Design and Manufacturing of a VTOL Micro Aerial Vehicle

by

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Submitted to the Department of Mechanical Engineering on March 9, 2001 in partial fulfillment of the requirements for the Degree of Master of Science in Mechanical Engineering.

Abstract

The miniaturization of electronic guidance and navigation systems has opened a new frontier in aviation. The opportunity now exists to develop miniature aerial vehicles capable of autonomous flight. Such vehicles could perform reconnaissance missions in environments that are unsafe to humans. Their small size minimizes their chance of being detected in hostile areas and allows them mobility not afforded by typical full size vehicles and humans.

The purpose of this thesis was to develop a 10” VTOL Micro Aerial Vehicle capable of high-speed forward flight. The innovative design incorporates a pair of counter-rotating, variable pitch propellers enclosed in a protective duct structure. Independent control of propeller pitch provides thrust and yaw control for the vehicle while in a hover mode. Translational control is provided by four independently actuated control surfaces which are placed in the area of high-speed flow aft of the propeller disks. In its forward flight mode, the duct acts as the primary lifting surface and the control surfaces act as movable canard wings. Power is provided by an off the shelf 0.25 cubic inch displacement 2-stroke glow-fuel engine. A volume of 9 cubic inches has been set
aside in the main fuselage to house the Draper Micro Electromechanical integrated navigation package. A payload sensor bay with a volume of 3 cubic inches has also been incorporated into the design.

Initial tests of this vehicle configuration indicate that it is a mechanically feasible design, which produces close to 75% of the thrust required to hover. Further development will focus on design refinements to create a lighter, more robust mechanical drive train, and aerodynamic analysis that will guide the redesign of the propeller.

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Title: Senior Member of the Technical Staff

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ACKNOWLEDGMENT

June, 2001

I would like to thank all those involved in completion of this thesis: To The Draper Labs, for providing me with this unique experience and a productive learning environment. I would like to thank Ed McCormack and John Mahoney of the machine shop for their great mechanical intuition and experience and to Pete Kerrebrock for his expertise and enthusiasm. Thank you to Kyrilian Dyer for all those long hours, late nights, and scary time standing behind a spinning propeller blade. To Paul Eremenko and Sean George for invaluable advise and great friendship. I would especially like to thank my advisor John Plump for trusting me with a great responsibility. It has been an honor to work with you, your humor and encouragement have kept me motivated through all the technical problems along the way. Of course I would also like to thank my family, Mom, Dad and Olaina who have always stood by me. You’ve helped me achieve and become the person I am.

This thesis was prepared at The Charles Stark Draper Laboratory, Inc., under Internal Company Sponsored Research Project 3019, Navigating Under the Canopy.

Publication of this thesis does not constitute approval by Draper of the findings or conclusions contained herein. It is published for the exchange and stimulation of ideas.

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1 Introduction

From the conception through construction of the Perching Unmanned Aerial Vehicle (PUAV) project, much knowledge and insight has been gained in the specifics of miniature scale aeronautical systems. The technical hurdles ranged control surfaces sustaining low Reynolds numbers and bi-directional flows, to high drive train rpm’s and material strength and weight issues. This paper will attempt to convey the design methodology involved in the evolution toward a complete vehicle system.

1.1 Customer Needs statement

The ever-changing global political environments around the world today have created a variety of new tactical battle needs. To modernize the role of the typical foot soldier, the US armed forces seek to increase the information available to individual soldiers on the battlefield. Part of this vision includes the use of Micro Aerial Vehicles (MAVs) that would be carried by soldiers. In order for our troops to operate safely in hostile areas that are nowadays more often urban, small surveillance craft are needed. These vehicles would allow ground troops to inspect an area for chemicals, explosives or hostile personnel before humans move into it. Their small size would reduce their chances of being detected, and allow them access to spaces that their full scale counterparts could not access. Small surveillance craft would also allow foot soldiers to familiarize themselves with the area while they are out of harm’s way. This could include recognizing landmarks, understanding civilian movements and visualizing expected terrain. Alternative missions scenarios for MAVs could include surveillance and remote sensing with the MAV being deployed from a series of larger, “parent” autonomous vehicles.
Currently several medium to full scale autonomous vehicles are deployed in combat situations. However, there are several limitations to these designs. The US armed forces states that the biggest improvements need to come in the areas of real time, “timely intelligence gathering” and range/duration improvements. MAVs present the ability to produce timely intelligence gathering by their proximity to the operating troops. Unlike large scale remotely piloted vehicles, MAVs require very little logistical support and the data they gather is on the front line where events can change rapidly. Particularly VTOL MAVs present the opportunity to stop and “perch” during a mission to gather intelligence on a particular area in detail.

Because these MAVs would be used in hostile environments their survivability and usability must be very high. The vehicle structure must be tough and durable, while the physical system itself retains a high degree of usability. It is desired that the final product be usable by any untrained field soldier. At the time of vehicle conception the Draper Laboratory had defined some basic outlines for such a vehicle, they were:

- 10 inch maximum linear dimension
- VTOL capability
- 20 minute flight duration
- Camera video down-link
- Capable of hovering flight in winds less than 5 mph
- Capable of transitioning to high speed (40 mph +) forward flight
One MAV operating scheme in the context of an overall military operation is shown in Figure 1-1. The vehicle is deployed as part of a cascading series of autonomous vehicles. Here the MAV acts as a continued extension of a battle groups arms providing finer detailed information at a much more local level. Presently, reconnaissance information obtained by battle group commanders often takes too long to get to the individual foot soldiers, or is not given at all. The MAV would provide local reconnaissance information to the forward deployed units of the battle group, helping to complete the dissemination of knowledge.

At the troop level the MAV can be used in both urban and rural settings as shown in Figure 1-2 and Figure 1-3.
Figure 1-2 MAV use in urban setting
Typically a MAV is described as being $10^6$ times less massive than a full-scale aircraft as can be seen in Figure 1-4. This figure also displays one of the major contributors to the complexity of MAVs. The lower axis shows how the Reynolds number scales with the size of the vehicle. As the order of magnitude drops dramatically, the flow regime will be significantly different from those seen on full-size aircraft. This can necessitate the use of different airfoils, and cause other undesired aerodynamic characteristics such as boundary layer separation, or unsteady flows.
1.2 **Industry overview**

Although computer guidance systems have been in use since the days of the Apollo program, it was not until the age of miniature electronics that they became widespread. The advancement of miniature navigation hardware, gyroscopes and accelerometers as well as the widespread use of GPS has enabled a trend toward smaller autonomous vehicles. However aerodynamics do not simply scale with the size of the aircraft, and the configurations of such vehicles have become less traditional in the attempt to improve their capabilities. The small size of these vehicles also increase the frequency of their dynamic modes making oscillations or response to disturbances often orders of magnitude faster. Consequently many of these vehicles need computer control systems to be stable in flight.

The past ten years have seen several MAVs with a wide range of configurations, sizes and mission requirements. Some military contractors have developed prototype
MAVs as demonstrators. Lockheed Martin Aeronautics pioneered a fixed wing MAV called Black Widow. This design has been popularized in the university MAV competitions and was used by the University of Notre Dame. The US Navy developed fixed wing vehicles such as Bluebird and then began to experiment with VTOL capabilities with their AROD and Arkytas vehicles. Micro Craft of San Diego is in the process of building a small ducted vehicle with VTOL capabilities. This concept is based on a General Dynamics design dating back to 1958.
2 Vehicle Concept Development

2.1 System requirements

The challenge of this project was to develop a MAV that met the needs of the military defined by The Draper Laboratory. A balanced design approach was used to address multiple layers of complexity in parallel. Vehicle robustness and usability were considered, with gross weight and system complexity balanced with the desired capabilities.

Physical requirements

Basic size requirements were dictated by the Military’s needs for a surveillance aircraft that could be carried with the standard issue equipment of a foot soldier and used without special training. The vehicle was to be capable of fitting in a ten inch cube. Although weight was not a specific factor it is apparent that the necessity to remain light is driven by the challenge of the vehicles hover capability. Power plant type and configuration were not specified allowing for the exploration of several options.

Performance requirements

A typical flight duration of between 10-20 minutes was required. Twenty minutes was seen as the final goal with ten minutes being an achievable prototype duration. The vehicle must be capable of vertical take off and landing as well as flying rapidly from one location to the next. This was interpreted as the ability of the vehicle to transition between a dash (40+ mph horizontal flight) and loiter flight mode. The vehicle would therefore have two distinct flight modes, and not a third “perturbed hover” mode that is a transitional phase between the two extremes. The vehicle defined at this stage would
consist of only flight capable hardware, no autonomy and control issues were to be addressed at this time.

2.2 Analysis of existing technology

Vertical Take Off and Landing (VTOL) aircraft are not uncommon in the aerospace industry. In their most basic form (the balloon) they have been in existence even before the airplane. As part of the initial design conceptualization, a specific effort was made to assess all approaches to VTOL technology. It was important to start the design from the ground up considering every possibility. This would allow for the coalescence of the best alternatives.

Several existing VTOL designs were considered to glean the best characteristics from each. The most common examples of VTOL aircraft are helicopters, although even these come in many different forms. The VS-300 flown by Igor Sikorsky in 1939 (Figure 2-1) was a single rotor aircraft with a tail rotor to counter-act the torque produced by the main rotor.

![Figure 2-1 Sikorsky VS-300 (1939)](image)

The Russian Kamov design bureau is famous for its contra-rotating propeller designs that eliminate the need for a tail rotor. Helicopters are true VTOL vehicles and
have exceptional maneuvering characteristics, but their horizontal speed is much less fuel efficient than fixed wing aircraft. To compensate for this lack of efficiency, tilt-rotors and tilt-wings have been built. These combine the VTOL and maneuvering capabilities of the helicopter with the horizontal flight speed and range of a conventional aircraft. Unfortunately this often involves complex mechanisms and aerodynamics.

An overview was done of existing Unmanned Aerial Vehicles (UAVs) and MAVs. Since the late seventies there have only been a few VTOL capable UAVs built. Some of these include the Cypher (Figure 2-2), QH-50 and CL 327. Several versions of Non-VTOL MAVs have been built, including Lockheed Martin's Microstar (Figure 2-3) and Aero Virenments Black Widow (Figure 2-4).
There have also been attempts at transitioning ducted vehicles. Micro Craft of San Diego, California is in the process of developing a transitioning ducted fan vehicle that will use the duct as a wing in horizontal flight. The genesis of this transitioning VTOL concept dated back to an unmanned vehicle built by General Dynamics in 1958 (Figure 2-5).

![Micro Craft VTOL vehicle](image)

**Figure 2-5 Micro Craft VTOL vehicle**

Positive and negative system characteristics were extracted during the overview of these existing technologies. The results are summarized in Table 2-1. This condensation helped outline the possible directions that VTOL design implementation could go. The high level mapping allowed informed decisions to be made as to the basic avenues to be considered. This assured that the final design would be well grounded, and helped to keep in perspective past lessons learned.
Table 2-1 Design Concepts

<table>
<thead>
<tr>
<th>Tilt Rotor</th>
<th>Fixed Rotor Transitioning</th>
<th>Helicopter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pros:</strong></td>
<td><strong>Pros:</strong></td>
<td><strong>Pros:</strong></td>
</tr>
<tr>
<td>• Increased range</td>
<td>• Simple Design</td>
<td>• Proven VTOL design</td>
</tr>
<tr>
<td>• Good forward flight</td>
<td>• Efficient use of available volume</td>
<td>• good Hover maneuverability</td>
</tr>
<tr>
<td>maneuverability</td>
<td>• Reasonable Hover and forward flight</td>
<td></td>
</tr>
<tr>
<td></td>
<td>capabilities</td>
<td></td>
</tr>
<tr>
<td><strong>Cons:</strong></td>
<td><strong>Cons:</strong></td>
<td><strong>Cons:</strong></td>
</tr>
<tr>
<td>• Multiple propellers</td>
<td>• Complex transition between flight modes</td>
<td>• Low range capability</td>
</tr>
<tr>
<td>• Complex miniaturization</td>
<td>• Complex stability and</td>
<td>• complex rotor mechanism</td>
</tr>
<tr>
<td>• Limited Hover capabilities</td>
<td>aerodynamic characteristics</td>
<td></td>
</tr>
<tr>
<td>• Unstable transition and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vortex ring state</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong></td>
<td><strong>Example:</strong></td>
<td><strong>Example:</strong></td>
</tr>
<tr>
<td>Boeing V22 Osprey.</td>
<td>Micro Craft transitioning VTOL vehicle.</td>
<td>Most modern helicopters,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hind A4 to Robinson.</td>
</tr>
</tbody>
</table>

2.3 Conceptual Layout

![Concept vehicle operation and layout](image)

Figure 2-6 Concept vehicle operation and layout

The overarching concepts were weighed and combined to create a composite design. The basic vehicle shape and operational concept can be seen in Figure 2-6. The
compiled characteristics of this design are listed in Table 2-2. This design concept was then fleshed into a block layout that incorporated all the characteristics listed in the table.

