]_T \) and \( \frac{1}{6}[\overline{2}11]_{\text{CTB}} \) dislocations are strongly attracted to each other. This attractive force may be calculated from which the stress necessary to overcome the attractive interaction can be estimated.

The radial component of the interaction force is given by:

\[
F_R = \frac{\mu}{2\pi R} (\vec{b}_1 \cdot \hat{t}) (\vec{b}_2 \cdot \hat{t}) + \frac{\mu}{2\pi (1-\nu) R} \left[ (\vec{b}_1 \times \hat{t}) \cdot (\vec{b}_2 \times \hat{t}) \right]
\] (78)

where:

\[
\vec{b}_1 = \frac{a}{6}[\overline{2}11]
\]

\[
\vec{b}_2 = \frac{a}{2}[101]
\]

\( \hat{t} \) = tangent vector

\( R \) = distance between the dislocations

\[
\rho = \frac{|\vec{b}_1|}{2 <110>} = 2.5 \times 10^{-8} \text{ cm}
\]

\( \mu \approx 3 \times 10^{11} \text{ dynes/cm}^2 \)

\( \nu \approx 0.3 \)
Thus
\[ F_R = |-366| \text{ dynes/cm} \]

The stress needed to overcome this attractive force is given by \[ \tau = 1.5 \times 10^{10} \text{ dyne/cm}^2 \]. This indicates that slip dislocations will have great difficulty penetrating the coherent twin boundary.

6.5 **Dislocation Pileups**

With such high stresses needed to cut through a twin boundary, it should not be surprising to see dislocations piled up at such intersections (Fig. 56). Mahajan and Chin\(^{39}\) have made similar observations in their work on copper, but in our case we have a pileup of perfect dislocations, not partials. This is due to the fact that the governing dislocation reaction at the coherent twin boundary in this material is different from their study on copper; however, Mahajan and Chin have suggested that dislocation pileups would provide the stress concentration needed for slip dislocations to penetrate a twin.

Let us analyse the dislocation configuration in a pileup in more detail (Fig. 58)\(^{73}\). A system or array of straight dislocations covering a distance L may be described by a continuous density function \( D(x) \). This array is stable in the presence of an external applied shear stress \( \tau_A \) if:
Fig. 59  Schematic of dislocation pileup
\[ \frac{\mu b}{2\pi k} \int_{-\infty}^{\infty} \frac{D(x)}{x' - x} \, dx + \tau_A(x') = 0 \quad (79) \]

where

\[-L \leq x' \leq L\]

\[ k=1 \quad \text{for screw dislocations} \]
\[ k=1-\nu \quad \text{for edge dislocations} \]

The other condition is defined by the normalization of \( D(x) \):

\[ \therefore \int_{-\infty}^{\infty} D(x) \, dx = nb \quad (80) \]

The result is:

\[ D(x) = \frac{2nb}{\pi L} \left( \frac{L-x}{x} \right)^{1/2} \quad 0 \leq x \leq L \quad (81) \]

and

\[ D(x) = 0 \quad x > L, \quad x < 0 \quad (82) \]

and

\[ L = \frac{n\mu b}{\pi \tau_A} \quad (83) \]

Consider the case in Fig. 56 with a pileup of dislocations
of Burger's vector $\frac{1}{2}[110]$ on the $(111)$ plane designated
Set I dislocations, intersecting the twin lying on
the $(111)$ plane. For this example, the dislocations are
approximately of screw character, thus $k \approx 1$.
Therefore

$$\tau_A = \frac{n \mu b}{\pi L} \quad (84)$$

where:

$$n = 26 \text{ dislocation in pileup}$$

$$L = 0.84 \ \mu m$$

Thus:

$$\tau_A \approx 10^9 \text{ dynes/cm}^2$$

In other words, this is the externally applied stress
needed to maintain the observed dislocation configuration.
At the head of the pileup however, this stress is magnified
due to the presence of the other dislocations and the
applied shear stress. Therefore the piled up group exerts
in its glide plane a shear stress given by:

$$\tau \approx n \tau_A \quad (85)$$

With twenty-six dislocations, $\tau \approx 3 \times 10^{10} \text{ dynes/cm}^2$.
Recall that the stress needed to overcome the obstacle
due to the twin-slip interaction as described by the
interface dislocation reaction was estimated to be $1.5 \times 10^{10}$
dynes/cm$^2$. What this means is that the stress concentration
present at the coherent twin boundary is great enough to allow the penetration of the twin by some slip dislocations. Given the mechanism of twin-slip interaction described in this work, the minimum number of slip dislocations needed in a pileup for distributing the twinning strain and relaxing the stress concentration at the twin-CTB intersection is defined by the condition \( \tau = \tau_A \). In this case \( \tau_A \approx 10^9 \text{ dynes/cm}^2 \) and \( \tau = 10^{10} \text{ dynes/cm}^2 \); therefore one would expect to see at least ten dislocation in a pileup before twin penetration by some perfect dislocations may occur.
V. CONCLUSIONS

