STALLED FLOW CHARACTERISTICS
FOR AXIAL COMPRESSORS

by

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Submitted to the Department of Mechanical Engineering
on May 12, 1983 in partial fulfillment of the
requirements for the Degree of Master of Science in
Mechanical Engineering

ABSTRACT

This thesis describes a study of stalled flow performance of multi-
stage compressors. The study is focused not only on the performance in
rotating stall, but also on the so-called axisymmetric performance curve,
in particular, the impact on stall inception and stall recovery. A new
compressor characteristic is envisioned which provides this axisym-
metric pumping performance over the entire compressor flow range, in-
cluding low forward and reversed flow; this axisymmetric characteristic
is required in any rotating stall model. It is possible for the axisym-
metric performance to rise above the stall point pressure rise, thus
indicating greater unstalled pressure rise potential. In addition, the
axisymmetric characteristic for forward flow parallels diffuser per-
formance. It is also argued that the recovery point measured in post-
stall compressor tests results from a compression system instability,
rather than from an unstable rotating stall flow. If so, recoverability
from rotating stall may be improved by altering system parameters.
Furthermore, the full-span rotating stall characteristic is extrapolated
beyond the measured recovery point. On this basis, the compressor stall
point is viewed as a bifurcation, where a change in flow mode exists
(perhaps analogous to the critical point in the axial compression of
thin shells). An application for the extended rotating stall charac-
teristic is in a compression system model.

An axisymmetric (actually two-dimensional) reversed flow model is
presented, and is shown to be in reasonable agreement with available
high backflow compressor data. A technique is also developed to predict
the axisymmetric curve over the entire flow map. Finally, calculations
are carried out using the rotating stall model of Moore and the axi-
symmetric characteristic developed herein. Using this characteristic,
the compressor exit geometry is predicted to have little effect on the
performance in rotating stall and the point of recovery.

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BIOGRAPHICAL NOTE

Steven Gordon Koff was born on December 30, 1960 in Cincinnati, Ohio to Bernard L. and Sonia E. Koff. He graduated from Walnut Hills High School in Cincinnati in June 1978. In May 1981, he received a Bachelor of Science degree in Mechanical Engineering from The University of Michigan with Summa Cum Laude status. He enrolled in the Graduate Mechanical Engineering Department of the Massachusetts Institute of Technology in September 1981.

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for their continual support.
ACKNOWLEDGEMENTS

The author is greatly indebted to Prof. E.M. Greitzer, who supervised this work. His experience, accessibility and patience made this thesis possible. In addition, the author thanks Mr. R.E. Davis, Mr. J.T. Lewis and Mr. R.C. Parsley of Pratt and Whitney Aircraft for their helpful suggestions and insight. The numerous discussions with Capt. R.N. Gamache, and his differential equation solver program (DESOLV) are also appreciated. The remarks of Mr. A.H. Eastland, Mr. P.L. Lavrich and Dr. C.S. Tan are acknowledged. Furthermore, the author is grateful for Prof. E.E. Covert's comments.

Special thanks are extended to Pratt and Whitney Aircraft, Governments Products Division, for supporting this project.

The author also thanks Mrs. C. Callahan for her skilled work in constructing most of the figures in this thesis.
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NOMENCLATURE

A or $A_c$  compressor cross-sectional area

$AR$  area ratio for a planar diffuser

$a$  stall cell speed relative to a blade row

$a_x$  axial blade row gap size

$B$  Greitzer's dimensionless parameter, given in Appendix D

$b$  fractional size of stalled or unstalled region in compressor annulus

$C$  absolute flow velocity

$C'$  slope of compressor characteristic, $\psi_{TS}(\phi)$

$C_p$  pressure recovery coefficient for a planar diffuser shown in Fig. 3.7 = $(p_2 - p_1)/(1/2 \rho U_1^2)$

$C_p(x)$  local pressure coefficient in Fig. 3.7 = $[p(x) - p_1]/(1/2 \rho U_1^2)$

$C_x$  axial flow velocity

$c$  blade chord plus axial gap contribution = $l + a_x/\cos \gamma$

$D$  mean compressor diameter

$F(\phi)$  axisymmetric static pressure rise at $\phi$ per blade row

$F_s(\phi)$  backflow compressor characteristic, $\psi_{TS}(\phi)$

$F_u(\phi)$  uninstalled compressor characteristic, $\psi_{TS}(\phi)$

$f$  nondimensionalized stall cell speed in absolute frame = $V_s/U$

$f_o$  stall cell speed calculated from the small disturbance approximation in [5], given by equation (5.4)

$g$ or $g(\theta)$  axial velocity perturbation at compressor inlet

$g'(\theta)$  $dg/d\theta$

$h$  in Chapter 4 only, $h$ = blade span
otherwise, $h$ or $h(\theta)$ = circumferential velocity perturbation at compressor inlet
\[ h'(\theta) = \frac{dh}{d\theta} \]

average value of \( h(\theta) \) over a complete cycle

\( h_{avg} \)

viscous lag constant

\( k \)

empirical lag parameter

\( L_c \)

effective compressor length

\( \lambda \)

blade chord

\( m \)

empirical compressor exit geometry parameter

\( N \)

number of compressor stages

\( P \)

axial force in shell buckling test from Fig. 3.17

\( P_{cr} \)

axial force at the critical point in Fig. 3.17

\( P_{cycle} \)

period of the limit cycle solution of \( g(\theta) \) and \( h(\theta) \)

\( p \)

static pressure

\( P_T \)

total pressure

\( \Delta P_{TS} \)

exit static to inlet total pressure rise for forward flow; and exit total to inlet static pressure rise for reversed flow

\( \Delta P_{TT} \)

exit total to inlet total pressure rise

\( R \)

mean compressor radius = \( D/2 \)

\( T' \)

slope of throttle characteristic, \( \psi_{TS}(\phi) \)

\( t \)

in Chapter 4 only, \( t = \) blade pitch otherwise, \( t = \) time

\( U \)

mean rotor speed

\( V_s \)

stall cell speed in absolute frame

\( w \) or \( w(\theta, t) \)

relative flow velocity

\( x \)

axial coordinate in compressor

\( \alpha \)

in Chapter 4 only, \( \alpha = \) flow angle in absolute frame otherwise, \( \alpha = \) fraction of compressor annulus occupied by stall cell
\( \beta \)  
flow angle relative to a blade row

\( \beta_{1,cr} \)  
critical inlet flow angle

\( \beta_{LE} \)  
camber line angle at blade leading edge

\( \beta_{TE} \)  
camber line angle at blade trailing edge

\( \beta_{LE,p} \)  
pressure side metal angle of blade leading edge

\( \beta_{LE,s} \)  
suction side metal angle of blade leading edge

\( \gamma \)  
blade stagger angle referred to the axial direction

\( \bar{\gamma} \)  
mean compressor stage blade stagger angle  
\[ = \frac{1}{2}(\gamma_{\text{rotor}} + \gamma_{\text{stator}}) \]

\( \delta \)  
referring to Fig. 3.17, only \( \delta = \text{crosshead displacement in shell buckling test} \)

otherwise, \( \delta = \text{difference in rotating stall performance and axisymmetric pressure rise, defined by equation (5.1)} \)

\( \delta \)  
crosshead displacement speed in shell buckling test in Fig. 3.17

\( \theta \)  
circumferential coordinate in compressor annulus

\( \theta_c \)  
blade camber angle

\( \lambda \)  
given by equation (2.8)

\( \mu \)  
wake parameter shown in Fig. 4.2

\( \nu \)  
jet velocity ratio = \( \frac{w_1}{w_2} \)

\( \rho \)  
density

\( \tau \)  
given by equation (8.5)

\( \tau' \)  
\( \tau \) evaluated at \( k = 1 \)

\( \tau_{\text{form}} \)  
rotating stall formation time scale, defined in equation (3.1)

\( \tau_{RS} \)  
rotating stall propagation time scale, defined in equation (2.4)

\( \tau_s \)  
blade passage stalling rate time scale, defined in equation (2.3)
\( \tau_{\text{surge}} \) surge cycle time scale defined in Section 3.1

\( \phi \) average of the local axial velocity coefficient, \( \phi \), in annulus

\( \phi \) in Moore's analysis, \( \phi = \) local axial velocity coefficient defined by
\[ \phi = \delta + \gamma \]
o otherwise, \( \phi = \) axial velocity coefficient = \( C_x/U \)

\( \phi^* \) \( \phi \) at the design conditions

\( \phi_{\text{avg}} \) average value for \( \phi \) encountered by a blade passage in the transition region between stalled and stalled flow

\( \psi \) rotating stall performance, \( \psi_{TS} \), at \( \phi \)

\( \psi_c(\phi) \) axisymmetric compressor characteristic, \( \psi_{TS}(\phi) \)

\( \psi_{TS} \) or \( \psi \) nondimensionalized exit static to inlet total pressure rise = \[ \frac{\rho_{\text{exit}} - \rho_{i\text{inlet}}}{\rho U^2} \]

(\( \cdot \)) average value

Subscripts

1 blade row inlet conditions

2 jet/wake conditions

3 blade row exit conditions

i compressor inlet

e compressor exit

IGV inlet guide vane

OGV outlet guide vane

\( S_0 \) stator vane just before OGV

\( R_1 \) rotor row just before stator vane \( S_0 \)

\( u \) unstalled

\( s \) stalled or backflow

\( v \) or valley axisymmetric valley coordinate
INTRODUCTION

The occurrence of a so-called non-recoverable stall has been a serious problem with military gas turbine engines. When these high performance engines undergo transients such as throttle accelerations and decelerations, maneuvers causing inlet flow distortions or a "hardlight" in the afterburner, the compressor can be driven past its stall limit. In this case, a fluid flow instability is encountered where one of two possible stall patterns have been observed in the compressor, namely, a surge cycle or a rotating stall.

A surge cycle is characterized as an unsteady and nearly axisymmetric event which involves large oscillations in flow and includes unstalled compressor operation for a portion of the cycle, i.e. recovery. Consequently, the surge is a desirable stalled state because of this recovery portion. In contrast, a rotating stall state occurs in steady compressor operation at low mass flow and low pressure rise. Possible consequences of operation under these conditions can even include engine failure, since the continual addition of fuel in the combustor under these decreased mass flows would lead to turbine over-temperatures. Recovery from rotating stall may be accomplished by limiting the fuel addition in the combustor which effectively opens the downstream throttle to the compressor. However, this process may not open the throttle enough for recovery, and in this case an engine shutdown must be implemented.
Ideally, the designer would like to provide enough stall margin for the compressor to avoid any stalled engine operation. This is not always possible considering the increasing demand for greater performance in military engines, and especially and the wide range of transients encountered. Thus, as the stall limit of the compressor is exceeded, recoverability from the stalled state must be examined. It is therefore essential to understand the post-stall behavior of the compressor in order to deal with the problem of engine recoverability.

Detailed experimental studies by Greitzer [1]*, Day and Cumpsty [2] provided descriptions of rotating stall in a multi-stage compressor. The flow pattern is non-axisymmetric and consists of a low or reversed flow region known as a stall cell which propagates circumferentially at constant speed in an annulus of high mass flow. A first attempt to model compressor operation in rotating stall was provided by Day, Greitzer and Cumpsty [3], using essentially a parallel compressor concept. The stalled and unstalled flow zones were viewed as two compressors working in parallel, one operating in the unstalled condition, and the other operating at zero mass flow. Davis, Lewis and Parsley [4] have implemented a parallel compressor model for the fully compressible case to predict stall performance on their high speed test compressor. However, difficulties arise in applying this technique mainly because it is based on experimental observations rather than fundamental principles. Moore [5] [6] provided a new and fundamental

*Bracketed numbers are references.
approach to rotating stall. His theory solves for the permissible flow field in the compressor by specifying the axisymmetric pumping performance of the compressor and by setting the upstream and downstream boundary conditions.

Moore's model requires an axisymmetric compressor characteristic as an input over the entire flow regime, including low forward flow and reversed flow. This axisymmetric characteristic is well defined in the regions of un stalled and reversed flow, since an axisymmetric flow occurs in these areas. However, Moore assumes a characteristic in the low forward flow region of the compressor map since the actual axisymmetric pumping performance remains unknown under these flow conditions. This occurs primarily because the axisymmetric curve has not been directly measured in this region since it is inherently unstable. In this case, the non-axisymmetric flow mode of rotating stall exists during steady-state operation.

This thesis deals with a theoretical investigation of rotating stall in low rotor speed compressors. Stalled compressor characteristics are the primary focus since they provide insight into the stall phenomenon. Chapter II gives the initial impetus for the study of compressor performance characteristics that follows by discussing two modeling approaches for rotating stall. Compressor characteristics are described in detail in Chapter III. Here, a method is developed for obtaining the unknown axisymmetric characteristic from transient compressor data. Also, possible variations of the axisymmetric curve for different compressor configurations, and their connection with the
stall inception point are considered. Rotating stall characteristics are also discussed with theories involving compressor recovery. Chapter IV provides a prediction method for the complete axisymmetric curve. Lastly, in Chapter V, a practical application of the Moore model is illustrated, which utilizes the axisymmetric characteristic obtained from Chapter III.
II
MODELING APPROACHES FOR ROTATING STALL

This chapter provides the initial motivation for the extensive discussions regarding compressor characteristics that follow in subsequent chapters. It also gives necessary background for predicting stall performance by describing two models for rotating stall: the parallel compressor concept first proposed by Day, Greitzer and Cumpsty [3], and an approach by Moore [5][6]. Motivation for the analysis, along with a derivation including the assumptions and required inputs are presented for each modeling procedure. The assumptions and necessary "givens" are outlined in detail so that one may apply these modeling approaches to a specific test compressor. The differences and similarities of the models are also described.

2.1 Parallel Compressor Model

The parallel compressor model was first employed by Day, Greitzer and Cumpsty [3] in an attempt to quantify rotating stall performance. This model originated from the experimental observation that a compressor operating in rotating stall appears to have two distinct regions around the annulus in the r-θ plane as shown in Fig. 2.1. One region consists of high mass flow or un stalled blade passages, while the remaining portion of the annulus contains low or reversed flow. This pattern extends axially through a multi-stage machine. The approach assumes that the un stalled and stalled regions each have constant flows, i.e. the flow rates do not depend on the circumferential or
radial position within the two regions. The Day, Greitzer and Cumpsty model makes the following additional assumptions:

1. Compressibility effects are neglected.
2. Radial flows are neglected.
3. The flow in the stall cell is identically zero.
4. The dimensionless exit static to inlet total pressure rise, $\psi_{TS}$, at zero flow is the empirical constant 0.11 per stage.
5. At the compressor exit, the average static pressure in the stalled region is equal to the average static pressure in the stall cell.
6. At the compressor inlet, the average total pressure in the unstalled portion is equal to the average total pressure in the stall cell.

Assumptions (3), (4), (5) and (6) are justified in reference [3]. Assumption (1) is used throughout this thesis except where noted. Rotating stall is not well understood even for incompressible flow, so that this approach seems to be a reasonable starting point in studying stall stagnation. The author believes that conclusions drawn from the incompressible study can be a useful adjunct to compressible results from high speed compressors.

An extended and more practical version of the parallel compressor model would include information about the compressor in the form of the unstalled and reversed flow pumping characteristics, and the upstream flow field. This approach can be formulated without assumptions (3) and (4) but would include the additional assumptions:

- Upstream flow field of the compressor behaves as a potential flow.
- Size of the stall cell is given.
Appendix A provides the derivation. The necessary givens are:

1. Unstalled compressor characteristic, $\psi_{TS} = F_u(\phi)$.
2. Backflow compressor characteristic, $\psi_{TS} = F_s(\phi)$.
3. Downstream throttle setting or far upstream flow rate (i.e. the average flow).
4. Size of the stall cell, $\alpha$.

Using assumptions (1), (5) and (6) of the Day, Greitzer and Cumpsty model, the rotating stall performance can then be determined.

The problem with this modeling procedure is that both the downstream throttle setting and the size of the stall cell must be specified as inputs. In an actual compressor test, a given throttle setting will also determine the size of the stall cell. These two parameters are treated as independent quantities in this extended version of the parallel compressor model, however they are coupled in the actual case. In order to use this approach for prediction purposes, it would be necessary to calculate the stall cell size by some procedure, and determine the unstalled and backflow pumping characteristics from compressor design parameters. Much work has been devoted to obtaining the unstalled performance, while predicting the reversed flow characteristic has not been accomplished. A simplified prediction method for backflow will be discussed subsequently.

The required calculation of the stall cell size remains as the primary barrier to applying the parallel compressor model as a prediction scheme for rotating stall. This difficulty arises since the modeling approach is not derived from fundamental laws, but instead based on
particular observations.

2.2 **Analysis of Moore** [5][6]

Prof. F.K. Moore of Cornell University developed a new approach to the problem of predicting compressor performance in rotating stall. Since it is found experimentally that the state of the compressor in rotating stall is determined by the downstream throttle setting and by the compressor geometry, Moore attacked the problem from this point of view. His theory finds the permissible flow field in the compressor by setting the average mass flow (i.e. imposing the downstream throttle), and restricting only that the pressure field far upstream and far downstream of the compressor is circumferentially uniform.

The flow field is divided into three regions, namely, the entrance region to the compressor (upstream), the compressor itself, and an exit portion (downstream). The entrance region is treated as a potential flow. For a two-dimensional inlet region, this implies the following relation between the circumferential and axial velocity perturbations at the compressor inlet, $h(\theta)$ and $g(\theta)$, respectively:

$$h(\theta) = -\frac{1}{\pi} \int_0^{2\pi} g'(\xi) \ln|\sin\frac{\theta-\xi}{2}| d\xi.$$  \hspace{1cm} (2.1)

However, this expression can be approximated by

$$h'(\theta) = -g(\theta).$$  \hspace{1cm} (2.2)

Comments and justification regarding this simplification are given in [6].

In Moore's model, the pressure rise across the compressor at a given circumferential position is expressed as a local axisymmetric
pressure rise plus a dynamic correction term since the flow is unsteady relative to the blade rows. This dynamic term is a one-dimensional correction, and the derivation is provided in Appendix B. This pressure rise across a blade passage is determined from the one-dimensional form of the momentum equation:

\[ \left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{blade passage}} = F(\phi) - \tau \frac{\partial \phi}{\partial t}. \]  \hspace{1cm} (B.4)

\( F(\phi) \) is the axisymmetric pressure rise at \( \phi \), and \( \tau \) is found from the inertial considerations in the momentum equation and is modified by the empirical constant, \( k \). Or by equation (B.5),

\[ \tau = \frac{2c}{U \cos \gamma} k. \]  \hspace{1cm} (B.5)

For \( k = 1 \), only inertial effects are included for the pressure rise across a blade passage. For \( k > 1 \), lag phenomena can be included in the compressor model.

Moore does not discuss how specific lag processes can be included in the lag parameter, \( k \). However, a lag process will be discussed in detail here. For example, when \( \phi \) changes rapidly as in the transition regions between the stall cell and unstalled flow, the axisymmetric blade passage performance lags the local flow coefficient, \( \phi \). As a result, the actual axisymmetric pressure rise may be approximated very simply by \( F(\phi - K \frac{\partial \phi}{\partial t}) \), where \( K \) is a viscous lag constant. This viscous lag effect can be justified by examining the time durations
associated with the blade passage stalling rate, and the propagation rate of a stall cell.

