But nature is a stranger yet,
The ones that cite her most
Have never passed her haunted house,
Nor simplified her ghost.

To pity those that know her not
Is helped by the regret
That those who know her, know her less
The nearer her they get.

Emily Dickinson
Abstract

Vague Objects

by

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Peter Unger’s puzzle, the problem of the many, is an argument for the conclusion that we are grossly mistaken about what kinds of objects are in our immediate surroundings. But it is not clear what we should make of Unger’s argument. There is an epistemic view which says that the argument shows that we don’t know which objects are the referents of singular terms in our language. There is a linguistic view which says that Unger’s puzzle shows that ordinary singular terms and count nouns are vague. Finally, there is an ontological view which says that the puzzle shows that there are vague objects.

The epistemic view offers the simplest solution to the problem of the many, but runs foul of a different problem, the problem of vague reference. The problem of vague reference is that given the presuppositions of the epistemic view there are too many too similar objects that might be the reference of a name such as ‘Kilimanjaro’ for it to be plausible that the name has a determinate reference. The linguistic view, spelled out in terms of semantic indecision and supervaluation, offers the same solution to the problem of the many and to the problem of vague reference. But it leaves no room for de re beliefs about ordinary material objects. The ontological view offers a solution to the problem of the many that avoids the
problem of vague reference and the problem of *de re* beliefs. For these reasons it is preferable to the other two.

However, ontological vagueness has met strong objections. It has been argued that it is a fallacy of verbalism, that it is inconsistent and that once formulated in a consistent way it is not distinguishable from the linguistic view. These objections can be met, but not without cost. To avoid the charge of being inconsistent, friends of the ontological view have to give up the law of excluded middle.

A positive account of vague parthood has two parts. First, parthood is not primitive but dependent on other primitive facts. The most important of the primitive facts are facts about to what kinds objects belong and how objects are causally related. Second, sometimes the primitive facts fail to determine of two objects whether one is part of the other. Given a notion of vague parthood, a notion of vague object can be defined roughly in the following way: An object $O$ is vague iff there is an object $a$ such that it is indeterminate whether $a$ is part of $O$. 
Acknowledgments

In my first term at MIT I took a course on vagueness with Vann McGee, and in the second term I took a course by Judith Thomson where she discussed, among other things, material constitution. I thought that these two issues were intimately related, but I had only a vague idea about the nature of this relation. I decided to work on it and wrote my second year paper on vagueness and parthood under the supervision of Judy and Vann. Now, with the help of Judy, Vann, and Steve Yablo I have written a whole monograph on vagueness. They have all been extremely generous with their time and thoughts. I owe them special thanks.

I have also been helped by other members of the philosophy department at MIT. Special thanks go to Patrick Hawley who read the whole manuscript and made many helpful suggestions and to Matti Eklund who read a large part of the manuscript and sent me written objections to several points I make. I’d also like to mention Sylvain Bromberger, Andy Egan, Agustín Rayo, Carolina Sartorio and Gabriel Uzquiano.

Finally, I’d like to thank my wife Anna Sveinsdóttir for her constant encouragement. To her and our daughter Ása is this dissertation dedicated.
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1

Introduction

Vagueness comes into philosophy as a problem maker. We have our theories of what exists, how language has meaning and what we can know and believe, and then someone suggests that if we look closer we will see that there are no ordinary material objects, we never refer to anything, and we never believe anything of any particular object. The suggestion is incredible, but many philosophers have argued that we have no alternative short of accepting it. But they are mistaken, or so I argue. There are ordinary material objects, but they are vague, we can refer to them with ordinary names such as 'Kilimanjaro' and 'Toni Morrison' and we can believe all sorts of things about them, for instance, about Kilimanjaro that it is the tallest mountain in Africa and about Toni Morrison that she is a pretty good writer. But what does it mean to say that there are vague objects? The present essay is an attempt to give a detailed answer to this question.

There are at least two problem of vagueness that go all the way back to antiquity: the sorites paradox and The Ship of Theseus. The story has it that Eubulides of Miletus is the author of the sorites paradox, as well as several other
paradoxes, but we don't know much more about him although he is reported to have been famous enough to be commented on by a comic poet, important enough to have carried on a controversy with Aristotle, and clever enough to have gotten the better of Aristotle in their exchange. We don't know what the controversy was about, although one speculation has it that Eubulides used a sorites like reasoning to challenge Aristotle's definition of a virtue as the mean between two extremes.¹

The word 'sorites' comes from the Greek word 'σωρος' which means heap and the original sorites paradox calls into question the division of objects that satisfy the predicate 'heap' and those that do not satisfy it. The paradox is simple. We convince ourselves that the following thesis is true: Never does a single grain of wheat turn something that is not a heap into a heap. This is called the sorites premiss. From the sorites premiss and the premiss that a single grain of wheat does not make a heap, it follows that if we start with a single grain of wheat and then add one at a time, we will never get a heap. And that does not seem right. As a conclusion about heaps this is not disturbing, but similar reasoning can be repeated with many predicates in our language, and that makes the paradox more serious.

The sorites paradox shows that our predicates have borderline cases of application. There are clear cases of objects that satisfy predicates such as 'heap' or 'bald', and there are clear cases of objects that do not satisfy them, and then there are the borderline cases, objects that fall in-between. There is a contemporary problem of vagueness that is related to the sorites paradox but differs from it in important ways. We might call this new problem 'the problem of vague objects', and it is this

problem that is the focus of the present essay. The two problems are similar in many respects; just as Tom is a borderline case of a bald man, so is Sparky a borderline case of a pebble that is part of Kilimanjaro, and as a result, competent speakers aware of all available evidence, are unwilling to classify the sentences

(1) Tom is bald,
(2) Sparky is part of Kilimanjaro,

as either true or false. These sentences are vague or indeterminate as I shall also say. In the case of (1) we can explain the vagueness by reference to semantic indecision. This is a linguistic view. We say that the facts that determine meaning are facts about thoughts and practices of speakers, and these facts just don't fix whether Tom is in the extension of 'bald'. If we help ourselves to supervaluation as our theory of truth then we have a solution to the problem of vague predicates and we won't even have to give up classical logic.

When it comes to sentence (2) we might consider two alternative explanations of why it is vague. We might say that Kilimanjaro, the mountain itself, is vague, or we might say that the name 'Kilimanjaro' is vague. The latter explanation, that the name 'Kilimanjaro' is vague, is similar to the above explanation of the vagueness of (1); the thoughts and practices of the speakers do not determine whether the name 'Kilimanjaro' refers to a landmass of which Sparky is a part or to some other landmass of which Sparky is not a part. But the predicate 'bald' and the name 'Kilimanjaro' are not quite on a par when it comes to vagueness. There are at least two differences that are worth mentioning. First, there is a difference in how we might resolve issues of vagueness. In the case of the predicates 'bald' or 'heap' we might resort to an arbitrary stipulation, we can stipulate that fewer than $n$ hairs
make for a bald head and that it takes at least \( k \) grains to make a heap, where \( n \) and \( k \) are somewhat arbitrarily chosen natural numbers. We might add some clauses about the distribution of the hairs and the arrangement of the grains, but, in principle at least, this should be possible. When it comes to singular terms or count nouns things are different. It is hard to imagine a similar stipulation in the case of the name ‘Kilimanjaro’ for the stipulation would not only need to say which pebbles are parts of the mountain, but also which pebbles would be parts of it under various counterfactual circumstances. The problem is clearer if we consider the general term ‘mountain’ instead of the name ‘Kilimanjaro’, or if we consider count nouns such as ‘chair’ and ‘table’. It is impossible to formulate any convention, no matter how arbitrary, that will decide for all chairs, whether such and such a molecule is part of a chair or not.\(^2\)

The second difference is that while there are men of which it is determinate that they satisfy the predicate ‘bald’ and others that determinately do not satisfy the predicate, according to the linguistic view there are no objects that are determinately the reference of the name ‘Kilimanjaro’, although there are many objects that are determinately not the reference of the name. This may not be intolerable as a conclusion about mountains, but since any ordinary material object will have a borderline part, reference to ordinary material objects will never be determinate. A description such as ‘The present pope’, a demonstrative such as ‘That thing over

\(^2\) This difference was noted by Quine in his paper “What price bivalence?”. Quine says: “We were able to stipulate an arbitrary minimum to the number of grains in a heap, and a maximum to the number of hairs on a bald head, but we are at loss to frame a convention for the molecular demarcation of the surface of a table. Words fail us.” (“What price bivalence?”, *Theories and Things*, Harvard University Press 1981, pp. 34-35).
there', a proper name such as 'Toni Morrison' and a pronoun such as 'I', will all be vague in this respect. We might be able to live with this as a conclusion about language, but the same kind of reasoning that shows that we are unable to single particular objects out as the referents of singular terms also shows that we are unable to have *de re* beliefs about ordinary material objects. We can not even have *de re* beliefs about ourselves. And that I find worrisome.

Alternatively, we might explain the vagueness of (2)

(2) Sparky is part of Kilimanjaro,

by saying that the mountain itself is vague. This may seem appealing in many ways; an object such as Kilimanjaro does, intuitively, not have a precise boundary and if there are vague objects then reference to ordinary material objects can be determinate and we can have *de re* beliefs about them. But the idea that there are vague objects has been found very perplexing. I believe, however, that this is mainly because the idea has taken on perplexing forms, not because it need be perplexing.

The view that there are vague objects is usually formulated in one of three ways. First, in terms of there being borderline parts or there being no fact of the matter about something. Here is an example.

... I shall classify a concrete object o as vague (in the ordinary sense in which Everest is vague) if, and only if, (a) o has borderline spatio-temporal parts and (b) there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts, nor non-parts of o.\(^3\)

What we have here is a definition of 'vague object' in terms of borderline parts and there being no fact of the matter about something. But how are we to understand the expressions 'is a borderline part' or 'there is not fact of the matter'? If we explain why there is no fact of the matter why, say, \( a \) is part of \( b \) in terms of semantic indecision then what was meant to be a definition of ontological vagueness ends up being a definition of linguistic vagueness. What is lacking is an explanation of why there is no fact of the matter about something that is distinctively ontological.

The second way in which the ontological view has been formulated is in terms of vague identity. Insofar as I can make sense of the idea that identity might be vague it is not helpful in explaining the vagueness of a sentence such as

\[
(2) \quad \text{Sparky is part of Kilimanjaro.}
\]

The idea that identity is vague is better seen as a response to the Ship of Theseus. This is how Plutarch of Chaeronea described it around the turn of the second century A.D.

The ship in which Theseus and the youth of Athens returned had thirty oars, and was preserved by the Athenians down even to the time of Demetrius of Phalerus, for they took away the old planks as they decayed, putting in new and stronger timber in their place insomuch that this ship became a standing example among the philosophers, for the logical question of things that grow; one side holding that the ship remained the same, and the other contending that it was not the same.\(^4\)

If someone pointed to the ship at the time of Theseus' travel and gave it the name 'T' and then, some years later, pointed at the ship that was being preserved in the dock in Athens and gave it the name 'D', then it seems unclear whether \( T = D \). Although

the idea that identity is vague may seem initially plausible in the light of The Ship of Theseus, I don’t think it withstands scrutiny. But, be that as it may, my concern is not the Ship of Theseus but the indeterminacy of sentences such as (2) above. And if it is the indeterminacy of such sentences that we want to explain then what is at stake is parthood rather than identity.

The third way in which the ontological view has been formulated is in terms of fuzzy objects. Friends of fuzzy objects are right in focusing on the notion of parthood given that it is the indeterminacy of sentences such as (2) above that we want to explain. The theory of fuzzy objects is an adaptation of the theory of fuzzy sets to account for borderline cases of material objects. In the theory of fuzzy sets membership comes in degrees. Similarly, in the theory of fuzzy objects, parthood comes in degrees. Sparky, for instance, might be part of Kilimanjaro to a degree 0.7438. But fuzzy objects are not vague objects, they are precise objects with a strange feature, namely that there are entities that are parts of them to a degree. Consider for instance the following analogy. Suppose we have been assigning grades on a pass/fail basis, but then change to a scale from 0 to 10 where 0 stands for no comprehension and 10 stands for perfect comprehension. To assign grades on such a scale may describe the abilities of students better than the simple pass/fail classification, but it does not bring in vagueness in any way. Similarly, one can argue that having parthood come in degrees rather than being a matter of all-or-nothing allows us to give a more truthful description of the world, but it is not to bring in vagueness in any way.

We have briefly seen three formulations of the view that there are vague objects. In the first formulation the view is not clearly distinguished from the linguistic view, the second formulation seems to be directed at a different problem than the one we are concerned with, and in the third formulation what we get is not a thesis about vague objects at all. This situation is rather perplexing. On the one hand, we have philosophers arguing for the view that there are vague objects and against alternative views, but on the other hand, no one seems to be able to formulate the preferred view in a clear and plausible way. But the situation need not be this perplexing. The present proposal is a combination of two theses: (i) an object is vague if and only if it is indeterminate whether another object is part of it and (ii) we can quantify into indeterminately contexts. The first locates the vagueness in the parthood relation, the latter marks the vagueness as ontological rather than linguistic.

The present essay is an attempt to develop this proposal in some detail and to answer some criticisms. In Chapter Two I explain how vague parthood allows us to solve the problem of the many, and in Chapter Three I answer some criticism that ontological vagueness has met. Chapters Four and Five then develop the view in more detail by giving an account of the notions of parthood, vague object and material constitution. Finally, in Chapter Six, I consider the logical properties of a language with a relation term, ‘part of’ that stands for a vague relation and singular terms referring to vague objects.
2

Vagueness and The Problem of the Many

In his paper “The problem of the many” Peter Unger gives an argument for the conclusion that we are grossly mistaken about what kinds of objects are in our immediate surroundings. But it is not clear what we should make of Unger’s argument. Some say it shows something about our knowledge of the external world, namely that we don’t know which objects are the referents of singular terms in our language. This is the epistemic view. Others say it shows something about language, namely that the singular terms that we use to refer to ordinary material objects are vague. This is the linguistic view. Finally, there are those who say that Unger’s argument shows something about the nature of material objects, in particular that it is a reductio of the premiss that objects must be precise. This I call the ontological view, and it is this view that I will favor.

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1. The problem of the many

1. Before I turn to solutions to the problem I give a reconstruction of Unger’s argument that is more explicit about certain formal features of the argument than Unger’s original formulation. Unger’s argument is a *reductio* argument, it begins with the supposition that there is a single cloud in the sky and then concludes that there must either be a host of clouds there or none. As a conclusion about clouds this is not very troubling, but it should be clear that the same kind of argument can be repeated for almost any ordinary material object, whether it is a mountain or a person or what have you.

Unger’s argument is not only a *reductio* argument, it is also what I will call ‘an indifference argument’ since a crucial step in the argument turns on two or more objects being indifferent in a special way.\(^7\) We begin with the supposition that there is a single cloud in the otherwise blue sky and the following thesis about clouds.

Cloud Thesis: A cloud is an aggregate of droplets distributed in a suitable way.\(^8\)

There are two worries that one might have about Cloud Thesis. The first concerns the notion of aggregation that figures in it, the second is that we should not say that clouds are identical to aggregates of droplets but only constituted by such aggregates at any time. At this point we shall not worry about the notion of aggregation that figures in Cloud Thesis. When Unger introduced the problem and

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\(^7\) My treatment of Unger’s argument as an indifference argument draws on a discussion by Stephen Makin in his *Indifference Arguments* (Blackwell 1993).

\(^8\) What ‘in a suitable way’ means here is just that the droplets are distributed so as to form a cloud; we could say that they are cloudishly arranged.
coined the phrase ‘the problem of the many’ he did not offer much in a way of explanation of the relation between droplets and clouds. What he says is that the only likely candidates [for being a cloud] will be concrete complexes composed, at least in the main, not merely of some water droplets but of a great many droplets that are “suitably grouped together”.

And this seems fair enough. The second worry can also be postponed since the difference between identity and constitution will not be relevant in the first run of the problem. We shall come back to the notion of aggregation and constitution, but for now I shall follow Unger and not worry about what these come to in the end.

Consider now two different aggregates, A1 and A2, that are identified by their constituent droplets and whose sole difference lies in the presence of one particular droplet in A1 and its absence from A2. Then the difference between A1 and A2 is so minute that there should be no reason why A1 is a cloud rather than A2, or vice versa. I shall say that A1 and A2 are indifference candidates, because they are candidates for being the cloud and it is a matter of indifference which one is the cloud. Now, if A1 is a cloud, then it must also be the cloud in the sky, since we are assuming that there is only one cloud there, and similarly for A2. Let p be the proposition that A1 is the cloud and q be the proposition that A2 is the cloud. Then there is no reason why p should be true rather than q, or vice versa. It is a matter of

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9 “The problem of the many”, p. 415.

10 If we say that clouds are constituted by such aggregates then the many, which will turn out to be the problematic ones, will not be clouds but constituters of clouds. Instead of there being too many candidates for being the cloud in the sky, there will be too many candidates for constituting the cloud in the sky. This conclusion is, however, problematic only if we assume that the constitution relation is one-one. I say more about this when I get to the ontological view later in this chapter and when I discuss material constitution in Chapter Five.
indifference which of $p$ and $q$ is true, or as I shall also say, $p$ and $q$ are indifferent.

This gives us what I call Indifference Premiss.

**Indifference Premiss:** $p$ and $q$ are both plausible but indifferent.

What is the notion of indifference that we have here? Let's start by looking at what Peter Unger says where he is introducing the problem of the many.

... it seems clear that no matter which relevant concrete complex is deemed fit for clouddom, that is, is deemed a cloud, there will be very many others each of which has, in any relevant respect, a claim that is just as good.11

When Unger says that there are many candidates that have *just as good* a claim for being a cloud, he is not just saying that there are many candidates about which we have equally good reasons to assert, say, belief or judge that it is a cloud. He wants to say something stronger: for any reason for one aggregate of droplets being a cloud, there is an equally good reason for other aggregates' being a cloud. What I shall mean by 'indifferent' is then the following:

$p$ and $q$ are indifferent iff for any non-question begging reason for one's being true there is an equally good reason for the other's being true.

It may be difficult to say exactly what a non-question begging reason is but some examples may help. A typical question begging reason is the proposition that $p$ is true, whereas a non-question begging reason is, for instance, a proposition about the distribution of droplets in the sky, or some general proposition about the natures of clouds, or the conjunction of some such propositions.

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11 "The problem of the many", p. 415.
From the Indifference Premiss, understood in this way, it is tempting to conclude that either both \( p \) and \( q \) are true or neither is, which would suggest the following thesis:

**Duplication Thesis:** If \( p \) and \( q \) are indifferent then either both are true or neither is.

The Duplication Thesis is in fact a consequence of the indifference of \( p \) and \( q \) and the principle of sufficient reason. According to the principle of sufficient reason, if \( p \) and \( q \) have a different status, then there must not only be a reason why their status is different but a sufficient reason. But the Indifference Premiss, as I am suggesting it should be understood, says that there is no such reason. Hence, \( p \) and \( q \) must have the same status.

Even in the absence of the principle of sufficient reason we might want to accept Duplication Thesis on the grounds of the indifference of \( p \) and \( q \). We might find it counterintuitive that complexes such as \( A_1 \) and \( A_2 \) that differed so minutely with respect to their constituent matter differed so greatly with respect to cloudhood. Still, it should be born in mind that the mere indifference does not commit us to the Duplication Thesis.

From the Indifference Premiss and the Duplication Thesis, along with the following principle about truth:

\[
\begin{align*}
&\text{if } p \text{ is true then } (p \text{ is true } \leftrightarrow S) \\
&\text{if } p \text{ is false then } (p \text{ is false } \leftrightarrow \neg S) \tag{12}
\end{align*}
\]

we get the following conclusion:

12 It is necessary to have ‘\( \models \)’ rather than ‘\( \rightarrow \)’ since if there is a vague term in \( p \) the proposition that \( p \) says that \( S \) might lack a truth value.
Either both A1 and A2 are clouds or neither is.

Since we are assuming that there is exactly one cloud in the sky it must be that neither A1 nor A2 is the cloud. This alone is not alarming, but since A1 and A2 are as good candidates as any for being a cloud, by parity of reasoning we can generalize the above conclusion.

Conclusion: Either there is a host of clouds in the sky or none.

And since we want to say that there is exactly one cloud in the sky, this conclusion is unacceptable. Something must give.

2. Two solutions

1. Philosophers have offered three kinds of solution to this problem: epistemic, linguistic and ontological. The epistemic solution is in many respects the simplest one. On this view all that the argument shows is that we don’t know where the boundaries of the cloud are and, therefore, don’t know which aggregate of droplets is the cloud. Friends of the epistemic view insist that insofar as

   Indifference Premiss: p and q are both plausible but indifferent,

is a plausible premiss, the notion of indifference must be epistemic, i.e. the

Indifference Premiss should read along the following lines:

   Epistemic Indifference: p and q are both plausible but there is no reason to assert (say, judge, believe, ...) that p is true rather than to assert (say, judge, believe, ...) that q is true.

If the Indifference Premiss is read along the lines of Epistemic Indifference, then the Duplication Thesis has no force and Unger’s argument is not compelling. Two propositions can obviously have equal epistemic status and yet one be true and the
other false; if the police can't tell of two suspects which one is guilty it does not follow that both suspects are guilty or neither is. And similarly, if we can't find a reason why one aggregate of droplets should be a cloud rather than some other, it does not follow that either both are clouds or neither is.

According to the epistemic view all that the problem of the many shows is that we use certain singular terms and count nouns in our language despite ignorance about their reference and extension. And this is not an alarming conclusion. Moreover, since the candidates among which we are too ignorant to choose are indifferent in an important respect, one might suggest that this ignorance is, after all, harmless.

But even if the epistemic view offers a solution to the problem of the many, a different problem will make trouble for it. This new problem is not an ontological problem but a problem about language, I call it 'the problem of vague reference'.

2. According to the epistemic view a name such as 'Kilimanjaro' refers determinately to some object – there is a fact of the matter about what it refers to – we just don’t know which object it is. Now, suppose a skeptic comes along who challenges the claim that the name 'Kilimanjaro' has a determinate reference. First, the skeptic suggests two different candidates for being the mountain, say K1 and K2. These candidates are aggregates of gravel and soil, and they seem to be just as good a candidate for being Kilimanjaro as any other aggregate. Moreover, these two candidates differ only minutely; let's say that the only difference is a single pebble, call it 'Sparky', that belongs to one but not the other. Second, the skeptic suggests the following thesis about reference:
Reference Thesis: A name $n$ in a language $L$ refers to $e$ if and only if the thoughts and practices of the speakers of $L$ determine that $n$ refers to $e$.

Now, the skeptic challenges us to explain what fact about our thoughts and practices, however broadly construed, could determine that the name ‘Kilimanjaro’ referred to K1 rather than K2. And there just does not seem to be any such fact. The problem is generated by the combination of three theses, (i) that there are indifference candidates such as K1 and K2, (ii) that an ordinary name such as ‘Kilimanjaro’ has determined reference and (iii) that what determines reference are thoughts and practices of speakers.

I should stress that this problem is not a problem for formal semantics, i.e. we can, for instance, still give a Tarski style definition of truth where we have clauses such as:

$$\text{if } x = \text{‘Kilimanjaro’} \text{ then } x \text{ refers to Kilimanjaro},$$

for all the singular terms and clauses such as:

$$\text{if } x = \text{‘mountain’} \text{ then } x \text{ refers to mountains},$$

for all the count nouns in the language. But, unless we can solve the problem of vague reference we can’t hope to use the notion of reference as a basis for serious attempt to understand linguistic behavior. Consider, for instance, the following passage from the opening of Strawson’s *Individuals*.

