A STUDY OF

SOME CHARACTERISTICS OF THE JET-PIPE VALVE

by

JOHN FREDERICK DUNN, JR.

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JET-PIPE VALVE

by
John Frederick Dunn Jr.

Submitted to the Department of Mechanical
Engineering on May 13, 1957 in partial
fulfillment of the requirements for the
degree of Doctor of Science

ABSTRACT

A discontinuous-potential-flow description of the
flow field is investigated in an attempt to predict
analytically the blocked-ram pressure—versus-displacement
characteristics of a two-dimensional jet-pipe valve.
Equations are formulated which relate the significant
geometrical dimensions of the valve configuration and
other flow-field parameters to a transformed configuration
of the flow field. Use of these relations in any practical
solution procedure requires the use of a digital computer
in an elaborate and costly program of numerical computation
that appears unwarranted at this time. An electric analog
is employed to demonstrate that, for two cases examined,
the proposed potential-flow description gives reasonably
accurate results.

An alternative approach that is less direct and less
amenable to rigorous technical defense is employed to
develop semi-empirical formulas that express, with accuracy
sufficient for most design purposes, the pressure-displace-
ment relationships for the jet-pipe valve over a practical
range of distributor-block-and-nozzle configurations.

Experimental data demonstrates the applicability of
the developed semi-empirical relations and offers support
for the discontinuous-potential-flow representation of the
flow field.

Thesis Supervisor: John A. Hrones
Title: Professor of
Mechanical Engineering
Cambridge, Massachusetts
May 13, 1957

Professor Joseph H. Keenan
Chairman, Departmental Committee on Graduate Students
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In partial fulfillment of the requirements for the degree of Doctor of Science in Mechanical Engineering at the Massachusetts Institute of Technology, I present this thesis entitled, "A Study of Some Characteristics of the Jet-Pipe Valve".

Respectfully submitted,

[Signature]
John F. Dunn, Jr.
Acknowledgements

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PREFACE

The basic physical principles governing the operation of the jet-pipe valve have been known for many years. As early as the 1880's Beauchamp Tower(1)*, in England, used a device having the essential attributes of the present-day jet-pipe valve in an ingenious mechanism for stabilizing a naval-gun platform. The valve was first commercially produced in Germany and since has found important application in the industrial control field. Though some work has been done which permits one to study the performance of systems utilizing jet-pipe valves(2),(3), the characteristics of the valves themselves have always been experimentally determined. Hence, in spite of its relatively long history, the details of its operation have not been fully investigated and practically all design modifications have been made on the basis of intuition and experimentation.

Because requirements placed upon modern control systems are widely varied and are tending to become more and more demanding, the ability to understand, to predict, and to select the characteristics of each system component is becoming more of a necessity. This paper reports the results of some theoretical and experimental jet-pipe valve investigations carried out in order to obtain a better insight into

*Numbers in parentheses refer to similarly numbered references in the bibliography at the end of this paper.
the mechanisms by which valving is accomplished and to provide some of the groundwork for predicting, with reasonable accuracy, the influence of various parameters upon the valving characteristics.


John F. Dunn, Jr.
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A, B, C, ... H; A', B', C', ... H'  Points in the complex plane
K                Jet width
M, M', M''            Constants during integration
P                Pressure
U                Absolute velocity at the nozzle exit
V                Absolute velocity at any point
W                Complex potential
a                Plate "length"
B                Receiving opening width
h                Portion of flow
k                Scale factor
r                Radius
u                x-component of velocity
v                y-component of velocity
X                Horizontal physical coordinate
Y                Vertical physical coordinate
Z                Complex physical coordinate
α, α', β, β'           Flow angles
δ, δ'           Displacements
ξ                Hodograph function
ρ                Mass density
θ                General angle
ϕ                Velocity potential
ψ                Stream function
CHAPTER I

INTRODUCTION

THE VALVE AND ITS OPERATION

Physically the jet-pipe valve is an extremely simple device consisting of two major components--a nozzle and a distributor block, Fig. 1. The nozzle converts the pressure-potential energy of supply fluid into the kinetic energy of a jet and directs this jet toward the distributor block. In the distributor block there are two closely spaced holes, which, in a typical application, are connected to opposite sides of a ram. When the jet impinges upon the distributor block its momentum is altered. The pressure forces which result from this sustained momentum change are highest directly under the jet; hence, if the jet is directed more toward one hole than the other, the pressure on one side of the ram is higher and a force tending to cause motion is exerted on the ram piston.

The jet-pipe valve has several features which might make its use as a control element attractive. The nature of its construction, characterized by absence of tight clearances and by flow passages that are relatively large, suggests both economy of manufacture and a high degree of reliability. Silting and sticking problems are usually of little concern. Actuation forces are low and little reserve force is needed to overcome friction. For use as the first stage of a two-stage valve, particularly where only signals at very low power
SCHEMATIC DIAGRAM OF A JET PIPE SERVO WITH FORCE FEEDBACK

FIG 1
levels are available, the jet-pipe valve can offer distinct advantages. The use of the jet-pipe valve with shop-air supplies and with hot-gas generators, which seldom are sources of clean fluid unless special pains are taken to provide good filtration, also seems feasible.

The disadvantages of the jet-pipe valve must also be mentioned in order to accurately complete the picture. When connected to a constant pressure supply, the jet pipe requires the same flow of fluid whether called upon to move a load or not. Thus, there is a continuous and constant power dissipation. (On the other hand, when hooked up to a constant flow supply, such as a gear pump running at constant speed, the jet pipe eliminates the necessity for pressure-regulating valves, by-pass valves, etc., other than those used as safety devices.) Because the jet-pipe valve may be classified as open-centered, it is load sensitive. To hold a load stationary, the nozzle must be moved from its centered position in order to develop a holding force. Its use to drive the final element in a highly loaded system, where simple feedback is to be employed, is questionable. The turbulence of the jet emerging from the nozzle and of any fluid surrounding the nozzle can introduce undesirable noise into a system.

THE PROBLEM

To intelligently incorporate a valve into a system, one must be able to predict or know beforehand, at least approximately, the relationship between load pressure, load flow,
and valve displacement. The fact that the fluid in the region between the nozzle and the distributor block is not confined by rigid physical boundaries, however, makes accurate analysis of the jet-pipe valve extremely difficult; the characteristics of the flow field are not easily determined. The complexity of the problem increases manifoldly if compressible fluid flow is considered because normal operation would involve a sonic nozzle with a pressure roughly half that of the supply pressure at the exit plane. Resulting shock phenomena would necessitate consideration of several parameters in addition to those required for a reasonable treatment of incompressible flow.

The present study is restricted to examination of the flow field that is established in a two-dimensional jet-pipe valve for the blocked-ram condition. Only the flow of fluids which can be assumed to be incompressible is considered. These restrictions are made in an effort to reduce the problem to one which is reasonably amenable to analytic treatment.
CHAPTER II

ANALYTICAL INVESTIGATION

VISUALIZATION OF THE FLOW FIELD

Visualization of the flow field that would be established in a two-dimensional jet-pipe valve, assuming blocked-ram conditions and an ideal fluid, leads to a theoretical model which appears susceptible to analytic treatment. In such a model, the fluid can be pictured as separating from the nozzle and emerging as a jet of liquid which streams toward the distributor block. Impingement of the jet upon the distributor block causes the jet to widen and then to divide—some of the fluid going to each side. When a side stream passes over a receiving opening, it dips into the opening since the fluid in the chamber cannot provide the non-uniform pressure distribution that would cause the stream to move directly sideward. Downward motion of the side stream continues until the reacting pressure forces cause an upward deflection, and at the outer edge of the receiving hole the fluid separates and emerges as a jet. This kind of flow regime is illustrated in Fig. 2a, where it is also assumed that no mixing occurs between fluid coming from the nozzle and fluid already in the receiving chambers.

