Adaptive Modeling of GPS Receiver Clock for Integrity Monitoring During Precision Approaches

by

Sean G. Bednarz

Bachelor of Science in Aerospace Engineering
University of Notre Dame, May 2002

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Abstract

The FAA is developing the Local Area Augmentation System (LAAS) to replace ground-based navigation aids as the primary means of aircraft precision approaches. Stringent requirements on the integrity of the system have caused significant delays and raised questions about the adopted approach to integrity monitoring. The system architecture must be capable of detecting when the position estimation error exceeds the alert limit, with a probability of missed detection less than $10^{-7}$ for Category I approaches. This thesis offers a new approach to integrity monitoring that exploits an overlooked resource: the receiver clock.

Conventional GPS positioning requires the estimation of four parameters. In addition to three position components, the receiver clock offset from GPS time must be found. Clock-aided positioning involves removing this fourth unknown by using a stable atomic frequency standard to keep time. This not only frees up one extra pseudorange, but it changes the structure of the problem from four-parameter to three-parameter estimation. The result is a drastic improvement in the vertical dilution of precision (VDOP), which leads to enhanced accuracy of vertical position estimates. Laboratory data shows accuracy improvements ranging from 34% to 44%, while field tests performed in a moving vehicle show 28% to 49% improvement using clock-aided positioning.

Clock-aiding can provide an even greater benefit in integrity monitoring. The error in a clock bias estimate obtained from pseudorange measurements is a good predictor of the error in the corresponding vertical position estimate. Thus, we can estimate four parameters as before, and exploit the knowledge that we know part of the answer (clock bias) to serve as a quality check on the position estimate. Using an adaptive clock model based on precise carrier phase measurements, the clock bias error can be estimated and a corresponding Vertical Protection Level (VPL) established. This approach automatically adjusts for the measurement quality, accounting for atmospheric disturbances, changing multipath conditions and other error sources. This thesis establishes the potential of clock-aided integrity monitoring, showing that a VPL based on clock bias prediction error predicts error peaks, adjusts to changing conditions and bounds the vertical position error for a modest data set.
Acknowledgements

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Lincoln Laboratory gave me the opportunity to work with a great group of people in the Air Traffic Control Systems group. In particular, Dr. Pratap Misra offered guidance along every step of the way. What is presented here is an extension of his novel ideas on GPS integrity monitoring. Dr. Timothy Hall also lent his expertise to the project on many occasions. My work at Lincoln was funded by a grant from the Federal Aviation Administration.

I would also like to thank my faculty advisor, Dr. Jonathan How, for his direction in the writing process.

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Finally, I thank my girlfriend of seven years, Nicole, for her patience and support while I spent such a great deal of time writing this thesis.
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<th>Description</th>
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<tbody>
<tr>
<td>APV</td>
<td>Approach with Vertical Guidance</td>
</tr>
<tr>
<td>C/A</td>
<td>Coarse/Acquisition (code)</td>
</tr>
<tr>
<td>CORS</td>
<td>Continuously Operating Reference Stations</td>
</tr>
<tr>
<td>DGPS</td>
<td>Differential GPS</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution of Precision</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth-centered, Earth-fixed</td>
</tr>
<tr>
<td>EDOP</td>
<td>East Dilution of Precision</td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aviation Administration</td>
</tr>
<tr>
<td>GLS</td>
<td>GNSS Landing System</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HDOP</td>
<td>Horizontal Dilution of Precision</td>
</tr>
<tr>
<td>HMI</td>
<td>Hazardously Misleading Information</td>
</tr>
<tr>
<td>ILS</td>
<td>Instrument Landing System</td>
</tr>
<tr>
<td>LAAS</td>
<td>Local Area Augmentation System</td>
</tr>
<tr>
<td>LAL</td>
<td>Lateral Alert Limit</td>
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<tr>
<td>LPL</td>
<td>Lateral Protection Level</td>
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<tr>
<td>NDOP</td>
<td>North Dilution of Precision</td>
</tr>
<tr>
<td>NGS</td>
<td>National Geodetic Survey</td>
</tr>
<tr>
<td>NOAA</td>
<td>National Oceanic and Atmospheric Administration</td>
</tr>
<tr>
<td>NSE</td>
<td>Navigation System Error</td>
</tr>
<tr>
<td>PDOP</td>
<td>Position Dilution of Precision</td>
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<tr>
<td>RAIM</td>
<td>Receiver Autonomous Integrity Monitoring</td>
</tr>
<tr>
<td>RMS</td>
<td>Root-mean-square</td>
</tr>
<tr>
<td>SSE</td>
<td>Sum of Squared Errors</td>
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<tr>
<td>TDOP</td>
<td>Time Dilution of Precision</td>
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<tr>
<td>UPS</td>
<td>Uninterrupted Power Source</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>URE</td>
<td>User Range Error</td>
</tr>
<tr>
<td>VAL</td>
<td>Vertical Alert Limit</td>
</tr>
<tr>
<td>VDOP</td>
<td>Vertical Dilution of Precision</td>
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<tr>
<td>VPL</td>
<td>Vertical Protection Level</td>
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<tr>
<td>WAAS</td>
<td>Wide Area Augmentation System</td>
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Chapter 1

Introduction

1.1 Problem Background and Motivation

The Global Positioning System (GPS) was approved in 1994 as a primary means of navigation for civil aircraft during phases of flight from en route through non-precision approach. With the elimination of Selective Availability and the introduction of the Wide Area Augmentation System (WAAS), GPS can now be used as the sole means of navigation for en route flight through Approach with Vertical Guidance (APV). Once the Local Area Augmentation System (LAAS) is introduced, the system will provide service for precision approaches as well. While the purpose of LAAS is to serve as the navigation system for CAT I through CAT III approaches and landing, it still remains to be seen exactly what level of service it will be able to offer.

1.1.1 WAAS and LAAS

GPS is a satellite-based radionavigation system that provides position, velocity and time information to users throughout the world. Although it was developed by the Department of Defense for the U.S. military, it has found utility in a variety of applications. The Federal Aviation Administration (FAA) recognized the value of a global navigation system for civil aviation and began developing two augmentation systems to provide the accuracy required for each phase of flight. The Wide Area Augmentation System (WAAS) has already been declared operational and provides en route, terminal area and
APV instrument approach service [20]. It is a space-based system with nationwide coverage, allowing for both lateral and vertical guidance throughout these flight phases. The Local Area Augmentation System (LAAS) is a ground-based system that is still in development. It will provide even greater accuracy than WAAS and is scheduled to serve all categories of precision approach and landing.

These systems will eventually replace ground-based navigation aids, which have limited coverage and are very costly. The improved en route navigation capabilities provided by WAAS will allow for more direct flight routes and shorter flight times [17]. LAAS will eliminate the need for expensive instrument landing systems (ILSs), with one ground station providing precision approach capability to all runways at an airport.

Despite the obvious potential benefits, there are some obstacles that must be overcome before WAAS and LAAS can replace current navigation aids. The integrity and availability requirements imposed on these systems by the FAA are very difficult to meet. Any architecture must provide availability of service greater than 99.9% for all precision approaches, while meeting stringent integrity standards (probability of an integrity loss no greater than $10^{-7}$ per CAT I approach). WAAS has been unable to meet the requirements for CAT I approaches, so new categories with looser requirements have been defined. Integrity monitoring is proving difficult for LAAS as well, causing significant delays in the program [7].

1.1.2 Clock-aiding

Most GPS receivers utilize an inexpensive quartz oscillator to keep time. In order to compensate for the clock bias between GPS time and receiver time, an extra pseudorange measurement is needed to solve for the additional unknown. It is widely believed that the only benefit of a perfect receiver clock—one that keeps GPS time—is that it eliminates the need to waste one pseudorange measurement. The truth is that utilizing a perfect clock to measure ranges rather than pseudoranges changes the structure of the problem from four-parameter estimation to three-parameter estimation. The improved structure allows for increased accuracy in position estimation as well as increased availability.
Alternatively, the concept of clock-aiding can be extended to tackle the more difficult problem of integrity monitoring. The error in a clock bias estimate obtained using conventional four-parameter estimation is a good predictor of the error in the corresponding vertical position estimate. This knowledge can serve as the basis for an integrity monitoring algorithm for LAAS precision approaches, where requirements on the vertical accuracy are the most difficult to meet. While there may be no such thing as a perfect clock, there is still much benefit to be gained from highly stable frequency standards, such as rubidium oscillators.

1.2 Prior Research

Misra [15] first proposed the idea that the vertical position accuracy of GPS estimates could be improved by taking advantage of the frequency stability of atomic clocks. In that paper, it is shown that while the horizontal dilution of precision \( HDOP \) is essentially unchanged, the vertical dilution of precision \( VDOP \) can be drastically reduced by using clock-aided positioning. This is said to be a result of the strong correlation between the vertical position error and the clock bias error. The paper also suggests the use of a quadratic model of the receiver clock bias based on recent estimates.

Kline [11] expands on this principle by using the precise carrier phase measurements to develop an improved clock model. His paper shows the reduction in \( VDOP \) that results from clock-aided positioning and compares the vertical positioning accuracy of conventional and clock-aided DGPS estimates during flight tests conducted as part of the study.

Zhang [24] explores the use of an adaptive low-pass filter to estimate the bias of rubidium clocks. He shows that clock-aided positioning offers better accuracy than conventional positioning through static tests in both standalone and differential GPS modes. The use of a barometer and a gyro as additional augmentation devices is also examined in field tests using a moving car.
1.3 Thesis Overview

LAAS will certainly provide the position accuracy necessary to meet the requirements that have been set for Category I precision approaches. The challenge lies in recognizing when the error in a particular estimate is unacceptable and alerting the pilot. This is known as integrity monitoring, and it is the most challenging problem facing LAAS developers today. It has been shown that clock-aided positioning can reduce the vertical position error in GPS estimates, improving availability of service. The more important benefit of clock-aiding is its utility in integrity monitoring. This thesis will provide the background necessary to understand the important concepts, as well as the theoretical basis and empirical data that support these conclusions.

The Global Positioning System is introduced in Chapter 2. The purpose of this thesis is to introduce an integrity monitoring algorithm, and only the relevant topics in GPS positioning are discussed. There is a wealth of information on GPS available, and the author directs the reader to the textbook by Misra and Enge [17] for a comprehensive description. Chapter 2 of this thesis gives a brief overview of the system architecture before positioning and error mitigation techniques are detailed. Since single-epoch pseudorange positioning is used in civil aviation, it is the focus of the chapter, and carrier phase positioning is not discussed. Finally, the effect of receiver clocks on GPS positioning is also examined.

