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A MUSIC SYNTHESIZER
USING FREQUENCY MODULATION

by

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Departmental Committee on Theses
ABSTRACT

A MUSIC SYNTHESIZER
USING FREQUENCY MODULATION

by

James Gerard Breen

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Title: Instructor in Math and Educational Division

The purpose of this project is to produce a working music synthesizer for the MIT Artificial Intelligence Laboratory's LOGO group. This synthesizer has great potential as an educational tool, and can be used as such not only in the field of music, but in the fields of math, science and computer technology as well.

Although the scope of this project proved to be too great to be accomplished in a term, much was accomplished towards the final goal. The hardware to control the real time organ inputs was designed, built and tested. Research into the method of Synthesis developed by John Chowning was carried out, and a somewhat unsuccessful one voice implementation of his method was written and experimented with on a PDP 11/45.

Chowning's results indicate that further development of the software involved in this project would be well worthwhile. The final outcome would be useful to both the composer and the student.
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The LOGO Music System

The Frequency Modulation (FM) Music Synthesizer is the basis of a complex music system in the MIT Artificial Intelligence Laboratory's LOGO group's Learning Lab. It has the potential to be used in the teaching of music, math, science, and computer technology. The system consists of the group's PDP 11/45 computer which supports LOGO, a T2500 micro processor, a set of organ keyboards, pedals, and stop switches, and an audio system for output.

The use of this system as a tool for teaching music has been debated by some, but it is felt that such a system will have a tremendous impact on the development of music projects. Such a system will enable a student to create traditional instrument sounds, along with new sounds of his own design, and use them to enhance his compositions.

The realization of these sounds comes from the use of FM to produce complex tones. The shape of these tones is accomplished by experimentation, along with a good understanding of the science and mathematics of frequency modulation and acoustics. The better a student's understanding of these fundamentals, the more versatile will be the system to him.

The use of the music system is accomplished in one of two ways. The LOGO system is already equipped with a set of instructions to control an already existing Music Box. For the new music system, these instructions have to be changed to accept the additional inputs required to achieve frequency modulation, and then LOGO can supply programmed music inputs to the modulator. The other input is through a set of organ inputs, which give the system the added
ability to be used in real time. This second feature allows the student to use the system as a musical instrument, hearing his results as he creates, rather than after considerable delay.

To achieve this real time capability, a system had to be developed to read the organ keyboards, pedals, and switches. This was accomplished with a hardware scanning circuit, which checks the inputs, looking for those that have been pressed or released. When such an input is discovered, information describing which input changed, and how, is passed on to the computer for processing. Complete detail of this scanning circuitry is included in appendix A.

The information that is thus received by the computer, is translated to a form usable by the FM processor, by a software subroutine which will be referred to as Input Translation. This subroutine will use note tables, and amplitude envelope tables to handle this task. The FM processor then uses these inputs to calculate which entries of the sine table it should output to an amplifier. The amplifier is a digital process which multiplies the frequency modulated signal by the amplitude envelope, and outputs it to a first in first out (FIFO) stack. The contents of this stack are outputted to a Digital to Analog (D/A) Converter at a constant rate by a twenty kilohertz clock signal. The analog signal that is thus produced is passed through a low-pass filter and then is amplified and played through a conventional hi fidelity audio system. Figure 1 shows the graphical representation of the music system.
Figure 1. The LOGO Music System
Frequency Modulation using the Chowning Method

The initial idea for this project came from a paper by Steve Saunders of Carnegie-Mellon University, entitled "FM The Easy Way". His research was a simplification of the method developed by John Chowning in 1972, which used frequency modulation of two sine waves in the audio range to produce complex tones. Saunders' preliminary results showed that this method could be reduced to a modulation by a triangle wave. This process, which eliminates several computational steps, causes the implementation of this synthesizer to be practical.

Chowning's reasoning behind using frequency modulation for music synthesis was based on the fact that FM causes a signal to become spread out in the frequency domain. This widening of the signal's bandwidth causes the signal's fundamental frequency to change, but at the same time creates harmonics of that new frequency. This method of adding harmonics is much faster than taking a signal and adding in its harmonics one at a time, thus saving considerable amounts of computation time. This savings allows a greater number of voices to be played. In addition, the parameters used in specifying the modulation can be easily changed, thus allowing the artist to more easily imitate conventional instrument sounds.