DISCONTINUOUS POTENTIAL FLOW

Sharp separation and discontinuity in a flow field usually can best be approximated analytically by the classical theory of discontinuous potential flow. In this theory a
ψ = h₂ KU
ψ = h₁ KU
Z = x + iy

ξ = u - iv

Where V = Velocity = u + iv
Unit of Velocity = U ft/sec
ξ (Hodograph) Plane

\[ \frac{1}{2} \ln \left( \frac{1 - P_1}{P_2} \right) \]

In ξ Plane

ψ = h₁ K

W = φ + iψ
W (Potential) Plane

ψ = h₂ K

FIG. 2

COMPLEX PLANES DESCRIBING THE PROBLEM OF THE TWO-DIMENSIONAL JET-PIPE VALVE
discontinuity is assumed to originate at the sharp edge of a solid boundary. It is assumed that the flow is incompressible, steady, and inviscid, and that the flow everywhere has the same fluid energy. As a consequence of the assumptions that the flow is inviscid and that the motion everywhere has the same fluid energy, no vortices can exist in the flow field. In reality, it is assumed that all vorticity is restricted to the constant pressure discontinuities of infinitesimal thickness originating at solid boundaries and lying outside that part of the flow field to be analyzed.

Such problems were first analyzed by Helmholtz\(^{(4)}\), who in 1868 determined the form of the free streamlines for flow through a Borda mouthpiece. The method of Helmholtz has been refined and extended by many mathematicians\(^4\). Now the general approach is outlined in many theoretical hydrodynamics texts, i.e., Milne-Thompson\(^{(5)}\) and Lamb\(^{(6)}\).

The theory has been applied by Von Mises\(^{(7)}\) to a large class of problems in which two-dimensional liquid jets are discharged into air or other low density media. In such cases the free surfaces can usually be accurately represented by a discontinuity at constant pressure and Von Mises' results, in general, correlate very well with experimental data.

\(^{4}\text{See Bibliography}\)
There are cases, however, where the discontinuity is not easily defined and where application of the theory of discontinuous potential flow is questionable. Wake flow is one such case. Kirchhoff\(^{(8)}\) who first suggested the application of the theory to wake-type flow, considered the problem of a lamina held oblique to the flow in an infinite stream. Rayleigh\(^{(9)}\) completed the solution and obtained results which avoided many of the paradoxes that arose with a Dirichlet description of the flow (infinite velocities at the sharp edges, zero drag coefficient, etc.) but which quantitatively were still far from reality. The fact that a streamline of discontinuity in a non-viscous fluid can be shown to be in unstable equilibrium offers a possible explanation. Experimentally it can be observed to become wavy and then turbulently to break up. Real wakes, hence, cannot accurately be described as motionless zones of "dead" fluid.

A discrepancy between theory and observation also exists for the case of jet flowing into a medium of the same density. Ehrich\(^{(10)}\), in an interesting doctoral thesis, demonstrated that the discrepancy is small, however, for valving orifices where the confined geometry "inhibit(s) the irregular type wake associated with flat plates in a free stream". Indeed, Lee and Blackburn\(^{(11)}\) successfully employed Von Mises' results to a valve-spool design that greatly reduced axial flow forces.
APPLICATION OF DISCONTINUOUS POTENTIAL FLOW

In applying a potential flow description to the flow field of a two-dimensional jet-pipe valve, the assumption that gravity has no influence on the flow is made to simplify the mathematics. Normally, negligible error is incurred by this assumption. The further assumptions are made that the nozzle is far enough up-stream of the distributor block so that flow conditions at the distributor block have negligible influence on the flow emanating from the nozzle and that the flow from the nozzle is uniform and parallel and at atmospheric pressure. Fig. 2a represents the flow geometry to be studied.

The general solution to problems in potential flow is completely determined if the complex potential function \( W \), which combines the potential and stream functions in the form \( W = \phi + i \psi \), can be expressed in terms of the physical coordinate \( z = x + iy \). The shape of the flow field in the \( W \)-plane usually can be drawn readily, since the stream function is a constant along a streamline. Figure 2e shows the shape of the complex potential plane for the jet-pipe valve; it must be noted that the point-by-point location of the streamlines is not known.

The complex potential plane is related to the physical plane by the relation

\[
\frac{dW}{dz} = \zeta = u - iv. \tag{2.1}
\]

\* \( \frac{\partial \phi}{\partial x} = u; \frac{\partial \phi}{\partial y} = v; \frac{\partial \psi}{\partial y} = u; \frac{\partial \psi}{\partial x} = -v \); where \( u = x \)-component of velocity, \( v = y \)-component of velocity
As in the $W$-plane, the flow field is usually simple in shape in the $\zeta$ (hodograph or conjugate velocity) plane. There, the traces of all straight boundaries are straight lines and all constant-pressure discontinuities are arcs of circles. Usually the simple geometries of the transformed flow field in these planes can be related to one another without difficulty via conformal mapping techniques. Once such relation is established, integration of Eq. (2.1) yields the solution to the problem. In the case under consideration, however, the hodograph, Fig. 2b, is complicated and, as will be shown in Chapter III, the difficulties encountered in seeking the requisite relations by exact mathematical treatment render such an approach impractical.

With the neglect of the influence of gravity the energy equation for the flow field assumes the form

\[
\frac{P}{\rho} + \frac{V^2}{2} = \frac{U^2}{2} = \frac{P_s}{\rho}
\]  

(2.2)

where $P$ is the pressure at any point in the flow field,

$\rho$ is the density of the fluid,

$V$ is the fluid velocity at any point in the flow field,

$U$ is the fluid velocity at the exit of the nozzle,

and $P_s$ is defined as $\frac{\rho U^2}{2}$.

The dimensionless velocity $V/U$ at any point in the flow field becomes:

\[
\frac{V}{U} = \sqrt{1 - \frac{P}{P_s}}
\]  

(2.3)
If the unit of velocity is taken as $\frac{U}{\text{ft/sec}}$, $\zeta$ has the magnitude $\sqrt{1 - \frac{P}{P_s}}$ at any point in the flow fluid and the hodograph representation of the flow field is confined within a unit circle, Fig. 2b. The $\ln \zeta$ plane takes the form shown in Fig. 2c.

The polygonal boundaries of the flow field in the $\ln \zeta$ and $W$ planes can each be mapped, with a one-to-one correspondence, onto the real axis of an intermediate $t$-plane with the aid of Schwarz-Christoffel transformations. The interiors of the polygons are mapped onto the upper half of the $t$-plane. These transformations may be expressed in the forms

$$\ln \zeta = - \int_{t=0}^{t} \frac{(t-t_{P})(t-t_{P'})}{\sqrt{(t-t_{G})(t-t_{G})(t-t_{P})(t-t_{P})(t-t_{G})(t-t_{G})}} \ dt + \frac{i\pi}{2}$$

and

$$dW = \frac{t_{H}t_{H'}K}{\pi} \frac{dt}{t(t-t_{H})(t-t_{H'})},$$

(2.5)

derivation of which is shown in Appendix A.

