Real Time Estimation of Ship Motions Using Kalman Filtering Techniques

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Abstract—**The estimation of the heave, pitch, roll, sway, and yaw motions of a DD-963 destroyer is studied, using Kalman filtering techniques, for application in VTOL aircraft landing.**

The governing equations are obtained from hydrodynamic considerations in the form of linear differential equations with frequency dependent coefficients. In addition, nonminimum phase characteristics are obtained due to the spatial integration of the water wave forces.

The resulting transfer matrix function is irrational and nonminimum phase. The conditions for a finite-dimensional approximation are considered and the impact of the various parameters is assessed.

A detailed numerical application for a DD-963 destroyer is presented and simulations of the estimations obtained from Kalman filters are discussed.

INTRODUCTION

THE PRESENT STUDY started as part of the effort directed toward designing an efficient scheme for landing VTOL aircraft on destroyers in rough seas. A first study [12] has shown the significant effect of the ship model used on the thrust level required for safe landing.

In a landing scheme, it would be desirable to have accurate ship models capable of providing a good real time estimation and possibly prediction of the motions, velocities, and accelerations of the landing area, resulting in safer operations and with reduced thrust requirements.

The modeling of the vessel dynamics is quite complex and a substantial effort is required to reduce the governing equations to a finite dimensional system of reasonable order.

The first part of this paper studies the equations of motion as derived from hydrodynamics, their form and the physical mechanisms involved, and the form of the approximations used.

The second part describes the modeling of the sea, which proved to be a crucial part of the overall problem.

In the third part, the Kalman filter studies are presented and the influence of the various parameters is assessed.

The Appendix provides some basic hydrodynamic data and the particulars of the DD-963 destroyer.

PREVIOUS WORK

There are several applications of state-space methods to ship related problems, such as for steering control [7] and dynamic positioning [2], [6]. In these studies, the complexity of modeling the wave induced motions is recognized, caused by the structure–fluid interaction that introduces memory effects.

The procedure used in the literature is to combine the inertia model of the vessel with transfer functions based on empirical data. Efforts to predict the ship motions a few seconds ahead in time using both frequency and time domain techniques demonstrated the importance of using accurate ship and sea spectrum models [5], [9], [16], [21], [23].

A study of the feasibility of landing VTOL aircrafts on small destroyers [12] demonstrated the significant effect on accurate ship motion estimations on automatic landing performance.

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OVERVIEW

The dynamics of a floating vessel are complicated because it moves in contact with water and in the presence of the free water surface, which is a waveguide, introducing memory and damping mechanisms [14].

For control purposes, it is necessary to estimate the motions, velocities, and accelerations of the vessel using a few noisy measurements. This can be best achieved by a Kalman filter, which makes optimal use of *a priori* noise information and the model of the system, to reconstruct the state [11]. The theory and application of the Kalman filter can be found in [1], [11] and has been used for numerous applications in many fields.

The Kalman filter requires a ship model in state-space form, preferably of the lowest possible order, so as to reduce the computational effort. This is no easy task, because, as shown in this paper, the transfer functions between ship motion and sea elevation are irrational nonminimum phase functions. For this reason, the major part of this paper is devoted to developing systematic techniques of deriving state-space models using hydrodynamic data.

To the authors' knowledge, this is the first time that such a procedure has been applied systematically. It is important to note that computer programs developed to solve numerically the ship motion problem such as described in [24], predict very accurately ship motions, as found by comparison with model and full scale data [15], [22].

The amount of data obtained for a single vessel is overwhelming, since the six motions of the vessel are obtained at a number of wave frequencies, ship speeds, and wave directions. The present study is the outcome of a two-year study on developing efficient ship models in state-space form and it is hoped that future applications will find the procedure established here of significant help.

Two recent publications [4], [21], using the results pre-

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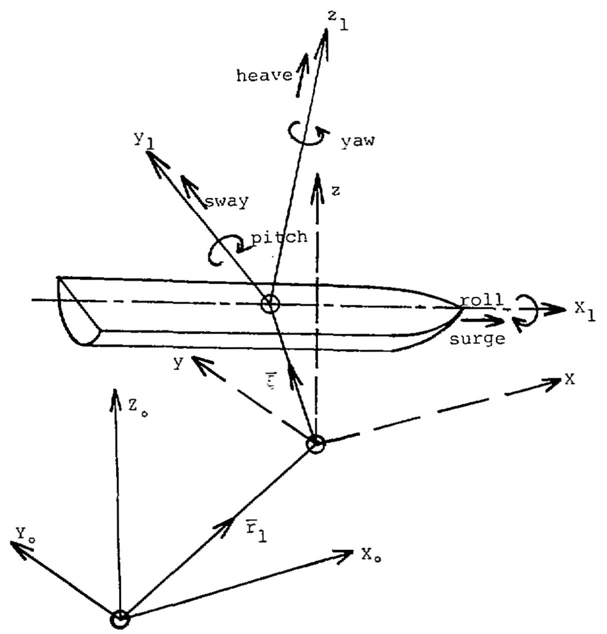

Fig. 1. Reference systems.

sented here, showed the significant insight and accuracy gained by developing the ship model directly from the hydrodynamic data.

EQUATIONS OF MOTION

A. Definitions

The rigid body motions of a ship in 6 degrees of freedom are shown in Fig. 1. We define the $x_1 z_1$ plane to coincide with the symmetry plane of the ship, with the z_1 axis pointing vertically upwards when the vessel is at rest, and the y_1 axis so as to obtain an orthogonal right-hand system, while the origin need not coincide with the center of gravity. The $x_0 y_0 z_0$ system is an inertial system with $x_0 y_0$ fixed on the undisturbed sea surface, while the $x\ y\ z$ system is moving with the steady speed of the vessel, i.e., it follows the vessel but it does not participate in its unsteady motion. Then the linear motions along the x_1, y_1, and z_1 axes are surge, sway, and heave, respectively. In order to define the angular motions, we normally require Euler angles. In the present case, though, we consider small motions, so that the tensor of angular displacements can be replaced by a vector of small angular displacements, which are roll, pitch, and yaw around the x_1, y_1, and z_1 axes, respectively.

The characteristics of a ship are its slender form, i.e., $L/B \gg 1$ and $L/T \gg 1$, where L is the length, B the beam, and T the draft. Also, the ship is symmetric about the xz plane and near symmetric about the yz plane. For this reason

$$I_{yz} = I_{zy} = 0$$

$$I_{xy} = I_{yx} = 0.$$

The value of I_{xz} is typically small compared with I_{xx} and I_{yy}. The justification of using the linearity assumption is as follows: the excitation consists of wave induced forces, which include fluid inertia forces and hydrostatic forces. It is well established that the waveheight to wavelength ratio is small, since, at a typical upper value of 1/7, the wave breaks and loses all its energy [13]. As a result, the major part of the wave force is a linear function of the wave elevation and can be obtained by a first order perturbation expansion of the nonlinear fluid equation, using the waveheight to wavelength ratio as the perturbation parameter. [13]

The wave spectrum, as will be shown later, has a frequency range between typically 0.2 and 2 rad/s. Given the large mass of the vessel, the resulting motions, within this frequency range, are of the order of a few feet, or a few degrees, so that the equations of motion can be linearized.

The only motion that requires attention is roll, because due to the slender form of the ship, the rolling motion may become large, in which case nonlinear damping becomes important.