The blade passage stalling rate time scale, \( \tau_s \), is derived first. \( \tau_s \) is assumed to be the time required to shed a vortex through the blade passage, so that \( \tau_s \) can be approximated by:

\[
\tau_s \approx \frac{\text{blade chord}}{\text{flow through velocity}} = \frac{\ell \cos \gamma}{|\phi_{avg}| U}, \tag{2.3}
\]

where

\[ \phi_{avg} = \text{average value for } \phi \text{ encountered by the blade passage in the transition region between unstalled and stalled flow.} \]

The rotating stall propagation time scale, \( \tau_{R/S} \), is defined as,

\[
\tau_{R/S} \equiv \frac{\text{stall cell size}}{\text{stall cell speed}} = \frac{\pi Db}{a},
\]

where

\[ b = \text{fractional size of the stalled or unstalled region,} \]

\[ a = \text{stall cell speed relative to a blade row, or} \]

\[ a = \pi U \text{ for a stator, and} \]

\[ a = (1-f)U \text{ for a rotor.} \]

The rotor row is the limiting case since \( a_{\text{rotor}} > a_{\text{stator}} \), which results in \( (\tau_{R/S})_{\text{rotor}} < (\tau_{R/S})_{\text{stator}} \). Thus, \( \tau_{R/S} \) is defined as \( \tau_{R/S} = (\tau_{R/S})_{\text{rotor}}, \) or

\[
\tau_{R/S} = \frac{\pi Db}{(1-f)U}. \tag{2.4}
\]
The ratio of \( \tau_s \) to \( \tau_{\text{R/S}} \) is

\[
\frac{\tau_s}{\tau_{\text{R/S}}} = \frac{(1 - f) \cos \gamma}{\pi \phi_{\text{avg}} (D/\ell)} \cdot \frac{1}{b}.
\] (2.5)

By considering only realistic values for \( D/\ell, f, \gamma \) and \( \phi_{\text{avg}} \), the stall cell size, \( b \), has the largest effect on the ratio, \( \tau_s/\tau_{\text{R/S}} \), since \( 0 < b \leq 1 \). For this case, \( \tau_s/\tau_{\text{R/S}} \) is plotted against the stall cell size, \( b \), in Fig. 2.2 for a representative case (\( D/\ell = 12, f = 0.30, \gamma = 35^\circ, \phi_{\text{avg}} = 0.3 \)). Also notice that \( \gamma \) and \( \phi_{\text{avg}} \) are not independent in the actual compressor because as \( \phi \) increases (i.e. \( \phi^* \) decreases), then \( \phi_{\text{avg}} \) decreases. From Fig. 2.2, for stall cells occupying less than about 10% of the compressor annulus, \( \tau_s \) and \( \tau_{\text{R/S}} \) become the same order of magnitude. In other words, the blade passages are not given sufficient time to completely stall as a stall cell which encompasses less than 10% of the annulus passes through them. Data from Day [8], and Das and Jiang [9] suggest rather small transition regions between the unstalled and stalled flow zones, which remain less than the approximate 10% size when \( \tau_s/\tau_{\text{R/S}} \to 1 \). Therefore, the viscous lag effect is substantiated, because the blade passages will not respond fast enough as \( \phi \) varies rapidly in the small transition regions.

In order to account for the viscous lag process, \( F(\phi) \) in equation (B.4) is replaced by \( F(\phi - K \frac{\partial \phi}{\partial t}) \) so that

\[
\left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{blade row}} = F(\phi - K \frac{\partial \phi}{\partial t}) - \tau^t \frac{\partial \phi}{\partial t}.
\]
where

\[ \tau' = \tau \bigg|_{k=1} \]

Expanding \( F(\phi - K \frac{\partial \phi}{\partial t}) \) in a Taylor series gives

\[
F(\phi - K \frac{\partial \phi}{\partial t}) = F(\phi) - \frac{dF}{d\phi} \cdot K \cdot \frac{\partial \phi}{\partial t} + \frac{1}{2} \frac{d^2F}{d\phi^2} \cdot K^2 \left( \frac{\partial \phi}{\partial t} \right)^2 \ldots
\]

To first order in \( K \),

\[
\left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{blade row}} = F(\phi) - (\tau' + K \frac{dF}{d\phi}) \cdot \frac{\partial \phi}{\partial t}
\]

(2.6)

Section 3.1 shows that \( \frac{dF}{d\phi} \) is positive for most of the transition region. Then, \( \tau \) can replace \( \tau' + K \frac{dF}{d\phi} \) in equation (2.6) with \( k > 1 \) thus, equation (8.4) can account for the viscous lag process. Moore concluded that \( k = 2 \) approximates the actual situation in a multi-stage compressor [5].

In the exit region, a linearized description of the flow field is used [10]. Moore considers two extreme compressor exit geometries, which are described by the empirical parameter, \( m \):

- \( m=1 \) corresponds to a sudden expansion,
- \( m=2 \) corresponds to a long straight duct.

It is important to realize that \( m \) is not rigidly bounded by \( 1 \leq m \leq 2 \), because a linearized exit flow is assumed for the non-linear flow associated with a rotating stall.
A pressure match for the three regions is performed by imposing circumferentially uniform pressures far upstream and far downstream. This results in the following differential equation for the inlet flow perturbations as a function of $\theta$:

$$g'(\theta) = \frac{1}{\lambda}[\psi_c(\phi) - \psi + mfh] \quad (2.7)$$

where,

$$\lambda = \frac{2k}{(D/c)} \left[ \frac{N}{\cos \gamma} - \left( \frac{2N}{\cos \gamma} + \frac{1}{\cos \gamma_{IGN}} + \frac{1}{\cos \gamma_{OGV}} \right) f \right] \quad (2.8)$$

Additional conditions are needed with equations (2.1) or (2.2), (2.7) and (2.8) to find the rotating stall performance, $\psi$. Periodicity must occur in the annulus, i.e.

$$g(\theta) = g(\theta + 2\pi), \ h(\theta) = h(\theta + 2\pi). \quad (2.9)$$

Continuity requires that

$$\int_{0}^{2\pi} g(\theta)d\theta = 0, \quad (2.10)$$

and since the upstream flow field is assumed to be irrotational, Kelvin's theorem implies that

$$\int_{0}^{2\pi} h(\theta)d\theta = 0 \quad (2.11)$$

at the compressor inlet.
Therefore, equations (2.1) or (2.2), (2.7) and (2.8) with the constraints (2.9), (2.10) and (2.11) provide the rotating stall performance.

A list of all the assumptions made in the analysis are:

1. Potential flow in the entrance region.
2. The Hilbert transform of equation (2.1) can be approximated by equation (2.2).
3. Straight entrance duct.
4. Radial flows are neglected.
5. Compressor blading has 50% reaction, i.e. the blading is symmetric.
6. In the compressor model, all lag processes are described by a first order lag as in equation (B.4).
7. The pressure rise across a compressor blade passage is unaffected by the neighboring blade passages.

Assumptions (2) and (3) could be relaxed to obtain a more elegant solution, while greatly complicating the numerical solution procedure. Assumption (5) is not necessary but is included for simplicity. Assumption (6) relates to the approximation of the Taylor series expansion for $F(\phi - K \frac{3\phi}{3t})$ in equation (2.6) when modeling the viscous lag phenomenon. In this series expansion, only terms to first order in the viscous lag constant, $K$, were included in equation (2.6), since $K$ is assumed small enough to neglect any higher order terms. If $K$ is not small enough, additional terms could be added in equation (B.4), which models the blade row performance. Assumption (7) also results from the one-dimension dynamic correction term in the compressor model.

The required inputs to Moore's model are:
1. Compressor parameters: \( N, D/c, \gamma, \gamma_{IGV}, \gamma_{OGV} \).

2. Axisymmetric compressor characteristic over the entire flow regime.

3. Empirical compressor exit geometry parameter, \( m \).

4. Empirical lag parameter, \( k \).

5. Downstream throttle setting or the average flow coefficient, \( \phi \).

The complete axisymmetric characteristic is partitioned into three regions in Fig. 2.3. Region I is the stalled pumping characteristic with the division line drawn between regions I and III at the compressor stall point. This is the point at which the axisymmetric flow becomes unstable in the compressor annulus, resulting in either a surge or a non-axisymmetric, rotating stall. Further discussion of the stall point is provided in Sections 3.2 and 3.3. Region II is axisymmetric reversed flow, where the compressor behaves as a throttling device, i.e. at larger backflows the compressor suffers greater pressure drops. Region III is designated as low forward flow, and for a portion of the region the characteristic is shown dashed because axisymmetric flow is unstable for these flow rates. This unstable behavior could result from an overall compression system (compressor, plenum, throttle) instability, or from a compressor flow field instability, or from both of these factors. According to Fig. 2.3, the stable, steady-state mode for most of Region III is the non-axisymmetric, rotating stall. The axisymmetric characteristic in this region has not been measured experimentally because of its unstable nature, and thus has remained undetermined. The axisymmetric curve will be discussed further in later
chapters, along with identification of the unknown portion and prediction methods.

2.3 A Comparison of the Two Approaches

The most apparent difference in these two modeling procedures is that no information about the resultant flow field in rotating stall is needed by the Moore model, whereas the parallel compressor concept requires the stall cell size or the pressure level as an input. Also, the parallel compressor model approximates the actual axial flow coefficient, $\phi$, around the annulus in rotating stall as a square wave. The unstalled and stalled regions have constant flow coefficients, $\phi_u$ and $\phi_s$, respectively, as shown in Fig. 2.4. This may be an adequate representation since the transition regions between the stalled and unstalled flow are steep as experimentation suggests. However, Moore's approach makes no such approximation, and a representative axial velocity profile in rotating stall, calculated by the method described in Chapter V, is shown in Fig. 2.5. This profile indicates steep transition regions, but with non-constant flow coefficients in the unstalled portion and in the stall cell. This observation is discussed further in Chapter V.

Moore's approach can be thought of as an extension of the parallel compressor model. Each blade passage in the Moore model is treated as an individual compressor working in parallel with the other blade passages, or "compressors" around the annulus. The pressure contribution due to each "compressor" (axisymmetric part plus dynamic part) is added around the annulus considering the upstream and downstream boundary conditions. Both models divide the compressor annulus
into a grid-like form. The simple parallel compressor model involves a coarse grid of only two segments, whereas Moore's approach includes a fine grid of many divisions. Consequently, the Moore model should provide greater detailed information about compressor performance in rotating stall than does the two-segment parallel compressor concept.

Summary

Both the simple parallel compressor model and the Moore analysis can be used to model the rotating stall phenomenon. The primary constraint with the two-segment parallel compressor concept involves inputting the stall cell size. The Moore model eludes this difficulty, but requires an axisymmetric characteristic which remains unknown under certain compressor flow conditions. However, the work presented in the next chapter eradicates this barrier. Also, Moore's analysis includes viscous lag processes in his compressor model. These effects may be important, since it is shown that for stall cells less than about 10% of the compressor annulus, the blade passages are not given sufficient time to completely stall by the time scaling analysis. Both models demand a prediction of the reversed flow characteristic, and a proposed method is provided in Chapter IV. The Moore model could be considered as a multi-segment parallel compressor, and should more closely approximate the actual case than does the two-segment parallel compressor model.
III
COMPRESSOR PERFORMANCE CHARACTERISTICS

This chapter describes the steady-state compressor characteristics for two distinct flow modes: axisymmetric flow and full-span rotating stall flow. The unknown portion of the axisymmetric characteristic mentioned in the previous chapters is estimated from transient compressor data. From the surge cycle experiments by Greitzer [1], it is possible to obtain the axisymmetric characteristic in the low forward flow region for his three-stage test compressor. Other compressor geometries, i.e. different unstalled characteristics, are considered with their respective axisymmetric curves. Certain compressor stall behavior can be extracted from these characteristics. In the last section, the rotating stall flow mode is discussed. Hypotheses regarding recovery from stall are proposed which leads to the extrapolation of the full-span rotating stall characteristic for stall cell sizes less than 30% of the annulus area. Also, possible methods which may improve recoverability are provided.

3.1 Unknown Portion of the Axisymmetric Characteristic

The axisymmetric characteristic in region III of Fig. 2.3 has remained undetermined since the compressor will invariably operate in a steady-state rotating stall mode for these flow rates, because the axisymmetric flow is unstable. In order to directly measure the steady-state axisymmetric curve for region III, it is necessary to maintain a constant low mass flow around the annulus, and then record the axisymmetric pressure rise. This technique might be implemented
for portions of the characteristic by placing high-loss screens at the inlet and exit of the test compressor, which act to suppress rotating stall [3]. However, these screens may restrict the rotating stall mode for only a small portion of the low forward flow region for some compressor geometries. Day, Greitzer and Cumpsty [3] showed that part-span stall can be induced by the addition of a screen at the compressor exit. Their screen postponed the full-span rotating stall flow, however an axisymmetric flow was not obtained in their case. Similar behavior is observed when the rear stages choke in a multi-stage compressor at part speed. They act like an exit screen for the front stages which are driven into a part-span stall mode. Thus, the axisymmetric characteristic may not be attainable directly by adding screens to the compressor, so that another procedure for obtaining the unstable, axisymmetric pumping performance must be formulated.

An alternative, indirect method for providing this characteristic involves inertially correcting transient compressor data. More specifically, this approach "subtracts off" the accelerations (or decelerations) of the flow during a compressor surge cycle, which is assumed to be axisymmetric. This surge cycle consists of highly accelerating and decelerating flows in the low forward flow region, an axisymmetric reversed flow compressor blow-down, and an axisymmetric plenum pressurization. The inertial correction process results in a quasi-steady axisymmetric characteristic.

The calculation described is possible, since the individual blade row performance can respond quickly with the changes in flow rate encountered in a surge cycle. For example, the time duration in order
to stall or un stall a blade passage is assumed to be on the order of the time needed to shed a vortex through the blade passage as provided in Section 2.2 by

$$\tau_s = \frac{\ell \cos \gamma}{|\phi_{avg}|U}.$$  \hspace{1cm} (2.3)

From the surge cycle experiments by Greitzer [1], a representative surge time scale, \(\tau_{\text{surge}}\), is assigned as the time difference at which the data were recorded, or \(\tau_{\text{surge}} = 25\) msec. This proves sufficient since the highly accelerating and decelerating portions of the cycle persisted for about 125 msec. In other words, at least 5 data points were recorded in these highly transient regions. This is evident in Fig. 3.1, which shows a sample trace of nondimensionalized compressor mass flow versus time from [1]. The data points are shown in the figure, and the time elapsed between each point is 25 msec. From Greitzer's test rig (\(l = 1.8\) in., \(\gamma = 30^\circ\), \(\phi_{avg} = 0.3\)) at the rotor speed of the sample mass flow plot of \(U = 249\) ft/sec, the estimated blade row response time from equation (2.3) is \(\tau_s = 1.7\) msec. Thus, a quasi-steady characteristic can be determined by inertially correcting compressor surge cycle data, since the surge time scale, \(\tau_{\text{surge}}\), is an order of magnitude greater than the blade passage response time, \(\tau_s\).

In order to deduce a quasi-steady axisymmetric characteristic from surge data, it is necessary to have an axisymmetric flow for at least a portion of the highly transient region of the surge cycle. As the compressor state passes transiently through the low forward flow region, a stall cell would tend to form since rotating stall is
the stable, steady-state mode for these flow rates. However, when the rate of mass flow change through the compressor is larger than the rate of rotating stall formation, an axisymmetric flow would be maintained for a portion of the transient surge region. A rotating stall formation time scale, $\tau_{\text{form}}$, is difficult to quantify since little is known about the dynamics of stall cell formation. However, $\tau_{\text{form}}$ is assumed to be on the order of 2 rotor revolutions, which is justified to some extent by Greitzer in [11]. Or,

$$\tau_{\text{form}} \approx \frac{2\pi D}{U}$$  \hspace{1cm} (3.1)

An axisymmetric flow will be preserved over part of the transient region when the surge time scale, $\tau_{\text{surge}}$, is less than $\tau_{\text{form}}$. For Greitzer's compressor ($D = 1.70$ ft) at $U = 249$ ft/sec from the sample mass flow plot in Fig. 3.1, equation (3.1) estimates that $\tau_{\text{form}} = 43$ msec. Thus, Greitzer's surge data may include axisymmetric flow since $\tau_{\text{surge}} < \tau_{\text{form}}$. But, it is still necessary to examine the time resolved data to check this condition, because of the uncertainty of $\tau_{\text{form}}$.

These time scale arguments maintain that $\tau_{\text{surge}}$ must be bracketed as $\tau_s < \tau_{\text{surge}} < \tau_{\text{form}}$ to obtain an axisymmetric characteristic by inertially correcting compressor surge data. Also, it is important to re-iterate that $\tau_s$ and $\tau_{\text{surge}}$ are more closely approximated than the uncertain $\tau_{\text{form}}$.

When $\tau_{\text{surge}}$ is bracketed properly, the unstable portion of the axisymmetric characteristic is found by adding a pressure correction term, due to the fluid acceleration in the compressor, to the transient
pressure rise of a surge cycle. Both the quasi-steady axisymmetric curve and the surge data pressure rise are referred to ambient pressure with a compressor exit static to compressor inlet total pressure rise, $\Delta P_{TS}$. The surge cycle data were recorded as plenum pressure referred to ambient which closely approximates an exit static to inlet total pressure rise in the forward flow case. But for reversed flow, the plenum pressure corresponds to an exit total to inlet static pressure rise [12] as derived in Appendix C. Fig. 3.2 shows the compressor model as a duct of length $L_c$ and area $A_c$ with the pressure rise across the compressor occurring over the actuator disk [11]. Assuming an axisymmetric surge cycle so that the duct flow can be modeled as a one-dimensional flow with negligible compressibility effects and of constant cross-sectional area, then $C_x = C_x(t)$. The inertial force associated with the slug of fluid in the duct is

$$F_{\text{inertial}} = \rho A_c L_c \frac{dC_x}{dt}$$

where

$$F_{\text{inertial}} = \Delta P_{\text{inertial}} \cdot A_c.$$ 

By eliminating $F_{\text{inertial}}$ and re-arranging yields

$$\left( -\Delta P \right) \left( \frac{1}{2} \rho U^2 \right)_{\text{correction}} = \frac{2L_c}{U} \cdot \frac{d\phi}{dt}$$

Therefore,

$$\psi_{TS}^{\text{axisymmetric}} = \psi_{TS}^{\text{surge data}} \quad + \quad \frac{2L_c}{U} \cdot \frac{d\phi}{dt} \quad (3.2)$$

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Fig. 3.3a shows a sample of Greitzer's surge data with the inertially corrected characteristics provided by equation (3.2). Quasi-steady characteristics 1 and 2 are found from correcting the accelerating and decelerating portions of the surge cycle, respectively. This correction term of equation (3.2) involves $d\phi/dt$ of the measured surge data. $d\phi/dt$ is calculated simply by a central difference and is performed on two of the cycles. Although this is a crude approach, $d\phi/dt$ does compare favorably for different cycles, and the central difference approach seems adequate in calculating the first derivative of the nondimensionalized compressor mass flow.