By combining Eqs. (2.1), (2.4), and (2.5), there is obtained
\[
\frac{dz}{e^{\ln \xi}} = \frac{dW}{\xi}
\]

\[
= \frac{t_H t_H' K}{\pi} \left( \frac{dt}{t(t-t_H)(t-t_H') e^{\sqrt{\int_0^t \frac{(t-t_C)(t-t_C')}{(t-t_H)(t-t_H')(t-t_G)(t-t_G')} dt}} + i \frac{2\pi}{t(t-t_H)(t-t_H') e^{\sqrt{\int_0^t \frac{(t-t_C)(t-t_C')}{(t-t_H)(t-t_H')(t-t_G)(t-t_G')} dt}}} \right)
\]

which relation together with Eq. (2.4) can be used directly to formulate the following expressions of the significant physical geometrical dimensions of the distributor-block-and-nozzle combination in terms of the t-transformed flow field.

\[
b = \text{Re} \int_{t=t_G}^{t=t_C} dz = \text{Re} \int_{t=t_G}^{t=t_C} \frac{dW(t)}{e^{\ln \xi(t)}} \tag{2.7}
\]

\[
o = \text{Im} \int_{t=t_G}^{t=t_C} dz = \text{Im} \int_{t=t_G}^{t=t_C} \frac{dW(t)}{e^{\ln \xi(t)}} \tag{2.8}
\]

\[
b = \text{Re} \int_{t=t_G'}^{t=t_C'} dz = \text{Re} \int_{t=t_G'}^{t=t_C'} \frac{dW(t)}{e^{\ln \xi(t)}} \tag{2.9}
\]

\[
o = \text{Im} \int_{t=t_G'}^{t=t_C'} dz = \text{Im} \int_{t=t_G'}^{t=t_C'} \frac{dW(t)}{e^{\ln \xi(t)}} \tag{2.10}
\]
$$a = \text{Re} \int_{t = t_G}^{t = t_G'} dz = \text{Re} \int_{t = t_G}^{t = \infty} \frac{dW(t)}{e^{\ln \zeta(t)}}$$

$$+ \text{Re} \int_{t = -\infty}^{t = t_G'} \frac{dW(t)}{e^{\ln \zeta(t)}}, \quad (2.11)$$

$$\alpha = \text{Im} \int_{t = t_G}^{t = t_G'} dz = \text{Im} \int_{t = t_G}^{t = \infty} \frac{dW(t)}{e^{\ln \zeta(t)}}$$

$$+ \text{Im} \int_{t = -\infty}^{t = t_G'} \frac{dW(t)}{e^{\ln \zeta(t)}}, \quad (2.12)$$

$$S = \text{Re} \int_{t = t_G}^{t = \infty} dz + b + \frac{1}{2} - \frac{\pi}{2} = \text{Re} \int_{t = t_G}^{t = \infty} \frac{dW(t)}{e^{\ln \zeta(t)}}$$

$$+ b + \frac{1}{2} - \frac{\pi}{2}, \quad (2.13)$$

$$i \tau = \int_{t = t_G}^{t = \infty} d(\ln \zeta) = \int_{t = t_G}^{t = \infty} \frac{(t-t_D)(t-t_{D1}) dt}{(t-t_G)(t-t_{G1})(t-t_{G1})(t-t_{D1})},$$

$$\text{and}$$

$$i \mu = \int_{t = 0}^{t = t_G} d(\ln \zeta) = \int_{t = 0}^{t = t_G} \frac{(t-t_D)(t-t_{D1}) dt}{(t-t_G)(t-t_{G1})(t-t_{G1})(t-t_{D1})}.$$ 

$$\quad (2.14)$$

$$\quad (2.15)$$

These expressions may be considered to translate a given t-plane flow-field configuration into a physical jet-pipe.
valve geometry, and to impose restrictions upon the acceptable t-plane configurations. The simultaneous solution of these nine equations will permit the determination of nine of the quantities \( t_C, t_G, t_D, t_{D''}, t_F, t_{F''}, t_G, t_G', t_H, \) and \( t_H' \) in terms of any one of the others. Because of the nature of the Schwarz-Christoffel transformations, the value of any one of these ten quantities can arbitrarily be selected.

Just as the above equations relate a given t-plane flow-field configuration to physical dimensions of the jet-pipe valve, other equations can be formulated which similarly relate the t-plane configuration to other flow-field parameters. Particularly useful are equations relating the chamber pressures \( P_1 \) and \( P_2 \) and the flow angles \( \beta_1 \) and \( \beta_2 \) to \( t \). These equations may be given as:

\[
\frac{3}{2} \ln \left(1 - \frac{P_2}{P_3}\right) = \int_{t = t_G}^{t = t_F} d(\ln \xi)
\]

\[
= - \int_{t = t_G}^{t = t_F} \frac{(t-t_D)(t-t_{D''})}{\sqrt{(t-t_G)(t-t_G')(t-t_{F})(t-t_{F'})}(t-t_G)(t-t_G')} \, dt
\]

(2.16)
\[ \frac{1}{2} \ln \left( 1 - \frac{P_l}{P_s} \right) = \int_{t = t_G}^{t = t_P} \frac{(t-t_D)(t-t_{D1})}{\sqrt{(t-t_C)(t-t_{C1})(t-t_P)(t-t_{P1})(t-t_G)(t-t_{G1})}} \, dt \]

(2.17)

\[ \int_{t = 0}^{t = t_H} \frac{(t-t_D)(t-t_{D1})}{\sqrt{(t-t_C)(t-t_{C1})(t-t_P)(t-t_{P1})(t-t_G)(t-t_{G1})}} \, dt \]

(2.18)

and \[ \int_{t = 0}^{t = t_{H1}} \frac{(t-t_D)(t-t_{D1})}{\sqrt{(t-t_C)(t-t_{C1})(t-t_P)(t-t_{P1})(t-t_G)(t-t_{G1})}} \, dt \]

(2.19)
CHAPTER III

SOLUTIONS, ANALYTICAL AND EMPIRICAL

TRIAL AND ERROR TECHNIQUES

As has previously been indicated in Chapter II, exact mathematical treatment of the potential-flow representation of flow in a two-dimensional jet-pipe valve is not practical. The essential difficulty is that Eqs. (2.7) through (2.15), which describe the t-plane configuration that corresponds to a particular distributor-block-and-nozzle geometry, can not be integrated explicitly. As a result, the appropriate values of \( t_C, t_{Ci}, t_D, t_{Di} \), etc. can not be directly determined. The utilization of trial-and-error techniques in conjunction with numerical, step-by-step, integration appears, at present, to be the only feasible method of handling these equations.

The first step in such a procedure is to select arbitrary values of \( t_C, t_{Ci}, t_D, t_{Di} \), etc. and to perform numerically the integrations indicated in Eqs. (2.7) through (2.15) and to assess whether these equations are satisfied. If they are not, the values of \( t_C, t_{Ci}, t_D, t_{Di} \), etc. are altered and the process repeated. Successive trials are made until values are found for \( t_C, t_{Ci}, t_D, t_{Di} \), etc. for which the equations are simultaneously satisfied. Since, in the general case, nine independent variables are involved, such a procedure is an extremely
lengthy and laborious one. Practical considerations dictate the use of a digital computer and the cataloging of results for each trial.