B. Simple Derivation

We derive the equation of motion for a simple two-dimensional object to demonstrate the overall procedure.

Let us assume that we wish to derive the motion of a two-dimensional cylinder subject to wave excitation, allowed to move in heave only.

The incoming wave of amplitude a_0 and frequency ω_0 will cause a force on the cylinder, and, therefore, heave motion. Due to the linearity of the problem, the following decomposition can be used, which simplifies the problem considerably.

a) Consider the sea calm and the ship forced to move sinusoidally with unit heave amplitude, and frequency ω_0, and find the resulting force.

b) Consider the ship motionless and find the force on the cylinder due to the incoming waves and the diffraction effects (diffraction problem).

c) In order to find the heave amplitude, within linear theory, we equate the force found in a) times the (yet unknown) heave amplitude, with the force found in b).

The force in b) can be decomposed further for modeling purposes, again due to linearity; one part is due to the undisturbed incoming waves and the other part due to the diffracted waves. The first is called the *Froude-Krylov* force and the second the *diffraction force*. The total force is called the *excitation force* [13].

The force in a) due to linearity can be also decomposed; the first part is simply the hydrostatic force. The second part is a dissipative force, caused by the fact that the refraction waves carry energy from the ship to infinity. For this reason, we define a damping coefficient B so that the dissipative force will be $-Bx'$, where x' is the heave velocity. The third part is an inertia force, caused by the fact that the heaving ship causes the fluid particles to move in an unsteady motion, so that we define an "added" mass A and the inertia force becomes $-Ax''$, with x'' the heave acceleration. If we denote the undisturbed incoming wave elevation amidhsips as $\eta(t)$

$$\eta(t) = a_0 e^{i\omega_0 t}. \tag{1}$$

Where the real part of all complex quantities is meant, here and in the sequel, then the excitation force will be

$$F = F_0 e^{i\omega_0 t} a_0. \tag{2}$$

Where F_0 is complex (to take into account the phase difference with respect to the wave elevation), the equation of motion becomes

$$Mx'' = F - Ax'' - Bx' - Cx. \quad (3)$$

Where M the mass of the cylinder; the motion is also sinusoidal so with x_0 complex

$$x(t) = x_0 e^{i\omega_0 t}. \quad (4)$$

A very important remark is that F_0, A, B depend on the frequency of the incoming wave ω_0. This can be easily understood by the fact that, at various frequencies, the heaving cylinder will produce waves with different wavelength. We rewrite, therefore, (3) as

$$-Mx_0\omega_0{}^2 e^{i\omega_0 t} = \{F_0(\omega_0)a_0 + [A(\omega_0)\omega_0{}^2 - i\omega_0 B(\omega_0) - C]x_0\}e^{i\omega_0 t}. \quad (3a)$$

By dropping $e^{i\omega_0 t}$, we can rewrite (3a) as

$$\{-[M + A(\omega_0)]\,\omega_0{}^2 + i\omega_0 B(\omega_0) + C\}x_0 = F_0(\omega_0)a_0. \quad (4)$$

The motion of a cylinder in water, therefore, results in an increase in the mass and a damping term. Equation (4) is used because of its similarity to a second order system. It is strictly valid, though, only for a monochromatic wave.

Ultimately, we wish to obtain the response in a random sea, so (4) must be extended for a random sea. This can be done by obtaining the inverse Fourier transform of (3a), i.e.,

$$\int_{-\infty}^{\infty} K_\alpha(t-\tau)x''(\tau)\,d\tau + \int_{-\infty}^{\infty} K_u(t-\tau)x'(\tau)\,d\tau + Cx(t) = \int_{-\infty}^{\infty} K_f(t-\tau)\eta(\tau)\,d\tau \quad (5)$$

where K_α, K_u, and K_f is the inverse Fourier transform of $-\omega^2 [M + A(\omega)]$, $i\omega B(\omega)$, and $F_0(\omega)$, respectively. The random undisturbed wave elevation is denoted by $\eta(t)$. Equation (5) is not popular with hydrodynamicists, because the effort required to evaluate the kernels K_a, K_u, and K_f is by far greater than that required to find the added mass, damping, and excitation force. For this reason, (5) is rewritten in a hybrid form as follows:

$$-[M + A(\omega)]\,X''(t) + B(\omega)x'(t) + Cx(t) = F(\omega)\eta(t). \quad (6)$$

This is an integro-differential equation (or differential equation with frequency dependent coefficients), whose meaning is in the sense of (5).

C. Strip Theory

The evaluation of $A(\omega)$, $B(\omega)$, and $F(\omega)$ is not an easy task for complex geometries, such as the hull of a ship. The strip theory can be used to approximate these quantities by subdividing the ship in several transverse strips. Due to the elongated form of the ship, and for high frequencies, each strip has small interactions with the other strips, except near the ends. Usually these end effects are small, so that instead of solving the overall three-dimensional problem, we can solve several two-dimensional problems (one for each strip) and sum up the partial results [3], [10].

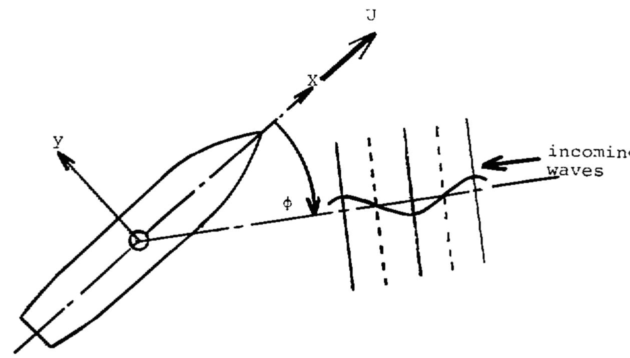

Fig. 2. Angle of incoming waves.

D. Relation Between Added Mass and Damping

The added mass and damping coefficients are not independent of each other, because their frequency dependence is caused by the same refraction waves. If we define

$$T(\omega) = \omega^2 \left[A(\omega) + \frac{B(\omega)}{i\omega} \right] \quad (7)$$

then $T(\omega)$ is an analytic function [14]. As a result, $A(\omega)$ and $B(\omega)$, which are real, are related by the Kramers-Krönig relations, in order to describe a causal system [14].

E. Speed Effects

The effect of the forward speed is to couple the various motions by speed dependent coefficients. Simplified expressions for the added mass, damping, and exciting force can be derived, with a parametric dependence on the speed U [15].

F. Frequency of Encounter

An additional effect of the ship speed is the change in the frequency of encounter. If the incident wave has frequency ω and wavenumber k, then the frequency of encounter ω_e is

$$\omega_e = \omega + kU\cos\phi \quad (8)$$

where ϕ is the angle between the x axis of the ship and the direction of wave propagation (Fig. 2). In deep water, the dispersion relation for water waves is

$$\omega^2 = kg \quad (9)$$

with g the gravity constant, so that we can rewrite (10) as

$$\omega_e = \omega + \frac{\omega^2}{g} U\cos\phi. \quad (10)$$

A very important consideration in the difference between frequency of encounter and wave frequency is the following:

the ship motions, within linear theory, will be of frequency ω_e, so that the refraction waves are of frequency ω_e and the added mass and damping can be written as $A(\omega_e)$ and $B(\omega_e)$.