Atomic oscillators are at the heart of clock augmentation, and the relevant theory is covered in Chapter 3. The basic components of an atomic clock are described, as well as several common types of oscillators. The cost, size and performance details of these oscillators are discussed and measures of frequency stability defined.

Chapter 4 gives a description of integrity monitoring. The problem is introduced, and some of the most common approaches are explained. Included are receiver autonomous integrity monitoring (RAIM) algorithms and the current LAAS architecture. The integrity requirements for aircraft precision approaches are listed there as well.
Once the necessary background has been provided, the implementation of positioning and integrity monitoring using clock augmentation is explained. Chapter 5 begins by discussing the technical details of clock-aided positioning and integrity monitoring and then offers an adaptive clock model based on both code and carrier phase measurements.

In order to verify the theoretical assertions regarding clock-aiding, data collection was performed in both the benign environment of a laboratory and the hostile environment of a moving vehicle. Chapter 6 describes the experimental setup used for each setting, as well as the empirical results. The analysis is based on the implementation details given in the previous chapter.

Finally, the thesis ends with conclusions based on theoretical and empirical evidence. The potential benefits of clock augmentation, as well as the challenges involved are reviewed. Remaining issues and future research efforts are addressed as well.
Chapter 2

Global Positioning System

2.1 Introduction

The Global Positioning System (GPS) is a space-based radionavigation service that was developed by the United States Department of Defense. It was declared operational in 1995 and has benefited a diverse and ever expanding group of users around the world. While it was developed to meet the needs of the US military, it has become an essential tool for surveyors, civil aviators, scientists and emergency service providers, to name just a few. Recreational use is also on the rise, as PDAs and cell phones now double as GPS receivers. With recent renovations completed, such as the Wide Area Augmentation System (WAAS), and future improvements on the way, the increased accuracy and availability of GPS will certainly expand the list of applications.

2.2 System Architecture

The baseline GPS satellite constellation consists of twenty-four satellites orbiting the earth in six nearly-circular orbital planes, each with an inclination of 55° and a semi-major axis of 26,559.8 km. Four satellites are unevenly distributed in each plane. With this constellation, most users will have six to eight satellites in view at any given time.
Each satellite broadcasts navigation and ranging data using spread spectrum signaling on each of two different radio frequencies (L1 and L2). The navigation data contains information that is necessary to calculate the position of each satellite. The coarse/acquisition (C/A) ranging code is a binary sequence unique to each satellite that allows the user to estimate the transit time of the signal and thus the range to the satellite. Once the navigation and ranging data are available, a position estimate can be made.

2.3 Positioning

There are two signals available for use in position estimation. The code phase measurements provide course but unambiguous estimates of the ranges, called pseudoranges, to each satellite in view. Since the C/A signal structure is known, the code can be replicated by the receiver and the time shift necessary to align it with the received signal measured. This provides instantaneous range estimates, but the precision is not sufficient for some applications (meter-level).

Carrier phase measurements make precision estimates possible by comparing the phase of the received signal with the phase of the replicated signal. Since the carrier phase can be measured to within a tiny fraction of a cycle, the precision of the position estimates are generally much better than meter-level. In fact, post-processing of relative positioning data can give millimeter-level precision. There is a difficulty that arises using carrier phase positioning, however. The carrier signal from a satellite consists of an integer number of cycles plus some fractional cycle. Since only the fractional cycle can be measured, there is an integer ambiguity that makes it impossible to determine the range without additional information. Integer ambiguity resolution is not an easy problem. There are several methods available, but each requires the collection of carrier data over some time interval before the integers can be fixed with any degree of certainty. Even then, it is difficult to know if the correct integers have been obtained, and even a momentary loss of lock can require the process to be restarted. Pseudorange positioning will be the focus of this chapter. The equations shown are based on the text by Misra and Enge [17].
2.3.1 Pseudorange Errors and Error Mitigation

The pseudorange measurements obtained using the C/A code contain errors from several sources. While GPS satellites use very stable atomic clocks to keep time, each satellite clock still has some offset from GPS time. This affects the apparent transit time of the signal and, consequently, the pseudorange observation. Fortunately, this error can be accounted for using parameters broadcast by the satellite in a quadratic model of the clock error. The receiver clock also introduces a bias into the pseudorange measurement, but this bias is common to all pseudoranges and can be removed by treating it as an additional unknown in the equations.

There are propagation delays due to refraction of the GPS signal within the earth’s atmosphere that also contribute to the error. Ionospheric and tropospheric effects can be modeled and removed, although not perfectly. There will generally be some residual atmospheric error due to mis-modeling or atmospheric disturbances. Using a dual frequency receiver or picking up pseudorange corrections from a nearby reference station are two ways to lessen the residual error.

Multipath reflections are another significant source of error. These occur when a GPS signal reaches the receiver antenna through more than one path. Because the signals were chosen to have autocorrelation functions with sharp peaks at zero and little correlation for nonzero shifts, many of these reflections are mitigated [17]. Another error source is receiver noise. This is the term given to measurement noise introduced by the antenna, cables, receiver or other radio frequency sources broadcasting in the L1 or L2 band.

Taking all of these errors into consideration, each pseudorange \( \rho^{(k)} \) observation can be written as

\[
\rho^{(k)} = r^{(k)} + c(\delta t - \delta t^{(k)}) + I^{(k)} + T^{(k)} + \epsilon^{(k)},
\]

(2.1)
where $r^{(k)}$ is the actual range from the receiver to the $k^{th}$ satellite, $c(\delta t - \delta^{(k)})$ is the difference in clock offsets between the receiver and satellite clocks multiplied by the speed of light, $I^{(k)}$ and $T^{(k)}$ are the signal delays caused by the ionosphere and troposphere, respectively, and $\epsilon^{(k)}$ represents the combined effect of modeling errors and un-modeled error sources.

Fortunately, the most significant error sources are both spatially and temporally correlated. Atmospheric delays, satellite clock error and ephemeris error all change slowly over time and are similar for users in the same vicinity [17]. This is the basis for differential GPS (DGPS). A reference station at a surveyed location can determine precise ranges to each satellite in the constellation at any given time. If the reference receiver makes pseudorange observations, these can be compared with the known ranges to determine the pseudorange errors. These are broadcast to GPS users in the area who can apply them as corrections to their own measurements.

DGPS can offer drastic reduction in pseudorange error compared to standalone positioning, which relies on empirical models to characterize the effects of the atmosphere and satellite clock. However, as the baseline between the reference and rover receiver increases, spatial decorrelation of the atmospheric effects reduces the benefit that this service provides. With a baseline of 10 km, the 2 - 5 meter pseudorange error that is typical of standalone receivers will generally be reduced to less than 1 meter using DGPS [14].

Since the multipath and receiver noise are not correlated between the reference and user receivers, these effects cannot be eliminated using DGPS. Instead, they introduce additional noise to the user signal through the broadcast corrections. Much of this noise can be eliminated by smoothing the pseudoranges using the very precise carrier phase measurements. Although the carrier phase measurements are ambiguous, their change from one epoch to the next can easily be tracked. These delta pseudoranges are measured with centimeter precision (Figure 2.1). They can be used to improve the course code
phase pseudoranges through the use of a recursive filter. A filter of length \( N \) epochs, is set up as follows

\[
\bar{\rho}(t_i) = \frac{1}{N} \rho(t_i) + \frac{N-1}{N} [\rho(t_{i-1}) + (\Phi(t_i) - \Phi(t_{i-1}))],
\]

where \( \bar{\rho}(t_i) \) is the smoothed pseudorange at time \( t_i \) based on a weighted combination of the current pseudorange measurement, \( \rho(t_i) \), the carrier phase change, \( (\Phi(t_i) - \Phi(t_{i-1})) \), and the previous filter estimate, \( \bar{\rho}(t_{i-1}) \), all of which have units of length.

Figure 2.1 Carrier and code delta pseudoranges with trend removed
2.3.2 Positioning With Pseudoranges

The unambiguous nature of the code phase observations allows a standalone receiver to immediately estimate position using data from a single epoch. Including all possible error sources, the pseudorange to each satellite as measured by the GPS receiver can be written in the form of Eq. (2.1). After applying the appropriate corrections to remove atmospheric effects and the satellite clock offset, the corrected pseudorange, \( \rho_c^{(k)} \), takes the form

\[
\rho_c^{(k)} = r^{(k)} + b + \tilde{e}^{(k)},
\]

(2.3)

where \( \tilde{e}^{(k)} \) represents the residual errors, and the receiver clock bias term \( c\delta t \) has been replaced by \( b \) to simplify the notation. In order for the user position to show up explicitly in this expression, a substitution is made for the range. Thus,

\[
\rho_c^{(k)} = \|x^{(k)} - x\| + b + \tilde{e}^{(k)},
\]

(2.4)

where \( x \) is the user position at the current receiver time, and \( x^{(k)} \) is the position of the \( k^{th} \) satellite at the time of signal transmission. Both three-dimensional position vectors are referenced to an earth-centered, earth-fixed (ECEF) coordinate frame defined at the receiver time. Each satellite in view contributes one equation in terms of the unknown position vector and clock bias. Thus, for \( K \) satellites in view, there are \( K \) equations and only four unknowns—three position components and clock bias. However, this set of equations is non-linear in terms of the position. The approach that is commonly used is to linearize the equations about an initial position estimate and iterate until the corrections are small [17].

Given initial estimates \( x_0 \) and \( b_0 \), the corresponding pseudorange is

\[
\rho_0^{(k)} = \|x^{(k)} - x_0\| + b_0.
\]

(2.5)
The difference between the measured and estimated pseudoranges is then

\[
\delta \rho^{(k)} = \rho^{(k)} - \rho^{(k)}_0
\]

\[
= -1^{(k)} \cdot \delta x + \delta b + \varepsilon^{(k)}
\]

where \(1^{(k)}\) is a unit line-of-sight vector from the receiver antenna to the \(k\)th satellite, and \(\delta x\) and \(\delta b\) are corrections to the initial position and clock bias estimates, respectively.

Collecting these equations and putting them in matrix notation, we get

\[
\begin{bmatrix}
\delta \rho^{(1)} \\
\delta \rho^{(2)} \\
\vdots \\
\delta \rho^{(k)}
\end{bmatrix}
= G
\begin{bmatrix}
\delta x \\
\delta b
\end{bmatrix} + \varepsilon,
\]

where \(G\) is a matrix based on the geometry of the satellites in view, defined as

\[
G =
\begin{bmatrix}
(-1^{(1)})^T & 1 \\
(-1^{(2)})^T & 1 \\
\vdots & \vdots \\
(-1^{(k)})^T & 1
\end{bmatrix}
\]

Since there are typically more than four satellites in view, Eq. (2.7) represents an over-determined system \((K > 4)\), and a least-squares solution is found. The corrections to the initial estimates are

\[
\begin{bmatrix}
\delta x \\
\delta b
\end{bmatrix} = (G^T G)^{-1} G^T \delta \rho.
\]

Another iteration is then performed using the improved estimates given by
\[ \hat{x} = x_0 + \delta \hat{x} \]  \hspace{1cm} (2.10)

\[ \hat{b} = b_0 + \delta \hat{b} \]

This procedure is repeated, with each iteration bringing better estimates of the position and clock bias, until the corrections become very small.