Given the various strengthening mechanisms proposed in this study, it is clear that some have a more dominant role than others (Table 8). From the summary presented in Table 8 it can be seen that stacking fault intersections and twins provide the major obstacles to dislocation motion, Lomer locks also act as strong impediments to slip; however, they do not form frequently.

While the fcc→hcp transformation has always been considered important to the mechanical behavior of cobalt based alloys, its role has never been stated explicitly. The electron microscopy done in this study has defined in specific terms the contributions of the fcc→hcp transition to the work hardening rate of a cobalt based alloy. Stacking fault formation and twin formation may be considered, from a structural standpoint, as precursors to hcp formation. The mechanism by which stacking faults and twins contribute to the flow stress was shown in this study. Localization of slip along stacking faults and the presence of dislocation pileups at twin-twin intersections are examples of the role stacking faults and twins play in deformation.

The occurrence of fcc twinning as opposed to hcp formation carries with it an important implication concerning mechanical behavior. Both twins and hcp platelets can act as major obstacles to slip. In the hcp structure there is only
Table 8

Relative contribution to the flow stress
by various strengthening mechanisms

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>$\tau$ (dynes/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stacking faults</td>
<td>$5 \times 10^3$</td>
</tr>
<tr>
<td>Stacking fault intersections</td>
<td>$2 \times 10^{10}$</td>
</tr>
<tr>
<td>Lomer dislocation loop</td>
<td>$2.3 \times 10^{10}$</td>
</tr>
<tr>
<td>Solid Solution effects</td>
<td>$5 \times 10^8$</td>
</tr>
<tr>
<td>Dislocations on intersecting planes</td>
<td>$17 \times 10^8$</td>
</tr>
</tbody>
</table>
| Lattice friction                       | $\tau \approx 3.6 \times 10^9$
|                                        | $.2\%$ Twin nucleation|
| Twin-slip interaction                  | $1.5 \times 10^{10}$  |
one primary slip system, and thus ductility is severely limited. The fcc structure, on the other hand, with five operative slip systems is far more ductile. Therefore fcc twinning provides a means of obtaining strength without sacrificing ductility in cobalt based alloys.

Since the importance of stacking faults and twins in the work hardening behavior of this alloy has been clearly established, it follows that the stacking fault energy is a crucial parameter to consider. The lower the stacking fault energy, the higher is the stacking fault density and subsequently higher is the twin density. Of the various solute elements in this alloy the magnitude of the stacking fault energy is most sensitive to the presence of nickel. In this context nickel must be considered as having an important role in strengthening. Nickel being an austenitic stabilizer tends to increase the stacking fault energy. The stacking fault density and the associated work hardening rate, may thus be controlled by varying the amount of nickel in this alloy.

Finally, it should be pointed out that second phase strengthening can also be an important mechanism by which an increase in the flow stress of cobalt alloys may be obtained. Given that slip is localized along stacking faults it follows that carbide precipitation on stacking faults can provide a strong impediment to dislocation motion.
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Phase Transformations in a Wrought Co-Cr-Mo-C Alloy

A.1. Introduction

The results of VanderSande et.al.\textsuperscript{1} in their study of wrought H.S.21 (Table 9) indicated an improvement in room temperature mechanical properties with certain heat treatments. The material was press forged and hot rolled at 1150°C, given a solution treatment at 1230°C for four hours and gas quenched. Following this, the alloy was given aging treatments at 750°C for various times including 2, 5, 20 and 50 hours. A microstructural examination of the heat treated material indicated that aging yielded a greater density of fcc stacking faults than existed in just the solutionized specimen, and produced extensive transformation to a heavily faulted hcp phase. At the shorter aging times (less than 20 hours), the heavily faulted hcp phase existed along with the fcc phase which contained stacking faults. They noted that the amount of hcp phase and the fcc stacking fault density increased with aging time. At the long aging times (50 hours) another hcp 'phase' was observed, which contained a very low stacking fault density and a large number of non-planar
Table 9

<table>
<thead>
<tr>
<th>Element</th>
<th>Composition (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr</td>
<td>26.7%</td>
</tr>
<tr>
<td>Mo</td>
<td>5.5%</td>
</tr>
<tr>
<td>Ni</td>
<td>0.01%</td>
</tr>
<tr>
<td>Fe</td>
<td>0.1%</td>
</tr>
<tr>
<td>C</td>
<td>0.15%</td>
</tr>
<tr>
<td>Co</td>
<td>Balance</td>
</tr>
</tbody>
</table>
precipitates. It appeared as if the hcp "equilibrates" or "recrystallizes" simultaneously with precipitate coarsening.