Greitzer's time resolved data suggested traces of rotating stall towards the end of the highly transient regions as indicated in Fig. 3.3a, although the data are not straightforward to interpret. Consequently, the inertially corrected curves in these regions may not be correct since equation (3.2) is derived for axisymmetric flow.

Equation (3.2) is also applied for three other surge cycles from Greitzer's three-stage compressor shown in Figs. 3.3b, 3.4 and 3.5. The surge cycles in Figs. 3.3a and 3.3b were recorded at the same mean rotor speed, but they are slightly different because the throttle setting was not the same for these two data sets. Three sets of inertially corrected characteristics from surge cycles at various mean rotor speeds in Figs. 3.3a, 3.4 and 3.5, are plotted together along with the unstalled compressor performance in Fig. 3.6. Two unique sets of quasi-steady curves seem to result from the three distinct surge cycles. Again, the portions of the quasi-steady characteristics that
are assumed invalid due to a non-axisymmetric flow are shown in the figure. However, the corrected characteristic on the decelerating portion of the surge cycle in Fig. 3.4 seems to deviate slightly from the other corrected curves for this portion. This may be attributed to the large decelerations of the surge in Fig. 3.4 which results in a large surge data correction, thus prone to greater error in the inertially corrected curve.

It was expected that the surge cycle of the lowest rotor speed, U = 194 ft/sec, would provide more of the axisymmetric characteristic calculated from the inertial correction process than the higher speed cases. This would come about since $\tau_{\text{form}}$ increases for lower rotor speeds as evidenced in equation (3.1), and also the accelerations are slightly larger in the lowest rotor speed case ($\tau_{\text{surge}}$ would decrease slightly). Both of these factors would tend to maintain an axisymmetric flow for a longer portion of the highly transient surge regions. Although it is unclear whether this can be deduced from the corrected characteristics in Fig. 3.6.

For the three rotor speed cases in Fig. 3.6, 1.4 $\leq \tau_s \leq$ 2.2 msec, which implies that the blade passage response time is an order of magnitude lower than the surge time scale, $\tau_{\text{surge}} = 25$ msec. Recall that $\tau_{\text{surge}}$ is a weak function of rotor speed, but this relationship is ignored by choosing a constant $\tau_{\text{surge}}$. In Fig. 3.6, 35 $\leq \tau_{\text{form}} \leq$ 55 msec, for the three cases. Therefore, $\tau_{\text{surge}}$ is bracketed properly between $\tau_s$ and $\tau_{\text{form}}$ for each rotor speed case from Greitzer's data.
It remains necessary to specify a procedure which yields an axisymmetric compressor characteristic from the corrected curves of Fig. 3.6. In this context, it is helpful to establish an analogy between axisymmetric compressor behavior and planar diffuser performance. Koch [13] inferred that compressor stall is often triggered by boundary layer growth at the compressor casing or hub (wall stall), rather than by flow separation from the cascades (blade stall). In this case, compressor stall parallels planar diffuser stall. As diffusers begin to stall, they do not exhibit any sudden drop in pressure rise. In fact, their pressure recovery coefficient, $\tilde{C}_p$, continues to increase when the first stall mode, transitory stall, is encountered as seen in Fig. 3.7 [14]. Similarly, if one could suppress the rotating stall mode in a compressor and maintain axisymmetric flow, the compressor might operate along its axisymmetric characteristic with no sudden decrease in pressure rise. However, one might suspect that a blade stall could initiate at a lower compressor flow rate than when the wall stall occurred. In this case, a sudden drop in pressure rise would be expected since thin airfoil stall exhibits this property. But blade stall seems improbable after a wall stall formed, because the core flow associated with the wall stall must be larger than the average flow at that stalled state. The core flow would tend to provide favorable incidences on the cascades, thus seemingly eliminating the chance of a blade stall. Therefore, planar diffuser stall could be analogous to axisymmetric compressor stall, since an axisymmetric stall is most likely composed of only end wall stall.
If the axisymmetric characteristic is diffuser like, it should be single-valued with respect to mass flow. No hysteresis exists in the planar diffuser until a jet flow occurs at very low performance shown in Fig. 3.7. But, the diffuser analogy is primarily concerned with the transitory stall regime, so that the jet flow mode seen at very large opening angles has little significance. From the analogy, the author maintains that the axisymmetric curve in low forward flow is free of any hysteresis, i.e. it is independent of the rate of change of mass flow.

Assuming that no hysteresis occurs in the axisymmetric characteristic, the inertially corrected curves for both the accelerating and decelerating regions of a completely axisymmetric surge cycle, should degenerate into a single quasi-steady, axisymmetric characteristic. For a surge cycle which is not totally axisymmetric as in Greitzer's surge data, the unstable axisymmetric curve is constructed by connecting the valid portions of the inertially corrected characteristics. From Greitzer's data, the corrected curves for the decelerating portion of the surge cycles should coincide with the stalled performance for $\phi > 0.48$ (see Fig. 3.6). The slight error in the corrected characteristics is believed to be introduced from the inertial correction process. The axisymmetric pumping characteristic is assumed to smoothly continue the stalled curve past the stall point as the quasi-steady curves suggest. These corrected curves drop off in pressure rise around $\phi \sim 0.4$, and the unstable axisymmetric performance is drawn accordingly in Fig. 3.6. The complete axisymmetric characteristic for Greitzer's compressor is given in Fig. 3.8.
3.2 Complete Axisymmetric Curves for Other Compressors

This section will illustrate the axisymmetric performance for different compressor geometries. Due to a lack of surge cycle data, the calculation procedure for the unstable axisymmetric characteristic posed in the last section has not been implemented for other compressors. However, many features of the axisymmetric curve proposed for Greitzer's test compressor can be generalized for various compressor builds.

The proposed axisymmetric characteristic for Greitzer's compressor exhibits the following properties:

1. No sudden drop in pressure rise exists.

2. The complete characteristic is smooth, i.e. the first derivative is continuous.

3. The stall point is at the maximum pressure rise on a $\psi_{TS}(\phi)$ plot. The author chooses to call this maximum point the axisymmetric "peak" point.

4. A relative minimum point of the characteristic occurs at shut-off, i.e. zero mass flow. The author refers to this minimum point as the axisymmetric "valley" point.

5. The unstable portion between the axisymmetric peak and valley is steeply, positively sloped.

Properties (1), (2) and (5) are believed to be true in general for the axisymmetric characteristic of any multi-stage compressor. The planar diffuser analogy presented in the previous section allows for the generalization of properties (1) and (2). Property (5) encourages an unstable compression system (compressor, plenum, and throttle) [11], as well as an unstable axisymmetric flow for a stable system. This is consistent with the experience of post-stall compressor tests, where a rotating stall (or surge) is always observed in region III (Fig. 2.3) except near
shut-off.* Hence, this unstable portion of the axisymmetric characteristic should be steeply, positively sloped for every compressor. Properties (3) and (4) are not true in general. The axisymmetric curve will still consist of a peak and a valley, but the compressor stall point could be before the maximum point and the valley point may not occur at zero flow.

Many compressors appear to have stall points on the strongly, negatively sloped portions of their characteristics. The unstable region of their axisymmetric curves must rise in performance above the observed stall point, which maintains that the maximum point, or the peak of the characteristic occurs at a greater pressure rise than the compressor stall point, as in Fig. 3.9. In this case, a compressor instability first exists at the stall inception point. As the stall point is approached on the speedline, the axisymmetric flow in the compressor annulus becomes unstable since the flow velocity perturbations in the annulus become amplified [7]. The compressor stalls in a non-axisymmetric fashion, which locally provides a zero-slope on the characteristic, thus driving the system dynamically unstable.** If an axisymmetric flow could be sustained in the compressor past its stall point, the system would not go unstable until point A is reached in Fig. 3.9. Thus, compressors of this type have stall points which are governed by a compressor instability. Compressor and system stability are addressed in the next section regarding the recovery point of a stall stagnation.

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*The axisymmetric stall can become stable near zero flow when the characteristic flattens or is negatively sloped.

**Appendix D gives the compression system stability criteria [11].

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The notion that the maximum axisymmetric performance can be at a higher pressure rise than the compressor stall point is supported by some data of Greitzer [15]. He placed a screen at the compressor inlet. This maintained axisymmetric flow past the stall point without the screen, by suppressing the rotating stall mode. Fig. 3.10 illustrates that the compressor performance with the screen increased beyond its previous stall point in a smooth, continuous fashion. The screen provided a portion of the previously unstable axisymmetric characteristic for the compressor, proving that the stall point can occur before the maximum point of the axisymmetric curve.

The planar diffuser analogy can give additional insight into this phenomenon involving the axisymmetric characteristic. Recall from Section 3.1 that as the diffuser entered the transitory stall mode its pressure recovery coefficient, \( \bar{C}_p \), continued to rise, shown in Fig. 3.7. This diffuser behavior parallels the axisymmetric compressor performance, in that compressor stall inception can occur before the peak of the axisymmetric characteristic. For diffusers, gross unsteadiness occurs in the transitory stall regime in a cyclic manner as described by Kline [14]. This cycle involves the formation of a stall region on a side wall and then the elimination of this stall zone. In spite of this stalled flow, the diffuser performance peaks! No sudden drop in \( \bar{C}_p \) is experienced as shown in Fig. 3.7, but rather smooth, continuous behavior results. Similarly, near the compressor stall point, unsteadiness can be experienced in the annulus before the axisymmetric peak. However, this unsteadiness in the
compressor annulus could trigger a flow mode change into a rotating stall state, which usually results in a sudden drop in pressure rise. If the rotating stall mode was not induced, the compressor might operate in the axisymmetric flow mode. Thus, no sudden drop-off in pressure rise would be experienced in this case, only operation along the axisymmetric characteristic would be observed.

The behavior seen on Day's three-stage test compressor [8], in the highest rotor and stator stagger angle case (low $\phi^*$ case), can be accounted for by the preceding observations. In this rig, Day's measured rotating stall performance occurs at nearly the same pressure rise as the stall point seen in Fig. 4.9. Fig. 3.11 shows a sketch of the compressor characteristics for Day's $\phi^* = .35$ configuration. In order to model this compressor with Moore's approach, an axisymmetric characteristic similar to the one shown in Fig. 3.11 must be specified. Moore's model will predict a rotating stall curve somewhere between the peak and the valley pressure rise of the axisymmetric performance. Since the peak of the characteristic is well above the stall point pressure rise, the Moore model can predict the rotating stall performance at the same pressure level as the stall point. If the axisymmetric characteristic were assumed to drop-off at the stall point as shown in Fig. 3.11, Moore's predicted rotating stall line would underestimate the actual performance. Thus, Moore's model predicts that a relatively high pressure rise in rotating stall compared with the stall point can be achieved when the peak of the unstable axisymmetric characteristic occurs well above the stall point pressure rise.
As seen from Day's data [8] in Fig. 4.9, the valley of the axisymmetric curve must be at some low positive flow, and not at zero flow. Greitzer's compressor of Section 3.1 had the valley of its characteristic at shut-off, and as explained earlier, this property is not generally true for other compressor builds. The blade stagger angle appears to affect the position of this valley along the flow axis as Day's experiments [8] suggest. He obtained an axisymmetric flow near shut-off for his low and intermediate design flow coefficient compressors, $\phi^* = 0.35$ and $\phi^* = 0.55$, respectively. The low $\phi^*$ case includes a negatively sloped portion of the axisymmetric characteristic near zero flow, shown in Fig. 3.11. The intermediate $\phi^*$ compressor has a flat, stable characteristic near shut-off, displayed in Fig. 3.12, which is similar to Greitzer's compressor ($\phi^* = 0.65$) in Fig. 3.8. Rotating stall was observed throughout the low forward flow region to shut-off on Day's high $\phi^*$ ($\phi^* = 1.0$) build. The author believes that the axisymmetric characteristic for this compressor is positively sloped at shut-off and therefore remains unstable. The valley of its axisymmetric curve would lie in the reversed flow regime as shown sketched in Fig. 3.13.

In order to confirm this behavior, an axisymmetric reversed flow characteristic could be measured in the laboratory by blowing flow backwards through the test compressor. This procedure allows the shut-off point to be approached from the backflow side and it would determine where the axisymmetric flow becomes unstable. Another method which would provide the axisymmetric performance in this region involves inertially correcting surge cycle data as explained in the preceding section. The
compressor parameters, namely, the blade stagger angle and the design flow coefficient, shift both the peak and the valley of the complete axisymmetric characteristic.

Properties (1), (2) and (5), discussed at the beginning of this section, are believed to be preserved in the general case. As a result, the general shape of the axisymmetric curve proposed in Section 3.1 for Greitzer's test compressor should apply to any multi-stage compressor. However, the compressor stall point could occur before the maximum axisymmetric pressure rise, while the valley of the characteristic could be in either the low forward flow or the reversed flow regions of the compressor map.

3.3 Complete Rotating Stall Characteristics

Even though the unstable axisymmetric characteristic may never be completely observed in post-stall compressor tests, it does disclose certain features regarding stall, and is useful in modeling post-stall behavior, as in Moore's analysis. Just as this unstable axisymmetric curve is an extension of the stalled and reversed flow characteristics, the full-span rotating stall characteristic can be extrapolated for stall cells smaller than 30% of the annulus and for stall cells approaching 100% of the annulus. These extended characteristics provide a different view of recoverability from rotating stall, and they also can be included in models studying the compression system response. Furthermore, schemes for possible improved recoverability are provided.
The recovery point from rotating stall parallels the stall point in that both involve a flow mode change in the compressor annulus. The author chooses to define the "true stall point" as the point at which the unstalled axisymmetric flow becomes unstable in the annulus, independent of the particular compression system. Similarly, the "true recovery point" is defined as the point when the stall cell shrinks past its critical size needed to sustain the rotating stall mode. Moore also mentions the existence of the true recovery point [6], but this idea was formulated by the author and Moore, independently.

The true stall point may or may not be measured in a compressor test since the compression system could become unstable first, and then induce a compressor instability. For example, the compressor in Fig. 3.14 stalls at the peak of its axisymmetric characteristic because the system could be dynamically unstable with the steep downstream throttle line as shown. However, the true stall point, which is a function of compressor geometry only, could exist beyond the measured stall point, and remain hidden from the compressor test. Since the unstable system could drive the compressor flow unstable at the measured stall point, it may appear as if the true stall point is measured in the experiment, when actually it is not.

Present analyses predict that a compressor flow instability will exist at

\[
\frac{d\psi_{TS}}{d\phi} = 0.
\]  

(3.3)

An explanation for this criterion is provided in [7]. Moore's model also predicts that stall inception occurs when equation (3.3) is satis-
fied [5]. Notice that Fig. 3.14 is inconsistent with this stall inception criterion, and only one of these conceptions regarding the stall point might be correct. As discussed in Section 3.2, many compressors appear to have stall points on negatively sloping portions of their characteristics, which are identified as their true stall points. This observation is also inconsistent with the criterion of equation (3.3). In addition, it was mentioned in Section 3.1 that compressor stall often results from the formation of a wall stall. However, the criterion specified by equation (3.3) is derived from two-dimensional theories which are unable to address a wall stall situation. Therefore, the stall inception criterion derived from present analyses cannot be expected to hold in an actual compressor test because of the inconsistencies cited with observed phenomenon.

For the same logic regarding the stall inception point, the measured recovery point from rotating stall may not coincide with the true recovery point. The well-known correlation proposed by Day, Greitzer and Cumpsty maintains that the critical stall cell size is approximately 30% of the annulus at recovery [3]. This point may be governed by a system instability rather than a local, compressor instability. All of the low speed stall data known to the author have positively sloped rotating stall characteristics \( \psi_{TS} \) vs. \( \phi \) at the measured recovery point. This includes the data collected by Day [8]. Every one of his compressor builds exhibited this property at recovery. However, his three-stage, high \( \phi \) build seemed to have a slightly negatively sloped rotating stall line at recovery. These compressor data are shown in Fig. 3.15. Close examination of his three-stage
rig in rotating stall reveals that the data point just before recovery and at recovery form a positive slope here, when Day's line which connects the data points is ignored. This supports the observation of positive sloping rotating stall curves at the recovery point. Thus, the system stability criteria may be violated at the measured recovery point, because of the positively sloped characteristic.

A negatively sloped rotating stall curve at recovery would support that recovery is due to a compressor instability, since the compression system remains stable under these conditions. This is consistent with the author's statement in Section 3.2, commenting that the compressor stall point on a negatively sloped portion of the unstalled characteristic results from the unstable axisymmetric flow in the compressor annulus. However, the rotating stall line is positively sloped at recovery, which maintains that the compression system could always be unstable at recovery. There seems to be no strong evidence on the basis of characteristics which can support that the recovery point is governed by a compressor instability.

One possible mechanism for the true recovery point involves the response lag time of a blade passage subjected to the unsteady, rotating stall flow. Section 2.2 discussed the time scales of the stall cell propagation and the blade passage stalling rate. The plot of $\tau_s/\tau_{R/S}$ versus $b$ in Fig. 2.2, implied that the cascades may not respond fast enough for stall cells, or unstalled regions, of less than approximately 10% of the annulus. In other words, the blade passages are not given sufficient time to completely stall as the stall cell size drops below
its critical value. This critical stall cell size depends on the compressor geometry, and on the basis of Fig. 2.2, it remains lower than the 30% size seen at recovery in laboratory tests. Day recorded uninstalled regions occupying a minimum of about 10% of the annulus near shut-off in rotating stall [8]. This evidence suggests the possibility of stall cell sizes on the order of 10%, especially based on the unsteadiness mechanism for true recovery. From this unsteadiness mechanism, true recovery would occur for stall cell sizes less than the classic 30% size. Therefore, the positively sloped characteristics and the unsteadiness mechanism both support that the measured recovery point is due to a compression system instability rather than the unstable nature of the rotating stall flow.

It should be re-iterated that this unsteadiness mechanism for true recovery is only one possible explanation. An alternate view of true recovery would deal with the flow field incompatibility between the stalled and the uninstalled zones. Again from Day's data which reported a 10% size for uninstalled regions near shut-off, one may suspect that the flow field for a 10% stall cell size might also be compatible. This is pure conjecture since there is very little known regarding the critical stall cell size for true recovery. The Moore model predicts a recovery point which must result from a compressor flow instability because his analysis does not include any compression system considerations. However, the physical mechanism for Moore's predicted recovery point is unclear to the author. It may be due to the unsteady effects associated with Moore's parameter \( \tau \) (given by equation (B.5)), or due to
a flow field incompatibility. Further examination of Moore's recovery prediction is clearly necessary.

There is evidence which supports that recovery from rotating stall is governed by a system instability. In this context, the true recovery point, which depends only on compressor geometry, must lie along the extrapolated, full-span rotating stall characteristic. This curve is constructed with the following assumptions:

1. The slope of the rotating stall characteristic increases just after the measured recovery point.

2. There is only one compressor operating point for each stall cell size, i.e. no hysteresis exists in the complete rotating stall characteristic.