### 2.3.3 Accuracy and Dilution of Precision

The covariance of the estimates obtained from the iterative least squares process is given by

\[ \text{Cov} \left[ \begin{bmatrix} \hat{x} \\ \hat{b} \end{bmatrix} \right] = \sigma^2 (G^TG)^{-1}, \]  \hspace{1cm} (2.11)

where \( \sigma^2 \) is the variance of the user range error. Thus the covariance of the position and bias estimates are only influenced by the user range error variance and a term which depends solely on the satellite geometry. If we let

\[ H = (G^TG)^{-1}, \]  \hspace{1cm} (2.12)

then we can represent the variances of each position component as

\[ \sigma_x^2 = \sigma^2 H_{11}, \]

\[ \sigma_y^2 = \sigma^2 H_{22}, \]

\[ \sigma_z^2 = \sigma^2 H_{33}, \]

\[ \sigma_b^2 = \sigma^2 H_{44}, \]  \hspace{1cm} (2.13)

where \( H_{11}, H_{22}, H_{33}, \) and \( H_{44} \) are the diagonal entries of \( H \). Defining the position dilution of precision (PDOP) and time dilution of precision (TDOP) as
\[ PDOP = \sqrt{H_{11} + H_{22} + H_{33}} \]  \hspace{1cm} (2.14)

\[ TDOP = \sqrt{H_{44}} \]  \hspace{1cm} (2.15)

the root-mean-squared position and clock bias estimation errors can be written as

\[
\text{RMS Position Error} = \sigma \cdot PDOP \\
\text{RMS Clock Error} = \sigma \cdot TDOP .
\]  \hspace{1cm} (2.16, 2.17)

The dilution of precision parameters can be redefined on the basis of local east-north-up (ENU) coordinates by introducing the ENU line-of-sight vector, defined as

\[
\mathbf{1}_{ENU}^{(k)} = \begin{bmatrix}
\cos EL^{(k)} \sin AZ^{(k)} \\
\cos EL^{(k)} \cos AZ^{(k)} \\
\sin EL^{(k)}
\end{bmatrix},
\]  \hspace{1cm} (2.18)

where \( EL^{(k)} \) and \( AZ^{(k)} \) are the elevation and azimuth angles of the \( k^{th} \) satellite. Defining a geometry matrix based on the ENU line-of-sight vectors and proceeding as before with Eq. (2.12) and Eq. (2.13) produces parameters denoted \( EDOP, NDOP, VDOP \) and \( TDOP \), which represent the east, north, vertical and time DOPs, respectively. The horizontal dilution of precision, \( HDOP \), is the vector sum of \( EDOP \) and \( NDOP \). Now the position error can be broken down into horizontal and vertical components using the following relationships:

\[
\text{RMS Horizontal Error} = \sigma \cdot HDOP \\
\text{RMS Vertical Error} = \sigma \cdot VDOP .
\]  \hspace{1cm} (2.19, 2.20)

The values of the dilution of precision parameters depend on the number of satellites in view and their distribution around the receiver. \( HDOP \) is generally smaller than \( VDOP \), because satellites below the horizon are not visible. The cumulative probability distribution functions of the DOPs available to users around the globe with the 24-
satellite baseline GPS constellation and a 5-degree elevation mask are shown in Figure 2.2.

![Figure 2.2: Cumulative probability distribution functions of VDOP and HDOP available with the 24-satellite baseline GPS constellation and a 5-degree elevation mask.](image)

**2.4 Receiver Clocks**

It is a common misconception that the only cost of using an inexpensive receiver clock is that one pseudorange measurement must be sacrificed to determine its bias [14]. In fact, if a perfect clock could be used, the structure of the problem would be changed from four-parameter estimation to three-parameter estimation, and vast improvements would be seen in the vertical accuracy [15].
Figure 2.3 shows that vertical positioning accuracy is strongly correlated with the clock bias estimation error. Such a relationship does not exist with the horizontal positioning accuracy, as seen in Figure 2.4. This is due to the fact that the user has satellites visible on all sides but cannot see those below the horizon. Since all pseudoranges are equally affected by the clock bias, and there are visible satellites above the user but not below, the effect is similar to a change in altitude [15].
**Figure 2.3** Vertical position error correlation with clock bias error

**Figure 2.4** Horizontal position error correlation with clock bias error
There is no such thing as a perfect receiver clock, but there is still benefit to be gained from very stable frequency standards, such as cesium and rubidium oscillators. In order to remove the bias from pseudorange measurements to perform a three-parameter estimation, it is not necessary to keep perfect GPS time; rather, it is only necessary that the clock bias be predictable. A predictable clock is one whose bias can be accurately estimated based on its behavior during a recent time interval. If the frequency changes smoothly (no frequency jumps or significant changes in frequency drift rate over short intervals), then the bias can be modeled using a quadratic fit of the following form

\[ \hat{b}(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)^2, \]  

where \( b_0, b_1 \) and \( b_2 \) are the coefficients based on recent clock bias estimates obtained from conventional four-parameter estimation [15]. The accuracy of this approach is limited because it relies on code phase measurements, but the model can be improved by incorporating carrier phase-based estimates of the bias drift rate [11]. Using such an adaptive clock model will allow the user to benefit from three-parameter estimation as long as the model accurately describes the clock’s present behavior.

### 2.4.1 Effects on Positioning

Once the clock model is used to estimate the current bias and remove it from the pseudorange measurements, Eq. (2.3) becomes

\[ \rho^{(k)}_c = r^{(k)} + \delta b + \tilde{e}^{(k)}. \]  

The clock bias term has been replaced with \( \delta b \), which is the error resulting from mismodeling of the clock. The error distributions can be represented as

\[ \tilde{e}^{(k)} \sim N(0, \sigma^2) \]  

\[ \delta b \sim N(0, \alpha \sigma^2), \]  

37
where $\sigma^2$ is the variance of the user range error (URE), which has been observed to be less than 1 m for DGPS. The variance of the clock modeling error has been represented as a constant $\alpha$ multiplied by the URE variance. The value of this constant depends on the behavior of the clock, as well as the quality of the model. An erratic clock will be difficult to model well and will have a high value of $\alpha$. Alternatively, a poor clock model will introduce error and drive up $\alpha$ as well. At the other end of the spectrum, a value of $\alpha = 0$ corresponds to a perfectly predictable clock and a perfect model. Note that both distributions are considered Gaussian, zero-mean.

The vector form of Eq. (2.22) is

$$\mathbf{p}_c = \mathbf{r} + \mathbf{b} \cdot \mathbf{1} + \bar{\mathbf{e}}, \quad (2.24)$$

where $\mathbf{1}$ is a $K \times 1$ vector of unit entries, not to be confused with the direction cosine vectors that make up the geometry matrix (denoted $\mathbf{1}^{(i)}$). We can collect the errors into one term, $\xi$, and represent the combined error distribution as

$$\xi \sim N(0, \Sigma), \quad (2.25)$$

where

$$\Sigma = \sigma^2 (\mathbf{I} + \alpha \mathbf{1} \mathbf{1}^T) \quad (2.26)$$

and $\mathbf{1}$ is a $K \times K$ identity matrix. Proceeding as before, an iterative least-squares approach is again employed. The new linearized equations become

$$\delta \mathbf{p} = \mathbf{A} \cdot \delta \mathbf{x} + \xi, \quad (2.27)$$

where $\mathbf{A}$ is defined as
and $\xi$ represents the combined effect of pseudorange errors and clock modeling. Multiplying both sides of Eq. (2.27) by $\Sigma^{-1/2}$ gives

$$\Sigma^{-1/2}\delta\rho = \Sigma^{-1/2} A \cdot \delta x + \Sigma^{-1/2} \xi,$$

(2.29)

which can be written in terms of newly defined variables as

$$\delta\tilde{\rho} = \tilde{A} \cdot \delta x + \tilde{\xi}.$$

(2.30)

The least-squares solution for $\delta x$ is

$$\delta\hat{x} = (A^T \tilde{A})^{-1} A^T \delta\tilde{\rho}.$$

(2.31)

Noting that

$$\Sigma^{1/2} \Sigma^{1/2} = \Sigma$$

$$\Sigma^{-1} \Sigma = I,$$

(2.32)

both $\delta\hat{x}$ and its covariance can be represented in terms of the original $A$ matrix and $\delta\rho$ vector as

$$\delta\hat{x} = (A^T A)^{-1} A^T \delta\rho$$

(2.33)

$$Cov(\delta\hat{x}) = (A^T \Sigma^{-1} A)^{-1}.$$

(2.34)
New dilution of precision parameters consistent with Eq. (2.16) can be defined by noting that Eq. (2.34) is equivalent to the expression

\[ \text{Cov}(\delta \mathbf{x}) = \sigma^2 (A^T (I + \alpha \mathbf{I})^{-1} A)^{-1}. \] (2.35)

Thus the new \( H \) matrix is

\[ H = (A^T (I + \alpha \mathbf{I})^{-1} A)^{-1}, \] (2.36)

and \( PDOP, HDOP \) and \( VDOP \) are defined in the same manner as in the case of conventional GPS estimation. Since clock bias is no longer estimated, there is no \( TDOP \) defined in the case of clock-aided positioning. Figure 2.5 compares the traditionally defined four-parameter estimation DOPs to the new three-parameter estimation DOPs for clock-aided positioning with a perfect receiver clock (\( \alpha = 0 \)).

![Figure 2.5 Conventional DOPs vs. clock-aided DOPs for a perfect clock](image-url)
As expected, there is only a slight improvement in $HDOP$. On the other hand, $VDOP$ has drastically improved. Figure 2.6 shows the case of a more realistic clock, with $VDOP$ plotted for various values of $\alpha$. Even for $\alpha = 1$, which corresponds to clock error as bad as the user range error, there are significant gains to be made using clock-augmentation.