On the contrary, although the exciting force varies with frequency ω_e also, its value is the same as if the frequency were ω, plus the speed dependent terms, i.e.,

$$F(t) = a_0 F(\omega) e^{i\omega_0 t} \tag{11}$$

with a_0 the incident wave amplitude.

G. Equation of Motion

We write the equations of motion neglecting the surge motion as a second order quantity. This is in agreement with experiments [15]. Within linear theory and using the ship symmetry, *the heave and pitch motions are not coupled with the group of sway, roll, and yaw motions.* This is not to imply that the motions are independent, because they are excited by the same wave, so there is a definite relation both in amplitude and phase.

1) Heave-Pitch Motions

$$\left\{\begin{bmatrix} M & 0 \\ 0 & I_y \end{bmatrix} + \begin{bmatrix} A_{33} & A_{35} \\ A_{53} & A_{55} \end{bmatrix}\right\} x_v'' + \begin{bmatrix} B_{33} & B_{35} \\ B_{53} & B_{55} \end{bmatrix} x_v' + \begin{bmatrix} C_{33} & C_{35} \\ C_{53} & C_{55} \end{bmatrix} x_v = \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \eta. \tag{12}$$

2) Sway-Roll-Yaw Motion

$$\begin{bmatrix} M M_{24} & M_{26} \\ M_{42} I_x & I_{xz} \\ M_{62} I_{xz} & I_z \end{bmatrix} + \begin{bmatrix} A_{22} & A_{24} & A_{26} \\ A_{42} & A_{44} & A_{46} \\ A_{62} & A_{64} & A_{66} \end{bmatrix} x_u'' + \begin{bmatrix} B_{22} & B_{24} & B_{26} \\ B_{42} & B_{44} & B_{46} \\ B_{62} & B_{64} & B_{66} \end{bmatrix} x_u' + \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{44} & 0 \\ 0 & 0 & 0 \end{bmatrix} x_u = \begin{bmatrix} F_2 \\ F_4 \\ F_6 \end{bmatrix} \eta. \tag{13}$$

where A_{ij}, B_{ij}, and C_{ij} are the added mass, damping, hydrostatic coefficient matrices, respectively; F_j is the exciting forces and η the wave elevation

$$x_v = \{x_3, x_5\}^T \tag{14}$$

$$x_u = \{x_2, x_4, x_6\}^T. \tag{15}$$

The frequency and velocity dependence is not written explicitly, but is understood, as described in the previous sections.

H. Heave-Pitch Approximation

We start with the heave and pitch motions approximation. As it is obvious from (4), it involves two stages:

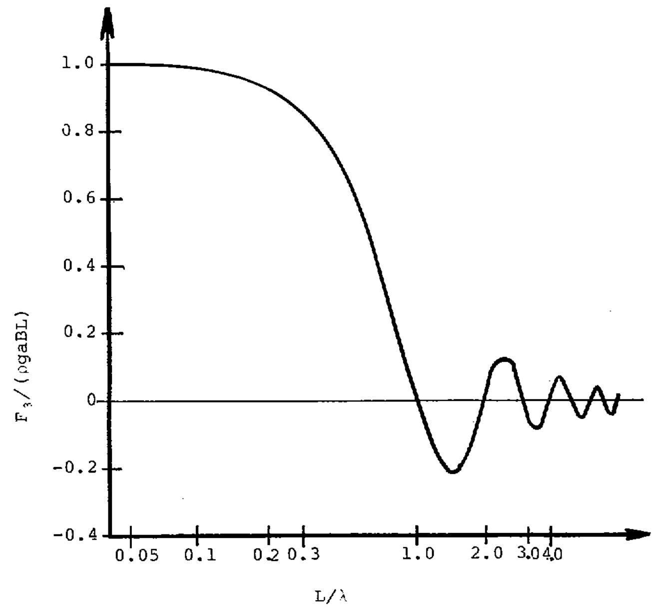

Fig. 3. Heave excitation force F_3 on a rectangular cylinder ($L \times B \times T$) as a function of the ratio between the ship length L and the wavelength.

a) approximation of the exciting force, and
b) approximation of the added mass and damping coefficients.

Data are provided by the hydrodynamic theory for both components and for the whole wave frequency range.

I. Exciting Force Approximation

Fig. 3 shows the exciting heave force on a box-like ship [22]. This figure is important to demonstrate the infinite sequence of zeros of the amplitude of the heaving force along the frequency axis.

The transfer function between the wave elevation and the heave force can not be represented as a ratio of polynomials of finite degree as evidenced by Fig. 3. Similar plots can be obtained for the pitch moment. Within the wave frequency range, though, only the first zero is important, while the remaining peaks are of minor significance. This is not true for other types of vehicles such as the semi-submersible, but for ships it is valid for both heave force and pitch moment, so it will be used to simplify considerably the modeling procedure.

As it was mentioned before, the exciting force changes with frequency ω_e, but its amplitude is determined on the basis of the frequency ω. The following variables must be included in an appropriate modeling of the exciting forces

1) frequency ω
2) speed V
3) wave angle ϕ

$$F_3(t, a_0, \phi, U) = F_3(\omega, \phi) a_0 e^{i\omega_e t} \tag{16}$$

$$F_5(t, a_0, \phi, U) = \left\{ F_5(\omega, \phi) + \frac{U}{i\omega} f_3(\omega, \phi) \right\} e^{i\omega_e t} \cdot a_0 \tag{17}$$

where a_0 is the wave amplitude and f_3 is the heave diffraction

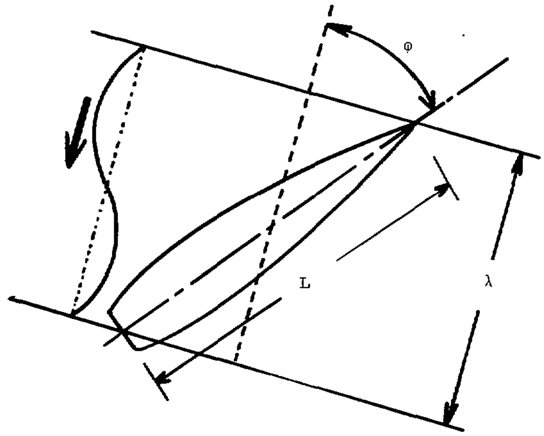

Fig. 4. The wavelength for heave and pitch is seen as $\lambda/\cos\phi$ and as $\lambda/\sin\phi$ for sway, roll, and yaw.

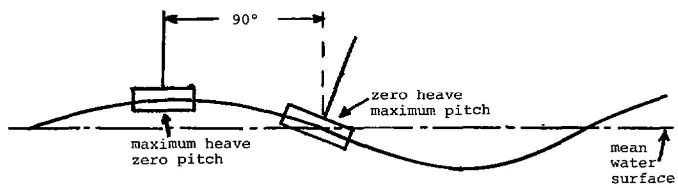

Fig. 5. Phase difference between heave and pitch in long waves (low frequencies).

force. Equations (16), (17) show that the heave force does not depend on the ship speed, whereas the pitch moment does, in a linear fashion.

In order to approximate $F_3(\omega, \phi)$, $F_5(\omega, \phi)$, and $f_3(\omega,\phi)$, we use the plots in Fig. 4, which show the approximate influence of the wave angle on the excitation forces.

