# Hardware \& Software Architecture for Multi-Level Unmanned Autonomous Vehicle Design 

by

Jesse H. Z. Davis

# Submitted to the Department of Electrical Engineering and Computer Science in Partial Fulfillment of the Requirements for the Degrees of Bachelor of Science in Electrical Engineering and Master of Engineering in Electrical Engineering and Computer Science at the Massachusetts Institute of Technology 

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#### Abstract

1.0 Abstract

The theory, simulation, design, and construction of a radically new type of unmanned aerial vehicle (UAV) are discussed. The vehicle architecture is based on a commercially available non-autonomous flyer called the Vectron Blackhawk Flying Saucer. Due to its full body rotation, the craft is more inherently gyroscopically stable than other more common types of UAVs. This morphology was chosen because it has never before been made autonomous, so the theory, simulation, design, and construction were all done from fundamental principles as an example of original multi-level autonomous development.


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### 4.0 Overview

One of the latest pushes to advance aerial technology has been towards creating robust unmanned autonomous vehicles, UAVs. The applicability of these devices is widespread and ranges from military to commercial to consumer utility. Uses include, for example, possibilities of reconnaissance and supply delivery for the military, hazardous or difficult terrain management for commercial businesses, and advanced technological hobbies and toys for civilians. With such a large market for the devices, research in the UAV field has been rapidly increasing, and interestingly, over a wide range of scale factors. The military has been working towards developing full-scale autonomous stealth reconnaissance and troop transport multi-mode aircraft; NASA has been holding design competitions for meter-scale UAVs for extraterrestrial exploration; Sikorsky Corporation has released its mid-sized Cypher I UAV; several academic institutions around the country and the world have been developing hobby helicopter UAV systems, and there is even research going on at Stanford into centimeter-scale devices (mesiocopters).

With such a large breadth of research and, in some cases, development also comes many new technological and scientific needs. UAV research has sparked advances in other fields such as material science for stronger lighter weight components, fuel cell, advanced battery, and non-gasoline engine technology for lighter weight, greater efficiency, and longer range possibilities, and microelectronics, micro-sensor, and microelectromechanical systems (MEMS) for smaller and more robust control. Since the technology transfer from UAV research promises great strides in many disciplines, and the value of their development affects so many levels of society, the amount of attention and support that UAVs have been receiving should not be surprising.

One particular area of UAV research that has not been adequately explored is the vehicle morphology. When the requirement of a large cockpit space is eliminated, what other flight mechanisms or craft designs may be employed to most efficiently enhance performance and reduce power and control requirements? For foot-scale craft, is a simple downsizing of the larger scale manned aircraft the best solution? To answer the first question, many different types of designs have been tested or simulated including small airplanes and craft's with coaxial rotors, multiple rotors, ducted fans, or tilt-rotors, but no definitive best solution has yet been found. To answer the second question, the flight dynamics of smaller scale craft can be very different from those for larger craft, and as such, it is most likely not a simple downsizing that will prove to be most effective.

It is the goal of this thesis to present a radically new type of small scale UAV that will have expandable multi-level autonomous capabilities. This vehicle design has never before been made autonomous, and even a user controlled version of the craft has only been invented within the past two years. It is the hope of the author that this particular UAV design will lay a foundation for developing this and other radical vehicle morphologies as yet unexplored. In the subsequent sections of this paper, first the mathematical theory of the craft will be examined, second the theoretical control algorithms derived will be simulated on a state space model of the craft, and third the design and construction of a complete user interface and on-board automating system will be explained. This progression should provide the reader a complete picture of the development cycle necessary to create a new type of UAV.

### 5.0 Theory

In order to develop the theory of operation of the rotating rotorcraft, the basic morphology must first be understood. As can be seen in Figure 5.1 below, the craft exhibits a bicycle wheel structure with an inner hub at the center and three spokes to an outer hoop.


Figure 5.1: Rotating Rotorcraft Sketch
The hub is de-spun from the rest of the craft by a yaw control motor mounted between the hub and spokes so that the hub can serve as a stable platform for the autopilot. Each spoke has on it a motor with propeller that is oriented in such a way as to provide most of its thrust normal to the plane of the hoop. (The motors are actually set all at a slight angle towards the direction of rotation of the hoop in order to aid the rotation. This will be termed the advance angle of the propellers, and explained later.) There are several different theoretical approaches that aid in the understanding, control, and engineering of the craft. The main aspects to be considered here are the full non-linear state-space description of the craft as well as a simple control and steering methodology.

### 5.1 Conservation of Angular Momentum

At the most basic level, it must first be shown that the hoop will actually spin as shown and described above (even without the advance angle of the propellers). If the craft is initially at rest, and the motors are then turned on, it is apparent from the Jesse Davis Professor Charles Coleman

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conservation of angular momentum that the angular momentum of the entire closed system must be zero:

$$
\begin{equation*}
\frac{\partial}{\partial t} A=0 \& A(0)=0 \rightarrow A(t)=0 \tag{5.1}
\end{equation*}
$$

Since, as drawn in Figure 5.1, the motors all spin the same direction, there are three angular momentum vectors from the propellers' rotation oriented in the negative z direction (into the paper for the top view, and down for the side view of Figure 5.1). In order for the total angular momentum to be zero, with the motors firmly mounted to the frame, the entire outer assembly must rotate in the opposite direction creating an angular momentum vector in the positive z direction to exactly counter the propellers' rotation (ignoring frictional losses in the motor).

### 5.2 State-Space Description

The next, much more complex, theoretical step to be taken is to develop the statespace description of the entire system. There are several sections of the state-space model. The sections that will be discussed are as follows: motor actuation, craftreferenced three-dimensional translation, craft-referenced three-dimensional rotation, and Earth-referenced three-dimensional translation and rotation.

### 5.2.1 DC Voltage Driven Motor

Each motor has states associated with it due to the inductance of the motors and the inherent back voltage feedback loop of a voltage driven motor. (The choice to drive the motors with voltage control rather than current control is made based on the ease of creating pulse-width modulated (PWM) voltage signals and also to take advantage of the inherent speed stabilizing back voltage feedback.)

The fundamental concept that is used in a DC motor is that a current carrying wire in a magnetic field is subject to a force. Also, a current will be induced in any wire loop subjected to a magnetic field, and this current will in turn cause the wire loop to experience an opposing force. Controlling either the magnetic field around these loops or the current flowing through them results in control of the torque and rotation of anything attached to loops. See Figure 5.2 below:


Figure 5.2: Simplified DC Motor Model
From fundamental electromagnetic physics, the torque on the wire loops is given by:

$$
\begin{equation*}
T=N R_{l} l K i_{f} i_{a} \tag{5.2}
\end{equation*}
$$

where N is the number of loops in the magnetic field, $\mathrm{R}_{1}$ is the radius of the loops in the field, 1 is the length of the cylindrical wire loop windings into the page, $i_{\mathrm{f}}$ is the current flowing in a wire wrapped around the magnetic core, K is a proportionality constant between $i_{f}$ and the magnetic field $B$, and $i_{a}$ is the current in the wire loops. (See References 9 and 14.) In a voltage controlled motor, the voltage is applied across wire loops so as to cause the current $\mathrm{i}_{\mathrm{a}}$, and in this case, all of the other variables in Equation (5.2) can be lumped into a motor proportionality constant, $\mathrm{K}_{\mathrm{m}}$. If the torque is applied to Jesse Davis
some motor inertia, $\mathrm{I}_{\mathrm{m}}$, plus some load inertia, $\mathrm{I}_{\mathrm{l}}$, an angular speed, b , will result. The relationship between torque and angular speed is given by:

$$
\begin{equation*}
T=K_{m} i_{a}=\left(I_{m}+I_{l}\right) \frac{\partial b}{\partial t} \tag{5.3}
\end{equation*}
$$

The motor itself can be modeled as a series connected inductance, L, and resistance, $R_{\Omega}$, and thus if a voltage, V , is across the motor terminals, the relationship between the voltage and current $\mathrm{i}_{\mathrm{a}}$ will be:

$$
\begin{equation*}
V=i_{a} R_{\Omega}+L \frac{\partial i_{a}}{\partial t} \tag{5.4}
\end{equation*}
$$

However, the applied voltage, $\mathrm{V}_{\mathrm{a}}$, is not necessarily equal to the actual voltage, V , across the motor due to the back voltage induced in the loops by the magnetic field. The back voltage, $\mathrm{V}_{\mathrm{e}}$, is given by:

$$
\begin{equation*}
V_{e}=K_{e} b \tag{5.5}
\end{equation*}
$$

Thus, Equation (5.4) should be re-written as:

$$
\begin{equation*}
V_{a}-V_{e}=i_{a} R_{\Omega}+L \frac{\partial i_{a}}{\partial t} \tag{5.6}
\end{equation*}
$$

Combining Equations (5.3), (5.5), and (5.6), the full relationship between $V_{a}$ and $b$ can be seen to be:

$$
\begin{equation*}
V_{a}=K_{e}+\frac{R_{\Omega}\left(I_{m}+I_{l}\right)}{K_{m}} \frac{\partial b}{\partial t}+\frac{L\left(I_{m}+I_{l}\right)}{K_{m}} \frac{\partial^{2} b}{\partial t^{2}} \tag{5.7}
\end{equation*}
$$

From Equation (5.7), it is simple to see that there are two states associated with a voltage controlled motor, and in fact, for realistic motors, this is generally an over-damped second order relationship.

### 5.2.2 Craft-Based Three-Dimensional Translational Motion

The three-dimensional translational motion states are derived straight from
Newton's Second Law of Motion:

$$
\begin{equation*}
\sum F=m \nabla^{2} r \tag{5.8}
\end{equation*}
$$

The only translational forces on the craft result from the three propellers and gravity, so the analysis here is quite straightforward. (The force of gravity will be included only in the Earth-based reference in section 5.2.4. This choice is made in order that the reference frames are not confused. In the Earth-based reference frame, the force of gravity is always oriented on the negative z-axis, but in the craft based frame, the orientation is not so clearly defined.) To slightly complicate matters, the propellers are tilted out of the plane of the craft, by an angle ' $d$ ', in order to increase the torque around the out of plane axis, thus increasing the hoop's rotational speed. This out of plane tilt is depicted in Figure 5.3 below:


Figure 5.3: Propeller Mounting Advance Angle
In order to create the equations of motion, a reference frame must be established. The first reference frame that will be established is a craft-based reference frame, and in
section 5.2.4, a translation between craft-based and Earth-based reference frames will be developed. Figure 5.4 shows the orientation of the craft-based reference frame:


Figure 5.4: Craft-Based Reference Frame
Let the propeller depicted on the x -axis be propeller 1 , the propeller immediately counterclockwise in the diagram be propeller 2, and the last propeller be propeller 3. The one special feature about this reference frame is that it is completely fixed only to the hub and not to the hoop, so the hoop can have a z-axis rotation in the reference frame. The angle of rotation of the hoop in the craft-based reference frame will be defined as $\psi_{o}^{C}$.

From elementary thrust theory, the normal force exerted by a spinning propeller is

$$
\begin{equation*}
f^{C}=\frac{1}{2} C_{t} \rho n^{2} b^{2} \pi R_{\text {prop }}^{4} \tag{5.9}
\end{equation*}
$$

where the ' C ' superscript denotes the force in a craft-based reference frame, $\mathrm{C}_{\mathrm{t}}$ is the propeller's coefficient of thrust, $\rho$ is the density of air, n is the gear ratio between the propeller and the motor, $b$ is the rotational speed of the motor (which is the same $b$ as in Equation (5.7)), and $\mathrm{R}_{\text {prop }}$ is the propeller radius. (See Reference 8.) From Figure 5.3, the advance angle of the propellers is $d$, so the $z$-axis component of the forces from each of the three propellers will be:

$$
\begin{equation*}
f_{z(1,2,3)}^{C}=f_{(1,2,3)}^{C} \cos (d) \tag{5.10}
\end{equation*}
$$

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The x - and y -axis components of the propeller forces, before any z -axis hoop rotation, will be:

$$
\begin{gather*}
f_{x(1)}^{C}=f_{(1)}^{C} \sin (d) \cos \left(\frac{\pi}{2}\right)=0  \tag{5.11}\\
f_{y(1)}^{C}=f_{(1)}^{C} \sin (d) \sin \left(\frac{\pi}{2}\right)=f_{(1)}^{C} \sin (d)  \tag{5.12}\\
f_{x(2)}^{C}=f_{(2)}^{C} \sin (d) \cos \left(\frac{7 \pi}{6}\right)  \tag{5.13}\\
f_{y(2)}^{C}=f_{(2)}^{C} \sin (d) \sin \left(\frac{7 \pi}{6}\right)  \tag{5.14}\\
f_{z(3)}^{C}=f_{(3)}^{C} \sin (d) \cos \left(\frac{11 \pi}{6}\right)  \tag{5.15}\\
f_{z(3)}^{C}=f_{(3)}^{C} \sin (d) \sin \left(\frac{11 \pi}{6}\right) \tag{5.16}
\end{gather*}
$$

where the angles are measured from the x -axis counter-clockwise. (The angles are $\frac{\pi}{2}$
ahead of the propeller locations at $0, \frac{2 \pi}{3}$, and $\frac{4 \pi}{3}$.)

The state-equations in the craft-based reference frame are thus:

$$
\begin{gather*}
\frac{\partial \overrightarrow{s^{C}}}{\partial t}=\left[\begin{array}{c}
\frac{\partial x^{C}}{\partial t} \\
\frac{\partial y^{C}}{\partial t} \\
\frac{\partial z^{C}}{\partial t}
\end{array}\right]=\left[\begin{array}{c}
v_{x}^{C} \\
v_{y}^{C} \\
v_{z}^{C}
\end{array}\right]  \tag{5.17}\\
\frac{\partial \overrightarrow{v^{C}}}{\partial t}=\frac{1}{m} \overrightarrow{f^{C}} \tag{5.18}
\end{gather*}
$$

where $v_{x}^{C}, v_{y}^{C}$, and $v_{z}^{C}$ are the velocities of the craft along the $\mathrm{x}-, \mathrm{y}-$, and z - craft-based axes, and $\overrightarrow{f^{C}}$ is specified by Equations (5.10)-(5.16) and rotated through $\psi_{o}^{C}$ :

### 5.2.3 Craft-Based Three-Dimensional Rotational Motion

As for the rotational motion of the craft in the craft-based reference frame, there are four primary angles of concern. The angles that need to be tracked are the pitch, roll, and yaw angles of the hoop, as well as the yaw angle of the hub. (The pitch and roll of the hub are the same as for the hoop by construction.) Let the roll be an angle $\theta^{C}$ around the x-axis, the pitch be an angle $\phi^{C}$ around the y-axis, the hoop yaw be an angle $\psi_{o}^{C}$ around the z-axis, and the hub yaw be an angle $\psi_{i}^{C}$ around the z-axis. Furthermore, let

$$
\begin{gather*}
\frac{\partial \theta^{C}}{\partial t}=\omega_{\theta}^{C}  \tag{5.20}\\
\frac{\partial \phi^{C}}{\partial t}=\omega_{\phi}^{C}  \tag{5.21}\\
\frac{\partial \psi_{o}^{C}}{\partial t}=\omega_{\psi_{o}}^{C}  \tag{5.22}\\
\frac{\partial \psi_{i}^{C}}{\partial t}=\omega_{\psi_{i}}^{C} \tag{5.23}
\end{gather*}
$$

be the rotational velocities of the hoop around the $\mathrm{x}-, \mathrm{y}$-, and z -axes and the hub around the $z$-axis. These last four equations also make up the first four rotational motion state equations.

From Euler's laws of angular motion it is known that:

$$
\begin{equation*}
\sum M=J \frac{\partial \omega}{\partial t}+\omega \times J \omega \tag{5.24}
\end{equation*}
$$

where M are the moment vectors, J is a moment of inertia matrix, and $\omega$ is the vectored angular velocity. This leads to state equations for rotational motion of:

$$
\begin{equation*}
\frac{\partial \omega}{\partial t}=J^{-1}\left(\sum M-\omega \times J \omega\right) \tag{5.25}
\end{equation*}
$$

This equation will have two parts to it, the first to deal with the hub and the second to deal with the hoop. All variables for the hub will have a subscript ' i ', and all variables for the outer hub will have a subscript ' $o$ '. With a rigid-body assumption, and thus entirely independent axes of motion, the moment of inertia matrices are given by:

$$
J_{i}=\left[\begin{array}{ccc}
I_{x x} & 0 & 0  \tag{5.26}\\
0 & I_{y y} & 0 \\
0 & 0 & I_{z z_{i}}
\end{array}\right]
$$

and

$$
J_{o}=\left[\begin{array}{ccc}
I_{x x} & 0 & 0  \tag{5.27}\\
0 & I_{y y} & 0 \\
0 & 0 & I_{z z_{o}}
\end{array}\right]
$$

The two different $\omega$ vectors are given by:

$$
\overrightarrow{\omega_{i}^{C}}=\left[\begin{array}{c}
\omega_{\theta}^{C}  \tag{5.28}\\
\omega_{\phi}^{C} \\
\omega_{\psi_{i}}^{C}
\end{array}\right]
$$

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and

$$
\overrightarrow{\omega_{o}^{C}}=\left[\begin{array}{c}
\omega_{\theta}^{C}  \tag{5.29}\\
\omega_{\phi}^{C} \\
\omega_{\psi_{o}}^{C}
\end{array}\right]
$$

The moments that will be exerted on the craft will have three different sources: one from the changing rotational velocities of the propellers, another from the torque exerted on the craft by the forces caused by the propellers, and another from the yaw motor's changing rotational velocity. Moments from the first source will be labeled with a subscript ' 1 ', from the second source ' 2 ', and the third source ' 3 '. Since the $x$ - and $y$ axis rotational motion of the hoop and hub are locked together, the x - and y -components of all of the moments will act on both the hoop and hub as if they are one body. Additionally, the x - and y -axis rotational state equations for the hoop and hub must be the same. Since the z -axis rotational motion of the hoop and hub can be different, however, care must be taken to specify which sections of the craft each of the z -axis moments will act on.

The z-axis component of the moments from the first and second sources will act only on the hoop. The reason for this is that the source of these moments is from bodies connected only to the hoop, namely the motors and propellers. The z-axis component of the moment from the third source will act on both the hoop and the hub, however. This is because the yaw motor will be connected to both sections of the craft, and will therefore affect both sections. These precise effects will be discussed shortly.

As for the actual equations, the moment caused by the first source, the changing rotational velocities of the propellers, will be:

$$
\begin{align*}
& \overrightarrow{M_{1(1)}^{C}}=\left[\begin{array}{c}
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(1)}}{\partial t} \sin (d) \cos \left(\frac{\pi}{2}+\psi_{o}^{C}\right) \\
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(1)}}{\partial t} \sin (d) \sin \left(\frac{\pi}{2}+\psi_{o}^{C}\right) \\
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(1)}}{\partial t} \cos (d)
\end{array}\right]  \tag{5.30}\\
& \overrightarrow{M_{1(2)}^{C}}=\left[\begin{array}{c}
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(2)}}{\partial t} \sin (d) \cos \left(\frac{7 \pi}{6}+\psi_{o}^{C}\right) \\
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(2)}}{\partial t} \sin (d) \sin \left(\frac{7 \pi}{6}+\psi_{o}^{C}\right) \\
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(2)}}{\partial t} \cos (d)
\end{array}\right]  \tag{5.31}\\
& \overrightarrow{M_{1(3)}^{C}}=\left[\begin{array}{c}
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(3)}}{\partial t} \sin (d) \cos \left(\frac{11 \pi}{6}+\psi_{o}^{C}\right) \\
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(3)}}{\partial t} \sin (d) \sin \left(\frac{11 \pi}{6}+\psi_{o}^{C}\right) \\
\left(I_{m}+n I_{p}\right) \frac{\partial b_{(3)}}{\partial t} \cos (d)
\end{array}\right] \tag{5.32}
\end{align*}
$$

for motors (1), (2), and (3), giving a total of:

$$
\overrightarrow{M_{1}^{C}}=\left[\begin{array}{c}
\left(I_{m}+n I_{p}\right) \sin (d)\left(\frac{\partial b_{(1)}}{\partial t} \cos \left(\frac{\pi}{2}+\psi_{o}^{C}\right)+\frac{\partial b_{(2)}}{\partial t} \cos \left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+\frac{\partial b_{(3)}}{\partial t} \cos \left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right.  \tag{5.33}\\
\left(I_{m}+n I_{p}\right) \sin (d)\left(\frac{\partial b_{(1)}}{\partial t} \sin \left(\frac{\pi}{2}+\psi_{o}^{C}\right)+\frac{\partial b_{(2)}}{\partial t} \sin \left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+\frac{\partial b_{(3)}}{\partial t} \sin \left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) \\
\left(I_{m}+n I_{p}\right) \cos (d)\left(\frac{\partial b_{(1)}}{\partial t}+\frac{\partial b_{(2)}}{\partial t}+\frac{\partial b_{(3)}}{\partial t}\right)
\end{array}\right]
$$

where $I_{m}$ is the moment of inertia of the motor and motor gear, and $I_{p}$ is the moment of inertia of the propeller and propeller gear.

The moments from the second source, the torque from the forces caused by the propellers, will all take the form:

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$$
\begin{equation*}
\overline{M_{2(1,2,3)}^{C}}=\overline{r_{(1,2,3)}^{C}} \times \overline{f_{(1,2,3)}^{C}} \tag{5.34}
\end{equation*}
$$

The forces have already been defined, but the position vectors for the motors, $\overline{r_{(1,2,3)}^{C}}$, have not yet been determined. From Figure 5.4, the position of the motors is apparent:

$$
\begin{gather*}
\overrightarrow{r_{(1)}^{C}}=\left[\begin{array}{c}
R_{m} \cos \left(\psi_{o}^{C}\right) \\
R_{m} \sin \left(\psi_{o}^{C}\right) \\
0
\end{array}\right]  \tag{5.35}\\
\overrightarrow{r_{(2)}^{C}}=\left[\begin{array}{c}
R_{m} \cos \left(\frac{2 \pi}{3}+\psi_{o}^{C}\right) \\
R_{m} \sin \left(\frac{2 \pi}{3}+\psi_{o}^{C}\right) \\
0
\end{array}\right]  \tag{5.36}\\
\overrightarrow{r_{(3)}^{C}}=\left[\begin{array}{c}
R_{m} \cos \left(\frac{4 \pi}{3}+\psi_{o}^{C}\right) \\
R_{m} \sin \left(\frac{4 \pi}{3}+\psi_{o}^{C}\right) \\
0
\end{array}\right] \tag{5.37}
\end{gather*}
$$

where $R_{m}$ is the radius from the hub center to propeller center. Using Equation (5.34) to combine Equations (5.10)-(5.16) and (5.35)-(5.37), the total moment from the second source is:
$\overrightarrow{M_{2}^{C}}=\left[\begin{array}{c}R_{m} \cos (d)\left(f_{(2)}^{C} \sin \left(\psi_{o}^{C}\right)+f_{(2)}^{C} \sin \left(\frac{2 \pi}{3}+\psi_{o}^{C}\right)+f_{(3)}^{C} \sin \left(\frac{4 \pi}{3}+\psi_{o}^{C}\right)\right) \\ -R_{m} \cos (d)\left(f_{(1)}^{C} \cos \left(\psi_{o}^{C}\right)+f_{(2)}^{C} \cos \left(\frac{2 \pi}{3}+\psi_{o}^{C}\right)+f_{(3)}^{C} \cos \left(\frac{4 \pi}{3}+\psi_{o}^{C}\right)\right) \\ R_{m} \sin (d)\left(f_{(1)}^{C}+\left(f_{(2)}^{C}+f_{(3)}^{C}\right) \sin \left(\frac{\pi}{2}\right)\right)\end{array}\right]$

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Lastly, the moment caused by the third source, the yaw motor's changing rotational velocity, will be a positive moment for the hoop and a negative moment for the hub. This orientation is chosen so that the yaw motor will counter-act the frictional forces between the hub and hoop which would tend to draw the hub into a rotation with the hoop. Since the hub needs to be de-spun in order to mount an effective autopilot on it, the yaw motor must counter-balance this pull. The moment from the yaw motor on the hoop will be:

$$
\overrightarrow{M_{3_{o}}^{C}}=\left[\begin{array}{c}
0  \tag{5.39}\\
0 \\
\left(I_{m, y a w}+I_{z z_{o}}+I_{z z_{i}}\right) \frac{\partial b_{y a w}}{\partial t}
\end{array}\right]
$$

and correspondingly on the hub will be:

$$
\overrightarrow{M_{3_{i}}^{C}}=-\left[\begin{array}{c}
0  \tag{5.40}\\
0 \\
\left(I_{m, y a w}+I_{z z_{o}}+I_{z z_{i}}\right) \frac{\partial b_{y a w}}{\partial t}
\end{array}\right]
$$

where $\mathrm{I}_{\mathrm{m} \text {, yaw }}$ is the moment of inertia associated with the yaw motor.
Combining all the moments of Equations (5.33), (5.38), (5.39), and (5.40), the total moment on the hoop will be:

$$
\overrightarrow{M_{o}^{C}}=\left[\begin{array}{c}
\left(I_{m}+n I_{p}\right) \sin (d)\left(\frac{\partial b_{(1)}}{\partial t} \cos \left(\frac{\pi}{2}+\psi_{o}^{C}\right)+\frac{\partial b_{(2)}}{\partial t} \cos \left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+\frac{\partial b_{(3)}}{\partial t} \cos \left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) \\
\left(I_{m}+n I_{p}\right) \sin (d)\left(\frac{\partial b_{(1)}}{\partial t} \sin \left(\frac{\pi}{2}+\psi_{o}^{C}\right)+\frac{\partial b_{(2)}}{\partial t} \sin \left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+\frac{\partial b_{(3)}}{\partial t} \sin \left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) \\
\left(I_{m}+n I_{p}\right) \cos (d)\left(\frac{\partial b_{(1)}}{\partial t}+\frac{\partial b_{(2)}}{\partial t}+\frac{\partial b_{(3)}}{\partial t}\right)+\left(I_{m, y a w}+I_{z z_{o}}+I_{z z_{i}}\right) \frac{\partial b_{y a w}}{\partial t} \\
+R_{m} \cos (d)\left(f_{(1)}^{C} \sin \left(\psi_{o}^{C}\right)+f_{(2)}^{C} \sin \left(\frac{2 \pi}{3}+\psi_{o}^{C}\right)+f_{(3)}^{C} \sin \left(\frac{4 \pi}{3}+\psi_{o}^{C}\right)\right)  \tag{5.41}\\
-R_{m} \cos (d)\left(f_{(1)}^{C} \cos \left(\psi_{o}^{C}\right)+f_{(2)}^{C} \cos \left(\frac{2 \pi}{3}+\psi_{o}^{C}\right)+f_{(3)}^{C} \cos \left(\frac{4 \pi}{3}+\psi_{o}^{C}\right)\right) \\
+R_{m} \sin (d)\left(f_{(1)}^{C}+\left(f_{(2)}^{C}+f_{(3)}^{C}\right) \sin \left(\frac{\pi}{2}\right)\right)
\end{array}\right.
$$

where the x - and y -components act on both the hoop and hub, and the z -component acts only on the hoop. The total z-component moment that acts on the hub is given by Equation (5.40).