To carry out the Frequency Modulation process, four parameters must be specified. It is the relationship between these inputs that specify the shape and pitch of the produced tone. The four variables are:
(5)

1) m the modulating frequency in radians / second
2) c the carrier frequency in radians / second
3) A(t) the amplitude envelope
and 4) I the modulation index.

Chowning's method for synthesis combines these terms to find e, the instantaneous amplitude of the outputted waveform.

\[ e = A(t) \sin (ct + I \sin (mt)) \]

This formula requires that for every value of e, the product of the modulation index (which for some instruments is a function of time) and the sine of the modulating frequency be computed. Since multiplication is a very slow operation (compared to addition), this would be a difficult process to use. The production of low grade high fidelity output (10kHz bandwidth) would necessitate the computation of twenty thousand values of e per second.

Saunders' research was aimed at easing this computational burden. He simplified the problem to one of reducing Chowning's equation to a series of additions and table lookups to produce e. Saunders changed the inner sine function to a triangle function, changing the argument of the outer sine function to:

\[ \text{freq} (t) = c + m^*I^*\text{tri}(mt) \]

By letting the frequency at time zero (starting frequency or \text{freq}(0)) equal c, this reduces to:

\[ \text{freq} (t) = \text{freq} (t-1) + \text{INC} \]

where the increment (INC) is a constant (INC = m^2*I).

This gives a constantly increasing function, so to change that to a triangle function, it is only necessary to change the sign of
INC whenever it exceeds its limits. These limits are also constants which can be evaluated ahead of time. The values of these limits are:

\[ \text{Upper Limit} = c + m \times I \]
\[ \text{Lower Limit} = c - m \times I \]

Observation shows that even in the outer loop, the multiplications are minimal. The constant \((m \times I)\) which is used to determine the limits is also used to calculate the increment \((m \times (m \times I))\).

To be able to get a desired waveform using this method, it is necessary to know how to get the characteristics you seek. To begin with, we define the ratio of the carrier to modulating freq:

\[ \frac{c}{m} = \frac{N1}{N2} \]

If \(N1\) and \(N2\) are both Integers, and if the common factors have all been removed, then the fundamental frequency (pitch) is defined:

\[ \text{pitch} = \frac{c}{m} = \frac{N1}{N2} \]

This is true only if there is modulation (ie. \(I \neq 0\)), otherwise the pitch is equal to the carrier frequency.

The peak frequency deviation (d) is the amount the carrier varies around its average. This is defined as:

\[ d = m \times I \]

The bandwidth, then, is:

\[ BW = 2(d+m) \text{ (approximately)} \]
SOFTWARE CONSIDERATIONS

The software for the T2500 microprocessor must be efficient so that the greatest number of voices can be handled by the system. The structure of the program consists of a short MAIN LOOP which calls subroutines, and causes interruptions when inputs are available. The subroutines consist of the input translator, and the INNER LOOP which handles the actual frequency modulation. In order to have the system work, it also is necessary to have several tables which are used to set up the inputs, and to produce the final output. These tables include a SINE table, a (set of) note table, and a set of Amplitude Envelopes to shape the note for attack, sustain, and decay.

When the system gets a zero from its READY bit from the Scanning Circuit, this should cause an interrupt to occur. The software then takes the eight bit address and the value of the input, and put that information in a temporary register. A LOW pulse is then returned on the ACKNOWLEDGE line, and the input translator is called.

The input translator takes the address and value, and does a test to find out if the input is a note input, or a control input. It also has to test the value to see if the input is to be activated or deleted. If it is to be activated, then the subroutine must assign that note to a voice generator, and attach the appropriate parameters. The parameters are found by using the note and amplitude tables, along with the appropriate subroutines to process them. A possible use of the organ stops would be to de-
termine which subroutine is appropriate. If the input is to be
deactivated, then the note must be found among the voice generators
and tagged to begin decay.

The inner loop will be the subroutine that carries out the
modulation algorithm. The inner loop must produce one value for
each voice and send it to the Digital Amplifier. Then it goes
around the circle again, continually outputting values to be sent
towards their final destination.