### Table 2-2 Composite design concept

<table>
<thead>
<tr>
<th>PUAV</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pros:</strong></td>
<td></td>
</tr>
<tr>
<td>• Transitioning vehicle with limited helicopter thrust augmentation</td>
<td></td>
</tr>
<tr>
<td>• Capability to alter blade angle for forward flight efficiency and hover speed</td>
<td></td>
</tr>
<tr>
<td>• Ducted rotor for increased flight range and operator safety</td>
<td></td>
</tr>
<tr>
<td><strong>Cons:</strong></td>
<td></td>
</tr>
<tr>
<td>• Increased mechanical complexity due to collective</td>
<td></td>
</tr>
<tr>
<td>• Complex stability and aerodynamic characteristics</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong></td>
<td></td>
</tr>
<tr>
<td>Final configuration of fixed ducted vehicle with ability to change thrust direction and flight mode</td>
<td></td>
</tr>
</tbody>
</table>

#### 2.4 Conceptual Operation

The resulting vehicle was a marriage of several proven design technologies. It incorporated a toroidal outer shell that would act both as a ring wing in horizontal flight, a duct for improving propeller thrust in hover, and a rotor guard for safe in-field operations. The concept of variable collective pitch would be borrowed from helicopter design to give variable thrust at constant engine rpm and the ability to reverse thrust direction. Keeping engine rpm constant is especially important with small two-stroke engines which are optimized for a narrow bandwidth of operational speeds. Control surfaces downstream of the propeller down-wash would enable translation during hover, and would act as canards during horizontal flight. Maneuvering from one flight mode to
another by means of thrust reversal is aggressive and more complex, however it allows for a fast transition between flight modes, seen as a boon in hostile environments.
3 Preliminary Vehicle Design

The conceptual layout developed in chapter two was then detailed on several levels outlined in the following chapter. The feasibility of each major design decision is supported by simulation or analysis whenever possible.

3.1 Feasibility Analysis

A preliminary feasibility analysis was performed on the vehicle concept in its early stages. This ensured that the rough estimate sizes and components under consideration would be reasonable choices. To reduce the time and complexity of building a prototype, off-the-shelf technologies were implemented wherever possible. The model hobby industry supplied much of the basic hardware that would otherwise require considerable re-development time. Stock model two-stroke engines were considered along with electric motors. A preliminary overview of the power density available for each propulsion method indicated that electric motors would require a prohibitively large battery pack for sustained flight. Model engines in the size range of .15 cid to .32 cid were considered for their relatively small size and power output in the range of 400-650 watts (.5-.85 bhp).

Other stock items to be used were RC radio receivers, servo actuators, NimH batteries and model helicopter rotor assembly parts. A review of the market was done to select the smallest lightest components that would perform adequately. A summary of the components selected is in Table 3-1. Figure 3-1 below is a graphic showing the percentage of total vehicle weight that each element represents.
Table 3-1 Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Qty</th>
<th>Part</th>
<th>Weight (grams)</th>
<th>total weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Servos</td>
<td>4</td>
<td>S-90 (FMA)</td>
<td>9.07</td>
<td>36.28</td>
</tr>
<tr>
<td>Micro Servo</td>
<td>2</td>
<td>LS-3.0</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Receiver</td>
<td>1</td>
<td>Tetra FM (FMA)</td>
<td>14.17</td>
<td>14.17</td>
</tr>
<tr>
<td>Fuel (20 min dur.)</td>
<td>1</td>
<td>380 ml</td>
<td>380</td>
<td>380</td>
</tr>
<tr>
<td>Engine</td>
<td>1</td>
<td>.25 FX (OS)</td>
<td>249.47</td>
<td>249.47</td>
</tr>
<tr>
<td>Batteries</td>
<td>3</td>
<td>Lithium Rec. Bat</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>Variable Pitch Mech.</td>
<td>1</td>
<td>1</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Vehicle</td>
<td>1</td>
<td>structure</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

| Total (grams)      | 1135.92 |

Figure 3-1 Mass Percentages

3.2 Helicopter Rotor Dynamics

The PUAV design had a simple two bladed unhinged rigid rotor capable of collective pitch adjustments. Because of the rotor’s small radius (.120m, (4.73 inches)) it was possible to make the assumption of a rigid, untwisted blade, greatly simplifying the
analysis. It was intended that accurate simulation of the single rotor model would enable proper design of the control surface scheme necessary for flight control. Coupled with the simulation of these control surfaces and the aerodynamics of the vehicle body itself a complete simulation would be achieved allowing for the implementation of initial control algorithms.

Basic Momentum theory and Blade Element theory were employed to derive expressions for the rotor induced velocity and power. Most schemes express the induced velocity as a function of propeller thrust and visa versa leading to an iterative solution. The desire in this simulation was to develop a rotor model that was both accurate to the ducted design, and able to predict multiple configurations (i.e. reverse collective pitch). The result is a hybrid of blade element theory and use of lookup tables generated by X-Rotor software.

As shown in (Figure 3-2), the propeller induced velocity $V_i$ is the sum of the incoming air relative velocity $V_e$ and the air velocity at the propeller disk $V_t$. 
The velocity at the propeller disk is calculated from:

\[ V_i = \sqrt{\frac{TN_{ge}}{2 \rho A}} \]  
\[ \text{Eq. 3-1} \]

Where \( N_{ge} \) is the Ground effect parameter defined as:

\[ N_{ge} = \frac{1}{1 + K_{pe} \left( \frac{R}{h} \right)^2} \]  
\[ \text{Eq. 3-2} \]
Here $K_{ge}$ is the ground effect parameter and its value is .125, and the altitude of the craft is given by $h$.

In order to obtain a non iterative solution for Thrust, X-rotor was utilized to generate a table of $C_T$ values for varied advance ratios and collective pitch settings. The Preliminary PUAV rotor design was input into X-rotor and the program run for a range of propeller rpm’s form 4,000 to 15,000 and several craft vertical velocities. The assignment of rpm and vertical velocity individually allowed for the definition of specific advance ratios. This prohibits the X-rotor program from creating unreasonable rotor rpm’s in the case where only an advance ratio is defined. Rotor rpm’s to be simulated were chosen from the expected capabilities of the model engines rated at 4,000-18,000 rpm. Data was extrapolated into large advance ratio regimes. A truncated version of the current $C_T$ Vs. $\gamma$ and $\Omega$ table is shown in Table 3-1.

**Table 3-2 Advance ratios**

<table>
<thead>
<tr>
<th>Collective</th>
<th>6.0E+00</th>
<th>2.0E-02</th>
<th>2.0E-04</th>
<th>4.5E-06</th>
<th>2.4E-04</th>
<th>2.0E-02</th>
<th>2.0E+00</th>
<th>8.0E+00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3.1E-04</td>
<td>4.7E-03</td>
<td>6.3E-03</td>
<td>7.9E-03</td>
<td>8.0E-03</td>
<td>8.7E-03</td>
<td>9.1E-03</td>
<td>8.8E-03</td>
</tr>
<tr>
<td>4</td>
<td>4.2E-02</td>
<td>3.3E-02</td>
<td>2.9E-02</td>
<td>2.6E-02</td>
<td>2.3E-02</td>
<td>2.4E-02</td>
<td>2.3E-02</td>
<td>2.0E-02</td>
</tr>
<tr>
<td>6</td>
<td>8.3E-02</td>
<td>6.1E-02</td>
<td>5.2E-02</td>
<td>4.4E-02</td>
<td>3.8E-02</td>
<td>4.0E-02</td>
<td>3.8E-02</td>
<td>3.0E-02</td>
</tr>
<tr>
<td>8</td>
<td>1.3E-01</td>
<td>8.9E-02</td>
<td>7.5E-02</td>
<td>6.2E-02</td>
<td>5.3E-02</td>
<td>5.5E-02</td>
<td>5.2E-02</td>
<td>4.1E-02</td>
</tr>
<tr>
<td>10</td>
<td>1.7E-01</td>
<td>1.2E-01</td>
<td>9.8E-02</td>
<td>8.0E-02</td>
<td>6.8E-02</td>
<td>7.1E-02</td>
<td>6.6E-02</td>
<td>5.2E-02</td>
</tr>
<tr>
<td>12</td>
<td>2.1E-01</td>
<td>1.4E-01</td>
<td>1.2E-01</td>
<td>9.8E-02</td>
<td>8.3E-02</td>
<td>8.6E-02</td>
<td>8.1E-02</td>
<td>6.3E-02</td>
</tr>
<tr>
<td>14</td>
<td>2.1E-01</td>
<td>1.9E-01</td>
<td>1.8E-01</td>
<td>1.7E-01</td>
<td>1.6E-01</td>
<td>1.7E-01</td>
<td>1.7E-01</td>
<td>1.3E-01</td>
</tr>
</tbody>
</table>

The advance ratio $\gamma$ is defined as:

$$\gamma = \frac{V_c}{\omega D} \text{ Eq. 3-3}$$

Thrust is then defined as:
\[ T = \rho C_T \omega^2 D^4 \]  \hspace{1cm} \text{Eq. 3-4}

In this manner the iterative formulation problem was overcome. The continued development of an accurate \( C_T \) table will enable reliable rotor predictions to be made.

**Rotor Torque**

An expression for rotor torque was developed from blade element theory\(^1\). This begins with a formulation of rotor coefficient of power. This includes terms for both profile drag and induced drag on the rotor. The rotor coefficient of power is derived as:

\[ C_p = \left( \frac{\sigma C_d}{8} + \frac{k C_T^{3/2}}{\sqrt{2}} \right) \]  \hspace{1cm} \text{Eq. 3-5}

Where \( C_d \) is found from a lookup table of \( C_d \) versus Collective pitch angle of attack. Power is then defined as :

\[ P = \frac{V_{tip}^3}{2} \rho C_p R^2 \]  \hspace{1cm} \text{Eq. 3-6}

\( V_{tip} \) is the rotor tip velocity in meters per second. Finally Torque is found from:

\[ \tau = \frac{P}{\omega} \]  \hspace{1cm} \text{Eq. 3-7}

**Rotor Model Results**

The completed rotor model was capable of predicting thrust, induced velocity and torque for any applicable rotor rpm and positive collective pitch settings. Reverse collective pitch settings could later be studied in order to implement transition of the vehicle into horizontal flight. The results framed an expected induced velocity range that could be applied to the control surface model to predict the control force necessary to
counteract the torque produced by the spinning rotor. At the expected engine rpm under load (12,000) the simulation produced a propeller induced velocity of 30-40 ft/s with collective pitch deflections of 10-15 degrees. The predicted thrust for this operating regime was 1.25 Kg. With a vehicle weight estimate of 1.13 Kg the vehicle would have a margin of 10% for translation maneuvers in hover.

3.3 Preliminary Design

In the proceeding section the preliminary design will be presented in stages. Each stage is critiqued and the technical issues encountered during design and manufacturing are discussed.

Airframe

Using the basic design outline generated from the trade study a preliminary vehicle design was created. The primary goal was to keep the drive train simple by using only one variable pitch propeller. It was conceived that the torque generated by the rotor could be counteracted by the stators inside the duct. With the drive train in line with the axis of the vehicle the structural concept was to link a rigid center body to a strong outer duct. Although the outer duct needed to be rigid, it still needed to be as light as possible. In an effort to use heavier structural components only where needed, it was determined that the inner surface of the duct should be separate from the outer surface. Composite materials, (preferably carbon fiber) would be used wherever practical. Therefore the inner surface could be made thicker and reinforced with stringers in the fore and aft. The thin pliable outer shell would then provide only the shape of the airfoil in horizontal flight. It would slip on and off to provide access to internal fuel tanks and servos.
With an estimated diameter of 25.4 cm (10 inches) and a span of 15.24 cm (6 inches) the surface area of the duct would be approximately 774 square cm. The thickness of a single layer of carbon fiber was measured at .76 mm (.030 inches) resulting in a duct material volume of 59 cubic cm. Assuming a composite lay-up with 70% fiber and 30% matrix by volume, the density of the duct carbon material was estimated as 1.64 grams per cubic centimeter. This density multiplied by the estimated duct volume gave a theoretical duct weight of 97 grams. This value was well within the target weight of 180 grams.

**Drive Train**

An initial concept for a single propeller variable pitch mechanism that fit directly over the engine was constructed. The goal was to use off the shelf hardware wherever possible and keep the parts as small as possible. The initial design borrowed heavily from existing tail rotor designs on RC helicopters. Figure 3-3 shows a CAD model of the mechanism. Torsional stiffening rods were placed along the sides of the mechanism to help carry the load from the rotor hub to the actuation ring.

![Figure 3-3 Variable pitch mechanism](image)

**Control Surfaces**

Directional control of the vehicle would be provided by the four movable control surfaces inside the duct. These could be biased to take out the torque of the spinning
rotor blades and directional control could be applied over that. In forward flight the canards would aid in the production of lift, while the internal control surfaces acted like rudders and elevators. These surfaces would be mounted on a sliding surface so that their center of pressure coincided with the actuation axis during both forward flight and hover flight. (Figure 3-4) They would switch configuration due to the change in pressure drag force direction in hover and forward flight modes.

Figure 3-4 floating actuator
The center body of the aircraft would be supported at one end by the control surfaces and at the other end by the canard. The reaction torque from the rotor would be transferred through the canard into the duct. Therefore the canard structure should be reinforced. The preliminary design would employ a reinforced rib structure in the canard that carried across the length of the vehicle (Figure 3-5). These spars and ribs would be made out of balsa wood with carbon as the outer shell.

### 3.4 System Modeling

To gain an understanding of the flight dynamics of the vehicle a system simulation was undertaken. Many other VTOL MAVs use only one propeller and incorporate stator vanes to counter-act the torque of the rotor. In order to properly size the control surfaces
a simulation of the aerodynamic forces was developed. At the same time this model helped to determine the maximum thrust that could be expected from the rotor blades.

Control Dynamics

In this design configuration four control surfaces internal to the duct were used to control Roll, Pitch and Yaw. It was desired that these control surfaces act as control surfaces as well as stator vanes to take out the rotational torque of the spinning rotor. As a preliminary design guide to the feasibility of this approach, a control surface model was developed to predict the torque that could be produced downstream of the propeller induced velocity by control vanes.

Basic Geometric Layout

The four control surfaces of PUAV formed a cross pattern opposite the engine cylinder head. The model layout can be seen in Figure 3-6. Figure 3-7 shows a more schematic approach with the relevant moment arms and centers of gravity and pressure labeled.
Figure 3-6 Concept vehicle layout (duct removed)
Figure 3-7 Schematic dynamic force diagram

As can be seen from Figure 3-7, the torque’s caused by the drag forces cancel and the lift forces produce torque’s proportional to the magnitude of the vector form the vehicle Cg to the Cp of the control surface.