**Figure 2.6** Conventional $VDOP$ vs. clock-aided $VDOP$ for various values of $\alpha$
Chapter 3

Atomic Clocks

3.1 Introduction

Timekeeping is a science that has existed for thousands of years. The earliest clocks estimated time based on the position of the sun or stars. These required adjustments based on latitude and could only be used during certain times of day. When the weather was bad, they couldn’t be used at all. Other devices such as graduated candles and hourglasses provided time measurement that wasn’t tied to weather conditions, but these weren’t always convenient to use. Even with the invention of mechanical clocks around the end of the thirteenth century, it was still not possible to measure minutes reliably. Four hundred years later, pendulum clocks provided the capability to count seconds. It was only in the twentieth century that the vibration of quartz crystal oscillators was employed as a frequency standard, allowing for precise time measurement.

Precision requirements have changed drastically since the clock was invented, and the evolving timekeeping technology has reflected those changes. In ancient times, farmers watched the movement of the stars to roughly determine when to plant and harvest their crops. Sailors of the eighteenth century realized the need for a clock to keep time to within three seconds per day to determine longitude with the required precision, leading to the invention of the maritime chronometer [6]. More recently, computers, communication systems and electrical power distribution networks have relied heavily on
the capability of quartz oscillators for their operation. But even the best quartz oscillators
don’t meet the needs of today’s most demanding applications. Instead, systems such as
GPS rely on atomic standards to meet their precision timekeeping requirements.

3.2 Atomic Oscillator Theory

When atoms undergo energy level transitions, they generally emit or absorb photons
ranging from radio waves to ultraviolet light. Since the frequency of certain transitions is
known for various elements, these can be used as frequency standards for atomic clocks.
A quartz oscillator is frequency or phase-locked to the atomic transition frequency,
generating a stable electronic signal that can be used for precision time keeping.

Each atomic oscillator shares some common components, but there are also important
differences that distinguish them from one another. The basic mechanism consists of a
chemical element, a heater, a containment device, an atom state-selection device, an
electromagnetic radiation source or a resonator, a quartz oscillator and the electronics [2].

Elements that are not already gaseous are vaporized using the heater and stored in the
containment device. Magnets or an illuminator are then used to select atoms in the
desired state. Electromagnetic radiation emitted at the atom’s hyperfine transition
frequency causes a change of state. A detector counts the atoms that have undergone a
state change, and tunes the radiation generator to maximize these transitions [22]. The
quartz oscillator is locked to the generator and provides the output signal for the clock.

3.3 Atomic Frequency Standard Types

There are several different chemical elements used in atomic clocks, but all of them share
some important characteristics [2]. All are Group IA elements, which means that each
has a single active electron. This is desirable because it provides a narrow resonance and
a high signal to noise ratio. They are also readily available, and can be vaporized at
temperatures produced by a practical heater. Finally, these elements have hyperfine transition frequencies in the approximate range of 1 GHz to 10 GHz.

The most common elements used are hydrogen, cesium and rubidium. Hydrogen masers are the most stable of the three in both the short and long-term, but they are large, heavy and expensive. They also require a great deal of power. These factors limit their use to stationary environments. The same restrictions apply to cesium oscillators, which do not have the short-term stability of hydrogen masers but exhibit excellent long-term stability. These too are generally used for stationary applications. The precision oscillator of choice for mobile applications is the rubidium oscillator. It is small, light and less expensive than the others. In addition, it requires less power and warms-up quickly. While it ages a few parts in $10^{10}$ per year, the short-term stability of high performance rubidium oscillators often exceeds that of standard cesium oscillators [2].

### 3.4 Performance Measures

There are several standard performance measures used to assess the capabilities of atomic clocks. Frequency aging refers to the change in a clock’s frequency due to internal changes in the oscillator [2]. This measure does not consider external factors, such as environmental conditions, power fluctuations or vibration. Frequency drift represents the combined effect of aging and external factors specific to an application. Both measure long term stability, but frequency aging is more commonly used in atomic oscillator evaluation.

Short-term stability is described using a statistical measure similar to variance, called Allan Variance. The bias-corrected variance of a sample is defined as

$$
\sigma^2 = \frac{1}{N - 1} \sum_{i=1}^{N} (x_i - \bar{x})^2 ,
$$

(3.1)
where \( \sigma^2 \) is the variance, \( x_i \) is the \( i \)th element of the random sample, \( \bar{x} \) is the sample mean and \( N \) is the sample size. Because this measure diverges for some noise processes, the Allan Variance is used in its place. It is defined as

\[
\sigma_a^2 = \frac{1}{2(N-1)} \sum_{i=1}^{N-1} (x_{i+1} - x_i)^2 ,
\]

(3.2)

where \( \sigma_a^2 \) is the Allan Variance, \( x_{i+1} - x_i \) is the difference between successive readings and \( N \) is the sample size. This measure is easy to calculate and converges for the noise processes effecting precision oscillators [2]. Table 3.1 compares the short-term stability performance of the most common precision oscillators.

<table>
<thead>
<tr>
<th>Oscillator</th>
<th>Allan Deviation @ 1 sec</th>
<th>Allan Deviation @ 1 day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>( 10^{12}/10^{-13} )</td>
<td>( 10^{-10} )</td>
</tr>
<tr>
<td>Rubidium</td>
<td>( 10^{11}/10^{-12} )</td>
<td>( 10^{-12}/10^{-13} )</td>
</tr>
<tr>
<td>Cesium</td>
<td>( 10^{11}/10^{-12} )</td>
<td>( 10^{-13}/10^{-14} )</td>
</tr>
<tr>
<td>Hydrogen Maser</td>
<td>( 10^{12}/10^{-13} )</td>
<td>( 10^{-14}/10^{-15} )</td>
</tr>
</tbody>
</table>

Table 3.1: Allan Deviation of common precision oscillators [2]

### 3.4 Oscillator Selection

In order for clock-aided positioning or integrity monitoring to be a feasible option for aircraft navigation, it is necessary to rely on a low cost, portable clock that has good short-term stability. In addition, the oscillator must be insensitive to vibration and temperature changes. It is also desirable to keep power consumption to a minimum. A rubidium oscillator is ideal for clock-aided positioning and integrity monitoring for LAAS precision approaches based on these requirements. Not all rubidium oscillators exhibit the required short-term frequency stability, however, and any clock should be evaluated before attempting to implement a clock-aided GPS algorithm. Section 6.2.2 describes the AR-61A rubidium frequency standard used in this study.
Chapter 4

Integrity Monitoring

4.1 Introduction

Navigation systems must be able to provide timely warnings when they fail to meet their stated accuracy requirements [23]. This concept is referred to as integrity monitoring and is of critical importance in safety-of-life applications such as aircraft navigation. While GPS satellites provide some level of integrity monitoring on their own, it is not sufficient for many applications. Even when there are no satellite failures, the quality of a position estimate can still be unacceptable. This may be due to poor satellite geometry at the user location, excessive multipath noise or mis-modeled atmospheric effects. There are two approaches that are generally taken to guard against these occurrences [3]. The first is called receiver autonomous integrity monitoring (RAIM). It relies on exploiting a redundant measurement set to perform a self-consistency check. There are many different approaches to RAIM, but most were designed to detect single-satellite failures. The second approach uses a network of local ground stations to monitor satellite signals and broadcast information about their quality to local users. These rely on accurately characterizing the distribution of the pseudorange errors based on recent measurements of stationary GPS receivers at known locations.
4.2 Receiver Autonomous Integrity Monitoring

Most RAIM schemes rely only on current measurements. These are called snapshot schemes and are desirable because they don’t depend upon assumptions about the history of the system. Instead, they take advantage of the fact that there are typically more satellites in view than are necessary to estimate position. There are four unknowns in GPS position estimation: three position components and clock bias. For the baseline GPS constellation, it is common to have seven to nine satellites visible. Thus, there are usually three to five extra satellites in view. The redundancy of the measurement set can be leveraged to determine the quality of the position estimate.

Two classic approaches to RAIM are known as the range comparison method and the least-squares-residuals method. The range comparison method was introduced by Lee [12] in 1986. It consists of using four measurements to estimate position and then predicting the magnitudes of the remaining pseudoranges based on this estimation. The residual differences between measured and predicted pseudoranges are combined into a test statistic, which can be compared to a decision boundary to determine if there is an integrity failure. The least-squares-residual method suggested by Parkinson and Alexrad [19] uses the position estimate obtained from all satellites in view to predict the magnitudes of the pseudoranges. The residuals are calculated, and the sum of their squares is used as the test statistic. This value is called the sum of squared errors (SSE) and is a nonnegative scalar. Both of these schemes were developed at a time when single-satellite failures were of main concern.

4.2.1 Maximum Separation of Solutions

Brown and McBurney [5] suggested a method that is also best suited for single-satellite failures. The basic premise involves calculating position estimates using subsets of the satellites in view. Given \( N \) pseudorange measurements, \( N \) subsets are formed by omitting one satellite at a time from the full set [3]. The subset solutions are compared with each other, and the maximum separation is used as a test statistic. A tight cluster of solutions indicates a consistent set, while scattered solutions indicate a possible failure.
This method is excellent for detecting single-satellite failures, but it is not suitable for the task of LAAS integrity monitoring.

Misra and Bednarz [16] have developed a modified algorithm that may meet the demanding requirements of LAAS precision approaches once a richly redundant constellation becomes available (i.e. GPS + Galileo). The modifications involve the selection of measurement subsets in a way that does not degrade the satellite geometry but still shows a correlation between maximum solution separation and maximum position error. Computer simulations have shown the potential of the algorithm, but more analysis is necessary to determine whether it fails less than once per ten million approaches.