In order to appraise the difficulties one might expect to encounter in programming the problem on a digital computer, a hand solution was computed. The symmetrical case was selected for this trial in order to reduce the number of variables and simplify the problem as much as possible. In this symmetrical case $t_C = -t_C$, $t_D = -t_D$, etc. and only four equations are needed to determine the valve geometry, i.e. Eqs. (2.7), (2.8), (2.11), and (2.15). Values of $t_C$ and $t_G$ were arbitrarily selected and the locus of points $t_D$ vs. $t_P$ which satisfied the condition imposed by Eq. (2.15) and the restrictions indicated by Fig 2(d), i.e. $t_G < t_P < t_D < t_C$, was then determined. A point on this locus was then arbitrarily selected, as was a value of $t_H$. The integrations indicated by Eqs. (2.7), (2.8), and (2.11) were then performed.

Results were discouraging in several respects. Approximately two man-weeks were required to arrive at the "solution" for a single trial. Integration proved to be exceedingly difficult and reasonable accuracy was maintained only with a great deal of effort. In spite of the precautions taken, the momentum equation applied in the vertical direction as a check showed an unbalance of approximately
ten percent — probably the result of accumulated error. The resulting "valve configuration" proved to be an impractical one with the top of the receiver plate depressed well below the surface of the distributor block, represented by a line GG in Fig. 2.

In spite of the difficulties encountered, it is felt that computer treatment of the potential-flow representation of flow in a jet-pipe valve is possible if great care is exercised in programming and if frequent checks are made. However, the labor and expense necessary for such treatment hardly appear warranted at this time.

**THE ELECTRICAL ANALOG**

Because the equations describing the potential flow of a liquid in two dimensions and those describing electrical current flow in a two dimensional conductor are mathematically identical*, it is possible to set up

*Potential flow of a liquid:

\[ \frac{\partial \phi}{\partial x} = u; \quad \frac{\partial \phi}{\partial y} = v; \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \]

*Current Flow:

\[ \frac{\partial V}{\partial x} = -\frac{J_x}{\sigma}; \quad \frac{\partial V}{\partial y} = -\frac{J_y}{\sigma}; \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \]

where \( V \) = voltage

\( J \) = current density

\( \sigma \) = resistivity
an electrical analog and to solve the problem of the jet-pipe-valve flow-field by a trial-and-error process. Unless information in addition to that which is normally available at the start of the solution is known, however, the process probably will not converge to the solution for the specific valve configuration in which one is interested. Without this advance information the method offers little hope for practical application unless it can be mechanized and performed by machine.

The method was applied for two different cases of the jet-pipe-valve problem. This was done to demonstrate that, for at least two isolated cases, the potential-flow representation describes the conditions that actually prevail in the jet-pipe-valve flow-field with sufficient accuracy to obtain reasonable results. Experimentally determined values for the flow angles $\beta_1$ and $\beta_2$ (see Fig. 2) were used.

The apparatus for the electrical analog study is shown schematically in Fig. 3. An enlarged model, with scale factor $k$, very roughly approximating the shape of the flow field was cut from electrical resistance paper. All dimensions except those actually known or assumed to be known ($kK$, $k\delta$, $ka$, $\beta_1$ and $\beta_2$) were generously proportioned since the actual boundaries were not known. Silver conducting paint was applied in those places indicated in Fig. 3. It was applied in such a manner
APPARATUS FOR ELECTRIC ANALOG STUDY

FIG. 3
that the distances, measured along the edge of the model, from A to B and from C to D were equal. The apparatus was connected as illustrated and lines of constant voltage were plotted with the aid of the probe, the galvanometer, and the voltage divider.

Lines of constant voltage corresponded to lines of constant flow potential. Since the edges of the model AB, CD, FE and IJ were ultimately to correspond to free streamlines at atmospheric pressure, the voltage gradient should have been constant and have had the same value along each of these edges. Likewise, the edge GF should have had a constant voltage gradient and the edge HI another constant voltage gradient since these edges were to correspond to velocity discontinuities at constant pressures $P_1$ and $P_2$ respectively. The model was reshaped in such a manner as to make these conditions more nearly true, always keeping the edges AB and CD equal in length by applying conducting paint.

The fact that the flow should everywhere have been continuous aided in shaping the contour. At points H and G the streamlines HI and GF should have been tangential to HG. At points I and F the flow should have been perpendicular to the reference line. Since $\beta_1$ and $\beta_2$ were experimentally determined, the distance HI and GF, measured along the reference line should have been very nearly equal to $k$ times the length $b$, of the receiving opening for which $\beta_1$ and $\beta_2$ were determined; this
parameter, however, was allowed to float. The sum of the
distances AJ and DE should have been equal to BC by
continuity.

After converging upon a model which satisfied the
requirements imposed upon it, the pressure in each chamber,
relative to supply pressure, was found by utilizing the
analogy between voltage gradient and fluid velocity and by
applying Bernoulli's equation. A check was made by
applying the momentum equation in the horizontal and
vertical directions.

The results for the two cases considered are shown
in Figs. 4 and 5. The chamber pressures indicated are
about 15 per cent higher than those measured experimentally.
It must be remembered, however, that an error of only
seven or eight per cent in determining the magnitude of
the velocities along the discontinuities could account for
such discrepancies in pressure. In both cases, the
momentum equation for the y-direction indicated incomplete
convergence and suggested either steeper angles for the
fluid leaving the chambers or lower chamber pressures.
The limitations of the apparatus, i.e. the variations
in the resistance of the paper used to form the model
and the inaccuracies inherent in measuring distances
along curved surfaces, precluded the possibility of
attaining much better accuracy without making substantial
scaling changes.
DEVELOPMENT OF SEMI-EMPIRICAL EXPRESSIONS

As an alternative to the more direct and more easily technically substantiated attacks, a semi-empirical approach was used to develop expressions which enable one to predict, with accuracy sufficient for most design purposes, the pressure-displacement relationships for a considerable range of practical receiver and nozzle configurations. Such expressions, besides constituting valuable design guides, may provide points of departure for computer treatment of the more rigorous representation. These expressions are composed of terms which are based upon the solutions of two simpler but somewhat related problems: the problem of determining the pressure distribution along an infinite flat-plate struck by a liquid jet and the problem of determining the manner in which a jet divides when it impinges upon the edge of a "thin" plate.

The potential flow problem of a jet striking a finite plate is defined by Figure 6. The steps in the mathematical treatment of this problem are outlined in Appendix B. The results of such treatment are presented in the desired form in Figure 7.

The problem of a jet striking an infinite plate is actually the limiting case of the problem illustrated in Figure 6. The details of the mathematics involved in its solution are outlined in Appendix C and a dimensionless plot of pressure versus displacement is given in Figure 8.