Very often, when two people are talking, one of them, the speaker, refers to or mentions some particular or other. Very often, the other, the hearer, knows what, or which, particular the speaker is talking about; but sometimes he does not.\(^{13}\)

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If the notion of reference is to serve in an account of communication, we should say that when the hearer does not know which individual the speaker refers to or when the hearer identifies a different individual from the one intended by the speaker, then communication is defective; we have a case of mis-communication or even lack of communication.

Now, suppose mountains are aggregates of clods of earth and that on some beautiful day down in Tanzania I point to where the land rises and say “That is Kilimanjaro”. Since at the same location there are so many aggregates of clods of earth that differ only minutely I have no means of identifying one as opposed to some other complex as the bearer of the name ‘Kilimanjaro’, i.e. I can’t have mentioned any particular object. And even if I could identify one aggregate, it is unlikely that my interlocutor would single out the same aggregate as the referent of the name. The same goes for our talk about other ordinary material objects, clouds, people, etc. If reference is central to communication, then almost all communication will turn out to be defective.

3. What the skeptic has done is to follow up on the epistemic solution to the problem of the many with a different indifference argument. This new indifference argument is not about clouds or mountains, or other material objects, but about ordinary names in our language. Friends of the epistemic view have three options in this situation. They might insist that our words have determinate meanings in either of two ways. First, they might reject Reference Thesis, second, they might suggest that there are, after all, some facts about the thoughts and practices of speakers that determine the reference of names such as ‘Kilimanjaro’. The third
option is to accept that our words are vague. The first option is hardly plausible. What Reference Thesis says is that whatever the connection is between our language and the world it has to be established by our thoughts and practices. And what else is there to establish such a connection? Nothing comes to mind. The second option does not seem promising either. How could thoughts and practices of speakers determine of every single grain of sand whether it is part of Kilimanjaro or not? Or of every drop of water whether it belongs to the North Sea or to the Atlantic Ocean? Our thoughts and practices are not that fine grained. The remaining alternative is to accept the claim that names are vague, but that undermines the epistemic solution to the problem of the many. The epistemic solution maintains that a name such as 'Kilimanjaro' has a determinate reference, but that we just do not know which object is the referent of the name. Now friends of the epistemic view seem forced to retract the first part, i.e. they have to give up the thesis that a name such as 'Kilimanjaro' has a determinate reference.

4. The situation is now like this. Although the epistemic solution works well for the problem of the many, it can't solve the problem of vague reference. So, we have to look for a different solution to the problem of the many. According to the linguistic view the problem of the many shows that names such as 'Kilimanjaro' and 'Puffy' and general nouns such as 'mountain' and 'cloud' are vague in the sense that they don't have any determinate reference and extension. One challenge now is to give an account of the truth of a sentence such as "There is exactly one cloud in the sky" when, intuitively, there is only one cloud there. It is not enough just to say that our language is vague, because for something to be a solution it must be able to
retain what seems intuitively beyond doubt, for instance that in certain circumstances it is determinately true that there is exactly one cloud in the sky.

The linguistic view consists of two parts.\textsuperscript{14} The first is an explanation of why there are vague words in a language like English, the second is a systematic account of the truth conditions of complex sentences in a vague language. The explanation of why we have vague words in our language consists of two basic claims: First, a claim about what kind of facts determine meaning, and second, the claim that these facts leave meaning underdetermined. In the case of singular terms, the first part is given by something like Reference Thesis from above.

\textbf{Reference Thesis:} A name \( n \) in a language \( L \) refers to \( e \) if and only if the thoughts and practices of the speakers of \( L \) determine that \( n \) refers to \( e \).

The claim that meaning is underdetermined by the relevant facts is then cashed out in terms of which objects are candidate referents and to what extent the relevant facts might distinguish between these as the actual referents of names in \( L \). This is how David Lewis puts it:

\begin{quote}
The reason it's vague where the outback begins is ... [that] there are many things, with different borders, and nobody has been fool enough to try to enforce a choice of one of them as the official referent of the word 'outback'. Vagueness is semantic indecision.\textsuperscript{15}
\end{quote}

The second part of the linguistic view is an account of truth conditions for complex sentences and identity sentences containing vague constituents, whether these are common nouns, names or predicates. The sentences

\textsuperscript{14} My discussion of the linguistic view is largely based on the writings of Vann McGee, in particular his paper "'Kilimanjaro'".

\textsuperscript{15} David K. Lewis, \textit{On the Plurality of Worlds}, p. 212.
Kilimanjaro is the tallest mountain in Africa,

If Sparky is part of Kilimanjaro then Sparky is part of Kilimanjaro,

are certainly true. But friends of the linguistic view can not explain the truth of the first by saying that the referent of the name 'Kilimanjaro' is denoted by the description 'the tallest mountain in Africa', as standard semantics has it, since they don't believe that there is such a thing as the referent of the name 'Kilimanjaro' or the denotation of the description. The truth of the second sentences does not follow from the standard truth table definition of '→' since both conjuncts are indeterminate. Moreover, any truth table definition of '→' won't be able to distinguish between the above sentence and

If Sparky is part of Kilimanjaro then Tom is bald,

where Tom is a borderline case of a bald person.

To account for the truth of sentences with vague count nouns and singular terms some friends of the linguistic view have used supervaluation. Supervaluation is a two stage process. First, one identifies ways in which words in the language might admit of precisification, then one uses these precisifications to define a notion of truth for the language in question. A precisification is a classical model, it is a model in which every singular term has a determinate reference and every general term has a determinate extension. In each such precisification the name 'Kilimanjaro' has a unique reference though it may differ from one precisification to another what the reference is. But the notion of a precisification is not enough, one needs a notion of an acceptable precisification.\textsuperscript{16} The distinction between an arbitrary

\textsuperscript{16}The notion of an acceptable precisification, as opposed to the unqualified notion of precisification, cannot be defined within the supervaluation framework but must be taken as
and an acceptable precisification is that the latter has to meet certain intuitive
constraints on what the semantic values of the terms in the language can be. These
constraints are basically twofold. First, there are constraints that one might call
'correspondence constraints'. For instance, a precisification that has as the reference
of the name 'Kilimanjaro' half of Tanzania is not acceptable since half of Tanzania is
determinately not what the name 'Kilimanjaro' refers to. In such a precisification
the correspondence between the name and the world has gone wrong. Second,
there are coherence constraints. In each acceptable precisification the object that is
the referent of 'Kilimanjaro' must also satisfy the description 'the tallest mountain in
Africa'. This is because the sentence "Kilimanjaro is the tallest mountain in Africa"
is definitely true and must, therefore, be true in each acceptable precisification.17

17 Quine's thesis about inscrutability of reference might suggest that only the latter
constraints, i.e. the coherence constraints, are tractable. The argument might go something
like this: Suppose we have a class of precisifications where the reference of the name
'Kilimanjaro' meets the intuitive reference constraints. Now, we might define a permutation
function for this class of precisification in such a way that in the permutated precisifications
'Kilimanjaro' did not refer to a landmass in Tanzania but, say, to some number or other. The
rest of the expressions in the language would be permuted so that for any precisification in
the original class there would be a permuted precisification such that any sentence in the
original precisification and the corresponding sentence in the permuted precisification
agreed in truth value. If this was possible, then we could not distinguish the two classes of
precisifications in terms of how they assigned truth values, and, hence, could not say that the
precisifications in the first were acceptable while those in the second were not. But are such
permutations possible? Hilary Putnam has offered a proof to the effect that they are possible
("Appendix" in Reason, Truth and History, Cambridge University Press 1981), but the proof
seems to be defective. The problem is that to guarantee that a permutation preserves the
truth conditions for modal sentences an individual has to exist at the same worlds as its
image exists. For ordinary physical objects this is a serious constraint. (This objection to
Putnam's alleged proof is due to Vann McGee.)
Now we get to the second stage, namely how all this bears on the truth values of sentences of English. The entire class of acceptable precisifications is taken to be a model of English in the following way: If a sentence is true in every acceptable precisification, then it is true in English, if it is false in every acceptable precisification then it is false in English, and if it is true in some but false in other precisifications, then it is indeterminate.

If we go back to the problem of the many we can see that according to the linguistic view the dilemma posed by the Duplication Thesis,

**Duplication Thesis:** If $p$ and $q$ are indifferent then either both are true or neither is,

is solved by going for the second horn; neither $p$ nor $q$ is true. But, neither $p$ nor $q$ is false either, instead they are indeterminate. In some acceptable precisifications $p$ is true and $q$ false, in some others $q$ is true but $p$ false.

5. Supervaluation enables friends of the linguistic view to account for the truth values of ordinary English sentences when some constituent expressions are vague. In particular, it allows that the sentence "There is exactly one cloud in the sky" to come out true when, intuitively, there is only one cloud there despite the challenge of the problem of the many. The reason why the sentence is true is that in each acceptable precisification there is exactly one object that is a cloud in the sky, and so in each such precisification the sentence "There is exactly one cloud in the sky" is true.

What the problem of the many shows is that in different precisifications different objects satisfy 'is a cloud in the sky' and, therefore, different objects will be
the referent of the name ‘Puffy’. This leads to the conclusion that the sentence “There is exactly one cloud in the sky” can be true while it is not true of any object that it is the cloud in the sky, for different precisifications disagree about which object it is.¹⁸ Let’s say that a sentence is definitely true if it is true in all precisifications and write ‘Def S’ if ‘S’ is such a sentence. The initial assumption of the problem of the many is that it is definitely true that there is exactly one cloud in the sky, i.e.

(1) \( \text{Def (}\exists x \text{ x is the cloud in the sky)} \)

is true. The supervaluation account of vague singular terms tells us that this sentence can be true while

(2) \( \exists x \text{ Def (x is the cloud in the sky)} \)

is not true. What the problem of the many shows is that (2) can’t be true, but if truth conditions of complex sentences are given by supervaluation then (2) does not need to be true in order for it to be true that there is exactly one cloud in the sky. All that is needed is that (1) be true and in the supervaluation framework (1) and (2) are not equivalent.

We have seen how the linguistic view solves the problem of the many, but how does it respond to the problem of vague reference? The problem of vague reference is a redactio of the conjunction of three assumptions: (i) reference of ordinary names such as ‘Kilimanjaro’ is determined, (ii) there are indifference

¹⁸ The sentence “There is something that is the cloud in the sky” is definitely true, since it is true in each precisification, and yet it is not true of any thing that it is the cloud in the sky. Here, truth behaves like necessity; the sentence “Someone is the tallest boy in the class” may be necessary true (at least if by tallest we meant ‘taller or equally tall to any other boy in the class’) and yet it is not necessary of any boy in the class that he is the tallest one.
candidates such as K1 and K2, and (iii) insofar as reference is determined, it must be determined by the thoughts and practices of speakers of the language. But this is not a problem for friends of the linguistic view since they don’t accept the first assumption. The problem of vague reference was a problem for friends of the epistemic view since they wanted to maintain all three assumptions and, hence, had to find a way to refute the skeptic that challenged their claim that a name such as ‘Kilimanjaro’ had a determined reference. The challenge was that there are indifference candidates, say K1 and K2, between which thoughts and practices of speakers of the language in question are unable to decide as the reference of the name ‘Kilimanjaro’. In order to refute the skeptic friends of the epistemic view had to find a fact that fixed whether the name ‘Kilimanjaro’ referred to K1 or K2. By contrast, friends of the linguistic view do not face this problem since they accept the skeptic’s conclusion, and they accept it precisely because they believe there is no such fact. Our thoughts and practices don’t determine whether our name ‘Kilimanjaro’ refers to one rather than the other. It is undecided.

‘Vagueness is semantic indecision’ is the central thesis of the linguistic view and what makes the linguistic view worthy of being called ‘a solution’ to the problem of the many and the problem of vague reference is that it can account for how ordinary sentences have the truth values we expect them to have despite this indecision.

6. So far so good for the linguistic view: we have a solution to the problem of the many and the problem of vague reference is solved by the same means. But if we shift the focus from clouds and language to beliefs, a different problem arises, I call
it 'the problem of de re beliefs'. In the paper "The problem of the many" Unger
spells out the problem under the heading 'the problem of having an object in
mind'.\textsuperscript{19}

We commonly suppose that, regarding various existing ordinary things, several
nearby stones, for example, we can think of each of them, or have each in mind. ... But if there are millions of "overlapping stones" before me ... how am I to think of
a single one of them, while not then equally thinking of so many others, with each
of which "it" might so readily be confused? The presumed \textit{relations} between us,
and our minds, and ordinary material complexes look to be in deep trouble.

And a little later Unger adds:

I suggest that up until now, at least, not one of us has ever really thought of any
existing stone or table or human hand.\textsuperscript{20}

Now, what shall one make of this last problem? It isn't a contradiction, so
philosophers can live their professional life with it, and some may be willing to live
with it in their spare time as well. But this problem shows that if the linguistic view
is correct, we need a major revision of how we think of relations between us and
ordinary material objects.

Let's look closer at the problem. If I am entertaining a belief about a particular
object, if I have an object in mind as Unger puts it and am believing that it is \( F \), then
my belief can not be represented by "I believe that there is something that is \( F \)",
but rather by "There is something such that I believe of it that it is \( F \)". I can believe that
something is \( F \) without believing of anything in particular that it is \( F \). If my cookie
disappears then I may believe that someone took my cookie without believing of

\textsuperscript{19} "The problem of the many", part A of section 12.

\textsuperscript{20} "The problem of the many", p. 456.
anyone in particular that he or she took my cookie. This general belief can be represented in the following way:

I believe $\exists x \ (x \text{ took my cookie})$.\(^{21}\)

But once I have investigated the case and found out who the guilty one is my belief will be more specific; we say that I believe of someone in particular that he took my cookie. In other words, I will have a \textit{de re} belief about the thief, and this \textit{de re} belief can be represented in the following way:

$\exists x \ (\text{I believe } x \text{ took my cookie})$.\(^{22}\)

Similarly, I may believe that some cloud is the most beautiful cloud ever; i.e. the following might be true:

\[^{21}\text{This formulation is neutral with respect to the debate between those who take belief to be a sentential attitude and those who take it to be a propositional attitude. What is embedded can either be a canonical representation of a sentence or a proposition.}\]

\[^{22}\text{Here I use the difference in scope to mark the distinction between \textit{de dicto} and \textit{de re} beliefs. But it should be clear that the distinction between \textit{de re} and \textit{de dicto} beliefs is not just the difference between wide and narrow scope. This is best seen in the case where we have intermediate scope. The sentence “I believe the dude took my cookie” is ambiguous between the following forms:}\]

\[(i) \quad \text{I believe } \exists x \ (x = y\text{Dude}(y) \land x \text{ took my cookie})\]

\[(ii) \quad \exists x \ (\text{I believe } x = y\text{Dude}(y) \land x \text{ took my cookie})\]

\[(iii) \quad \exists x \ (x = y\text{Dude}(y) \land \text{I believe } x \text{ took my cookie})\]

Here, the \textit{de dicto} reading might be represented by (i) but the \textit{de re} reading might be represented by either (ii) or (iii). For present purposes we need not decide which formulation represents best the \textit{de re} reading, on either reading \textit{de re} beliefs will be beyond human capacity.

Alternatively, one might want to represent my \textit{de re} belief that the dude took my cookie in the following way:

$\exists x \ (\text{I believe } \Phi(x) \land x \text{ took my cookie})$

where ‘$\Phi$’ stands for some description, perhaps purely qualitative, that uniquely singles out the individual in question. But this won’t help since as long as the description is something I can comprehend, there will be many indifference candidates that satisfy it, and, hence, the belief won’t be singular.
I believe \( \exists x (x \text{ is a cloud} \land \forall y (y \text{ is a cloud} \rightarrow x \text{ is more beautiful than } y)) \).

But if I have the *de re* belief of Puffy that it is the most beautiful cloud ever, then my attitude cannot be described in the above way. In order to get the content of my belief right, we must force the choice between different candidates, i.e. we must place the existential quantifier outside the scope of my belief like this:

\( \exists x (x = \text{Puffy} \land \forall y (y \text{ is a cloud} \rightarrow x \text{ is more beautiful than } y)) \).

The question now is whether friends of the linguistic view can accommodate *de re* beliefs such as the ones that I have been talking about and seem to be ordinary everyday beliefs. The problem for the linguistic view (and the epistemic view as well) is that since there are so many candidates that differ so minutely it would be incredible if my cognitive powers were able to distinguish among them. Peter Unger says that it would not only be incredible, it would be impossible.

But why is there a problem here? Why can’t supervaluation come to rescue here as it did before? The reason is that the referring entities that we were considering before were names in a language, names such as ‘Puffy’ and ‘Kilimanjaro’, but in the case of reports of *de re* beliefs the referring is not done by names but by variables. And while supervaluation can give an account of vague names, it can’t make room for vague variables. Variables are mere place holders for objects and they just can’t be vague. The only choice for friends of the linguistic view is to give up the idea that people can have *de re* beliefs about such objects as clouds, mountains and people, including beliefs about themselves, and try to explain why we have the illusion that we do have *de re* beliefs about all sorts of things.
A friend of the linguistic view might say that what we ordinarily think of as *de re* beliefs are just beliefs that can be reported by a sentence of the form “A believes that *a* is F”. This way of going about *de re* beliefs may actually seem to have independent motivation. Consider for instance Quine’s skepticism about *de re* beliefs. Quine analyzed *de re* beliefs in terms of *de dicto* beliefs in roughly the following way: To say of someone, say Paul, that he believes of *a* that it is *F*, is just to say that Paul believes that a sentence of the form “*a* is *F*” is true. Here, *de re* beliefs are not a special kind of beliefs but, rather, a special way of describing beliefs.23 As we saw in relation to the problem of vague reference, a sentence of the form “*a* is *F*” need not be singular in the sense that some particular object is said to be *F*, and if the premisses of the problem of the many are right then such sentences about ordinary material objects will never be singular.24 The distinction between *de re* and *de dicto* beliefs will not be a distinction between singular and general beliefs but merely between beliefs that can be reported by sentences that have a singular term in the subject place and beliefs that can be reported by sentences that have a general term in the subject place.

This last problem may not show that the linguistic view is untenable, but, as I said earlier, if it is right then some serious revisions in the way we think of our

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24 More precisely, “*a* is *F*” will not be singular if ‘*a*’ is used to talk about ordinary material objects, i.e. if ‘*a*’ is a name such as ‘Kilimanjaro’ or ‘Puffy’. The situation may be different if ‘*a*’ is a numeral or a name of an abstract object, although these also face a challenge about vagueness. For instance, supposing that numbers are sets, we can’t say whether what Fermat referred to when he used the numeral ‘3’ was a Zermelo’s {0, 0} or von Neuman’s {0, {0}, 0, {0}}, or some other set. But the reasons for holding that the numerals are vague are different from the problem of the many.
relations to the external world are required. I will not go into the prospects of undertaking such revisions, but instead move on and prepare the ground for the ontological view.

3. The complex problem of the many

1. In order to spell out the ontological view, as I want to construe it, we have to take back certain simplifications that I made earlier. When I laid out the problem of the many at the beginning of this chapter I said that I would come back to the notions of aggregation and constitution. Now is the time to fulfill that promise. One of the premisses that the problem of the many relied on was Cloud Thesis.

Cloud Thesis: A cloud is an aggregate of droplets distributed in a suitable way.

But this thesis is unclear and obviously so since no attempt has been made to explain what an aggregate is. Does an aggregate survive a replacement of parts? What region of space does an aggregate occupy? What a friend of the ontological view wants to know, in particular, is whether an aggregate has all its parts determinately. So, what is this notion of aggregation?

When philosophers talk about aggregates (or compounds or complexes etc.) in this context they often seem to have in mind some sort of mereological sum along the lines of Classical Extensional Mereology. This notion is, arguably, insufficient for the purposes of explaining the relation between ordinary objects and their parts, such as clouds and droplets, since it does not say anything about temporal and
modal properties of objects. This shortcoming can be dealt with in two ways. First, one might suggest that the parts of objects are themselves temporal and modal, time-world slices, and then have a principle of composition that was neither tensed nor modalized. Alternatively the parts might be three dimensional objects, i.e. objects that do not have a part at another time or another world, and instead have a tensed and modalized principle of composition.

2. Those who want to take the first alternative and stick to the principle of mereological sum in its simplest form (as defined in the Leonard and Goodman calculus) as a principle governing material composition might be motivated by simplicity considerations. But simplicity is soon lost. In order to make room for intuitive temporal and modal properties one has to accept time-world slices as the basic constituents of the world, and to do that is to trade simplicity at one level in for complexity at another. Besides, if the basic parts are time-world slices the definition of mereological sum has to be changed in order to allow for sums of non-actual entities as well as actual ones. This means that unless one is a modal realist, the


26 It would not be fair to say that Leonard and Goodman themselves were motivated by such considerations. Their concern in "The calculus of individuals and its uses" was more of a logical or semantic nature than metaphysical. This should be clear from the following remark from the closing paragraph of the paper: "We have performed the important service of divorcing the logical concept of an individual from metaphysical and practical prejudices, thus revealing that the distinction and interrelation of classes and wholes is capable of a purely formal definition, and that both concepts, and indeed all the concepts of logic, are available as neutral tools for the constructional analysis of the world". (The calculus of individuals and its uses", The Journal of Symbolic Logic, Vol. 5, No. 2 1940, p. 55).
definition of mereological sum must allow that what does not exist has a sum. Not
only will there be such an object as the sum of the actual present slice of me and
Julius Caesar’s nose, there will also be such an object as the sum of the actual present
slice of me and some daughter that Caesar might have had had his life been
different. This may not be inconsistent, but I’m not sure I know what it means, and
whatever one can say in favor of it, it won’t be that it is particularly simple.

A different defect of the view under consideration is that it obscures the
dependency relation between what is actual and what is possible. Intuitively, what
is possible must depend on what is actual. When I say that my desk could have
been made of slightly different material I am saying of an actual object that it could
have been different, and whether my claim is true or false is going to depend on
what this actual object is like. It is not that all the possible worlds are given in
advance and the actual world is just one among them, differing from the others
simply in that it happens to be the one we inhabit. When it comes to metaphysical
possibility and necessity it seems to me (though not to David Lewis) that the realm
of possible worlds must be defined with reference to the actual world.27 Consider
the account Kripke gives of possible worlds towards the end of his first lecture in
Naming and Necessity.

An analogy from school ... will help to clarify my view. Two ordinary dice (call
them die A and die B) are thrown, displaying two numbers face up. For each die,
there are six possible results. Hence there are thirty-six possible states of the pair
of dice, as far as the numbers shown face-up are concerned, though only one of
these states corresponds to the way the dice actually will come out. ...

27 This goes counter to modal realism, at least as David Lewis has argued for it. But even if
my claims about possible worlds are contentious I will not defend them here.
Now in doing these school exercises in probability, we were in fact introduced at a tender age to a set of (miniature) ‘possible worlds’. The thirty-six possible states of the dice are literally thirty-six ‘possible worlds’...

... But when we talk in school of thirty-six possibilities, in no way do we need to posit that there are some thirty-five other entities, existent in some never-never land ...

And a little later he adds:

‘Possible worlds’ are little more than the miniworlds of school probability blown large.\(^{28}\)

If actual material objects are sums of time-world slices then what exists at the actual world is in part determined by what exists at other possible worlds, which means that the merely possible worlds are just as real as the actual one. They are just somewhere in the never-never land, to borrow Kripke’s words.