Now that the total moment on each section of the craft is established, Equations (5.25)-(5.29) can be used to finally derive the state equations for $\overrightarrow{\omega_{i}^{C}}$ and $\overrightarrow{\omega_{o}^{C}}$. However, a transformation will have to be developed between the rotational velocities in a rotated hoop-based frame, $\overrightarrow{\omega_{i}^{r}}$ and $\overrightarrow{\omega_{o}^{r}}$, and an un-rotated hub-based frame, $\overrightarrow{\omega_{i}^{C}}$ and $\overrightarrow{\omega_{o}^{C}}$. (The only difference between the rotated and un-rotated craft-based frames is that the rotated craft-based frame is rigidly attached to the hoop, while the un-rotated craft-based frame is rigidly attached to the hub.) The rotated craft-based frame will have rotational velocities of:

$$
\begin{align*}
& {\left[\frac{1}{I_{x x}}\left(\left(I_{y y}-I_{z z_{o}}\right) \omega_{\phi}^{C} \omega_{\psi_{o}}^{C}+\sin (d)\left(I_{m}+n I_{p}\right)\left(\frac{\partial b_{(2)}}{\partial t} \cos \left(\frac{7 \pi}{6}\right)+\frac{\partial b_{(3)}}{\partial t} \cos \left(\frac{11 \pi}{6}\right)\right)\right)\right.} \\
& \frac{\partial \overrightarrow{\omega_{i}^{r}}}{\partial t}=\frac{1}{I_{y y}}\left(\left(I_{z z_{o}}-I_{x x}\right) \omega_{\theta}^{C} \omega_{\psi_{o}}^{C}+\sin (d)\left(I_{m}+n I_{p}\right)\left(\frac{\partial b_{(1)}}{\partial t}+\frac{\partial b_{(2)}}{\partial t} \sin \left(\frac{7 \pi}{6}\right)+\frac{\partial b_{(3)}}{\partial t} \sin \left(\frac{11 \pi}{6}\right)\right)\right) \\
& \frac{1}{I_{z z_{i}}}\left(\left(I_{x x}-I_{y y}\right) \omega_{\theta}^{C} \omega_{\phi}^{C}-\left(I_{m, y a w}+I_{z z_{o}}+I_{z z_{i}}\right) \frac{\partial b_{y a w}}{\partial t}\right) \\
& +\frac{1}{I_{x x}}\left(R_{m} \cos (d)\left(f_{(2)}^{C} \sin \left(\frac{2 \pi}{3}\right)+f_{(3)}^{C} \sin \left(\frac{4 \pi}{3}\right)\right)\right) \\
& -\frac{1}{I_{y y}}\left(R_{m} \cos (d)\left(f_{(1)}^{C}+f_{(2)}^{C} \cos \left(\frac{2 \pi}{3}\right)+f_{(3)}^{C} \cos \left(\frac{4 \pi}{3}\right)\right)\right) \\
& +0 \\
& \frac{\partial \overrightarrow{\omega_{o}^{r}}}{\partial t}=\left[\begin{array}{c}
\frac{1}{I_{x x}}\left(\left(I_{y y}-I_{z z_{o}}\right) \omega_{\phi}^{C} \omega_{\psi_{o}}^{C}+\sin (d)\left(I_{m}+n I_{p}\right)\left(\frac{\partial b_{(2)}}{\partial t} \cos \left(\frac{7 \pi}{6}\right)+\frac{\partial b_{(3)}}{\partial t} \cos \left(\frac{11 \pi}{6}\right)\right)\right) \\
\frac{1}{I_{y y}}\left(\left(I_{z z_{o}}-I_{x x}\right) \omega_{\theta}^{C} \omega_{\psi_{o}}^{C}+\sin (d)\left(I_{m}+n I_{p}\right)\left(\frac{\partial b_{(1)}}{\partial t}+\frac{\partial b_{(2)}}{\partial t} \sin \left(\frac{7 \pi}{6}\right)+\frac{\partial b_{(3)}}{\partial t} \sin \left(\frac{11 \pi}{6}\right)\right)\right) \\
\frac{1}{I_{z z_{i}}}\left(\left(I_{x x}-I_{y y}\right) \omega_{\theta}^{C} \omega_{\phi}^{C}+\left(I_{m, y a w}+I_{z z_{o}}+I_{z z_{i}}\right) \frac{\partial b_{y a w}}{\partial t}+\cos (d)\left(I_{m}+n I_{p}\right)\left(\frac{\partial b_{(1)}}{\partial t}+\frac{\partial b_{(2)}}{\partial t}+\frac{\partial b_{(3)}}{\partial t}\right)\right.
\end{array}\right) \\
& +\frac{1}{I_{x x}}\left(R_{m} \cos (d)\left(f_{(2)}^{C} \sin \left(\frac{2 \pi}{3}\right)+f_{(3)}^{C} \sin \left(\frac{4 \pi}{3}\right)\right)\right) \\
& -\frac{1}{I_{y y}}\left(R_{m} \cos (d)\left(f_{(1)}^{C}+f_{(2)}^{C} \cos \left(\frac{2 \pi}{3}\right)+f_{(3)}^{C} \cos \left(\frac{4 \pi}{3}\right)\right)\right)  \tag{5.43}\\
& \left.+\frac{1}{I_{z z_{o}}}\left(R_{m} \sin (d)\left(f_{(1)}^{C}+\left(f_{(2)}^{C}+f_{(3)}^{C}\right) \sin \left(\frac{\pi}{2}\right)\right)\right)\right]
\end{align*}
$$

The un-rotated craft-based frame has the same z -axis as the rotated craft-based frame; therefore the z -components of the two rotational velocities will be the same. The x - and y -components of the un-rotated craft-based frame must be rotated by $\psi_{o}^{C}$, however, thus:

$$
\overrightarrow{\omega_{i, o}^{r}}=\left[\begin{array}{c}
0  \tag{5.44}\\
0 \\
\omega_{z_{i, o}}^{C}
\end{array}\right]+\left[\begin{array}{ccc}
\cos \left(\psi_{o}^{C}\right) & -\sin \left(\psi_{o}^{C}\right) & 0 \\
\sin \left(\psi_{o}^{C}\right) & \cos \left(\psi_{o}^{C}\right) & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
\omega_{x}^{C} \\
\omega_{y}^{C} \\
0
\end{array}\right]
$$

Solving for $\overrightarrow{\omega_{i, o}^{C}}$ gives:

$$
\overrightarrow{\omega_{i, o}^{C}}=\left[\begin{array}{ccc}
\cos \left(\psi_{o}^{C}\right) & \sin \left(\psi_{o}^{C}\right) & 0  \tag{5.45}\\
-\sin \left(\psi_{o}^{C}\right) & \cos \left(\psi_{o}^{C}\right) & 0 \\
0 & 0 & 1
\end{array}\right]_{\overrightarrow{\omega_{i, o}^{r}}}
$$

which is simply the rotated rotational velocity multiplied by an inverse rotation of $\psi_{o}^{C}$.
The full form of $\overrightarrow{\omega_{i, o}^{C}}$ is omitted to conserve space.

### 5.2.4 Earth-Based Three-Dimensional Translational and Rotational Motion

Now that the craft-based reference frame has been fully understood, a conversion between the craft-based frame and an Earth-based frame must be established. The three major differences between the craft-based frame and the Earth-based frame are the difference in origin, the difference in rotational orientation, and the addition of a constant z-axis force of gravity in the Earth-based frame. The difference in origin is very easy to add in since it is simply a translational offset and will not affect the state equations at all. Also, the force of gravity is easy to add in since it is simply a constant z-directed force. The most difficult conversion is that of the rotation angles.

Let $\theta^{E}$ be the Earth-based x-axis angle, $\phi^{E}$ be the Earth-based y-axis angle, $\psi_{i}^{E}$ be the Earth-based z-axis angle relating to the hub, and $\psi_{o}^{E}$ be the Earth-based z-axis angle relating to the hoop. If the translational offset is subtracted from the craft-based reference frame, these angles are those through which the craft-based frame would have Jesse Davis
to be rotated in order to arrive at the Earth-based frame. Since the order of rotation matters, the convention that will here be established is x -axis rotation first, y -axis rotation second, and z-axis rotation third when rotating a vector from the craft-based frame to the Earth-based frame. These rotations can be encapsulated in a $3 \times 3$ matrix given by:

$$
R_{C}^{E}=\left[\begin{array}{ccc}
\cos \left(\psi_{i}^{E}\right) & -\sin \left(\psi_{i}^{E}\right) & 0  \tag{5.46}\\
\sin \left(\psi_{i}^{E}\right) & \cos \left(\psi_{i}^{E}\right) & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{ccc}
\cos \left(\phi^{E}\right) & 0 & \sin \left(\phi^{E}\right) \\
0 & 1 & 0 \\
-\sin \left(\phi^{E}\right) & 0 & \cos \left(\phi^{E}\right)
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \left(\theta^{E}\right) & -\sin \left(\theta^{E}\right) \\
0 & \sin \left(\theta^{E}\right) & \cos \left(\theta^{E}\right)
\end{array}\right]
$$

In order to rotate any given vector from the craft-based frame to the Earth-based frame, it must be multiplied by the rotation matrix $R_{C}^{E}$, and to rotate from the Earth-based frame to the craft-based frame, it must be multiplied by $R_{C}^{E-1}$ (which also happens to be $R_{C}^{E T}$ ). For the translational motion vectors $\overrightarrow{f^{C}}, \overrightarrow{s^{C}}$, and $\overrightarrow{v^{C}}$ from Equations (5.17), (5.18), and (5.19) this multiplication by $R_{C}^{E}$ is all that need be done. Applying $R_{C}^{E}$ to these vectors, and adding in the z -axis constant force of gravity, the Earth-based translational state equations become:

$$
\frac{\partial \overrightarrow{s^{E}}}{\partial t}=\left[\begin{array}{c}
\frac{\partial x^{E}}{\partial t}  \tag{5.47}\\
\frac{\partial y^{E}}{\partial t} \\
\frac{\partial z^{E}}{\partial t}
\end{array}\right]=\left[\begin{array}{c}
v_{x}^{E} \\
v_{y}^{E} \\
v_{z}^{E}
\end{array}\right]
$$

$$
\begin{align*}
& {\left[\left(f_{(1)}^{C}+f_{(2)}^{C}+f_{(3)}^{C}\right) c(d)\left(c\left(\psi_{i}^{E}\right) s\left(\phi^{E}\right) c\left(\theta^{F}\right)+s\left(\psi_{i}^{E}\right) s\left(\theta^{F}\right)\right)+\left(f_{(1)}^{c}\left(\frac{\pi}{2}+\psi_{o}^{C}\right)+f_{(2)}^{c} c\left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+f_{(3)}^{C}\left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) s(d) c\left(\psi_{i}^{E}\right) c\left(\phi^{E}\right)\right.} \\
& \left.\frac{\partial \overrightarrow{\partial^{E}}}{\partial t}=\frac{1}{m}\left(f_{(1)}^{C}+f_{(2)}^{C}+f_{(3)}^{C}\right) c(d)\left(s\left(\psi_{i}^{E}\right) s\left(\phi^{E}\right) c\left(\theta^{\theta}\right)-c\left(\psi_{i}^{E}\right) s\left(\theta^{F}\right)\right)+\left(f_{(1)}^{C} c\left(\frac{\pi}{2}+\psi_{o}^{C}\right)+f_{(2)}^{C}\right)\left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+f_{(3)}^{C}\left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) s(d) s\left(\psi_{i}^{E}\right) c\left(\phi^{E}\right) \\
& -g n+\left(f_{(1)}^{C}+f_{(2)}^{C}+f_{(3)}^{C}\right) c(d) d\left(\phi^{E}\right) d\left(\theta^{\sigma}\right)-\left(f_{(1)}^{C} c\left(\frac{\pi}{2}+\psi_{o}^{C}\right)+f_{(2)}^{C} c\left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+f_{(3)}^{C}\left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) s(d) s\left(\phi^{E}\right) \\
& \left.+\left(f_{(1)}^{C} S\left(\frac{\pi}{2}+\psi_{o}^{C}\right)+f_{(2)}^{C} s\left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+f_{(3)}^{C} s\left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) s(d)\left(c\left(\psi_{i}^{E}\right) s\left(\phi^{E}\right) s\left(\theta^{E}\right)-s\left(\psi_{i}^{E}\right) c\left(\theta^{E}\right)\right)\right] \\
& +\left(f_{(1)}^{C} s\left(\frac{\pi}{2}+\psi_{o}^{C}\right)+f_{(2)}^{C} s\left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+f_{(3)}^{C} s\left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) s(d)\left(s\left(\psi_{i}^{E}\right) s\left(\phi^{E}\right) s\left(\theta^{E}\right)+c\left(\psi_{i}^{E}\right) c\left(\theta^{E}\right)\right)  \tag{5.48}\\
& \left.+\left(f_{(1)}^{C} s\left(\frac{\pi}{2}+\psi_{o}^{C}\right)+f_{(2)}^{C} s\left(\frac{7 \pi}{6}+\psi_{o}^{C}\right)+f_{(3)}^{C} s\left(\frac{11 \pi}{6}+\psi_{o}^{C}\right)\right) s(d) c\left(\phi^{E}\right) s\left(\theta^{E}\right) \quad\right]
\end{align*}
$$

where the shorthand $c\left({ }^{*}\right)$ replaces $\cos \left({ }^{*}\right)$ and $\mathrm{s}\left({ }^{*}\right)$ replaces $\sin \left({ }^{*}\right)$, g is the acceleration due to gravity, and $m$ is the craft's total mass. The $z$-axis angle of the hub is used since it is ultimately the orientation of the hub which will be controlled, and also since the craftbased reference frame is really a hub-based reference frame.

As for the rotational motion, by association with Equations (5.20)-(5.23), the first rotational motion state equations in the Earth-based frame are:

$$
\begin{align*}
& \frac{\partial \theta^{E}}{\partial t}=\omega_{\theta}^{E}  \tag{5.49}\\
& \frac{\partial \phi^{E}}{\partial t}=\omega_{\phi}^{E}  \tag{5.50}\\
& \frac{\partial \psi_{o}^{E}}{\partial t}=\omega_{\psi_{o}}^{E}  \tag{5.51}\\
& \frac{\partial \psi_{i}^{E}}{\partial t}=\omega_{\psi_{i}}^{E} \tag{5.52}
\end{align*}
$$

The conversion of the rotational velocity vectors from the craft-based frame to the Earthbased frame is slightly more complicated than for the translational motion. The same
rotation that was applied to the translational vectors cannot be applied again. The reason for this is that the rotational axes must be separated from each other in order to correctly convert their components. If the order of rotation is x -axis, y -axis, z -axis when rotating from the craft-based frame to the Earth-based frame, then the x-axis component of $\overrightarrow{\omega_{i, o}^{E}}$ will be the same as the x-axis component of $\overrightarrow{\omega_{i, o}^{C}}$. However, the y-axis component of $\overrightarrow{\omega_{i, o}^{E}}$ must be un-rotated through the x -axis Earth-based rotation angle, and the z -axis component of $\overrightarrow{\omega_{i, o}^{E}}$ must be un-rotated through both the x- and y-axis Earth-based rotation angles. The conversion becomes:

$$
\overrightarrow{\omega_{1, o}^{C}}=\left[\begin{array}{c}
\omega_{\theta}^{E}  \tag{5.53}\\
0 \\
0
\end{array}\right]+\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \left(\theta^{E}\right) & \sin \left(\theta^{E}\right) \\
0 & -\sin \left(\theta^{E}\right) & \cos \left(\theta^{E}\right)
\end{array}\right]\left[\begin{array}{c}
0 \\
\omega_{\phi}^{E} \\
0
\end{array}\right]+\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \left(\theta^{E}\right) & \sin \left(\theta^{E}\right) \\
0 & -\sin \left(\theta^{E}\right) & \cos \left(\theta^{E}\right)
\end{array}\right]\left[\begin{array}{ccc}
\cos \left(\phi^{E}\right) & 0 & -\sin \left(\phi^{E}\right) \\
0 & 1 & 0 \\
\sin \left(\phi^{E}\right) & 0 & \cos \left(\phi^{E}\right)
\end{array}\right]\left[\begin{array}{c}
0 \\
0 \\
\omega_{\psi_{i, o}^{E}}^{E}
\end{array}\right](
$$

giving

$$
\overrightarrow{\omega_{i, o}^{C}}=\left[\begin{array}{ccc}
1 & 0 & -\sin \left(\phi^{E}\right)  \tag{5.54}\\
0 & \cos \left(\theta^{E}\right) & \cos \left(\phi^{E}\right) \sin \left(\theta^{E}\right) \\
0 & -\sin \left(\theta^{E}\right) & \cos \left(\phi^{E}\right) \cos \left(\theta^{E}\right)
\end{array}\right] \overrightarrow{\omega_{i, o}^{E}}
$$

Solving Equation (5.54) for $\overrightarrow{\omega_{i, o}^{E}}$ gives:

$$
\overrightarrow{\omega_{i, o}^{E}}=\left[\begin{array}{ccc} 
& &  \tag{5.55}\\
1 & \tan \left(\phi^{E}\right) \sin \left(\theta^{E}\right) & \tan \left(\phi^{E}\right) \cos \left(\theta^{E}\right) \\
0 & \cos \left(\theta^{E}\right) & -\sin \left(\theta^{E}\right) \\
0 & \frac{\sin \left(\theta^{E}\right)}{\cos \left(\phi^{E}\right)} & \frac{\cos \left(\theta^{E}\right)}{\cos \left(\phi^{E}\right)}
\end{array}\right] \overline{\omega_{i, o}^{C}}
$$

By this equation, there will be two different $\omega_{x}^{E}$ and $\omega_{y}^{E}$, one for the hub and one for the hoop. The $\omega_{x}^{E}$ and $\omega_{y}^{E}$ that will be concentrated on are those for the hub. This is done because it is ultimately the orientation of the hub that will be controlled, so knowing its rotational velocities is of primary importance. Finally, the complete state-space description of the craft in both the craft-based and Earth-based reference frames has been shown.

### 5.3 Altitude, Attitude, and Yaw Control

The actual implementation of the craft and user interface is going to take the form of a two joystick remote control mechanism that can operate in two separate modes. In a user controlled mode, one joystick will control roll and pitch, and the other will control yaw motor speed and thrust; in a computer controlled mode, one joystick will control roll and pitch, and the other will control yaw speed of the hub and vertical speed. In the computer controlled mode, the user's commands will be processed by a controller so that the on-board computer ultimately controls the craft. To this end, three essentially separate motions will need to be controlled: vertical speed, yaw speed of the hub, and attitude (pitch and roll).

### 5.3.1 System Analysis

### 5.3.1.1 Motor Voltages and Forces

The yaw speed control is essentially separated from the other two controlled dynamics because the yaw motor is the main actuator of craft-based z-axis torque. From Equation (5.42) and (5.45):

$$
\begin{equation*}
\frac{\partial \omega_{i_{z}}^{C}}{\partial t}=\frac{1}{I_{z z_{i}}}\left(\left(I_{x x}-I_{y y}\right) \omega_{\theta}^{C} \omega_{\phi}^{C}-\left(I_{m, y a w}+I_{z z_{i}}+I_{z z_{\theta}}\right) \frac{\partial b_{y a w}}{\partial t}\right) \tag{5.56}
\end{equation*}
$$

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Since $\mathrm{I}_{\mathrm{xx}} \sim \mathrm{I}_{\mathrm{yy}}$, and also since $\omega_{\theta}^{C} \sim \omega_{\phi}^{C} \sim 0$ around any stable operating point for which we are designing the controller, it is clear that the moment imparted by the yaw motor is dominant. Furthermore, the yaw motor moment doesn't appear in either the x - or y -axis component of $\overrightarrow{\omega_{i, o}^{C}}$; hence the yaw control can be fully actuated with the yaw motor alone while not affecting any other aspect of the craft.

The reason the altitude and attitude control are essentially independent requires more development. Since the three propellers will provide both lift and steering of the craft, it initially does not seem reasonable that the altitude and attitude control can be separated. The method by which the motors provide lift is quite obvious; if higher voltages are applied to the motors, the motors will turn the propellers faster, the propellers will provide more lift, and the craft will rise along its z -axis. (As well as spin slightly faster around its z-axis due to the advance angle d.) In order to steer the craft, however, it seems reasonable that different voltages will have to be applied to each motor. In addition, since the motors are rotating along with the rest of the hoop, these voltages will have to be adjusted depending on the yaw of the hoop.

In order to steer the craft, some torque vector in the craft-based $x-y$ plane will have to be created around which the craft will then pivot. Looking at Equations (5.42) and (5.43) again, noting that the rotation of Equation (5.45) will linearly combine $x$ - and y -components, and assuming that the advance angle of the propellers, d , will be small, the primary components of the x - and y -axis moments are due to forces exerted by the propellers at certain lever arms away from the origin. As an engineering assumption, these torques will be considered the dominant torques experienced by the craft along its x - and y -axes, and so these are the torques that will be controlled in order to steer the Jesse Davis Master's Thesis, Electrical Engineering Professor Charles Coleman
craft. If there is a goal torque steering vector, $\overrightarrow{G^{C}}$, in the craft-based x-y plane, the necessary conditions on the propeller forces are:

$$
\begin{equation*}
f_{(1)}^{C} \sin \left(\psi_{o}^{C}\right)+f_{(2)}^{C} \sin \left(\psi_{o}^{C}+\frac{2 \pi}{3}\right)+f_{(3)}^{C} \sin \left(\psi_{o}^{C}+\frac{4 \pi}{3}\right)=\frac{G^{C} \sin (g)}{R_{m} \cos (d)} \tag{5.57}
\end{equation*}
$$

and

$$
\begin{equation*}
f_{(1)}^{C} \cos \left(\psi_{o}^{C}\right)+f_{(2)}^{C} \cos \left(\psi_{o}^{C}+\frac{2 \pi}{3}\right)+f_{(3)}^{C} \cos \left(\psi_{o}^{C}+\frac{4 \pi}{3}\right)=-\frac{G^{C} \cos (g)}{R_{m} \cos (d)} \tag{5.58}
\end{equation*}
$$

where g is the z -axis angle of the goal steering vector. The only unknowns in these two equations are the forces, so if $f_{(3)}^{C}$ is held to a constant, F , for the moment, the resulting forces required will be:

$$
\begin{gather*}
f_{(1)}^{C}=F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g\right)}{R_{m} \cos (d)}+\frac{G^{C} \sin \left(\psi_{o}^{C}-g\right)}{\sqrt{3} R_{m} \cos (d)}  \tag{5.59}\\
f_{(2)}^{C}=F-\frac{2 G^{C} \sin \left(\psi_{o}^{C}-g\right)}{\sqrt{3} R_{m} \cos (d)}  \tag{5.60}\\
f_{(3)}^{C}=F \tag{5.61}
\end{gather*}
$$

It is clear that if $f_{(3)}^{C}$ were better chosen, the equations would become much more symmetric. Choosing

$$
\begin{equation*}
f_{(3)}^{C}=F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g+\frac{\pi}{2}\right)}{\sqrt{3} R_{m} \cos (d)} \tag{5.62}
\end{equation*}
$$

Equations (5.59) and (5.60) become:

$$
\begin{equation*}
f_{(1)}^{C}=F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g\right)}{R_{m} \cos (d)} \tag{5.63}
\end{equation*}
$$

$$
\begin{equation*}
f_{(2)}^{C}=F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g-\frac{\pi}{2}\right)}{\sqrt{3} R_{m} \cos (d)} \tag{5.64}
\end{equation*}
$$

where F is a thrust offset and is chosen such that the forces are never negative:

$$
\begin{equation*}
F \geq \frac{G^{C}}{R_{m} \cos (d)} \tag{5.65}
\end{equation*}
$$

Since Equation (5.9) gives a relationship between propeller thrust and motor rotational speed, and Equation (5.7) gives a relationship between motor rotational speed and applied voltage, the voltages that need to be applied to the motors in order to steer the craft can be determined.

In Equation (5.7) there are three terms, one is a multiple of $b$, one is a multiple of $\frac{\partial b}{\partial t}$, and one is a multiple of $\frac{\partial^{2} b}{\partial t^{2}}$. Looking at the coefficients of these terms, it is clear that the $\frac{\partial^{2} b}{\partial t^{2}}$ can be neglected since a motor's inductance will be at least three orders of magnitude less than its resistance. The $\frac{\partial b}{\partial t}$ term cannot necessarily be neglected, however. Motor's have an intrinsic mechanical time constant associated with the spin up of the rotor once a voltage is applied, and for the simple model developed in 5.2.1, this time constant is given by:

$$
\begin{equation*}
\tau=\frac{R_{\Omega}\left(I_{m}+n I_{p}\right)}{K_{e} K_{m}} \tag{5.66}
\end{equation*}
$$

If this time constant is fast enough, and the hoop's rotation is slow enough, the rotor's rotational speed can track the applied voltage without delay, and hence the $\frac{\partial b}{\partial t}$ term may
be neglected. If the motor time constant is too slow, however, there will be an effective time lag between the applied voltage sinusoid and the rotor's rotational speed sinusoid required to precisely steer the craft. This time lag can be interpreted as an angular lag if $\omega_{\psi_{o}}^{C}$ is known, and so the steering will always actually occur at a constant angle offset from the commanded steering.