In order to model the DD963 destroyer, the Massachusetts Institute of Technology (M.I.T.) Five Degrees of Freedom Sea-keeping Program [24] was used to derive hydrodynamic results. The following model was derived to model shape of the heave force at $V = 0$ and $\phi = 0$ (no speed, head seas)

$$F_3(s) = \frac{\alpha_1 \eta}{\left[1 + 2J\dfrac{s}{\omega_a} + \dfrac{s^2}{\omega_a{}^2}\right]^2} \tag{18}$$

where $J = 0.707$, α_1 a constant to be determined from hydrodynamic data, η the wave elevation, and ω_a the corner frequency. Remembering the analysis above concerning the dependence of the force on ω and Fig. 4, we can derive

$$\omega_a = \sqrt{\frac{2\pi g}{L\cos\phi + B}} + \frac{2\pi}{L\cos\phi + B}\,U\cos\phi \tag{19}$$

where L is the ship length and B the beam.

Before we establish a relation similar to (18) for the pitching moment, we must consider Fig. 5, where it is shown that *for long waves, the heave force and the pitching moment are 90° out of phase*. This means that the transfer function between heave and pitch is a nonminimum phase one, because the amplitude is constant, while the phase is 90°. We choose to attribute the nonminimum phase to pitch. Also, the pitch angle tends to the wave slope for large wavelengths, so that the pitching moment can be written as

$$F_5 = \alpha_2 \frac{1 - s/\omega_0}{1 + s/\omega_0} \cdot \frac{s^2\cos\phi}{\left[1 + 2J\dfrac{s}{\omega_a} + \dfrac{s^2}{\omega_a{}^2}\right]^2} \cdot \eta \tag{20}$$

where α_2 is a constant to be determined, ω_a is the same (for simplicity) as in (19), and ω_0 is an artificial frequency to model the nonminimum phase. It will be chosen to be equal to the wave spectrum modal frequency as indicated in Section II.

J. Added Mass and Damping

By using (7), we can rewrite the equations of motion as

$$\begin{bmatrix} T_{33}s^2 + Ms^2 + C_{33} & T_{35} + UsT_{33} + C_{35} \\ T_{53}s^2 - UsT_{33} + C_{35} & I_y \cdot s^2 + T_{55}s^2 - U^2T_{33} + C_{55} \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \eta. \tag{21}$$

Here we construct a simplified model where

$$T_{ij} = A_{ij} + \frac{B_{ij}}{i\omega} \tag{22}$$

with A_{ij} and B_{ij} to be evaluated from the hydrodynamic data. Then (23) can be written, after we define

$$y_1 = x_3 \qquad y_2 = \mathring{x}_3$$
$$y_3 = x_5 \qquad y_4 = \mathring{x}_5$$
$$y = \{y_1, y_2, y_3, y_4\}^T \tag{23}$$

in the form

$$\begin{bmatrix} \mathring{y}_2 \\ \mathring{y}_4 \end{bmatrix} = \begin{bmatrix} A_{33} & A_{35} \\ A_{53} & A_{55} \end{bmatrix}^{-1} \cdot \left\{ - \begin{bmatrix} C_{33} & B_{33} & C_{35}' & B_{35} \\ C_{53}' & B_{53} & C_{55}' & B_{55} \end{bmatrix} y + \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \eta \right\} \tag{21a}$$

where

$$C_{35}' = C_{35} + UA_{33}$$
$$C_{53}' = C_{35} - UA_{33}$$
$$C_{55}' = C_{55} - U^2 x A_{33}. \tag{24}$$

K. Exciting Force Approximation for Sway-Roll-Yaw

An infinite-dimensional form is obtained for the exciting force in sway, roll, and yaw motions. The same considerations apply, therefore, as discussed in the previous section. Using the M.I.T. Five Degrees of Freedom Sea-keeping Program, the following finite dimensional approximation was found in the case of $U = 0$ and $\phi = 90°$ for sway, roll, and yaw

$$F_j(s) = \frac{A_j s^2}{\left(\dfrac{s}{\omega_j}\right)^2 + 2J_j \dfrac{s}{\omega_j} + 1}, \qquad j = 2, 4, 6 \tag{25}$$

where

$$\omega_2 = 0.60 \qquad J_2 = 0.72$$
$$\omega_4 = 0.76 \qquad J_4 = 0.70$$
$$\omega_6 = 0.96 \qquad J_6 = 0.35 \tag{26}$$

and A_2, A_4, and A_6 are obtained from hydrodynamic data

$$A_2 = 310,$$
$$A_4 = 2120$$
$$A_6 = 11\,300. \tag{27}$$

We redefine the value of ω_2, ω_4, and ω_6 so that it will be valid for angles ϕ other than 90°, and speed other than 0

$$\omega_j' = \left(\omega_j + \omega_j^2 \frac{U}{g} \cos\phi\right) \sin\phi \tag{28}$$

where $j = 2, 4$, and 6 and ω_j is given above. Also

$$A_j = A_{j_0} \cdot \sin\phi$$
$$J_j = J_{j_0} \cdot \sin\phi. \tag{29}$$

It should be noted that the sway, roll, and yaw forces are proportional to the wave slope, i.e., 90° out of phase with respect to the wave amplitude. This means that they belong to the same group with pitch, and the same nonminimum phase transfer function

$$\frac{s - \omega_0}{s + \omega_0}$$

must be used for all three of them, when the total system (all 5 degrees of motion) is considered.

L. Added Mass and Damping

The amplitude of the transfer function between the wave elevation and the rolling motion has a very narrow peak so that the coefficients can be approximated as constant [15]

$$A_{44} = A_{44}^0 \qquad B_{44} = B_{44}^0$$
$$A_{42} = A_{24}^0 \qquad B_{42} = B_{24}^0$$
$$A_{46} = A_{46}^0 + \frac{U}{\omega^2} B_{24}^0$$
$$B_{46} = B_{46}^0 - UA_{24}^0 \tag{30}$$

using the value of ω at the roll peak. It should be noted that roll involves a significant nonlinear (viscous) damping, which is approximated by introducing an additional "equivalent" damping coefficient B_{44}^* [3].

Similarly, we calculate the sway and yaw coefficients at the same frequency

$$A_{22} = A_{22}^0 \qquad B_{22} = B_{22}^0$$
$$A_{26} = A_{26}^0 + \frac{U}{\omega^2} B_{22}^0$$
$$B_{26} = B_{26}^0 - UA_{22}^0$$
$$A_{62} = A_{26}^0 - \frac{U}{\omega^2} B_{22}^0$$
$$B_{62} = B_{26}^0 + UA_{22}^0$$
$$A_{66} = A_{66}^0 + \frac{U^2}{\omega^2} A_{22}^0$$
$$B_{66} = B_{66}^0 + \frac{U}{\omega^2} B_{22}^0 \tag{31}$$

$$[s^2 \{A_{ij} + M_{ij}\} + s\{B_{ij}\} + \{C_{ij}\}] \begin{Bmatrix} X_2 \\ X_4 \\ X_6 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_4 \\ F_6 \end{Bmatrix}, \qquad \begin{matrix} i = 2, 4, 6 \\ j = 2, 4, 6 \end{matrix} \tag{32}$$

where $C_{ij} = 0$ except for C_{44}, which is the roll hydrostatic constant, i.e., $C_{44} = \Delta \cdot (GM)$ with Δ the ship displacement and (GM) the metacentric height.

