The Implications of Potential Vorticity Homogenization for Climate and Climate Sensitivity

by

Daniel Bernard Kirk-Davidoff

Submitted to the Department of Earth, Atmospheric and Planetary Sciences in partial fulfillment of the requirements for the degree of

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Abstract

Meridional heat fluxes carried by transient atmospheric eddies greatly moderate the earth’s climate by cooling the tropics and warming polar regions. It has long been known that these eddies arise due to the baroclinic instability of the mean tropospheric flow, and long been suspected that the observed meridional temperature gradient might be due to a process of baroclinic neutralization. In this thesis we consider the implications for climate and climate sensitivity of a neutral state proposed by Lindzen (1993), in which the role of baroclinic eddies is to remove potential vorticity (PV) gradients in the troposphere up to a height sufficient to neutralize the troposphere to the eddies’ further growth. We show that an assumption of fixed PV gradients in the extratropical troposphere, combined with an assumption of a moist adiabatic temperature structure in the tropics, a constraint on surface static stability, and overall radiative equilibrium, suffices to constrain the earth’s zonal mean climate.

We analyze observations of PV and convective available potential energy (CAPE) to evaluate the reliability of these assumptions. We find that CAPE variations are small on interannual time scales, and that the gross distribution of zonal mean PV varies little over the seasonal cycle, and less from year to year. We then use these assumptions to construct a model of the earth’s climate. By comparing the model climate with the observed climate, and varying certain of the model’s assumptions to resolve differences, we are able to draw some conclusions about the causes of several features of the observed climate. We show that the downward slope of the zonal mean tropopause is attributable to the Brewer-Dobson circulation of the stratosphere, and we demonstrate the necessity of a strong boundary layer inversion for maintaining the warmth of polar regions. The model exhibits a strong poleward amplification of temperature change, due to its limited cross-isentropic dynamic heat transport. This suggests that the excessive vertical diffusion of heat and moisture in GCMs might be responsible for the trouble those models have in reproducing equable paleoclimates.

Thesis Supervisor: Richard S. Lindzen
Title: Professor of Meteorology
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Chapter 1

Introduction

Atmospheric and oceanic heat fluxes deliver about 6 PW to regions poleward of 30° in each hemisphere (Trenberth and Solomon, 1994). If this flux is divided by the surface area poleward of that line, the implied downward surface flux works out to 50 W m⁻², nearly twice the annual mean solar input at the polar surface, and so represents the crucial moderating influence on the earth's climate. Without them, the earth's polar regions would be far colder than they currently are, or ever have been. Of these fluxes, the atmosphere delivers approximately two thirds. Where these fluxes peak, at around 40° latitude in each hemisphere, they are carried almost entirely by transient atmospheric eddies (Peixoto and Oort, 1992).

Several theories have been advanced to describe the interaction of atmospheric eddies with climate. Sellers (1969), in designing a one-dimensional energy balance climate model, assumed that the eddies behaved as a diffusive term acting on the meridional temperature distribution. The eddy diffusion term could then be coupled with a radiation parameterization to determine the earth's zonal mean climate. Stone (1978) and Lindzen and Farrell (1980) theorized that eddies act to keep the atmosphere marginally stable to their own growth by eliminating the change in sign in potential vorticity (PV) gradient at the earth's surface which is a necessary condition for baroclinic instability (Charney and Stern, 1962). In Stone's approach, this translated directly into a neutral temperature gradient which could be approached but not much exceeded, while Lindzen and Farrell used the heat fluxes implicit in the neutralization to adjust the temperature gradient to an equilibrium value. General circulation models (GCMs) make the implicit assumption that the energy-carrying eddies must be explicitly resolved in order to be correctly modeled.

Each of these approaches has disadvantages. Modeling eddies as a diffusive influence on surface temperature begs the question, what sets the value of the diffusivity? We know from the geological record that climate states have existed in which the earth's mean meridional temperature gradient was much smaller than it currently is. Such climates require larger meridional energy fluxes (Rind and Chandler, 1991) to support warm polar temperatures; clearly the relationship between heat fluxes and temperature gradient was different in the Eocene (50 Ma) than at present. Theories of eddy heat fluxes which require them to remove the change in sign of the PV gradient are confronted with the fact that in the real atmosphere, such a change in sign is almost universally observed, since the negative meridional temperature gradient at the surface implies a negative PV gradient there (Bretherton, 1966). Finally, because GCMs attempt to include all physics relevant to climate, it can be difficult to identify the processes most responsible for a certain effect. A possibly better
measure of our understanding of a physical process is our ability to predict the behavior of interest using a model which contains only the mechanisms we think relevant.

In this thesis we discuss a new approach towards the connection of baroclinic instability theory with climate dynamics. We will consider the hypothesis that the primary climatological role of atmospheric eddies is to mix PV in the free troposphere. Because the distribution of PV determines the slopes of isentropes with respect to one another, the degree of PV mixing is a strong constraint on the temperature distribution of the troposphere, which in turn, through radiative interactions, has a strong influence on the surface temperature structure. This approach is inspired by the work of Lindzen (1993), who showed that the troposphere could be made neutral to linear growth of baroclinic disturbances if PV gradients were removed up to a certain height (dependent on the width of the subtropical jet), and associated this height with the tropopause. Sun and Lindzen (1994) demonstrated that the temperature structure of the mid-latitude troposphere could be well approximated by assuming small PV gradients, and using the resulting relationship among the slopes of isentropes to integrate the tropospheric temperatures from boundaries at the earth’s surface and at the the poleward edge of the tropics. This integration makes possible an extension of the one dimensional convective adjustment model to two dimensions. In a convective adjustment model, one assumes that saturation moist entropy is homogenized in the vertical through the action of cumulus convection. Here we assume that PV is partially homogenized along isentropes by the action of baroclinic eddies. As with the moist adjustment model, we can couple this procedure for determining the atmospheric temperature with a radiative code in order to define a climate which is in overall radiative equilibrium and has a prescribed gradient of PV. As we will show, several additional assumptions are necessary to define that climate uniquely. Variation of these assumptions allows us to test hypotheses about the origins of several features of the observed climate, such as the meridional variation of the tropopause height, and the temperature structure of polar regions.

Before we proceed to combine the constraints of PV homogenization and radiative equilibrium, we will present results from two observational studies. The temperature of a PV-homogenized troposphere is strongly dependent on the tropical boundary condition. We will analyze the annual and interannual variations of the tropical mean vertical temperature structure to determine whether the use of a moist adiabatic adjustment adequately captures the dependence of observed temperature variations on surface temperature. We next investigate the seasonal and interannual variability of the PV structure of the extratropical troposphere. We will show that seasonal variations of the mean PV gradient are fairly small, and that interannual variations are very small.

After a review of the theory of PV mixing, and a discussion of the stability properties of a PV homogenized troposphere, we proceed to define the conditions necessary to define a unique climate with a troposphere with prescribed PV gradients. This gives us a new tool with which to explore the interaction of radiation and dynamics in the atmosphere. Some of the questions we will address are:

- How does the degree of PV homogenization relate to the magnitude of meridional energy fluxes?
- What is responsible for the decline in tropopause height from the tropics to the poles?
- What role does the polar temperature inversion play in setting the pole-to-equator temperature difference?
• How should a uniform change in atmospheric opacity influence the pole-to-equator temperature gradient?

• How important are oceanic fluxes of energy in setting the pole-to-equator temperature gradient?

The thesis is organized as follows. In Chapter 2 we discuss observations of variations in the tropical mean vertical temperature structure. We wish to establish the degree to which the use of a moist adiabatic adjustment captures the variations of the tropical mean temperature structure. In Chapter 3 we consider observations of potential vorticity and its meridional gradient, and investigate their variations over time. In Chapter 4 we review the theory of PV homogenization, and consider the extent to which the homogenization of PV in the troposphere acts to stabilize the troposphere to the linear growth of baroclinic disturbances. In Chapter 5 we show that a fixed extratropical PV gradient, combined with moist adiabatic adjustment in the tropics, and a number of other conditions, suffices to predict a unique climate. Through a process of modifications and perturbation, we use the resulting model to address the questions listed above, to consider the implications of PV homogenization for climate sensitivity, and to address some questions brought up by the paleoclimatic record. Finally, Chapter 6 summarizes our findings.
Chapter 2

Observations of Saturation Equivalent Potential Temperature in the Tropics

2.1 Introduction

One aim of this thesis is the extension of the convective adjustment model to two dimensions. Before we develop that theme, we will review the observational support for the one dimensional convective adjustment model in the region where its use is best justified. In this chapter we explore the mean vertical temperature structure of the tropical latitudes, and evaluate the extent to which it may be explained by moist adiabatic adjustment. In particular, we ask how much the convective available potential energy (CAPE) of the mean tropical atmosphere varies over various time scales. CAPE depends on the departure of the atmosphere's vertical temperature structure from a moist adiabat. Thus, if we find that CAPE varies little on long time scales, we can anticipate that the use of a moist adiabat to approximate the temperature structure of the tropics will not grossly misrepresent the vertical profile of temperature change during climate fluctuations.

Compared to temperatures in temperate and polar latitudes, the temperature at a given height in the tropics (but above the trade inversion) varies little in the horizontal. Thus, we choose for the moment to approximate the tropical temperature structure by a single vertical temperature profile. In equilibrium, this profile participates in the mean energy balance of the non-convecting regions of the tropical atmosphere as follows:

$$\rho w \frac{\partial \theta}{\partial z} = \dot{Q}$$  \hspace{1cm} (2.1)

where $\dot{Q}$ is the radiative heating rate, $w$ is the vertical velocity, $\rho$ is the density, and $\theta$ is the potential temperature, all averaged over non-convecting regions of the tropics. For equilibrium, $\rho w$ must equal the ascending mass flux in convecting regions. If $\dot{Q}$ changes, either $\rho w$ or $\frac{\partial \theta}{\partial z}$ must change to match it. Sun and Lindzen (1993) argue that the total tropical convective mass flux is tied to surface evaporation, and so cannot easily change to compensate for changes in $\dot{Q}$. Thus, changes in the tropical troposphere's radiative structure must be balanced by changes in its lapse rate. On the other hand, Emanuel et al. (1994), noting observations that show that convective activity in a given location is generally proportional to large scale forcing of CAPE, argue that it is the convective mass flux which
adjusts to hold $\frac{90}{92}$, and thus CAPE, fixed. More recently, Robe and Emanuel (1996) have shown that in a moist non-hydrostatic primitive equation model, CAPE does depend on the vertical structure of radiative cooling, but has no dependence on the magnitude of the cooling.

This debate about the variations of CAPE has its roots in an earlier argument about just how much CAPE the mean tropical atmosphere holds. Xu and Emanuel (1989), on the basis of a series of soundings from the western equatorial Pacific, claimed that temperature soundings in the deep tropics are indistinguishable from a moist adiabat, the profile of temperature with height traced out by the reversible, adiabatic ascent of a parcel of air from the top of the sub-cloud layer to the top of the troposphere. In that case, such a parcel would experience no acceleration due to buoyancy forces during its ascent, so that the convective available potential energy, or CAPE, of the sounding would be zero. They argued that reversible ascent was the parcel path relevant to deep convection because it represents the maximum actually attainable buoyancy for non-precipitating cloud parcels. Lucas et al. (1994a) point out that although actual clouds rarely achieve reversible values of condensate loading, the best way to explain the large difference in observed updraft velocities between continental and tropical marine convection, despite similar values of environmental CAPE when condensate loading is ignored, is to take some condensate loading into account in calculating parcel buoyancy. If this is done, they note, continental profiles which appear to have the same CAPE as tropical profiles, are revealed to have larger CAPE, because they tend to have relatively large values of parcel buoyancy over a relatively small height range, when compared to marine soundings.

Williams and Rennó (1993) added that Xu and Emanuel's (1989) observation of zero CAPE was sensitive to the assumptions made about the phase of the water substance carried by the hypothetical ascending parcel. If the energy of fusion released by the freezing of cloud droplets in the parcel were taken into account, mean tropical soundings did indeed have significant CAPE. Further, they found many tropical stations which exhibited significant CAPE regardless of the method of calculation. Fu et al. (1994) found that high values of CAPE can be maintained over monthly time scales, if surface winds are divergent, while strong deep convective activity can be maintained in regions of zero CAPE, if surface winds are convergent. Emanuel et al. (1994), following Arakawa and Schubert (1974), then proposed the quasi-equilibrium hypothesis, acknowledging that some CAPE must always exist for convection to take place at all, but arguing that the tropical atmosphere, while not in strict equilibrium with respect to moist convection, is in a quasi-equilibrium state, where CAPE is small, and its rate of change is small compared to the terms acting to produce or destroy it. Thus, in this view, the CAPE of a convecting region has little dependence on surface temperature on time scales longer than a few days (the time required for moist convection to act to restore moist convective equilibrium). They emphasized the entropy disequilibrium between the tropical sea or land surface and the atmosphere's boundary layer as the source of energy for organized tropical disturbances.

Observations supporting this view were reported by Wang and Randall (1994), who showed that during GATE, precipitation rates were proportional to the non-convective rate of production of the generalized convective available potential energy (GCAPE, which measures the maximum amount of kinetic energy which can be released by the vertical reshuffling of a column of the atmosphere), but inversely proportional to the GCAPE itself. That is, precipitation occurred in an environment of relatively low GCAPE, and occurred at a rate proportional to the production of GCAPE by advection, radiation, and surface heat and moisture fluxes. High GCAPE only occurred when precipitation was low for some
period of time. The extent to which variations in the the forcing of GCAPE depend upon variations in large scale forcing acting above the boundary layer is unclear, so the question of whether convective mass flux can respond to changes in CAPE due to temperature changes well above the boundary layer is not addressed by this study.

Observations which appear to contradict the quasi-equilibrium hypothesis have been reported by Williams (1992, 1994) and Kent et al. (1995). Williams (1992, 1994) has shown that parameters of the global electric circuit are correlated with tropical temperature variations on the annual and interannual (1969–1974) time scales. He argues that this reflects a positive dependence of global average CAPE on surface temperature, since deep convection taking place in an environment with high values of CAPE generally supports larger updraft velocities. Faster updrafts can support a larger ice burden, which is correlated with higher lightning activity, and thus a more active global electric circuit.¹

Kent et al. (1995) have shown that tropical mean cloudiness was greater in the years 1986, 1987, 1989, and 1990 than in the years 1984, 1985 and 1988, while tropical mean surface temperatures were also warmer in the first group of years. They also found that cloud height increased with increasing temperature over that period. If larger CAPE is associated with stronger updrafts and greater ice burdens in deep convective storms, this finding might indicate that warmer mean surface temperatures are associated with increased CAPE in convecting regions. However, it might also be that warmer mean surface temperatures are associated with a larger total area covered by active convection, and thus increased cloudiness, without any change in mean CAPE.

The degree to which moist convective equilibrium is maintained in the tropics also has important implications for the paleoclimate record. Perhaps the clearest sign of climate change in the tropics during the last glacial maximum (LGM, 18 ka) is the nearly uniform lowering of the mountain snow line by some 900 m (Rind and Peteet, 1985). This indicates a temperature reduction at the altitude of the snow line (currently about 5000 m) of some 5–6 K, assuming that tropical lapse rates have not radically changed since that time. The height of the snowline depends not only on temperature, but also on relative humidity, and precipitation patterns. The tropics are generally thought to have been drier at the LGM (Rind and Peteet, 1985), implying a higher snowline for a given temperature, so the temperature change is probably a lower limit. Changes in foraminifera distribution in the tropical oceans have been interpreted to imply mean cooling of not more than 1 K at the LGM (CLIMAP project members, 1981), but if tropical mean CAPE is maintained at some nearly constant level, then those regions must have been some 2.5 K cooler (Betts and Ridgway, 1992). Sun and Lindzen (1993) resolved this apparent discrepancy using a model of convection which assumed that the convective mass flux responded not to the energy available for convection in the atmosphere, but rather to the net surface evaporation. In their model, a larger vertical temperature gradient (which implies weaker subsidence heating) is balanced by reduced radiative cooling in a moister atmosphere, and yields a 1 km lower freezing line for a change in surface temperature of only 1 K. This led them to conclude that the atmosphere is able to attain an equilibrium state with much more

¹Lucas et al. (1994b), citing a wide range of observational studies, assert that differences in updraft strength between oceanic and continental convection cannot be attributed to differences in CAPE. However, while they compare convection in several different geographic regions with similar CAPE and varying updraft velocities, they do not compare similar storms in a given region with varying CAPE. Certainly, the experience of weather forecasters is that the potential for hail in a given location and season is greater for larger values of CAPE (Cotton and Anthes, 1989).
CAPE than is currently observed, at a lower mean temperature. Recently, some studies of
different tropical paleoclimate indicators, such as ratios of noble gas concentrations stored
in aquifers (Stute et al., 1995) have indicated that tropical temperatures over land, even at
low altitudes, were some 5 K cooler during the LGM than at present. This finding poses
some interesting questions. At the present time, it is unclear whether the aquifer results
contradict the results from CLIMAP, or instead simply indicate that land temperatures
were cooler relative to ocean temperatures at that time. For instance, stronger seasonality
in the tropics might lead to a larger reduction of the apparent tropical land temperature
than occurred over the oceans, especially if the aquifer data have any seasonal bias.

To investigate the extent to which CAPE is held constant in the tropics when surface
temperature changes, we analyzed a data set consisting of monthly mean radiosonde mea-
surements from over eighty stations in the tropics (30 °S to 30 °N latitude) for the years
1964 through 1991. The stations are indicated by 's in Figure 2-1. Time series of CAPE,
and of its components due to variations of surface temperatures and to variations in upper-
air temperatures were calculated, and these time series were analyzed to test the degree
to which variations in temperatures aloft act to cancel the changes in CAPE which would
result from surface temperature variations alone.

2.2 Data Analysis Methods

The data for this study are drawn from a global archive of monthly average radiosonde
reports maintained by the National Center for Atmospheric research (Jenne et al., 1975).
The archive includes data from the years 1950 through 1991. Temperature, dew point,
and winds are reported at the surface and at nine other pressure levels, from 850 to 30
mb. Data prior to 1964 were discarded, as the number of active stations drops off sharply
before this date. Data from levels above 200 mb is also sporadic, and so was ignored. The
time of day of soundings used to construct the monthly mean varies among the stations (0
UTC, 12 UTC, or both), and sometimes even from year to year for a particular station.
Unfortunately, information about the time of day of a station’s sounding is recorded only
sporadically in the archive, so that the impact of changes in the time of day of sounding
launch on the interannual variations of temperature could not be well assessed. However,
since the time of day of sounding launch at the various stations does not seem to have varied
in any systematic way over the 27 years of recorded data, and since we are interested in
variations of quantities averaged over the whole tropics, the bias introduced by changes in
the time of day of sounding launch should be small. At most stations, the time of day of
sounding launch seems to have been chosen to correspond to local daytime conditions. This
introduces a warm bias at the surface, which tends to increase the mean value of CAPE,
but should not bias the temperature variations significantly.

Elliott and Gaffen (1991) have discussed the reliability of radiosonde humidity archives.
The most serious problem with the humidity record was caused by poor shielding of hygris-
tors during the years 1965-1973. This caused systematically low relative humidity readings
during the day, when sunlight would heat the hygrometer housing, lowering the relative hu-
midity to which the hygrometer was exposed. Our results are not sensitive to relative humidity
aloft (which makes only a small contribution to the density variations there), where errors
were worst, but are sensitive to surface relative humidity. To avoid contamination of trends
through this systematic error, we use only the years 1974 through 1991 to examine trends
in temperature, dew point and CAPE.
Figure 2-1: Locations of tropical sounding stations, indicated by *'s. Surface $\theta_e$ (equivalent potential temperature) in each grid box was derived from the sounding data by an objective analysis scheme. Those grid boxes which have mean surface $\theta_e > 332$ K for Januarys, Aprils, Julys, or Novembers, 1964-1991, are shown unshaded. For reference, note that a parcel of air with $T = 22^\circ$C, $T_d = 19.9^\circ$C, and $p = 1013.15$ mb has $\theta_e = 332$ K.
Surface temperatures were reported at varying pressures, while upper air temperatures were all recorded at standard levels. To make surface temperatures comparable with one another, potential temperatures were computed. In the following text, “surface temperature” will refer to the potential temperature calculated from the surface sounding level. A monthly mean climatology was constructed for each station. Suspicious data points (those temperatures or dew points which differed by more than 3 K from the climatology for a given month) were removed, and the monthly mean climatology was reconstructed. Only those stations with data in at least twelve of the years 1970-1985 were used. Monthly temperature anomaly time series were then computed by subtracting the monthly mean temperatures from the actual temperatures.

The monthly mean climatologies and anomalies of temperature and dew point at each pressure level were then placed on separate 10 degree by 10 degree grids using a simple objective analysis scheme. For each grid square, a weighted mean of each variable is constructed, by averaging all stations within the grid square’s 10 degree latitude band, and the two adjacent bands, each station being weighted by a factor of \( \exp(-r^2/d^2) \), where \( r \) is the distance from the stations to the grid square and \( d = 1000 \text{km} \). \( d \) was chosen to be large enough so that in a cluster of stations, ‘interior’ stations are not underweighted with respect to ‘exterior’ stations. Since separate gridded time series were made for the seasonal cycle and for the anomalies from this cycle, long term fluctuations in the reassembled temperature and dew point time series are not skewed by the changing geographic distribution of active sounding stations over the years.

Time series of temperature and dew point at the various pressure levels were assembled by averaging and adding together the area-weighted gridded monthly means and anomalies. \( \theta_e \) was then calculated on the surface grid using the formula:

\[
\theta = T \star \left( \frac{p_0}{p} \right)^{R_d/(C_{pd}+C_{tr})} \left( \frac{H-r R_o/(C_{pd}+C_{tr})}{L_v} \right)^{1/((C_{pd}+C_{tr})T)},
\]

(2.2)

where \( T \) is the temperature, \( r \) is the mixing ratio of water vapor, \( p \) is the pressure, \( p_0 = 1013.15 \text{ mb} \), \( R_d \) is the gas constant of dry air, \( R_o \) is the gas constant of water vapor, \( C_{pd} \) is the heat capacity of dry air, \( C_l \) is the heat capacity of liquid water, and \( L_v \) is the latent heat of evaporation of water, and equals \( L_v = (C_{pu} - C_l) \star T \), where \( C_{pu} \) is the heat capacity of water vapor, and \( L_v = 2.501 \times 10^6 \text{ J/kg} \) (Emanuel, 1994). At levels above the surface, the saturation equivalent potential temperature, \( \theta^* \), was computed. This quantity is equal to \( \theta_e \) if \( r \) is set equal to \( r^* \), the saturation water vapor mixing ratio, and thus depends only on temperature and pressure.

Several tropical mean surface temperature time series were constructed, using different screening criteria. Time series were constructed for the whole tropical region (30°S to 30°N), and for the following subregions: grid points with climatological monthly mean \( \theta_e \) > 332K (these will be referred to as “warm regions”), grid points with \( \theta_e < 332K \) (“cool regions”), ocean stations with \( \theta_e > 332K \), and land stations with \( \theta_e < 332K \) (warm regions are shown unshaded in Figure 2-1). Finally, time series for “summer” regions were constructed by averaging over all stations with latitudes in the range 15°sin((m + 8)\pi/6) ± 15°, where \( m \) is the month. The surface time series of the summer region was averaged over regions with \( \theta_e \) > 332K.

Figure 2-2 shows a mean sounding of \( \theta^* \) averaged over the whole tropical region, along with two values of surface \( \theta_e \): one averaged over the tropics, and the other averaged over warm regions. Figure 2-3 shows time series \( \theta_e \) at the surface (3d), and \( \theta^* \) aloft (3a-c), averaged over the whole tropics. Figure 2-4 shows the same variables averaged over the
Figure 2-2: Tropical mean (30°S to 30°N, 1964-1991) sounding of $\theta_e^*$ (saturation equivalent potential temperature). Mean surface values of $\theta_e$ are also shown, averaged over the whole tropics, and over warm regions ($\theta_e > 332$ K). Mean surface values of $\theta_e$ over warm land regions and warm ocean regions are very similar to those over all warm regions (341.67 K and 342.44 K, respectively), while the mean value of $\theta_e$ over cool regions (where $\theta_e < 332$ K) is 320.36.
region from the equator to 20°N only.

Variations of the tropical mean conditional stability were investigated by constructing time series of CAPE using the temperature and dew point time series described above. For the purposes of this paper, CAPE will be defined as the buoyant force experienced by a parcel of air ascending from the earth’s surface to the 200 mb level:

\[
\text{CAPE} = \int_{200\text{mb}}^{1000\text{mb}} R_d(T_{pp} - T_{pa}) \, d\ln p, \tag{2.3}
\]

where \( R_d \) is the gas constant of dry air, and \( T_{pp} \) and \( T_{pa} \) are the density temperature of the rising parcel and of the ambient air, respectively. The density temperature is expressed:

\[
T_p = T_v \frac{1 + r/\epsilon}{1 + r_T} \tag{2.4}
\]

(Emmanuel, 1994), where \( r \) is the water vapor mixing ratio, \( r_T \) is the net water mixing ratio, including condensed phases, and \( \epsilon \) is the ratio of the gas constants of dry air and of water vapor. In this work, several approximations are made. \( r_T \) is taken equal to \( r \), so that \( T_{pa} = T_v \), the virtual temperature, since no information on the concentration of condensed water in the ambient air is available from soundings. \( T_{pp} \) is calculated assuming pseudoadiabatic ascent: all condensate is assumed to rain out immediately on condensation, and all condensate is assumed to be liquid. An alternative assumption of reversible ascent was also tried (all condensate is carried along with the parcel). In this case, CAPE fluctuations are 10% smaller, for typical tropical conditions, than for the irreversible case. The integral in Equation 2.3 was computed using the trapezoid rule.

