AXIAL AND TRANSVERSE MOMENTUM BALANCE IN SUBCHANNEL ANALYSIS

by

JOHN GEORGE BARTZIS

Diploma in Mechanical & Electrical Engineering
National Technical University of Athens, Greece (1970)

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Signature of Author ..........................................................

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Certified by .................................................. Thesis Advisor

Accepted by .................................................. Chairman, Departmental Committee on Graduate Students

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ABSTRACT

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Submitted to the Department of Nuclear Engineering, January 1975, in partial fulfillment of the requirements for the Degree of Master of Science.

In this thesis an attempt has been made to obtain general axial and transverse momentum equations valid for both lumped parameter (LP) and distributed parameter (DP) techniques.

The axial momentum equation can include rod bowing and blockage effects. The conditions under which this general axial momentum equation reduces to existing codes (COBRA III C and VELASCG) utilizing LP and DP techniques has been identified. In addition values have been proposed for certain parameters required to utilize the general equation for LP techniques.

However emphasis is given to the transverse momentum equation which is where the most significant uncertainties exist. A general equation is obtained by using the moment-of-momentum equation in the Φ-direction. The existence of a blockage and a slight bowing can be accomodated in this model.

Conditions under which this equation can be reduced to the existing models are given. In this case a theoretical approach has been used to calculate the input parameters for existing simple models, e.g. COBRA III C.

Experiments are also proposed to examine the validity of the more general models under conditions requiring this increased analytic complexity, e.g. blockages.

Thesis Supervisor: Neil E. Todreas

Title: Associate Professor of Nuclear Engineering
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SYMBOLS

$A_i, A_j$  
Subchannel Area

$A_{ij}, A_{ij1}, A_{ij2}$  
Control Surfaces for the transverse Momentum Equation.

$A_{i1}, A_{i2}$  
Fraction of the control surfaces $A_{ij1}$ and $A_{ij2}$, belonging to subchannel $i$.

$A_{j1}, A_{j2}$  
Fraction of the control surfaces $A_{ij1}$ and $A_{ij2}$, belonging to subchannel $j$.

$C$  
Constant as defined in Eq. (3.63).

$C_f$  
Constant as defined in Eq. (3.71) and (1.20)

$C_g$  
Constant for gravity term in the transverse momentum balance as defined in Eq. (3.20)

$C_i$  
Constant as defined in Eq. 2.74.

$c_{ij}, c_{ji}$  
Constants as defined in Eq. (2.69).

$D$  
Fuel rod diameter

$e_1, e_2, e_x$  
Unit vectors in the radial plane of an orthogonal system.

$e_y, e_z$  
Unit vectors in the radial plane for Cartesian coordinates.

$e_r, e_\phi$  
Unit vectors in the radial plane for cylindrical coordinates.

$f_i, f_j$  
Mean friction factors for the subchannels $i$ and $j$ respectively.
\( f(r,\phi) \)  
Shape velocity function as defined in Eqs. (4.3) and (4.4).

\( g \)  
Gravitational acceleration.

\( \dot{n}_i \)  
Subchannel flow rate

\( n \)  
Normal unit vector.

\( \rho \)  
Time smoothed pressure

\( \overline{p}_i \)  
Mean pressure for subchannel \( i \)

\( \hat{p}_i \)  
Quantity as defined in Eq. 2.39.

\( p'_i \)  
Quantity as defined in Eq. 2.61

\( \bar{p}_i \)  
Asymptotic pressure for subchannel \( i \) as defined in 3.1.1.2

\( \overline{p}_{ij} \)  
Mean pressure along the gap \( S_{ij} \).

\( P/D \)  
Pitch ratio

\( q(\phi) \)  
Quantity as defined in Eq. 4.

\( \overline{r}_i, \overline{r}_j, \overline{r}_i \)  
Mean radii as defined in Equations (3.18).

\( S_i \)  
The perimeter of the subchannel

\( S_i \)  
Total length of the artificial boundary of the subchannel \( i \).

\( S_{ij} = S_{ji} \)  
Gap length.
$S_{im}$  
Length which separates the rod (or wall) m and the subchannel i

$S_{i1}, S_{j1}, S_{i2},$  
$S_{j2}, S_n, S_\phi,$  
$S_{w1}, S_{w2}$  
Lengths shown in Figs. 3.4 and 3.5

$\overline{S_{i1}}, \overline{S_{j1}}$  
Mean Lengths as defined in Eq. (3.67).

$u$  
Axial velocity as defined in Eq. (2.20b) or (2.23a).

$\overline{u}$  
Time smoothed axial velocity

$u'$  
Axial velocity fluctuation.

$u^*$  
Effective axial velocity for the Transverse Momentum Equation as defined in Eq. (3.83).

$u_i$  
Subchannel axial velocity for LP techniques.

$\tilde{u}_i$  
Subchannel axial effective velocity for the axial momentum equation.

$u'_i$  
Quantity as defined in Eq. (2.58).

$u^*_{i*}$  
Effective axial velocity along the artificial boundary $S_f$, as defined in Eq. (2.49b).

$u^*_{ij}$  
Effective gap velocity for the axial momentum equation as defined in Eq. (2.49a).

$\overline{u}_{ij}$  
Mean gap velocity
$u_{i1}^*, u_{j1}^*, u_{l}^*$
Effective axial velocities for the transverse momentum Equation as defined in Eq. (3.17).

$\bar{u}(\phi)$
Mean axial velocity along a $S_{\phi}$ line.

$v'$
Effective specific volume as defined in Eq. (2.53a).

$v_1, v_2$
Velocity components in the radial plane for an orthogonal coordinate system (Cartesian: $v_1=v_x, v_2=v_y$. Cylindrical: $v_1=v_r, v_2=v$)

$v_n$
Normal velocity as shown in Figs. 2.1 and 3.6.

$v_{\phi m}^{i} (r, \phi)$
Component of the velocity $v_{\phi}$, due to cross flow $W_{mi}$ within the surface $A_{i1}$.

$v_{\phi D}^{i} (r, \phi)$
Component of the velocity $v_{\phi}$, due to cross flows only, as defined in Eq. (3.28).

$v_{\phi secondary}$
Component of the velocity $v_{\phi}$, due to the secondary flow.

$W$
Velocity as defined in 2.20e.

$\hat{W}$
Time smoothed velocity

$W_{ji}$
Diversion cross flow between the gap $S_{ji}$.

$W_{i*}$
Cross flow as defined in Eq. 2.41b.

$W(\phi)$
Cross flow past $S_{\phi}$ line

$x_1, x_2$
Coordinates in the radial plane of an orthogonal system (Cartesian: $x_1=y, x_2=z$. Cylindrical: $x_1=r, x_2=\phi$).
\( \alpha \)  
Factor as defined in Eq. 3.84.

\( \alpha' \)  
Factor as defined in Eq. 3.24c

\( \alpha_{i}, \alpha_{j} \)  
Factors as defined in Eqs. 3.24a and 3.24b.

\( \overline{\beta_{m}} \left( \gamma_{i} \theta \right) \)  
Quantity as defined in Eq. 3.29.

\( \overline{\beta_{m}}, \overline{\beta_{m'}} \)  
Quantities as defined in Eqs. 3.32

\( \overline{\beta_{i}}, \overline{\beta_{j}} \)  
Quantity as defined in Eq. 3.36.

\( \cos \beta \)  
Constants as defined in Eqs. 3.43a and 3.43b.

\( \Delta P_{o e}, \Delta P_{o e} \)  
The cosine of the angle formed by the \( g \) vector and the positive x-axis.

\( \Delta P_{g}, \Delta P_{g} \)  
Pressure losses due to change of the shear stress \( \tau_{\phi x} \) with respect to \( z \), as defined in Eqs. (3.25b) and (3.81b).

\( \Delta P_{g'}, \Delta P_{g} \)  
Pressure losses due to the presence of the gravitational field as defined in Eqs. (3.25c) and (3.81c).

\( \Delta P_{o e}, \Delta P_{o e} \)  
Lateral pressure losses as defined in Eqs. (3.25a) and (3.81a)

\( \Delta P_{e}, \Delta P_{e} \)  
Pressure losses due to change of \( A_{ijkl} A_{ijk} \) with respect to \( x \) as defined in Eqs. (3.25d) and (3.81d).

\( \varepsilon_{i}, \varepsilon_{j} \)  
Constants as defined in Eq. (3.45a) and (3.45b)

\( \varepsilon_{m}, \varepsilon_{m} \)  
Local eddy diffusivities.
\( \bar{\varepsilon}_{m\phi} \) Mean eddy diffusivity as defined in Eq. (3.62).

\( \bar{S} \) Lateral resistance factor as defined in Eq. (3.69).

\( \eta_{i1}, \eta_{j1} \) Constants as defined in Eq. (3.43a) and (3.43b).

\( \bar{\rho} \) Time smoothed coolant density

\( \bar{\rho}_i \) Mean subchannel density

\( \bar{\rho}_{ij} \) Mean gap density

\( \bar{\rho}_i, \bar{\rho}_j, \bar{\rho}_1 \) Mean densities as defined in Eq. (3.19)

\( \tau_x = (\tau_{xx}, \tau_{x1}, \tau_{x2}) \) Axial Stresses in a orthogonal coordinate system. (Cartesian: \( \tau_{x1} = \tau_{x\gamma} \) \( \tau_{x2} = \tau_{x\zeta} \) (Cylindrical \( \tau_{x1} = \tau_{x\r} \) \( \tau_{x2} = \tau_{x\phi} \)

\( \tau_\phi = (\tau_{\phi x}, \tau_{\phi\r}, \tau_{\phi\phi}) \) Transverse stresses in cylindrical coordinates.

\( \tau_{x1}^{(L)}, \tau_{x\phi}^{(L)} ; \tau_{x1}^{(T)}, \tau_{x\phi}^{(T)} \) Laminar or turbulent stress components.

\( \tau_{x\r}, \tau_{\phi\r} \) Normal stress components as shown in Fig. (2.4) and Fig. (3.6) respectively.

\( \bar{\tau}_{xx} \) Mean axial shear stress as defined in Eq. 2.32.

\( \phi_{i1}, \phi_{j1} \) Angles shown in Figs. 3.4 and 3.5
Quantities defined in Eqs. 3.16.

Quantity defined in Eq. 3.18.
CHAPTER I

INTRODUCTION

1.1 General

The solution of the energy equation inside a subchannel of the nuclear reactor core presupposes the knowledge of the velocity field in the coolant. For this purpose the two main approximate techniques have been developed.

1. Lumped Parameter Techniques (LP)
2. Distributed Parameter Techniques (DP).

The two techniques appear to approach the problem from different directions.

The DP techniques as presently formulated are restricted to fully developed flow, i.e. no diversion cross flow, unbaffled bare rods, bundle cross sections independent of axial coordinates, one phase flow (Ref. 14 p.14).

The LP techniques have the advantage that they can treat much more complex cases such as the 3-D problem, change of the cross section with respect the axial direction, forced as well as one phase flow and non steady as well as steady flow. However they have the inherent disadvantage that the entire subchannel region must be represented by average properties.
A number of codes have appeared in the literature based on the lumped parameter approach such as HAMBO(I5), THINC I (I6), THINC IV(I7), COBRA III(I8), COBRA IIIC(I8), THI3D(I9) and HERA-IA(I3). Some of the codes include the capability to treat transients i.e. the COBRA codes. Some are developed mainly for a particular type of the reactor i.e. the THINC codes for LWR's and THI3D and HERA-IA for LMFBR's.

With respect to treating the widest spectrum of problems COBRA IIIC appears to be the most advanced presently available. Further COBRA IIIC also includes inertial terms in the transverse momentum equation.

The most advanced technique utilizing the distributed parameter technique seems to be Eifler-Nijsing's approach which treats the 2-D case. Their momentum solution has been coded as VELASCO(I1).

1.2 Axial momentum equation

Looking at the LP techniques and the VELASCO code we can see that both methods have the common characteristic that they are both integral methods with respect to the axial momentum equation. In both techniques, the above equations are integrated within a radial surface which however is different for each technique. For LP techniques the area is the subchannel cross-section. In VELASCO the surface is limited by two constant \( \phi \)-lines and the zero shear stress line (see § 2.4 and Fig.2.2). Within the area considered, in LP techniques the average values of the various quantities are con-
sidered, while in VELASCO the shape if the various quantities are assumed known.

The above common characteristic can permit us to examine both in a unified way. Specifically integral conservation equations can be obtained which can be reduced to those of either technique. These general equations will be valid for any 3-D problem and will allow the shapes of the various quantities to be introduced into the analysis. Therefore, they can be useful for extending techniques to more complex cases.

One of the accomplishments of the thesis will be the derivation of the general axial momentum equation and a step toward improvement of the treatment of the inertial terms by taking into consideration the velocity shape.

It is worthwhile to point out here the attempt toward improvement of the axial momentum equation for LP techniques which has been by Ruhani (7).

I.2.1 Ruhani's Approach for Axial momentum Equation.

Ruhani starts with the Navier-Stokes equation for turbulent compressible flow in the axial direction in Cartesian coordinates and integrates it over the subchannel area. The integration is obtained by assuming that the average velocity gradient near the gap can be approximated by (Ref. 7, Eq.2.14).

\[
\int_{\Omega} \frac{\partial u_i}{\partial y_{ij}} \approx \frac{\bar{u}_{ij} - \bar{u}_i}{\partial y_{ij}} \tag{4.1}
\]
where $u_i$ is the mean velocity of the subchannel i

$\bar{u}_{ij}$ is the mean gap velocity between the subchannels i & j

$\Delta v_{ij}$ is the transverse distance along which the axial velocity changes from $u_{ij}$ to $u_i$.

The derived axial momentum equation is as follows (Ref. 7, Eq. 2.22) according to our notation:

$$\frac{dP_i}{dx} + \frac{1}{A_i} \left[ F_{i}^{fric} + F_{i}^{mix} + \frac{\dot{m}_i}{A_i} \frac{d\gamma'}{dx} + \sum_j W_{ij} (\bar{u}_{ij} - u_i) \right] = \bar{P}_i \cos \beta \tag{1.2}$$

where $\bar{P}_i$ is the mean pressure of the subchannel i

$A_i$ is the area of the subchannel

$W_{ij}$ is the cross flow along the gap

$\bar{P}_i$ is the subchannel mean density

$\gamma'$ is the effective specific volume for momentum (Ref. 3, p.A-5, Eq. A-I2)

$\dot{m}_i$ is the mass flow rate at the elevation of the subchannel.

$g$ is the gravity acceleration

$\cos \beta$ is the cosine of the angle between and the positive x-axis.

Notice that the axial momentum equation in COBRA IIIC which is given in Ref. 8, Eq.A-I5, p.A-5 can be written

$$\frac{1}{A_i} \frac{\partial \dot{m}_i}{\partial t} - 2 \dot{u}_i \frac{\partial \bar{P}_i}{\partial t} + \frac{\partial \bar{P}_i}{\partial x} + \frac{1}{A_i} \left[ F_{i}^{fric} + F_{i}^{mix} + \right.$$


\[ + \frac{m_i^2}{A_i} \frac{d}{dx} \left( \frac{v_i'}{A_i} \right) + \sum_j W_{ij} \left( u_{ij}^* - 2u_i \right) = \]

\[ = \bar{s}_i g \cos \beta \]  
\( (1.3) \)

For steady state and \( \frac{\partial A_i}{\partial x} = 0 \), Eq. 1.3 is reduced to (1.2) if we substitute the gap velocity \( \bar{u}_{ij} \) for the effective velocity \( u_{ij}^* \), which in COBRA IIIC CODE is given as

\[ u_{ij}^* = \frac{u_i + u_j}{2} \]  
\( (1.4) \)

The suggested correlation for \( \bar{u}_{ij} \) given by Ruhani is

\[ \bar{u}_{ij} = \frac{c_{ij} u_i + c_{ji} u_j}{2} \]  
\( (1.5) \)

where \( c_{ij} \) and \( c_{ji} \) will have some value around or less unity.

1.2.2 Present approach for the axial momentum equation

In this approach, the Navier-Stokes equation in axial direction for compressible or incompressible flow (turbulent or laminar) in orthogonal general curvilinear coordinates or Cartesian coordinates, will also be used. We will see that approximation (1.1) is not necessary. Transient conditions and axial variations of the flow area (\( \frac{\partial A_i}{\partial x} \neq 0 \)) will be considered as in COBRA IIIC. In addition the momentum equation will be generalized to take care of bowing of the rods.
1.3 Transverse Momentum Equation

The main emphasis of this thesis will be given to the transverse momentum equation which the largest uncertainties in approximations exist. Such an equation is not yet used for DP techniques because present DP techniques consider no diversion cross flow.

Before explaining our approach let us review the key characteristics of existing models. The transverse momentum equation gives the pressure difference between two adjacent subchannels i and j and is also an integrated equation over a surface which partly belongs to the subchannel area Ai and partly to the subchannel area Aj. This pressure difference balances the friction and the inertial forces. A good discussion of the forms of the transverse momentum equation for the various codes is given in Ref. 20, p. 1.4. Here we will review briefly the transverse momentum equations used in COBRA IIIC, THINC IV and TH13D which appear to be the most advanced code models and Ruhani's approach which has been the starting point of our approach.
I.3.1 COBRA IIIC Model (Ref. 7, p. A-6)

The transverse momentum equation in COBRA IIIC has the form

\[ \bar{p}_j - \bar{p}_i = \frac{4}{s} \left[ \frac{\partial W_{ji}}{\partial t} + \frac{\partial (u^* W_{ji})}{\partial x} \right] + \frac{b}{s} F_{ji} \tag{1.6} \]

where \( W_{ji} \) is the diversion cross flow
\( s \) is the gap
\( l \) is the transition length for the pressure difference
\( \frac{b}{s} F_{ji} \) is the friction and form losses
\( u^* \) is the effective velocity given by equation I.4

Equation (I.6) is obtained by making the transverse momentum balance on the volume shown in Fig. I.1.

I.3.2 THINC IV Model

The THINC IV code developed for LWR's is formulated in rectangular coordinates and therefore is applicable to square subchannels.

Eq. 27 of Ref. I4 (p. 3-10) according to our notation can be written
Fig. I.I Control volume for transverse momentum equation.

COBRA IIIC Model.
\[
\frac{\Delta \left( \rho_o u_o v_{ji}' \right)}{\Delta x} + \left( p_j' - p_i' \right) \frac{1}{\Delta y} + p_o F_y | v_{ji}' | v_{ji}' = 0 \tag{1.7}
\]

where \( \Delta y \) is the centroid distance

\( \rho_o \) is the unperturbed density

\( u_o \) is the unperturbed velocity

\( v_{ji}' \) is the velocity at the gap

and \( p_j' - p_i' \) is the perturbed pressure difference.

Because the unperturbed pressure is the same in both sub-channels \( i \) and \( j \),

\[
p_j' - p_i' = \overline{p_j} - \overline{p_i} \tag{1.8}
\]

Multiplying \( (1.7) \) by \( \Delta y \) and writing \( \frac{\Delta \left( \rho_o u_o v_{ji}' \right)}{\Delta x} \) in differential form we obtain

\[
\overline{p_j} - \overline{p_i} = \Delta y \cdot \frac{d \left( \rho_o u_o v_{ji}' \right)}{dx} + p_o \Delta y F_y | v_{ji}' | v_{ji}' \tag{1.9}
\]

Call

\[
\Delta p_{ll}^{\text{THINC}} = \rho_o \Delta y F_y | v_{ji}' | v_{ji}' \tag{1.10a}
\]

\[
W_{ji} = \rho_o s v_{ji}' \tag{1.10b}
\]

\[
\left( \frac{\rho}{s} \right)^{\text{THINC}} = \frac{\Delta y}{s} \tag{1.10c}
\]

Eq. 1.9 becomes

\[
\overline{p_j} - \overline{p_i} = \left( \frac{\rho}{s} \right)^{\text{THINC}} \frac{\partial}{\partial x} \left( u_o W_{ji} \right) + \Delta p_{ll}^{\text{THINC}} \tag{1.11}
\]
Comparing equations 1.6 and 1.11 we see that both equations are similar. The difference is that THINC IV instead of $u^*$, the unperturbed axial velocity $u_0$ is used. The density for $W_{ji}$ is the unperturbed density while in COBRA IIIC the mean density of the donor subchannel is used. In THINC IV $\ell$ always has the value of the subchannel centroid in contrast with COBRA IIIC where $\ell/s$ is an input. In the COBRA IIIC code if you do not specify a value, $\ell/s$ takes the value 2.00.

Another difference is also in the friction term. In COBRA IIIC the friction factor has a constant value and is given as input, whereas in THINC IV it is a function of the lateral Reynolds number i.e.

$$f = AR_e^{-0.2} \quad (1.12)$$

### 1.3.3 THI3D Model

The THI3D code which has been developed to analyze mainly LMFBR's, is a computer program for steady state single phase thermal hydraulic subchannel and assembly analysis.

The THI3D cross flow approximation includes

1. The model without inertial terms i.e.

$$\Delta P_{ji} \approx \Delta P_{fri} \quad (1.13)$$

2. The COBRA IIIC model.

3. A new model in which the lateral acceleration losses are included in the lateral losses term i.e.

$$\Delta P_{ll} = \Delta P_{ac} + \frac{c}{s}F_{ji} \quad (1.14)$$
where $\Delta p_{\alpha}$ is the acceleration lateral losses and are given in Ref. 19, App. C, Eq. C-2.

Other characteristics of the THI3D new model are:
1. The parameter $l$ equals always the centroid distance
2. The effective velocity $u^*$ is the same as in COBRA IIIC model.
3. The density for the diversion cross flow $W_{ji}$ is taken as the mean value of the densities of the subchannels $i$ and $j$.

Comparing THI3D new model and the COBRA IIIC model, we can see that THI3D new model seems to be better in the cases that acceleration losses are important. In the cases that these losses have little effect on the transverse momentum balance, COBRA IIIC model appears to be better since we have the flexibility to select any value for $l/s$.

13.4 Ruhani's Approach (Ref. 7)

Ruhani's approach is the same in principle as in the axial momentum equation. The Navier-Stokes equations in the $y$ direction is taken and is integrated over a surface adjacent to both subchannels under consideration. The surface is shown in Fig. 3 (shaded area). The lines $y_i$ and $y_j$ must be located such that the pressure along these lines is $\overline{p}_i$ and $\overline{p}_j$ respectively. Ruhani suggests
\[ | \gamma_i | = | \gamma_j | = \alpha D \]  \hspace{1cm} (1.15a)

where \( D \) is the rod diameter, and

\[ \alpha = 0.5 \sin \frac{n}{4} \]

for subchannels between rods placed on a triangular array

\[ \alpha = 0.5 \sin \frac{n}{6} \]

for subchannels between rods placed on a square array and also side subchannels for any array.

The idea is that if we know the exact shapes of the lateral velocities \( V_y \) and \( V_z \) in advance, the integration of the transverse momentum equation would give the correct pressure difference. Ruhani assumes the following shapes for the velocities.

\[ V_y(x,y,z) = Q(y) \left[ B^2 - Z^2 \right] \overline{V_{gap}}(x) \]  \hspace{1cm} (1.16)

where \( B \) is shown in the Fig. 1.2

\( \overline{V_{gap}} \) is the mean velocity along the gap

\( Q(y) \) is a proportionality factor depending on the position \( y \).

and \( V_z(x,y,z) = V_y(x,y,z) \frac{Z}{B} \tan \phi \)  \hspace{1cm} (1.17)

where the angle \( \phi \) is shown in Fig. 1.2

For the axial velocities we assume linear variation
Fig. I.2 Control surface for Rubani's Approach.
from the gap position i.e.

$$
\bar{u}(y) = \bar{u}_{ij} + \left( \bar{u}_{ij} - u_i \right) \gamma / \alpha_i D \quad \text{for } y < 0 
$$

$$
\bar{u}(y) = \bar{u}_{ij} - \left( \bar{u}_{ij} - u_j \right) \gamma / \alpha_j D \quad \text{for } y > 0
$$

(1.18)

It is evident from Eq. 1.18 that it is assumed that the mean velocity along the $y_i$ and $y_j$ lines, is equal to the average subchannel velocity. An analogous relationship was previously pointed out as applicable for pressures.

From the friction terms only the lateral friction term is considered and given by

$$
\Delta P_{el} = \frac{1}{4} \int C_f \bar{\rho} \bar{V}_{gap}^2 
$$

(1.19)

where $C_f$ reflects the change in $v_y$ along $y$ for the surface considered.

$C_f$ can be found easily by using Eq. 1.16 and has the value

$$
C_f = \frac{2}{9} \int_0^{Q_i} B_i^3 \, dy + \frac{2}{9} \int_0^{Q_j} B_j^3 \, dy
$$

(1.20)

Everything now is available to permit integration of the momentum equation and find the pressure difference.
The result depends on the assumptions made for \( Q_i \) and \( Q_j \).

The assumption that Ruhani makes for \( Q_i \) is that all parts of the subchannel area contribute to the diversion cross flow equally, i.e.

\[
\frac{dW(y)}{dA} = \text{const.} = \frac{W_{ij}}{A_i}
\]

where \( dA \) is shown in Fig. 1.2.

\( W(y) \) is the cross flow past line \( y \). (Fig. 1.2)

1.3.5 Khan's Model

It is evident from the previous discussion that in both COBRA IIIC and the THINC IV models and in Ruhani's approach the area of integration for the transverse momentum balance is constant along the whole length of the subchannel. Although such a simplification does not cause problems in the most cases of the reactor analysis, in the case of blockages this approach appears to fail, as the EIR experiment has shown (Ref. 23, 24). In order to be able to analyze the EIR experiment, Khan (25) has considered that the length in the Rowe's model is a function of \( x \). This function depends on the particular case we have to analyze.

The transverse momentum equation applied to a control volume between \( x \) and \( x + dx \) written according to our notation (Ref. 26, Eq. 5).

\[
\bar{P}_j - \bar{P}_i = \Delta P_{ll} + \rho \left[ \bar{u}_{ij}(x+dx) \bar{v}_{gap}(x+dx) - \bar{u}_{ij}(x) \bar{v}_{gap}(x) \right] \frac{d \bar{P}}{dx} + \dot{\bar{u}}_{ij}(x+dx) \bar{v}_{gap}(x+dx) \frac{d \bar{P}}{dx} \]  

(1.22)
Putting this equation in a differential form with respect to \( x \) we obtain

\[
\vec{P}_j - \vec{P}_i = \frac{d}{dx} \left( \frac{\ell}{s} \, u^* \, W_{ji} \right) + \Delta P_{\ell \ell} \quad (1.24)
\]

W.D. Brown, E.U. Khan and N.E. Todreas have tried to find the suitable function \( l(x) \) which fit the data for the various cases of the EIR experiment after a distance downstream of the blockages. This work has showed that a polynomial up to second order with respect to \( x \) for \( \ell \) has to be used to correlate the data. This means that analyzing a case with high \( W_{ji} \), the dependence of \( \ell/s \) as a function of \( x \) (and consequently a function of \( W_{ji} \)) cannot be neglected.

### 1.3.6 Present Approach

Chapter III presents an approach to the problem which is based on the same idea as discussed in the previous sections, i.e. to integrate momentum equation over a suitable selected surface. As we will see, we can integrate an equation without knowing in advance the lateral velocities. Our desire to use quantities familiar in distributed parameter techniques (e.g. \( v_\rho \) field instead of \( v_y \) and \( v_z \) field) with as much simplicity and physical representation of rod arrays as possible lead us to use the moment of momentum equation.

