Potential Airport Capacity Gains from the Optimal Assignment of Aircraft Types to Runways

by

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B.S., Aeronautical Science and Engineering (1999)
University of California

Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

Large commercial airports worldwide still experience demand in excess of capacity which leads to considerable delays. As an operational solution to alleviate delays, this thesis presents a model that aims at increasing runway arrival capacity by optimally allocating aircraft to runways such that the overall inter-arrival time between successive aircraft is minimized and the total capacity thus maximized. A mathematical model is presented that consists of two parts: an optimization based on analytic formulations and deterministic assumptions, followed by a Monte Carlo simulation using the results from the first part to account for the probabilistic nature of some variables.

If taken by themselves, the optimization results appear very promising and might also suggest the advisability of developing an algorithm that incorporates our optimal runway allocation scheme into existing air traffic automation systems. However, the simulation suggests that the benefits obtained from the optimization scheme are reduced considerably when stochasticity is taken into consideration. In light of all the practical factors and model simplifications we have identified—and especially in view of the gain reduction under probabilistic assumptions—we conclude that the expected capacity benefits from implementing this optimization model may be quite negligible at most major airports. Thus, an effort to implement the model on a wide scale cannot be recommended, based on the findings of this thesis. Exceptions may exist at airports that have unusually non-homogeneous traffic mixes.

Thesis Supervisor: Amedeo R. Odoni
Title: T. Wilson Boeing Professor of Aeronautics and Astronautics and of Civil and Environmental Engineering
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Most of all, I wish to thank my mother, Johanna, and my brother, Bal, without whom my achievements would have been impossible.
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Chapter 1

Introduction

Increase in air travel during the late 20th century and resulting delays have challenged the aviation industry worldwide to improve the capacity of the airspace system. Air travel around the globe has passed the 1.5 billion passenger per year mark and is expected to increase over the next decade by an average of 5% per year [20].

Seeking possible solutions that rely heavily on the expansion of airfields and on the construction of new runways, run counter to efforts to preserve land resources and reduce noise and particle emissions. Therefore, in addition to being very costly, these solutions are also extremely difficult to implement, often requiring two decades or more between conception and completion. For these reasons, operational solutions that do not require new land and facilities are eagerly sought out by civil aviation authorities and decision-makers—especially, when these solutions have minimal negative environmental impacts, or, better, also provide some mitigation of noise and/or emission effects.

Proposing an operational solution to alleviate capacity constraints, which in turn reduces delays, this thesis presents a model that seeks to increases runway capacity by finding a systematic way for optimizing the allocation of aircraft classes to arrival runways such that the overall inter-arrival time between successive aircraft is minimized and therefore the total capacity for the airport increased.

In Chapter 2, we present a brief review of the extensive literature on runway capacity and on sequencing of runway operations, with some emphasis on models
that significantly contributed to the improvement of the national airspace system.

Next, to provide the reader with the necessary background to fully comprehend the runway capacity problem, Chapter 3 describes the airspace system and its components, in particular airports, air traffic management, regulations, and flight operations and procedures. The concept of capacity is introduced and, as various measures for capacity exist, defined in the context of our work with focus on the runway system.

Since the air traffic flow transitions from three to two dimensions when arriving at the runway threshold, runways most frequently constitute the bottleneck in the dynamic movement of aircraft in and out of airfields. In addition, the arrival phase of any flight demands greatest attention from the flight crew and the air traffic controllers, which frequently complicates the practical implementation of theoretical models or solutions to the capacity problem. It is therefore crucial, and beneficial, to have a practical application in mind when conceiving the development of a theoretical optimization scheme.

Our optimization model is presented in Chapter 4. A mathematical formulation based on some of the best known capacity models is given, the solution process described, and some of the difficulties in converging the solutions explained. The optimization model considers arrival queues of various final-segment lengths, while all four standard aircraft weight classes are being part of the traffic mix. The model is then expanded to account for a simultaneous arrival to either two or three independent runways. By means of a simple example, we illustrate a phenomenon that can be seen from the numerical results as well, which suggests that capacity gains can be obtained by “homogenizing”, as much as possible, the traffic mix on each runway.

Chapter 5 takes the model a step further, expanding it by a Monte Carlo simulation to account for the stochasticity of some of the variables that have thus far been treated in deterministic fashion. The simulation thus aims at providing more realistic answers, and seeks to determine how the gain found in the optimization erodes as we depart from strictly deterministic considerations.

At first glance, the results in section 6.1 appear very promising. Taken by themselves, they might even suggest the advisability of developing an algorithm that in-
corporates our optimal runway allocation scheme into existing air traffic automation systems, like the Center TRACON Automation System (CTAS) and its associated components.

However, in light of all the practical factors and model simplifications we have identified—and especially in view of the gain reduction under probabilistic assumptions shown in the second half of Chapter 6.2—we conclude that the expected capacity benefits from implementing this optimization model may be quite negligible at most major airports. Thus, an effort to implement the model on a wide scale cannot be recommended, based on the findings of this thesis.

Nonetheless, exceptions to our conclusions may exist at airports that have unusually non-homogeneous traffic mixes where our optimization scheme might become applicable. Chapter 7 will offer interpretations and suggestions with respect to possible further research to which our work might prove beneficial. With this, we will then close the report.
Chapter 2

Research and Literature Review

2.1 Historical Notes

Airport capacity and delay problems have been studied by researchers since the end of World War II. Interest in the topic peaked in the 1990s and up to September 2001 when traffic demand increased greatly. Among early researchers, the focus was primarily on the arrival queue assuming, for the main part, a Poisson arrival distribution. In 1960, Blumstein [4] analyzed capacity for landing traffic using discrete and uniform distributions of the arrival speed to compute the mean landing time and thus capacity. He concluded that runway capacity is increased by interposing takeoff operations into landing traffic, by reducing the separation between arriving aircraft, by increasing the arrival velocity, by shortening the final segment of the queue, and by equally dividing the traffic load between multiple runways if such are in use. These points are still very relevant: while much has changed, researchers today still address the very same questions. Subsequent to his publication, Blumstein’s model was further improved and generalized. In 1972, Hockaday and Kanafani [19] examined the interspersion of departing traffic into successive arrivals, while Odoni [31] treated some of the parameters in the arrival model as random variables instead of constants. Finally, in 1981, Swedish [39] presented an extension to two or more runways. Seeking to minimize delay while increasing capacity, Dear [7] introduced what he called Constrained Position Shifting (CPS). In his model, arriving aircraft were, unlike the first-come
first-serve (FCFS) discipline, re-sequenced by up to a maximum number of positions to allow for shorter separation distances and hence a higher arrival rate. Psaraftis [32] followed up on that idea and introduced dynamic programming to minimize either the overall landing time or, alternatively, total passenger delay. In 1987, Trivizas [40] applies combinatorial search methods to runway scheduling with mixed landings and takeoffs on single and multiple runways. Another advanced algorithm is provided by Muhtarremoglu [27]. Although some of these reports address the aircraft sequencing problem, they are nonetheless quite relevant to the runway allocation problem treated in this thesis and are listed here for the sake of completeness.

2.2 Recent Work

In the early 1990s, the Federal Aviation Administration (FAA) undertook initiatives [11] to increase capacity by examining independent parallel approaches to close runways, more accurate range devices with faster update rates known as precision runway monitor (PRM) systems, reduced separation, flight management system (FMS) guided short transition, and use of existing traffic collision avoidance systems (TCAS) for traffic separation.

Additionally, efforts have been made to partially automate the air traffic management system and to provide assistance to the air traffic controller. In 1991, Hazelton [18] developed and tested the artificial-intelligence based “Tower Chief” to assist controllers in scheduling runway configurations. In 1995, Erzberger at the NASA Ames Research Center designed a landing scheduler to “assign arrival aircraft to a favorable landing runway and schedule them at times that minimize delays” [8], while Long et al. presented a dynamic queuing network model to alleviate congestion in the National Airspace System (NAS) [23]. Finally, in 1997, NASA Ames developed, tested, and successfully implemented a knowledge based Final Approach Spacing Tool (FAST) [33] to assist air traffic controllers in their tasks at busy times.

Since then, researchers and officials have been striving to reconcile the often contradictory objectives of maintaining safety on the one hand while achieving high
efficiency on the other. Integrated models have emerged that focus beyond isolated components of airports and that take into account the dynamic characteristics of capacity and stochastic aspects of airport operations [38][2]. With respect to the work presented herein, the concepts presented in this last three paragraphs are most pertinent and will be explained further in section 3.4
Chapter 3

The Capacity Problem

3.1 The Airspace System

With 500,000 to 1 million operations per year and, in some cases, 200 or more operations per hour, the busiest airports worldwide, located in the United States of America, Europe and Asia, still experience demand in excess of capacity which in turn causes delays on the ground and in the air. This problem has been at the forefront of concerns of aviation authorities all around the globe. To effectively address the capacity problem, however, it is essential to understand the underlying air space and Air Traffic Management (ATM) system. The current chapter will provide an overview of the National Airspace System (NAS), in particular its airfields, ATM, and applicable regulations. This is followed by an explanation of how commercial carriers (i.e. airlines) commonly conduct flight operations. Finally, we will introduce the concept of airport capacity and its relation to runway usage which is the main topic of this thesis.