In order to find this angular offset, Equation (5.7) must be manipulated. First, the $2^{\text {nd }}$ order term is ignored as concluded in the paragraph above. Second, the Fourier transform of the equation is taken giving:

$$
\begin{equation*}
\frac{b^{f}}{V_{a}^{f}}=\frac{1 / K_{e}}{i \omega \tau+1} \tag{5.67}
\end{equation*}
$$

where i equals $\sqrt{-1}$, the superscript ' $f$ ' denotes a Fourier transform variable, and $\omega$ is the Fourier transform frequency. The phase delay, $p_{d}$, of this equation at a frequency of $\omega_{\psi_{o}}^{C}$ is given by:

$$
\begin{equation*}
p_{d}=\frac{\arctan \left(\tau \omega_{\psi_{o}}^{C}\right)}{\omega_{\psi_{o}}^{C}} \tag{5.68}
\end{equation*}
$$

which translates into an angular delay, $\delta_{\psi_{o}}$, of:

$$
\begin{equation*}
\delta_{\psi_{o}}=\arctan \left(\tau \omega_{\psi_{o}}^{C}\right) \tag{5.69}
\end{equation*}
$$

The last thing that needs to be determined is the amplitude change between $V_{a}$ and $b$. This will simply be the magnitude of Equation (5.67) which is given by:

$$
\begin{equation*}
\left|\frac{b^{f}}{V_{a}^{f}}\right|_{\omega_{\psi_{o}}^{C}}=\frac{1 / K_{e}}{\sqrt{\left(\omega_{\psi_{o}}^{C} \tau\right)^{2}+1}} \tag{5.70}
\end{equation*}
$$

Using Equations (5.9), (5.62)-(5.64), (5.69), and (5.70), the required applied voltages in order to steer the craft to some goal torque $\overrightarrow{G^{C}}$ are:

$$
\begin{gather*}
V_{a(1)}=K_{e} \sqrt{\frac{2\left(\left(\omega_{\psi_{o}}^{C} \tau\right)^{2}+1\right)}{C_{t} \rho n^{2} \pi R_{p r o p}^{4}}} \sqrt{F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g+\delta_{\psi_{o}}\right)}{R_{m} \cos (d)}}  \tag{5.71}\\
V_{a(2)}=K_{e} \sqrt{\frac{2\left(\left(\omega_{\psi_{o}}^{C} \tau\right)^{2}+1\right)}{C_{t} \rho n^{2} \pi R_{p r o p}^{4}}} \sqrt{F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g+\delta_{\psi_{o}}-\frac{\pi}{2}\right)}{\sqrt{3} R_{m} \cos (d)}}  \tag{5.72}\\
V_{a(3)}=K_{e} \sqrt{\frac{2\left(\left(\omega_{\psi_{o}}^{C} \tau\right)^{2}+1\right)}{C_{t} \rho n^{2} \pi R_{p r o p}^{4}}} \sqrt{F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g+\delta_{\psi_{o}}+\frac{\pi}{2}\right)}{\sqrt{3} R_{m} \cos (d)}} \tag{5.73}
\end{gather*}
$$

It is clear that each of these equations has three quantities which must be specified in order for the craft to be manipulated: F , the force offset, $\mathrm{G}^{\mathrm{C}}$, the goal torque magnitude, and $g$, the goal torque angle. As was seen through the development of these equations, the force offset F , does not affect the steering torque in any way. For any value of F , if $\mathrm{G}^{\mathrm{C}}$ and g remain constant, the resulting torque will always be $\overrightarrow{G^{C}}$. (In the case if the radicand of the second radical were to go negative, the radical should be taken of the absolute value of the radicand, and a negative sign should be appended to the entire equation. This would correspond to rotating the propellers in the direction opposite to normal.)

The other part that is of concern is whether or not the total force in the z-direction is independent of $\mathrm{G}^{\mathrm{C}}$ and g. Adding together Equations (5.62)-(5.64), the total force is:

$$
\begin{equation*}
\sum_{i} f_{(i)}^{C}=3 F+\frac{G^{C} \cos \left(\psi_{o}^{C}-g+\delta_{\psi_{o}}\right)}{R_{m} \cos (d)} \tag{5.74}
\end{equation*}
$$

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which is obviously not independent of $\mathrm{G}^{\mathrm{C}}$ or g . However, if the average of Equation (5.74) is taken over a full cycle of $\psi_{o}^{C}$, the resultant average force is:

$$
\begin{equation*}
\overline{\sum_{i} f_{(i)}^{C}}=3 F \tag{5.75}
\end{equation*}
$$

which is, of course, independent of $\mathrm{G}^{\mathrm{C}}$ and g . Controlling the average force will be sufficient as long as the actual vertical movement variation is not significant throughout a rotation. Using only the z-component of Newton's Second Law of Motion from Equation (5.8), solving the differential equation for the z-position gives:

$$
\begin{equation*}
z=\frac{3 F}{2 m} t^{2}+c_{1} t+c_{2}-\frac{G^{C} \cos \left(\psi_{o}^{C}-g+\delta_{\psi_{o}}\right)}{m R_{m} \cos (d) \omega_{\psi_{o}}^{C^{2}}} \tag{5.76}
\end{equation*}
$$

if $\omega_{\psi_{o}}^{C}$ is assumed to be a constant, where $\mathrm{c}_{1}$ and $\mathrm{c}_{2}$ are arbitrary integration constants.
Thus, as long as

$$
\begin{equation*}
\frac{G^{C}}{m R_{m} \cos (d)} \square \omega_{\psi_{o}}^{C^{2}} \tag{5.77}
\end{equation*}
$$

the variation in z-position will be minimal. In order to satisfy this constraint, the craft must be heavy, the steering cannot be too forceful, and the rotation rate of the craft must be high. Plugging some likely numbers into this equation, it is not hard to see that this constraint can easily be met. (Likely numbers are $\omega_{\psi_{o}}^{C} \sim 10 \pi, \mathrm{~m} \sim 1, \mathrm{R}_{\mathrm{m}} \sim .2, \mathrm{G}^{\mathrm{C}}<\sim 1$, and d $\sim 5$ degrees.)

In the previous paragraphs it has been determined that F and $\overrightarrow{G^{C}}$ are essentially independent of each other. Furthermore, 3F has been shown to be the primary vertical force acting on the craft, and $\overrightarrow{G^{C}}$ has been shown to be the primary craft-based x-y plane
torque acting on the craft. Since $F$ and $\overrightarrow{G^{C}}$ can be independently set, the vertical motion and rotational motion are de-coupled from each other. The proposition that the altitude and attitude can be independently controlled has thus been proven.

### 5.3.1.2 Pitch, Roll, and Yaw

In order to steer the craft, a pitch, roll, yaw sensor will be placed on the hub. For any given attitude, the sensor will give a constant reading, and for any given reading, the craft must be at a given attitude. Since the order of rotation in rotation matrices matters, so that a single $\theta^{E}, \phi^{E}$, and $\psi_{i, o}^{E}$ combination can give rise to six different attitudes depending on what order the rotations are made in, the question arises as to what the sensor is actually measuring. Furthermore, there is another question as to how to translate between the rotation matrix angles, $\theta^{E}, \phi^{E}$, and $\psi_{i}^{E}$, and the sensor's pitch, roll, and yaw measurements, $\theta_{m}^{E}, \phi_{m}^{E}$, and $\psi_{m_{i}}^{E}$. (The subscript 'm' will denote a measured variable.) It is in fact the sensor's measured angles that will be controlled, and not the Earth-based rotation matrix angles. This will make any combination of angles unique to a given attitude so there can be no confusion about the orientation of the craft at any time.

The sensor assumes there is a front end of the craft and a vector which points from the center of the craft through the front, and also from the center of the craft straight up. For the purpose of this discussion, it is assumed that before any rotation, the front pointing vector is co-linear with the Earth-based x -axis, and the up pointing vector (the normal vector) is co-linear with the Earth-based z-axis. To follow this discussion, Figure 5.5 below will help with orientation.


Figure 5.5: Craft Measured Roll and Pitch angles
For any attitude, the front pointing vector is projected onto an Earth-based stationary $x$-y plane. The angle in the $x$ - $y$ plane measured clockwise from the $x$-axis is defined as the yaw, $\psi_{m_{i}}^{E}$. The normal vector is then rotated around the stationary Earthbased z-axis by an angle of $\psi_{m_{i}}^{E}$, which in a sense de-yaws the measurement of the pitch and roll. (In mathematics, angles are measured counter-clockwise; on a compass, which is what measures yaw, angles are measured clockwise.) This un-rotated normal vector is then projected onto Earth-based stationary y-z and z-x planes. The angle in the y-z plane measured counter-clockwise from the z-axis is the roll, $\theta_{m}^{E}$, and the angle in the z-x plane measured clockwise from the z -axis is the pitch, $\phi_{m}^{E}$. (Positive roll would be leaning to the right, and positive pitch would be leaning backward.)

Now that the angles have all been explained, a translation between the $\theta^{E}, \phi^{E}$, and $\psi_{i}^{E}$ rotation matrix angles, and the sensor's measurements, $\theta_{m}^{E}, \phi_{m}^{E}$, and $\psi_{m_{i}}^{E}$ needs to be established. Firstly, since the z-axis rotation angle, $\psi_{i}^{E}$, is the last rotation to occur Jesse Davis
when rotating from the craft-based frame to the Earth-based frame, the measured yaw angle will simply be:

$$
\begin{equation*}
\psi_{m_{i}}^{E}=2 \pi-\psi_{i}^{E} \tag{5.78}
\end{equation*}
$$

which is just the z-axis rotation angle measured clockwise instead of counter-clockwise. In order to find a translation between the other angles, using the description from the paragraph above, the easiest method is to start with a unit normal z-directed vector, rotate it using rotation matrices by $\theta^{E}$ and $\phi^{E}$, and then find the projected angles on the y-z and z-x planes. The rotated normal vector becomes:

$$
\left[\begin{array}{ccc}
\cos \left(\phi^{E}\right) & 0 & \sin \left(\phi^{E}\right)  \tag{5.79}\\
0 & 1 & 0 \\
-\sin \left(\phi^{E}\right) & 0 & \cos \left(\phi^{E}\right)
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \left(\theta^{E}\right) & -\sin \left(\theta^{E}\right) \\
0 & \sin \left(\theta^{E}\right) & \cos \left(\theta^{E}\right)
\end{array}\right]\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]=\left[\begin{array}{c}
\cos \left(\theta^{E}\right) \sin \left(\phi^{E}\right) \\
-\sin \left(\theta^{E}\right) \\
\cos \left(\theta^{E}\right) \cos \left(\phi^{E}\right)
\end{array}\right]
$$

Since all of the components of this vector are known, simply taking inverse tangents of the appropriate component ratios should give the correct angles of the projections onto the $y-z$ and $z-x$ planes. This process reveals:

$$
\begin{equation*}
\phi_{m}^{E}=2 \pi-\phi^{E} \tag{5.80}
\end{equation*}
$$

and

$$
\begin{equation*}
\theta_{m}^{E}=\arctan \left(\frac{\tan \left(\theta^{E}\right)}{\cos \left(\phi^{E}\right)}\right), \text { or } \theta^{E}=\arctan \left(\tan \left(\theta_{m}^{E}\right) \cos \left(\phi_{m}^{E}\right)\right) \tag{5.81}
\end{equation*}
$$

where the sign of $\theta_{m}^{E}$ is adjusted to be the sign of $\theta^{E}$. (Like the yaw angle, the pitch angle is adjusted as in Equation (5.80) because of the direction of measurement.) These translations can be applied at any time in order to switch between the two sets of angles.

The total tilt, ${ }^{\Gamma}$, of the normal vector from the Earth-based z-axis, and the induced yaw, ${ }^{\Upsilon}$, of the normal vector from the Earth-based x -axis can now be determined in terms of the measured angles. Figure 5.6 diagrams exactly what $\Gamma$ and $\Upsilon$ are meant to be. Figure 5.6 is simply Figure 5.5 (in which the craft has undergone a pitch and roll but no yaw) with a different labeling of angles.


Figure 5.6: Craft Tilt and Induced Yaw of the Normal Vector
Using ' $z$ ' as the projected $z$-component of the rotated unit normal in Figure 5.6, the projected components of the normal onto the three axes will be:

$$
\begin{gather*}
\hat{n}_{Z}=z  \tag{5.82}\\
\hat{n}_{y}=-z \tan \left(\theta_{m}^{E}\right)  \tag{5.83}\\
\hat{n}_{x}=-z \tan \left(\phi_{m}^{E}\right) \tag{5.84}
\end{gather*}
$$

If there was also some yaw of $\psi_{m_{i}}^{E}=2 \pi-\psi_{i}^{E}$, the actual projected components would be:

$$
\left[\begin{array}{ccc}
\cos \left(\psi_{m_{i}}^{E}\right) & -\sin \left(\psi_{m_{i}}^{E}\right) & 0  \tag{5.85}\\
\sin \left(\psi_{m_{i}}^{E}\right) & \cos \left(\psi_{m_{i}}^{E}\right) & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
-z \tan \left(\phi_{m}^{E}\right) \\
-z \tan \left(\theta_{m}^{E}\right) \\
z
\end{array}\right]=z\left[\begin{array}{c}
-\cos \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)+\sin \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right) \\
-\cos \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)-\sin \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right) \\
1
\end{array}(\right.
$$

The other known fact about these components is that they make up three sides of a rectangular box of which a unit vector is the diagonal, thus:

$$
\begin{equation*}
1=z \sqrt{1+\left(\cos \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)+\sin \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)\right)^{2}+\left(-\cos \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)+\sin \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)\right)^{2}} \tag{5.86}
\end{equation*}
$$

which gives:

$$
\begin{equation*}
z=\frac{1}{\sqrt{1+\tan \left(\phi_{m}^{E}\right)^{2}+\tan \left(\theta_{m}^{E}\right)^{2}}} \tag{5.87}
\end{equation*}
$$

Now $\Gamma$ and $\Upsilon$ can be found to be:

$$
\begin{align*}
& \Gamma=\arccos \left(\frac{1}{\sqrt{1+\tan \left(\phi_{m}^{E}\right)^{2}+\tan \left(\theta_{m}^{E}\right)^{2}}}\right)  \tag{5.88}\\
& \Upsilon=\arctan \left(\frac{\tan \left(\theta_{m}^{E}\right)+\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)}{\tan \left(\phi_{m}^{E}\right)-\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)}\right) \tag{5.89}
\end{align*}
$$

The only problem that might be encountered with these definitions is that there is a discontinuity in the arctangent function, and incorrect results might be given under certain situations. Specifically, assuming $\left|\theta_{m}^{E}\right| \leq \frac{\pi}{2}$, and $\left|\phi_{m}^{E}\right| \leq \frac{\pi}{2}$, the adjustments that should be made are as follows:
if $\theta_{m}^{E}>0$, and $\phi_{m}^{E}>0$ :

$$
\begin{equation*}
\Upsilon=\pi+\arctan \left(\frac{\tan \left(\theta_{m}^{E}\right)+\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)}{\tan \left(\phi_{m}^{E}\right)-\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)}\right) \tag{5.90}
\end{equation*}
$$

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if $\theta_{m}^{E}<0$, and $\phi_{m}^{E}>0$ :

$$
\begin{equation*}
\Upsilon=\pi+\arctan \left(\frac{\tan \left(\theta_{m}^{E}\right)+\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)}{\tan \left(\phi_{m}^{E}\right)-\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)}\right) \tag{5.91}
\end{equation*}
$$

if $\theta_{m}^{E}>0$, and $\phi_{m}^{E}<0$ :

$$
\begin{equation*}
\Upsilon=2 \pi+\arctan \left(\frac{\tan \left(\theta_{m}^{E}\right)+\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)}{\tan \left(\phi_{m}^{E}\right)-\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)}\right) \tag{5.92}
\end{equation*}
$$

and if $\theta_{m}^{E}<0$, and $\phi_{m}^{E}<0$ :

$$
\begin{equation*}
\Upsilon=\arctan \left(\frac{\tan \left(\theta_{m}^{E}\right)+\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\phi_{m}^{E}\right)}{\tan \left(\phi_{m}^{E}\right)-\tan \left(\psi_{m_{i}}^{E}\right) \tan \left(\theta_{m}^{E}\right)}\right) \tag{5.93}
\end{equation*}
$$

It is assumed that the propellers on the craft can only provide appreciable lift when they are spinning in one direction. Therefore, if the craft tilts off the Earth-based zaxis, the offset force, F, required by the motors will increase. Since there will be some maximum voltage, $\mathrm{V}_{\text {max }}$, that could be applied to the motors, there must also be some maximum tilt, and thus some maximum pitch and roll. In order for the craft to maintain a constant altitude, the average sum of the forces, given by Equation (5.75), projected onto the Earth-based z-axis must equal the crafts weight:

$$
\begin{equation*}
\overline{\sum_{i} f_{(i)}^{C}} \cos (\Gamma)=3 F \cos (\Gamma)=m g \tag{5.94}
\end{equation*}
$$

The offset force, F, in order to maintain a constant altitude, must therefore be:

$$
\begin{equation*}
F=\frac{m g}{3 \cos (\Gamma)} \tag{5.95}
\end{equation*}
$$

For some maximum allowable voltage, $\mathrm{V}_{\mathrm{max}}$, and some maximum steering vector magnitude, $G_{\max }^{C}$, that must always be able to be applied, using Equation (5.71), the maximum offset force is given by:

$$
\begin{equation*}
F_{\max }=\frac{V_{\max }^{2} C_{t} \rho n^{2} \pi R^{4}}{2 K_{e}^{2}\left(\left(\omega_{\psi_{o}}^{C} \tau\right)^{2}+1\right)}-\frac{G_{\max }^{C}}{R_{m} \cos (d)} \tag{5.96}
\end{equation*}
$$

which gives a maximum tilt of:

$$
\begin{equation*}
\Gamma_{\max }=\arccos \left(\frac{m g}{3 F_{\max }}\right) \tag{5.97}
\end{equation*}
$$

and if the maximum pitch and maximum roll are the same, these maximum angles are:

$$
\begin{equation*}
\theta_{m, \text { max }}^{E}=\phi_{m, \text { max }}^{E}=\arctan \left(\frac{9 F_{\max }^{2}}{2 m^{2} g^{2}}-\frac{1}{2}\right) \tag{5.98}
\end{equation*}
$$

### 5.3.2 Controller Design

What is then left to provide are the actual control laws for the altitude, attitude, and yaw. As was discussed in the beginning of this system analysis section, the roll and pitch angles, $\theta_{m}^{E}$ and $\phi_{m}^{E}$, the vertical speed, $\nu_{z}^{E}$, and the yaw speed of the hub, $\omega_{\psi_{m_{i}}}^{E}$, are the primary variables for which control needs to be determined. It should be noted that all of these variables are in the Earth-based reference frame.

Before the control can be derived, the sensors that will be used to determine the current state of the craft must be examined. If the pitch, roll, yaw speed, and vertical speed are to be controlled, these variables must also be measured. Commercially available units exist that are complete pitch, roll, yaw sensors, and the derivatives of these variables can be determined in software on the on-board control computer. Altitude
sensors can also be easily found commercially, the cheapest of which is an ultrasonic implementation.

The final variables that will need to be measured or determined are the yaw angle and speed of the outer hub in the craft-based frame, $\psi_{o}^{C}$ and $\omega_{\psi_{o}}^{C}$. The yaw angle and speed of the hoop are needed because of their importance in the motor voltage Equations (5.71)_(5.73) that determine the steering of the craft. In order to measure these variables, a potentiometer will be mounted with one end attached to the hub and the other end attached to the hoop. The yaw speed can then be determined by passing the potentiometer tap voltage through a comparator and measuring the time between positive or negative transitions of the comparator output signal. The yaw angle can be determined either by multiplying the yaw speed by time if the yaw speed is fairly constant, or can be more accurately determined by passing the potentiometer tap voltage through an analog-todigital converter. It is apparent that there will be a "front" to the craft, thus in order to steer properly, the craft based yaw angle should be referenced from the front of the craft. In fact, in implementation, the easiest thing to do would be to set the comparator's transition at the front end of the craft.

There are several ways to approach the control of the craft. A full non-linear system could be derived from the state-space description of the craft, a linearized version of the state-space model could be used to create a linear controller, engineering approximations could be used to create a simplified system for which controllers could be derived, or other approaches could be taken. For the purposes of this paper, engineering approximations will be made from which vastly simplified controllers can be derived.

These control methodologies will then be simulated in the subsequent simulation section with the full state-space model of the craft in order to verify proper operation.

### 5.3.2.1 Attitude Control

In order to steer the craft, the torque vector $\overrightarrow{G^{C}}$ must be determined from a desired roll and pitch, $\theta_{d}^{E}$ and $\phi_{d}^{E}$, and a current measured roll and pitch, $\theta_{m}^{E}$ and $\phi_{m}^{E}$. (In order not to confuse variables, the subscript ' $d$ ' will denote a desired variable, and as previously noted, the subscript ' $m$ ' will denote the measured variable.) The magnitude of this vector, $\mathrm{G}^{\mathrm{C}}$, should reasonably depend on the difference between the current and desired attitude. The angle of this vector, g , will be referenced from the front of the craft as defined in the previous paragraph. In order to simplify the control, it would be desirable to separate the pitch and roll control. Since the craft is almost perfectly symmetrical around pitch or roll angles (and in fact is perfectly symmetrical if averaged over time), the same controller for pitch angle can be used as for roll angle. This simple observation provides a means of separating the control.

If the roll and pitch controllers are separate but equal, the output of the controllers can be imagined as orthogonal control vectors, $\vec{\theta}=\theta_{d}^{E}-\theta_{m}^{E}$ and $\vec{\phi}=\phi_{d}^{E}-\phi_{m}^{E}$, whose sum is the craft steering vector, $\overrightarrow{G^{C}}$. See Figure 5.7 below:


Figure 5.7: Roll and Pitch Components of Steering Control Vector Mathematically, $\mathrm{G}^{\mathrm{c}}$ and g will thus be given by:

$$
g=\left\{\begin{array}{c}
G^{C}=\sqrt{|\vec{\theta}|^{2}+|\vec{\phi}|^{2}} \\
\arctan \left(\frac{|\vec{\phi}|}{|\vec{\theta}|}\right), \text { for } \vec{\theta}>0, \vec{\phi}>0  \tag{5.100}\\
\pi+\arctan \left(\frac{|\vec{\phi}|}{|\vec{\theta}|}\right), \text { for } \vec{\theta}<0 \\
2 \pi+\arctan \left(\frac{|\vec{\phi}|}{|\vec{\theta}|}\right), \text { for } \vec{\theta}>0, \vec{\phi}<0
\end{array}\right.
$$

The primary equation relating the goal torque vector to the attitude angles of the craft is simply:

$$
\begin{equation*}
\overrightarrow{G^{C}}=J_{r o t} \frac{\partial^{2} \vec{\Theta}}{\partial t^{2}} \tag{5.101}
\end{equation*}
$$

where $J_{\text {rot }}$ is a rotated $2 \times 2$ inertia matrix given by:

$$
J_{\text {rot }}=\left[\begin{array}{cc}
\cos \left(\psi_{o}^{C}\right) & -\sin \left(\psi_{o}^{C}\right)  \tag{5.102}\\
\sin \left(\psi_{o}^{C}\right) & \cos \left(\psi_{o}^{C}\right)
\end{array}\right]\left[\begin{array}{cc}
I_{x x} & 0 \\
0 & I_{y y}
\end{array}\right]\left[\begin{array}{cc}
\cos \left(\psi_{o}^{C}\right) & \sin \left(\psi_{o}^{C}\right) \\
-\sin \left(\psi_{o}^{C}\right) & \cos \left(\psi_{o}^{C}\right)
\end{array}\right]
$$

and $\vec{\Theta}$ is a vector angle given by:

$$
\vec{\Theta}=\left[\begin{array}{l}
\theta^{C}  \tag{5.103}\\
\phi^{C}
\end{array}\right]
$$

Using the magnitudes of $\overrightarrow{G^{C}}$ and $\vec{\Theta}$, and assuming that there is a single scalar moment of inertia, $\mathrm{J}_{\text {scalar }}$, about an axis co-linear with $\overrightarrow{G^{C}}$ and $\vec{\Theta}$, the Laplace transform of Equation (5.101) gives:

$$
\begin{equation*}
\frac{|\vec{\Theta}|}{\left|\overrightarrow{G^{C}}\right|}=\frac{1}{J_{\text {scalar }} s^{2}} \tag{5.104}
\end{equation*}
$$

which is the primary equation for which a control law must be determined.
It is apparent from Equation (5.104) that since there are two poles at zero in the torque-angle system, the system is inherently unstable with a zero degree phase margin. In order to create a stable system, phase must somewhere by added by a controller. An ideal controller for this purpose is a lead controller which takes the form:

$$
\begin{equation*}
\text { Lead }=A \frac{\alpha_{c} \tau_{c} s+1}{\tau_{c} s+1} \tag{5.105}
\end{equation*}
$$

A lead controller adds about

$$
\begin{equation*}
\Phi_{\text {center }}=\arcsin \left(\frac{\alpha-1}{\alpha+1}\right) \tag{5.106}
\end{equation*}
$$

degrees of phase at a frequency of

$$
\begin{equation*}
\omega_{\text {center }}=\frac{1}{\tau \sqrt{\alpha}} \tag{5.107}
\end{equation*}
$$

The overall gain of the lead controller, A, is adjusted in order to make the cross-over frequency of the entire system equal to that where the maximum phase of the lead controller occurs. (A standard upper limit on $\alpha$ is 10. Since a lead controller amplifies high frequency signal components by a factor of $\alpha$ more than low frequency signal components, this choice is usually made so as not to overly amplify any high frequency noise that might corrupt the controller. The maximum phase for an $\alpha$ of 10 is about 55 degrees.)

There is a limit as to how far the bandwidth of the system can be increased, however. The torque vector is created by the three motors that all have poles associated with them as well. Taking the Laplace transform of the motor characteristic Equation (5.7) gives:

$$
\begin{equation*}
\frac{b}{V_{a}}=\frac{1 / K_{e}}{\frac{L\left(I_{m}+I_{l}\right)}{K_{e} K_{m}} s^{2}+\frac{R_{\Omega}\left(I_{m}+I_{l}\right)}{K_{e} K_{m}} s+1} \tag{5.108}
\end{equation*}
$$

which is a two pole system. (One of the poles of the motor system comes from the electrical time constant associated with the inductance and resistance, and the other pole comes from the mechanical time constant of Equation (5.66).) Since every pole subtracts 90 degrees of phase, and there are already two poles at zero from Equation (5.104), the cross-over frequency of the system must be set at least a decade before the first pole of Equation (5.108) in order to create a stable system with decent phase margin. Taking all these factors into consideration, Equations (5.104)-(5.108) can be used to easily determine the parameters of a lead controller: $\mathrm{A}, \tau$, and $\alpha$.

### 5.3.2.2 Vertical Speed Control

As for the altitude control, a similar simplification will provide much help. Again using Newton's Second Law of Motion:

$$
\begin{equation*}
3 F \cos (\Gamma)=m \frac{\partial v_{z}^{E}}{\partial t} \tag{5.109}
\end{equation*}
$$

where the F is the same as in the motor voltage Equations (5.71)-(5.73). Taking the Laplace transform of Equation (5.109) and rearranging gives:

$$
\begin{equation*}
\frac{v_{z}^{E}}{F}=\frac{3 \cos (\Gamma)}{m s} \tag{5.110}
\end{equation*}
$$

This system only has one pole at the origin, but since F is generated by manipulating motor voltages, the poles from Equation (5.108) will also still be of concern.

In order to control $v_{z}^{E}$, there are now a few different options. Since the pole at the origin adds only -90 degrees of phase, a simple proportional controller could be used, as long as the bandwidth is kept well below the first pole of the motor system. In order to increase the bandwidth as high as possible, however, a lead controller could also be used with a center frequency in between the two pole frequencies of the motor system. The second choice is obviously preferable since it will give higher bandwidth leading to faster control, and a lead controller can be derived in exactly the same way as in the previous section.