Someone might respond to my complaints about time-world slices by saying that talk about such slices is not meant to be taken literally; such talk does not commit one to there really being such entities. A claim like this is often heard in the context of modal talk, and it is, I think, often quite legitimate. In Kripke’s framework we do, for instance, quantify over possible worlds and yet we want to say that there are no such worlds, that they do not really exist. The plausibility of such excuses depends on the way we use our possible worlds. The problem for friends of time-world slices is that they make these entities defining constituents of actually existing objects; an actual object is the sum of such-and-such time-world slices, as one’s library is, for instance, the collection of such-and-such books. And

just as the books must really exist if there is to be a library, so the time-world slices must really exist if there are to be any actual objects. And this makes the excuse that talk about possible worlds is just a manner of speaking unavailable to friends of time-world slices.

3. The oddities, and obscurities, that time-world slices bring with them should push us into the direction of a tensed and modalized principle of composition. To extend the notion of mereological sum from Classical Extensional Mereology to take into account tense and modality, I shall introduce a notion of fusion. A description of the form “The fusion of $x_1, \ldots, x_n$” is (i) a rigid designator, (ii) it denotes an object at a time in a world only if all of $x_1, \ldots, x_n$ exist at that time in that world, and (iii) all of $x_1, \ldots, x_n$ are parts of the fusion of $x_1, \ldots, x_n$ and the fusion has no parts that do not overlap one of $x_1, \ldots, x_n$. In a formal framework fusion would be represented by a function $f$ that takes as its argument a plurality of objects, $x_1, \ldots, x_n$, and returns as its value a single object, $y = f(x_1, \ldots, x_n)$.

In order for this notion of material composition to make room for intuitive temporal and modal properties of material objects it has to be complemented by a notion of constitution. Since aggregates have all their parts essentially, but ordinary objects such as clouds and mountains can survive replacements of parts, ordinary material objects cannot be fusions of basic elements. Clouds can’t be fusions of droplets and mountains can’t be fusions of clods of earth. Instead, we shall say that

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29 According to this what gets fused is a plurality of objects. We might as well have used a notion of fusion according to which what gets fused is a set of objects. Nothing important hinges on defining fusion in one way rather than the other. This notion of fusion resembles Judith Thomson’s notion of all-fusion in “The statue and the clay”.
clouds are, at any time, constituted by fusions of droplets and that mountains are, at
any time, constituted by fusions of clods of earth. Having to rely on a notion of
constitution is a departure from simplicity, but such a departure cannot be avoided
as long as one accepts that some material objects can survive replacements of parts.

4. Going back to the problem of the many, we can see that Cloud Thesis needs to
be replaced. Instead of saying that a cloud is identical to an aggregate of droplets we
shall say that a cloud is, at any time, constituted by such an aggregate.\(^{30}\) In other
words, we replace the initial Cloud Thesis with the following:

Cloud Constitution Thesis: A cloud is, at any time, constituted by an aggregate
of droplets distributed in a suitable way.

If we substitute Cloud Constitution Thesis for Cloud Thesis then the conclusion of
the problem of the many won’t be that there are either many clouds in the sky or
none. Instead, it will be that there are either many cloud constituters in the sky that
do not completely overlap\(^{31}\) or there will be no cloud constituter there. Whether this
conclusion is problematic depends on what the constitution relation is like. If we
assume that a cloud and anything that constitutes it must overlap completely then
we have a problem since it will not be possible that there are different cloud

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\(^{30}\) Unger acknowledges that something like this might be appropriate in a section titled "The
problem of identity through time" towards the end of his paper, but he thinks that this only
makes things worse.

\(^{31}\) Complete overlap can be defined in two ways; (i) in terms of spatial occupation, and (ii) in
terms of basic elements. According to the first way, two objects overlap completely if and
only if any region occupied by one is also occupied by the other. According to the second
way, two objects overlap completely if and only if exactly the same basic elements are parts of
them. For the present purposes it does not matter which way we go. On either definition
complete overlap is reflexive and transitive.
constituters that do not completely overlap. In other words, having replaced Cloud Thesis with Cloud Constitution Thesis we will have a problem of the many only if we have some principle like the following:

**The Pairing Principle:** If \( a \) and \( b \) each constitutes a cloud at \( t \) and \( a \) and \( b \) do not overlap completely, then \( a \) and \( b \) constitute distinct clouds at \( t \).

I call this last principle 'the pairing principle', since it suggests that even if clouds are not identical to fusions of droplets a cloud can, at any time, be paired with such a fusion in a non-arbitrary way.

Stating the problem of the many in this more complex way does not make much difference from the point of view of the epistemic or the linguistic views. It is, however, crucial for the form of the ontological view that I will defend. My basic thesis is that parthood is a vague relation but some philosophers might worry that vague parthood leads to vague identity. If all objects are either basic entities, i.e. non-composed objects, or fusions of such entities, then vague parthood will lead to vague identity. But if ordinary objects are not identical to fusions of basic entities but merely constituted by such fusions then an ordinary object, say a cloud, can be vague with respect to parthood without being indeterminately identical to something, say a fusion of droplets.

4. The ontological view

1. So far I have considered two solutions to the problem of the many, the epistemic view and the linguistic view. The epistemic view offers a straight and a rather simple solution to the problem, but runs foul of a different problem, the problem of vague reference. The linguistic view, on the other hand, accepts the
conclusion of the problem of the many and it also accepts the conclusion of the problem of vague reference. But it blocks any inconsistent consequences by giving a definition of truth according to which it can be true that there is exactly one cloud in the sky while it is not true of any object that it is the cloud in the sky. However, when it comes to the problem of *de re* beliefs, friends of the linguistic view have no option short of biting the bullet and admitting that we need to reconsider seriously our relations to the external world.

I now turn to a third kind of a solution to the problem of the many, the ontological view. Under this heading are several proposals that all have in common that they give an affirmative answer to the question: Is there vagueness in the world? Previous ontological proposals have mainly taken three forms. First, philosophers have suggested that an object is vague if and only if it has borderline parts or if there is no fact of the matter whether something is part of it. Others have suggested that identity might be vague, and still others have thought that the theory of fuzzy sets might be adapted to account for the vagueness of objects. To say that an object has

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32 Some philosophers find it very strange to call things that are not representations vague. If "n is vague" means "n does not have determined reference" then, of course, 'vague' can only be applied to things that are representations. When I say that an object is vague the notion of vagueness is not linguistic, and moreover, it is not the linguistic notion of vagueness applied to non-linguistic entities. That would not make any sense. Rather, to say that an object is vague is, on my view, to say that there isn’t a fact of the matter whether something is part of it. In this context, vagueness contrasts with determinacy rather than precision. Others, such as Terence Parsons and Peter Woodruff say that an object is vague when there isn’t a fact of the matter whether it is identical to something else. Again, vagueness contrasts with determinacy rather than precision.


borderline parts or that there is no fact of the matter whether something is part of it is just to report the fact of vagueness not to explain anything. What is needed in addition, and what is usually lacking, is an explanation of what it is for something to be a borderline part or why there is no fact of the matter whether one thing is part of another. To say that an object is vague just in case it has a borderline part is good as far as it goes, it just does not go very far.

According to the vague identity proposal, the cloud in Unger's story would be indeterminately identical to one or another of the precise aggregates of droplets. This is then taken to open the possibility of there being only one cloud in the sky while there are numerous aggregates of droplets there. I find this unattractive, not because of any logical difficulties but because I simply don't know what it might mean to say of an object that it is indeterminately identical to some object, for then, it seems, there would be an object that was indeterminately itself.

The idea that the theory of fuzzy sets might be applicable isn't much better, although its problems are different. The theory of fuzzy sets is a generalization of ordinary set theory. Instead of objects being either members or non-members of sets, they can also be members to a degree. Similarly, in what one might call a theory of fuzzy objects, objects are not either parts or non-parts of other objects but

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36 The logical implications of vague identity have been the subject of much discussion, most of which draws on a one page paper by Gareth Evans. I shall not add to that bulk. I think that it is generally agreed now that one can have logic with vague identity. Helpful discussions of Evans' argument can be found in Vann McGee's "Kilimanjaro", and in Richard Heck's Jr., "That there might be vague objects (so far as concerns logic)" The Monist, Vol. 81, no. 2, pp. 274-296.
also parts to a degree. So, for instance, if Sparky was part of Kilimanjaro to a degree 0.7 then it would not be fully a part of Kilimanjaro but it would be more of a part of the mountain than was Spooky who was only part of Kilimanjaro to a degree 0.01. But this I don’t find attractive either since it is not properly a theory of vague objects but of incredibly precise objects with a surprising structural feature.

2. I’ll say more about fuzzy objects later, but now I turn to my own view. Instead of vague identity or fuzzy objects, my suggestion is that parthood is a vague relation. This has two components; (i) facts about parthood are not primitive but determined by certain other facts about the objects in question, and (ii) sometimes the primitive facts which determine facts about parthood fail to determine of something whether it is a part or a non-part of something else. That is when we have an instance of vague parthood. There are no degrees, there are just three statuses; a is part of b, a is not part of b, and it is indeterminate whether a is part of b.

Now, if clouds don’t have all their parts determinately, as we are allowing, then the problem of the many does not arise since there will be no indifference candidates. Suppose we have two fusions of droplets, A1 and A2 that overlap Puffy more or less. Now we might ask: Might either A1 or A2 constitute Puffy? Suppose it is indeterminate whether the droplet Sprinkle is part of Puffy and that Sprinkle is part of A1 but not part of A2. Offhand it seems that this should rule out A1 and A2 as constituters of Puffy since they and Puffy disagree with respect to parthood; what is indeterminately part of Puffy is either determinately or determinately not part of A1 and A2. And since any fusion is precise, this has nothing to do with A1 and A2 in particular. Shall we then conclude that no fusion of droplets constitutes Puffy?
We have to be careful about our notion of constitution. It is determinately true that Puffy is constituted by droplets, but if this means that there is a fusion of droplets that constitutes Puffy and if Puffy is vague then constitution had better not be a precise relation, since then it will at best be indeterminate whether Puffy is constituted by some fusion or other of droplets. The way to resolve this is to distinguish between two notions of constitution, 'partial constitution' and 'exact constitution'. The notion of partial constitution should entail two principles that are relevant for our present concern. If c partially constitutes O then:

- everything which is determinately part of O has a part that overlaps some part of c at t,
- no part of c is determinately not part of O at t.

Moreover, the two principles above should be the only extensional constraints on partial constitution, although not the only constraints. Other constraints will have to do with modal properties of the constituter and the constitutee. The basic idea behind the notion of exact constitution is then this: c exactly constitutes O just in case c partially constitutes O and it is determinate that anything that is not part of c is not part of O. This definition of exact constitution turns out to be too simple, as I discuss in Chapter Five, but it suffices for now.

If we accept the above constraints as the only extensional constraints on partial constitution then there will be no extensional constraints that rule out the possibility that two fusions that do not completely overlap both partially constitute a single object. If we now go back to the Pairing Principle,

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37 Full definition of the two notions of constitution are given in Chapter Five.
The Pairing Principle: If $a$ and $b$ each constitutes a cloud at $t$ and $a$ and $b$ do not overlap completely, then $a$ and $b$ constitute distinct clouds at $t$.

and if the notion of constitution here is partial constitution, we can see that the possibility of many non-overlapping constituters renders it simply false. It will not be true that if two fusions each of which constitutes a cloud do not overlap completely then they must constitute two distinct clouds.

If we accept the notion of partial constitution and reject the Pairing Principle then we can maintain that there is only one cloud in the sky, that there is only one cloud candidate in the sky, namely the cloud, and that we can know full well what entity it is that we refer to when we point to the sky and say “Let’s call that cloud ‘Puffy’”. Of course there will be all sorts of things we don’t know about the cloud; we don’t know where it will drift off to, we don’t know how far away it is and we don’t know which droplets are parts of it. But this ignorance is no barrier to our ability to single out a unique object as the referent of the name ‘Puffy’ and to believe of it that it is the most beautiful cloud ever.

My view is that parthood and constitution are vague relations, but I have not offered any details of the view. So, rather than saying that since parthood and constitution are vague we can solve the problem of the many, my arguments might be taken to support the following conditional proposition: If parthood and constitution are vague, then we can solve the problem of the many. In the remainder of this chapter I will address the charge that the conditional proposition is simply false, that one can reformulate the problem in such a way that even with vague parthood and constitution one cannot avoid the problem.

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38 I give a detailed account of parthood in Chapter Four and in Chapter Five I define material constitution.
5. Is the ontological view really a solution?

1. It is clear that the ontological view allows us to reject the conclusion of the problem of the many, as I formulated it, since it rejects the Pairing Principle.

   **The Pairing Principle:** If \( a \) and \( b \) each constitutes a cloud at \( t \) and \( a \) and \( b \) do not overlap completely, then \( a \) and \( b \) constitute distinct clouds at \( t \).

   and once we give up the Pairing Principle we don’t have to accept that any fusion is the fusion of droplets that constitutes the cloud. There will be various fusions of droplets in the sky roughly where the cloud is, but since in general there is not a one-one correlation between clouds and fusions of droplets that are constituters of clouds, we don’t have to accept that if either of \( A_1 \) and \( A_2 \) constitutes the cloud the other doesn’t.

   But one might worry that the solution was too provisional and that an analogous problem could be raised by shifting the focus. The charge would be that although fusions such as \( A_1 \) and \( A_2 \) would not be problematic, there might be some other indifference candidates by mention of which the problem could be restated. The new indifference candidates might be objects that differed only in that one was a little less vague than the other; there might, for instance, be a droplet, call it ‘Sprinkle’, such that it was indeterminate whether it was an indeterminate part of the first candidate but not indeterminate whether it was an indeterminate part of the second one. In other words, although precise objects are not indifference candidates in a way that poses a problem, there might be vague indifference candidates that were problematic.
The basic point of the problem of the many is that in great many cases one can draw indifferent boundaries, and it does not matter whether these boundaries are precise or not. Consider, for instance, the theory of fuzzy objects. Even though proponents of fuzzy objects can reject the original indifference candidates as problematic, it seems that they will still be committed to all sorts of indifference candidates, say, cloud like objects that are slightly differently fuzzy, objects that differ only in that something is part of one to degree 0.7 but part of the other to degree 0.70001. Why should the first be the cloud rather than the second? This is why Vann McGee says that the theory of fuzzy objects, which strikes him as pretty implausible to begin with, appears even worse on a closer inspection.39

2. If we see the problem from this point of view, then chances for a straight solution seem rather bleak; no matter how we draw boundaries the problem of the many can be reformulated with respect to those very boundaries. But this point of view is a deceptive one. Ordinarily we don't individuate objects by drawing boundaries but as instances of kinds and as the subjects of some contingent truths. We say: “I shall call the cloud in the sky 'Puffy'”. Here an object is individuated as belonging to the kind 'cloud' and as satisfying the predicate 'is in the sky'. Similarly, we may say, “the man by the bar is drinking champagne” or “the planet closest to Sun is Mercury”. In no cases do we draw a boundary, not explicitly at least.40


40 One might wonder whether we are any better off if we individuate objects as instances of kinds and subjects of contingent truths rather than by drawing boundaries for the following reason: There might exists many kinds: mountains, mountains*, mountains^ etc., that are so similar that we don’t know which one our word ‘mountain’ refers to, and, therefore, when we
But what we do ordinarily isn’t the final court, we need to ask whether it could be that for each way of individuating an object, there is an equivalent way of individuating it that involves the drawing of a boundary. It is not clear how one could answer this question. On the one hand, the problem of the many might be taken to show that it is impossible to individuate ordinary material objects by drawing boundaries around collections of basic elements. On the other hand, one might be presented with the following dilemma. Either ordinary macroscopic objects are wholly made of some more basic elements in which case they can be individuated in terms of those basic elements, i.e. by drawing a boundary. This is the first horn of the dilemma and taking it leads to the problem of the many. Or, and this is the second horn, such objects are mereologically simple, i.e. they are not composed of basic elements. But surely, ordinary objects are composed of basic elements, clouds are made of droplets, mountains of clods of earth, and all these are made of atoms. So, it looks as if we are stuck with the problem and the best we can do is to look for ways to alleviate the consequences. But the dilemma is a false one. It presupposes that all objects must have their parts determinately, either absolutely or to a degree. My point is that once we give that presupposition up, we can have a straight solution to the problem.

individuate an object by saying “It is the tallest mountain in Africa” there might, after all, be many candidates among which we were unable to distinguish. My reply is two twofold. First, even if there are many candidates because there are many slightly different kinds among which we are unable to distinguish, this would not be as bad as the original problem of the many since the problematic many do not belong to the same kind. Second, it is not clear that there are all these different kinds, or even if there are many kinds of mountain like things that differ only minutely from the kind mountain, it is not clear that there is a similar multiplicity of kinds of living beings. And although this last possibility may still leave us something of a problem it is much less serious than the initial problem of the many.
3

Does Ontological Vagueness Make Sense?

In the previous chapter we saw that with a vague parthood relation we can solve the problem of the many without running into other related problems. In particular, we do not face the problem of vague reference or the problem of de re beliefs. This is not to say that the ontological view comes for free. The idea that there are vague objects has faced some strong objections, three of which I shall look at in the present chapter. The first objection is that the idea that there are vague objects is a fallacy of verbalism; vagueness and precision are said to apply to language, and representations in general, but not to objects. The second objection is that the ontological view is inconsistent, and the third objection is that once the ontological view is spelled out in a consistent way then it is not clearly distinct from the view that language is vague.

1. Is the ontological view a fallacy of verbalism?

1. In the 1920s Russell wrote that the idea that objects might be vague was a fallacy of verbalism. Representations such as names, reports or pictures could be
vague, but objects could not. But Russell also maintained that it was the same fallacy to say of objects that they were precise. Objects where just not the kind of things that were either vague or precise. But what was Russell’s notions of vagueness? Russell defined vagueness in terms of precision or accuracy.

The exact definition [of accuracy] is as follows: one system of terms related in various ways is an accurate representation of another system of terms related in various other ways if there is a one-one relation of the terms of the one to the terms of the other, and likewise a one-one relation of the relations of the one to the relations of the other, such that, when two or more terms in the one system have a relation belonging to that system, the corresponding terms of the other system have the corresponding relation belonging to the other.41

In short, a representation is accurate if there is a one-one relation between it and what it represents. A representation is then vague when the relation is one-many. In the case of singular terms vagueness contrasts not with accuracy but precision. To be precise for a name is for there to be one-one correspondence between it and some object in the world. Russell then adds: “Maps, charts, photographs, catalogues, etc. all come within this definition in so far as they are accurate”. In short, we can say that accuracy is isomorphism. Examples of vague representations that he offers are “a photograph which is so smudged that it might equally represent Brown or Jones or Robinson” and, he says, “a small-scale map is usually vaguer than a large scale map, because it does not show all the turns and twists of the roads, rivers, etc. so

that various slightly different courses are compatible with the representation that it
gives". 42

We need not go into the details of Russell’s view, what matters for our
purposes is the idea that precision (and accuracy) is a one-one relation between a
representation and what is represented. Given this idea there is no room for vague
objects, except in so far as these objects are representations. We might perhaps sum
up Russell’s view in the following way: Objects are only vague as representations, in
and of themselves objects cannot be vague.

2. Russell was right that talk about vague objects is dubious given the notion of
vagueness he had in mind. I am, however, inclined to regard such talk as simply
false rather than a fallacy of verbalism. If an object was vague in and of itself then
that object would have to bear a one-many relation to itself. But how could that be?
Some philosophers have suggested that identity might be a vague relation and
perhaps they might accept Russell’s notion of vagueness and yet maintain that
objects could, after all, be vague in and of themselves. I shall, however, side with
Russell on this point; objects can’t be vague in and of themselves. However, if we
can say that an object can represent itself, and if identity is a one-one relation, then it
seems in the spirit of Russell’s view to say that objects are, after all, precise.

Now, how does Russell’s criticism apply to my notion of vagueness? There is a
sense in which my view is in agreement with Russell’s. On my view no object is
vague in and of itself since vagueness with respect to parthood requires at least two

objects; an object is vague only with respect to some other object. But, and perhaps more to the point, it is not clear that Russell’s concerns are at all relevant for my notion of vagueness since my notion does not contrast with accuracy or precision, but with determinacy.

3. Even if Russell’s expressed concerns about the idea that there are vague objects do not affect my view there is a cause for concern here, a concern that has to do with the fallacy of verbalism. If relations are sets of ordered pairs and if the parthood predicate is vague then it may be indeterminate which set of ordered pairs it stands for, i.e. the relation between the predicate and sets of ordered pairs will be one-many. But to conclude from the vagueness of the predicate that there must be a set of indeterminate ordered pairs would be to commit the fallacy of verbalism. We can make sense of an ordered pair where the first element is some smudged photograph and the second is a collection of objects, say Brown, Jones and Robinson. Similarly, we can make sense of an ordered pair where the first element is the parthood predicate and the second element is a collection of different sets of ordered pairs. But we can’t maintain that there are ordered pairs of which it is indeterminate which objects they have as members. Here, indeterminacy seems to apply to representations but not to the objects represented.

Things are, however, more complicated than the above illustration suggests. Sometimes it does seem appropriate to attribute indeterminacy to things other than representations. Some philosophers, going all the way back to Aristotle, have argued that the future is indeterminate, i.e. that facts about the present fail to determine the future. Whatever the merits or defects of this view are, it is at least generally
accepted that it is not a fallacy of verbalism. Now, if it is not a fallacy of verbalism to say that the future is indeterminate because the present does not determine it, then why should it be a fallacy to say that parthood is indeterminate because the primitive facts on which it depends do not determine it? On the face of it these might seem on a par but on closer examination we can see that they are not.

Suppose that the determination relation is spelled out in terms of logical consequence. If the future is indeterminate then there is no singular proposition about some future event that follows from a set of proposition describing the past and the present, i.e. the principle

$$\neg \exists \Gamma (\Gamma \models f \lor \Gamma \models \neg f) \rightarrow \forall f$$

is true, where 'f' is a singular proposition about the future, \( \Gamma \) is a set of propositions describing the past and the present and '\( \forall \)' reads 'it is indeterminate whether'. The question here is whether the above principle makes sense, and in particular, whether accepting it would make one guilty of the fallacy of verbalism. I think that most philosophers who do not accept it would say that it is simply false rather than a fallacy.

But if accepting the above principle is not a fallacy of verbalism, why should the result of substituting a proposition about parthood for the one about the future be such a fallacy, i.e. why should there be anything wrong with accepting

$$\neg \exists \Gamma (\Gamma \models p \lor \Gamma \models \neg p) \rightarrow \forall p,$$

where 'p' is a singular proposition about parthood? Here, things are not as simple as they might seem. If we combine the view that relations are sets of ordered pairs with the view that the only actual events are past and present events, as the
Aristotelian view would have it, then it follows that there will be no relations between present events and future events. No future event can be a member of binary relations since there are no future events. They don’t exist. There may be facts about the future, for instance the fact that either there will be or there won’t be a sea battle tomorrow. But as of now, there is no future sea battle that can be a member of any ordered pair. And that is why it can be indeterminate whether there will be a sea battle tomorrow without there being indeterminate ordered pairs. When it comes to parthood things are different. If it is indeterminate in the ontological sense whether Sprinkle is part of Puffy and if binary relations are sets of ordered pairs, then, since both Sprinkle and Puffy exists, there would exist an indeterminate ordered pair. And that would be absurd.