3.1.1 Airfields

Large airports are complex, dynamic systems comprised of terminals, maintenance and servicing facilities, air traffic control facilities, movement and non-movement areas, and last but not least the surrounding airspace. Terminals accommodate pas-
sengers, luggage, and cargo, as they transition from and to the aircraft and constitute the interface to the "outside" world. Maintenance and servicing facilities include repair, cleaning, catering, fire and accident response, push-back, cargo, and transport services. Air traffic control (ATC) organizes the traffic of all vehicles on ground and in the air, commonly housing its command center in a control tower on the field. Finally, movement and non-movement areas are the collective terms for runway, taxiway, tarmac, or apron, gates, and stands respectively.

To avoid impeding the flow of traffic, close collaboration and good coordination of all mentioned components are crucial. Some of them, most notably the runway system, are particularly prone to become "bottlenecks" and are therefore interesting from an optimization point of view. For an in-depth discussion on airport in general the reader is referred to de Neufville and Odoni [6].

3.1.2 Air Traffic Management

Air traffic management (ATM) is the all-embracing term to describe the combination of "procedures and regulations, human air traffic controllers, automation systems, communication systems, surveillance systems, and navigation systems" [6] as implemented at varying levels of sophistication by a nation’s aviation administration. The operational side that directly interacts with the flight crew is commonly referred to as Air Traffic Control (ATC). All aircraft are required to comply with ATC instructions from the moment they start the engine until they dock at the destination gate, except when in distress or emergency. ATC thus has great control over the flow of air traffic and can, at least to a certain degree, reconcile air traffic demand with the actual capacity. ATC is broken up into three principal groups (see Table 3.1).

Tower, departure, and approach controllers usually sit inside one or more airport control towers. However, in busy areas, like New York, approach and departure traffic is handled by Terminal RAdar CONtrol (TRACON), a separate entity located off the airfield that controls traffic in and out of several airports at once (viz. JFK, La Guardia, Newark, and many small fields). TRACON always uses radar, while ATC Towers and ARTCC use radar if installed, but still track and observe manually in
<table>
<thead>
<tr>
<th>Name</th>
<th>Principal Responsibility</th>
</tr>
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<tbody>
<tr>
<td>ATC Tower</td>
<td>Control of traffic on the airport up to a radius of a few miles out. Taxiing and take off instructions, clearances, and advice. Separation between landing and departing aircraft.</td>
</tr>
<tr>
<td>TRACON</td>
<td>Separation and movement of aircraft departing, landing, and maneuvering in the airport environment, usually below 17,000 feet.</td>
</tr>
<tr>
<td>ARTCC</td>
<td>Separation, air traffic clearances, and advice during the en route portions of the flight.</td>
</tr>
</tbody>
</table>

Table 3.1: ATC Facilities

certain areas. The two facilities most closely related to airport traffic and capacity are thus TRACON and Tower Control.

### 3.1.3 Aviation Regulations

Aviation regulations pertinent to airport capacity are laid out primarily in the Code of Federal Regulations Title 14, Aeronautics and Space (14 CFR), traditionally also known as Federal Aviation Regulations (FAR) [28], the Aeronautical Information Manual (AIM) [12], FAA Order 7110.10 (Flight Services) [13], FAA Order 7110.65 (Air Traffic Control) [14], the Pilot Controller Glossary (P/CG) [15], FAA Directive 8260.3b, Terminal Instrument Procedures (TERPS), with changes 1–19 [9], and the FAA Advisory Circular AC150-5300-13, Airport Design, with changes 1–7 [10].

Some sections of these documents are of particular interest in later chapters. They are summarized in Table 3.2.

### 3.1.4 Flight Operations

There are two principal rules under which flights are conducted: Visual Flight Rules (VFR) and Instrument Flight Rules (IFR). Under the former, the pilot navigates by means of visual reference, pilotage, and dead reckoning, while in the second case, the flight is conducted using radio-navigation in combination with inertial reference systems and global positioning systems. Additionally, there are two principal meteorological situations commonly defined by aviation administrators: Visual Meteorolog-
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<tr>
<th>Code</th>
<th>Section</th>
<th>Title</th>
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<tbody>
<tr>
<td>14 CFR</td>
<td>§91</td>
<td>General Operating And Flight Rules</td>
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<tr>
<td>14 CFR</td>
<td>§97</td>
<td>Standard Instrument Approach Procedures</td>
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<td>§3-2-3 and §3-2-4</td>
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<td>§4-4-1 through §4-4-16</td>
<td>ATC Clearances and Separation</td>
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<td>§5-4-1 through §5-4-24</td>
<td>Arrival Procedures</td>
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<td>P/CG</td>
<td>letter A</td>
<td>Aircraft [Weight] Classes</td>
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<td>§2-1-9</td>
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<td>§5-5-4</td>
<td>Minima</td>
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<tr>
<td>ATC</td>
<td>§5-9-5</td>
<td>Approach Separation Responsibility</td>
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<td>§5-9-6 through §5-9-8</td>
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Table 3.2: Regulatory Documents

Ical Conditions (VMC, “out of clouds”), and Instrument Meteorological Conditions (IMC, “in the clouds”). Essentially all scheduled air carrier operations occur under Instrument Flight Rules regardless of meteorological conditions. In the U.S.A., IFR flight regulations are listed in FAR§91 and §97.

A typical flight originates at a stand or gate, where the aircraft is loaded. From there, the airplane is then pushed back onto the tarmac, proceeds via taxiways to the active runway, takes off, flies a departure procedure to cruising altitude, navigates to the destination, descends into the terminal area, intercepts the final approach course, lands, and taxis to the destination gate or stand where it is unloaded. Of this cycle, take-off, final arrival, and landing are the phases at which both pilot and controller experience the highest workload, at which approximately 60% of all accidents occur, and at which the greatest limitations in terms of capacity can be found.

For each flight, an IFR flight plan is filed that contains point of origin, proposed departure time and route, destination airport, and estimated time of arrival (ETA). Theoretically, one could thus conclude that the time schedule of any flight, in particular the arrival time, is exactly known. In reality this is not so. Irregularities in operation, like ground delays, but most of all weather and its associated winds effectively introduce strongly stochastic elements into every air traffic flow. As it affects runway capacity the most, the arrival phase will next be described in detail.
Before a flight crew leaves cruising altitude about 100–130 Nautical Miles away (1NM = 1.852km), they commonly obtain the current airport information for the destination via Automatic Terminal Information Service (ATIS). ATIS is recorded at least hourly by the ATC facility responsible for the particular area, and contains information pertaining weather, unusual conditions, but also active runway(s). About 30–35 NM out, the flight is handed off to approach control which issues a clearance for a particular standard arrival procedure (STAR).

A STAR is made up of several feeder routes towards one or more arrival fixes called Initial Approach Fix (IAFs, see Figure 3-1). The IAF is the place where multiple arrival streams are merged into one queue per active landing runway. This point is emphasized here because, as will be seen later, some optimization methods rely on position shifting of airplanes, which should occur before the IAF.

From the IAF, the two most common arrival procedures are use of an Instrument Landing System (ILS) approach, and the visual approach. In an ILS approach, the
aircraft continues at level altitude to intercept the localizer (a horizontal guidance beam along the extended centerline of the runway) roughly 8NM out. Once established on the localizer, the flight crew then initiates a descent along the glide slope (that is the slanted vertical guidance beam) down to the runway touch-down zone. The point where the glide slope intercept occurs constitutes the Final Approach Fix (FAF). While flying the ILS, the aircraft is thus confined to a narrow path which makes speed adjustment between two subsequent aircraft critical. There are 5 categories of the ILS approaches: I, II, IIIa, IIIb, and IIIc. In increasing order, they refer to increasingly worse visibility and effectively result in larger spacing between arriving airplanes, resulting in longer approach times and often causing delays.

Although the crew is always responsible to see and avoid traffic, separation distance between two aircraft is, particularly in bad visibility situations, the responsibility of, and enforced by ATC. Minimum separation distances have been established in response to wingtip wake vortices generated by aircraft. These vortices are directly proportional to the generated lift, which in turn is a function of weight, speed, and configuration. Slow flying, heavy aircraft have thus historically caused the upset of trailing planes to an extent where the latter have become uncontrollable. The minimum separation ranges from 2.5NM to 6NM, depending on the types of leading and trailing airplanes involved. These distances will be tabulated and used to compute the runway capacity in Chapter 4.

In contrast to the ILS approach, the visual approach procedure authorizes the flight crew to proceed visually and clear of clouds to the airport. The pilot must have either the airport or the preceding aircraft in sight. A pilot’s acceptance of a visual approach releases ATC from its responsibility of traffic separation and is thus issued only in good weather. Visual approaches are known to increase capacity as the crews tend to select the shortest path possible and often intercept the final approach course at a point closer to the runway than the FAF.

Note that both procedures are IFR procedures and that the visual approach is unrelated to flying under VFR. Additionally, in both cases airlines commonly require flight crews to have the aircraft “fully configured” 1000 feet above runway touchdown.
zone elevation (TDZE). Fully configured in this context means having landing gear extended, having landing flaps set, and maintaining landing reference speed \( V_{REF} \), see 14 CFR §25.125) adjusted for wind and configuration. Most glide slopes are set at an angle of 2.5° – 3.0°. Consequently, one can assume airplanes fly at a constant speed for at least the last 3NM, a fact that will also become significant in chapter 4 where the optimization model is introduced.

The final point to be mentioned with respect to arrivals is the inter-dependence of runways according to their geometric layout (see Figure 3-2). Simultaneous (independent) parallel approaches can only be flown onto runways that are spaced at least 4300ft apart. If the distance is less than 4300feet, but greater than 2500feet, staggered approaches have to be flown except if Precision Radar Monitor (PRM) is used, in which case simultaneous close parallel approaches can be flown. Our model will consider independent runways only.