### 5.3.2.3 Yaw Speed Control

The final controller that needs to be derived is for the yaw speed of the hub. The yaw speed of the hub is equal to the yaw speed of the hoop minus the speed of the yaw motor, so in order to control the yaw speed of the hub, the speed of the yaw motor must be controlled. The equation for the speed of the motor is simply given by Equation Jesse Davis

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(5.108) again! This is just a simple two pole system, and with a lead controller, the crossover frequency can now be pushed past the second characteristic motor pole. In fact, with a lead controller, the gain and cross-over frequency can be made indefinitely high. There are, of course, limitations to making the gain too high, but these limitations are only imposed by implementation issues, not by the system itself. For the purposes of simulation, a lead controller was placed at a frequency about an order of magnitude greater than the lead of the vertical speed controller.

### 6.0 Simulation

Simulation is used only to verify the theory. Although the theory has been proven mathematically to be true, it is enlightening to observe the theoretical results by carrying out numerical simulations. The simulation of the controllers and the simulation of actual motor voltage commands are performed below. The full state-space model is much more complicated than the models that were used to derive the controllers, so it is important to verify that the simplification assumptions were valid. Also, since the dynamic voltages to the motors that will be used in an actual implementation will be discretized, it is important to understand the effects this will have on the system.

### 6.1 Controlled System Step Responses

In order to verify the controllers, simulations were performed using a Matlab ode solver. Within the Matlab ode file, all of the state equations for the Earth-based frame, as well as all of the control laws in the Earth-based frame were included. Using the ode solver, the state equations were stepped forward in time from an initial zero-state starting point, and by creating time dependent control goals, the operation of the controllers could be tested.

The lead controlled vertical speed and yaw speed step responses are shown below in Figures 6.1 and 6.2 respectively. The lead controllers were empirically tuned in order to get the best responses. As can be seen in Figure 6.1, there is an overshoot in the vertical speed step response, but the settling time is quite fast so this overshoot is ignorable. The yaw speed step response has no overshoot, and the reason for this is that there is 90 degrees more of phase margin in the yaw speed system than in the vertical speed system. The other important feature of these responses is that they have a zero
steady-state error, so that any commanded values will be perfectly met after transients settle.


Figure 6.1: Vertical Speed Step Responses


Figure 6.2: Yaw Speed Step Responses

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As for the step responses of the pitch and roll, Matlab was unable to be used to accurately lead control the steering vector. The reason for this was several discontinuities encountered when evaluating the derivative of the commanded steering vector to compute the lead control. The theoretical design of a lead controller for the attitude in section 5.3.2.1 should be correct, but simulations were unable to be carried out to prove this fact. Instead, a simple proportional controller was implemented. This approach worked because a linear damping factor was added to each of the rotational velocities, and this moved one of the poles at zero slightly off zero. This allowed for a proportional controller to be used to make a zero-crossing between these first two poles at a much lower bandwidth than would be expected with the full lead controller. The resulting proportionally controlled step responses of the pitch and roll are shown below in Figure 6.3:


Figure 6.3: Pitch and Roll Step Responses

Overall, the controller design for the vertical speed and yaw speed seems to be flawless, whereas the controller design for the attitude could use significant improvement. One important fact that may also be causing problems with the pitch and roll step responses is the non-linearity of the actual system. It is possible that the assumptions made in order to simplify the system and derive a simple control law ignore some fundamental properties of the system that cannot be overlooked. While the proportional control can be seen to work for a slightly altered system, the controller for the steering remains an area for future research.

### 6.2 Time Discretized Dynamic Motor Control

The other simulation that needs to be carried out, besides the step responses of the controlled system, is of the actual steering torque vector during operation. As was seen in the system analysis section of the theory, the necessary forces from the motors as they spin around the hub are sinusoidal in order to properly steer the craft. Since a computer will eventually be the mechanism that provides the PWM signals to the motors, it would be difficult to continuously vary the PWM signals as the equations require. Instead, the computer will most likely have to discretize the PWM variation into a certain number of segments per rotational period. This will create a discretized sinusoidal torque from each of the three steering motors, and the effects of this discretization should be simulated in order to verify that proper steering can still be commanded.

A Matlab file was created that calculated the total torque vector from three discretized sinusoidal torque vectors with a variable number of discretizations. The inputs to the file are the number of discretizations per period, the magnitude of the goal torque vector, $\mathrm{G}^{\mathrm{C}}$, and the angle of the goal torque vector, g . The file plots the RxF torque from
each of the three motors over hoop rotation angle, the total varying torque vector over time, and also finds the average torque vector. The complete code for the file can be found in Appendix A. An example of the torque from each motor over hoop rotation angle is given in Figure 6.4 below:


Figure 6.4: Example RxF Torques from Each Motor During Steering
It was discovered that both the magnitude and angle of the actual average torque vector vary significantly with the number of discretizations commanded. The angle variation was found to be a positive (counter-clockwise) angle offset to the commanded angle, and the magnitude variation was found to be a less than unity multiple of the commanded magnitude. These variations are captured in Figures 6.5 and 6.6 below:


Figure 6.5: Error Angle of Actual Average Torque Vector vs. Discretizations


Figure 6.6: Magnitude Error Factor vs. Discretizations
Once the discovery of the offset angle and magnitude error were made, the file was altered to pre-correct for these errors by subtracting the angle offset from the commanded Jesse Davis

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angle, and pre-scaling the commanded magnitude by the inverse of the magnitude error factor.

The other major variation that occurred with the number of discretizations was the actual calculated torque variation over time. (The magnitude did not vary significantly in this regard.) This variation gives a sense of how close the actual torques are to the average torque over hoop rotation angle. For a commanded $G^{C}=1$ and $g=0$, Figures 6.7 and 6.8 below give an example torque variation over time and total angular variation of torques as a function of discretizations:


Figure 6.7: Polar Plot of Torque Vector Variation Resulting from Discretizations


Figure 6.8: Total Angle of Torque Variation Over Time vs. Discretizations The number of discretizations that were chosen for the actual implementation in the computer code was based on a need to balance steering precision with computer processing time. The higher the number of discretizations, the better the precision of the steering, but also the more processor time required to update the PWM signals. Based on Figure 6.8 above, 20 discretizations per revolution were chosen.

### 7.0 Design and Construction

Once the theory and simulations were complete, the task turned to building a working stabilization system. There are two main sections to the system: an on-board, stabilizing computer, and an off-board user interface and controller. The sub-sections of the on-board unit are a pitch, roll, yaw sensor, an altimeter, an outer hoop rotation rate sensor, a receiver for the user's commands, a central processor, and motor driver circuitry. The sub-sections of the off-board unit are a user interface, a command serializer, and a transmitter for sending commands to the craft. The primary goal behind the design of the system was modularity. This methodology simplifies the design, construction, and debugging, and for a complex system, breaking the problem into parts aids greatly. The remainder of this section will describe in detail the actual design and construction of both the hardware and software needed to create a working system. Unforeseen problems that arose during this process will be discussed, and solutions that were implemented will be described.

### 7.1 User Interface and Off-Board Controller

The main sections of the off-board unit are the user interface, a command serializer, and a transmitter. The user interface is implemented in a simple and widely used method of using a two-axis potentiometer as a joystick. A two-axis potentiometer joystick is a single unit in which there are two potentiometers mounted in such a way that a single stick moved in orthogonal directions will create varying resistances in two orthogonal potentiometers. Voltages from the potentiometers center-tap can then be interpreted as user input commands. The complete block diagram of the off-board system is shown in Figure 7.1 below:


Figure 7.1: Off-board System Block Diagram
The "A2D" is an analog-to-digital converter that takes in one of four center tap voltages from an analog "4:1 Mux" and gives a parallel digital output; the "A2D Controller" is needed in order to control the inputs and outputs of the analog-to-digital converter; the "Serializer" takes in parallel data and sends out a single serial signal; the "Serializer Controller" is needed in order to control the serializing procedure of the data input to the serializer; the "Bit Timer" is used to set the baud rate (bits/second) of the serialized data; the "Byte Timer" is used to set the byte transmission frequency bytes/second; and the "Clock" is used to give the system a time reference. The signals on the interconnecting wires will be explained in the sections below.

A single micro-controller chip could be used to incorporate all parts of the offboard system except for the transmitter giving just a two-chip solution. An ideal microcontroller for this might be something in the PIC12C series, the PIC16C7 series, or the PIC16F88 from MicroChip. The implementation used here, however, is accomplished with VHDL (much like Verilog) programmable PLAs. This decision was made in order to demonstrate hardware modularity (the on-board computer uses mostly software modularity), and was also made due to a lack of resources. The design of each section, as well as the input/output signals from and to the section will be presented individually. Jesse Davis

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### 7.1.1 Multiplexer and Analog-to-Digital Converter

Inputs: center-tap from each of four potentiometers on the two joysticks data_select 0,1: two bit data selector from A2D Controller nread: active-low conversion enable signal from A2D Controller

Outputs: nconv_done: active-low conversion complete signal to A2D Controller data_in 1-5: five parallel digital lines to Serializer

The only two parts of Figure 7.1 that were combined into a single chip solution were the multiplexer and analog-to-digital converter. The chip that was used for this was the AD7824 which is a four-channel, 8 -bit ADC. The fastest cycle time for switching input channels and converting to a stable output is 2.5 us corresponding to 400 kHz . This frequency turns out not to be the limiting factor to define the number of bytes per second that the controller can send at top speed, but it is an important upper limit to note for the proper use of this ADC.

The inputs to the four analog channels of the ADC are the center-taps from the joystick potentiometers. The channel selection and active-low conversion enable are controlled by the A2D Controller. As will be described in a subsequent section, the A2D Controller waits for the Serializer to finish sending the data from the ADC before switching input channels and requesting another conversion. The outputs of the ADC are the parallel digital converted data which is sent directly to the Serializer for transmission, and the conversion complete signal sent to the A2D Controller. The reason all eight bits are not sent to the Serializer is in order to conserve the number of bits per byte of transmission, and also because the full resolution should not be necessary to still have
very precise control of the craft. The active-low conversion complete signal is sent to the A2D Controller which then alerts the Serializer Controller that new data is ready to be sent by the Serializer.

### 7.1.2 Byte Timer

Inputs: no signal inputs from any other section
Outputs: tx_trigger: active-high data transmission trigger to A2D Controller

The Byte Timer has the purpose of setting the frequency of transmission of bytes from the off-board unit. The choice of byte frequency can be made rather arbitrarily, with the main three restrictions being that the command updates should be fast enough to control the craft precisely, slow enough that the on-board computer doesn't get overloaded with data, and slow enough that bytes don't overlap each other. Since the baud rate was chosen to be about 2400 (for reasons that will be explained shortly), and there are a total of 10 bits to a standard serial byte transmission, the maximum frequency of non-overlapping bytes per second is 240 Hz . In order not to push this maximum, the choice was made to update each of the four command channels at a rate approximately equal to the inherent update rates of the Pitch, Roll, Yaw Sensor and Altimeter.

The implementation of the Byte Timer was accomplished with a 555 timer setup in an astable oscillation mode as in Figure 7.2 below:


Pin Out


Byte Timer Circuitry

Figure 7.2: Byte Timer Implementation
The analysis of operation of this circuit can be found in any 555 data sheet, and will thus only be abbreviated here. The capacitor C 2 charges through Ra and Rb and discharges through Rb , and this charging and discharging sets up the frequency and duty cycle of oscillation. With the trigger input, $\operatorname{pin} 2$, tied to the threshold input, pin 6, the capacitor charges up and triggers its own discharge. Since the 555 automatically stops discharging the capacitor at a certain fraction of Vcc , the capacitor then charges again, and again triggers its own discharge in a repeating cycle. The charge and discharge waveform is squared to give the tx_ trigger output.

As given by the 555 datasheet, the resistors and capacitors should be chosen such that:

$$
\begin{gather*}
C_{1}=.01 u F  \tag{7.1}\\
f_{\text {osc }}=\frac{1.44}{\left(R_{a}+2 R_{b}\right) C_{2}}  \tag{7.2}\\
D=\frac{R_{a}+R_{b}}{R_{a}+2 R_{b}} \tag{7.3}
\end{gather*}
$$

where $f_{\text {osc }}$ is the frequency of oscillation of the output square wave, and D is the squarewave duty ratio (the time the square wave is high). Using these formulas, the actual
values that were chosen are: $\mathrm{C} 1=.01 \mathrm{uF}, \mathrm{C} 2=.1 \mathrm{uF}, \mathrm{Ra}=39 \mathrm{k} \Omega$, and $\mathrm{Rb}=39 \mathrm{k} \Omega$. In testing, the oscillation frequency was about 145 Hz . (Note: the duty ratio of the signal doesn't matter since it is only the frequency of low to high transitions that affects the rest of the system.)

### 7.1.3 A2D Controller

Inputs: clk: system clock
nconv_done: active-low conversion complete signal from ADC
txdone: active-high data transmission complete signal from Serializer Controller tx_trigger: active-high data transmission trigger from Byte Timer

Outputs: data_select 0,1: two bit data selector to ADC and Serializer
nread: active-low conversion enable signal to ADC
data_ready: active-high data ready signal to Serializer Controller

The A2D Controller controls the selection and conversion of the analog joystick data via the ADC. The logic behind the controller is very direct. When triggered by the rising edge of tx_trigger from the Byte Timer, the controller initiates a conversion of one of the four analog inputs by issuing a two bit data_select signal, to choose the analog input, and pulling low the active-low ADC read trigger, nread. The A2D Controller then goes into a wait state until the analog-to-digital conversion is complete, as signaled by the nconv_done line being pulled low by the ADC. At the end of a conversion, the A2D Controller alerts the Serializer Controller that new digital data is on the bus between the ADC and Serializer. Again the A2D Controller goes into a wait state until it is signaled by the Serializer Controller, which eventually pulls txdone high, signifying that the
transmission of the data is complete. The A2D Controller then updates the data_select bus to select the next analog input, and resets to its initial state to wait for the trigger from the Byte Timer again.

The controller is implemented as a finite state machine in a VHDL programmed PLA. As was already mentioned, instead of using a single micro-controller and programming in a more powerful language, such as C, several PLAs were used in order to demonstrate hardware modularity. Also, finite state machines are particularly easy to write in VHDL, and for all three programmed parts, the A2D Controller, the Serializer, and the Serializer Controller, finite state machines were straightforward architectures. The complete VHDL code for these three off-board components is included in Appendix B.

### 7.1.4 Serializer

Inputs: clk: system clock
txbit: bit transmit signal from Serializer Controller
txdone: active-high data transmission complete signal from Serializer Controller data_in 0-9: ten parallel digital lines from ADC, A2D Controller, and three switches

Outputs: data_out: serialized data output to Transmitter

The Serializer takes ten parallel digital inputs and time multiplexes them into a single serial output. The Serializer is controlled by two signals from the Serializer Controller, txbit and txdone. Most of the time, the Serializer is in a wait state. The rising edge of the bit transmit command, txbit, triggers the Serializer to read an internal register
bit_count which decides which bit, $0-9$, the Serializer should place on the data_out line. The Serializer then updates bit_count and returns to the wait state. When the transmission complete signal, txdone, is asserted, the Serializer resets the bit_count and returns to the wait state. Since both the Serializer and Serializer Controller are hard-coded to send eight bit bytes plus a start and stop bit, and also since the bit_count of the Serializer is inherently initialized to zero when power is applied, no byte truncation or bit duplication should occur. The txdone signal is used to ensure that if there is a communication mistake, however, only a single byte will be affected and communication will be reset before the next byte is sent.

Each byte of data sent by the Serializer has the same format. There is a start bit, eight data bits, and a stop bit. The format of the eight data bits is given in Figure 7.3 below, with the least significant bits sent first:


Figure 7.3: User Command Data Format
The most significant bit is a mode bit. There will be two modes of operation of the craft, a completely user controlled mode and an on-board computer aided mode. The user controlled mode will simply interpret the commands from the user straight into the motor signals, and the computer aided mode will act as a flight stabilizer so the user can simply steer the craft instead of also having to balance it. The next two bits are the selector bits that select which of the four commanded parameters the following data is in reference to. The next bit, bit 4, determines the directionality of the data that follows. If, for example, a

30 degree pitch is commanded, this bit will decide whether the pitch is forward or backward. For the thrust parameter in the user controlled mode, this bit will be an additional data bit because there will not be the possibility of negative thrust. Finally, the last four bits are the actual data bits. Four bits, including the positive or negative bit, will give 31 total possible commands per parameter. (The thrust will have 32 commands.)

The Serializer was implemented on a PLA programmed in VHDL with a finite state machine architecture. The interesting aspect of the Serializer implementation was the method of catching the rising edge of the bit transmit signal, txbit. Since the transmission complete signal, txdone, is independent of txbit, txbit could not be used as the Serializer's clock input because the txdone signal would never be read. Thus the system clock had to be used as the Serializer's clock. If txbit lasted for more than one clock cycle, however, the Serializer might send a subsequent bit far sooner than expected, thus garbling the transmission. In order to trigger on only the rising edge of txbit, a flag was used in the VHDL code to indicate whether or not txbit had gone low since the last time it had been high. If txbit was read high but had not gone low since the last time it was read high, the same high txbit signal must be being read twice. If txbit was read high and had been read low since the last time it was read high, txbit must have just gone high, and the rising edge was caught. A code snippet of this logic is immediately below. For the full VHDL code see Appendix B.

```
if rising_edge(clk) then
    if (txdone = '1') then
        data_out <= '0';
        bit count <= "0000";
        flag
    elsif (txbit = '0') then
        flag <= '0';
```