The derived equation will be quite general so that it will include all the previous models and all terms invol-
ved in lateral momentum balance. The equation will be applicable for both compressible and incompressible fluids, one phase or two phase flow, laminar or turbulent flow. Also a subchannel geometry representing a slight bowing of rods is permitted.

In the remaining Chapters our interest will be concentrated on deriving the general equation reducing it to existing models and developing on our model together with methods to evaluate the various constants. introduced.
CHAPTER II

AXIAL MOMENTUM BALANCE IN SUBCHANNEL ANALYSIS

2.1 Development of the general momentum equation

2.1.1 Differential form

Consider an orthogonal system one axis of which is the x-axis. (i.e. Cartesian, cylindrical, or else)

Call $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_x$, the unit vectors of the $x_1$, $x_2$, $x$ coordinates respectively. Thus, the coolant velocity vector $\vec{W}$ is represented by

$$\vec{W} = \vec{u} \vec{e}_x + \vec{V}_1 \vec{e}_1 + \vec{V}_2 \vec{e}_2 \quad (2.1)$$

where $\vec{u}$, $\vec{V}_1$, $\vec{V}_2$, are the time smoothed components of the coolant velocity.

For incompressible fluids, the momentum balance equation in the x-direction, is given in Ref. 1, Eq. 5.2.3, p.159

$$\rho \frac{D \vec{u}}{D t} = -\nabla \tau_x + \rho g \cos \beta \quad (2.2)$$

where $\cos \beta = \vec{e}_x \cdot \vec{e}$

$g$ is the gravity acceleration

$$\tau_x = \tau_{xx} \vec{e}_x + \tau_{x1} \vec{e}_1 + \tau_{x2} \vec{e}_2 \quad (2.3)$$
and \( \tau_{xx}, \tau_{x1}, \tau_{x2} \) are the fluid stresses\(^(*)\)

which are the sum of viscous and Reynolds (turbulent) stresses i.e.

\[
\tau_x = \tau_x^{(l)} + \tau_x^{(t)}
\]
\[
\tau_{xx} = \tau_{xx}^{(l)} + \tau_{xx}^{(t)}
\]
\[
\tau_{x1} = \tau_{x1}^{(l)} + \tau_{x1}^{(t)}
\]
\[
\tau_{x2} = \tau_{x2}^{(l)} + \tau_{x2}^{(t)}
\]

where \((l)\) means "laminar" and \((t)\) "turbulent".

The normal viscous stress \( \tau_{xx}^{(l)} \) is given by the following equation (Ref. 2, Eq. 3-34, p. 64)

\[
\tau_{xx}^{(l)} = \rho - \left[ 2 \mu \frac{\partial \bar{u}}{\partial x} - \frac{2}{3} \mu \nabla \bar{u} \right] \quad (9.5)
\]

The shear stresses can be found from the transformation from the expressions in Cartesian coordinates i.e.

\[
\tau_{x1}^{(l)} = -\mu \left[ \frac{1}{h_1} \frac{\partial \bar{u}}{\partial x_1} + h_1 \frac{\partial}{\partial x} \left( \frac{\bar{V}_1}{h_1} \right) \right] \quad (9.6b)
\]
\[
\tau_{x2}^{(l)} = -\mu \left[ \frac{1}{h_2} \frac{\partial \bar{u}}{\partial x_2} + h_2 \frac{\partial}{\partial x} \left( \frac{\bar{V}_2}{h_2} \right) \right] \quad (9.6c)
\]

where \( h_1, h_2 \) are the scale factors for the particular coordinate system (Ref. 3, Eq. I.3.4, p. 24)

For Cartesian coordinates \( h_1 = h_2 = 1 \)

and

\(^(*)\) \( \tau_{xx} \) is a normal stress
\[ \tau_{xy}^{(l)} = -\mu \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \]
\[ \tau_{xz}^{(l)} = -\mu \left( \frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{v}}{\partial x} \right) \]  
\[ (2.7a) \]

For cylindrical coordinates \((r, \phi, x)\) \(h_I \equiv h_r = 1\), \(h_2 \equiv h_\phi \equiv r(\text{Ref. 3, p. II6})\) and
\[ \tau_{x\phi}^{(l)} = -\mu \left( \frac{\partial \bar{v}_\phi}{\partial x} + \frac{1}{r} \frac{\partial \bar{u}}{\partial \phi} \right) \]  
\[ (2.7b) \]

The Reynolds stresses, for Cartesian coordinates have the form (Ref. 1, Eqs 5.2-5.6, p. I59)
\[ \tau_{xx}^{(t)} = \sigma \bar{u}'^2 \]
\[ \tau_{xy}^{(t)} = \sigma \bar{u}' \bar{v}' \]
\[ \tau_{xz}^{(t)} = \sigma \bar{u}' \bar{v}' \]  
\[ (2.8a) \]

For the cylindrical coordinates (Ref. 4, Eqs 143-144, p. 2-31, 2-32)
\[ \tau_{x\phi}^{(t)} = \sigma \bar{u}' \bar{v}' \]  
\[ (2.8b) \]
\[ \tau_{x\phi}^{(t)} = \bar{\rho} u' v_{\phi}' \quad (2.8 b) \]

In the general case it is evident that
\[ \tau_{x}^{(t)} = \bar{\rho} u' w' \quad (2.9) \]

where
\[ w' = u' e_x + v_1' e_1 + v_2' e_2 \quad (2.10) \]
is the velocity fluctuation and \( \bar{\rho} u' w' \) the time smoothed quantity.

The acceleration term \( \frac{D\bar{u}}{Dt} \) for Cartesian coordinates has the form (Ref. I, p. 84)
\[ \frac{D\bar{u}}{Dt} = \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v}_y \frac{\partial \bar{u}}{\partial y} + \bar{v}_z \frac{\partial \bar{u}}{\partial z} \quad (2.11a) \]

and for cylindrical coordinates (Ref. I, p. 85)
\[ \frac{D\bar{u}}{Dt} = \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v}_r \frac{\partial \bar{u}}{\partial r} + \frac{\bar{v}_\phi}{r} \frac{\partial \bar{u}}{\partial \phi} \quad (2.11b) \]

In the general case, as proved in APPENDIX A, the term \( \frac{D\bar{u}}{Dt} \) can be written (Eq. A-8)
\[ \frac{D\bar{u}}{Dt} = \frac{\partial \bar{u}}{\partial t} + \bar{w} \nabla \bar{u} \quad (2.12) \]
which in expanded form is

\[
\frac{D\bar{u}}{Dt} = \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \frac{\mathbf{V}_1}{h_1} \frac{\partial \bar{u}}{\partial x_1} + \frac{\mathbf{V}_2}{h_2} \frac{\partial \bar{u}}{\partial x_2}
\]  

(2.11c)

Finally we note that

\[
\rho \bar{W} \nabla u = \nabla (\rho \bar{u} \bar{W}) - \nabla \rho \bar{W}
\]  

(2.13)

With these preliminaries accomplished, we will now proceed to develop the x-direction momentum equation (2.2). First we substitute the result of Eq. 2.13 into Eq. 2.12 to obtain

\[
\rho \frac{D\bar{u}}{Dt} = \rho \frac{\partial \bar{u}}{\partial t} + \nabla (\rho \bar{u} \bar{W}) - \bar{u} \nabla \rho \bar{W}
\]  

(2.14)

Now the continuity equation is

\[
\frac{\partial \rho}{\partial t} + \nabla \rho \bar{W} = 0
\]  

(2.15)

Substituting Eq. 2.15 into Eq. 2.14 yields

\[
\rho \frac{D\bar{u}}{Dt} = \rho \frac{\partial \bar{u}}{\partial t} + \nabla (\rho \bar{u} \bar{W}) + \bar{u} \frac{\partial \rho}{\partial t}
\]

or

\[
\rho \frac{D\bar{u}}{Dt} = \nabla (\rho \bar{u}) + \nabla (\rho \bar{u} \bar{W})
\]  

(2.16)
Thus utilizing Eq. 2.16 we can express Eq. 2.2 as

\[
\frac{\partial}{\partial t}(\bar{\rho} \bar{u}) + \nabla (\bar{\rho} \bar{u} \bar{w} + \bar{\tau}_x) = \bar{\rho} g \cos \beta
\]  

(2.17)

It is pointed out here that Eq. 2.17 is valid for every fluid in which density fluctuations are neglected. In case of fluid fluctuations have to be taken into account things are slightly different as show below. In Ref. 5, (Eq. 4-5, 4-6, p. 82), for Cartesian coordinates the time smoothed momentum equation and the continuity equations including density fluctuations are as follows

\[
\frac{\partial}{\partial t}(\bar{\rho} \bar{u} + \bar{\rho} \bar{u'}) + \frac{\partial}{\partial x} \left( \bar{\rho} \bar{u}^2 + \bar{u} \bar{\rho} \bar{u'} \right) + \\
+ \frac{\partial}{\partial y} \left( \bar{\rho} \bar{u} \bar{v} + \bar{u} \bar{\rho} \bar{v'} + \bar{v} \bar{\rho} \bar{u'} \right) + \\
+ \frac{\partial}{\partial z} \left( \bar{\rho} \bar{u} \bar{v} + \bar{u} \bar{\rho} \bar{v'} + \bar{v} \bar{\rho} \bar{u'} \right) + \\
+ \nabla \tau^{(l)} = 0
\]

(2.18)

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x} \left( \bar{\rho} \bar{u} + \bar{\rho} \bar{u'} \right) + \frac{\partial}{\partial y} \left( \bar{\rho} \bar{v} + \bar{\rho} \bar{v'} \right) + \\
+ \frac{\partial}{\partial z} \left( \bar{\rho} \bar{v} + \bar{\rho} \bar{v'} \right) = 0
\]

(2.19)
Für the above equations the viscosity appearing in \( \nu \) is considered to have negligible fluctuation. It is desirable to work with more compact expressions of Eqs 2.18 and 2.19. To achieve this end we can generalize the forms of Eqs 2.17 and 2.15 (originally time smoothed, no density fluctuations) by introducing the following definitions

\[
\begin{align*}
\varrho & = \bar{\varrho} \\
\mathbf{u} & = \bar{\mathbf{u}} + \frac{\mathbf{e} \mathbf{u}'}{\varrho} \\
\mathbf{v}_1 & = \bar{\mathbf{v}}_1 + \frac{\mathbf{e} \mathbf{v}_1'}{\varrho} \\
\mathbf{v}_2 & = \bar{\mathbf{v}}_2 + \frac{\mathbf{e} \mathbf{v}_2'}{\varrho} \\
\mathbf{W} & = \bar{\mathbf{W}} + \frac{\mathbf{e} \mathbf{W}'}{\varrho}
\end{align*}
\]  

It is evident from Eq. 2.19 that continuity equation 2.15 will become

\[
\frac{\partial \varrho}{\partial t} + \nabla (\varrho \mathbf{W}) = 0
\]  

(2.21)
For the momentum equation, neglecting terms such as
\[
\frac{1}{\rho} \frac{g' u'^2}{\rho}, \quad \frac{1}{\rho} \frac{g' u' v'_i}{\rho}, \quad \frac{1}{\rho} \frac{g' u' v'_a}{\rho}
\]
we end up with
\[
\frac{\partial}{\partial t} (\rho u) + \nabla (\rho u w + \tau_x) = \rho g \cos \beta \quad (2.22)
\]
The same equation is also obtained from the time smoothed equation 2.17 by making the good approximation
\[
u \approx \bar{u} \quad (2.23a)
\]
i.e. \[
\frac{g' u'}{\rho} \ll \bar{u} \quad (2.23b)
\]
In this case Eq. 2.22 is equivalent with that suggested by Schlichting (Ref. 2, p. 658, Eq. 23.7) for the two dimensional problems.

Equation 2.22 is general and holds for compressible or incompressible flow, laminar or turbulent. Also, it has the advantage that it can be easily integrated over a subchannel as we will see below.

2.1.2 Integral form

Consider a subchannel \( i \) with gaps and rods and walls (Fig. 2.1a). Let \( J \) be the number of gaps, each denoted by \( j \) having wetted length within the subchannel \( S_{ij} \) and \( M \) the number of rods and walls, each denoted by \( m \) having wetted length within the subchannel \( S_{im} \).
Fig. 2.1 Control volume for the axial momentum balance.
To be more general we can allow also for an artificial boundary within the fluid with total length \( S_i^* \).

It is evident that the perimeter of the channel is

\[
S_i = S_i^* + \sum_{j=1}^{J} S_{ij}^* + \sum_{m=1}^{M} S_{im}^*
\]  

(2.24)

Let us integrate Eq. 2.22 over the control volume bounded by the areas \( A_i(x) \) and \( A_i(x+dx) \) (Fig. 2.Ib). The equation becomes

\[
\frac{\partial}{\partial t} \iiint \rho u dV + \iiint \nabla (\rho u W + \tau_x) \, dV = g \cos \beta \iiint \rho dV
\]  

(2.25)

According to the divergence theorem

\[
\frac{\partial}{\partial t} \iiint \rho u dV + \oiint (\rho u W + \tau_x) n \, dS = g \cos \beta \iiint \rho dV
\]  

(2.26)

where \( S \) is the surface of the control volume and \( n \) is the unit vector perpendicular to the surface as shown in Fig. 2.Ib.

It is evident that

\[
\iiint \rho u dV = dx \oiint \rho u dA = dx \, m_i
\]  

(2.27)
where $\dot{m}_i$ is the mass flow rate along the x-axis.

Also

$$\int\int\int \rho \, dV = dx \int\int \rho \, dA = dx \overline{\rho}_i A_i \quad (2.28)$$

where $\overline{\rho}_i$ is the mean fluid density within the subchannel.

Taking into consideration Eqs. 2.27 and 2.28,

Eq. 2.25 becomes

$$\frac{\partial \dot{m}_i}{\partial t} + \frac{1}{dx} \int\int\int (\rho u W + \tau_x) \mathbf{n} \, dS =$$

$$= \overline{\rho}_i A_i \cos \beta \quad (2.29)$$

Now we proceed to simplify the surface integral term.

First we deal with the surfaces $A_i(x)$ and $A_i(x+dx)$:

$$\int\int (\rho u W + \tau_x) \mathbf{n} \, dS = \int\int (\rho u^2 + \tau_{xx}) \, dA_{i(x)+A_{i(x+dx)}} -$$

$$- \int\int (\rho u^2 + \tau_{xx}) \, dA_{i(x)} \quad (2.30)$$

Define the weighted velocity as

$$\overline{u}_i = \frac{\int\int \rho u^2 \, dA_{A_i}}{\int\int \rho u \, dA_{A_i}} = \frac{\int\int \rho u^2 \, dA}{\int\int \rho u \, dA} \quad (2.31)$$

and the mean spatial value $\overline{\tau}_{xx}$ of $\tau_{xx}$

$$\overline{\tau}_{xx} = \frac{1}{A_i} \int\int \tau_{xx} \, dA \quad (2.32)$$
Then
\[
\frac{\int (\varphi u^2 + \tau_{xx}) \, dA}{A_i(x+dx)} - \frac{\int (\varphi u^2 + \tau_{xx}) \, dA}{A_i(x)} = \frac{d}{dx} \left( m_i(x+dx) \, \hat{u}_i(x+dx) - m_i(x) \, \hat{u}_i(x) \right) + \frac{d}{dx} \left( \tau_{xx} \, A_i \, (x+dx) - \tau_{xx} \, A_i(x) \right)
\]
\[
= \frac{d}{dx} \left( m_i \, \hat{u}_i \right) + \frac{d}{dx} \left( \tau_{xx} \, A_i \right) \quad (2.33)
\]

Now dealing with the surfaces comprising Si, we note from figures 2.1b and 2.1c that
\[
W \, \hat{n} \, ds = W \, \hat{n} \, \frac{dldx}{\cos \theta} =
\]
\[
= (u \sin \theta + v_n \cos \theta) \, \frac{dldx}{\cos \theta} = (u + g \theta + v_n) \, dldx \quad (2.34)
\]
where \(v_n\) is the velocity component perpendicular to \(dldx\)
\(\theta\) is the angle formed by the direction of \(v_n\) and \(n\)
(Fig. 2.1b)

Similarly
\[
\tau_x \, \hat{n} \, ds = (\tau_{xx} + g \theta + \tau_{xn}) \, dldx \quad (2.35)
\]

Eq. 2.29 becomes
\[
\frac{\partial m_i}{\partial t} + \frac{\partial (\tilde{u}_i m_i)}{\partial x} + \frac{\partial (\tau_{xx} A_i)}{\partial x} + \\
+ \phi \left[ \left( p u^2 + \tau_{xx} \right) t g \theta + p \nu n + \tau_{xn} \right] dl = \overline{\sigma_i} g A_i \cos \beta
\]  

(2.36)

Taking into consideration Eq. 2.24 and carrying the provision that can be non zero, Eq. 2.36 becomes (since \( u = 0 \) along Sim)

\[
\frac{\partial m_i}{\partial t} + \frac{\partial (\tilde{u}_i m_i)}{\partial x} + \frac{\partial (\tau_{xx} A_i)}{\partial x} + \phi \left[ \tau_{xx} t g \theta \right] dl + \\
+ \phi \tau_{xx} t g \theta dl + \int_{s_i} p \nu n dl + \sum_{j=1}^{J} \int_{s_{ij}} p \nu n dl + \\
+ \int_{s_i^*} \tau_{xn} dl + \sum_{j=1}^{J} \int_{s_{ij}} \tau_{xn} dl + \sum_{m=1}^{M} \int_{s_{im}} \tau_{xn} dl = \\
= \overline{\sigma_i} g A_i \cos \beta
\]  

(2.37)

One of the factors that makes Eq. 2.37 very complicated is the appearance of \( t g \theta \). Therefore, we try to isolate terms which include \( t g \theta \) as much as possible i.e. define:
\[ \hat{u}_i = \begin{cases} \frac{\phi_{s_i} \rho u^t g \theta d\ell}{\phi_{s_i} \rho u t g \theta d\ell} & \text{for } \rho u^t g \theta \text{ not zero everywhere.} \\ 0 & \text{otherwise.} \end{cases} \] (2.38)

\[ \hat{p}_i = \begin{cases} \frac{\phi_{s_i} \tau_{xx} g \theta d\ell}{\phi_{s_i} \rho u t g \theta d\ell} & \text{for } \rho u^t g \theta \text{ not zero everywhere.} \\ 0 & \text{otherwise.} \end{cases} \] (2.39)

The above formation helps us to substitute more familiar quantities in the denominators. Recall that the continuity equation is given by Eq. 2.15. Integrating this over the control volume of the Fig. 2.Ib. and working similarly as we did for the momentum equation we find that the continuity equation becomes

\[ \frac{\partial (\bar{\rho}_i A_i)}{\partial t} + \frac{\partial \bar{m}_i}{\partial x} + \sum_{i=1}^{\Xi} W_{ij} + W_{i*} + \]

\[ + \frac{\phi_{s_i} \rho u t g \theta d\ell} = 0 \]

(2.40)
where \[ W_{ij} = \int_{s_{ij}} \rho v_n dl \] (2.41a)

is the diversion cross flow

and \[ W_i^* = \int_{s_i^*} \rho v_n dl \] (2.41b)

is the flow across the artificial boundary within the fluid.

Then from Eq. 2.38 and 2.40 we find

\[
\int_{s_i} \rho u^2 + g \theta dl = - \int_{s_i} \frac{\partial P_i A_i}{\partial t} - \int_{s_i} \frac{\partial m_i}{\partial x} - \\
- \sum_{j=1}^{J} \hat{u}_i W_{ij} - \hat{u}_i W_i^* \tag{2.42b}
\]

Also it is not difficult\(^(*)\) to prove that

\[
\int_{s_i} \tau g \theta dl = - \frac{\partial A_i}{\partial x} \tag{2.43}
\]

Then Eq. 2.37 becomes

\[
\frac{\partial m_i}{\partial t} + \frac{\partial (\hat{u}_i m_i)}{\partial x} + \frac{\partial (\tau_{xx} A_i)}{\partial x} - \hat{u}_i \frac{\partial (P_i A_i)}{\partial t} - \hat{u}_i \frac{\partial m_i}{\partial x} + \\
- \sum_{j=1}^{J} \hat{u}_i W_{ij} - \hat{u}_i W_i^* - \hat{p}_i \frac{\partial A_i}{\partial x} + \int_{s_i^*} \rho u v_n dl + \\
+ \sum_{j=1}^{J} \int_{s_{ij}} \rho u v_n dl + \int_{s_i^*} \tau_{xx} dl + \sum_{j=1}^{J} \int_{s_{ij}} \tau_{xx} dl + \\
+ \sum_{m=1}^{M} \int_{s_{im}} \tau_{xx} dl = \bar{P}_i A_i \cos \beta \tag{2.44}
\]

\(^(*)\) Substitute for \(tg \theta = \frac{dS}{dY}\) where \(dY\) is the projection of \(dS\) in \(A_1\) (Fig. 2.1b).
Making the reasonable assumptions that

1. $\tau_{xx} \approx P_i$ \hspace{2cm} (2.45)

2. $\frac{\partial A}{\partial t} \approx 0$ \hspace{2cm} (2.46)

Eq. 2.44 simplifies to

\[
\frac{\partial m_i}{\partial t} \approx \hat{u}_i A_i \frac{\partial \bar{p}_i}{\partial t} + \frac{\partial (\bar{u}_i m_i)}{\partial x} + A_i \frac{\partial \bar{p}_i}{\partial x} + \\
+ (\bar{P}_i - \bar{P}_i) \frac{\partial A_i}{\partial x} - \hat{u}_i \frac{\partial m_i}{\partial x} - \hat{u}_i \sum_{j=1}^{J} W_{ij} - \\
- \hat{u}_i W_{i*} + \int_{S_{i*}}^{\mu} \sum_{j=1}^{J} \int_{S_{ij}}^{\mu} \sum_{m=1}^{M} \int_{S_{im}}^{\mu} \sum_{j=1}^{J} \int_{S_{ij}}^{\mu} \tau_{xn} \, dl = \\
= \bar{p} A_i g \cos \beta \hspace{2cm} (2.47)
\]
In cases that we can assume that

\[
\int_{S_{ij}, S_{i^*}} g_{uvn} dl \approx 0 \tag{2.48}
\]

when \( W_{ij}, W_{i^*} = 0 \)

we define the following quantities

\[
u_{ij}^* = \frac{\int_{S_{ij}} g_{uvn} dl}{W_{ij}} \tag{2.49a}
\]

\[
u_{i^*}^* = \frac{\int_{S_{i^*}} g_{uvn} dl}{W_{i^*}} \tag{2.49b}
\]

Then Eq. 2.47 becomes

\[
\begin{align*}
\frac{\partial m_i}{\partial t} & - \dot{\hat{u}}_i A_i \frac{\partial \hat{g}_i}{\partial x} + \frac{\partial (\dot{\hat{u}}_i m_i)}{\partial x} + A_i \frac{\partial \hat{P}_i}{\partial x} + \\
& + (\hat{p}_i - \hat{p}_i) \frac{\partial A_i}{\partial x} - \dot{\hat{u}}_i \frac{\partial m_i}{\partial x} + (\nu_i^* - \dot{\hat{u}}_i) W_{i^*} + \\
& + \sum_{j=1}^{\Xi} (\nu_{ij}^* - \dot{\hat{u}}_i) W_{ij} + \int_{S_{i^*}} \tau_{x_n} dl + \sum_{m=1}^{M} \int_{S_{im}} \tau_{x_n} dl + \\
& + \sum_{j=1}^{\Xi} \int_{S_{ij}} \tau_{x_n} dl = \\
& = \overline{g}_i A_i \cdot g \cdot \cos \beta \tag{2.50}
\end{align*}
\]
2.2 Generalized Equation for Lumped Parameter Approaches

In this generalized approach we specifically treat the situation where \( \frac{\partial A_i}{\partial x} \neq 0 \) i.e. a situation where the sub-channel geometry is axially distorted by rod bowing or blockage.

In this case:

1. \( \tilde{u}_i = u_i \) \hspace{1cm} (2.51*)

since the subchannel is to be characterized by a single velocity.

Also we take

2. \( \xi_i = 0 \) \hspace{1cm} (2.52)

From Eq. 2.51 the term in Eq. 2.47 or 2.50 becomes

\[
\begin{align*}
\frac{\partial (\tilde{u}_i m_i)}{\partial x} &= u_i \frac{\partial m_i}{\partial x} + m_i \frac{\partial u_i}{\partial x} = \\
&= u_i \frac{\partial m_i}{\partial x} + m_i \frac{\partial}{\partial x} \left( \frac{m_i v'}{A_i} \right) = \rho u_i \frac{\partial m_i}{\partial x} + \\
&+ m_i^2 \frac{\partial}{\partial x} \left( \frac{v'}{A_i} \right)
\end{align*}
\] (2.53)

(*) The quantity \( u_i \) is the mean axial velocity for single phase flow, but in two phase flow is defined (Ref. 8, Eq. A-I2)

\[
\bar{u}_i = \frac{\dot{m}_l u_l + \dot{m}_g u_g}{\dot{m}}
\]

where \( \dot{m}_l, u_l \) are the mass flow rate and mean velocity for the liquid,
and \( \dot{m}_g, u_g \) are the mass flow rate and mean velocity for the gas.
Recall from §1.2.2. that $u'$ is the effective specific volume and is given by the expression (Ref. 8, p. 54)

$$u' = \frac{(1-x)^2}{\rho_4(1-\alpha)} + \frac{x^2}{\rho_g x} \quad (2.53a)$$

where $x$ is the steam quality

$\alpha$ is the void fraction

$\rho_4$ is the liquid density

$\rho_g$ is the gas density

Substituting for $\frac{\partial u_i}{\partial x}$ the expression 2.53 and taking into consideration the fact that $\frac{\partial m}{\partial x}$ according to Eq. 2.40 is given by

$$\frac{\partial m_i}{\partial x} = - \frac{\partial (\bar{\rho}_i A_i)}{\partial t} + \sum_{j=1}^{J} W_{ij} - W_{ix} - \int \rho u_i g \theta dl \quad (2.54)$$

equation 2.50 becomes

$$\frac{\partial \bar{m}_i}{\partial t} - 2 u_i A_i \frac{\partial \bar{\rho}_i}{\partial t} + \bar{m}_i \frac{\partial}{\partial x} \left( \frac{u'}{A_i} \right) + A_i \frac{\partial \bar{P}_i}{\partial x} +$$

$$+ (\bar{P}_i - \bar{P}_i) \frac{\partial A_i}{\partial x} + \sum_{j=1}^{J} \left( u_{ij}^* - 2 u_i \right) W_{ij} - (2 u_i - \hat{u}_i) \int \rho u_i g \theta dl$$

$$= \bar{P}_i A_i \cos \beta \quad (2.55)$$
The above equation has the drawback that it is dependent on complex quantities i.e.

\[ \hat{u}_i, \hat{p}_i, \oint \rho u + g \theta \, dl \quad \text{and} \quad u_{ij}^* \]

In order to make Eq. 2.55 useful for applications we have to approximate the above quantities by more familiar ones.