To summarize this section: Approach procedures to, and airspace surrounding an
airfield greatly influence the traffic flow to a runway. The combination of systems, regulations, and procedures forces all arrival traffic into a small number of queues in which aircraft are spaced by a prescribed minimum separation. ATC faces the difficult task of merging the stochastic arrival flow into this small number of queues, while at the same time satisfying the objectives of maintaining fairness in terms of arrival sequencing, separating traffic safely, allocating a runway suitable to ground operations, and minimizing overall delay.

3.2 System Capacity

The capacity of a nation's airspace system depends on the level of technical expertise and available equipment, but also on policy decisions and the related legal framework. In Europe, for example, capacity constraints sometimes stem from strict noise-emission and particle-emission regulations, or from restrictive flow management along airways. In the United States, on the other hand, capacity limitations are frequently found on the runways. Despite these differences, runway capacity is an important factor in determining the overall system capacity both in the United States and in Europe.

3.3 Runway Capacity

Runway capacity is defined in four ways: maximum throughput capacity, practical hourly capacity, sustained capacity, and declared capacity [6]. Maximum throughput capacity (or saturation capacity) is defined as the expected number of movements that can be performed in one hour on a runway system without violating ATM rules, assuming continuous aircraft demand. In contrast, practical hourly, sustained, and declared capacity all depend on some qualitative level of service—that is delay—in their respective definition. Because it is the least subjective measure, maximum capacity is what will be understood henceforth whenever reference to capacity is made.
<table>
<thead>
<tr>
<th>Airport</th>
<th>Runways</th>
<th>Annual Movements</th>
<th>Hourly Capacity</th>
</tr>
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<tbody>
<tr>
<td>Genève Cointrin</td>
<td>1</td>
<td>170,000</td>
<td>40</td>
</tr>
<tr>
<td>London Heathrow</td>
<td>3</td>
<td>460,000</td>
<td>90</td>
</tr>
<tr>
<td>Chicago O’Hare</td>
<td>7</td>
<td>930,000</td>
<td>200</td>
</tr>
</tbody>
</table>

Source: FAA / Eurocontrol

Table 3.3: Runway Capacity, 2003

Factors that affect capacity include geometric layout and configuration of the runways, weather, taxiway system, runway exits, ATC infrastructure and equipment, legal and environmental constraints, aircraft mix of weight classes, and the required separation of traffic. Although airports have only one geometric runway layout that is given when they are built, runways can be used in various configurations according to the prevailing winds and operational or environmental considerations. A single runway, for example, only leaves two options in terms of configuration (use in one or the other direction), but still allows for departing traffic to be interspersed into the arrival flow at various rates. In contrast, multiple runways allow for various configurations and usage—that is landing–only, take–off only, or mixed traffic. Table 3.3 shows three international airports, their number of runways, and their respective annual movements and throughput capacity.

The numbers given for hourly capacity in Table 3.3 most often refer to an equal split of arrivals and departures. However, this is not necessarily occurring continuously in reality. Many airlines still schedule what is known as “banks” of arrivals—a practical solution for transfers—that leads to a wave–like pattern of arrivals and departures. However, looking at an individual runway, aircraft accelerate after take–off and, for the most part, take different routes, resulting in a departure capacity that is usually higher than the arrival capacity.

Gilbo at the Volpe National Transportation systems Center has, for a decade, been developing algorithms to optimize the dynamic sequencing of arrivals and departures under the given constraints of demand and weather [16][17]. Attempts to avoid demand in excess of capacity have led, as part of Collaborative Decision Making, to a Ground Delay Program (GDP) similar in nature to the Central Flow Management
(CFM) used in Europe by which aircraft are kept on the ground until it is reasonably assured that they will not have to hold (in the air) above the destination airport. Naturally, an increased arrival capacity reduces holding time delay as long it is supported by the infrastructure on the field, viz. as long as there are enough gates, stands, and ground transportation.

3.4 Increasing Arrival Capacity

With this background, we turn the focus to the core problem: the increase of arrival capacity onto runways, and the fundamental question of whether physical or operational changes are the most effective for this purpose.

3.4.1 Physical Measures

If the available number of runways on an airfield has been identified as the limiting element, the immediate solution that comes to mind to increase capacity is to build one or more additional runways. Yet, many countries, especially in Europe, are hesitant to pave vast land resources. Moreover, building another runway inevitably brings up the question of where and how. Sometimes, the environmental or financial cost of building another runway is greater than the potential gain. Capacity increases may also be obtained by modifying existing runways. For example, high-speed turn-offs onto taxiways are a common solution for reducing the runway occupancy time of the leading aircraft, which is the criterion for issuing landing clearance to the next plane on final approach. If physical measures are infeasible, as is often the case, it is worthwhile to take a look at operational options.

3.4.2 Operational Measures

As indicated in Chapter 2, much research has already been done in the quest of finding operational procedures that increase airport capacity. However, this section will discuss the relevance of some of this research to the work presented here. The
three main options in maximizing throughput capacity seem to narrow down to:

(a) Sequencing the traffic flow such that the resulting average separation distance, and thus the average inter-arrival time, is minimized, resulting in an increased throughput.

(b) Reducing separation distances between airplanes.

(c) Allocating aircraft to individual runways (when more than one are available) such that the overall throughput is maximized.

Re-sequencing aircraft as opposed to applying a First Come First Serve (FCFS) discipline is known as Position Shifting (PS). If the number of positions by which an aircraft can be shifted is limited, the resulting regime is referred to as Constrained Position Shifting (CPS). PS methods inherently harbor two difficulties: fairness and feasibility. In practice, PS methods show limited gain in throughput, however, they have the potential to substantially reduce the delay [8].

Alternatively, capacity may be increased by reducing the minimum separation between arriving aircraft. NASA Langley’s reduced separation algorithm, the Aircraft VORtex Spacing System (AVOSS), has achieved an average 6% capacity gain at Dallas Fort Worth, one of the busiest airports in the world [30].

Finally, as part of the Center–TRACON–Automation–System (CTAS) program, NASA Ames researchers have developed a Final Approach Spacing Tool (FAST) to help air traffic controllers allocate traffic to runways efficiently while they are busy with their main task of traffic separation. Runway allocation, that is the decision which runway an inbound aircraft should land on, is done by the approach controller when demand is high. FAST emulates, and learns from, expert controllers to create its database. It then finds a runway assignment algorithm that considers overall system delay, scheduled time of arrival, as well as aircraft and engine type. FAST has achieved a capacity improvement of 13.3% on average [34] and has been upgraded to aFAST (active FAST) to provide speed, heading, and altitude commands.

In conjunction with the FAST algorithm, one wonders if the capacity could possibly be further increased by allocating aircraft to the runways based on their weight.
classes with the objective of reducing the average time separation between consecutive landing aircraft. This thesis addresses precisely this question. In particular, it seeks to determine whether a theoretical gain could even be achieved. That is, if the minimum separation is never violated, the aircraft approach speed given, and the mix of overall arrival traffic known, is there a better mix for each runway—still meeting the constraining overall distribution—such that the total capacity for the airport is increased.

Chapter 4 will provide the mathematical optimization model used, while Chapter 6 will present the resulting gain.
Chapter 4

Optimization of Arrival Traffic

4.1 Problem Statement

For the purposes of Wake Turbulence Separation Minima, ATC classifies aircraft as Heavy, Large, and Small (see P/CG, AIM, Order 7110.10, and Order 7110.65P) as follows:

(a) **Heavy**- Aircraft capable of takeoff weights of more than 255,000 pounds whether or not they are operating at this weight during a particular phase of flight.

(b) **Large**- Aircraft of more than 41,000 pounds, maximum certificated takeoff weight, up to 255,000 pounds.

(c) **Small**- Aircraft of 41,000 pounds or less maximum certificated takeoff weight.

The question we pose is the following: Assuming a constant nominal approach speed, and further assuming that we know the probability distribution (that is the mix of the classes) of the arriving aircraft for the entire airport, does a systematic method exist to allocate traffic to more than one active runways such that the average throughput capacity is maximized. To answer this question, we will now introduce a mathematical model.
4.2 Optimization Model

In accordance with FAA Order 7110.65, the separation distances shown in Table 4.1 apply on final approach. There is a separate column for the Boeing 757, which would technically fall into class "large", but has more restrictive separation requirements due to its unusually large vortex generation. Thus, instead of the three basic classes listed in FAA Order 7110.65, we are effectively dealing with four. Moreover, Table 4.2 gives the nominal final approach speeds that were used in this thesis for each class. Final approach speed, commonly measured in knots (kts., nautical miles per hour), is a function of weight and configuration, and is then corrected for wind and gust. Here, all approach speeds have been taken near their lower limits to give the most conservative result in terms of capacity gain.

<table>
<thead>
<tr>
<th>NM</th>
<th>Trailing Traffic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
</tr>
<tr>
<td>Small</td>
<td>2.5</td>
</tr>
<tr>
<td>Leading Large</td>
<td>4.0</td>
</tr>
<tr>
<td>Traffic B757</td>
<td>5.0</td>
</tr>
<tr>
<td>Heavy</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Table 4.1: Wake Separation

<table>
<thead>
<tr>
<th>Knots</th>
<th>Small</th>
<th>Large</th>
<th>B757</th>
<th>Heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>130</td>
<td>140</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Final Approach Speed

4.2.1 Assumptions

The model in the next section is based on assumptions that directly result from Chapter 3 and standard concepts of probabilistic analysis and queuing theory. They can be summed up as follows:

1. Every aircraft belongs to one of 4 classes: Small (S), Large (L), Boeing 757 (B), or Heavy (H).
2. At a particular distance from each runway threshold, there is a point we call “gate”, by which all traffic assigned to that particular runway has merged into one queue.

