Wideband Processing and Analysis of Lightning in the Earth-Ionosphere Waveguide

by

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Submitted to the Department of Electrical Engineering and Computer Science
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Abstract

A new vertical electric field antenna has been installed at our West Greenwich, RI Schumann resonance field station, coupled to a high-bandwidth charge amp and a computerized sampling system. Techniques are given for the processing of lightning transients. Digital signal processing techniques for the removal of power line harmonic noise are presented and evaluated. Additionally, a wideband (3 Hz - 24 kHz) simulation model has been developed. This model gives evidence that slow tail waveforms are generated primarily due to the variation of waveguide phase velocity with frequency and the depletion of spectral energy in the waveguide cutoff region. Preliminary formulas for distance estimation have been developed from the simulations, based on slow tail separation time and spectral ratios. These methods are evaluated for consistency against real events.

Thesis Supervisor: Earle R. Williams
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Chapter 1

Introduction

The observational study of atmospheric electrodynamics, the propagation of electromagnetic (EM) radio waves in the atmosphere, began in the 1930’s [Watson Watt et al., 1937]. Theoretical treatments of extremely low frequency (ELF) propagation began to be expanded upon in the 1950’s [Schumann, 1952]. Since the early 90’s, interest in ELF propagation has been renewed [Williams, 1992]. Barr provides a good history of atmospheric electrodynamic research from the 1950’s to the present [Barr et al., 2000].

This thesis will discuss the methodology behind wideband (3 Hz - 24 kHz) sampling and signal processing, present a new wideband simulation, and preliminarily explore the validity of the simulated results in the estimation of lightning source distance.

1.1 Why High Bandwidth?

Previously, researchers have often restricted their inquiry to isolated regions of the EM spectrum, namely ELF (3-300 Hz), and very low frequency (VLF: 3k-30k Hz). This was partly due to technical limitations, and partly due to personal interest and highly specific theoretical models. Since high-speed data acquisition and digital signal processing (DSP) techniques have recently come to maturity, it is now possible to tackle a much wider bandwidth. Capturing wideband EM signals gives a more
complete picture of lightning processes. This allows for the development and testing of more complex models, and ultimately a deeper understanding of atmospheric phenomena.

1.2 Wideband Problems

1.2.1 Anthropogenic Noise

A major problem that occurs with high bandwidth systems, is that anthropogenic noise becomes more apparent. The major source of this noise is EM contamination from power lines, as well as military RF transmissions and electric trains. The noise present at our EM observation station in West Greenwich, RI, will be assessed in Chapter 3.

1.2.2 Data Overload

Another problem is a large increase in the volume of data. For instance, a system sampling at 200 Hz will have 200 samples per second, while a system sampling at 48 kHz will have 240 more data values per second. This is not a prohibitive problem, since data storage has become less expensive over the years. However, for a system that is in a remote location and cannot be frequently maintained, a large volume of data can present difficulties. Additionally, large amounts of data can lead to the proverbial searching for a needle in a haystack.

1.2.3 DSP and Thresholding

DSP techniques come to the rescue for both of these problems. For the data problem, various thresholding schemes can be implemented to determine which events are worth committing to disk. The simplest scheme is a voltage threshold, however transients can be obscured by noise. DSP techniques provide many possible solutions to remove the anthropogenic noise. A summary of digital filtering techniques particularly suited to this task will be presented in Chapter 4.
Chapter 2

Apparatus

2.1 System Overview

The atmospheric observation system referred to in this thesis is located at the Alton Jones (Greenwich, RI) campus of the University of Rhode Island. The measurement station was originally set up by Charles Polk, and has been used to measure N-S and E-W horizontal magnetic fields ($H_{NS}$, $H_{EW}$) and vertical electric field ($E_Z$) ([Polk, 1982], [Heckman et al., 1998].

2.1.1 A New Antenna

In the fall of 2002, a new $E_Z$ antenna was installed to supersede the old one. The old antenna mast, which was installed by Polk in the late 1960’s, had a problem of being susceptible to mechanical oscillations caused by the wind. It was used up until July 2002, when a lightning strike shattered the antenna ball. The new antenna tower consists of five stacked ceramic insulator columns, donated by the Boston Edison Company, and measures 7 meters tall. The insulators are significant for their great mechanical stability, and they also serve to isolate the pre-amplifier at the top from the space charge noise near the Earth’s surface.
2.1.2 The Charge Amplifier

The actual antenna electrode is made of an inverted stainless-steel salad bowl, which sits at the top of the tower. It is very similar to the system described by Ogawa [Ogawa et al., 1966]. Inside this sealed hemisphere is a custom charge amplifier, designed by Michael Stewart of Thunderstorm Technology. Regular antennas pick up electric field, while this antenna’s preamplifier measures the charge induced on the top electrode by the vertical electric field. The electrical output is therefore linearly proportional to the vertical electric field, and has a flat response over the desired bandwidth. This makes it superior to the traditional ultra-high impedance systems used by Ogawa, Polk, and others.

2.1.3 The Post-Amplifier System

At the base of the antenna tower is a metal lock box which contains more custom electronics. An amplifier with wideband specifications has a switch-adjustable gain. Also available to be switched in and out of the system is a 3 Hz “DC-blocking” highpass filter, as well as a 20 kHz Butterworth anti-aliasing filter. A very long (approx. 600 ft) cable leads from the electronics box to the hut which contains the observation and data-logging equipment. Phase and magnitude plots of the custom
Figure 2-2: Transfer functions of system electronics (charge amp and post gain amps), courtesy of M. F. Stewart.

electronics are provided in Figure 2-2.

2.1.4 The Analog-to-Digital Converter

The analog-to-digital converter (ADC) that was chosen for this new wideband system is a Data Translations model DT3005. It operates at 16-bits and a has a maximum throughput of 500 kS/s. A 16-bit converter was necessary, as the noise from the power lines is considerably greater than the desired atmospheric signals. Each bit of an ADC provides approximately 6 dB (20 log₁₀ 2) of dynamic range; therefore a 16-bit converter has 96 dB of dynamic range.
2.1.5 The Data-Acquisition Software

This author developed the data-acquisition software, using Microsoft Visual Studio 6.0 and the DTx-EZ ActiveX controls provided by Data Translations. The software was designed to continuously monitor the incoming $E_z$ signal and capture supra-threshold transients, while providing displays of various spectral regions.

Figure 2-3: Screenshot of the data acquisition software.

1Unfortunately, Data Translations currently provides hardware drivers only for Microsoft Windows operating systems. The company does not support the Linux operating system, which is superior to Windows for scientific research due to its excellent stability and customization options. This author had major development problems with unexplained software instability while writing large files to disk, and strongly recommends against the Windows platform for research purposes. It is important for researchers to have precise knowledge and control over what is occurring at the system level, and this is simply not possible on a proprietary Windows system. Therefore, it is recommended that researchers should not purchase hardware that is not fully supported on the Linux platform, unless they are experienced Linux hackers and plan on writing their own device drivers.
2.1.6 Sampling Rate and Anti-Aliasing

24 kHz was determined to be an adequately large bandwidth of the sampled transients, since VLF energy is strongly attenuated in the waveguide and the peak radiation from lightning lies at lower frequencies (5-15 kHz). By the Shannon-Nyquist sampling theorem, this bandwidth requires a sampling rate of 48 kHz. Since the ADC can sample at frequencies up to 500 kHz, it was decided to have the software drive the ADC at 392 kHz, or 8x-oversampled. This oversampling process gives the analog anti-aliasing filter extra room in the frequency domain to properly attenuate any frequencies above the sample rate. Any energy that exists above the Nyquist rate (half the sampling rate) will be aliased and contaminate the spectrum. For instance, sampling at 48 kHz, a frequency of 25 kHz (Nyquist + 1 kHz) will be aliased as 23 kHz (Nyquist - 1 kHz). See [Oppenheim and Shafer, 1989] for a more in-depth discussion.
Chapter 3

Characterization of Anthropogenic Noise

3.1 Power Line Harmonics

Electromagnetic fields from power lines are the main offenders which obscure the atmospheric data in the ELF/VLF band selected for this study. In North America, this energy is present at 60 Hz, while overseas it is usually present at 50 Hz. The power line noise present at the Rhode Island station contains harmonics all the way up to the range of 10 kHz, making it extremely pervasive (See Figure 3-1). The RI power line noise is especially problematic because the exact frequencies and amplitudes vary over time, and the harmonics do not appear to be exact integer multiples of the fundamental. This frequency skewing could be explained by nonlinear distortion, either mechanical or electromagnetic. For most harmonics, a filter bandwidth of 3 Hz is sufficient to remove the noise relative to the signal floor. The fundamental harmonic requires a wider bandwidth (5 Hz) to remove the noise relative to the signal.

3.2 Other Sources of Noise

There are other sources of noise which are present in the electric spectrum. Peaks occur at approximately 15.975, 19.575, 21.75, 23.8, and 24 kHz (See Figure 3-1).
Figure 3-1: Exemplary noise seen at the ADC, taken from a 4 second waveform which had no visible lightning transients. Note the 60 Hz harmonics (up to 10 kHz), and RF interference (16-20 kHz).
The exact source of these peaks is unknown, though they are most likely from radio frequency (RF) communication stations.
Chapter 4

Digital Signal Processing

This chapter will discuss the theory behind the DSP techniques that were applied in the processing of wideband lightning waveforms, as well as their application to the task of removing power line harmonics. For more detailed information regarding digital filters, it is highly recommended to consult the following references: [Oppenheim and Shafer, 1989] and [Orphanidis, 1996]. The actual procedures used in the event sampling and processing are discussed in Chapter 7.

4.1 Advantages of Digital Signal Processing

Digital filtering presents many advantages over analog filtering methods. The most obvious advantage is one of space and manufacturing cost; many filters can be implemented in software on a single computer or microprocessor, performing the equivalent tasks of many racks full of analog equipment. In addition, DSP software is reprogrammable, which makes the hardware reusable. Software can even be designed so as to be self-adapting to changes in the input, making DSP highly versatile. Digital filters are extremely stable compared to analog circuits, which are subject to component drift due to temperature fluctuations. Digital filters can also accurately process ultra low frequencies, which is problematic with analog hardware. Perhaps the most useful advantage of digital filtering is that they can be designed to be linear-phase (see Sec. 4.6), meaning that there will be absolutely no distortion of the signal due
to uneven phase lags—something which is difficult to achieve with analog circuitry.

4.2 Infinite Impulse Response vs. Finite Impulse Response

There are two basic classes of digital filters: Infinite Impulse Response (IIR), and Finite Impulse Response (FIR). IIR filters often called “recursive” filters, since they rely on feedback of past output. The transfer function of an IIR filter will always have a denominator, or poles. Because they employ the power of feedback, IIR filters provide much more computational efficiency than FIR filters, making them useful for realtime applications. However, there is also a possible danger of instability whenever feedback is employed. Additionally, feedback requires time to reach steady-state (see Sec. 4.5). IIR filters are the digital version of active analog systems. The magnitude response transitions of IIR filters in the frequency domain tend to be very smooth, which may not always be desirable, depending on the application.

While IIR filters use the power of feedback, FIR filters could be said to work by brute force. FIR filters can be designed to approximate virtually any transfer function to any degree of accuracy, however the tradeoff for precision is that more computation required. It is also possible approximate the response of an IIR filter by windowing the IIR to create a FIR. FIR filters are also known as “non-recursive” filters, as they rely only on previous input. This means that the transfer function will have no denominator, as there are no poles in the system.

4.2.1 Choices for This Application

For the task of filtering the RI power line noise, IIR filters were chosen for their ability to create a smooth and precise magnitude response with minimal computations. Also, IIR filters have a closed form that is a function of cutoff frequency, which means a minimum of computations is necessary to change the cutoff frequency for reuse. In comparison, FIR filters require calculation of hundreds of coefficients specific to
Figure 4-1: Bode plot and pole-zero diagram of the peaking comb filter (M=6).

the cutoff frequency. FIR filters are used in this thesis for tasks such as envelope extraction, due to their potential to be designed with linear phase\(^1\).