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```
elsif ((txbit = '1') and (flag = '0')) then
    flag <= '1';
    case bit_count is
    ...
end if
```


### 7.1.5 Serializer Controller

Inputs: clk: system clock
data_ready: active-high data ready signal from A2D Controller per_counter 0-3: four-bit baud period counter from Bit Timer

Outputs: en_nres: active-high counter enable, active-low counter reset to Bit Timer txbit: bit transmit signal to Serializer txdone: active-high data transmission complete signal to A2D Controller and Serializer

The Serializer Controller controls the data transmission of the off-board unit. It enables and resets the Bit Timer which sets the baud rate, it triggers the Serializer to send bits at the correct times, and it signals the A2D Controller to update the data after a transmission is complete. There are several stages to the operation of the Serializer Controller. After the Bit Timer is reset and enabled, the Serializer Controller waits until the time required between each bit is counted by the Bit Timer. At the end of this time, the Serializer Controller triggers the Serializer to send a bit, and restarts the Bit Timer. After ten bits have been sent, the Serializer Controller alerts both the Serializer and the A2D Controller that the transmission is complete. This transmission complete signal resets the Serializer's bit_count so that the next bit to be sent will be the data 0 bit, and it Jesse Davis Master's Thesis, Electrical Engineering Professor Charles Coleman
also tells the A2D Controller to trigger the ADC for new data when the Byte Timer triggers it.

The Serializer Controller was implemented as a finite state machine in VHDL. The full code is given in Appendix B.

### 7.1.6 Bit Timer

Inputs: en_nres: active-high counter enable, active-low counter reset from Serializer Controller
clk: system clock
Outputs: per_counter 0-3: four-bit baud period counter to Serializer Controller

The only purpose of the Bit Timer is to act as a binary counter that can be reset at any time and enabled and disabled to count. This is precisely what the LS163 IC can do, and additionally, the LS163 can be daisy-chained to provide slower and slower counting. Since the speed of the system clock was 1.8432 MHz , and the baud rate required was 2400, the clock had to be divided by 768 by the Bit Counter. This required three LS163's where bits 8 and 9 of the total counter where used as the baud period timer by the Serial Controller. $(768=512+256$.) The complete implementation of the Bit Timer is shown in Figure 7.3 below:


Figure 7.4: Bit Timer Implementation

### 7.1.7 Transmitter

Inputs: data_out: serialized data output from Serializer
Outputs: antenna_output: output to 50 Ohm matched antenna for RF transmission

The Transmitter and Receiver pair, along with their antennas, are meant to replace a wire connected between the input of the transmitter and output of the receiver. This project was in no way related to the complex field of RF engineering, and a simple black box solution was thus required. The Transmitter and Receiver that were chosen, the TXM-418-RM and RXM-418-RM from Linx Technologies, were chosen for their simplicity, small size, and long range capability. The Transmitter has only a data input and antenna output aside from power supply requirements. The operation of the

Transmitter and Receiver does not require any tuning or any supporting hardware, and as a result the entire RF implementation was finalized within five minutes.

### 7.1.8 Clock

Inputs: no signal inputs from any other section
Outputs: clk: system clock

The Clock also was a very simple aspect of the system to implement. Instead of attempting to build an oscillator that would become the system clock, a clock IC was used that had only the clock output pin besides power supply requirements. The chip used was the P1100-HC from Pletronics which held a stable clock output of 1.8432 MHz .

The off-board computer was prototyped and then hand laid and hand soldered onto a smaller controller board. A nine volt battery was used as a power supply, and a linear five volt regulator created the power buses for the hand held circuit. A picture of the final implementation is in Figure 7.5 below:


Figure 7.5: Off-Board Controller Final Implementation

### 7.2 On-Board Computer

The on-board computer is also broken into several parts. The architecture of the on-board computer uses a master controller to collect data from several sensors, as well as command data from the off-board controller, and process these data into PWM signals sent to four motor drivers to steer the craft. This architecture is captured by Figure 7.6 below:


Figure 7.6: System Block Diagram
As discussed in the control section of the theory, an Altimeter, Pitch, Roll, Yaw Sensor, and an Outer Hoop Rotation Rate Sensor are the minimum necessary sensors needed to fly the craft. The CPU Data Interface is required because of some very specific issues with the communication protocols available on the CPU. Since each of these sensors needs to be interfaced in an entirely different manner, the CPU code demonstrates a wide variety of the functionality of the particular CPU chosen.

The Altimeter and Pitch, Roll, Yaw Sensor are both polled by the CPU for data, while the Outer Hoop Rotation Rate Sensor and Receiver both continuously transmit data to the CPU. The Altimeter and Outer Hoop Rotation Rate Sensor data transmissions interrupt the processor when they are received since the sensors are inherently asynchronous as will be described. The Pitch, Roll, Yaw Sensor and Receiver data inputs
are serial and are buffered in software by background processes. The formatting of each set of data is also very different. The Outer Hoop Rotation Rate Sensor signal is a square wave with a period equal to the period of rotation of the hoop around the hub; the Altimeter signal is a pulse with a width proportional to the altitude of the craft; the Pitch, Roll, Yaw Sensor has a manufacturer specified seven byte response format; and the command data from the user that comes in from the Receiver has a custom format given in Figure 7.3 and explained in the Serializer section of the off-board controller above.

Each section of the on-board computer is explained in detail below. Since most of the components were purchased and not designed and built as part of the project, descriptions of the communications protocols and basic operating characteristics about the specific devices are all that will be provided. For more detailed information, see the manufacturers' data sheets. As for the CPU section, all of the work necessary was done in software, so the general structure of this code will be given, and the full text of the code will be provided in Appendix C.

### 7.2.1 Outer Hoop Rotation Rate Sensor

Inputs: no signal inputs from any other section
Outputs: rotation_clock: square wave signal with period matched to hoop rotation around hub to CPU

The primary purpose of the Outer Hoop Rotation Rate Sensor is to determine how fast the hoop is spinning with respect to the hub. The sensor also provides a reference for the angle between the front of the craft (the front of the hub). Both of these data are needed in order to properly control the steering voltage signals to the motors, and thus in
order to properly control the craft. (One other piece of information that may be given by this sensor is the actual angle between the front of the craft and the location of the motors, and not just a reference for it. Depending on the particular parts used in the implementation, this information either will or will not be accurately available.)

In order to determine the rate of rotation in a simple manner, the wrap around on the center tap of a potentiometer will be detected. With one end of the potentiometer mounted on the hub, and one end of the potentiometer mounted on the hoop, the voltage on the center tap of the potentiometer should ideally be a saw-tooth wave with the same frequency as the rotation frequency of the hoop. In the implementation given here, this saw-tooth wave is squared to give a square wave clock with clean transitions that can easily be detected by the CPU. The total circuit is provided in Figure 7.7 below:


Pin Out


Outer Hoop Rotation Rate Sensor Circuitry

Figure 7.7: Outer Hoop Rotation Rate Sensor Implementation
Since the wrap around is linked to the physical spinning of the hoop, the angular placement of the potentiometer can be made such that the wrap around corresponds to a certain orientation between hoop and hub. If a full turn potentiometer is used, instead of the more standard partial turn potentiometers, the actual location of the hoop in relation to the hub can be determined at any time. In the current implementation, a partial turn potentiometer was used so that the only useful information given by the sensor is an angular rate and angular reference and not any angles themselves. It is hoped that, other Jesse Davis

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than at start-up, the rotation rate of the hoop will vary only slowly in relation to its own speed so that the angular data can be created from the angular rate data and angular reference.

### 7.2.2 Altimeter

Inputs: alt_query: negative transition query signal from CPU Data Interface
Outputs: echo: output relating to altitude of craft to CPU

The altimeter that was chosen is an ultrasonic range finder that is accurate between three centimeters and three meters. The range finder works by sending out a burst of ultrasonic tones and waiting for the echo to return. The length of time of the echo then determines how far away the nearest object is since the speed of sound in air is fairly constant under normal conditions. The specific ranger that was used, the Devantech SRF04, requires a falling transition trigger, alt_query, and then responds with a signal, echo, that has a high time proportional to the distance of the nearest object. The high time of the echo signal is specified at 36 milliseconds when no ultrasonic echo is recorded by the ranger. There is also a mandatory specified 10 millisecond delay between the end of the echo signal and another falling transition of alt_query. This corresponds to a maximum querying rate of 21.74 Hz , so in order not to push the limit, the altimeter was queried at 20 Hz . Data taken on the ranger at four centimeter increments from four centimeters to over three meters is shown in Figure 7.8 below:


Figure 7.8: Distance vs. Echo Time of the Ultrasonic Altimeter
As can be seen in Figure 7.8, the response of the device is remarkably linear across its operating regime. In fact, the r-squared value from a linear regression is $99.997 \%$ ! Based on the linear regression, the slope of the data is $74.016 \mathrm{us} / \mathrm{cm}$ with a .048 us offset. Since the resolution of the CPU clock that measures the echo time is only about 25 us , the offset is negligible, and the minimum altitude resolution is about .34 cm . Also, rounding 74.016 to 74 corresponds to less than a millimeter error over the entire range, so to simplify calculations in the CPU code, $74 \mathrm{us} / \mathrm{cm}$ will be used. As for the repeatability of the sensor, the measured maximum deviation in microseconds anywhere in the range was about 25 , corresponding to only .34 cm of repeatability error at worst. This error is negligible on the order of scale of the craft, and so the sensor's data can be used with confidence.

### 7.2.3 Pitch, Roll, Yaw Sensor

Inputs: pry_query: formatted one byte serial request from CPU Data Interface Outputs: pry_data: formatted seven byte serial response to CPU Data Interface

The Pitch, Roll, Yaw Sensor used, a MicroStrain 3DM-DH, uses several orthogonal accelerometers and magnetometers to determine the attitude of the craft. The detailed inner workings of the sensor are complicated and unnecessary to present here. The manufacturer supplied specifications are an accuracy of $\pm .7$ degrees in pitch and roll, and $\pm 1.5$ degrees in yaw each with resolutions of less than .1 degrees. The usable range of the pitch and yaw is $\pm 180$ degrees, and the usable range of the roll is $\pm 70$ degrees. The pitch, roll, and yaw data is updated at a maximum rate of about 30 Hz , and is communicated using an RS-232 serial protocol at 9600 baud.

The greatest benefit of the sensor is that it requires no initialization routine which would cause a startup delay and also complicate the CPU code. The one setup that could be done is to tare all the pitch, roll, and yaw values, but these tares can be handled in the CPU so that the only request from the CPU will be for pitch, roll, and yaw data. Also, the Pitch, Roll, Yaw Sensor requires the same voltage range as does the CPU, so only one power supply needs to be used. With the ease of integration and communication, the precision and accuracy of the sensor, as well as the physical size of the sensor package, the MicroStrain sensor is ideal for the craft. (A new version of the MicroStrain sensor has just been released which also includes angular rate gyroscopes. This new sensor is unaffected by the movement of the craft itself, and will work in any dynamic as well as static environment. The sensor used in this project is meant only for a static environment,
but the accelerations experienced by the craft should be small compared to the effects of gravity, so the sensor deviation will be minimal. For a future design, the new sensor, the 3DM-G should be used.)

### 7.2.4 Receiver

Inputs: antenna_input: input from 50 Ohm matched antenna for RF reception Outputs: user_data: formatted one byte serial data to CPU Data Interface detect: analog data detect signal to CPU

The receiver's main function is to receive digital serial data from the transmitter on the off-board controller. Since the receiver will always be picking up noise from the environment, even if the transmitter is turned off, one additional requirement of the receiver is to also have an output that alerts the CPU to the presence or absence of a strong carrier signal from the transmitter. This detect output is essentially a data validation output, and if it is not active, the data that is being sent out of the receiver must be spurious.

The receiver used has exactly and only these two outputs. As mentioned in the Transmitter section of the off-board controller above, RF engineering was not a major part of the design work of this project, and a black box solution was sought. The TXM-418-RM and RXM-418-RM from Linx Technologies were chosen as the Transmitter/Receiver pair, and they were both matched with an ANT-418-CW-RH antenna giving a manufacturer specified line-of-sight range of over 500 feet. The main drawback to the communications link is that the Linx components are specified to only handle data rates up to 10 K baud. In testing, the maximum safe error-free standard baud
rate was found to be 2400 . As was mentioned in the Byte Timer section of the off-board controller above, this limits the maximum byte rate to 240 Hz . If a higher speed system were desired, with a higher user command update rate, a new Transmitter/Receiver pair would need to be found.

The only unexpected issue with the Receiver hardware implementation was with the detect output. The detect output is not a standard digital output, and its voltage varies by only a very slight amount between on and off states. As a result, a conversion of the detect output into a digital signal was required. Since the CPU has eight analog-to-digital converter inputs, however, no special off-CPU hardware was needed, and the conversion could be handled in software on the CPU. The proper loading of the detect pin was still required, however, and it was discovered that a $1 \mathrm{M} \Omega$ resistor was required from the detect pin to ground in order to significantly differentiate the on and off outputs.

### 7.2.5 CPU Data Interface

Inputs: pry_data: formatted seven byte serial response from Pitch, Roll, Yaw Sensor user_data: formatted one byte serial data from Receiver alt_query_clock: 4 x speed PWM altimeter query signal from CPU pry_query_fromCPU: one byte serial request to Pitch, Roll, Yaw Sensor in the serial protocol of the CPU to the CPU Data Interface

Outputs: alt_query: PWM altimeter query signal to Altimeter pry_query: one byte serial request to Pitch, Roll, Yaw Sensor pry_data_toCPU: formatted seven byte serial response from Pitch, Roll, Yaw Sensor altered to the serial protocol of the CPU to the CPU
user_data_toCPU: formatted one byte serial data from the Receiver altered to the serial protocol of the CPU to the CPU

The CPU Data Interface was not an original component of the system design, and was discovered to be necessary after extensive testing of the CPU. The CPU has two dedicated programmable hardware serial ports, but the hardware interface for these ports requires more hardware design work to be done than seems reasonably necessary for this project. The other serial possibilities are 14 general purpose programmable I/O pins on the CPU, so these were chosen to be used instead. The interesting aspect of the serial communication from these pins, however, is that the pins can only be driven between 0 and 5 volts opposed to the standard -12 and 12 volts, and also the voltages are inverted from standard RS-232 protocol. In other words, instead of 5 volts corresponding to 12 volts and 0 volts corresponding to -12 volts, 5 volts corresponds to -12 volts and 0 volts corresponds to 12 volts.

The three serial inputs of the CPU Data Interface are thus simply inverted to become the correct serial outputs. Since the only bi-directional serial communication occurred with the Pitch, Roll, Yaw Sensor, and it was discovered that this sensor could detect serial communications between 0 and 5 volt levels, only an inverter is required, and no more hardware intensive level changer. The inverter must sustain $\pm 12$ volt inputs, however, but this is not a difficult constraint to meet. All this leads to the fact that the digital signal processing that the CPU Data Interface provides gives $p r y_{-} q u e r y=\overline{p r y}$ _query_fromCPU and pry_data_toCPU $=\overline{p r y_{-} d a t a}$.

Since the user command data coming from the receiver is specifically formatted in the off-board controller, there is a choice as to whether or not to put the data through an inverter on the on-board computer or to invert it before transmission. Since the CPU would require that a high voltage level be held across the RF channel unless data was being received, it was decided to invert the data on the on-board computer since holding a non-zero level can be difficult over an RF link. Thus another conversion that the CPU Data Interface provides is user_data_toCPU $=\overline{\text { user_data }}$.

The other interfacing that the CPU Data Interface provides is for the altimeter querying. As specified in the Altimeter section of the off-board controller above, the querying rate of the Altimeter is 20 Hz . This specification is precisely met by a PWM signal from the CPU. In order to use the built-in software serial communication routines for the other data I/O lines, however, the minimum frequency PWM signal from the CPU is about 125 Hz . In order to provide a proper trigger for the Altimeter, a frequency reduction is thus required. It was chosen to program the CPU to give a 160 Hz alt_query_clock signal that is frequency divided 8 times by a three bit counter on the CPU Data Interface. The CPU Data Interface uses the alt_query_clock signal as a clock input that cycles a finite state machine through eight possible states. In only one of these states, the altimeter query signal, alt_query, is brought high, and the negative transition of this signal triggers the Altimeter to take a measurement. The high time requirement of the alt_query signal specified by the Altimeter manufacturer of 10 microseconds is also easily met by this 6.25 millisecond high time trigger.

The CPU Data Interface is implemented on a VHDL programmed PLA. The three necessary inverters are made as non-clocked, non-latched inverters so that the
communication between the CPU and Altimeter would be completely independent of the other CPU communication. The complete VHDL code for the CPU Data Interface is provided in Appendix B.

### 7.2.6 Motor Drivers 1-3 and Yaw Motor Driver

Inputs: PWM 1-3, Yaw: PWM signals for motors from CPU
Outputs: Power amplified PWM signals to motors

The final sections of the on-board computer to be described besides the actual CPU are the Motor Drivers for the three steering motors and the one yaw motor. The main requirement on the Motor Drivers is that they be able to create sharp transitions for a high power output PWM signal. The PWM frequency required will be specified by the types of motors to be used in the construction of the craft. Since the actual craft construction is not a part of this project, the specific motors for implementation were not chosen, so a definite PWM frequency could not be chosen. In order to supply a relatively constant voltage to a motor, a PWM frequency should be at least 10 times higher than the inherent electrical time constant frequency. For most standard miniature DC motors, a PWM frequency of 40 KHz is more than ample, and so this is the PWM frequency that was chosen. The switching time of the PWM signal should be at least about 100 times faster than the PWM period, so the Motor Drivers need to be able to switch the full system voltage, $\mathrm{V}_{\text {max }}$, in 250 nanoseconds.

As for the current sourcing requirement of the Motor Drivers, some thrust theory will need to be done in order to determine power requirements for the craft. From

Bernoulli's Law, pressure is dependent on velocity of a fluid, and it can be determined that:

$$
\begin{equation*}
P_{o}=P+\frac{1}{2} \rho v_{i n}^{2} \tag{7.4}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{o}}$ is the ambient non-moving air pressure well above the propeller, P is the air pressure just above the propeller, $\rho$ is the air density, and $\mathrm{v}_{\text {in }}$ is the downward velocity of the air just above the propeller. The pressure just below the propeller will be $P+\Delta P$ and will be moving with a velocity of $\mathrm{v}_{\mathrm{in}}$, and the pressure well below the propeller will again be $\mathrm{P}_{\mathrm{o}}$ and will be moving with a velocity $\mathrm{v}_{\mathrm{out}}$. Using these observations gives the following equality:

$$
\begin{equation*}
P_{o}+\frac{1}{2} \rho v_{\text {out }}^{2}=P+\frac{1}{2} \rho v_{\text {in }}^{2}+\frac{1}{2} \rho v_{\text {out }}^{2}=P+\Delta P+\frac{1}{2} \rho v_{\text {in }}^{2} \tag{7.5}
\end{equation*}
$$

and solving for $\Delta P$ yields:

$$
\begin{equation*}
\Delta P=\frac{1}{2} \rho v_{o u t}^{2} \tag{7.6}
\end{equation*}
$$

In order to create this pressure differential, a thrust, $f^{C}$, is required by the propeller:

$$
\begin{equation*}
f^{C}=\pi R_{\text {prop }}^{2} \Delta P=\frac{1}{2} \rho \pi R_{\text {prop }}^{2} v_{\text {out }}^{2} \tag{7.7}
\end{equation*}
$$

This thrust can also be written in terms of a mass flux, $\Phi$, as:

$$
\begin{equation*}
f^{C}=\Phi v_{\text {out }}=\rho \pi R_{\text {prop }}^{2} v_{\text {in }} v_{\text {out }} \tag{7.8}
\end{equation*}
$$

Equating Equations (7.7) and (7.8) gives:

$$
\begin{equation*}
v_{\text {out }}=2 v_{\text {in }} \tag{7.9}
\end{equation*}
$$

In order produce this required thrust, the propeller must be supplied with some amount of power. This required power, $\mathrm{P}_{\text {pow, }}$ is:

$$
\begin{equation*}
P_{\text {pow }}=\frac{1}{2} \Phi v_{\text {out }}^{2}=\frac{1}{2} \rho \pi R_{\text {prop }}^{2} v_{\text {in }} v_{\text {out }}^{2}=\frac{1}{4} \rho \pi R_{\text {prop }}^{2} v_{\text {out }}^{3} \tag{7.10}
\end{equation*}
$$

Rearranging Equation (7.7) in order to find the required $v_{\text {out }}$ for a given thrust gives:

$$
\begin{equation*}
v_{\text {out }}=\sqrt{\frac{2 f^{C}}{\rho \pi R_{\text {prop }}^{2}}} \tag{7.11}
\end{equation*}
$$

and substituting Equation (7.11) into Equation (7.10), the power required for a given thrust is:

$$
\begin{equation*}
P_{p o w}=\sqrt{\frac{f^{C 3}}{2 \rho \pi R_{p r o p}^{2}}} \tag{7.12}
\end{equation*}
$$

If the maximum voltage in the system is $\mathrm{V}_{\max }$, the motors have an efficiency of $\eta$ for converting electrical power into mechanical power, and the maximum thrust is $f_{\text {max }}^{C}$, the maximum current, $\mathrm{I}_{\text {max }}$, required by the Motor Drivers will be:

$$
\begin{equation*}
I_{\max }=\frac{1}{\eta V_{\max }} \sqrt{\frac{f_{\max }^{C}{ }^{3}}{2 \rho \pi R_{p r o p}^{2}}} \tag{7.13}
\end{equation*}
$$

If a 4 inch propeller blade is used, the maximum thrust is limited to twice the weight of the craft, the mass of the craft is 700 grams, the efficiency of the motors is $50 \%$, and the maximum system voltage is 20 volts, the maximum necessary continuous current required from the Motor Drivers is 6 amps . There are several possible power transistors that will supply this necessary power. Three possible suppliers of such transistors are Mospec Semiconductor, Toshiba Semiconductor, and Fairchild Semiconductor. The largest problem with finding these transistors will be matching both
the high current requirement and fast switching requirement, but transistors close to the specifications certainly exist. Two possible transistors for this purpose are the BU326 from Mospec or the KSC3552 from Fairchild.

One significant problem with these high power output transistors is that they have extremely low beta values, so the base current may have to be up to one-tenth of the collector current. This means that for a 6 amp output, the signal driving the transistor has to source 600 milliamps! This high driving current requires that a pre-amplifier also be used between the power amplifying transistors and the CPU. This pre-amplifier, besides supplying the necessary driving current for the output stage transistors, needs to be highspeed and also needs to convert the 0 to 5 volt digital signal from the CPU into a 0 to $\mathrm{V}_{\text {max }}$ volt signal. One possible amplifier that will do this is the LM4765 from National Semiconductor.

Since the motors that will be used on the actual craft have not been chosen, the components used in the Motor Drivers remain unspecified here. A general topology of the Motor Drivers might be as shown in Figure 7.9 below, however:


Pin Outs


Figure 7.9: Motor Driver Implementation
This topology uses the pre-amps (probably implemented as opamps) as high current output comparators, and the NPN transistors in an emitter follower configuration to give the high current gain. The resistors R1 and R2 should be chosen so that the comparator threshold is 2.5 volts, half way between 0 and 5 volts. This will require that

$$
\begin{equation*}
\frac{R_{2}}{R_{1}}=\frac{2.5}{V_{\max }-2.5} \tag{7.14}
\end{equation*}
$$

The resistor R3 is used as a current limiter so the Pre-Amps don't get under-loaded. R3 should be chosen such that:

$$
\begin{equation*}
R_{3} \geq \frac{V_{\max }}{I_{\max }} \beta=\eta \beta V_{\max }^{2} \sqrt{\frac{2 \rho \pi R_{\text {prop }}^{2}}{f_{\max }^{C}}} \tag{7.15}
\end{equation*}
$$

where $\beta$ is the current gain of the output driving transistor. The resistor R 4 is also used as a current limiter, and is chosen so that the transistors don't get under-loaded. R4 should be chosen such that:

$$
\begin{equation*}
R_{4} \geq \frac{V_{\max }}{I_{\text {rated }}} \tag{7.16}
\end{equation*}
$$

where $\mathrm{I}_{\mathrm{rated}}$ is the current rating of the transistor. The actual implementation of the Motor Drivers must be done after specifying appropriate motors, so the Motor Driver topology presented here may require alterations, however, this should be at least a good starting point.

### 7.2.7 CPU

Inputs: detect: analog data detect signal to CPU
pry_data_toCPU: formatted seven byte serial response from Pitch, Roll, Yaw Sensor altered to the serial protocol of the CPU from the CPU Data Interface
user_data_toCPU: formatted one byte serial data from the Receiver altered to the serial protocol of the CPU from the CPU Data Interface
echo: output relating to altitude of craft from Altimeter rotation_clock: square wave signal with period matched to hoop rotation around hub from Outer Hoop Rotation Rate Sensor

Outputs: alt_query_clock: 4 x speed PWM altimeter query signal from CPU
pry_query_fromCPU: one byte serial request to Pitch, Roll, Yaw Sensor in the serial protocol of the CPU to the CPU Data Interface

PWM 1-3, Yaw: PWM signals for motors to Motor Drivers

The CPU is the heart of the on-board computer. It is the main data collector, processor, and controller of the on- and off-board system. The CPU that was chosen was the TattleTale 8v2 from Onset Computers. This model was chosen for its wide range of data I/O capabilities and built-in protocols, its large program memory, its C programmability, its hardware floating point processing, and its ease of interfacing. At the beginning of the project, the processing requirements of the CPU were unknown, and so another reason this particular CPU was chosen was to be sure the system would be able to succeed using it. At the end of the project, it turns out that the CPU is indeed overkill for what it has been used for, and a much cheaper implementation might use a microcontroller such as a PIC chip from MicroChip. Since there was a very demanding time restriction, the implementation of an operational system took higher precedence than implementing an optimized cost solution.

The particular CPU code that was written will not be explained because a lot of the code is machine dependent. It is the logic flow of the code that is academically important for this project, and so this is all that will be presented. A complete copy of the actual code that was used to program the CPU can be found in Appendix C. (The compiled code took up 102 kilobytes of the TT8's 256 kilobyte capacity.) The code can be broken up into three major conceptual sections: data querying and collection, control decision, and PWM motor driving output. The logic flow of the main execution loop is depicted in Figure 7.10 below:


Figure 7.10: CPU Main Execution Loop Logic Flow Diagram

In addition to the main execution loop, there are three possible interrupts that can take control of the CPU at any time. These interrupts control the data collection from the Outer Hoop Rotation Rate Sensor, the data collection from the Altimeter, and the changing of the PWM signals to the Motor Drivers. The logic flow of the interrupts is depicted in Figure 7.11 below:


Figure 7.11: CPU Interrupt Logic Flow Diagram
In Figures 7.10 and 7.11, the diamond blocks represent conditional branches and the rectangular blocks represent unconditional processes. Also, "pry" stands for Pitch, Roll, Yaw, "user data" refers to the user commands received by the RF Receiver, and the "rotation rate" refers to the Outer Hoop Rotation Rate Sensor. The interrupts can occur at any point during CPU operation and the Altimeter and Outer Hoop Rotation Rate Sensor interrupts can also interrupt each other. When an interrupt is encountered by the CPU, the
complete current state of the CPU is stored, the interrupt is handled and reset, the stored CPU state is reloaded, and normal code execution continues.

The overall architecture of the code should be easy to ascertain from Figures 7.10 and 7.11. The CPU collects data from the sensors if there is any data to collect, and if any of the data collection times-out, the CPU declares "bad data" on the reading. Specifically, the user data is asynchronous but buffered by a serial port. This allows the CPU to check the buffer for data, and if there is data available, collect it and flush the buffer without having to worry that the user transmissions are non-polled and asynchronous. When the CPU goes to collect the user data out of the serial port, it first checks to see whether or not there is a positive detect signal from the RF Receiver. The Receiver will spew bad data at the CPU if there is no RF signal, but the data can be invalidated by a negative detect signal. If there is a negative detect signal, the CPU declares "bad data" for the user commands.

The operation of the Pitch, Roll, Yaw Sensor is completely different. It is polled a hard-coded number of times a second, and so the CPU must keep track of the last time the sensor was queried, and also some sort of time count. When it is time to query the sensor, the CPU sends a query and waits for all of the response data to arrive. If the response is taking too much time, the CPU times-out the response and declares "bad data" for the pitch, roll, and yaw data.

Since the collection of the data from the Outer Hoop Rotation Rate Sensor and Altimeter are performed in interrupt routines which can be treated essentially as background processes, there is no explicit data collection from these two sensors in the main execution loop. There is, however, a time-out for both of these sensors in the main
execution loop because under normal operation, both interrupts should occur periodically. The rotation rate interrupt occurs at the same rate that the craft spins, and after start-up, this should be at least twice a second. As for the Altimeter interrupt, since the Altimeter is queried by a 20 Hz PWM signal, the Altimeter interrupt should also occur at a rate of 20 Hz . If new data has not been recorded by these sensors in under a hard-coded time-out period, "bad data" is declared by the CPU.

Since the data collection for the Altimeter and Outer Hoop Rotation Rate Sensor is performed in interrupts that can interrupt each other, data may occasionally be corrupted if this occurs. The Outer Hoop Rotation Rate Sensor requires the time between low to high transitions to be measured, and the Altimeter requires the high time of an echo signal be measured. Based on the expected rate at which these two interrupts will occur, and also the resolution of the CPU timers ( 25 us), data from the Outer Hoop Rotation Rate Sensor should not be significantly affected if its collection is interrupted by the Altimeter interrupt. The Altimeter echo signal requires a much higher accuracy, however, so if the Altimeter data collection is interrupted by the Outer Hoop Rotation Rate Sensor interrupt, the Altimeter data may be faulty. In order to circumvent this problem, the Outer Hoop Rotation Rate Sensor interrupt invalidates the next Altimeter measurement after which the data collection returns to normal operation.

The final version of the complete on-board electronics package that was built weighs about 104 grams not including any power supply or motor driver circuitry that will have to be implemented along with the actual craft. Aside from connectors, the entire
on-board unit could be made about the size of two packs of cards. A picture of the final on-board computer is given in Figure 7.12 below:


Figure 7.12: On-Board Computer Final Implementation

### 7.3 Known Hardware and Software Issues

Due primarily to a lack of time, there are a few remaining unresolved issues with the hardware and software implementations. The two largest problems will be discussed in brief here with the hopes of exposing areas of future development. These problems may require some redesigning or restructuring of the system in order to fix, but it is likely that no complete system overhaul will be needed in order to resolve the problems.
(The only other problems that remain with the electronics package are not design related. The complete electronics package was prototyped and tested successfully, but when the electronics were miniaturized and hand-soldered onto a breadboard problems arose with the altimeter query signal and the off-board controller. From the testing that has been done, it is likely that the CPU Data Interface chip is faulty or has a bad connection to the Altimeter, and it seems that the analog-to-digital converter chip in the
off-board controller might also be faulty or have a bad connection with the A2D Controller. Since the prototypes worked perfectly, these two problems are considered of minor importance, and could easily be fixed by either finding the faulty chips or connections or rebuilding the electronics as described in the design and construction section.)

The first major issue is that the altitude, attitude, and yaw controllers have not been implemented in the on-board CPU code. Since the controllers have already been derived and simulated, this will hopefully not be a major addition. Different components of the voltages to the motors must be compensated using different controllers, however, and so it seems that the controllers must be implemented in software and not more easily in off-CPU hardware. In order to implement the lead controllers in software a miniature ordinary differential equation computing engine must be implemented. Though this can be accomplished rather easily, it is unclear whether this method will be sufficient to successfully control the dynamics of the craft.

The second major remaining issue has to do with the interrupt processing capabilities of the TT8. The main interrupts that the TT8 has to handle are from the Outer Hoop Rotation Rate Sensor, the Altimeter, and the periodic interrupts required to dynamically change the motor voltages as the motors rotate around the hub. The most frequent of these interrupts is the periodic interrupt to change motor voltages, and the period of this interrupt is set in the code to be 20 times faster than the Outer Hoop Rotation Rate Sensor interrupt. When the hoop is spinning slowly with respect to the hub $(<2 \mathrm{~Hz})$, the operation of the CPU is perfect. When the hoop spins fast, however, the CPU stops collecting data from the off-board controller and the Pitch, Roll, Yaw Sensor - the
non-interrupt-driven sensors. The likely cause of this problem is that the CPU becomes overloaded with the periodic interrupt requests to change the motor voltages. Probably the best solution to this problem would be to design a second CPU into the system. This second CPU would take steering commands from the main data collection CPU and translate them into dynamic motor voltages. The second CPU could also be programmed with the compensation and control methodologies for the system. Since specialized PID control chips exist that would have this capability, both of the remaining implementation problems could be solved at the same time if this additional CPU were implemented.

### 8.0 Conclusion

The theory, simulation, and design of a control system for a radically new type of flying device have been demonstrated. Control methodologies were developed from the theoretical physical dynamics of the craft, and a complete wireless user interface system and on-board computer were constructed and programmed to implement these methodologies. Furthermore, it has been demonstrated by means of numerical simulation that the control methodologies derived are viable if the actual craft were to be built.

Finally, all remaining issues with the current system implementation have been described and suggestions for solution have been given. The primary work that remains to be done in order to create a fully developed flying craft is to build the airframe along with mounted motors and propellers, and interface these mechanics with the existing electronics.

Besides simply completing the construction of the craft, there are many possibilities for future work on this craft design. First, the hoop has not been aerodynamically designed in order to maximize the downward thrust of the propellers. Second, the air flow around the propellers themselves has not been investigated, but since each propeller is constantly in the backwash of the preceding propeller, it is possible that an entirely new propeller design would make for a more efficiently controlled craft. Third, the efficiency of the design has not been investigated, but for any realistic use, this would be an imperative study. Fourth, the scalability of the design has not been investigated, so no realistic physical size limitations either large or small have been discussed. Fifth, a non-linear state-space model based control methodology has not been derived for this type of craft, only a vastly simplified control methodology was used.

Sixth, GPS could be added to the craft, and more complicated types of autonomous behavior could be developed. There are undoubtedly more possibilities for future study surrounding this project, the ones listed here are merely to point out how broad of an academic arena can be opened by this new type of craft design.

The possibilities for future uses of this craft make a similarly open ended list. Constellations of the craft could be used in wide area dynamic remote surveillance systems; the craft could be part of a Russian doll type tiered multi-vehicle reconnaissance or delivery system; the hoop of the craft could be enlarged and hollowed to be filled with lighter-than-air gasses in order to create a hybrid light-weight, highly maneuverable flying device. The field of UAV research is rapidly gaining more and more attention, and with this attention comes a responsibility to deliver meaningful advances in aerial capabilities. Trying new types of vehicle design, regardless of their eventual implementation into mass production, is a necessary expansion of these aerial capabilities. Real progress rarely comes from simply re-inventing the already invented. Instead, pushing back the boundaries of what has been done and what can be done will prove to advance human knowledge at a far more rapid pace. Creativity and an open mind are vital and indispensable keys to unlocking the potential of the future.

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### 10.0 Appendices

### 10.1 Appendix A: Matlab Simulation Code

### 10.1.1 State-Space Description and Step Responses