Assumption (1) encourages an unstable compression system at the measured recovery point, which is consistent with the author's hypothesis of recovery. All known rotating stall experiments suggest the validity of assumption (2). For a given compressor test, one unique rotating stall characteristic is always observed for increasing or decreasing flow rates. Any hysteresis in the experiment results from a flow mode change (stall inception and recovery) rather than some double-valued rotating stall curve. Equivalently, experiments maintain that a rotating stall state is independent of the path in which it formed. From these two properties, an extrapolated rotating stall curve is formulated in Fig. 3.16. True recovery is shown in the figure for a stall cell size of 10%. Assumption (2) provides the continuous nature of the compressor characteristic at the true stall point. However, a discontinuous slope can exist at this point because a flow mode change occurs. In this sense, the true stall point can be treated as a
bifurcation where the axisymmetric flow mode degenerates to the non-axisymmetric, rotating stall flow mode.

The author believes that a connection exists between general instabilities encountered in nature and the fluid flow instability of a compressor stall. For instance, it is helpful to consider the elastic instability of an axially loaded, thin shell, since this can provide additional insight for the proposed qualitative shape of the extended rotating stall characteristic. Fig. 3.17 shows a typical axial compression test of a thin, cylindrical shell structure. The shell is loaded by a constant crosshead speed, \( \dot{\delta} \), while the axial force, \( P \), remains variable. The axial force versus crosshead displacement plot obtained in such an experiment consists of: a uniform compression until the critical load, \( P_{cr} \), is reached (path A in Fig. 3.17), a highly transient, snap-through region (path B) in which the buckled configuration initiates, and the post-buckling behavior (path D).

It was accepted that this \( P \) versus \( \delta \) plot (path A→B→D) completely described the shell buckling phenomenon. In the 1930's, theoretical post-buckling solutions were presented which showed the existence of the actual buckling behavior, path C, instead of the transient path B observed in the experiments. Since the testing device loaded the shell with a constant crosshead speed, it was unable to reverse the crosshead displacement during the test. As a result, the actual curve, path C, could not be measured by this testing technique, and remained as a hidden portion of the actual post-buckling behavior (path A→C→D) for many years.
Parallels between the initiation of the buckling phenomenon and compressor stall onset can be identified. The actual buckling curve (path A+C+D) in Fig. 3.17 is continuous even at the bifurcation point where a change of mode shape occurs, namely, from a uniform cylindrical shell to a buckled configuration consisting of many half-sine waves along the shell circumference and shell axis. A discontinuous slope is evidenced at the bifurcation point because of the mode change. This behavior is similar to the proposed, extrapolated rotating stall characteristic in Fig. 3.16. The transient portion, path B of Fig. 3.17, is seen in experimentation only because of the particular testing procedure. This is directly analogous to the transient regions observed in compressor tests along the throttle lines at stall inception and recovery from rotating stall shown in Fig. 3.18. Certain compressor behavior is lost because of the way in which the compressor is tested, i.e. the compression system. Just as path C of Fig. 3.17 was missed in experimentation, it is likely that the extended rotating stall line in Fig. 3.16 is overlooked due to the particular compression system.

System parameters should affect the measured recovery point, since it may be caused by a compression system instability. In this context, compressor stall recoverability could be enhanced by the following:

1. Increasing the rotating stall characteristic slope beyond \( C' > 0 \) (where \( C' \) is the slope of the compressor characteristic).
2. Increasing the throttle characteristic slope.
3. Increasing Greitzer's dimensionless parameter, \( B \).

These methods are specified from Appendix D in order to produce an un-
stable compression system at the lowest mass flow along the rotating stall curve. Method (1) involves altering the compressor geometry properly. This might be accomplished by correlating data on rotating stall lines for various compressor configurations, or by applying a model which predicts rotating stall performance. Method (2) could be implemented by adding components into the basic compressor, plenum, throttle system, such as an additional compressor or fan. Method (3) has been documented by Greitzer in [11] and [1]. These three points should lead to better stall recoverability on the basis that a system instability causes recovery from rotating stall.

Any theoretical compression system analysis, requires a complete compressor characteristic over the entire flow regime. The rotating stall curve has been extended beyond the recovery point, and by the same procedure, it can be extrapolated near shut-off. Fig. 3.19 gives the complete rotating stall line for various compressor characteristics, based on actual data. In Fig. 3.19, compressor builds, #1 and #2, have less of their rotating stall curves extrapolated past the measured recovery point than does compressor #3. However, the stall cell size at recovery for #3 is about 45% of the annulus, as opposed to approximately 30% for #1 and #2. On this account, a longer extended characteristic would be expected for #3, since this curve is single-valued with respect to stall cell size. The complete rotating stall line along with the uninstalled and reversed flow curves provide the compressor characteristic needed in a theoretical system study.
Summary

Compressor performance characteristics were discussed in detail since they provide insight into the problem of stall stagnation. Moore's rotating stall model requires a complete axisymmetric characteristic which remained unknown in the low forward flow regime. A method is presented for obtaining this unstable portion of the axisymmetric curve from compressor surge cycle data. Also, it was shown that planar diffuser performance is analogous to axisymmetric compressor behavior, and that the axisymmetric peak can occur at a greater pressure rise than the compressor stall point. This last observation could account for the rotating stall behavior recorded by Day on his low $\phi^*$ compressor. Furthermore, the general shape of the axisymmetric curve as in Fig. 3.8 from Greitzer's test rig is proposed for any compressor configuration.

The compressor stall point parallels the recovery point from a rotating stall. The true stall point may or may not be measured in a post-stall compressor test while the true recovery point is most likely not measured in such a test. The measured recovery point at the well-known stall cell size of approximately 30% of the annulus seems to be due to a system instability rather than a compressor instability. This is concluded because measured rotating stall curves have positive slopes at recovery, and because the true recovery point may be governed by the unsteadiness of the rotating stall flow. The rotating stall characteristic is extrapolated beyond the measured recovery point in order to locate the proposed, true recovery position.

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An analogy with the initiation of elastic buckling of axially loaded, thin shells further supports the proposed shape of the extended rotating stall line. In addition, three schemes are provided which may enhance the recoverability from a stall stagnation. Lastly, these complete rotating stall curves can be used in a compression system model. Understanding the nature of the complete performance characteristics for axisymmetric flow and rotating stall flow discloses certain features of post-stall compressor behavior.
IV

PREDICTION OF THE COMPLETE AXISYMMETRIC CHARACTERISTIC

All present rotating stall models require compressor performance characteristics to be specified. Hence, the prediction of these characteristics is necessary to predict stall stagnation behavior. This chapter presents a procedure to predict the complete axisymmetric characteristic. An analytical prediction scheme is proposed for axisymmetric backflow, and is compared to one-stage and three-stage compressor data. Also, a correlation is provided for the valley of the axisymmetric curve, along with a discussion concerning the axisymmetric peak. Obtaining the unstalled compressor characteristic is not considered in this thesis since there are adequate methods for this purpose in existence. The concepts expressed in Sections 4.1 and 4.2 are integrated in Section 4.3 to produce a complete characteristic.

4.1 Axisymmetric Reversed Flow

Prediction of compressor backflow is needed in both the two-segment, parallel compressor model and Moore's model. This characteristic can be measured in the laboratory in the following ways:

1. By forcing flow backwards through a compressor in normal operation, and maintaining an axisymmetric flow.

2. From a surge cycle which includes an axisymmetric plenum blowdown portion, i.e. a backflow portion.

In reversed flow operation, the compressor behaves as a throttling device in that at higher backflows, the compressor suffers greater
pressure drops. Since the pressure rise in forward flow is in the same direction as the pressure drop in backflow, i.e. $p_e > p_i$ in the figure included in Appendix C, the reversed flow characteristic is plotted as a positive pressure rise in a compressor map, as in Fig. 2.3. As mentioned in Section 3.1, when a compressor is tested in the familiar compression system (compressor, plenum, throttle), the plenum pressure referred to ambient represents an exit total to inlet static pressure characteristic for backflow.

Day observed in his high $\phi^*$ configuration that a rotating stall cell with a large degree of reversed flow is two-dimensional in nature with negligible radial velocities [8]. An attempt is made to model the planar backflow case.

Fig. 4.1 shows a multi-stage compressor subjected to a two-dimensional backflow. The last rotor row (first from the rear) experiences a very high incidence flow angle on its trailing edge. Consequently, the flow is assumed to be separated and is modeled by an incompressible, free streamline potential flow theory, derived by Cornell [16]. His analysis is of a two-dimensional cascade of infinitely thin, flat plates as shown in Fig. 4.2. Here, the flow separates from the blade leading edge, and divides into high velocity jets and wakes of zero velocity at station 2. No losses are incurred from stations 1 to 2, so that a static pressure drop occurs according to Bernoulli's equation. The jets and wakes are then completely mixed from 2 to 3 by maintaining constant momentum in the $\theta$-direction. This results in a static pressure rise, and a
blade row leaving angle, $\beta_3$. All the losses in this stalled cascade flow are absorbed in the mixing process, which provides a total pressure drop across the blade row, $p_{T1} - p_{T3}$. Fig. 4.3 illustrates the case of two-dimensional backflow. Here, a static pressure drop, $p_1-p_3$, occurs across the cascade which results from the large incidence angle on the blade row by the inlet flow at station (1).

The flow between stations (1) and (2) is found using the free streamline theory. As shown in [16], the hodograph solution of the problem leads to two simultaneous equations for the jet velocity ratio, $\nu = \frac{w_1}{w_2}$, and the jet angle, $\beta_2$. These are:

$$2\cos\beta_1 \cos(\beta_2 - \gamma) = \frac{1}{\nu} \cos \gamma + \nu \cos(2\beta_1 - \gamma),$$  \hspace{1cm} (4.1)$$

$$\frac{\pi \ell}{\nu t} = \cos \beta_1 \cos(\beta_2 - \gamma) \ln \left\{ \frac{1+\cos(\beta_2 - \gamma)}{1-\cos(\beta_2 - \gamma)} \cdot \frac{1-2\nu \cos(\beta_1 - \gamma) + \nu^2}{1+2\nu \cos(\beta_1 - \gamma) + \nu^2} \right\}$$

$$+ \left[ \nu \sin(2\beta_1 - \gamma) + \frac{1}{\nu} \sin \gamma \right] \tan^{-1} \left\{ \frac{2\nu \sin(\beta_1 - \gamma)}{1 - \nu^2} \right\}$$

$$- \frac{\beta_1 - \gamma}{|\beta_1 - \gamma|} \left[ \pi \cos \beta_1 \sin(\beta_2 - \gamma) + \frac{\pi}{\nu} \sin \gamma \right].$$  \hspace{1cm} (4.2)$$

For the backflow case involving high incidence angles on the cascade, there is a critical inlet flow angle, $\beta_{1cr}$, where equations (4.1) and (4.2) fail to yield a solution for $|\beta_1| < |\beta_{1cr}|$. This critical angle is approximately $80^\circ$ for a solidity, $\frac{\lambda}{c} \approx 1$. $\beta_{1cr}$ increases as the solidity is reduced. The author can provide no
mathematical reason for the inconsistency of the two equations under these circumstances. However, this inconsistency may result since Cornell's hodograph solution set the static pressure at the jet/wake station equal to the far downstream static pressure. This condition is not consistent with the observed wake behind a cylinder in a uniform flow, where the wake static pressure is less than the far downstream static pressure. Near the critical inlet angle, the jet angle coincides with the stagger angle of the cascade, which is the maximum possible flow turning, $|\beta_1 - \beta_2|$. In general, for $|\beta_1| < |\beta_{cr}|$, $\beta_2 > \gamma$ for a rotor row, while $\beta_2 < \gamma$ for a stator row.* Since the maximum turning occurs near the critical inlet flow angle, it is assumed that for inlet flow angles greater than the critical value, $|\beta_1| > |\beta_{cr}|$, the maximum possible turning is achieved, i.e. the jet angle is the blade stagger angle. Thus, for the case of $|\beta_1| > |\beta_{cr}|$, equation (4.1) is solved with $\beta_2 = \gamma$.

It may be inconvenient to solve equations (4.1) and (4.2), simultaneously for $\nu$ and $\beta_2$. An approximation is recommended for solidities of about 1.0 or greater, i.e. for $\frac{\nu}{c} \geq 1.0$ [16]. This involves specifying that the jet angle is the stagger angle of the cascade ($\beta_2 = \gamma$), which is used in place of equation (4.2). However, Cornell's theory neglects the blade camber angle, so that a more realistic condition on the jet angle would maintain that it coincides with the pressure surface metal angle of the blade leading edge, $\beta_{LE}$. For thin blade profiles, the camber line angle at the leading edge could be used as the jet leaving angle. By solving equation (4.1) for $\nu$ with $\beta_2 = \gamma$,

*For a rotor: $\beta_1 > 0$, $\gamma < 0$ For a stator: $\beta_1 < 0$, $\gamma > 0$
the approximate relations for $\frac{\theta}{t} \approx 1.0$ are:

$$\nu = \frac{\cos \beta_1 + \sqrt{\cos^2 \beta_1 - \cos(2\beta_1 - \gamma) \cos \gamma}}{\cos(2\beta_1 - \gamma)}$$  \hspace{1cm} (4.3)

$$\beta_2 = \beta_{LE}$$  \hspace{1cm} (4.4)

There are two solutions for the jet velocity ratio, $\nu$, however one root will be negative or greater than 1.0 and can be discarded ($0 < \nu \leq 1.0$). The negative radical is usually chosen since $\cos(2\beta_1 - \gamma) < 0$ for most inlet flow angles in backflow.

The size of the wake is determined from the continuity relation between stations 1 and 2 of Fig. 4.3, so that

$$\mu = 1 - \nu \frac{\cos \beta_1}{\cos \beta_2}.$$  \hspace{1cm} (4.5)

Since $p_1 = p_2$, the Bernoulli equation gives the static pressure drop from 1 to 2 as

$$p_1 - p_2 = \frac{1}{2} \rho w_1^2 \left[ \frac{1 - \nu^2}{\nu^2} \right].$$  \hspace{1cm} (4.6)

The flow from 2 to 3 is assumed to be completely mixed, and since no net forces act in $\theta$-direction, the $\theta$-momentum equation on the control volume shown in Fig. 4.3 maintains that,

$$w_2 \sin \beta_2 = w_3 \sin \beta_3.$$  \hspace{1cm} (4.7)
The continuity equation for stations 2 and 3, together with equation (4.7) give the exit flow angle of the cascade as

\[ \tan \beta_3 = \frac{1}{1-\mu} \tan \beta_2. \]  (4.8)

The flow velocity at station 3 is found by rearranging equation (4.7), or,

\[ w_3 = \frac{w_1}{\nu} \frac{\sin \beta_2}{\sin \beta_3}. \]  (4.9)

The static pressure rise due to the mixing process is found from the x-momentum equation as,

\[ p_2 - p_3 = -\left[ \frac{\mu}{1-\mu} \right] \rho w_3^2 \cos^2 \beta_3. \]  (4.10)

Thus, the nondimensional static pressure drop across the blade row is obtained by combining equations (4.6) and (4.10):

\[ \frac{p_1 - p_3}{\frac{1}{2} \rho U^2} = \left[ \frac{1 - \nu^2}{\nu^2 \cos^2 \beta_1} - \frac{2\mu}{1-\mu} \right] \phi^2. \]  (4.11)

Cornell's analysis is for a constant area annulus in the axial direction, but practical compressors do not have these constant area annuli. Fig. 4.4 shows a typical non-constant area annulus, which can be modeled as a constant area from stations 1 to 2, where the Cornell approach applies. The entire area change across the blade
row, $A_3 - A_1$, is accounted for in the downstream mixing calculation. This procedure is carried out for a constant density throughout the compressor, and is valid for low rotor speeds. Equations (4.1) through (4.6) apply for this case, however the downstream mixing process must be re-formulated. Since there are no net forces acting in $\theta$-direction during the mixing, equation (4.7) is also valid here. The continuity equation for the flow between $2$ and $3$ with equation (4.7) provide $\beta_3$:

$$\tan \beta_3 = \frac{h_3}{h_1(1-\mu)} \tan \beta_2. \quad (4.12)$$

The x-momentum equation on the control volume shown in Fig. 4.4 determines the pressure rise due to the mixing:

$$\int_{(a)}^{(b)} p(l) \cos \theta \, dl + p_2 h_1 - p_3 h_3 = \rho \left[ h_3 w_3^2 \cos^2 \beta_3 - h_1(1-\mu) w_2^2 \cos^2 \beta_2 \right]. \quad (a)$$

The pressure force term along the path from (a) to (b) is approximated by

$$\int_{(a)}^{(b)} p(l) \cos \theta \, dl \approx \frac{1}{2} (p_2 + p_3)(h_3 - h_1). \quad (b)$$

The mixing pressure rise then becomes

$$p_2 - p_3 = -\frac{2h_3}{h_1 + h_3} \left[ \frac{1}{1-\mu} \left( \frac{h_3}{h_1} \right) - 1 \right] \rho w_3^2 \cos^2 \beta_3. \quad (4.13)$$

Therefore, the nondimensional static pressure drop across a cascade of
non-constant area is found from equations (4.6) and (4.13):

\[
\frac{p_1 - p_3}{\frac{1}{2} \rho u^2} = \left[ \frac{1 - \nu^2}{\nu^2 \cos^2 \beta_1} \right] \phi_1^2 - \frac{4h_3}{h_1 + h_3} \left[ \frac{1}{1 - \mu} \left( \frac{h_3}{h_1} \right) - 1 \right] \phi_3^2. \tag{4.14}
\]

Now that the performance of a single blade row has been determined for a planar backflow, the reversed flow characteristic of a multistage compressor can be obtained. A marching procedure is used from the rear to the front of the compressor using Fig. 4.1. This technique is outlined for the constant area annulus case as follows:

1. **Required inputs**
   - Blade geometry: $\beta_{LE}$ of $S_0$ approximates the stalled leaving flow angle from $S_0$. $\gamma$ and $\beta_{LED}$ are needed for each blade row with separated flow except the IGV row. $\gamma$ and $\beta_{LE}$ are specified for the IGV row. If $c/t \geq 1.0$, equations (4.3) and (4.4) may be used instead of equations (4.1) and (4.2). However, if this condition is not valid, (4.1) and (4.2) must be solved with the blade row solidity, $c/t$.
   - The reversed axial flow coefficient, $\phi$, must be specified.

2. **Procedure**
   - A purely axial flow is imposed on the trailing edge of the OGV which is turned in an unstalled manner through the OGV and the first stator row from the rear, $S_0$. The static pressure drop is calculated from the overall turning in the OGV and $S_0$.
\[
\begin{bmatrix}
\frac{\Delta p}{1/2 \rho u^2} \\
\end{bmatrix}_{\text{OGV}+S_0} = \phi^2 \tan^2 \beta_{\text{LE}_{S_0}}.
\]

(4.15)

where the exit flow angle from $S_0$ is approximated $\beta_{\text{LE}_{S_0}}$.