The CAPE time series were divided into surface and upper-air components (CAPE\(_s\) and CAPE\(_u\)), whose sum differs from CAPE by a constant. They are expressed:

\[
\text{CAPE}_{s} = \int_{200\text{mb}}^{1000\text{mb}} R_d(T_{pp} - \bar{T}_{pa}) \, d\ln p, \tag{2.5}
\]

and

\[
\text{CAPE}_{u} = \int_{200\text{mb}}^{1000\text{mb}} R_d(\bar{T}_{pp} - T_{pa}) \, d\ln p, \tag{2.6}
\]

where \( \bar{T}_{pa} \) is the mean over the time series of the ambient density temperature, and \( \bar{T}_{pp} \) is the mean over the time series of the parcel’s density temperature at a given pressure level. An increase in CAPE\(_s\) indicates that surface temperature or dew point has increased, while an increase in CAPE\(_u\) indicates that temperatures aloft have fallen, so that an unchanged surface parcel is more buoyant. If CAPE\(_s\) and CAPE\(_u\) are anticorrelated, this indicates that warming or cooling at the surface tends to be accompanied by warming or cooling aloft. If moist convection acts to restrict variations of CAPE to some equilibrium value, then a linear fit of the dependence of CAPE\(_u\) on CAPE\(_s\) should have a slope equal to \(-1\). On the other hand, in an atmosphere in which temperature variations were constant with height, the slope of this dependence would be about \(-0.35\), for deviations about a typical tropical sounding.
Figure 2-3: Time series of $\theta_e$ at the surface (d), and $\theta_e^*$ at various levels aloft (a-c), averaged over the tropics ($30^\circ$S to $30^\circ$N).
Figure 2-4: As in the previous Figure, but for the region from the equator to 20°N.
Table 2.1: Correlation coefficients of time series of temperature at the surface and at various levels aloft. Surface temperatures averaged globally 30 °S to 30 °N (Tropics), over regions in the tropics with climatological monthly mean temperature between 21 °C and 31 °C (Warm), over tropical ocean regions (Ocean) and land regions (Land) with temperatures between 21 °C and 31 °C, and over tropical regions with temperatures below 21 °C (Cool). Upper air temperatures are always averaged over the whole tropical region.

2.3 Results

The results of this investigation support the hypothesis that CAPE variations are small in the tropics, and that temperatures aloft are tied to temperatures in convecting regions, for time scales of a year or more. On shorter time scales, neither hypothesis appears to be true on the large spatial scales investigated here.

We turn first to an analysis of the correlation of tropical mean surface temperature, dew point, and $\theta_e$, with temperatures and $\theta_e^*$ aloft. If a region at the surface can be found whose mean $\theta_e$ correlates well with $\theta_e^*$ values aloft, then we can reasonably ask if the amplitudes of the variations at the surface and aloft are in the proportion predicted by the quasi-equilibrium hypothesis. If the correlation is poor, quasi-equilibrium clearly fails. For reference, correlations of $R = 0.60$ and $R = 0$ are significantly different at the 95% confidence level for the seasonal cycle (assuming $N = 12$), while correlations of $R = 0.80$ and $R = 0.60$ are significantly different at the 95% confidence level for interannual variations (assuming that the twelve month running mean of a 27 year time series has $N = 27$ independent data points).

Tables 2.1 and 2.2 show coefficients of correlation between tropical mean temperatures aloft, and surface temperatures and dew points, respectively, averaged over the various surface regions defined in the previous section. Table 2.2 also shows the correlation between
<table>
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<th>850 mb</th>
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<th>500 mb</th>
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<td>0.67</td>
<td>0.62</td>
<td>0.51</td>
<td>0.24</td>
</tr>
<tr>
<td>Cool</td>
<td>0.83</td>
<td>0.57</td>
<td>0.64</td>
<td>0.53</td>
<td>0.42</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 2.2: Correlation coefficients of time series of surface (1000 mb) dew point temperature and temperature at various levels. Surface temperatures averaged as above. Surface temperature time series are averaged over the same regions as surface dew points, so that ocean surface dew point temperatures are correlated with ocean surface temperatures, but with 850 mb temperatures averaged over the whole tropics.
<table>
<thead>
<tr>
<th>Surface Region</th>
<th>850 mb</th>
<th>700 mb</th>
<th>500 mb</th>
<th>300 mb</th>
<th>200 mb</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Timeseries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tropics</td>
<td>0.52</td>
<td>0.45</td>
<td>0.48</td>
<td>0.43</td>
<td>0.35</td>
</tr>
<tr>
<td>Warm</td>
<td>0.28</td>
<td>0.44</td>
<td>0.37</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>Ocean</td>
<td>0.55</td>
<td>0.56</td>
<td>0.44</td>
<td>0.39</td>
<td>0.42</td>
</tr>
<tr>
<td>Land</td>
<td>0.09</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.29</td>
</tr>
<tr>
<td>Cool</td>
<td>0.36</td>
<td>0.22</td>
<td>0.18</td>
<td>-0.09</td>
<td>-0.15</td>
</tr>
<tr>
<td><strong>Seasonal Variations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tropics</td>
<td>0.40</td>
<td>0.11</td>
<td>0.22</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>Warm</td>
<td>0.06</td>
<td>0.13</td>
<td>-0.16</td>
<td>0.21</td>
<td>0.38</td>
</tr>
<tr>
<td>Ocean</td>
<td>0.43</td>
<td>0.32</td>
<td>-0.13</td>
<td>-0.02</td>
<td>0.26</td>
</tr>
<tr>
<td>Land</td>
<td>-0.02</td>
<td>0.08</td>
<td>-0.07</td>
<td>0.32</td>
<td>0.38</td>
</tr>
<tr>
<td>Cool</td>
<td>0.35</td>
<td>0.20</td>
<td>0.18</td>
<td>-0.38</td>
<td>-0.52</td>
</tr>
<tr>
<td><strong>Interannual Variations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tropics</td>
<td>0.77</td>
<td>0.88</td>
<td>0.83</td>
<td>0.73</td>
<td>0.52</td>
</tr>
<tr>
<td>Warm</td>
<td>0.80</td>
<td>0.90</td>
<td>0.85</td>
<td>0.77</td>
<td>0.56</td>
</tr>
<tr>
<td>Ocean</td>
<td>0.90</td>
<td>0.83</td>
<td>0.77</td>
<td>0.72</td>
<td>0.59</td>
</tr>
<tr>
<td>Land</td>
<td>0.54</td>
<td>0.76</td>
<td>0.71</td>
<td>0.64</td>
<td>0.45</td>
</tr>
<tr>
<td>Cool</td>
<td>0.61</td>
<td>0.68</td>
<td>0.58</td>
<td>0.49</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 2.3: Correlation coefficients of time series of $\theta_c$ at the surface and $\theta_c^*$ at various levels aloft. Surface $\theta_c$ averaged as in Table 1.

temperature and dew point at the surface. Table 2.3 shows correlation coefficients for the relationship of $\theta_c^*$ aloft and surface $\theta_c$. Correlations are shown for the complete time series, for the seasonal cycle (data averaged over all occurrences of each month), and for interannual variations (detrended twelve month running means).

On interannual time scales, temperatures aloft are well correlated with surface temperature, dew point and $\theta_c$. Correlation coefficients ($R$) of 0.8 are typical for surface values averaged over the whole tropical region, or over warm or ocean regions, as defined above. On these time scales, surface temperature and dew point variations are essentially equally well correlated with upper-air temperature variations. Temperatures aloft are better correlated with surface temperature and dew point variations in warm regions ($R \approx 0.8$) than with variations in cool regions ($R \approx 0.6$). Since warm regions are more likely to support deep convection, this fact lends support to the argument, based on the quasi-equilibrium hypothesis, that tropical temperatures aloft should be controlled by equivalent potential temperatures averaged over convecting regions. Above the boundary layer, separate regions of the tropics are well correlated with one another. For instance, detrended temperatures at the 500 mb level over ocean regions and over land regions have $R = 0.8$. At the surface the correlation decreases to $R = 0.4$. Standard deviations of the time series for land and ocean regions are nearly identical, so that the tropical mean interannual temperature variations do not reflect large anomalies in any single region.

In an atmosphere whose vertical temperature structure is constrained to lie along a moist adiabat, $\theta_c^*$ variations at all levels would be approximately the same size as variations in $\theta_c$ at the surface, and would be perfectly correlated (Emanuel et al., 1994). Figure 2-5 shows the first Empirical Orthogonal Functions (EOFs) (Peixoto and Oort, 1992) of
Figure 2-5: Empirical Orthogonal Functions of tropical mean perturbation temperature and $\theta_e$ ($\theta_e^*$ aloft). (a) and (b) show the first EOF’s for the 12 month running means, and (c) and (d) show the first EOF’s for the annual cycle of temperature and $\theta_e$, for various averaging regions. The first EOF’s of the 12 month running mean time series explain about 85% of the variance of those series, while the first EOF’s of the annual cycles explain about 65% of their variance.
<table>
<thead>
<tr>
<th>Region</th>
<th>850 mb</th>
<th>700 mb</th>
<th>500 mb</th>
<th>300 mb</th>
<th>200 mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Timeseries</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North</td>
<td>0.88</td>
<td>0.86</td>
<td>0.86</td>
<td>0.87</td>
<td>0.44</td>
</tr>
<tr>
<td>South</td>
<td>0.98</td>
<td>0.94</td>
<td>0.91</td>
<td>0.84</td>
<td>0.44</td>
</tr>
<tr>
<td>North vs. South</td>
<td>-0.81</td>
<td>0.64</td>
<td>0.60</td>
<td>-0.55</td>
<td>0.10</td>
</tr>
<tr>
<td>Summer</td>
<td>0.47</td>
<td>0.52</td>
<td>0.58</td>
<td>0.39</td>
<td>0.23</td>
</tr>
<tr>
<td>Seasonal Variations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North</td>
<td>0.89</td>
<td>0.90</td>
<td>0.93</td>
<td>0.93</td>
<td>0.64</td>
</tr>
<tr>
<td>South</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.95</td>
<td>0.77</td>
</tr>
<tr>
<td>North vs. South</td>
<td>-0.88</td>
<td>-0.89</td>
<td>-0.96</td>
<td>-0.84</td>
<td>-0.30</td>
</tr>
<tr>
<td>Summer</td>
<td>0.43</td>
<td>0.46</td>
<td>0.63</td>
<td>0.35</td>
<td>0.18</td>
</tr>
<tr>
<td>Interannual Variations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North</td>
<td>0.75</td>
<td>0.87</td>
<td>0.89</td>
<td>0.85</td>
<td>0.55</td>
</tr>
<tr>
<td>South</td>
<td>0.92</td>
<td>0.82</td>
<td>0.75</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>North vs. South</td>
<td>0.69</td>
<td>0.87</td>
<td>0.86</td>
<td>0.69</td>
<td>0.67</td>
</tr>
<tr>
<td>Summer</td>
<td>0.79</td>
<td>0.90</td>
<td>0.82</td>
<td>0.80</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 2.4: Correlation coefficients of time series of $\theta_e$ at the surface and $\theta_e^*$ at various levels aloft. Data at all levels averaged over the region from 20°S to the equator (South), and over the region from the equator to 20°N. 200 mb data coverage was poor in the North region. The rows labeled “North vs. South” show the correlations of $\theta_e^*$ in one region with $\theta_e^*$ in the other region, at each level. The rows labeled “Summer” show data averaged at all levels over the region $0^\circ + 15^\circ \times \sin((m + 8)/6\pi) \pm 15^\circ$.

the vertical profiles of temperature and $\theta_e^*$ ($\theta_e$ at the surface) perturbations about their pressure-level mean values, for interannual (5a,b) and seasonal (5c,d) variations. Note the sharp distinction between variations over annual and interannual time scales. Over the annual cycle, temperature perturbations decrease with height (5d), while for interannual variations, temperature perturbations increase with height (5b). However, the increase of temperature perturbation with height is not so great as to imply constant $\theta_e^*$ perturbations: this variable also decreases with height above 700 mb (5a). This shows that the tropical free troposphere cannot be in convective equilibrium with any single surface region, since if it were, the $\theta_e^*$ perturbations of the free troposphere would be everywhere equal to $\theta_e$ perturbations in that surface region (Emanuel et al., 1994). Note that the dotted line in Figure 2-5a, indicating the relationship of ocean surface $\theta_e$ with $\theta_e^*$ aloft, is nearly vertical up to 700 mb, suggesting that the lower part of the free troposphere is in a state of quasi-equilibrium with warm ocean regions. Of course, we expect the trade wind boundary layer, which extends up to about 800 mb (Kloesel and Albrecht, 1989), to be in equilibrium with the surface (Lindzen and Nigam, 1987), but the 700 mb level is above the top of the inversion about 80% of the time in the deep tropics (Kloesel and Albrecht, 1989), so high correlations between the surface and 700 mb suggest communication between these levels by convection which penetrates the trade inversion. Alternatively, the high correlations could be due to horizontal propagation of temperature perturbations from regions where the trade inversion is high. Data with more vertical detail would be required to distinguish between these possibilities.

Table 2.5 shows the correlation coefficients and linear regression slopes of CAPE as a
<table>
<thead>
<tr>
<th>Surface Region</th>
<th>CAPE versus $T_{\text{surface}}$</th>
<th>CAPE$_u$ versus CAPE$_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>slope ($\text{J kg}^{-1}\text{K}^{-1}$)</td>
</tr>
<tr>
<td>Full Timeseries</td>
<td>0.53</td>
<td>560 ± 310</td>
</tr>
<tr>
<td>Tropics</td>
<td>0.41</td>
<td>650 ± 470</td>
</tr>
<tr>
<td>Warm</td>
<td>0.41</td>
<td>940 ± 670</td>
</tr>
<tr>
<td>Ocean</td>
<td>0.67</td>
<td>510 ± 200</td>
</tr>
<tr>
<td>Land</td>
<td>0.96</td>
<td>480 ± 20</td>
</tr>
<tr>
<td>Cool</td>
<td>0.93</td>
<td>549 ± 40</td>
</tr>
<tr>
<td>North</td>
<td>0.98</td>
<td>560 ± 10</td>
</tr>
<tr>
<td>South</td>
<td>0.41</td>
<td>640 ± 460</td>
</tr>
<tr>
<td>Summer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seasonal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tropics</td>
<td>0.61</td>
<td>480 ± 220</td>
</tr>
<tr>
<td>Warm</td>
<td>0.43</td>
<td>600 ± 410</td>
</tr>
<tr>
<td>Ocean</td>
<td>0.32</td>
<td>1100 ± 890</td>
</tr>
<tr>
<td>Land</td>
<td>0.71</td>
<td>480 ± 160</td>
</tr>
<tr>
<td>Cool</td>
<td>0.98</td>
<td>480 ± 10</td>
</tr>
<tr>
<td>North</td>
<td>0.94</td>
<td>530 ± 30</td>
</tr>
<tr>
<td>South</td>
<td>0.99</td>
<td>550 ± 10</td>
</tr>
<tr>
<td>Summer</td>
<td>0.44</td>
<td>570 ± 390</td>
</tr>
<tr>
<td>Interannual</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tropics</td>
<td>0.17</td>
<td>1720 ± 1620</td>
</tr>
<tr>
<td>Warm</td>
<td>0.26</td>
<td>990 ± 870</td>
</tr>
<tr>
<td>Ocean</td>
<td>0.48</td>
<td>720 ± 450</td>
</tr>
<tr>
<td>Land</td>
<td>0.08</td>
<td>3860 ± 3810</td>
</tr>
<tr>
<td>Cool</td>
<td>0.37</td>
<td>1160 ± 880</td>
</tr>
<tr>
<td>North</td>
<td>0.18</td>
<td>1240 ± 1160</td>
</tr>
<tr>
<td>South</td>
<td>0.64</td>
<td>800 ± 340</td>
</tr>
<tr>
<td>Summer</td>
<td>0.25</td>
<td>940 ± 830</td>
</tr>
</tbody>
</table>

Table 2.5: Correlations and linear regression slopes of CAPE as a function of surface temperature, and of CAPE$_u$ as a function of CAPE$_a$. Slopes are the mean of the linear regression slopes of each variable as a function of the other, error margins in the slopes are half the difference of the two linear regression slopes. Surface data averaged as in Table 1, upper air data averaged over the whole tropics, except for "Summer", which is calculated as in Table 4.
function of surface temperature, and of CAPE\textsubscript{u} as a function of CAPE\textsubscript{s}. CAPE and CAPE\textsubscript{u} are calculated with respect to the various surface regions described above. Since in each case the first time series is regressed against a time series which has non-zero errors, the linear regression slope is calculated by first regressing the first time series against the second, and then the second time series against the first. The slope is then taken to be the mean of the slope calculated in these regressions, and the error bounds listed are equal to one half of the difference of the two slopes. If we are finding the slope of the relationship of \(x\) and \(y\), where both have errors, the slope determined in this way will always be equal to or larger in magnitude than the slope of the linear dependence of \(y\) on \(x\), if we assume that \(x\) is known precisely. If the correlation between \(x\) and \(y\) is perfect, the slopes determined in these ways will be identical.\textsuperscript{2} The errors determined by taking the difference of the slopes are intended to quantify the scatter of the data points about the chosen linear fit, rather than to be a rigorous estimate of the uncertainty of the slope. However, comparison with the method for determining a linear fit to data with errors in both coordinates given by Press et al. (1992) shows that the errors in the slope are comparable, except when the correlation is very high (greater than 0.9). In that case the errors are underestimated by the method given here.

For detrended interannual variations, CAPE's dependence on surface temperature varies greatly, depending on the region over which surface data are averaged. If surface conditions are averaged over the whole tropics, or over all warm regions, CAPE\textsubscript{s} and CAPE\textsubscript{u} are highly anticorrelated, and the linear regressions of CAPE\textsubscript{u} versus CAPE\textsubscript{s} give slopes indistinguishable from \(-1\). Thus, CAPE calculated for these regions is almost uncorrelated with surface temperature. However, Figure 2-5b shows that the high degree of cancellation between CAPE\textsubscript{s} and CAPE\textsubscript{u} on interannual time scales does not imply that changes in \(\theta^*_e\) aloft are equal to changes in \(\theta_e\) at the surface. Rather, an average of \(\theta^*_e\) changes aloft is equal to \(\theta_e\) changes at the surface, averaged over all warm regions.

For cool regions, CAPE\textsubscript{u} is poorly correlated with CAPE\textsubscript{s}, so that CAPE variations are well correlated with surface temperature. For ocean regions, CAPE\textsubscript{u} and CAPE\textsubscript{s} are highly correlated, but the slope of CAPE\textsubscript{u} with respect to CAPE\textsubscript{s} is less than one, so that CAPE here also exhibits some dependence on surface temperature. The slope values are sensitive to the assumptions made about the nature of parcel ascent in convecting towers. Assuming reversible ascent in the CAPE calculation decreases the amplitude of CAPE\textsubscript{s} variations by a factor of 0.88, while leaving CAPE\textsubscript{u} variations unchanged, so that the slopes of CAPE\textsubscript{u} versus CAPE\textsubscript{s} are increased. For interannual variations, and surface temperature and dew point averaged over warm regions, the slope increases from 0.92 to 1.03.

These results are displayed graphically in Figures 2-6 and 2-7. Figure 2-6 shows plots of CAPE\textsubscript{s}, CAPE\textsubscript{u} and CAPE\textsubscript{w} plotted as functions of time, while Figure 2-7 shows scatter plots of CAPE\textsubscript{w} as a function of CAPE\textsubscript{s}. Figures 2-6a and 2-7a show interannual variations, while Figures 2-6b and 2-7b shows the annual cycles of these variables. Surface data for these plots are taken from warm regions, and the data for the seasonal variations is taken from summer regions only. Note the clear anticorrelation of CAPE\textsubscript{s} and CAPE\textsubscript{u} over interannual time scales, and the relatively small variations of CAPE. The best fit slope to the dependence of CAPE\textsubscript{u} on CAPE\textsubscript{s} is \(-0.89\pm0.11\), and \(R = -0.89\), as shown in Table 2.5. Assuming reversible ascent of surface parcels leads to a slope \(-0.98\pm0.12\) (these numbers

\textsuperscript{2} Let \(x_i\) and \(y_i\) be vectors with mean values of zero. The linear regression slope of \(y_i\) on \(x_i\) is \(a = \frac{\sum x_i y_i}{\sum x_i^2}\), while the linear regression slope of \(x_i\) on \(y_i\) is \(b = \frac{\sum x_i y_i}{\sum y_i^2}\). The ratio \(a/(1/b) = \frac{(\sum x_i y_i)^2}{(\sum x_i^2 \sum y_i^2)}\), which must be between 0 and 1. Thus, the mean of the two slopes \((a + 1/b)/2\), must be steeper than \(a\).
Figure 2-6: (a) 12 month running means of CAPE (solid), CAPE_s (dashed) and CAPE_u (dotted) anomalies. Surface data average over regions with $\theta_e > 332K$. (b) Annual cycle of CAPE (solid), CAPE_s (dashed) and CAPE_u (dotted) anomalies. The CAPE values are calculated using data averaged over the summer region: $0^\circ + 15^\circ \times \sin((m + 8)/6\pi) \pm 15^\circ$. Surface data again averaged over regions with $\theta_e > 332K$. 
Figure 2-7: (a) Scatter plot of running means of CAPE_u and CAPE_s. Correlation coefficient of CAPE_u and CAPE_s: r = -0.89. Linear regression fits: CAPE_u = -0.75CAPE_s, and CAPE_s = -0.92CAPE_u. Surface data average over regions with \( \theta_s > 332K \). (b) Scatter plot of seasonal cycle of CAPE_u and CAPE_s. Correlation coefficient of CAPE_u and CAPE_s: r = -0.61. Linear regression fits: CAPE_u = -0.20CAPE_s, and CAPE_s = -1.85CAPE_u. As in the previous Figure, the annual cycle is calculated from data are averaged over the summer region, and surface data are averaged over regions with \( \theta_s > 332K \).
Trends in Temperature and CAPE

<table>
<thead>
<tr>
<th>Surface Region</th>
<th>Temperature K (100 yr)$^{-1}$</th>
<th>Dew Point K (100 yr)$^{-1}$</th>
<th>CAPE$^s$ J kg$^{-1}$ yr$^{-1}$</th>
<th>CAPE$^u$ J kg$^{-1}$ yr$^{-1}$</th>
<th>CAPE J kg$^{-1}$ yr$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropics</td>
<td>1.9 ± 0.9</td>
<td>2.0 ± 1.1</td>
<td>15.9 ± 8.3</td>
<td>−16.3 ± 7.1</td>
<td>−0.4 ± 3.3</td>
</tr>
<tr>
<td>Warm</td>
<td>2.1 ± 0.9</td>
<td>2.3 ± 1.2</td>
<td>18.8 ± 9.2</td>
<td>−16.3 ± 7.1</td>
<td>2.4 ± 3.7</td>
</tr>
<tr>
<td>Ocean</td>
<td>2.4 ± 1.1</td>
<td>5.2 ± 1.2</td>
<td>37.0 ± 9.7</td>
<td>−16.3 ± 7.1</td>
<td>20.7 ± 3.9</td>
</tr>
<tr>
<td>Land</td>
<td>2.6 ± 1.1</td>
<td>0.7 ± 1.2</td>
<td>10.2 ± 9.1</td>
<td>−16.3 ± 7.1</td>
<td>−6.2 ± 4.7</td>
</tr>
<tr>
<td>Cool</td>
<td>1.7 ± 0.9</td>
<td>1.6 ± 1.0</td>
<td>12.4 ± 7.1</td>
<td>−16.3 ± 7.1</td>
<td>−3.9 ± 4.2</td>
</tr>
</tbody>
</table>

Table 2.6: Seventeen year trends (1974-1991) in surface temperature and dew point, CAPE$^s$, CAPE$^u$, and CAPE computed by linear regression of their 12 month running means, sampled every 18 months. Errors are calculated by taking the standard deviation of the residual time series to be the error of each data point. The results are then averaged over all 18 possible starting times.

Differ slightly from those in Table 2.5, because in this case the time series have not had their trends removed. The standard deviation of CAPE (69 J kg$^{-1}$) is only half as large as those of CAPE$^u$ (133 J kg$^{-1}$) or CAPE$^s$ (151 J kg$^{-1}$). Further, the standard deviation of the linear regression of CAPE onto CAPE$^s$ (33 J kg$^{-1}$) is only one fifth as large as that of CAPE$^s$, indicating a high degree of cancellation between CAPE$^s$ and CAPE$^u$.

Table 2.6 shows the trends in surface temperature and dew point, CAPE$^s$, CAPE$^u$, and CAPE for the years 1974 through 1991. In this case, since the time series are regressed against time, which is known precisely, error ranges are calculated by taking the standard deviation of the detrended time series to be the error in each data point. Before the trends are calculated, the 12 month running mean time series is computed, and this series is sampled every 18 months, to reduce the autocorrelation in the series before errors are calculated. Trends and errors are averaged over all eighteen possible initial months. For the tropics as a whole, the trends in CAPE$^s$ and CAPE$^u$ nearly cancel, so that the trend in CAPE is small compared to either. For warm regions, and for warm ocean and land regions, there is a positive trend in CAPE over the period, while for cool regions the trend in CAPE is negative. Thus, the variability on long time scales essentially mirrors that on the interannual time scale. The tropics as a whole, and tropical warm regions show little or no variability of CAPE, since changes in CAPE$^u$ balance changes in CAPE$^s$, while warm ocean regions show a positive trend in CAPE, because the trend in CAPE$^u$ is not large enough to balance the increase in CAPE$^s$. The trends in surface CAPE$^s$ are due somewhat more to trends in dew point than to trends in temperature temperature. This is consistent with the results of Gutzler (1992), who analyzed 15 years of daily sounding data from several stations in the Western Pacific, and found trends in specific humidity of about 1 g kg$^{-1}$ (10 y)$^{-1}$, which corresponds to a dew point trend of about 1 K (10 y)$^{-1}$, but temperature trends of only 0.2 K (10 y)$^{-1}$. Note again that an assumption of reversible adiabatic ascent in the CAPE calculation would lead to 10% smaller trends in CAPE$^s$, and thus somewhat smaller trends in CAPE as well.

Figure 2-3 shows $\theta^*_e$ time series at various levels aloft, and $\theta_e$ at the surface, averaged from 30°S to 30°N, while Figure 2-4 shows the same variables averaged over the region from the equator to 20°N. Note the increasing dominance of the interannual variability as height increases in Figure 2-3, and the decreasing amplitude of the annual cycle with height. Note also that the northern time series have maxima in the northern summer, while southern
time series have maxima in the southern summer.

For variations over the seasonal cycle, the surface \( \theta_e \) and upper air \( \theta^* \) variations are much less coherent. In fact, Tables 2.1-2.3 show no significant correlation between temperature, dew point or \( \theta_e \) averaged over any surface region and temperatures or \( \theta^* \) aloft. To find out what drives seasonal temperature variations in the free troposphere, three new averaging regions are introduced. Table 2.4 shows the correlation coefficients of surface \( \theta_e \) with \( \theta^* \) aloft, averaged over the regions from 20°S to the equator (south), and from the equator to 20°N (north). Also shown are the correlations, at each level, of \( \theta^* \) in the North with \( \theta_e \) in the south. The statistics from the north and south regions reveal that the low correlations over the annual cycle shown in Tables 2.1-2.3 are due to the averaging over the two hemispheres. Within each half of the tropics, correlations of annual variations of \( \theta^* \) with surface \( \theta_e \) are very high, while \( \theta^* \) in the South is strongly anticorrelated with \( \theta_e \) in the North. However, the standard deviation of the variations decreases sharply with height, from over 5 K for the annual cycle at the surface, to about 1.7 K above 700 mb.