3. The entire system consists of either a single, dual, or triple server arrangement, corresponding to 1, 2, or 3 runways, each of which is associated with its own queue and final approach gate.

4. Demand meets or exceeds capacity of the server, that is to say that there are no gaps in the queue and capacity is determined solely by the throughput on the runway threshold.

5. Aircraft arrive at the gate of each runway and are, independently, in random order, served in a FCFS discipline without being re-sequenced or side-stepped to other runways.

6. Passing the gate, aircraft maintain minimum separation and constant approach speed.

7. The probability distribution of the 4 classes of aircraft, i.e. the traffic mix, is known.

8. All active runways considered are devoted entirely to landing traffic.

4.2.2 Mathematical Formulation of the Capacity Model

With the assumptions made in the preceding section in mind, we will first present a capacity model based on Blumstein’s formulation [4]. We will then solve an optimization model aimed at maximizing total throughput capacity when more than one active runways are in use.

Considering first a single runway that is used exclusively for landing, we define the following quantities:

\[ r := \text{the length of the common final approach path.} \]
\( v_i := \) speed on final approach assuming, as a reasonable approximation, that aircraft \( i \) maintains a constant speed throughout the approach.

\( o_i := \) the runway occupancy time, i.e., the time that elapses from the instant when the aircraft touches down on the runway to the instant when it leaves the runway at one of the runway exits.

Now we consider the case in which an aircraft of type \( i \) is landing, followed immediately by another aircraft of type \( j \), then:

\[ s_{i,j} := \text{the minimum separation required by ATC between the two aircraft while they are both airborne.} \]

\[ t_{i,j} := \text{the minimum time interval (in the sense of not violating any ATM separation requirements) between the successive arrivals at the runway of the type} \]
\[ \text{\( i \) and type} \ j \text{\ aircraft.} \]

Then, applying Blumstein's and Odoni's formulation [4][6]:

\[
 t_{i,j} = \begin{cases} 
 \max(\frac{r+s_{i,j}}{v_j} - \frac{r}{v_i}, o_i) & v_i > v_j \\
 \max(\frac{s_{i,j}}{v_j}, o_i) & v_i \leq v_j 
\end{cases} \tag{4.1}
\]

Because separation distances are set such that runway occupancy time is never the constraint, we henceforth disregard \( o_i \).

Next, if for any pair of aircraft in the queue, \( p_i \) denotes the probability that the leading aircraft will be of type \( i \), and \( p_j \) denotes the probability that the trailing aircraft will be of type \( j \), then, assuming FCFS, the probability that we find an aircraft of class \( i \) followed by one of class \( j \) is given by \( p_i \cdot p_j \). Hence, the expected inter-arrival time can be stated as:

\[
 E(t_{i,j}) = \sum_{i=1}^{C} \sum_{j=1}^{C} p_i p_j t_{i,j} \tag{4.2}
\]
where \( C \in \{1, 2, 3, 4\} \) represents the set of weight classes in which 1 stands for “small”, 2 for “large”, 3 for “B757”, and 4 for “heavy”. This numbering convention in replacing the class names will be implied for the remainder of this thesis.

Expanding (4.2) yields

\[
E(t_{i,j}) = \{p_1^2t_{1,1} + p_1p_2t_{1,2} + p_1p_3t_{1,3} + p_1p_4t_{1,4} \\
p_1p_2t_{1,2} + p_2^2t_{2,2} + \cdots + \cdots \\
\cdots + \cdots + p_3^2t_{3,3} + \cdots \\
\cdots + \cdots + \cdots + p_4^2t_{4,4}\} \\
= \mathbf{p}^T T \mathbf{p}
\]

For any time matrix \( T \), the last term is a positive scalar since the probability vector is one-dimensional and probabilities and times take on strictly positive values. Typical time matrices, measured in units of hours, are for 3 and 10 miles, respectively

\[
T_3 = 10^{-2}. \begin{bmatrix} 2.50 & 1.92 & 1.78 & 1.66 \\ 4.69 & 1.92 & 1.78 & 1.66 \\ 5.85 & 3.24 & 2.85 & 2.66 \\ 7.00 & 4.15 & 3.00 & 2.66 \end{bmatrix} \quad T_{10} = 10^{-2}. \begin{bmatrix} 2.50 & 1.92 & 1.78 & 1.66 \\ 6.30 & 1.92 & 1.78 & 1.66 \\ 7.85 & 3.62 & 2.85 & 2.66 \\ 9.33 & 4.87 & 3.33 & 2.66 \end{bmatrix}
\]

Here, equation (4.1) has been applied to obtain \( T_3 \) and \( T_{10} \), using the values given in Tables 4.1 and 4.2. We thus find the expected hourly throughput (arrival) capacity, \( K \), to be the reciprocal of \( E(t_{i,j}) \), that is

\[
K = \frac{1}{E(t_{i,j})}
\]  

(4.4)

4.2.3 A Simple Numerical Example

Before expanding the capacity model to multiple runways, we first present a simple example involving a single runway and two aircraft classes to illustrate how the
clustering of aircraft can affect throughput. We compare two queues containing 10 aircraft each, whereof 5 are of a smaller, and the other 5 of a larger class. In the first queue, planes arrive at the gate in random order, while in the in the second queue, the 10 aircraft arrive sorted according to their classes: first all 5 larger aircraft followed by all 5 smaller aircraft. (Fig 4-1).

Figure 4-1: Two Simple Arrival Queues

From (4.1), the matrix that yields the minimum overall time, $T^*$, is obtained when the separation between two aircraft remains the same if their order is reversed, i.e. $s_{2,1} = s_{1,2} = s$, and if the approach speed, $V_{REF}$, of the larger aircraft is only minimally higher, viz. $v_2 = v_1 + \delta = v + \delta$; that is:

$$T^* = \begin{bmatrix}
\frac{s}{v} & \frac{s}{v + \delta} \\
\frac{r + \delta}{v} & \frac{r}{v + \delta}
\end{bmatrix}$$

Rearranging entry $(1,1)$ and $(2,1)$ in $T^*$ as $\frac{s}{v + \delta} + \frac{\delta s}{v(v + \delta)}$ and $\frac{s}{v + \delta} + \frac{\delta s + \delta r}{v(v + \delta)}$, respectively, we may write

$$T^* = \begin{bmatrix}
\frac{s}{v + \delta} + \frac{\delta s}{v(v + \delta)} & \frac{s}{v + \delta} \\
\frac{s}{v + \delta} + \frac{\delta s}{v(v + \delta)} + \frac{\delta r}{v(v + \delta)} & \frac{s}{v + \delta}
\end{bmatrix} = \begin{bmatrix}
t + \alpha_1 & t \\
t + \alpha_1 + \alpha_2 & t
\end{bmatrix}$$

where $\alpha_1 = \frac{s\delta}{v(v + \delta)}$ and $\alpha_2 = \frac{r\delta}{v(v + \delta)}$. The expected arrival time for the 10th plane after the first has landed is thus:

In Queue 1

$$E_1(t_{10}) = 9 \left( .5^2(t + \alpha_1) + .5^2t + .5^2(t + \alpha_1 + \alpha_2) + .5^2t \right)$$

$$= 9t + 4.5\alpha_1 + 2.25\alpha_2$$
In Queue 2

\[ E_2(t_{10}) = 4t + (t + \alpha_1 + \alpha_2) + 4(t + \alpha_1) \]
\[ = 9t + 5\alpha_1 + \alpha_2 \]

For our proposition to hold that the arrival time for all aircraft in Queue 2 is less than the arrival time for all aircraft in Queue 1, we need \( E_1(t_{10}) - E_2(t_{10}) > 0 \). From the preceding two equations we obtain:

\[ E_1(t_{10}) - E_2(t_{10}) = -0.5\alpha_1 + 1.25\alpha_2 \]
\[ = \frac{\delta}{v(v + \delta)} \cdot (1.25r - 0.5s) > 0 \]

This last inequality holds whenever \( r > 0.4s \). In addition, Table 4.1 gives a maximum separation distance of 6NM. Hence, for \( E_2(t_{10}) < E_1(t_{10}) \), we need \( r > 2.4NM \). However, since aircraft need to be fully configured at 1000 ft, we may safely assume that \( r \geq 3NM \) and thus conclude that, on average, the last aircraft in Queue 2 arrives earlier than the last aircraft in Queue 1.