The accuracy of Eq. 2.55 will depend then on the accuracy of these approximations. We will next suggest approximate schemes for \( \hat{u}_i, \hat{p}_i, \oint \rho u + g \theta \, dl \) and \( u_{ij}^* \)

under the assumption that there is a distortion of \( M' \) rods and \( J' \) is the number of the gaps for which \( \theta \neq 0 \) (i.e. \( M-M' \) rods and \( J-J' \) gaps have \( \theta = 0 \))

2.2.1 The quantity \( \hat{u}_i \)

Going back to the definition of \( \hat{u}_i \) (Eq. 2.36) and taking into consideration that \( u = 0 \) at the wall we obtain

\[ \hat{u}_i = \frac{\sum_{j=1}^{J'} \int_{S_{ij}'} \rho u + g \theta \, dl}{\sum_{j=1}^{J'} \int_{S_{ij}'} \rho u + g \theta \, dl} \]  

(2.56)

Making the approximation that \( u \) along all gaps for which \( \theta \neq 0 \) can be represented by an average velocity \( u_i' \) such that

\[ \sum_{j=1}^{J'} \int_{S_{ij}'} \rho u + g \theta \, dl \approx u_i' \, \sum_{j=1}^{J'} \int_{S_{ij}'} \rho u + g \theta \, dl \]  

(2.57)
where
\[ u_i' = \frac{\sum_{j'=1}^{J'} s_{ij'} \bar{u}_{ij'}}{\sum_{j'=1}^{J'} s_{ij'}} \quad (2.58) \]

and \( \bar{u}_{ij'} \) is the mean axial velocity along the gap \( j' \). Then according to (2.57), Eq. 2.56 becomes
\[ \hat{u}_i \approx u_i' \equiv \frac{\sum_{j'=1}^{J'} s_{ij'} \bar{u}_{ij'}}{\sum_{j'=1}^{J'} s_{ij'}} \quad (2.59) \]

### 2.2.2 The quantity \( \hat{P}_i \)

From the definition 2.39 and the approximation 2.45
\[ \hat{P}_i \approx \frac{\sum_{j=1}^{J'} s_{ij} \bar{p}_{ij} + \sum_{m=1}^{M'} s_{im} \bar{p}_{im}}{\sum_{j=1}^{J'} s_{ij} + \sum_{m=1}^{M'} s_{im}} \quad (2.60) \]

Following the same method as in the case of \( \hat{u}_i \), we can define the quantity
\[ p_i' = \frac{\sum_{j=1}^{J'} s_{ij} \bar{p}_{ij} + \sum_{m=1}^{M'} s_{im} \bar{p}_{im}}{\sum_{j=1}^{J'} s_{ij} + \sum_{m=1}^{M'} s_{im}} \quad (2.61) \]

Similarly, we can approximate
\[ \hat{P}_i \approx p_i' = \frac{\sum_{j=1}^{J'} s_{ij} \bar{p}_{ij} + \sum_{m=1}^{M'} s_{im} \bar{p}_{im}}{\sum_{j=1}^{J'} s_{ij} + \sum_{m=1}^{M'} s_{im}} \quad (2.62) \]
2.2.3 The quantity $\phi \rho u + g \theta dl$

Working similarly as in the two previous cases the above integral reduces to

$$\phi \int_{s_i} \rho u + g \theta dl = \frac{1}{\sum_{i=j}^j} \int_{s_i} \rho u + g \theta dl$$  \hspace{1cm} (2.63)

This is well approximated by

$$\int_{s_{ij'}} \rho u + g \theta dl \approx \frac{1}{\sum_{i,j'}} \int_{s_{ij'}} \rho u + g \theta dl = \frac{1}{\sum_{i,j'}} \int_{s_{ij'}} \rho u + g \theta dl$$  \hspace{1cm} (2.64)

where

$$(tg \theta)_{ij'} = \frac{1}{s_{ij'}} \int_{s_{ij'}} g \theta dl$$  \hspace{1cm} (2.65)

Thus

$$\phi \int_{s_i} \rho u + g \theta dl \approx \frac{1}{\sum_{i,j}} \sum_{i,j'} \frac{1}{s_{ij'}} \int_{s_{ij'}} \rho u + g \theta dl$$  \hspace{1cm} (2.66)

2.2.4 The quantity $u_{ij}^*$

According to the definition of $u_{ij}^*$ (2.49) a good approximation will be

$$u_{ij}^* \approx \overline{u}_{ij}$$

This approximation is based on the fact that the distribution of the axial velocity along the gap is reasonably flat except for a region of small length at the walls relative to
the total gap length. As the \( P/D \) ratio decreases, this approximation becomes less valid.

Up to this point we have succeeded to approximate

\[ \hat{u}_i, \hat{p}_i, \hat{d}_i, \hat{g}_{ij} \text{ and } u_{ij}^* \]

with more realistic quantities which are functions of

\[ u_{ij}, p_{ij}, \text{ and } (t_{ij}^*)^{ij} \]

The quantity \((t_{ij}^*)^{ij}\) will be an input coming from the structural mechanics calculation. For the quantities \(p_{ij}, u_{ij}\) relations involving the unknowns \(\bar{p}_i, \bar{p}_j, u_i, u_j\) etc. will be necessary as shown in the next sections.

2.2.5 **Estimation of** \(\bar{p}_{ij}\)

\(\bar{p}_{ij}\) strictly speaking will be a function of \(p_i, p_j\) and to less extent of the geometry of the subchannels and \(W_{ij}\).

A first approximation will be

\[ p_{ij} \approx \frac{p_i + p_j}{2} \quad (2.68) \]

2.2.6 **Estimation of** \(\bar{u}_{ij}\)

For the estimation of \(\bar{u}_{ij}\) we will take Ruhani's suggestion (Ref. 7, Eq. 2.13 p.9)

\[ \bar{u}_{ij} = \frac{1}{2} \left( c_{ij} u_i + c_{ji} u_j \right) \quad (2.69) \]

where \(c_{ij}, c_{ji}\) will have some values around or less than unity.

Thus we have succeeded in developing forms of both the momentum equation (2.55) and the continuity Equation (2.40) having
only the quantities \( \tilde{P}_i, \bar{u}, W_{ij} \) as unknowns provided that the integrals involving the shear stresses are also functions of these quantities, i.e. the correlations used by COBRA IIIC (Ref. 8, APP. c)

2.2.7 Reduction to COBRA IIIC Code model

Take now the case for which

1. \( \hat{u}_i = 0 \) \hspace{1cm} (2.70a)
2. \( \sum_i \tilde{g}_{i} \bar{u} + \bar{g} \theta \text{dl} = 0 \) \hspace{1cm} (2.70b)

Making also the approximation

3. \( \bar{P}_i = \tilde{P}_i \) \hspace{1cm} (2.71)

Thus, the Eq. 2.55 reduces to

\[
\frac{\partial m_i}{\partial t} - 2u_i A_i \frac{\partial \bar{g}_i}{\partial t} + m_i \frac{\partial}{\partial x} \left( \frac{\nu'}{A_i} \right) + A_i \frac{\partial \bar{P}_i}{\partial x} +
\]

\[
+ \sum_{j=1}^{J} \left( u_{i}^{*} - 2u_i \right) W_{ij} - \sum_{j=1}^{J} \sum_{s=1}^{S} \bar{c}_{x n} \text{dl} +
\]

\[
+ \sum_{m=1}^{M} \sum_{s=1}^{S} \bar{c}_{x n} \text{dl} = \tilde{g}_{i} A_i \bar{g} \cos \beta \]  \hspace{1cm} (2.72)

Eq. 2.72 is essentially the same given in COBRA IIIC (Eq. I.3)

From (2.70a) and (2.70b) it is evident that COBRA IIIC is valid where we have no distortion of the rods or at least the terms included the distortion effect are negligible.
2.3 Improvements on Lumped Parameter Techniques

The essential simplification in lumped parameter techniques is that an approximation of 2.5I is valid, i.e.

$$\bar{u}_i = \frac{\mathcal{C}_i}{\mathcal{U}_i}$$  \hspace{1cm} (9.51)$$

where the quantity $\bar{u}_i$ is defined in Eq. 2.30 i.e.

$$\bar{u}_i = \frac{\int \int p u^2 dA}{\int \int p u dA}$$  \hspace{1cm} (9.31)$$

The quantity $\bar{u}_i$ appears only in the term $\frac{\partial (\bar{u}_i \bar{m}_i)}{\partial x}$ (i.e Eq. 2.44).

The importance of this term becomes less as the diversion of cross flow becomes smaller. The error in the approximation 2.5I is expected to be more severe in the case where we have large enthalpy gradients and two phase flow. In most cases the contribution of $\frac{\partial (\bar{u}_i \bar{m}_i)}{\partial x}$ is small compared with the other terms and therefore the error using approximation 2.5I does not affect severely the overall error in the axial momentum equation. However for the cases for which the above term is important or a higher accuracy is needed, one has to try to find more accurate approximations.

2.3.1. Approximations for the term $\bar{u}_i$

The quantity $\bar{u}_i$ reflects the radial variation of the axial velocity distribution in the momentum equation. The idea is that if it is possible to know beforehand the $\varphi$ and $u$ distributions we can calculate $\bar{u}$ as a function of $u$ or equivalently use a relation of the form:

$$\bar{u}_i = c_i u_i$$  \hspace{1cm} (9.73)$$
where the proportionality factor must be known.

Combining 2.31 and 2.73

\[ C_i = \frac{1}{u_i} \frac{\int_A \rho u^2 dA}{\int_A \rho u dA} \]  \hspace{1cm} (2.74)

Ci will be primarily a function of

1. Reynolds number
2. Geometry of the channel
3. Diversion cross flow
4. Position of the rod within the bundle
5. Steam quality (if two phase flow)
6. Distance from the entrance
7. The change in Ai(x) with x
8. Temperature distribution within the subchannel

If we knew accurately the function Ci in terms of these quantities and provided that we have the correct expression of the shear stress terms, the resulting solution of the axial momentum equation would be as accurate as the solution obtained by distributed parameter techniques.

Thus the contribution of each above parameters ought to be examined analytically or experimentally. It is evident however that some simplification can be made for special cases. For example take the case

a. One phase flow (or homogeneous two-phase flow)
b. An elevation far from the entrance
c. \[ \frac{\partial A_i}{\partial x} = 0 \]
It is evident in such a case that consideration of effects on C_i of parameters 5, 6, 7 can be omitted. Parameter 8 affects mainly the density within the subchannel. Because does not change dramatically within the subchannel for single phase flow and appears both in numerator and denominator in Eq. 2.74 we can approximate C_i as

\[ C_i = \frac{1}{A_i} \iint \left( \frac{u}{u_i} \right)^2 dA \]

which means that the effect of parameter 8 can be neglected as well.

Thus, for subchannel at a point x, far from the entrance in one phase flow (or homogeneous two phase flow) the factor C_i is primarily a function of

1. Reynolds number
2. Subchannel geometry
3. Diversion cross flow
4. Position of the rod within the bundle

Consider now further the case

a. The subchannel into consideration is far from the bundle boundaries.

b. The diversion cross flow is sufficiently small (i.e. bare rods and not severely high enthalpy gradients)

In this case we expect the parameters 3 and 4 become less important and we expect the value of C_i to be close to
that given by an infinite array and fully developed flow. This value we will try to estimate in the next paragraph. We expect this value to be mainly a function

I. Reynolds number

2. Subchannel Geometry

2.3.2 Infinite array

Experimental and analytical results of velocity distribution for triangular and rectangular arrays often have appeared in literature. We will pick the expressions that are given in Ref. 10 for illustrative calculations. The expressions and the method of calculation of Ci is given in APPENDIX B. The results are given in figures 2.2a, 2.2b, 2.3a, 2.3b. Figures 2.2 show Ci as a function of P/D ratio for various Reynolds numbers. Figures 2.2 show the dependence of Ci vs Re for various P/D ratios. The diagrams show that as Re and P/D is increased Ci goes to unity.

Also they show that for P/D > 1.30 Ci primarily depends on Reynolds number and slightly on the geometry. Only in small pitch ratios does the geometry also become important. The importance of the Reynolds number decreases for higher Reynolds number.

2.3.3 Diversion Cross-Flow Effects

When the diversion cross-flow becomes large enough a departure from the previously estimated value of Ci is expected. This difference is expected to be higher for higher diversion
Figure 2.2a Square array, C1 vs P/D.
Figure 2.2b. Triangular array. $C_l$ vs $P/D$. 

- $Re = 5 \times 10^3$
- $10^4$
- $5 \times 10^4$
- $10^5$
cross flow. It is worth to point out here that the estimation of $C_i$ becomes more important as we go for higher cross flows because the term $\frac{\partial (\bar{u}_i m_i)}{\partial x}$ in the axial momentum equation becomes more important under these conditions.

The dependence of versus $W_{ij}$ can be examined experimentally. In the experiments proposed in Chapter V for determining parameters entering in the transverse momentum equation the estimation of $C_i$ is taken into consideration.

2.4 Distributed Parameter Techniques

We select as a characteristic example for this class of problems the model embodied in the Code VELASCO developed by Eifler-Njörsing (Ref. II)

In this code the subchannel is defined as shown in Fig. 2.4, by the origin line $S_0$ (usually a portion of a gap length close to $F/2$), the line of zero shear stress $S_n^*$, and the line $S_\phi^*$ as a function of $\phi$ within the zone $\tau$ considered. Hence the correspondences in nomenclature between the general control surface and the VELASCO control surface are

$$S_i^* = S_\phi^* + S_n^*$$

$$S_i m = S_{rod}(\phi)$$

$$S_{ij} = S_0$$

The essential simplifications are

1. $\frac{\partial}{\partial t} = 0$  \hspace{1cm} (9.76)
Fig. 2.4 Control surface in VELASCO.
2. \[ \frac{\partial u}{\partial x} = \frac{\partial m}{\partial x} = \frac{\partial u}{\partial x} = \frac{\partial A_\phi}{\partial x} = 0 \quad (2.77) \]

3. \[ \frac{\partial P}{\partial x} = \frac{dP}{dx} \quad (\text{i.e. pressure independent of coordinates } r, \phi) \quad (2.78) \]

4. \[ \nu_n|_{s_n^*} = 0 \quad (2.79) \]

5. \[ \cos \beta = 0 \quad (2.80) \]

Assumption 2.43 is not necessary here and therefore we use Eq. 2.44 or 2.47 at the axial momentum equation which under the above simplification become

\[ A(\phi) \frac{dP}{dx} + \int_{S_0} g u v_n d\ell + \int_{S_\phi} g u v_n d\ell + \int_{S_\phi^*} \tau_{x_n} d\ell + \int_{S_0} \tau_{x_n} d\ell = 0 \quad (2.81) \]

where \( A(\phi) \) is the subchannel area. Eq. 2.81 for \( \phi = 2\pi \)

becomes

\[ \int_{S_{rod(\phi=2\pi)}} \tau_{x_n} d\ell = -A(\phi=2\pi) \frac{dP}{dx} \quad (2.82) \]

Call \( \tau_{x_n} = -\tau \quad (2.83a) \)

(The minus appears because \( \tau \cdot \tau = -1 \) )
and \( \tau_{R, av} = \frac{1}{2 \pi R} \int_{\text{Seqd}(\phi=2\pi)} \tau_R \, dl \) \hspace{1cm} (2.83b)

Then Eq. 2.32 becomes

\[
\frac{d\rho}{dx} = \frac{\eta \tau_{R, av}}{d_h} \tag{2.94}
\]

where \( d_h = \frac{4A(\phi=2\pi)}{2\pi R} \) \hspace{1cm} (2.85), the hydraulic diameter, for the whole channel \((\phi = 2\pi)\).

Substitute Eq. 2.84 into Eq. 2.81 and defining

\[
\tau_\phi \equiv \tau_{\phi x} + \phi \nu \nu_\phi \tag{2.86}
\]

\[
d_{h,1} = \frac{4A(\phi)}{S_{\text{rod}(\phi)}} \tag{2.87}
\]

take the direction going into the subchannel from as positive(Fig. 2.4), Eq. 2.81 becomes

\[
\int_{\text{Seqd}(\phi)} \tau_R \, dl = \int_{\text{Seqd}(\phi)} \tau_\phi \, dl - \int_{\phi} \tau_\phi \, dl + \frac{4A(\phi) \tau_{R, av}}{d_h} \tag{2.88}
\]

Call now

\[
T(\phi) \equiv \frac{\tau_R}{\tau_{R, av}} \tag{2.89a}
\]

\[
\chi(\phi) = \frac{S_{\text{rod}(\phi)}}{Pe}
\]

where Pe the length of the rod within the zone considered.
\[
Y(\phi) = \frac{y}{s^*_\phi}
\]  \hfill (2.89c)

(\(y\) represents a point on \(s^*_\phi\) the distance of which is \(y\) from the wall)

\[
Y_M(\phi) = \frac{s^*_\phi}{R}
\]  \hfill (2.89d)

\[
DHI(\phi) = \left( \frac{\gamma A(\phi)}{s \cdot \theta(\phi)} \right) \frac{1}{dh} \frac{1}{P_e}
\]  \hfill (2.89e)

\[
DU = \frac{P_e}{nR}
\]  \hfill (2.89f)

Then Eq. 2.88 divided by \(\tau_{Rav} P_e\) gives

\[
\int_0^X T \, dx = DHI + \frac{Y_M}{nDU} \left\{ \int_0^1 \frac{\tau}{\tau_{Rav}} \, dY - \left[ \int_0^1 \frac{\tau}{\tau_{Rav}} \, dY \right]_{X=0} \right\}
\]  \hfill (2.90)

which is the Eq. 7 of Ref. II, p. 3.
CHAPTER III

TRANSVERSE MOMENTUM EQUATION

3.1 General Equation

3.1.1 Overall Approach

To find a transverse momentum equation between two subchannels \( i \) and \( j \) we select again a surface \( A_{ij} \) (Fig. 3.1) which should be a surface including the gap with artificial boundaries. In particular the artificial boundaries would be selected so that the pressures at those boundaries has values equal to the mean values of the corresponding subchannels. The idea is again to integrate the transverse momentum equation over the control volume \( A_{ij} dx \). However, this integration can be easily done only for rectangular coordinates, because the directions \( y \) and \( z \) are fixed and the simplification (A-8) is valid (see Appendix A).

However, as the P/D ratio becomes smaller the \( V \)-field becomes more parallel to rod surface. Thus, for cylindrical rods the \( \phi \)-component of \( V \) dominates in the formation of pressure difference between the subchannels. The idea, therefore, adopted here is to use cylindrical coordinates and take the transverse momentum equation balance in the \( \phi \)-direction. However, the difficulty that arises is that the \( \phi \)-direction is not a fixed direction and the simplification (A-8) is not possible. We overcome this difficulty by using the moment of momentum equation in the \( \phi \)-direction. (Ref. 9, p. 125-126, Eqs. 5-39 and 5-40).
We will use a cylindrical rod and the reference axis will be the center line of the rod. This means that bowing of the rod is not permitted. We relax this restriction by extending the application to the cases that bowing is small enough so that we neglect the additional terms in moment-of-momentum equation due to change of the direction.

The \( \phi \)-component of moment-of-momentum equation is similar to Equation (2.22) i.e.

\[
\frac{\partial}{\partial t} (r \phi v_\phi) + \nabla \left( r \phi v_\phi W + r \tau_\phi \right) = M_g 
\]  

(3.1)

where \( M_g \) is the gravity term

\( v_\phi \) is the \( \phi \) component of the velocity \( W \)

\[
\tau_\phi = \tau_{\phi\phi} \xi_\phi + \tau_{\phi\gamma} \xi_\gamma + \tau_{\phi x} \xi_x 
\]  

(3.2)

Again

\[
\tau_\phi = \tau^{(L)}_\phi + \tau^{(T)}_\phi 
\]  

(3.3)

where \( \tau^{(L)}_\phi \), \( \tau^{(T)}_\phi \) are the laminar and turbulent stresses respectively.

In cylindrical coordinates these stresses are expressed (Ref. 1, p. 89)
Fig. 3.1 Possible control surface for the transverse momentum equation.

\[ F_{\phi \phi} = \rho v_{\phi}^2 + \tau_{\phi \phi} \]
\[ F_{\phi v} = \rho v_{\phi} v_{v} + \tau_{\phi v} \]
\[ F_{\phi x} = \rho v_{\phi} v_{x} + \tau_{\phi x} \]

Fig. 3.2 Infinitesimal control volume for the transverse moment-of-momentum equation.
\[ \tau_{\phi x}^{(l)} = -\mu \left( \frac{\partial v_\phi}{\partial x} + \frac{1}{r} \frac{\partial u}{\partial \phi} \right) \quad (3.4a) \]

\[ \tau_{\phi r}^{(l)} = -\mu \left( r \frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \phi} \right) \quad (3.4b) \]

\[ \tau_{\phi \psi}^{(l)} = p - \mu \left[ 2 \left( \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} \right) \right] - \frac{2}{3} \nabla \cdot \mathbf{w} \quad (3.4c) \]

\[ \tau_{\phi}^{(l)} = \frac{\rho v_\phi w'}{2} \quad (3.4d) \]

Essentially Eq. (3.1) comes from a torque balance of the momenta and stresses acting on the control volume of Fig. 3.2.

The gravity term \( M_g \) can be easily expressed, if we know a) the angle \( \theta \) between the gravity acceleration vector \( g \) and the \( x \)-axis and b) the angle \( \phi_r \) which is formed by the axis \( \phi = 0 \) and the projection of vector \( g \) in the plane perpendicular to \( x \)-axis at the particular position \( x \). (Fig. 3.3) since we know the angle \( \phi \) for any position \( r, \phi \).

It is evident from the Fig. 3.3 that
Fig. 3.3 Gravity vector in the radial plane.
\[ M_q = - \rho g \sin \beta \ r \sin (\phi - \phi_0) \]  \tag{3.5}

Thus, the Equation 3.1) becomes

\[ \frac{\partial}{\partial t} (r \rho \nu_\phi) + \nabla (r \rho \nu_\phi w + r \nu \phi) = \rho g \sin \beta \ r \sin (\phi_0 - \phi) \]  \tag{3.6}

It should be recalled that \[ \beta = \omega_t \times \mathbf{\Omega} \]

The problem now remains what should be selected for the transverse surface \( A_{ij} \).

3.1.1.1 Selection of the Transverse Surface

The surface \( A_{ij} \) can be selected as shown in Fig. 3.4 and 3.5. In the case of inner subchannels (see Fig. 3.4) we have two ways to select \( A_{ij} \), one adjacent to rod 1, called \( A_{ij1} \) and one adjacent to rod 2, called \( A_{ij2} \). The axial momentum balance will be examined within \( A_{ij1} \). Later we will examine the benefit of using \( A_{ij2} \) along with \( A_{ij1} \).

3.1.1.2 Selection of \( \phi_{i1} \)

It is evident that our desire is to select \( \phi_{i1} \) (Fig. 3.4 and 3.5) in such a way that the pressure (mean) along \( S_{i1} \) is equal or at least very close to the mean sub-channel pressure \( \overline{P_i} \), in order to have the ability to use
\[ A_{ij} = A_{ij1} + A_{ij2} \]
\[ A_{ij1} = A_i + A_j \]
\[ A_{ij2} = A_i + A_j \]

Fig. 3.4 Control surface for the transverse momentum equation. Inner subchannels.
\[ S_{W1} = S_{W1}^I + S_{W1}^{II} \]

Fig. 3.5 Control surface for the transverse momentum equation. Side subchannels.
Fig. 3.6 Control volume for the transverse momentum equation.
Fig. 3.7 Characteristic regions for the pressure shape model.

\[ A^* : \text{Transition region} \]
\[ A^{**} : \text{Asymptotic region} \]
the resulting transverse momentum equation together with the axial momentum equation. Recall that

\[
\bar{P}_i = \frac{l}{A_i} \int_{A_i} p \, dA
\]

(3.7)

It is evident that the local pressure \( p \) is not uniform within the subchannel. In the region near a gap the pressure distribution affected by the pressure of the subchannel adjacent to that gap. This means that \( p \) and hence \( \bar{P}_i \) according to Eq. (3.7) is a function of the surrounding subchannels. This also means that the angle defined this way is not a characteristic of the subchannel \( i \) but rather a function of the pressure of other subchannels as well.

In addition such a pressure \( \bar{P}_i \) which is affected by all surrounding subchannels cannot be the driving force of the diversion crossflow along a particular gap \( s_{ij} \).

The driving force of such a crossflow will be the pressure distribution along that gap rather than the pressure distribution within the whole subchannel. Therefore it seems better to define \( \phi_{ii} \) in a different way.

Thus, we accept the following model for the pressure profiles. We will divide the subchannel into the transition regions \( A^* \) and the asymptotic region \( A^{**} \) as shown in Fig. 3.7. Within \( A^{**} \) we will work with the mean pressure of this region. This mean pressure is a good representation
of the actual region pressure for a trullv flat pressure profile. We will call it "the asymptotic pressure" of the subchannel i, denoted by $p_{ias}$. Within the regions $A^*$ the pressure changes from $p_{ias}$ to the gap pressure $\bar{p}_{ij}$. The area $A^*$ can be different for the various gaps bounding a given subchannel and can change along x-axis.

The angle $\phi_i$ will be such that the mean pressure along the $S_i$ will be equal to the asymptotic pressure $p_{ias}$. The value of $\phi_i$ to satisfy the condition has to come from the analysis. Since the transverse momentum equation will be developed for a control volume bounded by $\phi_i$, the pressure containing terms will involve asymptotic pressure differences rather than the mean pressure differences which are used in the axial momentum equation.

A easy way to overcome this mismatch of pressures between momentum equations is to approximate $\bar{p}_i$ within the axial momentum equation as

$$\bar{p}_i \approx p_{ias} \quad (3.8)$$

In order to examine the validity of this approximation let us make, for this purpose only, the additional simplifications:

a. The transition regions have the same area $A^*$ for all the gaps, i.e., $A_i = A**(i) + JA^*$ \((3.9a)\)

b. The pressure within $A^*$ changes approximately linearly, i.e. $\bar{p}_{ij} \approx \frac{p_{ias} + p_{jas}}{2} \quad (3.9b)$


and

$$\int_{A^*} p d A \approx \frac{\bar{p}_{ij}}{2} + \frac{P_{ias}}{A^*} \hspace{1cm} (3.9c)$$

Then according to (3.9c) from (3.7) we have

$$\bar{P}_i = \frac{1}{A_i} \left[ P_{ias} A^* + \sum_{j=1}^{J} \frac{P_{ij} + P_{ijs}}{2} \right] \hspace{1cm} (3.10)$$

Substituting (3.9b) into (3.10) and defining

$$P_{i}^* = \frac{1}{J} \sum_{j=1}^{J} P_{ij} \hspace{1cm} (3.11)$$

and taking into consideration (3.9a) we end up with

$$\bar{P}_i = P_{ias} + \frac{J A^*}{4 A_i} \left( P_{i}^* - P_{i} \right) \hspace{1cm} (3.12)$$

only in cases for which
\[ p_{ias} \gg \frac{J A^*}{4 A_i} \left( p_i^* - p_{ias} \right) \quad (3.13a) \]

approximation (3.8) can be valid.

Restriction (3.13a) can be expressed in a slightly different way if we take into consideration how the pressure appears in the axial momentum equation. Thus, looking at Equation 2.50 we see that the pressure terms are:

\[ \left( p_i - p_i^* \right) \frac{\partial A_i}{\partial x} \quad \text{and} \quad A_i \frac{\partial p_i}{\partial x} \]

The most important term appears to be the second one i.e. \( A_i \frac{\partial p_i}{\partial x} \). Thus the restriction (3.13a) is changed to

\[ \frac{\partial p_{ias}}{\partial x} \gg \frac{J}{4} \frac{\partial}{\partial x} \left[ \frac{A^*}{A} \left( p_i^* - p_{ias} \right) \right] \quad (3.13b) \]

Even in cases that conditions (3.13a) and (3.13b) are not strictly met it is possible to obtain good results, if we take into consideration that in most of the problems the contribution of the transverse momentum is relatively small.
3.1.1.3 Selection of $S_n$

It is common to select the line $S_n$ such that along the line is zero or at least negligible. Because the main contribution to Equation (3.6) comes from the inertial terms, the selection of the location of $S_n$ is not too restrictive. The restriction comes only from the desire that the $V_\phi$-field be represented as correctly as possible. Thus $S_n$ could be also a line near the middle of the gap, such that $\tau_{xn}=0$ along it or the line perpendicular to the gap at its midpoint.