As discussed in the introduction, since seasonal temperature variation are larger at the surface than aloft, we expect that CAPE variations will be highly correlated with temperature variations. Table 2.5 confirms this. Over the annual cycle, CAPE\(_u\) is nearly perfectly anticorrelated with CAPE\(_s\) in each hemisphere, but the slopes of its linear regression onto CAPE\(_s\) are only around \(-0.3\) in each hemisphere, so that CAPE depends strongly on surface temperature. Finally, Table 2.4 shows correlation coefficients of surface \( \theta_e \) and \( \theta^* \) aloft when both are averaged over summer regions. The correlations are not as strong as within the north and south tropical regions, but are a bit better than those for the regions shown in Table 2.3. Again, as shown in Table 2.5, and plotted in Figures 2-6b and 2-7b, seasonal variations of CAPE\(_s\) are far larger than those of CAPE\(_u\), and the two series are only poorly (\( R = -0.42 \)) correlated. Thus the annual cycle of CAPE is nearly identical with that of CAPE\(_s\). Further, Figures 2-5c and 2-5d show that both seasonal temperature and \( \theta^* \) variations decrease sharply with height in the lower troposphere, even when the averaging region is made to follow the summer north and south.

### 2.4 Discussion

#### 2.4.1 Interannual Variability

These results show that interannual variations of CAPE in the tropics are small compared to those which would result if temperature perturbations were constant with height, and, for surface conditions averaged over warm regions, indistinguishable from those which would result if temperatures aloft were forced to lie along a pseudo-adiabat originating at the surface. However, the EOF analysis shows that the temperature variations themselves are distinguishable from such a pattern. Although interannual temperature perturbations do increase with height, they do not increase as much as they would if \( \theta^* \) were held constant, as the quasi-equilibrium hypothesis of Emanuel et al. (1994) predicts. Further, there are certainly convecting regions (e.g. warm ocean regions) over which large fluctuations of CAPE, with about half the magnitude of the corresponding variations of CAPE\(_s\) alone, occur on interannual time scales.

Moist convective quasi-equilibrium does seem to hold sway in the lower part of the troposphere. Figure 2-5a shows that up to 700 mb, \( \theta^* \) variations aloft are correlated with, and the same size as, surface \( \theta_e \) variations averaged over warm ocean regions. This suggests that temperatures in the lower free troposphere are controlled by relatively shallow,
continuous convection, while temperatures at 500 mb and higher are more influenced by deep convection from a few very warm locations, such as the Pacific warm pool, and equatorial Africa and South America. If surface $\theta_e$ is averaged over the region 10°S to 10°N, and 100°W to 160°W, the mean $\theta_e$ equals 347 K, warm enough to allow communication with the upper troposphere. (As shown in Figure 2-2, $\theta_e^*$ at 200 mb reaches 350 K. Surface parcels with $\theta_e$ much lower than this will not be able to ascend to that level via moist convection.) However, the time series of interannual $\theta_e$ variations in this region is actually more poorly correlated with $\theta_e^*$ at 200 mb ($R = 0.29$) than is the time series of $\theta_e$ averaged over warm regions ($R = 0.56$), or averaged over the whole tropical belt ($R = 0.52$). Very sparse data over some important regions of deep convection (the central equatorial pacific, and the continental interiors of South America and Africa) probably doom any attempt, using radiosonde data, to isolate the surface region which best accounts for temperatures at 200 mb.

The increase in $\theta_e^*$ towards the tropopause, shown in Figure 2-2 will be shown in Chapter 5 to be important for another reason. Namely, the reduced lapse rate implied by this increase helps to explain the rapid fall of the tropopause at the edge of the tropics.

The radiosonde data add another piece to the puzzle of tropical temperatures at the last glacial maximum. Our results are consistent with the prediction by Betts and Ridgway (1992), based on an assumed moist adiabatic lapse rate, that a 5.5 K cooling at the snowline implies a 2.5 K cooling at the surface. Linear regression of the time series of temperature at 500 mb, and at the surface, averaged over warm ocean regions predicts, for a temperature change of 5.5 K at 500 mb, a cooling of $3.3 \pm 0.6$ K at 1000 mb (where the error is determined as in Table 2.5). Betts and Ridgway point out that the lowering of the sea level by some 120 m at the last glacial maximum implies an increase of about 0.5 K in surface temperature, for constant $\theta_e$ at 1000 mb. Subtracting 0.5 K gives an ocean surface temperature change of $2.8 \pm 0.6$ K, which is in agreement with their estimate of 2.5 K cooling at the surface. If we use the time series of temperature from land regions, we get a prediction of $3.9 \pm 1.0$ K cooling at 1000 mb. This is about 1 K less cooling than was reconstructed from aquifer data by Stute et al. (1995), but a 1 K larger cooling than that predicted for the 1000 mb level by Betts and Ridgway (1992), and shows that if the aquifer-based temperature reconstruction is correct, the atmosphere over lowland Brazil, at least, must have been significantly more stable with respect to moist convection than it is at present.

### 2.4.2 Annual Cycle

Emanuel (1995), following Held and Hou (1980), offers the following description of the tropical free troposphere. It should have very little vertical gradient in $\theta_e^*$, and should have a meridional gradient in $\theta_e^*$ determined by a requirement that there be no gradient of absolute vorticity at the tropopause. $\theta_e^*$ at a fixed point in the tropical free troposphere should then exhibit an annual cycle, with its maximum at the time when the region of maximum zonally averaged surface entropy is directly below the fixed point. This annual cycle should be smaller than that of the surface entropy directly below, since the gradient in $\theta_e^*$ permitted by the constant vorticity constraint is small. As surface regions cool, due to seasonal variations in insolation, meridional mixing of the free troposphere keeps the free troposphere warm, so that if the local surface entropy becomes too small relative to $\theta_e^*$ aloft to allow deep convection, the surface boundary layer becomes separated from the free troposphere by a trade inversion. This picture is consistent with observational studies by Firestone and Albrecht (1986), who showed that the frequency and strength of strong trade
inversions in tropical soundings decreases as one moves equatorward.

The observed annual cycle of $\theta_e^*$ in the free troposphere agrees with this picture in its gross features. As discussed above, the southern and northern halves of the tropics exhibit strong annual cycles in $\theta_e^*$ at all levels, with temperatures maximizing during the local summer. Above 700 mb, the vertical gradient of the standard deviation of $\theta_e^*$ is relatively small, although mean $\theta_e^*$ increases with height in each half of the tropics as in Figure 2-2. $\theta_e^*$ aloft is generally closer to $\theta_e$ at the surface during the warm part of the cycle than during the cool part (compare the warm and cool extremes of the annual variations in Figure 2-4a and b). This suggests that the boundary layer is indeed decoupled from the free troposphere during the local winter.

However, the EOFs shown in Figure 2-5c show clearly that the magnitude of tropical mean $\theta_e^*$ variations decreases dramatically from 850 mb to 500 mb. The small annual cycle of the tropical mean temperature at 500 mb, compared to the annual cycle at the surface, remains puzzling. Of course, we expect the annual cycle aloft to be small relative to the annual cycle at any given spot in the tropics, since convecting regions shift around following (but lagging) the latitude of maximum insolation, and since in the local winter the surface gets cut off from the free troposphere by the trade inversion. Nevertheless we expect that by averaging over the regions where convection is occurring at a given time we would isolate the surface region which determines the temperature of the free troposphere. Yet the various surface regions (whole tropics, warm regions, ocean regions, land regions, summer regions) all have annual variations in $\theta_e$ which range from 2.5 (summer regions) to 7.5 (land regions) time larger than that of 500 mb $\theta_e^*$. Two possible explanations suggest themselves. It could be that surface screening criteria do not adequately isolate convecting regions. One way to test this hypotheses would be to use the ISCCP data (Rosso and Schiffer, 1991), along with much finer resolution surface temperature data than were used in this study, to find those surface regions with active deep convection, and see whether the $\theta_e$ variations averaged over those regions are more in accord with 500 mb $\theta_e^*$ variations. Alternatively, it could be that while the location of the very warmest surface regions, whose very deep convective activity presumably controls the temperature structure of the upper troposphere, may vary, the temperature of those regions has a smaller annual cycle than the larger region of moderate convective activity which combine to influence the lower free troposphere. Tests of these hypotheses lie outside the scope of the present work.

2.5 Comparison with Other Investigations

Sun and Oort (1995) used radiosonde data to investigate the correlation of tropical mean temperature and humidity at each height with temperature at the surface. They showed that the first EOF of temperature accounted for 79% of the total variance of the horizontal mean temperature. Because they normalized temperature by its standard deviation at each height, they did not show the increase with height of temperature variations shown in our Figure 2-5a.

Williams' (1992) investigation of the global electric circuit shows that the monthly mean magnetic field for the fundamental mode of the Schumann Resonance was well correlated with tropical mean surface temperature for the period 1969 through 1974. He argues that magnetic field strength should depend strongly on CAPE, since the field strength is correlated with lightning intensity, lightning intensity is proportional to ice lofting, which depends on updraft velocity in convecting towers, which in turn has an upper limit of $(2 \text{ CAPE})^{1/2}$.
(though the relationship between actually attained updraft velocities and this upper limit is unclear, as we noted earlier). While the radiosonde data reproduce the tropical average surface temperature curve shown in his Figure 2-5, none of the regions over which CAPE was calculated produces a time series of CAPE which resembles the time series of the mean magnetic field. In particular, the time series of CAPE for warm land surface regions, where lightning is most common, was uncorrelated \( R = -0.15 \) with tropical average surface temperature during the years 1969-1974. The large annual cycle in lightning flash frequency reported in Williams (1994) certainly correlates well with the annual cycle in CAPE. However, on these time scales, surface \( \theta_e \) variations are indistinguishable from CAPE variations, since upper-air temperature have such a small annual cycle. Thus, our results do not confirm that the correlation between lightning intensity and surface temperature are due to a link between CAPE and surface temperature. However, the sparseness of our data do not allow us to rule out the possibility that the CAPE experienced by lightning-producing storms bears some relationship with tropical mean surface temperatures.

Kent et al. (1995) averaged cloudiness and temperature over the deep tropics (20°S to 20°N) and, comparing several warm years to several cool years, found a strong dependence of cloudiness on temperature over that region. The radiosonde record, on the other hand, shows that CAPE has very weak dependence on temperature in this region \( R = 0.24 \) for detrended interannual variations, and moreover, that CAPEu and CAPEc cancel each other out as well as in any other subregion of the tropics. They are well correlated, with \( R = -0.86 \), and their linear regression slope, calculated as in Table 2.5, is \(-0.97 \pm 0.15\). These results seem very much in keeping with those of Wang and Randall (1992), who found that increased precipitation accompanied large rates of GCAPE generation, and not necessarily high values of GCAPE. The increased cloudiness of the warmer years must represent increased convective mass flux due to increased areal coverage of convecting clouds, rather than increased vertical velocity of updrafts within the clouds. This is consistent with the modeling study of Robe and Emanuel (1996), who used a nonhydrostatic cloud model to show that the response of a convecting regime to increased radiative cooling in the atmosphere was to increase cloud coverage, while keeping CAPE constant.

The relatively large variations in CAPE over ocean regions found in this study are in qualitative agreement with the results of Brown and Bretherton (1997). They calculated interannual variations in tropical convective stability by comparing free tropospheric temperature variations derived from MSU observations with surface equivalent potential temperature over deep tropical ocean regions, and found that surface variations of \( \theta_e \) were much larger than the variations of \( \theta_e \) aloft implied by the MSU temperature record. However, the MSU’s channel 2 actually reports a weighted mean of temperature from the surface to 100 mb, which peaks around 500 mb. In fact, the correlation between temperature averaged from 1000 mb to 700 mb with MSU channel 2 is only about \( R = 0.5 \) in the deep tropics (Spencer and Christy, 1992). As demonstrated by the EOF analysis above, temperature variations at and above the 500 mb level are small compared to those at 850 and 700 mb. Thus, the use of MSU-derived temperatures gives unrealistically small fluctuations in mean free tropospheric temperature, and thus unrealistically large fluctuations in CAPE.

2.6 Conclusions

The tropical radiosonde record of temperature and dew point variability shows that for time scales longer than a year, over 70% of the variance of horizontally averaged temperatures
in the tropical free troposphere can be accounted for by the temperature variations of a moist adiabat originating at the surface with temperature and dew point characteristic of climatologically warm surface regions. The high correlation and equal magnitudes of CAPE_s and CAPE_n variations on interannual and longer time scales indicate that the tropical atmosphere cannot sustain a build-up of CAPE with respect to convecting regions for more than a few months. This finding suggests that at the time of the last glacial maximum, the warmer regions of the tropics were indeed some 3.2 ± 0.6 K cooler than they presently are. However, the radiosonde data also help to reconcile evidence of large reductions in land temperature at that time with very small changes in sea surface temperature in the subtropical gyres, since they show that temperature trends in non-convecting surface regions may oppose temperature trends aloft, and in regions of active convection. Finally, the stability of CAPE values on the interannual and decadal time scales shows that the use of the moist-adiabatic adjustment is preferable to an assumption of constant $T$ or $\theta$ perturbations in climate modeling studies concerned with these time scales.

Annual variations of tropical CAPE require a more detailed theory than currently exists. While Emanuel's (1995) theory of thermally direct circulations seems to account for the gross features of the upper tropospheric temperature structure, it does not fully explain the relationship of upper tropospheric temperature to surface conditions. In particular, we were not able to confirm that tropical mean temperatures aloft are in equilibrium with the mean moist entropy of convecting regions. Thus, it remains unclear whether, on time scales of a year or less, moist convection is free to bring the free troposphere into moist convective equilibrium with surface conditions in convecting regions.
Chapter 3

Observations of Potential Vorticity Gradients on Extratropical Isentropes

3.1 Introduction

As we will discuss in Chapter 4, there are certain dynamical situations, essentially when mixing by eddies is more important than diabatic heating or friction, in which we expect that gradients of Ertel potential vorticity (PV) will be small along isentropes. As we will further discuss, the degree of homogenization of PV on isentropes, and the size of the region over which homogenization takes place, have important consequences for the stability of the atmosphere with respect to baroclinic eddies. In Chapter 5 we will show that a given distribution of PV on isentropes can be used to constrain the earth's zonal mean climate. In this chapter we will examine observations of potential vorticity and its meridional gradient, so as to form a firmer basis for our theoretical discussions.

Hoskins et al. (1985), Morgan (1994), and Sun and Lindzen (1994) have all noted that potential vorticity gradients taken along isentropes located within the troposphere are generally much smaller than those along isentropes at or above the tropopause. Morgan (1994) took the zonal mean of PV along north-south lines whose origin lay at the location of maximum PV gradient, and found that its gradient was comparable to the $\beta$ component of the gradient both south of (below) and north of (above) the position of maximum gradient. Sun and Lindzen (1994), using zonal mean temperature and wind data, claimed that the PV gradient along isentropes in the troposphere was actually indistinguishable from zero. In this chapter, we will reexamine the zonal mean PV gradients using monthly climatologies taken from the NCAR/NCEP reanalysis project (Kalnay et al., 1996. hereinafter referred to as the reanalysis data). This seasonal analysis will enable us to address the dependence of the observed PV gradients on the strength of the pole-to-equator temperature gradient.

We will also look at interannual variations of potential vorticity gradients, to learn how they relate to interannual temperature variations. Hou (1997) investigated the dependence of the austral winter temperature gradient on the momentum export of the Hadley cell. He found a very strong correlation between Hadley cell momentum export and the temperature difference between the polar and mid-latitude regions. We will reanalyze his data to see how PV gradients varied when temperature differences varied. In particular, we wish to test the consistency of the observations with the hypothesis that stronger momentum export
from the tropics leads to more vigorous extratropical baroclinic eddies, which in turn lead to stronger mixing of PV on extratropical isentropes, and thus to reduced temperature differences between the polar regions and the middle latitudes.

3.2 Seasonal variations of PV gradients on isentropes

Sun and Lindzen (1994) pointed out that while the gradient of PV is very difficult to evaluate locally in the atmosphere, owing to the calculations of second derivatives on sparse data which are required, it can be more reliably calculated over a large meridional region, because the PV gradient is related to the slopes of isentropes. If a small change in PV gradient changes isentropic slopes over a large region, and if the isentropes have a fixed pressure at one end of that region, then the pressure at the other end of the region may be measurably affected. We will extend their study by introducing a quantitative measure of the match between a θ distribution predicted by a given PV gradient and the observed θ distribution, and by analyzing seasonal and interannual variations in the PV gradients thus determined.

In order to assess the degree of PV homogenization in the real atmosphere, we will calculate the distribution of PV from 10-year monthly mean climatologies of temperature and wind. We will then compare the predictive value of three assumptions about how temperature in the free troposphere is determined: that the potential vorticity gradient along isentropes is zero, that it is constant, or that it is a constant multiple of the PV gradient due to β. We will show that a constant PV gradient along isentropes yields the best match with observed tropospheric temperatures. Finally, we will discuss the seasonal cycle in the PV gradient in terms of a varying region of total PV homogenization.

3.2.1 Analysis Methods

PV in isentropic coordinates is:

\[ P = -g(f + \zeta) \left( \frac{\partial p}{\partial \theta} \right)^{-1}, \]  

(3.1)

where \( P \) is the PV, \( g \) is the acceleration of gravity, \( f \) is the Coriolis parameter, \( \zeta \) is the relative vorticity, \( p \) is the pressure, and \( \theta \) is the potential temperature. Differentiating with respect to \( y \) gives,

\[
\frac{\partial P}{\partial y} = -g \frac{\partial(f + \zeta)}{\partial y} \left( \frac{\partial p}{\partial \theta} \right)^{-1} + g(f + \zeta) \left( \frac{\partial p}{\partial \theta} \right)^{-2} \frac{\partial^2 p}{\partial y \partial \theta}.
\]

(3.2)

The term \( -g \beta \left( \frac{\partial p}{\partial \theta} \right)^{-1} \), where \( \beta = \frac{\partial f}{\partial y} \), will be referred to hereinafter as the β contribution to the PV gradient.

To calculate the distribution of PV in the troposphere, we use temperature and wind data recorded on pressure surfaces. The reanalysis data has a resolution of 2.5 degrees in the horizontal, and a vertical resolution of about 100 mb. We note that \( \left( \frac{\partial p}{\partial \theta} \right)^{-1} = \frac{\partial \theta}{\partial p} \), so that on pressure surfaces \( P = -g(f + \zeta) \left( \frac{\partial \theta}{\partial p} \right) \). If we note that \( \left( \frac{\partial \theta}{\partial p} \right) \frac{\partial \theta}{\partial \theta} = \frac{\partial}{\partial p} \), and that \( \frac{\partial p}{\partial \theta} \bigg|_{\theta} = \frac{-\partial \theta}{\partial \theta} \frac{\partial p}{\partial \theta} \), it becomes clear that Equation 3.2 can be expressed in pressure
Figure 3-1: January zonal mean PV (a), PV neglecting local vorticity (b), meridional poleward PV gradient (c), and meridional poleward PV gradient neglecting local vorticity (d). Contour labels are in PVU ($10^6 m^2 s^{-1} K kg^{-1}$) in a and b, and in PVU/(1000 km) in c and d. 12 year mean data from NCAR/NCEP reanalysis project, Kalnay et al., 1996.
coordinates as:

$$
\frac{\partial P}{\partial y} \bigg|_{\theta} = -g \left( \frac{\partial \theta}{\partial p} \right) \left[ (\beta + \frac{\partial \zeta}{\partial y} - \frac{\partial \zeta}{\partial p} (\partial \theta/\partial y)) + (f + \zeta) \left( \frac{\frac{\partial^2 \theta}{\partial p^2} y}{\partial y \partial \theta} - \frac{\frac{\partial \theta}{\partial p}}{\partial p} \right)^2 \right] \tag{3.3}
$$

We then use centered differences to evaluate PV and its derivative along $\theta$ surfaces using wind and temperature data on pressure surfaces. Because we intend to use the observed PV distributions to constrain the earth’s zonal mean climate, we limit our discussion to zonal mean PV and its gradients. In the following text and figures, zonal mean temperature and wind were used to calculate PV. Comparison of zonal mean PV with the PV of the zonal mean temperature and wind did not reveal significant differences. Figure 3-1 shows that relative vorticity makes only a small contribution to the observed distribution of zonal mean potential vorticity and its gradient, so we will assume that $\zeta \ll f$, and ignore $\zeta$ in the rest of these calculations.

To quantify the PV gradient’s variations in an easily measurable way, we also use the following alternative approach to the evaluation of the PV gradient. If we assume some arbitrary gradient of PV along isentropes, we can use the resulting PV field to predict the potential temperature field. We can then compute the root mean square difference of the temperature predicted using the assumed PV distribution from the observed temperature distribution. The PV gradient can then be varied, and the process repeated, until the RMS difference between the observed and predicted temperature is minimized.

Setting $\zeta = 0$, and substituting $\frac{\partial P}{\partial y} = -\gamma \frac{\partial}{\partial y} \left( \frac{\partial P}{\partial \theta} \right)$ in Equation 3.2 yields:

$$
(1 - \gamma) \frac{\partial f}{\partial y} \left( \frac{\partial p}{\partial \theta} \right) = f \frac{\partial^2 p}{\partial y \partial \theta} \tag{3.4}
$$

Integration yields

$$
\frac{\partial p}{\partial \theta} = \frac{\partial p}{\partial \theta} \bigg|_{y_0} \left( \frac{\sin(\phi)}{\sin(\phi_0)} \right)^{(1-\gamma)}, \tag{3.5}
$$

where $\phi$ is latitude, $y = a \phi$, and $a$ is the earth’s radius. Note that when $\gamma = 1$, Equation 3.6 requires that the lapse rate of $\theta$ with $p$ be constant along isentropes, while $\gamma < 1$ implies that the lapse rate becomes less stable as latitude increases, and $\gamma > 1$ implies that the lapse rate becomes ever more stable as latitude increases.

If we make the alternate assumption that $\frac{\partial P}{\partial y} = c$, where $c$ is a constant, then by integration we get:

$$
P - P_{y_0} = c(y - y_0). \tag{3.7}
$$

Substituting the definition of $P$ and rearranging terms gives

$$
\frac{\partial P}{\partial \theta} = \frac{gf}{gf_{y_0} \left( \frac{\partial p}{\partial \theta} \bigg|_{y_0} \right)^{-1} - c(y - y_0)}. \tag{3.8}
$$

Since $f = 2\Omega \sin(\phi)$, when $c = 0$, Equation 3.8 reduces to Equation 3.6, when $\gamma = 0$.

We can integrate either Equation 3.6 or 3.8 to find the pressure on each $\theta$-surface, and
then use linear interpolation to find $\theta$ on each $p$-surface in some region of the troposphere. To do this, we need to know the location in $(\phi, p)$-space of the intersection of each isentropes with the lower and equatorward boundaries of the region, as well as the value of $\left.\frac{\partial p}{\partial \theta}\right|_{\theta_0}$ at each of these intersections. We choose a lower boundary at 925 mb, and an equatorward boundary at 22.5°, and obtain the boundary information from the reanalysis data. The value of $\left.\frac{\partial p}{\partial \theta}\right|_{\theta_0}$ is obtained by upward differencing, so for a lower boundary at 925 mb, the prediction depends only on $\theta$ observed at 925 mb and 850 mb in the extratropics, and on observed $\theta$ observed at and above 925 mb at the edge of the tropics.

### 3.2.2 Results

The seasonal cycle of the gradients of PV on isentropes of the zonal mean atmosphere is shown in Figure 3-2. It is clear that while the magnitude of the gradients varies somewhat over the year, the qualitative distribution is very similar from season to season: We find high PV gradients in the region below 850 mb, due to the increase in static stability as one follows an isentrope from the planetary boundary layer into the free troposphere, small PV gradients relative to the $\beta$ contribution in the lower free troposphere (around 700 mb), and gradients ranging from one to five times the $\beta$ contribution in the upper troposphere and stratosphere. The high PV gradients of the upper troposphere appear to conflict with the results of Morgan (1994), who found that PV gradients were generally on the order of $\beta$ until quite near the tropopause. The difference is due to the temporal and zonal variations of the tropopause height. These variations result in a zonal mean profile of PV gradients which has a broader region of relatively high values near the tropopause than would be observed at a particular place and time. Morgan’s averaging procedure made the location of maximum PV gradient the origin of his meridional axis, and so maintained a sharp spike in the PV gradient at the tropopause.

Figures 3-3 and 3-4 show contours of $\theta$ predicted using Equation 3.6 and Equation 3.8, respectively. The predictions are clearly worthless in the stratosphere, but seem to be useful below 200 mb. Figure 3-5 shows contours of $\theta$ predicted for PV gradients equal to zero everywhere. The match with observations is clearly inferior to that obtained for moderate PV gradients.

We can quantify the comparison of the accuracy of the various $\theta$ predictions by summing the squares of the differences of the predicted and observed $\theta$ values. Tables 3.1-3.2 show the root mean square differences, averaged over the region between 300 mb and 850 mb, and between 22.5° and 70° in each hemisphere, between observed temperatures and temperatures predicted by assuming $PV_\theta = -\gamma \beta \left(\frac{\partial p}{\partial \theta}\right)^{-1}$, and by assuming $PV_\theta = c$. These results verify the impression obtained from Figures 3-3-3-5 that assuming a zero PV gradient throughout the troposphere gives a significantly worse prediction of $\theta$ than assuming a gradient of the order of the $\beta$ contribution to the PV gradient. They reveal a distinct seasonal cycle in the PV gradient, so that the PV gradient which best explains the structure of $\theta$ is smallest in July and largest in January in both hemispheres. Finally, they show that a constant PV gradient produces a better prediction of $\theta$ than does a PV gradient set proportional to the $\beta$ contribution.