Model Parameters

The control surfaces were modeled as symmetric NACA 0009 airfoils. Data sheets\textsuperscript{vi} indicated $C_D$ and $C_L$ values for various angles of attack and Reynolds numbers. However rotor induced velocity data indicates that the controls surfaces would see a flow velocity of approximately 35 ft/s. With a chord of one inch this translates into a Reynolds number of approximately 18,000. NACA charts plot airfoil characteristics for flows on the order
of Re 3e6. This significant discrepancy was overcome by approximating maximum lift and stall characteristics. From this an extrapolated $C_D$ lookup table was created for the NACA 0009 airfoil in a low flow regime.

Control Surface lift and drag are calculated as.

$$L = \frac{1}{2} \rho V^2 AC_L \quad \text{Eq. 3-8}$$

$$D = \frac{1}{2} \rho V^2 AC_D \quad \text{Eq. 3-9}$$

The cross product of a force and the position vector from the airfoil $C_P$ to the vehicle $CG$ gives the total torque vector. Thus each force vector $F_{lx}$ is crossed with each position vector $R_{CG-CPx}$. In the vehicle simulation, a vector of four control vane angles of attack is fed into a lookup table. This function finds the force of lift and drag for each of those control angles. These angles are multiplied by a direction vector according to the control surface they represent. The force of drag always acts in the Z direction, while the lift force will act in the negative X and positive Y directions alternately depending on which surface is producing the force and the angle of attack it is at.

**Control Model Results**

The most important finding in this analysis has been the realization that the propeller induced velocity represents a low flow regime. Thus the controls are in a primarily laminar flow, and stall at significantly lower angles of attack. Figure 3-8 and Figure 3-9 show these results for constant control deflections of 6 degrees and for a constant inflow velocity of 37 ft/s. A final important result is that the control deflections will likely not be able to produce enough torque to counter act torque produced by the rotor. Much of
this is due to the low flow conditions of the propeller induced velocity, and the relatively small size of the control surfaces which are constrained by the vehicle geometry.

3.5 *Horizontal Flight Analysis*

The stability of the vehicle in during horizontal flight was analyzed. In this flight mode the annular duct acts as the main lifting surface while the four internal control surfaces provide directional control. The propeller has reversed collective to the opposite direction and is acting as a pusher propeller.

![Six Degree Deflection Graph](image)

*Figure 3-8 (6) Degree control deflection*
Modeling the duct accurately was difficult without the use of a wind tunnel. In lieu of those facilities data was gathered from third party analysis of ducts from other research papers. Details on the equations used for analysis were provided with the aid of co-workers. From acquired duct data the best L/D point was extracted as 4 at an angle of attack of 18 degrees. At this operating point the coefficient of lift was 1 and the coefficient of drag was .25. This estimates the duct as a NACA 0012 airfoil with an aspect ratio of .19.

Figure 3-9 (37) ft/s constant inflow velocity
The vehicle top speed is estimated as the point where the drag of the duct is equal to the thrust. This gives the equation:

\[ D = T = \frac{1}{2} \rho v^2 S C_D = 10N \]  \hspace{1cm} \text{Eq. 3-10}

the value of ten newtons is derived from the estimated vehicle weight of 1 Kg, and the surface area of the duct is estimated from a chord of six inches and a radius of five inches. This calculation yields a top speed of 41 m/s (80 knots).

Similarly the cruise speed of the vehicle is defined as the trim point between lift and weight. This gives the equation:

\[ L = W = \frac{1}{2} \rho v^2 S C_L = 10N \]  \hspace{1cm} \text{Eq. 3-11}

Therefore the vehicle cruise speed was found to be 20 m/s (40 knots).

The vehicles stall speed was calculated using the lift to drag ratio at the \( C_1 \) maximum point. With a L/D of 1.8 at an angle of attack of 38 degrees the use of Eq. 3-11 results in a stall speed of 16.7 m/s (33knots). Using the vehicles estimated cruise speed in Eq. 3-10 and solving for thrust results in a value of 2.5 newtons of thrust required to cruise. The vehicles stall speed at the maximum lift condition is used in Eq. 3-10 to find the thrust required at this operating point.

To determine the characteristics of the transition maneuver a simulation was run with the canards set at an angle to produce the maximum lift from the duct. The time domain simulation was then evolved and the characteristic curve describes the vehicle’s motion through transition. This simulation resulted in a transition curve with a radius of 120 feet and that would subject the vehicle to no more than 3 G’s.
3.6 Summary

The use of simulation models assisted in verifying the basic design and brought us to the conclusion that the control surface layout and its interaction with the propulsion system was not entirely feasible. Both the propulsion system and the control method would need to be redesigned for more stable and robust operation. Four movable canards of 6 square inches each will be needed to maintain control authority of the vehicle through transition and in both flight modes. This size requirement would prohibit the canards from fitting within the duct. As an alternative the use of counter rotating blades was proposed. Although this would increase the complexity of the design, this configuration could produce the desired torque cancellation effects and keep vehicle size at a minimum. The concept of using the control surfaces as anti-torque stators and producing directional control by biasing them was thus deemed unlikely to work. Calculations of the vehicles aerodynamic performance did yield useful results for the basic operating parameters in flight. An annular wing of 10" diameter and 6" chord with a NACA 0012 section profile would generate enough lift to sustain forward flight at a cruise speed of 50 mph for a 3 pound vehicle. The maximum speed would be 60 mph and stall speed would be 40 mph. The transition from hover to horizontal flight would take approximately 120 feet of vertical distance and would produce forces of 3 G’s on the vehicle airframe.
4 Alpha Design Cycle

The completion of initial design validity allowed for the initialization of the first of two design cycles. This alpha design phase incorporated the concept of counter rotating blades and further decreased the weight of the vehicle by decreasing the part count and increasing the use of composite materials. Modeling done in section 3.3 had pointed to a vehicle target weight of 3-4 pounds. Component layout and positioning details were finalized so that stability margins in forward flight could be estimated. Specific actuators, engines, bearings and gears were ordered. The airframe was detailed at this stage (Figure 4-1) since it would likely be an integral part of the structure supporting the drive train.

4.1 CAD Design

![Figure 4-1 Preliminary alpha conceptual layout](image)

Figure 4-1 Preliminary alpha conceptual layout
Airframe

The airframe of the vehicle would consist of the duct, center body, and fore body. The duct would act as a rigid support for the engine and center body. The fore end of the main shaft running axially through the vehicle would secure into the fore body. This would effectively use the duct as a rigid member in the drive train. The stress loop of forces produced by the operating engine and aerodynamics would be closed along the path from the engine through the supporting spars and the duct and back into the main shaft. (Figure 4-2). Since the vehicle was relatively small in comparison to the drive train it was believed that vibration effects would be of too high a frequency to present a problem.

Figure 4-2 Alpha design airframe
The fore body pods would house the servo actuators used to control the four independent canards as well as the batteries used to power the radio system (Figure 4-3). The fore body itself was reserve space for a MEMS IMU developed at The Draper Laboratory for MAV navigation.

![Figure 4-3 Fore body pods](image)

**Figure 4-3 Fore body pods**

**Drive train**

One of the most important constraints on the drive train was the need to fit it within the six linear inches of the duct chord. This meant that the stack up of drive train components (differential mechanism, variable collective and engine) must amount to no more than six inches. Iterations on stack up calculations led to the conclusion that the optimum arrangement (minimum linear spacing) would be to lay the engine on its side and run the drive train along the side of the duct and then back in to a differential drive.
Another contributing factor in this decision was that the complexity of any gears, and gear ratio changes could be built into the duct, which at 12% had a maximum thickness of 0.75” and was virtually empty. Detailed reading on counter rotating propellers also suggested that for maximum thrust an optimum vertical spacing between the rotor disk was a ratio of 0.1 vertical spacing to rotor diameter.

The first iteration of the drive train used a rigid differential encased in a gear box mounted in the center of the vehicle. This design is shown in (Figure 4-4). However, as component weights escalated every gram of weight became critical. Much of the structure that surrounded the gearbox was unnecessary.

![Preliminary differential gearbox design](image)

**Figure 4-4 Preliminary differential gearbox design**

In light of weight issues and the ideal spacing of the counter rotating blades, the design was revised to that shown in (Figure 4-5). This new lightweight design used a
minimum amount of material and left the gears open. To cut down on size the propeller hubs were made essentially part of the gear. Bearings on the hubs were counter bored inside the gears to reduce vertical stack up distances. The end result was a set of counter rotating propellers with a spacing of \(0.15 \text{ H/D}\).

![Propeller hub/gear assembly](image)

**Figure 4-5 Revised alpha gearbox design**

A primary technical issue was how to reliably transmit power from the engine outside the duct and back into the gearbox. Two competing alternatives were reviewed. The first was to use a belt drive system along the outside of the duct. The second alternative was to use two sets of beveled gears. A trade study of several miniature belt
drives showed the best belt option to be a HTD curvilinear profiled timing belt. These belts are neoprene rubber with fiber fillings and haploid curved grooves.

![HTD Curvilinear and Trapezoidal Tension Member at Pitch Diameter](image)

**Figure 4-6 HTD Belt stress concentrations**

This enables the tooth load to be carried over much of the face of a pulley tooth and thus the belts are capable of higher rpm’s and loads (Figure 4-6). Even with these advanced belt drives the vehicle requirements of 0.25 N/m torque at 10,000 rpm would push the belt to the limit. (Figure 4-7) A study of gear drives was undertaken.

![Design Horespower Per Inch of Belt Width](image)

**Figure 4-7 EPS, HTD belt selection table**
Computational Validation

Lewis form factor calculations were performed to determine the maximum load of individual gear teeth at the expected operating rpm and torque. The Lewis calculations are a means of computing the bending stress in gear teeth. (Figure 4-8) shows a simple cross section diagram of a gear tooth. The bending moment in this tooth is due to the tangential load \( W_t \). With a tooth length of \( l \) and cross sectional dimensions \( F \) and \( t \) as shown the bending stress can be described as:

\[
\sigma = \frac{M}{I/c}
\]  
Eq. 4-1

where

\[
\frac{I}{c} = \frac{Ft^2}{6}
\]  
Eq. 4-2

using geometry and substitution the stress can be described in terms of a gears diametrical pitch as:
where the Lewis form factor

\[ Y = \frac{2xP}{3} \]

Eq. 4-4

Figure 4-8 Forces on miter gear mesh

The force \( F_t \) and the other forces acting on the gear were then calculated for the case of the miter gears to be used in the drive train. Figure 4-9 shows the geometry of the gears and gear forces considered. Reference\(^x\) gives the forces on the gear as:

\[ F_u = F_n \cos \alpha \]
\[ F_1 = F_n \sin \alpha \]

Eq. 4-5

The radial force \( F_1 \) can then be divided into axial and radial forces

\[ F_a = F_1 \sin \delta \]
\[ F_r = F_1 \cos \delta \]

Eq. 4-6

Table 4-1 Lists the relevant geometric and dynamic information for the set of gears that was analyzed.
Figure 4-9 Directions of forces on gear mesh

Table 4-1 Lewis factor analysis case

<table>
<thead>
<tr>
<th>Gear specifications</th>
<th>Case Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus (mm)</td>
<td>Speed (rpm)</td>
</tr>
<tr>
<td>0.5</td>
<td>Torque (Nm)</td>
</tr>
<tr>
<td>Pitch Diameter (mm)</td>
<td>Power (watts)</td>
</tr>
<tr>
<td>32</td>
<td>Diametrical Pitch (mm)</td>
</tr>
<tr>
<td>Pressure Angle (deg)</td>
<td>Pitch Line Velocity (m/s)</td>
</tr>
<tr>
<td>20</td>
<td>Kv</td>
</tr>
<tr>
<td>gamma (deg)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>tooth height (mm)</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Face Width (mm)</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Number of Teeth</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Lewis Form Factor (Y)</td>
<td></td>
</tr>
</tbody>
</table>

The bending stresses calculated from the Lewis formula are then compared to the maximum yield stress for the gear material. In this case the stock gears are made of brass, whose yield stress is 44 (kpsi) MPa. Maximum bending stress was also calculated as the product of several modifiers used as estimators for different physical
properties. The equations for this semi empirical approach were supplied by the manufacturer. Reference 2 gives the stress as:

\[ \sigma_F = F_{tm} \frac{Y_F Y_c Y_B Y_C}{\cos \beta_m m b} \frac{R_a}{R_{a-.5b}} \left( \frac{K_M K_N K_O}{K_L K_{FX}} \right) K_r \]  

Eq. 4-7

The resulting stress \( \sigma_F \) was calculated with three different configurations of Eq. 4-7 resulting in values of between .29-.44 Mpa. These calculations indicated that a .8 module 20mm PD brass miter gear could withstand the forces produced during operation by a factor of approximately 100. Results are listed in Table 4-2. The module .8 gear was the smallest gear available off the shelf that had a large enough face to be bored out to accommodate the smallest bearing available off the shelf. Based on these considerations, the gear train was chosen as the best means of transmitting power.

**Table 4-2 Gear force and pressure results**

<table>
<thead>
<tr>
<th>Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ft</td>
<td>0.012434</td>
</tr>
<tr>
<td>Fr</td>
<td>0.004508</td>
</tr>
<tr>
<td>Fa</td>
<td>1.43E-05</td>
</tr>
<tr>
<td>bending stress(KgF/mm2)</td>
<td>0.298424</td>
</tr>
<tr>
<td>bending stress2(KgF/mm2)</td>
<td>0.06908</td>
</tr>
<tr>
<td>bending stress3(KgF/mm2)</td>
<td>0.441742</td>
</tr>
</tbody>
</table>

**Variable pitch mechanism**

Two factors drove the design of the variable pitch mechanism. First, the design must create a minimum blockage inside the duct, allowing as much air to flow by as possible. This constraint would drive a design that had the smallest radius about the vehicle's center axis. Secondly, space within the duct was limited and must create a minimum obstruction to airflow through the duct. This constraint would drive the two
variable pitch servo actuators to one end of the duct where they could be packaged together.