Based upon experimental observation the semi-
$\zeta = u - iv$

Where $V = \text{Velocity} = u + iv$
Unit of Velocity = U ft/sec

$\xi$ (Hodograph) Plane

$W = \phi + i\psi$

$\psi = h_1K$

$\psi = h_2K$

$\psi = 0$

$\psi = O$

$z = x + iy$

$\psi = h_1KU$

$\psi = h_2KU$

$\psi = 0$

$\xi = \zeta$

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\[ \frac{\delta'}{K} = \frac{h_1}{2} \sin a_1 - \frac{h_2}{2} \sin a_2 - \frac{h_1 \cos a_1}{\pi} \ln \frac{\tan a_2}{\tan a_1} - h_1 \sin a_1 - h_2 \sin a_2 \]

\[ \frac{a}{K} = 1 + \frac{h_1 \cos a_1}{\pi} \ln \frac{(1 + \cos a_1)(1 + \cos a_2)}{(1 - \cos a_1)(1 - \cos a_2)} - h_1 \sin a_1 - h_2 \sin a_2 \]

\[ h_1 = \frac{\cos a_2}{\cos a_1 + \cos a_2} \]

\[ h_2 = \frac{\cos a_1}{\cos a_1 + \cos a_2} \]

\[ a_1 \text{ AND } a_2 \text{ AS FUNCTIONS OF } \frac{a}{K} \text{ AND } \frac{\delta'}{K} \text{ FOR A JET STRIKING A PLATE} \]

FIG 7
FLAT PLATE PRESSURE DISTRIBUTION

FIG. 8
empirical expressions that have been developed combine the results obtained from analysis of the two above mentioned problems in a form that permits estimating of upper and lower bounds for chamber pressure as a function of distributor-block geometry and nozzle displacement. The proposed "bounding functions" are:

\[
P_{\text{min}} = (P_{fp})_{\text{avg}} + \left[ \frac{\rho U^2}{2} - (P_{fp})_{\text{avg}} \right] \frac{2hK \sin \alpha}{b} \quad (3.1)
\]

and

\[
P_{\text{max}} = (P_{fp})_{\text{avg}} + \left[ \frac{\rho U^2}{2} - (P_{fp})_{\text{avg}} \right] \frac{2hK}{b} \quad , \quad (3.2)
\]

where

\[
P = \text{average chamber pressure},
\]

\[
(P_{fp})_{\text{avg}} = \text{average pressure on a solid flat plate being struck by the jet over an area which corresponds to the receiver opening},
\]

\[
\rho = \text{mass density of the fluid},
\]

\[
U = \text{fluid velocity at the nozzle exit},
\]

\[
K = \text{jet width},
\]

\[
b = \text{width of the receiver opening},
\]

\[
h = \text{portion of the jet which would flow toward the side, corresponding to the chamber being considered, of the isolated receiver plate},
\]

and

\[
\alpha = \text{the angle, measured from the horizontal, which would be assumed by the portion of the jet flowing to the side of the isolated receiver plate}.
\]
The quantity $\rho U^2/2$ is assumed to be equivalent to the supply pressure $P_s$. The quantity $\rho U^2 h \tan \alpha / b$ represents the pressure that would be established in the receiving chamber if the y-momentum of the fluid flowing to the side of the isolated receiver plate, and the quantity $\rho U^2 h K / b$ represents the pressure that would be established if the total-momentum were recovered.

The significance of these relations for large receiver openings will now be discussed. Under such circumstances the flow configuration near the receiver plate is very much like that which would be found if the receiver plate were isolated (because chamber pressures are very low). The postulation of a uniform pressure in the chamber and a flow discontinuity at constant pressure subjects the fluid to a constant upward component of acceleration. For this constant acceleration to be maintained across the entire receiver opening the fluid would have to leave at an angle very nearly equal to the entering angle (approximately equal to $\alpha$). Thus the pressure in the chamber for the case of large receiving openings might be expressed as

$$P \approx \frac{\rho U^2 h \tan \alpha}{b} + \frac{\rho U^2 h K \sin \alpha}{b} \quad (3.3a)$$

or

$$P \approx 2 \frac{\rho U^2 h \sin \alpha}{b} \quad (3.3b)$$
With large receiver openings it can also be demonstrated that \((P_{fr})_{avg} \approx \rho U^2 h \sin \alpha / b\) for near center operation with narrow receiver plates and for a much broader band of operation with wider plates (see overlays of experimental curves in Chapter IV). The proposed relation for \(P_{min}\), Eq. (3.1), in this instance is thus of reasonable form and almost certainly represents a minimum because each term in the proposed expression is actually somewhat smaller than the corresponding term of Eq. (3.3a).

Under the conditions outlined in the preceding paragraph, Eq. (3.2) is also of correct form and almost certainly represents a maximum because it merely represents Eq. (3.3a) with the leaving angle assumed to be 90°.

As the receiving openings are reduced in size, the chamber pressures increase. This causes the flow pattern near the receiver plate to look more nearly like that which would exist for a solid plate and the angles of the exiting jets to decrease. The proposed equations, (3.1) and (3.2), postulate such a tendency.

Although Eqs. (3.1) and (3.2) cannot be rigorously defended, it has been experimentally verified that they can be applied with good accuracy to receiver configurations of practical significance for a substantial range of operation. Dimensionless plots of Eqs. (3.1) and (3.2) for various cases are presented in Chapter IV as overlays of experimental data.
CHAPTER IV

EXPERIMENTAL INVESTIGATION

OBJECTIVES

In order to partially examine the validity of the assumptions made in describing the flow in the jet-pipe valve by the discontinuous potential flow pattern illustrated in Fig. 2 and in order to obtain data for possible use in developing semi-empirical expressions for pressure-displacement characteristics, an experimental investigation was carried out. Flow patterns were observed and receiving chamber pressures were measured for various nozzle displacements and for various receiver geometries. A number of photographs were taken of the flow patterns.

DESCRIPTION OF THE APPARATUS

A two-dimensional model of the jet-pipe valve was constructed in such a manner that the receiving openings and the receiver-plate width could easily be varied and that the flow could easily be observed. A half-size outline drawing of the model is shown in Fig. 9 and the model is pictured in the photograph of Fig. 11b. The nozzle and receiver components were aluminum while the sidewalls were lucite. The model was one inch in depth. The nozzle opening was 1/4 inch wide.

The arrangement of the test apparatus is shown
TWO-DIMENSIONAL MODEL OF THE JET-PIPE VALVE

FIG. 9
FIG. 11. PHOTOGRAPHS OF THE APPARATUS
schematically in Fig. 10 and is pictured in the photograph of Fig. 11a. Water from a six inch main was supplied to the nozzle through one inch pipes and a flexible hose. Flow meters in the supply line were used to measure the volumetric flow rate which was controlled with gate valves.

Pressure readings were read on mercury and water manometers. Each chamber was connected, through a common manifold, to one mercury and one water manometer. The desired manometer was selected with a 3-way petcock which also was used to provide damping of the manometer.

MEASUREMENTS

The receiver openings were adjusted with the aid of accurately sized cylindrical rods. Three different receiver openings, 1/4, 1/2 and 3/4 inch, were used. The settings were probably accurate to within 0.002 inch.

Two calibrated flowrators, one having a capacity of 20 gpm and the other a capacity of 2000 cu.in./min. were employed to measure the volumetric rate of flow. The flowrators were calibrated by measuring the time required for either 100, 200, or 300 pounds to flow into a drum set on a scale platform. The time required was repeatable to within two seconds for time intervals in excess of three minutes. Weight measurements were accurate to within 1/2 pound.