The seasonal cycle of the optimal PV gradient is rather sensitive to the level of the lower boundary. Because PV gradients near the surface are quite large (see Figure 3-2), if the lower boundary is taken to be the surface, rather than 925 mb, the optimal values of the PV gradient become much larger (varying from 1.5 to 2.5 times the $\beta$ contribution, and
Figure 3-2: Poleward meridional gradient of PV on isentropes, calculated from zonal mean $\theta$, divided by the $\beta$ contribution to PV, for January, April, July and October. Contours are labeled in multiples of the $\beta$ contribution.
Figure 3-3: Zonal mean $\theta$ from observations (solid contours), and $\theta$ predicted using Equation 6, with $\gamma = 1.2$ (dashed lines), for January, April, July and October.
Figure 3-4: Zonal mean $\theta$ from observations (solid contours), and $\theta$ predicted using Equation 7, with $c = 0.12$ PVU/(1000 km) (dashed lines), for January, April, July and October.
Figure 3-5: Zonal mean $\theta$ from observations (solid contours), and $\theta$ predicted using Equation 6, with $\gamma = 0$ PVU/(1000 km) (dashed lines), for January, April, July and October.
Figure 3-6: Contours of $\frac{\partial \theta}{\partial \phi}$ (solid, labeled in K/mb) and $\theta$ (dashed, labeled in K) plotted against latitude and pressure for January and July, from the reanalysis 12 year climatology.

The seasonal cycle is the same in both hemispheres: PV gradients are minimized in local summer. However, we are primarily interested in the PV distribution above the boundary layer, since it is there that there is a theoretical expectation of low PV gradients (see Chapter 4).

To assess the robustness of the seasonal cycle in the PV gradient in each hemisphere, we analyzed the $\theta$ distribution for the years 1984 and 1994, in addition to the 1982-1994 climatology. Tables 3.3 and 3.4 show that the seasonal cycle in the PV gradient which best predicts $\theta$ is consistent from year to year, varying little among the years 1984 and 1994, and the 1982-1994 mean.

Lindzen (1994b) showed that an atmosphere with constant $N$, and a constant shear of the zonal wind has a PV gradient equal to about three times the $\beta$ contribution, for a typical mid-latitude values of $N$ and the temperature gradient. Since $N$ really doesn't vary very much in the atmosphere (see Peixoto and Oort, 1992), the much smaller gradients observed here require an explanation. It turns out that the seemingly subtle distinction between an atmosphere with $\frac{\partial p}{\partial \phi}$ approximately constant on isentropes, and one with constant $N$ makes a large difference to the PV gradient. In isentropic coordinates, $N^2 = -\frac{\partial^2 \rho}{\partial \phi^2}$. For $N^2$ constant, $\rho$ must be proportional to $\frac{\partial p}{\partial \phi}$. If temperature decreases polewards along isentropes, $\rho$ must also decrease, and so must $\frac{\partial p}{\partial \phi}$. By Equation 3.2, this implies a positive PV gradient. In the real atmosphere, $N$ decreases gradually from the lower to the mid troposphere, and this gradual decrease results in a much smaller PV gradient than would be observed otherwise. Equation 3.6 shows that for PV gradients equal to the $\beta$ contribution, $\frac{\partial p}{\partial \phi}$, and thus also its inverse, are constant along isentropes. For gradients larger than the $\beta$ contribution, where $\gamma > 1$ in Equation 3.6, we expect $\frac{\partial p}{\partial \phi}$ to decrease in magnitude with latitude, and
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Table 3.1: RMS differences of zonal mean θ from observations and θ predicted using $\frac{\partial P}{\partial \phi} = -xg\beta(\partial P/\partial \theta)^{-1}$. The equatorward boundary is at 22.5°, and the lower boundary is at 925 mb. The differences are averaged over the region 22.5° - 70°, and 300 mb - 850 mb. * denotes minimum RMS difference for each month.
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Table 3.2: As in Table 1, but \( \theta \) is predicted using \( \frac{2\pi}{\theta} = x(0.1\text{ PVU})/(1000\text{ km}) \) on isentropes
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Table 3.3: PV gradient which gives best fit to observed monthly mean $\theta$ from the NCEP 1982-1994 climatology, and the years 1984 and 1994. Region as in Table 1, and $\frac{\partial P}{\partial y}$ is set equal to $-xg\beta(\partial P/\partial \theta)^{-1}$.

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Table 3.4: PV gradient which gives best fit to observed monthly mean $\theta$ from the NCEP 1982-1994 climatology, and the years 1984 and 1994. Region as in Table 1, and $\frac{\partial P}{\partial y}$ is set equal to $z(0.1\text{ PVU})/(1000\text{ km})$.   

53
to increase in magnitude. Figure 3-6 shows that $\theta$ is nearly constant along isentropes between about 500 mb and 850 mb, but that contours of $\theta$ regularly cross contours of $\frac{\partial\theta}{\partial p}$ at lower pressures.

Another way to predict $\theta$ using the PV distribution is suggested by Figure 3-2, which shows that the observed PV gradient, rather than being homogeneous in pressure and latitude, takes on different values in different regions. The PV gradient tends to be near zero in the region around 700 mb, is large below about 850 mb, and comparable to the $\beta$ term, or larger in the upper troposphere. We can view this PV distribution as resulting from a seasonal variation in the size of the region over which the PV gradient is very small. Table 3.5 shows the accuracy of predictions of $\theta$ when the meridional PV gradient is assumed to be zero over a region between 750 mb and a varying upper limit, and equal to $-1.5 g \beta (\partial p/\partial \theta)^{-1}$ elsewhere. The results echo the results in Table 3.1 and 3.2. A larger region of zero gradients (thus a smaller mean gradient) gives a better prediction of $\theta$ for the northern hemisphere summer, while a smaller region of zero gradients (thus a larger mean gradient) for December, January and February in both hemispheres, though the cycle is less noisy in the southern hemisphere.

3.2.3 Interannual Variations of PV gradients

Hou (1997) investigated the dependence of midlatitude temperature gradients on the strength of the momentum flux carried by the Hadley cell from the tropics to the midlatitudes. He found that increased momentum transport was linked to a smaller temperature difference between the midlatitudes and the polar regions, and proposed that the connection was due to the prompting of increased eddy activity in the midlatitudes by the enhanced momentum transport. The stronger eddy activity would then carry more heat from the midlatitudes to the polar region. We can reanalyze the data used by Hou using the procedures described above, to ascertain whether the decreased temperature gradient was associated with a significant change in the PV gradient.

Figure 3-7a reproduces Figure 22 of Hou (1997). It shows that in the austral winter (average over June, July and August) of 1988, the temperature gradient between the midlatitudes and the polar regions was reduced by 2K to 3K, depending on pressure, compared to same months averaged over the four years 1985, 1986, 1987 and 1989. Hou shows that the momentum flux due to the Hadley cell was elevated in that year, and showed that the winter time momentum flux, and high latitude temperatures were highly correlated over the period 1985–1989. He concludes that anomalously warm polar seasons are due to increased heat fluxes between temperate and polar latitudes. The increased heat fluxes in turn are assumed to be a result of increased eddy activity, driven by a poleward shift of the subtropical Rossby-wave source, which is observed to accompany stronger Hadley cell momentum transport.

We wish to determine whether the 'homogenization' of temperature gradients caused by increased Hadley cell momentum transport is accompanied by increased PV homogenization. Figure 3-7b supports this connection: it shows that the polar PV in the neighborhood of the largest temperature anomaly is somewhat decreased in 1988 relative to the other years in the study. Accompanying this change in PV is a reduction of the poleward PV gradients in lower troposphere around 70°. This change is small and localized; if we predict the $\theta$ distribution by assuming that $\frac{\partial\theta}{\partial y} = -xg\beta (\partial p/\partial \theta)^{-1}$, and adjust $x$ to get the best fit over the region from 300 mb to 850 mb, and from 45° to 80° S latitude, we find that the optimal value of $\beta$ is the same for the austral winter of 1988 as for the mean of the winters.
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Table 3.5: RMS differences of zonal mean $\theta$ from observations and $\theta$ predicted using $\frac{\partial p}{\partial y} = 0$ between 650 + x mb and 750 mb, and $\frac{\partial p}{\partial y} = -1.5 g \beta (\partial p/\partial \theta)^{-1}$ elsewhere. The equatorward boundary is at 22.5°, and the lower boundary is at 925 mb. The differences are averaged over the region 22.5° - 70°, and 300 mb - 850 mb. * denotes minimum RMS difference for each month.
Figure 3-7: (a) Austral winter zonal mean $\theta$ difference between 1988 and the mean of the years 1985, 1986, 1987 and 1989 (K). Data from the GEOS-1 DAS reanalysis, courtesy of A. Hou. (b) PV difference for the same years (PVU). (c) Poleward PV gradient difference for the same years (PVU/1000 km).

If we now ask how the change in polar PV is related to the change in temperature, it becomes apparent that connection is not simple. The polar PV was reduced in 1988 because the static stability was reduced at that time: the temperature anomaly decays with height away from the surface. Yet the Antarctic wintertime surface is generally very close to thermodynamic equilibrium (Carroll, 1982). To sustain the increased surface temperatures of the winter of 1988, there must have been an increase in the downward flux of energy to balance the increased upwelling infrared radiation due to the warmer surface temperatures. Since the polar lapse rate was anomalously negative, and the wind was not significantly higher it is unlikely that sensible heat fluxes towards the surface were larger that year. Thus the net downward radiative fluxes must have been larger, despite the decrease of the temperature anomaly with height. The most likely cause of the increased downward radiative flux was an increase in the opacity of the polar atmosphere, due either to increased water vapor or cloud fraction, or both. Thus, as far as can be determined using Hou's (1997) data, the increased PV mixing, in itself, cannot have been directly responsible for the warming.

3.3 Discussion

In the previous section we showed that the mean tropospheric PV gradient is clearly distinguishable from zero. It was also shown that the PV gradient has a seasonal cycle which is of opposite phase in each hemisphere, so that in the southern hemisphere, the PV gradient is at a minimum in the winter, while in the northern hemisphere, it is at a maximum in the winter. Similarly, we found that if the PV gradient were assumed to take a value of 1.5β in most of the troposphere, but to be zero in a region of varying size in the lower troposphere, then the optimal size of the homogenized region was largest in the northern hemisphere summer, and the southern hemisphere winter.

This is an interesting situation, since explanations for either phase of the cycle are easy to imagine. We might rationalize a summertime minimum of the PV gradient by supposing that the gradients are dominated by radiative forcing of the temperature distribution. Since summertime polar lapse rates are generally less stable than wintertime lapse rates, we might expect that \( \frac{\partial \theta}{\partial p} \) would tend to become larger towards the poles in the winter, implying a larger PV gradient in that season. On the other hand, since eddy activity is greater in winter than in summer, we might expect that PV would be more thoroughly homogenized by adiabatic mixing in the winter: than in the summer, leading to a wintertime minimum of the PV gradient. That the cycles have opposite phases in each hemisphere indicates that the radiative and adiabatic forcing of PV are of roughly comparable magnitudes. In the northern hemisphere, the annual variation of the forcing of PV gradients is in some sense larger than the variation in mixing, so that the PV gradients are largest in the winter, while in the southern hemisphere, the variations in mixing outweigh the variations in forcing of PV gradients, so that PV gradients are largest in the summer.

Our analysis of Hou's (1997) data lends some support to this view. Hou provided strong evidence that meridional mixing was enhanced in the austral winter of 1988 relative to the years 1985-86 and 1989, consistent with a reduction of the temperature gradient between the midlatitudes and the polar region. We found that this reduction was accompanied by a reduction in the PV gradient between those years. Thus, in at least some situations, more strongly forced eddies, in the presence of comparable radiative forcing, are observed to lead
to smaller atmospheric gradients of PV.

3.4 Conclusions

The results presented above show that the potential vorticity distribution of the atmosphere varies in detail, but not in overall nature from season to season. This seems to indicate that the forcing of the PV gradient and the degree of mixing vary in phase with each other. Despite a change in polar temperature of 30 K from winter to summer, and a doubling of the midlatitude temperature gradient (from -4.7 K/1000 km to -9.9 K/1000 km in the northern hemisphere), the PV gradient which best explains the potential temperature distribution of the free troposphere varies by only about 25% over the year. Since the "climate change" represented by the transition from summer to winter is at least as large as any known variation in the earth's annual mean climate, our results suggest that the change in the PV distribution for small climate changes should be very small indeed. This suggests the possibility of constraining the earth's climate by assuming a given PV distribution. In Chapter 5 we will explore this possibility in detail. Note that this reasoning in no way precludes the possibility that small changes in the PV distribution could have important effects on the stability of the tropospheric flow.
Chapter 4

The Theory of PV Homogenization

4.1 Introduction

In this chapter we will discuss the interaction of atmospheric eddies and the potential vorticity distribution of the atmosphere's mean flow. We first review the current theoretical understanding of the equilibrium PV distribution of an atmosphere with vigorous eddies, and relatively weak diabatic effects. We then consider the implications of the degree of homogenization of PV for the stability of the troposphere with respect to the linear growth of baroclinic eddies. Finally, we discuss the interaction of non-linear equilibration and PV homogenization, in light of the modeling results of Solomon (1997).

4.1.1 PV as a quasi-conserved tracer

We first review the work of Rhines and Young (1982), who described some of the circumstances under which PV should be homogenized along isentropes. They also described some alternative equilibrium distributions of PV, in which the gradient of PV is due entirely to the meridional variation of the planetary vorticity. We will discuss the relevance of these alternative scenarios to the real atmosphere.

Rhines and Young confine their arguments to steady flows, and to regions of the atmosphere or ocean within which diabatic heating is negligible, and they use the quasi-geostrophic equations of motion. They then assume that PV fluxes will be proportional either to the horizontal gradient of PV, if the local eddy contribution to vorticity is large compared to the planetary contribution, or to the vertical gradient of angular momentum, if the planetary contribution dominates. They base this distinction on the work of Rhines and Holland (1979) who show that for large scale flows, with $\zeta < f$, horizontal eddy transport of PV is equivalent to a body force in the momentum equation, which in turn could be modeled by a diffusive vertical flux of momentum. In either case, for steady flow in a region within a closed streamline, or between two closed streamlines, there can not be any flux of PV through the boundaries of the region, and the absence of any source of PV (since diabatic effects are small) means that the flux of PV must be zero everywhere.

If the local eddy vorticity is comparable to the planetary vorticity, and the PV flux depends on the local gradient of PV, this gradient must be zero within such a region. If the planetary vorticity dominates the total PV, and the PV flux depends on diffusive vertical
momentum transport, then the vertical convergence of the momentum flux must be zero. If this derivative is expressed as \((\nu U_z)_z\), and if we set \(\nu_z = 0\), then \(U_{zz}\) must equal zero. In the notation of Rhines and Young, the quasi-geostrophic PV is expressed as

\[ \bar{q} = \beta y + (F \bar{\psi})_z, \]

where \(\bar{\psi}\) is the basic state stream function, \(F = f_0^2 / N^2(z)\), \(f_0\) is the coriolis parameter, and \(N\) is the Brundt-Väisälä frequency. Taking the meridional derivative, we get

\[ \bar{q}_y = \beta + (F \bar{U}_z)_z. \]

Thus, if \(F_z = 0\),

\[ \bar{q}_y = \beta. \]

This result seems attractive, since, as we saw in Chapter 3, the mean PV gradient which best explains the observed temperature structure is indeed close to the \(\beta\) contribution. However, several of the conditions for it are not met in the atmosphere. Although, as we showed in Chapter 3, the zonal mean local vorticity is indeed small compared to the planetary vorticity, the eddies which transport PV in the atmosphere often do have \(\zeta\) comparable to \(f\). \(F\) is not constant in the vertical, and most importantly, the assumption of Fickian vertical diffusion of zonal momentum is difficult to justify. Although moist convective transport of momentum might be parameterized as a diffusive process, the dominant extratropical tropospheric momentum transfer processes, such as Hadley cell momentum transport, and Rossby and gravity wave flux divergence, result in forces which are not directly dependent on the vertical momentum gradient. If we cannot assume that eddies act to remove the curvature of the vertical profile of zonal momentum, the derivation of \(\bar{q}_y = \beta\) presented above cannot be valid.

Returning to the case where \(\zeta\) is comparable to \(f\), we note that the dependence of eddy fluxes of PV on the mean PV gradient was more recently investigated by Pavan and Held (1996), using a two layer, damped, quasi-geostrophic model. Because their model included Newtonian cooling, which restored the mean flow to an equilibrium jet structure, their results are complementary to, but not directly comparable with Rhines and Young’s (1982) theoretical work. Pavan and Held found that PV fluxes in their model were well explained by a direct dependence on the local PV gradient. This evidence supports an expectation of PV homogenization in regions where the change in the diabatic forcing of PV in response to a small change in the distribution of PV is small compared to the change in eddy fluxes of PV.

Pavan and Held did find that eddies in their model were unable to remove the PV gradient associated with the thermal equilibrium jet towards which the model was relaxed. They concluded that baroclinic adjustment models, which depend on eddies to modify the model’s mean flow so that eddy growth is inhibited, were a poor description of the processes occurring in their model, and thus were likely to be a poor description of nature. However, the discretization of their model into only two layers makes the homogenization of PV artificially difficult, since the static stability of the model cannot be altered. Further, in the real troposphere, radiative cooling is fairly uniform in the vertical direction (Olaguer et al., 1992). Thus the diabatic PV forcing, which depends on the vertical gradient of the diabatic heating (Haynes and Ward, 1993), is relatively small in the tropospheric interior. So the real atmosphere might have regions of PV homogenization, which a two level model, having no unforced “interior” could not be expected to reproduce.
If the equilibrium gradients of PV are constrained by mixing to be small in regions where the diabatic forcing of PV is weak, the value of PV itself must be specified in regions where diabatic forcing is strong. A simple extension of Rhines and Young’s analysis leads to the following argument. Consider an isentrope which extends from the lower tropical troposphere to the polar tropopause. If \( \phi \) represents latitude along the isentrope, assume that from \( \phi_0 \) to \( \phi = \pi/2 \), diabatic forcing of PV is weak. Assume further that equatorward of \( \phi_0 \), deep convection is the dominant determinant of static stability, and thus of PV. We anticipate that PV on this isentrope will be constant at a value imposed by moist convective equilibrium at \( \phi_0 \) up to the tropopause. This analysis assumes that mixing operates freely between the tropics and the extratropics, but that the tropopause represents an effective barrier to mixing, so that PV need not be continuous across the tropopause. In fact, as we saw in Chapter 3, zonal mean gradients of PV on isentropes are small between the boundary layer and about 650 mb, rise to about the \( \beta \) contribution, then increase towards the tropopause. This indicates that some mixing across the tropopause does occur, increasing the PV, and thus the PV gradients, of the upper troposphere. In addition, the height of the tropopause, which separates high-PV stratospheric air from low-PV tropospheric air, varies around latitude circles. Thus the zonal mean of PV at a pressure level slightly below the mean tropopause will likely include some regions where that pressure level is above the tropopause. This will tend to increase the zonal mean PV as one approaches the zonal mean tropopause. Since isentropes also approach the tropopause in the poleward direction, gradients of PV in the upper troposphere will tend to be increased due to the zonal variations of the tropopause height.

4.1.2 PV as a governor of dynamics

It is potential vorticity’s central role in the dynamics of geophysical fluids that make its conservation properties interesting. The conservation of potential vorticity by parcels moving within a fluid with a PV gradient provides the “restoring force” which allows the propagation of Rossby waves in a geophysical fluid. Flow geometries which allow these waves to obtain energy from the mean flow make possible the linear instabilities which are, at least in part, responsible for the eddies and storms of the real atmosphere.

Charney and Stern (1962) showed that sufficient conditions for stability in a quasi-geostrophic flow are that the meridional gradient of the basic state PV never vanishes along isentropes, and that the potential temperature at any rigid horizontal surface be constant. Bretherton (1966) added that any meridional temperature gradient on the lower boundary of such a flow is equivalent to a PV gradient of the same sign in an infinitesimal region adjacent to the surface. Since the extratropical zonal mean poleward temperature gradient is always negative at the surface, and since the poleward PV at the tropopause is always positive, the Charney-Stern condition is always violated on global scales in the earth’s atmosphere. Yet atmospheric eddies do not grow indefinitely; some other mechanism must restrain their growth. Either eddies must grow until they attain sufficient amplitude that nonlinear processes dissipate eddy energy as quickly as it is converted from the mean flow through linear growth, or the eddies must modify the mean flow in such a way that eddy growth rates are reduced to a level which matches dissipation. For the latter situation, which we will refer to as neutralization, small changes in the flow’s mean state should produce large changes in eddy growth rate, so that perturbations of the mean flow result in the rapid growth of eddies which act to return the mean flow to the neutralized state.

Lindzen (1993) proposed the following mechanism whereby eddies could act to neutralize
the troposphere with respect to their own growth. He noted that in the Eady (1949) model, an atmosphere with a rigid lid and no interior PV gradient can be made stable to eddies with any zonal wavenumber if the meridional domain is small enough and the lid is high enough. He proposed that in the real atmosphere, the part of the meridional walls in the Eady model might be played by the meridional variations of the zonal mean jet. Ioannou and Lindzen (1986) showed that this meridional confinement was the primary effect of a jet-like mean flow on the structure of linear modes growing within such a flow. The height of the tropopause could then be derived by requiring the atmosphere to be marginally neutral to the growth of the waves with the smallest zonal wavenumber.

As we will discuss, linear stability analyses of idealized flows somewhat more realistic than the flows assumed in Lindzen (1993) seems to indicate that the atmosphere is not fully neutralized by this mechanism. Specifically, they indicate that the tropopause height required for neutrality is probably much higher than that observed in the mid-latitude troposphere. Nevertheless, it is clear that the tendency of eddies to mix PV on isentropes acts to stabilize the atmosphere. Furthermore, the quasi-geostrophic beta-plane channel model of Solomon (1997) produces an equilibrated flow with a significant region of PV homogenization, and PV distribution elsewhere which is only slightly sensitive to variations in thermal forcing. This flow appears to be stable with respect to the linear growth of small disturbances. It is not immediately apparent which aspects of Solomon's flow, or of her model's damping and dissipation, stabilize it as compared to the flows investigated in the next section, and by Harnik and Lindzen (1997). However, Solomon's results confirm that the mixing of PV in the lower troposphere contributes to the stabilization of the atmosphere.

4.2 Stabilization of the Troposphere by Removal of PV Gradients and Adjustment of Tropopause height

We now review the evidence relevant to the hypothesis that the troposphere is close to a state that is neutral with respect to the linear growth of baroclinic disturbances, because PV gradients in the troposphere are sufficiently reduced and the tropopause is sufficiently high, that Rossby waves propagating near the surface and the tropopause are unable to interact to form a single disturbance. We examine results from a hierarchy of models, beginning with a simple extension of Lindzen's (1994a) modified Eady model, continuing with a review of Harnik and Lindzen's (1997) linear analysis of a troposphere with a finite PV gradient, and concluding with some results from Solomon's (1997) multilayered quasi-geostrophic channel model.

4.2.1 Modified Eady Model with Finite Scale Height

In this section, we investigate the stability properties of a dry atmosphere on a $\beta$-plane, having finite scale height, but no interior gradient of potential vorticity, confined by rigid surfaces on the top and bottom, and having a zonal, jet-like mean wind. The dependence of the growth rate of small normal mode disturbances on the height of the atmosphere's lid and on the disturbances' total wavenumber will be found, and the lid height will be compared to the tropopause of the real atmosphere. We begin by rederiving the governing equations of the modified Eady model (Lindzen, 1994a), further modified by the elimination of the boussinesq approximation- that is, $H$, the scale height, is now taken to be finite.
Development of Theory

We will follow the notation of Lindzen (1994a). The linearized quasi-geostrophic equation for conservation of pseudo-potential vorticity in log-p coordinate, where \( z = -H \log(p/p_s) \), is (Pedlosky, 1987)

\[
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q' + v' \frac{\partial q}{\partial y} = 0
\]

(4.1)

where \( v' \) and \( q' \) are defined

\[
v' = \frac{1}{f_0} \frac{\partial \Phi'}{\partial x}
\]

(4.2)

\[
q' = \frac{1}{f_0} \left( \frac{\partial^2 \Phi'}{\partial x^2} + \frac{\partial^2 \Phi'}{\partial y^2} \right) + e^{-z/H} \frac{\partial}{\partial z} \left( \frac{f_0}{N^2} e^{-z/H} \frac{\partial \Phi'}{\partial z} \right)
\]

(4.3)

and \( \frac{\partial q}{\partial y} \) is defined by:

\[
\frac{\partial q}{\partial y} = \beta - e^{z/H} \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} e^{-z/H} \frac{\partial U}{\partial z} \right)
\]

(4.4)

where \( q \) is the basic state pseudo-potential vorticity, \( \Phi' \) is the perturbation geopotential, \( y \) is the meridional direction, \( z \) is the height, \( U \) is the basic state zonal wind, \( f_0 \) is the coriolis parameter at the center of the \( \beta \)-plane channel, and we have assumed for the present that the basic state flow has no meridional curvature.

If we set \( \frac{\partial q}{\partial y} = 0 \), we can find the vertical gradients of basic state static stability and vertical shear which just cancel the PV gradient due to \( \beta \) and \( H \). Note that this results in a constraint on the slopes of isentropes which is, for constant \( H \), identical to that which results from setting the meridional derivative of Ertel’s potential vorticity to zero along isentropes. This was pointed out by Charney and Stern (1962), but the details of the correspondence bear a little attention. Setting \( \frac{\partial q}{\partial y} = 0 \), we get

\[
\beta = e^{z/H} \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} e^{-z/H} \frac{\partial U}{\partial z} \right)
\]

Integration, assuming that \( H \) is a constant, yields,

\[
\int_0^z \frac{\beta}{f_0} e^{-z'/H} dz' = \left[ \frac{f_0^2}{N^2} e^{-z/H} \frac{g}{f_0 T} \frac{\partial T}{\partial y} \right]_0^z.
\]

Expanding \( N^2 \), we get:

\[
\frac{\beta}{f_0} H \left( e^{z/H} - 1 \right) = \left[ \frac{f_0}{g} e^{-z/H} \frac{\theta}{\theta T} \frac{\partial T}{\partial y} \right]_0^z,
\]

\[
\frac{\beta}{f_0} H \left( e^{z/H} - 1 \right) = \left[ e^{-z/H} \frac{\partial \theta}{\partial y} \right]_0^z
\]

\[
e^{-z/H} \left| \frac{\partial z}{\partial y} \right|_0^z - \left| \frac{\partial z}{\partial y} \right|_0^z = \frac{\beta}{f_0} \left( H - H e^{z/H} \right)
\]
\[
\frac{\partial p}{\partial y} \bigg|_{\theta} = \frac{\partial p}{\partial y} \bigg|_{\theta, \rho} + \frac{\beta}{f_0} (p - p_0),
\]

which is just the relation derived in Chapter 3, for \( \gamma = 0 \).