- The relative inlet flow angle, $\beta_1$, into $R_1$ is calculated from the velocity triangle

$$
\tan \beta_1 = \tan \beta_{\text{LE}_{S_0}} + \frac{1}{\phi}
$$

(4.16)

- A separated flow is assumed for the subsequent blade rows in Fig. 4.1. For $\ell/t \geq 1.0$, $\nu$ for the rotor row, $R_1$, is determined from equation (4.3) with $\gamma_{R_1} = -|\gamma_{R_1}|$ and $\beta_1$ given by (4.16) ($\beta_1 > 0$).

$\beta_{\text{LE}_p} = -|\beta_{\text{LE}_p}|$ for $R_1$ in equation (4.4). $\nu$ is found from (4.5), and the static pressure drop across $R_1$ is calculated from (4.11). The exit flow angle from the cascade, $\beta_3 < 0$, is given by (4.8), which provides the inlet flow angle in the next stator row from the velocity triangle:

$$
\tan \alpha_3 = \tan \beta_3 - \frac{1}{\phi}
$$

(4.17)

The same steps from the rotor calculations are repeated for the stator rows except that the inlet flow angle is negative, or $\alpha_3 < 0$, with $\gamma > 0$ and $\beta_{\text{LE}_p} > 0$.

- The compressor performance for the given reversed flow coefficient, $\phi$, is obtained by adding up the static pressure drop in each blade row. Varying $\phi$ generates a complete backflow characteristic.
A similar procedure can be formulated for the case of low solidity or a non-constant annulus area.

This procedure is applied to two compressors: the single stage tested by Turner and Sparkes [17], and the three-stage rig tested by Greitzer [1]. Both have constant annulus area. Turner and Sparkes generated a reversed flow curve by blowing flow backwards through their compressor consisting of an inlet guide vane, a rotor and a stator vane. The compressor blading is provided in Table 4.1. In the prediction, the flow through the stator vane is assumed stalled and leaves the cascade at the camber line angle of the stator leading edge. Separated flow occurs in the rotor row and the inlet guide vane, and equations (4.3) and (4.4) are substituted for (4.1) and (4.2) since $l/t > 1.0$.

The blade metal angle on the pressure surface of the leading edge, $\beta_{LE}$, was not recorded so that the camber line angle of the blade leading edge is used instead as the jet angle, $\beta_2$. The predicted characteristic is shown with the measured backflow curve in Fig. 4.5. These results compare favorably, especially for larger backflows.

At shut-off, a pressure rise is measured, however the two-dimensional prediction method does not account for this shut-off head. It is suspected that radial effects produce this axisymmetric pressure rise at zero flow, so that the separated flow theory could not be expected to predict this behavior.

A reversed flow characteristic is obtained for Greitzer's three-stage compressor [1] from surge cycle data. A one-dimensional correction is applied to the surge data near shut-off, as the flow begins to accelerate towards the unstalled condition. This is described in detail...
in Section 3.1. The compressor blading for the rig is presented in Table 4.2. The marching technique through a multi-stage compressor is used with the high solidity ($&/t \geq ~1.0$) approximation. As in the Turner and Sparkes case, $\beta_{LE_p}$ was not recorded for Greitzer's compressor, however $\gamma < \beta_{LE_p} < \beta_{LE}$, and for thin profiles, $\beta_{LE_p} \approx \beta_{LE}$. Two predicted characteristics for the cases of $\beta_2 = \gamma$ and $\beta_2 = \beta_{LE}$ are shown with the backflow curve corrected from surge data in Fig. 4.6. The prediction at least qualitatively follows the corrected characteristic. The jets/wakes pattern may not repeat throughout the blade rows in the actual case, since the jets and wakes probably do not completely mix in the blade row gaps as assumed in the theory. The first stage subjected to backflow may exhibit this separated pattern, however subsequent cascades are less likely to behave in this orderly fashion. This may account for the good agreement seen in the one-stage case, while much error in the prediction can be induced in a multi-stage compressor. As in predicting stalled compressor performance, empiricism can be incorporated into the theory to improve the prediction of multi-stage machines under large backflows. Thus, further experimentation is needed studying multi-stage compressors operating with axisymmetric reversed flows.

The influence of the blading stagger angle was investigated on the backflow performance prediction for Greitzer's test rig. Fig. 4.7 shows the predicted characteristics for blade stagger angles from $20.0^\circ$ to $45.0^\circ$ referred to the axial direction. It is evident that higher stagger angles provide a greater resistance to reversed flow by the proposed prediction technique. This observation seems reasonable,
however, experimental verification is still necessary to substantiate this claim.

The proposed backflow prediction technique can at best approximate the case of high backflow, believed to be essentially two-dimensional in nature. Both compressors studied in reversed flow operation produce a pressure rise at zero flow, which could not be predicted by the theory. In order to predict the complete backflow characteristic, additional information must be specified concerning axisymmetric compressor operation near the shut-off point. The next section provides an empirical approach for axisymmetric flow near shut-off.

4.2 Peak and Valley of the Axisymmetric Performance

The author maintains that the unstalled and reversed flow portions of the axisymmetric curve along with the axisymmetric peak and valley are of greater importance than the unstable, large positively sloped portion in predicting rotating stall performance. This is mainly due to the fact that the blade passages experience the large positively sloped portion for the shortest duration in a rotating stall. Some evidence for this reasoning will be provided in Chapter V. Moore concluded in [6] that his recovery prediction is independent from the positively sloped portion of the axisymmetric characteristic. Consequently, an understanding of the axisymmetric peak and valley is useful in modeling stall.

An unsuccessful attempt was made to predict the unstable portion of the axisymmetric curve by applying Cornell's free streamline theory [16], used for the backflow case in Section 4.1. This planar method
probably failed since wall stall rather than blade stall exists in the low forward flow region, as discussed in Section 3.1. Nevertheless, Appendix E provides this prediction approach.

Section 3.2 demonstrated that the axisymmetric peak may not coincide with the compressor stall point. This peak could be at a greater pressure rise when the compressor stalls on the negatively sloped portion of its characteristic. As mentioned previously, it may be important to identify the axisymmetric peak for such a compressor. Since the backflow model discussed in Section 4.1 can only approximate the higher reversed flow cases, additional information is required near shut-off to predict a backflow characteristic. A correlation involving the axisymmetric valley point can provide this information.

The valley of the axisymmetric characteristic will be discussed first. This valley occurs near the compressor shut-off, which remains as a unintelligible regime for compressor operation. Day recorded axisymmetric stalls at zero flow, which produced a pressure rise across the compressor in this state [8]. A two-dimensional flow description (x-θ) can not account for an axisymmetric shut-off head, so that some type of radial recirculation is expected at this state. Fig. 4.8 presents a sketch of a possible schematic recirculation pattern in an annulus near axisymmetric zero flow. It shows a centrifuged flow in the rotor row which would be driven towards the compressor exit, because of the blade stagger angle. But the adverse pressure gradient present in this state redirections this flow in the stator vane, backwards into the preceding rotor row. Analytically modeling
this flow field would be extremely difficult because of its complex, three-dimensional nature. However, a correlation is proposed for the axisymmetric valley region, which then specifies the compressor shut-off condition.

The axisymmetric valley correlation is based on data from Day [8] and Greitzer [1]. Fig. 3.8 gives the axisymmetric curve for Greitzer's compressor obtained from surge cycle data, while Figs. 4.9, 4.10 and 4.11 show possible axisymmetric characteristics for Day's data on various compressor builds. These assumed characteristics are sketched according to the criteria developed in Section 3.2, and are for illustrative purposes only. Day's performance curves shown in Figs. 4.9 4.10 and 4.11 exhibit an axisymmetric flow region near shut-off. When this portion does not include the minimum point, as in the three-stage rig in Fig. 4.9, the measured curve is extended to show an assumed valley point. Clearly, this is not a data point, but the assumed valley point will be checked whether it is consistent with the correlated value.

The minimum point, or the valley coordinates \((\phi_v, \psi_v)\) are correlated by two compressor parameters: the number of stages, N, and the design flow coefficient, \(\phi^*\). The pressure level, \(\Delta p_{TS}/\rho U^2\), per stage of the valley point seems to be constant for each compressor configuration, while the flow coefficient of the minimum point, \(\phi_v\), depends on both N and \(\phi^*\). The axisymmetric valley point is correlated as follows:

\[
\phi_v = (-.23\phi^* + 0.093)(N-1) + 0.075, \quad N \leq 4 \tag{4.18}
\]

\[
\psi_v = 0.10N \tag{4.19}
\]
Equation (4.18) was obtained through a systematic trial and error approach, and fits the available data fairly well, while equation (4.19) was found simply by averaging the data. Fig. 4.12 compares the correlation with the measured and assumed axisymmetric valley points. Compressor rigs B, F, H and J have assumed valley points. However, since the measured axisymmetric curve near shut-off was fairly flat for compressors H and J, \( \psi_v \) for H and J in Fig. 4.11 could be accurate. Fig. 4.12 shows that the data set in Table 4.3 is represented fairly well by the proposed correlation, equations (4.18) and (4.19). Nevertheless, this correlation should only be regarded as a rough, first guess of the axisymmetric valley point, since no information concerning the flow field is included in equations (4.18) and (4.19).

The axisymmetric peak will now be briefly discussed. As stated previously, the peak of the axisymmetric characteristic may occur at a greater pressure rise than the compressor stall point. As a result, predicting the unstalled performance may not provide the peak value. A method which might obtain the axisymmetric peak involves extending an unstalled flow prediction procedure past the anticipated stall point [18]. These unstalled prediction programs tend to be accurate near the design flow conditions, but their accuracy remains questionable for off-design predictions. Thus, the proposed axisymmetric peak prediction scheme needs to be compared with measured data to check its validity.

4.3 Predicting a Complete Axisymmetric Characteristic

A prediction technique for the axisymmetric characteristic over the
entire compressor flow map is outlined as follows:

1. The backflow calculation provided in Section 4.1 results in curve A in Fig. 4.13.

2. Equations (4.18) and (4.19) give the coordinates of the axisymmetric valley point, shown as point B in Fig. 4.13.

3. An un stalled prediction method provides curve C along with the anticipated stall point (point D) and the axisymmetric peak point (point E).

4. The large positively sloped portion (curve F) is just sketched in between the peak and the valley since it is of less importance than the other regions. Chapter V provides support for this notion.

5. The backflow characteristic is completed by sketching curve G which connects the axisymmetric valley with the large reversed flow asymptote (curve A). The axisymmetric curve is then defined over the entire flow range by curve C, curve F, curve G and curve A of Fig. 4.13.

Summary

The prediction of stall performance depends on specifying the compressor characteristics. A simplified backflow model is proposed for the case of high reversed flow. This gives good results for a one-stage compressor, however it overestimates the backflow performance for a three-stage rig. The flow field in a multi-stage compressor in backflow may not be repeatable throughout the blade rows as the model suggests, thus refinements in the backflow theory may be necessary for multi-stage compressors. Also, a correlation for the axisymmetric valley point is introduced, since the two-dimensional backflow model is unable to account for the axisymmetric, shut-off pressure rise observed in experimentation. The axisymmetric peak might be obtained from an un stalled compressor performance cal-
ulation, along with the stalled characteristic. Predicting the positively sloped, unstable portion of the axisymmetric curve may not be necessary because the blade passages experience this flow regime for the shortest period during a rotating stall. As a result, the remaining unstable axisymmetric region can be sketched in once the axisymmetric peak and valley are defined as in Fig. 4.13; therefore a complete axisymmetric compressor characteristic is obtained. Additional evidence involving the unimportance of the positively sloped axisymmetric portion in predicting rotating stall is given in Section 5.2.
APPLICATION OF THE MOORE MODEL

This thesis has been primarily concerned with compressor performance characteristics, and the insight they provide for the problem of stall. This chapter uses the characteristics obtained for Greitzer's rig in Section 3.1, and applies them in the Moore model to predict the rotating stall performance. The numerical solution procedure is discussed first. The results of the study are then presented in Section 5.2. Two different axisymmetric characteristics are used in the calculation procedure, and compared on the basis of the resulting predictions. Also, the effect of changing the compressor exit geometry parameter, m, is studied.

5.1 The Numerical Solution Procedure

Section 2.2 discussed Moore's analysis [5][6] and stated that solving equations (2.2), (2.7) and (2.8) together with the constraints (2.9), (2.10) and (2.11) results in the rotating stall performance, \( \Psi \). This section will outline this calculation. A list of the assumptions and the required inputs to the model are provided in Section 2.2. A further variable to be used is defined by

\[ \delta = \Psi - \Psi_c(\phi = \phi). \]  
(5.1)

\( \delta \) represents the difference in the rotating stall pressure rise and the axisymmetric performance at the specified average flow coefficient, \( \phi \). The unknowns in the problem are:
1. Rotating stall performance, $\psi$(or $\delta$).
2. Stall cell rotational speed, $f$.
3. Axial flow velocity perturbation, $g(\theta)$.
4. Circumferential flow velocity perturbation, $h(\theta)$.

The iterative solution procedure followed is:
1. Specify the average flow coefficient, $\phi$.
2. Guess $\delta$ and $f$.
3. Solve the set of non-linear differential equations,

$$g'(\theta) = \frac{1}{\lambda}[\psi_C(\phi) - \psi + mfh],$$

$$h'(\theta) = -g,$$

simultaneously for $g(\theta)$ and $h(\theta)$, with $\lambda$ given by equation (2.8)
4. Check if the calculated $g(\theta)$ and $h(\theta)$ satisfy the constraint conditions,

$$g(\theta) = g(\theta + 2\pi),$$

$$\int_0^{2\pi} h(\theta) d\theta = 0.$$  \hspace{1cm} (2.11)

If these are not satisfied, return to step 2 and re-guess $\delta$ and $f$. If these constraints are satisfied, then the problem is solved.

The differential equations, (2.7) and (2.2), of step 3 are solved numerically by Hamming's modified predictor corrector method. Note that this procedure solves the equations as an "initial value" problem so that $g(0)$ and $h(0)$ must also be specified. A solution for $g(\theta)$ and $h(\theta)$ is obtained from the non-linear equations when repeating cycles, or limit cycles, are generated by the differential equation solver (DESOLV [12]). The limit cycle solution seems to be independent of the...
initial values, \( g(0) \) and \( h(0) \), although this was not studied extensively.

The other constraint conditions listed in Section 2.2, namely,

\[
h(\theta) = h(\theta + 2\pi) \quad \text{and} \quad \int_0^{2\pi} g(\theta)d\theta = 0,
\]

are a consequence of the two constraints in step 4, (2.9) and (2.11).
Since,

\[
h'(\theta) = -g,
\]

if \( g(\theta) \) is periodic in \( 2\pi \), then \( h(\theta) \) must also have a period of \( 2\pi \).

\( h(\theta) \) is a "smoother" function than \( g(\theta) \), since \( g(\theta) \) is the derivative of \( -h(\theta) \). Hence, \( g(\theta) \) is chosen to satisfy the periodicity requirement, because this criterion is more strict than satisfying periodicity of \( h(\theta) \). By integrating equation (2.2) from \( \theta=0 \) to \( \theta=2\pi \), it is easily shown that if \( h(\theta) \) is periodic in \( 2\pi \), equation (2.10) is always satisfied. Thus, only the two conditions (2.9) and (2.11), in step 4 must hold for \( g(\theta) \) and \( h(\theta) \).

Before the constraints, (2.9) and (2.11), can be checked, the limit cycle solution for \( g(\theta) \) and \( h(\theta) \) must be identified from the output of the differential equation solver, which contained about 7 cycles. The solution develops into a limit cycle quite rapidly (in 2 or 3 cycles), so that the 7 cycles are sufficient. The details of the procedure are as follows:

1. \( g(\theta) \) is used instead of \( h(\theta) \) for the procedure, because of the previous arguments concerning equation (2.2).
2. The first cycle from the output of DESOLV is always ignored.

3. The period of the next 3 cycles are compared; and if they are within an arbitrarily decided tolerance of about 0.1%, then the second and third cycles, of the 3 cycles compared, are used in step 4. If the periods of the 3 cycles are not within the 0.1% tolerance, the first of the 3 cycles is ignored, and the next 3 cycles are selected to satisfy the 0.1% tolerance. If all of the cycles in the output array for \( g(\theta) \) are used without satisfying the 0.1% tolerance, a new \( \delta \) and \( f \) are guessed and DESOLV produces another solution of \( g(\theta) \). This procedure is then repeated by returning to step 1.

4. The 2 cycles obtained from step 3 are then compared to determine if the waveforms of both cycles are identical. This is implemented by checking that

\[
g(\theta) = g(\theta + \bar{\bar{P}}) \text{ where,}
\]

\[
\bar{\bar{P}} = \text{average of the cycle 1 period and the cycle 2 period,}
\]

for 30 evenly distributed points within the cycle. This condition must hold for each of the 30 points within a tolerance of approximately 4%. Most of the points compare much more closely than the 4% allowance. However, since \( g(\theta) \) can vary rapidly for portions of the cycle, a small error in \( \theta \) can result in a large error for \( g(\theta) \) in these regions. Thus, it turns out that this 4% tolerance is consistent with the 0.1% tolerance on the period of the cycle. If the 4% criterion is met, the first of these 2 cycles compared here is used as the representative limit cycle solution for \( g(\theta) \) and \( h(\theta) \). If the condition described does not hold for all 30 points, the first of the 3 cycles from step 3 is ignored, and the next 3 cycles are compared as described in step 3.

After a limit cycle solution is obtained for \( g(\theta) \) and \( h(\theta) \) from the procedure just described, the constraints (2.9) and (2.11) must be checked. The period of the cycle, \( P_{\text{cycle}} \), must lie within a 0.1% tolerance of \( 2\pi \), or

\[
|P_{\text{cycle}} - 2\pi| \leq 0.005.
\]  

(5.2) 

\( h(\theta) \) is integrated from \( \theta = 0 \) to \( \theta = P_{\text{cycle}} \) by the trapezoidal ap-
proximation, and the absolute value of this integral must be less than 0.003, or
\[ \left| \int_{0}^{P_{\text{cycle}}} h(\theta) \, d\theta \right| < 0.003. \quad (5.3) \]

Note that the amplitude of \( h(\theta) \) in a rotating stall mode is on the order of 0.5. If the conditions in (5.2) and (5.3) are both met, a solution pair, \( \delta \) and \( f \), has been obtained for the specified average flow coefficient, \( \phi \). If these conditions are not satisfied, \( \delta \) and \( f \) must be re-guessed as described in the iterative solution procedure. Once a solution pair is found for a given \( \phi \), a rotating stall characteristic can be generated by varying \( \phi \) and repeating the calculations.

From the computations, it is found that the period of the cycle, \( P_{\text{cycle}} \), is strongly dependent on \( f \) and only weakly dependent on \( \delta \). Conversely, the average value of \( h(\theta) \), \( h_{\text{avg}} \), is strongly dependent on \( \delta \) and only weakly dependent on \( f \). In another way,
\[
\left| \frac{\partial P_{\text{cycle}}}{\partial \delta} \right| \ll 1 \text{ and } \left| \frac{\partial h_{\text{avg}}}{\partial f} \right| \ll 1.
\]

Recovery is attained when the amplitude of the disturbance is very small, which implies that the compressor is operating axisymmetrically at the average flow, \( \phi \). This condition is obtained in the calculations when \( \delta = 0 \) with \( f = f_0 \). \( f_0 \) is the stall cell speed calculated from the small disturbance approximation in [5], and is given by
\[ f_0 = \frac{1}{2 + \cos \gamma \left[ \frac{m}{2k} \left( \frac{D}{c} \right) + \frac{1}{\cos \gamma_{IGV}} + \frac{1}{\cos \gamma_{OGV}} \right]} \]  \hspace{1cm} (5.4)

Similarly, a flow mode change can also be predicted at the other end of the rotating stall characteristic, towards shut-off, with the same criterion which defined the recovery point, i.e. \( \delta = 0 \) with \( f = f_0 \).