Vander Sande et al. noted certain alignments between fcc stacking faults and precipitate rows in adjacent hcp regions. They suggested that if a region of hcp formed by intrinsic faulting on every second plane grows into an area already traversed by stacking faults on other planes, this is equivalent to the process of stacking fault intersection. The intersected fault planes then become planes of high vacancy concentration in the hcp region. These vacancies would promote diffusion and would accommodate part of the mismatch between the lattice and precipitates. However Vander Sande et al. pointed out that this model does not satisfactorily explain all cases of precipitate row alignment observed. Also, they noted that the spacing of the precipitate rows and their orientation in relation to the fcc-hcp interface suggest that the precipitates may, in fact, be associated with the fcc-hcp interface. In the following discussion a model will be presented which will attempt to explain the nature of the fcc-hcp transformation and carbide precipitation in wrought H.S.21.
A.2 Results

In order to provide any viable hypothesis of the phase transformations it was first necessary to document more thoroughly the phase transformations occurring upon aging at 750°C. Heat treatments that had been examined previously were 2, 5, 20 and 50 hours. The program of heat treatments initiated here, include 10, 15, 18, 18 1/2, 19, 20, 23, and 25 through 45 hours in five hour intervals. It should be noted that all samples were first given a solutionizing treatment for four hours at 1250°C. Transmission electron microscopy was carried out on all the samples and the following discussion will concern itself with the observations made.

The shorter time heat treatments resulted in the extensive formation of arrays of stacking faults in the fcc phase. With increasing time a further transformation to a heavily faulted hcp structure occurred as indicated by selected area diffraction. The hcp regions exist as bands with long straight boundaries adjacent to the fcc phase. It will be recalled that the fcc and hcp structures are interrelated, each being obtained by an appropriate stacking sequence of close packed planes of atoms. Thus the fcc-hcp boundaries are straight with matching close packed planes at the interface.
While the density of stacking faults in the fcc region and the amount of band like hcp phase increased with aging time, no precipitates were observed until after 23 hours of heat treatment at 750°C when a low stacking fault density hcp phase formed. The precipitates were of the $M_{23}C_6$ variety.

Several features were noted concerning the carbide precipitation:

1) There are two distinctive size distributions of the same carbide in each of the microstructures. One distribution contains very fine particles (average size of 500 A in width) and the other contains somewhat coarser particles (1450 A).

2) The finer sized carbides are in linear arrays and are shown to be associated with stacking faults (Figure 59).

3) The coarser particles have a random distribution in the matrix (Figure 60) and are also located at the boundary of the new form of hcp (Figure 61).

The present work confirmed the observations of Vander Sande et al.. In other words, in addition to the heavily faulted hcp which had formed earlier, this new polymorph
Fig. 59  Precipitation on stacking faults

Fig. 60  Carbide precipitation in the hcp phase
Fig. 61  Precipitation at the hcp #1 - hcp #2 boundary

Fig. 62  Cellular type precipitation morphology at the fcc-hcp interference
formed coincident with the nucleation of precipitates. In fact it was noted that the precipitation occurred only in this "recrystallized" type of hcp. Also unlike its predecessor, the new form of hcp was characterized by both curved and straight boundary segments instead of long straight boundaries consisting of matching close packed planes of atoms.

A.3 Analysis of the Phase Transformations

We have essentially three transformations to consider in this alloy in the times (10-45 hrs) and temperature (750°C) under discussion.

1) The initial fcc→hcp transition producing a heavily faulted band-like hcp phase.

2) The polymorphism exhibited by the hcp structure.

3) The nature of the carbide precipitation in this alloy.

The initial fcc→hcp transition producing the heavily faulted band like structure may, upon initial examination, be termed a martensitic transformation. When one considers, however, the high temperatures involved, some diffusion must be occurring, and thus the resulting hcp product is not strictly martensitic in nature (e.g. $D_{Cr}$ in Co at 1000°C is $10^{11}$ cm²/sec). However, at the times being considered (less than 20 hours) the diffusion rate is not
high enough to affect the coherency; with the $\{111\}_{fcc} \parallel \{0001\}_{hcp}$. On this basis, it is suggested that the mechanism of the fcc hcp transformation is similar to that of pure cobalt.