The only way around this for friends of vague objects is to reject the view that relations are sets of ordered pairs. It might actually seem that that is exactly what any sane view should suggest. An ordered pair is an abstract object dependent on the existence of sets. Should the same be true of, say, the fatherhood relation? My being the father of my daughter should not depend on the existence of any set, and my bearing the fatherhood relation to my daughter should only depend on my being her father and, hence, also not depend on the existence of any set. But perhaps we should understand the view that binary relations are sets of ordered pairs as a claim about truth conditions of relational propositions such as “aPb”. The claim would be that “aPb” is true just in case the ordered pair <a,b> falls within the extension of the predicate ‘P’. But if this is the role of ordered pairs then there is no problem for vague relations. If ‘P’ stands for a vague relation then the following is a partial specification of the truth conditions for sentences containing ‘P’:
If \(<a,b>\) is a member of the determinate extension of ‘P’ then “\(aPb\)” is true.

If \(<a,b>\) is a member of the determinate anti-extension of ‘P’ then “\(aPb\)” is false.

The determinate extension of ‘P’ will be a set of ordered pairs, and so will the determinate anti-extension of ‘P’ be. But there is no need to identify the relation with any set of ordered pairs. In fact, nothing one way or the other needs to be said about the ontological status of relations. So, it seems that the view that there are vague relations need not commit us to indeterminate ordered pairs, and the ontological view can be cleared of the charge of making the fallacy of verbalism.

2. Is the ontological view consistent?

1. Now I turn to the charge that the ontological view is inconsistent. If Puffy is vague at time \(t\), i.e. if there is at least one droplet of which it is indeterminate that it is part of Puffy, then there is no set that is the set of all and only those droplets that are parts of Puffy at \(t\). And this claim is far from innocent; given some rather weak assumptions its negation can be proved.

From the following two premisses:

(1) There is at least one droplet that is part of Puffy,

(2) There are fewer than \(n\) droplets that are parts of Puffy,\(^{44}\)

\(^{43}\) This is on the assumption that sets are not vague, i.e. that set membership is a precise relation.

\(^{44}\) If we take premiss (2) to be equivalent to:

For any \(k\), if \(k\) is the number of droplets that are parts of Puffy then \(n\) is larger than \(k\),

then it is trivially true by falsity of the antecedent for any number \(n\), since there is no number which satisfies the definite description ‘the number of droplets that are parts of Puffy’. So,
where $n$ is some sufficiently large number, one can derive, using only classical logic and modest principles of set theory, the following proposition:

$$ (3) \quad \exists x \forall y (y \text{ is an element of } x \leftrightarrow y \text{ is a droplet that is part of Puffy}). $$

The inference from (1) and (2) to (3) is fairly straightforward.\(^{45}\) The set-theoretic principles that are needed are:

**Extensionality:** There are no distinct sets with the same members.

**Singletons:** For any $x$ there is a set with $x$ as its only member.

**Pairwise Union:** Any two sets have a union.

To see how (3) follows from (1) and (2) along with classical logic and these principles of set theory, it is convenient to consider an object that is smaller than the average cloud. Suppose we have an object $o$ which consists wholly of droplets but has not more than three droplets. We assume that the only droplets that are candidates for being part of $o$ are $d_1$, $d_2$, and $d_3$, and that $o$ is vague with respect to these droplets, i.e. it is indeterminate whether at least one of them is part of $o$. Then we assume for reductio that there is no set that is the set of all and only the droplets that are parts of $o$.

rather than taking (2) to state that there is a number $n$ that is larger than some number $k$, we can take (2) two be equivalent to a large existential sentence

$$ \exists x_1 \exists x_2 \ldots \exists x_n [x_1 \neq x_2 \land x_1 \neq x_3 \ldots x_1 \neq x_n \land x_2 \neq x_3 \ldots x_2 \neq x_n \land \ldots x_{n-1} \neq x_n \land$$

$$ \neg (x_1 \text{ is part of Puffy} \land x_2 \text{ is part of Puffy} \land \ldots x_n \text{ is part of Puffy})].$$

And this sentence suffices for the following derivation.

\(^{45}\) If premiss (2) said that there were at most finitely many droplets that were parts of Puffy, rather than have it stipulate some large finite upper bound, one needs mathematical induction to derive (3) from (1) and (2). But mathematical induction might seem suspicious in this context. The thought would be that mathematical induction works only for precise concepts, but not for imprecise ones. However, having (2) stipulate some large upper bound seems strong enough.
1. \( \neg \exists x \forall y (y \in x \leftrightarrow y \in o) \)
2. \( \forall x \exists y \neg(y \in x \leftrightarrow y \in o) \)

The next two lines are the assumptions that something is part of \( o \) and that anything that is part of \( o \) must be a member of the set of \( \{d_1, d_2, d_3\} \).

3. \( \exists x (x \in o) \)
4. \( \forall x (x \in o \rightarrow x \in \{d_1, d_2, d_3\}) \)

What follows is then just simple predicate logic.

5. \( \exists y \neg(y \in \{d_1, d_2, d_3\} \leftrightarrow y \in o) \) UI on line 2
6. \( (-d_1 \in o \lor -d_2 \in o) \lor -d_3 \in o \) PL from line 5

This tells us that there are at most 2 droplets that are parts of \( o \).

7. \( \exists y \neg(y \in \{d_1, d_2\} \leftrightarrow y \in o) \) UI on line 2
8. \( (-d_1 \in o \lor -d_2 \in o) \lor d_3 \in o \) PL from line 7
9. \( -d_1 \in o \lor -d_2 \in o \) PL from lines 6 and 8

Then we repeat the steps taken in lines 7 - 9 with \( \{d_1, d_3\} \) and \( \{d_2, d_3\} \) in place for \( \{d_1, d_2\} \), which gives us

10. \( (-d_1 \in o \lor -d_2 \in o) \land (-d_1 \in o \lor -d_3 \in o) \land (-d_2 \in o \lor -d_3 \in o) \)

This tells us that \( o \) has at most one droplet as a part. But we can continue.

11. \( \exists y \neg(y \in \{d_1\} \leftrightarrow y \in o) \) UI on line 2
12. \( -d_1 \in o \lor (d_2 \in o \lor d_3 \in o) \) PL from 11

From the first two conjuncts on 10 we get

13. \( -d_1 \in o \lor -(d_2 \in o \lor d_3 \in o) \)
14. \( -d_1 \in o \) PL from 12 and 13

Now we can repeat the steps taken in lines 11 - 14 for \( \{d_2\} \) and \( \{d_3\} \) in place for \( \{d_1\} \) on line 11. From this we can conclude

15. \( -d_1 \in o \land -d_2 \in o \land -d_3 \in o \)
But now lines 4 and 15 allow us to conclude that nothing is part of 0

16. $\neg \exists x \ ( xPo )$
17. $\exists x \ ( xPo ) \land \neg \exists x \ ( xPo )$ From 3 and 16.
18. $\exists x \ \forall y \ ( y \in x \leftrightarrow yPo )$ From 17 and 1 by reductio.

What the above argument shows is that an object of which only three objects might be parts can't be vague in the sense that its parts do not form a set. Similar reasoning will establish the corresponding solution for any object that is composed of at most n objects where n is some sufficiently large number; i.e. if Puffy is composed of no more than, say, octovigintillion droplets at any time, then an argument analogous to the one above will prove the following proposition:

(3) $\exists x \ \forall y \ ( y \text{ is an element of } x \leftrightarrow y \text{ is a droplet that is part of Puffy}).$

What proposition (3) says, or so it seems, is that there exists a set that is the set of all and only the droplets that are parts of Puffy. And this contradicts my view.

There are three ways we might try to resolve this contradiction. First, we might argue that (3) did not, after all, contradict the claim that there is no set which is the set of all and only the droplets that are parts of Puffy by claiming that whenever a sentence had an indeterminate constituent it was subject to some sort of paraphrasing that avoided the contradiction. Second, we might give up set theory. Third, we might give up classical logic.

The first way might be motivated by the observation that the embedded biconditional in (3) has the notion of parthood on the right side, and since we are allowing that parthood might be vague, we might need an interpretation of the '↔' symbol which allowed that the whole proposition be true while a constituent proposition is indeterminate. But I don't see how this could serve our purposes.
here. We can prove (3) and the negation of (3) is a direct consequence of my view. That (3) must be true, given (1) and (2) along with modest set theory and classical logic, is not something an interpretation of the biconditional can call into question, rather any interpretation of the biconditional must accommodate the truth of (3) given the initial conditions.

The second way, i.e. giving up set theory, does not look promising either. The problem is that the challenge might be repeated in terms of collections or pluralities. So, abandoning sets would not be enough, one would also have to abandon collections or pluralities. And having done that most of mathematics would have to be abandoned as well, and the price has become too steep.

The remaining option is to give up classical logic. In particular, blocking the derivation of (3) from (1) and (2) requires giving up the law of excluded middle. If the law of excluded middle is not valid then, instead of being provably true, (3) might be indeterminate. What the above demonstration shows is not that vague constitution is inconsistent but that accepting vague constitution comes at a price. But how high a cost is this? I think that if one is drawn to vague objects, for whatever reasons, one should not be committed to the law of excluded middle. If it is indeterminate whether a is part of b then one should not expect the sentence "aPb v ¬aPb" to be true.

3. Can the ontological view be distinguished from the linguistic one?

1. Mark Sainsbury has argued that there is no deep thesis about ontological vagueness that is clearly distinct from the thesis that all vagueness has its roots in language, and presumably other forms of representations. I shall call this
Sainsbury's thesis. This may seem like an unlikely thesis since part of the thesis that there are vague objects is just the claim that objects themselves are vague, and this claim is clearly at odds with the linguistic view. But the point is different. It is clear what the intended reading of the basic claim of the ontological view is, namely that there are vague objects. But this claim might have an unintended reading, and if there was such a reading that explained vagueness in linguistic terms, then one could accept all the basic claims of the ontological view, and reap all the benefits that come with it, without committing oneself to ontological vagueness. What would remain to distinguish the linguistic view, which we get from the unintended reading, from the intended ontological view, would be the bare claim that there are vague objects. A claim that nothing would hinge on.

Sainsbury begins his paper “Why the world cannot be vague” with the following report:

Whether or not the world is vague appears to be a deep issue. This paper reports the results of a search for a clear and substantive thesis to the effect that the world itself, as opposed to our language or concepts, is vague. Such a thesis of “ontic vagueness” ought to reflect the apparent depth of the issue, and so its truth or falsehood should be a matter of controversy. I did not find such a thesis. 46

The problem that Sainsbury has in finding a thesis of ‘ontic vagueness’ that reflects the depth of the issue is that once he has found a thesis that is clearly incompatible with the linguistic view, that thesis is provably wrong. In the previous section I defended the ontological view against one charge of inconsistency, now I consider whether it is clearly distinguishable from the linguistic view.

Sainsbury has put his finger on a real problem, although not a problem that is unsolvable for friends of ontological vagueness. To see what the problem is consider how ontological vagueness is commonly defined. Take, for instance, the following definition offered by Trenton Merricks.

There is [ontological] vagueness ... if, for some object and some property, there is no determinate fact of the matter whether that object exemplifies that property.47

Or this definition given by Michael Tye.

I shall classify a concrete object \( o \) as vague ... if, and only if, (a) \( o \) has borderline spatio-temporal parts and (b) there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts, nor non-parts of \( o \).48

The problem with these definitions is that saying that an object has borderline parts or that there is no fact of the matter whether some object has some property does not give us ontological vagueness. To say this is just to report the fact of vagueness. The contrast between the ontological and the linguistic views will show, if at all, in how we explain this fact, i.e. how we explain what it is to be a borderline part or why there is no fact of the matter. We can then put Sainsbury’s thesis in the following way: There is no explanation of vagueness that is clearly ontological rather than linguistic. I won’t contest Sainsbury’s claim concerning the formulations of the ontological view that he considers, instead I will argue that my thesis is clearly distinct from a thesis about linguistic vagueness.

2. The question we want to answer is this: Does my thesis admit of a linguistic interpretation? There isn’t going to be any quick answer to this question, nor perhaps will the answer be conclusive in the end. But, it should at least be clear that accepting Sainsbury’s thesis is very costly. On the face of it the two views seem readily distinguishable in terms of certain logical properties of the indeterminacy operator ‘V’. If we hold the linguistic view then ‘V’ will most naturally be a sentential operator, whereas on the ontological view it should attach to closed sentences as well as open formulas inside the scope of quantifiers. For instance, on the ontological view, the sentence

\[(1) \exists y \forall x P y\]

comes out true when the value of ‘x’ is Sparky and the value of ‘y’ is Kilimanjaro. But if vagueness is linguistic, for instance if the vagueness of “Sparky is part of Kilimanjaro” is due to the vagueness of the name ‘Kilimanjaro’, then sentences where ‘V’ attaches to open formulas inside the scope of a quantifier as in (1) should be expected to be false; even if singular terms in the language are vague, variables can not be vague. The reason for this is that variables are mere place holders for objects and not entities in a language for which the thoughts and practices of speakers must determine a meaning.

We might also distinguish the ontological view from the linguistic one in terms of restrictions on the inference from

\[\forall a P b\]

where ‘a’ and ‘b’ are some singular terms, to

\[(1) \exists x y \forall x P y.\]
Such an inference is more restricted on the linguistic view than it is on the ontological view. The reason for this is that the inference is only valid when 'a' and 'b' are precise names, and on the ontological view a name such as 'Kilimanjaro' will be precise while vague on the linguistic view.

It seems then that the ontological view has two features that distinguish it from the linguistic view. First, 'V' attaches to open formulas as well as closed sentences and, second, existential generalization on a sentence of the form "V aPb" is valid.

3. There is, however, a complication here. In the sentence

(2) \( \exists x \ (V x P(Kilimanjaro)) \),

'V' attaches to an open formula inside the scope of an existential quantifier and this ought to come out true on the linguistic view. It just says that there is something of which it is indeterminate whether it is part of Kilimanjaro. But in (2) the question about indeterminacy should not be seen as a question about vague objects or singular terms but as a question about vague predicates. In this case one might treat 'P(Kilimanjaro)' as a vague one place predicate. The predicate 'red' is similar. If 'red' is vague it ought be true that there is something of which it is indeterminate whether 'red' applies to it. In other words,

(3) \( \exists x \ (V Red(x)) \)

ought to come out true on the linguistic view. But if (3) can be true on the linguistic view, why not (1)? The answer is that if 'P' is a vague predicate then (1) can come out true on the linguistic view. We might read (1) as: "It is indeterminate of some
ordered pair whether it is in the extension of ‘P‘. So, it is not without qualifications that the logic of indeterminacy appropriate for the linguistic view is sentential logic rather than predicate logic.

In order to restore the distinction, in this respect, between the linguistic and the ontological views it might seem that we had only one alternative, namely to make use of second order logic and quantify over properties and relations as well as objects. And this might seem problematic. However, I don’t think that embracing second order logic is our only way of avoiding Sainsbury’s thesis.

If the linguistic theory explains the fact of vagueness by reference to semantic indecision and accounts for the truth conditions of sentences by supervaluation, then even if ‘P’ is a vague predicate an inference from

$$\forall aPb$$

to its existential generalization

(1) $$\exists xy \forall xPy$$

will not be valid. The inference is valid only when the names ‘a’ and ‘b’ are precise and on the linguistic view names of ordinary material objects are not precise while on the ontological view they are precise.

4. There is still one gap that has to be filled before we can reject Sainsbury’s thesis. This gap has to do with the metatheory for the quantifiers. A substitution instance of (1) is

$$\forall (\text{Sparky})P(\text{Kilimanjaro}),$$
and this is a true proposition. This means that if the quantification in (1) is substitutional rather than objectual then (1) will be true on the linguistic view and the inference from
\[ \forall a P b \]
to its existential generalization
\[ (1) \quad \exists x y \forall x P y, \]
will be valid. In order to distinguish the two we must add that quantification is objectual rather than substitutional. The question then is whether we can in the end distinguish these two kinds of quantification.

In “Ontological relativity” Quine has doubts about the availability of such a distinction. Quine introduces a notion of inseparability for uncovered models in the following way: The named objects are inseparable from the nameless ones if, and only if, all properties of the nameless objects that we can express are shared by the named objects. 49 When it comes to vagueness, we need to modify the above definition of inseparability in the following way: instead of talking about ‘all properties’ we should restrict our attention to ‘all determinate properties’, i.e.

Nameless objects are inseparable from named objects iff all properties that the nameless objects have determinately and we can express are also had determinately by the named objects.

The idea here is that we can only separate two objects if it is determinate that one has a property that the other lacks. Now, a friend of the linguistic view will accept (i) that the objects outnumber the names and (ii) that the unnamed objects are

49 Ontological Relativity, p. 65.
inseparable from the named ones in our sense of that term. If they weren’t inseparable, they would differ in a way that allowed speakers of the language in question to resolve linguistic vagueness. And if the unnamed objects are inseparable from the named ones then, Quine maintains, we will be unable to “distinguish objectively between referential [i.e. objectual] quantification and a substitutional counterfeit”.50 If Quine is right that we can’t distinguish between these two forms of quantification then perhaps Sainsbury’s thesis might still stand: It would be impossible to formulate the ontological view in such a way as to exclude interpretations that explained the vagueness as a linguistic rather than an ontological phenomenon.

But we have a good reason to reject Quine’s conclusion. Vann McGee has argued that if we look at counterfactual circumstances we can show that Quine’s worry is unfounded. The basic idea is that even if named and nameless objects are actually inseparable, that is an accidental feature of this world. Two objects may be inseparable in this world but readily separable in some other world.

Suppose that there is some property – call it F – that no individual that exists in the actual world possesses in w, but that at least one individual that exists in w but not in the actual world possesses in w. If quantification is objectual, (∃x) F(x) will be true in w; if quantification is substitutional, it will be false.51 McGee’s point is based on two assumptions that are worth mentioning. First, that names don’t change their designation when we move from one world to another and, second, that it is impossible to name counterfactual individuals. And since

50 Ontological Relativity, p. 67.
these assumptions are very plausible Sainsbury's thesis isn't convincing. In fact, I think that McGee's assumptions are true in which case Sainsbury's thesis is not only not convincing but simply false. I shall, however, not defend the stronger claim here.
4

Vague Parthood

The basic idea behind my notion of vague parthood is simple. First, facts about parthood are non-primitive in the sense that for two objects to be related as part to whole there must be some other primitive facts which determine that the objects are so related. Second, sometimes the primitive facts fail to determine of some two objects whether one is part of the other. That is when we have an instance of vague parthood.

The idea that there are primitive facts that determine non-primitive facts is common in philosophy; it is, for instance, behind the linguistic view of vagueness. According to the linguistic view there are primitive facts about thoughts and practices of speakers that determine the non-primitive facts about meaning. Vagueness is then explained as the result of the primitive facts’ not fixing whether, say, the name ‘Kilimanjaro’ refers to this or that landmass in Tanzania. The structure of the ontological view of vagueness is similar; vagueness is explained by the primitive facts’ not fixing whether Sparky is part of Kilimanjaro. The task of
spelling out the ontological view consists in specifying what the primitive facts are and what the determination relation is.

1. Vague parthood

1. I regard the parthood relation a three place relation, \( x \) is part of \( y \) at time \( t \). However, in what follows I will ignore the place for time except in definitions and elsewhere where time is specially relevant. My basic thesis about parthood entails that

\[
(1) \quad \exists xy \forall xPy,
\]

is true, where \( \forall \) stands for 'it is indeterminate whether' and \( P \) stands for 'is part of'. The variables \( x \) and \( y \) could, in principle at least, range over anything that can enter into the parthood relation, be it abstract objects, events or whatever else comes to mind. However, I shall focus on the special case where \( x \) and \( y \) range over material objects.

In Chapter Two we saw that if we can have (1) we can solve the problem of the many. The question that now needs to be answered is what makes a proposition of the form "\( \forall aPb \)" true. Without an answer to this question we do not have a thesis to defend. What we need is to complete the schema

\[
\forall aPb \text{ iff }...
\]

where \( a \) and \( b \) are precise singular terms. There are two approaches that one might take here. On the one hand, one might look for a general account of the indeterminacy operator that will give the truth conditions for sentences of the form "\( \forall S \)". Truth conditions for sentences of the form "\( \forall aPb \)" will then follow as a
special instance of this general account. On the other hand, one might think that what is needed is an account of parthood, i.e. in order to understand why it might be indeterminate whether one object is part of another a proper understanding of the parthood relation is needed. Friends of the epistemic and the linguistic views adopt the former strategy, I shall go for the latter.

2. What form an analysis of parthood for material objects takes depends on the underlying conception of material objects. To see this it is instructive to begin by looking at a view of material objects such as Quine's. According to Quine, a material object is just the material content of a region of space-time, and on this view parthood on material objects reduces to parthood on regions of space-time: one object is part of another just in case the region of space-time that the first occupies is included in the region of space-time that the latter occupies. This suggests something like the following principle as an account of the parthood relation on material objects:

\[(Q) \forall x y (xPy \iff \text{the region of space-time occupied by } x \text{ is part of the region of space-time occupied by } y).\]

If we also accept the thesis, which I think is reasonable, that regions of space are precise, then it follows that the only possibility for parthood to be vague is that there be objects of which it is indeterminate which region they occupy. Perhaps quantum

52 This is only a generic view, not a precise thesis. Quine has not told us how small a material object can be and, more fundamentally, he has not told us how to individuate regions of space-time. But the details need not concern us here. Some of the relevant questions are taken up in Richard Cartwright's "Scattered objects". The paper first appeared in *Analysis and Metaphysics*, (Keith Lehrer ed., Dordrecht 1975) but is reprinted in his *Philosophical Essays*, (MIT Press 1987).
mechanics tells us that there are such objects, but whichever explanation of vagueness is appropriate in the context of subatomic particles such an explanation does not help us explain why the sentence “Sparky is part of Kilimanjaro” is vague. We can, therefore, conclude that as long as we are concerned with ordinary material objects Quine’s view of material objects requires that parthood is precise and whatever vagueness we encounter must be explained in either linguistic or epistemic terms.

Quine’s view of material objects is simple, and for that reason some find it attractive. But if simplicity is our motivation, there is yet a simpler view that might tempt us. This is the view we get if we cut Quine’s view down to three dimensional space. On the resulting view objects are said to be the material content of regions of space instead of space-time. In other words, material objects are three dimensional occupiers of space instead of four dimensional occupiers of space-time. But this view has certain problems. One problem is what David Lewis calls ‘the problem of temporary intrinsics’.

Persisting things change their intrinsic properties. For instance shape: when I sit, I have a bent shape; when I stand, I have a straightened shape. Both shapes are temporary intrinsic properties; I have them only some of the time.53

To explain how a change from having a bent shape to having a straightened shape is possible is the problem of temporary intrinsics. Lewis’s favored solution to this problem is to bring in the fourth dimension, i.e. make time a defining constituent of material objects. And this gives us back Quine’s view. The solution seems artificial to some, others find it natural, but there is no doubt that it does solve the problem.

However, the resulting view, i.e. Quine’s, has still nothing to say about modal properties of objects; it does not offer anything to bear on the most mundane modal questions such as: Could this statue survive a replacement of a small part? Could this quantity of water survive a replacement of a part?