For a flow of approximately 10 gpm, a search of the velocity profile of the jet with a pitot tube at a distance
of approximately one inch from the nozzle (no distributor block) showed the velocity midway between the lucite sidewalls to be about 5 per cent higher than the average velocity computed from the volumetric rate of flow. The velocity near the sidewalls was lower than the average velocity. The velocity was substantially uniform along a path parallel to the sidewalls. Because of this uniformity in the direction of nozzle displacement and because boundary layer effects are less at higher flows, it was concluded that the average velocity describes the jet with acceptable accuracy.

Pressure readings could easily be taken to within 1/2 mm by simultaneously closing the petcocks when a reading was desired. Fluctuations in the flow, however, injected a degree of uncertainty even though damping was employed to reduce their effects; therefore, a minimum of three readings were averaged for each flow condition. In general, the resulting readings of pressure head were repeatable to well within 1 per cent of the velocity head at the nozzle exit.

The displacement of the nozzle was determined by measuring from rods fixed to the model (Fig. 11) with a depth micrometer. The hydraulic center, where both chamber pressures were equal, was graphically determined and used as a "zero" reference. Micrometer readings were accurate to within 0.001 inch.
FLOW GEOMETRY

In general, the experimentally observed flow patterns were very similar in nature to that pictured in Fig. 2a. As anticipated the side-jet angle increased when the chamber-pressure increased and decreased when the chamber-pressure decreased. Photographs of the two situations analyzed with the electric analog \(a/K = 1/2, b/K = 2, \theta/K = 0\), and \(a/K = 1/2, b/K = 2, \theta/K = 0.2\) are presented in Fig. 12 for purposes of comparison. The flow patterns did change radically, however, whenever the pressure in either chamber approached a value roughly comparable to the maximum average pressure that would be predicted for a flat-plate pressure distribution. When this pressure was reached the side jets became unstable, oscillating between their last stable position and positions described by smaller angles. Further increases in nozzle displacement caused the "frequency" of oscillation to increase and the angle at the lower extreme of the range of oscillation to assume smaller values. Finally a displacement would be reached at which the oscillation would cease and a new stable flow-pattern would be established. This new pattern corresponds closely to the pattern obtained when a jet strikes a solid plate. The condition of side-jet instability and the condition for flat side-jets are illustrated in the photographs of Fig. 13a and b, respectively.

From the photographs in Figs. 12 and 13 it can be
FIG. 12. PHOTOGRAPHS OF THE FLOW FIELD
(a) Unstable - $a/K = 1/2$, $b/K = 2$, $S/K = 0.7$

(b) Flat Side Jets - $a/K = 1/2$, $b/K = 1-1/2$, $S/K = 0.7$

FIG. 13 PHOTOGRAPHS OF THE FLOW FIELD
seen that impingement upon the receiver does not influence the jet very far upstream of the distributor-block. The assumption to this effect made in Chapter II thus seems to be justified.

PRESSURE VERSUS DISPLACEMENT

Figures 14 through 18 show, in dimensionless form, the experimental relationship between chamber pressure and nozzle displacement for various receiver geometries. The overlay for each of these curves shows the corresponding semi-empirical relationships discussed in Chapter III.

The correspondence between the experimental data and the relationships expressed by Eqs. (3.1) and (3.2) is, for the most part, good. As might be anticipated, the actual pressure-displacement relationship is somewhat sensitive to the jet velocity of the fluid. It is noticed, however, that the influence of jet velocity is most substantial where the curves for Eqs. (3.1) and (3.2) are the most at variance and very small where they nearly coincide. At the lower velocities viscous surface tension, and gravitational effects probably had some influence on the experimental results.

A sharp change in slope in the experimental pressure-displacement curves occurred, almost without exception, when the flow pattern closely corresponding to that pictured in Fig. 2a became unstable. The pressure
\[
\left( \frac{P_{fp}}{P_s} \right)_{avg} + 2h \left[ 1 - \left( \frac{P_{fp}}{P_s} \right)_{avg} \right]
\]

\[
\left( \frac{P_{fp}}{P_s} \right)_{avg} + 2h\sin\alpha
\]

OVERLAY—THE SEMI-EMPIRICAL EXPRESSIONS AND THEIR BASIC COMPONENTS
FIG. 14

PRESURE VS. DISPLACEMENT FOR a/K=1/2, b/K=1

DIMENSIONLESS NOZZLE DISPLACEMENT, 8_K

DIMENSIONLESS CHAMBER PRESSURE, P_R

MEAN SEMI-EMPIRICAL CURVE

FLUID NOZZLE VELOCITY
V_n = 8.3 FT/SEC — x —
V_n = 13.6 FT/SEC — △ —
V_n = 19.8 FT/SEC — o —
V_n = 25.9 FT/SEC — ▽ —
V_n = 34.9 FT/SEC — ▽ —
OVERLAY — THE SEMI-EMPirical EXRESSIONS AND THEIR BASIC COMPONENTS
DIMENSIONLESS CHAMBER PRESSURE, $\frac{p}{p_0}$

DIMENSIONLESS NOZZLE DISPLACEMENT, $\frac{\delta}{K}$

PRESSURE VS. DISPLACEMENT FOR $\frac{a}{K}=1/2, \frac{b}{K}=2$

FIG. 15
OVERLAY—THE SEMI-EMPIRICAL EXPRESSIONS AND THEIR BASIC COMPONENTS
\[ \left( \frac{P_{fp}}{P_s} \right)_{\text{avg}} \left[ \frac{1 - \left( \frac{P_{fp}}{P_s} \right)_{\text{avg}}}{\left( \frac{P_{fp}}{P_s} \right)_{\text{avg}}} \right] h + \left( \frac{P_{fp}}{P_s} \right)_{\text{avg}} \left[ 1 - \left( \frac{P_{fp}}{P_s} \right)_{\text{avg}} \right] h \sin \alpha \]

OVERLAY—THE SEMI-EMPIRICAL EXPRESSIONS AND THEIR BASIC COMPONENTS
Figure 17: Pressure vs. Displacement for $a/K=1, b/K=2$

- Mean semi-empirical curve
- Data points for different nozzle velocities:
  - $V_n = 8.3$ FT/SEC
  - $V_n = 13.6$ FT/SEC
  - $V_n = 19.8$ FT/SEC
  - $V_n = 25.9$ FT/SEC
  - $V_n = 34.9$ FT/SEC
\[ \left( \frac{P_{fp}}{P_s} \right)_{\text{avg}} + \left[ 1 - \left( \frac{P_{fp}}{P_s} \right)_{\text{avg}} \right] \frac{2}{3}h \sin \alpha \]

OVERLAY—THE SEMI-EMPIRICAL EXPRESSIONS AND THEIR BASIC COMPONENTS
in the "high" pressure chamber would then tend to level off or to increase less rapidly than predicted by the semi-empirical expressions, while the pressure in the "low" pressure chamber would tend to decrease less rapidly.
CHAPTER V
DISCUSSION OF RESULTS

The analytical and experimental investigation reported in this paper indicates that the proposed discontinuous-potential-flow model adequately describes, over a useful range of operation, the flow field in a two-dimensional jet-pipe valve operating under blocked-ram conditions. For a variety of practical distributor-block-and-nozzle configurations, the experimentally observed flow patterns qualitatively agree with the theoretical representation. Furthermore, the results of the electrical analog study, although too limited to be conclusive, strongly indicate that there is good quantitative agreement.