With both servo actuators on one side of the propeller disks, the challenge was to create a means of actuation that could operate through or around the spinning propellers. The option of multiple links or guide wires was discussed but dismissed because of their complexity and weight. What was proposed was the concept of a non-rotating shaft on which the propeller hubs spun. If this shaft were hollow then the mechanical actuation linkage could drive through this shaft from one side of the propeller disks. Figure 4-10 and Figure 4-11 display the drive train/servo actuation mechanism.

![Diagram of variable pitch mechanism]

**Figure 4-10 Cut-away view of variable pitch mechanism**
The basic design of a variable pitch mechanism was borrowed from typical RC model designs. As can be seen in Figure 4-12 the design incorporates a brass carrier that rides on the non-rotating shaft. The interface between the brass carrier and the aluminum shaft sees only slow linear movement when the propeller pitch is varied. The rotational motion is accommodated by two 5mm ID, 10mm OD 2mm width bearings that slip over the brass carrier. The inner race of these bearing is stationary while the outer race is bonded to the actuator arm. The actuator arm piece and the bearings are held in place by a small star retaining ring.
Figure 4-12 Exploded variable pitch actuator
The exact dimensions of the non-rotating shaft and the variable pitch actuating links was determined by the amount of variable pitch required. Aerodynamic models suggested that the propellers would need a maximum of ±15° in both flight modes. To determine the linear actuation space required to create this 15° of rotation, a simple model of the mechanism was created. The variable pitch mechanism was modeled as a four bar linkage system as shown in Figure 4-13.

![Four bar mechanism](image)

**Figure 4-13 Four bar mechanism**

The link labeled R, represents the actuating arm of the propeller blade clamp. This part is molded plastic and came off-the-shelf so its length was fixed at 15.2mm. The link labeled C represents the actuator arm that rides on the brass carrier. The length of this arm could be varied during the design phase to change the gain of the variable pitch mechanism. The link labeled L represents the link arm that would be manufactured to fit between the blade clamp and the actuator mounted on the brass carrier. The formulae governing the relationship between the three linkages, the angle of variable pitch θ, the angle of φ of link L and travel distance of the carrier x, are listed below.
\[ x = L \cos \varphi + R \cos \theta \]  \hspace{1cm} \text{Eq. 4-8}

\[ R \sin \theta = L \sin \varphi + c \]  \hspace{1cm} \text{Eq. 4-9}

\[ x = \sqrt{L^2 - (R \sin \theta - c)^2} + R \cos \theta \]  \hspace{1cm} \text{Eq. 4-10}

Iterations through these equations resulted in a reasonable value for the length C and a range of motion \( x \). The resulting distance \( x \) was 10.39 mm with a linkage \( L \) of length 10mm and a linkage \( C \) of length 10mm. This allowed for a variable pitch of ±40°, leaving some margin beyond the minimum requirements for variable pitch derived in aerodynamic simulation. In the design of the main shaft, adequate space was left for the carrier to traverse the length \( x \) on each side of the propeller disks.

**Control Surfaces**

The Alpha vehicle design incorporated four independently movable canards below the duct. In hover they would act as flight control surfaces and would aid in the transition to forward flight. Their aerodynamic shape was chosen based on calculations of the maximum output torque of the servos to be used. FMA S90 servos had the highest torque to weight ratios of all micro servos on the market (19 in/oz). Eighty percent of this torque value was taken as a maximum actuation torque and the center of rotation of the canard was found based on this.

The moment generated by an airfoil is given by:
\[ M = \frac{1}{2} \rho v^2 VC_m \]  

Eq. 4-11

The canards would see two different flow velocities based on the vehicle's flight mode. In Forward flight, the vehicle would be traveling at approximately 55 mph. Based on simulated calculations of propeller induced velocity, the canards would see approximately 30 mph flows in hover. The airfoil would need to be symmetric from top to bottom because it would see flow in both directions. The center of actuation for the canards was placed where the moment arm would not exceed 80% of the maximum servo torque in either flight mode. The servo actuators for each canard were mounted in pods that protrude from the duct. These pods not only house the servos, but were used to house the batteries, and act as landing gear.

4.2 Weight and balance issues

The stability profile in both flight modes was analyzed by predicting the position of the CG and Cp in hover and horizontal flight. The CAD model of the vehicle allowed for the prediction of the center of mass of each major weight component and its weight. These values were entered into a spreadsheet and the CG, Cp calculations were performed. Table 4-3 and Table 4-4 show the each individual component and its relative weight and position to the vehicle CG. The moment arm for each element was calculated. Early vehicle layouts gave a negative stability margin in horizontal flight. This occurred because the engine was located at the trailing edge of the duct airfoil and represented the largest single contributor to weight. At this stage of design all electronic components were located in the duct and had been moved as far forward as possible. Moving the engine to the leading edge and the propellers to the trailing edge could not completely correct the problem of a negative stability margin because the moment arm...
would not be sufficiently large. It was decided that the second largest contributor to weight (the batteries for the RC electronics) could be moved far forward as their profile was small and could be packaged easily. The challenge was to develop an aerodynamic means of positioning the batteries in forward of the nose of the vehicle. Concepts for this ranged from thin arching booms from the duct leading edge ending in a battery, to support rings in front of the nose and pods protruding from the duct.
Table 4-3 Structural weight distributions

PUAV WEIGHT BUDGET
All c.g. locations relative to front nose

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Duct chord = 6.00 inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>Mass</td>
</tr>
<tr>
<td></td>
<td>grams</td>
</tr>
<tr>
<td>Duct Composite</td>
<td>100.00</td>
</tr>
<tr>
<td>Fuselage Composite</td>
<td>20.00</td>
</tr>
<tr>
<td>Canard Surfaces</td>
<td>10.00</td>
</tr>
<tr>
<td>Structural Subtotal</td>
<td>0.287</td>
</tr>
</tbody>
</table>
Table 4-4 Propulsion and Avionics weight distributions

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<thead>
<tr>
<th>Item</th>
<th>Wt (lbs)</th>
<th>Wt Frac</th>
<th>Wt %</th>
<th>Wt Fr%</th>
<th>CG</th>
<th>Av Frac</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td>270.00</td>
<td>0.595</td>
<td>5.00</td>
<td>2.977</td>
<td>-1.241</td>
<td>0.918</td>
</tr>
<tr>
<td>Collective 1</td>
<td>10.00</td>
<td>0.022</td>
<td>0.088</td>
<td>-0.241</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>Collective 2</td>
<td>10.00</td>
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<td>0.209</td>
<td>-5.741</td>
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<td></td>
</tr>
<tr>
<td>Fuel Tank</td>
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<td>3.759</td>
<td>1.243</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Propulsion Subtotal</strong></td>
<td><strong>0.970</strong></td>
<td><strong>4.518</strong></td>
<td><strong>1.646</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Servo 1</td>
<td>9.07</td>
<td>0.020</td>
<td>0.060</td>
<td>0.759</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>Servo 2</td>
<td>9.07</td>
<td>0.020</td>
<td>0.060</td>
<td>0.759</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>Servo 3</td>
<td>9.07</td>
<td>0.020</td>
<td>0.060</td>
<td>0.759</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>Servo 4</td>
<td>9.07</td>
<td>0.020</td>
<td>0.060</td>
<td>0.759</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>Collective Servo</td>
<td>9.07</td>
<td>0.020</td>
<td>0.120</td>
<td>-2.241</td>
<td>0.100</td>
<td></td>
</tr>
<tr>
<td>Avionics batteries</td>
<td>83.00</td>
<td>0.183</td>
<td>0.500</td>
<td>0.092</td>
<td>3.259</td>
<td>1.943</td>
</tr>
<tr>
<td><strong>Avionics Subtotal</strong></td>
<td><strong>0.283</strong></td>
<td><strong>0.451</strong></td>
<td><strong>2.090</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The concept of a thin boom protruding forward of the duct leading edge and housing the batteries at the end soon evolved into a multi-use pod. The pods would extend farther than the avionics section allowing them to be used as landing stilts in hover flight. They would house the batteries at the far end successfully moving the CG forward of the Cp in horizontal flight. They would also house the servo actuators for the canards allowing the center body to be minimally small.
Table 4-5 Payload distribution and total CG calculation

<table>
<thead>
<tr>
<th>Payload</th>
<th>Instrumentation package</th>
<th>Camera</th>
<th>Main Ballast (Lead)</th>
<th>Payload Subtotal</th>
<th>SYSTEM TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>120.00</td>
<td>30.00</td>
<td>0.00</td>
<td>0.265</td>
<td>1.500</td>
</tr>
<tr>
<td></td>
<td>0.066</td>
<td>0.000</td>
<td>0.250</td>
<td>0.397</td>
<td>2.259</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.250</td>
<td>0.000</td>
<td>1.350</td>
<td>1.350</td>
</tr>
<tr>
<td>CG Location =</td>
<td>3.759</td>
<td>2.284</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage Location =</td>
<td>19.27</td>
<td>7.031</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From duct lip</td>
<td>0.241</td>
<td>7.756</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duct lip position</td>
<td>4.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required CG Position</td>
<td>4.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost parameter</td>
<td>-0.241</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of the CG calculation are summarized in Table 4-5. They indicate a vehicle with a 19% stability margin, where the CG is located ¼ inch in front of the leading edge of the duct. The leading edge is considered the composite Cp for the vehicle including the canards as contributors to lift.
5 Manufacturing

Manufacturing for this vehicle involved two major areas of concentration. The mechanical drive components were primarily metal and machined on traditional mills and lathes. The airframe and all other structural components were constructed from graphite composites and were made either by traditional wet lay-up on female molds and vacuum bagging processes or a hybrid compression lay-up using male and female molds.

5.1 Composite Design

Composite materials are becoming more prevalent in aerospace design today. They offer lighter weights with comparable stiffness to their metallic competitors. They were specifically useful in this vehicle design because of the ability to form a complex shape from a mold. This allowed for the production of the complex geometry's of the duct, canards and faired supports that would otherwise be formed out of metals. In some cases only a lightweight faired form was needed to cover an opening or produce an airfoil surface (i.e. the outside shell of the duct). Because the life cycle of the vehicle was predicted to be short, cyclic fatigue was not a concern. Heat was also not a major obstacle in the composite design of this vehicle because of the relatively low operating temperatures of the small IC engine. Therefore little research was done into the type of resin to be used.

Factors

Initial iterations on the construction of the duct were more flexible than expected. This was because the ducts curvature was mainly in the radial direction. A more complex curvature with curves in different axes would create strength in multiple directions. It was calculated that the airframe should weigh less than 300 grams to meet the necessary
take-off weight. This dictated a duct weight of less than 180 grams. However, it was believed that the stiffness of the duct could be increased by altering the geometry and construction technique while maintaining the same weight. The first alteration was to the shape of the inner duct. Aerodynamic considerations suggested that the leading edge of the duct should be molded as one piece so that the airflow over this surface was as smooth as possible. After implementing the necessary changes in the mold it was found that the duct was significantly stronger. The complex curve created by making the leading edge part of the duct body dramatically increased the shape’s overall stiffness. This reinforced the concept that the inner duct could carry all the dynamic loads on the vehicle simply by making it a more complex, and stiffer geometry. The same wrap-over design was then implemented in the trailing edge, again increasing overall stiffness. Figure 5-1 summarizes the design iterations pursued. Figure 5-1A displays the first duct iteration with poor vacuum bagging and no front or rear lip. Figure 5-1B displays a carbon duct with a leading edge lip only. Figure 5-1C displays a duct constructed from one ply of 4 oz. fiberglass, this design was too flexible to be implemented. Figure 5-1D displays the final iteration on the duct with leading and trailing edges as well as integrated mounts for pods. This design was the final iteration used in the Beta vehicle design.
Lessons Learned

The type of fiber used was also a significant factor in the strength of the duct. To conform to the complex curves of the new geometry a light weight fabric was needed. However the patterns of the weave played an important role in the overall permeability of the duct. The first choice of fabric was a simple $0-90^\circ$ fabric with one over one under weave. This is a weave where the fibers in the zero degree and 90 degree directions are woven over and under every strand (Figure 5-2). This provided the maximum conformability for the fabric but because only one layer could be used to keep weight down, it resulted in a part with several visible holes. Since this would not be aesthetically feasible, a tighter weave fabric was needed.
The most challenging aspect of the geometry was the leading edge. It was difficult to get fabric to conform to the complex shape, and stay tight to the mold surface to produce a void free part. When a good fit was produced it was extremely difficult to remove the part from the mold without damaging the mold. Experiments were done with twill fabrics which weave the fabric at 45 degree angles (Figure 5-3).

Different ducts were produced with heavier weight fabrics up to 9 oz per square foot. The final solution was to cut the fabric of the leading edge as a circular piece and place it in the leading edge. Then peel ply and a silicone tube was placed into the leading edge. This allowed the vacuum bag to apply pressure to the tip of the leading edge without actually being pushed into the sharp corner of the leading edge (Figure 5-4).
5.2 Mold Design

Traditional Composite female molds can be made from male plugs coated in fiberglass or dipped in Urethane, or molds machined directly out of metals or hard plastics. When first approached with the challenge to create the composite airframe of this vehicle these were the options considered. The male plug could be made in a rapid prototyping machine. Stereolithography (SLA) machines or 3D printers would both have the resolution capabilities to create viable parts. (A 3D printer builds layers of deposited powder, in this case paper, much like a SLA.) The male parts could then be cast into female molds.

As in most projects, time was a driving factor, in an effort to minimize the steps required to make composite parts it was decided to use the rapid prototyping capabilities to move directly from CAD design of a male part to design and manufacturing of the
mold. Thus the SLA machine was used to create the molds directly. In determining the
gometry of the mold the build time and the overall strength of the mold were considered.
The mold would need to endure the force of the vacuum to be pulled on the part as well
as point loads generated from tools while trying to remove the final composite part.
Build time increases directly with the height of the part and the amount of material in a
layer. Therefore the attempt was to keep the molds thin, using gussets and flanges to
increase strength.

