

AGGREGATE ADVERTISING RESPONSE MODELS:
THE STATE OF THE ART

John D. C. Little
Sloan School of Management
M.I.T.

WP 1048-79

February 1979

ABSTRACT

Aggregate advertising response models relate product sales to advertising spending for a market as a whole. Although many models have been built, they frequently contradict each other and considerable doubt exists as to which models best represent advertising processes.

An increasingly rich literature of empirical studies helps resolve these issues. Illustrative examples are used to demonstrate various aspects of advertising response including dynamics over time, shapes of long run response functions, and competitive interactions.

A review of aggregate models developed on apriori grounds brings out similarities and differences among those of Vidale and Wolfe, Nerlove and Arrow, Little, and others and identifies ways in which the models agree or disagree with observed phenomena. A Lanchester-like structure is shown to generalize many of the features of these models and to conform to some but not all of the empirical observations. Standard econometric models are criticized for their structural forms. Future work must join better models with more powerful statistical calibration methods.

Ignorance of advertising response phenomena, inability to make good measurements, and lack of a theory to organize existing knowledge contribute to great waste in advertising. Contradictions abound. For example, advertising partisans in one company declare that certain markets should receive more advertising because "the brand is strong there and we should take advantage of its momentum" and then, a few minutes later, propose that other markets should also receive more because "industry sales are strong there and our share is low", which, freely translated, means "the brand is weak there and we don't have any momentum."

One often sees media scheduled in intensive "flights" so that "the message can be heard through the noise", but, if someone asks why not make the flight half as long and twice as intense or twice as long and half as intense, no good answer can be given.

In one company the brand managers push to spend their budgets early in the calendar year. Is this because of product seasonality? Or a belief in the effectiveness of campaigns lasting six months? No, it is because corporate management has a reputation for calling back unspent monies to improve earnings in the fourth quarter. Brand management responds by spending all its money in the spring. One might suspect that management in this company is not quite sure what it is getting for its advertising dollars.

In most companies, advertising strategy is subject to intermittent upheavals. Sometimes this happens brand by brand - each year one or two products undergo an agonizing reappraisal. At other times a whole division will go through a convulsion. Perhaps these strategy shifts are appropriate, but rarely is there any clear reason why the re-examination should be taking place for one brand and not another.

After a substantial change, marketing management watches sales carefully and, more often than not, expresses satisfaction. Yet, though a major strategy shift offers a unique opportunity for measurement (say, by holding out some control markets) such steps are virtually never taken.

Advertising also is full of fads. Clearly a company's ads are conspicuous. (They had better be!) Everybody from the president's wife to the newest clerk voices an opinion. Clever copy becomes a conversation piece overnight. ("We try harder", "I can't believe I ate the whole thing".) Innovations perceived as successful are quickly imitated by others, rightly or wrongly. (Low key testimonials, comparison advertising and humor have been up and down over the past few years. Mature authority figures seem to be undergoing a revival at the moment.) It is an exciting world of good showmanship where strategy changes are conceived, packaged, and sold with many of the appeals that characterize advertising itself.

And, to a great extent, this is as it should be. Good strategy requires imagination and style and always will. At the same time, strategy emerges best from a foundation of reliable facts and sound analysis. These are not easy to come by.

The management science/operations research fraternity has nibbled at advertising issues. Moderate heartburn has been a fairly common result. Yet, there have certainly been successes, one or two of which have been widely publicized. See, for example, Weinberg [1960] and Ackoff and Emshoff [1975]. Other workers have often found these studies hard to duplicate, perhaps because marketing situations differ from company to company, or more likely, because studies to date simply do not supply enough knowledge to provide an adequate foundation for imitation. Quantitative understanding of advertising processes has made some headway but the job is far from done

and the available material needs pulling together. This paper takes on part of that job by examining aggregate response models.

A basic OR/MS goal is to find good models. But what is a good model? It depends. We should tailor a model to fit the job at hand. Lilien [1975] calls this "model relativism". Urban [1974] expresses the same thought when he says the model builder should state the purposes of his model in advance. All right, we want advertising response models that will be useful for

- tracking and evaluating advertising performance,
- diagnosing market changes,
- incorporation into decision models.

Although we shall not address decision models per se, they should contain response models with the necessary phenomena to assist meaningfully on

- annual budgets,
- geographic allocation, and
- allocation over time.

Two other important areas are media and copy. These enter our discussion but will not be treated with the detail required for incorporation into decision models.

In focusing on the response model rather than the decision model, we differ from the many writers who seek to characterize optimal policies once the response model is given. For an extensive review of this literature, see Sethi [1977].

Attainment of our goals requires dynamic models that relate advertising spending to sales. We confine attention to established products since they blot up most of the money and since new products use special models. As already mentioned, we focus on macro or aggregate models rather than models of individual customer behavior. The reasons are two: First, most micro models so far have been thin on either empirical data or marketing control variables (especially advertising) or both. Second, the most convincing data sources available to companies for calibrating advertising models today are aggregate in nature (historical time series at a national or market level and field experiments). This is not to play down the importance of modeling individual customer response to advertising (see, for example, the media selection models of Little and Lodish [1969], Gensch [1969], Zufryden [1973], and Starr [1978]). Rather it is to say that the catalog of advertising effects presented here comes almost entirely from aggregate data and so is inadequate to resolve most micro-modeling questions. We note, however, that micro models will have to reproduce the empirical macro effects reported here.

1. Controversies, Confusions, and Contradictions

The advertising models in the OR/MS literature are not especially consistent with each other nor with such measurements and data as are available. We identify three areas of controversy: shape, dynamics, and interactions.

1.1 Shape. By shape we mean the shape of a curve showing sales response to advertising under steady state conditions. In other words, if a set of different advertising rates were tested with other market influences held fixed, and brand sales were measured each time after the

market came to equilibrium, what would a plot of sales rate vs. advertising rate look like? Is such a relationship linear? Many econometric analyses implicitly assume it to be. What are sales with zero advertising? A good many theoretical models imply sales would be zero. Is response S-shaped? Most existing models do not permit such a possibility, and yet many media schedules contain "flights" whose justification seems to be based on belief in a threshold or S-shape in the curve. Do large amounts of advertising depress sales? So claim some writers but few models accomodate it.

1.2 Dynamics. How fast do sales respond when advertising is increased?

In the process of calibrating marketing models, the author has often asked marketing managers the following question. "What percent of the long run response to an advertising increase would you expect to obtain in the first year?" A typical answer would be 60% and the range might run from 30% to 80%. It will be interesting to compare these values with the data in the next section.

How fast do sales decay when advertising is decreased? Strong marketing men turn pale when advertising cuts are proposed, even if only for test purposes. "We might lose the brand franchise," they say. Their pallor may be role-playing because companies under financial stress regularly cut budgets drastically, apparently believing that the brand will survive.

Still another question is: Does hysteresis ever exist? In other words, are there circumstances under which sales would increase with increased advertising and stay there after withdrawal of advertising? Or, in the opposite direction, could a competitor take away sales and share by increasing advertising, and the brand find it difficult to regain position? Very few marketing models exhibit such a phenomena, but some people believe it to exist in practice.