Due to the special form of the matrix C, a zero-pole cancellation results from a direct state-space representation of the equation above. After some manipulations, the following representation can be obtained which avoids zero-pole cancellation problems:

$$\overset{\circ}{X} = TX + UF$$

where

$$X = \{X_2, \overset{\circ}{X}_4, X_4, X_6\}^T$$

$$F = \left\{ \int F_2, F_2, F_4, \int F_6, F_6 \right\} \tag{33}$$

where $\int F$ indicates the time integral of F and $T = \{t_{ij}\}$ and $U = \{u_{ij}\}$, with

$$t_{11} = \frac{r_{12}}{r_{22}} P_{21} - P_{11}, \; t_{12} = \frac{r_{12}}{r_{22}}, \; t_{13} = \frac{r_{12}}{r_{22}} P_{22} - P_{12},$$

$$t_{14} = \frac{r_{12}}{r_{22}} P_{23} - P_{13}$$

$$t_{41} = \frac{r_{32}}{r_{22}} P_{21} - P_{31}, \; t_{42} = \frac{r_{32}}{r_{22}}, \; t_{43} = \frac{r_{32}}{r_{22}} P_{22} - P_{32},$$

$$t_{44} = \frac{r_{32}}{r_{22}} P_{23} - P_{33}$$

$$t_{21} = -P_{21} t_{11} - P_{23} t_{41}$$

$$t_{22} = -P_{21} t_{12} - P_{23} t_{42} - P_{22} \tag{34}$$

$$t_{23} = -P_{21} t_{13} - P_{23} t_{43} - r_{22} C_{44}$$

$$t_{24} = -P_{21} t_{14} - P_{23} t_{44}$$

$$t_{3i} = 0 \text{ except for } t_{32} = 1$$

$$U_{ij} = 0 \text{ except for } U_{11} = r_{11} - \frac{r_{12} r_{21}}{r_{22}}$$

$$U_{14} = r_{13} - \frac{r_{12} r_{23}}{r_{22}}, \; U_{41} = r_{31} \frac{r_{32} r_{21}}{r_{22}}$$

$$U_{44} = r_{33} - \frac{r_{32} r_{23}}{r_{22}}, \; U_{21} = -P_{21} U_{11} - P_{23} U_{41}$$

$$U_{22} = r_{21}, \; U_{23} = r_{22}, \; U_{24} = -P_{21} U_{14} - P_{23} U_{44}$$

$$U_{25} = r_{23} \tag{35}$$

where

$$R = \{r_{ij}\} = [A + M]^{-1}, \qquad P = \{P_{ij}\} = RB. \tag{36}$$

SEA MODELING

The process of wind waves generation can be described simply as follows: due to the inherent instability of the wave-air interface, small wavelets appear as soon as the wind starts blowing, so the surface becomes rough and a significant drag force develops, which becomes zero only when the average wind speed (which causes the major part of the drag) equals the phase wave velocity. As a result, the steady-state condition of the sea develops slowly by creating waves whose phase velocity is close to the wind speed. Since the process starts with high frequencies, we conclude that a young storm has a spectrum with a peak at high frequency. We usually distinguish between a *developing storm* and a *fully developed storm*.

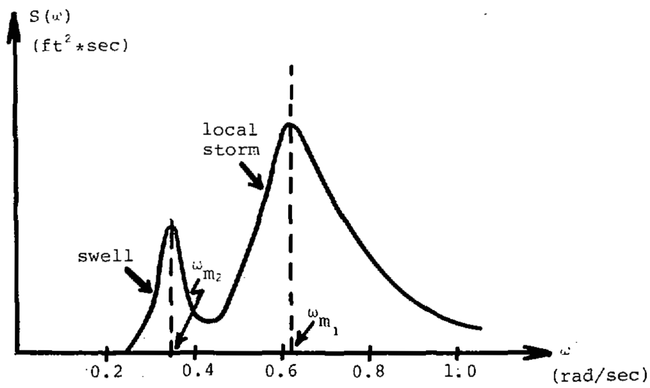

Fig. 6. Typical form of a wave spectrum containing swell.

As soon as the wind stops blowing, then the water viscosity dissipates the higher-frequency waves so that the so called *swell* (decaying seas) forms, which consists of long waves (low-frequency content), which travel away from the storm that originates them. For this reason, swell can be found together with another local storm. The spectrum of a storm usually contains one peak (except if swell is present when it contains two peaks) and the peak frequency ω_m is called the *modal frequency* (Fig. 6). Also, the intensity of the storm is required, which can be described in a number of ways: Beaufort Scale, Sea State, Wind Average Velocity, and Significant Waveheight. The most convenient one is the significant waveheight H defined as the statistical average of the 1/3 highest waveheights. For a narrow-band spectrum of area M_0

$$H \cong 4\sqrt{M_0}. \tag{37}$$

From our discussion on sea storm generation, we conclude that it is important to model a storm by both H (intensity) and ω_m (duration of storm). For this reason, the *Bretschneider Spectrum* will be used defined as

$$S(\omega) = \frac{1.25}{4} H^2 \frac{\omega_m^{\,4}}{\omega^5} \exp\left\{ -1.25 \left(\frac{\omega_m}{\omega} \right)^4 \right\}. \tag{38}$$

The spectrum was developed by Bretschneider for the North Atlantic, for unidirectional seas, with unlimited fetch, infinite depth, and no swell. It was developed to satisfy asymptotic theoretical predictions and to fit North Atlantic data. It was found to fit reasonably well in any sea location. Also, by combining two such spectra, we can model the swell as well. Its main limitations are unidirectionality and unlimited fetch. For the application studied, the unlimited fetched assumption is valid, while the model was found to be relatively insensitive to the wave direction, so the Bretschneider spectrum is an adequate approximation of the sea elevation.

TABLE I
SEA SPECTRUM COEFFICIENTS

α	$\gamma(\alpha)$	$\beta(\alpha)$
0.00	0.9538	1.8861
0.10	1.0902	1.6110
0.20	1.1809	1.3827
0.30	1.2717	1.2116
0.40	1.3626	1.0785
0.50	1.4539	0.9718
0.60	1.5448	0.8845
0.70	1.6360	0.8116
0.80	1.7272	0.7498
0.90	1.8182	0.6968
1.00	1.9095	0.6509
1.10	2.0008	0.6106
1.20	2.0918	0.5750
1.30	2.1833	0.5434
1.40	2.2744	0.5150
1.50	2.3657	0.4895
1.60	2.4567	0.4664
1.70	2.5481	0.4454
1.80	2.6395	0.4262
1.90	2.7306	0.4085
2.00	2.8218	0.3923

As it has already been mentioned, the forward speed of the vessel causes a shift in the frequency of encounter. The spectrum, now, can be defined for ship coordinates as follows:

$$S(\omega_e) = \left[\frac{S(\omega)}{d\omega_e/d\omega}\right]_{\omega=f(\omega_e)} \tag{39}$$

where

$$\omega = f(\omega_e) = \frac{-1 + \sqrt{1 + 4\omega_e \dfrac{U\cos\phi}{g}}}{2\dfrac{U}{g}\cos\phi}. \tag{40}$$

See [17] for a detailed description of obtaining $f(\omega_e)$.