Figure 5-5 Multi-part Mold

Further developments of more complex parts necessitated the use of multiple part
molds. These molds had several sections screwed together during lay-up that allowed for
easier removal after vacuum bagging. (Figure 5-5)
Lessons Learned

The small size of the vehicle created a need for several small composite fairing parts to support the center body. Because of their small size it was not always practical to use a conventional female mold with vacuum bagged lay-up.

The concept of compression molding lent itself to the quick production of small detailed parts. The complex aerodynamic geometry’s and mating features on the fairings could be produced directly from CAD models on the SLA machine. A male and female part would be made and pressed together with the fiber sandwiched in between (Figure 5-5). This required no peel ply, bleeder cloth or vacuum bags and produced a clean strong part. (Figure 5-6). The attractiveness of the compression molding technique spread its use to several components. However as the part sizes became larger it became difficult to remove the compression pieces without damaging or destroying them.

The SLA resin parts were too brittle to withstand the tools used to remove them. What was needed was a flexible compression part that would bend and deform when removed. The solution was to
use urethane resin was used as a male plug. The plug was formed by pouring urethane into the female mold. The resulting part would leave no space for the carbon fiber, but would conform because of its flexibility. Experiments were tried with different hardness urethanes and 40 durometer resin was found to have the best flexibility while maintaining its shape. (Figure 5-7)

Figure 5-7 Urethane male mold insert
6 Testing

Testing of PUAV followed a path of envelope expansion aimed at gaining insight into both the aerodynamic mechanical properties of the vehicle under dynamic operating conditions. Concurrent design changes would be implemented as testing revealed more information.

6.1 Ducted rotor tests

A properly shaped duct will typically cause an increase in propeller performance because of the reduction of tip vortex shedding off the propeller blades, therefore the shape of the duct inlet was a primary concern in the design of the duct. It was not clear what effect the inlet geometry would have on the flow entering the duct in hover. For optimal forward flight the duct would have a symmetric airfoil cross section. This would produce the maximum lift while flying horizontally. However, in hover, the trailing edge of the duct in horizontal flight becomes the inlet. It was believed that a sharp inlet might separate flow entering the duct and produce a large separation bubble that extended far into the duct disturbing the flow onto the propeller disk and reducing it’s ability to generate lift. This separation bubble could likely be unstable and thus cause oscillatory disturbances to thrust making the vehicle harder to control. Unfortunately the ideal shape for a duct inlet would be a bell shaped inlet. This geometry would not be practical for this design because the vehicles dual flight mode performance would require it to experience flows in both directions.

In an effort to quantify the effect of the duct shape some preliminary test were conducted using an electric motor. Figure 6-1 shows the test set-up involved. A counter balanced arm was placed over the edge of the table with one end on the scale and the
extended end supporting the electric motor and duct. This allowed the motor and propeller to be well out of ground effect. A single two bladed untwisted constant chord propeller was run between 4,000-8,000 rpm at fixed pitch with the duct and without it. Rpm was measured using a strobe. Thrust was measured on the scale. The conclusion derived from these results was that a sharp or a blunt inlet geometry made little difference to the thrust produced by the duct. Thrust produced with and without the duct was essentially the same.

![Diagram](image)

**Figure 6-1 Electric motor thrust test setup**

This result was unexpected as ducts usually increase the performance of propellers. One explanation for this result is that the shape of the inlet whether blunt or sharp was not a bell curve and thus both cases reduced the power of the propeller. It is also possible that the blades tested were not close enough to the sides of the duct and the
resulting interference caused a reduction in the propeller lift instead of an increase. It was determined that the relative thrusts in each case were close enough to justify the NACA 0012 section for use as a horizontal toroidal wing as well a hovering duct.

6.2 Electric motor tests

The next objective for testing was to develop a quantitative concept of the yaw control and thrust that the counter-rotating configuration would create. An overview was done of several off-the-shelf electric motors as well as some custom windings. The conclusion of this was that most stock motors for under $300 could not fulfill estimated power requirements of 625 Watts for a 30-60 second duration. However for short burst of time some stock high performance RC motors existed that could produce acceptable power. An Aveoxxii 4-wind motor was installed in the vehicle in place of the internal combustion engine. This would allow for safe quiet indoor testing. An electric motor has much less vibration, and a continuous rpm range that would allow us to perform safer initial tests.

An 18 cell 24 volt rechargeable battery pack for the electric motor was tethered to the vehicle. The vehicle was suspended from a fish weighing scale as detailed in Figure 6-2. If the vehicle created enough thrust to lift off and was unstable then it would not be able to roll over and damage itself. Even if the vehicle could not produce enough thrust, simply subtracting the displayed weight from the known vehicle weight would give the thrust at that time.
Initial test with both propeller blades produced very little thrust. It is believed that this is due to the lower propeller not being at the proper relative angle of attack to the first blade. This effectively makes the second propeller disk a thrust sink. When the lower blade was removed the vehicle could swing up, suggesting it was capable of producing enough thrust to hover. However flight test with the electric motor were inconclusive due to the inability to run for long enough to strobe the propellers and the inability to produce enough torque to attain 10,000 rpm at any angle of attack greater than 5 degrees.

### 6.3 IC engine tests

With the idea that increased torque would enable the vehicle to lift off the decision was made to move ahead and install the IC engine. The engine used was an off-
the-shelf OS .25 model engine. Its rated capacity is .82 bhp and 18,000 rpm, under load it was expected to run at 10-12,000 rpm. It was anticipated that the IC engine would produce significant vibration disturbances that were not seen with the electric motor. This is due to the periodic nature of the cyclic combustion. Not only does the engine produce periodic vibrations in the vertical (piston travel) direction, the drive shaft also produces oscillating torques. What was not anticipated was the difficulty in starting the engine. As the engine shaft rotates, the torque reacting the shaft varies greatly. As the piston passes through top dead center the highest torques are seen and immediately thereafter the torque is virtually zero. This oscillating torque effect produced large oscillating torques in the shafts shearing gears and keys in the drive train. These oscillations caused a resonant vibration between the engine unit and the vehicle airframe. As can be seen in (Figure 6-3) the system became a two mass vibrating system without any stiffeners in place.

\[ \omega = \sqrt{\frac{k}{m}} \]

Figure 6-3 Two mass oscillating system
IC engine tests were conducted with and without the carbon duct in place. Testing was conducted with the drive train mounted to a test stand as shown in (Figure 6-4). The stand is allowed to pivot freely in the horizontal direction on its bearings. The propellers will produce thrust horizontally pulling on the wire anchored to the end of the drive train. This wire loops through a small pulley and is attached to a ten pound weight placed on a scale.

![Figure 6-4 IC engine ducted test stand](image)

The scale is tarred with the weight in place, and thrust measurements are taken as negative weight values when the propellers pull up on the weight. RPM is measured through the use of a reflective IR sensor mounted on the drive train looking at the flywheel. Half of the flywheel is wrapped in black tape while the other half is left as
polished aluminum. Each revolution of the flywheel produces a step wave pulse signal from the IR sensor. This is read on the oscilloscope and the frequency is calculated. RPM is then 60 times the frequency readout on the scope. Measurements of thrust and RPM were taken for a variety of collective settings and two major throttle settings. The goal was to slowly expand the envelope of drive train performance and to gain an understanding of the relationship between the upstream collective pitch and the downstream collective pitch. It was known that the downstream blades would see a higher induced velocity than the upstream blades, but exactly how much and what effect this would have on collective setting were unknown. Consequently, collective for the upstream blades was kept constant while the downstream collective was increased from 0° to 30° of pitch. This process was repeated for three upstream collective settings at 0°, 12° and 24°. The results of this test are listed in Table 6-1 and the trend plotted in Figure 6-5.

**Table 6-1 Thrust, RPM, collective settings and throttle setting (unducted)**

<table>
<thead>
<tr>
<th>RPM</th>
<th>Upstream Collective (deg)</th>
<th>Downstream Collective (deg)</th>
<th>Thrust (kg)</th>
<th>Throttle Setting (%)</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>9000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>150</td>
</tr>
<tr>
<td>9000</td>
<td>0</td>
<td>15</td>
<td>0.14</td>
<td>30</td>
<td>150</td>
</tr>
<tr>
<td>7200</td>
<td>0</td>
<td>30</td>
<td>0.4</td>
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<td>120</td>
</tr>
<tr>
<td>9600</td>
<td>0</td>
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<td>0.36</td>
<td>60</td>
<td>160</td>
</tr>
<tr>
<td>7800</td>
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<td>0.03</td>
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<td>130</td>
</tr>
<tr>
<td>8400</td>
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<td>0.3</td>
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</tr>
<tr>
<td>7020</td>
<td>12</td>
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<td>30</td>
<td>117</td>
</tr>
<tr>
<td>9600</td>
<td>12</td>
<td>30</td>
<td>0.6</td>
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<td>160</td>
</tr>
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<td>110</td>
</tr>
<tr>
<td>7200</td>
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<td>30</td>
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</tr>
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<td>6800</td>
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<td>died</td>
</tr>
<tr>
<td>-</td>
<td>25</td>
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<td>0.75</td>
<td>60</td>
<td>?</td>
</tr>
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<td>8940</td>
<td>25</td>
<td>30</td>
<td>0.7</td>
<td>95</td>
<td>149</td>
</tr>
</tbody>
</table>
As expected an increase in downstream collective increased the overall thrust generated. However, an increase in upstream collective past the level of the downstream collective resulted in a net decrease in thrust. This supports the conclusion that the downstream collective sees a higher inflow velocity and will in fact produce negative lift if not set at an angle equal or slightly greater than the upstream blades. The test also provided useful information on the capabilities of the engine and collective servos. At a 30% throttle setting and full collective, the engine would stall due to the torque of the blades. At maximum throttle the collective servos were not capable of sustaining maximum collective pitch. Although the un-ducted rotor test results did not yield the necessary 1000 grams of thrust for hover, the measured 750 grams was sufficiently close to suggest that the addition of the duct would produce the extra 250 grams required. Knowledge gained from the un-ducted rotor testing allowed for further improvements to
the rotor simulation. The maximum Cl value was limited to .8 and the lift slope curve of the propeller blades was modified until the model accurately matched the data produced during un-ducted testing. The resulting simulation settings were capable of predicting the un-ducted data, and provided valuable insight into methods to improve performance. It was predicted that the blades would stall above 18° pitch. Through the use of the simulation it was calculated that the 9.45” propeller could produce 1000 grams of thrust at 18° pitch and 10,000 rpm using approximately 550 watts.

For the tests conducted on the ducted propeller the pitch settings would be fixed by small shims. This would allow for the testing of the engine capabilities independent of the servo capabilities. Also it guaranteed a constant pitch setting that could not be easily monitored given the addition of the duct. Ducted engine tests were to be run through the same data points as the un-ducted test. However, a sampling of various data points at different collective settings and a full range of throttle settings demonstrated a limit to engine rpm and consequently thrust. It was found that the engine was incapable of attaining an rpm greater than 6000. The maximum thrust was limited to 650 grams. Careful study of testing video tapes suggested the possibility of separation vortices forming on the sharp inlet of the duct. Tufts places around the inlet facing in the direction of flow would rapidly oscillate from an inward position to outward and back during operation. The engine was incapable of higher rpm’s once it passed the 50% throttle setting. Further increase in throttle would result either engine flooding or no further increase in rpm.
Figure 6-6 Typical model IC engine performance curves

These results and the study of model IC engine performance curves led to the conclusion that the engine was torque limited. A graph of typical IC engine performance (Figure 6-6) shows a peak in power at approximately 85% of maximum rpm\textsuperscript{iv}. Ducted rotor test revealed that the limit in engine rpm was due to an inability to produce adequate torque. Likewise, in order to reach the operating point of 750 watts suggested by simulations the propeller must see a higher applied torque from the engine.
7 Beta Design

Development of an Alpha vehicle design revealed several opportunities for improvement. While ongoing changes were being made to the drive train on the Alpha vehicle the Beta concept was being developed to accept the finalized working Alpha drive train.

7.1 Integrated inner duct

The overriding concept in the Beta design was greater robustness and lighter weight. By reducing parts count and reliance on post production, part joining operations weight was decreased dramatically. The internal duct assembly was reduced to one part with integrated mounting points for the front pods. The elimination of the previous pod mounting points allowed continuous circumferencial access to the leading edge of the duct. This area could now be occupied by the fuel tank (Figure 7-1). The tank would be a one piece silicone rubber tube bladder. Its position in the leading edge would help maintain the CG as far forward as possible. The complex geometry of the integrated pod mounts made the part difficult to manufacture but further increased the stiffness of the leading edge.
The Beta design enabled a second revision of the fore body. This came late enough in the manufacturing phase to be able to identify improvements to be made in assembly as well as fabrication. The foremost priority was to lighten the fore body structure which up until this point was made of SLA resin and weighed approximately 500 grams. This was 40% of the overall vehicle target weight. Experience gained with composites and SLA molds had advanced enough to allow for the creation of all the fore body parts out of composite shells. This would cut the weight more than 40% using one layer carbon lay-ups throughout most of the structure. The main body which would eventually house the avionics package was constructed from a four piece SLA mold that allowed us to create a one piece body (Figure 7-2) with integrated canard roots. The final weight of this piece was 25 grams compared to the 45 grams of the SLA main body.
Together with new composite pods and canards the fore body structure would weigh 300-350 grams, a 35% reduction in weight.