Finally, how does advertising effectiveness change with time and how can we model it?

1.3 Interactions. Is it better to advertise where sales are strong or weak? This is a classical argument, certain to draw proponents to each side. One can be sure that every model contains one or more, often inconspicuous, assumptions relating to this question, and so does any statistical analysis. In a similar vein, are advertising effects additive with other marketing variables, e.g., price, promotion, and competitive actions, or multiplicative, or do they interact in more complicated ways? All shades of assumptions appear in the model building and statistical literature. They are certainly not all consistent with one another.

2. Basic Phenomena: What do the data say?

Measurements must eventually resolve the issues just raised and tell us which advertising phenomena are real and which are only folklore. In this spirit, we present a collection of empirical examples of certain major effects. These will help sort out the models in the next section.

2.1 Upward Response

Advertising increases sales, or such is the intent. Figures 2.1-2.3 show instances of sales before and after the introduction of substantial new advertising dollars. In each case the sales rate increases within a month or two. Observe that this time span is inevitably shorter than the judgements reported in the previous sections.

Figure 2.4, taken from data of Bloom, Jay and Twyman [1977], is particularly interesting because it shows a jump in sales due, not to an increase in spending, but to a change in copy. Thus "advertising rate" is not necessarily the same as "spending rate". Notice again that sales response almost immediately. A similar copy change effect appears in the results of Pekelman and Tse [1976].

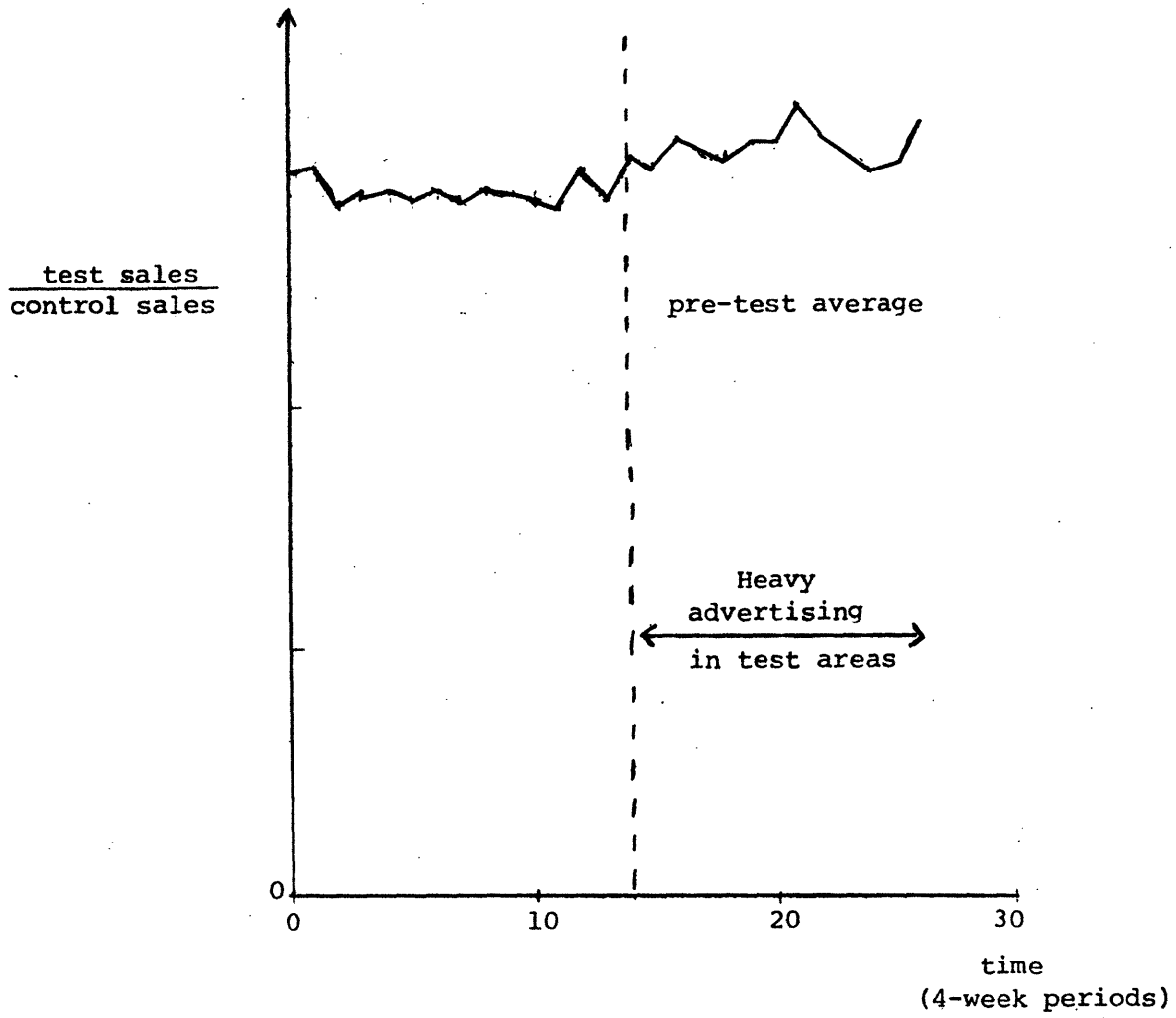


Fig. 2.1 Sales response to increased advertising. Ratio of sales in test areas to sales in control areas vs. time.