Despite the limitation of using a set of only 10 airplanes and two classes, the result suggests an interesting and important phenomenon that will become quite evident from the numerical results as we proceed: The optimum capacity is reached if aircraft have the freedom to homogenize into their respective classes. As analogy, one could think of the queues in terms of fluid dynamics, where a perturbed flow carries more energy—just like a disturbed queue causes longer inter-arrival times on average.

\[ \text{4.2.4 Formulation of the Optimization Problem} \]

We will now expand our model to an airfield with multiple runways in use. Let the number of “active” runways be equal to \( R \). If all runways are assigned the same traffic mix, then the capacity of the R runways found by multiplying (4.4) by R is:
\[ K = \frac{R}{E(t_{i,j})} \] (4.5)

However, for any particular overall mix of traffic at the airport, the throughput may be increased by allocating aircraft to runways such that the traffic mix on each runway may become more uniform than the overall mix for the entire airport. Formulating a corresponding mathematical model, the following constraints apply:

The sum of all aircraft in one particular class on all active runways has to match the overall number of aircraft in that very same class. *(Conservation).*

The sum of the probabilities of the 4 classes on one particular runway have to sum to 1. *(Conditional Probability).*

The probability that an aircraft of class \( c \) is found in the overall mix, \( p_c \), is constrained to the interval \( p_c \in [0, 1] \). Similarly, the conditional probability that, in the mix, an aircraft of class \( c \) is found *given that* it is on runway \( r \), \( x_c = x_c(p_c|R = r) \), is constrained to the interval \( x_c \in [0, 1] \). *(Definition of Probability).*

Expressed mathematically, the model can be stated as follows:

If

\[ C := \text{the total number of classes in the set of classes} \] (4.6)

\[ R := \text{the total number of runways in the set of runways} \] (4.7)

\[ c \in \{1, 2, \ldots, C\} \subset \mathbb{Z} \] (4.8)

\[ r \in \{1, 2, \ldots, R\} \subset \mathbb{Z} \] (4.9)

\[ p = [p_1 \ p_2 \ \ldots \ \ p_c \ \ldots \ p_C]^T \] (4.10)

\[ x = [x_{r,1} \ x_{r,2} \ \ldots \ x_{r,c} \ \ldots \ x_{r,C}]^T \] (4.11)

Then
find \( x_{r,c} \) for all \((r,c)\) combinations that

\[
\text{maximize: } K = \sum_{r=1}^{R} K_r = \sum_{r=1}^{R} \frac{1}{E_r(t)}
\]

\[
\text{subject to: } 0 \leq p_c \leq 1 \text{ and } 0 \leq x_{r,c} \leq 1 \quad \forall c, r
\]

\[
\sum_{c=1}^{C} p_c = 1 \quad \text{and} \quad \sum_{c=1}^{C} x_{r,c} = 1 \quad \forall r
\]

\[
\sum_{r=1}^{R} K_r x_{r,c} = p_c \sum_{r=1}^{R} K_r \quad \forall c
\]

Therefore, for 4 classes and 2 runways:

\[
\text{maximize: } K = \frac{1}{x_1^T T x_1} + \frac{1}{x_2^T T x_2}
\]

\[
\text{subject to: } 0 \leq \{ p, x_1, x_2 \} \leq 1
\]

\[
\sum_{c=1}^{C} \{ p, x_1, x_2 \} = 1
\]

\[
x_1 (x_2^T T x_2) + x_2 (x_1^T T x_1) = p (x_1^T T x_1 + x_2^T T x_2)
\]

Note that \( x^T T x \) is a scalar, and that 4 equations are hidden in the last constraint.

The 3 runway case is a straightforward expansion of (4.17) through (4.20). We have now found a mathematical formulation that we can implement into a solver.

### 4.2.5 Method and Implementation

Solving the problem (4.13)–(4.16) has proved far from trivial in some cases, due to severe difficulties with getting the solution to converge. The following discussion explains the methods applied, and difficulties encountered, to produce the results listed in Chapter 6.

First, the probabilities, \( p_c \), have been discretized into steps of 0.1 and each combination of \( p = [p_1 \ p_2 \ p_3 \ p_4] \) has been called a “case”. This discretization is sufficient
in determining capacity gain and allows for a case to be held fixed while solving the
system for the solution vector, $\mathbf{x}$, that represents the mix on each runway.

Next, expanding the left hand side of equations (4.16) and (4.20) for two and three
runways to

$$L.H. = x_{1,1}x_{2,1}^2t_{1,2} + x_{1,1}x_{2,1}x_{2,2}t_{1,2} + \cdots + x_{2,1}x_{1,4}^2t_{4,4}$$

and

$$L.H. = x_{1,1}x_{2,1}x_{3,2}^2t_{1,1}^2 + x_{1,1}x_{2,1}x_{2,2}t_{2,1}x_{3,2}x_{3,1}t_{2,1} + \cdots + x_{3,4}x_{1,4}x_{2,4}^2t_{4,4}^2$$

illustrates that the system consists of a hyperbolic objective function with a cubic or
quintic constraint in $C \cdot R$ variables depending on the number of classes and runways.
Although there is no effective method for solving the general nonlinear problem [5],
the following techniques have helped in arriving at solutions.

First, to obtain equations of simpler forms, constraints and objective function
have been rearranged, viz. to minimize $E(t)$ instead of maximizing $K$ or minimizing $-K$.
Second, several approaches were taken to obtain a suitable initial guess
for $\mathbf{x}$ insofar as it greatly affects convergence. Initial vectors that contain nearly
equally distributed classes almost never converge while initial vectors that use an
extreme value as predictor for the dominating class on one of the runways converge
quickly (viz. $\mathbf{x}_0 = [0.01, 0.9, 0.01, 0.01]$ could be such a vector if $p_2 > 0.3$.)
Third, several non-linear solvers (e.g. Matlab®, LOQO), and two "brute force" FORTRAN
codes were tried but failed to produce solutions. Last, the output from a Sequential
Quadratic Programming (SQP) routine revealed negative eigenvalues of the Hessian
of the respective Lagrangian function, suggesting non-convexity of the constraints.

The inspiration that eventually led to a breakthrough in numerical convergence
came from various sources [5][36][1][29], but mostly from Scales who wrote: "The
intersection of a set of non-linear equality constraints is in general a curved hypersur-
face for which feasible directions from any point will not exist. When the constraints
are inequalities, feasible directions may well exist ..." The equality constraints have
thus been relaxed to inequalities hoping that, in the limit, the solution would merge
with the constraint boundary. This is precisely what happened.

Having obtained satisfactory solutions, the length of the final segment has been varied from $3 \, NM$ to $10 \, NM$, motivated by the following considerations. While separation is determined by FAA regulations and aircraft type, approach speed and length of final segment in equations (4.1) and (4.3) vary. Moreover, we only assumed a reasonable approach speed, but have not yet determined a reasonable final-segment length. On the one hand, the queue forms at about $10NM$ from the runway threshold, which would justify to choose $r = 10 \, NM$. On the other hand, aircraft fly at constant speed only after reaching $V_{REF}$ plus the prescribed adjustment, which occurs no later than $3NM$ (see section 3.1.4), possibly suggesting $r = 3 \, NM$. To account for both of these factors, the two endpoints and one intermediate point in the range of final-segment lengths have been selected and computed individually, that is all cases were computed for $3NM$, $5NM$, and $10NM$, respectively.

Before discussing the results in Chapter 6, we shall now turn to the simulation that has been performed based on the deterministic solutions found.
Chapter 5

Simulation

5.1 Deterministic vs. Probabilistic Assumptions

In Chapter 4, we have presented a mathematical model that serves as a reasonable description to compute the runway arrival capacity to an airfield if the approach speeds are constant, the separation invariable, and the mix assumed to be exact. These deterministic assumptions, however, detach the model somewhat from realism. Real operations, in particular real arrival procedures, are comprised of stochastically varying properties for which we would need to make allowance.

For this reason, a Monte Carlo simulation has been designed that will treat some parameters as random variables. The simulation will indicate how the results obtained from the deterministic model will deteriorate under probabilistic considerations. First, although the ideal mix might be known, aircraft arrive in random order wherefore the class has to be treated as random variable. Second, all approach speeds listed in table 4.2 are merely target values and change with aircraft weight, wind, and configuration and can thus not be taken as constants. Third, despite their striving for tightest possible separation, even the most skilled air traffic controllers still exceed minimum separation by $0.25 - 0.5$ nautical miles [35]. We should thus assume class, speed, and separation to be random variables.
5.2 Mathematical Formulation

The following three subsections will derive the probability mass or probability density functions, \( f(x) \), and the corresponding cumulative distributions, \( F(x) \), for each random variable mentioned above. From \( F(x) \), we will then obtain an inverse function, \( F^{-1}(x) \), for which a uniformly distributed random number is drawn as input to retrieve the original distribution \( f(x) \). \( F^{-1}(x) \) is listed at the end of each subsection as it has been used in the simulation code. Also note that, by definition, \( F(x) = \int_{-\infty}^{x} f_X(\gamma)d\gamma \).

5.2.1 Random Weight Class

Each case, that is each permutation of the four probabilities constitutes a new, discrete probability mass function. That is, if \( \mathbf{p} = [p_{\text{small}} \ p_{\text{large}} \ p_{\text{b757}} \ p_{\text{heavy}}] \), and if we number the classes from 1 to 4 as has been done in chapter 4, then:

\[
f(x) = \begin{cases} 
p_{\text{small}} & x = 1 
p_{\text{large}} & x = 2 
p_{\text{b757}} & x = 3 
p_{\text{heavy}} & x = 4 
0 & x \notin \{1, 2, 3, 4\} 
\end{cases}
\]

\[
F(x) = \begin{cases} 
0 & x < 1 
p_S & 1 \leq x < 2 
p_S + p_L & 2 \leq x < 3 
1 - p_H & 3 \leq x < 4 
1 & 4 \leq x 
\end{cases} \quad (5.1)
\]

For example, if \( \mathbf{p} = [2.4.1.3] \), then \( f(x) \) and \( F(x) \) are as given in Figure 5-1.