4.3 Comb Filters

A comb filter is a system which affects the magnitude of a series of harmonics spanning the entire spectrum. This type of filter is so named due to the shape of its transfer function resembles the teeth on a comb. There are two varieties of comb filters, peaking (a.k.a. bandpass) and notching. The name ”comb” by itself usually implies the peaking sort, which is generally used for synthesis and signal enhancement applications. The FIR comb filter transfer function is

\[ H(z) = 1 + z^{-M} \]  

(4.1)

where

\[ M = \frac{\pi}{\omega_0}, \quad (\omega_0 = \frac{\pi}{M}) \]  

(4.2)

The notching comb filter simply involves a sign change from the peaking type:

\[ H(z) = 1 - z^{-M} \]  

(4.3)

\(^1\)Linear phase filtering is discussed in Section 4.6.
Figure 4-2: Bode plot and pole-zero diagram of the notching comb filter (M=6).

As one can see from Figures 4-1 and 4-2, the comb filter transfer functions express the Mth roots of unity, resulting in M zeros evenly spaced around the unit circle. Due to this fact, it is only possible to design a comb filter where the fundamental frequency is an exact integer fraction of the sample rate ($\omega_{SR} = 2\pi$).

4.3.1 Feedback-sharpened Comb Notch

Notice from the above figures that the comb filters presented so far do not have a very sharp frequency response. The comb filter becomes much more useful when it is sharpened with feedback\(^2\), which is equivalent to adding poles behind each zero. The closer the poles are to the unit circle ($a \rightarrow 1$), the more narrow the notches become.

$$H(z) = \frac{1 - z^{-M}}{1 - a^M z^{-M}}$$ (4.4)

4.3.2 Renormalization and Bandwidth

For practical use, the filter should be renormalized so that it presents unity gain in the passbands. Additionally, a conversion factor is necessary to go from 3-dB bandwidth to pole magnitude. This can be derived through the bilinear transform of an equivalent analog system [Orphanidis, 1996, p. 407]. The transfer function from

\(^2\)Recall that feedback denotes an IIR system.
Figure 4-3: Bode plot and pole-zero diagram of sharpened comb notch (M=6, a=0.9).

Equation 4.4 now becomes

\[
H(z) = b \frac{1 - z^{-M}}{1 - az^{-M}}
\]  

(4.5)

where

\[
a = \frac{1 - \beta}{1 + \beta}, \quad b = \frac{1 + a}{2}, \quad \beta = \tan \frac{M \Delta \omega}{4}
\]

4.3.3 Disadvantages of the Comb Filter

The comb notch filter is the most obvious way to filter periodic noise that has a regular harmonic structure, such as power line harmonics. It can be a very elegant solution, especially for realtime processing, since it removes regularly spaced harmonics across the entire spectrum with a trivial amount of processing.

However, it is not always the best solution. As was demonstrated earlier, only frequencies which are integer fractions of the sample rate can be filtered. It could also be a problem that the bandwidths and amplitude of the notches are not independently adjustable.

Another potential problem is the case in which the noise does not contaminate the entire spectrum, such that certain notches in the comb series may be removing desirable energy. For applications where preserving frequencies at DC (\(\omega = 0\)) or Nyquist (\(\omega = \pi\)) is an issue, it is possible to temper the comb filter by adding another transfer function in series, which cancels the pole and zero at the undesirable notch
frequency. In theory, this technique could be applied to neutralize any of the harmonic
cutoffs, however it will reduce the simplicity of implementation. This issue was not a
problem with the RI system, but is mentioned for completeness.

The biggest caveat of the comb filter is that it produces noticeable echoes in the
time domain. The reason for this becomes apparent after examining Equation 4.3;
the output is the sum of the input and a *delayed copy* of itself. For applications that
deal only with the frequency domain, or with very short time waveforms (below half
the fundamental cutoff period), this phenomena would not be a problem.

For the removal of the RI power line noise, the comb filter is not very useful
since the noise harmonics are not regularly spaced at exact multiples of the ~60 Hz
fundamental. This requires an increase of the bandwidth, and therefore a lack of
precision as desired energy will be removed as well as harmonics. Additionally, the
echoes are a problem since half the period of 60 Hz is 8.3 ms, which is shorter than
the length of a slow tail sferic. However, the comb filter is still useful for noise removal
prior to event thresholding (see Chap. 7), where accuracy is not as important.

### 4.4 The IIR Notch Filter

The single notch filter is highly useful, in that many filters with independent frequency
and bandwidth can be used in series. However, like the comb filter the attenuation
depth is not adjustable. The simplest way to create a digital notch filter is with a zero
of magnitude one and a pole of magnitude less than one, both at the desired angular
frequency. The closer the pole approaches to the unit circle, the more narrow the
notch bandwidth becomes. Since this results in an recursive system, it will become
unstable\(^3\) if the pole magnitude goes beyond the unit circle. In order to make this
filter system real, a conjugate pole/zero pair must be added, symmetrically reflected
across the real axis. This results in the second-order IIR system [Orphanidis, 1996]:

\[^3\text{In practice, stability should only be an issue when implementing an extremely narrowband filter on a hardware system with fixed precision, as the filter coefficients may be quantized in such a way as to create instability. This was not a problem with the RI system, but would become an issue if a stand-alone DSP system were to be implemented on fixed-point hardware.}\]
Figure 4-4: Bode plot and pole-zero diagram of the renormalized IIR notch filter $(\omega_0 = \frac{\pi}{2}, \Delta f = 10kHz)$.

\[ H(z) = \frac{1 - \beta + z^{-2}}{1 - b\beta z^{-1} + b^2 z^{-2}}, \quad \beta = 2 \cos \omega_0, \quad (b < 1) \quad (4.6) \]

where $b$ is given by Equation 4.8.

4.4.1 Renormalization and Bandwidth

The above transfer function is still not suitable for practical applications; one will find that the gain goes beyond unity at the edges of the passband, as it is dependent on pole magnitude. This problem will be exacerbated if many notches are to be used in series. The solution is to renormalize for unity gain at DC, giving the final transfer function

\[ H(z) = b \frac{1 - 2 \cos \omega_0 z^{-1} + z^{-2}}{1 - 2bz^{-1} + (2b - 1)z^{-2}} \quad (4.7) \]

The coefficient $b$ is a function of the 3-dB bandwidth ($\Delta \omega$), and is derived from the bilinear transform of a canonical second-order analog notch system [Orfanidis, 1996, pp. 366,584].

\[ b = \frac{1}{1 + \tan \Delta \omega / 2}, \quad \Delta \omega = \frac{\omega_0}{Q} = \frac{2\pi \Delta f}{f_s} \quad (4.8) \]
4.5 Steady-state

There is one caveat to using IIR filters. It takes a certain number of samples for the feedback to reach a steady-state. During this period the filter will “leak” the frequencies that it is supposed to attenuate. This is problematic when the signal to be filtered is approximately the same length as the steady-state time constant. It is also possible that the filter will “ring” when presented with a large discontinuity in frequency.

4.5.1 Effective Samples and Time-constant

The effective number of samples needed for the leakage/ringing to die down to a given percentage can be estimated using the magnitude of the largest filter pole:

\[
    n_{eff} = \frac{\ln(\epsilon)}{\ln(R)} \quad [\text{samples}] \quad (4.9)
\]

where \( \epsilon \) is the desired percentage of remaining energy, and \( R \) is the radius of the largest filter pole [Orphanidis, 1996, p. 238]. Note that when using the notch filter, \( R \) is equivalent to \( b \) from Equation 4.5. The filter time-constant in seconds is simply the number of effective samples over the sampling frequency:

\[
    \tau = \frac{n_{eff}}{f_s} \quad [\text{seconds}] \quad (4.10)
\]

For example, the width of the power line harmonics at the level of the desired signal is approximately 3 Hz. This value was found empirically by filtering noisy signals and observing the FFT lobes. When using a 3 Hz wide notch filter, \( R = b \approx 0.9998 \). Therefore the 0.1% time-constant for a 48kHz signal is \( \tau = \frac{\ln(0.001)}{\ln(0.9998) * 48e3} \approx 733 \text{ ms} \) – over six times larger than the captured buffer size! This value only increases as bandwidth gets smaller. This phenomena of pre-steady state “leakage” is troublesome when working with time signals that are short, relative to the steady-state time constant.
4.5.2 Forcing Steady State

In order to solve the steady-state / leakage problem, an algorithm was created which generates an artificial sine wave with the center frequency of the notch. This sine wave is prepended to the signal to be filtered, with care to match the phase and amplitude such that ringing will not occur.

The phase matching is performed by way of a linear-phase bandpass filter, followed by a zero-crossing analysis to find the starting point where the phase is zero. (see Matlab scripts in Appendix B). The amplitude of the sine wave is approximated by the FFT bin coefficients (see Section 4.7.2). The amplitude can only be approximated because the actual noise frequency is not a perfect multiple of the sample rate.

4.6 Phase Accuracy

In the processing of radio atmospherics, it is ideal to preserve the phase of the signal such that further mathematical analysis is accurate. In particular, there is interest in extracting the current moment of a lightning transient, using measurements of $E_Z\omega$ and the normal mode equation [Huang, 1998]. Phase is critical for this extraction.

The term ”group delay” is used to describe the amount of sample delay a system causes at each frequency, and is defined by

$$
grd(X(\omega)) = -\frac{dX(\omega)}{d\omega} \quad (4.11)$$

Like analog systems, IIR systems have non-ideal phase responses which cause group delay (a.k.a. dispersion). By the use of linear-phase FIR filters, signals can be processed with no group delay. Linear-phase filters have an ideal phase response which causes absolutely no dispersion. The only tradeoff is a fixed sample delay (a.k.a. latency), which is equal to half the length of the filter impulse response. The Matlab function \textit{fir1} makes the design of linear-phase filters effortless.
4.6.1 Forward-backward Filtering

One way to cancel group delay from IIR filters is to time-reverse the filtered signal and filter it a second time. This results in a magnitude-squared filter response. In theory the effect is to nullify the phase response, however in practice the result is not always ideal. It was determined empirically that in some cases this procedure will result in an attenuation of high frequencies. The exact cause of this is unknown to the author, however it may be due to finite signal lengths. In general, it is recommended to avoid this technique.

4.6.2 Allpass Filter Equalization

A better way to correct for group delay is to design an allpass filter which compensates to give a more desirable group delay. The Matlab Filter Design Toolbox has a command called \texttt{iirgrpdelay} which numerically fits an allpass filter to a given group delay response. This technique could easily be used to neutralize the dispersive effect of analog circuitry in a signal path, if the exact phase response of the analog system was known.

4.6.3 Summary for This Application

It was empirically observed\textsuperscript{4} that the IIR notch filter has a negligible phase response in the passbands, when the bandwidth is less than 5 Hz. A larger bandwidth is not necessary for the task of power line removal (see Chap. 3). Therefore no allpass equalization was actually performed. Linear-phase filters are used to envelope the signals for analysis (see Chap. 7).

Ideally, an allpass phase equalizer should be incorporated into the data acquisition software, designed to compensate for the antenna and the analog hardware phase response. This was not done as there were difficulties in obtaining the phase response of the antenna in series with the analog hardware. This will need to be implemented

\textsuperscript{4}This was done by plotting the group delay in Matlab.
if current moment extraction (via the normal mode equations) is to be performed in the future.

4.7 The Discrete Fourier Transform

The Discrete Fourier Transform (DFT) and its inverse transform (IDFT) are the backbone of many DSP techniques. In a nutshell, the DFT finds a collection of sine waves which form a basis for the reconstruction of an arbitrary signal. This section will briefly detail the use of the DFT in the context of this research.