```
function dstatedt = state_eqns_alt(t,state);
%the pitch and roll control is not working properly
dstatedt = zeros(34,1);
% the following command can be run at a matlab prompt once the file state_eqns.m
% is saved into the directory from which matlab is launched:
%
% ode45('state_eqns_alt',[0 50],zeros(1,34),odeset('OutputSel',[15]))
%
% syntax: ode45('function',[time_start time_stop],["initial condition vector"],options)
% the OutputSel selects which states are plotted, state 12 is z position
% state = (b, a, i, c, tz, wx, wy, wz, wi, x, y, z, vx, vy, vz, i1, i2, i3, i4, b1, b2, b3, b4)
% 1, 1 2 3 4 5 5 6 7 7 8 9 1011 12 13 14 15 16 17 18 19 20 21 22 23
% b,a,i,c are angles around x,y,z earth-based axes
% tz is the angle around the craft-based z-axis
% wx,wy,wz are angular velocities of the hoop in craft-based frame
% wi is the angular velocity of the hub in the craft-based frame
%x,y,z are position of craft in earth-based frame
% vx,vy,vz are velocities of craft in earth-based frame
% i1,i2,i3 are propeller motor currents
% b1,b2,b3 are propeller angular velocities
% i4,b4 are the yaw motor current and angular velocity
b = state(1)*pi/180;
a = state(2)*pi/180;
i = state(3)*pi/180;
c = state(4)* pi/180;
tz = state(5)*pi/180;
wx = state(6)* pi/30;
wy = state(7)*pi/30;
wz = state(8)*pi/30;
wi = state(9)*pi/30;
x = state(10);
y = state(11);
z = state(12);
vx = state(13);
vy = state(14);
vz = state(15);
i1 = state(16);
i2 = state(17);
i3 = state(18);
i4 = state(19);
b1 = state(20)*pi/30;
b2 = state(21)*pi/30;
b3 = state(22)*pi/30;
b4 = state(23)*pi/30;
db4dt = state(24)*pi/30;
f1 = state(25);
f2 = state(26);
f3 = state(27);
offset = state(28);
Vc4 = state(29);
steering = state(30);
wxr = state(31)*pi/30;
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wyr = state(32)*pi/30;
wzr = state(33)*pi/30;
wir = state(34)*pi/30;
ke = .022; % motor constant, value from 6.302
kei = .08; % yaw motor, bigger motor
km = ke; % motor constant
kmi = kei;
L = 6.162e-3; % motor inductance, value from 6.302
Li}=3\textrm{e}-2;%\mathrm{ yaw motor, bigger motor
R = 7.384; % motor resistance, value from 6.302
Ri=12; % yaw motor, bigger motor
Im = 3.147e-6; % motor inertia, value from 6.302
lp = 3.147e-5; % propeller inertia
n=3; % propeller to motor gear ratio
Imp = Im+n*Ip; %total loaded motor inertia
Imi = 8e-6; % yaw motor inertia, bigger motor
tau = R*lmp/ke/km; %motor mechanical time constant
delta = atan(tau*wz); %angular offset in voltage
Ct = .025; % thrust coeffecient
p=1.275;% density of air at OC at sea level
r=.15; % distance from center of craft to center of each propelle
% craft constants
% must specify: M, Ixx, lyy, Izz, d, g, nx, ny, nz
Mh = .7; % mass of hoop w/o center section
Mi = .5; % mass of center pod including yaw motor
M = Mh+Mi; % total mass of craft
Rh = .25; %radius of hoop
Rp = .05; %radius of pod
Rprop = .04; %radius of propellers
Ixx = .5*Mh*Rh^2; % moment of inertia xx of hoop, .5*Mh*Rh^2
lyy = lxx;
Izzo = Mh*Rh^2; % moment of inertia zz of hoop, Mh*Rh^2
Izzi = .4*Mi*Rp^2; % moment of inertia of center pod, 2/5*Mi*Ri^2
Imizz = Imi+Izzo+Izzi; %total loaded yaw motor inertia
d = 10*pi/180; % advance angle of propellers, but must convert to radians
g=9.8; % standard force of gravity
nx =.01; % linear air damping of wx
ny =.01; % linear air damping of wy
nz = .0075; % linear air damping of wz
niz = .0001; %linear friction between wi and wz
ni =.001; %linear air damping of wi
air_damp_xy = .1; % linear air damping of xy translation
air_damp_z = .15; % linear air damping of z translation
y2v_const = kei; %constant that multiplies yaw velocity to get voltage
t2v_const = ke*sqrt(2*(wz^2*tau^2+1)/(Ct*p*n^2*pi*Rprop^4));
%t2v_const = 10;
%constant that multiplies sqrt of forces to get motor voltages
%for the lead compensators, not for the actual ode
dvcxdt = 1/M**sin(d)*(f2*}\operatorname{cos}(\mp@subsup{7}{}{*}\textrm{pi}/6)+f3*\operatorname{cos}(1\mp@subsup{1}{}{*}\textrm{pi}/6))
dvcydt = 1/M* sin(d)*(f1+f2**in(7*pi/6)+f3*sin(11*pi/6));
dvczdt = 1/M** cos(d)*(f1+f2+f3);
dvzdt = -g*M-dvcxdt*}\operatorname{sin}(\textrm{b})+dvcydt*\operatorname{cos}(b)*\operatorname{sin}(\textrm{c})+dvczdt**\operatorname{cos}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{cos}(c)
dwidt = 1/Izzi*((lxx-lyy)*wx*wy-Imizz*db4dt+niz*(wz-wi)-ni*wi);
dcdt = wx+wy*tan(b)*}\operatorname{sin}(\textrm{c})+\textrm{wi}\mp@subsup{}{}{*}\operatorname{tan}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{c})
dbdt = wy*}\operatorname{cos}(c)-w\mp@subsup{i}{}{*}\operatorname{sin}(c)
measured_yaw = 2*pi-i;
measured_pitch = 2*pi-b;
if (cos(b)==0)
measured_roll = pi/2;

```
```

else
measured_roll = atan(tan(c)/cos(b));
end
% motor constants
% must specify: ke, kei, kt, kti, L, Li, R, Ri, Imp, Im, Ct, p, r
% dynamic control
% must specify: Vc1, Vc2, Vc3, Vc
min_offset = 0;
max_steering_to_offset_ratio = .75;
pitch_gain = .05;
roll_gain = pitch_gain;
steering_tau = 50;
steering_alpha = 10;
steering_gain = 10;
direction_gain = .7;
yaw_dgain = 1000;
yaw_tau = .0015;
yaw_alpha = 10;
alt_dgain =.00025;
alt_tau = .015;
alt_alpha = 10;
dalt_dgoaldt = 0;
dyaw_dgoaldt = 0;
if (t < 5) %10
alt_dgoal = 0;%1;
yaw_dgoal = 0;
elseif (t < 10) %20
alt_dgoal = 0;%2;
yaw_dgoal = 0;
elseif (t<15) %30
alt_dgoal = 0;%-1
yaw_dgoal = 0;
elseif (t<20) %40
alt_dgoal = 0;
yaw_dgoal = 0;
else
alt_dgoal = 0;
yaw_dgoal = 0;
end
yaw_dgoal = yaw_dgoal*pi/30; %convert to rpm goal from rad/sec goal
dVc4dt = (yaw_alpha*yaw_dgain*(dwidt-dyaw_dgoaldt)+1/yaw_tau*(yaw_dgain*(wi-yaw_dgoal)-Vc4))*y2v_const;
doffsetdt = alt_alpha*alt_dgain*(dalt_dgoaldt-dvzdt)+1/alt_tau*(alt_dgain*(alt_dgoal-vz)-offset);
if (offset < min_offset)
offset = min_offset;
end
dpitch_goaldt = 0;
droll_goaldt = 0;
if (t < 5)
pitch_goal = 0*pi/180;
roll_goal =0*pi/180;
elseif (t < 80)
pitch_goal = 5*pi/180;
roll_goal = 5*pi/180;
elseif (t < 11)

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    pitch_goal = 0*pi/180;
    roll_goal = 0*pi/180;
    elseif (t < 13)
pitch_goal = 0*pi/180;
roll_goal = 0*pi/180;
else
pitch_goal = 0*pi/180;
roll_goal = 0*pi/180;
end
tilt_goal = acos(cos(pitch_goal)*}\operatorname{cos(roll_goal);
if (roll_goal == 0)
direction_goal = 0;
elseif (roll_goal > 0)
direction_goal = atan(tan(pitch_goal)/tan(roll_goal))-pi/2;
else
direction_goal = pi + atan(tan(pitch_goal)/tan(roll_goal))-pi/2;
end
pitch_steering = pitch_gain*(pitch_goal-measured_pitch);
roll_steering = roll_gain*(roll_goal-measured_roll);
if (t < 5)
dsteeringdt = 0; else
dsteeringdt = 2/sqrt(pitch_steering^2+roll_steering^2)*(pitch_steering*(pitch_gain*(dpitch_goaldt-(-
dbdt)))+roll_steering*(roll_gain*(droll_goaldt-1/(\operatorname{cos}(\textrm{b}\mp@subsup{)}{}{\wedge}2+\operatorname{tan}(\overline{\textrm{c}}\mp@subsup{)}{}{\wedge}2\mp@subsup{)}{}{*}(\operatorname{cos}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{sec}(\textrm{c}\mp@subsup{)}{}{\wedge}\mp@subsup{2}{}{*}dcdt+\operatorname{sin}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{tan}(\textrm{c}\mp@subsup{)}{}{*}dbdt)));
end
if (roll_steering == 0)
direction = pi/2;
elseif (roll_steering < 0)
direction = pi+atan(pitch_steering/roll_steering);
elseif (roll_steering > 0)
if (pitch_steering >= 0)
direction = atan(pitch_steering/roll_steering);
else
direction = 2*pi+atan(pitch_steering/roll_steering);
end
end
%dsteeringdt = 0;
%if (t < 5)
% steering=0;
%else
% steering = steering_gain*(tilt_goal - acos(cos(c)* cos(b)));
%end
%if (b == 0)
% direction = direction_gain*direction_goal;
%elseif (b>0)
% direction = direction_gain*(direction_goal-(atan(tan(measured_pitch)/tan(measured_roll)) - pi/2));
%else
% direction = direction_gain*(direction_goal-(pi + atan(tan(measured_pitch)/tan(measured_roll)) - pi/2));
%end
if (steering > max_steering_to_offset_ratio*offset)
steering = max_steering_to_offset_ratio*offset;
elseif (steering < -max_steering_to_offset_ratio*offset)
steering = -max_steering_to_offset_ratio*offset;
end
Vc1 = sqrt(offset + steering/r/cos(d)*}\operatorname{cos(tz-direction+delta))*t2v_const;
Vc2 = sqrt(offset + steering/sqrt(3)/r/cos(d)*}\operatorname{cos}(tz-direction+delta-pi/2))*t2v_const

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Vc3 = sqrt(offset + steering/sqrt(3)/r/cos(d)*cos(tz-direction+delta+pi/2))*t2v_const;
% motor equations
f1 = Ct*p*pi*r^4*n^2** 1^^2;
f2 = Ct* p*pi*r^4 ** n^2*b2^2;
f3 = Ct* p*pi*r^4*n^2**3^^2;
Ve1 = ke*b1;
Ve2 = ke*b2;
Ve3 = ke*b3;
Ve4 = kei*b4;
Va1 = Vc1-Ve1;
Va2 = Vc2-Ve2;
Va3 = Vc3-Ve3;
Va4 = Vc4-Ve4;
di1dt = (Va1-R**1)/L;
di2dt = (Va2-R*i2)/L;
di3dt = (Va3-R*i3)/L;
di4dt = (Va4-Ri*i4)/Li;
m1 = km*i1;
m2 =km*i2;
m3 =km*i3;
m4 = kmi*i4;
dm4dt = kmi*di4dt;
db1dt = m1/Imp;
db2dt = m2/lmp;
db3dt = m3/mp;
db4dt = m4/Imizz;
d2b4dt2 = dm4dt/Imizz;

```

```

df2dt = 2*Ct*p*pi**^^4*n^2*b2*db2dt;
df3dt = 2*Ct* p*pi* r}\mp@subsup{|}{}{*}\mp@subsup{4}{}{*}\mp@subsup{n}{}{\wedge}\mp@subsup{2}{}{*}b\mp@subsup{3}{}{*}db3dt
% craft equations
dwxrdt = 1/lxx*((lyy-
Izzo)*wy*wz+sin(d)*Imp*(db2dt*}\operatorname{cos}(\mp@subsup{7}{}{*}\textrm{pi}/6)+db3dt** cos(1\mp@subsup{1}{}{*}\textrm{pi}/6))+\mp@subsup{r}{}{*}\operatorname{cos}(d)*(f2**\operatorname{sin}(\mp@subsup{2}{}{*}\textrm{pi}/3)+f\mp@subsup{3}{}{*}\mp@subsup{\operatorname{sin}}{}{*}(\mp@subsup{4}{}{*}\textrm{pi}/3))-n\mp@subsup{x}{}{*}wx)
dwyrdt = 1/lyy*((Izzo-Ixx)* wx*wz+sin(d)*Imp* (db1dt+db2dt*sin(7* pi/6)+db3dt*
r*}\operatorname{cos}(\textrm{d}\mp@subsup{)}{}{*}(f1+f\mp@subsup{2}{}{*}\operatorname{cos}(\mp@subsup{2}{}{*}\textrm{pi}/3)+f\mp@subsup{3}{}{*}\operatorname{cos}(\mp@subsup{4}{}{*}\textrm{pi}/3))-n\mp@subsup{y}{}{*}wy)
dwzrdt = 1/lzzo*((lxx-lyy)*wx*wy+Imizz*db4dt+cos(d)* Imp*(db1dt+db2dt+db3dt)+r*}\operatorname{sin}(\textrm{d})*((f1+(f2+f3)*sin(pi/2))-nz*wz
niz*(wz-wi));
dwirdt = 1/Izzi*((Ixx-lyy)*wx*wy-Imizz*db4dt+niz*(wz-wi)-ni*wi);
dwxdt = cos(tz)*dwxrdt+sin(tz)*dwyrdt-sin(tz)**xr**wz+\operatorname{cos}(tz\mp@subsup{)}{}{*}wy\mp@subsup{)}{}{*}*wz;
dwydt = -sin(tz)*dwxrdt+cos(tz)*dwyrdt-cos(tz)**xr**wz-sin(tz)**wyr*wz;
dwzdt = dwzrdt;
dwidt = dwirdt;
dtxdt = wx;
dtydt = wy;
dtzdt = wz;
dtidt = wi;
%german's:
dcdt = wx+wy*tan(b)*}\mp@subsup{}{}{*}\operatorname{sin}(\textrm{c})+\textrm{wi}\mp@subsup{}{}{*}\operatorname{tan}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{c})
dbdt = wy*
dadt = wy* *in(c)/cos(b)+wz** cos(c)/cos(b);
didt = wy*sin(c)/cos(b)+wi*}\operatorname{cos(c)/cos(b);
dvcxdt = 1/M* *in(d)*(f2*}\operatorname{cos}(\mp@subsup{7}{}{*}\textrm{pi}/6)+f\mp@subsup{3}{}{*}\operatorname{cos}(1\mp@subsup{1}{}{*}\textrm{pi}/6))

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dvcydt = 1/M**sin(d)*(f1+f2*sin(7*pi/6)+f3*}\operatorname{sin}(1\mp@subsup{1}{}{*}\textrm{pi}/6))
dvczdt = 1/M** cos(d)*(f1+f2+f3);
dvxdt = dvcxdt**}\operatorname{cos}(a\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{b})+dvcyd\mp@subsup{t}{}{*}(\operatorname{cos}(a\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{c})-\operatorname{sin}(\textrm{a}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{c}))+dvczd\mp@subsup{t}{}{*}(\operatorname{cos}(\textrm{a}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{c})+\operatorname{sin}(\textrm{a}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{c}))
dvydt = dvcxdt*}\operatorname{sin}(\textrm{a}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{b})+dv\mp@subsup{c}{|\mp@code{dt}}{
dvzdt = -g*M-dvcxdt*}\operatorname{sin}(\textrm{b})+dvcydt**\operatorname{cos(b)*}\operatorname{sin}(\textrm{c})+dvczdt*\operatorname{cos}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{cos}(c)
%dvxdt = 1/M*(-
(f1+f2+f3)*}\operatorname{cos(d)*}\operatorname{sin}(\textrm{b})+(f2*\operatorname{cos}(\mp@subsup{7}{}{*}\textrm{pi}/6)+f3*\operatorname{cos(11*pi/6))*}\operatorname{sin}(\textrm{d}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{b})+(f1+f2**\operatorname{sin}(\mp@subsup{7}{}{*}\textrm{pi}/6)+f\mp@subsup{3}{}{*}\operatorname{sin}(1\mp@subsup{1}{}{*}\textrm{pi}/6)\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{d}\mp@subsup{)}{}{*}\operatorname{sin}
i)*}\operatorname{cos(b) - air_damp_xy*vx);
%dvydt = 1/M*((f1+f2+f3)*}\operatorname{cos(d)*}\operatorname{cos(b)*}\operatorname{sin}(\textrm{c})+(\textrm{f}\mp@subsup{2}{}{*}\operatorname{cos}(\mp@subsup{7}{}{*}\textrm{pi}/6)+f3*\operatorname{cos}(1\mp@subsup{1}{}{*}\textrm{pi}/6)\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{d}\mp@subsup{)}{}{*}(\operatorname{cos}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{c})
sin(i)*}\operatorname{cos}(\textrm{c}))+(f1+f\mp@subsup{2}{}{*}\operatorname{sin}(\mp@subsup{7}{}{*}\textrm{pi}/6)+f\mp@subsup{3}{}{*}\operatorname{sin}(1\mp@subsup{1}{}{*}\textrm{pi}/6)\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{d}\mp@subsup{)}{}{*}(\operatorname{cos}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{c})+\operatorname{sin}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{c})) - air_damp_xy*vy)
%dvzdt = 1/M}\mp@subsup{}{}{*}(
g*M+(f1+f2+f3)* cos(d)* cos(b)*}\operatorname{cos(c)+(f2*}\operatorname{cos}(\mp@subsup{7}{}{*}\textrm{pi}/6)+f\mp@subsup{3}{}{*}\operatorname{cos}(1\mp@subsup{1}{}{*}\textrm{pi}/6)\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{d}\mp@subsup{)}{}{*}(\operatorname{cos}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{c})+\operatorname{sin}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{c}))+(\textrm{f}1+\textrm{f}\mp@subsup{2}{}{*}\operatorname{sin}
7*pi/6)+f3*}\operatorname{sin}(1\mp@subsup{1}{}{*}\textrm{pi}/6)\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{d}\mp@subsup{)}{}{*}(\operatorname{sin}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{b}\mp@subsup{)}{}{*}\operatorname{cos}(\textrm{c})-\operatorname{cos}(\textrm{i}\mp@subsup{)}{}{*}\operatorname{sin}(\textrm{c})) - air_damp_\mp@subsup{z}{}{*}vz)
%dvxdt = cos(a)*temp_dvxdt+sin(a)*temp_dvydt;
%dvydt = -sin(a)*temp_dvxdt+cos(a)*temp_dvydt;
dxdt = vx;
dydt = vy;
dzdt = vz;
%this creates a fake ground at z = 0
if (z<=0)\&(vz<=0)
dzdt = 0;
if (dvzdt <= 0)
dvzdt = -vz*100;
end
end
% dstatedt = d(c, b, a, i, tz, wx, wy, wz, wi, x, y, z, vx, vy, vz, i1, i2, i3, i4, b1, b2, b3, b4)dt
% 1
dstatedt(1) = dbdt*180/pi;
dstatedt(2) = dadt*180/pi;
dstatedt(3) = didt*180/pi;
dstatedt(4) = dcdt*180/pi;
dstatedt(5) = dtzdt*180/pi;
dstatedt(6) = dwxdt*30/pi;
dstatedt(7) = dwydt*30/pi;
dstatedt(8) = dwzdt*30/pi;
dstatedt(9) = dwidt*30/pi;
dstatedt(10) = dxdt;
dstatedt(11) = dydt;
dstatedt(12) = dzdt;
dstatedt(13) = dvxdt;
dstatedt(14) = dvydt;
dstatedt(15) = dvzdt;
dstatedt(16) = di1dt;
dstatedt(17) = di2dt;
dstatedt(18) = di3dt;
dstatedt(19) = di4dt;
dstatedt(20) = db1dt*30/pi;
dstatedt(21) = db2dt*30/pi;
dstatedt(22) = db3dt*30/pi;
dstatedt(23) = db4dt*30/pi;
dstatedt(24) = d2b4dt2*30/pi;
dstatedt(25) = df1dt;
dstatedt(26) = df2dt;
dstatedt(27) = df3dt;
dstatedt(28) = doffsetdt;
dstatedt(29) = dVc4dt;
dstatedt(30) = dsteeringdt;
dstatedt(31) = dwxrdt*30/pi;
dstatedt(32) = dwyrdt*30/pi;
dstatedt(33) = dwzrdt*30/pi;
dstatedt(34) = dwirdt*30/pi;

```

\subsection*{10.1.2 Effects of Discretization of Dynamic Motor Voltages on Steering Control}
```

function [avg_length,avg_angle,torque_range_factor,angle_range] = steering(discretizations,r,angle)
%to execute: [l,a,f,r] = steering(25,2,0)
% discretizations - number of pwm changes per revolution
% r - length of desired torque vector
% angle - angle of desired torque vector
%r=2;
start = -2*pi;
stop = 2*pi;
step = .01;
min = 3;
max = min+2*r;
offset = (max-min)/2+min;
angle = angle*pi/180;
min_mot = 0;
t=[0-1.5302 -1.0726 -0.7675 -0.6357 -0.5230
0.2237 -0.2087 -0.1963 -0.1833 -0.1739 -0.1659
-0.1162 -0.1118 -0.1068 -0.1040}-0.1013 -0.0979 -0.0958 -0.0918 -0.0890 -0.0869 -0.0833 -0.0828 -0.0803
-0.0784 -0.0765 -0.0741 -0.0731 -0.0713 -0.0691 -0.0683 -0.0670 -0.0655 -0.0639 -0.0628];
n=[[$$
\begin{array}{llllllllllll}{0}&{0.6359}&{0.8266}&{0.9001}&{0.9354}&{0.9548}&{0.9667}&{0.9744}&{0.9798}&{0.9836}&{0.9864}&{0.9886}\\{0.9.9903}\end{array}
$$]
0.9916
0.9977 0.9979 0.9980
0.9990
offset_angle = t(discretizations); %empirical angle offset
angle = angle + offset_angle;
pre_amp = 1/n(discretizations); %empirical pre-amplification
r = ' `*pre_amp;
t1 = zeros(1,floor((stop-start)/step+1));
t2 = zeros(1,floor((stop-start)/step+1));
t3 = zeros(1,floor((stop-start)/step+1));
rx = zeros(1,floor((stop-start)/step+1));
ry = zeros(1,floor((stop-start)/step+1));
j=0;
disc_step = 2*pi/discretizations;
for i= start:step:stop
index = round((i-start)/step) + 1;
if ((i-start)/disc_step>j+1)
j=j+1;
end
t1_tmp = offset + r*sin(start+j*disc_step-angle+pi/2);
if (t1_tmp > max)
t1 (index) = max;
elseif (t1_tmp < min)
t1(index) = min;
else
t1(index) = t1_tmp;
end
t2_tmp = offset + r/sqrt(3)*sin(start+j*disc_step-angle+pi);
if (t2_tmp > max)
t2(index) = max
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```
        elseif (t2_tmp < min)
        t2(index) = min;
        else
        t2(index) = t2_tmp;
    end
    t3_tmp = offset + r/sqrt(3)*}\operatorname{sin}(\mathrm{ start+j*disc_step-angle);
    if (t3_tmp > max)
        t3(index) = max
    elseif (t3_tmp < min)
        t3(index) = min;
        else
        t3(index) = t3_tmp;
        end
    %calculate r components
    rx(index) = t1(index)*}\operatorname{cos(i)+t2(index)*}\operatorname{cos(i+2*pi/3)+t3(index)*}\operatorname{cos(i+4*pi/3);
    ry(index) = t1(index)*}\operatorname{sin}(\textrm{i})+\textrm{t2}(\mathrm{ (index)*}\mp@subsup{}{}{*}\operatorname{sin}(\textrm{i}+\mp@subsup{2}{}{*}\textrm{pi}/3)+t3(\mathrm{ index)*}\mp@subsup{}{}{*}\operatorname{sin}(\textrm{i}+\mp@subsup{4}{}{*}\textrm{pi}/3)
end
figure(1);
hold off;
plot(rx,ry,'k');
axis([-r*1.25 r*1.25 -r*1.25 r*1.25]);
hold on;
mx = mean(rx);
my = mean(ry);
plot(mx,my,'ko'); %put a spot at the average
legend('variation','average');
title('polar plot of torque vector variation resulting from discretizations');
xlabel('calculated torque magnitude, Nm');
ylabel('calculated torque magnitude, Nm');
hold off;
i = start:step:stop;
i = i.*180/pi;
figure(2);
hold off;
plot(i,t1,'k-');
hold on;
plot(i,t2,'k-.');
plot(i,t3,'k:');
legend('motor 1','motor 2','motor 3',3);
title('actual discretized torque required by motors');
xlabel('angle, degrees');
ylabel('torque, Nm');
hold off;
avg_angle = atan(my/mx)*180/pi; %check to see if the angle was met
avg_length = sqrt(mx^2+my^2); %check to see if the length was met
torque_range_factor = max/min; %factor over which the motor speed must change per cycle from min to max
sorted = sort(ry);
miny = sorted(1);
maxy = sorted(length(sorted));
angle_range = (atan (maxy/mx)-atan(miny/mx))*180/pi;
figure(3);
angles =[\begin{array}{lllllllllllllll}{115.0743 95.9820}&{76.2684}&{64.8416}&{55.2586}&{48.5117}&{42.8720}&{38.5602}&{34.8101}&{31.8759}&{29.3216}\end{array}]
```





```
7.1856];
i = 2:50;
plot(i,angles,'k');
xlabel('discretizations');
ylabel('total angle of torque variation, degrees');
title('total angle of torqe variation over time vs. discretizations');
```

```
figure(4);
disc = 2:50;
tdegs = -t.*180/pi;
plot(disc,tdegs(2:50),'k');
xlabel('discretizations');
ylabel('error angle, degrees');
title('error angle vs. discretizations');
figure(5)
plot(disc,n(2:length(t)),'k');
axis([0 50 .6 1.05]);
xlabel('discretizations');
ylabel('magnitude error factor');
title('magnitude error factor vs. discretizations');
```


### 10.2 Appendix B: VHDL Code

### 10.2.1 A2D Controller

```
library ieee;
use ieee.std_logic_1164.all;
use work.std_arith.all;
--entity
entity a2dcontrol is
    port(clk, nconv_done, txdone, tx_trigger: in std_logic;
        nread, data_ready: out std_logic;
        data_select : buffer std_logic_vector(1 downto 0));
    attribute pin_avoid of a2dcontrol : entity is
    " 1213 24"; --gnd, vcc, and reserved
    attribute pin_numbers of a2dcontrol : entity is
    " clk:1 nconv_done:2 nread:22 data_ready:15 txdone:14" &
    " data_select(0):19 data_select(1):18 tx_trigger:11";
end a2dcontrol;
architecture behavioral of a2dcontrol is
    signal state : std_logic_vector(2 downto 0);
signal flag:std_logic;
begin
clocked: process (clk)
begin
    if rising_edge(clk) then
        case state is --mode 0 timing
        --read state
            when "000" =>
        nread <= '0';
        data_ready <= '0';
        data_select <= data_select;
        state <= "001";
```