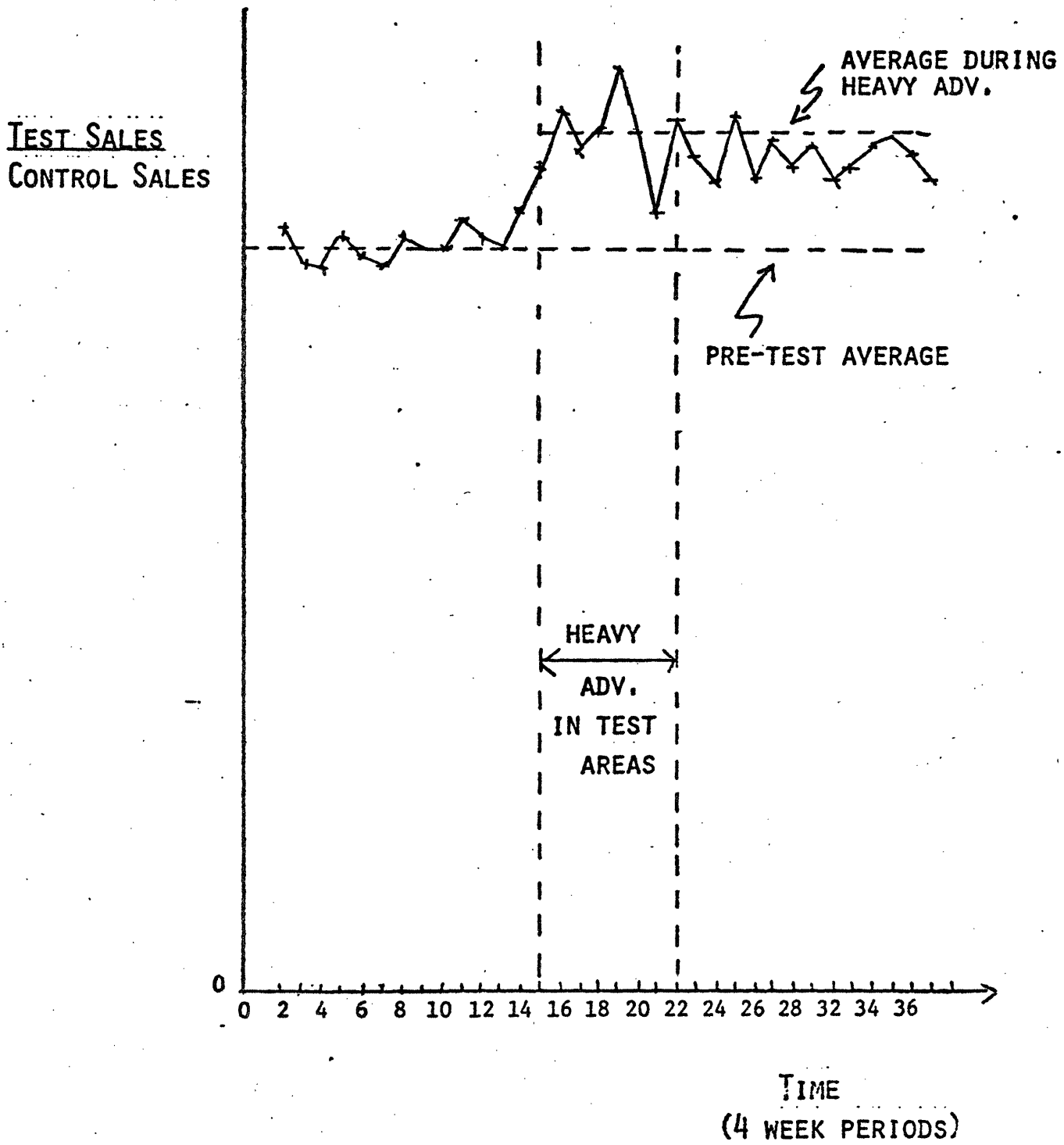


FIGURE 2.2 SALES RESPONSE TO INCREASED ADVERTISING AND ITS REMOVAL FOR A PARTICULAR PRODUCT.

Test Sales
Control Sales

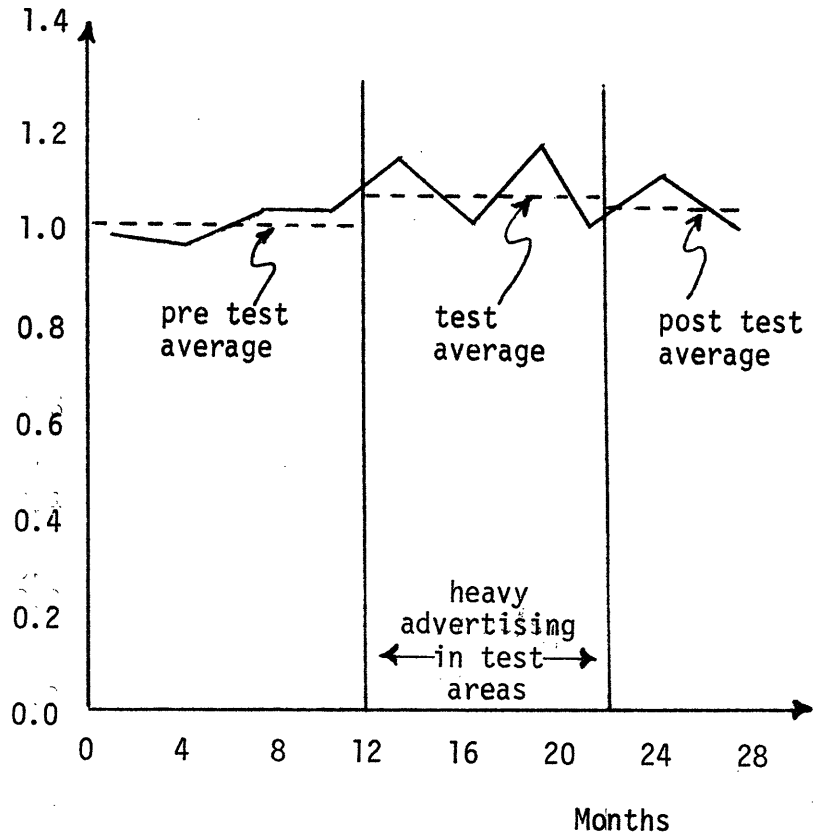


Figure 2.3 Sales response to increased advertising and its removal.