The DFT of a signal $x$ is defined by (taken from [Smith III, 2002])

$$X(\omega_k) = \sum_{n=0}^{N-1} x(t_n) e^{-j\omega_k t_n}, \quad k = 0, 1, 2, \ldots, N - 1 \quad (4.12)$$

and its inverse (IDFT) by

$$x(t_n) = \frac{1}{N} \sum_{k=0}^{N-1} X(\omega_k) e^{j\omega_k t_n}, \quad n = 0, 1, 2, \ldots, N - 1 \quad (4.13)$$

where

- $x(t_n) =$ input signal amplitude at time $t_n \ [sec]$
- $t_n = nT,$ the $n$th sampling instant \ [sec]
- $n =$ sample number \ [integer]
- $T =$ sample period \ [sec]
- $X(\omega_k) =$ complex spectrum amplitude of $x,$ at radian frequency $\omega_k \ [rad/sec]$
- $\omega_k = k\Omega = k$th frequency sample \ [rad/sec]
- $\Omega = \frac{2\pi}{NT} =$ radian frequency sampling interval \ [rad/sec]
- $f_s = 1/T =$ sampling rate \ [samples/sec, or Hz]
- $N =$ number of samples in both time and frequency \ [integer]

Note that the units of $x(t_n)$ and $X(\omega_k)$ are the same. In order to get a spectral amplitude of standard units \ [V/Hz], given an input of $x(t_n)$ in [V], $X(\omega_k)$ must be divided by the sampling rate, $f_s$.  

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4.7.1 Hermitian Symmetry

The DFT of a purely real time signal (no imaginary component) will have the following property which is known as Hermitian symmetry: (from [Smith III, 2002])

\[
X(-\omega_k) = \overline{X(\omega_k)}
\]  

(4.14)

This means that the negative frequency coefficients have complex conjugate symmetry across \( \omega_k = 0 \). Another way to state this property is that the real part of the DFT has even symmetry and the imaginary part has odd symmetry. Due to this property, it becomes apparent that a real signal can be reconstructed completely from its positive frequency DFT coefficients.

4.7.2 The Fast Fourier Transform

The Fast Fourier Transform (FFT) is a class of commonly used algorithms that greatly speed up the calculation of the DFT, namely for power-of-two and prime length signals. The details will not be discussed here, but a good overview is given in [Oppenheim and Shafer, 1989]. In the field of DSP, the term “FFT” is used interchangably with “DFT”; the FFT is nothing more than an algorithm for the speedy evaluation of the DFT. A common term for FFT coefficients is “bin”, which refers to the metaphorical storage of spectral energy.

All of the DFT computations used in this thesis were done with Matlab’s built-in FFT implementation, which computes the DFT coefficients by the standard form given in Equation 4.12.

4.7.3 Parseval’s Theorem

Parseval’s Theorem for the standard form of the DFT (Eqns. 4.12, 4.13) is expressed as

\[
\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2
\]  

(4.15)
This equation expresses the conservation of energy between the time and frequency domains. For completeness, it should be proven that any chosen FFT algorithm conforms to Parseval’s theorem, at least to the order of numerical precision available on a given computer. The Matlab FFT algorithm used in this thesis satisfies Parseval’s theorem to machine precision.

### 4.7.4 Zero-Padding and Interpolation

Zero-padding is often used with the FFT as a means of extending a signal length to the next power of two. It is also a method which can be used to interpolate, both in frequency and time. By zero-padding the time signal before transforming via FFT, the frequency resolution is enhanced via interpolation. Note that this does not increase the actual resolution of the signal, but rather enhances visibility. The frequency resolution per bin can be expressed as the sample rate divided by the length of the FFT:

\[
\text{Frequency resolution per bin} = \frac{f_s}{\text{length(FFT)}}
\]  

(4.16)

Zero-padding in the FFT domain is equivalent to ideal bandlimited (lowpass) interpolation in the time domain, which will make a signal appear smoother. By zero padding the DFT coefficients and inverse transforming, the waveform will appear to be smoothed and stretched in time. The zeros should be inserted at the midpoint between the largest positive frequency and the smallest negative frequency ($\omega = \pm \pi$).

In this thesis, zero padding was used to enhance the spectral features of short time signals, namely in the SR band. According to quantitative observations by [Smith III, 2002], zero-padding to at least five times the length of the signal is sufficient to give estimates spectral peaks with at least 1% accuracy. For accuracy and speed, it is useful to zero pad to the next power of two, nearest to five times the signal length.
Chapter 5

Waveguide Theory

5.1 Characteristics of Radio Atmospherics

This section will discuss the characteristics of radio atmospherics, in both the frequency and time domain. Refer to Figures 5-1 and 5-2 for illustrations of these characteristics.

5.1.1 Sferics

The name “sferic” is shorthand for an atmospheric radio wave caused by lightning. The onset of a lightning event consists of VLF oscillations. For this reason, the word “sferic” is also generally used as an adjective to describe VLF energy. Due to the low attenuation of the waveguide ($\alpha$) at VLF, sferic energy is observable even after large propagation distances. This is a result of the lossy ionosphere in the upper waveguide, characterized by the imaginary component of the complex eigenvalue $\nu$ (See Sec. 5.2.1). A typical attenuation rate at 10 kHz is approximately 2 dB/Mm.

5.1.2 Waveguide Cutoff

The waveguide cutoff is a feature of strong amplitude attenuation in the vicinity of 1.7 kHz. It is a result of transverse resonance of the TEM mode, which exists due to destructive interference of vertical standing waves in a parallel plate waveguide. The
waveguide cutoff frequency can be expressed as

\[ f_{\text{cutoff}} = \frac{c}{2h} \]  \hspace{1cm} (5.1)

where \( c \) is the speed of light and \( h \) is the waveguide height.

Because of this, the exact cutoff is a function of the ionospheric boundary height, which varies slightly due to diurnal variations [Sentman and Fraser, 1996]. With a typical value of 80-90 km, this formula yields a cutoff between 1.67 and 1.88 kHz. The prediction is only approximate because the TEM mode is actually quasi-TEM, due to the slightly absorbative nature of the ionosphere. Barr discusses the waveguide cutoff in his paper which compares waveguide mode theory to experimentally recorded spectra [Barr, 1970].

### 5.1.3 Slow Tail

The name “slow tail” comes from the characteristic signal shape of certain sferics, which were first observed by Watson-Watt [Watson Watt et al., 1937]. Slow tail energy is ELF energy below the waveguide cutoff, and exists due to the transverse electromagnetic (TEM) mode [Reising et al., 1996]. Slow tails are formed due to the dispersive nature of the atmosphere (frequency-dependence of propagation speed). The lower frequencies arrive later than the VLF sferic, forming a slow tail in the overall signal. The depletion of energy in the waveguide cutoff region also helps to set the slow tail apart from the sferic onset. An example sferic with a slow tail is shown in Figure 5-1.

### 5.1.4 Polarity

A positive event is said to be produced by a cloud-to-ground (CG) transfer of positive charge, while a negative event is said to be produced by a ground-to-cloud (GC) transfer of negative charge. Visual estimation of event polarity is not entirely accurate. The polarity of the initial sferic waveform is a general indicator of polarity of the source event, however this rule of thumb is not always accurate [Huang, 1998].
When the slow tail polarity matches the sferic polarity, it is likely that time-domain source polarity estimation will be more accurate. However, this method of estimation can only be confirmed with actual source polarity data such as from the National Lightning Database Network (NLDN).

### 5.1.5 Schumann Resonance

Schumann resonance (SR) is the phenomena of modal standing waves in the Earth-Ionosphere waveguide. The name comes after W. O. Schumann, the German who hypothesized its existence in 1952 [Schumann, 1952]. The fundamental SR occurs when the wavelength is approximately equal to the circumference of the earth. The average frequencies observed are 7.8, 14, 20, 26, 33, 39, and 45 Hz, though there are slight variations over a 24-hour period.

### 5.2 The Normal Mode Equations

The normal mode equations are a theoretical model for propagation of the TEM mode in a spherical waveguide cavity, and are credited to J. R. Wait [Wait, 1996]. Since
the equations only pertain to the TEM, they only provide a loose approximation for frequencies about the waveguide cutoff, where other modes should be present [Galejs, 1972].

The normal mode equations are presented here in a modified form, using ordinary frequency rather than angular frequency. While this thesis deals only with the vertical electric field, the equation for horizontal magnetic field is presented for completeness.

\[ E_z(f) = \frac{i[\nu(\nu + 1)] I(f) dS P_{\nu}^0(- \cos \theta)}{4ha^2 \epsilon_0 \sin \pi \nu} \text{ [volts/meter]} \] (5.2)

\[ H_{\phi}(f) = -\frac{I(f) dS P_{\nu}^1(- \cos \theta)}{4ha \sin \pi \nu} \text{ [amps/meter]} \] (5.3)

In the above equations, the variables \( a \) and \( h \) correspond to the radius of the Earth (6.371 Mm) and the height of the ionosphere (∼80 km) respectively. \( \theta \) corresponds to the great circle distance between the source and receiver, and is calculated by dividing distance over \( a \). \( I(f) dS \) is the current moment of the source, where \( I \) is in amperes and \( dS \) is the vertical distance in meters that the current flows. \( P_{\nu}^{0,1} \)
are complex Legendre functions. The numerical computation of complex Legendre functions is discussed in Appendix A, where frequency and time domain simulations of electric lightning transients are presented. The actual simulation process, based on Equation 5.2, is discussed in Chapter 6.

5.2.1 The Complex Eigenvalue: \( \nu \)

The variable \( \nu \) (Greek “nu”) is a function of frequency, and corresponds to the complex eigenvalue which describes the dispersive and dissipative characteristics of the Earth-ionospher waveguide, as they vary over frequency. It is most logical to express \( \nu \) as a function of real physical values: attenuation (\( \alpha \)) and phase velocity (\( C/V \)).

\[
\nu(f)(\nu(f) + 1) = (k\alpha S)^2
\]

\[
S = \frac{C}{V} - i(5.49 \frac{\alpha}{f})
\]  

where \( k \) is the wavenumber, \( S \) is the sine of the mode eigenangle, \( C \) is the velocity of light, \( V \) is the phase velocity of propagation, \( \alpha \) is the attenuation constant, and \( f = \omega/2\pi \) is the frequency [Jones, 1967].

Furthermore, it is notable that there are two complex roots to the quadratic equations expressed by Equation 5.4. The root that corresponds to the desired physical value of \( \nu \) is the one which yields a positive real part, and a negative imaginary part. This complex root is given by

\[
\nu(f) = \frac{-1 + \sqrt{1 + 4(k\alpha S)^2}}{2}
\]  

The quantities \( C/V \) and \( \alpha \) are derived from Jones’ numerical model that is based on a measured ionospheric profile ([Jones, 1967], [Ishaq and Jones, 1977]). The following equations are only good for ELF frequencies below 100 Hz.

\[
\frac{C}{V} = 1.64 - 0.1759 \ln f + 0.01791 \ln f^2
\]  

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5.3 A Wideband Specification for Complex Nu

A wideband (3 Hz to 24 kHz) specification for \( \nu(f) \), to be used with the normal mode equations, was created using numerical fits to experimental data for attenuation and phase velocity. The model of Ishaq and Jones (Eqns. 5.7, 5.8), which correlates nicely to measured data ([Mushtak and Williams, 2002 (in press)], was used up to 90 Hz. The rest of the data came from Chapman [Chapman et al., 1996, figs. 3-4], and also happens to match nicely with figures from Galejs [Galejs, 1972, figs. 7.12-7.13]. The values used for \( \nu \) were calculated according to Equations 5.4 through 5.6. Polynomial fits were performed in the log domain, in order to create better fitting over the entire frequency range. Additionally, the fits were scaled and shifted by Matlab in order to improved the fitting. These factors make the closed form equations a bit messy.