![Figure 7-2 Fore body mold](image)

**7.3 Pod redesign**

The alpha design used 4 screws to attach each pod to the duct. Each of these screws is stainless steel and weighs approximately .75 grams. With a total of 16 screws around the vehicle the weight contribution of these screws is 12 grams. The beta design would cut the weight of each pod mounting area by 8 grams and used half the number of screws. The pods are used as mounting points for the servo actuators that control the canards. The beta design would further refine the role of the pods by changing the actuation method from a friction string driven method to a bevel gear driven system. All components would be completely encased in the pod, presenting a smooth surface to the outside airflow. The pods would be completely one layer carbon fiber lay-ups with a one piece body, and one easy access hatch on the outside facing side (Figure 7-3).
7.4 Drive train

During testing of the Alpha vehicle several design changes were made to enhance the performance of the system. Two primary design improvements were the addiction of a one way clutch and the addition of a flywheel. Although the two stroke engine would run without a flywheel, the direction of its rotation was not consistent on every startup due to the lack of momentum. A small (55mm diameter, 60 gram) flywheel was attached to the starting cone for the engine. The addition of a one way Sprague clutch between the engine and the propellers was needed to prevent damage to the gear drive. When powered down, the IC engine requires significant torque to turn through top dead center.
This resistance to rotation is applied directly to the spinning propellers which at full operating speed will have a large rotational inertia. Without the addition of the clutch the torque applied to the drive system immediately after engine shut down was high enough to strip the brass gears. The one way Sprague clutch eliminated this problem.
7.5 Future Work

The Perching UAV design up to this time has been a prototype vehicle and as such is not well refined. Many changes were made on short notice to add parts to fix problems or increase functionality. As a consequence the design has evolved some complexity and that is in need of consolidation. Once the basic problem parameters have been identified and evaluated through testing of the prototype, redesign can incorporate these new findings.

Testing did not produce adequate thrust for hover. The primary cause for this is the engines inability to operate in its most efficient range. If it is assumed that torque for the engine is relatively constant, then the maximum power output will occur at the engines highest rpm, and the load on the engine must be tailored to its capabilities. Likewise the torque provided to the propellers must be matched to the required operating conditions. This would likely involve a gear reduction from the engine to the propellers. Allowing the engine to run near 18,000 rpm, and provide adequate torque to the propellers.

The design of the propellers could also be improved. Currently the propellers are stock R/C helicopter tail rotor blades. A higher overall propeller solidity would help to increase the thrust produced, and a larger propeller blade chord would produce less drag than an increased number of blades. If the restriction on vehicle diameter could be relaxed to 12” a larger diameter rotor would also improve the thrust generated by the propellers.

Un-ducted rotor test at maximum throttle and maximum rotor pitch demonstrated the servo’s inability to generate sufficient torque to keep the blades in the commanded
Higher torque servos should be implemented to adequately meet the actuation requirements of a larger higher torque rotor assembly. Although this improved performance usually comes at the price of a larger heavier servo, developments in the RC model industry during the production of this vehicle have produced lighter stronger servos. Therefore a substitute servo could be found that would not have a significantly increased weight or size.
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8 Conclusion

The design of a VTOL MAV was developed as well as the theoretical models that supported the design decisions made. The vehicle drive system and control surface actuators were successfully operated allowing for the collection of data on the thrust and rotor rpm's developed during operation. Although the maximum thrust produced did not meet the target weight of the vehicle, valuable insight was gained allowing for further improvements that could enable the production of adequate thrust.

8.1 Lessons Learned

In Vehicle systems design it is critical to be able to isolate complex subsystems in order to determine their individual characteristics. This system identification allows the designer to develop a virtual block diagram of the subsystems where inputs and outputs can be better understood and the interactions between these system “blocks” can be visualized. Of course schedule, budget and feasibility issues will limit the depth to which this can be accomplished. Over the course of this project it was the return to this subsystem identification method that allowed us to overcome the mechanical issues that plagued the initial versions of the vehicle. Specifically when developing a dynamic vehicle the core subsystem should be isolated and studied. In this project that subsystem was the drive train. This subsystem was the core of all power for the vehicle and contained the highest dynamic forces, fastest moving parts and highest mechanical complexity. Constructing the drive train as a test bed entity in itself allowed for a detailed study of the dynamic forces produced during start up and running. It also allowed us to measure thrust versus rpm and have better access to adjust the system.
When characterizing a dynamic subsystem as much instrumentation and measurement devices as are practical should be used. Although this sounds excessive, my experiences showed me that many dynamic issues are not readily apparent. Moreover during the rush of a system test there are many distracters (noise, fast moving parts, multiple controls) that reduce an individual's ability to absorb the full scope of interactions that may be taking place. Multiple instruments properly placed allow the designer to measure system characteristics such as vibration modes or rpm that may not be visually attainable. Recording devices such as video tapes allow you to carefully critique a series of events that often go unnoticed during the rush of a test.
Figure 8-1 Finished Vehicle
Appendix A:

MAV MODEL MEMORANDUM – SEAN GEORGE

1 Introduction
In this memo, we describe the dynamics of the MAV tail-sitter model. The MAV has an extremely rigid rotor rotating at a large angular rate, and therefore exhibits minimal flapping motion in comparison with conventional helicopters. This characteristic greatly simplifies the rotor dynamics, and allows the application of a fixed-wing flight dynamics approach to be implemented, with the inclusion of gyroscopic coupling terms. A conventional helicopter axis system has been used in order to more easily integrate Draper’s previous helicopter dynamics model into the MAV model. This memo was built upon the work conducted on the DSAAV, and much of the initial theoretical treatment can be found in [2].

Although the MAV rotor model represents a significant simplification to conventional helicopters, the small vehicle size and low velocity envelope add an equivalent complexity to the MAV aerodynamics. The validity of standard theoretical treatments, to estimate the relevant aerodynamic coefficients, comes into question at the range of Reynolds numbers which comprise the flight profile. In addition, the vehicle’s control authority relies completely on the interaction between the downwash produced by the rotor and stabilator surfaces at the base of the body. The MAV model, therefore, necessitates a better physical representation of the flow field surrounding the vehicle in order to account for the motion of the rotor downwash with relation to the control surface stabilators.

Section 2 of the memo defines the state and input variables used in the simulation. Section 3 describes the basic state derivative model. Section 4 contains the rotor and engine models, and Section 5 describes the aerodynamic forces and moments.

2 Variable definitions
We begin by defining the state variables and inputs to the simulation. Parameters and derived variables are defined in the sections in which they first appear.

The state variables are listed in Table 2-1. Note that a quaternion is used to keep track of the vehicle attitude. Relationships between quaternions and the more familiar Euler angles are described briefly in Appendix 2.

Table 2-2 lists the free variables which are inputs to the model. These include a disturbance (wind velocity) as well as control inputs.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_0$</td>
<td>Quaternion element</td>
</tr>
</tbody>
</table>
Table 2-1. State variable definitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>Quaternion element</td>
<td></td>
</tr>
<tr>
<td>$e_2$</td>
<td>Quaternion element</td>
<td></td>
</tr>
<tr>
<td>$e_3$</td>
<td>Quaternion element</td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>Forward velocity (body frame)</td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>Lateral velocity (body frame)</td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>Vertical velocity (body frame)</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Body roll rate</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>Body pitch rate</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>Body yaw rate</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>North position (local geographic)</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>East position (local geographic)</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>Down position (local geographic)</td>
<td></td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Angular rate of rotor</td>
<td></td>
</tr>
<tr>
<td>$m_f$</td>
<td>Fuel mass remaining</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-2. Input variable definitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V^L_w$</td>
<td>Wind velocity vector (local geographic)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{s,i}$</td>
<td>Stabilator Deflection Angles (i=1,4)</td>
<td></td>
</tr>
<tr>
<td>$U_{th}$</td>
<td>Engine throttle</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1. Vehicle parameter definitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_e$</td>
<td>Mass without fuel</td>
<td>0.02 slugs</td>
</tr>
<tr>
<td>$m_{full}$</td>
<td>Fuel capacity</td>
<td>0.001713 slugs</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
<td>32.174 ft/sec$^2$</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>Roll axis moment of inertia</td>
<td></td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>Pitch axis moment of inertia</td>
<td></td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>Yaw axis moment of inertia</td>
<td></td>
</tr>
</tbody>
</table>

In this section, we present the basic equations used in the simulation to compute state derivatives. Given components of the vehicle velocity in body-relative coordinates ($v^B$), we begin by converting to local-geographic coordinates ($v^L$), through the transformation matrix $T^L_B$ as follows.

$$T^L_B = \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{yx} & c_{yy} & c_{yz} \\ c_{zx} & c_{zy} & c_{zz} \end{bmatrix} = \begin{bmatrix} 1 - 2(e_1^2 + e_3^2) & 2(e_1 e_2 - e_0 e_3) & 2(e_1 e_3 + e_0 e_2) \\ 2(e_1 e_2 + e_0 e_3) & 1 - 2(e_2^2 + e_3^2) & 2(e_2 e_3 - e_0 e_1) \\ 2(e_1 e_3 - e_0 e_2) & 2(e_2 e_3 + e_0 e_1) & 1 - 2(e_1^2 + e_2^2) \end{bmatrix}$$
\[ \mathbf{v}^i = T^i_B \mathbf{v}^B = T^i_B \begin{bmatrix} u \\ v \\ w \end{bmatrix} \]

The aerodynamics of the vehicle depend on air-relative (fluid-relative), rather than ground-relative, velocity. We obtain the components \((u_f, v_f, w_f)\) of the vehicle’s air-relative velocity in body-relative coordinates as follows.

\[ V_f^B = \begin{bmatrix} u_f \\ v_f \\ w_f \end{bmatrix} = (T^L_B)^T (v^L - v^L_w) \]

Next we compute the total mass and weight of the vehicle.

\[ m = m_e + m_f \]
\[ W = mg \]

The forces and moments (about the center of gravity) in the body frame consist of contributions from the rotor (R, Section 4.1), the stabilators (S, Section 5.1), the rotor guard (RG, Section 5.2), and the fuselage (F, Section 5.3); as well as the weight and the Euler coupling terms due to the rotation of the coordinate axes fixed to the body. For the purposes of clarity the fuselage, rotor guard, and stabilator components will be labeled as aerodynamics (A). The moment balance equation can be simplified by accounting for several characteristics of the MAV vehicle:

1. Exhibits bilateral and biventral symmetry, therefore no inertial cross coupling.
2. Minimal influence of the rotor thrust on vehicle moments, since thrust line offset from the center of gravity is small.
3. The only gyroscopic inertia is produced by the rotor blades and the engine around the body z-axis (both angular momentum, \(H_{ZR}\), and acceleration, \(H'_{ZR}\), must be accounted for in the dynamics).

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
X_A \\
Y_A \\
Z_A
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & W
\end{bmatrix} + \begin{bmatrix}
rv - qw \\
pw - ru \\
qu - pv
\end{bmatrix}
\]

\[
\begin{bmatrix}
L \\
M \\
N
\end{bmatrix} = \begin{bmatrix}
L_A & L_R \\
M_A & M_R \\
N_A & N_R
\end{bmatrix} + \begin{bmatrix}
-qH_{ZR} \\
pH_{ZR} \\
-H_{ZR}
\end{bmatrix} + \begin{bmatrix}
qr(l_{yz} - l_{zz}) \\
rp(l_{zz} - l_{xx}) \\
pq(l_{xx} - l_{yy})
\end{bmatrix}
\]

Finally, the state derivatives are computed using the usual 6-degree-of-freedom (6-DOF) equations of motion as follows.
4 Rotor Model Section

In this section we describe the model for the rotor (and engine) dynamics contained in the simulation. Section 4.1 details the rotor dynamics and the calculation of the downwash produced by the rotor. The downwash estimation is based on momentum theory, but includes several corrections to allow for empirical tuning, and to account for the effects of non-uniform flow.

The engine model appears in Section 4.2. The MAV rotor is rigid (no flapping, leading, or lagging), therefore a simplified modeling approach is applicable, using fixed-wing flight dynamics with the addition of gyroscopic coupling terms to account for the engine and rotor rotation. The calculation of angular acceleration is important because both the engine angular momentum and torque need to be accounted for in the moment equations.

### 4.1 Rotor dynamics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_r$</td>
<td>Rotor location (below c.g.)</td>
<td>-0.24475 ft</td>
</tr>
<tr>
<td>$R_r$</td>
<td>Rotor radius</td>
<td>0.23958 ft</td>
</tr>
<tr>
<td>$r_{dir}$</td>
<td>Rotor direction (1=CW from top, -1=CCW from top)</td>
<td>-1</td>
</tr>
<tr>
<td>$C_{L,a,r}$</td>
<td>Lift curve slope of rotor blade</td>
<td>4.75</td>
</tr>
<tr>
<td>$\alpha_{L,0,r}$</td>
<td>AOA of rotor blade at zero lift</td>
<td>-0.035 rads</td>
</tr>
<tr>
<td>$b_r$</td>
<td>Number of rotor blades</td>
<td>3</td>
</tr>
<tr>
<td>$\theta_{col}$</td>
<td>Root pitch angle of rotor blades</td>
<td>0.2618 rads</td>
</tr>
<tr>
<td>$\theta_{twist}$</td>
<td>Twist angle of rotor blades</td>
<td>-0.1309 rads</td>
</tr>
</tbody>
</table>
We begin by deriving some auxiliary variables related to the rotor dynamics. The tip velocity, $v_{tip}$, equivalent flat plate area, $F_r$, and aggregate blade velocity normal to the rotor blade $w_b$, are computed below.

$$v_{tip} = \Omega R,$$

$$F_r = C_{D0,r} R b_c c_r,$$

$$w_b = w_f + \frac{2}{3} v_{tip} \left[ \theta_{col} - \alpha_{L0,r} + \frac{3}{4} \theta_{weiss} \right]$$

At low altitudes, there is a ground effect which increases the rotor thrust which is achieved for a given throttle setting. The ground effect is characterized through a factor, $\eta_{GE}$, which depends on both the height of the rotor hub above the ground, $h_r$, as well as the speed of the vehicle parallel to the ground.

$$h_r = -z - c_{\infty} z_r,$$

$$\eta_{GE} = \frac{1}{1 + K_{GE,r} \left( \frac{R_i}{h_r} \right)^2}$$

The thrust, $T$, produced by the rotor depends on the average downwash velocity, $v_i$, through the following equations.

$$T = \frac{1}{4} \rho b_c c_r v_{tip} C_{La,r} (w_b - v_i)$$

$$v_i = \frac{\eta_{GE} T}{2 \rho \pi R_i^2 \sqrt{u_f^2 + v_f^2 + (w_f - v_i)^2}}$$

Note that an iterative scheme is necessary to solve these equations. Next, the induced power, $P_{i,r}$, and profile power, $P_{p,r}$, dissipated by air resistance are used to compute the torque, $\tau_r$, acting on the rotor.

$$P_{i,r} = K_{ip} T (v_i - w_f)$$
Although a fixed stator configuration is being designed to cancel out the aerodynamic torque at conditions near hover, the rotor torque is still useful for calculating rotor angular acceleration and velocity. At this time there is no definitive mapping of the perturbation yaw torque generated by the MAV at conditions away from trim, however it is likely that the residual torque could be estimated by a relationship similar to the calculation used above, and therefore as a function of $\tau_r$.