However, at longer times, the effect of diffusion is to break down this coherency as the atoms shuffle across this interface. The polymorphism of the hcp phase may be considered in terms of the reactions:

\[
\text{hcp #1} \rightarrow \text{hcp #2} + \text{carbides} \\
\text{fcc} \rightarrow \text{hcp #2} + \text{carbides}
\]

or in more general terms

\[
\gamma_1 \rightarrow \gamma_2 + \beta
\]

The new $\gamma_2$ grain has a much lower alloy content than the original supersaturated $\gamma_1$ grain. The loss of supersaturation occurs discontinuously as the $\gamma_1/\gamma_2$ boundary advances and is manifested in the discontinuous nature of the carbide precipitation ($\beta$) which occurs only in the second hcp phase. It should be noted that the new form of hcp grows indiscriminantly into the matrix in that it grows both into the fcc and into the initial form of the hcp. Now let us consider each of these cases individually.

In the hcp #1 $\rightarrow$ hcp #2 transition (Figure 61), it is
proposed that the incoherent boundary acts as a diffusion pipe for solute atoms and consequently precipitation begins to occur at the boundary and concurrent with the growth of the new form of hcp. The incoherent boundary acts as a preferential nucleation site for precipitation. The boundary may be stationary for a while during the initial stages of growth of the precipitate; however, it is not pinned by these particles and it moves forward leaving behind the carbides. Secondary sites for carbide nucleation are stacking faults in the new hcp region and the particles are usually smaller or shorter aging times since they probably form after those that nucleated on the moving boundary (Figure 59).

For the fcc+hcp #2 transformation as indicated in Figure 62, the microstructure appears more cellular in nature with "cells" of hcp #2 and carbide. The almost planar interface is striking especially in Figure 64. Unlike the hcp #1 - hcp #2 interface, discrete carbide particles do not exist at the boundary. The mechanisms proposed for such cellular precipitation are still unclear in explaining the geometry of the interface. However, it has been suggested that in such cases, part of the free energy of the reaction is used in actually pulling the boundary along with the growing precipitate.
In more general terms the point to note in the present study is that in discontinuous precipitation the structural and compositional changes occur in regions immediately adjacent to the advancing interface. The parent phase remains unchanged until swept over by the interface and the transition is complete in regions over which the interface has passed. It is noteworthy that as early as 1955 Weeton and Signorelli had observed a "pearlitic" structure in cobalt alloys through optical microscopy, although no attempt was made at interpreting this structure.
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APPENDIX B

Calculation of the Equilibrium Phase Diagram for the Co-Cr-Mo System

Calculations were carried out using a package of computer programs developed by Manlabs, Inc. The development of computer based methods for phase diagram calculations was due largely to Kaufman\textsuperscript{1-6}. The basis of these methods is the explicit definition of the free energy of solution and compound phases in metallic systems in terms of lattice stability, solution and compound parameters. These formulations provide explicit mathematical descriptions of the excess free energies of all the solution phases in the binary system.

The model employed in these programs assumes the form of the free energy of a solution phase $\phi$ in the binary system A-B to be:

$$G^\phi = x_A G_A^\phi + x_B G_B^\phi + RT (x_A \ln x_A + x_B \ln x_B) + G_E^\phi$$

(1)

where $G_I^\phi$ is the free energy of pure I in the $\phi$ state and the excess free energy is given by:

$$G_E^\phi = x_A x_B [x_A \alpha^\phi(T) + x_B \beta^\phi(T)]$$

(2)
The interaction parameters, $\alpha$ and $\beta$, are functions of absolute temperature given by:

$$\alpha = \alpha_0 + \alpha_1 T$$  \hspace{1cm} (3)

$$\beta = \beta_0 + \beta_1 T$$  \hspace{1cm} (4)

(The program can also handle $T^2$ and $T^3$ terms in the interaction parameters; these terms are useful near $0^\circ K$.)

This model reduces to the regular solution model when $\alpha_0 = \beta_0 = \Omega$; $\alpha_1 = \beta_1 = 0$. It effectively allows $\Omega$ to be a linear function of composition. The first-order temperature dependence ($\alpha_1$ & $\beta_1$) corresponds to an entropy term in the bond energies of the quasichemical model, which is also allowed to vary linearly with composition. Phase boundaries are calculated by determining at a given $T$ where the chemical potentials,

$$\mu_i = \frac{\partial G}{\partial x_i}_{T,P,x_j}$$

of each species are equal in a pair of phases $T,P,x_j$.