Quine’s response to this latter problem is well known, he simply denies that quantified modal logic makes any sense. And in so far as it is a solution, it is effective even if not attractive. But there are also different responses in the literature. Just as David Lewis solved the problem of temporary intrinsics by making the temporal dimension a defining constituent of material objects, some philosophers have suggested that the present problem could be solved by making possible worlds a defining constituent of material objects. The idea is that a material object is not identified with the material content of a four dimensional space-time region but the material content of a five dimensional space-time-world region. If this idea makes sense, which I am not sure it does, it follows that we can answer questions such as: Could this statue survive a replacement of parts? But the answer depends on how we establish the cross-world identification and what limits there are on which cross-world regions count as statues.

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54 Some of his reasons are mentioned in section 3 of this chapter where I compare quantified modal logic to quantified logic of vagueness.

55 See for instance Vann McGee, “‘Kilimanjaro’”.

56 I pointed out certain difficulties for this view in chapter 2, section 3.

57 If there is no limit on which cross-world regions count as objects then whenever we have a statue there is at least one statue like thing that does survive a replacement of a small part and another statue like thing that does not survive such a replacement. The demonstrative ‘This statue’ will be vague in such a way that the question: Does this statue survive a replacement of a small part? will have to be answered by: It depends on which statue like object it is that you are referring to.
The number of cross-world regions that are statue like objects might be cut down in two ways. First, by reference to some principle about which cross-world regions count as objects and, second, by some principle about language. Let’s look briefly at these. The first option, to postulate some restrictions on which cross-world regions count as objects, requires restrictions on the notion of composition. Friends of five dimensional objects sometimes say that objects are mereological sums of time-world slices, and since each time-world slice is a mereological atom, there does not seem to be any reason why some atoms have a sum while others do not. This means that from the point of view of friends of five dimensional objects, the first option is counter intuitive.

The second option is to answer modal question by reference to some principle about language. We can call this ‘the linguistic view of modalities’. On this view, it will be a feature of the word ‘statue’ that it picks out an object that is a statue in this world and also a statue in all other possible worlds where it exists. Supposing that Alfred is a statue, we might still ask: Might Alfred have been an elephant? The intuitive answer is negative, and on the linguistic view this is explained by reference to a feature of the word ‘statue’, namely that whatever it applies to in this world is a statue in any other possible world where it exists. But to decide modal questions by reference to language has certain problems. A friend of the linguistic view of modalities would like to say something like this: “We might, of course, have had a different world, ‘statphant’, that applied to an object just in case it is a statue in this world and an elephant in other possible worlds.” But, on the linguistic view of modalities as I described it above, this is inconsistent. To say of a that it is a statue in the actual world but an elephant in w is inconsistent since it will be analytic that a is
a statue in the actual world just in case it is a statue in all worlds where it exists.

What friends of the linguistic theory of modalities have to say is that a statphant overlaps a statue in this world but overlaps an elephant in all other worlds where it exists.58

Let’s assume that friends of the linguistic view can answer such question as “Could Alfred have been an elephant like object?” in accordance to our intuitions. But there are other and more mundane modal questions that make problem for the linguistic view of modalities. Everyone should agree that Alfred survives replacement of some parts but not others, i.e. some parts of Alfred are contingent while others are essential. This means that

Alfred has an essential part,
is true. Suppose B is an essential part of Alfred. Then the sentence

B is part of Alfred,
ought not only be true but necessarily true. But on the linguistic view of modalities the notion of necessity is just the notion of analyticity, which means that the sentence “B is part of Alfred” should be analytic. But that is hard to make out even

58 One question for the linguistic view of modalities is this: How do we know that our word ‘statue’ means statue rather than statphant? Since the only statue like objects we encounter exist at the actual world, we cannot distinguish the two in terms of observable evidence. But this question may be answered by consulting intuition about modalities. Suppose we have a statue like object in front of us. If it is a statphant then the sentence “Had I worn my sneakers today instead of sandals, then this would have been an elephant like object” is true. But it is not true. However, there is more explaining to do and it is not clear that the linguistic view of modalities can accommodate this. We must explain why we have the intuition that it is not true that had I worn sneakers today there would have been an elephant like object in front of me. The explanation of this is simply that what footwear I have on does not alter the world in any significant ways, in particular it does not have any impact on the statue like objects that are in my vicinity. But this explanation is non-linguistic.
on a description theory of names. Suppose the name ‘B’ is a short for the description
‘the portion of clay which consists of such and such atoms’. From the sentence “B is
the portion of clay that consists of such and such atoms” it does not follow that it B
is part of Alfred. What is needed is that the name ‘Alfred’ is short for ‘the statue
that consists of such and such atoms’. But on a description theory of names we
could have ‘B’ be a short for the description ‘the portion of clay that consists of such
and such atoms’ and ‘Alfred’ be short for the description ‘the statue on the table’.59
But then it is not analytic that B is part of Alfred.

3. Suppose we give up a Quinean view about material objects. What should we
then say about (Q) as an analysis of parthood.

(Q) \( \forall xy (xPy \iff \text{the region of space-time occupied by } x \text{ is part of the}
\text{region of space-time occupied by } y) \).

Principle (Q) presupposes that material objects are four dimensional, i.e. occupiers
of space-time but once we give up the Quinean view this presupposition becomes
questionable. There is a simpler principle that retains the basic idea about parthood
from (Q) but which is indifferent to whether objects are three or four dimensional.
This is the following principle about parthood at a time.

(P) \( \forall xy (xPy \text{ at } t \iff \text{the region of space occupied by } x \text{ at } t \text{ is part of the}
\text{region of space occupied by } y \text{ at } t) \).

59 A different problem for the view under consideration has to do with quantification.
Suppose that Alfred is a statue. Then there is something the replacement of which Alfred
won’t survive, i.e.

\[ \exists x \text{ necessary } xP(\text{Alfred}) \]

ought to be true. But if necessity is analyticity, then quantification into modal contexts won’t
be legitimate. This is a point that Quine has repeatedly pressed.
This principle may very well be true, but if what we are after is an analysis of parthood then there is a problem with (P), namely that if our conception of material objects isn’t analyzable in terms of spatial occupancy, then (P) won’t serve as an analysis of parthood. The best way to see this is to notice that which region of space an object occupies might depend on what its parts are, rather than the other way around. This is not to suggest that (P) is false, rather that facts about spatial extension are not among the primitive facts that determine parthood. In other words (P) may be extensionally correct, but it isn’t informative in the way an analysis ought to be informative.

4. In order to arrive at an analysis of parthood we should look at ordinary objects and ordinary claims about their parts. In adopting this strategy we are presupposing that ordinary objects, mountains, clouds, bodies, etc. are properly instances of the kind ‘material object’ and this is not an unchallenged assumption; van Inwagen says that the only material objects are living organisms and Peter Unger has not been shy about concluding that even he himself does not exist. But such views are hard to defend. If we are told that someone has shown that we are almost always wrong about the extension of some firmly entrenched word or phrase, in this case ‘material object’, we might reply that the person probably misunderstood the word when arriving at this incredible conclusion. Or to put it a bit differently, it seems to be a condition on any analysis of the meaning of a word

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that it should preserve some of the fundamental intuitions that we associate with it, i.e. if an analysis requires too radical a revision, then it is probably wrong.\footnote{This principle is not immune to challenge. If it were, we would have an answer to radical skepticism, which I don't think we have. (See for instance C.B. Martin, "The new Cartesianism" \textit{Pacific Philosophical Quarterly}, 1986) It is rather a maxim about interpretation, and it should not be given up unless the reasons for doing so are forced on us. But I don't think that van Inwagen's or Peter Unger's reasons are forced on us. So, the conclusion, at this point at least, is that we should go on and assume that at least some of the things that we thought were paradigm cases of material objects are in fact material objects.}

Now, let's look at some ordinary claims about part-hood and see what we can glean from them. Take an example of a living organism, say my body, and some basic entities such as cells, atoms and subatomic particles. All agree that a cell that is inside my left leg is part of me, and it should also be agreed that a cosmic ray that travels through my leg isn't part of me at any time. But what about things that are smaller than cells but larger than cosmic rays, say hydrogen atoms? A hydrogen atom that is inside my leg will, other things being equal, be part of me. Such an atom might be part of some water molecule that is caught up in various physiological processes that are among the processes in virtue of which I am a living organism. But suppose such an atom travels through my leg like a cosmic ray. Is it part of me for the moment it is inside my leg? It doesn't seem to be part of me any more than the cosmic ray. So, we could have two hydrogen atoms, both inside my left leg at the same time, and one was part of me while the other wasn't. Why is this so? What can account for the different status of these two hydrogen atoms? Facts about location are not going to help, since both atoms are located inside my leg at some moment, they might even be inside the same cell. The difference seems to be that one atom is caught up in various causal processes that are definitely part of...
what makes my body a living organism while the other atom doesn’t have anything to do with such processes. It is causally irrelevant as far as my existence goes. But why should it matter how it is caught up in various causal processes? The answer is that that’s how people are; people are complex bodies of basic elements that are so causally related that the product is a living thing.

If we are right to say that an atom is part of me because it is caught up in certain causal processes, then its being part of me is explained by what kind of being I am and how the atom is caught up in various causal processes. This supports the basic thesis that facts about parthood are not primitive. But the above example does more than support the basic thesis, it is suggestive about what particular facts are relevant for determining parthood in the case of living organisms: If \( a \) is part of \( b \) then what determines that \( a \) is part of \( b \) are (i) facts about what kind of an object \( b \) is, and (ii) how \( a \) is causally related to \( b \). Different material objects, houses, clouds and mountains, are also bodies of basic elements that are causally related in their own way. But the causal relations that are important in the case of people are different from those that are important when it comes to houses, clouds or mountains.

It is fairly clear that there are intricate causal relations that hold between the basic parts of living organisms in virtue of which these parts make up a living organism. Even in the cases of many artifacts, for instance houses, it is clear that there are basic parts that have to be causally related in certain ways in order for there to be a house; the bricks, if it is a brick house, have to be stuck together, the walls must support the roof and so on. Otherwise we just have a heap of building materials instead of a house. But is the same true of mountains and clouds? I think it is, although the causal relations are simpler and perhaps not as well defined as in
the case of artifacts and living organisms. In order for clods of earth to make up a mountain they have to be stuck together, they have to be impenetrable to a certain degree and they have to form a certain shape. It is in virtue of the shape that we distinguish mountains from the flatlands, and it is in virtue of the forces between the parts that we distinguish mountains from clouds of dirt.

5. Now we get to the notion of determination. We need to get a grip on how to characterize this notion and, then, we need to explain under what circumstances the primitive facts may fail to determine of some two objects whether one is part of the other.

A proposition that says of some object a that it is part of some other object b, might be true, false or indeterminate. If Γ is a set of propositions describing the primitive facts on which parthood depends, then (i) if "aPb" is a consequence of Γ then "aPb" is determinately true, and (ii) if the negation of "aPb" is a consequence of Γ then "aPb" is determinately false. Finally, (iii) if there is no appropriate set Γ such that either "aPb" or its negation is a consequence of Γ then "aPb" is indeterminate. In this respect the relevant notion of determination is just the notion of logical consequence.

But what propositions should be in Γ? What makes Γ appropriate? In keeping with what I said earlier the set Γ should contain propositions describing two aspects of the objects in question. (i) Propositions about what kind of object b is, for instance, "b is a human body", or "b is a living organism". An object may belong to more than one kind, some particular human will, for instance, not only belong to the kind 'human' but also to the kinds 'animal' and 'living organism'. I shall use the
variables ‘k_1b’, ‘k_2b’, ‘k_3b’ etc. to range over propositions describing such facts. (ii) Γ should contain propositions describing how a is causally related to b, propositions about some other contingent relations such as “a is inside b”, or “a is attached to b” and other propositions describing the nature of such an attachment in more details as the case might be. I shall use the variables ‘c_1’, ..., ‘c_k’ to range over propositions describing such facts. But this is not enough. These propositions do not say anything about parthood and, therefore, nothing about parthood will follow from them. So, Γ should also contain (iii) whatever general principles about parthood that hold of objects falling under the kinds in question, such as: “cells are among the parts of human bodies”, “Cosmic rays are not parts of human bodies” and “If an object is causally related to an object of kind k in such-and-such a way then it is part of that object”. I will use the variables ‘g_1’, ..., ‘g_r’ to range over such general principles.

But there is no reason to believe that all facts about parthood are determined directly by the primitive facts, i.e. there might be facts about parthood that are determined only because some other facts about parthood are determined. It might be determined that d is part of e, but indeterminate whether c is part of d. Still, it will be determined that if c is part of d then it is part of e. For this reason, Γ should also contain whatever facts about parthood that are determined independently of whether a is part of b. I shall use the variables ‘f_1’, ..., ‘f_m’ to range over propositions describing such facts.

The idea now is that if it is determinate that a is (not) part of b, then there should be a sound derivation of the form:
When such a derivation is available I shall say that it is determinate that $a$ is (not) part of $b$, or that $a$ is a determinate (non-) part of $b$. I will call this kind of determination normic determination, (ND) for short.\(^{62}\)

Now that we have an analysis of parthood we can say more precisely what it is for parthood to be vague: parthood is vague when it is not determinate in the sense of the (ND) schema, either directly or indirectly. In other words, if it isn’t determinate in the sense of the (ND) schema of some object $a$ that it either is or isn’t part of some other object $b$, then it is indeterminate whether $a$ is part of $b$. This gives us the following indeterminacy principle:

\textbf{Indeterminacy Principle: } $\forall x \forall y (\neg xPy \iff \exists \exists \exists (\Gamma \vdash xPy \lor \Gamma \vdash \neg xPy)).$

The Indeterminacy Principle presupposes that indeterminacy contexts are referentially transparent. This contrasts with the linguistic view according to which such contexts are not transparent.\(^{63}\)

6. Now that we have my thesis about parthood spelled out in abstract terms, a concrete example will help us get a better feel for it. Suppose we want to know


\(^{63}\) I discussed the difference between the ontological and the linguistic proposal in this respect in some detail in Chapter Three.
whether a certain hair, call it 'Harry', is part of the cat Tibbles. First, we might notice that Tibbles' fur is definitely part of Tibbles, i.e.

\[ f_1 = \text{Tibbles' fur is part of Tibbles.} \]

The question now becomes whether Harry is part of Tibbles' fur. We have the general principles that a hair that is attached in the appropriate way to a fur (i.e. is not just glued onto it or stuck in chewing gum etc.) is part of the fur, and a hair that is not attached to a fur is not part of the fur, i.e.

\[ g_1 = \text{A hair is part of a fur iff it is appropriately attached to the fur.} \]

What would need spelling out is what 'appropriately attached to the fur' amounts to. Suppose that for a hair to be appropriately attached to a fur it is sufficient for it to bear relation R to the fur and necessary for it to bear the relation Q to the fur. Now, although we have a sufficient and a necessary condition for appropriate attachment, these conditions need not be jointly exhaustive, i.e. a hair may satisfy the necessary condition but not the sufficient condition. And assuming that Harry is such a hair, we will not have any premiss saying that Harry either is or isn't appropriately attached to the fur.

And, suppose further that there isn't anything else that might settle the issue whether Harry is part of Tibbles' fur, then it does not follow that Harry is part of the fur, nor does it follow that Harry is not part of the fur. And since the only way for Harry to be part of Tibbles is for it to be part of Tibbles' fur, it is indeterminate whether Harry is part of Tibbles. In this example, it is directly indeterminate whether Harry is part of the fur, but only indirectly indeterminate whether Harry is part of Tibbles.
7. So far my examples have involved parts of living organisms, and it seems that these, along with artifacts, are the material objects that best suit my purposes. There is a natural way for a hair to be attached to the fur of a cat, namely to be in the socket from which it grew, and so we might say that the hair is a natural part of the cat. Similarly, there is a natural way for a leg of a chair to be part of the chair and an arm of a statue to be a part of the statue. Of course there is difference between organisms and artifacts in this respect; the way in which it is natural for artifacts to have certain of their parts isn't a matter of natural law but, perhaps, a matter of convention or the intention of the maker. But despite this difference we can, in both cases, make a distinction between a contingent part and an accidental part.

Contingent parts of Tibbles are the hairs that are parts of the fur, the tail, the legs, and so on. These are parts that are not essential but we expect a cat to have them and we can, perhaps, explain why a particular cat has these parts by reference to some underlying nature. Such a nature might be given by the genetic makeup of the cat. Among the accidental parts of Tibbles are the atoms that make up Tibbles. These are accidental since there is nothing specific about Tibbles that explains why those atoms are parts of him rather than some other atoms. The reason why some atoms are parts of Tibbles while some other atoms are not is just that those atoms were in a particular location at a particular time, say in Tibbles' food a week ago.

But do mountains, for instance, have any natural parts? Perhaps the peak is such a part, if the mountain has one, and the slopes also. But once we get down to smaller things, such as pebbles and grains of sand, these will only be accidental parts of any mountain.
8. Determination of parthood along the lines that I have been suggesting resembles Hempel's model for deductive-nomological explanation (D-N for short). But there are important differences between the two models. First, instead of general laws and bridge laws in the D-N model, we have propositions which describe kinds ('\(k_{1b}'\), '\(k_{2b}'\)) and general principles true in virtue of the nature of the members of the kinds in question ('\(g_1'\), . . . , '\(g_r'\)); and second, in a D-N explanation the conclusion follows by standard first order logic from the premisses but in the (ND) schema the derivation of the conclusion is not as straightforward, as I will explain below. So, what a normic determination of parthood comes to in the end depends on what our notion of kind is, what general principles about parthood are available, and what the appropriate logic for the (ND) schema is.

2. Kinds and general principles

1. My account of parthood requires that objects belong to kinds and that there be certain general principles true of members of those kinds. But talk about kinds is often viewed with suspicion and prompts resistance from philosophers with nominalistic leanings. I believe, however, that such resistance is unwarranted in the present context.64 In our daily practices we do distinguish between all kinds of

64 We can even construe the notion of a kind in a purely nominalistic way along the lines of Sylvain Bromberger. The basic notion in Bromberger's account is that of a model. It is defined as follows:

\[ M \text{ is a model of } O \text{ relative to a quadruple } \langle Q_m, Q_o, P, A \rangle \text{ iff (a) } M \text{ and } O \text{ are numerically distinct, (b) in that quadruple } Q_m \text{ is a non-empty set of questions about } M, \text{ } Q_o \text{ is a non-empty set of questions about } O, \text{ } P \text{ pairs members of } Q_m \text{ with members of } Q_o, \text{ and } A \text{ is an algorithm that translates answers to any member of } Q_m \text{ into answers to the members of } Q_o, \text{ paired with it by } P, \text{ and translates correct answers to the former into correct answers to the latter.} \]
objects; there are kinds of stuff, for instance metals and minerals, and there are kinds of living things, for instance plants and animals, and so on. This is ought to be unproblematic. Then come the philosophers who, in order to make things more systematic, distinguish between kinds and natural kinds or real kinds and so on.

From the present point of view, where our concern is parthood rather than metaphysics of kinds, we need not make these philosophical moves. Perhaps it is indeterminate to a greater extent what the parts of instances of the mundane kinds, such as mountains, are than what the parts of, say, cats are. But that is just as we should expect things to be, intuitively it is indeterminate to a greater extent what the parts of Kilimanjaro are than what the parts of the cat Tibbles are. However, from the point of view of the (ND) schema the two kinds, mountain and cat, have the same role; all we need a notion of kind for is to yield certain general principles about parthood and, it seems, a notion of kind can do that without committing us to a certain metaphysical view about kinds. There are books in the library, and they are of a different kind from the shelves, and it is in general true of books that they have such parts as pages and spines. Similarly, there are chickens in the hut, and chickens and huts are of different kinds, and it is generally true that among the parts of chickens are wings and among the parts of huts are roofs.

When members of $Q_m$ are paired only with members of $Q_n$, with which they are identical except for the replacement of reference to $M$ by reference to $O$ and the answers to each question are identical except for similar replacement of reference, then $M$ is an exact model of $O$. This notion of exact model then serves to define first what he calls minimal natural kind and, then, quasi natural kind. Finally, Bromberger arrives at a notion of a biological kind by identifying a subset of the kind that is a quasi-natural kind and closing the set under the descendant relation. ("Natural kinds and questions", Poznan Studies in the Philosophy of the Sciences and the Humanities, Vol. 51 1997.)
2. I have been suggesting that the notion of a kind that we need for the (ND) schema is just the common sense notion of a kind that ought to be uncontroversial. But one might worry that this treatment of kinds only transfers whatever problems one might have with kinds over to the general principles that the kinds yield. More specifically, someone might agree that a notion of a kind can yield certain general principles about members of kinds and that such principles are constitutive of our understanding of parthood, but maintain that (i) if the notion of a kind is reduced to common sense then so must the general principles and (ii) such general principles are not exceptionless and, therefore, do not have the entailment properties that the (ND) schema requires them to have.

To make the worry clearer consider the kind 'chicken' and the general principle 'chickens have wings'. The assumption that there is such a kind should be unproblematic, and the general principle seems to be an appropriate generalization about the members of this kind. The question now is: What logical properties does this principle have? When a principle such as "Chickens have wings" figures in a scientific explanation (for instance in a D-N model) we treat it either as a universal generalization, "All chickens have wings" or as a statistical generalization, "Most chickens have wings". But construing the generalization in either of those ways in the context of the (ND) schema is going to be problematic. On the one hand, if it is a universal generalization then it is proven false by Chanticlear who is a chicken but was born without wings. On the other hand, if it is a probabilistic generalization it may be true, but then it won't follow that any particular chicken, for instance Chanticlear, either has or does not have wings.
Since we do not reject the principle "Chickens have wings" in the light of the fact that Chanticleer was born without wings – we say that he is an exception to the principle – we can not say that the principle is a universal generalization. And since we want to be able to draw inferences from it about particular chickens, we can not say that the principle is a statistical generalization. An alternative reading is needed. The required reading is no new reading, we find it, for instance, in Aristotle’s discussion of causal generalizations. 65 Aristotle distinguished three varieties of causes. First, there are causes that produce certain effects invariably and of necessity, second, those which produce certain effects for the most part (epi to polu as he said), and third, those that produce certain effects by chance. According to Aristotle, natural causes, as opposed to celestial ones, fall in the second category, i.e. produce their effects for the most part. But how are we to understand such causal generalizations, say the generalizations that rhubarb purges or that malathion kills weeds, if not as either universal or statistical generalizations. Here is a brief description of Aristotle’s account of causal generalizations:

For Aristotle, A and B are connected as cause and effect – so that we may assert the general truth that A’s cause B’s – just when A in a given set of prerequisite conditions [...] invariably produces B, “unless something prevents”. This means that the epi to polu character of the relationship of A’s as cause to B’s as effect has to do with the possibility of A’s causal efficacy [...] being interfered with. 66


66 “Defeating the inference from general to particular norms”, p. 279. Notice that there is nothing contradictory about the causal efficacy’s being interfered with in many or even the majority of cases.
This account of generalizations isn’t clearly incompatible with treating causal
generalizations as universal generalizations and the inferences to particular cases as
universal instantiation, for although such a generalization could not be of the form
“All A’s are B’s” it might have an unless clause, i.e. it might look like “All A’s are B’s
unless this-or-that is the case”. The question then is how the ‘unless’ clause might
be completed, i.e. what might interfere with the causal efficacy.