3.1.2 Development of the General Equation

Having selected the $A_{ij1}$, we integrate Eq. (3.6) over the volume formed by $A_{ij1}(x)$ and $A_{ij1}(x+dx)$ (Fig. 3.4 and 3.6).

Applying the divergence theorem we transform to surface integrals yielding

$$\frac{\partial}{\partial t} \iiint r \varphi v \, dV + \iint (r \varphi v \, w_n + r \tau \phi \, n) \, dS' =$$

$$= g \sin \beta \iiint \rho \, r \sin(\phi_0 - \phi) \, dV \tag{3.14}$$

Working similarly as with the axial momentum equation we find finally obtain
\[
\frac{\partial}{\partial t} \iint_{A_{ij}} \rho v_\phi \, dA + \frac{\partial}{\partial x} \left[ \iint_{A_{ij}} \rho v_\phi u \, dA \right] + \\
+ \frac{\partial}{\partial x} \left[ \iint_{A_{ij}} \rho \tau_{\phi x} \, dA \right] + \\
+ \oint_S \left[ \rho v_\phi v_n + \rho v_\phi u \frac{\partial y}{\partial x} + \rho \tau_n + \rho \tau_{\phi x} \frac{\partial y}{\partial x} \right] \, dl = \\
= g \sin \beta \iint_{A_{ij}} \rho \sin (\phi_0 - \phi) \, dA 
\] (3.14a)

where \( v_n \), \( \tau_n \) are the velocity and the stress components along the intersection line of \( x \)-plane and \( (\xi, \eta) \) plane. (See Fig. 3.6).

Let \( A_{ij} \) be the part of \( A_{ij} \) belonging to subchannel \( i \) and \( A_{j} \) the part belonging to subchannel \( j \).

It is evident that

\[ A_{ij} = A_i + A_j \] (3.15)
We introduce the following definitions:

\[
\psi_{i_1} = \iint_{A_{i_1}} r \rho v_\phi \, dA \quad (3.16a)
\]

\[
\psi_{j_1} = \iint_{A_{j_1}} r \rho v_\phi \, dA \quad (3.16b)
\]

\[
\psi_1 = \psi_{i_1} + \psi_{j_1} \quad (3.16c)
\]

\[
u_{i_1}^* = \frac{1}{\psi_{i_1}} \iint_{A_{i_1}} r \rho u v_\phi \, dA \quad (3.17a)
\]

\[
u_{j_1}^* = \frac{1}{\psi_{j_1}} \iint_{A_{j_1}} r \rho u v_\phi \, dA \quad (3.17b)
\]

\[
u_1^* = \frac{\left( \psi_{i_1} \nu_{i_1}^* + \psi_{j_1} \nu_{j_1}^* \right)}{\psi_1} \quad (3.17c)
\]

\[
\overline{r}_{i_1} = \frac{1}{A_{i_1}} \iint_{A_{i_1}} r \, dA \quad (3.18a)
\]

\[
\overline{r}_{j_1} = \frac{1}{A_{j_1}} \iint_{A_{j_1}} r \, dA \quad (3.18b)
\]

\[
\overline{r}_1 = \frac{\overline{r}_{i_1} A_{i_1} + \overline{r}_{j_1} A_{j_1}}{A_{i_1} + A_{j_1}} \quad (3.18c)
\]
\[ \bar{P}_{i1} = \frac{1}{A_{i1}} \iint_{A_{i1}} \rho \, dA \] (3.19a)

\[ \bar{P}_{j1} = \frac{1}{A_{j1}} \iint_{A_{j1}} \rho \, dA \] (3.19b)

\[ \bar{P}_1 = \frac{1}{A_{ij1}} \left( \bar{P}_{i1} A_{i1} + \bar{P}_{j1} A_{j1} \right) \] (3.19c)

\[ Cg_1 = \frac{1}{A_{ij1}} \iint_{A_{ij1}} r \sin (\phi_0 - \phi) \, dA \] (3.20)

Now we will develop equation 3.14a through the following assumptions which simplify the line integral along \( S \).

1) \[ \tau_{\phi \phi} \propto P \] (3.21a)

2) \[ v_n \bigg|_{S_n} \] is negligibly small \( (3.21b) \)

3) \[ t_g \theta \] along \( S_n \) is sufficiently small so that

\[ \int_{S_n} r (\tau_{\phi x} + \rho v_{\phi u}) t_g \phi \, dl \] is negligible. (3.21c)
4) The axial velocity gradient along $\phi_{i1}$ and $\phi_{j1}$ is sufficiently small so the integrals

\[
\int_{S_{i1}} r \tau_{\phi x} \theta y \, dl, \quad \int_{S_{j1}} r \tau_{\phi x} \theta y \, dl
\]

can be neglected. \hfill (3.21d)

As a result of these assumptions the line integral term of Equ. 3.14a along $S$ where per figure 3.4

\[
S = S_n + S_{i1} + S_{j1} + S_{w1}
\]

(3.22)

is reduced to the components shown in Table 3.1.

With the following definitions,

\[
\tau_w = \left( \tau_{\phi n} + \tau_{\phi x} \theta y \right) \bigg|_{S_{w1}} \tag{3.23}
\]

\[
\alpha_{i1} = \frac{(S_{i1} + R_{i1})^2 - R_{i1}^2}{\alpha} \tag{3.24a}
\]

\[
\alpha_{j1} = \frac{(S_{j1} + R_{j1})^2 - R_{j1}^2}{\alpha} \tag{3.24b}
\]

where $R_{i1}$ and $R_{j1}$ are lengths shown in Fig. 3.4, the terms of Table 3.1 are compacted and equation 3.14a becomes
Table 3.1 Evaluation of the line integral terms of Eq. 3.14a.

<table>
<thead>
<tr>
<th>Path of the line integral</th>
<th>$\int r \rho \varphi v_n dl$</th>
<th>$\int r \rho \varphi u g \theta dl$</th>
<th>$\int r \tau \phi n dl$</th>
<th>$\int r \tau \phi x t g \theta dl$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_m$</td>
<td>0 by assumption 2</td>
<td>0 by assumption 3</td>
<td>0, since $s_m$ selected such that $\tau_{\phi n}$ along $s_m \neq 0$</td>
<td>0 by assumption 3</td>
</tr>
<tr>
<td>$S_{ii}$</td>
<td>$\int r \rho \varphi^2 dl$</td>
<td>$(\tau \theta)_{ii} \int r \rho \varphi u d \theta$</td>
<td>$\int r \rho d \theta$ by $S_{ii}$ assumption 1</td>
<td>0 by assumption 4</td>
</tr>
<tr>
<td>$S_{w1}$</td>
<td>0 since $\varphi, v_n = 0$ dt $S_{w1}$</td>
<td>0 since $\varphi, v_n = 0$ dt $S_{w1}$</td>
<td>$\int r \tau_{\phi n} d \theta$ $S_{w1}$</td>
<td>$\int r \tau_{\phi x t} g \theta d \theta$ $S_{w1}$</td>
</tr>
<tr>
<td>$S_{ji}$</td>
<td>$\int r \rho \varphi^2 dl$</td>
<td>$(\tau \theta)_{ji} \int r \rho \varphi u d \theta$</td>
<td>$\int r \rho d \theta$ by $S_{ji}$ the assumption 1</td>
<td>0 by the assumption 4</td>
</tr>
</tbody>
</table>
\[
\frac{d\psi_i}{dt} + \frac{\partial(u_i^* \psi_i)}{\partial x} + \frac{\partial}{\partial x} \left[ \int_{A_{ij_l}} r \tau_\phi x \, dA \right] + \\
+ \int_{S_{i1}} r \rho v_\phi^2 \, dl - \int_{S_{j1}} r \rho v_\phi^2 \, dl + p_{ias} \alpha_{i1} - \\
- p_{jas} \alpha_{j1} + \int_{S_{w1}} r \tau_w \, dl + (tg\theta)_{i1} \int_{S_{i1}} r \rho v_\phi u d l + \\
+ (tg\theta)_{j1} \int_{S_{j1}} r \rho v_\phi u d l = c_{g1} \bar{g}_1 A_{ij_l} g \sin \beta \\
\] (3.25)

Another simplification can be made at this point by taking

\[
\alpha_{i1} = \alpha_{j1} = \alpha_1 \\
\] (3.24c)

In addition define

\[
\Delta P_{ll} = \frac{1}{\alpha_1} \left[ \int_{S_{i1}} r \rho v_\phi^2 \, dl - \int_{S_{j1}} r \rho v_\phi^2 \, dl + \\
+ \int_{S_{w1}} r \tau_w \, dl \right] \\
\] (3.25a)
\[ \Delta P_{al} = \frac{1}{\alpha_1} \frac{\partial}{\partial x} \left[ \sum_{A:ij1} r \tau_{\phi x} dA \right] \]  

(3.25b)

\[ \Delta P_{g} = -\frac{1}{\alpha_1} c_{g1} \bar{g}_1 A_{ij1} \bar{r}_i g \cos \beta \]  

(3.25c)

\[ \Delta P_{\theta} = \frac{1}{\alpha_1} \left[ (t \theta)_{i1} \int_{S_{i1}} r \rho v_{\phi} u d\ell + \right] \]  

\[ + (t \theta)_{j1} \int_{S_{j1}} r \rho v_{\phi} u d\ell \]  

(3.25d)
Then Equation (3.25) becomes

\[
\Delta p^1_{ji} = p_{jas} - p_{ias} = \frac{1}{\alpha_1} \left[ \frac{\partial \psi_1}{\partial t} + \frac{\partial (u_1^* \psi_1)}{\partial x} \right] \\
+ \Delta p^1_{\ell \ell} + \Delta p^1_{al} + \Delta p^1_g + \Delta p^1_\theta
\]  

(3.26)

The Equation (3.26) shows that the pressure difference between two adjacent subchannels depends on a number of factors that quantitatively are expressed by the terms appearing on the right side of the equation.

The physical meaning of each term is as follows.

1) \( \frac{1}{\alpha_1} \frac{\partial \psi_1}{\partial t} \) It reflects the change of the cross-flow with respect to time.

2) \( \frac{1}{\alpha_1} \frac{\partial (u_1^* \psi_1)}{\partial x} \) This term reflects both the cross-flow and axial flow changes along the x-axis.

3) \( \Delta p^1_{\ell \ell} \) It is defined by (3.25a) and it expresses the lateral friction and acceleration pressure losses.

4) \( \Delta p^1_{al} \) It is defined by (3.25b) and reflects the axial change in the lateral shear stress with
the axial flow. We call it "the axial friction losses".

5) $\Delta p_g^\prime$ It is defined by (3.25c) and expresses the pressure losses due to the presence of the gravity field.

6) $\Delta p_\theta^\prime$ It is defined by (3.25d) and reflects the axial flow "gain or loss" through the edges of $A_{ijkl}$.

Equation (3.26) interrelates the $u$ and $v_\phi$ distribution with the pressure difference. However in order to use the above equation with an axial momentum equation of the type (2.50) or (2.55) we have to express (3.26) as a relation between the pressure difference, axial velocities and diversion cross flows.

Therefore the next step will be an attempt to relate the $v_\phi$ field with the diversion cross flow.

3.1.2.1 Development of the term $\psi_1$ (in Eq. 3.26)

We distinguish two main components of the $v_\phi$ vector at a particular point $(r,\phi)$ which we assume are developed
independently of each other.

1) the component $V_{\phi D}$ due to the diversion cross flows.

2) the component $V_{\phi -sec}$ that is formed by the asymmetry of the subchannel shape. The net cross flow across any line $s_\phi$ due to this component is zero. This is the $V_\phi$ due to secondary flow.

Thus,

$$V_\phi (r, \phi) = V_{\phi D} (r, \phi) + V_{\phi -sec} (r, \phi) \quad (3.27)$$

Suppose that $I$ and $J$ are the number of subchannels surrounding the subchannels $i$ and $j$ respectively. We will assume also here, that the effect of each adjacent subchannel to the velocity $V_\phi$ is independent of the effects of the other subchannels, i.e.

$$V_{\phi D} (r, \phi) = \sum_{m=1}^{I} V_{\phi D m} \quad (3.28)$$

where $V_{\phi D m}$ is the component of $V_{\phi D}$ within $A_{im}$ due to the diversion cross flow $W_{im}$ and $m$ is a subchannel adjacent to subchannel $i$.

Since $V_{\phi D m}$ becomes larger as $W_{im}$ becomes larger, we express $V_{\phi D m}$ as follows.

$$\beta_{im} (r, \phi) = \beta_{im} (r, \phi) \frac{W_{mi}}{S_{mi}} \quad (3.29)$$

$\beta_{im} (r, \phi)$ is a dimensionless number, positive or negative, which will depend, generally, on the geometry of the sub-
channel, on the condition of flow and the fluid properties and strictly speaking on \(W_m\).

Thus,

\[
\rho V_{\phi}^{i_1} (r, \phi) = \sum_{m=1}^{I} p_{m}^{i_1} (r, \phi) \frac{W_m}{S_m} + \rho V_{\phi}^{i_1} (r, \phi)
\]

(3.30a)

Similarly

\[
\rho V_{\phi}^{j'_{1}} (r, \phi) = \sum_{m'=1}^{I} p_{m'}^{j'_{1}} (r, \phi) \frac{W_{m'}}{S_{m'}} + \rho V_{\phi}^{j'_{1}} (r, \phi)
\]

(3.30b)

Then from Equation (3.16a)

\[
\psi_{i_1} = \sum_{m=1}^{I} \frac{W_m}{S_m} \int_{A_{i_1}} r p_{m}^{i_1} (r, \phi) dA + \int_{A_{i_1}} \rho V_{\phi}^{i_1} (r, \phi) dA
\]

(3.31)

Define

\[
\overline{p}_{m}^{i_1} = \frac{1}{A_{i_1}} \int_{0}^{R_{\phi} + s_{\phi}} d\phi \int_{R_{\phi}}^{r_o} r^2 p_{m}^{i_1} (r, \phi) dr
\]

(3.32a)

where

\[
r dr d\phi = dA
\]

(3.33)

and \(R_{\phi}\) and \(s_{\phi}\) are lengths shown in Figures (3.4) and (3.5).
Similarly

\[
\overline{\beta}_{m'}^{j'} = \frac{1}{\overline{r}_{j'}^{i_1}} A_{i_1} \int_{0}^{\phi_{j'}^{i_1}} d\phi \int_{q_{j'}^{i_1}}^{R_{\phi} + S_{\phi}} r^2 \overline{\beta}_{m'}^{j'} (r, \phi) dr d\phi \quad (6.32b)
\]

Then

\[
\psi_{i_1} = \overline{r}_{i_1} A_{i_1} \left[ \sum_{m=1}^{I} \overline{\beta}_{m}^{i_1} \frac{W_{m'}^{m}}{S_{m'}^{m}} \right] + \\
+ \iint_{A_{i_1}} r \phi v_{\phi - sec} (r, \phi) dA \quad (3.34a)
\]

Similarly

\[
\psi_{j_1} = \overline{r}_{j_1} A_{j_1} \left[ \sum_{m'=1}^{I'} \overline{\beta}_{m'}^{j_1} \frac{W_{m'}^{m}}{S_{j_1}^{m'}} \right] + \\
+ \iint_{A_{j_1}} r \phi v_{\phi - sec} (r, \phi) dA \quad (3.34b)
\]

Expression (3.34b) is slightly different from (3.34a) because we have taken into consideration the fact that

\[W_{m'}^{m} = -W_{m'}^{m},\]

and the integral with respect to \( \phi \) for \( \psi_{j_1} \)

is from \( \phi_{j_1}^{i_1} \) to 0 instead of 0 to \( \phi_{j_1}^{i_1} \) as it is in the definition of \( \overline{\beta}_{m'}^{j_1} \).

There we have to put \( -\overline{\beta}_{m'}^{j_1} \) instead of \( \overline{\beta}_{m'}^{j_1} \).

The two negative signs cancel and we obtain the expression (3.34b).
Then $\psi_i$ becomes according to its definition (3.16c)

$$
\psi_i = \psi_{i1} + \psi_{i2} =
$$

$$
= \bar{r}_{i1} A_{i1} \left[ \sum_{m=1}^{I} \bar{p}^{i1} \frac{w_{mi}}{s_{mi}} \right] +
$$

$$
+ \bar{r}_{i1} A_{i1} \left[ \sum_{m' = 1}^{J} \bar{p}^{i1} \frac{w_{m'm'}}{s_{m'}} \right] +
$$

$$
+ \iint r F \nu_{\phi-sec}(\nu, \phi) \, dA
$$

(3.34c)

$\bar{p}^{i1}$, $\bar{p}^{i1}$ are also dimensionless numbers. They are expected to be dependent on selection of $A_{i1}$ and $A_{i1}$ and primarily a function of the geometry of the subchannels. Also they will be a function of the conditions of the flow and the fluid properties.

Looking for less complicated forms of $\psi_{i1}$ (and $\psi_{j1}$) than the form (3.34a), we can notice that the main contribution to $V_{\phi}$ with $m A_{ij1}$ comes from the diversion cross flow $W_{ij1}$. Therefore, as a first approximation we can neglect the terms in (3.34a) and (3.34b) with diversion cross flow other than $W_{ij1}$. Under these circumstances, we have
\[ \psi_{i1} \sim \bar{r}_{i1} A_{i1} \bar{\beta}_{i1} \frac{W_{ji}}{S_{ji}} + \iint_{A_{i1}} r \rho v_{\phi - sec}(r, \phi) dA \] (3.35a)

and

\[ \psi_{j1} \sim \bar{r}_{j1} A_{j1} \bar{\beta}_{j1} \frac{W_{ji}}{S_{ji}} + \iint_{A_{j1}} r \rho v_{\phi - sec}(r, \phi) dA \] (3.35b)

hence

\[ \psi_{i} \sim \bar{r}_{i} A_{i} \bar{\beta}_{i} \frac{W_{ji}}{S_{ji}} + \bar{r}_{j} A_{j} \bar{\beta}_{j} \frac{W_{ji}}{S_{ji}} + \iint_{A_{i1}} r \rho v_{\phi - sec}(r, \phi) dA \] (3.35c)

In this case we can define a quantity \[ \bar{\beta}_{i} \] as follows

\[ \bar{\beta}_{i} = \frac{\bar{r}_{j} A_{j} \bar{\beta}_{j} + \bar{r}_{i} A_{i} \bar{\beta}_{i}}{\bar{r}_{1} A_{i} \bar{r}_{j}} \] (3.36)
where

$$\bar{r}_i = (\bar{r}_{i_1} A_{i_1} + \bar{r}_{j_1} A_{j_1}) / A_{i_1} \quad (3.37)$$

Then from (3.35c)

$$\psi = \bar{r}_1 A_{i_1} B_i \frac{W_i}{s_{i_1}} + \iint \rho \varphi \sigma_{\text{sec}}(\gamma, \phi) \, dA$$

$$(3.35d)$$

Of course difficulty remains in the estimation of

$$\iint \rho \varphi \sigma_{\text{sec}} \, dA$$

$$_{A_{i_1}}$$

However we do not expect that

$$\iint \rho \varphi \sigma_{\text{sec}}(\gamma, \phi) \, dA$$

$$_{A_{i_1}}$$

has a significant effect of the value of $$\psi_1$$ because

$$\int_{S_\phi} p \nu_{\text{sec}} \, dl = 0$$
and therefore the secondary momentum

\[ \sum_{A_{ij}} r \phi v_{\phi-sec} (\gamma, \phi) \, dA \]

is expected to be a small number.

Therefore good approximations of (3.35a), (3.35b) and (3.35d) will be

\[ \psi_{i1} \approx \bar{r}_{i1} \, A_{i1} \, \bar{\beta}_{j1} \, \frac{W_{ji}}{S_{ji}} \]  \hspace{1cm} (3.38a) \]

\[ \psi_{j1} \approx \bar{r}_{j1} \, A_{j1} \, \bar{\beta}_{i1} \, \frac{W_{ji}}{S_{ji}} \]  \hspace{1cm} (3.38b) \]

\[ \psi_i \approx \bar{r}_i \, A_{ij1} \, \bar{\beta}_i \, \frac{W_{ji}}{S_{ji}} \]  \hspace{1cm} (3.38d) \]

However, an approximate estimation of the secondary momentum is straightforward as we can see in the next paragraph.

3.1.2.1.1 Development of the term \[ \sum_{A_{ij}} r \phi v_{\phi-sec} (\gamma, \phi) \, dA \]

Let us accept the Eifler-Nijsing approach (4) which includes secondary flow in one-phase fully developed flow. The secondary velocity "intensity" is proportional to \[ \frac{d}{d\phi} \sqrt{E_2} \]
To be more specific, Eifler-Nijsing's model is expressed

\[
V_\phi = 2 C_{sec} \phi_e \frac{d}{d\phi} \left( \sqrt{\frac{\tau_R}{\phi}} \right) f(r)
\]  
(3.39)

where \( \tau_R \) is the wall shear stress which is a function of \( \phi \). \( \phi_e \) is the angle between two wall shear stress extrema, i.e., \( \phi_e \) will be close to \( \pi/4 \) for square array and \( \pi/6 \) for triangular array.

\( C_{sec} \) is a proportionality constant. In VELASCO Code (11) p. 4, \( C_{sec} \) has the value 0.573.

\( f(r) \) reflects the assumption of the velocity shape which is approximated by a cosine shape (Ref. 11 p. 4).

At this point we will make a step further and we will assume that for the cases for which \( W_{j1} \neq 0 \), far from the inlet, a similar equation is applied where now \( C_{sec}, \phi_e, f(r) \) may be different. In absence of experimental verification and for one phase flow we take the same values for \( C_{sec}, \phi_e \) and \( f(r) \) as proposed by Eifler using for the case \( W_{j1} = 0 \).

Using the relation from a circular tube

\[
\sqrt{\frac{\tau_R}{\phi}} \sim \sqrt{\frac{f}{\phi}} \bar{u}(\phi)
\]  
(3.40)

where \( \bar{u}(\phi) \) is the mean axial velocity along a \( S_\phi \)-line (see Fig. 3.4), and \( f \) is the friction factor. Then
\[
\sum_{A_i} \int_{\Pi_i} \int_0^{R_{\phi_{i_i}}} \phi_{i_i} \left( \rho - \frac{d\bar{u}(\phi)}{d\phi} \right) \rho f(r) dr
d\phi = 2 \sec \phi_{i_i} \rho \int_{\phi_{i_i}}^{\phi_{i_{i+1}}} \frac{d\bar{u}(\phi)}{d\phi} \rho f(r) dr
d\phi
\]

(3.41a)

\[
\sum_{A_j} \int_{\Pi_j} \int_0^{R_{\phi_{j_j}}} \phi_{j_j} \left( \rho - \frac{d\bar{u}(\phi)}{d\phi} \right) \rho f(r) dr
d\phi = 2 \sec \phi_{j_j} \rho \int_{\phi_{j_j}}^{\phi_{j_{j+1}}} \frac{d\bar{u}(\phi)}{d\phi} \rho f(r) dr
d\phi
\]

(3.41b)

where \( \Pi_i \) and \( \Pi_j \) mean densities defined by (3.19).

Making the approximation

\[
\left. \frac{d\bar{u}(\phi)}{d\phi} \right|_{A_{i_{i_i}}} \approx \frac{\bar{u}(\phi_{i_{i+1}}) - \bar{u}_{i_{i_1}}}{\phi_{i_{i+1}} - \phi_{i_{i_1}}}
\]

(3.42a)

\[
\left. \frac{d\bar{u}(\phi)}{d\phi} \right|_{A_{j_{j_j}}} \approx \frac{\bar{u}(\phi_{j_{j+1}}) - \bar{u}_{i_{j_1}}}{\phi_{j_{j+1}} - \phi_{j_{j_1}}}
\]

(3.42b)
and defining the constants

\[
\eta_{i1} = \frac{1}{V_{i1}} A_{i1} \int_0^{R_{i1}} \frac{r^2 f(r)}{R_{i1}} \, dr
\]  \hspace{1cm} (3.43a)

\[
\eta_{j1} = \frac{1}{V_{j1}} A_{j1} \int_0^{R_{j1}} \frac{r^2 f(r)}{R_{j1}} \, dr
\]  \hspace{1cm} (3.43b)

Thus Equation (3.41a) and (3.41b) become respectively

\[
\iint_{A_{i1}} \rho \nu \nu \phi \, dA \approx \iint_{A_{i1}} \rho \nu \nu \phi \, dA 
\]

\[
\approx 2 \cos \frac{\phi_i}{\phi_{i1}} \sqrt{\frac{f_i}{8}} \left[ \bar{u}(\phi_{i1}) - \bar{u}_{i1} \right] \]  \hspace{1cm} (3.44a)

\[
\iint_{A_{j1}} \rho \nu \nu \phi \, dA \approx \iint_{A_{j1}} \rho \nu \nu \phi \, dA 
\]

\[
\approx -2 \cos \frac{\phi_j}{\phi_{j1}} \sqrt{\frac{f_j}{8}} \left[ \bar{u}(\phi_{j1}) - \bar{u}_{j1} \right] + 2 \cos \frac{\phi_j}{\phi_{j1}} \sqrt{\frac{f_j}{8}} \left[ \bar{u}(\phi_{j1}) - \bar{u}_{j1} \right] \]  \hspace{1cm} (3.44b)

\[
\iint_{A_{j1}} \rho \nu \nu \phi \, dA \approx 2 \cos \frac{\phi_j}{\phi_{j1}} \sqrt{\frac{f_j}{8}} \left[ \bar{u}(\phi_{j1}) - \bar{u}_{j1} \right] 
\]

\[
-2 \cos \frac{\phi_j}{\phi_{j1}} \sqrt{\frac{f_j}{8}} \left[ \bar{u}(\phi_{j1}) - \bar{u}_{j1} \right] \]  \hspace{1cm} (3.44c)
For the evaluation of $\bar{u}(\phi_{i})$ as a function of the mean velocity $u_{i}$ of the subchannel we make the reasonable approximation

$$\bar{u}(\phi_{i}) = \epsilon_{i} u_{i}$$  

(3.45a)

Similarly

$$\bar{u}(\phi_{j}) = \epsilon_{j} u_{j}$$  

(3.45b)

where $\epsilon_{i}, \epsilon_{j}$ will be mainly a function of $\phi_{i1}$ and the geometry of the subchannel.