It has been ascertained in this study that, with numerical methods and with trial and error techniques, analytic treatment of the theoretical model is possible. However, any practical attempt to carry out the vast number of computations required in such a treatment must make use of a digital computer. More than ordinary difficulty will attend such a procedure because extremely small intervals must be used in certain of the numerical integrations to maintain sufficient accuracy; very careful programming is required and provision must be made for frequent checks. The inevitable limitations on time
and funds precluded the extensive program called for by such an approach.

As has been indicated above, the potential flow representation appears applicable to useful configurations and ranges of operation; there are, however, realistic situations for which the model does not hold. Extremely small receiving openings do not disturb the flow under the jet sufficiently to cause a flow pattern like that of Fig. 2. Instead, the flow pattern approximates that for a jet striking a solid flat plate, and the receiving-chamber pressures approximate the mean pressures on the flat-plate areas corresponding to the receiving openings. In effect, small receiver openings behave as taps similar to those commonly used for pressure measurement. Figures 14 through 18 support the conclusion that average flat plate pressure gives increasingly good approximation to chamber pressure as receiving openings are reduced in size.

Even for larger receiving openings the postulated potential flow representation will not always hold. There is reached, as nozzle displacement is increased, a displacement magnitude beyond which such a model does not apply. Here, one "side jet" tends to enter its receiving chamber at a very steep angle, rendering tenuous the assumption of a potential flow discontinuity and of "dead" fluid in the chamber. Local dynamic forces could
conceivably permit a fluid pressure distribution approaching the flat-plate pressure distribution. Indeed, in the experimental work the slope of the pressure-displacement curves were found to alter abruptly and the flow pattern to change radically whenever the pressure in a receiving chamber approached the maximum obtainable average flat-plate pressure.

Because the semi-empirical expressions, Eqs. (3.1) and (3.2), were developed for valve configurations characterized by the model of Fig. 2, it is reasonable to expect their applicability to parallel that of the model. It is therefore not surprising that discrepancies between predictions of these expressions and experimental results were encountered when flow patterns were not amenable to the model.

Other methods of attack besides the ones described in this paper might be possible. The obvious one, however, which consists of application of the momentum equations, the continuity equation, and the energy equation to the overall flow field, was investigated and discarded because an insufficient number of equations could be formulated to mathematically describe the problem. Analyses which do not consider detailed representation of the flow field seem doomed to failure.

A great deal of work beyond that performed in this study must be done before the jet-pipe valve can be handled
with the same confidence and dexterity that now attend
the design of many other valves. Until then, the
attractive features of the jet-pipe valve — its
insensitivity to dirt and its simplicity of construction —
cannot be fully exploited. Experimental effort might
well be spent in investigating those flow regimes in
which the discontinuous-potential-flow representation
of Fig. 2 is no longer applicable. Further experimental
and analytical investigations should deal with steady-
state-load flows, submerged jets, dynamic characteristics,
operation with compressible media, flow in three-dimensions,
etc.

A discontinuous-potential-flow representation may
be useful in investigating the effects of chamfering
the outer edges of the receiving openings as is often
done in jet-pipe valves to reduce "frothing" and noise.
For such application Eqs. (2.14) and (2.15) would be
altered to correspond to other than vertical orientations
of the outer walls of the receiving chambers.

The possibility of constructing a theoretical model
which would permit analytic treatment of the flow field
in a three-dimensional jet-pipe valve appear remote,
although experimental investigations could yield valuable
design data and a basis for interpreting two-dimensional
analyses. It would appear prohibitively difficult to
formulate a model with which to treat flow of a compressible
medium, even in two-dimensions.
APPENDIX A. MAPPING OF THE lnT AND W-PLANES ONTO THE t-PLANE

The Schwarz-Christoffel transformation*, a mapping which sets up a one-to-one correspondence between points on a polygonal boundary in one plane and points on the real axis of another plane, is used to map the W-plane of Fig. 2e onto the t-plane of Fig 2d. Thus,

$$\frac{dW}{dt} = M \frac{1}{t(t-t_H)(t-t_{H'})}.$$  \hspace{1cm} (A.1)

By partial fraction expansion,

$$\frac{dW}{dt} = M \left[ \frac{1}{t_H t_{H'}} \frac{1}{t} - \frac{1}{t_H(t_{H'}-t_H)} \frac{1}{t-t_H} \right. $$

$$ \left. + \frac{1}{t_{H'}(t_{H'}-t_H)} \frac{1}{t-t_{H'}} \right] . \hspace{1cm} (A.2)

Integrating and arbitrarily setting W equal to zero when t = infinity (point B),

$$W = M \left[ \frac{1}{t_H t_{H'}} \ln t - \frac{1}{t_H(t_{H'}-t_H)} \ln(t-t_H) \right. $$

$$ \left. + \frac{1}{t_{H'}(t_{H'}-t_H)} \ln(t-t_{H'}) \right] . \hspace{1cm} (A.3)

*Ref 12, pp 550 - 562
As $t$ increases along the real axis by passing around the point $A = 0$, $\arg(t-o)$ decreases from $\pi$ to $0$. $W$ thus changes by $-\frac{M}{tH_tH} i\pi$.

In passing around the corresponding point $A$ in the $W$-plane, however, $W$ decreases by $-iK$. Hence,

$$-\frac{M}{tH_tH} i\pi = -iK$$

and

$$M = tH_tH \frac{K}{\pi} \quad (A.4)$$

Equation (A.1), therefore, can be written in the form

$$dW = \frac{tH_tH}{\pi} \frac{K}{t(t-t_H)(t-t_H^*)} \frac{dt}{t(t-t_H)(t-t_H^*)} \quad (A.5)$$

as was previously indicated by Eq. (2.5).

Similarly, the Schwarz-Christoffel transformation is used to map the $\ln\zeta$-plane of Fig. 2c onto the $t$-plane. Thus,

$$\frac{d(\ln\zeta)}{dt} = \frac{M^*}{\sqrt{(t-t_D)(t-t_D^*)(t-t_{DP})(t-t_{DP^*})(t-t_{D^*})(t-t_{D^*})}} \quad (A.6)$$

If $t = Re^{i\theta}$ and $R$ is allowed to approach infinity, Eq. (A.6), in the limit, becomes

$$d(\ln\zeta) = M^* i dt^* \quad (A.7)$$

From Figs. 2c and 2d, it then becomes apparent that
\[ \ln \zeta = r + i\theta = \pi \quad \theta = \pi \]

\[ -i\pi = \lim_{r \to \infty} \int d(\ln \zeta) = \int^{M'}_{M} i d\theta = M' i\pi. \quad (A.8) \]

Therefore,

\[ M' = -1. \]

Since \( \ln \zeta = \frac{i\pi}{2} \) when \( t = 0 \), Eq. (A.6) can be written in the form

\[ \ln \zeta = -\int_{t = 0}^{t = t} \frac{(t-t_P)(t-t_P')}{\sqrt{(t-t_C)(t-t_C')(t-t_F)(t-t_F')(t-t_G)(t-t_G')}} \, dt + \frac{i\pi}{2}, \quad (A.9) \]

as indicated by Eq. (2.4).
APPENDIX B. SOLUTION OF THE PROBLEM OF A JET STRIKING A PLATE OF FINITE LENGTH

Referring to Fig. 6, it is seen that the W-plane can be mapped onto the t-plane with the use of the Schwartz-Christoffel transformation. Thus,

\[
\frac{dW}{dt} = \frac{M''}{(t + \cos a_1)(t - \cos a_2)t} \cdot \tag{B.1}
\]