[Aristotle’s] answer is: any number of things; and which things they are is in general
a question which is open-ended; while we can perhaps enumerate a variety of
interfering causes, no complete enumeration is possible.67

If the list is truly open-ended then the hope for treating the causal generalizations as
standard first order universal generalizations with an ‘unless’ clause become bleak
because the ‘unless’ clause itself would have to be open-ended. That, however, does
not make it impossible to give a formal account of the consequence relation, it only
makes attempts to do it in the standard way hopeless. The appropriate logic for the
(ND) schema might be either default logic or infinitary logic. In the first case the
general principles would be treated as what are called ‘defaults’. A default is a rule
of inference of the form: “If x is an A and it is consistent to infer that x is a B, then x
is a B”. I say more about default logic below. Alternatively, we might allow infinite

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67 “Defeating the inference from general to particular norms”, p. 279. Karlsson then says that
if the list is truly open-ended then the probabilistic reading will be hopeless since there will,
in general, be no definite relation of probability that the A’s have in relation to the B’s. He is
right that the probabilistic reading is hopeless but that the list open ended is not enough to
make it hopeless. The list of natural numbers is open ended but there is still a definite
relation of probability for picking a pair of odd numbers to picking a pair of even numbers
when picking numbers at random. There is, however, no definite relation of probability for
picking a pair of odd numbers to a pair of prime numbers. What is needed in addition to the
open endedness to make a probabilistic reading hopeless is a certain irregularity and the list
of interfering factors is irregular in the required way.
disjunctions and formulate the general principles as “All As are Bs, unless ...” where the unless clause is allowed to be infinitely long. In either case the general principles that figure in the (ND) schema will have the required logical properties. 68

4. If the generalizations that figure in the (ND) schema are defaults, one might wonder what makes them truth-preserving, and if they are universal generalizations with an infinite unless clause, one might wonder what makes them true. Similar questions arises with respect to causal generalizations that admit of exceptions. Let's first look at why causal generalizations are truth preserving or true, and then come back to generalizations about parthood.

When causal generalizations are true, it is in virtue of certain causal mechanisms that they are true, and it is in virtue of those same mechanisms that we are licensed to draw certain inferences. And when our inferences fail it is not because we were mistaken about the causal mechanisms but because there are some interfering factors that prevented the mechanism from working properly. If we understand general principles about causality in this way then we can maintain (i) that they are true or false, (ii) that from such a principle that is true, say the principle that malathion kills weeds, one can infer that weeds sprayed with malathion will

68 The present issue is to explain when one thing is part of some other thing, but there is a related issue that is worth distinguishing from the present one, namely what is needed in order to answer the question: “Why is this part of that?” In order to answer the why question we only need what has been called ‘abnormic law’. An abnormic law is of the form “All As are Bs unless so-and-so” where the ‘so-and-so’ is replaced by a description of some relevant or common exceptions to the unqualified claim that all As are Bs. See Sylvain Bromberger, *On What we Know we don’t Know*, University of Chicago Press 1992.
die, and (iii) that a particular weed sprayed with malathion that does not die does not prove the principle false.69

So, we have a reading of general principles about causality that have the logical properties that we want general principles about parthood to have. But general truths about causality are quite different from general principles about parthood and one might worry whether the above reading of the former can also apply to the latter. I think it can. When a generalization such as "chickens have wings" is true, it is so not in virtue of some underlying causal mechanism but in virtue of a characteristic of chickens.

["chickens have wings"] may be meant as a truth about a characteristic of chickens - something that relates to what it is to be a chicken. This is not a defining characteristic, since a wingless thing may, after all, be a chicken. Rather it tells us how we may expect a chicken to be; and why may we have such an expectation? Because there is something about being chicken - about the chicken nature, if you like [...] that produces wings, albeit epi to polu, or subject to interference.70

Suppose we have a general principle G that says that As are Bs. We assume that this principle is true, but we are also allowing that it has exceptions. If we take G to be a universal generalization our question is: Why is it true? Since G has an infinite unless clause it might be trivially true because any putative counterexample could be taken care of by assuming that the relevant constraints are included in the unless clause. If it is not trivially true, but true nevertheless, the answer is more intricate. We can’t say it is true because all As are Bs, for that is not right, nor can we say it is

69 It is interesting to compare this account of general causal propositions to Wittgenstein’s remarks on movements of machines in §§ 193-194 of Philosophical Investigations.

70 “Defeating the inference from general to particular norms”, p. 281.
true because all As are Bs unless such and such is the case, since that is just to say that G is true because G is true. Instead, we say that G should be understood as saying something about a characteristic of As. If it is true, it is so because As have this characteristic. If we understand G as a default, our question is: Why is G truth preserving? We explain the legitimacy of the inference from something’s being an A to its being a B by reference to a characteristic of the As, namely that they are Bs unless something interferes.

But what is a characteristic? This question is not trivial. Suppose G says that chickens are winged animals. Now, if we consider a single chicken such as Chanticlear, it is clear that it is not a characteristic of him that he is a winged animal. Someone might suggest that in the present context characteristics are best understood as dispositions, past or present. The suggestion might then be that although Chanticlear does not have wings, nor a disposition to grow any, at an early stage in the development of the chicken something had a disposition to become a winged bird although that disposition was never manifest. To explain what characteristics are by reference to past or present dispositions may be appropriate in many cases but it won’t work as a general account of characteristics. Chanticlear may, for instance, by wingless due to some genetic mutation in which case at no point in the development of the bird was there anything that had a disposition to grow wings. Instead, we must understand talk about characteristics as normative talk about kinds of things. When we say that being winged is a characteristic of chickens we predicate it of the species, and it applies to individual chickens only indirectly.
If it is open to us to have general principles about parthood that are (i) true or false, (ii) license inferences to particular cases and (iii) admit of exceptions, then there should be no formal or logical problem with the (ND) schema. Moreover, if we think that the facts that are relevant for determining parthood are facts about kinds of objects, characteristics of objects falling under certain kinds and facts about contingent relations, such as causal relations, then the (ND) schema should seem at least plausible as an account of parthood for material objects. But the price one has to pay for such an account of parthood is a commitment to characteristics or natures of things.

3. Three worries

1. There are three worries that someone might have about my account of parthood that I will now address. The first has to do with the status of the general principles, the second worry is that my account won't give the intuitive conclusion when the parts are strange or unexpected in certain ways, and the third is that the explanation of vagueness given by the (ND) schema is linguistic rather than ontological.

The first worry derives from the analogy between my (ND) schema and the D-N model for scientific explanations. One problem for the D-N model has to do with the status of the bridge laws, and the similarities between such explanations and my (ND) schema might suggest that there was a similar problem with the latter. The problem for the D-N model is that the bridge laws in D-N explanations are intended only to connect two vocabularies, say a vocabulary of elementary physics and a vocabulary used to represent certain observations, but they actually carry with them
theoretical commitments. The bridge laws are not scientifically innocent, or at least not as innocent as friends of D-N explanations wanted them to be. A similar worry might arise with respect to the general principles that figure in my (ND) schema. The worry would be that since a principle like “Chickens have wings” is neither analytic nor a defining principle about chickens, it brings with it empirical weight, i.e. it has the status of a scientific law. This means that the principle is not a priori and our account of, say, which entities are parts of Chanticlear might be false.

This worry is unfounded because the role of the general principles in my (ND) schema is quite different from the role of bridge laws in a D-N model. The role of the general principles in my schema is not just to connect two vocabularies but to tell us something about the way the world is. The principle that chickens have wings isn’t just a principle about the general noun ‘chicken’ but an empirical principle about what chickens in general are like. And as any empirical principle it is fallible. This, however, does not mean that the account of parthood is misguided.

A comparison with the linguistic explanation of vagueness might make this clearer. The linguistic view maintains that facts about thoughts and practices of speakers determine meaning. But any account of what determines, say, the extension of the predicate ‘bald’ will have to take into account all sorts of fallible empirical evidence. But that is not a reason to doubt the general account of what facts determine meaning nor is it a reason to question the linguistic explanation of vagueness.71 Similarly, the general account of parthood in terms of which the

71 I’m not saying that there is no reason to question the linguistic explanation of vagueness, only that this is not a reason for questioning it.
ontological view is spelled out is not subject to doubt simply because any particular claim about the parts of some particular chicken is fallible.

2. The second worry is this: The schema may conform to our intuitions as long as we are considering ordinary parts such as arms of people and hairs of cats, but it won't do well when we imagine fantasy worlds where creatures have unusual parts. Take for instance Ursula LeGuin's story about the Catwings.\(^72\) The Catwings were a family of winged cats and the story made it clear that no other cats before or since had wings. Had the story not been a fiction, there would have been cats that had wings as parts. The question now is whether this can be accommodated by the (ND) schema. One thing that can certainly not be appealed to is the general principle "Cats have wings" since it is plainly false.

I think the (ND) schema can handle this case without any modification. The thing to notice is that the Catwings fall under other kinds than just 'cat'. They are also mammals, animals and living organisms. Now, the wings were not just strapped to the cats by some rubber strings, they were attached to the cats' bodies very much like their feet and tail. There were veins extending from the body into the wings, there were muscles that were attached to bones in the wings and also to bones in the body of the cats, and so on. Now we can appeal to a principle that says that an entity that is attached to a living organism in this way is part of that organism. And since the members of the Catwings family are living organisms then, given the principle about parts of living organisms, it follows that the wings

\(^{72}\) The example comes from Vann McGee. The story is serialized in the children's magazine *Spider*. 

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are parts of the cats. We might say that the wings are parts of the Catwings not because they are cats, but because they are living organisms.

Similarly, if I were to start to grow wings one day, it would not be because I am a human being that these wings would be parts of me, but because I am a living organism, and an outgrowth from a living organism that is sustained by the functioning of the organism, is part of that organism. That's how living organisms are.

3. The third worry is that the explanation of vagueness that we get from the (ND) schema is, in the end, linguistic rather than ontological. Vague parthood is explained in terms of failure of logical consequence, i.e. when neither ‘apb’ nor ‘¬apb’ is a consequence of a certain set of premisses, then it is indeterminate whether a is part of b. The worry now is this: Whether some object turns out to be vague, in the above sense, will depend on how it is described or referred to. When I say that the wings are parts of the Catwings because they are living organisms but not because they are cats, one might worry that the (ND) schema had the wings be determinate parts of the Catwings described as living organism but not determinate parts of them described as cats. This would, indeed, be bad.

This charge is similar to Quine’s criticism that quantified modal logic is caught up in a use-mention confusion, and it is instructive to compare the two. Quine invites us to consider the following sentences.

A cyclist is necessarily biped.

A mathematician is necessarily rational.
A mathematician is not necessarily biped.

A cyclist is not necessarily rational.

On modest assumptions about what it is to be biped and rational these are true. But what are we then to say about someone who is both a cyclist and a mathematician? Quine’s answer is that saying of someone, independently of how he is described, that he is necessarily one way but only contingently some other way does not make any sense.

Quine’s notion of necessity was the notion of logical truth extended to the notion of analyticity, and with this notion one cannot make any sense of an open sentence being nontrivially necessarily true of an object.73 So, for instance, Quine would not allow a necessity operator appearing inside the scope of a quantifier as in

$$\exists x \Box Fx,$$

except where ‘F’ is a tautological predicate such as ‘is or is not red’. Similarly, the linguistic notion of vagueness does not allow the ‘V’ operator attaching to an open formula inside the scope of a quantifier.74 But the fundamental claim of my view is that

$$(1) \exists x y \neg xPy$$

is true.

Quine insisted that if one was to make sense of quantified modal logic a different interpretation of ‘\(\Box\)’ was needed.

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74 This is not quite straightforward as I explained in chapter 2.
... the important point to observe is that granted an understanding of the modalities (through uncritical acceptance, for the sake of the argument, of the underlying notion of analyticity) and given an understanding of quantification ordinarily so called, we do not come out automatically with any meaning for quantified modal sentences ...75

Quine then added that to save quantified modal logic from the charge of use-mention confusion one would need Aristotelian essentialism, i.e. one could not just extend the notions of logical truth and analyticity to make sense of such necessity but would need something altogether different and, by his lights, unreasonable.76

Similarly, we can’t simply extend the linguistic notion of indeterminacy to make sense of ontological indeterminacy. We need something different. To continue the analogy with modal logic, we might say that we need something to do for the notion of ontological indeterminacy what Aristotelian essentialism was to do for metaphysical necessity before we can have quantified logic of indeterminacy. Without Aristotelian essentialism quantified modal logic would be mysterious. It would be a mystery why ‘□F’ was true of some objects and not others. The role of essentialism was to remove this mystery, or relocate it as Quine would probably say. Similarly, without some metaphysical underpinnings quantified logic of vagueness will be mysterious.

4. But what do we need in order to save quantified logic of vagueness from the charge of use-mention confusion without ending up in mysteries? Again, a look at


76 “Reference and modality”, second ed., p. 156
Quine’s animadversions on modal logic is helpful. Quantified modal logic can take a form that would not be objectionable by Quine’s lights. We can easily convince ourselves that the following formula is valid:

(2) \( \forall x (Gx \land \Box Fx) \rightarrow \Box \forall y (Gy \rightarrow Fy) \).

If ‘G’ stands for ‘is a cyclist’ and ‘F’ stands for ‘is biped’, then the formula says that if something is a cyclist and it is analytic that it is biped, then it is analytic that every cyclist is biped. Similarly, we can convince ourselves that the following is true:

(3) \( \forall x ((Gx \land \Box \forall y (Gy \rightarrow Fy)) \rightarrow \Box Fx) \).

This formula says that if anything x is a cyclist and it is analytic that anything that is a cyclist is also biped, then it is analytic that x biped. Combining (2) and (3) we get

(4) \( \forall x (Gx \rightarrow (\Box Fx \leftrightarrow \Box \forall y (Gy \rightarrow Fy))) \).

Adding (4) to the list of axioms of our first order predicate logic should be unobjectionable from the point of view of Quine except that it collapses modal distinctions altogether and makes quantified modal logic pointless. In order to get interesting modal logic, modal logic that is worth the trouble, one needs something different. One thing to do is to have (4) apply selectively to certain ‘canonical’ predicates for then, if (i) for each object there is at least one canonical predicate it satisfies and, (ii) for any two canonical predicates, ‘A’ and ‘B’

(5) \( \exists x (Ax \land Bx) \rightarrow \Box \forall y (Ay \leftrightarrow By) \).

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77 Here I follow John P. Burgess’ “Quinus ab omni nævo vindicatus”, in particular pp. 34-35.
is true, then principle (4) could serve as a basis for *interesting* quantified modal logic. But how could we select what predicates should be taken to be canonical? John P. Burgess offers the following answer:

> It seems that making the selection [...] would require reviving something like the ancient and mediaeval notion of ‘real definitions’ as opposed to ‘nominal definitions’ ... (p. 35)

In the context of vagueness, as opposed to modality, the role of ‘real definitions’ would be to determine which general principles were admitted into \( \Gamma \). That the principle “Chickens have wings” is an acceptable principle while “Cats have wings” is not acceptable, is not due to some linguistic or semantic facts. It is because that’s how chickens and cats are. That certain propositions about parthood turn out to be determinate while others are not, is likewise not a linguistic matter, but a matter of how certain objects are. But in order to select principles that are neither arbitrary nor trivial friends of vague objects must take on board the idea that objects have natures; it is in virtue of the nature of objects that certain principles are true and certain principles are false.\(^7\)\(^8\) So, just as the friends of quantified modal logic were pushed back to essentialism so are the friends of vague parthood forced

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\(^{78}\) What the natures of different things are is not the subject of philosophy. What the nature of, say, living organisms are is the subject of biology and as long as there is some ignorance concerning the nature of certain objects, parthood may be epistemically indeterminate. Perhaps there is no reason to believe that we will ever be able to get complete knowledge of the nature of organisms, or any object for that matter, and if that is so, there may always be an element of ignorance surrounding parthood. Moreover, insofar as speakers are ignorant about parthood it may be impossible for them to fix a definite extension of a parthood predicate, the result of which would be some linguistic vagueness. But this is in no conflict with the present view. The present view does not entail that there should be no epistemic or linguistic vagueness, only that vagueness of parthood is not only, and not primarily, a matter of imperfect knowledge or semantic indecision.
to accept natures of things, which, although unreasonable by Quine’s lights, I am happy to accept.

What makes the present thesis properly a thesis about ontological vagueness as opposed to linguistic vagueness is not just the failure of logical consequence. The mere failure of logical consequence makes it a theory about vagueness. What makes it a theory about ontological vagueness is which general principles are acceptable and their role in determining facts about parthood.

4. General principles and default logic

1. The idea that default logic might be applicable to account for the consequence relation in the (ND) schema needs explaining. Default logic has primarily been used to model common sense reasoning, to describe what one might call epistemically valid inferences and as a logical basis for artificial intelligence. What sets default logic apart from classical logic is the introduction of new inference rules, called ‘defaults’, which allow one to draw conclusions that one would otherwise not be allowed to draw. An example makes this clearer: Suppose we are reasoning about chickens. We have the rule of thumb that if we have no special reason to doubt that a particular chicken has wings we can infer that it has wings. Now we introduce a rule of inference which allows us to infer of any chicken that it has wings unless we have a reason to believe otherwise. The rule takes the following form:

\[
\begin{array}{c}
\Pi \vdash x \text{ has wings} \\
\hline
x \text{ has wings}
\end{array}
\]

where \( \Pi \) is a deductively closed set of sentences describing our epistemic status.

The rule reads thus: if for an arbitrary chicken \( a \) it is consistent with what we know that it has wings, then we can infer that it has wings. In other words, if "\( a \) has wings" is consistent with the set \( \Pi \), then we may infer the sentence "\( a \) has wings".

But the inference is non-monotonic since some additional information may have the consequence that we can no longer infer "\( a \) has wings". Suppose, for instance, that all we know about Chanticlear is that he is a chicken and that he lives on this or that farm, but we don't know anything about whether he has wings or not. Then, "Chanticlear has wings" will be consistent with our epistemic status, and the default allows us to conclude that Chanticlear has wings. But suppose we receive a picture of Chanticlear and learn that he has no wings after all, then we have to take back our earlier conclusion.

2. This is how default logic has been used, but this needs to be changed once we apply the idea of defaults to characterize ontological vagueness as opposed to epistemically valid reasoning.

A default theory \( T = (\Pi, D) \), is a pair where \( \Pi \) is a deductively closed set of sentences and \( D \) is a set of defaults. The first question we need to ask is: What sentences should the set \( \Pi \) contain? Since our concern is vague parthood as defined by the (ND) schema, the set \( \Pi \) should not describe our epistemic status but instead whatever can figure as a premiss in the (ND) schema except the general principles. The general principles will be the members of \( D \).
But it is not enough to say what the members of \( \Pi \) should be, we also need to know what general properties the resulting default theory has and that depends on what the defaults are like. In general a default has the form:

\[
\Pi \models p \\
\hline
q
\]

and such defaults can behave in strange ways. For instance, \( q \) might be the negation of \( p \) in which case the default allows one to infer the negation of the justification for the inference. But in respect of the (ND) schema we do not have to worry about such strange defaults. All the defaults will take the form:

\[
\Pi \models p \\
\hline
p
\]

A default of this form is called 'normal default', and a default theory where all the defaults are normal is called 'normal default theory'. Such a theory is relatively well behaved, it is, for instance, almost monotonic, i.e. the following can be proved:

Semi-monotonicity: Let \( T = (\Pi, D) \) and \( T' = (\Pi, D') \) be normal default theories such that \( D \subseteq D' \). Then, each extension of \( T \) is contained in an extension of \( T' \).\(^{80}\)

In a normal default theory the only time one might have to take something back is when additional information is obtained independently of the defaults as in the example above, where we learned that Chanticlear had no wings by getting a picture of him. This is why normal default theories are not monotonic but only

\(^{80}\) See Nonmonotonic Reasoning, p. 50.
semi-monotonic. But in the case of the (ND) schema, where the initial set of sentences does not describe our epistemic status but the primitive facts on which parthood depends, we do not have to worry about such cases. The set of primitive facts does not expand as we learn more things. It is fixed. The conclusion is, therefore, that if we treat the (ND) schema as a normal default theory, then it will not only be semi-monotonic but also monotonic.

Another important feature of normal default theories is that they have an extension, i.e. in applying the defaults we will reach a point where further application does not make any difference. However, in normal default theory there is nothing to prevent us from having a pair of defaults such as the following:

\[
\begin{align*}
&\text{(a) } \Pi \models p & \quad &\text{(b) } \Pi \models \neg p \\
& p & & \neg p
\end{align*}
\]

But if we have defaults like these, what the extension of the default theory is depends on which default is applied first, i.e. if we apply (a) first we may be able to infer \( p \). But then the set \( \Pi \) will be inconsistent with \( \neg p \) which means that we won’t be able to infer \( \neg p \). However, if we apply (b) first, we may be able to infer \( \neg p \), in which case we won’t be able to infer \( p \). In sum, even in a normal default theory the extension of the theory may depend on the order in which the defaults are applied.

To make the relevance of order clearer an example from ethics might help. Suppose we use default theory to model reasoning from the following moral norms.

One should not lie.

One should save life.

We might represent these norms as defaults in the following way:
Suppose that one encounters a situation where saving a life requires lying, i.e. the conjunction of “I save life” and “I do not lie” is inconsistent with a description of the circumstances. Now, if I begin by applying the default that tells me not to lie, then I will have to sacrifice life and the second default becomes inapplicable. However, if I apply the default that tells me to save life first, then I have to lie, so that the first default becomes inapplicable. What I end up doing depends then on in what order I apply the defaults.

A question we now need to ask is whether the extension of the predicate ‘P’ may depend on in which order we apply the defaults. There are two cases which we need to consider. The first is where the consequences of different defaults are inconsistent as is the case with (a) and (b) above. The second case is like the ethical case where the consequences are not inconsistent by themselves but $\Pi$ has a sentence that is the negation of the conjunction of the consequences of some defaults. I think it is clear that we do not have to worry about the first case. The defaults represent what I called earlier general truths or general principles about parthood, and these do not contradict each other.

The second case is not as clear. Could it be that $\Pi$ included the sentence “$-(aPb \land aPc)$” and that there were two defaults such that applying one before the other would allow us to conclude “$aPb$” but applying the second before the first would allow us to conclude “$aPc$”? The question is whether it could be that the only thing that prevented $a$ from being part of $c$ was that $a$ was part of $b$, and likewise, that the only thing that prevented $a$ from being part of $b$ was that $a$ was part of $c$. I
don’t know whether there is any clear answer to this question, but neither is it clear that much hinges on which way we answer it. If it is possible that the extension depends on the order of application, we can define a notion of minimal commitment as the intersection of all the extensions that we get by applying the defaults in different order and then identify the determinate extension and the determinate anti-extension of ‘P’ with the minimal commitment. After all, if it does depend on the order of application of the defaults whether a is part of b, and if there is no fact of the matter in which order the defaults should be applied, then it seem right to say that it is indeterminate whether a is part of b.

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81 It should be noted that the minimal commitment is need not be an extension of a normal default theory.

82 In the case of parthood it does not seem to be problematic that the order in which the defaults are applied affects the extension of the theory. The same is not true of the ethical case described above. In the case of parthood we can make do with the minimal commitment, i.e. the intersection of the extensions we get by applying the defaults in different order. In the moral case we cannot make do with the minimal commitment. The extension of the default theory in the moral case are sentences describing my actions. The minimal commitment might have neither the sentence “I lie” nor “I sacrifice life”, but the scenario might just be such that I cannot avoid both lying and sacrificing a life.
In this chapter I offer an analysis of the notions of vague object and material constitution. The former is rather straightforward once we have an account of vague parthood. The basic idea is that vagueness for objects is just vagueness with respect to parthood. The notion of material constitution turns out to be more difficult. In section two I turn to the notions of 'partial constitution' and 'exact constitution' that I mentioned briefly in Chapter Two. Now I give a detailed definition of them.