A rational approximation was found to (39) subject to (40) in the following form:

$$S_a(\omega_e) = \frac{1.25}{4\omega_m{}^5} H^2 B(\alpha) \frac{(\omega_e/\omega_0)^4}{\left[1 + \left(\dfrac{\omega_e}{\gamma(\alpha)\omega_0}\right)^4\right]^3} \tag{41}$$

where $S_a(\omega_e)$ is the approximate spectrum, and

$$\alpha = \frac{U}{g}\omega_m \cos\phi. \tag{42}$$

$B(\alpha)$, $\gamma(\alpha)$ functions are given in Table I.

The stable system whose output spectrum is given by (41) when excited by white noise has transfer function

$$H_a(s) = \sqrt{S_0}\,\frac{(s/\omega_0)^2}{\left[1 + 2J\dfrac{s}{\omega_0} + \left(\dfrac{s}{\omega_0}\right)^2\right]^3} \tag{43}$$

where

$$S_0 = \frac{1.25}{4\omega_m} H^2 B(\alpha) \tag{44}$$

$$\omega_0 = \gamma(\alpha)\omega_m \tag{45}$$

$$J = 0.707. \tag{46}$$

Important Remark

The Bretschneider spectrum $S_\beta(\omega)$ is a one-sided spectrum, and the following relation provides a two-sided spectrum $S(\omega)$:

$$S(\omega) = \begin{cases} \pi S_\beta(\omega), & \omega > 0 \\ \pi S_\beta(-\omega), & \omega < 0. \end{cases} \tag{47}$$

KALMAN FILTER AND SIMULATION

The heave-pitch approximation resulted in a 15 state system and the sway-roll-yaw approximation in a 16 state system. Given that 6 states describe the sea, the total system required for 5 degrees of freedom motion studies would contain 25 states. If the sea spectrum contains two peaks, then a 31 state model is required.

The heave-pitch group is not coupled with the sway-roll-yaw group so that the study of each group can be independent. This is not to indicate that in a total design the two groups must remain independent, since they are excited by the same sea.

A. Heave-Pitch Motions

It is assumed that the heave and pitch motions are measured. The gyroscopes can provide accurate measurements of angles, up to about 1/10°. The noise, therefore, is due to structural vibrations, which in the longitudinal direction can be significant due to the beam-like response of the vessel. As a result, the measurement noise was estimated based on data from ship vibrations. The same applies to the heave measurement noise.

A Kalman filter was designed for speed $U = 21$ ft/s and waves coming at 0° (head seas) with significant waveheight $H = 10$ ft and modal frequency $\omega_m = 0.73$ rad/s (sea state 5). The measurement noise intensity matrix was selected from ship vibration data to be

$$V = \begin{bmatrix} 0.75 & 0.0 \\ 0.0 & 0.0003 \end{bmatrix}.$$

The filter poles are within a radius of 1.3 rad/s. A typical simulation results are shown in Figs. 7 and 8. In these figures, exact knowledge is assumed for the significant waveheight and modal frequency. The accuracy of the filter is very good both for heave and pitch.

Subsequently, the same filter was used combined with a ship and sea model different than the nominal one, to investigate the sensitivity to the following parameters.

The influence of the significant waveheight is very small when the modal frequency is accurately known. On the contrary, the influence of the modal frequency is quite critical, particularly for pitch (see Figs. 9 and 10). The same conclu-

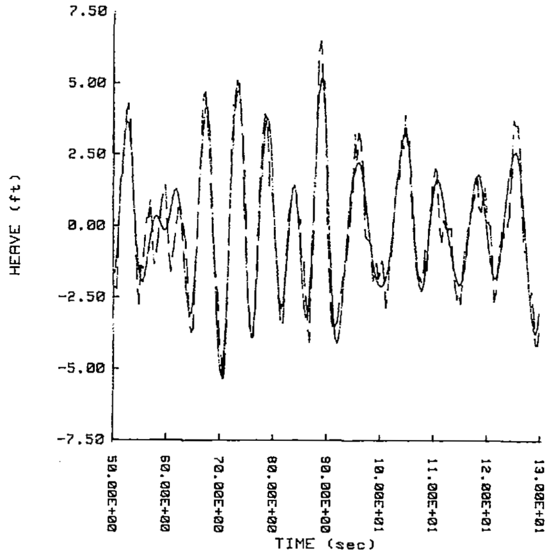

Fig. 7. Simulation results of the heave motion (solid line) and its Kalmal filter estimate (dotted line) based on noisy measurements.

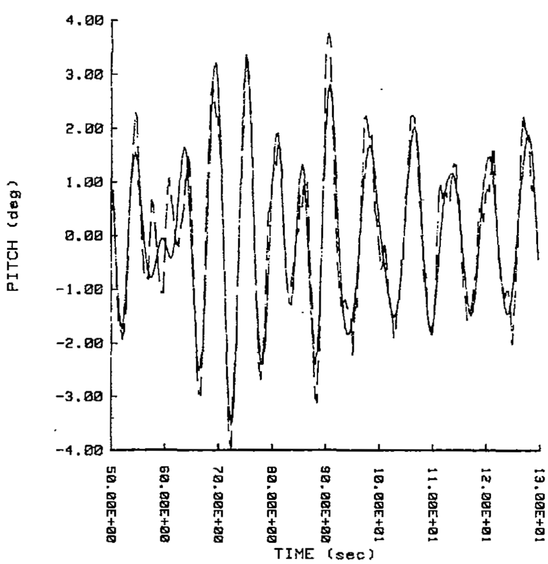

Fig. 8. Simulation results of the pitch motion (solid line) and its Kalman filter estimate (dotted line).

sion is reached when a double peak sea spectrum is used [19], [20].

The effect of the forward speed and wave direction was found to be unimportant particularly for heave, while for small changes in wave angle (±15°) the pitch prediction error was not affected significantly [19].

B. Sway-Roll-Yaw Motions

As in the case of heave and pitch, the measurement noise consists primarily of structural vibrations rather than instrument noise. For roll, such vibrations are quite small for a destroyer vessel and similarly, for sway and yaw, the vibrations are smaller than in the case of heave and pitch.

The noise intensity used was nonetheless similar to the heave and pitch noise, so as to bound the filter eigenvalues below 2 rad/s, which is the typical wave bandwidth.

A specific example has been worked out for a forward ship speed of 15.5 ft/s and waves at 45° and sea state 5 (significant waveheight of 10 ft and modal frequency of 0.72 rad/s. The

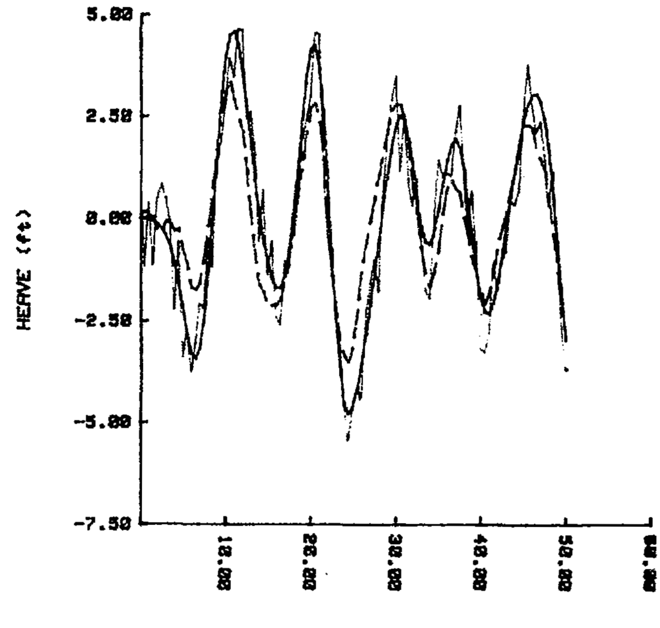

Fig. 9. Simulation results of the heave motion (solid line), the noisy measurement (light solid line), and the estimate (dotted line) when the modal frequency is in error (ω_m = 0.32 rad/s).