$\epsilon_{i1}$ is a number close to unity. In the first approximation we can take $\epsilon_{i1} \approx 1$  

(3.46)

In order to see how (3.44c) affects the transverse momentum equation, let us take the case

$$\bar{u}(\phi_{i}) = \bar{u}(\phi_{j}) , \quad \bar{p}_{i1} = \bar{p}_{j1} , \quad \bar{v}_{i1} A_{i1} = \bar{v}_{j1} A_{j1}$$

$$\frac{\phi_{e}^{i}}{\phi_{i1}} = \frac{\phi_{e}^{j}}{\phi_{j1}} \quad \text{and} \quad f_{i} = f_{j}$$

In this case

$$\int_{A_{i1}} g_{r} \mathbf{V}_{\phi-sec} \; dA = 0$$

which means that according to this model, this area integral becomes more important as the difference axial velocities and geometry of the subchannels increase and the enthalpy gradient between subchannels increases, (i.e., inner-side channels).
3.1.2.2 Development of the term $u^*_i$ (in Eq. 3.17c)

For the evaluation of $u^*_i$, we have to look at the definition of $u^*_i$ (Eq. 3.17a)

$$u^*_i = \frac{\iint_{A_{i1}} r p v \phi u dA}{\iint_{A_{i1}} r p v \phi dA} \quad (3.17a)$$

It is expected that $u$ will not change significantly within $A_{i1}$ and therefore we make the approximation

$$\iint_{A_{i1}} r p v \phi u dA \approx \left(\frac{1}{A_{i1}} \iint_{A_{i1}} u dA\right) \iint_{A_{i1}} r p v \phi dA \quad (3.47)$$

which gives

$$u^*_{i1} \approx \frac{1}{A_{i1}} \iint_{A_{i1}} u dA \quad (3.48a)$$

Similarly

$$u^*_{j1} \approx \frac{1}{A_{j1}} \iint_{A_{j1}} u dA \quad (6.48b)$$
In order to express $u^*_{i1}$ as a function of the familiar quantities $u_i$ and $u_j$, we expect that $u^*_{i1}$ given by (3.48a) can be well represented by the approximation

$$u^*_{i1} \approx \left( \overline{u}(\phi_{i1}) + \overline{u}_{i1} \right)/2 \quad (3.49a)$$

Similarly

$$u^*_{j1} \approx \left( \overline{u}(\phi_{j1}) + \overline{u}_{ij} \right)/2 \quad (3.49b)$$

where $\overline{u}(\phi_{i1}), \overline{u}(\phi_{j1})$ are given by (3.45) and $\overline{u}_{ij}$ by (2.69).

The quantity $u^*_1$ will be given by substituting expression (3.49) into (3.17c). We obtain

$$u^*_1 \approx \frac{1}{\psi_1} \left[ \psi_{i1} \frac{\overline{u}(\phi_{i1}) + \overline{u}_{i1}}{2} + \psi_{j1} \frac{\overline{u}(\phi_{j1}) + \overline{u}_{ij}}{2} \right] \quad (3.49c)$$

If $\psi_{i1}$ and $\psi_{j1}$ are not very different using (3.16c) we can further approximate $u^*_1$ as

$$u^*_1 \approx \frac{u^*_{i1} + u^*_{j1}}{2} = \frac{1}{2} \left( \overline{u}_{i1} + \frac{\overline{u}(\phi_{i1}) + \overline{u}(\phi_{j1})}{2} \right) \quad (3.49d)$$

In the cases that we have relatively high axial velocity gradients (e.g. blockages) and the approximation (3.47) is not good, better approximations for $u^*_{i1}$ can be
made.

One improved approximation is to express \( \bar{u}(\phi) \) as a Taylor series, i.e.

\[
\bar{u}(\phi) = \bar{u}_{ij} + \left( \frac{\partial \bar{u}(\phi)}{\partial \phi} \right)_{\phi=0} \phi + \frac{1}{2} \left( \frac{\partial^2 \bar{u}(\phi)}{\partial \phi^2} \right)_{\phi=0} \phi^2 + \ldots
\]

(3.50)

Making the approximation that

\[
\left. \frac{\partial \bar{u}(\phi)}{\partial \phi} \right|_{A_{ii}} \approx \frac{\bar{u}(\phi_{ii}) - \bar{u}_{ij}}{\phi_{ii}}
\]

(3.51a)

and taking only the two first terms in (3.50) we obtain

\[
\bar{u}(\phi) \approx \bar{u}_{ij} + \frac{\bar{u}(\phi_{ii}) - \bar{u}_{ij}}{\phi_{ii}} \phi
\]

(3.51b)

Putting Equation (3.51b) into the expression (3.17a) we obtain

\[
(\bar{u}_{i1}^* = \bar{u}_{ij} + \frac{\bar{u}(\phi_{ii}) - \bar{u}_{ij}}{\phi_{ii}}) \cdot \frac{\iint_{A_{ii}} \nu \phi \nu \phi \, dA}{\iint_{A_{ii}} \nu \phi \nu \phi \, dA}
\]

(3.52a)
Similarly
\[ u_{ij}^* = \overline{u}_{ij} + \frac{u(\phi_{ij}) - \overline{u}_{ij}}{\phi_{ij}} \frac{\oint_A r \rho v_\phi \phi \, dA}{\oint_A r \rho v_\phi \, dA} \]  \hspace{1cm} (3.526)

will be given again by (3.17c).
Of course better approximations can be made where are needed.

3.1.2.3. Development of the term \( \Delta P_{al}^i \) (Eq. 3.25b)

The term \( \Delta P_{al}^i \) is given by (3.25b), i.e.
\[ \Delta P_{al}^i = \frac{1}{\alpha_1} \frac{\partial}{\partial x} \left[ \oint_{A_{ij}} r \tau_{\phi x} \, dA \right] \hspace{1cm} (3.25b) \]

where \( \tau_{\phi x} \) is
\[ \tau_{\phi x} = -\mu \left( \frac{\partial v_\phi}{\partial x} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} \right) + \rho \nu' \overline{u}' \hspace{1cm} (6.53) \]

Adopting the concept of eddy diffusivity we can approximate
\[ \tau_{\phi x} \cong - \rho \left[ (\gamma + \epsilon_m) \frac{\partial \nu}{\partial x} + (\gamma + \epsilon_m) \frac{\partial u}{\gamma \partial \phi} \right] \] 

(3.54)

where \( \epsilon_m \) are the eddy diffusivities.

In the cases that the gradient of \( \nu \phi \) with \( x \) is small compared with the gradient of \( u \) with \( \phi \) we can approximate \( \tau_{\phi x} \) as:

\[ \tau_{\phi x} \cong - \rho (\gamma + \epsilon_m) \frac{\partial u}{\gamma \partial \phi} \] 

(3.55)

and

\[ \iint_{A_{ii}} \gamma \tau_{\phi x} \, dA = - \iint_{A_{ii}} \rho (\gamma + \epsilon_m) \frac{\partial u}{\gamma \partial \phi} \, dA \] 

(3.56)

Defining a proportionality factor \( \eta_{T}^{ii} \) such that

\[ \iint_{A_{ii}} \rho (\gamma + \epsilon_m) \frac{\partial u}{\gamma \partial \phi} \cong \eta_{T}^{ii} \frac{\bar{u}(\phi_{i}) - \bar{u}_{ij}}{\phi_{i}} \]

(3.57)

\( \eta_{T}^{ii} \) will be a number close to unity, depending mainly upon the \( \bar{u} \) -distribution, and the angle \( \phi_{i1} \).
As a first approximation we can take $\eta^{ii} \approx 1$ (3.58)

Working similarly with the area $A_{ij}$ we end up with

$$\Delta P_{al} \approx - \frac{1}{\alpha_1} \frac{\partial}{\partial x} \left\{ \eta^{ii} \left[ \int \int_{A_{ii}} \varphi \left( \nu + \varepsilon m_\phi \right) dA \right] \frac{1}{\phi_{ii}} \left[ \bar{u}(\phi_{ii}) - \bar{u}_{ij} \right] - \eta^{jj} \left[ \int \int_{A_{jj}} \varphi \left( \nu + \varepsilon m_\phi \right) dA \right] \frac{1}{\phi_{jj}} \left( \bar{u}(\phi_{jj}) - \bar{u}_{ij} \right) \right\}$$

(3.59)

Defining

$$\left[ \frac{\varphi \left( \nu + \varepsilon m_\phi \right)}{A_{ii}} \right] = \frac{1}{A_{ii}} \int \int_{A_{ii}} \varphi \left( \nu + \varepsilon m_\phi \right) dA$$ (3.60a)

$$\left[ \frac{\varphi \left( \nu + \varepsilon m_\phi \right)}{A_{jj}} \right] = \frac{1}{A_{jj}} \int \int_{A_{jj}} \varphi \left( \nu + \varepsilon m_\phi \right) dA$$ (3.60b)

equation (3.59) becomes

$$\Delta P_{al} \approx - \frac{1}{\alpha_1} \frac{\partial}{\partial x} \left\{ \eta^{ii} \frac{A_{ii}}{\phi_{ii}} \left[ \frac{\varphi \left( \nu + \varepsilon m_\phi \right)}{A_{ii}} \right] \left[ \bar{u}(\phi_{ii}) - \bar{u}_{ij} \right] - \eta^{jj} \frac{A_{jj}}{\phi_{jj}} \left[ \frac{\varphi \left( \nu + \varepsilon m_\phi \right)}{A_{jj}} \right] \left[ \bar{u}(\phi_{jj}) - \bar{u}_{ij} \right] \right\}$$

(3.61)
For \[ \bar{\varepsilon} (\bar{v} + \varepsilon_m \phi) \] one can approximate
\[ \bar{\varepsilon} (\bar{v} + \varepsilon_m \phi) \approx \bar{\varepsilon} (\bar{v} + \varepsilon_m \phi) \] (3.62)

For \( \varepsilon_m \phi \) one can substitute, as a first approximation, for one phase incompressible flow the correlations that are given for fully developed flow (see Ref. 4, p. 2-33).

\[ (\varepsilon_m \phi)_{ii} = \frac{1}{A_{ii}} \iint_{A_{ii}} C S_{\phi} \sqrt{\tau_R} \, dA \] (3.63)

where \( C \) is a constant,
\( S_{\phi} \) is shown in Fig. 3.4
\( \tau_R \) is the axial shear stress

The constant \( C \) in the code HERA-1A (Ref. 13, p. 8) which has the value 0.154.

Using the expression for a circular tube
\[ \sqrt{\frac{\tau_R}{\delta}} \approx \sqrt{\frac{f}{\delta}} \bar{u}(\phi) \] (3.40)

We take
\[ \iint_{A_{ii}} S_{\phi} \sqrt{\tau_R} \, dA \approx \sqrt{\frac{f_i}{\delta}} \iint_{A_{ii}} \bar{u}(\phi) \, S_{\phi} \, dA \] (3.64)
Thus from (3.63)

\[
\left( \overline{\varepsilon_{m\phi}} \right)_{ii} \approx \frac{1}{A_{ii}} \epsilon \sqrt{\frac{f_{ii}}{8}} \int_{A_{ii}} \overline{u}(\phi) s_{\phi} dA \quad (3.65a)
\]

Similarly

\[
\left( \overline{\varepsilon_{m\phi}} \right)_{jj} \approx \frac{1}{A_{jj}} \epsilon \sqrt{\frac{f_{jj}}{8}} \int_{A_{jj}} \overline{u}(\phi) s_{\phi} dA \quad (3.65b)
\]

A further approximation with respect to the integrals in the right side of the equations (3.65a) and (3.65b) can be made by taking

\[
\int_{A_{ii}} u s_{\phi} dA \approx \left( \frac{1}{\phi_{i1}^{\prime}} \int_{0}^{\phi_{i1}} \overline{u}(\phi) d\phi \right) \int_{A_{ii}} s_{\phi} dA \quad (3.66a)
\]

The above approximation is based on the fact that \( \overline{u}(\phi) \) is not expected to change dramatically within \( A_{ii} \). Similarly

\[
\int_{A_{jj}} u s_{\phi} dA \approx \left( \frac{1}{\phi_{j1}^{\prime}} \int_{0}^{\phi_{j1}} \overline{u}(\phi) d\phi \right) \int_{A_{jj}} s_{\phi} dA \quad (3.66b)
\]
Defining

\[
\overline{S_{i1}} = \frac{1}{A_{i1}} \int_{A_{i1}} S_{\phi} \, dA \quad (3.67a)
\]

\[
\overline{S_{j1}} = \frac{1}{A_{j1}} \int_{A_{j1}} S_{\phi} \, dA \quad (3.67b)
\]

and making the approximations

\[
\frac{1}{\phi_{i1}} \int_{0}^{\phi_{i1}} \overline{u}(\phi) \, d\phi \approx \frac{\overline{u}(\phi_{i1}) + \overline{u}_{ij}}{2} \quad (3.67c)
\]

\[
\frac{1}{\phi_{j1}} \int_{0}^{\phi_{j1}} \overline{u}(\phi) \, d\phi \approx \frac{\overline{u}(\phi_{j1}) + \overline{u}_{ij}}{2} \quad (3.67d)
\]

Equations (3.65a) and (3.65b) become

\[
(\overline{\epsilon m \phi})_{i1} \approx C \overline{S_{i1}} \sqrt{\frac{f_i}{g}} \frac{\overline{u}(\phi_{i1}) + \overline{u}_{ij}}{2} \quad (3.68a)
\]

\[
(\overline{\epsilon m \phi})_{j1} \approx C \overline{S_{j1}} \sqrt{\frac{f_j}{g}} \frac{\overline{u}(\phi_{j1}) + \overline{u}_{ij}}{2} \quad (3.68b)
\]
3.1.9.4 Development of the term $\Delta P_{\text{ll}}$ (Eq. 3.25a)

An exact calculation of $\Delta P_{\text{ll}}$ could be made if we knew the velocity profile along $\phi$, especially near the wall.

However such an experimental evaluation is extremely difficult. The easiest way is to use a correlation similar to those which are used for normal flow across a bundle of circular tubes. (Ref. 12, pp. 332-334).

$$\Delta P_{\text{ll}} = \zeta \frac{W_{ij}}{\overline{\rho_{ij}} L_{ij} S_{ij}^q} \quad (3.69)$$

where $\overline{\rho_{ij}}$ is the mean density along the gap, and $\zeta$ is the friction factor given by an expression of the form.

$$\zeta = A \text{Re}_L^m + B \quad (3.70)$$

where $\text{Re}_L$ is the Reynolds number for the lateral flow; $A$ and $m$ are constants dependent on the geometry of the array and $B$ is a constant reflecting the change in the temperatures through the bundle.

Such a correlation with $B=0$ is used in the Code THINC-IV for PWR rod bundles.

However the situation is different here than for pure normal flow because of the presence of axial flow. The difference comes mainly due to the fact that
1) The diversion cross flow does not remain constant going from one subchannel to the other.

2) A presence of a secondary flow in addition to the diversion cross flow effects the shear stress at the wall and the line integrals in (3.25a).

One way of approaching the problem will be to introduce in (3.69) an additional factor $C_f$ as is done in Ruhani's Approach (§4.3.4) which will reflect the above effects, i.e.

$$\Delta p_{\parallel} \approx \Delta p_{\parallel} = C_f \int \frac{W_{ij}^q}{2 \Phi_{ij} S_{ij}^2}$$  \hspace{1cm} (3.71)

The constant $C_f$ will be primarily a function of pitch ratio and has to be determined experimentally or theoretically. An approximate way to calculate $C_f$ is given in Chapter 4.

3.1.2.5 Development of the term $\Delta p^i_\theta$ (Eq. 3.25d).

The term $\Delta p^i_\theta$ is given by (3.25d) i.e.

$$\Delta p^i_\theta \approx \frac{1}{\alpha_i} \left[ \left[ (t \theta)_{i \theta} \right]_{i \Omega} S_{i \Omega} + (t \theta)_{j \Omega} S_{j \Omega} \right]$$  \hspace{1cm} (3.25d)
Because we have assumed that the $A_{ij}^{ij}$ changes axially mainly due to displacement of $S_{ii}$ and $S_{ji}$ along $x$ we can approximate (See Eq. (2.43))

\[ S_{ii} \left( t \theta \right)_{ii} \approx - \frac{\partial A_{ii}}{\partial x} \quad (3.72a) \]

\[ S_{ji} \left( t \theta \right)_{ji} \approx - \frac{\partial A_{ji}}{\partial x} \quad (3.72b) \]

For the line integrals we can make the approximation

\[ \int S_{ii} r \rho v_\phi u d\ell \approx \bar{u}(\phi_{ii}) \int S_{ii} r \rho v_\phi d\ell \quad (3.73) \]

where $\bar{u}(\phi_{ii})$ is the mean velocity along $S_{ii}$ and is given by (3.45a).

Taking into consideration (3.27)

\[ \int S_{ii} r \rho v_\phi d\ell = \int S_{ii} r \rho v_\phi \phi_{ii} d\ell + \int S_{ii} r \rho v_\phi - v_{\phi_{ii}} d\ell \quad (3.74) \]

Taking into consideration the fact that the axial velocity gradient along $\phi_{ii}$ is expected to be small and
also that \( \int_{S_{ii}} g_{\text{sec}} \phi \, dl = 0 \)
we can neglect the second term in (3.74).

Therefore,

\[
\int_{S_{ii}} r \rho g \phi \, dl \approx \int_{S_{ii}} r \rho g \phi_d \, dl \tag{3.75}
\]

Neglecting, in addition the effects of the other subchannels, i.e. \( g \phi_d \) given

\[
g \phi_d (r, \phi_{im}) \approx \beta_{ji} (r, \phi_{im}) \frac{W_{ji}}{S_{ji}} \tag{3.76}
\]

Thus (3.75) becomes

\[
\int_{S_{ii}} r \rho g \phi \, dl \approx \frac{W_{ji}}{S_{ji}} \int_{S_{ii}} r \beta_{ji} (r, \phi_{im}) \, dl \tag{3.77}
\]

Defining

\[
\beta_{ji}^\Lambda = \frac{\int_{S_{ii}} r \beta_{ji} (r, \phi_{im}) \, dl}{\int_{S_{ii}} rdr} \tag{3.78a}
\]

and similarly

\[
\beta_{ji}^* = \frac{\int_{S_{ji}} r \beta_{ji}^* (r, \phi_{jm}) \, dl}{\int_{S_{ji}} rdr} \tag{3.78b}
\]
\[ \int r \, dr = \int r \, dr = \alpha \]

(See Equations 3.24a, 3.24b), we end up with

\[ \Delta p^1 = - \left( \frac{\hat{\beta}}{\phi_{ii}^1} \bar{u}_i (\phi_{ii}^1) \frac{1}{S_{ii}} \frac{\partial A_{ii}^1}{\partial x} + \frac{\hat{\beta}}{\phi_{jj}^1} \bar{u}_j (\phi_{jj}^1) \frac{1}{S_{jj}} \frac{\partial A_{jj}^1}{\partial x} \right) \frac{W_{ji}^1}{S_{ji}} \]  

(3.79)

As we see the term \[ \Delta p^1 \] depends how \[ A_{ji} \] changes with \[ x \] and hence \[ W_{ji} \]. In the range of \[ W_{ji} \] that we can assume that \[ \frac{\partial A_{ji}^1}{\partial x} \] does not change significantly (i.e., low \[ W_{ji} \]) we can neglect the term \[ \Delta p^1 \].

3.1.3 Characteristics of Eq. (3.26)

Up to now we have succeeded in expressing the transverse momentum equation as a relation between pressures and familiar flow characteristic quantities i.e. \[ u_i, \bar{c}, W_{im} \text{ etc.} \] through a number of constants that need to be evaluated either experimentally or theoretically.

The main advantages of using Equation (3.26) are
1) Higher accuracy.

2) Effects of all the surrounding subchannels can be considered in the creation of the pressure difference between two particular subchannels;

3) The analysis has introduced only one component, \( V_\phi \), of the velocity field instead of two \((V_y, V_z)\) using Ruhani's approach. In addition \( V_\phi \) is a familiar quantity for DP techniques and therefore (3.26) will be the possible equation for extending the DP techniques to 3-D problems;

4) Secondary flow effects can be included albeit through a postulate of their shape and magnitude;

5) Better representation of the subchannel pressures;

6) Change in axial shear stress is included;

7) Finally, the Equation (3.26) is quite general and therefore can handle extreme cases, e.g. blockades.

Equation (3.26) is valid for both inner and side subchannels. However in the case of inner subchannels and LP techniques, the integral surface is better to be extended to \( A_{1j} \) instead of \( A_{1j1} \), for the following reasons:

a) In LP techniques the "unit cell" of calculations will be the subchannel which includes both \( A_{1I} \) and \( A_{12} \). Thus the pressure difference \( \rho_{i_{as}} - \rho_{j_{as}} \) depends not only on what happens in \( A_{1j1} \) but what happens in \( A_{1j2} \) as well.

b) The combination of both integral momentum equations in \( A_{1j1} \) and \( A_{1j2} \) will have the advantage of reducing the error due to the assumptions (3.21b) and (3.21c) made along \( S_n \).
We will see below how the extension to $A_{ij2}$ can be done.

3.1.4. Improved Transverse Momentum Equation for Inner Subchannels.

The improvement consists of applying the transverse moment-of-momentum equations in $A_{ij2}$ and adding the resultant equation to (3.26).

Applying the momentum equation in $A_{ij2}$ we will obtain the same equation as (3.26) but with substitution of the subscript $I$ by the subscript $2$. The same happens to all the definitions made, related to the above equation.

Adding by parts the two above mentioned equations we obtain:

$$
\Delta p_{ji} = p_{j,i}^{as} p_{i,a} = \frac{1}{\alpha_1 + \alpha_2} \left[ \frac{\partial (\psi_i + \psi_2)}{\partial t} + \frac{\partial (u_1^a \psi_i - u_2^a \psi_2)}{\partial x} \right] + 
$$

$$
+ \Delta p_{yy} + \Delta p_{al} + \Delta p_{y} + \Delta p_{\theta} \tag{3.80}
$$

where

$$
\Delta p_{\theta} = \frac{\alpha_1 \Delta p_{\theta}^1 + \alpha_2 \Delta p_{\theta}^2}{\alpha_1 + \alpha_2} \tag{3.81a}
$$

$$
\Delta p_{al} = \frac{\alpha_1 \Delta p_{al}^1 + \alpha_2 \Delta p_{al}^2}{\alpha_1 + \alpha_2} \tag{3.81b}
$$
\[ \Delta p_g \equiv \frac{\alpha_1 \Delta p_g^1 + \alpha_2 \Delta p_g^2}{\alpha_1 + \alpha_2} \]  

\[ \Delta p_\theta \equiv \frac{\alpha_1 \Delta p_\theta^1 + \alpha_2 \Delta p_\theta^2}{\alpha_1 + \alpha_2} \]  

Calling in addition

\[ \psi \equiv \frac{\psi_1 + \psi_2}{2} \]  

\[ \omega^* \equiv \frac{\psi_1 \omega_1^* + \psi_2 \omega_2^*}{\psi_1 + \psi_2} \]  

\[ \alpha \equiv \frac{\alpha_1 + \alpha_2}{2} \]  

Equation (3.80) becomes

\[
\Delta p_{ji} = p_{j\alpha s} - p_{i\alpha s} = \frac{1}{\alpha} \left[ \frac{\partial \psi}{\partial t} + \frac{\partial (\omega^* \psi)}{\partial x} \right] + \\
+ \Delta p_{el} + \Delta p_{al} + \Delta p_g + \Delta p_\theta
\]  

Equation (3.85) is similar to Equation (3.26). The difference is that instead of taking the quantities related to the flow within the \( A_{ijl} \) we take the average of the
quantities related to the flow within $A_{ij1}$ and within $A_{ij2}$.

The average of the quantities is given in the expressions (3.81a) to (3.84).

Before seeing how the suggested transverse momentum equations are reduced to the existing schemes, it is worth while to examine what is the form of these equations in the cases that the approximations (3.38d) for $\psi$ is applied.

These forms of the equations which will be much simpler than those using (3.34c) for $\psi_1$, are closer to the existing schemes. We will call them "the simplified transverse momentum equations".

3.1.5 The Simplified Transverse Momentum Equation.

The reader is reminded that the expression (3.38d) for $\psi_1$ is an approximation and was derived by assuming

1) The effects due to cross-flows other than $W_{ij}$ are negligible;

2) Secondary flow effects are also negligible.

Substituting (3.38d) into (3.26) we obtain

$$\Delta P_{ji}^1 = P_{jas} - P_{ias} = \frac{1}{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{\tau_{ij}}{S_{ij}} \bar{P}_{ij} W_{ij} \right) \right]$$

$$+ \frac{\partial}{\partial x} \left( \frac{\tau_{ij}}{S_{ij}} u_i^* W_{ij} \right) +$$

$$\Delta P_{e}^1 + \Delta P_{al}^1 + \Delta P_{q}^1 + \Delta P_{o}^1 \quad (3.86)$$
Equation (3.85) becomes

$$\Delta \rho_{ji} = \rho_{jas} - \rho_{ias} = \frac{1}{\alpha_i} \left[ \frac{\partial}{\partial t} \left( \frac{\bar{V}_i A_{ij} \bar{B}_i + \bar{V}_2 A_{ij} \bar{B}_i \bar{w}_i}{2 \alpha sj} W_{ji} \right) + \frac{\partial}{\partial x} \left( \frac{\bar{V}_i A_{ij} \bar{B}_i + \bar{V}_2 A_{ij} \bar{B}_i \bar{w}_i}{alpha sj} W_{ji} \right) \right] + \Delta \rho_{ll} + \Delta \rho_{al} + \Delta \rho_{g} + \Delta \rho_{O} \tag{3.87}$$

3.1.6. **Summary**

Before illustrating how the derived equations are reduced to existing schemes, let us make a summary of the results obtained so far.

The suggested transverse momentum equation will be (3.26) or (3.86). For inner subchannels and LP techniques Eq. (3.86) has to be used.

The quantity $\psi$ is given by (3.82). $\psi_1$ (and similarly $\psi_2$) is defined by the equation (3.16c). Introducing the concept of $\delta$ quantities (see equations 3.29, 3.32, 3.36), $\psi_1$ is expressed by (3.34c) in the general case. If only the $W_{ji}$ effects are included, $\psi_1$ is expressed by (3.35d) and is based on Eifler-Nijssing model which refers to one phase, fully developed incompressible flow. In most of the cases
the secondary flow effects can be neglected. In such cases \( \psi_1 \) is given by (3.38d) and the formes of the resultant equations are given in (3.86) and (3.87).

The quantity \( u_1^* \) (and similarly the quantity \( u_2^* \)) is defined by (3.17c). \( u_1^* \) can be approximated by (3.49c) or (3.49d). A better approximation will be given by utilizing (3.17c) directly and for the quantities \( u_{11}^* \) and \( u_{11}^* \) (3.52a) and (3.52b) respectively.

The factor \( \alpha \) is given by (3.84). \( \alpha_1 \) (and similarly \( \alpha_2 \)) is given by the equation (3.24).

For the lateral form and friction losses \( \Delta p_{ll} \), the familiar correlation (3.71) is applicable. The proportionality factor \( \zeta \), which for a bundle of circular tubes is given by (3.70), must be now multiplied by a factor \( C_f \) less than unity which reflects the presence of the axial flow.

If the axial friction losses have to be included, Eq. (3.61) is proposed, as a first approximation, to express these effects.

If \( \sin \beta \neq 0 \) in the same cases we have to include the gravitational effects. \( \Delta p_{g}^1 \) is given by (3.81c). \( \Delta p_{g}^2 \) (and similarly \( \Delta p_{g}^2 \)) is given by (3.25c). The proportionality factor \( C_{g1} \) is given by (3.20).