1. Vague object

1. From a notion of vague parthood it is relatively easy to get to the notion of vague object that I am interested in. The basic idea is that if it is indeterminate of some object whether it is part of another object then the latter object is vague. So, for instance, if it is indeterminate of some droplet whether it is part of a certain cloud then the cloud is vague. To capture this one might suggest the following definition:
An object $O$ is vague relative to entities $E$ at time $t$ iff it is indeterminate of at least one of the $Es$ whether it is part of $O$ at $t$.

Although this definition is faithful to the basic idea it does not quite succeed. The problem is this: Suppose (i) that we have two objects, $a$ and $b$ where $a$ is a fusion of droplets while $b$ is a cloud. Intuitively, $b$ is vague with respect to some droplets while $a$ is precise with respect to droplets and larger entities. Suppose further (ii) that parthood is governed by the following principle:

\[(P) \forall xy (xPy at t iff the region of space occupied by x at t is part of the region of space occupied by y at t),\]

i.e. suppose that (P) is an extensionally correct principle about parthood and that it is indeterminate whether $a$ is part of $b$, as well as whether $b$ is part of $a$;\(^83\) and (iii) that this indeterminacy arises because it is not determined which droplets are parts of $b$. Now, according to the above definition, $a$ turns out to be vague with respect to $b$ which conflicts with our intuition that $a$ is not vague with respect to droplets and larger entities.

2. There is a relatively easy fix for this problem, a fix that should be obvious once we consider in what settings questions about vagueness arise. These settings are of two sorts. First, we have cases where some objects are treated as precise and then

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\(^{83}\) In Leonard and Goodman's calculus of individuals identity is defined in the following way:

\[x = y \text{ iff } xPy \text{ and } yPx.\]

The scenario I am imagining would, therefore, in the Calculus of Individuals entail that identity was indeterminate. But I don't think that identity can be defined in this way. The definition is more plausible if we stick a necessary operator in front of the right hand side, i.e. identity would not be defined in terms of symmetric parthood but in terms of necessary symmetric parthood. However, I am not sure whether identity should be defined in terms of parthood at all rather than taken as a primitive.
we have some other objects whose precision is in question. Examples include pebbles and mountains, atoms and bodies, droplets and clouds. Second, there are cases where we have a hierarchy of objects and the ones higher in the hierarchy are less precise or more vague than those that are lower. A mountain is vague with respect to a pebble and the pebble is vague with respect to some atom, but, we want to say, the mountain is more vague than the pebble.

In both kinds of cases it is given that there is a basic level, i.e. a level of objects whose precision is not in question, though it may vary from case to case what is at the basic level. The availability of a basic level, whether it is given absolutely or merely relative to the case in question, that allows us to solve the above problem. The reason why we say that it is because of the vagueness of the cloud that it is indeterminate whether this or that fusion of droplets is part of the cloud is that there is a collection, $B$, of basic elements, say droplets, such that (i) there is some subcollection of $B$ such that the fusion of droplets is precise with respect to it, while the cloud is not, and (ii) there is no subcollection of $B$ with respect to which the cloud is precise but the fusion of droplets is not. This gives us two additional conditions for determining vagueness. If there are two objects, $a$ and $b$, such that it is indeterminate whether $a$ is part of $b$ and

\[
\text{some subcollection, } C, \text{ of the basic elements, } B, \text{ is such that at } t, a \text{ is not vague with respect to } C \text{ but } b \text{ is,}
\]

and,

\[
\text{there is no subcollection, } D, \text{ of the basic elements, } B, \text{ such that at } t, b \text{ is not vague with respect to } D \text{ but } a \text{ is,}
\]
then it is because of the vagueness of $b$ that it is indeterminate whether $a$ is part of $b$
at $t$. Adding these two extra conditions to our previous definition of vagueness
gives us what I believe is a satisfactory definition of 'vague object'.

**Vague Object**: An object $O$ is vague relative to entities $E$ at time $t$ iff
for some $x$ that is one of the $E$s there are some basic elements $B$, such that
(1) it is indeterminate whether $x$ is part of $O$ at $t$, &
(2) $\exists C (C \subset B$ and at $t$, $x$ is not vague with respect to $C$ and $O$ is vague
with respect to $C)$, &
(3) $\neg \exists D (D \subset B$ and at $t$, $O$ is not vague with respect to $D$ and $x$ is
vague with respect to $D$).$^{84}$

To get a better grip on this definition an example will help. Suppose that
Puffy is a cloud. A consequence of the definition is that although a cloud like Puffy
may be vague it will not be vague with respect to a number of objects, for instance
any droplets that are on the ground and sufficiently far away, and barring vague
identity, no object will be vague with respect to itself. Here vagueness is a relative
property; an object is vague *relative* to some entities at a certain time. Given a
definition of vagueness, we can define precision as the negation of vagueness.$^{85}$
Then, precision will be relative in the same way as vagueness is.

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$^{84}$ Here the symbol 'c' does not stand for the subset relation but for the subcollection
relation. A collection $A$ is a subcollection of $B$ if anything which is one of the $As$ is one of the
$Bs$.

$^{85}$ In Chapter Three in a response to a concern of Russell I said that vagueness, in my case,
contrasted with indeterminacy and not precision. Now I am suggesting that vagueness does
contrast with precision. Do I then fall prey to Russell’s charge of a fallacy of vebalism? No,
Russell’s notions of vagueness and precision are different from mine. Instead of talking about
precise objects I could talk about sharp objects or determinate objects. In my case vagueness
and precision are defined in terms of determinacy, in Russell’s case they are defined in terms
of isomorphism between a system as a representation and a system represented.
2. Vague constitution

1. I now turn to the problem of material constitution. We want to say that clouds are made up of or constituted by droplets and that a statue is made up of or constituted by some clay. To explain what this means is the problem of material constitution.

First, let’s recall why the problem of material constitution is a difficult one. A statue and the portion of clay it is made of have many properties in common: they have all the same basic parts, they are in the same location, have the same weight etc. So, why are they not identical? Well, they can’t be identical for three reasons: (i) the portion of clay existed long before the statue did, (ii) if a small part breaks off the statue and falls on the floor, the statue becomes a bit smaller but the portion of clay will be of the same size as before but becomes scattered, and (iii) we might smash the statue, in which case the clay still exists but the statue is no more. So, the two can go their separate ways and, therefore, cannot be identical. We seem forced to conclude that the two are distinct, but somehow intimately related.

In discussing the notion of constitution the natural starting point is Judith Jarvis Thomson’s definition of constitution. The motivation for Thomson’s definition comes mainly from considerations that fall under reason (ii) above and we might call ‘replacement arguments’. We have a statue on the table at 2 PM, call it Alfred, and at the same time we have a portion of clay on the table, call it Clay. But if at a later time a small piece of Alfred is broken off and dropped on the floor, it is still right to say that Alfred is wholly on the table but the same does not hold for Clay. Part of Clay is on the floor. So, what we say is that at 2 PM Clay constitutes
Alfred, but after the piece is broken off, Clay does not constitute Alfred although a proper part of Clay does.

Considerations along these lines suggest that there are parts that are essential to Clay but not to Alfred and, also, that there are no parts that are essential to Alfred but not to Clay. This leads Thomson to the following definition:

**Thomson’s Constitution:** $x$ constitutes $y$ at $t =_{df}$

1. $x$ is part of $y$ and $y$ is part of $x$ at $t$, &
2. $x$ has an essential part that has, at $t$, no parts essential to $y$, &
3. it is not the case that $y$ has an essential part that has, at $t$, no parts essential to $x$.

In this definition ‘$x$’ might be replaced by a name of a portion of clay and ‘$y$’ might be replaced by a name of a statue. Alternatively, ‘$x$’ might be replaced by a name of a fusion of some atoms and ‘$y$’ might be as before.87

2. How does a definition of constitution along the lines of Judith Thomson fare when it comes to vague objects? Suppose that a statue is vague with respect to some atoms. Then there is no fusion of atoms of which it is determinate both that it is part of the statue and that the statue is part of it. The problem is that if $b$ is vague with respect to some objects, then for any fusion $c$ of some of those object the sentence “$c$ constitutes $b$” will be either indeterminate or false.

In a way this does not at all look like a problem since if an object is vague with respect to some atoms then it is only to be expected that it is not constituted by any

87 Thomson’s definition of material constitution is not immune to challenges not having to do with vagueness, but I won’t go into those matters here. It does get the most important cases right, i.e. cases where an object such as a statue is constituted by a quantity of matter and in the case of vagueness, these are also the cases that we need to worry about.
precise fusion of such atoms. But in another way it does look like a problem since it squares ill with our intuition that a statue is made of atoms and nothing else, and that clouds are made of droplets and nothing else, and so on. We want our notion of constitution to reflect, or at least be consistent with what we mean when we say such things as: “A cloud is just an aggregate of droplets”, or “A cloud is made up of nothing but droplets”. These sentences are supposed to express truths and, therefore, it would be welcome to have a notion of constitution such that it could be true of, say, some fusion of droplets that it constituted a cloud. Moreover, once we are allowing that parthood might be vague it seems natural that this should be reflected in a notion of constitution that is defined in terms of parthood.

3. Instead of the notion of constitution offered by Judith Thomson I suggest that we adopt importantly different notions of partial and exact constitution. When we say that Puffy is constituted by a fusion of droplets, we don’t have any particular droplets in mind. All we mean is, roughly, that Puffy is made of droplets and the only building blocks of Puffy are droplets. When we reflect on the vagueness of Puffy we see that there is no single collection whose fusion is more plausibly the constituter of Puffy than is an indefinite number of others. This was the conclusion of the complex problem of the many in Chapter Two. The idea now is that many fusions might partially constitute Puffy, i.e. instead of constitution being

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88 Mark Johnston has suggested that no such fusion is the constituter of Puffy, but rather that one is its constituter in one precisification and some other in another precisification, and that this is so because the constitution relation is vague. (“Constitution is not identity”, Material Constitution, M. Rea ed., Rowman and Littlefield, 1997, p. 56.) But it isn’t clear that we can legitimately talk about precisifications in this context. I say more about this in Chapter Six.
a one-one relation it will be a many-one relation, i.e. for some given object there may be more than one object that constitutes it.

There are two obvious conditions that any candidate for being a constituter of Puffy must meet; it must not be deficient, i.e. anything which is determinately part of Puffy should overlap some part of the constituter and, second, it must not be excessive, i.e. the constituter must not have a part that is determinately outside Puffy. So, for instance if an object c constitutes Puffy at t then:

- everything which is determinately part of Puffy has a part that overlaps some part of c at t,
- no part of c is determinately not part of Puffy at t.

These conditions serve a similar purpose as condition (1) in Thomson’s definition, namely to guarantee that the constituter and the constitutee are appropriately bound. My definition of partial constitution then retains the last two clauses of Thomson’s definition. This gives us the following definition of partial constitution.

**Partial Constitution:** \( x \) partially constitutes \( y \) at \( t \) =df

1. everything which is determinately part of \( y \) has a part that overlaps some part of \( x \),
2. no part of \( x \) is determinately not part of \( y \) at \( t \),
3. \( x \) has an essential part that has, at \( t \), no parts essential to \( y \),
4. it is not the case that \( y \) has an essential part that has, at \( t \), no parts essential to \( x \).

But Partial Constitution is not enough. To say that a water molecule is partially constituted by two hydrogen atoms and an oxygen atom is not false but somewhat less than one might want. Any particular water molecule is exactly constituted by the fusion of some two hydrogen atoms and an oxygen atom. Similarly, a statue will be exactly constituted by the head, arms, torso and legs although it may be only
partially constituted by a fusion of atoms. To mark this distinction, we need a notion of exact constitution in addition to partial constitution.

Given the above notion of partial constitution we might try the following definition:

\[
x \text{ exactly constitutes } y \text{ at } t \equiv (1) \text{ } x \text{ partially constitutes } y \text{ at } t, \text{ & (2) for any } p \text{ that determinately does not overlap any part of } x \text{ at } t, \text{ the fusion of } x \text{ and } p \text{ does not partially constitute } y \text{ at } t.
\]

Even if this sounds promising, it does not deliver the goods. Suppose we have five atoms \( a_1 \) to \( a_5 \) and an object \( b \) and that (i) it is determinate of \( a_1 \) to \( a_3 \) that they are parts of \( b \) at \( t \), (ii) that it is indeterminate of \( a_4 \) and \( a_5 \) whether they are parts of \( b \) at \( t \), and (iii) of all objects which do not overlap one of the \( a_s \) it is determinate that they are not parts of \( b \) at \( t \). Then, \( b \) is partially constituted by the fusion of \( \{a_1, a_2, a_3, a_4, a_5\} \), but it is also exactly constituted by it. But the idea was that the difference between partial and exact constitution should track the distinction between vagueness and precision. In order to fix this we need to add to the definition of exact constitution the following requirement:

\[
\text{If } x \text{ exactly constitutes } y \text{ at } t \text{ then every part of } x \text{ is determinately part of } y \text{ at } t.
\]

This gives us the following definition of exact constitution:

\[
\text{Exact Constitution: } x \text{ exactly constitutes } y \text{ at } t \equiv (1) \text{ } x \text{ partially constitutes } y \text{ at } t, \text{ & (2) every part of } x \text{ is determinately part of } y \text{ at } t, \text{ & (3) for any } p \text{ that determinately does not overlap any part of } x \text{ at } t, \text{ the fusion of } x \text{ and } p \text{ does not partially constitute } y \text{ at } t.
\]

Let's consider the following example to make things clearer. Suppose we have a statue that is wholly made of atoms and that we can single out a collection of atoms
such that at $t$ each member of it is determinately part of the statue and everything which is determinately part of the statue overlaps some of the atoms. Then the statue is partially constituted by a fusion of some atoms at $t$. Now, if in addition it is determinate for any atom whatsoever whether it is part of the statue or not at $t$ then there is a fusion of some atoms that exactly constitutes the statue at $t$. But, if there is but a single atom such that at $t$, it is indeterminate whether it is part of the statue or not, then the statue may still be partially constituted by a fusion of some atoms at $t$ but it won’t be exactly constituted by any such fusion.

Now, if Puffy is vague with respect to some droplets it will be partially constituted by fusions of some droplets but it won’t be exactly constituted by the fusion of any collection of droplets. This has the advantage that we are not forced to accept, as an unexplainable fact, that some one fusion of droplets constitutes Puffy, while other fusions, that differ only by a droplet or so, do not constitute a cloud, let alone Puffy.

4. It is instructive to compare the present notions of partial and exact constitution and Thomson’s notion of constitution. Anything which has a constitution in Thomson’s sense will have an exact constitution, but not the other way around. In other words, $c$ may exactly constitute $b$ without $c$ constituting $b$ in Thomson’s sense. To see how this can be consider the following example. Let ‘$b$’ be a name of a portion of gas and let ‘$c$’ be a name of a fusion (or the fusion if there is only one) of atoms that make up the gas at some time $t$. Intuitively, the fusion of the atoms occupies just the space occupied by the atoms, but if the size of the region of space occupied by the gas is the volume of the gas then if pressure is around normal
atmospheric pressure the gas occupies a much larger region. This means that $c$ may well be part of $b$, but $b$ cannot be part of $c$, for we don’t want what is part of an object to be larger than the object itself.

Now, if $b$ is not part of $c$ then $b$ cannot be constituted, in Thomson’s sense, by $c$. But since the notion of partial constitution only requires that $c$ be part of $b$ and not also the other way around, and if we can assume that $c$ has an essential part that is not essential to $b$ while $b$ does not have any essential part that is not essential to $c$, then $c$ will partially constitute $b$. But suppose now that it is quite clear which atoms are parts of the portion of gas at $t$. This means that (i) $c$ partially constitutes $b$ at $t$, (ii) any part of $c$ is determinately part of $b$, and (iii) of any atom which does not overlap some part of $c$ it is determined that the fusion of it and $c$ does not partially constitutes $b$ at $t$. But then it follows that $c$ exactly constitutes $b$ at $t$.

Of course this does not show that Thomson’s definition of constitution must be rejected for one might reject the premiss that the volume of a portion of gas has much to do with the region of space occupied by the gas or one might say that the region of space occupied by a fusion of atoms is the region occupied by the portion of gas that it constitutes. But be that as it may, the example makes clear that my notion of exact constitution is not equivalent to Thomson’s notion of constitution.
Towards a Formal Framework for Vague Objects

In this chapter I look at some logical properties of a language that has a relation term, 'is part of', that stands for a vague relation and singular terms referring to vague objects – for instance the name 'Puffy' and the definite description 'the tallest mountain in Africa'. In Chapter Three I described the truth conditions of sentences of the form "aPb" or "¬aPb", now the task is to describe the truth conditions of complex sentences that have indeterminate sentences as constituents.

We imagine, as a heuristic device, that we begin with a classical language, L, and then add to this language a parthood predicate, 'P', the extension and the anti-extension of which are given by the (ND) schema. The resulting language, L_P, resembles a partially interpreted language since there will be pairs of objects that are neither in the determinate extension nor in the determinate anti-extension of 'P'.
1. The three valued tables

1. Given a determinate extension and determinate anti-extension for ‘P’ truth values for atomic sentences, including negations of atomic sentences, are straightforward. There are still unanswered questions about the indeterminate atomic sentences but these are not about their truth value – they are indeterminate – but about how to understand this value and what logical properties indeterminate sentences have.

For a classical bivalent language the classical truth tables tell us how to assign truth values to complex sentences given an assignment of truth values to atomic sentences. But once we are in the territory of vagueness, whether it is linguistic or ontological, the classical tables are inadequate. The simplest idea for assigning truth values to complex sentences of a vague language, given an assignment for the atomic sentences, is to use some three valued truth tables. But this will not do. A reason sometimes cited for not using the three valued truth tables is that assignment of truth values according to them conflicts with classical logic; a sentence of the form “p ∨ ¬p” may be indeterminate according to the three valued tables while it is a logical truth in classical logic. However, since the logic that the ontological view demands does not have the law of excluded middle, this reason does not clearly tell against the three valued tables.

But there is a different problem that makes the three valued tables fail. The problem is that these tables do not respect what Kit Fine calls ‘penumbral connections’ or ‘penumbral truths’. Fine gives an example of a blob whose color is

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a borderline case between red and pink. He then argues that there are various truths that such a blob gives rise to, for instance “The blob is either red or pink” or “If the blob is red, it isn’t pink” or “It is false that the blob is both red and pink”. And for these truths the three valued truth tables can not account. An example of a penumbral truth that is more relevant in the present context is the following:

Assuming that Sprinkle is on the border of Puffy so that the sentence “Sprinkle is part of Puffy” is indeterminate, the sentence

If Sprinkle is part of Puffy then Sprinkle is part of Puffy,

ought to be true. However, assuming that Sparky is on the border of Kilimanjaro, the sentence

If Sprinkle is part of Puffy then Sparky is part of Kilimanjaro,

ought to be indeterminate. But, as far as the truth tables are concerned, the above sentences are on a par.

The problem with the three valued tables is that assignment of truth according to them presupposes that the truth value of a complex sentence is a function of the truth values of its constituents. For classical bivalent languages this presupposition is right, but for vague or partially interpreted languages, it is false.

2. Supervaluation

1. Given that the three valued truth tables fail to describe the distribution of truth values over complex sentences we might consider supervaluation. Since supervaluation is just a formal device there is no question that we can use it to define a predicate, call it ‘T’, that applies to sentences of $L_p$ and behaves in many
ways like a truth predicate for the language. For instance, by specifying certain conditions on the acceptable precisifications we can ensure an approximate fit between some intuitive notion of truth and our new predicate. Where ‘S’ is an atomic sentence we can guarantee that the following conditionals hold.

If ‘S’ is determinately true in $L_p$, then $T(S)$,

if ‘S’ is determinately false in $L_p$, then $not-T(S)$,

if ‘S’ is neither determinately true nor determinately false in $L_p$, then neither $T(S)$ nor $not-T(S)$.

Moreover, ‘T’ will apply to many complex sentences just in case they are true, for instance any sentence of the form ‘$p \rightarrow p$’. The question now is whether our new predicate ‘T’ is properly called a truth predicate for $L_p$.

Supervaluation fits well into the linguistic account of vagueness but our concern is ontological vagueness and not linguistic. And that makes some difference. Combining the ontological view about vagueness and the linguistic notion of precisification does not automatically give us an account of truth conditions of complex sentences where one or more constituent is vague. In the ontological view there is no obvious way to understand the notion of precisification that lends itself to a supervaluation definition of truth or satisfaction. To get a better grip on this problem, consider the following suggestion by Mark Johnston. He argues that the problem of the many shows that constitution is a vague relation and, having come to that conclusion, he says:

Our cloud $c$ is not only not identical with any one of the $k_0$, $k_1$, $k_2$, [which are aggregates of droplets] but also it is not definitely constituted by any one of the $k_0$,
Rather, on one legitimate sharpening it is constituted by one of the ks, on another, another of the ks, and so on.\textsuperscript{90}

I agree with Johnston that the problem of the many shows that constitution is a vague relation, but what he says about sharpenings I do not find helpful. I actually think that it is confused. What is it that is being sharpened, or precisified as I prefer to say? Is it the cloud? If it is the cloud we need to know what a precisification of a cloud is. Or is it perhaps the constitution relation that has precisifications or the relation term ‘constitutes’? Johnston does not tell us.\textsuperscript{91}

2. One way of understanding the notion of a precisification in the context of vague objects is this: To precisify a vague object is to change the status of its borderline parts so that they become either determinate parts or determinate non-parts of the object. This is a clear notion of a precisification, and it is the notion that Michael Tye has in mind when he says:


91 E.J. Lowe suggests a solution to the problem of the many that is similar to Johnston’s. He says that “... we can say that it is neither determinately true nor determinately false that it is \( c \), as opposed to \( c_1 \) or \( c_2 \) or some other \( c_n \) that constitutes Tibbles at present – though it \( i s \) determinately true that just one of them does, because whichever candidate were chosen as occupying the role of constituter of Tibbles would exclude all others from the role”. (“The problem of the many and the vagueness of constitution”, Analysis, Vol. 55, No. 3 1995, p. 180). What does it mean to say that such and such would be the case if this or that candidate where \textit{chosen} to be the constituter of Tibbles? Is Lowe suggesting that it is up to people to choose which are the parts of Tibbles? It may be up to people to choose which are the parts of certain objects, for instance, in 1944 the people of Iceland decided by a public vote that Iceland should not be part of Denmark. But in the case of Tibbles I don’t think the notion of a choice has any role, at least not if vagueness is to be explained in ontological rather than linguistic terms.
[The view that a vague object is one that is capable of being made more precise]
has certain plausibility for the case of concrete objects such as Everest. For it is not
difficult to imagine circumstances in which Everest is made more precise.
Suppose, for example, that extremely powerful bombs are detonated around the
base of Everest and that as a result of the explosions Everest's base is much more
clearly defined than before. In these circumstances, Everest has fewer indefinite
spatio-temporal parts. So Everest is more precise.\(^9\)

Obviously, the kind of precisification that Tye talks about is not helpful for
specifying the truth conditions of complex sentences.\(^9\) When we want to say why
the sentence "If Sparky is part of Kilimanjaro then Sparky is part of Kilimanjaro" is
true we don't have to imagine what would be true if Kilimanjaro would be made
more precise by one or another way of dropping bombs around its base.