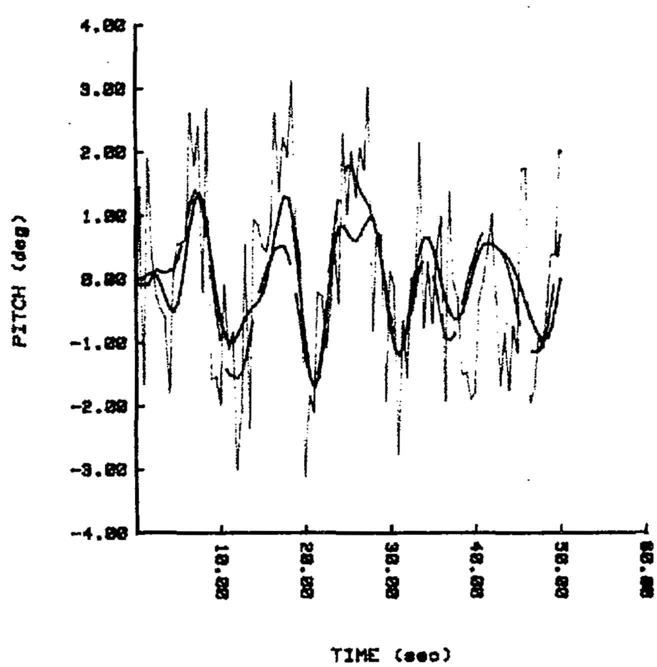

Fig. 10. Simulation results of the pitch motion (condition as in Fig. 9).

measurement noise intensity matrix was

$$\text{diag}\{0.1 \text{ ft}^2, 2 * 10^{-4} (\text{rad})^2, 2 * 10^{-4} (\text{rad})^2\}.$$

The simulation shows very good estimation as seen in Figs. 11, 12, and 13. Yaw is very small and the measurement noise is large relative to the yaw motion, nonetheless, the yaw estimation, based primarily on the roll and sway measurements, is very good.

Table II presents the results of a sensitivity study of the influence of the various parameters involved. The most critical parameter is again the modal frequency. The ship speed and

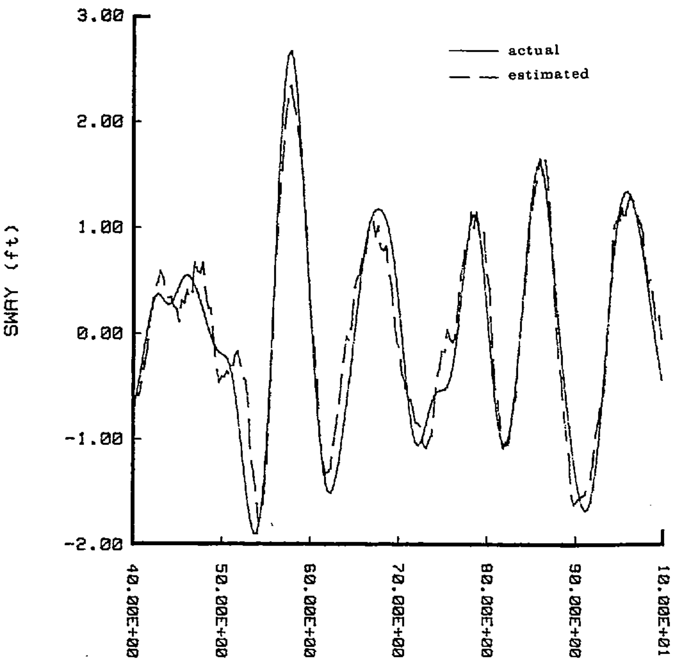

Fig. 11. Simulation results of the sway motion (solid line), and the Kalman filter estimate (dotted line) based on noisy measurements.

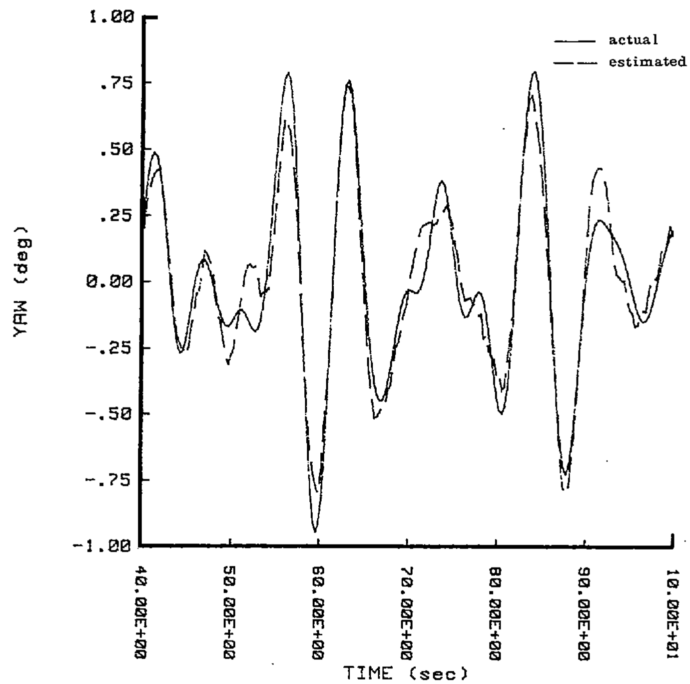

Fig. 13. Simulation results of the yaw motion (conditions as in Fig. 11).

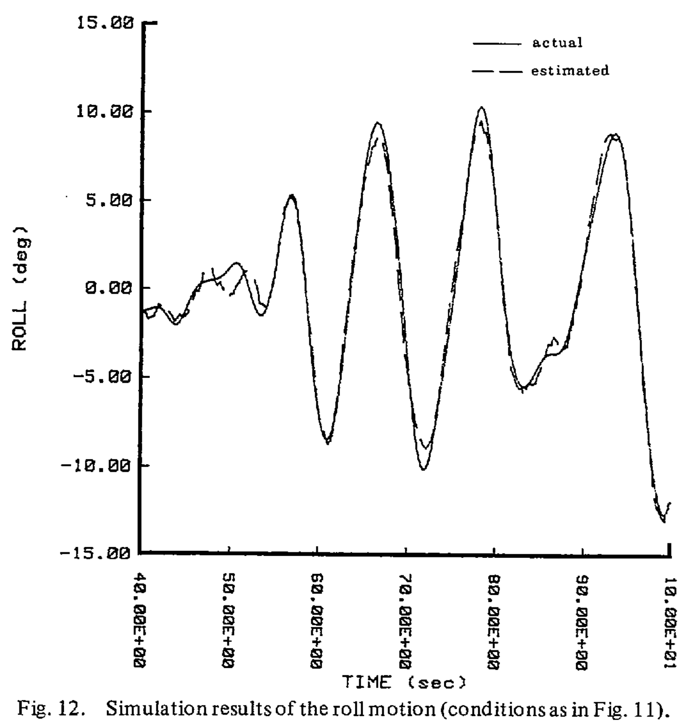

Fig. 12. Simulation results of the roll motion (conditions as in Fig. 11).