A version of a one voice inner loop was written by the author
for the PDP 11/45. The success of this endeavor was not considered
great, but several possibilities were tried to alleviate the prob-
lem. The assembly code is listed in Appendix B. The results of
that code were fine for unmodulated waves (modulating index I = 0)
were clean sine waves. The pitch was off because of lack of
compensation for the 20kHz clock pulse, but other than that the
output was flawless. The modulated waveform, on the other hand,
was very inharmonic, and disturbing to the senses. One possible
cause for that would be the lack of a Low Pass Filter on the
output. This is uncertain, since no filter was built to test the
hypothesis. The other observation that was made was that the
accuracy of the notes has to be improved, especially in the low
register. The percentage of error at the bottom of the scale was
great enough to be noticed due to the truncation of the frequency
at the decimal point. It is suggested that a fixed point, rather
than an integer model be used to alleviate that problem.

The Digital Amplifier is a subroutine that uses the formula:
\[ m \cdot \sin(x) = \sin(x) + \sin(x + \arccos(m/2)) \] (approx.)

to do the multiplication by the amplitude envelope. In the equation, \( m \) is equivalent to \( A'(t) \), not the modulating frequency.

The digital amplifier, then must do this addition for each voice, and then add the voices to output a single value for the FIFO.

The tables that are used are explained here in enough detail to allow the user to create and use his own without need for reproduction within this paper.

**SINE TABLE:** The sine table is a table with 1024 entries from zero to two \( \pi \). The table that was used on the PDP 11/45 used the following equation:

\[
\text{SINE}(N) = 77777 \times \sin(1024^N)
\]

The 77777 factor is there to give the table maximum accuracy, while still staying in the integer domain.

**NOTE TABLE:** The notes of a scale can be found by taking the A above middle C (440 Hz), and working from there. An octave is a factor of two, and a modern scale (even tempered) has all of its half tones evenly spaced. Thus to calculate the notes table, it is only necessary to multiply (or divide) each note to get its next higher (or lower) half tone by \( 2^{1/12} \).

The AMPLITUDE ENVELOPES which are used by the Digital Amplifier must be set up so that it is a list of the ARCCOS \( (A(t)/2) \), rather than the amplitude factor \( A(t) \) itself. By choosing appropriate values, the attack and decay of any instrument should be recreatable.
APPENDIX A

SCANNING CIRCUIT
(Hardware)

The purpose of the scanning circuit is to read all of the inputs of a conventional organ console, and inform the system of any changes made among those inputs. This is accomplished with the use of a 256-bit memory (1 bit width) which keeps track of the last state a key was in. An exclusive or function determines whether or not there was a change, and stops the scanner to inform the processor when there is. It passes the eight-bit address of the changed input, along with the new value of the input, to the computer. When the information is received, an acknowledge signal is returned to the scanner, and the hardware resumes its check.

The addresses of the inputs are determined by a large multiplexer circuit located on the scanner board. The organ console that was used for this project houses two keyboards of sixty-one keys each, a set of seventeen foot pedals, and a set of twenty-three stops. In addition to these 162 inputs, provisions were made to leave an additional thirty unused addresses for possible future expansion of the real-time inputs. These 192 addresses were implemented by using twenty-seven 8 line to 1 line data selectors/multiplexers, and one 4 line to 1 line data selectors/multiplexers.

The layout of the board is shown in figure A.1. The pin connectors J1, J2 and J3 are used to connect keyboard 1. J4, J5 and J6 are for keyboard 2, and J7 is for the connection to the T2500. The pedals and stops are connected by means of the end connectors.
at the bottom of the board.

The multiplexer circuit is shown in figure A.2. The operation of this circuit is controlled by the scanner circuitry shown in figure A.3. The bottom three bits of address control the twenty-four bottom level 8 line to 1 line multiplexers. The next three bits operate the next level of 8 to 1's, while the top two bits switch the 4 line to 1 line. With this configuration, every input is addressable by a unique eight bit number, and generation of that address will return the input's value.

The scanning circuit (figure A.3) takes an external clock pulse (64 kHz from the T2500) and generates the eight bit addresses to read the inputs from the multiplexer circuit. The counter has three positive NAND buffers to supply the proper TTL load ability to the twenty-four multiplexers it must serve. This causes the counting sequence of the bottom three bits to count down from seven, rather than up from zero. This is a harmless consequence, but is noted here for anyone trying to build and/or debug this circuit. The AND gate off the top two bits of the counter are there to inhibit the states of the form 11xxxxxx. This eliminates the scanning of the fourth input of the 4 to 1 multiplexer which has nothing connected to it.