Appendix F gives the Fortran code for the numerical solution procedure.

5.2 **Numerical Results**

The set of non-linear equations \{(2.2), (2.7), (2.8), (5.2), (5.3)\} are solved by the method discussed in Section 5.1 using characteristics from Greitzer's three-stage compressor. Two features examined are the influence of the shape of the unstable axisymmetric characteristic, and the effect of changing the compressor exit geometry parameter. The influence of the empirical lag parameter, \( k \), should also be investigated, but it is beyond the scope of this study.

Fig. 5.1 shows the predicted and measured rotating stall characteristics for this compressor. Two different axisymmetric curves are shown. Both have the same measured stalled performance, and the same inertially corrected backflow curve from the surge data of Section 3.1. However, the two possess quite different portions in the low forward flow region (i.e. the region in which axisymmetric flow is unstable). The curve labeled "Axi 1" in Fig. 5.1 closely resembles the proposed axisymmetric characteristic for Greitzer's machine (Fig. 3.8)
developed in Section 3.1. "Axi 2" represents the axisymmetric curve similar to that used by Moore in his analysis [6]. This curve involves a sudden drop in pressure rise at the stall point with a flat region through the shut-off point. The base case, depicted in Fig. 5.1, includes the following compressor geometry: \( m = 1.75, \ k = 2.72, \ N = 3, \ D/\lambda = 11.3, \ \bar{\gamma} = 28.4^\circ, \ \gamma_{IGV} = 25.3^\circ, \ \gamma_{OGV} = 23.3^\circ. \) The lag parameter, \( k, \) includes the inertial contribution of the axial blade gaps, so that \( D/\lambda \) is used for \( D/c \) in equation (2.8).

The predicted rotating stall characteristics using Axi 1 and Axi 2 are shown in Fig. 5.1. These curves are fairly similar for high flow coefficients in rotating stall, but deviate at lower flows. The Axi 2 prediction includes a flow mode change at \( \phi = 0.22, \) in which an axisymmetric stall is predicted for \( \phi \leq 0.22. \) Whereas, Axi 1 predicts rotating stall for low \( \phi. \) Even at \( \phi = 0.05, \) rotating stall was evident in the Axi 1 prediction, although the cycle calculations were not completely finished at this flow. This resulted because very small time steps were required in DESOLV, so that each iteration of \( \delta \) and \( f \) in the solution procedure was extremely slow. Greitzer's data maintains that rotating stall is experienced to shut-off. The author has never seen any data which would substantiate a flow mode change from full-span stall to axisymmetric stall at a flow coefficient greater than 0.1. Thus, the Axi 2 prediction seems unrealistic for low \( \phi, \) while the Axi 1 prediction is consistent with experimentation. At recovery, both predictions are consistent with the data.

Figs. 5.2 through 5.8 show the axial velocity distribution around the annulus for different average flows on both the Axi 1 and Axi 2
predictions. One important aspect to be noticed about these axial velocity profiles is that the flow in the unstalled and stalled zones is not constant within each region, as assumed in the two-segment parallel compressor model. However, it is not known at present which case describes the actual situation in a multi-stage compressor. The recovery point of the Axi 1 prediction is illustrated by Figs. 5.6 and 5.8, which correspond to $\phi = 0.47$ and $\phi = 0.48$, respectively. Notice that the predicted recovery points from Axi 1 and Axi 2 in Fig. 5.1 both correspond to flow coefficients where the axisymmetric characteristics are negatively sloped. Recall from Section 3.3 that Moore's model predicts stall inception at the zero slope point of the axisymmetric curve. This is the point where small perturbations from an axisymmetric flow become amplified, thus the author suspects that the Moore model must recover onto a negatively sloped portion of the axisymmetric characteristic.

The effect of the compressor exit geometry parameter, $m$, on the Axi 1 prediction of the rotating stall curve is illustrated in Fig. 5.9. Recall from Section 2.2 that $m=1$ corresponds to a sudden expansion, while $m=2$ corresponds to a long straight duct. This exit geometry parameter is varied from 1.25 to 2.00, but has very little effect on either the predicted rotating stall line or the predicted recovery point as seen in Fig. 5.9. This is contrary to Moore's results which employed axisymmetric characteristics similar to that of Axi 2. He concluded that $m$ has an appreciable effect on the predicted recovery point. The author believes that this conclusion came about from Moore's choice of the axisymmetric characteristic.
The predicted values for the stall cell rotational speed versus the average flow coefficient are shown in Fig. 5.10. The Axi 1 predictions for $f$ vary slightly with $\phi$, exhibiting a minimum around $\phi = 0.3$. Increasing $m$ from 1.75 to 2.00 results in a decrease of stall cell speed. The Axi 2 predictions match up closely with the Axi 1 values for $\phi > 0.22$, but for $\phi \leq 0.22$, Axi 2 predicts that $f = f_0$ with an axisymmetric stall.

Section 4.2 states that the steeply, positively sloped portion of the axisymmetric curve is relatively unimportant in predicting rotating stall performance, because the blade passages are subjected to this flow regime for the shortest duration in the rotating stall mode. The results from the Axi 1/Axi 2 study are consistent with this reasoning. Altering the unstable portion of the axisymmetric curve in Fig. 5.1 primarily results in a different mode change point from rotating stall to axisymmetric stall at the low flow coefficients. When a rotating stall is present for both the Axi 1 and Axi 2 predictions (at $\phi > 0.22$), the pressure performance and the axial velocity profiles are quite similar, even though the assumed unstable axisymmetric curves are very different. As a result, it can be assumed that small changes in the positively sloped axisymmetric characteristic have little or no effect on Moore's predicted rotating stall characteristic, as indicated in Section 4.2.

Summary

Moore's equations provided in Section 2.2 are solved by the numerical procedure discussed in Section 5.1. An iterative method is
used which involves guessing a solution pair, $\delta$ and $f$, and then checking if the constraints on the period, $P_{\text{cycle}}$, and the average value of $h(\theta)$, $h_{\text{avg}}$, given by relations (5.2) and (5.3), are satisfied. It was identified that

$$\left| \frac{\partial P_{\text{cycle}}}{\partial \delta} \right| \frac{\partial P_{\text{cycle}}}{\partial f} \ll 1 \quad \text{and} \quad \left| \frac{\partial h_{\text{avg}}}{\partial f} \right| \frac{\partial h_{\text{avg}}}{\partial \delta} \ll 1,$$

and that recovery occurs when $\delta = 0$ and $f = f_0$.

The Axi 1/Axi 2 study demonstrates that the predicted characteristics and the predicted recovery points are similar for high average flows in both the Axi 1 and Axi 2 cases. However, at the low flow coefficients, the Axi 1 prediction seems consistent with Greitzer's data, while the Axi 2 prediction seems unrealistic on the basis of all available rotating stall data. Also, the calculated axial velocity profiles in rotating stall have non-constant mass flows in both the unstalled and stalled regions, while the two-segment parallel compressor model assumes constant flows within these regions. Further examination of this discrepancy between the two models should be carried out. In addition, increasing the compressor exit geometry parameter, $m$, results in a decreased stall cell rotational speed. However, $m$ has very little or no effect on both the predicted rotating stall performance and the predicted recovery point, contrary to Moore's results. Lastly, as indicated in Section 4.2, the steeply, positively sloped portion of the axisymmetric characteristic seems to have little importance in predicting rotating stall behavior.
VI
SUMMARY AND CONCLUSIONS

1. The stalled flow performance of multi-stage, axial compressors has been examined, and resulted in a better understanding of compressor stall. This study has included, in particular, consideration of the so-called axisymmetric compressor characteristic, which corresponds to the pressure performance of the compressor while operating in an axisymmetrically stalled state. This characteristic is needed for any compressor stall prediction method. In addition, the study addressed full-span rotating stall characteristics.

2. The (previously undetermined) axisymmetric compressor characteristic in the low forward flow regime can be obtained by inertially correcting compressor surge cycle data.

3. The shape of the complete axisymmetric characteristic (over the entire compressor flow range) can be generalized for various compressor configurations.

4. The axisymmetric characteristic may rise in pressure performance above the compressor stall inception point.

5. Axisymmetric compressor performance in forward flow parallels two-dimensional diffuser behavior.

6. The compressor stall point parallels the recovery point from rotating stall.

7. The recovery point observed in post-stall compressor tests probably
results from a compression system instability, rather than from an instability in the rotating stall flow. If this is the case, recoverability from rotating stall could be enhanced by varying the system parameters.

8. The rotating stall characteristic is extrapolated beyond the measured recovery point to locate the point at which the rotating stall flow becomes unstable in the annulus. This extended curve implies that the stall point can be treated as a bifurcation. The author finds it helpful to view this bifurcation point as analogous to the critical point observed in the axial compression of thin shells.

9. An additional application for the extended rotating stall characteristic is in theoretical compression system models.

10. A two-dimensional, axisymmetric backflow model is applied to predict the performance of the only two compressors operating in reversed flow, for which the author has data. The predictions agree quite well with the measurements for a single stage compressor, but are less accurate in the case of a three-stage machine.

11. The stalled and reversed flow characteristics along with the axisymmetric peak and valley are of greater importance in predicting stall than the steeply positively sloped portion of the axisymmetric curve.

12. The axisymmetric characteristic over the entire compressor flow range can be predicted by three items: the proposed backflow
model, the empirical correlation for the axisymmetric valley point, and an unstalled compressor performance prediction.

13. By employing the proposed axisymmetric characteristic in Moore's analysis, it is found that the compressor exit geometry has very little effect on either the predicted rotating stall curve or the predicted recovery point. This is contrary to the conclusions obtained in [6], using a quite different axisymmetric characteristic.

14. The Moore model predicts non-constant mass flows within both the unstalled and stalled flow zones of a compressor annulus in rotating stall.
This study provided speculations concerning stall inception and stall recovery. These could be addressed in experimental work. First, the notion that the axisymmetric performance can rise above the compressor stall point could be checked in a case where the stall point occurs on a strongly negatively sloped portion of the unstalled characteristic, as in Day's three-stage, low \( \phi \) rig shown in Fig. 4.9. Inlet and/or exit screens may restrict the rotating stall mode past the original stall point and maintain axisymmetric conditions. In this case, the additional axisymmetric data can be compared to the assumed axisymmetric curve of Fig. 4.9. Secondly, the hypothesis that the measured recovery point from rotating stall is governed by a compression system instability, could be examined with low speed test compressor facilities. This would involve varying the system parameters, namely, the compressor characteristic slope, the throttle line slope and Greitzer's B number, and noting their effect on the recovery point. One way to implement this experiment would be to add components to the familiar compression system (compressor, plenum and throttle) such as another compressor or fan and another plenum. Similar studies could be implemented for stall inception when the compressor stall point lies on a slightly positively sloped portion of the unstalled performance.

Experimental effort can also be focused on obtaining complete axisymmetric characteristics for various compressor configurations, since they are required in modeling procedures. Compressor backflow
characteristics are important, and presently there is very limited data on this subject. The predicted effect of the blade stagger angle on backflow performance by the proposed reversed flow model, could be checked experimentally. Also, the validity of the axisymmetric valley point correlation could be examined. Furthermore, surge cycle experiments can provide the axisymmetric compressor behavior by applying the inertial correction technique discussed in Section 3.1. This could check the generalized properties, regarding the axisymmetric characteristic, presented in Section 3.2. Extending the uninstalled compressor performance calculation off-design to locate the axisymmetric peak point should also be investigated.
REFERENCES


18 Davis, R.E., Private Communication.

Fig. 2.1 Assumed rotating stall profile in compressor annulus [7]
\[
\frac{\tau_s}{\tau_{R/S}} = \frac{(1-f) \cos \gamma}{\pi |\phi_{avg}| (D/l)} \cdot \frac{1}{b}
\]

\(D/l = 12; \, \gamma = 35^\circ; \, f = 0.30; \, \phi_{avg} = 0.3\)

Fig. 2.2  Blade passage stalling time compared to rotating stall propagation time
Fig. 2.3 Complete axisymmetric characteristic partitioned into 3 regions
Fig. 2.4 Assumed axial velocity profile for the two-segment parallel compressor model
*For A Probe Stationary In The Absolute Frame

Fig. 2.5 Axial velocity profile calculated from Moore's analysis
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Fig. 3.2  Greitzer's compressor model [11]
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Fig. 3.3b Inertially corrected surge data from Greitzer's compressor (U = 304.9 ft/sec)
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Fig. 3.18 Typical rotating stall test showing the highly transient portions along the throttle lines
Fig. 3.19  Complete axisymmetric and rotating stall characteristics
Typical multi-stage compressor blading under planar backflow

Fig. 4.1
Fig. 4.2  Cornell's stalled cascade model [16]
Fig. 4.3  Cornell's stalled cascade model in the case of backflow
ACTUAL GEOMETRY:

MODEL OF ACTUAL CASE:

Control Volume

\[ h_1 = h_2 \]

Fig. 4.4 Backflow model for non-constant annulus area
Table 4.1  Compressor blading at mid-span for Turner and Sparkes' rig [17]

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$ (°)</th>
<th>$\theta_c$ (°)</th>
<th>$\beta_{LE}$ (°)</th>
<th>$\beta_{TE}$ (°)</th>
<th>$l/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IGV</td>
<td>15.2</td>
<td>30.4</td>
<td>-0.0</td>
<td>30.4</td>
<td>1.11</td>
</tr>
<tr>
<td>ROTOR</td>
<td>34.8</td>
<td>33.6</td>
<td>51.6</td>
<td>18.0</td>
<td>1.11</td>
</tr>
<tr>
<td>STATOR</td>
<td>33.6</td>
<td>35.9</td>
<td>51.5</td>
<td>15.7</td>
<td>1.11</td>
</tr>
</tbody>
</table>
Fig. 4.5 Predicted and measured reversed flow characteristics for the Turner and Sparkes compressor
Table 4.2  Compressor blading at mid-span for Greitzer's three-stage rig [1]

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$ (°)</th>
<th>$\theta_c$ (°)</th>
<th>$\beta_{LE}$ (°)</th>
<th>$\beta_{TE}$ (°)</th>
<th>$\lambda/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IGV</td>
<td>25.3</td>
<td>49.6</td>
<td>~0.0</td>
<td>—</td>
<td>1.24</td>
</tr>
<tr>
<td>ROTOR</td>
<td>26.3</td>
<td>38.8</td>
<td>49.2</td>
<td>10.4</td>
<td>1.24</td>
</tr>
<tr>
<td>STATOR</td>
<td>30.4</td>
<td>10.1</td>
<td>36.4</td>
<td>26.3</td>
<td>1.24</td>
</tr>
<tr>
<td>OGV</td>
<td>23.3</td>
<td>58.6</td>
<td>—</td>
<td>—</td>
<td>1.24</td>
</tr>
</tbody>
</table>
Fig. 4.6 Predicted backflow curves and inertially corrected surge data for Greitzer's compressor
Fig. 4.7  Effect of blade stagger angle on predicted backflow curves for Greitzer's rig
Fig. 4.8  Schematic flow description near axisymmetric shut-off
\[ \frac{\Delta p_{TS}}{p U^2} \]

\( \phi^* = 0.35 \)

- **A** Part-Span Stall, 4 Cells
- **B** Part-Span Stall, 12 Cells
- **C** Full-Span Stall, 1 Cell
- **D** Axisymmetric Swirling Flow
- **E** Part-Span Stall, 6 Cells
- **F** Part-Span Stall, 12 Cells
- **G** Part-Span Stall, 1 Cell
- **H** Axisymmetric Swirling Flow

---

**Assumed Axisymmetric Characteristic**

**Axisymmetric Valley Point**

**Fig. 4.9** Assumed axisymmetric characteristics for Day's low \( \phi^* \) rigs

123-
Fig. 4.10 Assumed axisymmetric characteristics for Day's intermediate $\phi^*$ rigs

A Part-Span Stall, 1 Cell
B Full-Span Stall, 1 Cell
C Axisymmetric Flow

--- Assumed Axisymmetric Characteristic
+ Axisymmetric Valley Point

$\frac{\Delta p_{TS}}{\rho u^2}$

$\phi^*$ = 0.55

1 Stage
2 Stages
3 Stages
4 Stages

Assumed Point
Fig. 4.11 Assumed axisymmetric characteristics for Day's high reaction rigs
Fig. 4.12  Correlation for the axisymmetric valley point
Table 4.3  Compressor builds used in the axisymmetric valley point correlation

<table>
<thead>
<tr>
<th>COMPRESSOR</th>
<th>N</th>
<th>φ*</th>
<th>(φ_v, ψ_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Day)</td>
<td>1</td>
<td>.35</td>
<td>.07, .09</td>
</tr>
<tr>
<td>B (Day)</td>
<td>3</td>
<td>.35</td>
<td>0.1(?), .28(?)</td>
</tr>
<tr>
<td>C (Day)</td>
<td>1</td>
<td>.55</td>
<td>.07, .08</td>
</tr>
<tr>
<td>D (Day)</td>
<td>2</td>
<td>.55</td>
<td>.05, .19</td>
</tr>
<tr>
<td>E (Day)</td>
<td>3</td>
<td>.55</td>
<td>.01, .31</td>
</tr>
<tr>
<td>F (Day)</td>
<td>4</td>
<td>.55</td>
<td>-.04(?), .40(?)</td>
</tr>
<tr>
<td>G (Greitzer)</td>
<td>3</td>
<td>.65</td>
<td>0.0, .295</td>
</tr>
<tr>
<td>H (Day)</td>
<td>1</td>
<td>.71</td>
<td>.08(?), .05(?)</td>
</tr>
<tr>
<td>I (Day)</td>
<td>2</td>
<td>.71</td>
<td>.01, .25</td>
</tr>
<tr>
<td>J (Day)</td>
<td>3</td>
<td>.71</td>
<td>-.05(?), .34(?)</td>
</tr>
</tbody>
</table>

(?) Assumed Coordinate
Fig. 4.13  Prediction process for a complete axisymmetric characteristic
Fig. 4.14 Prediction for the unstable portion of axisymmetric curve from Greitzer's rig

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Fig. 5.1  Predicted and measured rotating stall characteristics
Fig. 5.2  Calculated axial velocity profile using Axi 1, $\phi = 0.10$
Fig. 5.5 Calculated axial velocity profile using Ax 2, φ = 0.30
Fig. 5.6 Calculated axial velocity profile using Axi 1, $\phi = 0.47$
Fig. 5.7  Calculated axial velocity profile using Axi 2, $\phi = 0.47$
Fig. 5.8 Calculated axial velocity profile using Axil, $\phi = 0.48$. 