If change of \( A_{i,j} \) with respect to \( x \) has to be considered the term \( \Delta p_{\theta}^1 \) (3.81d) or \( \Delta p_{\theta}^1 \) (3.25d) is included. For \( \Delta p_{\theta}^1 \) (and similarly for \( \Delta p_{\theta}^2 \)) expression 3.79 is suggested.
3.2 Reduction to COBRA III C Model

Recall that the Transverse Momentum Equation in COBRA III C is given by Eq. (1.3) which according to our notation will be

\[
\overline{P}_{j} - \overline{P}_{i} = \frac{l}{s} \left[ \frac{\partial W_{ij}^{2}}{\partial t} + \frac{\partial (u^{*} W_{ij}^{2})}{\partial x} \right] + \Delta P_{ll} \quad (3.88)
\]

Our purpose at this point is to examine under what conditions Equations (3.86) or (3.87) are reduced to Equation (3.88).

3.2.1 Inner subchannels

In this case Eq. (3.87) is suggested. Take the case for which we can assume

1. \( \frac{\partial}{\partial t} \left( \frac{\overline{r}_{1} A_{ij} \overline{P}_{1} + \overline{r}_{2} A_{ij} \overline{P}_{2}}{2 S_{ij}} \right) = 0 \) \quad (3.89a)

2. \( \frac{\partial}{\partial x} \left( \frac{\overline{r}_{1} A_{ij} \overline{P}_{1} + \overline{r}_{2} A_{ij} \overline{P}_{2}}{2 S_{ij}} \right) = 0 \) \quad (3.89b)

3. \( \Delta P_{ad} = 0 \) \quad (3.89c)

4. \( \Delta P_{y} = 0 \) \quad (3.89d)

5. \( \Delta P_{\theta} = 0 \) \quad (3.89e)
6. \( P_{jas} = \bar{P}_j \); \( P_{ias} = \bar{P}_i \) \hspace{1cm} (3.89f)

7. \( \text{Cf} \beta \) independent of the Reynolds number.

\( \text{Cf} \beta \) is given in the correlation for \( \Delta P_{el} \) (Eq. 3.71)

Call in addition

\[
\frac{\ell}{S} = \frac{\bar{r}_1 A_{ij1} \bar{B}_1 + \bar{r}_2 A_{ij2} \bar{B}_2}{2 S_{ij} \alpha} \hspace{1cm} (3.90)
\]

Substituting Eq. 3.90 into 3.87 we obtain Eq. 3.88.

3.2.2 Side subchannels

In this case Eq. 3.86 is suggested. Similar suggestions are made here i.e.

1. \( \frac{\partial}{\partial t} \left( \frac{\bar{r}_1 A_{ij1} \bar{B}_1}{S_{ij}} \right) = 0 \) \hspace{1cm} (3.91a)

2. \( \frac{\partial}{\partial x} \left( \frac{\bar{r}_1 A_{ij1} \bar{B}_1}{S_{ij}} \right) = 0 \) \hspace{1cm} (3.91b)

3. \( \Delta P_{al} = \Delta P_{al} = 0 \) \hspace{1cm} (3.91c)

4. \( \Delta P_{y} = \Delta P_{y} = 0 \) \hspace{1cm} (3.91d)

5. \( \Delta P_{\theta} = \Delta P_{\theta} = 0 \) \hspace{1cm} (3.91e)

6. \( P_{jas} = \bar{P}_j \); \( P_{ias} = \bar{P}_i \) \hspace{1cm} (3.91f)
7. Again $C_{f3}$ is independent of the lateral Reynolds number.

Here the $\lambda/s$ parameter is given by

$$\frac{\ell}{s} = \frac{\bar{\nu}, A_{ij}, \bar{\beta}_i}{s_{ij}, \alpha_1} \quad (3.92)$$

3.2.3. Results.

It is evident from the previous discussion that Rowe's model can be valid only in the cases that assumptions (3.89) or (3.91) are good approximations. This means that $W_{j1}$ has to be sufficiently small so that (3.89c) and (3.89e) or (3.91c) and (3.91e) are good assumptions. The diversion cross flow $W_{j1}$ has also to be small enough so that it does not significantly effect the parameters $\bar{\beta}$ and the surface $A_{ij1}$ so that the assumptions (3.89a) and (3.89b) or (3.91a) and (3.91b) become reasonable. It is evident also that the gap length $s_{j1}$ can not change significantly along $s$.

In the cases that Rowe's model is valid, the parameter $\lambda/s$ can be given by (3.90) or (3.92), i.e.
\[
\frac{l}{s} = \begin{cases}
\frac{r_i A_{ij} \bar{B}_1 + r_j A_{jk} \bar{B}_2}{2 \sqrt{s_{ij} \alpha}} & \text{for inner subchannels} \\
\frac{r_i A_{ij} \bar{B}_1}{s_{ij} \alpha_1} & \text{for side subchannels}
\end{cases}
\quad (3.93)
\]

3.3 Reduction to Khan's Model.

Let us take the Equation for Khan's model (1.24), i.e.,

\[
\bar{p}_j - \bar{p}_i = \frac{d}{dx} \left( \frac{l}{s} \omega^* W_{ji} \right) + \Delta p_{ll} \quad (1.24)
\]

Consider now the Equations (3.86) and (3.87) in the steady state and make the approximations:

1. \( p_{jas} = \bar{p}_j \), \( p_{ias} = \bar{p}_i \) \quad (3.94a)

2. \( \phi_{jl} \) and \( \phi_{lj} \) do not change substantially along \( x \) so that

1a) \( \Delta p_\alpha \propto 0 \) \quad (3.94b)

1b) Parameter \( \alpha \) can be inserted into the derivatives of the Equations (3.86) and (3.87).
3. \( \Delta p_g = 0 \)  

4. \( \Delta p_{al} = 0 \)

If we define again the parameter \( l/s \) according to Eq. (3.93), Equations (3.86) and (3.87) are reduced to Equation (1.24).

It is evident that extending Khan's model to transients the transverse momentum equation will be

\[
\bar{p}_j - \bar{p}_i = -\frac{\partial (W_{ji} l)}{\partial t} + \frac{\partial}{\partial x} \left( \frac{l}{s} u^* W_{ji} \right) + \Delta p_{al}
\]

(3.95)

The Khan model seems to be good for higher \( W_{ji} \) 's so that \( \Delta p_{al} \) remains negligible. We can widen the spectrum of \( W_{ji} \) by adding to Eq. (3.95) the term \( \Delta p_{al} \).

3.4 Conclusions.

It is evident from the previous discussion that the concept of \( l/s \) in the transverse momentum equation can be valid in the cases that \( A_{ij} \) does not change significantly with \( W_{ji} \) (or \( x \)).

Thus for the cases where in addition the \( \beta \)'s are
not affected significantly by $W_{j1}$, Rowe's model expressed by Equation similar to 3.88 i.e.

$$p_{jas} - p_{ias} = \frac{\ell}{S} \frac{\partial W_{j1}}{\partial t} + \frac{l}{S} \frac{\partial (u^* W_{j1})}{\partial x} + \Delta p_{el} \quad (3.96)$$

can be satisfactory where $l/s$ is given by (3.93). Rowe's model is expected to work well for low $W_{j1}$ and therefore seems to be negligible.

In the case that $\beta$'s are affected significantly by $W_{j1}$, the Khan model can be valid, and the suggested equation will be Equation similar to 3.95 i.e.

$$p_{jas} - p_{ias} = \frac{\partial W_{j1}}{\partial t} + \frac{\partial}{\partial x} \left( \frac{l}{S} u^* W_{j1} \right) + \Delta p_{el} \quad (3.97)$$

$\ell/s$ is given again by (3.93). This model is expected to work well for higher $W_{j1}$. In the case that $W_{j1}$ becomes high enough so that $\Delta p_{el}$ or even $\Delta p_{\theta}$ becomes significant Equation (3.97) can be modified to
\[ p_{jas} - p_{ias} = \frac{\partial (W_{ji})}{\partial t} + \frac{\partial}{\partial x} \left( \frac{L}{S} u^* W_{ji} \right) + \Delta P_{el} + \Delta P_{al} + \Delta P_{\theta} \]  

(3.96)

Now in cases that \( W_{ji} \) has a significant effect on \( A_{ij} \) then the \( l/s \) parameter concept seems inapplicable and in this case Equations (3.86) or (3.87) have to be used.

In cases that the effects from the other cross flows than \( W_{ji} \) become important, one has to go to the Equations (3.26) or (3.85) or by (3.34c) when the secondary flow integral is neglected.

However, arriving at this point, the question remains how we can evaluate the various constants which have been used in our analysis. The constants have to be evaluated experimentally or theoretically. In the next chapter a theoretical approach will be made to calculate the main constants. Finally Chapter V will present a discussion of some existing experimental data and suggestions for new experiments to evaluate these constants.
CHAPTER IV

A THEORETICAL APPROACH OF EVALUATING TRANSVERSE MOMENTUM CONSTANTS

4.1 General

In this section we will try to evaluate the main constants we have introduced in Chapter III by making a number of assumptions. For simplicity we restrict ourselves to the case that "the simplified transverse momentum equations" (Eqns. 3.86 and 3.87) are valid, i.e., $\Psi_1$ can be well approximated by (3.35d) or (3.38d) which physically assumes that the pressure difference between two subchannels is principally due to the diversion cross flow between these two channels.

In addition we will assume

1) Normal geometry, i.e. $\frac{\partial A}{\partial x} = 0$

2) Steady state

3) One phase flow

4) Cylindrical fuel rods with equal diameter.

Our emphasis will be on the estimation of the $\lambda/s$ parameter (Eq. 3.93).

As we can see from (3.93) the value of $\lambda/s$ parameter depends on several parameters including $\bar{\beta}_1$ (and $\bar{\beta}_2$). However, according to its definition (3.36) $\bar{\beta}_1$ depends on
which are given by (3.32a) and (3.32b) for m=j and m'=1 (See Equations 3.32a and 3.32b). The definitions (3.32a) and (3.32b) show that \( \bar{p}_{j}^{i} \) and \( \bar{p}_{i}^{j} \) depend on the local \( \bar{p}_{j}^{i}(\gamma, \phi) \) and \( \bar{p}_{i}^{j}(\gamma, \phi) \) respectively. The local \( \bar{p}_{j} \)'s depend on the \( \nu_{\phi} \) field as is shown in Eq. (3.29).

4.2 A Representation of \( \nu_{\phi} \)-Field

In this analysis we will take also the line \( s_{n} \) perpendicular to the midpoint of the gap for inner subchannels.

Therefore,

\[
A_{i1} = A_{j1} = A_{i2} = A_{j2} \quad (4.1a)
\]

\[
\bar{r}_{i1} = \bar{r}_{j1} = \bar{r}_{i2} = \bar{r}_{j2} = \bar{r}_{i} = \bar{r}_{j} = \bar{r}_{2} \quad (4.1b)
\]

The \( \bar{r}_{i} \)'s are given by Equations (3.18). Under these circumstances the \( \nu_{\phi} \)-field and the \( u \)-field appear to be approximately symmetrical with respect to the axis of symmetry, the \( s_{n} \) line, for inner subchannels.

Thus, taking into consideration equations (3.29), (3.32a) and (3.32b) we obtain

\[
\bar{p}_{j}^{i} \approx \bar{p}_{j}^{i2} \quad (4.1c)
\]

\[
\bar{p}_{i}^{j} \approx \bar{p}_{i}^{j2} \quad (4.1d)
\]
\[ \bar{\beta}_1 \sim \bar{\beta}_2 \]  

(4.1e)

Let us consider the region \( A_{11} \) (Fig. 3.4). The cross flow past \( S_\phi \) (Fig. 3.4) will be

\[ W(\phi) = \oint \vec{v}_{\phi} \cdot (\gamma, \phi) \, dl \]  

(4.2a)

Taking into consideration (3.29) and considering only the effect of the \( j \)'th subchannel, Eq. (4.2a) becomes

\[ W(\phi) = \frac{W_{ij}}{S_{ji}} \oint \beta_{ji} \cdot (\gamma, \phi) \, dl \]  

(4.2b)

\[ \beta_{ji} \cdot (\gamma, \phi) \] can be expressed always as

\[ \beta_{ji} \cdot (\gamma, \phi) \equiv f(\gamma, \phi) \, q(\phi) \]  

(4.3)

where

\[ \frac{1}{S_\phi} \oint f(\gamma, \phi) \, dl = 1 \]  

(4.4)
which means that $f(r, \phi)$ is the shape function for the velocity along $S_\phi$.

$q(\phi)$ reflects the magnitude of the cross flow $W(\phi)$.

Combining (4.3), (4.4) with (4.2b) we find that

$$W(\phi) = q(\phi) \frac{S_\phi}{S_{ij}} W_{ji}$$  \hspace{1cm} (4.5)

or

$$q(\phi) = \frac{S_{ij}}{S_\phi} \frac{W(\phi)}{W_{ji}}$$  \hspace{1cm} (4.6)

It is evident that if we knew the exact shape of $V_\phi$ as a function of $\phi$, i.e. $f(r, \phi)$, and how $W_{\phi}$ changes with $\phi$, then the evaluation of $\beta_{ij}(r, \phi)$ would be straightforward.

4.2.1 The function $q(\phi)$ (Eq. 4.6)

Call $A(\phi)$ the surface between the lines $S_o$ and $S_\phi$ (Fig. 4.1). $A(\phi)$ for $\phi=\phi_i$, will be

$$A(\phi_{i1}) = A_{i1}$$  \hspace{1cm} (4.7)

In general the length $S_o$ which is the distance (AB) in Fig. 4.1 depends on the position of $S_n$ for inner subchromels. It is equal to the gap width for side subchannels.
Fig. 4.1 Region $A_{ii}$ of the subchannel $i$. 
The crossflow $W(0)$ across $S_o$ is reasonably assumed to be the following fraction of the crossflow, $W_{ji}$ across the whole gap $S_{ji}$

$$W(0) = \frac{S_o}{S_{ji}} \cdot W_{ji} \quad (4.8)$$

In our analysis we will take

$$\frac{S_o}{S_{ji}} = \begin{cases} \frac{1}{2} & \text{for inner subchannel} \\ 1 & \text{for side subchannel} \end{cases} \quad (4.8a)$$

If, now, we assume that the flow rate past (AD) (see Fig. 4.1) is negligible, the continuity equation in the volume $dx dA$ will give

$$dx \left[ W(\phi + d\phi, x) - W(\phi, x) \right] = -$$

$$- \left[ \overline{\rho u} (\phi, x) - \overline{\rho u} (\phi, x) \right]_{ji} \frac{dA}{dx} \quad (4.9)$$

which is equivalent to

$$\frac{\partial W(\phi, x)}{\partial A} = - \left( \frac{\partial \overline{\rho u}(\phi, x)}{\partial x} \right)_{ji} \quad (4.10)$$
where the subscript $j_1$ means change in axial velocity due to $W_{j_1}$ only.

From Equation (4.10) it is evident that in the case that we have only the gap $j$ for the subchannel $i$, then

$$\frac{\partial w}{\partial x}$$

will be the actual change in the axial velocity.

Integration of the Equation (4.10) will give us

$$W(\phi) = W(0) - \int \left[ \frac{\partial (\rho u)}{\partial x} \right]_{j_1}^{j} dA$$

(4.11)

Thus from (4.6) taking into consideration (4.8) we obtain

$$q(\phi) = \frac{S_{ji}}{S_{\phi}} \left[ \frac{S_{0}}{S_{ji}} - \frac{1}{W_{j_1}} \int \left[ \frac{\partial (\rho u)}{\partial x} \right]_{j_1}^{j} dA \right]^{A(\phi)}$$

(4.12)

which can be written also as

$$q(\phi) = \frac{S_{ji}}{S_{\phi}} \left[ \frac{S_{0}}{S_{ji}} - \frac{A(\phi)}{W_{j_1}} \left[ \frac{\partial (\rho u)}{\partial x} \right]^{A(\phi)}_{j_1} \right] j_{i}$$

(4.13)
where

\[
\left( \frac{\partial \bar{u}}{\partial A(\phi)} \right)_{A(\phi)} = \frac{1}{\lambda(\phi)} \int_{0}^{A(\phi)} \rho u \, dA
\]  

(4.14)

Equations (4.13) or (4.14) tell us that the knowledge of \( q(\phi) \) presupposes knowledge of \( \frac{\partial (\rho u)}{\partial x} \) and \( W_{ji} \). Experimental work to show how \( \frac{\partial (\rho u)}{\partial x} \) changes as a function of \( \phi \) and \( W_{ji} \) is seen to be necessary.

A correlation of the form

\[
\frac{\partial (\bar{u})}{\partial x} = c_{0} + c_{1} \left[ A(\phi)A_{i} \right] + c_{2} \left[ \frac{A(\phi)}{A_{i}} \right]^{2} + \cdots \]  

(4.15)

can be examined.

In the absence of experimental data we take

\[
\frac{\partial (\bar{u})}{\partial x} = c_{0} 
\]  

(4.16)

which means that each part of the subchannel contributes the same to the creation of \( W_{ji} \). This is substantially the same as Ruhani's assumption (§ 1.3.3):

\[
\frac{\partial W}{\partial A} = c_{0} = \frac{W_{ji}}{A_{i}} 
\]  

(4.17)
Eq. (4.17) is similar to (1.21). Then Eq. (4.12) becomes

\[
q(\phi) = \frac{S_j^i}{S_\phi} \left[ \frac{S_o}{S_j^i} - \frac{A(\phi)}{A_i} \right]
\]  

(4.18)

4.2.2 The function \( f(r,\phi) \) (in Eq. 4.3)

The exact evaluation of \( f(r,\phi) \) is a much more complicated problem. Approximate shapes can be taken by examining the velocity shapes for flow normal to rod bundles. Of course there is a difference between this case and the case of nuclear reactor rod bundles because for nuclear bundles \( W(\phi) \) is a function of \( \phi \). However we will not expect to have large differences to exceed the desirable accuracy.

In this case to get an idea of the magnitudes and the effect of \( f(r,\phi) \) on the final results we take \( f(r,\phi) \) independent of \( \phi \) and we take two shapes for \( f(r,\phi) \):

a) uniform velocity, i.e.

\[
f(r,\phi) = 1
\]  

(4.19)
b) a parabolic velocity distribution

\[
\begin{aligned}
I(y,\phi) &= 6 \frac{S_0}{S_j} \left( \frac{y-R}{S_\phi} \right) \left( 1 - \frac{S_0}{S_j} \frac{y-R}{S_\phi} \right) \\
\end{aligned}
\] (4.20)

where \( \frac{S_0}{S_j} \) is given by (4.8a)

Equations (4.19) and (4.20) are normalized to take into consideration that \( \frac{\partial V_\phi}{\partial y} \bigg|_{S_n} = 0 \) for inner subchannels, \( V_\phi \bigg|_{S_n} = 0 \) for side subchannels and the definition of Eq. (4.4) must be satisfied.

4.3. Calculation of \( \vec{\beta}_j \)

Knowing the \( \beta(r,\phi) \) the constant \( \vec{\beta}_j \) can be calculated immediately according to its definition (3.32a, i.e.,

\[
\vec{\beta}_j = \frac{1}{\nu_i A_{i1}} \int_r \int_y \vec{\beta}_{i1}(y,\phi) y^2 \, dy \, d\phi \\
\] (4.21)

where

\[
A_{i1} = \frac{R^2}{\nu_i} \left[ \left( \frac{R+S_\phi}{R} \right)^2 \right] \\
\] (4.22)
\[ \overline{v}_{i_1} = \frac{1}{A_{i_1}} \int_0^{\phi_{i_1}} \int_0^{R+S\phi} r^2 \, dr \, d\phi \] (4.23)

It is found

\[ \overline{v}_{i_1} = \frac{R}{3} \left[ \frac{1}{2} \left( \frac{R+S_0}{R} \right)^3 \left[ \frac{\tan \phi_{i_1}}{\cos \phi_{i_1}} + \ln \left[ \tan \left( \frac{\pi}{4} + \frac{\phi_{i_1}}{2} \right) \right] \right] - \phi_{i_1} \right] \]

\[ \left[ \left( \frac{R+S_0}{R} \right)^2 \tan \phi_{i_1} - \phi_{i_1} \right] \] (4.24)

4.4 Calculation of the parameter \( \overline{B}_i \) (Eq. 3.36)

From Eq. (3.36), taking into consideration Equations

\[ \beta_i = \frac{-\beta_i^+ + \beta_i^-}{2} \] (4.25α)

A good approximation in our case will be

\[ \beta_i^+ \approx \overline{B}_i^+ \quad \beta_i^- \approx \overline{B}_i^- \] (4.26)
Then $\overline{P}_{j}$ becomes

$$\overline{P}_{j} \approx \overline{P}_{i}^{i_1}$$  \hspace{1cm} (4.25b)

It is noticed here that approximation (4.26) is not as good as (4.1c) or (4.1d). The reason is that in one sub-channel the flow is converging and in the other one the flow is diverging. The main difference appears in velocity distributions, i.e. the function $f(r,\phi)$. For example, the velocity distribution becomes flatter in the accelerating flow and in decelerating flow changes continuously shape and in some point may be reversed (Ref. 21, p. 2).

4.5 Calculation of the parameter $l/s$

The $l/s$ parameter is given by (3.93) which in our case becomes

$$\frac{\ell}{S} = \frac{q \overline{r}_{i_1} A_{i_1}}{\alpha_{i} \overline{P}_{j}^{i_1}}$$  \hspace{1cm} (4.28)

for both inner and side subchannels where

$\overline{r}_{i_1}$ is given by Eq. (4.24)

$A_{i_1}$ is given by Eq. (4.22)

$\overline{P}_{j}^{i_1}$ is given by Eq. (4.21) and

$\alpha_{i}$ is given by Eq. (3.24), i.e.
\[
\alpha_1 = \frac{1}{2} \left[ \left( \frac{S_0 + R}{\cos \phi} \right)^2 - R^2 \right]
\]  
(4.29)

4.6 Estimation of the parameter \( C_f \)  
(Eq. 3.71)

4.6.1 Inner Subchannels

In order to proceed with an estimation of \( C_f \) we will assume that the main contribution to \( \Delta P_{el} \) comes from the friction losses.

Thus we approximate

\[
C_f = \frac{\Delta P_{el}}{\Delta P_{el}^*} \approx \frac{\Delta P_f}{\Delta P_f^*}
\]  
(4.30)

where \( \Delta P_{el}^*, \Delta P_f^* \) are the "lateral losses" and friction losses respectively for the case of pure normal flow to circular tubes with the same \( W_{f1} \) along the gap.

According to (3.25b)

\[
C_f = \frac{\int r \tau_{fr} dl}{\int r \tau_{fr}^* dl} = \frac{R \int \tau_{fr} dl}{R \int \tau_{fr}^* dl} = \frac{\int \tau_{fr} dl}{\int \tau_{fr}^* dl}
\]  
(4.30a)
where $\tau^*_{\phi r}$ is the shear stress corresponding to the circular tubes.

Assuming that $\tau_{\phi r} \sim [W(\phi)]^2$ \hfill (4.31) this implies

$$\frac{\tau_{\phi r}(\phi)}{\tau^*_{\phi r}(\phi)} = \left[ \frac{w(\phi)}{w^*(\phi)} \right]^2 = \left[ \frac{w(\phi)}{w(0)} \right]^2 = \left[ \frac{W(\phi)}{W(0)} \right]^2 \hfill (4.32)$$

Taking into consideration (4.8) and (4.5)

$$\frac{W(\phi)}{W(0)} = \frac{S_{\phi}^2}{S_0^2} q(\phi) \hfill (4.33)$$

and $q(\phi)$ is given in general by (4.12) or (in our analysis) by (4.18). Thus

$$\tau_{\phi r}(\phi) \approx \frac{S_{\phi}^2}{S_0^2} q(\phi) \tau^*_{\phi r}(\phi) \hfill (4.34)$$

and Eq. (4.30) becomes

$$c_{\xi} \approx \frac{\int \frac{S_{\phi}^2}{S_0^2} q^2(\phi) \tau^*_{\phi r}(\phi) \, dl}{\int \tau^*_{\phi r}(\phi) \, dl} \hfill (4.35)$$
Provided that we know \( q(\phi) \) and \( \tau_{fr}(\phi) \) the constant \( C_f \) can be calculated. The \( \tau_{fr} \) can be found from circular tube experiments.

However as a first approximation we take

\[
C_f \approx \left( \frac{S_\phi^2}{S_0^2} q^2(\phi) \right)_{\text{mean}} \tag{4.36}
\]

where \( \left( \frac{S_\phi^2}{S_0^2} q^2(\phi) \right)_{\text{mean}} \) is the mean value along \( S_w \), i.e.

\[
C_f \approx \frac{1}{S_w} \int_{S_w} \frac{S_\phi^2}{S_0^2} q^2(\phi) \, dl \tag{4.37}
\]

Because we have accepted symmetry of \( q(\phi) \) in our analysis (i.e. \( \overline{p_{ji}} = \overline{p_{ij}} \))

\[
C_f \approx \frac{1}{\phi_{ii}} \int_{0}^{\phi_{ii}} \frac{S_\phi^2}{S_0^2} q^2(\phi) \, d\phi \tag{4.38}
\]
4.6.2 Side Subchannels

The side subchannels (Fig. 3.5b) become more difficult to treat because we now have the outer wall shear stress.

The integral
\[ \int_{S_{w1}} r \tau_{\phi n} dl \]
represents the stress momentum over \( S_{w1} \).

Recall that for \( \theta = 0 \)

\[ \tau_{\phi n} = \tau_{n n} = \tau_{r r} (e_r n) + \tau_{\phi r} (e_{\phi r} n) + \tau_{\phi x} (e_{\phi x} n) \]  
(4.39)

From fig. 4.2 it is evident that:

\[ e_r n = \cos \phi \]  
(4.40a)

\[ e_{\phi n} = -\sin \theta \]  
(4.40b)

\[ e_{\phi x} n = 0 \]  
(4.40c)

Taking, in addition, into consideration the fact that

\[ \tau_{\phi \phi} \propto p \]  
(3.21a)
Fig. 4.2 Stress in the outer wall for a side subchannel.
and
\[ R = (S_0 + R) \left| \cos \phi \right. \] (4.41)

the integral
\[
\int_{S_{W_1}''} r \tau_{\phi \eta} dl
\]
becomes
\[
\int_{S_{W_1}''} r \tau_{\phi \eta} dl = (S_0 + R) \left[ \int_{S_{W_1}''} \tau_{\phi \gamma} dl - \int_{S_{W_1}''} p dt y_\phi dl \right] \] (4.42)

Thus, in this case
\[
C_f = \frac{R \left( \int_{S_{W_1}} \tau_{\phi \gamma} dl + (R + S_0) \int_{S_{W_1}''} (\tau_{\phi \gamma} - p^* t y_\phi) dl \right)}{R \left( \int_{S_{W_1}'} \tau_{\phi \gamma} dl + (R + S_0) \int_{S_{W_1}''} (\tau_{\phi \gamma} - p^* t y_\phi) dl \right)} \] (4.43)

where again $p^*$ is the pressure corresponding to the circular tubes.
Using again an expression similar to (4.34), i.e.

$$\left. \tau_{\phi r} \right|_{S_{W1}'} = \frac{S_{\phi}}{S_o} q^2(\phi) \left. \tau_{\phi r}^* \right|_{S_{W1}'} \quad (4.44a)$$

$$\left. \tau_{\phi r} \right|_{S_{W1}''} = \frac{S_{\phi}^2}{S_o^2} q^2(\phi) \left. \tau_{\phi r}^* \right|_{S_{W1}''} \quad (4.44b)$$

we obtain

$$C_f = \frac{\int_{S_{W1}'} \left( \frac{S_{\phi}}{S_o} q^2(\phi) \tau_{\phi r}^* \right) dl + \frac{R+S_o}{R} \int_{S_{W1}''} \left( \frac{S_{\phi}^2}{S_o^2} q^2(\phi) \tau_{\phi r}^* - p \nabla \phi \right) dl}{\int_{S_{W1}'} \tau_{\phi r}^* dl + \frac{R+S_o}{R} \int_{S_{W1}''} \left( \tau_{\phi r}^* - p^* \nabla \phi \right) dl} \quad (4.45)$$

The knowledge of $\tau_{\phi r}^*$, $p$, and $p^*$ are necessary to estimate $C_f$. However accepting the fact that the pressure drop along $S_{W''}$ cannot be different from pressure drop along $S_{W'}$ we assume that the terms
\[
\int_{S_{wl}}^{S_{wl}'} \frac{q}{S_0} q^2(q) \tau_{\phi r}^* \, dl
\]

and

\[
\int_{S_{wl}}^{S_{wl}'} \tau_{\phi r}^* \, dl
\]

are of the same magnitude with

\[
\frac{R+S_0}{R} \int_{S_{wl}''} \left( \frac{q}{S_0} q^2(q) \tau_{\phi r}^* - p^* t y \phi \right) \, dl
\]

and

\[
\frac{R+S_0}{R} \int_{S_{wl}''} (\tau_{\phi r}^* - p^* t y \phi) \, dl
\]

respectively.