A less dramatic idea of precisification for vague objects is this: A precisifica-
tion of a vague object, say Kilimanjaro, is a precise object, \(K\), that overlaps
Kilimanjaro in such a way that everything that is determinately part of Kilimanjaro
is part of \(K\) and everything that is determinately not part of Kilimanjaro is not part
of \(K\). The set of all acceptable precisifications will be the set of all the precise objects
that satisfy these two conditions. This is closer to the idea that Johnston seemed to
have in mind, and it is also closer to the linguistic notion since, according to this
idea, one does not have to do anything to the vague object when giving a
precisification of it. But on this account, a precisification of Kilimanjaro is an object
that is distinct from Kilimanjaro both numerically and qualitatively. And why

\(^{92}\) Michael Tye, "Vague Objects", p. 538.

\(^{93}\) Tye does not think that this idea of precisification is helpful in giving the truth conditions
of complex sentences. I borrow Tye's example here because he, unlike Johnston I'm afraid,
has a clear idea of what a precisification of a vague object might be.
should truths about Kilimanjaro be determined by truths about these different objects.

Johnston does not explain why truths about Kilimanjaro should depend on truths about all the different precisifications and although that, by itself, does not show that no such explanation is available, we have independent reason to believe that it isn’t. In the case of supervaluation over precisification of objects instead of words what we get is not a definition of truth in terms of truth in all precisifications but a definition of satisfaction in terms of satisfaction by all precisifications.

Satisfaction w.r.t. precisification: A vague object satisfies an open sentence just in case every acceptable precisification of the object satisfies the same open sentence.

Now, since each of the precisifications of Kilimanjaro is precise, they all satisfy the open sentences:

There is some set that is the set of all and only the atoms that are parts of _____.

_____ is precise.

_____ has no borderline parts.

And since each of the precisifications satisfies these open sentences, by our definition of satisfaction so does Kilimanjaro. But if vagueness is to be explained by a feature of Kilimanjaro itself, namely that it is a vague object, then Kilimanjaro should not satisfy these sentences. Moreover, none of the Ks satisfies the open sentence

_____ is a mountain,
since otherwise we would still have the problem of the many. But then it follows by
the above definition of satisfaction that Kilimanjaro does not satisfy it, i.e. it follows
that Kilimanjaro is not a mountain. And that is not right.

The question why truths about Kilimanjaro should be determined by truths
about all the different precisifications is analogous to a question sometimes asked
about the linguistic view: Why should truth for English be defined in terms of truth
for a class of different languages? The thought behind this question is that the
different precisifications are different languages and although these different
language resemble English in some important respects it is not clear that truth in
English should be defined in terms of truth in these other languages. After all, these
other languages also differ from English in important respects. But this thought is
mistaken. Different acceptable precisifications are not different languages, they are
different models of English. The idea behind supervaluation can be put like this: A
sentence in English is true if and only if it is true in all acceptable models of English.
It is still a non-trivial question why a sentence in a vague language should be
regarded true just in case it is true in all acceptable models of the language, and it is
a still further question why these models should be classical. But, whatever attitude
we take towards these questions we might notice that on the idea of precisification
that Johnston seems to have in mind, no comparable questions arise. The
precisifications in Johnston’s story are not different models, classical or otherwise,
they are just different objects.

94 Vann McGee and Brian McLaughlin discuss this question in an appendix to “Distinction
without a difference” (“Appendix: Answer to a question of Sanford”, *Southern Journal of
3. I don’t think there is any way of applying the notion of precisification to material objects for the purpose of giving truth conditions of complex sentences. However, this alone does not show that supervaluation will not be of help. If the parthood relation is vague, then a predicate ‘P’ that respects this vagueness will also be vague, i.e. there will be objects, say Sparky and Kilimanjaro, such that “(Sparky)P(Kilimanjaro)” will be neither true nor false. Now, someone might suggest that although vagueness is being explained in ontological terms, we focus on language when it comes to describe truth conditions of complex sentences; we precisify the predicate and use supervaluation to describe the truth conditions of complex sentences just the way friends of the linguistic view do. Can we do this? I don’t think there is any straightforward answer to this question. As I mentioned before, we can use supervatuation to define a predicate ‘T’ that resembles a truth predicate in many ways. But assuming that we have the predicate ‘T’ defined in terms of supervaluation over precisifications of the predicate ‘P’, we might ask: According to what conception of truth would ‘T’ actually be a truth predicate of \( L_p \)?

The conception of truth could hardly be a correspondence conception since, in defining the acceptable precisifications, we have explicitly ignored what the language is about and focused our attention just on the language itself.

3. A mixed batch

1. Our task is to explain how the truth values of complex sentences, whether ‘true’, ‘false’ or ‘indeterminate’, derive from the truth values of atomic sentences. We have seen that the three valued truth tables are not helpful, and although the
conclusion is not as straightforward in the case of supervaluation, there seems to be no clear way of defining a notion of a precisification that respects the intention that the vagueness is ontological while lending itself to a supervaluation definition of truth or satisfaction. How can we then explain how truth values of complex sentences derive from truth values of atomic sentences?

Let's attend to certain features of how the predicate 'P' is introduced, i.e. how we get from \( L \) to \( L_p \). First, the extension and the anti-extension of 'P' are given by the (ND) schema and the (ND) schema treats a negation of an atomic sentence about parthood as itself atomic. In other words, the truth of a sentence of the form "\( \neg aPb \)" is determined in a way analogous to the way in which the truth of a sentence of the form "\( aPb \)" is determined. This means that negation should not be treated as a one place logical connective. Second, truth of an atomic sentence about parthood, whether it has the negation sign or not, amounts to provability; i.e. such a sentence is true just in case it is derivable in the sense of the (ND) schema. In this respect, the notion of truth is similar to the intuitionistic notion of truth. But similarities with the intuitionistic notion do not cut deep since the notion of derivability in the case of the (ND) schema is more robust than the intuitionistic notion. Third, there are general sentences that are true independently of a derivation of their instances. These general sentences are treated as meaning postulates. This last condition opens up the possibility that truth is not compositional, a disjunction, for instance, might be true (derivable) without any of the disjuncts being true (derivable) simply by being an instance of a disjunctive meaning postulate.
2. We assume that $L$ contains sentences describing the primitive facts on which facts about parthood depend. Then we add to this language some general principles about parthood, for instance the principle that parthood is transitive, that if an atom is caught up in the physiological processes that make up a living organism then the atom is part of that organism, and so on. These are the general principles that figure in the (ND) schema. We call the resulting language $L_0$. But no singular sentences about parthood are true in $L_0$, i.e. the extension and the anti-extension of 'P' are empty. At the next stage, $L_1$, sentences about parthood that are derivable from the primitive facts along with the general principles about parthood are determined, for instance at $L_1$ the sentence “Tibbles’ fur is part of Tibbles” is true. At the next stage, $L_2$, this fact along with the principle about the transitivity of parthood determines (H).

\[
\text{(H)} \quad \text{If Harry is part of Tibbles’ fur then Harry is part of Tibbles.}
\]

We say that some facts about parthood are immediately settled by the primitive facts, others are settled by the primitive facts plus facts about parthood that are immediately settled, and so on. The set of true sentences of $L$ we call $\Pi$. The set of true sentences of $L_0$ we call $\Pi_0$ and so on. There will be certain sentences of the form "$aPb$" that are determined true or false when the premisses are restricted to the set $\Pi_0$. This expands the set of true sentences, we call the expanded set $\Pi_1$. The monotonicity of the (ND) schema guarantees that the new set $\Pi_1$ does not alter the truth value of any sentence that has already been classified as either true or false, i.e. $\Pi_0 \subseteq \Pi_1$. Once we have $\Pi_1$ instead of $\Pi_0$ there are more resources to make sentences about parthood either true or false, so that we reach a new set of true sentences, $\Pi_2$. We can imagine repeating the process indefinitely. If at some stage we reach a level
where $\Pi_n = \Pi_{n+1}$ we call $\Pi_n$ a fixed point. We call the set of sentences true at a fixed point $\Pi_f$ and this is the set of true sentences of $L_p$. At a fixed point the predicate ‘$P$’ has an extension and an anti-extension but if parthood is vague then the one is not the compliment of the other.

This framework gives us two notions of truth. A relative notion of truth at a stage, and an absolute notion of truth at a fixed point. The intuitive notion of truth in English corresponds to the absolute notion of truth, i.e. ‘true in English’ will correspond to ‘true at a fixed point’. The question whether the framework offers a satisfying account of truth for a language depends on two things. First, whether the process ever reaches a fixed point, and second, what properties the fixed point has. The first question is easily answered. From the monotonicity of the (ND) schema and the assumption that sentences about parthood form a set, it follows that the process does have a fixed point. The second question is more difficult, I am not even sure that it has any clear answer.

3. We assume that the distribution of truth values over sentences not containing the parthood predicate is as we would intuitively expect. We should notice, however, that the underlying logic does not have the law of excluded middle. The task now is to describe the truth conditions of complex sentences composed of sentences that are indeterminate. We have already seen how one such sentences is treated, namely

95 See Kripke's “Outline of a theory of Truth”, p. 704. That there is a fixed point also follows if we treat the (ND) schema as a normal default theory, i.e. it is a theorem that any normal default theory has an extension.
If Harry is part of Tibbles' fur then Harry is part of Tibbles.

What allows us to get the truth value of (H) right is that at the bottom level we adopt the required meaning postulates; in the case of (H) the principle that parthood is transitive. Once it is determined that Tibbles' fur is part of Tibbles, the transitivity of parthood allows us to derive (H). But what about

If Harry is part of Tibbles then Harry is part of Tibbles.

Here, the transitivity of parthood is not going to help. What we need is a definition of ‘→’.

Intuitively, any sentence of the form “p → p” ought to be true, no matter what the truth value of p is. This makes two things clear, first that ‘→’ can not be defined in terms of the standard truth table, or the three valued tables, second, that “p → p” is not equivalent to “p ∨ ¬p” since we allow that the latter be indeterminate when p is indeterminate. But given our notion of truth as provability, there is an intuitive way of defining ‘→’. We say that a sentence of the form ‘p → q’ is true just in case q is derivable from p, in symbols

\[ \vdash p \rightarrow q \text{ iff } p \vdash q. \]

More specifically, we can say that a sentence of the form “p → q” is true at a fixed point just in case q follows from the union of \( \Pi_f \) and \( \{p\} \), in symbols

\[ \Pi_f, p \vdash q \text{ iff } \Pi_f \cup \{p\} \vdash q, \]

where \( \Pi_f \) is the set of sentences true at a fixed point.

4. As I mentioned above a negation of an atomic sentence of the form “aPb” is itself an atomic sentence and this complicates things somewhat. The problem is that
when negation is applied to an atomic sentence about parthood the complexity of
the sentence does not increase but when negation is applied to a complex sentence
the complexity of the sentence does increase. The question that we need to answer
is: What are the logical properties of negation? From the logic governing the (ND)
schema it follows that negation has some familiar properties, for instance that a
sentence and its negation can never be true at the same time,
\[
\text{not } \vdash p \land \neg p.
\]
But what shall we say about sentences of the form "\(\neg(p \lor q)\)" or "\(\neg(p \land q)\)". One way
to give the truth conditions of negations of complex sentences is to use De Morgan’s
law and give the truth conditions in terms of negated atomic sentences.
\[
\neg(p \lor q) \leftrightarrow \neg p \land \neg q,
\]
\[
\neg(p \land q) \leftrightarrow \neg p \lor \neg q.
\]
The question then becomes how to define conjunction and disjunction.

5. Before I consider how we might define conjunction and disjunction I want to
consider one more example of a penumbral truth about parthood. Supposing that A
is a place on the border of the United States and Canada, the sentence
\[
(2) \quad \text{Either } A \text{ is part of the United States or } A \text{ is part of Canada}
\]
should be true. But can we make sense of this sentence’s being true in the present
framework? As long as we can assume, at the bottom level, the general principles
that there is no unclaimed space between adjacent countries the above sentence
should come out true. In other words, as long as we can assume at the bottom level
the following principle:
(B) If A is a place on the border of two countries then it is part of one of those countries,
then sentence (2) will come out true. It follows from (B) and the assumption that A is on the border of the United States and Canada. This has some interesting consequences. Since (2) is derivable at some stage, it is true, and yet neither disjunct is derivable at any stage, i.e. neither disjunct is true.

This situation is somewhat peculiar. The feature that is responsible for the provability of a disjunction without either disjunct being provable, is that the general principle that is involved is about entities that depend on each other. Which places belong to Canada depends partially on which places belong to the States, and vice versa. By contrast, living beings, such as cats and people, are not like this, i.e. what parts the cat Tibbles has does not depend on what parts some other entity, which is not part of the cat, has. Mountains and lakes are, perhaps, somewhere in-between as are parts of animals.

6. One explanation of this interdependence of countries is to consider countries as human constructs and principle (B) as a defining principle for countries.96 If we suppose that countries are human constructs, what our definition of disjunction will look like depends, in part, on what principles govern such constructs. Friends of

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96 Interdependence of objects can not, in general, be explained by reference to human constructs. Among interdependent objects are parts of animals; Tibbles' head and nect are distinct objects but wehre one ends depends on where the other begins, and I don't want to explain this interdependence by saying the the head and the neck are human constructs. The reason why I mention the thesis that countries are human constructs is simply that it lends itself well to my present purposes, in particular, assuming that countries are human constructs we can, in an intuitive way, give the truth conditions of complex sentences about countries by supervaluation in a way analogous to the way friends of the linguistic theory give truth conditions of such sentences.
constructivism in mathematics usually ascribe to intuitionistic logic, according to which a disjunction is true just in case at least one of the disjuncts is true. But it is not clear that what is appropriate in the context of constructivism in mathematics is also appropriate for other kinds of constructed entities such as countries. All that is needed in order for there to be a disjunction that is true while neither disjunct is true is that there be a principle like (B) above that is among the defining principles for countries. And assuming that countries are human constructs I don’t see why (B) might not be such a principle. More generally, if the constructs are not constructed one at a time but in pairs or bunches, then I don’t see any reason why this should not give rise to a disjunction being true about a pair or a bunch of objects without there being some one thing of which it is true. And if that is right, then intuitionistic logic will not do justice to the logical properties of a language that is about such objects.

However, although supervaluation over precisifications of material objects is doubtful, there is a way in which supervaluation might be used to account for truths about constructs such as countries. The idea would be that the constructed objects are incomplete in some respects; their properties depend on people’s decisions and deeds but these might not fix, for instance, whether a certain place belongs to the United States or to Canada, although they might fix that it belongs to one or the other. What is true about a construct would then be defined in terms of what would be true on any way of completing the construction. The slogan might be: Vagueness is ontological indecision. This might open the possibility to use supervaluation to define satisfaction, but the appropriate notion of supervaluation will be different.
from the linguistic notion since it will not be defined over a class of precisifications but over a class of completions.

In the case of supervaluation over completions we define satisfaction in the following way:

Satisfaction w.r.t. completion: An object \( O \) satisfies an open sentence \( \phi \) just in case every acceptable completion of \( O \) would satisfy \( \phi \).

In this definition '\( \phi \)' is not restricted to what we might call atomic contexts, i.e. '\( \phi \)' is not restricted to contexts such as

___ is part of Canada,

but could also stand for contexts such as

___ is part of Canada or ___ is not part of Canada,

___ is part of Canada or ___ is part of the United States.

Satisfaction in a model \( M \) of the language \( L_p \) is defined with respect to a class of models whose domain is different from the domain of \( M \). We call them 'acceptable completions of \( M \). If \( M_c \) is an acceptable completion of \( M \) then for any incomplete object \( O \) in \( M \) there is a different object \( O_c \) in \( M_c \) such that \( O_c \) is an acceptable completion of \( O \). Now we can think of the acceptable completions as possible worlds and the relation 'is an acceptable completion of' as analogous to the counterpart or trans world identity relation in modal logic. We can then interpret the '\( \Box \)' in modal logic as 'it is determinate that'. We say that it is determinate that \( a \) is part of \( b \) just in case it is true in all acceptable completions that \( a \) is part of the completion of \( b \). But notice that the accessibility relation, i.e. the relation which determines which worlds are accessible from which worlds is quite restrictive. Two
worlds (i.e. two completions) may be accessible from the actual world but they won’t be accessible from one another.

7. Rather than consider countries as incomplete human constructs, we might concede a little to the linguistic view and accept the claim that names such as ‘United States’ and ‘Canada’ are vague. Now, if the vagueness of the sentence “Place A is part of Canada” is explained by semantic indecision, truth for complex sentences, with this sentence as a constituent, will be given in terms of supervaluation over acceptable precisifications of the name ‘Canada’.

But can we concede to the linguistic view that, in some cases, vagueness is due to semantic indecision while maintaining that, in other cases, vagueness is due to the way objects themselves are? The answer depends on whether we can distinguish these cases in some systematic way. I think we can. There is an important disanalogy between names of, say, countries on the one hand, and proper names of people and other living beings, on the other. Notice that what the reference of the name ‘United States’ is depends, in part, on what the reference of the name ‘Canada’ is, but in the case of proper names of people and cats we don’t have such interdependence.

8. The situation is now like this. Indeterminacy of sentences such as

Sparky is part of Kilimanjaro

Specky is part of Toni Morrison
has to be explained in ontological terms; these sentences are vague because Kilimanjaro and Toni Morrison are vague objects. But there are other indeterminate sentences, such as

\[ \text{Place A is part of Canada,} \]

where we have a choice between an ontological explanation and a linguistic explanation. According to either explanation the source of the vagueness is indecision, ontological indecision if the explanation is ontological, semantic indecision if the explanation is linguistic. And either explanation lends itself to supervaluation.

In an evaluation, whether it is a precisification or a completion, we can define disjunction and conjunction by the three valued truth tables and ‘→’ by “\( \vDash p \rightarrow q \) iff \( p \vDash q \)”, and then use supervaluation to define the corresponding connectives for \( L_p \). But what about the quantifiers? We can define the existential quantifier in terms of disjunction, but defining universal quantification will not be as straightforward. Since in our framework truth amounts to provability, a universal generalization ought to be true just in case it is provable. As long as we are dealing with only finite domain this is fine; we can name all the members of the domain and then say that a universal generalization is provable just in case every instance of it is provable. But once we allow that the domain of discourse is infinite the simple equivalence between the provability of a universal generalization and the provability of all its instances does not hold. The problem is that we might run out of names so that although we prove all the instances we do not thereby prove the universal generalization itself.
To make the problem clearer, suppose we have an infinite domain and three names, \( a, b \) and \( c \). Now, we might be able to prove "\( Fa \land Fb \landFc \)" but that is no prove of "\( \forall x \ Fx \)". Even having infinitely many names does not suffice for a proof of all instances of a universal generalization being a proof of the generalization itself.

Suppose we run a hotel with infinitely many rooms and that we have one guest in each room. Imagine that the we have an infinite list of names, 'Tom', 'Dick', 'Harry' ... such that substituting each of these for 'x' in "\( x \) has ordered breakfast" makes a true sentence. Does this mean that the sentence "all the guests ordered breakfast" is true? No, for instance the guest in room 13 might not have a name at all. What is needed in order for a prove of all instances of a generalization amounting to a proof of the generalization itself is that the model is covered, i.e. that every object in the domain has a name.

The assumption that every object has a name is not a plausible assumption for a natural language if the domain is very large, let alone infinite. But at this point we can turn to mathematics. We can imagine a mathematical function that assigns a name to every object in the domain. These names should not be understood as names in a natural language, since, first, we will not be able to know what the reference of each name is and, second, each name is by definition precise. Let \( n_1, n_2, n_3 \) ... be such names. Now we can imagine adding the infinite list of names, \( n_1, n_2, n_3 \) ... , to \( L_p \), we call the resulting language \( L_{N'} \) and then define a universal generalization in the following way: A universal generalization is true in \( L_p \) iff every instance of it is true in \( L_{N'} \), or in symbols:

\[
\models_{L_p} \forall x \ Fx \iff \models_{L_{N'}} F_{n_1} \land F_{n_2} \land F_{n_3} \ldots
\]
were "Fn₁ \land Fn₂ \land Fn₃ ..." is an infinite conjunction where each name on the list n₁, n₂, n₃ ... appears following an instance of the predicate symbol 'F'.

9. Whether the names 'United States' and 'Canada' are vague names referring to precise objects or precise names referring to vague objects, truth conditions for complex sentences can be given by supervaluation over evaluations. In either case in an evaluation the logical connectives are defined in the following way:

\[
\begin{align*}
 p \lor q & \iff p \text{ or } q. \\
 p \land q & \iff p \text{ and } q. \\
 \models p \rightarrow q & \iff p \models q.
\end{align*}
\]

Negation of an atomic sentence will be treated as an atomic sentence and negation of disjunction or conjunction will be defined by De Morgan's law and the above definitions of disjunction and conjunction. But the acceptable evaluations will not be classical models, since in any such model there will be sentences that are neither true nor false, for instance the sentence "Sparky is part of Kilimanjaro" or "Specky is part of Toni Morrison" where Specky is a borderline case of an atom that is part of Toni Morrison. This means that the law of excluded middle will not be valid, i.e. \( L_p \) will not be classical.
Concluding Remarks

On the previous pages I have offered an account of the ontological view, I have distinguished it from two alternatives, the epistemic view and the linguistic view, and I have argued that the ontological view has some advantages over these alternatives. But the ontological view does not come for free and deciding whether to accept it or not will be a matter of balancing the advantages against the cost. So, what are the advantages and what is the cost?

The main advantage of the ontological view is that it offers a solution to the problem of the many without running into the problem of vague reference and the problem of \textit{de re} beliefs; i.e. the following three propositions are true according to the ontological view:

1. There are ordinary material objects.
2. Ordinary names in our language have determinate reference.
3. We can have \textit{de re} beliefs about ordinary material objects.
On the alternative views at least one of the above propositions is false. Along the way I have mentioned some cost:

(4) We have to give up the law of excluded middle. (Chapter Two, section 2)

(5) We have to accept kinds. (Chapter Four, section 2)

(6) We have to accept the thesis that objects have natures. (Chapter Four, section 2)

(7) We have to give non-standard semantics for the logical connectives. (Chapter Six, section 3).

In addition to these costs, one should bear in mind that the ontological view, as I have laid it out, offers only a limited solution to problems of vagueness; it has little or nothing to say about the sorites paradox and the Ship of Theseus. Now two questions arise: First, should we accept a limited solution to problems of vagueness? Second, is the cost of the ontological view too high?

I shall begin addressing the first question. I accept, as a general maxim, that similar problems should have similar solution. For instance, insofar as the liar paradox and Russell's paradox are similar problems they should receive similar solutions. But, as I mentioned in the introduction, I don't think that all problems of vagueness have more in common than the name, in particular, I don't think that the problem of the many, on the one hand, and the sorites paradox and the Ship of Theseus, on the other, are similar problems. If this is right then the fact that the ontological view offers only a limited solution doesn't count against it.

But is the cost of the ontological view too high? Any solution to the problem of the many that I know of is costly, and there isn't going to be any neutral
standpoint from which we can balance the cost against the benefits. So, rather than asking which view is least costly, we should ask whether there is any attractive standpoint from which the benefits of the ontological view are worth the cost. It is obvious from the above list of what the cost of the ontological view is that such a standpoint is unattractive by, for instance, Quine’s lights, if only for the fact that it is revisionary about logic. But I think there is an attractive standpoint from which the benefits of the ontological view outrun the cost; it is a broadly Aristotelian standpoint and it is non-revisionary about ordinary objects.
Bibliography