```
--allow a2d time to respond
```

    when "001" =>
    nread <= '0';
data_ready <= '0';
data_select <= data_select;
state <= "010";
--wait for data to be ready
when "010" =>
if (nconv_done = '1') then
state <= "010";
else
state <= "011";
end if;
nread <= '0';
data ready <= '0';
data_select <= data_select;
--wait for rising tx_trigger / let ser_send know data is ready
--and allow for valid data delay out of a2d
when "011" =>
if (tx_trigger = '1') then
if (flag = '1') then
data_ready <= '1';
state <= "100";
flag <= ' 0 ';
else
data_ready <= '0';
state <= "011";
end if
else
data_ready <= '0';
state <= "011";
flag <= '1';
end if;
nread <= '0';
data_select <= data_select;
--give ser_send time to respond with a txdone = '1'
when "100" =>
nread <= '0';
data_ready <= '1';
state <= "101";
data_select <= data_select;
--wait for ser_send to finish transmitting
when "101" =>
if (txdone = '1') then
nread <= '1';
data_ready <= '0';
state <= "110";
else
nread <= '0';
data_ready <= '1';
state <= "101";
end if;
data_select <= data_select;
--allow for address hold time of a2d (might not need this state)
when "110" =>
nread <= '1';
data_ready <= '0'
state <= "111"
data_select <= data_select;
--change data to be sent
when "111" =>
nread <= '1';
data_ready <= '0';

```
        state <= "000";
        data_select <= data_select + 1;
            --when others go to 000 state
            when others =>
        nread <= '1';
        data_ready <= '0';
        state <= "000";
        data_select <= data_select;
    end case;
    end if;
end process;
end behavioral
```


### 10.2.2 Serializer

```
library ieee;
use ieee.std_logic_1164.all;
use work.std_arith.all;
entity serializer is
port(clk, txbit, txdone : in std_logic;
        data_in : in std_logic_vector(9 downto 0);
        data_out: out std_logic);
    attribute pin_avoid of serializer : entity is
    " 13 "; --reserved
attribute pin_numbers of serializer : entity is
    " clk:1 txbit:2 txdone:3 data_out:19 data_in(0):14 data_in(1):15 " &
    " data_in(2):4 data_in(3):5 data_in(4):6 data_in(5):7 " &
    " data_in(6):8 data_in(7):9 data_in(8):10 data_in(9):11 ";
end serializer;
architecture behavioral of serializer is
signal bit_count : std_logic_vector(3 downto 0);
signal flag : std_logic;
begin
    clocked: process (clk)
begin
    if rising_edge(clk) then
        if (txdone = '1') then
            data_out <= '0';
            bit_count <= "0000";
        flag <= '0';
        elsif (txbit = '0') then
        flag <= '0';
```

```
elsif ((txbit = '1') and (flag = '0')) then
    flag <= '1';
    case bit_count is
        when "0000" =>
        data_out <= data_in(0); --start bit, tied high
        bit_count <= "0001";
        when "0001" =>
        data_out <= not data_in(1); --data 0
        bit_count <= "0010";
        when "0010" =>
        data_out <= not data_in(2); --data 1
        bit_count <= "0011";
        when "0011" =>
        data_out <= not data_in(3); --data 2
        bit_count <= "0100";
        when "0100" =>
        data_out <= not data_in(4); --data 3
        bit_count <= "0101";
        when "0101" =>
        data_out <= not data_in(5); --data 4
        bit_count <= "0110";
        when "0110" =>
        data_out <= not data_in(6); --select 0
        bit_count <= "0111";
        when "0111" =>
        data_out <= not data_in(7); --select 1
        bit_count <= "1000";
        when "1000" =>
        data_out <= not data_in(8); --mode
        bit_count <= "1001";
        when "1001" =>
        data_out <= data_in(9); --stop, tied low
        bit_count <= "1010";
        when "1010" =>
        data_out <= '0';
        bit_count <= "0000";
        when others =>
        bit_count <= "0000";
        end case;
    end if;
end if;
end process;
end behavioral;
```


### 10.2.3 Serializer Controller

```
library ieee;
use ieee.std_logic_1164.all;
use work.std_arith.all;
--entity
entity ser_send is
port(clk, data_ready: in std_logic;
        per_counter : in std_logic_vector(3 downto 0);
        en_nres, txbit, txdone: out std_logic);
attribute pin_avoid of ser_send : entity is
    " 1213 24"; --gnd, vcc, and reserved
    attribute pin_numbers of ser_send : entity is
    " clk:1 data_ready:2 txdone:18 en_nres:20 " &
    " per_counter(0):5 per_counter(1):6 per_counter(2):7 " &
    " per_counter(3):8 txbit:14";
end ser_send;
```

--the period counter might have to be frigged with to get the data rate
--equal to that when the computer is txmitting. in this case, you could
--use per_counter inputs as certain inputs that when all 1 's will be the
--right period, but may not be sequential bits from the counter. this will
--make transition from computer control to joystick control easier
architecture behavioral of ser_send is
signal bit_count : std_logic_vector(3 downto 0);
signal state : std_logic_vector(1 downto 0);
begin
clocked: process (clk)
begin
if rising_edge(clk) then
case state is
--startup
when "00" =>
state <= "01";
txdone <= '0';
txbit <= ' 0 ';
en_nres <= '0';
bit_count <= "0000";
--wait for data to be ready
when "01" =>
if (data_ready $=$ ' 1 ') then
state <= "10";
txdone <= ' 0 ';
txbit <= '1';
bit_count $<=$ bit_count +1 ;
else
state <= "01";
txdone <= '1';
txbit <= '0';
bit_count <= "0000";

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```
        end if;
        en_nres <= '0';
        --count period and bits
        when "10" =>
        if (bit_count = "1010") then
        state <= "11";
        txdone <= '1';
        en_nres <= '0';
        txbit <= '0';
        bit_count <= "0000";
        elsif (per_counter = "1111") then
        bit_count <= bit_count + 1;
        txbit <= '1';
        state <= "10";
        txdone <= '0';
        en_nres <= '0';
        else
        bit_count <= bit_count;
        txbit <= '0';
        state <= "10";
        txdone <= '0';
        en_nres <= '1';
        end if;
        --allow time for a2d to give back data_ready = 0
        when "11" =>
        state <= "01";
        txdone <= '1';
        en_nres <= '0';
        txbit <= '0';
        bit_count <= "0000";
        --same as 00
        when others =>
        state <= "01";
        txdone <= '0';
        en_nres <= '0';
        txbit <= '0';
        bit_count <= "0000";
        end case;
    end if;
end process;
end behavioral;
```


### 10.2.4 CPU Data Interface

```
library ieee;
use ieee.std_logic_1164.all;
use work.std_arith.all;
entity alt_querrier is
port(clk, serial_in_1, serial_in_2, serial_in_3 : in std_logic;
    querry, serial_out_1, serial_out_2, serial_out_3 : out std_logic);
attribute pin_avoid of alt_querrier : entity is
    " 13 "; --reserved
```

attribute pin_numbers of alt_querrier : entity is

```
    " clk:1 querry:22 " &
    " serial_in_1:2 serial_out_1:21" &
    " serial_in_2:3 serial_out_2:20" &
    " serial_in_3:4 serial_out_3:19";
```

end alt_querrier;
architecture behavioral of alt_querrier is
signal state : std_logic_vector(2 downto 0);
begin
serial_out_1 <= not serial_in_1;
serial_out_2 <= not serial_in_2;
serial_out_3 <= not serial_in_3;
clocked: process (clk)
begin
if rising_edge(clk) then
case state is
when "000" =>
state <= "001";
querry <= ' 0 ';
when "001" =>
state <= "010";
querry <= '0';
when "010" =>
state <= "011";
querry <= ' 0 ';
when "011" =>
state <= "100";
querry <= '0';
when "100" =>
state <= "101";
querry <= ' 0 ';
when "101" =>
state <= "110":
querry <= '0';
when "110" =>
state <= "111";
querry <= '0';
when "111" =>
state <= "000";
querry <= '1';
when others =>
state <= "000";
querry <= '0';
end case;
end if;
end process;
end behavioral;

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### 10.3 Appendix C: CPU Code

### 10.3.1 Header File

```
#include <tt8.h>
#include <tat332.h>
#include <sim332.h>
#include <tpu332.h>
#include <dio332.h>
#include <qsm332.h>
#include <tt8pic.h>
#include <tt8lib.h>
#include <userio.h>
#include <stdio.h>
#include <stdlib.h>
#include <stddef.h>
#include <math.h>
#include <float.h>
//constants
#define PI 3.141593
#define G 9.8
#define CRAFT_MASS 1//mass of total craft
#define HOVER_THRUST CRAFT_MASS*G
#define USPERCM 74 //microseconds per centimeter for alt echo
#define MAX_ALT_ECHO_TIME 39000 //39ms, data sheet says 36ms
#define MIN_ALT_ECHO_TIME 100 //100us, data sheet says 100us
#define MOTOR_PWM_HI_VOLTS 20 //20 volt battery pack
#define MOTOR_PWM_HI_MIN 20 //minimum amount of hi time on pwm
#define MOTOR_PWM_HI_MAX 180 //maximum amount of hi time on pwm
#define MOTOR_PWM_HI_STARTUP 50 //startup amount of hi time on pwm
#define MOTOR_STARTUP_TIME 1000 //bring motors to startup speed in 1000 ms
#define MOTOR_MIN_THRUST 3 //this can be changed or calibrated
#define MOTOR_MAX_STEERING 2.5 //don't want max steering to be larger than min thrust
#define MAX_STEER_TO_THRUST_RATIO . }75\mathrm{ //don't want steering compared to thrust too large
#define OUT_VOLTSSQPERNEWTON . 5 //this will have to be calculated and calibrated
//should theoretically be equal to 2*Ke^2/(p*r^4*PI)
#define OUT_VOLTSSQPERNEWTONMETER .5 //this will also have to be calculated and calibrated
//should theoretically be equal to 2*Ke^2/(p*r^5*PI)
#define OUT_MOTOR_KE . }2\mathrm{ //Ke
//where Ke is the back emf motor constant, p is the density of air, and r is the radius
//from the center of the vectron to the center of the motors
#define USR_MAX_DATA 15 // 0b1111 = 15
#define USR_DAT2DEGREES 2 //max 30 degrees / max data of Ob1111=15
#define USR_DAT2NEWTONS . }3//.5\textrm{kg*}10\textrm{m}/\mp@subsup{\textrm{s}}{}{\wedge}2=5, max thrust data = 31, max thrust ~10
#define USR_STABLE_PITCH 0.Of
#define USR_STABLE_ROLL 0.Of
#define USR_STABLE_ALT_SPEED 0.Of
#define USR_STABLE_YAW_SPEED 0.Of
#define PRY_DAT2DEGREES 180/PI/10430 //this is given in the pry datasheet
#define WZ_MILLISEC2HZ 1000 /NWZ_MILLISEC2HZ/milliseconds = hertz conversion
#define WZ_MAX_HZ 10 //max allowable hz output for glitch protection
#define BAD_DATA -1000.0f
#define ALT_SPEED_GAIN 5 //will have to be determined in simulations
#define YAW__GAIN 5}///will have to be determined in simulation
#define PITCH_GAIN 5 //will have to be determined in simulations
```

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\#define ROLL_GAIN 5 //will have to be determined in simulations

```
//define timing vars
#define MOTOR_PWM_PERIOD 200 //4MHz/200 --> 20KHz PWM, 50us period
#define ALT PWM PERIOD 25000 //4MHz/25000 --> 160Hz PWM
#define ALT_PWM_HI_TIME 12500 //datasheet says min is 10us, will be 6.25ms
#define PRY_REQ_DELAY 35 //35ms between each pry request, ~28.57Hz
#define WZ_TIMEOUT_DELAY 10000 //10s timeout
#define ALT_TIMEOUT_DELAY 1000 //1s timeout since it's querried all the time
//define pins
#define TPU USR PIN 0
#define TPU_PRY`PIN 1
#define TPU PRY REQ PIN 2
#define TPU_ALT_QUE\overline{RY_PIN 3}
#define TPU_ALT_ECHO_PIN 4
#define PWM OUTPUT PIN1 5
#define PWM_OUTPUT_PIN2 6
#define PWM OUTPUT PIN3 }
#define PWM_Y_YAW_OUTPUT_PIN 8
#define TPU_WZ_PIN 9
#define A2D_USR_DETECT_PIN 0 //A2D 1, not TPU 1
//needed a 1 MOhm resistor to ground on usr detect pin
#define NUM_PRIMARY_PWM_CHANNELS 3
#define PWM_CHANNEL_OFFSET 5
```

\#define MOTOR_PWM_DISCRETIZATIONS $20 / / 20$ ~-> 10ms changes, 40 ~-> 5ms changes
//define pry command and response formats
\#define PRY_REQ 192 //Ob11000000-0xC0
\#define PRY RESPONSE BYTES 7
\#define PRY_VALID_HEADER 68 //0x44
//define buffer sizes for PRY and user data
\#define USR BITSPERBYTE 8
\#define USR_BUF_SIZE USR_BITSPERBYTE*16 //16 byte buffer
\#define PRY_BITSPERBYTE 8
\#define PRY_BUF_SIZE PRY_BITSPERBYTE*16 //16 byte buffer
\#define PRY_REQ_BITSPERBYTE 8
\#define PRY_REQ_BUF_SIZE PRY_REQ_BITSPERBYTE*4 //4 byte buffer
//this last buffer is never used
//define some macro functions
\#define USR_READY_FOR_UPDATE TSerByteAvail(TPU_USR_PIN) //is there usr data?
\#define GET_USR_BYTE TSerGetByte(TPU_USR_PIN) //get the usr data
\#define FLUS̄H_USR_DAT TSerInFlush(TPU_USR_PIN) //flush the usr data buffer

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```
#define PRY_READY TSerByteAvail(TPU_PRY_PIN) //is there pry data?
#define PRY_READY_FOR_UPDATE ((MilliSecs() - pry_req_timer) >= PRY_REQ_DELAY)
#define GET_PRY_BYTE TS̄erGetByte(TPU_PRY_PIN)//get the pry data
#define FLUSH_PRY_DAT TSerInFlush(TPU_PRY_PIN)//flush the pry data buffer
#define SEND_PRY_REQ TSerPutByte(TPU_PRY_REQ_PIN,PRY_REQ)
#define ECHO TPUGetPin(TPU_ALT_ECHO_PIN) //query echo
#define WZ TPUGetPin(TPU_WZ_PIN)
#define USR_DETECT_MILLIV ĀtoDReadMilliVolts(A2D_USR_DETECT_PIN)
#define PWM_READY_FOR_UPDATE (pit_counter != pit_hits)
#define UPDATE_PWM\
    if (PWM_READY_FOR_UPDATE) \
        {\
            TPUUpdateAllChans(); \
            pit_counter = pit_hits; \
        }
#define UPDATE_USR\
    if (USR_READY_FOR_UPDATE) \
        {\
            usr_time = get_usr_dat(&usr_dat, 0, usr_time); \
            change_output= 1; \
        }
#define UPDATE_PRY\
        if (PRY_READY_FOR_UPDATE)\
            {\
                        pry_req_timer = MilliSecs(); \
                            pry_time = get_pry_dat(&pry_dat, 0, pry_time); \
                            change_output =1;\
        }
#define TIMEOUT_WZ \
    if ((MilliSecs() - wz_time) >= WZ_TIMEOUT_DELAY) \
    {l
        wz_dat.wz = BAD_DATA; \
        wz_dat.dwzdt = BAD_DATA; \
        start_up_pwm = 1; \
        SetSteeringInterruptTimer(0); \
    }
#define TIMEOUT_ALT \
    if ((MilliSecs() - alt_time) >= ALT_TIMEOUT_DELAY)\
    {l
            alt_dat.alt = BAD_DATA; \
            alt_dat.daltdt = BA
    }
#define RESTART_PWM_AFTER_WZ_TIMEOUT \
    if ((wz_dat.wz != BAD_DATA) & (start_up_pwm == 1)) \
        {1
            start_up_pwm = 0; \
    }
#define UPDATE_OUTPUT\
    if (change_output == 1) {\
        state_time = state_estimator(&state_dstate, &pry_dat, 0, state_time); \
        controller(&state_dstate, &pry_dat,_/&usr_dat); \
        UPDATE_PWM; I
        change_output = 0; \
    }
```

```
struct USR_STRUCT {
    int mode; //mode0 = usr control, mode1 = hover, etc...
    float pitch_c; //all angles are measured in degrees
    float roll_c; //angles are 4-bit discretized and then converted
    float yaw_c;
    float thrust_c; //measured in newtons
    float dpitch_cdt; //change in degrees
    float droll_cdt; //by the transmitter
    float dyaw_cdt;
    float dthrust_cdt;
};
struct PRY_STRUCT {
    float pitch; //want high precision from sensor, sub degree
    float roll;
    float yaw;
    float dpitchdt; //want high precision in derivative too
    float drolldt;
    float dyawdt;
    float pitch_tare;
    float roll_tare;
    float yaw_tare;
};
struct ALT_STRUCT {
    float alt; //measured in cm, range ~3-300
    float daltdt:
};
struct WZ_STRUCT {
    float wz; //measured in Hz
    float dwzdt;
};
struct STATE_STRUCT {
    float tilt;
    float aparent yaw;
    float actual_altitude;
    float dtiltdt;
    float daparent_yawdt;
    float dactual_altitudedt;
};
struct OUTPUT_STRUCT {
    float thrust_offset; //want precision on voltage to be applied to motors
    float pitch_roll_steering;
    float yaw_steering;
    float steering_angle;
    short pwm_hi_time_1;
    short pwm_hi_time_2;
    short pwm_hi_time_3;
    short pwm_hi_time_yaw;
};
//for the pwm control
```



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| \#define | DetEither | $0 \times 0 \mathrm{c}$ |
| :---: | :---: | :---: |
| \#define | NoChangePAC | $0 \times 10$ |
| /* PAC Pin Action Control (Outputs) */ |  |  |
| \#define | NoChangematch | $0 \times 00$ |
| \#define | HighOnMatch | $0 \times 04$ |
| \#define | LowOnMatch | $0 \times 08$ |
| \#define | ToggleOnMatch | 0x0c |
| /* TBS Time Base/Directionality Control */ |  |  |
| \#define | InputChan | $0 \times 00$ |
| \#define | OutputChan | $0 \times 80$ |
| \#define | Cap1Match1 | 0x00 |
| \#define | Cap1Match2 | 0x20 |
| \#define | Cap2Match1 | 0x40 |
| \#define | Cap2Match2 | $0 \times 60$ |
| \#define | NoChangeTBS | $0 \times 100$ |
| //function headers void init_alt_meas(void); |  |  |
| ulong get_usr_dat(struct USR_STRUCT *usr_dat, short initialize_flag, ulong usr_time); void get_alt_dat(void); <br> ulong get_pry_dat(struct PRY_STRUCT *pry_dat, short initialize_flag, ulong pry_time); void get_wz_dat(void); |  |  |
| ulong state_estimator(struct STATE_STRUCT *state_dstate, struct PRY_STRUCT *pry_dat, short initialize_flag, ulong state_time); |  |  |
| void TPUInitPWM(short chan, short hi_time, short period); void TPUChangePWM(short chan, short hi_time, short period); void TPUUpdateAllChans(void); |  |  |
| void Init <br> void Set <br> void Ser | lay); <br> (int delay); <br> terrupt(void); |  |

### 10.3.2 C Code File

```
#include "main.h"
//THE USR DATA INPUT AND ALTIMETER NEED TO BE CALIBRATED
//THE OUTPUT PWM HI TIMES NEED TO BE CALIBRATED
I/IF THE PERIODIC INTERRUPT TIMER GOES TOO FAST, THINGS GO AWRY
ulong alt_time; //global so interrupt can access
ulong wz_time;
struct ALT_STRUCT alt_dat; //global so interrupt can access
struct WZ_STRUCT wz_dat;
struct OUTPUT_STRUCT output;
int alt_dat_valid;
double current_angle;
int change_output; //change flag, global so interrupts can access
int pit_hits;
int pit_counter;
void main(void)
{
    int start_up_pwm = 0;
    int i, j;
//initialize variables
```

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```
struct USR_STRUCT usr_dat; //these structs are defined in main.h
struct PRY_STRUCT pry_dat;
struct STATE_STRUCT state_dstate;
ptr usr_buf;
ptr pry_buf;
ptr pry_req_buf;
ulong pry_time = 0; //initialize timers
ulong pry_req_timer = 0;
ulong usr_time = 0;
ulong state_time = 0;
static ExcCFrame efp1;//initialize interrupt frame handler
static ExcCFrame efp2;
static ExcCFrame efp3;
//initialize tt8
InitTT8(NO_WATCHDOG, TT8_TPU); //initializes tt8 for action
SimSetFSys}(16000000); //sets system speed to 16MH
alt_time = 0;
wz_time = 0;
pit_hits = 0;
pit_counter = 0;
alt_dat_valid = 1;
current_angle = 0; //will be initialized to zero every time the wz transitions
change_output = 0;
InstallHandler(get_alt_dat, TPU_INT_VECTOR + TPU_ALT_ECHO_PIN, &efp1); //setup interrupt handling
InstallHandler(get_wz_dat, TPU_INT_VECTOR + TPU_WZ_PIN, &-fp2);
*PITR = 0; //disable periodic timer interrupt
InstallHandler(ServiceOutputSteeringInterrupt, PIT_INT_VECTOR, &efp3);
//initialize serial ports for usr and pry
if ((usr_buf = malloc(USR_BUF_SIZE+TSER_MIN_MEM)) == 0)
    return;///can't allocate serial input buffer for usr commands so quit
if (TSerOpen(TPU_USR_PIN,HighPrior,0,usr_buf,USR_BUF_SIZE,2400,'N',USR_BITSPERBYTE,1) != tsOK)
    return; //can't open channel for usr serial input
//the TSER_MIN_MEM is a tt8 minimum "scratchpad" memory requirement in the buffer.
//it is defined in tat332.h. all other capitalized variables are defined in main.h
//initialize serial ports for pry
if ((pry_buf = malloc(PRY_BUF_SIZE+TSER_MIN_MEM)) == 0)
    return; //can't allocate serial input buffer for pry data so quit
if (TSerOpen(TPU_PRY_PIN,HighPrior,0,pry_buf,PRY_BUF_SIZE,9600,'N',PRY_BITSPERBYTE,1) != tsOK)
    return; //can't open channel for pry serial input
//may need to adjust some of the pry serial settings once pry sensor comes in
if ((pry_req_buf = malloc(PRY_REQ_BUF_SIZE+TSER_MIN_MEM)) == 0)
    return; //can't allocate serial input buffer for pry data so quit
if
(TSerOpen(TPU_PRY_REQ_PIN,HighPrior,1,pry_req_buf,PRY_REQ_BUF_SIZE,9600,'N',PRY_REQ_BITSPERBYTE,1)
!= tsOK)
    return; //can't open channel for pry serial input
TPUGetPin(TPU_ALT_ECHO_PIN); //define as input pins
TPUGetPin(TPU_WZ_PIN);
TPUInterruptEnable(TPU_ALT_ECHO_PIN); //enable interrupts
TPUInterruptEnable(TPU_WZ_PIN);
SetInterruptMask(ALL_RUPTS_MASK); //tell main computer about interrupts
```

```
//initialize motors to startup speed
for (i=0; i <= MOTOR_PWM_HI_STARTUP; i++)
    {
        for (j = 0; j < NUM_PRIMARY_PWM_CHANNELS; j++)
            {
                if (i == 0)
                    TPUInitPWM(j + PWM_CHANNEL_OFFSET, i, MOTOR_PWM_PERIOD);
                else
                    TPUChangePWM(j + PWM_CHANNEL_OFFSET, i, MOTOR_PWM_PERIOD);
            }
        DelayMilliSecs(MOTOR_STARTUP_TIME/MOTOR_PWM_HI_STARTUP);
    }
//initialize data structures and operations, wz initialized behind the scenes sorta
usr_time = get_usr_dat(&usr_dat, 1, usr_time);
pry_time = get_pry_dat(&pry_dat, 1, pry_time);
init_alt_meas();
DelayMilliSecs(1);
//give altimeter time to respond, should be ~200us response + ~200us echo on ground
state_time = state_estimator(&state_dstate, &pry_dat, 1, state_time);
controller(&state_dstate, &pry_dat, &usr_dat);
pry_req_timer = MilliSecs();
//main operation loop, infinite
while (1)
    {
        UPDATE_PRY;
        UPDATE_USR;
        UPDATE_PWM;
        RESTART_PWM_AFTER_WZ_TIMEOUT;
        TIMEOUT_WZ;
        TIMEOUT_ALT;
        UPDATE_OUTPUT;
    } //end of while loop
    Reset(); //needs to be Reset() for ram programs ResetToMon() for flash
    return;
}
/*************************Sub-routines follow************************/
/******************************************************************/
/*
    function: Initiates altimeter measurements
    inputs: none
    outputs: none
*/
void init_alt_meas(void)
{
TPUInitPWM(TPU_ALT_QUERY_PIN,ALT_PWM_HI_TIME,ALT_PWM_PERIOD);
    return;
}
```

```
/**
    function: Collects the data from the user input
    inputs: *usr_dat - pointer to the current user data
            initialize_flag - tells routine whether this is the initialization
            usr_time - last time the user data was updated (from MilliSecs())
outputs: time - current time from MilliSecs() routine
*/
ulong get_usr_dat(struct USR_STRUCT *usr_dat, short initialize_flag, ulong usr_time)
{
//assume buffered by tt8 serial port
//need to know what tt8 expects for incoming serial data bits
/*
    bit value: mode <select> +/- <- data ->
    bit number:}\begin{array}{llllllllll}{8}&{7}&{6}&{5}&{4}&{3}&{2}&{1}
    lolllll
    since there is only positive thrust, bit 5 will be
        data in the thrust case
*/
ulong elapsed_time, time;
//setup variables to be their own masks to the incoming byte
int mode = 128;
int selector = 96;
int pve_nve = 16;
int data_c_tmp = 15;
uchar tmp_byte;
float data_c_tmp_f;
float pve_nve_f;
float elapsed_time_f;
float usr_dat2degrees_f = USR_DAT2DEGREES;
float usr_dat2newtons_f = USR_DAT2NEWTONS;
//find out how much time since last update
time = MilliSecs();
elapsed_time = time - usr_time;
if (4050<USR_DETECT_MILLIV) //this is a safety, just in case the transmitter shuts off
    tmp_byte = GET_USR_BYTE; //defined above main{}
else
    {
        //this will put the vectron into a stable hover
        usr_dat->mode = 0; //mode 0 will be auto mode, mode 1 will be user mode
        usr_dat->thrust_c = BAD_DATA; //30; //will control thrust for hover in the controller
        usr_dat->dthrust_cdt = BADD_DATA; //0; //see thrust note one line up
        usr_dat->yaw_c = BAD_DATA; //current_yaw;
        usr_dat->dyaw_cdt = BAD_DATA; //0;
        usr_dat->pitch_c = BAD_DATA; //USR_STABLE_PITCH;
        usr_dat->dpitch_cdt = BAAD_DATA; //0;
        usr_dat->roll_c = BAD_DATA; //USR_STABLE_ROLL;
        usr_dat->droll_cdt = BAAD_DATA; //0;
        return usr_time; //receiver not receiving
    }
mode &= tmp_byte; //as initialized mode is its own mask to get the mode
            //data out of the received byte, all others are same
mode >>= 7; //shift bit down to bottom
mode &= 1; //this will erase all but bottom bit in case 1's came in top
```

```
selector &= tmp_byte;
selector >>= 5;
selector &=3;//this will erase all but the bottom two bits
data_c_tmp &= tmp_byte; //no need to shift, already at bottom
pve_nve &= tmp_byte;
if (sēlector != 3) //if not thrust data, convert to +1 or -1
{
    if (pve_nve == 0) //let 0=-1, 1=1
pve_nve = -1;
                    data_c_tmp ^= 15; //invert bits
            }
    else
            pve_nve = 1;
}
else //it's thrust data so tack it on to data_c_tmp
    {
    data_c_tmp |= pve_nve;
}
//change all data to floating point data
data_c_tmp_f = data_c_tmp;
pve_nve_f = pve_nve;
elapsed_time_f = elapsed_time;
//all data is now collected so lets put it in the usr_dat struct
usr_dat->mode = mode; //need this syntax since usr_dat is a pointer
//each byte only has info for either pitch or roll or yaw or thrust
switch(selector)
    {
    case 0:
    if (initialize_flag == 0) //if this isn't an initialization, change d(x)dt vars
            {
            usr_dat->dpitch_cdt = 1000.0f*(pve_nve_f*data_c_tmp_f*usr_dat2degrees_f - usr_dat-
>pitch_c)/elapsed_time_f;
            }
    else
            {
                usr_dat->dpitch_cdt = pve_nve_f*data_c_tmp_f*usr_dat2degrees_f;
            }
    //elapsed time is in milliseconds so we need to multiply by }100
    usr_dat->pitch_c = pve_nve_f*data_c_tmp_f*usr_dat2degrees_f;
    //update data in usr_dat, must convert from binary number to degrees
    break;
    case 1:
    if (initialize_flag == 0)
            {
                    usr_dat->droll_cdt = 1000.0f*(pve_nve_f*data_c_tmp_f*usr_dat2degrees_f - usr_dat->roll_c)/elapsed_time_f;
            }
    else
            {
            usr_dat->droll_cdt = pve_nve_f*data_c_tmp_f*usr_dat2degrees_f;
            }
    usr_dat->roll_c = pve_nve_f*data_c_tmp_f*usr_dat2degrees_f;
    break;
    case 2:
    if (initialize_flag == 0)
            {
            usr_dat->dyaw_cdt = 1000.0f*(pve_nve_f*data_c_tmp_f*usr_dat2degrees_f - usr_dat-
>yaw_c)/elapsed_time_f;
            }
```

```
        else
            {
                usr_dat->dyaw_cdt = pve_nve_f*data_c_tmp_f*usr_dat2degrees_f;
            }
        usr_dat->yaw_c = pve_nve_f*data_c_tmp_f*usr_dat2degrees_f;
        break;
    case 3:
    if (initialize_flag == 0)
            {
            usr_dat->dthrust_cdt = 1000.0f*(data_c_tmp_f*usr_dat2newtons_f - usr_dat->thrust_c)/elapsed_time_f;
            }
    else
            {
                usr_dat->dthrust_cdt = data_c_tmp_f*usr_dat2newtons_f;
            }
        usr_dat->thrust_c = data_c_tmp_f*usr_dat2newtons_f;
        //update thrust, must convert from binary to newtons
    }
//no default case because selector can only be 0,1,2,3 due to the
//processing we did on it above
    return time;
}
/******************************************************************/
/*
    function: Collects the data from the pitch, roll, yaw sensor
    inputs: *pry_dat - pointer to the current pry data
        initialize_flag - tells routine whether this is the initialization
        pry_time - last time the pry data was updated (from MilliSecs())
    outputs: time - current time from MilliSecs() routine
*/
ulong get_pry_dat(struct PRY_STRUCT *pry_dat, short initialize_flag, ulong pry_time)
{
int bytes_received = 0;
unsigned int pitch_tmp, yaw_tmp, header;
int roll_tmp;
ulong elapsed_time, time; //find out how much time since last update
ulong time_out;
float tmp_f, elapsed_time_f, pry_dat2degrees_f;
time = MilliSecs();
elapsed_time = time - pry_time;
FLUSH_PRY_DAT; //macro defined in main.h
//this will be a polled operation
SEND_PRY_REQ; //macro defined in main.h
DelayMilliSecs(15); //15ms worst case delay between request and response from testing
for (bytes_received=0; bytes_received < PRY_RESPONSE_BYTES ; bytes_received++)
    {
    time_out = MilliSecs();
        while (!PRY_READY) //timed out wait for data to arrive
            {
                if (MilliSecs() - time_out > 5) //from testing 5ms is good time out period
                    {
                        if (bytes_received <= 2)
                            {
                //if we timed out with no bytes received then the pry isn't working
                pry_dat->roll = BAD_DATA;
                pry_dat->drolldt = BADD_DATA;
                pry_dat->pitch = BAD_DATA;
```

```
                pry_dat->dpitchdt = BAD_DATA;
                    pry_dat->yaw = BAD_DATA;
                    pry_dat->dyawdt = BADD_DATA;
                }
            return pry_time;
        }
        }
UPDATE_PWM;
switch(bytes_received)
    {
        case 0:
            header = GET PRY BYTE;
            if (header != PRY_VALID_HEADER) // defined in main.h
            return pry_time; //there was some error
            break;
            case 1:
            roll_tmp = GET_PRY_BYTE; //get roll msb
            roll_tmp *= 256; //shift msb up 8 bits
            break;
            case 2:
            roll_tmp += GET_PRY_BYTE; //get roll Isb
            break;
            case 3:
            pitch_tmp = GET_PRY_BYTE; //get pitch msb
            pitch_tmp *= 256; //shift msb up 8 bits
            break;
            case 4:
            pitch_tmp += GET_PRY_BYTE; //get pitch Isb
            break;
            case 5:
            yaw_tmp = GET_PRY_BYTE; //get yaw msb
            yaw_tmp *= 256; //shift msb up 8 bits
            break;
        case 6:
            yaw_tmp += GET_PRY_BYTE; //get yaw Isb
            break;
        }
    //shouldn't need to be a default case because for loop
    //will automatically time this out
}
```