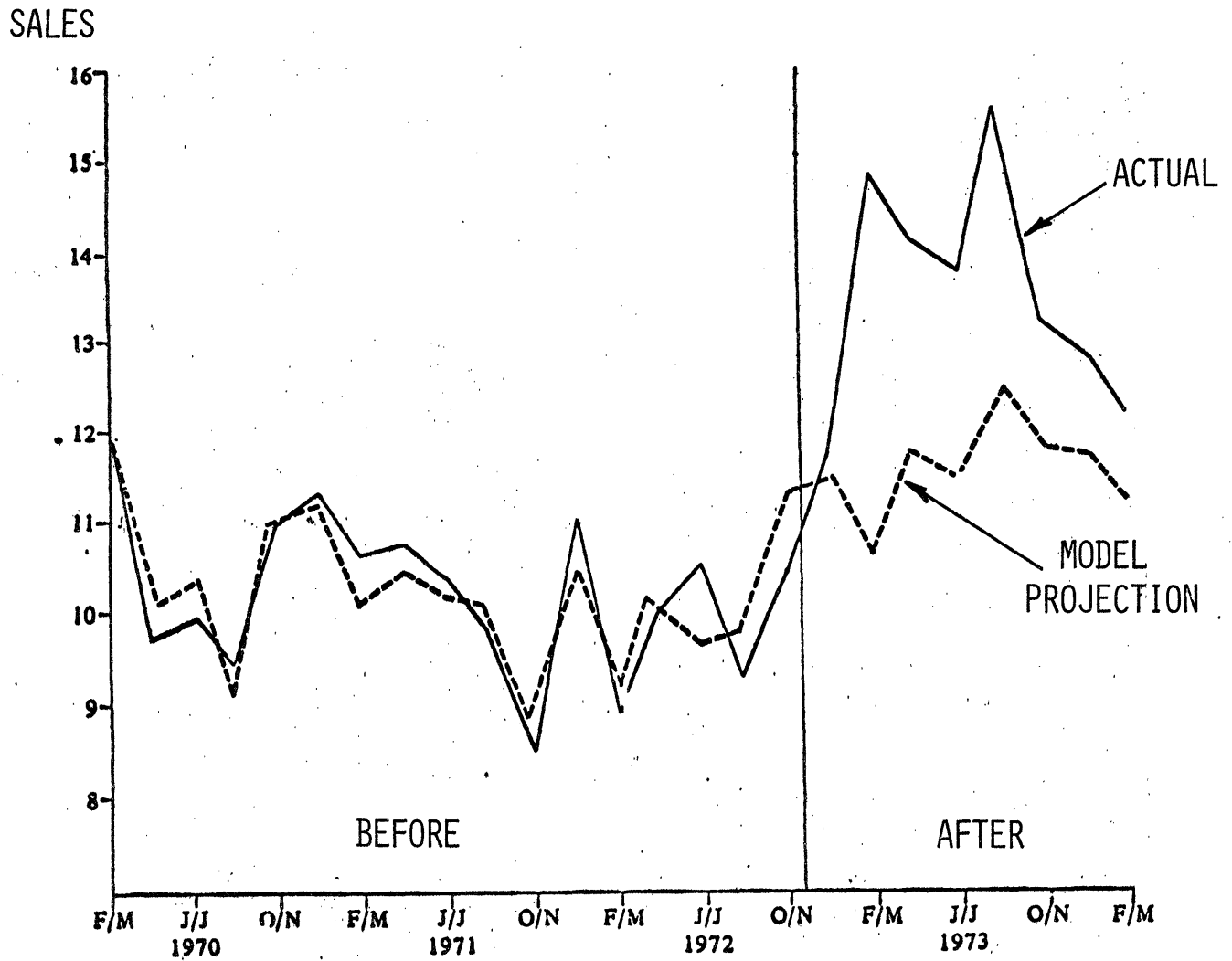


FIGURE 2.4

NEW ADVERTISING COPY INCREASED SALES
OF HORLICK'S SUBSTANTIALLY

Reference: from Bloom, Jay, and Twyman [1977]

2.2 Sales at the New Level

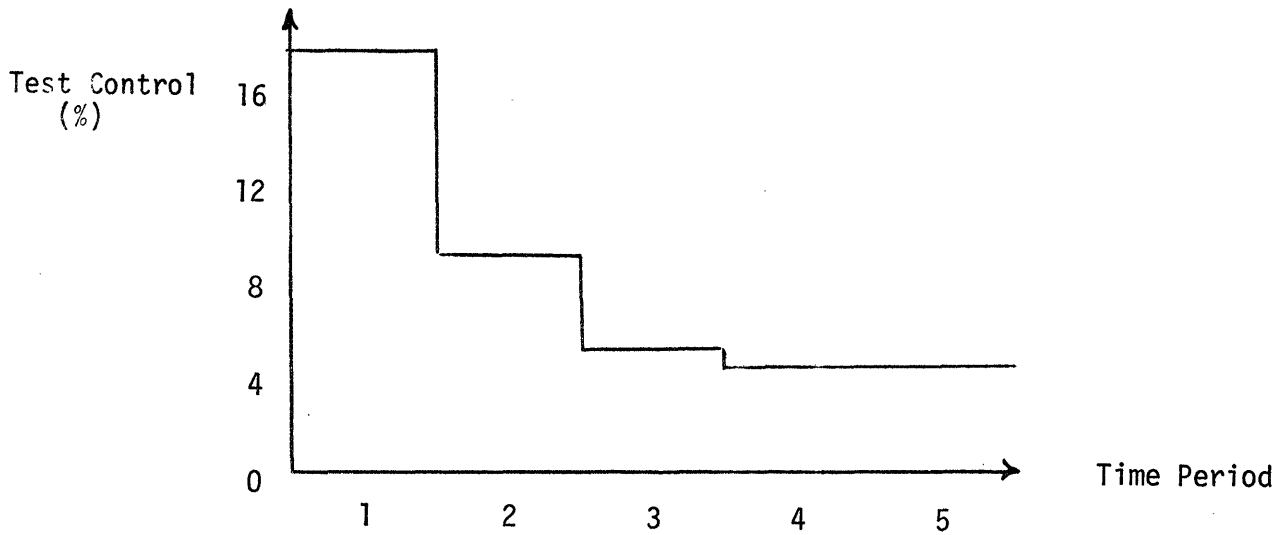
Figures 2.1-2.3 show sales leveling off under the new, higher spending rates. Whatever was going to happen in these cases appears to have happened before the advertising stopped. Haley [1978], however, has found a further effect, shown in Figure 2.5. The sales increase is there but its magnitude decreases with time. The leveling off appears to take place but at a value lower than the initial gain. Such an effect is common in the case of new products that are purchased frequently. In such cases people learn of the product through advertising and try it, thereby causing a sharp spurt in sales. Only a fraction of the triers become regular purchasers and so sales taper off to a lower rate. In this paper we deal with established brands, but an analogous process seems quite likely: Increased advertising leads a group of non-users to buy the product for reexamination or just for variety. Some of these customers continue to purchase, others not.

The copy-induced sales increase in Figure 2.4 also seems to fall off. This too may be a new-trier effect although many advertising people would say that the copy is wearing out.

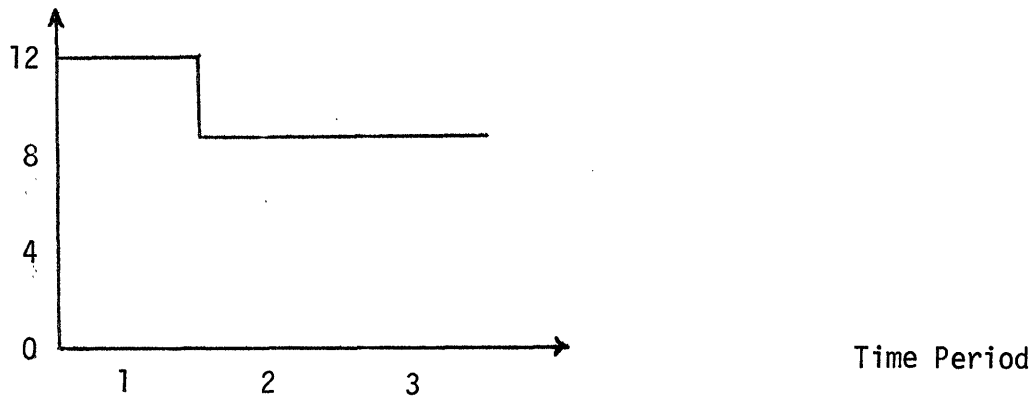
2.3 Downward Response.

Figures 2.2, 2.3, and 2.5c show sales response to decreased advertising. Notice that sales decay appears to take place more slowly than sales growth. This is particularly evident in Figure 2.2 and with more variance in 2.3. In these cases we are able to observe the same product under both increases and decreases of advertising.