The presence of stiff, hingeless rotor blades rotating at extreme angular rates allows the neglection of blade flapping dynamics, however the moments generated by the rotor on the hub must now be accounted for in the motion of the MAV. Generally, this means that flow tangential to the rotor disk, generated by gusts or forward flight, will cause a nonuniform thrust distribution on the rotor, inducing pitching and rolling moments. Experimental data is probably the best source of information for these effects, since they are difficult to quantify analytically in the appropriate flight conditions.

As a first order approximation, due to the fact that the rotor is shrouded by a substantial guard and that only small advance ratios are expected, only the roll moment generated by the change in local angle of attack on the advancing and retreating blades will be considered. In standard helicopters the response is for the rotor blades to initiate a flapping motion, in order to compensate for the unbalanced moments. The MAV’s rigid blades and hingeless hub do not allow the thrust line to be rotated, therefore a finite thrust offset will develop on the rotor disk. The simplified calculation for the rotor generated roll moment is completed under the assumption of constant blade circulation.

$$L_R = r_{dir} \frac{1}{2} \frac{T}{\Omega} u_i$$

$$M_R = r_{dir} \frac{1}{2} \frac{T}{\Omega} v_i$$

While the fidelity of the above equation should be critically examined, it does allow at least a functional assessment of the magnitude of the roll moments on the MAV rotor. The MAV rotor is rotating at an extremely high rate, meaning that the change in local angle of attack over the rotor disk is not significant even at relatively large forward velocities. In addition, the MAV rotates in flight, therefore the largest velocities tangent to the rotor disk will most likely come from gusts in hover. Additional experimental data will be required to assess cross-plane motion generated by gusts and forward translation.

The other important rotor aerodynamic coefficient can be easily accounted for using the force and torque equations shown above

$$Z_R = -T$$
\[ N_R = -r_{dir} \left[ \tau_e - \tau_b (v_i - w_f) \left( v_i - w_f \right) \right] \]

4.2 Engine Dynamics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{bhp} )</td>
<td>Engine brake power</td>
<td>125 ft-lb/s (170 W) (.227 hp)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Efficiency of engine</td>
<td>0.9</td>
</tr>
<tr>
<td>( \Omega_{\text{max}} )</td>
<td>Engine speed at maximum output</td>
<td>27500 rpm</td>
</tr>
<tr>
<td>( I_b )</td>
<td>Blade moment of inertia</td>
<td>( 3.583554 \times 10^6 ) \text{slug ft}^2</td>
</tr>
<tr>
<td>( I_e )</td>
<td>Engine moment of inertia</td>
<td>( 2.6877 \times 10^6 ) \text{slug ft}^2</td>
</tr>
<tr>
<td>( \dot{m}_{\text{max}} )</td>
<td>Fuel consumption rate at maximum output</td>
<td>0.66 lb/hr</td>
</tr>
<tr>
<td>( D_e )</td>
<td>Gear reduction ratio of rotor</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4-3. Engine parameter definitions.

The engine dynamics model is used to compute the fuel consumption rate, \( \dot{m}_f \), and the angular acceleration, \( \dot{\Omega} \), of the rotor. The engine torque is a function of the commanded throttle setting and the maximum engine torque. The fuel consumption rate is proportional to the power output of the engine, and the angular acceleration of the rotor is simply the total torque acting on the rotor shaft divided by the rotational inertia.

\[
\dot{\Omega} = \Omega D_e
\]

\[
\tau_e = U \cdot \eta \left( \frac{P_{bhp}}{\Omega_{\text{max}}} \right)
\]

\[
P_e = \tau_e \Omega_e
\]

\[
\dot{m}_f = -\dot{m}_{\text{max}} \left( \frac{P_e}{P_{bhp}} \right)
\]

\[
\dot{\Omega} = \frac{\tau_e D_e - \tau_f}{(I_e + I_b b_r)}
\]

The motion of the MAV vehicle is dependent on the angular momentum and acceleration induced by the engine. These quantities can be calculated using the engine dynamic equations computed above as follows

\[
H_{ZR} = r_{dir} (I_e D_e + b_r I_b) \Omega
\]

\[
\dot{H}_{ZR} = r_{dir} (I_e D_e + b_r I_b) \dot{\Omega}
\]

5 Aerodynamics Section

In this section we compute the aerodynamic forces and moments acting on the vehicle fuselage and stabilators. The flow field surrounding the vehicle is linked to the downwash produced by the rotor, calculated in the previous section. The attitude of the MAV vehicle is controlled by the interaction of the rotor downwash with the four stabilators, therefore a high fidelity estimation of the forces generated by the surfaces must be completed. Large angles of attack are possible, for both the MAV fuselage and stabilators, due to the combined vehicle velocity and rotor downwash conditions. In order to account for possible nonlinearities at these extreme flow conditions, a general model of the aerodynamic characteristics of the MAV has been adopted.
The effect of the rotor downwash can be approximated by assuming the vehicle is immersed in a uniform wake, and calculating the resultant change to the flow velocity using the assumption of superposition. Momentum theory dictates that the induced velocity in the far wake is accelerated to twice the rotor inflow, therefore empirical constants need to be used to account for the increase in downwash at both the fuselage and stabilators, relative to the rotor disk. These amplification constants should range from 1 to 2, and would most likely be calculated from wind tunnel estimates of the vehicle downwash distribution. Methods to approximate the downwash velocities and the geometry of the rotor wake will be discussed in the next section. For now, the inclusion of constant downwash amplification parameters at each relevant station is assumed in the force and moment calculations.

The main vehicle aerodynamic parameters are contained in Table 5-1, and a general description of important nomenclature and characteristics are given in Table 5-2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_F$</td>
<td>Fuselage normalization area (at max radius)</td>
<td>0.02182 ft$^2$</td>
</tr>
<tr>
<td>$D_F$</td>
<td>Fuselage maximum diameter</td>
<td>0.08333 ft</td>
</tr>
<tr>
<td>$k_{i,F}$</td>
<td>Fuselage downwash amplification factor</td>
<td>1.00</td>
</tr>
<tr>
<td>$S_{RG}$</td>
<td>Rotor guard normalization area</td>
<td>0.14748 ft$^2$</td>
</tr>
<tr>
<td>$D_{RG}$</td>
<td>Rotor guard diameter</td>
<td>0.43333 ft</td>
</tr>
<tr>
<td>$S_s$</td>
<td>Stabilator area (based on chord &amp; span)</td>
<td>0.01285 ft$^2$</td>
</tr>
<tr>
<td>$z_s$</td>
<td>Stabilator c.p. location (below c.g.)</td>
<td>0.2725 ft</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Stabilator radial c.p. location</td>
<td>0.0775 ft</td>
</tr>
<tr>
<td>$k_{i,s}$</td>
<td>Stabilator downwash amplification factor</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Table 5-1. Aerodynamic parameter description.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{A,F}$</td>
<td>Fuselage axial force coefficient</td>
</tr>
<tr>
<td>$C_{N,F}$</td>
<td>Fuselage normal force coefficient</td>
</tr>
<tr>
<td>$C_{M,F}$</td>
<td>Fuselage moment coefficient</td>
</tr>
<tr>
<td>$C_{D,S}$</td>
<td>Stabilator drag coefficient</td>
</tr>
<tr>
<td>$C_{L,S}$</td>
<td>Stabilator lift coefficient</td>
</tr>
<tr>
<td>$C_{A,RG}$</td>
<td>Rotor guard axial force coefficient</td>
</tr>
<tr>
<td>$C_{N,RG}$</td>
<td>Rotor guard normal force coefficient</td>
</tr>
<tr>
<td>$C_{M,RG}$</td>
<td>Rotor guard moment coefficient</td>
</tr>
</tbody>
</table>

Table 5-2. Aerodynamic input characteristics.

5.1 Fuselage Aerodynamics

The fuselage’s rotational symmetry means that it reacts identically to changes in angle of attack or sideslip. A convenient approach, then, is to calculate the static aerodynamic forces and moments in the normal and axial direction according to a complex angle of attack ($\alpha_F$), and then decompose into body coordinates using the cross flow orientation. The forces and moments acting on the fuselage, in body centered coordinates, may be written in a generalized form as:

\[
X_F = -\frac{1}{2} \rho V_F^2 S_F C_{N,F}(\alpha_F) \cdot \frac{u_f}{\sqrt{u_f^2 + v_f^2}}
\]

\[
Y_F = -\frac{1}{2} \rho V_F^2 S_F C_{N,F}(\alpha_F) \cdot \frac{v_f}{\sqrt{u_f^2 + v_f^2}}
\]

\[
Z_F = \frac{1}{2} \rho V_F^2 S_F C_{A,F}(\alpha_F)
\]

\[
L_F = \frac{1}{2} \rho V_F^2 D_F S_F C_{M,F}(\alpha_F) \cdot \frac{v_f}{\sqrt{u_f^2 + v_f^2}}
\]

\[
M_F = \frac{1}{2} \rho V_F^2 D_F S_F C_{M,F}(\alpha_F) \cdot \frac{u_f}{\sqrt{u_f^2 + v_f^2}}
\]

where

\[
\alpha_F = \tan^{-1}\left(\frac{\sqrt{u_f^2 + v_f^2}}{w_F}\right)
\]

\[
w_F = k_f v_f - w_f
\]
\[ V_F^2 = w^2_F + u_f^2 + v_f^2 \]

As mentioned above, a downwash amplification factor for the fuselage \((k_{i,F})\), has been used to account for the increase in the rotor inflow at the fuselage. Although the flow velocity over the fuselage is accelerating, a constant representing the average fuselage station has been used. This topic will be taken up later when a method to approximate the lateral flow velocity is derived. More elaborate modeling techniques, which include damping derivatives dependent on the rate of change in angle of attack, can be used in the future to account for lateral flow acceleration if the dynamics of the MAV proves sensitive to the amplification parameter.

5.2 Rotor Guard Aerodynamics

The MAV rotor guard’s primary functions are to protect the rotor blades from obstructions while in flight and to maintain loading on the rotor blades out to the tips. Ducting the propeller, however, can have the additional benefit of actually developing a thrust on the rotor guard itself, even in hover. This increase in thrust \((T_{DP})\) is a function of both the MAV’s velocity as well as the induced velocity of the rotor. The rotor guard can also act as a lifting surface, or ring airfoil, at angle of attack. References [3] and [4] discuss this topic, including ways to estimate the effects of rotor shrouds on the net forces and moments of a ducted propeller. This analysis is limited to hover or axial flight, as well as small angle perturbations from this condition, however experimental data could be used to predict the rotor guard’s influence over the complete flight envelope.

The forces and moments acting on the rotor guard, in body centered coordinates, may be written in a generalized form as:

\[
X_{RG} = \frac{1}{2} \rho V_{RG}^2 S_{RG} C_{N,RG}(\alpha_{RG}) \cdot \left( \frac{u_f}{\sqrt{u_f^2 + v_f^2}} \right)
\]

\[
Y_{RG} = \frac{1}{2} \rho V_{RG}^2 S_{RG} C_{N,RG}(\alpha_{RG}) \cdot \left( \frac{v_f}{\sqrt{u_f^2 + v_f^2}} \right)
\]

\[
Z_{RG} = \frac{1}{2} \rho V_{RG}^2 S_{RG} C_{A,RG}(\alpha_{RG}) - T_{DP}(v_i, V_{RG})
\]

\[
L_{RG} = \frac{1}{2} \rho V_{RG}^2 D_{RG} S_{RG} C_{M,RG}(\alpha_{RG}) \cdot \left( \frac{v_f}{\sqrt{u_f^2 + v_f^2}} \right)
\]

\[
M_{RG} = \frac{1}{2} \rho V_{RG}^2 D_{RG} S_{RG} C_{M,RG}(\alpha_{RG}) \cdot \left( \frac{u_f}{\sqrt{u_f^2 + v_f^2}} \right)
\]

where
\[ \alpha_{RG} = \tan^{-1}\left( \frac{w_f}{\sqrt{u_f^2 + v_f^2}} \right) \]

\[ V_{RG}^2 = w_f^2 + u_f^2 + v_f^2 \]

5.3 Stabilator Aerodynamics

The four stabilators provide the main means to control the motion of the MAV vehicle. This is accomplished by deflecting the canard stabilators and producing a moment of required magnitude, based on the induced downwash from the rotor. The input variable for the stabilators is the control deflection angle. The difficulty in modeling the stabilator aerodynamics is the large range in angle of attack which can be produced by the superposition of the rotor downwash and the flight velocity of the vehicle. An approach, similar to the fuselage, has been taken which defines the forces and moments produced by the stabilators in generalized functional form. The four stabilators can be broken up into two classes, pitch and roll stabilators, which are defined by the angular motion which they primarily control.