5.3.1 Attenuation Fit

The equation for the twelfth-order attenuation fit is given by

\[
\alpha(f) = 10^{a_{12} \hat{f}^{12} + a_{11} \hat{f}^{11} + a_{10} \hat{f}^{10} + a_9 \hat{f}^9 + a_8 \hat{f}^8 + a_7 \hat{f}^7 + a_6 \hat{f}^6 + a_5 \hat{f}^5 + a_4 \hat{f}^4 + a_3 \hat{f}^3 + a_2 \hat{f}^2 + a_1 \hat{f} + a_0} \tag{5.9}
\]

where

\[
\hat{f} = \log_{10} \frac{f - \mu_{a_1}}{\mu_{a_2}}
\]
and

\begin{align*}
a_{12} &= 0.083990 \\
a_{11} &= 0.183969 \\
a_{10} &= -0.738223 \\
a_9 &= -1.553159 \\
a_8 &= 2.341272 \\
a_7 &= 4.911758 \\
a_6 &= -2.910078 \\
a_5 &= -6.846465 \\
a_4 &= 0.612539 \\
a_3 &= 3.477203 \\
a_2 &= 0.537105 \\
a_1 &= 0.540817 \\
a_0 &= 0.203966 \\
\mu_{a_1} &= 2.2598 \\
\mu_{a_2} &= 1.3160
\end{align*}

\section*{5.3.2 Phase Velocity Fit}

The equation for the ninth-order phase velocity fit is given by

\[
\frac{V}{C}(\hat{f}) = b_9 \hat{f}^9 + b_8 \hat{f}^8 + b_7 \hat{f}^7 + b_6 \hat{f}^6 + b_5 \hat{f}^5 + b_4 \hat{f}^4 + b_3 \hat{f}^3 + b_2 \hat{f}^2 + b_1 \hat{f} + b_0 \tag{5.10}
\]

where

\[
\hat{f} = \log_{10} \frac{f - \mu_{b_1}}{\mu_{b_2}}
\]
\begin{align*}
b_9 &= -0.0050278 \\
b_8 &= -0.0109618 \\
b_7 &= 0.0245402 \\
b_6 &= 0.0427126 \\
b_5 &= -0.0458421 \\
b_4 &= -0.0305358 \\
b_3 &= 0.0755177 \\
b_2 &= -0.0137582 \\
b_1 &= 0.0527641 \\
b_0 &= 0.8161356 \\
\mu_{b_1} &= 1.7519 \\
\mu_{b_2} &= 1.1181
\end{align*}

Beginning at 4000 Hz (after the peak of the phase velocity fit), an exponential decay was inserted in order to properly model the asymptotic behavior. This exponential equation was fit manually, not via analytical least-squared-error methods. It is expressed as

\[
\frac{V'}{C} (f) = \left. \frac{V}{C} (f) \right|_{f=0} + \left( \frac{V}{C} (f = \text{start}) - 1 \right) e^{-\frac{f - \text{start}}{\tau}} + 1 \quad (5.11)
\]

where

\[
\text{start} = 4000 \text{ Hz}, \quad \tau = 1500 \text{ Hz}
\]
Figure 5-3: Fits for attenuation and phase velocity, versus frequency.

Figure 5-4: Real and imaginary values of wideband $\nu$. Note that $\nu$ is unitless, and the imaginary component is negative.
Chapter 6

Normal Mode Simulations

This chapter will detail the procedures involved in the simulation of wideband spectra and time waveforms for vertical electric fields at various source-receiver distances.

6.1 Calculation of Simulated Spectra

Using the normal mode equation for electric field (Eqn. 5.2) and the wideband specification for $\nu(f)$ given in Chapter 5, theoretical spectra were derived for source-receiver distances between 0.1 and 20 Mm. Plots of the results are shown in Appendix A, for two different forms of source current (see Sec. 6.5).

Similar but more narrowband simulations have been carried out by Wong [Wong, 1996] and Nickolaenko [Nickolaenko and Hayakawa, 1998]. Nickolaenko’s simulations do not show slow tails; he claims that additional energy (above a flat impulse spectrum) at $\sim$400 Hz is needed to overcome waveguide attenuation. Nickolaenko does not discuss the effect of phase velocity on the generation of slow tails. Other wideband calculations based on the normal mode equations were also carried out in the early days of ELF research [Johler and Berry, 1964], though their results did not reveal SR in the ELF range.
6.2 Legendre Function Evaluation

The values for the complex Legendre functions over frequency were calculated for intervals of 1 Mm (as well as for values of 0.1, 0.5, and 0.75 Mm), and stored in a lookup table. This process can be very time consuming, depending on the desired frequency resolution. A table with 0.1 Hz intervals took nearly two days to compute on a 1 GHz Pentium III, while a table with 0.5 Hz intervals took only an hour and a half. This difference hints that the bottleneck was most likely due to RAM limitations rather than processor speed.

The Legendre Matlab script provided in Appendix B was adapted from Fortran code provided by Vadim Mushtak. This code approximates the Legendre terms of the normal mode equations, either by way of zonal harmonic summation or asymptotic series; the exact algorithm used depends on the combination of frequency and angular distance. See the following references for further discussion on methods of approximating Legendre functions, and their relationship to SR: [Jones, 1970], [Jones and Burke, 1990], [Huang, 1998][chap. 2.2].

6.3 From Frequency to Time Domain

Using the IDFT, it is a simple task to transform the simulated spectra into simulated event waveforms. However, before inverse transforming, Hermitian symmetry\footnote{Hermitian symmetry is discussed in Chapter 4.7.1.} must be guaranteed in order to yield a purely real time signal. This is accomplished by taking the positive frequency $E_Z(f)$ coefficients and mirroring them across $f = 0$ to reconstruct complex conjugate negative frequencies.

There is much empirical evidence that the complex normal mode equations are naturally Hermitian symmetric. If $\nu$ is assumed to be Hermitian, it follows that the normal mode equations are Hermitian symmetric as well, since the normal mode equations are functions of $\nu$. Furthermore, it would not make physical sense to generate a complex time waveform, since real-world sferics are not complex.
6.4 Summary

The general procedure used for making these simulation plots is as follows:

I. Calculate the complex spectra $E_z(f)$ in units of $[V/m/Hz]$, using Equation 5.2 for each discrete frequency component.

II. Reconstruct complex conjugate (Hermitian) symmetry about $\omega_k = 0$. This will effectively double the length of the frequency vector. The Matlab expression for this is:

$$Ezf = [Ezf fliplr(conj(Ezf(2:length(Ezf)-1)))] ;$$

III. To get the electric field time series, inverse Fourier transform the DFT coefficients from Step II and take the real part. The resulting waveform should be scaled by the sample rate, in order to convert the units from $[V/m/Hz]$ to $[V/m]$.

6.5 Effect of Source Spectrum

6.5.1 Impulse Current Moment

The first set of simulations in Appendix A show the result of an impulsive current moment source, with a charge moment of 100 C-km. An impulse results in a source spectrum that perfectly white (flat over frequency). Therefore these simulated spectra are equivalent to the transfer functions of the atmosphere, scaled by the impulse charge moment $(QdS = \int I(f)dS = 100 \text{ C-km} = 4800 \text{ kA-km} / 48 \text{ kHz})$.

6.5.2 Exponential Current Moment

Actual lightning sources are not always well modelled by an impulse. Many sources instead have a continuing current, which can be approximated by an exponentially
decaying current moment [Sentman, 1996]. This exponential decay is of the form

\[ I(t) dS = I_0 dS e^{-\frac{t}{\tau}} \]  

(6.1)

where \( I_0 \) is in amperes, \( dS \) is in meters, and \( \tau \) is in seconds.

The spectrum of this exponential is given by [Huang, 1998] as

\[ I(f) dS = \frac{I_0 dS \tau}{2\pi f \tau j + 1} \]  

(6.2)

\( \tau = 1 \text{ ms} \) is a typical time scale for the continuing currents following positive cloud-to-ground (CG) flashes [Huang et al., 1999]. A value of 100 kA-km was chosen for \( I_0 dS \), in order to guarantee the same charge moment as that of the simulated impulse (100 C-km). Analysis of Equation 6.2 shows that as \( \tau \) approaches zero, \( I(f) dS \) approaches the charge moment \( Q dS \).

The waveform and spectrum of an exponential current moment, similar to the one used in the simulations, is shown in Figure 6-1. The spectrum has the shape of a lowpass filter. The frequency breakpoint occurs at \( f = \frac{1}{2\pi \tau} = 160 \text{ Hz} \), which is in the slow tail region.

Since the impulse simulations are transfer functions of the atmosphere, it follows that any source spectra can be imposed on the impulsive simulations by frequency domain multiplication of the impulsive spectra with the source spectra. Therefore the simulations resulting from this exponential current moment are simply the lowpass filtered versions of the impulsive simulations.

6.6 Polarity Issues

The polarity of the current moment does not have an effect on the source spectrum. Identical simulations will result for either polarity. The convention is that the first peak of the sferic (as well as the slow tail peak) reveals the polarity of the source charge moment. According to this convention, the simulation plots presented in Appendix A should come from negative charge moments.
Figure 6-1: Time waveform and spectrum of an exponentially-decaying current moment, with $\tau=1$ ms and $I_0dS=1$ kA-km. The spectrum is flat up until 160 Hz ($f = \frac{1}{2\pi\tau}$), at which point it begins to decrease strongly with frequency.
Chapter 7

Processing Methods for Lightning Transients

7.1 Sampling and Thresholding

The $E_Z$ signal from the antenna amplifier is 8x oversampled\(^1\) at 192 kHz, and then downsampled to 48 kHz by throwing out every eighth sample. This allows for a frequency reconstruction up to 24 kHz, more than sufficient to cover the sferic bandwidth. The gain of the post-amplifier is set to 2, in order to maximize sampling headroom while preventing clipping.

The $E_Z$ signal is obscured by power line harmonics that are approximately 60 dB greater in amplitude (see Fig. 3-1). Because of this, a comb filter is used as the foundation of the thresholding procedure. The comb filter is well suited for this realtime application due to its computational efficiency.

A continuous buffer system is used, which writes data to disk when a software trigger is generated. The amount of pre- and post-trigger samples are hardcoded in the software, though this could be defined through a user interface. Only raw data is captured to disk, leaving the actual filtering for more intricate offline (non-realtime) methods.

Events were initially thresholded by sferic amplitude, which resulted in capturing

\(^1\)(relative to the reciever high frequency limit of 24 kHz)
many nearby events, as the sferic of a nearby event is large. In order to capture only
events with strong ELF energy, the events should be thresholded by ELF amplitude.
This can be accomplished by triggering when the magnitude of an ELF envelope
exceeds a user defined constant. The ELF envelope can be obtained by taking the
absolute value of a lowpass filtered version of $E_Z(t_n)$. This sort of ELF filtering was
not implemented, due to major software problems with file saving under Windows
98. Instead, a last-resort manual trigger was used, based on a low bandwidth (3-100
Hz) visual observation of $H_{NS}$ on an oscilloscope.

## 7.2 Noise Removal

The noise removal is done offline, using Matlab. The power line noise is problematic in
that it does not have a perfectly regular harmonic structure, nor does its amplitude or
frequencies remain perfectly constant over time. This is most likely due to nonlinear
distortion occurring along the route from generator to atmospheric transmission. For
visual inspection, it is sufficient to remove harmonics at 60, 180, 300, and 540 Hz,
with respective 3-dB bandwidths of 5, 3, 3, and 2 Hz. It is also helpful to remove RF
noise at 23.99kHz with a bandwidth of 400 Hz, as this high frequency energy gives
the signal a jagged appearance. The source of this energy is either from a high-order
VLF mode or more likely a radio transmitter of undetermined source. Figure 7-1
shows a waveform before and after the filtering process.

The actual filter used for the noise removal is the IIR notch (Sec. 4.4), using the
forced steady state technique (Sec. 4.5). See Section 4.2.1 for details about why this
filter was chosen.

### 7.2.1 Filter Ringing

There are some cases in which the filtering still leaves residual power line noise.
This happens when the waveform has large bipolar excursions over a long time scale
(relative to 60 Hz). This discontinuity causes the 60 Hz filter to ring. An example
case is shown in Figure 7-2. The only apparent solution to this problem is to increase
Figure 7-1: A sampled waveform before and after filtering. The signal was filtered at 60, 180, 300, 540, and 23.99k Hz.
Figure 7-2: A sampled waveform demonstrating a large discontinuity which causes ringing. The top waveform is unfiltered. The middle waveform is filtered according to the general method given ($\Delta f = 3$ Hz). The bottom waveform is filtered with an increased 60 Hz notch bandwidth ($\Delta f = 15$ Hz). This event is analyzed in Chapter 8.

the bandwidth of the offending filter, which reduces the steady state time constant (see Chap. 4.5).