The memory unit is a 256 x 1 bit memory, which stores the compliment of the number at the input when the write enable goes low. The exclusive OR gate (XOR) that is connected between the memory's input and output is used to detect when an input changes state. If the output of the XOR gate is HIGH (both inputs different) then the circuit continues scanning (No Change). If the input
(12)
goes LOW (both inputs the same), then there is a state change. This LOW line will cause the AND gate from the clock to output LOW, causing the clock to be inhibited. The clock will continue to be inhibited until one of two events occur:

1) The 2500 reacts to its Ready Signal (the LOW bit from the XOR gate, which is delayed one clock pulse by the flip-flop to prevent a reaction from a glitch), and returns a LOW bit to the Write Enable, causing the new value to be written into memory (complimented) and resetting the system. or

2) The key is returned to its previous state before the 2500 reacts.

The latter form of reset is highly unlikely, due to the fast response time of the system.
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**Unused Gates**

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**Package Key**

- **7408**: Quad. 2-input pos. AND Gates
- **7437**: Quad. 2-input pos. NAND Buffers
- **7474**: Dual D-type Flip-Flop
- **74151**: 8 line to 1 line Data Selector
- **74153**: Dual 4 line to 1 line Selector
- **74193**: Synchronous 4 bit up/down counter
- **27LS00**: 256x1 bit memory
- **898-1-R-1.0k**: 1K pullup resistors

**Figure A.1: Scanner Board Layout**
64 inputs. A 1 indicates open switch, a zero indicates an activated switch. All Bottom Level Multiplexers are tied high with 1k pull-up resistors.
Figure A.3: Scanning Circuit
APPENDIX B

PDP 11/45 ASSEMBLY LANGUAGE CODE FOR FREQUENCY MODULATION

FM:    JSR PC,GETIN       ;GET VALUES FOR CARRYF,MODULF,MODINX,NSAMPL
      MOV CARRYF,F      ;CARRIER FREQ IN F
      CMP F,OLDCAR     ;IS IT A NEW NOTE?
      BEQ FM1         ;YES, DON'T RESET
      CLR E           ;NO, CLEAR PHASE
      MOV F,OLDCAR    ;REMEMBER NEW FREQUENCY

FM1:   MOV F,D           ;CARRIER FREQ IN D
      MOV D,C          ;CARRIER FREQ IN C
      MOV MODULF,B     ;MODULATING FREQ IN B
      MUL MODINX,B     ;INDEX * FREQ IN B
      ADD B,D         ;UPLIM = CARRIER*MOD*INDEX
      MOV D,UPLIM
      SUB B,C         ;LOWLIM=CARRIER-MOD*INDEX
      MOV C,LOWLIM
      MUL LNGTHF,B     ;INC=CARRIER*MOD*INDEX*LENGTH IN B
      MUL F,B
      MOV NSAMPL,D
      JSR PC,INLOOP
      BR FM

INLOOP: ADD B,F         
      TST B           
      BLE CHKLLO     
      CMP F,UPLIM    
      BLE ASSIGN     
      JSR PC,INLIM1  
      BR ASSIGN

CHKLLO: CMP F,LOWLIM
      BGE ASSIGN
      JSR PC,INLIM2

ASSIGN: ADD F,E
      BIC #174001,E
      MOV SINE(E),FIFO
      DEC D
      BNE INLOOP
      RTS PC

INLIM1: MOV UPLIM,C  
      BR INLIN
INLIM2: MOV LOWLIM, C

INLIM:  NEG B
        SUB C,F
        SUB F,C
        MOV C,F
        RTS PC
## APPENDIX C

### SINE TABLE

**SCALE** = 1

```
.MACRO S A,B,C,D,E,F,G,H
 A/SCALE
 B/SCALE
 C/SCALE
 D/SCALE
 E/SCALE
 F/SCALE
 G/SCALE
 H/SCALE
.ENDM
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NOTES TABLE

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F/NSCALE
G/NSCALE
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