Thus we can approximate

\[
\zeta_f \approx \frac{\int_{S_{wl}'}^{S_{wl}''} \frac{q}{S_0} q^2 \tau_{\phi r}^* \, dl}{\int_{S_{wl}'}^{S_{wl}''} \tau_{\phi r}^* \, dl}
\]

(4.46)

which is the same as in inner subchannel.
4.7 Estimation of $\bar{S}_{i1}$ (Eq. 3.67a)

Recall that

$$\bar{S}_{i1} = \frac{1}{A_{i1}} \iint_{A_{i1}} s_{\phi} \, dA$$  \hspace{1cm} (3.67a)

It is found that

$$\bar{S}_{i1} = \frac{R^2}{4A_{i1}} \left\{ \left( \frac{S_0 + R}{R} \right)^3 \tan \phi \cos \phi + \left[ \left( \frac{S_0 + R}{R} \right)^3 - 2 \left( \frac{S_0 + R}{R} \right)^2 \tan \phi_{i1} \right] \right\}$$

$$\cdot \ln \tan \left( \frac{\phi_{i1}}{2} + \frac{\pi}{4} \right) - 2 \left( \frac{S_0 + R}{R} \right)^2 \tan \phi_{i1} + \alpha \phi_{i1} \right\}$$

\hspace{1cm} (4.47)

Since $\phi_{i1} = \phi_{j1} = \phi_{i2} = \phi_{j2}$

$$\therefore \bar{S}_{i1} = \bar{S}_{i2} = \bar{S}_{j1} = \bar{S}_{j2}$$  \hspace{1cm} (4.48)
4.8 Estimation of $\eta_{i1}$ (Eq. 3.43a)

Recall that

$$\eta_{i1} = \frac{1}{\bar{v}_{i1}} \frac{\phi_{i1}}{A_{i1}} \int_{0}^{R} \int_{0}^{\pi} r^2 f(r) \, dr \, d\phi \quad (3.43a)$$

For $f(r)$ we can take the expressions used in VELASCO (Ref. 11), i.e.

$$f(r) = \cos \frac{n (r-R)}{S_\phi} \quad (4.49)$$

In this case we can find

$$\eta_{i1} = \frac{2R}{n^2 \bar{v}_{i1} A_{i1}} \int_{0}^{\pi} \left[ \left( \frac{S_\phi}{R} \right)^3 + 2 \left( \frac{S_\phi}{R} \right)^2 \right] d\phi \quad (4.50)$$

It is evident that in our case that we have assumed symmetry yielding

$$\eta_{i1} = \eta_{i2} = -\eta_{j1} = -\eta_{j2} \quad (4.51)$$
4.9 Analysis

Up to this point we have succeeded in being able to estimate the main constants that appear in the transverse momentum equation. In order to achieve this we have made a number of assumptions, the most severe of which seems to be the \( q(\phi) \) estimation.

We will try now to calculate the various proposed constants for both uniform and parabolic \( V_\phi \) distributions with various pitch ratios. Looking at the expression given for the various constants we see that their values will depend on what is to be assumed for \( \phi_{i1} \) which we have not yet specified. The calculations will be performed for a number of \( \phi_{i1} \) values.

A program has been constructed for this purpose. The method, the results and the listing of the program is given in APPENDIX C. In this APPENDIX the values of \( \ell/s, \bar{b}_i \), \( C_f \), \( \bar{S}_{i1} \), and \( \eta_{i1} \) are calculated as a function of \( \phi_{i1} \), \( p/D \) ratio and type of subchannel (square or triangular).

4.9.1 Results

The results are given in APPENDIX C and the discussion is given in \( \S \) C.1.3. We outline here the major conclusions.
1) $l/s$ is large for small $P/D$. Fig. 6.2 gives $l/s$ vs. $P/D$ for parabolic $v_\phi$ distribution.

2) $\phi_{i1}$ is around $15^\circ$ for $P/D = 1.05$ and goes very fast in the region of $30^\circ$ as $P/D$ increases.

3) $\phi_{i1}$ and $l/s$ are slightly lower for triangular arrays than for rectangular arrays.
CHAPTER V

PROPOSED EXPERIMENTAL WORK

5.1 Purpose

In the previous Chapter, the main constants for the transverse momentum equation have been evaluated. This was obtained by making a number of assumptions. The reason of making these simplifications was due to lack of experimental evidence. Therefore the performance of a number of experiments to permit testing and correction of these assumptions seems to be necessary.

5.2 Proposed Geometry

The geometry that appears to be attractive in the beginning is a two-subchannel (closed) channel, i.e. the two subchannels bounded completely by walls. We can make any combination of triangular, square and side subchannels. A variety of pitch ratios can be considered also. The two subchannel geometry have the advantage that effects from cross-flows other than \( W_{ji} \) do not exist. For example the expression (3.35a) for \( \psi_1 \) would be a correct expression. Also, in Eq. (4.12) the quantity \( \frac{\partial (\bar{p}u)}{\partial x} \) would equal the real \( \frac{\partial (\bar{p}u)}{\partial x} \).
This geometry has the disadvantage that the gaps other than $S_{ij}$ are replaced by walls. However we do not expect to have considerable changes in quantities like $\frac{\partial (pu)}{\partial x}$ and $V_\phi$ within the area $A_{ij}$.

It seems better to perform the first experiments in a square-square geometry. The reason is that the boundary $S_{1i}$ (Fig. 3.4) is farther from from the other gaps and the effects of using walls instead of open gaps will be less. With respect to size of the area a relatively large cross-section will be better for more accurate measurements.

Imposing, however, the restriction of using existing test section cross-sections at MIT, we propose use of the cross-sections used by P. Carasilesco (Ref. 27) or J. Kelly (Ref. 28). These cross-sections are shown in Figures (5.1) and (5.2) respectively. As we can see from these figures both cross-sections have a pair of open triangular subchannels. Therefore, they have to be isolated. However, it seems to be interesting to perform some measurements with open sub-channels and to compare with the corresponding measurements with the subchannels closed. For example it will be interesting to see the effect of this change on $\frac{\partial (pu)}{\partial x}$.
Fig. 5.1 Carajileskov's cross section.
Fig. 5.2 J. Kelly's cross section
5.3 Proposed Flow Pattern

It is evident that for the first experiments a single-phase flow has to be considered. The experiments can be later extended to two phase flow.

The next problem is the creation of the diversion cross flow. Diversion cross flows can be created by

1) Heating one of the simulated rods to produce an enthalpy gradient.

2) Closing one subchannel at a particular elevation by putting a blockage (See EIR experiment Ref. 23, 24).

3) Combination of the above two ways. Method 2 has the simplicity of having constant density and also there is experience (e.g. EIR experiment).

5.4 EIR Experiment

A schematic diagram of the experimental device and the test section is given in Fig. 5.3 (Ref. 24).

The test section consists of two rectangular subchannels connected by an adjustable gap formed by two oval rods (dumb-bell geometry). A movable filler plate was used as a partial blockage of one of the subchannels. The measurements were carried out in an open air rig. The inlet conditions for both subchannels could be regulated in such a way, that the static pressures as well as the mass flow were equal. The static pressure of each subchannel is
Fig. 5.3 Schematic diagram for the EIR experiment (Ref. 25, p. 176)
measured by means of 8 pressure taps, which were regularly distributed over the length. The mass flow distribution is found by integrating the velocity distribution which was measured by a pilot tube at the outlet of the test section. By displacing the blockage over the length of the test section one could find the entire pressure distribution and mass flow distribution downstream of the blockage. Indications of the mass flow distribution upstream of the blockage was gained by traversing the test section with a built in pitot tube.

The desirable gap length and hydraulic diameter are obtained by moving the oval rods and the walls opposite of the rods. The cases that are analysed are given in TABLE taken from Ref. 26, p. 11.

5.5 Conclusions from the EIR Experiment

As it is pointed out in Ref. 24, the following conclusions can be drawn from these experiments.

1) The upstream influence of a disturbance on the pressure difference and mass flow distribution could only be detected over a distance of about 8 hydraulic diameters.

2) A rearrangement of the mass flow distribution downstream of the disturbance takes place very slowly, whereas the pressure difference disappears within a short distance (also about 8 hydraulic diameters).
<table>
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<th>Case</th>
<th>$a$ (cm/ft)</th>
<th>$s$ (cm/ft)</th>
<th>$d_h$ (cm/ft)</th>
<th>$F_{tot}$ (cm²/ft²)</th>
<th>$F_{blocked}$ (\frac{k_n}{F_{total}})</th>
<th>$\dot{m}$ kg/(\text{m}^3/\text{sec})</th>
<th>$\dot{m}$ lb/(\text{m}^3/\text{sec})</th>
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</table>

$$\rho = 1.19 \text{ kg/ft}^3 = 0.0743 \text{ lb/ft}^3$$

\[\text{at 20}^\circ\text{C}\]

Table 5.1 EIR Experimental Parameters.
Looking upon the results given in References 23 and 24 and the analysis for the 3 first cases given in Table 5.1 made by Brown (Ref. 25) we can draw the following additional conclusions.

1) The analysis of these cases show that both cross flow velocity and gap axial velocity downstream blockage increase abruptly up to 4 to 6 hydraulic diameters and then start to decline (Fig. 2.5, p. 92, of the Ref. 25). This means that the inertial term in transverse momentum equation (Eqn. 3.95) can be zero somewhere in this region and also that the axial pressure losses (Eqn. 3.61) tend to be small somewhere in this region.

For this particular case

\[ \rho_{jas} \sim \rho_{ias} \sim \Delta P_{el} \]  

(5.1)

Thus the constant \( C_f \zeta \) in Eq. (3.71) could be measured immediately if we could obtain sufficiently accurate measurements.

For making measurements in this region in order to avoid a recirculation flow after the blockage a modification of the blockage shape may be necessary as shown in Fig. 5.4.
Fig. 5.4 Proposed Blockage Configuration for Measurements Near the Disturbance.

2) Another result shown from these experiments is that within the interval $8d_h$ upstream to (5 to 6) $d_h$ downstream ($d_h$ the hydraulic diameter at the test section) both the inertial and the frictional terms are both of the same sign. Therefore, the pressure difference $P_j - P_i$ in this region is relatively high. After this region the inertial term and friction term are of opposite signs thereby tending to eliminate the pressure difference. This explains the relatively fast disappearance of the big pressure differences at the blockage. If, really there is a region in which the pressure difference is zero, Equation (3.95) will become

$$-\frac{d}{dx} \left( \frac{L}{S} u^* W_j \right) \approx \Delta P_{\ell \ell}$$  \hspace{1cm} (5.2)
In this case, from measurements of $\Delta p_{\ell\ell}$ and $u^*$, the $\ell/s$ parameter is obtained.

3) Another conclusion is that the ratio of gap velocity to $\frac{u_i + u_j}{2}$ along $x$ $\frac{u_i}{(u_i + u_j)/2}$ takes values from 0.9 to 1.3 and varies with the distance (Ref. 25, p. 88). This means that

a) The value of unity for this ratio taken in axial momentum equation in COBRA III C is not valid in such high diversion cross flows.

b) If a correlation (2.72) for $u^*$ has to be used, the constants $C_{ij}$ and $C_{ji}$ have to be a function of diversion cross flow as well as geometry.

4) A last conclusion is that because the axial velocity gradient is very high, the constant $C_1$ (Eq. 2.75) seems to be affected significantly by $W_{ji}$. An estimation of $C_1$, therefore, has to be found as a function of $W_{ji}$. However the EIR experiment seems to have the following disadvantages.

1) The mass flow difference is given with a relatively high error.

2) The test-section geometry is not the same as triangular or square or side subchannel.

3) The given measurements are not adequate to calculate our parameters.

Therefore a more realistic geometry is needed and more detailed and accurate measurements have to be done.
More accurate measurements with respect to velocity field can be obtained by using the Laser Doppler Anemometer which is available at MIT. With this device a relatively high accuracy measurement of $\frac{\partial u}{\partial x}$ and the $V\phi$ field within $A_{ij}$ can be made.

5.6. Measurements

The quantities that have to be measured are

1) $V\phi$ velocity. It has to be measured within $A_{ij}$ for $\phi = \pm 30^\circ$.

2) u velocity. It will be measured along the r coordinate lines within $A_{ij}$ and in the rest of $A_i$ and $A_j$ as well.

3) Asymptotic measurements of $P_{ias}$ and $P_{jas}$ as and the detailed pressure field if possible.

5.7 Estimation of the Parameters

5.7.1 Parameters $\vec{\beta}_i$, $\ell/s$, $\Phi_i$

The value of the $\ell/s$ parameter or equivalently the behavior of the inertial term in the transverse momentum equation depends on the parameters $\vec{\beta}$ . Since $R_1 = R_2$ and $S_n$ is in the middle of the gap $\vec{\beta}_i$ is given by

$$\vec{\beta}_i = \vec{\beta}_2 = \frac{\vec{\beta}_i^1 + \vec{\beta}_i^2}{2}$$  \hspace{1cm} (5.3)
From Eq. (3.31)

$$\beta_i = \frac{1}{v_i A_i} \int \int \beta(r, \phi) \, dA$$  \hspace{1cm} (5.4)

and from Eq. (3.28)

$$\beta(r, \phi) = \frac{S_{ij}^*}{W_{ji}} V_{\phi_D}$$  \hspace{1cm} (5.5)

and

$$V_{\phi_D} = V_{\phi} - V_{\phi_{-sec}}$$  \hspace{1cm} (5.6)

The idea is that in order to estimate $\beta(r, \phi)$ the measurement of $V_{\phi}$ within $A_{ij}$ is necessary. Then taking into consideration the fact that we have to perform the integral of the form (5.4) the $V_{\phi_{-sec}}$ effect, especially for high $V_{\phi}$, will be negligibly small. Therefore we can substitute in (5.5) $V_{\phi}$ instead of $V_{\phi_{-sec}}$. Knowing $V_{\phi}$ we can calculate $\beta$'s and $l/s$ for various $\phi_{i1}$ as we
did in Chapter IV. We expect to achieve a curve similar to Figs. 0.1. The angle for which the flattening (the maximum) starts, will be the suggested $\phi_{i1}$. The suggested $\overline{\rho}^{'s}$ and $\ell/s$ will be those corresponding to this $\phi_{i1}$. The integral of $\rho V_f$ along the gap will give us the $W_{ji}$ and the integral of $\rho V_f$ along $S_f$ will give us $W(\phi)$, from which $q(\phi)$ is calculated from Equation (4.6). The quantity $q(\phi)$ can be calculated also by (4.12) in which case the measurement of $\frac{\partial(pu)}{\partial x}$ is needed which is much easier than the $V_f$ measurement. Based on this fact, a possibility can be examined to evaluate $\ell/s$ (or $\overline{\rho}^{'s}$) by measuring only $\frac{\partial(pu)}{\partial x}$.

Recall from (5.3) that

$$\overline{\rho}_i = \left( \overline{\rho}^{i1}_j + \overline{\rho}^{i1}_i \right) / 2$$

where $\overline{\rho}^{i1}_j$ (and similarly $\overline{\rho}^{i1}_i$) is given by (5.4) i.e.

$$\overline{\rho}^{i1}_j = \frac{1}{\overline{r}_i A_i}, \int A_i \rho(r, \phi) \, dA$$  \hspace{1cm} (5.4)$$

Taking into consideration the fact the $\rho(r, \phi)$ is given by (4.3) and

$$dA = rdrd\phi$$

we obtain
\[ \vec{P}_j = \frac{1}{r_{21}^2 A_{11}} \int_0^{\phi_{11}} q(\phi) d\phi \int_0^{\gamma + \phi} r^2 f(\gamma, \phi) d\gamma \quad (5.8) \]

It is evident from (5.8) that the knowledge of \( f(r, \phi) \) or more precisely of
\[ \int_0^{\gamma + \phi} r^2 f(\gamma, \phi) d\gamma \]

is necessary.

The ideal is as follows: Suppose that there exist one component \( \nu^{(\gamma)}_\phi (\gamma, \phi) \) with corresponding shape function \( f^{(\gamma)}(\gamma, \phi) \) such that
\[ \int_0^{\gamma + \phi} r^2 f(\gamma, \phi) d\gamma = 0 \quad (5.9) \]

This component \( \nu^{(\gamma)}_\phi \) has not effect on evaluating Eq. (5.8). Thus if we would substract all those components of \( \nabla \nu_\phi \) which satisfy (5.9) it would be possible to end up with an artificial velocity which will give the same results as the real velocity.
5.7.2 Estimation of $W(\phi)$, $\frac{\partial (\rho u)}{\partial x}$

The above quantities do not appear directly in the transverse momentum equations but they give us information on how the axial flow reacts to the diversion cross-flow. A plot of the quantities $W(\phi)/W_{ij}$ and $\frac{\partial (\rho u)}{\partial x}$ as a function of $A(\phi)$ or $A(\phi)/A_1$ would be very useful to understand these phenomena.

5.7.3 Estimation of $\varepsilon_{i1}$ (Eq. 3.40)

Measuring the axial velocity along $S_\phi$ for $\phi=\phi_{i1}$, finding its mean value and divided it by the mean velocity we get $\varepsilon_{i1}$. Working similarly we can find $\varepsilon_{j1}$.

5.7.4 Estimation of $u^*$ (Eq. 3.64)

The effective velocity $u^*$ is given by Eq. (3.64, i.e.,

$$u^* = \frac{u_{i1}^* \psi_i + u_{j1}^* \psi_j}{\psi_i + \psi_j}$$ (3.64)

which according to (3.16) becomes

$$u^* = \frac{\psi_{i1} u_{i1}^* + \psi_{j1} u_{j1}^* + \psi_{i2} u_{i2}^* + \psi_{j2} u_{j2}^*}{\psi_i + \psi_{i2} + \psi_j + \psi_{j2}}$$ (5.10)
The quantity $u_{i1}^*$ is given by (3.17a) which in this case becomes

$$u_{i1}^* = \frac{\int\int_{A_{i1}} \nabla \cdot v \phi u dA}{\psi_{i1}}$$ (5.1)

Measuring $\nabla \phi$ and $u$ we can calculate $u_{i1}^*$ and similarly the other $u_{ij}^*$ and then we can compare with the approximations given in § 3.1.3.2.

5.7. 5 Estimation of $C_f$ (Eq. 4.38)

Knowing $\frac{\partial (pu)}{\partial x}$ within $A_{ij}$, the quantity $q(\phi)$ can be calculated by (4.12). Then we can perform the integral (4.38) and find the quantity $C_f$.

5.7. 6 Estimation of $\zeta$ (Eq. 3.71) and $C$ (Eq. 3.63)

The quantities $\zeta$ and $C$ have to take values so that the pressure asymptotic difference between the subchannels as calculated equals those measured. For calculation of $\zeta$ a correlation such as (3.70) can be examined. In addition the existence of a point for which Eq. (5.1) is valid would help us to get an idea of the magnitude of $C_f \zeta$ at this particular lateral Reynolds number.
For the calculation of $C$ the value 0.185 given in Ref. 13 for fully developed flow can be examined.

5.7.7. Estimation of $C_1$ (Eq. 2.74)

$C_1$ can be calculated from Eq. (2.74) by using the measured axial velocity field within the subchannel $A_1$. Then a plot of $C_1$ vs $W_{j1}$ can be given.

5.8 Summary

Before finishing Chapter V which substantially is a proposal for experimental work, let us review the goals of this proposal.

It is proposed that a cross section such as given in Fig. 5.1 and 5.2 be used. The diversion cross flow is created by using a blockage as was done in the EIR experiment. The quantities that have to be measured are

1) Axial velocity distribution.

2) $V_\phi$ velocity distribution.

3) Asymptotic pressure difference and the pressure field if possible.

The measurement of the velocity field will be by a laser Doppler Anemometer available in MIT and the measurement of pressures by pitot tubes and static wall pressure taps.

By the above measurements the $l/s$ parameters and the
other parameters useful in the estimation of the various
terms in the Transverse Momentum balance, can be calculated.
The calculation of these constants will help us to examine:

1) The region of validity of the existing models
and the value of the suggested input parameters in this
region for these models, e.g. $\ell/s$ for COBRA IIIC.

2) In what regions the general equations as (3.96) or
(3.86) and (3.87) are needed and the proper expressions for
input parameters required by these general equations.
REFERENCES


APPENDIX A

Proof of Eq. 2.12

In this appendix proof of Equation (2.12) is presented.

The acceleration of a fluid particle is given by

(Ref. 6, Eq. 2.9b, p. 2-7).

\[
\frac{D \mathbf{w}}{Dt} = \nabla \left( \frac{\mathbf{w}^2}{2} \right) - \mathbf{w} \times (\nabla \times \mathbf{w}) + \frac{\partial \mathbf{w}}{\partial t} \tag{A-1}
\]

which for Cartesian system is reduced to

\[
\frac{D \mathbf{w}}{Dt} = \mathbf{w} \nabla \mathbf{w} + \frac{\partial \mathbf{w}}{\partial t} \tag{A-2}
\]

The x-component of \( \frac{D \mathbf{w}}{Dt} \) will be

\[
\left[ \frac{D \mathbf{w}}{Dt} \right]_x = \frac{\partial}{\partial x} \left[ \frac{u^2 + v_1^2 + v_2^2}{2} \right] - \left[ \mathbf{w} \times (\nabla \times \mathbf{w}) \right]_x + \frac{\partial u}{\partial t} \tag{A-3}
\]

Recall that

\[
\left[ \mathbf{w} \times (\nabla \times \mathbf{w}) \right]_x = v_1 \left[ \nabla \times \mathbf{w} \right]_2 - v_2 \left[ \nabla \times \mathbf{w} \right]_1 \tag{A-4}
\]

and

\[
\left[ \nabla \times \mathbf{w} \right]_1 = \frac{1}{\rho_2} \left( \frac{\partial u}{\partial x} - \rho_2 \frac{\partial v_2}{\partial x} \right) \tag{A-5a}
\]
\[
\left[ \nabla \times \mathbf{W} \right]_2 = \frac{1}{h_1} \left[ \frac{v_1}{h_1} \frac{\partial v_1}{\partial x} - \frac{\partial u}{\partial x_1} \right] \quad (A-5b)
\]

Substituting (A-5a) and (A-5b) into (A-4) we find
\[
\left[ \mathbf{W} \times (\nabla \times \mathbf{W}) \right]_x = \frac{\partial}{\partial x} \left( \frac{v_1^2 + v_2^2}{2} \right) - \frac{v_1}{h_1} \frac{\partial u}{\partial x_1} - \frac{v_2}{h_2} \frac{\partial u}{\partial x_2} \quad (A-6)
\]

Putting this result into (A-3) we end up with
\[
\left[ \frac{D\mathbf{W}}{Dt} \right]_x = u \frac{\partial u}{\partial x} + v_1 \frac{1}{h_1} \frac{\partial u}{\partial x_1} + v_2 \frac{\partial u}{h_2} \frac{\partial x_2}{\partial t} + \frac{\partial u}{\partial t} \quad (A-7)
\]

or
\[
\left[ \frac{D\mathbf{W}}{Dt} \right]_x = \mathbf{W} \cdot \nabla u + \frac{\partial u}{\partial t} \quad (A-8)
\]

This result was expected because x-axis is associated with the inertial system and additional terms as Conolis terms appeared for uni-inertial axes, here are zero.
APPENDIX B

Evaluation of Factor $C_i$ for Infinite Array in Fully Developed Turbulent Flow

Consider the characteristic triangle of the array as shown in Fig. B.1 on the next page.

The angle $\phi_o$ will be $\pi/4$ for square array and $\pi/6$ for triangular array.

Call $S$ the pith ratio and $A_o$ the flow cross section.

Then

$$A_o = R^2 \left( S^2 \frac{1}{2} g \phi_o - \phi_o \right) / 2 \quad (B-1)$$

The hydraulic diameter will be

$$D_e = \frac{4 A_o}{R \phi_o} \quad (B-2)$$

or

$$D_e / R = 4 \frac{A_o / R^2}{\phi_o} = \frac{1}{8} \frac{S^2 \frac{1}{2} g \phi_o - \phi_o}{\phi_o} \quad (B-2a)$$

Let us take the velocity distribution given in Ref. 10, i.e.
Triangular array: \( \phi_0 = \pi / 6 \)
Square array: \( \phi_0 = \pi / 4 \)

\[ \text{Surf}(bcde) = A_0 \]
\[ S = P / D \]

---

Fig. B.1 Characteristic Triangle for an Infinite Array.
\[ u^+(\phi, y) = 2.5 \ln \left( 1 + 0.4 y^+ \right) + 7.8 \left[ 1 - \exp \left( - \frac{y^+}{11} \right) \right] - \frac{y^+}{11} \exp \left( - 0.33 y^+ \right) \] 

\[ + 2.5 \ln \left[ \frac{1.5 \left( \frac{2 - (y/y_m)}{1 + 2 \left( 1 - (y/y_m) \right)^2} \right)}{1 + 2 \left( 1 - (y/y_m) \right)^2} \right] \] 

(B-3)

where

\[ u^+(\phi, y) = u(\phi, y) \sqrt{\frac{\tau_R}{\rho}} \] 

(B-4)

\[ y^+ = \frac{y}{\nu} \sqrt{\frac{\tau_R}{\rho}} \] 

(B-5)

g = the fluid density

\( \nu \) = the kinematic viscosity

\( \tau_R \) = the local wall shear stress

\( y_m \) = shown in Figure B.1

Equation B.3 has the simplicity that is approximately valid in the whole range of y. Other wave complicated distributions can be found in literature (e.g. in Ref. 11, p. 5).

If \( \tau_s \) is the mean shear stress in the rod surface, Eqs. (B-4) and (B-5) are written

\[ y^+ = \frac{y}{\nu} \sqrt{\frac{\tau_R}{\tau_s}} \sqrt{\frac{\tau_s}{\rho}} \] 

(B-6)
where \( \tau_s \) is defined as
\[
\tau_s = \frac{1}{\phi_0} \phi_0 \int_0^{\phi_0} \tau_R \, d\phi 
\] (B-7)

\[
u(\phi, \gamma) = \nu^+ \sqrt{\frac{\tau_R}{\tau_s}} \sqrt{\frac{\tau_s}{\phi^*}}
\] (B-8)

Using the equation
\[
\tau_s = \int \frac{\rho \overline{u}^2}{\rho} \quad \text{(B-9)}
\]

\[
\sqrt{\frac{\tau_s}{\phi^*}} = \overline{u} \sqrt{\frac{f}{\rho}}
\] (B-10)

Then (B-6) and (B-8) become
\[
\gamma^+ = \sqrt{\frac{f}{\rho}} \frac{\overline{u}}{\nu} \sqrt{\frac{\tau_R}{\tau_s}} = \sqrt{\frac{f}{\rho}} \frac{Re}{De} \sqrt{\frac{\tau_R}{\tau_s}}
\] (B-11)

\[
\nu(\phi, \gamma) \mid \overline{u} = \sqrt{\frac{f}{\rho}} \sqrt{\frac{\tau_R}{\tau_s}}
\] (B-12)
What remains is to give an expression for $\sqrt{\tau \tau_s}$.