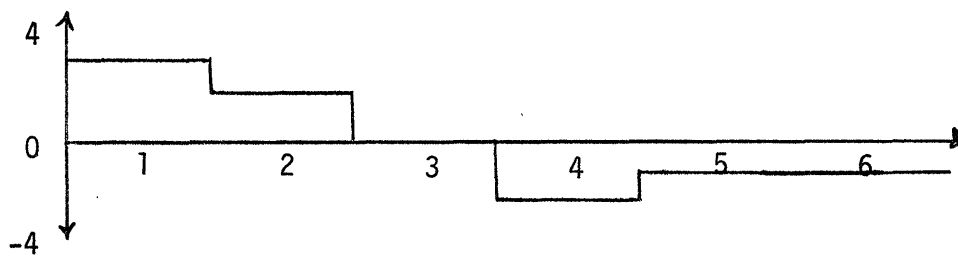
An explanation for decay time being longer than rise time is that the rise relates to the advertising communications process; i.e., hearing or seeing the advertising message, absorbing it and acting on it. Since



(a) Brand B: Sales effect of doubled advertising



(b) Brand D: Sales effect of 50% advertising increase



(c) Brand D: Sales effect of 25% advertising reduction

Figure 2.5 Brands B and D show the erosion of sales increases attained by heavier advertising. Brand D shows the relatively slow sales decay following decreased advertising. Data from Haley [1978].

nominal forgetting times for advertising are on the order of a month (Lodish [1971], Strong [1974]), it seems reasonable that an established product with good retail availability would show the positive effects of increased advertising within a short time. On the other hand, decay in the absence of advertising seems more a question of experience with the product. Using and liking a brand will have far more influence on a customer than advertising. Although sales decay will depend on competitive activity and other factors, it does not seem surprising (especially when facts stare us in the face) that decay is usually much slower than growth.

An essential point, however, is that a good model of sales response to advertising should permit different rise and decay rates.

2.4 Sales with Zero Advertising

Figure 2.6 shows the sales of a collection of never-advertised products. Many people do not realize this, but there are literally hundreds of unadvertised products selling happily away in every supermarket and department store. This will happen, for example, if distribution is assured. Thus, chain-store house-brands are guaranteed a place on the shelf. Stores also stock unadvertised "price brands" with unfamiliar names in order to offer the consumer a low cost choice. In other examples, vending machines look out on a captive market and frequently carry unadvertised and virtually unknown brands. Department stores stock certain items by function without fanfare, e.g., string, envelopes or thumbtacks. This does not mean that such products would not sell faster with advertising but rather that positive sales with zero advertising are quite reasonable.

We should not be surprised, therefore, that empirical studies of sales response often indicate that a substantial part of the market seems

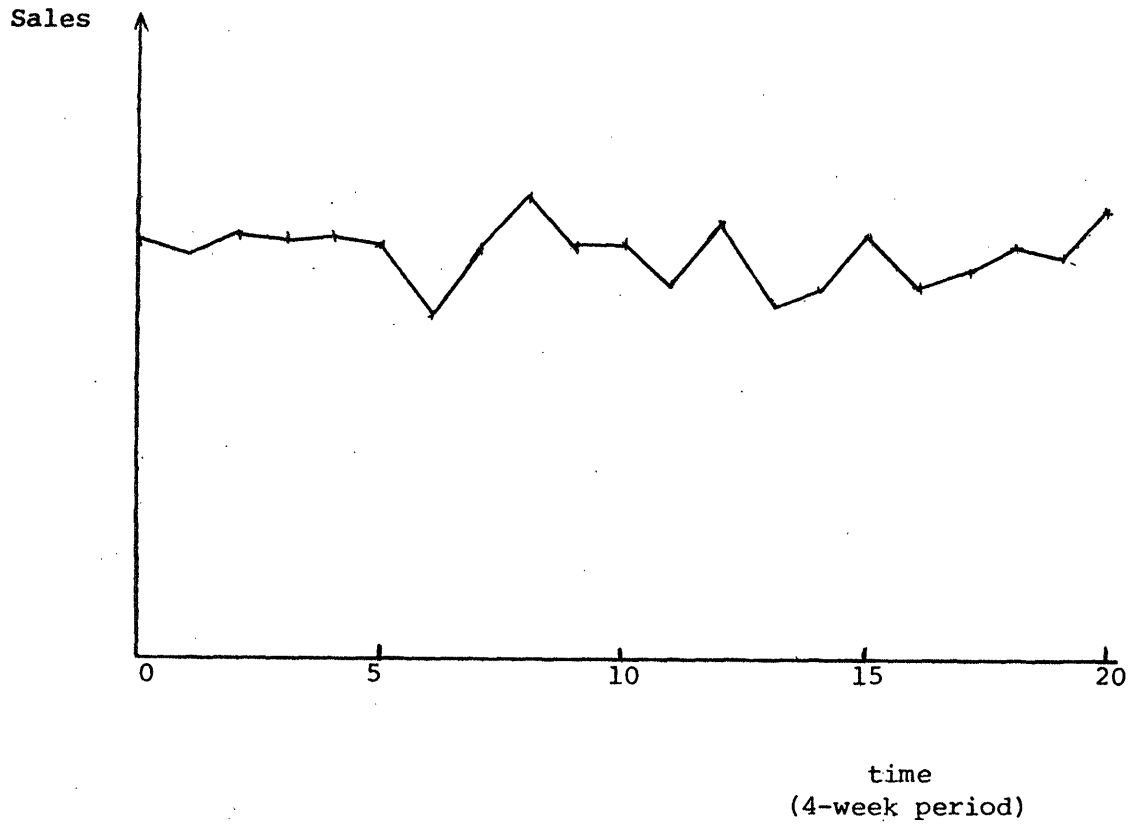


Fig. 2.6 Sales of a collection of unadvertised products.

not to be affected by advertising, at least over the medium run. This is noticeable in econometric studies with linear models where positive constant terms are common (e.g., Bass and Clark [1972]).