### 7.3 Separation Time Extraction

An algorithm was developed to extract the separation time of events, defined as the time between the peak arrival of VLF sferic energy and the peak of ELF slow tail energy [Wait, 1960]. The event waveform is linear-phase (phase accurate) filtered in these bands, and enveloped by taking the absolute value. By lowpass filtering the VLF waveform, an interpolated envelope is formed which gives a more reliable peak
The procedure is outlined as follows:

i. Extract the ELF envelope by linear-phase lowpass filtering the event at 1.5 kHz and take the absolute value.

ii. Extract the VLF envelope by linear-phase bandpass filtering the event between 20 and 23.9 kHz. Lowpass interpolate (ELF filter is suitable) and take the absolute value.

iii. Subtract the sample indices corresponding of the maximum of each envelope, to obtain the separation time in samples.

iv. Convert samples to milliseconds by scaling by $1000/samplerate$.

Since slow tail energy exists as ELF entirely below the waveguide cutoff, 1.5 kHz was chosen for the ELF envelope filter cutoff. The region between 20 and 23.9 kHz was chosen for the VLF envelope because this is the area where VLF energy is at a maximum in the simulated spectra. The peak of the VLF envelope corresponds very closely with the maximum peak of the sferic waveform.

### 7.4 Estimation of Source-Receiver Distance

#### 7.4.1 Slow Tail Separation Time

Watson-Watt, Herd, and Lutkin [Watson Watt et al., 1937] were the first to notice that the separation time between ELF and VLF increased with distance. Wait developed his own formula for slow tail separation, based on an exponential pulse current source and measurements of ionospheric conductivity [Wait, 1960]. This formula, known as Wait’s law, is given by

$$\sqrt{t_{sep}} = A + B\rho$$  \hspace{1cm} (7.1)

where $A$ is related to the source pulse width, and $B$ depends on ionospheric parameters. $\rho$ is distance in Mm. This formula was shown to match measured data for
Figure 7-3: ELF and VLF envelopes of a sferic event with a positive slow tail. The estimated distance is between 5 and 6 Mm.
distances less than 5 Mm [Wait, 1960]. However, when compared with the normal mode simulations, Wait’s law only matches for distances greater than 8 Mm (see Fig. 7-5).

A new formula for source-receiver distance estimation by separation time was created via analysis of simulated events (see Appendix A). The simulated separation times and a quadratic fit are shown in Figure 7-4.

By solving the quadratic, a formula for distance estimation is found:

$$\text{distance} = \frac{-B + \sqrt{B^2 - 4A(C - t_{sep})}}{2A}$$  \hspace{1cm} (7.2)

where distance is given in Mm and $t_{sep}$ is given in milliseconds.

The coefficients $A$, $B$ and $C$ depend on which current moment source is used in the model. For an impulse (flat-spectrum), the coefficients are

$$A = 7.69 \cdot 10^{-3}, \quad B = 4.87 \cdot 10^{-1}, \quad C = -4.61 \cdot 10^{-1}$$

For a continuing current (exponential decay), the coefficients are

$$A = 6.81 \cdot 10^{-3}, \quad B = 5.49 \cdot 10^{-1}, \quad C = -1.08 \cdot 10^{-1}$$

This formula is only suitable for distances between 1 and 18 Mm. Below 1 Mm, the separation method resulted in a negative separation time. Above 18 Mm, the antipodal wave overlaps and can obscure accurate separation time extraction.

### 7.5 Spectral Ratios

Two other estimation formulas were derived from the normal mode simulations, based on spectral ratios (see Fig. 7-6). These formulas come from the solution of the log-linear fits. The formulas come from the exponential simulation data, since these simulations resemble observed events more than the impulse simulations.
Figure 7-4: Graph of separation time between sferic (VLF) and slow tail (ELF) arrival, with quadratic fits. (It was problematic to plot both quadratics on one figure, however the impulse quadratic fits just as well as the continuing current quadratic.)
Figure 7-5: Graph of square-root of separation time vs. distance. According to Wait’s law [1960], this plot should be linear at distances less than 5 Mm. On the contrary, it is approximately linear at distances greater than 8 Mm.
7.5.1 VLF Ratio

A distance estimator is given based on the ratio of VLF and SR maxima:

\[
\text{distance [Mm]} = \frac{\log_{10}\left(\frac{\text{VLF}_{\text{max}}}{\text{SR}_{\text{max}}}ight) - 0.30}{-0.0781}
\]  

(7.3)

7.5.2 SR ratio

A distance estimator is given based on the ratio of SR maximum and waveguide cutoff minimum:

\[
\text{distance [Mm]} = \frac{\log_{10}\left(\frac{\text{SR}_{\text{max}}}{\text{CUT}_{\text{min}}}ight) + 0.3985}{0.732}
\]  

(7.4)

The consistency of the estimators given in this chapter, when applied to real events, will be evaluated in Chapter 8.
Figure 7-6: Summary of simulation measurements for an exponentially decaying source ($\tau=1$ ms). Slopes shown were calculated by a linear fit of log values.
Chapter 8

Wideband Characterization of Specific Lightning Transients

8.1 SNR

Some level of energy is always bouncing around inside the waveguide, due to the low level of attenuation at SR and sferic VLF. In order to show that measurements of a transient are accurate, it must be shown that the transient signal is well above the noise floor of the background energy. This is accomplished by performing a signal-to-noise ratio (SNR), which is a ratio of electric amplitude spectra between the transient and the background. The SNR level is especially important in the SR region; it is an indicator that the signal length is long enough to resolve SR frequencies. Signals that have a stable SR signal should have an SNR that is much greater than one, throughout the SR region.

Since the signal and noise waveform selections are usually not the same length, it is useful to zero-pad them so that they match in length. This way the FFTs are the same length and can be easily ratioed.
8.1.1 Fingerprinting

If the SNR shows that SR energy is unique to the transient, then the SR can be “fingerprinted” against known SR structure from the simulations. It is theoretically possible to also do VLF fingerprinting, however the normal mode equation does not provide the details of VLF modes.

8.2 Analysis of Sampled Events

The following section will examine the characteristics of three transient waveforms. They will be analyzed in the time and frequency domain, yielding a comparison of various methods for source-receiver distance estimation.

8.2.1 A Nearby Event

An positive polarity event is presented in Figure 8-1. In the time domain, this is distinguishable as a nearby (less than 1 Mm) event because of the very large sferic amplitude and the lack of a well-defined\(^1\) slow tail. The distance estimated via the slow tail separation (0.5 Mm) matches with this visual observation, as does the SR fingerprint. It is puzzling that the spectrum shows a distinguished waveguide cut-off (centered around 2.5 kHz), which should be the case for a more distant event (SR/CUT ratio yields 4 Mm). The VLF/SR ratio estimate is totally out of the ballpark at 19 Mm.

8.2.2 A Slow Tail Event

A positive polarity event with a slow tail is presented in Figure 8-2. Negative slow tail events are commonly found as well, but this particular event was chosen for its well developed SR structure. The slow tail is clearly evident in the time domain, which gives a visual estimate of approximately 5-9 Mm. The separation estimate

\(^1\)The ELF energy is nearly coincident with the sferic peak, making it indistinguishable in the time scale shown in Figure 8-1. The smaller ELF ripple seen after the sferic should not be confused with a negative slow tail peak.
Figure 8-1: Shown above is a nearby positive waveform and spectra, with distance estimators. Below is the SNR, and SR fingerprint comparison to a simulated event at 0.5 Mm distance with an exponentially-decaying current moment.
(5.3 Mm) concurs with this range. The SR fingerprint matches closely to that of an 8 Mm event, which is strengthened by the observation that the sferic amplitude is smaller than that of the slow tail. The SR/CUT ratio estimate (5.7 Mm) is in general agreement with the other observations, however the VLF/SR ratio (31 Mm) is again way off. Note that the SNR is moderately strong at ELF; it is greater than unity throughout the SR region, which is a good indicator that this event created its own SR wave after the slow tail. The approximate distance of this event suggests that it may have originated in Africa.

8.2.3 A Negative Event

A negative polarity event is presented in Figure 8-3. Visual observation suggests that this is a nearby event, due to the strong sferic and absence of slow tail. The separation estimate (0.61 Mm) supports this conclusion. It is puzzling that the SR fingerprint matches with 14 Mm. Due to the relatively small SNR values at ELF, it is likely that the fingerprint is actually from a previous event. Again there are discrepancies with the spectral ratio estimates; yielding 15 and 5.3 Mm for the VLF/SR and SR/CUT ratios, respectively.

8.3 Estimation Summary and Discussion

The visual observation of the time domain signal generally concurs with the slow tail separation estimate. The SR fingerprint method seems to work well when the SNR is strong at ELF; when the SNR is greater than ten at ELF, it means that the SR modes are unique for a given transient. For events with large slow tails, the SR/CUT ratio gives similar estimates that are similar the slow tail separation estimates.

\footnote{The ELF signal used for visually triggering events was from the N-S magnetometer coil in RI, which is responsive to African events.}
Figure 8-2: Shown above is a positive slow tail waveform and spectra, with distance estimators. Below is the SNR, and SR fingerprint comparison to a simulated event at 8 Mm distance with an exponentially-decaying current moment.
Figure 8-3: Shown above is a negative polarity waveform and spectra, with distance estimators. Below is the SNR, and SR fingerprint comparison to a simulated event at 0.5 Mm distance with an exponentially-decaying current moment.
Chapter 9

Conclusions and Future Work

9.1 Summary

9.1.1 Noise Removal

DSP methods have been developed and applied to the removal of anthropogenic noise, namely 60 Hz power line noise. This has enabled the identification of the SR modes, the ELF slow tail, the waveguide cutoff, and the VLF “sferics” region—all in a single measurement of a lightning transient.

9.1.2 Evidence for the source of slow tails

Some researchers have claimed that a continuing current with a time scale of 1 ms will enhance the generation of slow tails [Reising et al., 1996]. While an exponential continuing current enhances the slow tail energy relative to VLF, making it more apparent, this study has shown that it is not the fundamental cause for the slow tail.

The impulsive simulations clearly demonstrate that it is not the source spectra which causes slow tail, but rather the waveguide dispersion and the depletion of spectral energy in the cutoff region.

Some researchers have claimed that slow tails would result from a source spectrum with large slow tail energy relative to SR energy [Nickolaenko and Hayakawa, 1998]. The exponential currents used to simulate continuing current in the normal mode
simulations do exactly the opposite; the exponential has a red spectrum (decreasing with frequency) compared to the white (flat with frequency) spectrum of the impulse.

9.1.3 Wideband Distance Estimation

Formulas for distance estimation have been developed from the simulations, based on slow tail separation time and spectral ratios. Preliminary evaluation of these methods suggests that slow tail separation time is a good method for distance estimation. The performance of this separation estimator will be enhanced if even more realistic waveguide parameters are included in the model.

The spectral ratio estimators do not perform as well as was hoped, due to differences between the simulations and actual events. The SR/CUT ratio estimator shows some concurrence with the slow tail separation method; however its accuracy may be compromised by the presence of line noise harmonics in the cutoff, where less attention has been given to their removal.

The VLF/SR ratio method results in highly erroneous distance estimates. The huge discrepancies between real and simulated events suggest that the source spectrum of many events are not well modelled by a continuing current with $\tau = 1$ ms. In order to make the spectrum redder, the time constant should be increased.

9.2 Future Work

9.2.1 Higher Order Modes

A major problem is that the normal mode equations do not generate higher modes than the TEM mode. Rafalsky gives a method which estimates source-observer distance by counting modal oscillations in the spectrum that occur between harmonics of waveguide cutoff. The number of oscillations in each pattern indicates the distance, measured in wavelengths of the cutoff frequency [Rafalsky et al., 1995]. This method was not applicable to spectra recorded at the RI station, as these harmonic cutoffs are not immediately apparent.
However, the recorded spectra does appear to match the simulated modal calculations done by Cummer [Cummer, 2000]. For instance, Figure 9-1 shows the presence of VLF modes, where the normal mode simulations would only show a smooth curve. Cummer calculated a modal residue series based on equations given by Wait [Wait, 1996]. Since the normal mode equations do not yield realistic VLF structure, it seems that Cummer’s model would be more appropriate for wideband simulations.