### 4.3 LAAS Integrity Monitoring

The purpose of LAAS integrity monitoring is to protect against Hazardously Misleading Information (HMI), which is defined as a Navigation System Error (NSE) that exceeds the alert limit while no alert is given within the time to alarm [13]. The requirement is that the probability of such occurrences, called missed detections, is very small: less than $10^{-7}$ per CAT I approach. The LAAS integrity monitoring system must provide upper bounds on the lateral and vertical position estimation errors, called Lateral Protection Level ($LPL$) and Vertical Protection Level ($VPL$), each of which must satisfy this requirement. In other words,

\[
\Pr(\text{Vertical Position Error} > \text{VAL} | \text{VPL} < \text{VAL}) < 10^{-7} \tag{4.1}
\]

\[
\Pr(\text{Lateral Position Error} > \text{LAL} | \text{LPL} < \text{LAL}) < 10^{-7},
\]

where VAL and LAL represent Vertical Alert Limit and Lateral Alert Limit, respectively. The system should not be used for navigation whenever either protection level exceeds its corresponding alert limit, but it must be safe to use otherwise. Since it is much more difficult to meet the integrity requirements for vertical positioning, that will be the focus of the remainder of this thesis.
Under the current architecture, the LAAS ground stations will broadcast pseudorange corrections along with their estimated standard deviation. Multipath and thermal noise are the main components of the pseudorange error, and it is assumed that these error sources can be characterized or bounded by Gaussian random processes [13]. As a result, the pseudorange corrections are assumed to adhere to a zero-mean, Gaussian distribution. It is simple to set the protection level given the standard deviation of the pseudorange error, \( \sigma \), if it is indeed Gaussian. In this case, the vertical position error standard deviation is

\[
\sigma_v = \sigma \cdot VDOP, \tag{4.2}
\]

and the Vertical Protection Level is

\[
VPL = k \cdot \sigma_v, \tag{4.3}
\]

where \( k \) is chosen from Gaussian probability tables so that the \( VPL \) bounds the vertical position error with some required probability. Actually, the problem is more complicated, but this is the central idea. Unfortunately, this algorithm depends on an assumption that is not valid. In order to make the problem tractable, a Gaussian distribution was chosen to represent the pseudorange errors [14]. Empirical data shows that the distribution is not actually Gaussian, but is "nearly Gaussian," the latter having fatter "tails." Figure 4.1 compares the two distributions.
The approach taken to solve this dilemma is to find a Gaussian model that over-bounds the actual error distribution. This would allow a $VPL$ meeting the integrity requirement to be defined according to Eq. (4.3). There is a tradeoff between integrity and availability, however, and the cost of over-bounding is a loss of availability. In addition, it is difficult to prove with the required certainty that the integrity requirement can be met using a $VPL$ based on an a priori estimate of $\sigma$. The safety of approaching aircraft depends on whether or not all of the threats have been accounted for and the distributions are correct. There is no way to know whether changing multipath conditions, traveling ionospheric disturbances or other un-modeled effects have introduced additional error that has not been included in the model. It would be better to instead define the $VPL$ based on a conditional probability density function. Chapter 5 discusses the use of the clock bias estimation error as a conditioning variable.
4.4 Integrity Requirements for Precision Approaches

The LAAS integrity requirements are defined in terms of alert limits, time to alarm and probability of missed detection. Table 4.1 and Table 4.2 show the vertical and lateral requirements for CAT I through CAT III precision approaches [13]. It is immediately obvious that the vertical requirements are much more stringent than the lateral requirements.

<table>
<thead>
<tr>
<th>Performance Category</th>
<th>$VAL$ (meters)</th>
<th>Time to Alarm (seconds)</th>
<th>$P_v$(HMI) (per approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAT I</td>
<td>10.2</td>
<td>6</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>CAT II</td>
<td>5.3</td>
<td>1</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>CAT III</td>
<td>4.5</td>
<td>1</td>
<td>$10^{-9}$</td>
</tr>
</tbody>
</table>

Table 4.1 Vertical integrity requirements

<table>
<thead>
<tr>
<th>Performance Category</th>
<th>$LAL$ (meters)</th>
<th>Time to Alarm (seconds)</th>
<th>$P_l$(HMI) (per approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAT I</td>
<td>36.5</td>
<td>10</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>CAT II</td>
<td>17.3</td>
<td>1</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>CAT III</td>
<td>15.5</td>
<td>1</td>
<td>$10^{-9}$</td>
</tr>
</tbody>
</table>

Table 4.2 Lateral integrity requirements
Chapter 5

Clock-aided GPS Implementation

5.1 Introduction

The theory underlying clock-aided GPS positioning and integrity monitoring can be tested through a simple implementation. It is necessary to have access to observation data from two receivers that are in close proximity to one another, each of which must reference a stable frequency standard. One receiver must be stationary with precisely known position. It is designated the reference receiver and plays the role of a LAAS ground station. Because its location is known, pseudorange corrections can be calculated and “broadcast” to others in the area. If real-time broadcast corrections are not available, observations logged at each site are processed together to replicate a DGPS setup.

The second receiver is the user, which may be stationary or mobile. After applying corrections from the reference receiver, the user performs a conventional position/clock bias estimation based on pseudorange observations from a single epoch. In order to find the position estimation error, the actual position of the user must be known precisely from post-processing of carrier phase data or some other surveying technique. The error can then be found from

\[ E_{\text{XYZ}} = \hat{x} - x, \]  

(5.1)
where $\mathbf{E}_{XYZ}$ is the three-dimensional error vector representing the difference in the position estimate, $\hat{x}$, and the known position, $x$, in ECEF coordinates. The error can be resolved into its east-north-up components through the transformation

$$
\mathbf{E}_{ENU} = \mathbf{T} \cdot \mathbf{E}_{XYZ},
$$

(5.2)

where $\mathbf{T}$ is a 3 x 3 orthonormal transformation matrix which transfers the ECEF error vector, $\mathbf{E}_{XYZ}$, into the ENU error vector, $\mathbf{E}_{ENU}$ · The lateral and vertical error components are then defined as

$$
\mathbf{E}_{lat} = \sqrt{\mathbf{E}_{ENU,1}^2 + \mathbf{E}_{ENU,2}^2}
$$

(5.3)

and

$$
\mathbf{E}_{vert} = \mathbf{E}_{ENU,3}.
$$

(5.4)

The purpose of clock-aided GPS positioning is to reduce the magnitude of the vertical position error by reducing the dimensionality of the estimation problem. This involves precisely modeling the user clock bias (which now includes the effect of the reference receiver clock, since this bias is part of the broadcast corrections. This is why it is necessary for both receivers to use stable frequency standards.). Clock-aided integrity monitoring, on the other hand, involves detecting large values of the vertical error by comparing the pseudorange-based clock bias estimates with the precise clock model.
5.2 Clock-aided GPS Positioning

Assuming a short baseline between the user and reference station, both receivers should have the same satellites in view most of the time. The only exception should be low-elevation satellites that are masked by obstructions on the horizon. Pseudorange corrections are only available for the satellites common to both receivers, and one simple implementation of DGPS is to simply discard pseudoranges unique to the user receiver. The reason is that using low-elevation satellites without differential corrections is not likely to improve the position estimate and may actually degrade it. Assuming a common satellite set, the procedure for estimating position using DGPS with both conventional and clock-aided algorithms is explained in the following discussion.

First, the reference receiver estimates the actual range to each satellite. This is possible because the reference antenna position is known (from a survey), and the satellite positions can be predicted based on ephemeris parameters broadcast in the navigation message [Appendix A]. The corrections are given by

\[ \Delta \rho^{(k)} = \hat{r}_\text{ref}^{(k)} - \rho_\text{ref}^{(k)}, \]  

(5.5)

where \( \Delta \rho^{(k)} \) is the correction to the pseudorange from satellite \( k \), \( \hat{r}_\text{ref}^{(k)} \) is the estimated range from the reference station antenna to the satellite and \( \rho_\text{ref}^{(k)} \) is the pseudorange observed by the reference receiver. A user with access to these corrections can compute corrected pseudoranges as follows

\[ \tilde{\rho}_u^{(k)} = \rho_u^{(k)} + \Delta \rho^{(k)}, \]  

(5.6)

where \( \rho_u^{(k)} \) is the measured pseudorange and \( \tilde{\rho}_u^{(k)} \) is the corrected pseudorange to satellite \( k \). There is no need to apply any further corrections unless the baseline is long enough for decorrelation of the atmospheric delays to occur. In this case, the
tropospheric delay should be corrected for at both stations separately \[17\]. Also note that the multipath and receiver noise affecting the reference receiver are picked up by the user with the corrections. This is because such errors are uncorrelated at the two receivers. However, 100 second carrier-smoothing of the pseudoranges should be performed at both stations before corrections are calculated or applied. This will decrease the effect of noise. Also, it should be noted that there is generally some latency in the transmission of corrections. If the data is being post-processed to simulate DGPS, some delay should be included.

Once the pseudoranges have been corrected, the position and clock bias can be estimated using the iterative least-squares approach described by Eq. (2.5) – Eq. (2.10). Since the true position of the user is known, the vertical position error can be found by applying Eq. (5.1), Eq. (5.2) and Eq. (5.4).

The clock-aided estimate is calculated using a similar procedure. Corrections are calculated in the same fashion and applied to the user pseudoranges. A clock model is then used to estimate the current clock bias based on past estimates, and this bias is removed from each pseudorange according to the equation

\[
\hat{r}_u^{(k)} = \tilde{p}_u^{(k)} - b_{\text{pred}},
\]

where \(\hat{r}_u^{(k)}\) is the estimated range to satellite \(k\) based on the corrected pseudorange, \(\tilde{p}_u^{(k)}\), and the predicted clock bias, \(b_{\text{pred}}\), all of which have units of length. Again, an iterative least-squares solution is found. This time, however, only three parameters are estimated, so Eq. (2.27) - Eq. (2.31) are used. As before, the vertical position error can be calculated given the known position of the user antenna. Chapter 6 compares the vertical position accuracy of conventional and clock-aided positioning using DGPS.
5.3 Clock-aided GPS Integrity Monitoring

Four parameters are estimated using the conventional approach: three position components \((x, y, z)\) and clock bias \((b)\). If the clock bias is known, or can be modeled precisely, then it can serve as a quality check on the other parameters. In particular, the clock bias estimation error is a good predictor of the vertical position error due to the strong correlation between the two.

The first step to integrity monitoring using clock-augmentation is to develop a good model of the clock's behavior. This is described in section 5.4. Once this has been achieved, the implementation is simple:

(i) Predict the current clock bias based on the model
(ii) Estimate the current clock bias using conventional, four-parameter estimation
(iii) Calculate the clock bias estimation error: (ii) – (i)
(iv) Determine a Vertical Protection Level \((VPL)\) based on the bias estimation error

The computational load involved in this algorithm is very light. Since the clock bias is already estimated, the only additional steps are clock prediction and determination of a \(VPL\) based on the clock estimation error.