//if spurious data comes in, i don't care since by the next pry
//request it'll be all in the pry_buffer and i flush the buffer
//each time before gathering data. also, it seems that the spurious
//data that does sometimes arrive, arrives only after correct
//angle measurements, so we can just ignore it
//by this point, we know we have good data, so store it in the struct

```
pry_dat2degrees_f = PRY_DAT2DEGREES;
elapsed_time_f = elapsed_time;
if (initialize_flag == 0) //if this isn't an initialization, change d(x)dt vars
    {
        tmp_f = roll_tmp; pry_dat->drolldt = 1000.0f*(tmp_f*pry_dat2degrees_f - pry_dat->roll)/elapsed_time_f;
        tmp_f = pitch_tmp; pry_dat->dpitchdt = 1000.0f*(tmp_f*pry_dat2degrees_f - pry_dat->pitch)/elapsed_time_f;
        tmp_f = yaw_tmp; pry_dat->dyawdt = 1000.0f*(tmp_f*pry_dat2degrees_f - pry_dat->yaw)/elapsed_time_f;
    }
else
    { //initialize d_dt's to 0 because that's what we're initializing the vars to
    pry_dat->drolldt = 0;
    pry_dat->dpitchdt = 0;
    pry_dat->dyawdt = 0;
    tmp_f = roll_tmp; pry_dat->roll_tare = tmp_f*pry_dat2degrees_f;
    tmp_f = pitch_tmp; pry_dat-> pitch_tare = tmp_f*pry_dat2degrees_f;
    tmp_f = yaw_tmp; pry_dat->yaw_tare = tmp_f*pry_dat2degrees_f;
```

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```
    }
tmp_f = roll_tmp; pry_dat->roll = (tmp_f*pry_dat2degrees_f - pry_dat->roll_tare);
tmp_f = pitch_tmp; pry_dat->pitch = (tmp_f*pry_dat2degrees_f - pry_dat->pitch_tare);
tmp_f = yaw_tmp; pry_dat->yaw = (tmp_f*pry_dat2degrees_f}- pry_dat->yaw_tare)
FLUSH_PRY_DAT; //prepare for next pry data reception
return time;
}
```

```
|******************************************************************/
```

/*
function: Handles the wz interrupt and collects wz data
inputs: none
outputs: none
*/
void get_wz_dat(void)
\{
ulong elapsed_time, time; //find out how much time since last update
float elapsed_time_f, wz_millisec2hz_f, motor_pwm_discretizations_f;
int pit_delay;
if(WZ) //if we were triggered by a positive edge
if(
time $=$ MilliSecs();
elapsed_time = time - wz_time;
elapsed_time_f = elapsed_time;
wz_millisec2hz_f = WZ_MILLISEC2HZ;
motor_pwm_discretizations_f = MOTOR_PWM_DISCRETIZATIONS;
if (wz_time !=0)
\{
if (wz_millisec2hz_f/elapsed_time < WZ_MAX_HZ) //glitch protection
\{
wz_dat.dwzdt = 1000.Of*(wz_millisec2hz_f/elapsed_time_f -wz_dat.wz)/elapsed_time_f;
$w z$ _dat.wz = wz_millisec2hz_f/elapsed_time;
pit_delay $=(1000.0 f /$ motor_pwm_discretizations_f)/wz_dat.wz;
current_angle $=0 ; / /$ this sets the zero point for the steering output, the "front"
ServiceOutputSteeringInterrupt();
if (wz_dat.wz != 0)
SetSteeringInterruptTimer(pit_delay);
change_output = 1 ;
\}
$\}$
else //this was the first time so don't record anything it will be incorrect
\{
wz_dat.dwzdt = 0;
wz_dat.wz = 0;
\}
wz_time = time; //update time
\}
//unless i get a better potentiometer, can't use the a2d'ed wz pot dat as a current_angle help
alt_dat_valid = 0;
//since this handler may have interrupted the altimeter handler, this will make
//the next altimeter reading invalid

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```
//another way to make the interrupts independent of the wz_dat would be to
//read the wz potentiometer directly into an a2d pin and from this value
//calculate at what point we are in the rotation and then figure out a
//way to set the pwm using this data. this would involve an architectural
//restructuring of the pwm output code and also a new potentiometer that
//has a much shorter "dead" area
//this last disabling on wz time out will certainly cause the craft to crash
//if it is in mid-air when wz times out. a more intelligent system would have
//a backup sensor to determine whether or not it was a sensor malfunction that
//caused the time out, or if it was because the craft stopped rotating. if
//it was becuase the craft stopped rotating, a different kind of control law
//could take over for a non-rotational vectron, and if it was a sensor
//malfunction, we could use the last good data (which could be easily added
//to the wz struct) in order to then safely land the craft for recovery
/*
** THIS IS THE IMPORTANT PART, ACKNOWLEDGES AND TURNS OFF INTERRUPT TILL NEXT
*/
TPUClearInterrupt(TPU_WZ_PIN);
return;
}
/*
    function: Handles the altimeter interrupt and collects altimeter data
    inputs: none
    outputs: none
*/
void get_alt_dat(void)
{
    ulong elapsed_time, time; //use find out how much time since last update
    ulong echo_time;
    float elapsed_time_f, echo_time_f;
    float uspercm_f = USPERCM;
    if (ECHO) /lif echo is high, it just went high, so start the stop watch
    {
        StopWatchStart();
    }
else //echo is low, it just went low, so read the stopwatch and process
    {
        if (alt_dat_valid)
            {
                echo_time = StopWatchTime();
                time = MilliSecs();
                elapsed_time = time - alt_time;
                elapsed_time_f = elapsed_time;
                echo_time_f = echo_time;
                if ((echo_time_f > MIN_ALT_ECHO_TIME)&(echo_time_f < MAX_ALT_ECHO_TIME)) //defined in main.h
                {
            if (alt_time != 0)//if we're initializing, set daltdt to 0
                    {
                        alt_dat.daltdt = 1000.0f*(echo_time_f/uspercm_f - alt_dat.alt)/elapsed_time_f;
                    }
                else
                    {
```

```
                    alt_dat.daltdt = echo_time_f/uspercm_f;
                    }
                        //multiply by }1000\mathrm{ since elapsed_time is in milliseconds
                alt_dat.alt = echo_time_f/USPERCM;
                //this will be *very*}\mp@subsup{}{}{*}\mathrm{ close to actual height (within }~.5\textrm{cm}\mathrm{ )
                    alt_time = time; //update time
                    change_output = 1;
                }
            }
    else
        alt_dat_valid = 1; //make altimeter readings valid again
    //else, echo is either a glitch or ECHO just went high
    } //end else
/*
** THIS IS THE IMPORTANT PART, ACKNOWLEDGES AND TURNS OFF INTERRUPT TILL NEXT
*/
TPUClearInterrupt(TPU_ALT_ECHO_PIN);
return;
}
```

```
/******************************************************************/
```

/******************************************************************/
/*
/*
function: Initiates PWM output on specified channel
function: Initiates PWM output on specified channel
inputs: chan - channel number to set up
inputs: chan - channel number to set up
hi_time - amount of time spent hi during period
hi_time - amount of time spent hi during period
period - period of PWM signal
period - period of PWM signal
outputs: none
outputs: none
*/
*/
void TPUInitPWM(short chan, short hi_time, short period)
void TPUInitPWM(short chan, short hi_time, short period)
{
{
CHANPRIOR(chan, Disabled); // stop what its doing
CHANPRIOR(chan, Disabled); // stop what its doing
*CIER \&= ~(1 << chan); // don't want interrupts enabled
*CIER \&= ~(1 << chan); // don't want interrupts enabled
FUNSEL(chan, PWM); // channel function select
FUNSEL(chan, PWM); // channel function select
PRAM[chan][0] = OutputChan | NoChangePAC | (hi_time ? ForceHigh : ForceLow);
PRAM[chan][0] = OutputChan | NoChangePAC | (hi_time ? ForceHigh : ForceLow);
PRAM[chan][2] = hi_time;
PRAM[chan][2] = hi_time;
PRAM[chan][3] = period;
PRAM[chan][3] = period;
HOSTSERVREQ(chan, 2);
HOSTSERVREQ(chan, 2);
CHANPRIOR(chan, HighPrior); // set channel priority
CHANPRIOR(chan, HighPrior); // set channel priority
while (HOSTSERVSTAT(chan) \& 3) // wait for init
while (HOSTSERVSTAT(chan) \& 3) // wait for init
{}
{}
return;
return;
}//this routine was copied from the example files

```
}//this routine was copied from the example files
```

```
/******************************************************************/
/*
    function: Changes the PWM output on specified channel
    inputs: chan - channel number to set up
            hi_time - amount of time spent hi during period
        period - period of PWM signal
    outputs: none
*/
```

```
void TPUChangePWM(short chan, short hi_time, short period)
{
    *** NEED TO DO THIS WRITE COHERENTLY (AS DOUBLE WRITE)
*/
* (ulong *) &PRAM[chan][2] = ((ulong) hi_time << 16L) | (ulong) period;
return;
}//this routine was copied from the example files
```

```
|******************************************************************
```

|******************************************************************
/*
/*
function: Changes the PWM output on specified channel
function: Changes the PWM output on specified channel
inputs: chan - channel number to set up
inputs: chan - channel number to set up
hi_time - amount of time spent hi during period
hi_time - amount of time spent hi during period
period - period of PWM signal
period - period of PWM signal
outputs: none
outputs: none
*/
*/
void TPUUpdateAllChans(void)
void TPUUpdateAllChans(void)
{
{
int i;
int i;
for (i = 0; i < NUM_PRIMARY_PWM_CHANNELS; i++)
for (i = 0; i < NUM_PRIMARY_PWM_CHANNELS; i++)
{
{
HOSTSERVREQ(i + PWM_CHANNEL_OFFSET, 1); /* issue request */
HOSTSERVREQ(i + PWM_CHANNEL_OFFSET, 1); /* issue request */
while (HOSTSERVSTAT(i + PWM_CHĀNNEL_OFFSET) \& 3) /* await reply */
while (HOSTSERVSTAT(i + PWM_CHĀNNEL_OFFSET) \& 3) /* await reply */
{}
{}
}
}
HOSTSERVREQ(PWM_YAW_OUTPUT_PIN, 1); /* issue request */
HOSTSERVREQ(PWM_YAW_OUTPUT_PIN, 1); /* issue request */
while (HOSTSERVSTAT\(PWM_YAW_OŪTPUT_PIN) \& 3)/* await reply */
while (HOSTSERVSTAT\(PWM_YAW_OŪTPUT_PIN) \& 3)/* await reply */
{}
{}
return;
}
/**
function: initializes the periodic timer interrupt and r^2 steering
inputs: delay - number of ms per interrupt
outputs: none
*/
void InitOutputSteering(int delay)
{
*PITR = 0; /* set count to zero - disable timer */
SetSteeringInterruptTimer(delay);
return;
}

```
```

/******************************************************************/
/*
function: sets the periodic delay for the output steering interrupt
inputs: delay - number of ms per interrupt
outputs: none
*/

```

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```

void SetSteeringInterruptTimer(int delay)
{
*PITR = 0;
*PITR = 10*delay; /* delay of 10=10ms interrupt */
return;
} /* if delay = 0 stop */

```
```

/*
function: provides a full state and d(state)dt estimate of parameters
not encompased by alt_dat, pry_dat, or wz_dat
inputs: *state_dstate - points to state estimator struct
*pry_dat - points to pitch, roll, yaw data struct
initialize_flag - tells whether to initialize or not
state_time - the last time from MilliSecs() that the sub routine was called
outputs: time - an update for the state_time variable in the main routine
*/
ulong state_estimator(struct STATE_STRUCT *state_dstate, struct PRY_STRUCT *pry_dat, short initialize_flag, ulong
state_time)
{
ulong elapsed_time, time; //find out how much time since last update
ulong time_out;
float elapsed_time_f;
float tilt_tmp;
float aparent_yaw_tmp;
double pitch_rad = PI/180*pry_dat->pitch;
double roll_rad = PI/180*pry_dat->roll;
double cp = cos(pitch_rad);
double tp = tan(pitch_rad);
double cr = cos(roll_rad);
double tr = tan(roll_rad);
time = MilliSecs();
elapsed_time = time - state_time;
elapsed_time_f = elapsed_time;
//assume that pitch and roll only vary by up to 90 degrees from their original values
//yaw is positive to the right, CW. yaw is negative to the left, CCW.
if (pry_dat->roll > 0)
{
aparent_yaw_tmp = -1*(180/PI*atan2(tp,tr) - 90);
}
if (pry_dat->roll < 0)
{
aparent_yaw_tmp = -1*(180 + 180/PI*atan2(tp,tr) - 90);
}
tilt_tmp = 180/PI*acos(cp*cr);
if ((pry_dat->pitch = BAD_DATA)|(pry_dat->roll = BAD_DATA))
{
state_dstate->dtiltdt = BAD_DATA;
state_dstate->daparent_yawdt = BAD_DATA;
state_dstate->tilt = BAD_DATA;
state_dstate->aparent_yaw = BAD_DATA;
}
else

```

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```

    {
        if (initialize_flag == 0)
        {
        state_dstate->dtiltdt = (tilt_tmp - state_dstate->tilt)/elapsed_time_f;
        state_dstate->daparent_yawdt = (aparent_yaw_tmp - state_dstate->aparent_yaw)/elapsed_time_f;
        }
    else
    {
        state_dstate->dtiltdt = tilt_tmp;
        state_dstate->daparent_yawdt = aparent_yaw_tmp;
    }
    state_dstate->tilt = tilt_tmp;
    state_dstate->aparent_yaw = aparent_yaw_tmp;
    }
/*
don't correct altimeter for angle of tilt, the altimeter is not that directional
and will pick up an echo up to a 30 degree tilt anyway which is much more than
the vectron should ever be tilted. can correct beyond this tilt if i want to, but it's
fairly uneccessary. for a correction we would have to add:
put this at the top of the sub-routine:
float actual_altitude_tmp;
and then down here:
actual_altitude_tmp = alt_dat.alt*cos(roll_rad)*}\operatorname{cos(pitch_rad);
if (alt_dat.daltdt = BAD_DATA)
{
state_dstate->dactual_altitudedt = BAD_DATA;
state_dstate->actual_altitude = BAD_DATA;
}
else
{
if (initialize_flag == 0)
{
state_dstate->dactual_altitudedt = (actual_altitude_tmp - state_dstate->actual_altitude)/elapsed_time_f;
}
else
{
state_dstate->dactual_altitudedt = actual_altitude_tmp;
}
state_dstate->actual_altitude = actual_altitude_tmp;
}
the extra states are already part of the state_dstate struct
*/
return time;
}

```
```

/******************************************************************/
/*
function: services output steering periodic interrupt
inputs: none
outputs: none
*/

```
```

void ServiceOutputSteeringInterrupt(void)
{
int i;
short hi_time;
float motor_factor;
for (i = -1; ; < NUM_PRIMARY_PWM_CHANNELS - 1; i++)
{
if (i == 0)
motor_factor = 1;
else
motor_factor = sqrt(3);
hi_time = sqrt((output.thrust_offset + output.pitch_roll_steering*sin(current_angle*PI/180 -
output.steering_angle*PI/180 +
i*Pl/2)/motor_factor)*OUT_VOLTSSQPERNEWTONMETER)*MOTOR_PWM_PERIOD/MOTOR_PWM_HI_VOLTS;
if (hi_time < MOTOR_PWM_HI_MIN)
hi_time = MOTOR_PWM_HI_MIN;
if (hi_time > MOTOR_PWMM_HI_MAAX)
hi_time = MOTOR_PWM_HI_MAX;
switch(i)
{
case 0:
output.pwm_hi_time_1 = hi_time;
break;
case 1:
output.pwm_hi_time_2 = hi_time;
break;
case 2:
output.pwm_hi_time_3 = hi_time;
break;
default:
//add any other motors here
break;
}
TPUChangePWM(i + PWM_CHANNEL_OFFSET + 1, hi_time, MOTOR_PWM_PERIOD);
}
hi_time = output.yaw_steering*OUT_MOTOR_KE*MOTOR_PWM_PERIOD/MOTOR_PWM_HI_VOLTS;
if (hi_time < MOTOR_PWM_HI_MIN)
hi_time = MOTOR_\overline{PWM_}\mp@subsup{\mathbf{HI_MIN}}{}{\prime}
if (hi_time > MOTOR_PWM_HI_MAX)
hi_-time = MOTOR_PWM_\overline{HI_MAX;}
output.pwm_hi_time_yaw = hi_time;
TPUChangePWM(PWM_YAW_OUTPUT_PIN, hi_time, MOTOR_PWM_PERIOD);
current_angle += 360/MOTOR_PWM_DISCRETIZATIONS;
if (current_angle >= 360)
current_angle -= 360;
pit_hits++;
return;
}
/*******************************************************************/
/*
function: heart of auto-piolot, decides what the voltage outputs to the motors should be based on the sensor inputs
inputs: *state_dstate - points to state estimator struct
*pry_dat - points to pitch, roll, yaw data struct
*usr_dat - points to the user inputed data struct
outputs: none (essentially output)
*/

```

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```

void controller(struct STATE_STRUCT *state_dstate, struct PRY_STRUCT *pry_dat, struct USR_STRUCT *usr_dat)
{
float roll_error, pitch_error;
float tilt, daltdt, dyawdt,
if (usr_dat->mode == 1) //user controlled mode
{
if (usr_dat->roll_c > 0)
output.steering_angle = -1*(atan2(tan(usr_dat->pitch_c*PI/180),tan(usr_dat->roll_c*PI/180))*180/PI - 90);
else
output.steering_angle = -1*(180 + atan2(tan(usr_dat->pitch_c*PI/180),tan(usr_dat->roll_c*PI/180))*180/PI - 90);
if (usr_dat->thrust_c > MOTOR_MIN_THRUST)
output.thrust_offset = usr_dat->thrust_c;
else
output.thrust_offset = MOTOR_MIN_THRUST;
output.pitch_roll_steering = cos(usr_dat->pitch_c*PI/180)*}\operatorname{cos(usr_dat-
>roll_c*PI/180)/pow(cos(USR_DAT2DEGREES*US\overline{R_MAX_DATA*PI/180),2)*MOTOR_MAX_STEERING;}
if (output.thrust_offset*MAX_STEER_TO_THRUST_RATIO < output.pitch_roll_steering)
output.pitch_roll_steering = output.thrust_offset*MAX_STEER_TO_THRUST_RATIO;
output.yaw_steering = usr_dat->yaw_c;
}
else //computer controlled mode
{
if (pry_dat->roll = BAD_DATA) /lif one is bad, both will be bad
{
roll_error = 0; //going down in flames
pitch_error = 0;
}
else
if (usr_dat->roll_c = BAD_DATA)
roll_error = ROLL_GAIN*(USR_STABLE_ROLL-pry_dat->roll); //command a hover
else
roll_error = ROLL_GAIN*(usr_dat->roll_c - pry_dat->roll);
if (usr_dat->pitch_c = BAD_DATA)
pitch_error = PITCH_GAIN*(USR_STABLE_PITCH-pry_dat->pitch); //command a hover
else
pitch_error = PITCH_GAIN*(usr_dat->pitch_c - pry_dat->pitch);
}
output.pitch_roll_steering = sqrt(pow(roll_error,2)+pow(pitch_error,2));
//this output.pitch_roll_steering will have to be lead compensated as written
//about in Jesse Davis' thesis controller section
if (roll_error > 0)
output.steering_angle = -1*(atan2(tan(pitch_error*PI/180),tan(roll_error*P//180))*180/PI - 90);
else
output.steering_angle = -1*(180 + atan2(tan(pitch_error*PI/180),tan(roll_error*PI/180))*180/PI - 90);
if (state_dstate->tilt = BAD_DATA)
tilt = 0; //going down in flames
else
tilt = state_dstate->tilt;
if (alt_dat.daltdt = BAD_DATA)
daltdt = 0
else
daltdt = alt_dat.daltdt;
if (usr_dat->thrust_c = BAD_DATA)
output.thrust_offset = HOVER_THRUST/cos(tilt*P//180)+ALT_SPEED_GAIN*(USR_STABLE_ALT_SPEED-daltdt);
else

```
output.thrust_offset = HOVER_THRUST/cos(tilit*PI/180)+ALT_SPEED_GAIN*(usr_dat->thrust_c-daltdt);
//this output.thrust_offset will have to be dominant pole compensated as written
//about in Jesse Davis' thesis controller section
if (pry_dat->dyawdt = BAD_DATA)
dyawdt \(=0\);
else
dyawdt = pry_dat->dyawdt;
if (usr_dat->yaw_c = BAD_DATA)
output.yaw_steering = YAW_GAIN*(USR_STABLE_YAW_SPEED-pry_dat->dyawdt);
else
output.yaw_steering = YAW_GAIN*(usr_dat->yaw_c-pry_dat->dyawdt);
//this output.yaw_steering will have to be dominant pole compensated as written //about in Jesse Davis' thesis controller section
\}
return;
\}```