Ibragimov et al (See Ref. 10) suggested the following expression since they fit the experimental data well.

$$\frac{\tau_R}{\tau_S} = f_1 \left[ 1 - \exp \left( -f_2 \frac{Y_m}{Y_{ms}} \right) \right]$$  \hspace{1cm} (B-13)

where $Y_m$ is shown in Fig. B.1

$Y_{ms}$ is the mean value of $Y_m$

i.e.

$$Y_{ms} = \frac{1}{\phi_o} \int_0^{\phi_o} Y_m d\phi = R \left[ \frac{S}{\phi_o} \ln \tan \left( \frac{\phi_o}{2} + \frac{\pi}{4} \right) - 1 \right]$$  \hspace{1cm} (B-14)

$f_2$ is a parameter which is generally dependent on Reynolds number and $f_1$ is found from the normalizing condition (B-7).

For most cases of our interest the Reynolds number dependence can be neglected even for close spacing array (P/D = 1) (Ref. 10 p.393) In this case the following expression is given for $f_2$

$$f_2 = 7.7 \left[ \frac{A_o}{Y_{ms}} \right]^{-0.8} = 7.7 \left[ \frac{\left( \frac{s}{\phi_o} + \frac{\phi_0}{2} \right) - 1}{\frac{S}{\phi_o} \ln \tan \left( \frac{\phi_o}{2} + \frac{\pi}{4} \right) - 1} \right]^{0.8}$$  \hspace{1cm} (B-15)
The expression for \( f_1 \) is given by

\[
f_1 = \left[ \frac{1}{\phi_0} \int_0^{\phi_0} \left[ 1 - \exp \left( -f_2 \frac{\gamma_m}{\gamma_{m_0}} \right) \right] d\phi \right]^{-1}
\]  \hspace{1cm} (B-16)

Now that everything is defined we return to the expression of \( C_i \) given in Eq. (2.75) i.e.

\[
C_i \approx \frac{1}{A_i} \iint_{A_i} \left( \frac{u}{u_i} \right)^2 dA
\]  \hspace{1cm} (B-17)

One can use the Eq. (B-12) for \( u/u_i \) but because the above expressions are approximations it is possible that \[
\frac{1}{A_0} \iint u dA \neq \overline{u}
\] instead of being equal as we might expect. Therefore it is better to calculate \( u_i \) from

\[
u_i = \frac{1}{A_0} \iint_{A_0} u dA = \frac{\overline{u} \sqrt{\frac{r}{8}}}{A_0} \int_0^{\phi_0} \int_0^{\gamma_m} d\phi \sqrt{\frac{r}{2s}} \int_0^{u(\phi,y)(\gamma_0)} dy
\]  \hspace{1cm} (B-18)
\[ u_2^q = \frac{f}{A_o^2} \bar{u}^q \left[ \int_0^{\phi_o} d\phi \sqrt{\frac{\tau_{\phi}}{\tau_5}} \left( \int_0^{\gamma_m} u^+(\phi, \gamma) (R+\gamma) d\gamma \right)^2 \right] \quad (B-19) \]

The expression \( \frac{1}{A_o} \int_{A_o} u^q dA \) will be

\[ \frac{1}{A_o} \int_{A_o} u^q dA = \frac{f}{A_0} \bar{u}^q \left[ \int_0^{\phi_o} d\phi \frac{\tau_{\phi}}{\tau_5} \left( \int_0^{\gamma_m} [u^+(\phi, \gamma)]^2 (R+\gamma) d\gamma \right) \right] \quad (B-20) \]

Then \( C_i \) will be

\[ C_i = A_0 \cdot \frac{\int_0^{\phi_o} d\phi \frac{\tau_{\phi}}{\tau_5} \left( \int_0^{\gamma_m} [u^+(\phi, \gamma)]^2 (R+\gamma) d\gamma \right)}{\left[ \int_0^{\phi_o} d\phi \sqrt{\frac{\tau_{\phi}}{\tau_5}} \left( \int_0^{\gamma_m} u^+(\phi, \gamma) (R+\gamma) d\gamma \right)^2 \right]^2} \quad (B-21) \]
A program has been written to calculate $C_i$. The listing is given in pages 193 to 196. The results for various pitch ratios are given in pages 197 to 204. The curves $C_i$ vs Re and $C_i$ vs P/D are given in Chapter II (Figs 2.2 and 2.3).
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<tr>
<td>N1</td>
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<td>No of pitch ratios</td>
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| N2               |           | 1 for triangular  
|                  |           | 2 for square     |
| NX               |           | No. of intervals for subroutine SINTER |
| NY               |           | No. of intervals for Y in subroutine SINTER |
| NZ               |           | No. of intervals in subroutine CALF2. |
C C

MAIN PROGRAM

DIMENSION S0(30), RE0(15)
READ(5,80) N1, N2, NX, NY, N3, NR
80 FORMAT(6I5)
READ(5,70) CA, CB, CC
70 FORMAT(3F10.5)
READ(5,60) (SO(I), I=1,N1)
60 FORMAT(F5.3)
READ(5,90) (RE0(J), J=1,NR)
90 FORMAT(E10.3)
DO 200 J=1,NR
RE=RE0(J)
F=CA*EXP(CB*ALOG(RE))*CC
CF=SQR(F/8.)
IF(N2.EQ.2) GO TO 65
WRITE(6,180)
180 FORMAT(1H1, "TRIANGULAR ARRAY")
GO TO 75
65 WRITE(6,185)
185 FORMAT(1H1, "SQUARE ARRAY")
75 WRITE(6,190) RE, F
190 FORMAT(/,12X, "RE NUMBER =", E10.3, /, "FRICTION FACTOR =", F10.4)
181 FORMAT(1X, F6.4, 1X, F6.4, 1X, F5.3)
WRITE(6,181) CA, CB, CC
WRITE(6,170)
DO 100 I=1,N1
S=SO(I)
IF(N2.EQ.1) GO TO 115
X0=3.14159/4.
GO TO 125
100 X0=3.14159/6.
115 AO=(S*S*ТАN(X0)-X0)/2.
DE=AO/X0
YMS=(S/X0)*ALOG(TAN((2.*X0+3.14159)/4.))-1.
F2=7.7*EXP(-.8*ALOG(AO/YMS**2))
CALL CALFZ(S, YMS, X0, N3, F2, SUMZ)
F1=1./SUMZ
CALL SINTER(NX, NY, SS, X0, S, F1, F2, RE, DE, YMS, AO, CF, SUMX)
C=SS
C2=SUMX/(SS*SS)
WRITE(6,160) S, AO, C, C2, YMS, DE, CI
160 FORMAT(7F10.4)
100 CONTINUE
200 CONTINUE
STOP
END
SUBROUTINE CALF2(S,YMS,X0,N3,F2,SMZ)
DZ=X0/N3
Z=0.0
KZ=0
SZ=0.0
ZOLD=0.0
55 YM=S/COS(Z)-1.
CZ=(1.-EXP(-F2*YMS)/X0
IF(KZ.GT.0) GO TO 45
CZ0=CZ/2.
45 ZNEW=CZ
SZ=SZ+(ZNEW+ZOLD)/2.
Z=Z+DZ
IF(Z.GT.X0) GO TO 50
KZ=KZ+1
ZOLD=ZNEW
GO TO 55
50 SUMZ=(SZ-CZO)*DZ
RETURN
END
SUBROUTINE SINTER(NX,NY,SS,X0,S,F1,F2,RE,DE,YMS,A0,CF,SUMX)
DX=X0/NX
X=0.
XX=0.
SX=0.
XOLD=0.
SU=0.
XUOLD=0.

35 YM=S/COS(X)-1.
   TR=F1*(1.-EXP(-F2*YM/YMS))
   DY=YM/NY
   Y=0.
   KY=0
   SY=0.
   SYY=0.
   UOLD=0.
   WOLD=0.

15 YS=Y*RE*CF*SQRT(TR)/DE
   WS1=2.5*ALOG((1.+4.*YS)
   WS3=2.5*ALOG((1.+2.*(1.-Y/YM)/(1.+2.*(1.-Y/YM)*2))))
   IF(YS.LE.520.0) GO TO 40
   IF(YS.LE.1950.0) GO TO 41
   WS=WS1+WS3 +7.8
   GO TO 45

41 WS2=7.8*(1.-EXP(-YS/11.))
   WS=WS1+WS2+WS3
   GO TO 45

40 WS4=7.8*YS*EXP(-33*YS)/11.
   WS2=7.8*(1.-EXP(-YS/11.))
   WS=WS1+WS2+WS3-WS4

45 WNEW=WS*WS*(Y+1.)
   UNEW=WS*(Y+1.)
   SY=SY+(WOLD+WNEW)/2.
   SYY=SYY+(UNEW+UOLD)/2.
   Y=Y+DY
   IF(Y.GT.YM) GO TO 20
   KY=KY+1
   WOLD=WNEW
   UOLD=UNEW
   GO TO 15

20 SUMY=SY*DY
   SUMU=SYY*DY
   CX=SUMY*CF*CF*TR/A0
   CUX=SUMU*CF*SQRT(TR)/A0
   IF(KX.GT.0) GO TO 25
   CX0=CX/2.
   CUX0=CUX/2.

25 XNEW=CX
   XNEW=CUX
   SX=SX+(XOLD+XNEW)/2.
   SU=SU+(XUOLD+XNEW)/2.
   X=X+DX
   IF(X.GT.X0) GO TO 30
   KK=KK+1
XOLD=XNEW
XUOLD=XUNEW
GO TO 35
30 SUMX=(SX-CXO)*DX
SS=(SU-CUXO)*DX
RETURN
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APPENDIX C

Calculation of the Various Constants
For Transverse Equation

C-1 Parameter $l/s$

The parameter $l/s$ is calculated from the relation

$$(4.28)\ i.e.,$$

$$\frac{l}{s} = \frac{2}{\alpha_i} \frac{\bar{v}_{i1} A_{i1} \bar{B}_{j1}^{i1}}{s_{i1} A_{i1} \bar{B}_{j1}^{i1}} = 4 \frac{s_{o1}}{s_{i1}} \frac{\bar{v}_{i1} A_{i1} \bar{B}_{j1}^{i1}}{\alpha_i s_{o1}} \quad (C-1)$$

where according to (4.22), (4.24) and (4.29)

$$A_{i1} |_{R}^{2} = \frac{1}{2} \left[ \left( \frac{R+s_{o1}}{R} \right)^{2} \tan \phi_{i1} - \Phi_{i1} \right] \quad (C-2)$$

$$\bar{v}_{i1} |_{R} = \frac{1}{3} \left( \frac{R+s_{o1}}{R} \right)^{3} \left[ \frac{1}{\cos \phi_{i1}} + \ln \left[ \tan \left( \frac{n}{4} + \frac{\phi_{i1}}{2} \right) \right] \right] - \Phi_{i1} \quad (C-3)$$

The parameter $\bar{B}_{j1}^{i1}$ is given by (4.21) i.e.,
\[
\overline{\beta}_{i,j}^{11} = \frac{1}{r_{i,j} A_{i,j}^{11}} \int_0^{R+S_{\phi}} d\phi \int_0^R \beta_{i,j}^{11}(r,\phi) r^2 dr \tag{C-4}
\]

which according to (4.3) and (4.18)

\[
\overline{\beta}_{i,j}^{11} = \frac{1}{r_{i,j} A_{i,j}^{11}} \int_0^{R+S_{\phi}} \phi_{i,j} \left( \frac{S_{o}}{S_{ij}} - \frac{A(\phi)}{A_i} \right) \int_0^R f(r,\phi) r^2 dr \tag{C-5}
\]

Call

\[
I = \frac{S_{i,j}}{S_{\phi}} \int_0^{R+S_{\phi}} f(r,\phi) r^2 dr \tag{C-6a}
\]

and

\[
B = \frac{S_o}{S_{i,j}} - \frac{A(\phi)}{A_i} \tag{C-6b}
\]

Then

\[
\overline{\beta}_{i,j}^{11} = \frac{1}{r_{i,j} A_{i,j}^{11}} \int_0^{R} B I d\phi \tag{C-7}
\]
The integral I is dependent on the velocity shape. It has been calculated for both uniform and parabolic velocity distributions.

C-1-1 Uniform Distribution

In this case according to (4.19)

\[ \int (r, \phi) = 1 \] \hspace{1cm} (C-8)

Putting this expression to (C-6a) we find

\[ \frac{I}{R^3} = \frac{S_{ij}}{R} \left( 1 + \frac{S_\phi}{R} + \frac{(S_{ij} | R|^2)}{3} \right) \] \hspace{1cm} (C-9)

The calculation of the integral in Eq. (C-7) has been accomplished numerically. The listing of the calculational program together with the calculation of the other parameter is given at the end of this Appendix.

C-1-2 Parabolic Velocity Distribution

In this case (Eq. 4.20)

\[ \int (r, \phi) = 6 \frac{S_0}{S_{ij}} \left( \frac{r-R}{S_\phi} \right) \left( 1 - \frac{S_0}{S_{ij}} \frac{r-R}{S_\phi} \right) \] \hspace{1cm} (C-10)
Substituting in Eq. (C-6a) we find

$$\frac{i}{R^3} = 6 \frac{S_{ij}}{R} \frac{S_0}{S_{ij}} \left[ \frac{5}{3} - \frac{1}{3} \left( \frac{S_0}{S_{ij}} \right) \right] + \left( \frac{9}{3} - \frac{1}{2} \frac{S_0}{S_{ij}} \right) \frac{S_0}{R} +$$

$$+ \left( \frac{1}{4} - \frac{1}{5} \frac{S_0}{S_{ij}} \right) \left( \frac{S_0}{R} \right)^2 \right] \tag{C-11}$$

$\tilde{B}_{ij}$ is found from (C-7) numerically.

C-1-3 Results

The calculations have been done for $\phi = 1^\circ + 50^\circ$ for square array and $\phi = 1^\circ + 35^\circ$ for triangular array.

Also the calculations have been done using pitch ratios from 1.05 to 2.0.

The $\tilde{B}$'s and $\ell/s$ as a function of $\phi$, and pitch ratios for both square and triangular subchannels are given in pages 224 to 247. A graphical representation of $\ell/s(\phi)$ as a function of $\phi$ is given in Figs C-1. This figure show that $\ell/s$ increases to a maximum and then decreases. Additionally taken into consideration the fact that for the usual case of low diversion cross flow the friction terms in Eq. (3.75) are small compared with the inertial term and the velocity $u^*$ does not change substantially from $\phi=0$ to $\phi$ under consideration, the graph of $\ell/s$ vs $\phi$. 
is approximately the pressure profile in the transition region between assumed pressure plateaus. Thus the curve $\ell/s$ after the $(\ell/s)_{\text{max}}$ cannot represent the real situation due to

a) Possibility of flow separation as a reaction to tendency of the fluid to increasing to pressure in the diverting part (Ref. 22, p. 28).

b) Effects of the other subchannels at large angles.

c) Possibility of error in expressing the $\nabla \phi$-field.

d) Going to large angles a correction may be needed to the momentum equation to take into consideration radial change in pressure.

Therefore it seems reasonable to assume a flattening of the pressure after $\phi_{\text{max}}$, which means that the angle $\phi_{i_{11}}$ will be the angle $\phi_{\text{max}}$.

Thus corresponding to each P/D ratio, we have one single value for $\phi_{i_{11}}$ and one single value for $\ell/s$. These values for both triangular and rectangular array are given in figures C-2 and C-3. We see the curves $\ell/s$ vs P/D and $\phi_{i_{11}}$ vs P/D. In both these figures the parabolic velocity distribution is considered. However comparison with the uniform velocity distribution has shown:

a) The velocity shape becomes less important for low P/D ratios.

b) The angle $\phi_{i_{11}}$ does not depend significantly on the velocity profile.
Looking upon the change of $\phi_{14}$ with P/D we see that $\phi_{14}$ is small for low P/D which means that the more "closed" the subchannel, the more insensitive to "external" pressure effects which is very reasonable.

However, as we can see from figure C-2, $l/s$ (and consequently the pressure $\Delta p_{ij}$) is much larger for low P/D subchannels. This reflects the fact that a subchannel with low P/D (i.e. smaller gap) needs higher pressure differences to create the same condition of diversion flow as a subchannel with high P/D.

Also from the Diagrams we can see that for the same P/D ratio, the triangular array shows slightly less $l/s$ which means that the triangular array experiences more mixing than does the square array.

A final conclusive drawn from the Diagrams C-1 is that for low P/D the value of $l/s(\phi)$ for $\phi>\phi_{\text{max}}$ decreases faster than for high P/D. This means that the tendency for flow separation would be stronger for low P/D cases versus high P/D cases.

In the Tables C-1, C-2 the values of $l/s$ and $\phi_{14}$ are given as a function of the P/D and the array geometry for the parabolic velocity distribution.
C-2 Other Parameters

The calculation of the other parameters is straightforward. \( \eta_{i1} \) is calculated from Eq. (4.50), \( C_4 \) from Eq. (4.37) and \( S_{i1} \) from (4.47). The values of \( |\eta_{i1}| \) and \( S_{i1}/R \) are given as a function of \( \phi \) in the program output (pages 224 to 227). These values \( \eta_{i1} \) and \( S_{i1}/R \) corresponding to the suggested value of \( \phi_{i1} \) are given in Tables C-1 and C-2. Figure C-4 shows \( C_4 \) vs P/D for both triangular and square arrays.
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<th>Definitions</th>
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|                  | $=2$: Square array,  
|                  | $=3$: Side          |
| $N_3$            |           | Number of $S_{ij}$ ratios |
| $RL$             | $S_{ij}$  | (Eq. 4.8a) |
| $SURF$           | $A(\phi)/R^2$ | Subchannel area over square of the rod radius. |
| $RSMN$, $R$-MEAN | $F_{il}/R$ | (Eq. C.3) |
| $ETA$            |           | (Eq. 4.50) |
| $BETA-1$         |           | Eqs. (C.7) and (C.9)  
|                  |           | (i.e., uniform velocity distribution) |
| $BETA-2$         |           | Eqs. (C.7) and (C.11)  
<p>|                  |           | Parabolic velocity distribution |</p>
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<td>Eq. (C-1)(Uniform velocity distribution)</td>
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Table C.1  Calculated parameters for square array.

| $p/D$ | $\phi_{i1}$ | $A_{i1}/R^2$ | $\tau_{i1}/R$ | $\overline{P}_{ij}$ | $\ell/S$ | $C_F$ | $\overline{s}_{i1}/R$ | $|\overline{n}_{i1}|$ |
|-------|-------------|---------------|----------------|---------------------|---------|-------|----------------------|----------------|
| 1.05  | 16          | 0.0184        | 1.0336         | 0.782               | 3.087   | 0.974 | 0.0665               | 0.026           |
| 1.10  | 21          | 0.049         | 1.0664         | 0.796               | 2.140   | 0.949 | 0.1300               | 0.049           |
| 1.15  | 24          | 0.085         | 1.0985         | 0.809               | 1.723   | 0.931 | 0.1913               | 0.071           |
| 1.20  | 27          | 0.1312        | 1.1327         | 0.809               | 1.477   | 0.913 | 0.2555               | 0.091           |
| 1.25  | 28          | 0.1711        | 1.1631         | 0.828               | 1.312   | 0.904 | 0.3119               | 0.109           |
| 1.30  | 30          | 0.2261        | 1.1975         | 0.828               | 1.193   | 0.892 | 0.3750               | 0.127           |
| 1.35  | 31          | 0.277         | 1.2298         | 0.838               | 1.102   | 0.885 | 0.4334               | 0.143           |
| 1.40  | 32          | 0.3331        | 1.2628         | 0.845               | 1.030   | 0.878 | 0.4926               | 0.158           |
| 1.45  | 32          | 0.3776        | 1.2923         | 0.862               | 0.972   | 0.876 | 0.5451               | 0.171           |
| 1.50  | 33          | 0.4426        | 1.3265         | 0.865               | 0.923   | 0.870 | 0.6055               | 0.185           |
| P/D | $\phi_{i1}$ | $A_{i1}/R^2$ | $\bar{T}_{i1}/R$ | $\bar{\rho}_{j1}$ | $\ell/s(\phi)$ | $C_f$ | $\bar{s}_{i1}/R$ | $|\eta_{i1}|$ |
|-----|-------------|--------------|-----------------|-------------------|----------------|------|----------------|----------|
| 1.05 | 16          | 0.0184       | 1.0336          | 0.754             | 2.976          | 0.907 | 0.066          | 0.026    |
| 1.10 | 20          | 0.0457       | 1.0647          | 0.767             | 2.015          | 0.852 | 0.127          | 0.048    |
| 1.15 | 23          | 0.0800       | 1.096           | 0.766             | 1.597          | 0.813 | 0.187          | 0.069    |
| 1.20 | 25          | 0.1176       | 1.1277          | 0.770             | 1.355          | 0.788 | 0.246          | 0.088    |
| 1.25 | 26          | 0.154        | 1.157           | 0.782             | 1.195          | 0.774 | 0.302          | 0.106    |
| 1.30 | 27          | 0.1949       | 1.1883          | 0.789             | 1.079          | 0.761 | 0.358          | 0.122    |
| 1.35 | 28          | 0.240        | 1.2196          | 0.793             | 0.992          | 0.749 | 0.415          | 0.138    |
| 1.40 | 29          | 0.290        | 1.2517          | 0.794             | 0.923          | 0.738 | 0.473          | 0.153    |
| 1.45 | 30          | 0.345        | 1.2844          | 0.794             | 0.868          | 0.727 | 0.531          | 0.168    |
| 1.50 | 30          | 0.388        | 1.314           | 0.807             | 0.822          | 0.726 | 0.584          | 0.180    |
Figure C.1a. Square array.

$\psi_s(\phi)$ vs $\phi$

a: Parabolic velocity
b: Uniform velocity
Fig. C.1b  Triangular array. $\ell / D(\phi)$ vs $\phi$

a: Parabolic velocity
b: Uniform velocity
Fig. 4.8: $\theta/s$ vs $P/D$  
Parabolic velocity

- Square
- Triangular
MAIN PROGRAM

DIMENSION PHI(20),SO(10),SAO(10),SSO(10)
READ (5,10) N,N1,N2,N3,RL
10 FORMAT(4I5,F5.3)
READ (5,20) (PHI(I),I=1,N1)
20 FORMAT(F5.3)
READ (5,60) (SO(J),SAO(J),SSO(J),J=1,N3)
60 FORMAT(F5.3)
DO 200 J=1,N3
S=SO(J)
SA=SAO(J)
SS=SSO(J)
IF(N2<2) S,6,7
5 WRITE(6,65)
GO TO 34
6 WRITE(6,66)
GO TO 34
7 WRITE(6,67)
65 FORMAT(1H1,5X,'TRIANGULAR')
66 FORMAT(1H1,5X,'SQUARE')
67 FORMAT(1H1,5X,'SIDE SUBCHANNEL')
34 WRITE(6,70) SO(J)
70 FORMAT(///,'14HPITCH RATIO = F5.3')
WRITE(6,30)
30 FORMAT(///,5X,'PHO',7X,'SURF',6X,'R-MEAN',4X,'BETA1',5X,'BETA2',1
5X,'L/S=1',5X,'L/S-2',5X,'ETA',7X,'CIJ',7X,'S-MEAN')
DO 100 I=1,N1
PHO=PHI(I)*3.14159/180.
IF(N-2) 25,26,27
25 A0=1.732*S*S-3.14159/2.
PHS=3.14159/6.
GO TO 35
26 A0=4.*S*S-3.14159
GO TO 35
27 A0=2.*S*SA+SS*(SA-1.)-3.14159/2.
35 CONTINUE
CALL DINTER (PHO,N,N2,S,RL,SM1,SM2,SM3,AR1,SM4)
AR1=SUM1
AR2=SUM2
AR3=SUM3
AR4=SUM4
SURF = (S*S*SIN(PHO)/COS(PHO)-PHO)/2.
THO=.5*PHO+.25*3.14159
DHO=SIN(THO)/COS(THO)
RINT = (0.5*SIN(PHO)/(COS(PHO)*COS(PHO))*S*ALOG(DHO))*S*S*S/3.
RINT=RINT-PHO/3.
RSMN=RINT/SURF
SII= S*S*S*TAN(PHO)/COS(PHO)*(S*S*S-2.*S)*ALOG(TAN(THO))
SII=(SII-2.*S*S*TAN(PHO)+2.*PHO)/(SURF+4.)
BETA1=AR1/RINT
BETA2=AR2/RINT
ETA=AR3/RINT
CIJ=AR4/PHO
SPH=(S/COS(PHO))**2-1.
RAT1 = 4.0 * RINT * RL * BETA1 / (SPH * (S-1))
RAT2 = 4.0 * RINT * RL * BETA2 / (SPH * (S-1))
WRITE (6, 40) PHI(I), SURF, RSMN, BETA1, BETA2, RAT1,
1 RAT2, ETA, CIJ, SI1
40 FORMAT (5X, 11(F8.4, 2X))
100 CONTINUE
200 CONTINUE
STOP
END
SUBROUTINE DINTER(PH0, N, N2, S, RL, SUM1, SUM2, SUM3, AO, SUM4)
DX=PH0/N
GAP=(S-1)/RL
X=0.
K=0
S1=0.0
S2=0.0
S3=0.0
S4=0.0
F10LD=0.0
F20LD=0.0
F30LD=0.0
F40LD=0.0
A=(S*S*SIN(X)/COS(X)-X)/2.
SM=S/COS(X)-1.
Y=A/A0
B=RL*Y
C1=1.0*1.0*(1.0+SM+SM*SM/3.0)*GAP
C2=6.0*RL*1.0*(1.0-5.0-RL/3.0+SM*SM*SM*2.0+M/2.0+M/3.0)*GAP
C3=1.0*SM*(SM+2.0)*SM*2.0/3.0*1415926535.8979
C4=(B/RL)**2
IF(K.GT.0) GO TO 25
F1BEG=C1*B/2.
F2BEG=C2*B/2.
F3BEG=C3/2.
F4BEG=C4/2.
25 F1NEW=C1*B
F2NEW=C2*B
F3NEW=C3
F4NEW=C4
S1=(F10LD+F1NEW)/2.*S1
S2=(F20LD+F2NEW)/2.*S2
S3=(F30LD+F3NEW)/2.*S3
S4=(F40LD+F4NEW)/2.*S4
X=X+DX
IF(X.GT.PH0) GO TO 20
K=K+1
F10LD=F1NEW
F20LD=F2NEW
F30LD=F3NEW
F40LD=F4NEW
GO TO 15
20 SUM1=(S1-F1BEG)*DX
SUM2=(S2-F2BEG)*DX
SUM3=(S3-F3BEG)*DX
SUM4=(S4-F4BEG)*DX
RETURN
END
**SQUARE**

**ITCH RATIO = 1.050**

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