The error due to the assumption that \( H \) is constant is not trivial. In \( z \)-coordinates, the PV gradient is expressed:

\[
\bar{q}_y = \beta + \frac{f_0}{\frac{\partial T}{\partial z} + \frac{\partial H}{\partial z}} \left[ \frac{\partial^2 T}{\partial y \partial z} - \frac{1}{H} \frac{\partial T}{\partial y} \left( \frac{\kappa T}{\partial z} + \frac{\partial}{\partial z} \left( \frac{\partial T}{\partial z} + \frac{\kappa T}{\rho \bar{c}_p} \right) - \frac{\bar{c}_p}{\rho \bar{c}_p} \right) \right],
\]

(4.5)

where \( \kappa = R/C_p \). The assumption of constant \( H \) is equivalent to assuming \( \kappa = 1 \) in Equation 4.5, and results in an error of about \( \beta \), so that Equation 4.4 will give a gradient of \( 2\beta \) for a flow whose actual PV gradient is \( 3\beta \). Nevertheless, for simplicity’s sake we take \( H \) to be constant.

Integration of equation (4.4) yields

\[
-\beta H + (m_0 \epsilon_0 + \beta H) e^{z/H} = \frac{f_0}{N^2} \frac{\partial U}{\partial z}
\]

(4.6)

where \( \epsilon_0 = \frac{f^2}{N^2} \), and we have assumed that \( N^2 = N_0^2 \) and that \( \frac{\partial U}{\partial z} = m_0 \) at \( z = 0 \). If we assume for the moment that \( \frac{\partial U}{\partial z} = m_0 \) everywhere, we can solve the equation for \( N^2 \):

\[
N^2 = \frac{N_0^2}{e^{z/H} + \frac{\rho H}{\rho_0 m_0} \left( e^{z/H} - 1 \right)}.
\]

(4.7)

We now introduce a parameter \( \alpha \) which indicates the extent to which variations in the static stability cancels the contribution of \( \beta \) to the meridional gradient of PV, so that

\[
N^2 = \frac{N_0^2}{e^{z/H} + \frac{\rho H}{\rho_0 m_0} \left( e^{z/H} - 1 \right)}.
\]

(4.8)

We then substitute our equation for \( N^2 \) back into equation (4.6) and solve for \( \frac{\partial U}{\partial z} \), which gives

\[
\frac{\partial U}{\partial z} = m_0 e^{z/H} + \frac{\rho H}{\rho_0 m_0} \left( e^{z/H} - 1 \right).
\]

(4.9)

The resulting profiles of \( U \) and \( N \), as well as the temperature profiles implied by the variations in \( N \) are shown in Figures 4-1-4-4, for values of \( \alpha \) ranging from 0.0 to 1.0 in steps of 0.2, for a lid height of 10 km. Figures 4-1-4-3 show profiles for \( H = 8 \) km, with various constraints on \( N \) and \( U \). Figure 4-4 shows profiles for the \( H \sim \infty \) case. Note that the deviation from constant shear is about twice as large, and the deviation from constant Brunt-Väisälä frequency about six times larger for the finite \( H \) case than for the infinite \( H \) case. In Figure 4-1, the values of \( N^2 \) and \( \frac{\partial U}{\partial z} \) at the lower surface are varied so that their mean values over the layer remain constant at \( 1.05 \times 10^{-2} \) s\(^{-1}\) and \( 2.5 \times 10^{-3} \) s\(^{-1}\), respectively. For an upper boundary at 10 km, this produces fairly realistic profiles of \( N \) and \( U \). However, for higher lid heights, since \( N \) tends to fall off approximately exponentially,
Figure 4-1: Profiles of $U$, $\frac{\partial U}{\partial z}$, $N$, and $T$ for $\alpha$ varying from 0 to 1 in steps of 0.2. $H = 8$ km. Latitude is set at 30°. $N$ and $\frac{\partial U}{\partial z}$ are constrained to have fixed mean values, so that $U(25\text{km}) = 25\text{ m s}^{-1}$, and $(N^2)^{1/2} = 1.05 \times 10^{-2}\text{s}^{-1}$. 
Figure 4-2: As in previous Figure, but now $N(0\text{km}) = 1.4 \times 10^{-2}\text{s}^{-1}$.
Figure 4-3: As in previous Figure, but now $\frac{\partial U}{\partial z}(0) = 2.5\text{ m s}^{-1}\text{ km}^{-1}$, $N(0\text{ km}) = 1.4 \times 10^{-2}\text{ s}^{-1}$.
Figure 4-4: As in previous Figure, but now $H \sim \infty$, $U(25 \text{ km}) = 25 \text{ m s}^{-1}$, and $(\tilde{N}^2)^{1/2} = 1.05 \times 10^{-2} \text{s}^{-1}$.
maintaining a layer mean value of \( N = 1.05 \times 10^{-2} \, s^{-1} \) requires an ever more unrealistically high surface value of \( N \). When calculations of the dispersion relation for the modified Eady model are made, we will consider the results of using both constant layer-mean \( \sqrt{N^2} \) (\( \bar{N} \)), and constant \( N_0 \). As we shall see, the choice of \( N \) distribution has important consequences for the stability of the model troposphere. Figure 4-2 shows \( N, T, U \), and its derivative with \( N_0 \) and \( U(25 \, km) \) held constant, while Figure 4-3 shows profiles for \( N_0 \) and \( \frac{\partial U}{\partial z}(0) \) held constant.

If we assume solutions to equation (4.1) of the form

\[
\Phi' = \Phi'(z) \sin(\ell y) e^{ik(x-ct)}
\]  
(4.10)

where \( c \) is the phase speed, \( k \) the zonal wavenumber, and \( \ell \) the meridional wave number of the perturbation, and substitute into equation (4.1) the relations for \( N^2 \) and \( U \) derived above, we get a vertical structure equation for the behavior of a modal disturbance of the basic state flow:

\[
(U(z) - c) \left\{ -\frac{k^2 + \ell^2}{\epsilon_0} \Phi' + \frac{\alpha \beta}{\epsilon_0 m_0} \frac{d\Phi'}{dz} + \left[ e^{\epsilon H} + \frac{\alpha \beta}{\epsilon_0 m_0} \left( e^{\epsilon H} - 1 \right) \right] \frac{d^2\Phi'}{dz^2} \right\} = 0.
\]  
(4.11)

Note that as \( \beta \) goes to zero and \( H \) goes to infinity, we recover the governing equation of the standard Eady model:

\[
(U(z) - c) \left( -\frac{k^2 + \ell^2}{\epsilon_0} \Phi' + \frac{d^2\Phi'}{dz^2} \right) = 0.
\]  
(4.12)

As in the Eady model, we assume zero vertical velocity at the surface and at the lid, and set \( U = 0 \) at surface. This yields the conditions

\[
-c \frac{d\Phi'}{dz} - \Phi' \frac{dU}{dz} = 0 \text{ at } z = 0
\]  
(4.13)

\[
(U - c) \frac{d\Phi'}{dz} - \Phi' \frac{dU}{dz} = 0 \text{ at } z = h,
\]  
(4.14)

where \( h \) is the lid height. If we substitute Equation 4.9 for \( \frac{dU}{dz} \), we get:

\[
-c \frac{d\Phi'}{dz} - m_0 \Phi' = 0 \text{ at } z = 0
\]  
(4.15)

\[
U(h) \frac{d\Phi'}{dz} - m_0 \frac{e^{\epsilon H} + \frac{\beta H}{\epsilon_0 m_0} \left( e^{\epsilon H} - 1 \right)}{e^{\epsilon H} + \frac{\alpha \beta H}{\epsilon_0 m_0} \left( e^{\epsilon H} - 1 \right)} \Phi = 0 \text{ at } z = h,
\]  
(4.16)

where

\[
U(h) = m_0 \left\{ H \left( \frac{1 + \frac{\beta H}{\epsilon_0 m_0}}{1 + \frac{\alpha \beta H}{\epsilon_0 m_0}} - \frac{1}{\alpha} \right) \ln \left[ e^{\epsilon h/H} + \frac{\alpha \beta H}{\epsilon_0 m_0} \left( e^{\epsilon h/H} - 1 \right) \right] + \frac{h}{\alpha} \right\}.
\]  
(4.17)

Equations 4.11, 4.15, and 4.16 can be solved numerically, and a dispersion relation obtained. We can then apply the same techniques used by Ioannou and Lindzen (1986) to derive the effective meridional wavenumber imposed on a disturbance by the varying meridional shear of the basic state wind. In fact, because their approximation to the Eady
Figure 4-5: Imaginary part of phase speed plotted versus non-dimensional wavenumber for solutions of the governing equation, solid. Dashed line shows Ioannou and Lindzen's (1986) approximation to the Eady model dispersion relation (see text).
model dispersion relation,

\[ 4c^2 \approx \bar{u}^2(Y) \left( 1 - \frac{\pi^2 + d_e^2}{\pi^2 + d_c^2} \right), \]

where \( d \) is our \( \mu \), and \( d_e \) is \( \mu \) at the short wave cutoff, approximates the modified Eady model imaginary phase speed very well in the neighborhood of the short-wave cutoff (see Figure 4-5), their result, that the meridional wave number should be approximately equal to the reciprocal of the scale of the jet, will hold also for this modified Eady model. In that case, we can use a climatological value for the width of the subtropical jet to derive a meridional wavenumber, as in Lindzen (1994a).

**Numerical Solutions to Dispersion Relation**

Solutions to these equations were obtained using a shooting method (Press et al., 1986). The resulting dispersion relations are shown in Figure 4-6, for both the \( H \sim \infty \) and \( H = 8 \text{ km} \) cases. In each case, \( h \) is set to 10 km, \( \alpha \) is set equal to 0.5, \( N_0 \) is set to \( 1.05 \times 10^{-2} \text{s}^{-1} \), and \( m_0 \) is chosen so that \( U(h) = 25 \text{ m s}^{-1} \). The axes of the plots are \( \mu \) and \( c \), with the real and imaginary parts of \( c \) plotted separately, and where \( \mu = \sqrt{k^2 + \frac{\rho^2 N^2}{m_0} h} \). \( \bar{N} \) is the square root of the average over \( \xi \) from 0 to \( h \) of \( N^2 \), and is expressed:

\[ \bar{N} = N_0 \left( \frac{\epsilon_0 m_0}{\alpha \beta} \right)^{1/2} \left\{ \frac{1}{h} \ln \left[ \frac{e^x/H}{\epsilon_0 m_0} \left( \frac{e^x/H}{\epsilon_0 m_0} - 1 \right) \right] - \frac{1}{H} \right\}^{1/2} \]  

(4.18)

Except for a slight lengthening of the long-wave cut-off, and a nearly uniform reduction in the real part of the non-dimensional phase speed of about 0.09, the dispersion relations are identical for the \( H \sim \infty \) and \( H = 8 \text{ km} \) cases. In particular, the short wave cut-off is located at 2.4... in both cases, essentially identical with the value for the traditional Eady model. Note that this result only holds when \( \mu \) is non-dimensionalized with \( \bar{N} \) rather than \( N_0 \). \( \bar{N} \) turns out to be a good measure of the static stability that is actually experienced by the growing mode as it projects across the model atmosphere.

This similarity between the \( H = 8 \text{ km} \) and \( H \sim \infty \) cases diminishes when the lid height is increased. In the \( H = 8 \text{ km} \) case, as \( h \) increases, both the short- and long-wave cutoff moves to larger values of \( \mu \), while for the \( H \sim \infty \) case, the short-wave cut-off stays nearly constant near \( \mu \approx 2.4 \). This can be seen in Figures 4-7 and 4-9, which show, for \( H \sim \infty \), and \( H = 8 \text{ km} \), respectively, the unstable modes' growth rates as functions of the height of the model lid and of zonal wavenumber, assuming a meridional wavenumber equal to \( \frac{1}{0.15a} \). Figures 4-8 and 4-10 show the real, non dimensional phase speeds for the same cases. For these cases, \( m_0 \) and \( N_0 \) were continuously varied as \( h \) increased so as to hold the layer mean values of \( \frac{\bar{u}}{2h} \) and \( \bar{N} \) constant.

Figures 4-11 and 4-12 are the same as 4-9 and 4-10, except that now \( N_0 \) is held constant. Note that for the \( H = 8 \text{ km} \) cases, the stability characteristics differ considerably for the two \( N \) distributions. When \( N_0 \) is held constant as the height of the lid is increased, the longest zonal wavenumbers are not beyond the short-wave cut-off for any reasonable lid height, and growth rates exceed the nominal tropospheric damping rate of \( 0.2\text{d}^{-1} \) for some wavenumbers at lid heights well above 20 km. On the other hand, when \( N_0 \) is adjusted to keep \( \bar{N} \) constant as the lid height changes, the longest zonal wavenumbers are stabilized below 20 km, and the maximum growth rate (over all zonal wavenumbers) falls below \( 0.2\text{d}^{-1} \) by about 10 km. Comparison of Figures 4-1-4-3 shows that the former case is more realistic. Observations of \( N \) just above the boundary layer generally give values around \( 1.4 \times 10^{-2} \)
Figure 4-6: Dispersion relations compared for Modified Eady model with $H \sim \infty$: (a) real part of phase speed, (b) imaginary part. For $H = 8$ km: (c) real part of phase speed, (d) imaginary part.
Figure 4.7: Contours of growth rate, $k c_1$, solid, and non-dimensional wavenumber, dashed, plotted over height of the model lid, and zonal wavenumber, for $H \sim \infty$. Means of $U$ and $N^2$ are held constant for the various zonal wavenumbers and lid heights.
(Peixoto and Oort, 1992), and never find values higher than $1.8 \times 10^{-2}$, while for lid heights much above 10 km, requiring the tropospheric mean value of $N^2$ to remain constant results in unrealistically large values of $N$ at the surface. In either case, values of $N$ become very small near the top of the model. However, these small values of $N$ are difficult to distinguish from one another in observations, since all imply lapse rates near the dry adiabatic lapse rate.

It is interesting that, when $N_0$ is held constant as the lid height of the model is varied, the short wave cutoff remains very close to its classical Eady model value of $\mu = 2.4$, while when $N_0$ is adjusted to keep $\bar{N}$ constant, the cutoff increases as the height of the lid is raised, to about 3.0 when $h = 20$ km and $H = 8$ km. For these high values of the lid height, the static stability, which must fall off nearly exponentially with height in order to balance $\beta$, is very much larger in a thin layer near the surface than in the rest of the troposphere. Apparently, the penetration depth of the disturbances are not well estimated using the layer mean value $\bar{N}$, when the $N$'s value very near the lower surface is much larger than everywhere else in the fluid. For these high lid heights, the critical level is much closer to the surface than to the model's lid, and the amplitude of the disturbance is much larger at the lid than at the surface. For instability, waves traveling along the upper and lower boundaries of the flow must be detectable at the critical level. It may be that the ability of waves traveling along the lower boundary to influence the flow near the critical layer is guaranteed for such a low critical level, so that the influence of the upper wave at the critical level is most important in setting the short-wave cutoff. Then the appropriate $N$ with which to non-dimensionalize $\mu$ would be the square root of the average of $N^2$ over the region from the critical layer to the lid. This would lower $\mu$, which for large lid heights
Figure 4-9: Contours of growth rate, $k_c$, solid, and non-dimensional wavenumber, dashed, plotted over height of the model lid, and zonal wavenumber, for $H = 8$ km. Means of $U$ and $N^2$ are held constant for the various zonal wavenumbers and lid heights.
would tend to move it closer to the familiar $\mu = 2.4$ value.

In contrast to the results of Lindzen (1994a) for the $H \sim \infty$ case, we find that for reasonable tropopause heights (i.e. up to 15 km at 30° latitude), and for $H = 8$ km, the long-wave cutoff present in the modified Eady model is relevant for the longest waves. The model troposphere is neutral with respect to the growth of zonal wavenumber 1 and, if $N$ rather than $N_0$ is held constant as $z$ increases, zonal wavenumber 2.

Another interesting result is that, for $H = 8$ km there are some regions in $h - k$ space in which negative real phase speeds coincide with non-zero growth rates (see Figures 4-9 and 4-11 and 4-13). These negative phase speeds do lie within the constrains of the semi-circle theorem (Pedlosky, 1987), which states, for $U = 0$ at the lower boundary, that for growing modes,

$$ - \frac{\beta L^2}{2U_{MAX}^2 (\pi^2/4 + k^2)} \leq c_r \leq 1. $$

(4.19)

where $c_r$ is the non-dimensional real phase speed, $c_r/U_{MAX}$, $U_{MAX}$ is the maximum basic state wind attained over the domain. For small $k$, and $L$ equal to the circumference of the earth at 30° latitude, this gives a minimum value for $c_r/U_{MAX}$ of 0.09. Typical values of minimum real phase speeds in the parameter space I have investigated are about -0.02. Figure 4-14 shows plots of the eigenfunctions for two values of the zonal wavenumber. Each value of $k$ results in a finite imaginary phase speed, $c_i = 0.02$, but for $\mu = 2.66$, the real phase speed is positive, and for $\mu = 2.10$, the real phase speed is negative. In the latter case the wave amplitude decays monotonically away from the upper lid, while in the former case the amplitude goes through a minimum near the critical level, which lies at about 1.8
Figure 4-11: Contours of growth rate, $k c_i$, solid, and non-dimensional wavenumber, dashed, plotted over height of the model lid, and zonal wavenumber, for $H = 8$ km. Mean of $U$ and the surface value of $N^2$ are held constant for the various zonal wavenumbers and lid heights.
km for this case.

Summary

The height of the tropopause predicted by our modified Eady model, under the assumption that the troposphere should be just neutral to the growth of normal mode instabilities, depends on how static stability is specified. For a mean Brunt-Väisälä frequency of $1.05 \times 10^{-2} \text{ s}^{-1}$, the model troposphere is, for lid heights corresponding well to the tropopause height, neutral to the growth of modal instabilities. However, in order both to assure zero PV gradient and to maintain this mean frequency, Brunt-Väisälä frequencies near the surface need to be unrealistically high. On the other hand, if Brunt-Väisälä frequencies are fixed at observed values at the surface, the model troposphere is unstable, with maximum growth rates exceeding the nominal tropospheric dissipation time, for tropopause heights up to 25 km. In this case though, mean Brunt-Väisälä frequencies are significantly lower than observed mean values. Since layer mean static stability is, to a good approximation, the relevant stability parameter in this problem, this casts doubt on the conclusion that the real troposphere is far from a state neutral to the growth of normal mode instabilities.

To resolve this question we need to generalize our model to include a troposphere in which some meridional gradient of PV remains. Then we could test the stability of an atmosphere more closely resembling the real one in its distribution of static stability and vertical shear.
Figure 4-13: (a) Real part of phase speed for Modified Eady model, with $H = 8$ km, and lid height set to 13.7 km. (b) enlargement of (a) to show negative real phase speeds in region where the imaginary phase speed is not zero.
Figure 4-14: Eigenfunctions of Equation 11 for $\mu = 2.66$, which gives a positive real phase speed, and $\mu = 2.10$, which gives a negative real phase speed.
4.2.2 Models with finite tropospheric PV gradients

Harnik and Lindzen (1997) carried out the generalization mentioned above. They investigated growth rates for a model with no lid, but a large basic state PV gradient at the top of the troposphere, high PV in a model stratosphere, and variable PV gradients, from zero to $\beta$ in the troposphere. They confirmed that the degree of instability of a flow with a finite but small tropospheric PV gradient varies smoothly with changes in that gradient, decreasing as the gradient is decreased. However, they also found that the replacement of the rigid lid by a layer of strong PV gradients tended to destabilize the model troposphere. Indeed, even when PV gradients were reduced to zero in the troposphere, the tropopause height required for stability was approximately 18 km, considerably higher than the observed mid-latitude tropopause. These results tend to confirm the impression gained in the previous section that the real atmosphere differs substantially from an atmosphere which would be stable with respect to the linear growth of baroclinic waves.

However, the atmospheres upon which these stability analyses were performed are artificially confined by their analytically convenient structures. Experiments with a 17 layer, thermally forced, quasi-geostrophic channel model (Solomon, 1997) show that the equilibrated flow exhibits small or zero meridional PV gradient just above the boundary layer, increasing PV gradient towards the tropopause, and large gradients within the boundary layer. Linear stability analysis of the equilibrated flow indicates that it is indeed stable. In this model, neutralization of the flow proceeds as follows. Initially, larger wavenumber disturbances grow most readily, and act to homogenize the PV gradient of the lower troposphere. This modification of the mean flow tends to stabilize the flow with respect to the larger wavenumbers. Smaller wavenumber disturbances continue to grow for some interval, until the flow becomes stable with respect to them as well, though the particular mechanism of stabilization is not yet clear. The details of the equilibrated PV distribution (e.g. the maximum gradients in the boundary layer) are rather dependent on the details of the thermal forcing and the damping scheme.

4.3 Conclusions

The theoretical analysis of Rhines and Young (1982) treats the case where the balance between the diabatic forcing of the PV gradient and the mixing of PV across that gradient is shifted entirely to the side of mixing: equilibrium PV gradients are expected to be small because nothing acts to make them large. The real atmosphere is not so simple. Small PV gradients are indeed observed in the lower free troposphere, where mixing is large and the vertical gradient of radiative forcing (which directly forces PV) is small. But it is not obvious whether the observed increase in observed PV gradients from the 700 mb level to the 500 mb level is a consequence of increased forcing or reduced mixing. Thus we turn to stability analysis to see if the relatively small variations of the the observed PV distribution can be understood as a result of a neutralization process.

Linear stability analysis shows that the homogenization of PV gradients in finite regions of the troposphere does indeed tend to reduce the linear growth rates of small disturbances. Analytical approximations of the observed PV distribution do show significant instability to the linear growth of eddies, even when some damping is assumed. However, results from Solomon's (1997) modeling study suggest that the atmosphere can arrive at an equilibrium climate with a PV distribution which is robust to changes in the thermal forcing, resembles the observed mean flow, and is neutral to linear eddy growth. Thus it seems reasonable
to proceed to ask what the climate implications are of constraining the PV gradients to match observations. In the observations, large changes in meridional temperature gradient (i.e. from winter to summer) are accompanied by fairly small variations in PV gradients. If these varying gradients in and of themselves imply temperature changes much smaller than those they accompany in observations, then the exact nature of the real troposphere's equilibration process may not matter so much.
Chapter 5

The consequences of PV homogenization

In previous chapters we have discussed the observed zonal mean distribution of potential vorticity (PV) along isentropes in the earth's atmosphere. We have noted that one can predict the atmosphere's temperature fairly well by assuming that the meridional gradient of PV is everywhere equal to a constant, whose value varies from about 1.0 to about 1.3 \times 10^{-7} \text{ PVU m}^{-1} over the seasonal cycle. We have discussed the physical basis of this observation, and have suggested that the seasonal variation of the gradient which best predicts the atmosphere's temperature can be understood as a consequence of the varying size of the region over which PV is well mixed.

In this chapter, we will explore the extent to which these observations constrain the earth's climate. We have shown that fixing the PV gradient along isentropes results in an easily solvable differential equation in the pressure gradient of the potential temperature \( \frac{\partial \theta}{\partial p} \). Thus, if \( \frac{\partial \theta}{\partial p} \) is known at some location on every isentrope, it can be predicted everywhere. If the temperature at the surface is specified, \( \frac{\partial \theta}{\partial p} \) can be integrated to yield \( \theta \), and thus temperature, at every pressure and latitude.

This information can, with additional assumptions about the distribution of water vapor and cloud, be used to predict radiative fluxes. The surface temperature can then be adjusted so that the energy budgets of the surface and of the earth as a whole are balanced. Differences between the resulting model climate and the observed climate will point to flaws or omissions in our assumptions; by modifying those assumptions to reduce the differences, we can gain insight into the physical processes responsible for the observed climate.

In the following sections we will discuss an initial set of conditions about how various parts of the climate system function. We will explain the procedure by which those conditions are combined to produce a model climate consistent with all of them. We will compare this model climate with the observed climate, and determine which conditions must be modified to bring the derived climate closer to the observed climate. We will discuss these modifications, and the sensitivities of the derived climate to details of the various conditions, in terms of their implications for the physics of climate.

5.1 Deriving a Climate

Subject to twelve a priori plausible conditions it is possible to calculate a thermodynamically consistent climate which is also consistent with PV homogenization in the troposphere.
<table>
<thead>
<tr>
<th>#</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Potential vorticity in the extratropical troposphere has a gradient along isentropes equal to (-1.0g\beta_{\partial p}^{gg}).</td>
</tr>
<tr>
<td>2</td>
<td>Moist convection in the tropics acts to bring the atmosphere to a state of moist neutrality with respect to surface conditions. Thus temperatures at each latitude in the tropics will lie along moist adiabats originating at the surface at that latitude.</td>
</tr>
<tr>
<td>3</td>
<td>The large Rossby deformation radius, and the overturning Hadley cell force the tropics to have a single surface temperature.</td>
</tr>
<tr>
<td>4</td>
<td>Surface albedo, trace gas, relative humidity and cloud distributions are fixed.</td>
</tr>
<tr>
<td>5</td>
<td>The stratosphere is in radiative equilibrium.</td>
</tr>
<tr>
<td>6</td>
<td>Temperature must be continuous across the tropopause.</td>
</tr>
<tr>
<td>7</td>
<td>The atmosphere is everywhere in thermodynamic equilibrium, so that differential radiative cooling tending to drive the atmosphere away from the prescribed PV distribution in the extratropics, and from moist convective equilibrium in the tropics, is balanced by dynamic heating acting to restore these balances.</td>
</tr>
<tr>
<td>8</td>
<td>Deep convection takes place only in the tropics.</td>
</tr>
<tr>
<td>9</td>
<td>Energy transport across isentropes occurs only by deep convection.</td>
</tr>
<tr>
<td>10</td>
<td>The edge of the tropics is at the closest latitude to the equator such that, in equilibrium, temperatures decrease from the tropics to the extratropics.</td>
</tr>
<tr>
<td>11</td>
<td>Surface sensible heat fluxes are correlated with static stability near the surface.</td>
</tr>
<tr>
<td>12</td>
<td>The earth’s surface can be represented by an ocean mixed layer with a heat capacity corresponding to a given depth. Oceanic circulations transport 2 PW of heat from the tropics to polar regions in each hemisphere.</td>
</tr>
</tbody>
</table>

Table 5.1: Sufficient Conditions to Prescribe a Climate
Table 5.1 lists all twelve together. Conditions 1, 2 and 3 constrain tropospheric temperatures to be in a fixed relationship to surface temperatures and lapse rate. Condition 4 allows a radiative transfer code to be used to calculate surface radiative fluxes, and atmospheric heating rates. Conditions 5 and 6 allow the stratosphere’s temperature to be calculated, and the location of the tropopause to be determined. Conditions 7, 8, 9 and 10 allow calculation of surface turbulent heat fluxes. Condition 11 closes the system, by allowing $\partial \theta/\partial p$ at the extratropical surface to be internally derived. Finally, Condition 12 allows the various surface energy fluxes to be used to adjust the surface temperature until equilibrium is achieved.