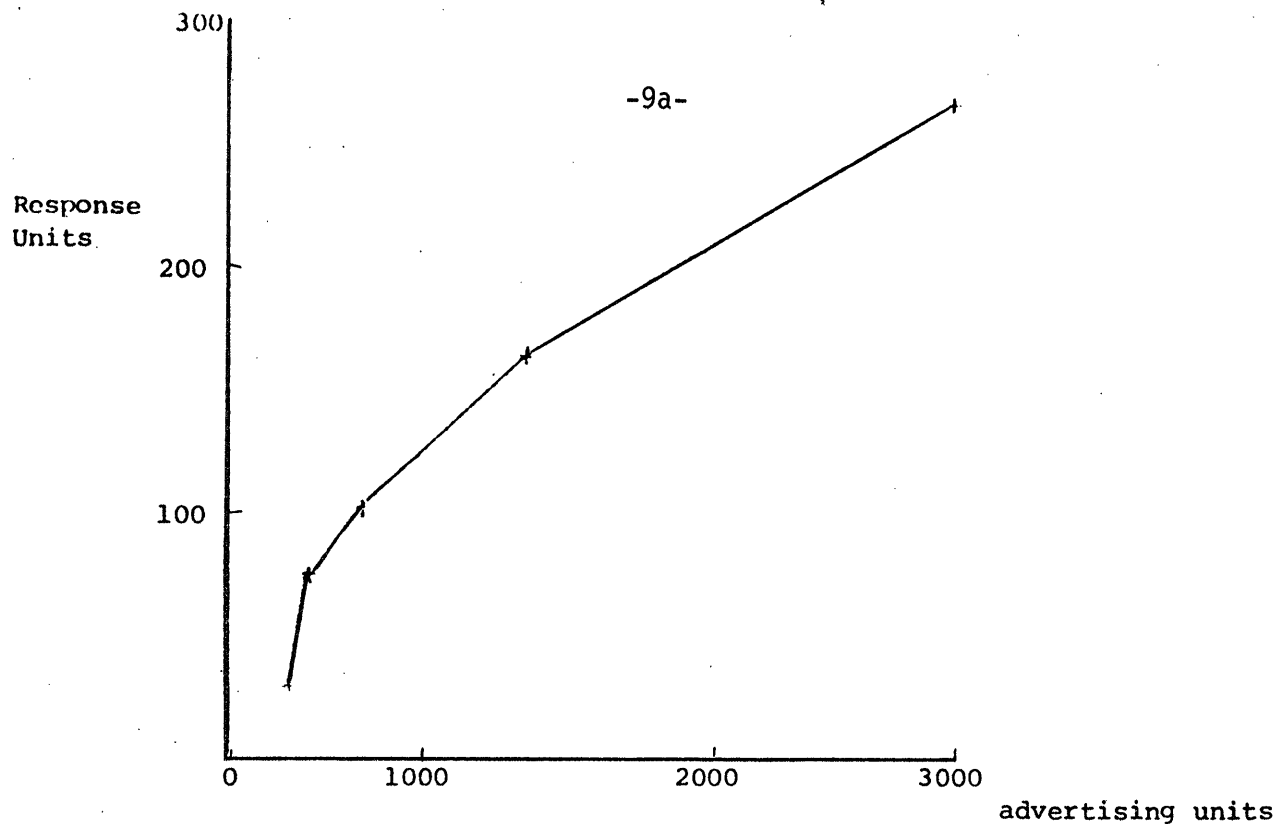
Thus an advertising response model should admit the possibility of sales with zero advertising (many do not).

2.5 Nonlinearity

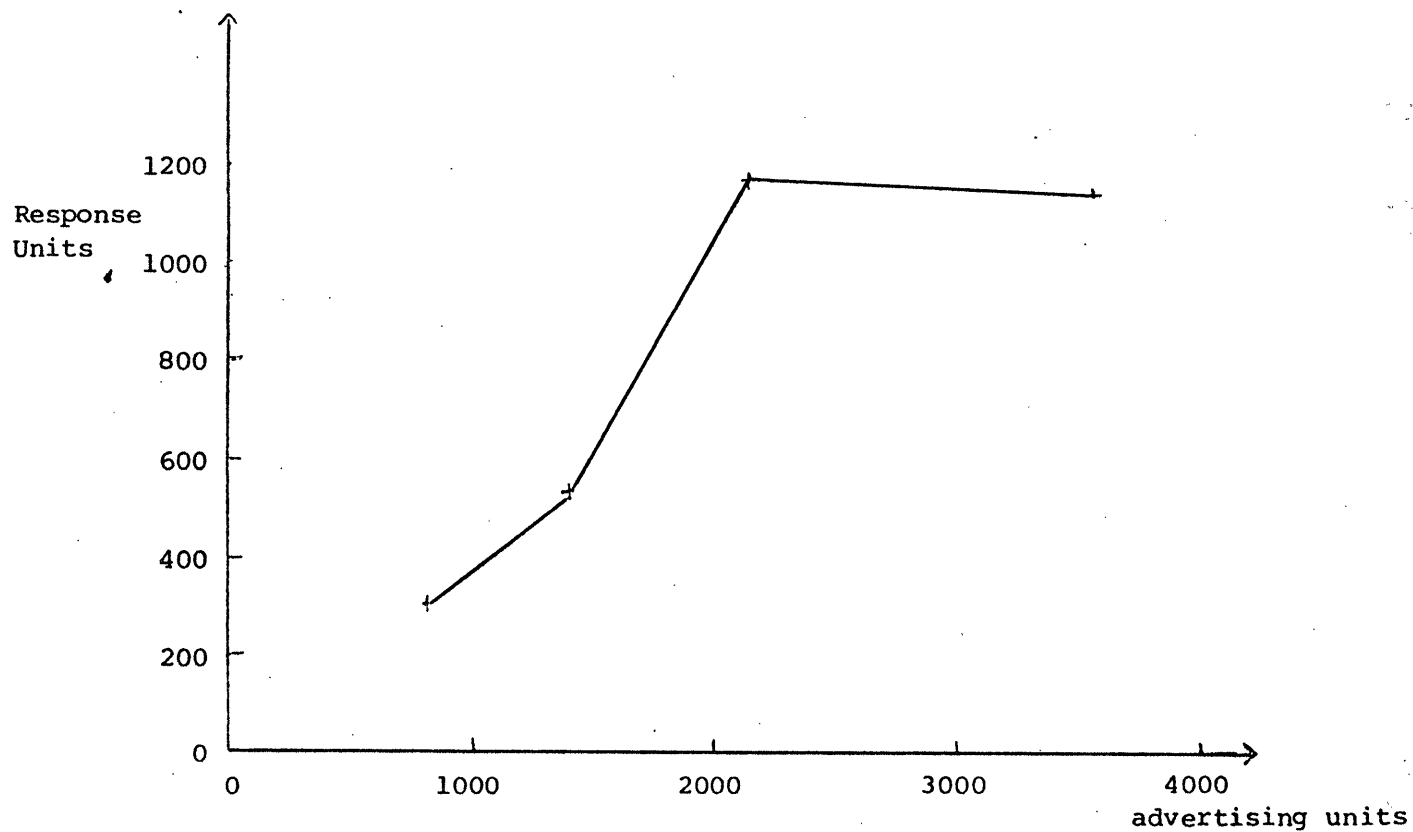
Suppose advertising is held constant and other market conditions do not change. After some time period the market can be expected to be in steady state. If this were done for a number of different advertising rates, we could make a plot of steady state sales vs. advertising.

What would the curve look like? We would not expect it to be linear, for this would have a variety of nonsensical consequences. (For example, a product with a fixed production cost per unit, would have an optimal advertising rate of either zero or infinity.) However, "not linear" covers many possibilities. We describe two important ones.

(a) Diminishing returns. Figure 2.7 displays a pair of empirical advertising response curves plotted from data of Benjamin and Maitland [1958]. Their data is particularly valuable because of the great range of advertising levels studied. In each case the slope of the curve decreases at high advertising levels, thereby showing concavity or diminishing returns. Less obvious is whether response is better modeled by an absolute ceiling (saturation level) or by a function that can surpass any prespecified level, albeit with increasing difficulty. Benjamin and Maitland choose the latter course; they take sales to be the lag of advertising. Such a function, however, does not make sense at zero advertising since $\log 0 = -\infty$.