Attention has to be given when trying to find the inverse \( F^{-1}(x) \). Here, it does not exist on the entire domain since we obtain multiple values for, say, \( F^{-1}(0.7) \), namely 3 and 4. When coding the equations, we thus have to implement a switch, which is effectively done by the correct choice of the relation operator, \( \leq \) or \( < \). Hence, \( F^{-1}(x) \)

46
becomes

\[ F^{-1}(r) = x = \begin{cases} 
1 & 0 \leq r < p_s \\
2 & p_s \leq r < p_s + p_L \\
3 & p_s + p_L \leq r < 1 - p_H \\
4 & 1 - p_H \leq r < 1 
\end{cases} \tag{5.2} \]

From the piecewise equation 5.2, any random number \( r \in (0, 1) \) will now yield one of the four classes and, ad infinitum, also match their original distribution.

### 5.2.2 Random Approach Speed

A similar procedure is applied to randomize the approach speed, except that the speed is a continuous variable that is modeled by a probability density function (PDF). We assume a triangular distribution in which the value underneath the top vertex, \( x = c \), represents the target speed, while the two base vertices represent the lower and upper extrema, \( x = a \) and \( x = b \), respectively. From elementary geometry, \( f(c) = \frac{2}{b-a} \). The triangular PDF is therefore

\[ f(x) = \begin{cases} 
\frac{2(x-a)}{(b-a)(c-a)} & a \leq x < c \\
\frac{2(b-x)}{(b-a)(b-c)} & c \leq x < b \\
0 & x < a, b \leq x 
\end{cases} \]

\[ F(x) = \begin{cases} 
0 & x < 0 \\
\frac{(x-a)^2}{(b-a)(c-a)} & 0 \leq x < c \\
1 - \frac{(x-b)^2}{(b-a)(b-c)} & c \leq x < b \\
1 & b \leq x 
\end{cases} \tag{5.3} \]
which, assuming a target approach speed of 150 knots with a ±5 knot deviation translates to

![Velocity PDF and CDF](image)

Figure 5-2: Speed Probability Density and Cumulative Distribution

Again, the CDF, F(x), is only a one-to-one function on each subinterval a < x < c and c < x < b, and has a cusp at x = c requiring a brief analysis at that point. We can find a piecewise inverse function on each subinterval such that \( F^{-1}(r) = x \), where \( r \) is the random number drawn. In this case, the piecewise function is

\[
F^{-1}(r) = x = \begin{cases} 
\pm [r(b-a)(c-a)]^{1/2} + a & a \leq x < c \\
\pm [(1-r)(b-a)(b-c)]^{1/2} + b & a \leq x < c 
\end{cases}
\] (5.4)

The correct sign is taken from the physical interpretation: \( a \) represents the minimum speed and \( b \) the maximum speed, respectively. This implies that we may only add values to \( a \) and only subtract values from \( b \). Hence, we choose the positive sign on the top square root and the negative on the bottom, such that \( x \to a \) as \( r \to 0 \), and \( x \to b \) as \( r \to 1 \). With this, we are assured that for \( r \in (0, 1) \), \( x \in (a, b) \). The fact that the intervals are open shall not be of our concern as we can approach infinitely close to the endpoints.

### 5.2.3 Random Wake Separation

To model separation, we assume another triangular distribution that reflects the controllers' tendency to space as closely to the target distance as possible. We let this target distance be \( (x = a) \), and the upper endpoint representing the maximum
sum of separation plus deviation that will occur \((x = b)\). Then

\[
f(x) = \begin{cases} 
\frac{2(b-x)}{(b-a)(b-c)} & c \leq x < b \\
0 & x < a, b \leq x 
\end{cases} \quad F(x) = \begin{cases} 
0 & x < 0 \\
1 - \frac{(x-b)^2}{(b-a)^2} & a \leq x < b \\
1 & b \leq x 
\end{cases}
\]

(5.5)

For a 2.5 \(NM\) separation with a 0.5 variation, the pdf and cdf are, respectively

![Graph](image)

Figure 5-3: Separation Probability Density and Cumulative Distribution

Analogous to equation (5.4):

\[
F^{-1}(r) = x = \pm [(b - a)(1 - r)]^{1/2} + b
\]

(5.6)

Again, we choose the negative sign on the square root such that values are only subtracted from \(b\) and \(x \to a\) as \(r \to 0\). This way, \(x \in (a, b)\) if \(r \in (0, 1)\).

### 5.2.4 Random Number Generator

Last but not least, a suitable number generator needs to be selected to feed the preceding three groups of equations with a uniformly distributed random number. The type of random number generator, like the selection of the random seed, may both significantly influence the simulation result [22][25][24]. The random number generator provided with the FORTRAN compiler we used produced unsatisfactory results and was consequently replaced by Mersenne Twister (MT19937) [26]. Developed by Matsumoto and Nishimura at Keiko University, Japan, the MT19937 algorithm pro-
duces a number with a period of $2^{19937}$ and is known to have passed all commonly applied statistical tests [21][37], including John Walker’s entropy (ENT) test and George Marsala’s “DieHard” test [3]. MT19937 has, as will be seen in section 6.1, produced good reproductions of the simulated distributions throughout.

5.3 Simulation Implementation

Two runs were conducted each for each combination of final-segment length and number of runways in turn. In the first scenario, 1,000 simulated aircraft were flown onto each runway while the arrival times for all aircraft were measured. The second scenario was a repetition of the first, but with 1,000,000 airplanes instead. The simulation was entirely written in FORTRAN and produced run times from 1 – 4 seconds in the first scenario to 33 – 160 minutes in the second. This completes the simulation section. We will now turn to the discussion of the results presented in Chapter 6.
Chapter 6

Results

6.1 Optimization Results

We will first give a brief account of the goodness of the results. In each permutation, the final solution was back substituted into the constraint function to compute the “residual” of the optimization, defined as $RES := \forall c \sum_{r=1}^{R} K_r x_{r,c} - p_c \sum_{r=1}^{R} K_r$. This residual has been in the order of $10^{-6}$, which is satisfactory insofar as the objective function varies in the same order of magnitude. The values of the residual to each combination of number of runways and final-segment length are given in Figure 6-1.

It has been our goal to maximize runway capacity with an arrival mix composed of four classes. Yet, numerical difficulties led to the calculation of simpler, two and three class cases first. These much simpler cases provided insight in some basic behavior of the functions, served as an intermediate step to the next level of numerical complexity, and allowed for a result verification between the two, three, and four class cases whenever they had an equivalent mix, e.g. $p_{(2)} = [0.3, 0.7]$ and $p_{(4)} = [0, 0.3, 0, 0.7]$. All solutions of the two and three class cases are contained in the set of the four class solutions. However, we will still present selected two and three class results to explain some of the mechanics in a less complex and more illustrative fashion.
OPTIMIZATION RESIDUAL
3, 5 and 10 NM Final;
2 and 3 Runways Respectively

Figure 6-1: Constraint Residual
6.1.1 Two-Class Cases

The two class cases can be solved analytically, and might even appear trivial. Nonetheless, these cases explain well the interaction between length of final segment, number of runways, and type of classes involved. Looking at Figure 6-2, we see how the lines connect at each endpoint. These two points represent the local maxima and are a logical consequence of the presence of only one class. For a single class, the non-optimized and optimized values are the same, but they simultaneously vary with approach speed or separation of that single class. The left end of the curve shows a capacity lower than the right end because class “heavy” is subject to a 4NM separation while class “large” is subject to a 2.5NM separation. The fact that the “heavy” aircraft flies 20 kts faster than the “large” plane has less bearing on capacity than the separation difference of 1.5 NM. In terms of inter-arrival time, a change in separation of only 1NM has the same effect as changing the approach speed by 28%, or 41 knots at a speed of 150 (refer to equation (4.1)).

Departing from either endpoint along the same curve, the non-optimized capacity drops as the classes start to mix. This happens as the separation in the now mixed traffic may increase to 5NM, namely whenever a “large” aircraft is trailing a “heavy” aircraft. At the same time, the optimized capacity increases on some given interval, resulting in a maximum gain somewhere in the interior region—here at \( p_L = 0.6 \). This maximum constitutes the point at which runway allocation is most effective, likely allowing one aircraft class to occupy a runway by itself, viz. form a homogenized flow. From Table 6.1, one can read that, with the exception of \( p_L = 0.6 \), the dominating class is essentially always occupying a runway by itself.

Adding a third runway, the capacity increases by approximately 50% over the two-runway configuration (Figure 6-3). In spite of this capacity increase, the gain achieved through optimization is not significantly greater than that seen in the two runway case, but extends over a slightly larger interval. We will later see a similar phenomenon in the four class cases.

Increasing the length of the final segment causes the total capacity to drop but
Figure 6-2: Two-Class Capacity and Gain: First Case
CAPACITY
2 Classes (Large & Heavy), 3 NM Final, 3 Runways

ABSOLUTE GAIN
2 Classes (Large & Heavy), 3 NM Final, 3 Runways

PERCENT GAIN

Figure 6-3: Two-Class Capacity and Gain: More Runways
the potential gain to rise. In Figure 6-4, the minimum reaches 70 aircraft/hour as opposed to the 75 seen initially in Figure 6-2. The reason for this drop is the extended time during which a leading, slower airplane possibly thwarts a trailing, faster one. For the very same reason, gain is potentially increased.

Last, we exchange class “heavy” with “B757”. Since the “B757” is assumed to have a 10 kts slower approach speed, the capacity of $p_B$ is approximately 5 aircraft/hour less than the capacity of $p_H$ (Figures 6-5 and 6-2). Moreover, despite segregating onto individual runways, the two classes “heavy” and “B757” fall into a constellation of speed and separation distances that only allows for a small gain to be possible (i.e. 1.5 aircraft/hour).