The surface temperature is adjusted by integrating the thermodynamic equation for the earth’s surface:

$$\rho DC_t \frac{1}{\Delta A} \frac{\partial T_s(\phi)}{\partial t} = F_{IR}(\phi) + F_{SO}(\phi) + F_{SH}(\phi) + F_{LH} + F_O(\phi)$$  \hspace{1cm} (5.1)

where $\rho$ is the density of liquid water, $D = 50$ m is the depth of the ocean mixed layer taken to constitute the surface of the model, $C_t$ is the heat capacity of liquid water, $T_s$ is the temperature of the mixed layer, $t$ is time, $\phi$ is latitude, $F$ denotes a downward directed energy flux per unit area, the subscripts $IR$, $SO$, $SH$, and $LH$ denote fluxes of infrared radiation, solar radiation, sensible heat and latent heat, respectively, and $F_O$ represents the meridional flux convergence of the poleward oceanic heat flux. The equation is integrated numerically until the sum of the terms on the left hand side of Equation 5.1 falls below 0.1 W m$^{-2}$ at every latitude. At that point, the surface temperature has converged to a unique function of latitude.

We now turn to a detailed discussion of the Conditions, their rationale, and how they are combined to integrate Equation 5.1.

**Condition 1** Potential vorticity in the extratropical troposphere has a gradient along isentropes equal to $-1.2 g \beta \partial \theta/\partial p$.

Condition 1 is motivated by observations, discussed in Chapter 3, that temperatures outside the tropics can be predicted reasonably well by assuming such a PV distribution. Although a strictly constant PV gradient gives a better temperature prediction for most of the troposphere, it gives unrealistic results near the poles. Setting the PV gradient equal to a function of $\beta$ does a better job there. Since the pole-to-equator temperature difference is a salient climate characteristic, we chose the PV distribution which predicts the polar temperatures better. We will use this Condition to predict the temperature of the extratropical troposphere, given an arbitrary surface temperature distribution.

As also discussed in Chapter 3, fixing the meridional gradient of potential vorticity along isentropes yields an equation for $\partial \theta/\partial p$ as a function of its value at some point on a given isentrope (Equation 3.6). This relationship can be integrated over potential temperature to obtain the pressure of each isentrope for all latitudes outside the tropics:

$$p(\phi, \theta) = \int_0^\theta \frac{\partial p}{\partial \theta} \left( \frac{\sin(\phi)}{\sin(\phi_t)} \right)^{(1-\gamma)} d\theta$$  \hspace{1cm} (5.2)

where $\phi_t$ is the latitude at which $\theta$ intersects either the lower boundary or the latitude at the edge of the tropics. Boundary conditions are continuity in potential temperature and its pressure derivative along the vertical boundary of the tropics, and along a fixed surface pressure at the horizontal boundary near the model earth’s surface.
To satisfy the boundary conditions, we need to know $\theta$ and $\frac{\partial \theta}{\partial p}$ along the vertical edge of the tropics ($\phi_t$), and $\theta$ and $\frac{\partial \theta}{\partial p}$ along the model free troposphere's lower boundary outside of the tropics. The first two are known from the condition that $\theta$ lies along moist adiabats in the tropics. The lower boundary of the free troposphere is taken to be at the 900 mb level, in order to assure continuity with the tropical region. In the model tropics, the lifting condensation level occurs at about this height, so that the static stability jumps from very low values below about 900 mb to moderate values above that level. To assure meridional continuity at the edge of the tropics in $\theta$ and $\frac{\partial \theta}{\partial p}$ both above and below that level, two distributions of $\frac{\partial \theta}{\partial p}$ are specified, one within the boundary layer, and one at the top of the boundary layer, from which Equation 5.2 is integrated. The value of $\frac{\partial \theta}{\partial p}$ within the boundary layer just outside the tropics is set equal to its value just inside the tropics, and then varied linearly with latitude until, at the poles, $\frac{\partial \theta}{\partial p}$ within the boundary layer is equal to $\frac{\partial \theta}{\partial p}$ just above the boundary layer. The calculation of $\frac{\partial \theta}{\partial p}$ at the top of the lower boundary is described below, but it too is set to be continuous across the edge of the tropics.

Equation 5.2 allows the calculation of $p$ on a given isentropic surface, given knowledge of the above boundary conditions. This is done for each value of $\theta$ represented at a grid point along the boundaries described above, resulting in a matrix of $p$ values in $\theta$-$\phi$ space (see Figure 5-1). This matrix is then interpolated linearly to find values of $\theta$ in $p$-$\phi$ space. Finally, temperatures on pressure surfaces are calculated from the $\theta$ values.

**Condition 2** Moist convection in the tropics acts to bring the atmosphere to a state of moist neutrality with respect to surface conditions. Thus temperatures at each latitude in the tropics will lie along moist adiabats originating at the surface at that latitude.

As with Condition 1, we use Condition 2 to predict the tropospheric temperature, given the temperature at the surface. Within the tropics (defined according to Condition 10), temperatures aloft are set so that they lie along moist adiabats originating at the surface at the same latitude. Specifically, the temperature at each pressure level is set so that
the pseudo-equivalent potential temperature ($\theta_{ep}$) is equal to $\theta_{ep}(p_{surface})$. The pseudo-equivalent potential temperature is a quantity which is conserved during the ascent of a parcel which conserves sensible heat, but loses all water condensed from vapor during the ascent. It is defined as

$$\theta_{ep} = T \left( \frac{1000}{p} \right)^{0.2854(1-0.28r)} \exp \left[ r(1+0.81r) \left( \frac{3.376}{T^*} - 2.54 \right) \right]$$

(Emanuel, 1994), where $T$ is the temperature (in K), $T^*$ is the temperature at which a surface parcel, lifted adiabatically, reaches saturation with respect to liquid water, $p$ is the pressure (in mb), and $r$ is the water vapor mixing ratio, in this case, that of a parcel lifted adiabatically from the surface.

The temperature profile corresponding to fixed $\theta_{ep}$ is calculated as follows. Water vapor mixing ratio is determined in the model by fixing the relative humidity at observed levels, and then using the model temperature to calculate the saturation mixing ratio, $r^*$ (see Condition 4). $r$ is determined in this manner for the surface parcel. As the calculation of temperature proceeds upwards in the atmosphere, $r$ is held constant until $r^* = r$. Above this level, $r$ is set equal to $r^*$. To find the temperature at a given level for which $\theta_{ep}(p) = \theta_{ep}(p_{surface})$, the temperature at the next lower level is chosen as an initial guess. First $r$ and $\theta_{ep}$ are calculated for that temperature, pressure and $r^*$, then $T$ is increased by 0.01 K. The procedure is repeated until the temperature is found for which $\theta_{ep} = \theta_{ep|surface}$.

**Condition 3** The large Rossby deformation radius, and the overturning Hadley cell force the tropics to have a single surface temperature.

Condition 3 is a slight simplification of observations which show that zonal mean temperature variations from the equator to about 20 degrees are of the order of 1 K (see, for instance, Peixoto and Oort, 1992, p.145), and is consistent with the theory of Schneider (1977) and Held and Hou (1980), for regions near the equator. Accordingly, all surface grid points in the tropics are required to have the same temperature.

**Condition 4** Surface albedo, trace gas, relative humidity and cloud distributions are fixed.

Condition 4 allows us to supply information which cannot be derived from any of our other Conditions to a radiative transfer model (Chou and Suarez, 1994). That model can then be given the temperature information derived from Conditions 1, 2 and 3, and is then able to return information about radiative heating rates and heat fluxes within the atmosphere and at the earth’s surface.

The model of Chou and Suarez (1994) is a band model of thermal infrared radiation in the earth’s atmosphere which uses a mixture of k-distribution and one- and two-parameter scaling in different spectral regions and pressure regimes to get optimal accuracy and speed. The code produces heating rates with an accuracy of 0.2 K/day, as compared to line-by-line calculations. We use this model, along with the solar radiative transfer model of Chou (1986) to calculate net radiative heating rates and fluxes throughout the atmosphere.

The atmosphere is assumed to have a constant surface pressure of 1013.15 mb. The mixing ratio of carbon dioxide is set at 300 ppm (a round number, corresponding to observed levels from the first half of this century), and the ozone mixing ratio is set equal to a zonal mean climatology (Dütsch, 1978). The concentration of water vapor, as a function of height
and latitude, is determined by assuming a fixed distribution of relative humidity (Peixoto and Oort, 1992). Similarly, for the purposes of calculating solar radiation, cloud height, depth and fractional coverage are all supplied from the ISCCP satellite cloud climatology (Drake, 1993). For infrared radiative calculations, a simplified cloud distribution is used. Two cloud layers are set: one extending from 800 mb to 500 mb, and the other from 400 mb to 200 mb, or the tropopause, which ever is lower. Cloud fraction for the lower cloud varies from 0.15 at the equator to 0.35 at the poles, while the fractional coverage of the upper cloud ranges from 0.5 at the equator to 0.4 at the poles. The two cloud are assumed to overlap randomly. These distributions were chosen to arrive at a climate in reasonable agreement with observations, but are themselves in reasonable agreement with the ISCCP climatology. Annual mean solar forcing, and annual mean climatologies of cloud properties, relative humidity, and surface albedo are used. Experiments with seasonally varying insolation, humidity and albedo show little difference in the annual mean climate. For simplicity, data from the northern and southern hemispheres are averaged at opposite latitudes, so that the relative humidity at 50° N is equal to the relative humidity at 50° S.

**Condition 5** The stratosphere is in radiative equilibrium.

Because net radiative heating rates in the stratosphere are generally observed to be smaller than in the troposphere by a factor of five or so (see Peixoto and Oort, 1992, Figure 13.2), designers of simple climate models have traditionally assumed that the stratosphere is in radiative equilibrium when calculating its temperature (e.g. Manabe and Strickler, 1964; Rennó et al., 1994).

At each latitude grid point, an initial guess of the tropopause height is made, and the temperatures at that level and above are adjusted to their radiative equilibrium values in the following way. The temperature at each level is increased or decreased by the rate predicted by the radiative transfer model for one model day. The radiative transfer model is then used to predict new cooling or heating rates, and the process is repeated until the maximum heating or cooling rate in the stratosphere is less than 0.1 K/day. The tropopause height is then adjusted under the following Condition.

**Condition 6** Temperature must be continuous across the tropopause.

Clearly, the potential temperature cannot, in equilibrium, be lower above the tropopause than below, since this arrangement would be statically unstable. A discontinuous increase in potential temperature across the tropopause is conceivable, but observations of the tropopause show that while lapse rates generally make a rather sudden jump at the tropopause, temperatures do not. We choose to impose this observation as Condition 6. This, along with Condition 5 fully determine the stratosphere’s temperatures and the tropopause height.

Recall that the methods used to calculate troposphere temperature have been carried out throughout the vertical domain of the model. Now the tropopause height at each latitude is free to be specified, and troposphere temperatures below that height are known from those completed calculations. The procedure for determining the tropopause height is shown schematically in Figure 5-2 and explained below. The new temperature at the assumed tropopause level is compared with the temperature computed for the troposphere by assuming moist convective neutrality, or a fixed PV gradient. Then the height of the tropopause is varied, and the stratospheric temperatures are once again set to their radiative equilibrium values. This process is repeated until the temperature determined at the
The tropopause height is set so that the tropopause temperature determined by tropospheric rules (by imposing PV homogenization) is equal to the tropopause temperature determined by stratospheric rules (radiative equilibrium).

![Diagram showing tropopause height determination](image)

**Figure 5-2:** Schematic depiction of tropopause height determination.
tropopause by radiative equilibrium is equal to the temperature determined there by the
tropospheric dynamics.

The adjustment of the tropopause height is performed by a bisection routine, which first
brackets the tropopause, and then searches the bracketed interval by successive bisections
of the interval. Since the tropopause height generally does not vary much from one latitude
grid point to the next, this process is generally completed after two or three guesses. At that
point, the stratosphere is in radiative equilibrium, so that the net radiative flux through
the tropopause is equal to the net radiative flux through the top of the atmosphere, and
the temperature is approximately continuous across the tropopause.

Note that the necessity of allowing the tropopause height to be internally set is associated
with Condition 11, which connects the surface static stability to surface turbulent heat
fluxes. It would be mathematically possible to set the tropopause height, and then to derive
the surface static stability required in order to minimize the temperature discontinuity at the
tropopause. We chose to fix the static stability, and let the tropopause height be constrained
by continuity because the stability studies described in Chapter 4 do not support the idea
that the tropopause height is tightly constrained by a requirement of baroclinic neutrality.

Condition 7 The atmosphere is everywhere in thermodynamic equilibrium, so that dif-
ferential radiative cooling tending to drive the atmosphere away from the prescribed PV
distribution in the extratropics, and from moist convective equilibrium in the tropics, is
balanced by dynamic heating acting to restore these balances.

Since interannual temperature variations are much smaller than variations on more rapid
time scales, it is evident that Condition 7 must be true for the annual mean climate. Even
over the seasonal cycle, the time rate of change of the atmosphere's temperature rarely
exceeds 0.2 K d⁻¹ (See Peixoto and Oort, 1992, Figure 13.3), which is much smaller than
typical radiative cooling rates of 1 K d⁻¹. Thus we choose to assume local thermodynamic
equilibrium.

Condition 8 Deep convection takes place only in the tropics.

This Condition is clearly unrealistic. However, we will use it to examine the importance
of moist convection in the extratropics for the global mean climate.

Condition 9 Energy transport across isentropes occurs only by deep convection.

The point of this Condition is that the energy lost to radiation from the portion of the
atmosphere contained between two isentropes must be balanced by energy lost, either as
sensible or latent heat, from the region at the surface bounded by these isentropes. Condition 9
can be justified on the following basis. Viewed in isentropic coordinates, any diabatic
heating or cooling in the atmosphere corresponds to a vertical motion. By requiring a
steady circulation (Condition 7) and conservation of mass, we can derive the rates of turbu-
lient sensible heating of the atmosphere near the surface, given the heating rate everywhere
else. Of course, we do not know the full diabatic heating rate throughout the atmosphere,
because our model cannot predict the rate of latent heat release in the atmosphere. How-
ever, if we assume that all latent heat released on a given isentrope derives from water vapor
evaporated from the region at the surface, where it is intersected by that isentrope, we can
easily calculate the total energy flux from the surface. As we will show, the net energy loss
from the surface region bounded by two adjacent isentropes must, under this condition,
approximately equal the net radiative energy loss from the atmospheric region bounded by those isentropes. The assumption that latent heat release takes place on the same isentrope from which it evaporated is supported by the work of Yang and Pierrehumbert (1994), who found that water vapor concentrations in the extratropics can be modeled well by assuming chaotic mixing of water vapor along isentropes.

This procedure does not account for the heat lost on tropospheric isentropes which never intersect the surface, henceforth referred to as middleworld isentropes, after Hoskins (1991). Radiative cooling on these isentropes must be balanced by warming due to deep convective motions originating in the planetary boundary layer. We assume (Condition 8) that all this deep convection occurs within the model tropics.

Diabatic heating in the atmosphere consists of radiative heating rates, which have already been calculated, heat release or loss associated with the phase changes of water, and sensible heat fluxes occurring at the earth’s surface, or delivered by convecting parcels. Other diabatic effects in the atmosphere, such as frictional heating, are assumed to be negligible. We will call the net diabatic heating rate of the atmosphere \( Q \). The radiative component of \( Q \) is calculated using the radiative transfer model. The component of \( Q \) due to convective motions will be ignored for the moment. This leaves unknown the flux of sensible heat from the earth’s surface to the atmosphere, the cooling due to the latent heat of evaporation in regions where rain falls through subsaturated air (which we will ignore), and the release of latent heat due to condensation in clouds. The latter diabatic heating term will be assumed to be associated with water vapor evaporated at the earth’s surface where it is intersected by the isentrope on which condensation takes place. If we take \( Q = Q_r + Q_{LH} \), where \( Q_r \) is the diabatic heating due to radiation, and \( Q_{LH} \) is the diabatic heating due to latent heat release, then this condition implies that

\[
F_{LH} = \frac{1}{A_{\theta_b}^2} \int_{\phi_a}^{\pi/2} \int_{p_{\theta_a}}^{p_{\theta_b}} \frac{-Q_{LH}}{g} dp \, 2\pi a^2 \cos \phi \, d\phi
\]

(5.3)

where \( A_{\theta_a}^2 \) is the area on the earth’s surface bounded by the isentropes \( \theta = \theta_a \) and \( \theta = \theta_b \), \( \phi \) is latitude in radians, \( \phi_a \) is the latitude where the isentrope \( \theta_a \) intersects the surface, and \( p_{\theta_a} \) and \( p_{\theta_b} \) are the pressures of the two isentropic surfaces at a given latitude \( \phi \).

In isentropic coordinates

\[
\dot{\theta} = Q/\Pi,
\]

(5.4)

where \( \Pi = \left( \frac{p}{\rho g} \right)^{R/C_p} \), and mass conservation requires that in a steady state, the mass-weighted integral of \( \dot{\theta} \) over an isentropic surface vanish. Thus, we can determine \( \dot{\theta} \) near the earth’s surface in a given region by integrating \( \dot{\theta} \) over the isentrope intersecting the surface in that region, and setting the local value of \( \dot{\theta} \) so that the net mass flux through the isentrope is zero.

We can now find the flux of energy from the earth’s surface to the atmosphere near the surface. We first integrate the energy flux necessary to conserve mass flux along the isentropes intersecting the earth’s surface within a given grid box, bounded by the isentropic surfaces \( \theta = \theta_a \) and \( \theta = \theta_b \). We then divide this energy flux by the area of the surface region bounded by the intersection of these isentropes with the earth’s surface. This gives

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the sensible energy flux from the surface to the air:

\[ F_{SH} = \frac{1}{A_{\phi_a}^b} \int_{\phi_a}^{\pi/2} \int_{p_{\phi_a}}^{p_{\theta_a}} \frac{Q}{g} \left( \frac{p_0}{p} \right)^{R/C_p} dp \right] 2 \pi a^2 \cos \phi d\phi. \tag{5.5} \]

The model requires only the sum \( F_{LH} + F_{SH} = F_i \) to permit integration of Equation 5.1, so we add Equations 5.3 and 5.5, to obtain

\[ F_i = \frac{1}{A_{\phi_a}^b} \int_{\phi_a}^{\pi/2} \int_{p_{\phi_a}}^{p_{\theta_a}} \left( \frac{Q_r}{g} + \frac{Q}{g} \left( \frac{p_0}{p} \right)^{R/C_p} - 1 \right) dp \right] 2 \pi a^2 \cos \phi d\phi \tag{5.6} \]

Now, by the conservation of energy, we know that in a steady state, the integral of the radiative cooling of the atmosphere must equal the turbulent flux of energy from the earth's surface. Thus,

\[ \int_0^{\pi/2} F_i 2 \pi a^2 \cos \phi d\phi = \int_0^{\pi/2} \int_0^{p_s} \frac{Q_r}{g} dp 2 \pi a^2 \cos \phi d\phi \tag{5.7} \]

Since the factor \( \left( \frac{p_0}{p} \right)^{R/C_p} \) increases monotonically with height and is always greater than one, and since radiation generally acts to cool the atmosphere, it is clear from Equations 5.5 and 5.6 that more energy must flow from the surface into the region between \( \theta_a \) and \( \theta_b \) than leaves the region by radiation. In the real atmosphere, the extra energy is returned to the surface by frictional heating, thus dissipating the kinetic energy generated from potential energy by the direct overturning circulation. We parameterize this heating by letting

\[ F_i = \frac{1}{A_{\phi_a}^b} \int_{\phi_a}^{\pi/2} \int_{p_{\phi_a}}^{p_{\theta_a}} \left( \frac{Q_r}{g} \right) dp \right] 2 \pi a^2 \cos \phi d\phi \tag{5.8} \]

which precisely expresses Condition 9 in the absence of convection. The difference between \( F_i \) calculated by Equation 5.6 and by Equation 5.8 is typically on the order of 1%. If we integrate Equation 5.8 over all values of \( \theta \) (i.e. set \( \theta_a = 0 \), and \( \theta_b = \infty \)), we recover Equation 5.7.

Now, for the middleworld isentropes \( A_{\phi_a}^{\infty} \) is undefined; so since deep convection is most active in the tropics, we add \( F_c = \left( A_{\phi_a}^{\infty}/A_{\phi_a}^b \right) F_i \) calculated for these isentropes to the values of \( F_i \) already calculated for the tropical latitudes. \( F_c \) is shown as a dashed line Figure 5-7. Note that the radiative cooling on the middleworld isentropes makes up about 85% of the total atmospheric radiative cooling, and that this percentage is insensitive to the details of clouds or humidity specification.

**Calculating the surface latent heat flux**

The surface turbulent heat flux can be separated into latent and sensible components, if we assume that both these fluxes are modeled well by their respective aerodynamic flux laws:

\[ F_{SH} = C_S C_p v (T_s - T_a) \tag{5.9} \]

and

\[ F_{LH} = C_L L v (q_s - q_a) \tag{5.10} \]
where $T_s$ is the surface temperature, $T_a$ is the air temperature at some small height above the surface, $q_s$ is the saturation specific humidity of the surface (which we assume to be wet), $q$ is the specific humidity at some small height above the ground, $v$ is the wind speed just above the ground, $C_p$ is the heat capacity of air, $L$ is the latent heat of evaporation of water, and $C_L$ and $C_S$ are empirically determined constants. If we assume that $q$ corresponds to some fixed relative humidity, that $C_L = C_S$, that $T - T_a$ is reasonably small, and note that $F_{SH} + F_{LH} = F_i + F_c$, then we can write

$$F_{LH}/F_{SH} = \frac{L(T_s)}{C_p} q^*(T_s) \frac{1 - H}{T_s - T_a}$$  \hspace{1cm} (5.11)$$

and

$$F_{SH} = (F_c + F_i)/\left(1 + \frac{L(T_s)}{C_p} q^*(T_s) \frac{1 - H}{T_s - T_a}\right),$$  \hspace{1cm} (5.12)$$

where $q^*$ is the saturation specific humidity and $H$ is the relative humidity of air near the earth’s surface. The ratio $F_{SH}/F_{LH}$ is termed the Bowen ratio, and the above analysis is due to Penman (1948). The Bowen ratio decreases approximately exponentially with temperature, from about 1.5 at 0 C to about 0.2 at 35 C. Thus we can retrieve a net sensible heat flux, $F_{SH}$, after all the turbulent heat fluxes due to both moist convection and mixing along isentropes have been calculated.

**Condition 10** *The edge of the tropics is at the closest latitude to the equator such that, in equilibrium, temperatures decrease from the tropics to the extratropics.*

In observations, zonal mean temperatures have only a single maximum, and decrease monotonically from there to the poles. Since Condition 2 implies that convection is always taking place in the tropics, it is possible that net cooling of the surface in the tropics, due to radiative and turbulent energy fluxes, can exceed the net cooling just outside the tropics. Thus, even though the downward solar radiative flux is greater in the tropics than just outside the tropics, the equilibrium temperature might be higher just outside the tropics than inside. Condition 10 avoids such a situation by requiring the location of the edge of the tropics to be adjusted so that temperatures fall monotonically from the tropics to each pole.

**Condition 11** *Surface sensible heat fluxes are correlated with static stability near the surface.*

Condition 11 is based on the observation that surface turbulent heat fluxes in the real atmosphere have a strong dependence on the static stability of the boundary layer (see for instance, Garratt, 1992). Where there are strong turbulent heat fluxes from the surface to the atmosphere, the static stability tends to be nearly zero, and when the static stability is very high, turbulent heat fluxes are either small or directed downwards. Specifically, the polar boundary layer, where surface turbulent fluxes tend to carry heat from the atmosphere to the surface, is generally very stable, often exhibiting a strong temperature inversion (Warren, 1996), while the tropical marine boundary layer is typically close to dry adiabatic up to the lifting condensation level (Sarachick, 1985, Betts and Ridgway, 1989, many others).

Noting these observations, we assume that in the zonal mean, the sensible heat flux should be proportional to the geostrophic winds above the boundary layer, multiplied by a function of stability which is zero when the lapse rate of temperature is zero, and equal to one.
when $\frac{∂θ}{∂p}$ is equal to its value at the edge of the tropics. The constant of proportionality is set so that the sensible heat fluxes are continuous at the edge of the tropics. To implement this scheme, an initial guess of the distribution of $\frac{∂θ}{∂p}$ is made and all the above-described steps are taken to calculate $F_{SH}$. Then $\frac{∂θ}{∂p}$ is recalculated so that it agrees with these fluxes. This process is repeated until $F_{SH}$ no longer changes with each iteration. Convergence generally proceeds rapidly, in two to four steps.

The equation relating $\frac{∂θ}{∂p}$ to $F_{SH}$ is:

$$F_{SH}(\phi) = \frac{U_s(\phi)}{U_s(\phi_t)} \left[ \frac{\exp \left( \frac{R(\phi_t) - R(\phi)}{10.0 \frac{∂θ}{∂p}} \right)}{1 - \exp \left( \frac{R(\phi_t) + \frac{∂θ}{∂p}}{10.0 \frac{∂θ}{∂p}} \right)} \right]$$

(5.13)

which can also be expressed in terms of $\frac{∂θ}{∂p}(\phi)$:

$$\frac{∂θ}{∂p}(\phi) = \frac{∂θ}{∂p}(\phi_t) \times \left\{ 1 - 10.0 \log \left[ \frac{U_s(\phi_t)F_{SH}(\phi)}{U_s(\phi_t)F_{SH}(\phi_t)} (1 - X) + X \right] \right\}$$

(5.14)

where $X = \exp \left( -\frac{R(\phi_t)}{10.0 \frac{∂θ}{∂p}(\phi_t)} \right)$. $U_s$ is the geostrophic wind at the lowest model layer, assumed to be proportional to the mean surface wind speed, $R$ is the atmosphere’s gas constant, $C_p$ is the heat capacity of dry air, $φ_t$ is the latitude at the edge of the tropics, and $-\frac{R(\phi_t)}{10.0 \frac{∂θ}{∂p}(\phi_t)}$ is equal to $\frac{∂θ}{∂p}$ when $\frac{∂T}{∂p} = 0$. These equations appear cumbersome but express two simple ideas. First, that as static stability increases, upward sensible heat fluxes should decrease and eventually become negative. Second, that the rate of decrease of the heat fluxes should itself decrease as static stability increases, reflecting the suppression of buoyant heat fluxes by stable conditions.