5.3.1 Pitch Stabilators

The pitch stabilators are oriented so that their main spar is parallel to the Y-axis. They are designed mainly to produce pitch moments in order to trim the vehicle in the X-Z plane. The angle of attack of the pitch stabilators is dependent on \( w_s \) and \( u_s \), the z and x-axis flow velocities at the stabilator position; as well as the individual stabilator deflection angles \( \alpha_{S_{i=1/2}} \). The two local relative velocities can be calculated, assuming a constant flow incidence across the stabilator span, using results from momentum theory as

\[ w_s = k_{i,s} v_i - w_f \]

\[ u_s = u_f + z_s q - r_s r \]

where \( k_{i,s} \) represents the downwash amplification parameter at the stabilator c.p. position \((z_s, r_s)\). The components dependent on the MAV’s angular velocity are used to account for the effect of vehicle motion on the apparent local flow angle at the stabilator. Once the local velocities have been computed, the forces and moments generated by the pitch stabilators can be calculated from information on the aerodynamic characteristics of the stabilator airfoils.

\[ X_s^p = -\frac{1}{2} \rho |V_s| S_s (C_{L,s}(\alpha_s^p) w_s + C_{D,s}(\alpha_s^p) \mu_s) \]

\[ Z_s^p = -\frac{1}{2} \rho |V_s| S_s (C_{L,s}(\alpha_s^p) \mu_s - C_{D,s}(\alpha_s^p) w_s) \]

\[ L_s^p = r_s \cdot Z_s^p \]

\[ M_s^p = z_s \cdot X_s^p \]
\[ N_S^p = -r_S \cdot X_S^p \]

where

\[ \alpha_S^p = \alpha_{S,i=1,2} + \tan^{-1} \left( \frac{u_S}{w_S} \right) \]

\[ V_S^2 = w_S^2 + u_S^2 \]

**5.3.2 Roll Stabilators**

The roll stabilators are oriented so that their main spar is parallel to the X-axis. The aerodynamic calculations are identical to the pitch stabilators except that the angle of attack is dependent on \( w_s \) and \( v_s \), the z and y-axis flow velocities at the stabilator position. Estimations of the y-axis velocity follow much the same treatment as above

\[ w_s = k_{i,s} v_i - w_f \]

\[ v_s = v_f - z_s p + r_s r \]

The induced downwash at the roll stabilators shares an identical amplification factor since they are positioned at the same vehicle station. The forces and moments are computed in much the same way as the pitch stabilators, except that the dominate effect is to induce a roll moment on the MAV vehicle.

\[ Y_S^R = -\frac{1}{2} \rho |V_S| S_S \left( C_{L,S} (\alpha_S^R) w_S + C_{D,S} (\alpha_S^R) v_S \right) \]

\[ Z_S^R = -\frac{1}{2} \rho |V_S| S_S \left( C_{L,S} (\alpha_S^R) v_S - C_{D,S} (\alpha_S^R) w_S \right) \]

\[ L_S^R = -z_S \cdot Y_S^R \]

\[ M_S^R = -r_S \cdot Z_S^R \]

\[ N_S^R = r_S \cdot Y_S^R \]

where

\[ \alpha_S^R = \alpha_{S,i=1,2} + \tan^{-1} \left( \frac{v_S}{w_S} \right) \]

\[ V_S^2 = w_S^2 + v_S^2 \]

**5.3.3 Stabilators in Ground Effect**

Stabilators can be subjected to the same ground effects which were accounted for with the induced downwash of the rotor. In fact, due to their position on the MAV vehicle, it is likely that the forces and moments generated during landing will be extremely sensitive to the air flow reflected off of the ground. This air cushion, while exhibiting beneficial effects with respect to power consumption at launch, has the potential to make landing the vehicle an extremely difficult task. The ground induced flow will be calculated in a manner similar to the rotor ground effect parameter shown in Section 4. In this case, it is the perturbation velocity (\( \delta v_{GE,s} \)) that needs to be calculated to account for the upwash created by the ground reflected flow.
\[ \delta z_S = -z - c_{\alpha S} \delta z_S \]
\[ \delta v_{GES} = K_{GES} \left( \frac{R_z}{\delta z_S} \right)^2 \cdot k_{iS} v_i \]

In this formulation we have included two constants, the stabilator downwash amplification factor and a stabilator ground effect parameter, in order to leave the ability to tune the ground interference with future experimental data. An additional computation is required to account for the direction of the rotor downwash with respect to the local ground normal. Perturbation velocities must be added to each component of the velocity at the stabilator, and should be computed using the normal to the rotor disk \((\hat{n}_r)\).

\[ \delta u = \delta v_{GES} \cdot \hat{n}_{x,r} \]
\[ \delta v = \delta v_{GES} \cdot \hat{n}_{y,r} \]
\[ \delta w = -\delta v_{GES} \cdot \hat{n}_{z,r} \]

Each individual component should be added to the local stabilator velocity computed with the vehicle out of ground effect. This has the potential to greatly affect the angle of attack seen by the stabilators when the MAV vehicle is close to the ground.

Finally, the aerodynamic forces and moments are obtained by combining the effects of the fuselage and the four stabilators as follows

\[
\begin{bmatrix}
X_A \\
Y_A \\
Z_A
\end{bmatrix} = \begin{bmatrix}
X_F + X_{S,i=1,2}^P \\
Y_F + Y_{S,i=1,2}^R \\
Z_F + Z_{S,i=1,2}^P + Z_{S,i=1,2}^R
\end{bmatrix}
\]

\[
\begin{bmatrix}
L_A \\
M_A \\
N_A
\end{bmatrix} = \begin{bmatrix}
L_F + L_{S,i=1,2}^R \\
M_F + M_{S,i=1,2}^P \\
N_{S,i=1,2}^P + N_{S,i=1,2}^R
\end{bmatrix}
\]

6 Wake Characteristics Section

In this section an attempt is made to estimate the important characteristics of the downwash wake, including both geometry and velocity distribution. The slipstream from the rotor is extremely important to the aerodynamics of the MAV, since it controls both the dynamic pressure as well as the angle of attack of all components below the rotor disk. In addition, the control effectiveness and rate limits on the vehicle will most likely be heavily related to the motion of the wake. Section 5 was predicated on the ability of the simulation to calculate the local velocity at both the fuselage and the stabilators, including the contribution from the rotor. These calculations were focused on the usage of amplification parameters at each station on the MAV, which could be used to account for the lateral acceleration of the rotor downwash over the vehicle. Although values were given for the two vehicle stations in question, certainly one has to provide an explanation for these constants based on physical principles. The downwash of a rotorcraft vehicle is an extremely complicated flow field, for which only a first order approximation has been completed here. It is hoped that even though the fidelity of the approximations can be
questioned, they at least provide a physically based means to check the sensitivity and robustness of the MAV aerodynamics to changes in the wake flow field.

6.1 Velocity Amplification Parameters

The variation of the rotor induced velocity with axial distance along the MAV body can be approximated using vortex theory. In this scheme the rotor wake is viewed as a vortex helix which contracts as the flow moves away from the rotor. This contraction causes a subsequent acceleration, which can be calculated by integrating the contribution of the vortex helix to the axial velocity using the Biot-Savart law [5]. The induced velocity, in terms of the downwash induced at the rotor disk, can be written as

\[
\frac{v_i(z)}{v_i(0)} = 1 + \frac{z}{\sqrt{R_r^2 + z^2}}
\]

This equation is just the definition for the amplification parameter \( k_i \), which has been used earlier, therefore the induced velocity can be calculated at any station along the MAV. For the MAV fuselage we have assumed a constant, average induced velocity, although from the equation above it is evident that a significant range of velocities will exist along the vehicle. More elaborate techniques could perhaps be utilized to account for the acceleration of the flow, however dynamic force and moment coefficients would have to be made available in order to assess these effects. It is clear, at least, that the velocity over the stabilators is well approximated by a constant value at the mid-chord.

Calculation of the wake geometry requires knowledge of the fully developed slipstream radius \( R_{\infty} \). This is a trivial calculation in hover, however during climbing maneuvers the induced velocity decreases with increasing axial translation, reducing the full-developed wake contraction. The contraction ratio can be estimated from the climbing velocity and the induced velocity using the following equation

\[
\frac{R_{\infty}}{R_r} = \sqrt{\frac{1 - \left( \frac{w_f}{v_i} \right)}{2 - \left( \frac{w_f}{v_i} \right)}}
\]

Given the two extreme cross-sections of the wake boundary, an expression for the variation in the wake radius can be estimated using the variation of the induced velocity shown above, coupled with the continuity equation. The area of the wake is inversely related to the induced velocity, therefore, assuming the wake retains a circular cross-section.

\[
\frac{R(z)}{R_{\infty}} = \sqrt{\frac{v_i(\infty)}{v_i(z)}} = \sqrt{\frac{2v_i(0)}{v_i(z)} + \frac{D_F^2}{4R_{\infty}^2}}
\]

Note that the equation above includes the effect of the fuselage, by including the area of the circular body in the calculation of the radius. The wake can then be viewed as an annulus of air which contracts along the MAV body in response to both the acceleration of rotor slipstream as well as the expansion of the fuselage area.

6.2 Sensitivity to Wake Position
The calculations made above are important because they estimate the position and geometry of the rotor wake along the entire length of the MAV fuselage. The control effectiveness of the stabilators is directly related to the downwash velocity in the wake, both due to the dynamic pressure and angle of attack. In strong gusts or flight tangent to the rotor disk, if the wake angle becomes sufficiently large to pull one or more stabilators out of the downwash stream control authority will be compromised. As vehicle velocity is increased, for more aggressive maneuvers, the downwash wake alters its position relative to the fuselage. The amplification parameters can be used to keep track of whether the specific stabilator is in the rotor downwash, by setting the parameter to zero if the wake angle becomes sufficiently large. The wake outer boundary (wind-side of the vehicle) on the x and y-axis at the stabilator fuselage position can be calculated based on parameters already define above:

\[ d_x = R(z_r + z_s) - \frac{2\mu_f}{v_f(1 + k_i_s)} - 2w_f \times (z_r + z_s) \]
\[ d_y = R(z_r + z_s) - \frac{2v_f}{v_f(1 + k_i_s)} - 2w_f \times (z_r + z_s) \]

These parameters are important because they provide an estimate of when a particular stabilator will become detached from the downwash stream created by the rotor. It is evident, given the span of the stabilator and the predicted wake contraction in hover, that the stabilators will almost certainly be partially outside the rotor slipstream for most of the flight conditions. This will have to be accounted for in the forces and moments generated by the stabilators. Pitch stabilators are affected by \( d_y \) and roll stabilators by \( d_x \), given by the orientation of their spar.

Appendix 1. Items for Future Development

Helicopters exhibit complicated aerodynamic characteristics due to the unsteady flow conditions generated by their rotor downwash, as well as the interaction between the helicopter blades and the surrounding air. In an effort to simplify the dynamic model of the MAV vehicle many assumptions have been made which could be detrimental to the fidelity of the simulation. This appendix lists some of the issues and categories which need to be resolved in future developments of the MAV model. Wind tunnel testing is the most likely source to evaluate the fidelity of the assumptions made by the model, as well as providing a means for empirical tuning.

I. Rotor induced flow field
   1) Non-uniformity of rotor induced velocities both spanwise and longitudinally
   2) Fuselage blockage of downwash and corresponding pseudo ground effect
   3) Finite rotation of the wake
   4) Contraction of the wake and tip vortex interference effects

II. Rotor blade lift and drag
    1) Tip and root losses which limit lift producing region
    2) Variation in chord and twist angle along blades
    3) Three term drag polar for rotor blades
    4) Mapping of yaw torque from rotor/stator combination

III. Vertical flight conditions
1) Vortex ring state at moderate rates of descent (25% of induced velocity)
2) Windmill brake state at large rates of descent (200% of induced velocity)
3) Interaction of landing gear with ground

IV. Input time lags and damping
1) Effect of rapid throttle change on thrust
2) Effect of throttle change on induced rotor velocities (and stabilators)

V. Stabilator aerodynamics
1) Model the stabilator’s tendency to pitch away from the stall condition
2) Control blanking of stabilator(s) by fuselage at slow forward flight

In order to address the above complexities more elaborate modeling techniques would be necessary. This would require an increased amount of relevant wind tunnel and experimental data in order to effectively tune parameters. In addition, an approach which introduced a numerical scheme to compute rotor blade and local velocity characteristics using blade element methods could be used if the simplified model is not adequate. This discretized approach could effectively address most of the factors listed above because it would make fewer assumptions about average blade characteristics.

Appendix 2. Relations between quaternions and Euler angles

This appendix relates the quaternion notation in Section 3 to the more familiar Euler angle notation. When using Euler angles, it is important to specify the order in which rotations about the three body axes are done. Here we assume the body rotates first about the yaw axis (ψ), then the pitch axis (θ), and finally the roll axis (φ). The conversion from Euler angles to quaternions is well known and is reproduced below; the remaining equations are derived from this conversion.

\[
\begin{bmatrix}
\phi \\
\theta \\
\psi
\end{bmatrix} =
\begin{bmatrix}
\sin \frac{\psi}{2} & \cos \frac{\phi}{2} & \cos \frac{\theta}{2} \\
\cos \frac{\psi}{2} & \sin \frac{\phi}{2} & \sin \frac{\theta}{2} \\
\sin \frac{\psi}{2} & -\sin \frac{\phi}{2} & \cos \frac{\theta}{2}
\end{bmatrix}
\]

\[
T_B^L =
\begin{bmatrix}
\cos \psi \cos \theta & \cos \psi \sin \phi - \sin \psi \cos \phi & \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\
\sin \psi \cos \theta & \sin \psi \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \sin \phi & \sin \theta \sin \phi & \sin \theta \cos \phi \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

References

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References


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