(a) Radio equipment in newspapers.



(b) Service recruiting in newspapers

Fig. 2.7 Response to advertising. Benjamin and Maitland [1958].

(b) S-shape. Controversy surrounds the question of whether steady-state sales response to advertising is S-shaped, i.e., whether, at low levels of advertising, increases are increasingly effective up to some point after which diminishing returns set in.

As mentioned earlier, many advertising schedules today contain "flights" or "pulses". A theory that might justify flights is that response is S-shaped, e.g., small advertising rates do little good but medium rates are effective. Published empirical evidence of such relationships is hard to find. We offer, however, Figure 2.8 taken from Rao [1978]. Rao and Miller [1975] have run time series regressions in different geographic areas having different average advertising rates to estimate advertising effectiveness by individual area. Generally they find lower effectiveness where average advertising has been very low or very high. They then fit these results cross-sectionally across areas and calculate the S-shaped curve shown in Figure 2.8.

On the direct question of the efficacy of pulses (as opposed to whether steady-state response is S-shaped), Ackoff and Emshoff [1975] report good results from pulsing, although they do not present statistical measures of quality. Sethi [1971] reports a Milwaukee Advertising Laboratory experiment that seems to show good short run but poor long run effects. In any case, considering current practice, Rao and Miller's work, and the importance of the issue, we argue that advertising models should accommodate S-shaped curves.

Before leaving the empirical evidence on steady state response, we present certain provocative results from McDonald [1970]. He has analyzed panel data that contained not only product purchases but also media usage. Figure 2.9 shows a sales measure plotted against an advertising measure.

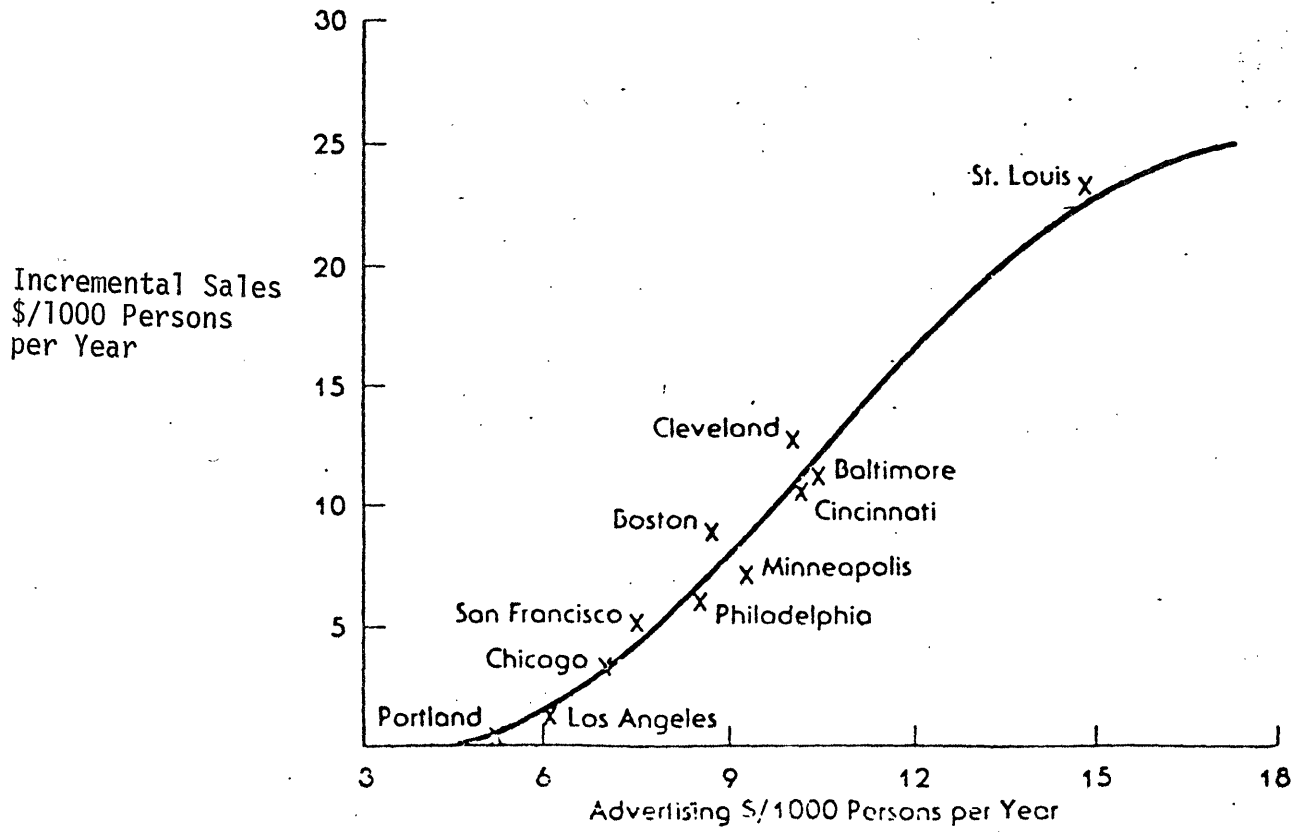


Figure 2.8 S-shaped curve of sales response to advertising from Rao [1978].