**Condition 12** The earth’s surface can be represented by an ocean mixed layer with a heat capacity corresponding to a given depth. Oceanic circulations transport 2 PW of heat from the tropics to polar regions in each hemisphere.

Since we generally use yearly mean solar forcing, the mixed-layer depth is significant only in its relationship to the time step: the combination of $D = 50$ m and $Δt = 1$ month generally gives a smooth integration. A shallower mixed layer or a longer time step generally result in temperatures which oscillate about their equilibrium values. For seasonally varying forcing, the depth of the mixed layer will have a strong impact on the seasonal climate variations.

An ocean heat flux of 2 PW is consistent with recent estimates of ocean heat transport (Trenberth and Solomon, 1994). The accompanying flux convergence is expressed as $F_O$ in Equation 5.1, and has the prescribed distribution:

$$F_O = \begin{cases} 
T_0 \omega (-\cos(3.857φπ/180) - 0.102)/1.1561 & \text{for } |φ| \leq 70° \text{ latitude} \\
0 & \text{for } |φ| > 70° \text{ latitude} 
\end{cases}$$

(5.15)

where $T_0$ is the maximum poleward heat flux (which in this scheme is forced to occur at 23° latitude and according to Condition 12 equals 2 PW), $ω$ is a geometric factor equal to $\frac{3}{8} \frac{1}{a^2} = 1.5286 \times 10^{-14} \text{m}^{-2}$, $a$ is the earth’s radius, and $φ$ is latitude. The cut-off at 70° latitude accounts for the Antarctic continent, which prevents any oceanic heat flux south of that latitude in the southern hemisphere, and for the negligible annual mean heat flux.
observed north of that latitude in the northern hemisphere (Aagaard and Greisman, 1975). The local area weighted mean of $F_0$ is zero, of course.

5.1.1 Procedure for Obtaining an Equilibrium Climate

The procedure for integrating Equation 5.1 is as follows. Following Condition 12, we represent the earth's surface by an ocean mixed layer of fixed heat capacity, discretized into an array with 3 degree latitude resolution (60 points, from -88.5° to 88.5°). It is initialized with an arbitrary initial temperature distribution, and the lapse rate $\frac{\partial T}{\partial \phi}$ is given an arbitrary initial distribution outside the tropics. The locations of the boundaries between the tropics and extratropics in each hemisphere are also chosen arbitrarily.

Next, the extratropical lapse rate at the top of the boundary layer must be adjusted until it is everywhere in agreement with the sensible heat flux, as required by Condition 11 and expressed by Equation 5.14. This is done by calculating all the fluxes based on the initial surface temperature and lapse rate distribution, then setting the lapse rate to the values predicted by Equation 5.14 and recalculating the fluxes. This process is repeated until the new lapse rate values are in good agreement with the old.

The fluxes $F_{IR}$, $F_{SO}$, $F_{SH}$, and $F_{LH}$ are derived from the surface temperature distribution as follows. The atmosphere is discretized into a pressure-latitude grid, with 3 degree latitude resolution (60 points, from -88.5° to 88.5°), and 15 mb pressure resolution (68 points, from 8 mb to 1013 mb). Temperatures in the tropical troposphere are derived according to Condition 2. This provides the boundary conditions for the calculation of the extratropical troposphere's temperature according to Condition 1. The water vapor mixing ratio is then calculated using the prescribed relative humidities provided by Condition 4. Using the radiative transfer code, we calculate the stratosphere's radiative equilibrium temperature and the height of the tropopause, according to Conditions 5 and 6. As noted above in the discussion of Condition 5, if we had good reason to fix the height of the tropopause, it would be possible to do this, and adjust the surface static stability to minimize the temperature discontinuity at the tropopause.

Once the extent and temperature of the stratosphere are known, the temperature and radiative flux and heating rate are known at each point in the atmosphere. The radiative fluxes predicted at the surface can now be applied directly to the surface mixed layer via Equation 5.1. Next the surface turbulent heat fluxes must be calculated. The integral in Equation 5.8 is approximated in the model by summing $\frac{\partial T}{\partial \phi} \delta p a^2 \cos \phi \delta \phi$ over all those grid points whose potential temperature is closest to, but not greater than the potential temperature of each point along the surface. To assure stable integration, the resulting fluxes need to be smoothed somewhat. This is accomplished by averaging the summation over several hundred evaluations, in each of which the surface temperature at each latitude is varied by an amount $r \frac{\partial T_s}{\partial \phi}$, where $r$ is a random number with zero mean and a range of $[-1, 1]$, and $T_s$ is the surface temperature. $F_s$ is calculated in this manner and $F_c$, the convective flux which balances the radiative loss of energy from middleworld isentropes, is allocated evenly over the tropics. Equations 5.11 and 5.12 are used to split up the net turbulent heat flux into sensible and latent components, $F_{SH}$ and $F_{LH}$.

Once the boundary layer lapse rate has converged, the radiative, turbulent, and oceanic heat fluxes can be applied to Equation 5.1, and the surface temperature adjusted. In the tropics, the temperature is adjusted to its mean value, according to Condition 3. Then the new surface temperature distribution is used to generate a new set of fluxes. This process is repeated until equilibrium is achieved. Finally, the locations of the edges of
Figure 5-3: Equilibrium surface downward energy fluxes. Solar fluxes, $F_{SO}$, solid; infrared fluxes, $F_{IR}$, dashed; Fluxes along isentropes, $F_i$, dotted; Convective fluxes, $F_c$, dot-dashed.

the tropics are varied, and the whole process repeated until the equilibrium temperature declines monotonically from the tropics to the poles (Condition 10).

5.2 Results and modifications

In equilibrium, the earth’s surface is warmed by solar radiation and by infrared radiation emitted by the atmosphere. It is cooled by outgoing infrared radiation, and by latent and sensible heat fluxes to the atmosphere, which balance the net loss of heat from the atmosphere to space. The net infrared flux is always upwards, since the surface is assumed to be a perfect emitter and is generally warmer than the air above it. Convergence of oceanic poleward heat fluxes warms the regions poleward of 35.27° latitude in each hemisphere, and cools the region between those latitudes. Figures 5-3 show equilibrium values of these fluxes subject to the twelve Conditions described above. Note that at each latitude (except for the poles), the sum of the solar (solid) and infrared (dashed) fluxes is positive, so that the net radiative forcing acts to warm the surface, while turbulent fluxes act to cool the surface. At the poles, the solar and infrared fluxes exactly balance, and turbulent fluxes are zero.

The equilibrium surface temperature for this climate, along with the observed annual mean 1000 mb temperature are shown in Figure 5-4, while contours of potential temperature along with the location of the tropopause are plotted against latitude and pressure in Figure 5-5. Observations of potential temperature are shown in Figure 5-6.

The fluxes $F_i$ are shown as a solid line in Figure 5-7. Convective fluxes, $F_c$ are shown as a dashed line. The fluxes go to zero at the poles because $\phi_a$ in Equation 5.8 is nearly equal to $\pi/2$, so that the integral over latitude goes to zero. The fluxes go to zero in the tropics  

---

1The net flux is also upward in the polar region, where an inversion generally exists. The atmosphere is transparent enough that the warmer, but less emissive atmosphere emits less radiation downwards than the surface emits upwards.
Figure 5-4: Solid: Equilibrium surface temperature for initial twelve Conditions. Dashed: Observed annual mean 1000 mb temperature, from NCAR/NCEP reanalysis.

Figure 5-5: Model equilibrium temperature as a function of $p$ and $\phi$. Contour interval is 10 K. The solid line is the tropopause, as determined using the WMO definition: the lowest level at which the lapse rate decreases to 2 K km$^{-1}$, provided that the average lapse rate between this level and all higher levels within 2 km does not exceed 2 K km$^{-1}$. 
Figure 5-6: Observed zonal mean April temperature as a function of $p$ and $\phi$. The bold line is the tropopause, as determined using the WMO definition. NMC reanalysis, courtesy of P. Newman.

Figure 5-7: Surface turbulent fluxes, $F_1$. Solid: Fluxes balancing radiative cooling on those isentropes which intersect the surface. Dashed: Convective fluxes, $F_c$, necessary to balance cooling on middleworld isentropes. By Condition 8 they occur in the tropics only.
Figure 5-8: Atmospheric meridional energy flux, \( T_A \) (dashed), oceanic meridional energy flux, \( T_O \) (dotted), and their sum (solid).

because surface temperature gradients are small, so that for latitudes in the tropics, \( \phi_a \) and \( \phi_b \), \( \theta_a \) and \( \theta_b \), will differ by a small value, so that the volume of air contained between those isentropes will also be small.

Atmospheric fluxes of energy through a given latitude can be calculated by integrating the net downward radiative energy flux at the top of the atmosphere from the south pole to that latitude, as in Oort and Vander Haar (1976). They wrote:

\[
T_A = - \int_{-\pi/2}^\phi (F_{TA} - F_{OA}) 2\pi a^2 \cos \phi d\phi, \tag{5.16}
\]

where \( T_A \) is the meridional energy flux, \( F_{TA} \) is the net upward radiative flux at the top of the atmosphere, \( F_{OA} \) is the net upward flux from the ocean to the atmosphere and \( a \) is the earth’s radius. Equilibrium meridional energy fluxes are shown in Figure 5-8. The maximum flux is 6 PW, which is in good agreement with observed values (Trenberth and Solomon, 1994).

5.2.1 Comparison of Predicted Climate with Observations

The temperature distribution predicted under the cited Conditions bears some resemblance to the observed distribution. Tropical temperatures are predicted to be 302 K, while observations show mean temperatures of about 300 K. Tropospheric temperatures are in approximately the correct relationship to surface temperatures.

However, three features of the predicted equilibrium climate are clearly unrealistic. First, the tropics extend up to 59° latitude, so that temperatures in the midlatitudes are 15 K to 30 K too warm. Second, temperatures at the Poles are about 70 K too cold. Finally, we will show that when the first two problems are corrected, the predicted tropopause height is essentially constant from the equator to the poles at about 200 mb, while in observations, it falls from about 100 mb at the equator to about 350 mb at the poles, with an abrupt
Figure 5-9: Temperatures after 5 time steps of model integration, with no convection scheme.

change at the edge of the tropics.

To understand the cause of the excessive extent of the tropics, we temporarily relax Condition 10. Instead, we fix the edge of the tropics at 21° latitude in each hemisphere, and attempt to integrate Equation 5.1. The surface temperature distribution after some thirty time steps (where $\Delta t = 1$ month, and $D = 50m$) is shown in Figure 5-9. Note that the tropics are now considerably cooler than the extratropical regions immediately adjacent to the tropics. A plot of net surface heating ($F_{SO} + F_{IR} + F_i$) versus surface temperature (Figure 5-10) shows that the integration is unstable, and the temperatures just outside the tropics are not approaching any equilibrium.

This instability arises because the regions outside but adjacent to the tropics have no strong negative feedback on their temperature. As the surface temperature rises, temperatures in the boundary layer and, to a lesser degree, the free troposphere increase. Since relative humidity is fixed, water vapor mixing ratios increase as well. This leads to increased downwelling radiation which more than balances the increase in upwelling radiation due to the surface’s warmer temperature. The atmosphere’s increased radiative cooling is balanced by fluxes of heat from the tropics, because most of the atmosphere above points adjacent to the tropics lies in middleworld. Thus surface temperatures fall in the tropics and rise without limit in the nearby extratropics. Clearly, some strong negative feedback has been neglected, which removes this instability in the real atmosphere. An obvious candidate for such a feedback is moist convection. We investigate this possibility below.

The flatness of the tropopause could in principle be due either to stratospheric or tropospheric causes. In fact, changes to our original Conditions involving both the stratosphere and the troposphere are necessary to produce a realistically shaped tropopause. It will be shown that the crucial physics missing from the model are the Brewer-Dobson circulation of the stratosphere and the reduced lapse rate of the tropical upper troposphere.

The polar temperature problem has to do with the way Condition 1 binds temperatures aloft very tightly to temperatures at the surface. As this problem provides a convenient
Figure 5-10: $F_{IR}$ (solid), and $F_{SO} + F_{IR} + F_i$ (dashed) plotted versus surface temperature at 25° latitude. As temperature increases, downwelling radiation, and net downward energy flux both increase, so that there is no stable equilibrium temperature.

way to examine several aspects of the climate dynamics inherent in our Conditions, we will address it first.

5.2.2 Decoupling the Polar Troposphere from the Polar Surface

Figure 5-3 shows that the annual mean incoming solar radiation at the poles is only 26.5 W m$^{-2}$. A blackbody in equilibrium with this flux would only reach a temperature of $T = (26.5 \text{ W m}^{-2}/\sigma)^{1/4} = 147 \text{ K}$ (where $\sigma$ is the Stefan-Boltzmann constant). The monthly mean temperature at the earth's poles rarely falls below about 230 K, which implies a loss of at least 160 W m$^{-2}$ in infrared radiation emitted by the surface. Since static stability is generally high in polar regions, making large sensible heat fluxes difficult to attain, downwelling infrared radiation from the atmosphere becomes the crucial factor determining surface temperatures in polar regions.

Downwelling infrared radiation is a function of the temperature and radiative properties of the atmosphere near the surface. Because temperatures are so low in polar regions, the atmosphere contains very little water vapor. In order to emit nearly as much radiation downwards as the surface emits upwards, the polar atmosphere must either have a very high emissivity in the infrared, and be only slightly cooler than the surface or, if the emissivity is substantially less than one, exhibit a strong temperature inversion at the surface. Since observed polar cloud fractions preclude atmospheric emissivities near unity, an inversion turns out to be necessary to support polar temperatures as warm as observations.

When temperatures aloft are tightly coupled to temperatures at the surface, via Conditions 1 and 11, the polar equilibrium temperature tends to get very cold, because as the surface temperature decreases, temperatures aloft decrease too (see Figure 5-11), reducing the change in the surface radiative budget expected from the blackbody equation. We note that in the real atmosphere, temperatures fall much less rapidly at 850 mb than at the surface as the year proceeds from summer to winter, and hypothesize that temperatures above the boundary layer in the polar regions are not tightly bound with surface temperatures, but are better correlated with temperatures in a region away from the poles, where the static stability is lower, and eddies originating at the surface can communicate with them.
Figure 5-11: Predicted temperature difference between surface and 850 mb level plotted versus surface temperature at the North Pole, as adjustment proceeds. The initial state is warm, the equilibrium state cold (solid). Observations of temperature difference between 1000 mb and 850 mb levels at the North Pole, from the reanalysis data (dashed).

To examine the implications of this assumption we modify Condition 11 so that the lapse rate in the polar 25° of latitude is set to make the potential temperature at the top of the polar boundary layer equal to the potential temperature at the surface at 65° latitude, so that

\[ \frac{\partial \theta}{\partial p}(\phi) = \frac{\theta(64.5°) - \theta(\phi)}{p_s - 900 mb} \frac{24°}{\phi - 64.5°}. \]

Thus, this new constraint on the polar surface lapse rate is essentially an arbitrary constraint on the slopes of isentropes in the boundary layer of that region. It allows the surface to cool radiatively, without requiring the free troposphere to cool along with the surface. Although it takes the form of an active constraint, it is intended to take account of the lack of a strong dynamical connection between the polar surface and the polar free troposphere, separated as they are by an inversion layer and a relatively transparent atmosphere. Thus cloud and cloud height become very important. The equilibrium surface temperature profile implied by this new version of Condition 11 is shown in Figure 5-12, along with the observed annual mean 1000 mb temperature from the reanalysis data. We maintain this modification in the studies described below.

5.2.3 Introducing a Convection Parameterization

We saw in a previous section that the climate resulting from our original Conditions was too warm in the subtropics, and had a tropical region which extended much farther poleward than is observed in the real atmosphere. Experimentation suggests that Condition 8 is largely responsible this problem. Instead of restricting deep convection to the tropics, we now introduce a simple convection parameterization as follows.

Moist convection occurs when conditional instability exists and sufficient dynamic forcing exists so that parcels of air in the boundary layer can be lifted to their level of free convection, at which level their density is lower than that of the surrounding air. It persists when sufficient convergence of heat and moisture exist to maintain conditional instability against the convective redistribution of heat. We will assume that sufficient heat and
Figure 5-12: Equilibrium surface temperature for modified polar static stability (see text, solid); Observed annual mean surface temperature, from NCEP/NCAR reanalysis data (dashed).

moisture convergence exist, and determine the probability of convection on the basis of the degree of conditional stability alone. We use the degree of conditional stability because we expect that, although the zonal and monthly mean vertical temperature profile for a given latitude may be conditionally stable, there will nevertheless be particular locations and times at that latitude where vertical profiles are conditionally unstable, so that the zonal mean convective energy flux will not be zero. These episodes of conditional instability will occur when fluctuations in surface temperature and temperature aloft lead to warmer than average surface temperature and cooler than average temperatures aloft. Thus, the probability that moist convection will occur should depend on the variance of the local surface temperature as well as on the degree of conditional stability of the mean state.

The convection parameterization works as follows. The proportion of radiative cooling balanced by convection, \( \alpha \), is determined for each grid point in the atmosphere, using the following formula:

\[
\alpha = \begin{cases} 
1 & \text{for } \theta_m \geq \theta \\
\exp \left( -\frac{(\theta_m - \theta)^2}{c_1 \left| \frac{\partial \theta_m}{\partial y} \right|} \right) & \text{for } \theta_m < \theta
\end{cases} 
\] (5.17)

where \( \theta \) is the potential temperature at a given latitude and pressure, \( \theta_m \) is the potential temperature at that latitude and pressure of a moist adiabat originating at the surface at the same latitude, and \( c_1 \) is a tunable parameter which relates local temperature variance to the local temperature gradient. \( c_1 \) has units of meters, and so represents a radius over which the mean temperature gradient is mixed to produce local temperature variance. Thus, the product \( c_1 \left| \frac{\partial \theta_m}{\partial y} \right| \) represents the variability of the temperature at a given latitude. If \( c_1 = 0 \text{m} \), local temperature variance is zero and convection only occurs when conditional instability exists (i.e. when \( \theta \leq \theta_m \)). If \( c_1 > 0 \text{m} \), sufficient temperature variance exists so that convection can sometimes take place, even when \( \theta > \theta_m \). The net downward convective energy flux
Figure 5-13: Equilibrium Temperature plotted against latitude for three values of the parameter $c_1$: $c_1 = 0$ m (solid); $c_1 = 3200$ km (dashed); $c_1 = 4800$ km (dotted).

Through the earth’s surface $F_c$ can then be determined at each latitude:

$$F_c = \int_p^0 \frac{\alpha Q_r}{g} dp$$  \hspace{1cm} (5.18)

Next, $\alpha$ is scaled so that the integral of $F_c$ over the earth’s surface is equal to the integral of $Q_r$ in middleworld, plus the integral of $\alpha Q_r$ in underworld. We then substitute $(1 - \alpha)Q_r$ for $Q_r$ in Equation 5.8, so that the fraction of radiative cooling not accounted for by convection is left to be balanced by surface fluxes along the isentrope on which the radiative cooling occurs. Finally, we modify Condition 10 so that the edge of the tropics is fixed at $20^\circ$ latitude (equilibrium climate away from the poles is not sensitive to the location of the edge, within the range $17^\circ - 30^\circ$). We can now integrate Equation 5.1 and find the climate consistent with our present conditions.

Figure 5-13 shows equilibrium temperature structure for three values of $c_1$. When $c_1 = 0$ m, we see that the equilibrium temperature is essentially flat for all latitudes equatorward of $50^\circ$ latitude. This is because, for this value of $c_1$, convection is an all-or-nothing phenomenon. The lapse rate outside the model tropics is tied to the lapse rate in the tropics by PV homogenization: when $\gamma = 1$, $\frac{\partial \theta}{\partial p}$ must be constant along $\theta$ surfaces. If temperatures outside the tropics are cooler than temperatures within the tropics, then if the temperature profile in the tropics is of marginal conditional stability, the cooler extratropical temperature profile will be conditionally stable, though it has the same lapse rate (Emanuel, 1994). Because the temperature of these regions would, in the absence of convection, tend to rise indefinitely (see Figure 5-9), they warm until they reach a surface temperature which makes the vertical temperature profile marginally conditionally unstable, convection switches on, and the temperature equilibrates.

Figures 5-13 show that as $c_1$ increases, the region of flat temperatures contracts. We choose to fix $c_1$ at a value just large enough so that temperatures fall at the edge of the tropics. Figure 5-13 shows that when $c_1$ is made half again as large as that critical value, the
resulting equilibrium climate is only slightly different. This insensitivity to $c_1$ is due to the high static stability and cold temperatures of polar regions, which prevent moist convection there for a wide range of $c_1$ values. Thus, the fluxes $F_c$ must always have a distribution which ranges from zero at the poles to large values at the equator, so that only the details of the equilibrium temperature structure, and not the pole-to-equator temperature difference, change as $c_1$ is varied. Figure 5-14 shows the ratio of $F_c$ to $F_c + F_i$, and demonstrates that moist convection dominates surface fluxes everywhere, but is relatively small in mid-latitudes. The total surface flux near the poles is very small, leading to the jumpy ratio there. The dominance of convective surface fluxes over surface fluxes balancing isentropic energy budgets does not indicate that eddy motions on isentropes are insignificant carriers of energy fluxes. Rather the meridional energy fluxes due to synoptic eddies are implicit in the constraint of PV homogenization.

5.2.4 Stratospheric Energy Transport

In the global mean, the stratosphere\footnote{We will ignore the atmosphere above the stratosphere, since its influence on the climate of the troposphere and of the surface is negligible.} is warmed by the absorption of solar radiation by ozone, and the absorption of terrestrial and tropospheric infrared radiation by carbon dioxide, ozone and other trace gases, and is cooled by infrared radiation emitted by the same group of gases. Indeed, for the portion of the stratosphere with $\theta > 380$ K (the level of the tropical tropopause), these are the only sources and sinks of energy, so that in the global mean, that portion of the stratosphere must be in radiative equilibrium (Olaguer et al. 1992, Holton et al., 1995). However, it has long been known (Brewer 1949, Dobson, 1956), that the observed distribution of trace gases in the stratosphere requires there to exist some overturning circulation, such that material is transported upwards in the tropics, polewards in the midlatitudes, and downwards in the polar regions. Equation 5.4 shows that such a circulation implies dynamic cooling of the tropical stratosphere, and warming of the polar stratosphere. Since this circulation is thermally indirect (the rising tropical air is colder than the sinking polar air), it must be driven by some mechanical forcing. Haynes et al. (1991) show that the circulation can best be explained by the action of a westward body
Figure 5-15: Model equilibrium temperature, as a function of $p$ and $\phi$, using convection scheme and revised polar surface lapse rate scheme described in text. Contour interval is 10 K. The solid line is the tropopause, as determined using the WMO definition: the lowest level at which the lapse rate decreases to 2 K km$^{-1}$, provided that the average lapse rate between this level and all higher levels within 2 km does not exceed 2 K km$^{-1}$.

force on the middle levels of the stratosphere. The force is presumably due to interaction of Rossby and gravity waves, propagating upwards from troposphere, with the stratosphere’s mean flow. However, we must note that although Rossby wave fluxes into the stratosphere are sharply restricted during the summer by mean easterly winds, temperatures in the summertime polar regions are significantly warmer than radiative equilibrium (Olaguer et al. 1992). Modeling studies by Hamilton et al. (1995) using GFDL’s SKYHI GCM predict little radiative disequilibrium in the polar summer stratosphere and upper troposphere, and so model temperatures there are up to 10 K too warm. Thus, a complete understanding of the mechanisms forcing the stratosphere’s overturning circulation does not yet exist.

While this general picture of the stratospheric circulation is well accepted, the magnitude of the overturning circulation, and of the radiative heating associated with it, is only poorly known. Olaguer et al. (1992) point out that trace gas concentrations in two-dimensional models of the stratosphere are sensitive to changes in radiative cooling rates on the order of 0.2 K d$^{-1}$, and that this is smaller than the error in heating rate of available radiative transfer models. We will show that the height of the tropopause is similarly sensitive to imposed dynamic heating and cooling of this magnitude.

The tropopause height predicted by our model has very little latitude dependence. The zonal mean temperature structure of the model, now including a convection scheme and the revised polar surface lapse rate scheme, with the tropopause superimposed, is shown in Figure 5-15, while in Figure 5-6, we see the observed zonal mean temperature distribution. The differences are striking. The model tropopause is 10 K too warm in the tropics, and about 100 mb to low, while in the polar regions it is some 10 K too cold, and 150 mb too high. We find that the tropopause height is fairly insensitive to variations in the tropospheric lapse rate, as did Held (1982), but that these differences can be corrected by relaxing Condition 5,
that the stratosphere is in radiative equilibrium. Thus, if we modify Condition 5 to include the effects of the observed overturning circulation of the stratosphere, by imposing dynamic cooling in the tropical stratosphere and warming in the polar stratosphere, and following Olaguer et al., (1992), require that this heating and cooling be balanced, so that it averages to zero on pressure surfaces over the globe, we obtain the imposed dynamic heating and cooling profile is shown in Figure 5-16.

When this is done, we find that the cooling necessary to raise the tropical tropopause to observed heights is -0.3 K d$^{-1}$, and the warming needed to lower the polar tropopause to observed heights is 0.3 K d$^{-1}$, which are in reasonable agreement with observations (c.f Olaguer et al., 1992, Figure 2), which show radiative warming in the tropics, and cooling in polar regions, balancing these motions. However, although the height of the tropopause is now closer to observed values at the equator and at the poles, Figure 5-17a shows that the decline in tropopause height from the tropics to the polar regions is now predicted to be much more gradual than in the observations. We find that this can be corrected by relaxing Condition 2, that tropical temperatures assume a moist adiabatic profile. In the real tropical troposphere, the lapse rate gets smaller as one approaches the tropopause (see Chapter 2 of this thesis, and Reid and Gage, 1996). If we impose a lapse rate of -2 K km$^{-1}$ above 200 mb, we find that the tropopause height falls off much more rapidly at the edge of the tropics, as shown in Figure 5-17b. This suggests that the observed downward slope in the height of the tropopause from the tropics to the poles, is due first of all to the stratosphere's indirect overturning circulation, and is sharpened at the edge of the tropics, relative to what we would expect for a moist adiabatic troposphere, by the reduced temperature gradient of the tropical upper troposphere.