### 9.2.2 Improved Distance Information

Actual distance information from the NLDN would be helpful in testing if the slow tail separation time of the simulations match that of real data. The acquisition of NLDN data would be simplified if a global positioning system (GPS) were included in the sampling system. However, the current GPS system is out of service, as it has been struck by lightning twice in the past few months. Also, it would be ideal if the magnetic channels ($H_{NS}, H_{EW}$) were sampled at high bandwidth simultaneously with the $E_Z$ channel. This would allow for better distance estimation via the wave impedance method [Huang et al., 1999].
Figure 9-2: Shown is a spectrogram of a 15 Mm continuing current simulated event. Note the “grainy”, undefined appearance of the low frequencies. Also apparent are vertical artifacts from FFT sidelobes. However, the expected progression over time from sferic to slow tail to SR is still apparent even with this poor frequency resolution.

9.2.3 Time-Frequency Analysis

In the analysis of wideband events that are changing over time, it would be insightful to observe exactly how each frequency changes over time. This is impossible to see with a single FFT over a large time window; such a large FFT can only reveal interference patterns. A spectrogram is most commonly made using the Short Time Fourier Transform (STFT), which is essentially the result of many overlapping DFT windows. When using the STFT, there is a tradeoff between frequency and time resolution [Oppenheim and Shafer, 1989]. As a result, a spectrum with detailed time resolution will have poor frequency resolution and look extremely blurry in the ELF region (see Fig 9-2). The use of the Continuous Wavelet Transform (CWT) is recommended as a possible solution to this resolution problem as the CWT divides a signal into logarithmic frequency bands, compared to the linear bin spacing of the Fourier Transform.
9.2.4 More Accurate Noise Removal

The noise removal used in this thesis would be improved if the power line harmonic frequencies could be more accurately identified than by FFT bin frequency. The fact that the sample rate is a multiple of 60 Hz is of no consequence, since it turns out the harmonics do not occur at integer frequencies. There is a DSP method to get a precise estimate of frequency, even when harmonics are not coincident with FFT bins. This is done by analyzing the modulo periodicity of the FFT phase [Sprenger, 1999]. There was not enough time to implement this method as it was discovered late in the project. As it is, the currently used method of noise removal is sufficient for time domain analysis.

Another possible method of noise removal, which would be immune to the problem of filter ringing, is to subtract an amplitude and phase matched sine wave of the undesired frequency. This method was attempted, but the frequency estimates were not accurate enough to give better results than the IIR notch filter.

9.2.5 Phase Equalization and Wideband Extraction of the Current Moment of Lightning

As was discussed in Section 4.6, allpass phase equalization will be necessary if current moment extraction is to be performed using the normal mode equations and the measured electric field. The design of the equalizer requires an accurate measurement of the phase response of the antenna and analog hardware.
Bibliography


Appendix A

Normal Mode Simulation Plots

The following pages show simulated $E_z$ spectra and time waveforms, calculated using the normal mode equation for electric field (Eqn. 5.2), as well as the extended model for wideband $\nu(f)$. The height of the ionosphere is approximated as 80 km.

As the distance increases, one can see the effect of dispersion between sferic and slow tail energy. Another distinct feature is the antipodal “around the world” wave, due to waves from the same event which travel the longer path around the Earth. The relative magnitude of this antipodal wave grows as the event source becomes more distant.

The first section shows simulations for an impulsive current moment. The second section shows simulations for an exponentially decaying current moment. The relative charge transferred in both simulations is the same, just delivered over a different time scale.

A.1 Impulsive Current Moment

This section shows simulations for an impulsive current moment of 48 kA-km. This results in a flat source spectrum of 100 C-km. The simulation methods are given in Chapter 6.
Calculated $E_z(f)$ spectrum @ 0.1 Mm
$SR_{\text{max}} = 0.000477, \text{CUT}_{\text{min}} = 0.00134, VLF_{\text{max}} = 0.0063$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 137 V/m)
Calculated $E_z(f)$ spectrum @ 0.5 Mm

$SR_{max} = 0.00023$, $CUT_{min} = 0.00034$, $VLF_{max} = 0.0023$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 61.6 V/m)
Calculated $E_z(f)$ spectrum @ 0.75 Mm

$SR_{\text{max}} = 0.000184$, $CUT_{\text{min}} = 0.000182$, $VLF_{\text{max}} = 0.0017$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 48.1 V/m)
Calculated $E_z(f)$ spectrum @ 1.0 Mm
$SR_{\text{max}} = 0.000156$, $CU_{\text{min}} = 0.000103$, VLF $\text{max} = 0.0014 [V/m/Hz]$

Simulated $E_z(t)$ waveform (max @ 38.8 V/m)

$SR_{\text{max}} = 0.000156$, $CU_{\text{min}} = 0.000103$, VLF $\text{max} = 0.0014 [V/m/Hz]$
Calculated $E_z(f)$ spectrum @ 2.0 Mm

$SR_{\text{max}} = 9.63 \times 10^{-5}$, $CUT_{\text{min}} = 1.34 \times 10^{-5}$, $VLF_{\text{max}} = 0.00073 \, [V/m/Hz]$

Simulated $E_z(t)$ waveform (max @ 20.4 V/m)
Calculated $E_z(f)$ spectrum @ 3.0 Mm
$SR_{\text{max}} = 6.94 \times 10^{-5}$, $CUT_{\text{min}} = 2 \times 10^{-6}$, $VLF_{\text{max}} = 0.00048 \text{ [V/m/Hz]}$

Simulated $E_z(t)$ waveform (max @ 12.6 V/m)
Calculated $E_z(f)$ spectrum @ 4.0 Mm
$S_{R,max} = 5.63\times10^{-5}$, $CUT_{min} = 3.17\times10^{-7}$, $VLF_{max} = 0.0033$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 8.28 V/m)
Calculated $E_z(f)$ spectrum @ 5.0 Mm

$SR_{\text{max}} = 4.97 \times 10^{-5}$, $CUT_{\text{min}} = 5.23 \times 10^{-8}$, $VLF_{\text{max}} = 0.00025$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 5.65 V/m)
Calculated $E_z(f)$ spectrum @ 6.0 Mm

$SR_{max} = 4.23e-005$, $CUT_{min} = 8.84e-009$, $VLF_{max} = 0.00019$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 3.99 V/m)
Calculated $E_z(f)$ spectrum @ 7.0 Mm

SR$_{\text{max}}$ = 4.11e-005, CUT$_{\text{min}}$ = 1.52e-009, VLF$_{\text{max}}$ = 0.00014 [V/m/Hz]

Frequency [Hz]

Simulated $E_z(t)$ waveform (max @ 2.88 V/m)

Time [ms]
Calculated $E_z(f)$ spectrum @ 8.0 Mm

$SR_{max} = 3.65 \times 10^{-5}$, $CUT_{min} = 2.67 \times 10^{-10}$, $VLF_{max} = 0.00011$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 2.11 V/m)
Calculated $E_z(f)$ spectrum @ 9.0 Mm

$SR_{\text{max}} = 3.71 \times 10^{-05}$, \( \text{CUT}_{\text{min}} = 4.73 \times 10^{-11} \), \( VLF_{\text{max}} = 9.1 \times 10^{-05} \) [V/m/Hz]

Frequency [Hz]

Simulated $E_z(t)$ waveform (max @ 1.58 V/m)
Calculated $E_z(f)$ spectrum @ 10.0 Mm

$SR_{\text{max}} = 3.83 \times 10^{-5}$, $\text{CUT}_{\text{min}} = 8.5 \times 10^{-12}$, $VLF_{\text{max}} = 7.4 \times 10^{-5}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 1.19 V/m)
Calculated $E_z(t)$ spectrum @ 11.0 Mm

$SR_{\text{max}} = 3.58e-005$, $\text{CUT}_{\text{min}} = 1.55e-012$, $\text{VLF}_{\text{max}} = 6.1e-005$ [V/m/Hz]

---

Simulated $E_z(t)$ waveform (max @ 0.918 V/m)

---

SR = $3.58 \times 10^{-5}$, CUT = $1.55 \times 10^{-12}$, VLF = $6.1 \times 10^{-5}$ [V/m/Hz]
Calculated $E_z(t)$ spectrum @ 12.0 Mm

$SR_{max} = 3.33\times10^{-5}$, $CUT_{min} = 2.85\times10^{-13}$, $VLF_{max} = 5.1\times10^{-5} [V/m/Hz]$

Simulated $E_z(t)$ waveform (max @ 0.716 V/m)
Calculated $E_z(f)$ spectrum @ 13.0 Mm

$SR_{max} = 3.66e-005$, $CUT_{min} = 5.32e-014$, $VLF_{max} = 4.4e-005$ [V/m/Hz]

Frequency [Hz]

Simulated $E_z(t)$ waveform (max @ 0.567 V/m)
Calculated $E_z(f)$ spectrum at 14.0 Mm

$SR_{\text{max}} = 3.86 \times 10^{-5}$, $CUT_{\text{min}} = 1.01 \times 10^{-14}$, $VLF_{\text{max}} = 3.8 \times 10^{-5} \text{ [V/m/Hz]}$

Frequency [Hz]

Simulated $E_z(t)$ waveform (max @ 0.456 V/m)

Time [ms]
Calculated $E_z(t)$ spectrum @ 15.0 Mm

$SR_{\text{max}} = 4.53\times10^{-5}$, $CUT_{\text{min}} = 1.95\times10^{-15}$, $VLF_{\text{max}} = 3.4\times10^{-5}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.375 V/m)
$SR_{\max}^H = 5.17 \times 10^{-5}$, $CUT_{\min} = 3.87 \times 10^{-16}$, $VLH_{\max} = 3.3 \times 10^{-5} \text{ [V/m/Hz]}$

Calculated $E_z(f)$ spectrum @ 16.0 Mm

Simulated $E_z(t)$ waveform (max @ 0.316 V/m)
Calculated $E_z(f)$ spectrum @ 17.0 Mm
$S_{\text{R, max}} = 5.7 \times 10^{-5}$, $CUT_{\text{min}} = 7.95 \times 10^{-17}$, $VLF_{\text{max}} = 3.3 \times 10^{-5}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.277 V/m)
Calculated $E_z(f)$ spectrum @ 18.0 Mm

$SR_{\text{max}} = 6.14 \times 10^{-5}$, $CUT_{\text{min}} = 1.74 \times 10^{-17}$, $VLF_{\text{max}} = 3.6 \times 10^{-5}$ [V/m/Hz]

Frequency [Hz]

Simulated $E_z(t)$ waveform (max @ 0.259 V/m)
Calculated $E_z(t)$ spectrum @ 19.0 Mm

$SR_{\text{max}} = 7.2 \times 10^{-5}$, $CUT_{\text{min}} = 4.39 \times 10^{-18}$, $VLF_{\text{max}} = 4.7 \times 10^{-5}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.279 V/m)
Calculated $E_z(f)$ spectrum @ 20.0 Mm

$SR_{\text{max}} = 7.84 \times 10^{-5}$, $CUT_{\text{min}} = 1.06 \times 10^{-17}$, $VLF_{\text{max}} = 0.00035 \text{ [V/m/Hz]}$

Simulated $E_z(t)$ waveform (max @ 2.76 V/m)
Close-up of slow tail zone @ 0.1 Mm
Separation Time: -0.083 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 0.5 Mm
Separation Time: -0.021 ms