### 6.1.2 Three-Class Cases

Two points are noteworthy in the three class cases. First, the trend noticed earlier that introducing a “contaminant” into the flow will deteriorate capacity is confirmed. The curves in Figures 6-6 and 6-2 are essentially the same for $p_S = 0$. Yet, as soon as $p_S \rightarrow .3$, capacity drops by as much as 20%. Second, a larger difference in separation and speed causes a greater fluctuation in capacity. We thus note that, among the four classes, “small” has the greatest adverse effect on capacity as it differs most from the other three classes. This observation will be confirmed later when looking at the overall data.
Figure 6-4: Two-Class Capacity and Gain: Longer Final
Figure 6-5: Two-Class Capacity and Gain: Similar Classes
CAPACITY
3 Classes (Small, Large & Heavy), 3 NM Final, 2 Runways

Figure 6-6: Three Class Capacity

ABSOLUTE & PERCENT GAIN
3 Classes (Small, Large & Heavy), 3 NM Final, 2 Runways

Figure 6-7: Three Class Gain
6.1.3 Four-Class Cases

We now arrive at the complete set of solutions to all cases given in the optimization problem introduced in section 4.2.4. Computing all permutations in four classes, two or three runways, and 3 NM, 5 NM, or 10 NM final-segment length, respectively, resulted in 1716 solutions to be analyzed. Although all of these have been examined, only a very small subset will be displayed here for illustration and discussion. The other cases are available upon request from the author. The most important results are now presented in a qualitative sense.

Heterogeneous vs. Homogeneous Queues

We mentioned that disturbing a queue has an adverse effect on capacity. The following two figures will visualize that this is a general trend and will show that capacity tends to be lowest when the mix is approximately evenly distributed. Looking at the first graph on the top left in Figure 6-8, where \( p_S = 0.2 \) and \( p_L = 0 \), the minimum is located where both classes, “B757” and “heavy”, are near 50%. Inspecting all subsequent graphs in ascending order of \( p_L \), the minimum is moving to the left along the capacity curves, implying that it is also moving along an equal distribution. Then, as soon as class “large” is increasing, starting at about \( p_L = 0.4 \), “large” begins to dominate the mix, the distribution is no longer even, and capacity increases again. Figure 6-9 allows an equivalent interpretation in terms of gain, and Appendix B will give additional evidence.

A corollary to the previous paragraph is that class “small” becomes a serious obstacle as its contribution to the total traffic mix approaches 50%. Its significantly lower approach speed and necessity for a large separation distance effectively retard the remaining traffic. Because of this, a mix with a significant “small” share has a greater potential in maximizing capacity. The resulting gain may reach as much as 25 aircraft per hour (Figure 6-10). “Small” traffic is therefore undesired unless we are able to isolate it onto its own runway. In fact, many large airfields do have an extra, usually shorter runway designated to “small”, general aviation traffic.
CAPACITY

4 Classes (Small, Large, B757 & Heavy), 5 NM Final, 2 Runways, probability Small = 0.2

Figure 6-8: Four Class Capacity
ABSOLUTE & PERCENT GAIN
4 Classes (Small, Large, B757 & Heavy), 5 NM Final, 2 Runways, probability Small = 0.2

Figure 6-9: Four Class Gain
MAXIMUM POTENTIAL GAIN vs. PROBABILITY SMALL
3, 5 and 10 NM Final;
2 and 3 Runways Respectively

Figure 6-10: Potential Gain, Class Small
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Table 6.2: Four-Class Capacity: Numerical Results, $p_S = 0.2$

To observe how traffic naturally homogenizes onto an individual runway during the optimization, we inspect a sample of the numerical results. Table 6.2 shows how class "small", with a probability of 0.2, almost entirely disappears from runway 1, while in Table 6.3, at $p_S = 0.5$, it essentially occupies runway 1. In general, every segregation of a class onto a separate runway has an associated capacity gain, although the latter might be very small.

**Capacity and Gain vs. Final Segment**

Last, we will relate capacity to the length of the final segment. As foreshadowed, capacity varies inversely with final-segment length (Figure 6-11), while gain varies directly with final-segment length (Figure 6-12). Physically explained, the shorter the final segment, the shorter the time span at which an aircraft may potentially be blocked by slow-moving, preceding traffic. However, if such a situation arises, the potential gain is larger for the very same reason. A typical procedure that makes use of this idea is the visual approach as it effectively shortens the final path and noticeably increases capacity.

Summing up part one of Chapter 6, we find that the final overall result of this optimization has produced a **minimum average gain of approximately 3 aircraft per**
AVERAGE CAPACITY vs. LENGTH OF FINAL
1, 2, 3 and 4 Classes;
2 and 3 Runways Respectively

Figure 6-11: Average Capacity, All Classes
Table 6.3: Four-Class Capacity: Numerical Results, $p_S = 0.5$

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Figure 6-12: Average Gain, All Classes
Figure 6-13: Random Number Histogram, 3 NM; 1,000,000 Draws

hour and a maximum average gain of 12 aircraft per hour (Figure 6-12). The following section will determine how this number decays as we depart from the deterministic assumptions.

6.2 Simulation Results

Any simulation requires a minimum frequency of hits, or points, to return the distribution of the probability function it tries to model. Hence, keeping track of the random numbers drawn, the corresponding histogram has to match closely the original probability distribution. If it does not, the cause might be a low frequency or flawed random number generator (see Chapter 5).

Having used a number generator that has been tested extensively, Figure C-1 in Appendix C suggests that the 1000 numbers drawn were insufficient. Based on the the ragged line, any attempt to interpret a distribution remains inconclusive, but after 1,000,000 numbers, all distributions gained credibility (Figure 6-13). Base on this observation, we only relied upon results from the simulations that used 1,000,000 draws. Histograms for all random numbers and relevant probability density functions are shown in Figures 6-13, 6-14, and 6-15. In each histogram, the effective range of
the represented variable has been divided into 21 bins and the points (hits) counted. 1,000,000 numbers have been drawn for each class, speed, separation, and runway. The columns in the uniform distribution 6-13 thus show three vertical lines per bin, and the point count reflects the number of runways involved. In contrast, the triangular distributions show only one vertical line per bin, but the point count still reflects the number of runways involved. No histogram is shown for the randomized classes as their permutations are quite numerous. However, the mean difference between the optimized and the simulated class distributions was $8.3042 \cdot 10^{-7}$ with a standard deviation of $9.6761 \cdot 10^{-8}$.

Looking at the results of the simulation, it becomes clear that by randomizing the three variables class, speed, and separation the gains made in the deterministic optimization were reduced—the question is to what extent. Moving from left to right in Figure 6-16, along the horizontal axis, the graph depicts in sequential order the capacity values for the following scenarios: base case (bse), optimized (opt), random speed (p.v.), random separation (p.s.), and random speed and random separation (p.v.&s.). We notice that, with the introduction of probabilistic assumptions, the average initial capacity gain of approximately 10 aircraft decreases by roughly 60% . Looking at 2 classes only, the result is even less promising, namely a decrease over the
Figure 6-15: Random Separation Histogram, 3 NM; 1,000,000 Draws

_base case_ by one aircraft. However, we gain confidence in confirming that randomizing speed and separation affects all cases in similar fashion.

It was mentioned earlier that the minimum potential gain in our model occurs at a final-segment length of $3NM$. In order to give the most conservative estimate, we shall henceforth focus on the $3NM$ case exclusively. In doing so, we determine that _the expected capacity improvement has diminished to 1 aircraft per hour with four classes, and a negative gain of 1 to 3 aircraft per hour if only two classes are present_. This small positive or even negative gain can alternatively be seen on the continuous capacity curves which show that, after the simulation, indeed only few profitable points remain (Figure 6-18 and 6-19).

### 6.2.1 Further Improvement of the Simulation Model

In an attempt to make the model even more realistic, we introduce the following last adjustment: Most airline procedures require their flight crew to correct the approach speed for wind and gust, resulting in a more stable approach and landing. The common formula used for this is $Approach\, Speed = V_{REF} + \frac{1}{2}Wind + Gust$ (measured in knots), but not more than $V_{REF} + 20$ kts. This formula says that, given a standard approach speed of $V_{REF} + 5$ kts, the variation is $[-5, +15]$ kts. The revised
OVERALL AVERAGE CAPACITY
with VARYING PROBABILISTIC ASSUMPTIONS
1,000,000 Simulated Arrivals per Runway

Figure 6-16: Total Average Capacity After 1st Simulation; 1,000,000 Draws
3NM FINAL AVERAGE CAPACITY
with VARYING PROBABILISTIC ASSUMPTIONS
1,000,000 Simulated Arrivals per Runway

2 Runways

3 Runways

Figure 6-17: 3 NM Average Capacity After 1st Simulation; 1,000,000 Draws
CAPACITY, 3 NM FINAL
4 Classes (Small, Large, B757 & Heavy), 2 Runways
probability Small = 0.1

Figure 6-18: Capacity Changes After 1st Simulation, 3 NM, 2 Runways
CAPACITY, 3 NM FINAL
4 Classes (Small, Large, B757 & Heavy), 3 Runways
probability Small = 0.1

Figure 6-19: Capacity Changes After 1st Simulation, 3 NM, 3 Runways
model now contains a non-isosceles triangle as velocity-PDF as shown in Figure 6-20. Interestingly, running the revised simulation we obtain a slight improvement as the average approach speed increases, but the results worsen with the random separation. At best, we remain with an expected gain of 3 aircraft per hour for 2 runways, and a loss of 5 aircraft per hour on 3 runways. This concludes the discussion of the results and establishes a basis to make a sound judgment in terms of possible future use of the model. The next chapter will discuss this option, provide an analysis, conclusion, and suggestions for possible future research.
Figure 6-21: 3 NM Average Capacity After 2\textsuperscript{nd} Simulation; 1,000,000 Draws
Chapter 7

Conclusions and Further Research

In this chapter some general conclusions are summarized based on a review of the results presented in Chapter 6. The chapter also includes a brief discussion of the limitations of our model and of possible directions for future research.