The calculations involved in deriving a radiative equilibrium profile for the stratosphere at each latitude are time consuming. To allow more rapid evaluation of the climate implications of varying assumptions, we modify Condition 5 in the following way. Rather than requiring the stratosphere to be in radiative equilibrium at each point, its temperature is
Figure 5-17: Model equilibrium temperature contours plotted against $p$ and $\phi$. The solid line is the tropopause, as determined using the WMO definition. (a) Equilibrium climate including simulated stratospheric overturning. (b) Equilibrium climate including simulated stratospheric overturning, and reduced lapse rate in upper tropical troposphere.
Figure 5-18: Model equilibrium surface temperature difference between stratosphere in local radiative equilibrium, and stratosphere in over-all radiative equilibrium, with fixed lapse rate.

assumed to increase linearly with pressure from the tropopause upwards at a rate of .2 K mb⁻¹. The stratosphere's mean temperature is then adjusted so that the net radiative heating in the stratosphere is zero. Then the height of the tropopause is found according to Condition 6, using the procedure explained above. The difference in equilibrium temperature for the two versions of Condition 5 is shown in Figure 5-18. The temperature difference within the troposphere is everywhere less than 0.5 K.

5.3 Sensitivities of Modeled Climate to details of Conditions

In the following sections we discuss the sensitivity of a climate constrained by the conditions described above to changes in those conditions. Figure 5-19 shows equilibrium values for a standard set of model parameters, as outlined above. The stratosphere is set to a constant lapse rate, the convection parameter c₁ is set equal to 3200 km, and the PV gradient parameter γ is set equal to 1. In the following discussions this will be referred to as the standard climate. Figure 5-20 shows equilibrium θ contours as a function of pressure and latitude for the same parameters.

5.3.1 Solar Constant

We begin our perturbations by varying the surface solar forcing in discrete latitude bands. This will serve as a primer to the feedbacks which operate in the model climate. Figure 5-21 shows the results for four different latitude bands. Clearly, the sensitivity of the model to forcing at the surface varies greatly as a function of latitude. If the forcing is applied in the tropics only, the response is relatively small, and decays to zero by about 60 ° latitude in either hemisphere. Warming occurs outside the tropics because Condition 1 requires continuity of temperature in the free troposphere at the tropical boundary. Figure 5-22a shows that as tropical surface temperatures rise in response to the perturbation in solar forcing, temperatures aloft rise along with them, by Condition 2, and this acts to warm the extratropical troposphere. Downwelling IR radiation increases, and warms the
extratropical surface. The effect diminishes with distance from the tropics, because the isentropes originating in the tropics become further and further from the surface as one moves polewards, and so have less and less radiative impact on the surface.

When the perturbation is applied in midlatitudes (Figure 5-21b), temperatures increase somewhat more where the forcing is applied, and the response extends throughout the tropics. In this case, a warmer extratropical surface radiates heat to isentropes which originate in the tropics. Their radiative cooling rates are reduced, which tends to reduce the heat removed from the tropical surface via Conditions 9 and 8. This in turn allows the tropics to warm. Of course, the warmer atmosphere also acts to warm the surface where the perturbation was applied, and this tends to increase the total warming there, so that the temperature anomaly is a little larger than it was when the perturbation in solar heating was applied to the tropics only.

When the solar forcing is perturbed further polewards (Figure 5-21c), the results are dramatically different. Equilibrium warming is about 10 times larger than for perturbations in other locations. The essential difference is that in this region, temperatures are too cool to allow moist convection, so that the following feedback cycle is established. As the surface temperature increases, temperatures aloft increase, because the lapse rate aloft is held constant by the requirement of fixed PV gradients, and the temperature itself is connected to surface temperature by continuity (See Figure 5-22c). This results in increase downwelling radiative flux at the surface, and increased radiative cooling aloft. Because there are only very weak moist convective fluxes at these high latitudes, the increased radiative cooling aloft is balanced by surface fluxes further towards the equator. Thus temperatures at the surface are freed to rise together with temperatures aloft. Equilibrium is only achieved when the surface's larger emissivity results in enough net increase in upwelling IR flux at the surface to balance the initial perturbation to the solar heating.

Perturbation of the solar heating polewards of 70° results in warming in that region only. This is due to the modification of Condition 11 described above. Because the potential
Figure 5-21: Plots of equilibrium temperature change accompanying a localized change in the downward solar energy flux at the surface. Temperature change is shown solid, and the change in solar forcing is shown dashed. Amplitude of solar forcing is always 1 PW, and is shown to scale. (a) Solar perturbation located between 20° N and 20° S. (b) Perturbation located between 30° and 50° in each hemisphere. (c) Perturbation located between 50° and 70° in each hemisphere. (d) Perturbation located between 70° and 90° in each hemisphere.
Figure 5-22: Propagation of changes in surface temperature to the troposphere due to fixed PV gradient. Temperature at the surface is increased by 2 K in the regions shown in the title of each plot.
\[
\begin{array}{|c|c|c|c|}
\hline
\gamma & T(\text{Equator}) \text{ (K)} & T(\text{Pole}) \text{ (K)} & T(\text{Equator}) - T(\text{Pole}) \text{ (K)} \\
\hline
0.0 & 303.6 & 247.1 & 56.5 \\
1.0 & 300.1 & 252.0 & 48.1 \\
1.2 & 299.3 & 254.0 & 45.3 \\
1.4 & 298.4 & 257.2 & 41.2 \\
\hline
\end{array}
\]

Table 5.2: Temperatures at the Equator and averaged over the North and South Poles, for various values of the tropospheric PV gradient.

temperature at the top of the polar boundary layer is held equal to the potential temperature at a point equatorwards of 70°, the increased solar radiation can be balanced by an increase in outgoing infrared radiation as the surface warms, and the atmosphere above maintains a constant temperature.

5.3.2 PV gradient

Equilibrium climates corresponding to a variety of assumed PV gradients are summarized in Table 5.2. The gradient is expressed in terms of $\gamma$, from Equation 5.2. Increasing PV gradient leads to a reduction in the pole-equator temperature difference. Figure 5-23, which shows the temperature change between a climate with $\gamma = 1.4$ and one with $\gamma = 1.2$ as a function of pressure and latitude, makes the mechanism clear. An increased PV gradient implies an increase in the static stability with latitude. Since Condition 11 results in very little change in surface lapse rate for this change in $\gamma$, increased static stability aloft means that temperatures increase more aloft than they do at the surface. Warmer temperatures aloft result in stronger downwelling IR radiation, which tends to warm the surface of the polar regions. Simultaneously, the warmer polar atmosphere aloft experiences stronger radiative cooling, as shown in Figure 5-24. This cooling must be balanced by warming from regions to the south, according to Conditions 7, 8 (as modified above), and 9. Stronger turbulent heat fluxes from the tropical surface result in cooler equilibrium temperatures there.

At the end of Chapter 4 we posed the question, would the observed seasonal variability of the PV gradients imply changes in temperature structure on the order of temperatures seasonal variability? In fact, the temperature changes implied by seasonal changes in PV homogenization (variations from $\gamma = 1.1$ to $\gamma = 1.4$) reach a maximum value of less than 5 K at the poles, far smaller than the seasonal variation of approximately 30 K at the poles. Indeed, complete elimination of tropospheric PV gradients only cools the poles by 5 K compared to the equilibrium climate for $\gamma = 1$.

However, these results raise some troubling questions. Consider a “climate change” in which the atmosphere moves from an equilibrium state in which $\gamma = 1.2$ to one in which $\gamma = 1.0$. To the extent that this transformation is caused by increased eddy activity, leading to reduced PV gradients, we expect it to be accompanied by increased meridional heat transport, and thus smaller temperature gradients. Yet we have seen that smaller values of $\gamma$ lead to larger values of the equilibrium pole-equator temperature difference. Indeed, for this change, maximum meridional energy fluxes in the atmosphere decrease from 5.8 PW to 5.4 PW. This contradiction can be resolved if we presume that changes in $\gamma$ occur in conjunction with other changes which tend to reduce the pole-to-equator temperature
Figure 5-23: Contours of temperature difference (K) between equilibrium climates with $\gamma = 1.2$ and with $\gamma = 1.4$ as a function of pressure and latitude.

Figure 5-24: Contours of heating rate difference (K $d^{-1}$) between equilibrium climates with $\gamma = 1.4$ and with $\gamma = 1.2$ as a function of pressure and latitude.
Figure 5-25: Temperature change from standard climate when $\frac{\partial \theta}{\partial p}$ is held constant in the extratropics at a value equal to 1.05 times its equilibrium value in the standard run (solid), and 0.95 times its equilibrium value in the standard run (dashed).

difference. We explore this possibility in the following two sections.

5.3.3 Boundary layer $\frac{\partial \theta}{\partial p}$

Figure 5-25 shows the changes in equilibrium surface temperature when Condition 11 is relaxed so that $\frac{\partial \theta}{\partial p}$ is held constant at 1.05 and 0.95 times its equilibrium value for the standard climate, outside the tropics. The warming for increased surface stability is slightly greater (5.5 K) than the cooling for decreased stability (4.3 K). When $\frac{\partial \theta}{\partial p}$ becomes larger in the extratropics, temperatures there tend to increase, because warmer temperatures at top of the boundary layer (due directly to the change in $\frac{\partial \theta}{\partial p}$) lead to increased downwelling IR radiation. This is accompanied by increased radiative cooling in the extratropical atmosphere, which in turn is communicated to the tropics via Conditions 9 and 8, so that temperatures there fall.

We next examine the results of a simultaneous reduction in PV gradients and increase in the magnitude of extratropical $\frac{\partial \theta}{\partial p}$. Figure 5-26 shows that the polar surface experiences a net warming of about 3.5 K, with little temperature change at the equator, a change accompanied by an increase in the maximum meridional atmospheric heat transport from 5.8 PW to 5.9 PW. This relatively large change in the surface meridional temperature gradient for a small change in the meridional energy fluxes is due to the reduced change in temperature in the upper troposphere, from which most radiation leaving the atmosphere is emitted. The reduction of the temperature perturbation with height above 750 mb is associated with the reduction in PV gradients: smaller values of PV at the pole imply reduced static stability there. However, it is the small increase of the temperature perturbation from the surface up to 750 mb which results in the increased downwelling IR radiation at the surface which initiates the surface warming. In its absence, we would return to the situation discussed in the last section, where the decrease in static stability associated with a reduction of the PV gradient results in a cooling of the polar regions.
Figure 5-26: Contours of temperature change (K) between a case where $\gamma$ is set equal to 1.0 and $\frac{\partial T}{\partial p}$ is set equal to 1.05 times its equilibrium value in the standard case, and a case where $\gamma = 1.2$.

Clearly, then, the reason for the unphysical association of increased PV mixing with reduced energy transport lies in the isolation of the degree of PV mixing from other parameters. For instance, there could be an association between changes in PV mixing and change in surface static stability. Condition 11 allows the surface lapse rate to depend only on the surface sensible heat fluxes. Although that qualitative relation certainly exists in the real atmosphere, the results of this section show that if increased PV mixing is to result directly in a smaller meridional temperature gradient, the surface lapse rate must also depend on eddy heat fluxes in the free troposphere. Consider the free troposphere just above the polar boundary layer. As the convergence of eddy heat fluxes there increases, we expect warming, and thus an increase in the static stability of the boundary layer. To the extent that a reduction in PV in the same location, implying a reduction in the meridional PV gradient, is accompanied by an increase in eddy heat fluxes, we should expect reduced PV gradients to be accompanied by increased static stability. Unfortunately, we do not currently have a theory which describes this dependence. Until that is developed, our model is limited to diagnostic rather than prognostic uses.

As we will show in the following section, changes in cloud parameters made simultaneously with a change in the PV gradient can also result in a climate which has both smaller meridional temperature gradients and smaller PV gradients than the standard climate. This association could arise in the real atmosphere if increased PV mixing were associated with an increase in moisture transport in such a way that the polar cloud fraction increased also.

5.3.4 Cloud Parameters

Figure 5-27 shows the equilibrium surface temperature change for a variety of cloud parameter changes. The pattern of temperature change for an increase of 0.05 in the low-level cloud fraction supplied to the infrared radiative code yields a much larger temperature change at the poles than at the equator, for similar reasons to those given above, though
<table>
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<th>T(Equator) (K)</th>
<th>T(Pole) (K)</th>
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<td>253.7</td>
<td>46.9</td>
</tr>
<tr>
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<td>q, ∂T/∂q fixed</td>
<td>300.6</td>
<td>253.6</td>
<td>47.0</td>
</tr>
<tr>
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<td>56.5</td>
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<td>251.0</td>
<td>53.2</td>
</tr>
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<td>300.1</td>
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<td>49.5</td>
</tr>
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<td>300.4</td>
<td>255.2</td>
<td>45.2</td>
</tr>
</tbody>
</table>

Table 5.3: Temperatures at the Equator and averaged over the North and South Poles, for various concentrations of CO₂ and various other parameter choices.

in this case all locations warm. If the cloud fraction supplied to the solar radiative code is increased by the same amount, the temperature change is once again greater at the poles, but the ratio between the polar and equatorial cooling is smaller, because the increase in albedo due to the increased cloud fraction results in a larger change in the solar energy flux at the equator, where the flux is largest, than at the poles. Lowering the cloud base and top height supplied to the infrared radiative code has a more modest effect. Lowering the cloud by 25 mb results in a warming of 0.9 K in polar regions, since the lowered cloud is slightly warmer, and therefore emits more radiation towards the surface. It also results in cooling near the equator and in mid-latitudes of 0.2 K to 0.3 K due to the increased net radiative cooling of the cloud, which must be balanced by cooling at the equator. Thus, as shown in Figure 5-28, when the cloud base is lowered, turbulent fluxes are initially reduced in the mid-latitude and tropical regions, while downwelling infrared radiation is increased everywhere. The resulting net surface flux cools the tropics and warms the poles. Once equilibrium has been reached, the warmer polar surface emits enough radiation to balance the increased radiative cooling of the clouds, and the fluxes relax towards their equilibrium values for the standard climate.

Figure 5-29 shows the change in equilibrium potential temperature, as a function of pressure and latitude, when a reduction of the tropospheric PV gradient is accompanied by an increase in cloud fraction. As with the simultaneous change in PV homogenization and surface lapse rate, a net warming of the polar regions results. In this case, however, the warming decays monotonically from the surface to the tropopause. The similarity to the polar warming investigated by Hou (1997) is striking, and confirms our speculation in Chapter 3 that an increase in PV homogenization accompanied by an increase in atmospheric opacity could account for the temperature pattern found in that case.

5.3.5 Carbon Dioxide concentration

Table 5.3 shows results from a series of runs which test the sensitivity of the climate constrained by our various assumptions to changes in CO₂, when various other parameters are held fixed or varied. Doubling CO₂ results in a warming of 4 K to 5 K at the poles, with a warming of only 0.3 K at the equator.

It is commonly observed that a large part of the sensitivity of various climate models
Figure 5-27: (a) Surface temperature change when the fractional cloud coverage of lower cloud layer supplied to the infrared section of the radiative transfer code is increased by 0.05 at all latitudes. (b) Surface temperature change when the fractional cloud coverage supplied to solar radiation code is increased by 0.05 at all latitudes. (c) Surface temperature change when cloud base pressure and cloud top pressure supplied to infrared code is lowered by 25 mb.
Figure 5-28: Initial change in surface fluxes (W/m²) when cloud base is lowered by 25mb in equilibrated model. Dotted: solar. Dashed: infrared. Dot-Dashed: total turbulent. Solid: net surface flux.

Figure 5-29: Contours of temperature change (K) between a case where $\gamma$ is set equal to 1.0 and infrared cloud fraction is increased by 0.05, and a case where $\gamma = 1.2$, with the standard cloud fraction.
to changes in carbon dioxide depends on the water vapor feedback (e.g. Lindzen, 1990). To investigate the degree to which relative humidity boosts sensitivity in this model, we modify Condition 4, so that instead of holding relative humidity constant, we hold absolute humidity fixed above 900 mb at the standard climate levels, while relative humidity remains fixed below that level. In this case the temperature increase at the poles is reduced to about 1.7 K, while the tropical warming increases to 0.5 K. It does not matter much whether the relative or absolute humidity is fixed in the tropics, since the temperature changes there are small in any case.

As for an increase in cloud fraction, increasing CO₂ increases the infrared opacity of the atmosphere, so that downwelling radiation increases everywhere. But the increased radiative cooling associated with this is balanced preferentially by tropical and subtropical surface regions, which are in dynamic communication with the polar troposphere via moist convection and isentropic mixing. Thus, when absolute humidity in the atmosphere is held fixed, while CO₂ is doubled, the equilibrium radiative cooling of the polar atmosphere is smaller than when relative humidity is held fixed, and the convective cooling at the tropics is proportionately smaller, which allows the tropics to warm more in the fixed absolute humidity case than in the fixed relative humidity case.

The sensitivity of the model climate to doubled CO₂ is not much affected by other parameter settings. Warming is somewhat reduced if the tropospheric PV gradient is set to zero. In that case, isentropes curve more steeply upwards as the approach the pole, so that static stability can decrease to balance the increasing planetary vorticity. This sharper curves means that fewer tropical isentropes reach the polar troposphere, and so more of the increased radiative cooling of the extratropical atmosphere must be balanced by increased cooling of the extratropical. This tends to reduce the net warming.
Figure 5-31: Change in surface fluxes (W/m$^2$) between equilibrium climates with CO$_2$ = 600ppm and with CO$_2$ = 300ppm. Solid: solar. Dashed: infrared. Dot-Dashed: moist convective. Dotted: non-convective turbulent.

5.3.6 Ocean heat fluxes

Figure 5-33 shows the change in surface temperature when the ocean heat transport is changed from 2 PW to either 1.5 PW or to 2.5 PW. As usual, the effect is amplified towards the poles. The actual change in ocean surface flux is shown (dotted) in Figure 5-32. The change reaches a minimum value of -5.8 W m$^{-2}$ at 49° in each hemisphere, and a maximum value of 7.25 W m$^{-2}$ at the equator. The local changes in temperature are approximately in the same proportion to the local change in ocean forcing as the changes observed for perturbations of the surface solar flux. The changes in ocean heat fluxes are mostly balanced by changes in the turbulent surface heat fluxes. Comparison of merdional heat fluxes shows a slight compensating reduction in atmospheric heat fluxes when ocean heat fluxes are increased. For an increase in peak meridional ocean heat fluxes from 2 PW to 2.5 PW, peak total energy fluxes increase only from 7.1 PW to 7.4 PW, due to a slight decrease in atmospheric energy fluxes. This compensating reduction in equilibrium atmospheric fluxes could be larger in the real atmosphere, if an increase in ocean fluxes were accompanied by a larger reduction in surface static stability than occurs in the model according to Condition 11.

5.4 Discussion

Our results help rationalize the oft-noted fact that paleoclimatic variations have generally been larger at the poles than at the equator. In our model, such a pattern arises for most perturbations of parameters and boundary conditions. The large polar and small tropical responses to various forcing mechanisms result from the assumption that the radiative cooling on a given isentrope can be balanced only by surface heat fluxes on that isentrope or by deep convection. Given the finding by Sun and Held (1996) that GCMs tend to exaggerate vertical correlation of temperature and water vapor variations, this suggests
Figure 5-32: Change in surface fluxes (W/m²) between equilibrium climates with the ocean heat transport \((T_0)\) equal to 1.5 PW and with \(T_0 = 2\) PW. Solid: solar. Dashed: infrared. Dot-Dashed: total turbulent (convective plus nonconvective). Dotted: oceanic.

Figure 5-33: Change in surface temperature for an increase in the meridional ocean heat flux from 2 PW to 2.5PW (solid), and for a decrease from 2 PW to 1.5 PW (dashed).
that some of the difficulty that GCMs have in reproducing this response pattern (Rind and Chandler, 1991) might have to do with excessive cross-isentropic energy transport in the extratropics.

In contrast with the results of the baroclinic neutralization model of Stone (1978), where eddies seek to restore midlatitude temperature gradients to the gradient corresponding to the "critical shear" a two-layer model, neutralization via PV homogenization does not have the effect of tightly constraining the midlatitude temperature gradient. This means that variations temperature outside the tropics are not so strongly coupled with tropical temperature variations. Polyak and North (1997) recently investigated the linear interdependence of zonal mean surface temperature data from observations and from the GFDL GCM. They found that the GCM greatly overestimated the dependence of each latitude band on distant latitude bands, and underestimated the coherence of tropical variations. This lends some conceptual support to our approach, since in our simulated climate, perturbations of the tropical energy budget at least result in temperature perturbations which decay away from the tropics. In the "critical shear" conception of baroclinic neutralization, the perturbation would extend undiminished to the poles.

The sensitivity studies discussed above show just how unfortunate it is that we do now have a good theory with which to predict cloud height and fraction in a two dimensional model. A mere five percent increase in cloud fraction produced a ten degree increase in polar temperatures. Changes of this magnitude occur regularly over the seasonal cycle (Drake, 1993). Thus the lack of a theory of how clouds fraction might change in a different climate, together with our problems relating the surface lapse rate to meridional heat fluxes aloft, precludes the use of the model we have described in a prognostic mode.

5.5 Conclusions

The major findings of this chapter are as follows.

- We found that the maintenance of relatively warm temperatures at the poles is a result of the decoupling of the free troposphere from the surface there. If temperatures aloft were tied to temperatures at the polar surface, the whole region would become very cold. But polar temperatures aloft must be coupled to other surface regions in order to supply the heat fluxes needed to balance radiative cooling aloft.

- The tropopause height distribution in a climate constrained by our Conditions is due to the overturning circulation of the stratosphere. Upwelling at the equator and downwelling at the poles are necessary in order to explain the tropopause height at either location; in their absence the tropopause height is approximately constant at 200 mb.

- Cross-isentropic fluxes of heat and moisture are necessary to explain the observed climate of the midlatitudes; in their absence temperatures there climb far too high. The use of a moist convection scheme allows sufficient cooling of the surface for equilibrium to be attained at temperatures in reasonable agreement with observations.

- Changes in the degree of PV mixing must be accompanied by changes in either surface lapse rate or changes in the opacity of the atmosphere. Otherwise we get the counter-intuitive result of increased PV mixing being accompanied by reduced mixing of heat.
The model has a strong polar feedback, which is the result of a combination of a radiative feedback and a dynamical feedback. The radiative feedback comes from the high atmospheric opacity needed to explain the polar annual mean temperature. The dynamical feedback is due to the fact that the atmosphere above the pole can warm, and increase its radiative cooling rate, without incurring any energy debt for the polar surface. The debt is paid by the regions which are in communication with the polar troposphere via deep convection or isentropic motions.
Chapter 6

Summary

This thesis has addressed the role played by the mixing of potential vorticity on isentropes in the determination of the earth's climate and that climate's sensitivity to various perturbations. We have discussed how a fixed PV gradient communicates information about the state of the tropical troposphere to the extratropical troposphere. This led us to investigate the appropriateness of the moist adiabatic adjustment as a description of the tropical troposphere. Using monthly mean radiosonde observations, we found that for interannual variations, the mean tropical sounding does vary as a moist adiabat originating in the warm ocean regions of the tropics would. However, for seasonal time scales, temperature variations aloft are much smaller than those predicted by a moist adiabat originating at the surface.

Returning to the PV gradients themselves, we have shown that a significant region of PV homogenization exists around the 700 mb level in each hemisphere and in all seasons. Gradients increase as one moves higher in the troposphere, so that the optimal PV gradient with which to predict the atmosphere's temperature structure is generally larger than the $\beta$ contribution to the PV gradient. We found that a seasonal cycle in this optimal PV gradient exists in both hemispheres, but with the same phase, so that mean PV gradient are smallest in the northern hemisphere summer, but in the southern hemisphere winter. Interannual variations of the PV gradient were found to be small, an order of magnitude less than the $\beta$ contribution.

Reviewing the evidence about the effect of the PV gradient structure on the baroclinic instability of the troposphere, we noted the work of Harnik and Lindzen (1997), who showed that fractional reduction of the tropospheric quasi-geostrophic PV gradient from $\beta$ to 0 tended to stabilize the troposphere. However, the flows we investigated were generally far from stability for parameter choices close to observations. We also discussed the modeling studies of Solomon (1997), who showed that in a multi-level quasi-geostrophic $\beta$-plane model, equilibration proceeded through the removal of PV gradients in the lower troposphere, resulting in a vertical profile of PV gradients qualitatively similar to observations. The model's equilibrium state appears to be neutral to the linear growth of eddies, but it is unclear which aspects of its mean flow, or of the model physics, are responsible for its stabilization relative to the analytical models which were far from stability. We speculate that the fact that Solomon's model allows vertical variation of the PV gradient may allow neutralization for flows with mean PV gradients similar to observed mid-latitude flows.

Emboldened by observations showing relatively small variations in PV gradients from season to season, and modeling results showing that the pattern of PV gradients observed in
the atmosphere tends to reduce the baroclinic instability of the atmosphere, we next sought
to compile a minimum list of assumptions, which together with an assumption of a fixed
tropospheric PV gradient, would constrain a model climate. This was accomplished, and
the distinct differences between this climate and the earth’s observed climate led us to make
some modifications to the initial assumptions. In doing so, we showed that the meridional
slope of the tropopause height is due largely to the stratosphere’s overturning circulation.
We were also able to demonstrate the climatological importance of deep convection for
the midlatitudes, and to demonstrate the necessity of a strong boundary layer inversion,
decoupled dynamically from the rest of the troposphere, for maintaining the warmth of the
polar regions.

Sensitivity studies of the resulting model showed that variations in the PV gradient alone
led to the result that reduced PV gradients were accompanied, in equilibrium, by larger
surface temperature gradients. We showed that this result was due to the lack of coupling
between surface lapse rates and tropospheric eddy fluxes. When reduced PV gradients
were accompanied by static stabilization of the boundary layer, equilibrium temperature
gradients were shown to be reduced.

One of the assumptions used to constrain the model climate, that energy fluxes only
cross isentropes via moist convection, was shown to lead to a strong polar amplification of
climate sensitivity. It was speculated that excessive vertical diffusion of heat and moisture
in GCMs might be responsible for the trouble those models have in reproducing equable
paleoclimates.

The model has several weaknesses which make its prognostic use impossible at this time.
We have found no good way of predicting cloud parameters from other model parameters,
nor do we have a theory for the variations of relative humidity which might accompany a
change in climate forcing. Finally, we have yet to derive a connection between the ability of
eddies to mix PV in the mean troposphere, and their ability to affect boundary layer static
stability. The specific nature of these issues suggests a reasonable focus for future research.
References


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