5.3.1 Vertical Protection Level

The current \(VPL\) is based on the prior probability function of the pseudorange error. A simplified equation can be written as

\[
VPL = k \cdot \sigma \cdot VDOP ,
\]

(5.8)

where \(\sigma\) is the standard deviation of the pseudorange errors and \(k\) is a constant used to adjust the \(VPL\) so that it meets the requirement for missed detections [Chapter 4.3].
VPL can now be improved using a conditional probability distribution function. The clock bias estimation error is defined as

$$\Delta b = \hat{b} - b_{pred},$$

(5.9)

where $\hat{b}$ is the bias estimate from conventional positioning using pseudoranges, and $b_{pred}$ is the bias predicted from the clock model. If the clock bias estimation error and vertical position error can be characterized by zero-mean, Gaussian distributions, the conditional distribution can be shown to be Gaussian with mean and standard deviation defined as

$$E \{ \Delta x_v | \Delta b \} = \alpha \frac{\sigma_{\Delta x_v}}{\sigma_{\Delta b}} \Delta b,$$

(5.10)

$$\text{Var} \{ \Delta x_v | \Delta b \} = \sigma_{\Delta x_v}^2 (1 - \alpha^2),$$

(5.11)

where $\Delta x_v$ is the vertical error in the position estimate, and $\alpha$ is the correlation coefficient [14]. It is expected that the vertical error will be bounded by its mean plus or minus some constant times the standard deviation, where the probability of the error exceeding this bound is determined by the constant. Thus, one possible VPL could be defined as the

$$VPL = \max \left| E \{ \Delta x_v | \Delta b \} \pm k \cdot \sqrt{\text{Var} \{ \Delta x_v | \Delta b \}} \right|$$

(5.12)

$$= \max \left| \alpha \frac{\sigma_{\Delta x_v}}{\sigma_{\Delta b}} \Delta b \pm k \cdot \sigma_{\Delta x_v} \sqrt{1 - \alpha^2} \right|. $$
Noting that

\[ \sigma_{a_y} = \sigma \cdot VDOP \]  
\[ \sigma_{\Delta b} = \sigma \cdot TDOP, \]  

Eq. (5.12) can be rewritten as

\[ VPL = \max \left| \alpha \frac{VDOP}{TDOP} \Delta b \pm k \cdot \sigma \cdot VDOP \sqrt{1 - \alpha^2} \right|. \]  

The correlation coefficient, \( \alpha \), depends solely on the Geometry matrix, and can be calculated at each epoch as follows

\[ A = (G_L^T \cdot G_L)^{-1} \]  
\[ \alpha = \frac{A_{34}}{\sqrt{A_{33} \cdot A_{44}}}, \]

where \( G_L \) is the local Geometry matrix, which is defined as

\[
G_L = \begin{bmatrix}
(-1_{ENU}^{(1)})^T & 1 \\
(-1_{ENU}^{(2)})^T & 1 \\
\vdots & \vdots \\
(-1_{ENU}^{(k)})^T & 1 \\
\end{bmatrix}.
\]

5.4 Clock Model

The idea of using clock-augmentation for integrity monitoring hinges on the ability to precisely model the behavior of the clock. It is essential to have an accurate estimate of the bias at each and every epoch for such a scheme to work. This requires a good model and a well-behaved clock.
As mentioned in Chapter 3, a clock is required to have excellent short-term stability to be used in integrity monitoring. Long-term effects such as frequency drifts are easily accounted for by performing a quadratic fit to the bias estimates of some previous interval. Frequency jumps, or changes in drift rate over short intervals, however, are not so easily predicted. Figure 5.1 shows such a frequency jump. No clock model would have predicted such an occurrence based on recent behavior, and an integrity monitoring algorithm relying on knowledge of the bias would likely run into problems.

![Figure 5.1 Quartz oscillator frequency jump](image-url)
Figure 5.2 compares the short-term behavior of several types of precision oscillators. It is evident from the figure that the temperature-controlled crystal oscillator (TCXO) does not meet the standards for clock-aided integrity monitoring. Fortunately, the other three oscillators all show sufficient short-term stability. As long as these clocks retain this characteristic when subjected to vibration, temperature changes and other environmental effects experienced during flight, any of them could be used in this algorithm. It should be noted that Figure 5.2 only shows that the particular cesium, rubidium and oven-controlled crystal oscillators examined in this study are sufficient for use in clock-aided GPS algorithms. This does not mean that all oscillators of these types will work. The short-term stability of any clock should be evaluated before implementing clock-aided GPS positioning or integrity monitoring.

**Figure 5.2** Short-term behavior of various precision oscillators (TCXO: temperature-controlled crystal oscillator; OCXO: oven-controlled crystal oscillator)

Once a good clock has been selected and shown to meet the stability requirements, the challenge of precisely modeling its behavior remains. Misra [15] suggests using a quadratic clock model to account for both frequency offset and frequency drift. Such a model would be based on bias estimates from some recent time interval, and would adapt to the changing characteristics of the clock. Kline [11] suggests incorporating bias rate estimates obtained using carrier phase measurements, which are much more precise than
the pseudoranges. An accurate clock model can be defined on the basis of these principles.

The Doppler shift measured in the carrier tracking loop of a GPS receiver is equivalent to the range rate of the user. It is a projection of the satellite velocity relative to the user velocity onto the line-of-sight vector [17]. As a result, it is possible to calculate velocity and clock bias rate from the Doppler observations at a single epoch. Differentiating Eq. (2.1) gives

\[ \dot{\rho}^{(k)} = \dot{r}^{(k)} + (\dot{b} - \dot{b}^{(k)}) + \dot{f}^{(k)} + \dot{T}^{(k)}, \]  

which can be rewritten as

\[ \dot{\rho}^{(k)} = (v^{(k)} - v) \cdot \mathbf{1}^{(k)} + (\dot{b} - \dot{b}^{(k)}) + \dot{f}^{(k)} + \dot{T}^{(k)}, \]  

where \( (v^{(k)} - v) \) is the relative velocity between the user and satellite \( k \), and \( \mathbf{1}^{(k)} \) is the line-of-sight unit vector from the baseline midpoint to satellite \( k \). Since the atmospheric effects change slowly over short time intervals, \( \dot{f}^{(k)} \) and \( \dot{T}^{(k)} \) are small. We are interested in the relative clock bias rate of the user and reference receivers, so we take a single-difference of the range rates and rewrite Eq. (5.19) as

\[ \dot{\rho}_{ur}^{(k)} = -\mathbf{1}^{(k)} \cdot \mathbf{v}_{ur} + \dot{b}_{ur} + \epsilon_{ur}^{(k)}, \]  

where the subscripts indicate that each term is now relative to a reference receiver. This step has eliminated common error terms, and the satellite velocity and clock bias rate terms have dropped out. The linearized equation is then

\[ \dot{\rho} = G \begin{bmatrix} v_{ur} \\ b_{ur} \end{bmatrix} + \epsilon_{ur}, \]  

64
where $G$ is the previously-defined geometry matrix. The relative clock bias rate is then estimated from the least-squares solution of Eq. (5.22) as

$$
\begin{bmatrix}
\dot{v}_{ur} \\
\dot{b}_{ur}
\end{bmatrix} = (G^T G)^{-1} G^T \dot{p}.
$$

Thus a precise clock bias rate estimate can be obtained using Doppler measurements. Eq. (5.21) is only valid for very short baselines, and a more general algorithm should be applied in most cases [Appendix B]. Figure 5.3 compares the smooth estimates obtained from Doppler measurements with the noisy ones obtained from conventional four-parameter code phase measurements.

**Figure 5.3** Code phase clock bias estimates vs. accumulated Doppler estimates of clock bias drift rate
Although we now have precise estimates of the bias rate, there is still an ambiguity that prevents us from knowing the clock bias (the initial bias is still unknown; hence the offset in Figure 5.3). In order to overcome this obstacle, the precise but ambiguous Doppler-based estimates are combined with the noisy but unambiguous code-based estimates using a recursive filter. The filter of length $N$ epochs, is implemented as follows

$$\bar{b}(t_i) = \frac{1}{N} b(t_i) + \frac{N-1}{N} \left[ \bar{b}(t_{i-1}) + (\dot{b}(t_{i}) - \dot{b}(t_{i-1})) \Delta t \right],$$

(5.24)

$$\bar{b}(t_1) = b(t_1)$$

where $\bar{b}(t_i)$ is the current filter estimate, and the subscripts have been dropped for simplicity. Finally, a quadratic fit is performed on the filter estimates over some time interval, accounting for frequency offset and drift. The clock bias at the next epoch can now be predicted using these quadratic coefficients. Figure 5.4 shows the clock bias prediction error that results from this scheme using a 10 minute fit interval and a 5 second prediction interval (i.e. a quadratic fit is performed over 10 minutes of data and then used to predict ahead 5 seconds). The RMS error for the data shown is 42 cm.
While it is evident that this model is good for short prediction intervals, its accuracy decreases as the prediction interval grows. Figure 5.5 and Figure 5.6 show the error that results from prediction intervals of 1 minute and 5 minutes, respectively. The RMS error for the 1 minute fit interval is 0.16 m, while for the 5 minute interval it is 0.70 m.
Figure 5.5 Clock bias prediction error for 1 minute prediction interval

Figure 5.6 Clock bias prediction error for 5 minute prediction interval
Chapter 6

Clock-aided GPS Evaluation

6.1 Introduction

The algorithms for clock-aided GPS positioning and integrity monitoring were evaluated at Lincoln Laboratory in Lexington, MA. The data collection process was broken up into two phases. First, a stationary setup was used to examine the nominal behavior of the atomic clock and determine the potential benefits of the algorithm. Next, the receiver, clock and antenna were mounted in a moving vehicle to test the system in a more hostile environment. Each setup is described in this chapter, and the corresponding data is presented.

6.2 Data Collection Setup

6.2.1 GPS Receiver

The GPS receiver that was used for data collection is the Lexon-GGD, which is manufactured by Javad Navigation Systems. It collects dual-frequency data on 20 channels at a sampling rate of 1 Hz. In addition, it is WAAS enabled and has an external frequency input. To be consistent with the LAAS architecture, only the code observations on the L1 frequency band were used. WAAS was disabled, and instead a LAAS-like setup [Section 6.2.4] was employed to correct the raw pseudoranges measured by the receiver. The receiver has 512 MB of internal memory but is also
capable of real-time data recording via a serial, USB or Ethernet connection. The retail value of this package is approximately $13,000.

### 6.2.2 External Frequency Standard

Clock-aided GPS requires a portable clock with good short-term stability, low power consumption and low sensitivity to vibration and temperature changes. Based on these requirements, the AR-61A was chosen. It is a fully militarized rubidium standard with a vibration isolator, available from Accubeat for around $10,000. The size, weight and performance details are listed in Table 6.1.