ELF waveform (lowpassed at 1.5 kHz)
Close-up of slow tail zone @ 0.75 Mm
Separation Time: 0.042 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 1.0 Mm
Separation Time: 0.104 ms
Close-up of slow tail zone @ 2.0 Mm
Separation Time: 0.521 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 3.0 Mm
Separation time: 1.042 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 4.0 Mm
Separation Time: 1.583 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 5.0 Mm
Separation Time: 2.146 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 6.0 Mm
Separation Time: 2.729 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 7.0 Mm
Separation Time: 3.313 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 8.0 Mm
Separation time: 3.938 ms

ELF waveform (lowpassed at 1.5 kHz)
Separation Time: 4.563 ms

Close-up of slow tail zone @ 9.0 Mm

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 10.0 Mm
Separation Time: 5.188 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone at 11.0 Mm
Separation Time: 5.833 ms

ELF waveform (lowpassed at 1.5 kHz)
Close-up of slow tail zone @ 12.0 Mm
Separation Time: 6.500 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 13.0 Mm
Separation Time: 7.188 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 14.0 Mm
Separation Time: 7.875 ms

ELF waveform (low passed at 1.5 kHz)
Close-up of slow tail zone @ 15.0 Mm
Separation Time: 8.583 ms

ELF waveform (lowpassed at 1.5 kHz)
Close-up of slow tail zone @ 16.0 Mm

Separation Time: 9.292 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 17.0 Mm
Separation Time: 10.042 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 18.0 Mm
Separation Time: 10.792 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone @ 19.0 Mm
Separation Time: 11.563 ms

ELF waveform (lowpassed at 1.5kHz)
Close-up of slow tail zone at 20.0 Mm
Separation Time: 11.042 ms

ELF waveform (lowpassed at 1.5 kHz)
Slope $\approx 0.716$ decade / Mm

Slope $\approx -7.45 \times 10^{-2}$ decades / Mm

Figure A-1: Summary of simulation measurements for an impulsive source. Slopes shown were calculated by a linear fit of log values.
A.2 Exponential Current Moment

This section shows simulations for an exponential current moment of the form

\[ IdS(t) = I_0 dSe^{-\frac{t}{\tau}} \]

where \( \tau \) is 1 ms, and \( I_0 \) is 100 kA-km. This results in a charge moment approximately equal to that of the impulse charge moment (100 C-km). The aim of the exponential is to simulate a continuing current. More discussion is given in Chapter 6.
Calculated $E_z(t)$ spectrum @ 0.1 Mm

$SR_{\text{max}} = 0.000409$, $CUT_{\text{min}} = 0.000127$, $VLF_{\text{max}} = 0.0001$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.249 V/m)
Calculated $E_z(t)$ spectrum @ 0.5 Mm

$SR_{\text{max}} = 0.000198$, $CUT_{\text{min}} = 2.91 \times 10^{-5}$, $VLF_{\text{max}} = 3.4 \times 10^{-5}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.153 V/m)
Calculated $E_z(t)$ spectrum @ 0.75 Mm

$SR_{\text{max}} = 0.000158$, $CUT_{\text{min}} = 1.56 \times 10^{-5}$, $VLF_{\text{max}} = 2.6 \times 10^{-5}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.198 V/m)
Calculated $E_z(t)$ spectrum @ 1.0 Mm

$SR_{\text{max}} = 0.000133$, $CUT_{\text{min}} = 8.87e-006$, $VLF_{\text{max}} = 2.1e-005$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.252 V/m)
Calculated $E_z(t)$ spectrum @ 2.0 Mm

$SR_{max} = 8.31e^{-005}$, $CUT_{min} = 1.16e^{-006}$, $VLF_{max} = 1.1e^{-005}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.187 V/m)
Calculated $E_z(t)$ spectrum @ 3.0 Mm,

$$\text{SR}_{\text{max}} = 6.26 \times 10^{-5}, \text{CUT}_{\text{min}} = 1.75 \times 10^{-7}, \text{VLF}_{\text{max}} = 7.6 \times 10^{-6} \text{[V/m/Hz]}$$

Simulated $E_z(t)$ waveform (max @ 0.117 V/m)
Calculated $E_z^r(f)$ spectrum @ 4.0 Mm

$SR_{\text{max}} = 5.68 \times 10^{-5}$, $CUT_{\text{min}} = 2.79 \times 10^{-8}$, $VLF_{\text{max}} = 5.4 \times 10^{-6}$ [V/m/Hz]

Simulated $E_z^r(t)$ waveform (max @ 0.0737 V/m)
Calculated $E_z(t)$ spectrum @ 5.0 Mm

$SR_{\text{max}} = 5.02 \times 10^{-5}$, $CUT_{\text{min}} = 4.61 \times 10^{-9}$, $VLF_{\text{max}} = 4 \times 10^{-6}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.0519 V/m)
Calculated $E_z(f)$ spectrum @ 6.0 Mm

$SR_{\text{max}} = 4.26 \times 10^{-5}$, $CUT_{\text{min}} = 7.8 \times 10^{-10}$, $VLF_{\text{max}} = 3 \times 10^{-6} \text{ [V/m/Hz]}$

Simulated $E_z(t)$ waveform (max @ 0.0386 V/m)
Calculated $E_z(f)$ spectrum @ 7.0 Mm

$SR_{\text{max}} = 4.12e-005$, $CUT_{\text{min}} = 1.35e-010$, $VLF_{\text{max}} = 2.3e-006$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.0286 V/m)
Calculated $E_z(f)$ spectrum @ 8.0 Mm

$SR_{\text{max}} = 3.66 \times 10^{-5}$, $CUT_{\text{min}} = 2.36 \times 10^{-11}$, $VLF_{\text{max}} = 1.8 \times 10^{-6}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.0214 V/m)

Frequency [Hz]

Simulated $E_z(t)$ waveform (max @ 0.0214 V/m)

Time [ms]
Calculated $E_z(f)$ spectrum @ 9.0 Mm

$SR_{\text{max}} = 3.73 \times 10^{-5}$, $CUT_{\text{min}} = 4.18 \times 10^{-12}$, $VLF_{\text{max}} = 1.5 \times 10^{-6}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.0163 V/m)
Calculated $E_z(f)$ spectrum @ 10.0 Mm

$SR_{\text{max}} = 3.86 \times 10^{-5}$, $CUT_{\text{min}} = 7.52 \times 10^{-13}$, $VLF_{\text{max}} = 1.2 \times 10^{-6}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.0127 V/m)
Calculated $E_z(f)$ spectrum @ 11.0 Mm
$S_{R_{\text{max}}} = 3.6 \times 10^{-5}$, $CUT_{\text{min}} = 1.37 \times 10^{-13}$, $VLF_{\text{max}} = 9.9 \times 10^{-7}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.01 V/m)
Calculated $E_z(t)$ spectrum @ 12.0 Mm

$SR_{\text{max}} = 3.34e-005$, $CUT_{\text{min}} = 2.52e-014$, $VLF_{\text{max}} = 8.3e-007$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.008 V/m)
Calculated $E_z(f)$ spectrum @ 13.0 Mm

$SR_{\text{max}} = 3.67 \times 10^{-5}$, $CUT_{\text{min}} = 4.71 \times 10^{-15}$, VLF $\text{max} = 7.1 \times 10^{-7}$ [V/m/Hz]

Frequency [Hz]

Voltage [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.00652 V/m)
Calculated $E_z(f)$ spectrum @ 14.0 Mm

$SR_{\text{max}} = 3.89 \times 10^{-5}$, $CUT_{\text{min}} = 8.93 \times 10^{-16}$, $VLF_{\text{max}} = 6.2 \times 10^{-7}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.00541 V/m)
Calculated $E_z(f)$ spectrum @ 15.0 Mm

$SR_{\text{max}} = 4.57 \times 10^{-5}$, $CUT_{\text{min}} = 1.73 \times 10^{-16}$, $VLF_{\text{max}} = 5.6 \times 10^{-7}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.00459 V/m)
Calculated $E_z(f)$ spectrum @ 16.0 Mm

$SR_{\text{max}} = 5.21 \times 10^{-5}$, $CUT_{\text{min}} = 3.42 \times 10^{-17}$, VLF $\text{max} = 5.3 \times 10^{-7}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.00401 V/m)
Calculated $E_z(f)$ spectrum @ 17.0 Mm  
$SR_{\text{max}} = 5.76 \times 10^{-5}$, $\text{CUT}_{\text{min}} = 7.03 \times 10^{-18}$, $\text{VLF}_{\text{max}} = 5.3 \times 10^{-7} \text{ [V/m/Hz]}$  

Simulated $E_z(t)$ waveform (max @ 0.00365 V/m)
Calculated $E_z(f)$ spectrum @ 18.0 Mm

$SR_{\text{max}} = 6.19 \times 10^{-5}$, $CUT_{\text{min}} = 1.53 \times 10^{-18}$, $VLF_{\text{max}} = 5.9 \times 10^{-7} \text{ [V/m/Hz]}$

Simulated $E_z(t)$ waveform (max @ 0.00353 V/m)
Calculated $E_z(t)$ spectrum @ 19.0 Mm

$SR_{\text{max}} = 7.22 \times 10^{-5}$, $CUT_{\text{min}} = 3.77 \times 10^{-19}$, $VLF_{\text{max}} = 7.7 \times 10^{-7}$ [V/m/Hz]

Simulated $E_z(t)$ waveform (max @ 0.00389 V/m)
Calculated $E_z(f)$ spectrum @ 20.0 Mm
$SR_{\text{max}} = 7.82e^{-005}, CUT_{\text{min}} = 9.5e^{-019}, VLF_{\text{max}} = 5.4e^{-006} [\text{V/m/Hz}]$

Simulated $E_z(t)$ waveform (max @ 0.0431 V/m)
Figure A-2: Summary of simulation measurements for an exponentially decaying source. Slopes shown were calculated by a linear fit of log values.
Appendix B

Matlab Scripts

B.1 IIR Notch Filter with Forced Steady State

function out = notchic(in, freq, bw, samplerate, z); M
%usage: notchic(in, freq, bw, samplerate, z) M
% z is an optional phase index M
%Values in Hz. (Q = freq/bw) M
M
N = length(in); M
M
dw = 2*pi*bw/samplerate; M
M
w0 = 2 * pi * freq / samplerate; M
M
beta = 1 / (1 + tan(dw/2)); M
M
b = beta * [1 -2*cos(w0) 1]; M
M
a = [1 -2*beta*cos(w0) 2*beta-1]; M
M
%is there a better way to get the magnitude? M
FFTSIZE = N/2 - 1; M
M
mag = abs(fft(in)); %could zero-pad here to interpolate FFT to more detail? M
M
gain = mag(freq*length(in)/samplerate + 1); M
M
if (nargin == 4) ˆM
%z not provided, find the sine phase ˆM
nyq = samplerate/2; ˆM
order = 5000; %is this enough for very small bandwidths? ˆM
ˆM
low = (freq−1/2)/nyq; %assume frequency is spectral max within 1 Hz ˆM
if low<0 ˆM
    low = 0; ˆM
end ˆM
high = (freq+1/2)/nyq; ˆM
if high>1 ˆM
    high = 1; ˆM
end ˆM
d = fir1(order,[low high]); %BPF ˆM
c = conv(d,in); ˆM
c = c(order/2+1:length(in)+order/2); ˆM
z = findzero(c); %% ˆM
end ˆM
ˆM
total = round(log(.001)/log(beta)); %0.1 percent time constant ˆM
period = samplerate/freq; ˆM
k = 1:period; ˆM
s = gain*sin(w0*k − z*w0); ˆM
ql = 0; c = ceil(total/period); ˆM
for i=1:c ˆM
    ql = [ql s]; ˆM
end ˆM
ql = [ql in ’]; ˆM
ˆM
out = filter(b,a,ql); ˆM
out = out(length(out)−length(in) + 1:length(out)) ’; ˆM

B.1.1 Finding the Sine Phase Start

function [j] = findzero(in) ˆM
%(needed by notchic.m) ˆM
\[ \hat{M} \]
prev_sign = sign(in(1)); \hat{M}
j = 2; \hat{M}
while (j < length(in)) \hat{M}
    next_sign = sign(in(j)); \hat{M}
    if (next_sign > prev_sign) \hat{M}
        break; \% found sine phase start \hat{M}
    end \hat{M}
    prev_sign = next_sign; \hat{M}
    j = j + 1; \hat{M}
end \hat{M}
\hat{M}
\% linear interpolate to find zero-crossing \hat{M}
m = in(j) - in(j-1); \% get slope \hat{M}
j = j - in(j-1)/m; \% find x-intercept: x = -b/m \hat{M}