AXIAL VELOCITY COEFFICIENT

0.00 0.20 0.40 0.60 0.80 1.00 1.20 1.40 1.60 1.80 2.00 2.20
Fig. 5.9 Effect of compressor exit geometry parameter on predicted rotating stall lines
Fig. 5.10  Predictions for stall cell rotational speed
Appendix A  Parallel compressor formulation

Reference frame relative to stall cell.

Let $\alpha =$ fraction of annulus occupied by stall cell

$1-\alpha =$ fraction of annulus occupied by uninstalled flow

$C_{x_u} =$ uninstalled axial velocity

$C_{x_s} =$ stalled and reversed axial velocity

Given:

- $C_{x_1}$ and $C_{x_4}$ ($C_{x_1} = C_{x_4}$)
- $\alpha$
- Uninstalled compressor characteristic, or $\frac{(p_3-p_T_2)u}{\rho U^2} = F_u(\phi)$.  \hspace{1cm} (A.1)
- Backflow characteristic, or $\frac{(p_3-p_T_2)_{s}}{\rho U^2} = F_s(\phi)$.  \hspace{1cm} (A.2)
The analysis:

The continuity equation from stations 1 to 2 gives

\[(1-\alpha)\phi_1 = \phi + \alpha\phi_s, \text{ where} \]

\[\phi = \frac{C_{x1}}{U}.\]

Assume that at station 3 the average static pressure in the stall cell is equal to the average static pressure in the unstalled flow, as justified in [3], or

\[p_3 = (p_3)_s = (p_3)_u.\]  \hspace{1cm} (A.4)

Then the x-momentum equation between stations 3 and 4 becomes

\[
\frac{p_4 - p_3}{pU^2} = (1 - \alpha)\phi_1^2 - \alpha\phi_s^2 - \phi^2.
\]  \hspace{1cm} (A.5)

Assume also that at station 2 the average total pressure in the stall cell is equal to the average total pressure in the unstalled flow, again as justified in [3], or

\[p_{T_2} = (p_{T_2})_s = (p_{T_2})_u.\]  \hspace{1cm} (A.6)

Assuming a potential flow from 1 to 2 maintains that

\[p_{T_1} = p_{T_2}.\]  \hspace{1cm} (A.7)
Combining equations (A.1), (A.2), (A.4), (A.6) and (A.7) results in

\[ \frac{p_3 - p_{T_1}}{\rho U^2} = F_u(\phi_u), \quad \text{and} \]

\[ F_u(\phi_u) = F_s(\phi_s). \quad \text{(A.9)} \]

Solve equations (A.3) and (A.9) simultaneously for \( \phi_u \) and \( \phi_s \). The compressor performance in rotating stall is defined as

\[ \psi = \frac{p_4 - p_{T_1}}{\rho U^2} = \frac{p_4 - p_3}{\rho U^2} + \frac{p_3 - p_{T_1}}{\rho U^2} \quad \text{(A.10)} \]

Therefore, substituting equations (A.5) and (A.8) into (A.10) gives

\[ \psi = F_u(\phi_u) + (1 - \alpha)\phi_u^2 - \alpha \phi_s^2 - \phi^2 \]
Appendix B  Moore's compressor model [5]

The pressure rise across a blade passage is the axisymmetric pressure rise plus a dynamic correction term, or

\[
\left(\frac{\Delta p}{\frac{1}{2} \rho U^2}\right)_{\text{blade passage}} = \left(\frac{\Delta p}{\frac{1}{2} \rho U^2}\right)_{\text{axisymmetric}} + \left(\frac{\Delta p}{\frac{1}{2} \rho U^2}\right)_{\text{unsteady}} \tag{B.1}
\]

The unsteady term is found from the one-dimensional form of the momentum equation along \(z\), relative to the blade passage (see figure):

\[
\frac{dp}{dz} = -\rho \frac{\partial w}{\partial t}.
\]

Integrating yields

\[
(\Delta p)_{\text{unsteady}} = -\rho c \frac{\partial w}{\partial t},
\]

with the axial gap contribution lumped into the chord parameter, \(c\).
Substituting for $w$ with

$$w = \frac{C_x}{\cos \gamma},$$

and non-dimensionalizing the equation results in the unsteady pressure rise term,

$$\left( \frac{\Delta P}{\frac{1}{2} \rho U^2} \right)_{\text{unsteady}} = -\frac{2c}{U \cos \gamma} \frac{\partial \phi}{\partial t}. \quad (B.2)$$

The axisymmetric pressure rise at $\phi$ is defined as

$$\left( \frac{\Delta P}{\frac{1}{2} \rho U^2} \right)_{\text{axisymmetric}} = F(\phi) \quad (B.3)$$

Substituting equations (B.2) and (B.3) into (B.1) results in

$$\left( \frac{\Delta P}{\frac{1}{2} \rho U^2} \right)_{\text{blade passage}} = F(\phi) - \tau \frac{\partial \phi}{\partial t}, \quad (B.4)$$

where

$$\tau = \frac{2c}{U \cos \gamma} k, \quad \text{and} \quad (B.5)$$

$k = \text{empirical lag parameter discussed in Section 2.2},$

$c = \text{blade chord + axial gap contribution} = \ell + \frac{a_x}{\cos \gamma}.$

Moore's differential equation, equation (2.7), is written as function of $\theta$, so that the time derivative, $\frac{\partial \phi}{\partial t}$, must be transformed into the spacial derivative, $\frac{\partial \phi}{\partial \theta}$. By assuming a
steady disturbance pattern, \( \phi \) takes the form of
\[
\phi = \phi(R\theta - at),
\]
in the figure provided. From the chain rule for differentiation,
\[
\frac{\partial \phi}{\partial t} = \frac{d\phi}{d(R\theta - at)} \cdot \frac{\partial (R\theta - at)}{\partial t} = \frac{\partial \phi}{\partial (R\theta)} \cdot (-a)
\]

Or,
\[
\frac{\partial \phi}{\partial t} = -\frac{a}{R} \frac{\partial \phi}{\partial \theta}.
\]  \( \text{(B.6)} \)

Substituting equation (B.6) into (B.4) yields
\[
\left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{blade passage}} = F(\phi) + \frac{aT}{R} \frac{\partial \phi}{\partial \theta}.
\]  \( \text{(B.7)} \)

The pressure rise per stage of the compressor is
\[
\left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{stage}} = \left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{rotor}} + \left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{stator}}.
\]  \( \text{(B.8)} \)

Equation (B.7) is substituted into (B.8) by noting that

\[ a = V_s \text{ for a stator, and} \]
\[ a = -(U - V_s) \text{ for a rotor.} \]
Also, by assuming that the stator and rotor rows have identical geometry, i.e. a 50% reaction case, equation (8.8) becomes

\[
\left( \frac{\Delta p}{\frac{1}{2} \rho U^2} \right)_{\text{stage}} = 2F(\phi) - \frac{4U}{D} \tau \left( \frac{1}{2} - f \right) \frac{\partial \phi}{\partial \theta}.
\]
Appendix C  Measured compressor data notation for an unsteady, one-dimensional flow

\[
\frac{p_{\text{atm}}}{C_x} \rightarrow \text{(Forward Flow)} \rightarrow \text{(Compressor)} \rightarrow \frac{p_p}{C_x} \equiv \text{plenum pressure}
\]

Compressor model [11]

The plenum pressure referred to ambient, \( p_p - p_{\text{atm}} \), is the measured quantity.

The analysis:

The momentum equation between stations 1 and 2 in the figure above for a one-dimensional, unsteady flow is

\[
(p_e - p_1) - (p_2 - p_1) = \rho L_c \frac{dC_x}{dt}. \tag{C.1}
\]

1. Backflow case, \( C_x < 0 \)

\[
p_1 = p_{\text{atm}}, \quad \text{and} \quad \frac{p_2}{C_x} = p_p = p_2 + \frac{1}{2} \rho C_x^2, \tag{C.2}
\]

or

\[
p_2 = p_p - \frac{1}{2} \rho C_x^2. \tag{C.3}
\]

Substituting equations (C.2) and (C.3) into (C.1) yields

\[
(p_{Te} - p_1)_{\text{compressor}} = (p_p - p_{\text{atm}}) + \rho L_c \frac{dC_x}{dt}. \tag{C.4}
\]
Therefore, equation (C.4) states that the measured quantity, \( p_p - p_{atm} \), corresponds to a compressor exit total to compressor inlet static pressure rise for backflow, and also that the correction term derived in Section 3.1 applies in the backflow case.

2. Forward flow case, \( C_x > 0 \)

\[
\begin{align*}
    p_{T_1} &= p_{atm} = p_1 + \frac{1}{2} \rho \, C_x^2, \text{ or} \\
    p_1 &= p_{atm} - \frac{1}{2} \rho \, C_x^2, \text{ and} \\
    p_2 &= p_p. \hspace{2cm} (C.5) \\
    (C.6)
\end{align*}
\]

Substituting equations (C.5) and (C.6) into (C.1) gives

\[
\frac{(p_e - p_{T_1})}{t_{compressor}} = (p_p - p_{atm}) + \rho L C \frac{dC_x}{dt}. \hspace{2cm} (C.7)
\]

Therefore, equation (C.7) states that the measured quantity, \( p_p - p_{atm} \), corresponds to a compressor exit static to compressor inlet total pressure rise for the forward flow case.
APPENDIX D

COMPRESSION SYSTEM STABILITY CRITERIA [11]

Let \( C' \) = Slope of the compressor characteristic on a \( \psi_{TS} \) versus \( \phi \) plot.

Let \( T' \) = Slope of the throttle characteristic on a \( \psi_{TS} \) versus \( \phi \) plot.

\[
B = \text{Greitzer's dimensionless parameter} = \frac{U}{2a_{\text{sound}}} \sqrt{\frac{V_p}{A_{\text{c}c}}}
\]

where, \( a_{\text{sound}} \) = speed of sound
\( V_p \) = plenum volume

1. Criterion for static stability: \( C' < T' \)

2. Criterion for dynamic stability: \( C' < \frac{1}{B^2 T'} \)

Both the static and dynamic criteria must hold for a stable system. Dynamic stability is usually the most restrictive as shown in [11].
APPENDIX E

AN ATTEMPT AT PREDICTING THE UNSTABLE PORTION OF THE AXISYMMETRIC CHARACTERISTIC

The positively sloped, unstable region of the axisymmetric characteristic is most likely unimportant in predicting stall. However, a planar flow description in this region is implemented in order to predict the axisymmetric performance. This modeling procedure involves Cornell's free streamline theory for stalled cascades in the low forward flow regime, using the D-factor stall criterion [19]. A wall stall criterion could also be invoked, but is not included in the present analysis. A marching process is invoked similar to the backflow prediction scheme in Section 4.1.

The two-dimensional modeling technique was used on Greitzer's three-stage compressor. The procedure for each blade row is outlined as follows:

1. The flow coefficient, $\phi$, and the inlet flow angle, $\beta_1$, is specified from the previous blade row calculation.

2. The D-factor is calculated from the measured unstalled leaving flow angle, $\beta_3$, as

$$D = 1 + \left[ \frac{|\tan \beta_3 - \tan \beta_1|}{2(\xi/t)} - \frac{1}{\cos \beta_3} \right] \cos \beta_1.$$ If $D > 0.6$, the blade row is assumed stalled.

3. The stalled pressure rise is obtained from Cornell's separated flow model as in Section 4.1, for the forward flow case.
4. The unstalled pressure rise is approximated including a simple total pressure loss calculation based on the measured blade row efficiency:

\[
\frac{\Delta p_{TT}}{\frac{1}{2} \rho U^2} \text{ loss in row} = \frac{2\phi (\tan \beta_1 - \tan \beta_3)(1 - n_{row})}{1 + \cos^2 \beta_1 [1 + (\frac{1}{\phi} - \tan \beta_3)^2]}.
\]

The process is repeated for each compressor cascade, which gives the axisymmetric pressure rise at the given flow coefficient.

The prediction is shown in Fig. 4.14 with the excepted axisymmetric characteristic for Greitzer's rig from Section 3.1. In the predicted curve, the blade rows are stalled at the low flow coefficients until \( \phi = .40 \) where the "jump" in the characteristic results from the stator rows unstalling. All of the cascades unstall in the next "jump" at \( \phi = .45 \). This two-dimensional prediction underestimates the expected curve which leads one to believe that the flow field is not planar but rather three-dimensional in the low forward flow region. Thus, the procedure outlined in this section remains as an unsuccessful attempt at predicting the positively sloped, unstable portion of the axisymmetric characteristic.
APPENDIX F  Fortran code for the numerical solution procedure in Section 5.1 (including a sample input file)
PROCESS SC(AXIS,SYMBO)  
C **************************************************  
C ******************************************************  
C ***  
C ***  THIS PROGRAM COMPUTES A ROTATING STALL COMPRESSOR  
C ***  CHARACTERISTIC USING MOORE'S ANALYSIS WITH AN AXISYMMETRIC  
C ***  PUMPING CHARACTERISTIC DEVELOPED BY KOFF AND GREITZER.  
C ***  
C ***  
C ***  
C ***  INPUTS:  
C ***  
C ***  A(k) = EMPIRICAL LAG PARAMETER  
C ***  A(k) = EMPIRICAL COMPRESSOR EXIT GEOMETRY PARAMETER  
C ***  COMP(2,100) = AXISYMMETRIC COMPRESSOR CHARACTERISTIC  
C ***  DCRA = MEAN COMPRESSOR DIAMETER TO BLADE CHORD RATIO  
C ***  GR = ROTOR BLADE STAGGER ANGLE (DEG.)  
C ***  GS = STATOR VANE STAGGER ANGLE (DEG.)  
C ***  GIGV = INLET GUIDE VANE STAGGER ANGLE (DEG.)  
C ***  GOGV = OUTLET GUIDE VANE STAGGER ANGLE (DEG.)  
C ***  N = NUMBER OF COMPRESSOR STAGES  
C ***  NPFCH = NUMBER OF POINTS WHICH DEFINE THE AXISYMMETRIC CURVE  
C ***  PHI = AVERAGE FLOW COEFFICIENT  
C ***  X0 = INITIAL VALUE OF THETA  
C ***  A(0) = INITIAL VALUE OF AXIAL VELOCITY PERTURBATION  
C ***  A(0) = INITIAL VALUE OF CIRCUMFERENTIAL VELOCITY PERTURB.  
C ***  
C ***  ITERATION PARAMETERS:  
C ***  
C ***  DELTA = DIFFERENCE IN ROTATING STALL PERFORMANCE AND  
C ***  AXISYMMETRIC PRESSURE RISE  
C ***  F = NONDIMENSIONALIZED STALL CELL SPEED IN ABSOLUTE FRAME  
C ***  
C ***  SUBROUTINES:  
C ***  
C ***  EVAL CONTAINS THE SET OF NON-LINEAR DIFFERENTIAL  
C ***  EQUATIONS  
C ***  
C ***  DESOLV (START) DIFFERENTIAL EQUATION SOLVER  
C ***  PROGRAMMED BY GAMACHE  
C ***  
C ***  INPUTS:  
C ***  
C ***  A(5), B(5), C(5) = RUNGE-KUTTA COEFFICIENTS  
C ***  ALPHA1 = LOWER ERROR BOUND  
C ***  ALPHA2 = UPPER ERROR BOUND  
C ***  DELX = INCREMENT OF THETA  
C ***  NEQN = NUMBER OF DIFFERENTIAL EQUATIONS  
C ***  NPOINT = NUMBER OF POINTS STORED IN OUTPUT ARRAYS  
C ***  PRTVAL = THETA INTERVAL BETWEEN STORED POINTS IN  
C ***  OUTPUT ARRAYS  
C ***  
C ********************************************************************************
FILE: STALL FORTRAN A
VM/SP CONVERSATIONAL MONITOR SYSTEM

C ******************************************************
COMMON/B1/ALPHA1,ALPHA2,PRTVAL,XOUT(2500),YOUT(10,2500),NPOINT
COMMON/B2/DELX,X0,Y0(10),YSTART(10,4),YSTDOT(10,4),NEQN
COMMON/B3/A(5),B(5),C(5)
COMMON/B4/AK,AM,DCRAT,CRAT,GMR,GIVR,GOVR,CRAD,COMP(2,100),NPCH,
& PHI,PSIPI,F,DELTAN,PHEINT
COMMON/B5/IFLI
DIMENSION THETAP(20),NLOC(20),PER(20),ER(3),SUMY(2),YAVG(2)
DIMENSION BETA(200),G(200),PHE(500),CYCLE(500),GOUT(500),HOUT(500)
DIMENSION OUTFHI(100),OUTPSI(100),AXIPHI(100),AXIPS1(100)

C
C INITIALIZE CONSTANTS AND COMPRESSOR PARAMETERS
C
READ(10,100) A,B,C,NEQN
100 FORMAT(3(5F10.5),/I2)
READ(10,200) NPOINT,PRTVAL,AK,AM,DCRAT,CRAT,GR,GS,GIGV,GOGV,N
200 FORMAT(I10,F10.3/2F10.3/6F10.3,I12)
READ(10,300) NPCH,((COMP(I,J),I=1,2),J=1,NPCH)
300 FORMAT(I2/(2F10.4))
X0=0.0
Y0(1)=0.0
Y0(2)=0.0
C
C INITIAL ESTIMATE OF PHI
C
CRAD= .017453
GMR=(GR+GS)*CRAD/2.
GIVR=GIGV*CRAD
GOVR=GOGV*CRAD
FO=1./(2.+(AM*DCRAT/2./AK+1./COS(GIVR)+1./COS(GOVR))*COS(GMR)/N)
C
C SET THE AVERAGE FLOW COEFFICIENT
C
90 WRITE(6,400)
400 FORMAT('SET THE AVERAGE FLOW COEFFICIENT, PHI')
READ(5,*) PHI
C
C SET FLAGS
C
IFLI=0
IFL4=0

C
C CALCULATE AXISYMMETRIC PRESSURE RISE AT PHI
C
IJ=1
20 IF(PHI-COMP(1,IJ)) 21,22,23
21 PSIPHI=COMP(2,IJ-1)+(COMP(2,IJ)-COMP(2,IJ-1))*(PHI-COMP(1,IJ-1))/
& (COMP(1,IJ)-COMP(1,IJ-1))
& GO TO 22
22 PSIPHI=COMP(2,IJ)
C
C INITIALIZE DELX, G, H AND GUESS F, DELTA
C
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C
FILE: STALL FORTRAN A

VM/SP CONVERSATIONAL MONITOR SYSTEM

11 DELX=0.1
   IFL1=0
   ALPHA1=0.00002
   ALPHA2=0.001
   X0=0.0
   Y0(1)=0.0
   Y0(2)=0.0
   WRITE(6,500) FO

500 FORMAT(' GUESS VALUES FOR F AND DELTA'/' APPROXIMATE F = ',F5.3)
   READ(5,*) F,DELTA
   RPSI=PSI+DELTA
   WRITE(20,500) PHI,FO,F,DELTA,RPSI

600 FORMAT(///3X,'PHI',4X,'=',F8.4/' APPROX F =',F8.4/4X,'F',5X,'=',
   &   F8.4/2X,'DELTA',3X,'=',F8.4/' R/S PSI',2X,'=',F8.4/)
   C
   C DETERMINE G AND H AS FUNCTIONS OF THETA
   C
   86 CALL DESOLV
   IF(IFL1 .EQ. 0) GO TO 84
   IF(IFL4 .NE. 0) GO TO 85
   WRITE(6,240)