7.1 Conclusions

At first glance, the results in section 6.1 appear very promising. For example, Figure 6-12 suggests that the overall capacity gain when operating with four classes of aircraft and three runways could be as high as 12 aircraft per hour (or 14.2%); and that with two classes the gain amounts to 5 aircraft per hour (or 6.0%). Better still, from Figure 6-10 one might determine that if “small” aircraft constitute 40% of the traffic, then the throughput capacity increases by 25 aircraft per hour (or 31.8%).

However, very few major airports, if any, have a traffic mix with 40% of the aircraft belonging to the “small” class. In fact, Eurocontrol and FAA data show a “small” contribution in the range of 2% – 17% at the busiest airports of Europe and of the United States. Additionally, a number of large airports have an additional, shorter runway designated for small (general aviation) traffic. This is a practical way to achieve some of the gains indicated by our optimization results of Chapter 6. These results suggest that gains can be obtained by “homogenizing”, as much as possible, the traffic mix on each runway.
The simulation in the second part of Chapter 6 suggests that the benefits obtained from the optimization scheme are reduced considerably when stochasticity is taken into consideration: in the simulation results, expected gain in throughput is at most 3 aircraft per hour (Figure 6-21, 4 classes, 2 runways). In fact, the capacity after the simulation becomes less than the baseline capacity. (This comparison, however, is not quite fair, because the baseline results are obtained by treating separations and approach speeds as constant.) Moreover, by treating approach speed and separation as random variables we still do not account for all factors that may adversely affect the optimized result. Even if one does not take into consideration the significant influence of wind or weather conditions, there are at least three more variables that would most likely reduce the achievable gain.

First, pilots are reluctant to reduce spacing and tend to give themselves extra margins of maneuverability. Often, and in spite of a controller’s attempt to apply minimum separation, pilots use information regarding surrounding traffic from ATC communications or the Traffic Collision Avoidance System (TCAS) screen, to increase the in-trail distance to the preceding traffic at their discretion to what appears safe. In a more realistic model, this behavior would need to be accounted for.

Second, we have not considered airline preferences, which might be an important factor in runway allocation. For example, it is often inefficient to assign an aircraft to a runway on the West side of an airfield if the scheduled gate lies on the East side. Crossing active runways or taxiing along unnecessarily long routes causes delays and is usually avoided. This assignment restriction would pose an additional constraint to our model.

Third, even in the absence of a runway preference, if we were allocating traffic according to our optimization scheme, we might not be able achieve the computed optimal distribution on each runway. The time window for which the overall mix is valid might not be long enough to allow for enough traffic in each of the four classes to compose a mix that reflects our computed optimal distribution.

Taken by themselves, the optimization results shown in the first half of Chapter 6 might suggest the advisability of developing an algorithm that incorporates our
optimal runway allocation scheme into existing air traffic automation systems, like the Center TRACON Automation System (CTAS) and its associated components.

The Traffic Management Advisor (TMA) currently allocates aircraft to runways according to TRACON acceptance rate, meter fix acceptance rate, gate acceptance rate, and miles-in-trail restrictions (separation). These allocations may be overruled during final approach by the controller using the Final Approach Sequencing Tool (FAST). Using feeder gate, aircraft engine type, runway category, occupation level, delay reduction, odd-aircraft-type in stream, and other parameters, FAST makes a Knowledge Based Runway Allocation (KBRA). An additional decision criterion could have been implemented that would consider the optimal assignment mixes derived from our optimization model.

However, in light of all the practical factors and model simplifications we have identified above—and especially in view of the of the gain reduction under probabilistic assumptions—we conclude that the expected capacity benefits from implementing this optimization model may be quite negligible at most major airports. Thus, an effort to implement the model on a wide scale cannot be recommended, based on the findings of this thesis. Exceptions may exist at airports that have unusually non-homogeneous traffic mixes.

### 7.2 Recommendations

Further recommended research in this field would mainly focus on two points. First, the model presented possesses the serious limitation of assuming a constant speed for each aircraft throughout the common final approach path. This is an assumption which, to our knowledge, is shared by all analytical models currently in use and does not quite reflect the real speed profiles of landing aircraft. In practice, during the entire final segment, flap settings are increased according to flap schedules and speed continuously decreases accordingly. Only at the prescribed point mentioned earlier, commonly 1000ft height above threshold, can the speed be assumed constant at $V_{REF}$ plus a correction factor. A model based on a continuously varying speed profile
until about 3 n. miles out would certainly predict runway capacity more accurately and might conceivably indicate that an increase is achievable through optimization of assignments.

A second systematic way of potentially achieving some improvements in the maximum throughput capacity is through the introduction of a position shifting method. As suggested by various researchers, position shifting might increase capacity by a few percent. This small gain in capacity may, in turn, result in significant delay reductions at very busy airports. However, concerns have been raised as to the extent to which implementation of position shifting would be feasible in practice. NASA’s aFAST already uses an advanced scheduling algorithm that makes use of position shifting at particularly viable locations like the merging points of downwind, base, or long-final legs. It might be worthwhile to consider optimum traffic assignment targets, such as the ones developed in Chapters 4 and 6, when performing such position shifting.
Appendix A

Acronyms
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<td>Active Final Approach Spacing Tool</td>
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<td>Air Route Traffic Control Center</td>
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<td>CDF</td>
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<td>CFR</td>
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<td>CMF</td>
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<td>CPS</td>
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Table A.2: Acronyms

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Appendix B

Selected Optimization Figures
CAPACITY
4 Classes (Small, Large, B757 & Heavy), 5 NM Final, 3 Runways, probability Small = 0

Figure B-1: Capacity, 4 Classes, 5 NM, 3 Runways, $p_S = 0$
ABSOLUTE & PERCENT GAIN

4 Classes (Small, Large, B757 & Heavy), 5 NM Final, 3 Runways, probability Small = 0

Figure B-2: Gain, 4 Classes, 5 NM, 3 Runways, $p_s = 0$
CAPACITY
4 Classes (Small, Large, B757 & Heavy), 10 NM Final, 2 Runways, probability Small = 0.3

Figure B-3: Capacity, 4 Classes, 10 NM, 2 Runways, $p_S = 0.3$
ABSOLUTE & PERCENT GAIN
4 Classes (Small, Large, B757 & Heavy), 10 NM Final, 2 Runways, probability Small = 0.3

Figure B-4: Gain, 4 Classes, 10 NM, 2 Runways, $p_S = 0.3$
CAPACITY

4 Classes (Small, Large, B757 & Heavy), 3 NM Final, 3 Runways, probability Small = 0.1

Figure B-5: Capacity, 4 Classes, 3 NM, 3 Runways, \( p_S = 0.1 \)
ABSOLUTE & PERCENT GAIN
4 Classes (Small, Large, B757 & Heavy), 3 NM Final, 3 Runways, probability Small = 0.1

Figure B-6: Gain, 4 Classes, 3 NM, 3 Runways, $p_s = 0.1$
Appendix C

Selected Simulation Figures
RANDOM NUMBER DISTRIBUTION
Used to Randomize Class, Velocity, and Separation
1,000 Simulated Arrivals per Runway

Figure C-1: Random Number Histogram, 1,000 Draws
CAPACITY CHANGES AFTER SIMULATION
4 Classes (Small, Large, B757 & Heavy), 3 NM Final, 2 Runways, probability Small = 0.1

Figure C-2: Capacity Changes After 1st Simulation, 3 NM, 2 Runways
CAPACITY CHANGES AFTER SIMULATION
4 Classes (Small, Large, B757 & Heavy), 3 NM Final, 3 Runways, probability Small = 0.3

Figure C-3: Capacity Changes After 1st Simulation, 3 NM, 3 Runways
CAPACITY CHANGES AFTER SIMULATION
4 Classes (Small, Large, B757 & Heavy), 5 NM Final, 2 Runways, probability Small = 0.1

Figure C-4: Capacity Changes After 1st Simulation, 5 NM, 2 Runways
CAPACITY CHANGES AFTER SIMULATION
4 Classes (Small, Large, B757 & Heavy), 5 NM Final, 3 Runways, probability Small = 0.3

Figure C-5: Capacity Changes After 1\textsuperscript{st} Simulation, 5 NM, 3 Runways
CAPACITY CHANGES AFTER SIMULATION
4 Classes (Small, Large, B757 & Heavy), 10 NM Final, 2 Runways, probability Small = 0.1

Figure C-6: Capacity Changes After 1st Simulation, 10 NM, 2 Runways
CAPACITY CHANGES AFTER SIMULATION
4 Classes (Small, Large, B757 & Heavy), 10 NM Final, 3 Runways, probability Small = 0.3

Figure C-7: Capacity Changes After 1st Simulation, 10 NM, 3 Runways
Bibliography