TABLE II
SENSITIVITY OF THE RMS ERROR OF SWAY, ROLL, AND YAW MOTION, TO CHANGES IN THE PARAMETERS OF THE SHIP-SEA MODEL. C (SWAY) INDICATES A CALIBRATION COEFFICIENT IN THE SWAY MEASUREMENT (AND SIMILARLY FOR THE OTHER MOTIONS), TO HANDLE SYSTEMATIC ERRORS IN THE MEASUREMENT.

Parameter Changed	Error Sway (ft)	Error Roll (deg)	Error Yaw (deg)
Basic Case	0.241	0.56	0.0776
U=20 ft/s	0.245	0.568	0.0963
ω_m=0.52 rad/s	0.314	0.91	0.0858
ϕ=60°	0.296	0.624	0.112
C(sway)=0.9	0.255	0.586	0.081
C(roll)=0.9	0.247	0.708	0.0808
C(yaw)=0.9	0.242	0.56	0.0777
C(sway)=0.0	0.518	1.21	0.1408
C(roll)=0.0	0.376	4.08	0.158
C(yaw)=0.0	0.242	0.563	0.0785
RMS values of the motions (nominal case)	0.60	4.56	0.227
Measurement noise intensity	$(.316\ ft)^2$	$(.81°)^2$	$(.81°)^2$

the wave direction are not critical for the estimation error. This is a very important conclusion as far as the wave direction is concerned, because in reality, seas are directional and very difficult to measure, or even model appropriately.

The influence of systematic measurement errors was studied by using a calibration factor. This factor is defined to be the ratio of the measurement fed to the filter over the actual measurement, thus introducing a systematic error. If C is the calibration factor, then the systematic error as a percentage of the actual measurement is $100 \cdot (1 - C)$. In the case of a 10-percent error, the most significant change was found in the case of the roll motion. In the case of a calibration factor 0 (indicating a disconnected measurement), significant errors resulted, especially for roll in the case of disconnected roll measurements (Table II).

CONCLUSIONS

A satisfactory approximation of the ship motion equations as provided by hydrodynamic theory has been achieved. The approximation is valid within the wave frequency and for seas described by the Bretschneider spectrum, whose major limitations are a) unidirectional seas, and b) unlimited fetch, deep water.

The resulting two groups of motions, i.e., heave-pitch and sway-roll-yaw can be approximated separately requiring 15 and 16 states, respectively. If both must be used, a 25 state system is required.

The model depends parametrically on the ship speed, the wave angle, and the significant waveheight and modal frequency.

The Kalman filter is designed using as measurement noise intensity, values from ship vibration studies. For sway-roll-yaw, the vibration levels are small, nonetheless, to bound the filter eigenvalues below 2.0 rad/s similar values were used, as for heave-pitch.

It should be remembered that heave and pitch are related by a nonminimum phase transfer function, resulting in reduced filter accuracy. Actually heave is 90° out of phase for low frequencies with respect to all the other motions.

A sensitivity analysis of the filter performance indicates that the most critical parameter is the spectrum modal frequency. It should be remembered that a sea spectrum may contain more than one peak, in which case it is essential to obtain an accurate estimate of both peak frequencies.

Of particular interest is the fact that the wave direction does not have a significant influence on the estimation error. This means that although our modeling used a unidirectional Bretschneider spectrum, it can be applied in its present form for directional seas.

APPENDIX

The M.I.T. Sea-keeping Program [24] was used to derive the hydrodynamic data. The values used to develop the simple models for heave-pitch and sway-roll-yaw are given below.

All units are consistent such that the forces are obtained in tons, the moments in ton•feet, the linear motions in feet and the angular motions in radian.

Heave-Pitch Characteristics

M	$= 214$	C_{33}	$= 587$
A_{33}°	$= 281$	B_{33}°	$= 260$
A_{35}°	$= 15\,500$	B_{35}°	$= 15\,500$
C_{35}	$= 260$		
I_{55}	$= 3.76*10^6$	A_{55}	$= 4.20*10^6$
B_{55}	$= 3.8*10^6$	C_{55}	$= 9.53*10^6$
A_1	$= 550$		
A_2	$= 120\,000$.		

The principal ship characteristics are as follows:

L_{BP}	Length between perpendiculars	= 529 ft
B	Beam	= 55 ft
T	Draft	= 18 ft
VCG	Vertical center of gravity (from waterplane)	= −4.6 ft
GM	Metacentric height	= 4.16 ft
XCG	Longitudinal center of gravity (from admidships)	= 1.07 ft *AFT*
Δ	Displacement	= 6800 ton
C_b	Block coefficient	= 0.461

Sway-Roll-Yaw Characteristics (for zero speed)

$A_{44} = 22\,900$

$I_{44} = 104\,000$

$B_{44} = 887 + B_{44}^*$, where B_{44}^* is the "equivalent nonlinear damping [3]

$C_{44} = 28\,000$

A_{24}	$= -759$	B_{24}	$= -55.4$		
A_{46}	$= 182\,000$	B_{46}	$= 6{,}270$		
A_{22}	$= 223$	B_{22}	$= 10.6$		
A_{66}	$= 4.18*10^6$				
I_{66}	$= 3.8*10^6$				
B_{66}	$= 144\,000$				
A_{26}	$= 14\,600$	B_{26}	$= 423$		
A_2	$= 380$	M_{24}	$= M_{42}$	$= 988$	
A_4	$= 2400$	M_{26}	$= M_{62}$	$= -230$	
A_6	$= 23\,000$				

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E. Tiemroth, graduate student in the Ocean Engineering Department, has prepared some of the programs presented in this study and assisted in the debugging and running of several others.

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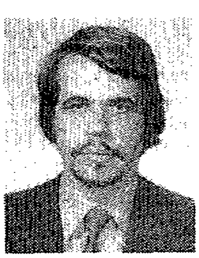

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He is the co-author of *Optimal Control* (McGraw-Hill, 1966), *Systems, Networks, and Computation: Basic Concepts* (McGraw-Hill, 1972); and *Systems, Networks, and Computation: Multivariable Methods* (McGraw-Hill, 1974). In 1974 he developed 65 color TV lectures on *Modern Control Theory* and accompanying study guides; these are distributed by the Center for Advanced Engineering Study, M.I.T., Cambridge, MA. In addition, he has authored or co-authored over 250 technical papers and reports. His research contributions span the areas of system, control, and estimation theory and its applications to the fields of defense, aerospace, transportation, power, manufacturing, economic and command, control, and communications systems.

In 1964 he was the first recipient of the American Automatic Control Council's Donald P. Eckman award for outstanding contributions to the field of automatic control. In 1969 he was the first recipient of the F.E. Terman award of the American Society for Engineering Education as the outstanding young electrical engineering educator. In 1980 he received the Education Award of the American Automatic Control Council for his outstanding contributions and distinguished leadership in automatic control education. In 1973 he was elected Fellow of the Institute of Electrical and Electronic Engineers and in 1977 Fellow of the American Association for the Advancement of Science. He has served on numerous committees of IFAC, AACC, and IEEE; he was president of the IEEE Control Systems Society from 1972 to 1974.

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