   240 FORMAT(' PHE WENT OUT OF BOUNDS!'/' DECREASE STEP SIZE? (YES=1)')
   READ(5,*) JST
   IF(JST .NE. 1) IFL1=0.
   IF(JST .NE. 1) GO TO 11

   C
   C DECREASES THE STEP SIZE 'DELX'
   C
   IFL1=0
   IFL4=1
   DELX=0.075
   ALPHA1=0.00001
   ALPHA2=0.0005
   GO TO 86

   85 IF(IFL1 .EQ. 1) GO TO 87
   WRITE(6,210) PHEINT
   WRITE(20,210) PHEINT

   210 FORMAT(///2X,'ERROR: PHE IS TOO SMALL FOR COMPRESSOR CHAR.:'/ &   ' PHE =',F8.4)
   GO TO 11

   87 WRITE(6,220) PHEINT
   WRITE(20,220) PHEINT

   220 FORMAT(///2X,'ERROR: PHE IS TOO LARGE FOR COMPRESSOR CHAR.:'/ &   ' PHE =',F8.4)
   GO TO 11

   C
   C DETERMINE THE PERIOD OF G
   C
   84 GCH=0.0
   SLPO=(YOUT(1,2)-YOUT(1,1))/(XOUT(2)-XOUT(1))
   I10=2
   I1=I10
   DO 30 I=1,14
   IFCH=1
   NCH=1
   C
   C
35 IF(GCH-YOUT(I1,1)) 31,32,33
31 IPCH=1
   IF(I1 .EQ. I10) GO TO 37
   IF(NCH .EQ. 1) GO TO 32
37 NCH=NCH+1
   GO TO 34
33 NCH=1
   IF(I1 .EQ. I10) GO TO 38
   IF(IPCH .EQ. 1) GO TO 32
38 IPCH=IPCH+1
34 I1=I1+1
   IF(XOUT(I1) .LE. XOUT(I1-1)) GO TO 70
   GO TO 35
32 SLP=(YOUT(I1,I1)-YOUT(I1,I1-1))/(XOUT(I1)-XOUT(I1-1))
   IF(SLP0*SLP) 60,61,62
60 IPCH=2
   NCH=2
   GO TO 34
61 WRITE(6,110)
110 FORMAT(//' ERROR: SLOPE OF G IS ZERO AT GCH')
   GO TO 99
62 THETAP(I)=XOUT(I1-1)+(XOUT(I1)-XOUT(I1-1))*(GCH-YOUT(I1,I1-1))
   & (YOUT(I1,I1)-YOUT(I1,I1-1))
   NLOC(I)=I1
   I10=I1
   NTNP-I
30 CONTINUE
   GO TO 71
70 NTNP=I-1
71 DO 63 I=2,NTNP
63 PER(I-1)=THETAP(I)-THETAP(I-1)
   ERLIM=0.005
78 IER=0
65 IER=IER+1
   IF((IER+3) .GT. NTNP) GO TO 76
   DO 64 I=1,2
64 ER(I)=ABS(PER(I+IER)-PER(I+IER-1))
   ER(3)=ABS(PER(IER)-PER(IER+2))
   DO 66 I=1,3
   IF(ER(I) .GE. ERLIM) GO TO 65
66 CONTINUE
   ICHEK=IER+1
   WRITE(6,120) ICHEK,PER(ICHEK)
   WRITE(20,120) ICHEK,PER(ICHEK)
120 FORMAT(//' THE PERIOD CONVERGES IN','I3',' CYCLES'/' PERIOD =', &
   F10.4)
C
C CHECK FOR PERIODICITY IN THE 'ICHEK' PERIOD
C
NDIV=30
APER=(PER(ICHEK)+PER(ICHEK+1))/2.
I2=NLOC(ICHEK)-1
IENT=NDIV*2+1
   DO 73 I=1,IENT
   BETA(I)=THETAP(ICHEK)+APER* (I-1)/NDIV
72 IF(BETA(I)-XOUT(I2)) 67,68,69
69 I2=I2+1
GO TO 72
68 G(I)=YOUT(I,J2)
GO TO 73
67 G(I)=YOUT(I,J2-1)+(YOUT(I,J2)-YOUT(I,J2-1))*(BETA(I)-XOUT(I2-1))
& / (XOUT(I2)-XOUT(I2-1))
73 CONTINUE
NNN=NDIV+1
DO 74 I=1,NNN
ERR=ABS(G(I)-G(I+NDIV))
IF(ERR .GE. ERLIM) GO TO 75
74 CONTINUE
WRITE(6,130)
WRITE(20,130)
130 FORMAT(' *** G IS PERIODIC ***')
GO TO 50
75 WRITE(6,140)
WRITE(20,140)
140 FORMAT(' G IS NOT PERIODIC YET, MORE ITERATION REQUIRED!')
GO TO 65
76 IF(ERLIM .GE. 0.05) GO TO 79
ERLIM=ERLIM+.005
WRITE(6,170) ERLIM
WRITE(20,170) ERLIM
170 FORMAT(' G IS NOT PERIODIC!! '// ERROR TOLERANCE INCREASED TO',
& F7.3)
GO TO 78
79 WRITE(6,160) NTHP
WRITE(20,160) NTHP
160 FORMAT(' *** G IS NOT PERIODIC, LONGER RUN TIME REQUIRED ***'/
& ' NO. OF PERIODS =',I3)
GO TO 11
C
C FIND THE AVERAGE VALUES OF G AND H
C
50 NL=NLOC(ICHEK)
THO=THETAP(ICHEK)
THF=THETAP(ICHEK+1)
PERIOD=PER(ICHEK)
SUMY(1)=(XOUT(NL)-THO)*(YOUT(1,NL)+GCH)/2.
HO=YOUT(2,NL-1)+(YOUT(2,NL)-YOUT(2,NL-1))*(GCH-YOUT(1,NL-1))
& *(YOUT(1,NL)-YOUT(1,NL-1))
SUMY(2)=(XOUT(NL)-THO)*(YOUT(2,NL)+HO)/2.
40 IF(XOUT(NL+1) .GT. THF) GO TO 41
DO 42 I=1,2
42 SUMY(I)=SUMY(I)+(XOUT(NL+1)-XOUT(NL))*(YOUT(I,NL+1)+YOUT(I,NL))/2.\nNL=NL+1
GO TO 40
41 SUMY(1)=SUMY(1)+(THF-XOUT(NL))*(GCH-YOUT(1,NL))/2.
HF=YOUT(2,NL)+(YOUT(2,NL+1)-YOUT(2,NL))*(GCH-YOUT(1,NL))
& *(YOUT(1,NL+1)-YOUT(1,NL))
SUMY(2)=SUMY(2)+(THF-XOUT(NL))*(HF+YOUT(2,NL))/2.
DO 43 I=1,2
43 YAVG(I)=SUMY(I)/PERIOD
ERROR=ABS(YAVG(2)) + ABS(PER(ICHK))-6.28319
WRITE(6,150) (YAVG(I),I=1,2),ERROR
WRITE(20,150) (YAVG(I),I=1,2),ERROR
150 FORMAT('/ GAVG =','F7.4/' HAVG =','F7.4/' ERROR =','F7.4)

C
C OUTPUT OPTIONS
C
NPL=0
82 INDE=NLOC(ICHK)+NPL-1
IF(XOUT(INDE) .GT. BETA(NENT)) GO TO 77
NPL=NPL+1
CYCLE(NPL)=(XOUT(INDE)-XOUT(NLOC(ICHK)-1))/6.283185
PHE(NPL)=PHI+YOUT(1,INDE)
GOUT(NPL)=YOUT(1,INDE)
HOUT(NPL)=YOUT(2,INDE)
GO TO 82

C
C RESET INITIAL CONDITIONS ON G AND H TO PREVIOUS STATE
C
77 Y0(1)=GOUT(1)
Y0(2)=HOUT(1)
WRITE(6,620)
620 FORMAT(' TABLE FOR CYCLE, G AND H ,OR, PLOT OF AXIAL VELOCITY'/
  & ' (TABLE = 1 , PLOT = 2 , NEW PHI = 3)')
READ(5,*) JTAB
IF(JTAB .EQ. 1) GO TO 80
IF(JTAB .EQ. 2) GO TO 81
IF(JTAB .EQ. 3) GO TO 99
GO TO 11
80 WRITE(20,610) (CYCLE(I),GOUT(I),HOUT(I),I=1,NPL)
610 FORMAT('/5X,'CYCLE','8X,'G' ,11X,'H'/(4X,F6.4,2(5X,F7.4)))
GO TO 11

C
C PLOTS AXIAL VELOCITY AROUND THE ANNULUS
C
81 CYCLE(NPL+1)=0.0
CYCLE(NPL+2)=0.2
PHE(NPL+1)=-0.2
PHE(NPL+2)=0.2
CALL PLOTS(IDUM,IDUM,30)
CALL AXIS(0.0,0,'CYCLES IN ANNULUS',-17,11,0,CYCLE(NPL+1),
  & CYCLE(NPL+2))
CALL AXIS(0.0,0,'AXIAL VELOCITY COEFFICIENT',26,6.90,PHE(NPL+1),
  & PHE(NPL+2))
CALL LINE(CYCLE,PHE,NPL,1,0,0)
CALL ENDP(15.,0.,999)
99 STOP
END

C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************

C*************************************************************************
C*************************************************************************
C*************************************************************************

-158-
FILE: STALL FORTRAN A VM/SP CONVERSATIONAL MONITOR SYSTEM

DO 2 J=1,6
Y(I,J)=0.0
YDOT(I,J)=0.0
AM(I,J)=0.0
AMDOT(I,J)=0.0
2 CONTINUE
1 CONTINUE
XVAR(1)=XINTL
DO 3 I=1,NEQN
YVAR(I,1)=YINT(I)
3 CONTINUE
KCNT1=2
KCNT2=1
NOCTS=0
NHSS=0
NDSS=0
NOPCC=0
IRETN=0
NCONVI=0
NCONVX=0

C
C ***** USE RUNGE-KUTTA METHOD TO COMPUTE STARTING VALUES.
C
19 CONTINUE
X(3)=XINTL
X(4)=X(3)+DELX
X(5)=X(4)+DELX
X(6)=X(5)+DELX
CALL START
IF(IFL1 .NE. C) RETURN
NOCTS=NOCTS+1
DO 4 I=1,NEQN
PDIFC(I)=0.0
DO 5 J=3,6
K=J-2
Y(I,J)=YSTART(I,K)
YDOT(I,J)=YSTDOT(I,K)
5 CONTINUE
4 CONTINUE
KCNT3=0

C
C ***** UPDATE OUTPUT ARRAYS (IF NECESSARY)
C
9 CONTINUE
KCNT3=KCNT3+1
IF(X(6)-XVAR(KCNT2).LT.PRTVAL) GO TO 8
XVAR(KCNT1)=X(6)
DO 7 I=1,NEQN
YVAR(I,KCNT1)=Y(I,6)
7 CONTINUE
KCNT1=KCNT1+1
KCNT2=KCNT1-1
IF(KCNT2.EQ.NPOINT) GO TO 20
8 CONTINUE

-160-
C ***** HAMMING'S MODIFIED PREDICTOR CORRECTOR METHOD.

J = 7
X(7) = X(6) + DELX
DO 10 I = 1, NEQN
PRE(I) = Y(I, 3) - 1.250 * DELX * (2.0 * YD(I, 6) - YD(I, 5) + 2.0 * YD(I, 4))
AM(I, 7) = PRE(I) - 0.9256196347 * PDIFC(I)
10 CONTINUE
TOTERR = 0.0
CALL EVAL(J, X, AM, AMDOT, NEQN)
IF (IFL1 .NE. 0) RETURN
DO 11 I = 1, NEQN
COR(I, 1) = 0.125 * (9.0 * Y(I, 6) - Y(I, 4) + 3.0 * DELX * (AMDOT(I, 7) + 2.0 * YD(I, 6)))
Y(I, 7) = COR(I, 1)
11 CONTINUE
IITER = 1
24 CONTINUE
CALL EVAL(J, X, Y, YDOT, NEQN)
IF (IFL1 .NE. 0) RETURN
DO 22 I = 1, NEQN
COR(I, 2) = 0.125 * (9.0 * Y(I, 6) - Y(I, 4) + 3.0 * DELX * (YDOT(I, 7) + 2.0 * YD(I, 6)))
PSERR = 0.0743801653 * ABS(PRE(I) - COR(I, 2))
CONV = ABS(COR(I, 2) - Y(I, 7))
IF (0.75 * CONV .GT. PSERR) IRETN = 2
Y(I, 7) = COR(I, 2)
22 CONTINUE
IF (IRETN .LT. 1) GO TO 23
IRETN = 0
IITER = IITER + 1
NCONVX = NCONVX - 1
IF (IITER .GE. 3) GO TO 26
GO TO 24
26 CONTINUE
NCONV = NCONV + 1
23 CONTINUE
DO 25 I = 1, NEQN
PDIFC(I) = PRE(I) - COR(I, 2)
ERROR(I) = 0.0743801653 * PDIFC(I)
TOTERR = TOTERR + ABS(ERROR(I))
25 CONTINUE

C C ****** CHECK STEP SIZE OF INDEPENDENT VARIABLE.

C IF (TOTERR .GT. ALPHA2) GO TO 13
IF (TOTERR .LT. ALPHA1) GO TO 12

C C ****** COMPUTE DEPENDENT VARIABLE DERIVATIVES AND REINITIALIZE
C ****** INFORMATION FOR THE NEXT PREDICTOR CORRECTOR STEP - STEP
C ****** SIZE REMAINS THE SAME.

C NOPCC = NOPCC + 1
CALL EVAL(J, X, Y, YDOT, NEQN)
IF (IFL1 .NE. 0) RETURN
DO 14 J=1,6
K=J+1
X(J)=X(K)
DO 15 I=1,NEQN
Y(I,J)=Y(I,K)
YDOT(I,J)=YDOT(I,K)
15 CONTINUE
14 CONTINUE
GO TO 9

C
C ***** TRUNCATION ERROR TOO LOW - DOUBLE STEP SIZE.

12 CONTINUE
NDSS=NDSS+1
DELX=DELX*2.0
IF(KCNT3.LT.3) GO TO 16
CALL EVAL(J,X,Y,YDOT,NEQN)
IF(IFL1 .NE. C) RETURN
X(6)=X(7)
X(4)=X(3)
X(3)=X(1)
DO 18 I=1,NEQN
Y(I,6)=Y(I,7)
Y(I,4)=Y(I,3)
Y(I,3)=Y(I,1)
YDOT(I,6)=YDOT(I,7)
YDOT(I,4)=YDOT(I,3)
YDOT(I,3)=YDOT(I,1)
18 CONTINUE
KCNT3=0
GO TO 9

16 CONTINUE
XINTL=X(7)
DO 17 I=1,NEQN
YINTL(I)=Y(I,7)
17 CONTINUE
GO TO 19

C
C ***** TRUNCATION ERROR IS TOO GREAT - HALVE STEP SIZE.

13 CONTINUE
NHSS=NHSS+1
DELX=DELX/2.0
XINTL=X(6)
DO 21 I=1,NEQN
YINTL(I)=Y(I,6)
21 CONTINUE
GO TO 19

C
C ***** RETURN TO THE MAIN CALLING PROGRAM.

20 CONTINUE
WRITE(20,100)
WRITE(20,200) NOCTS
WRITE(20,300) NDSS
WRITE(20,400) NHSS
WRITE(20,500) NOPCC
WRITE(20,600) NCONVX
WRITE(20,700) NCONVI
100 FORMAT(1X,/,'HNUMERICAL SOLUTION MESSAGES,/',I7)
200 FORMAT(5X,3EHNUMBER OF CALLS TO SUBROUTINE START: ,I7)
300 FORMAT(5X,3EHNUMBER OF TIMES STEP SIZE IS DOUBLED: ,I7)
400 FORMAT(5X,3EHNUMBER OF TIMES STEP SIZE IS HALVED: ,I7)
500 FORMAT(5X,3EHNUMBER OF ACCEPTED HAMMING PC POINTS: ,I7)
600 FORMAT(5X,4EHNUMBER OF TIMES CORRECTOR REQUIRES ITERATION: ,I7)
700 FORMAT(5X,4EHNUMBER OF FAILED CORRECTOR CONVERGENCES: ,I7,/) RETURN
END

C ******************************************************
C ******************************************************
C *** SUBROUTINE START Initializes DESOLV
C ***
C ******************************************************
C ******************************************************

SUBROUTINE START
DIMENSION YITM(10,7),YITMD(10,7),H(7),Q(10,5)
COMMON/B2/DELX,XINTL,YINTL(10),YSTART(10,4),YSTD(10,4),NEQN
COMMON/B3/A(5),B(5),C(5)
COMMON/B5/IFL1

C ***** INITIALIZE VARIABLES AND COMPUTE DEPENDENT VARIABLE
C ***** DERIVATIVES AT THE INITIAL CONDITION.

DO 1 I=1,NEQN
   YSTART(I,1)=YINTL(I)
   YITM(I,1)=YSTART(I,1)
   Q(I,1)=0.0
1 CONTINUE
   H(1)=XINTL
   J=1
   CALL EVAL(J,H,YITM,YITMD,NEQN)
   IF(IFL1.NE.0) RETURN
   DO 2 I=1,NEQN
      YSTD(I,1)=YSTD(1,1)
2 CONTINUE

C ***** BEGIN LOOP TO COMPUTE THREE ARRAYS OF DEPENDENT VARIABLES

DO 3 L=2,4
   H(1)=XINTL
   H(2)=XINTL+DELX/2.0
   H(3)=H(2)
   H(4)=XINTL+DELX
   H(5)=H(4)

C ***** BEGIN RUNGE-KUTTA CALCULATIONS

-163-
DO 4 J=2,5
  K=J-1
  CALL EVAL(K,H,YITM,YITMD,NEQN)
  IF(IFL1 .NE. 0) RETURN
  DO 5 I=1,NEQN
    YITM(I,J)=YITM(I,K)+DELX*A(J)*(YITMD(I,K)-B(J)*Q(I,K))
    Q(I,J)=Q(I,K)+3.0*A(J)*(YITMD(I,K)-B(J)*Q(I,K))-C(J)*YITMD(I,K)
  5 CONTINUE
  4 CONTINUE
  K=5
  CALL EVAL(K,H,YITM,YITMD,NEQN)
  IF(IFL1 .NE. 0) RETURN
C
C ***** REINITIALIZE VARIABLES FOR CALCULATION OF NEXT DEPENDENT
C ***** VARIABLE VECTOR ARRAY AND STORE EXISTING SOLUTION VECTOR
C
DO 6 I=1,NEQN
  Q(I,1)=Q(I,5)
  YITM(I,1)=YITM(I,5)
  YSTART(I,L)=YITM(I,5)
  YSTDOT(I,L)=YITMD(I,5)
  6 CONTINUE
  XINTL=XINTL+DELX
  3 CONTINUE
RETURN
END
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