---

**B.2 Distance Estimation**

\textbf{function} [d1, d2, d3, d4] = est_distance(in)
\textit{\% estimates distance of an event, via methods}
\textit{\% of spectral ratio and separation time}
\textit{\% samplerate = 48e3;}

\textbf{N} = length(in);  
freq_per_bin = samplerate / N;  
h = abs(fft(in)) / samplerate; h = h(1:N/2);  
ff = 0:(N/2-1); ff = ff * freq_per_bin;  

bins_per_freq = 1/freq_per_bin;  
SRmax = max(h(max([bins_per_freq*3 2]):bins_per_freq*100));  
CUTmin = min(h(bins_per_freq*1e3:bins_per_freq*2e3));  
VLFmax = max(h(bins_per_freq*3e3:bins_per_freq*23.9e3));  

r1 = VLFmax/SRmax;  
d1 = (log10(r1) - 0.30) / -0.0781;
r2 = SRmax/CUTmin;
d2 = (log10(r2) + 0.3985) / 0.732;

ms = separate(in); %separation time

a = 7.69e-3; b = 4.87e-1; c = -(4.61e-1 + ms); %impulse model
d3 = (-b + sqrt(b^2 - 4*a*c))/(2*a);

a = 6.81e-3; b = 5.49e-1; c = -(1.08e-1 + ms); %cont. current model
d4 = (-b + sqrt(b^2 - 4*a*c))/(2*a);

---

B.2.1 Separation Time Extraction

function ms = separate(in);

order = 1000;
samplerate = 48e3; nyq = samplerate/2;

ELF = fir1(order,1.5e3/nyq,'low');
elf = conv(ELF,in); elf = elf(order/2+1:length(in)+order/2);
elf = abs(elf);
[val index] = max(elf);
elf_max = index;

VLF = fir1(order,[20e3/nyq 23.9e3/nyq]);
vlf = conv(VLF,in); vlf = vlf(order/2+1:length(in)+order/2);
jvlf = abs(vlf); %jagged VLF envelope
vlf = conv(ELF,jvlf); vlf = vlf(order/2+1:length(in)+order/2);
vlf = abs(vlf); %smoothed VLF envelope
[val index] = max(vlf);
vlf_max = index;
[val index] = max(in);

ms = (elf_max - vlf_max) * 1000/samplerate; %separation time [ms]
B.3 Signal-to-Noise Ratio Plots

function signoise(sig, noise, grid);

samplerate = 48e3;
fftlen = 2^nextpow2(5*max([length(sig) length(noise)])) \text{%interpolate and speedup calc}
fftlen = fftlen*2;

sig = zeropad(sig,fftlen);
sig = abs(fft(sig));

noise = zeropad(noise,fftlen);
noise = abs(fft(noise));

N = fftlen;
fftlen = N/2 - 1;
h = sig ./ noise;
h = h(1:N/2);
out = h;

f = 0:fftlen; f = f * samplerate/(2*fftlen);
figure; loglog(f,h,'b-');
hold on; loglog([3 23e3],[1 1],':');
xlabel('Frequency (Hz)');
ylabel('SNR');
xlim([3 24e3]);

B.3.1 Zero-padding

function z = zeropad(in,total);
if size(in,1) < size(in,2)
in = in';
end

l = length(in);
if nargin<2
    total = 2*nextpow2(l*5);
end

z = [in' zeros(1,total - l)'];

\section*{B.4 Evaluating Complex Legendre Functions}

\textbf{function} \textbf{[P0am, P1am, KeyAlg, Nsum] = leg(theta, nuam, eps)}

\textit{%% adapted from Vadim Mushtak’s fortran code}
\textit{%% eps = .001 is sufficient.}

\textbf{thet = pi} - \textbf{theta};
\textbf{alg = 0}; \textbf{Nsum = 0};
\textbf{n = real(nuam+0.5) - sqrt(-1) * imag(nuam)};
\textbf{h = eps}; \textbf{t = thet}; \textbf{t0 = 3.1415 - t};
\textbf{e = h*h}; \textbf{r = real(n)^2 + imag(n)^2};

\textbf{y = 3.141593};

if ((t\cdot t\cdot r >= 61.4656) & (t0\cdot t0\cdot r >= 61.4656))
    \textbf{alg = 3}; \textit{%%}
    \textbf{y1 = 3.141593; \textit{%%pi}}
    \textbf{p0 = t * (-imag(n) + sqrt(-1)*real(n))};
    \textbf{p1 = y1 * (-imag(n) + sqrt(-1)*real(n))};
    \textbf{e2 = exp(p1-p0)/(1+exp(p1+p1))};
    \textbf{e1 = exp(p0+p0)};
    \textbf{p0 = (((0.4882812e-2 - 0.6408691e-3 / n)+0.78125e-2)/(n-0.125)+1.0)*e2/sqrt(n)};
    \textbf{p1 = (n+0.5)*p0};
    \textbf{s0 = sin(t)};
    \textbf{c1 = 1.0/sqrt(y1*s0)};
    \textbf{p0 = ((real(p0)-imag(p0)) + sqrt(-1) * (real(p0)+imag(p0))) * c1};
    \textbf{p1 = ((real(p1)+imag(p1)) + sqrt(-1) * (imag(p1)-real(p1))) * c1};
    \textbf{i1=0.0; c1=s0+s0};
    \textbf{sx = cos(t); cx = -s0; sl = 0.0; cl = 1.0};
    \textbf{z0 = 1.0+imag(c1) - sqrt(-1)*real(c1)};
\[ z_{11} = 1.0 - \text{imag}(e1) + \sqrt{-1}\text{real}(e1); \]
\[ t1 = z0; \]
\[ t2 = z_{11}; \]
\[ a = 1.0; \quad a1 = a; \quad a2 = 0.25; \]
\[ a2 = a2 + 2.0*i1; \]
\[ i1 = i1 + 1.0; \]
\[ cl1 = cl*cx - sl*sx; \]
\[ sl = sl*cx + cl*sx; \]
\[ cl = cl1; \]
\[ e2 = i1*c1*(n+i1); \]
\[ q = -a2/e2; \]
\[ q1 = (1.0-a2)/e2; \]
\[ a = a*q; \quad a1 = a1*q1; \]
\[ q = (\text{cl*real}(z0) + \text{sl*imag}(z_{11})) + \sqrt{-1}(\text{cl*imag}(z0) - \text{sl*real}(z_{11})); \]
\[ q1 = (\text{cl*real}(z_{11}) + \text{sl*imag}(z0)) + \sqrt{-1}(\text{cl*imag}(z_{11}) - \text{sl*real}(z0)); \]
\[ q = a*q; \quad q1 = a1*q1; \]
\[ t1 = t1 + q; \quad t2 = t2 + q1; \]
\[ Nsum = Nsum + 1; \]

\textbf{while } ((\text{real}(q1)^2 + \text{imag}(q1)^2) > e*(\text{real}(t2)^2 + \text{imag}(t2)^2)) \]
\[ a2 = a2 + 2.0*i1; \]
\[ i1 = i1 + 1.0; \]
\[ cl1 = cl*cx - sl*sx; \]
\[ sl = sl*cx + cl*sx; \]
\[ cl = cl1; \]
\[ e2 = i1*c1*(n+i1); \]
\[ q = -a2/e2; \]
\[ q1 = (1.0-a2)/e2; \]
\[ a = a*q; \quad a1 = a1*q1; \]
\[ q = (\text{cl*real}(z0) + \text{sl*imag}(z_{11})) + \sqrt{-1}(\text{cl*imag}(z0) - \text{sl*real}(z_{11})); \]
\[ q1 = (\text{cl*real}(z_{11}) + \text{sl*imag}(z0)) + \sqrt{-1}(\text{cl*imag}(z_{11}) - \text{sl*real}(z0)); \]
\[ q = a*q; \quad q1 = a1*q1; \]
\[ t1 = t1 + q; \quad t2 = t2 + q1; \]
\[ Nsum = Nsum + 1; \]
\textbf{end}
\[ p_0 = p_0 \ast t_1; \]
\[ p_1 = p_1 \ast t_2; \]

\textbf{else}

\[ x = t/2.0; \ u_1 = 0.25 - n \ast n; \ i = 0.0; \ c = \cos(x); \ s = \sin(x); \]
\[ z_1 = 0.0; \ z = 1.0; \ u = z; \]
\textbf{if} \ (x > 0.79)

\[ \text{alg} = 2; \]

\[ s_1 = 0.0; \ f = z_1; \ f_1 = f; \]

\[ u_1 = u_1 + (i + i); \]
\[ i = i + 1.0; \]
\[ p = c / i; \]
\[ s_1 = s_1 + 1.0 / i; \]
\[ u = p \ast u_1 \ast u; \]
\[ z_1 = z_1 - u; \]
\[ f_1 = f_1 - s_1 \ast u; \]
\[ b = \text{real}(u) \ast 2 + \text{imag}(u) \ast 2; \]
\[ u = u \ast p; \]
\[ z = z + u; \]
\[ f = f + s_1 \ast u; \]
\[ \text{Nsum} = \text{Nsum} + 1; \]

\textbf{while} \ (b > e \ast (\text{real}(z_1) \ast 2 + \text{imag}(z_1) \ast 2))

\[ u_1 = u_1 + (i + i); \]
\[ i = i + 1.0; \]
\[ p = c / i; \]
\[ s_1 = s_1 + 1.0 / i; \]
\[ u = p \ast u_1 \ast u; \]
\[ z_1 = z_1 - u; \]
\[ f_1 = f_1 - s_1 \ast u; \]
\[ b = \text{real}(u) \ast 2 + \text{imag}(u) \ast 2; \]
\[ u = u \ast p; \]
\[ z = z + u; \]
\[ f = f + s_1 \ast u; \]
Nsum = Nsum+1;

end

z1 = s*z1;
f1 = s*f1;
u = n+0.5;
y1 = real(u);
k = fix(11.0−y1);
if (k<0)
k = 0;
end

u = u+k;
u1 = log(u);
u = 1/u;
u1 = u1 − 0.5 * u;
u = u * u;
u1 = u1 − u*(1/12.0 − u*(1/120.0 − u/252.0));

if (k ≈= 0)
u = n+0.5+k;
for l=1:k
u=u−1;
u1=u1−1/u;
end
end

u1 = log(s/c) − 0.5772157−u1;
f1 = f1+z1*u1+0.5*(s/c+c/s)*z;
f = f+z*u1;
y = y+y;
u = −imag(n) * y + sqrt(−1) * real(n) * y;
u = exp(u);
u = (1.0−u) / (1.0+u);
u1 = −imag(u) + sqrt(−1) * real(u);
p0 = z*u1+4.0/y*f;
p1 = z1*u1+4.0/y*f1;
else
alg=1;
    u1=u1+(i+1);
    i=i+1.0;
    p=s/i;
    u=p*u1*u;
    z1=z1+u;
    b=real(u)*2 + imag(u)*2;
    u=u*p;
    z=z+u;
    Nsum=Nsum+1;

while (b > e*(real(z1)^2 + imag(z1)^2))
    u1=u1+(i+1);
    i=i+1.0;
    p=s/i;
    u=p*u1*u;
    z1=z1+u;
    b=real(u)^2 + imag(u)^2;
    u=u*p;
    z=z+u;
    Nsum=Nsum+1;
end

z1 = c*z1;
    u = -imag(u)*y + sqrt(-1) * real(u)*y;
        u = exp(u);
        u1 = 2.0*u / (1.0+u*u);
        p0 = z*u1;
        p1 = z1*u1;
end

P0am = -(real(p0) -sqrt(-1)*imag(p0));
P1am = -(real(p1) -sqrt(-1)*imag(p1));