The Manlabs programs are designed primarily for interactions between elements. They can also be used for compounds, provided that the edges A and B can be defined so that their ideal entropy of mixing is given by:

$$S^M = -R(x_A \ln x_A + x_B \ln x_B)$$  \hspace{1cm} (5)
which is the form assumed by the programs.

The free energy of a ternary solution phase is defined in a similar manner:

$$G = x_A G_A + x_B G_B + x_C G_C + RT (x_A \ln x_A + x_B \ln x_B + x_C \ln x_C) + G_E$$  \hspace{1cm} (6)

where the excess free energy is given by:

$$G_E = \frac{x_A x_B}{x_A + x_B} \left[ x_A \alpha(T) + x_B \beta(T) \right]$$

$$+ \frac{x_A x_C}{x_A + x_C} \left[ x_A \alpha(T) + x_C \gamma(T) \right]$$

$$+ \frac{x_B x_C}{x_B + x_C} \left[ x_B \beta'(T) + x_C \gamma'(T) \right]$$

$$+ x_A x_B x_C \Delta(\text{comp}, T)$$  \hspace{1cm} (7)

where $\alpha$ and $\beta$ are the A-B binary interaction parameters, $\alpha'$ and $\gamma$ correspond to A-C, and $\beta'$ and $\gamma'$ to B-C. All are linear in $T$, as before.

The ternary interaction parameter $\Delta$ is assumed to be zero throughout the present calculations. Therefore the ternary phase diagram is determined if the binary interaction coefficients are specified.
Consequently, the accuracy of the ternary phase diagram depends upon the accuracy of the calculated binary diagrams. Fortunately, there is enough experimental data for the Co-Cr, Co-Mo and Mo-Cr systems to provide explicit mathematical descriptions for the excess free energy expressions. In fact the observed and calculated phase diagrams as shown in Fig. 63-65 compare favorably. Thus we may progress to calculating the ternary phase diagram with some confidence.

For the purposes of the present study the ternary phase diagram was calculated primarily in the compositional range of this alloy. From the Co-Cr binary, the interactions of interest are: Sigma(Co₄Cr₆) - FCC, Sigma-HCP and FCC-HCP. From the Co-Mo system, the relevant interactions are: FCC-HCP, Mu(Co₅₃Mo₄₆₂ = Co₇Mo₆) - FCC, K(Co₇₅Mo₂₅ = Co₃Mo) - FCC, K-HCP and T(Co₈₁₈Mo₁₈₂) - FCC.

Three isothermal sections were calculated, at 900°K, 1200°K and 1500°K (Fig.66-68). The first two temperatures corresponded to the aging temperatures (650°C and 750°C) and the third to the solutionizing temperature. The enlargement of the fcc phase field with increasing temperature is consistent with the fact that the fcc phase is the thermodynamically stable phase at high temperatures. It should also be noted that if we approximate the real alloy system as a Co-30%
Fig. 63  Mo-Cr phase diagram
Fig. 64    Co-Cr phase diagram.
Fig. 65  Co-Mo phase diagram
Fig. 66  Computed Co-Cr-Mo phase diagram for 900°K
Fig. 67 Computed Co-Cr-Mo phase diagram for 1200°K
Fig. 68
Computed Co-Cr-Mo phase diagram for 1500°K
Cr-5% Mo alloy, after solutionizing at 1500 °K, this composition resides in the fcc-sigma phase field, according to the calculated phase diagram. However, with the presence of nickel, the fcc phase field is enlarged, and the sigma phase is thus less stable for this composition. Accordingly the sigma phase would not be a major second phase constituent, which is consistent with the microscopy observations. Finally, Fig. 69 shows an experimentally determined Co-Cr-Mo ternary phase diagram at 1500 °K. This diagram is very similar to the diagram computed in the present study.
Fig. 69  Observed Co-Cr-Mo phase diagram for 1500°K (Ref. 7)
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The author was born on December 31, 1952 in India. He moved with his parents to Los Angeles, California in 1957. After five years, he and his family moved to Ottawa, Canada where they have been ever since. Upon graduation from Lisgar Collegiate Institute in 1970, the author entered the University of Toronto. He obtained the Bachelor of Applied Science Degree in Metallurgy and Materials Science in 1974. In that year he began his doctoral program in the Department of Materials Science and Engineering at MIT. During his stay at MIT, the author has been involved with a number of activities including being a freshman advisor, teaching assistant for a number of courses over the last four years and a member of the Graduate Materials Committee. The most meaningful activity, however has been his involvement in the development and growth of a seminar course at MIT aimed at providing an educational alternative for the MIT student interested in relating professional training to development issues.