<table>
<thead>
<tr>
<th>Weight/Volume:</th>
<th>Lbs/In.³</th>
<th>4.6/47.5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aging:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From 2nd month</td>
<td>6E-11/month</td>
<td></td>
</tr>
<tr>
<td>From 1st year</td>
<td>6E-10/year</td>
<td></td>
</tr>
<tr>
<td>From 2nd Year</td>
<td>3E-10/year</td>
<td></td>
</tr>
<tr>
<td><strong>Allan Deviation:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 s</td>
<td>3E-11</td>
<td></td>
</tr>
<tr>
<td>10 s</td>
<td>1E-11</td>
<td></td>
</tr>
<tr>
<td>100 s</td>
<td>3E-12</td>
<td></td>
</tr>
<tr>
<td><strong>Power:</strong></td>
<td>Steady State @ 25°C</td>
<td>&lt;18 W @ 26 Vdc</td>
</tr>
</tbody>
</table>

*Table 6.1: AR-61A product details*

### 6.2.3 Antenna

A Choke Ring Antenna, made by Thales Navigation, was used to mitigate multipath reflections for better GPS observations. The antenna design is accepted by the International GPS Service (IGS) and is used in IGS and other high-end networks. It consists of five concentric rings set on a ground plate, a Dorne & Margolin C146-10 dipole element and a low-noise amplifier [21]. A low-loss antenna cable was used to connect the antenna to the GPS receiver.
6.2.4 Reference Station
The National Geodetic Survey (NGS), which is part of the National Oceanic and Atmospheric Administration (NOAA), maintains a network of reference stations that provides GPS observation data free of charge. The continuously operating reference stations (CORS) are located throughout the United States, as well as a few other select locations. Fortunately, there is a CORS station approximately 25 km away from Lincoln Laboratory. It is located at MIT Haystack Observatory in Westford, Massachusetts. In addition to its proximity, the Westford reference station, designated WES2, uses a hydrogen maser frequency standard. It is a prerequisite for clock-aided GPS that the reference receiver use a stable oscillator. Post-processing allowed data from this reference station to be used to calculate corrections to the observations taken at the Lincoln user station, thus creating a LAAS-like setup.

6.2.5 Laboratory Setup
The antenna was mounted on the roof of the four-story laboratory building, with a low-loss cable connecting it to the GPS receiver located at the workstation inside. Figure 6.1 shows the antenna setup.

![Laboratory antenna setup](image)
The indoor workstation consisted of the receiver, the rubidium oscillator, a power source and a computer used for real-time data logging. Figure 6.2 shows this configuration. Raw L1 pseudoranges measured by the receiver were transmitted to the desktop computer where they were stored. After a sufficient amount of data was collected, reference station observations corresponding to the same time period were downloaded from the NGS CORS website. The data were then processed together, as in the case of real-time DGPS systems like LAAS. Finally, results from conventional positioning were compared with those obtained using the clock-aided algorithm described in Chapter 5.

![Laboratory workstation setup](image)

**Figure 6.2** Laboratory workstation setup
6.2.6 Field Setup

Flight tests were not feasible at the time of the study, so another vehicle was needed to test the algorithm in a stressed environment. One of the laboratory vehicles was determined to be a good choice for this task. An old diesel truck with a damaged suspension was chosen because its excessive vibration posed a good challenge to the clock’s short-term frequency stability. The truck is shown in Figure 6.3.

![Truck antenna setup](image)

*Figure 6.3 Truck antenna setup*
The antenna was mounted securely to the bed of the truck, with the cable passing through a partially opened window. Inside, several uninterrupted power sources (UPSs) were used to power the receiver, the clock and a laptop computer. As before, data was collected and post-processed with reference station observations. Figure 6.4 shows the setup of the truck workstation.

![Figure 6.4 Truck workstation setup](image)

The first step in the field data collection scheme was to drive the truck around the roof of the laboratory parking garage (where it always had a clear view of the sky) for some period of time. This exposed the clock to inertial forces and vibration, as well as some temperature variations. Data was collected during this period and included in the adaptive clock model discussed in section 5.4. Next, the truck continued to take measurements as it remained parked for two hours, with the antenna stationary in a precisely-surveyed position. During this time, vibrations persisted as the engine was left running. Since the antenna position was now known, the error in position calculations
could easily be calculated, and the accuracy of the two different methods of position estimation compared. The roof of the parking garage is pictured on one of the days of data collection in Figure 6.5.

Figure 6.5 Parking garage roof
6.3 Results

6.3.1 Laboratory Positioning Results

Figure 6.6 compares the vertical position error from both conventional and clock-aided estimation for two hours of data collected in the laboratory. The improvement gained using clock-aided positioning is immediately evident from the figure. Conventional positioning results in a RMS vertical error of 0.99 meters, while the RMS error from clock-aided positioning is only 0.55 meters. Note that the pseudoranges were not smoothed at either the user or reference station in this first case.

Figure 6.6 Unsmoothed laboratory data
Figure 6.7 shows the vertical error of each method for the same time period, but this time the pseudoranges were smoothed at each end using a 100 second smoothing interval. Smoothing alone improves the error drastically, and the new RMS vertical error from conventional positioning decreases to 0.71 meters. Even so, this value is still worse than the RMS clock-aided positioning error from the unsmoothed case. Once clock-aided positioning is performed using smoothed pseudoranges, the RMS vertical error decreases to 0.47 meters.

Figure 6.7 Smoothed laboratory data
6.3.2 Field Positioning Results

Again, the vertical errors resulting from conventional and clock-aided positioning are compared in Figure 6.8, which corresponds to field data and unsmoothed pseudoranges. As before, there is a drastic improvement gained using clock augmentation. The RMS error decreases from 1.44 meters to 0.74 meters.

![Graph showing vertical errors over time](image)

Figure 6.8 Unsmoothed field data
Figure 6.9 shows the results of both positioning methods using data that has been smoothed at both stations with a 100 second smoothing interval. The RMS error decreases to 0.93 meters for conventional positioning, which is still worse than the clock-aided positioning using unsmoothed data. Once the smoothed data is processed using clock-aided positioning, the error decreases to 0.67 meters.
6.3.3 Integrity Monitoring Results

To first establish the potential of clock-aided integrity monitoring, it is necessary to show that a strong correlation exists between clock bias estimation error and vertical position error. Figure 6.10a shows the correlation for the laboratory data. Only one hour is shown so that the details of the picture are clear, but the figure is representative of behavior exhibited throughout a larger data set. As before, similar results are obtained in the case of the moving vehicle, so only one data set is presented in this section.

Using Eq. (5.17), the correlation coefficient can be calculated at each epoch based on the current satellite geometry. Figure 6.10b shows the variation of the correlation coefficient for the same one hour time period.

Figure 6.10 (a) Correlation between bias estimation error and vertical position error for laboratory data (b) Correlation coefficient for laboratory data
The expected value of the vertical position error is determined based on the equations developed in Chapter 5, which describe the conditional distribution of vertical position error given the clock bias error. Figure 6.11a compares the vertical position error with its expected value.

A Vertical Protection Level can be found at each epoch according to Eq. (5.15). This VPL is based on the correlation coefficient, the dilution of precision parameters and the clock bias estimation error determined from the adaptive clock model. It is equivalent to the expected value of the vertical position error plus some constant times its standard deviation, given the current estimate of the clock bias error. Figure 6.11b shows the VPL for a value of $k \cdot \sigma_{\Delta v}$ equal to 2.

Figure 6.11 (a) Vertical position error versus its expected value (b) VPL versus vertical position error magnitude for laboratory data
The \textit{VPL} follows the shape of the vertical position error, predicting all of the error peaks. The bound is not violated at any point in this sample. This can be seen more clearly in Figure 6.12, which shows the difference between the \textit{VPL} and the vertical position error. A negative value in this plot would indicate an error value greater than the \textit{VPL}. There are no such points in this data set.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6_12.png}
\caption{\textit{VPL} minus vertical position error}
\end{figure}
Chapter 7

Conclusions

7.1 Clock-aided Positioning

This thesis shows that clock-aided positioning offers significant reduction in vertical position error over conventional positioning. In laboratory tests, the reduction ranges from 44% using unsmoothed pseudoranges to 34% using carrier-smoothed pseudoranges. Similar results are seen for field tests, where there is a 49% reduction in vertical error for unsmoothed data and a 28% reduction for smoothed data. The field data is particularly promising, since it shows that rubidium standard clocks can meet the stability requirements of a clock-aided positioning algorithm when subjected to environmental conditions similar to those it would experience in flight. The improvement in vertical positioning accuracy is shown to be a result of the reduction in $VDOP$ that occurs when the structure of the problem is changed from four-parameter to three-parameter estimation.

7.2 Clock-aided Integrity Monitoring

It has been suggested that a strong correlation exists between the estimation error in clock bias and vertical position when conventional pseudorange positioning is used. This is verified with laboratory data, and a $VPL$ formula is developed on this basis. Data is presented to confirm that the $VPL$ bounds the vertical error for laboratory tests. While
only a modest data set is examined, the potential of clock-aided integrity monitoring for LAAS precision approaches is evident. The algorithm predicts error peaks, adjusts to changing conditions and does not depend heavily on assumptions regarding the nature of the pseudorange errors.

7.3 Future Work

The potential of clock augmentation for use in integrity monitoring has been shown, but there are some important issues remaining. First, it must be proven that a VPL such as the one suggested here can meet the strict integrity requirements set for precision approaches. This would involve much more data collection, as well as statistical analysis proving that missed detections can be kept below \(10^{-7}\) per CAT I approach. In particular, flight tests need to be performed to show that it continues to function properly when subjected to the conditions of flight.
Appendix A

A.1 Satellite Position

The steps for calculating the position of a GPS satellite given ephemeris data are detailed in the interface control document ICD-GPS-200 [18], which specifies technical requirements for the Navstar GPS space segment and navigation user interfaces. The algorithm is reproduced here; however, a few intermediate steps are presented in a slightly different manner for better clarity.

The ephemeris parameters transmitted by the satellites are:

\[ M_0 \] Mean anomaly at reference time
\[ \Delta n \] Mean motion difference from computed value
\[ e \] Eccentricity
\[ \sqrt{A} \] Square root of semi-major axis
\[ \Omega_0 \] Longitude of ascending node of orbit plane at weekly epoch
\[ i_0 \] Inclination angle at reference time
\[ \omega \] Argument of perigee
\[ \dot{\Omega} \] Rate of right ascension
\[ \dot{i} \] Rate of inclination angle
\[ C_{ac} \] Amplitude of cosine harmonic correction term to argument of latitude
\[ C_{as} \] Amplitude of sine harmonic correction term to argument of latitude
Given recent ephemeris data, the position of each satellite can first be found in an orbital frame whose origin is at the focus of the elliptical orbit. The x-axis points toward the perigee, with the y-axis rotated 90 degrees in the direction of orbital motion.