By partial fraction expansion

\[
\frac{dW}{dt} = \frac{M''}{\cos a_1 \cos a_2} \left[ -\frac{1}{t} + \frac{\cos a_2}{\cos a_1 + \cos a_2} \frac{1}{t + \cos a_1} \right. \\
+ \left. \frac{\cos a_1}{(\cos a_1 + \cos a_2) (t - \cos a_2)} \right] \tag{B.2}
\]

Integrating and arbitrarily setting \( W \) equal to zero when \( t \) equals infinity (point E),

\[
W = \frac{M''}{\cos a_1 \cos a_2} \left[ -\ln t + \frac{\cos a_2}{\cos a_1 + \cos a_2} \ln (t + \cos a_1) \right. \\
+ \left. \frac{\cos a_1}{\cos a_1 + \cos a_2} \ln (t - \cos a_2) \right] \tag{B.3}
\]
As \( t \) increases (along the real axis) by passing around the point \( t = -\cos a_1 \), \( \arg (t + \cos a_1) \) decreases from \( \pi \) to 0. Therefore \( \ln (t + \cos a_1) \) decreases by \( i\pi \) and \( \psi \), the imaginary part of \( W \), decreases by \( \left[ M''\pi/\left(\cos a_1 \cos a_2\right)\right] \left[\cos a_2/\left(\cos a_1 + \cos a_2\right)\right] \). In passing around the corresponding point B in the \( W \)-plane, however, \( \psi \) increases from 0 to \( \left[\cos a_2/\left(\cos a_1 + \cos a_2\right)\right] K \). Hence,

\[
-\frac{M''\pi}{\cos a_1 \cos a_2} \frac{\cos a_2}{\cos a_1 + \cos a_2} = \frac{\cos a_2}{\cos a_1 + \cos a_2} K
\]

and

\[
M'' = -\frac{\cos a_1 \cos a_2}{\pi} K \tag{B.4}
\]

Applying the continuity equation to the flow,

\[
h_1 + h_2 = 1. \tag{B.5}
\]

The momentum equation in the \( x \)-direction reduces to

\[
h_1 \cos a_1 = h_2 \cos a_2. \tag{B.6}
\]

Solving Eqs. (A.5) and (A.6) simultaneously,

\[
h_1 = \frac{\cos a_2}{\cos a_1 + \cos a_2} \quad \text{and} \quad h_2 = \frac{\cos a_1}{\cos a_1 + \cos a_2}. \tag{B.7}
\]
Realizing that

$$\frac{dw}{dz} = \zeta$$  \hspace{1cm} (B.8)

and that

$$t = -\frac{1}{2} (\zeta + \frac{1}{\zeta})$$  \hspace{1cm} (B.9)

Equations (B.2), (B.4), (B.7), (B.8) and (B.9) can be combined to give the relation

$$dz = -\frac{\zeta}{\pi} \left[ \frac{1 - \zeta^2}{\zeta^2 (\zeta^2 + 1)} - h_1 \frac{1 - \zeta^2}{2(\zeta^2 - 2 \cos a_1 \zeta + 1)} - h_2 \frac{1 - \zeta^2}{\zeta^2 (\zeta^2 + 2 \cos a_2 \zeta + 1)} \right] d\zeta.$$  \hspace{1cm} (B.10)

Integrating along the real axis from $\zeta = -1$ to $\zeta = +1$ it can be shown that

$$a_K = \left[ \begin{array}{c} 1 + \frac{h_1 \cos a_1}{\pi} \ln \frac{(1 + \cos a_1)(1 + \cos a_2)}{(1 - \cos a_1)(1 - \cos a_2)} \\ \ln \frac{(1 + \cos a_1)(1 + \cos a_2)}{(1 - \cos a_1)(1 - \cos a_2)} \\ -h_1 \sin a_1 - h_2 \sin a_2 \end{array} \right].$$  \hspace{1cm} (B.11)

With the use of the techniques of contour integration in the complex plane (12) and with the use of Eq. (B.11)
it can be shown that

\[
\delta' = \frac{h_1}{2} \sin \alpha_1 - \frac{h_2}{2} \sin \alpha_2 - \frac{h_1 \cos \alpha_1}{\pi} \ln \frac{\tan \alpha_2}{\tan \alpha_1}.
\] (B.12)

Results are plotted in Fig. 7.
APPENDIX C. THE PRESSURE DISTRIBUTION UNDER A JET NORMALLY STRIKING AN INFINITE PLATE

For a jet striking an infinite plate,

\[ a_1 = a_2 = 0 \quad (C.1) \]

and

\[ h_1 = h_2 = \frac{1}{2} \quad (C.2) \]

Equation (B.10) thus reduces to the form

\[ dz = \frac{K}{\pi} \left[ \frac{2}{\xi^2 + 1} - \frac{1}{\xi - 1} + \frac{1}{\xi + 1} \right] d\xi \quad (C.3) \]

Integrating for real \( \xi \) (where \( z = x \)) and letting \( x = 0 \) for \( \xi = 0 \) (stagnation line directly under center of jet by symmetry),

\[ x = \frac{K}{\pi} \left[ 2 \tan^{-1} \xi + \ln \frac{\xi + 1}{\xi - 1} \right] \quad (C.4) \]

\( \xi \) real

The Bernoulli Equation reduces to the form

\[ \frac{P}{\rho U^2} = \frac{P}{P_s} = (1 - \xi^2) \quad (C.5) \]
since

$$\zeta = \frac{V}{U} \text{ along the plate.} \quad (C.6)$$

Equations (C.4) and (C.5) thus relate pressure and displacement through $\zeta$. Results are plotted in Fig. 8.
BIBLIOGRAPHY


Additional Material


BIОGRAPHICAL NOTE

The author was born on May 13, 1930 in Passaic, New Jersey. He received most of his elementary and all of his secondary school training in that state and in June, 1947 was graduated from Fair Lawn High School. The following September he was admitted to the Massachusetts Institute of Technology for undergraduate study.

While enrolled in the Mechanical Engineering Cooperative Course, Machine Design Option, the author obtained six months plant experience as a student engineer at the Fairchild Engines Division of the Fairchild Engine and Aircraft Company located in Farmingdale, Long Island, New York. During the course of his studies at the Institute he was elected to the honorary fraternities Pi Tau Sigma and Sigma Xi. In his senior year he was awarded a partial tuition scholarship. The degree of Bachelor of Science was conferred upon him in June, 1951.

Immediately upon completion of his undergraduate studies the author entered the Graduate School of the Massachusetts Institute of Technology and was appointed a Research Assistant in the Department of Mechanical Engineering. His assignment as a Research Engineer to the Dynamic Analysis and Control Laboratory gave him an opportunity to become acquainted with various control systems and their components and resulted in the writing of a Master's Thesis entitled A Study of Permanent Magnet Torque Motors. In February, 1953, he was awarded the degree of Master of Science in Mechanical
Engineering.

The author continued his studies at the Institute, embarking upon a program leading toward the degree of Doctor of Science in Mechanical Engineering. His studies will be completed upon submission of this thesis.