\[ \mu = 3.986005 \times 10^{14} \frac{\text{meters}^3}{\text{sec}^2} \]  
WGS84 value of the earth's universal gravitational parameter

\[ \dot{\Omega}_e = 7.2921151467 \times 10^{-5} \frac{\text{rad}}{\text{sec}} \]  
WGS84 value of earth’s rotation rate

\[ A = (\sqrt{A})^2 \]  
Semi-major axis

\[ n_0 = \sqrt{\frac{\mu}{A^3}} \]  
Computed mean motion (rad/s)

\[ t_k = t - t_{oe} \]  
Time from ephemeris reference epoch

\[ n = n_0 + \Delta n \]  
Corrected mean motion

\[ M_k = M_0 + n t_k \]  
Mean anomaly

\[ M_k = E_k - e \sin E_k \]  
Kepler’s equation for eccentric anomaly (solved iteratively)

\[ v_k = \tan^{-1} \left( \frac{\sin v_k}{\cos v_k} \right) \]  
True anomaly

\[ = \tan^{-1} \left( \frac{\sqrt{1 - e^2} \sin E_k}{(1 - e \cos E_k)} \right) \]  
\[ = \tan^{-1} \left( \frac{(\cos E_k - e)/(1 - e \cos E_k)}{(1 - e^2 \sin E_k)/(1 - e \cos E_k)} \right) \]  
Eccentric anomaly

\[ E_k = \cos^{-1} \left( \frac{e + \cos v_k}{1 + e \cos v_k} \right) \]  
Eccentric anomaly
\[ \Phi_k = v_k + \omega \]  
**Argument of latitude**

\[ \delta u_k = C_{us} \sin 2\Phi_k + C_{uc} \cos 2\Phi_k \]  
**Argument of latitude correction**

\[ \delta r_k = C_{rs} \sin 2\Phi_k + C_{rc} \cos 2\Phi_k \]  
**Radius correction**

\[ \delta i_k = C_{is} \sin 2\Phi_k + C_{ic} \cos 2\Phi_k \]  
**Inclination angle correction**

\[ u_k = \Phi_k + \delta u_k \]  
**Corrected argument of latitude**

\[ r_k = A(1 - e \cos E_k) + \delta r_k \]  
**Corrected radius**

\[ i_k = i_0 + \delta i_k + \dot{i} t_k \]  
**Corrected inclination angle**

\[ \Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e) t_k - \dot{\Omega}_e t_{oe} \]  
**Corrected longitude of ascending node**

\[ x_k = r_k \cos v_k \]  
**Position in orbital plane**

\[ y_k = r_k \sin v_k \]

\[ z_k = 0 \]

### A.2 Satellite Velocity

While the velocity calculation is not described in the ICD, it is straightforward. The following steps show how to find the velocity of a satellite in the orbital plane given ephemeris data.

\[ v_x = - \frac{nA}{1 - e \cos E} \sin E \]  
**Velocity in Orbital Plane**

\[ v_y = \frac{nA\sqrt{1 - e^2}}{1 - e \cos E} \cos E \]

\[ v_z = 0 \]

### A.3 Transformation to ECEF

A position vector can be transformed from orbital coordinates to ECEF coordinates using the following relationship:
\[ x_{ECEF} = T x_0 , \]

where is a \( T \) \( 3 \times 3 \) transformation matrix. Taking the derivative of this equation gives an expression for the velocity vector transformation:

\[ v_{ECEF} = T v_0 + \dot{T} x_0 , \]

where \( \dot{T} \) represents the rate of change of the rotating ECEF coordinate system. The transformation matrix and its rate of change are defined as follows:

\[
T = \begin{bmatrix}
    a_1 & a_2 & a_3 \\
    b_1 & b_2 & b_3 \\
    c_1 & c_2 & c_3
\end{bmatrix},
\]

where

\[
a_1 = \cos \Omega_k \cos \omega - \sin \Omega_k \sin \omega \cos i
\]
\[
a_2 = -\cos \Omega_k \sin \omega + \sin \Omega_k \cos \omega \cos i
\]
\[
a_3 = \sin \Omega_k \sin i
\]
\[
b_1 = \sin \Omega_k \cos \omega + \cos \Omega_k \sin \omega \cos i
\]
\[
b_2 = -\sin \Omega_k \sin \omega + \cos \Omega_k \cos \omega \cos i
\]
\[
b_3 = -\cos \Omega_k \sin i
\]
\[
c_1 = \sin \omega \sin i
\]
\[
c_2 = \cos \omega \sin i
\]
\[
c_3 = \cos i
\]

and
\[ \begin{bmatrix} d_1 & d_2 & d_3 \\ e_1 & e_2 & e_3 \\ f_1 & f_2 & f_3 \end{bmatrix} \]

where

\[
\begin{align*}
d_1 &= \dot{\Omega}_k [-\sin \Omega_k \cos \omega - \cos \Omega_k \sin \omega \cos i] \\
d_2 &= \dot{\Omega}_k [\sin \Omega_k \sin \omega - \cos \Omega_k \cos \omega \cos i] \\
d_3 &= \dot{\Omega}_k [\cos \Omega_k \sin i] \\
d_1 &= \dot{\Omega}_k [\cos \Omega_k \cos \omega - \sin \Omega_k \sin \omega \cos i] \\
d_2 &= \dot{\Omega}_k [-\cos \Omega_k \sin \omega - \sin \Omega_k \cos \omega \cos i], \\
d_3 &= \dot{\Omega}_k [\sin \Omega_k \sin i] \\
f_1 &= 0 \\
f_2 &= 0 \\
f_3 &= 0
\end{align*}
\]

and

\[ \dot{\Omega}_k = (\dot{\Omega} - \dot{\Omega}_e). \]

Note that the assumption has been made that \( \omega \) and \( i \) change slowly over short time intervals, and there contributions have thus been omitted from \( \dot{T} \).
Appendix B

B.1 Standalone Clock Bias Rate

An expression for the bias rate of a GPS receiver clock can be found by differentiating Eq. (2.1). This gives

\[ \dot{\rho}^{(k)} = \dot{r}^{(k)} + (\dot{b} - \ddot{b}^{(k)}) + \dot{t}^{(k)} + \dot{T}^{(k)} + \epsilon^{(k)}, \]  

which can be rewritten as

\[ \dot{\rho}^{(k)} = (\mathbf{v}^{(k)} - \mathbf{v}) \cdot 1^{(k)} + (\dot{b} - \ddot{b}^{(k)}) + \dot{t}^{(k)} + \dot{T}^{(k)} + \epsilon^{(k)}. \]  

The atmospheric delays change slowly over time, so \( \dot{t}^{(k)} \) and \( \dot{T}^{(k)} \) are generally negligible. The satellite velocity and clock bias rate can be estimated from parameters broadcast in the navigation message, and Eq. (B.2) becomes

\[ (\dot{\rho}^{(k)} - \mathbf{v}^{(k)} \cdot 1^{(k)} + \dot{b}^{(k)}) = -1^{(k)} \cdot \dot{\mathbf{v}} + \dot{b} + \epsilon^{(k)}. \]  

(B.3)

Defining \( \dot{\rho}^{(k)} \) as

\[ \dot{\rho}^{(k)} = (\dot{\rho}^{(k)} - \mathbf{v}^{(k)} \cdot 1^{(k)} + \dot{b}^{(k)}). \]  

(B.4)
and collecting all of the measurements into a $K \times 1$ vector, Eq. (B.3) can be rewritten as

$$\hat{\rho} = G \begin{bmatrix} v \\ b \end{bmatrix} + \epsilon.$$  \hspace{1cm} (B.5)

The relative clock bias rate is then estimated from the least-squares solution of Eq. (B.5) as

$$\begin{bmatrix} v \\ b \end{bmatrix} = (G^T G)^{-1} G^T \hat{\rho}.$$ \hspace{1cm} (B.6)

**B.2 Relative Clock Bias Rate**

Eq. (B.2) can be written for Doppler observations at the user and reference receivers as

$$\dot{p}_u^{(k)} = (v^{(k)} - v_u) \cdot 1_u^{(k)} + (\dot{b}_u - \dot{b}^{(k)}) + \epsilon_u^{(k)} \hspace{1cm} (B.7)$$

$$\dot{p}_r^{(k)} = (v^{(k)} - v_r) \cdot 1_r^{(k)} + (\dot{b}_r - \dot{b}^{(k)}) + \epsilon_r^{(k)}. \hspace{1cm} (B.8)$$

Since $\dot{f}_{\text{rg}}^{(k)}$ and $\dot{f}_{\text{rg}}^{(k)}$ are negligible, they have been omitted from the equations. Taking a single difference of Doppler observations between the two receivers gives

$$\dot{p}_{ur}^{(k)} = (v^{(k)} - v_u) \cdot 1_u^{(k)} + (\dot{b}_u - \dot{b}^{(k)}) - (v^{(k)} - v_r) \cdot 1_r^{(k)} - (\dot{b}_r - \dot{b}^{(k)}) + \epsilon_{ur}^{(k)}, \hspace{1cm} (B.9)$$

where the subscripts indicate relative terms. The satellite clock bias terms cancel, and the equation can be written as

$$\dot{p}_{ur}^{(k)} = v^{(k)} \cdot (1_u^{(k)} - 1_r^{(k)}) - 1_{mid} \cdot v_{ur} + \dot{b}_{ur} + \epsilon_{ur}^{(k)}, \hspace{1cm} (B.10)$$
where \( \mathbf{1}_{\text{mid}} \) is the line-of-sight vector from the midpoint of the baseline to the \( k \text{th} \) satellite. Notice that Eq. (5.15) assumes that the term \( \mathbf{v}^{(k)} \cdot (\mathbf{1}_u^{(k)} - \mathbf{1}_r^{(k)}) \) is negligible. This is only valid for very short baselines, since the satellite velocities are large. In the general case (baselines greater than 100 meters), the satellite velocities would be calculated using the equations in Appendix A, and Eq. (B.10) would be rewritten as

\[
\hat{\mathbf{p}}_{ur}^{(k)} = -\mathbf{1}_{\text{mid}} \cdot \mathbf{v}_{ur} + \dot{\mathbf{b}}_{ur} + \mathbf{e}_{ur}^{(k)}, \tag{B.11}
\]

where

\[
\hat{\mathbf{p}}_{ur}^{(k)} = \hat{\mathbf{p}}_{ur}^{(k)} - \mathbf{v}^{(k)} \cdot (\mathbf{1}_u^{(k)} - \mathbf{1}_r^{(k)}). \tag{B.12}
\]

After collecting all of the Doppler observations into a \( K \times 1 \) vector, Eq. (B.11) becomes

\[
\hat{\mathbf{p}}_{ur} = \mathbf{G}_{\text{mid}} \begin{bmatrix} \mathbf{v}_{ur} \\ \dot{\mathbf{b}}_{ur} \end{bmatrix} + \mathbf{e}_{ur}. \tag{B.13}
\]

The bias rate is then found from

\[
\begin{bmatrix} \mathbf{v}_{ur} \\ \dot{\mathbf{b}}_{ur} \end{bmatrix} = (\mathbf{G}_{\text{mid}} \mathbf{T} \mathbf{G}_{\text{mid}})^{-1} \mathbf{G}_{\text{mid}} \mathbf{T} \hat{\mathbf{p}}_{ur}. \tag{B.14}
\]
References