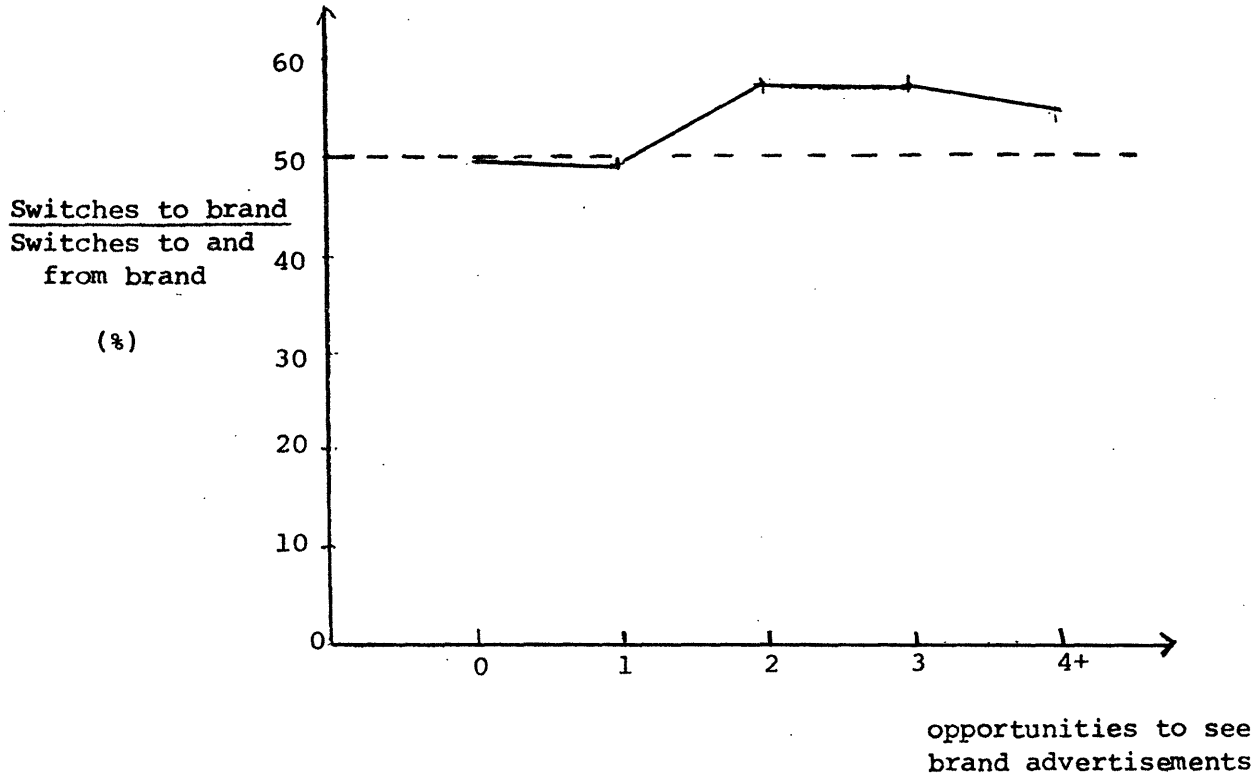


Fig. 2.9 Sales response to advertising. The sales measure is the number of switches to the advertised brand as a percentage of the total number of switches to and from the brand. Opportunities to see include only those in the last 4 days of the purchase interval. McDonald [1970].

The sales measure is the percentage of brand switches to the advertised product as a proportion of switches both to and from it. (Thus 50% would be expected in the absence of an advertising effect.) The advertising measure is the number of opportunities to see ads for the brand in the last four days of the customer's time interval between successive purchases. The curve is an aggregate over several product classes and many brands, all essentially supermarket items. The curve is not comparable to those presented earlier because it deals with individuals not market and because both time interval and sales measure are very specialized. However, the results are quite revealing, especially the S-shape, the seeming saturation after just a few exposures and the evidence of immediate advertising effects.

2.6 Impulse Response

A standard question about a dynamic system is, "What is its impulse response?" Thus, suppose we put a short burst of advertising into the market, say an expensive TV special, a multi-page four-color spread in a magazine, or a massive direct mail drop; what would be the resulting shape of the sales response over time?

Figure 2.10 shows an example of this. A test group of people was exposed and a control group not exposed to a sharp pulse of advertising. The ratio of test sales/control sales in the following months was recorded. A number of tests have been averaged to give the impulse response shown.

Another type of analysis, common in econometric studies, measures the effect of past advertising on current sales by regression. This yields an implied impulse response even though the advertising was not actually done in pulses. Figure 2.11, plotted from the results of Bass and Clarke [1972], displays such a case.

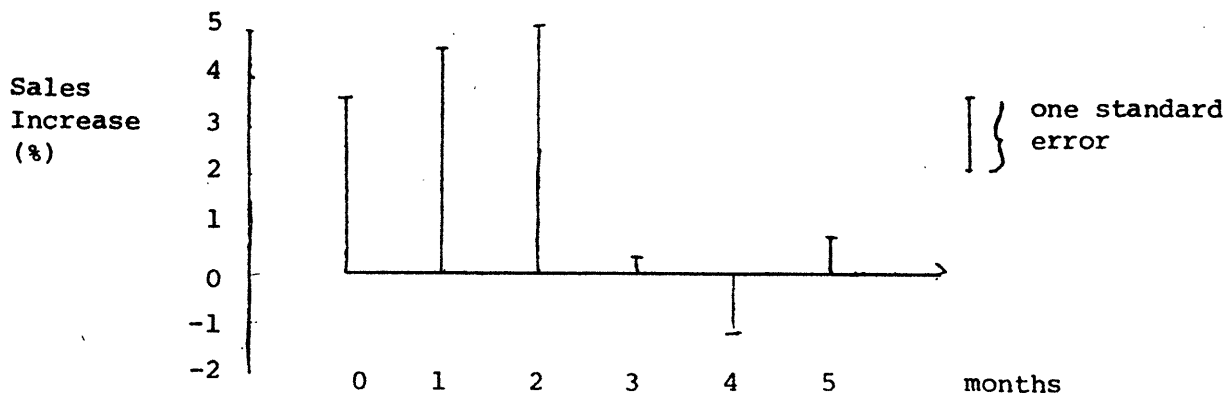


Fig. 2.10 Impulse response to a pulse of advertising for an infrequently purchased consumer durable.

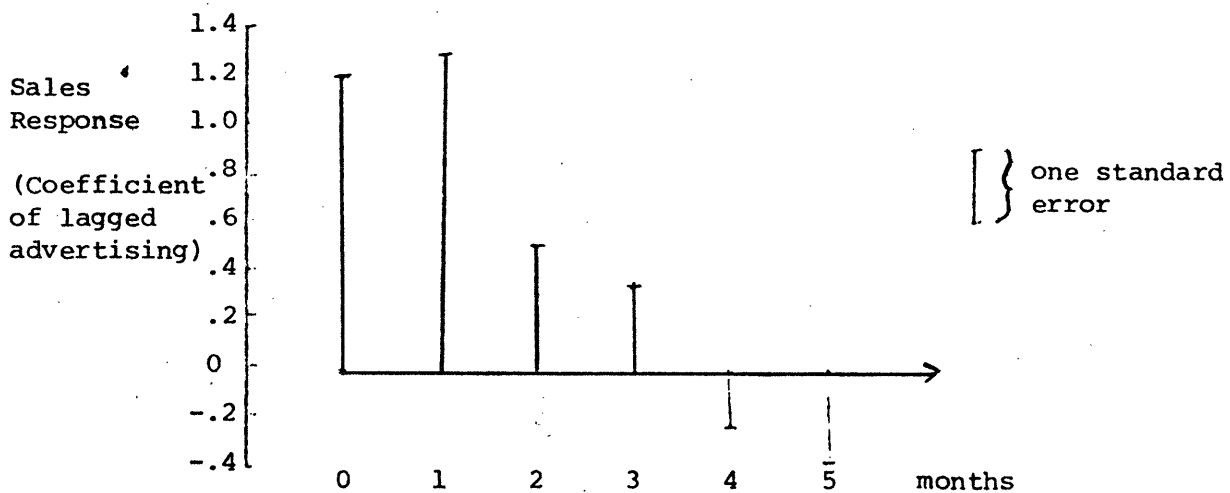


Fig. 2.11 Impulse response for a dietary weight control product. Bass and Clarke [1972].

Notice that Figures 2.9-2.11 corroborate earlier observations that response to advertising is relatively quick. The initial effect of a pulse takes place within 2 months. This is in line with the rise times in Figures 2.1-2.3. Ideally, impulse response measurements would also pick up long run effects in the tail. However, if the decay is as slow as those of Figures 2.2 and 2.3, the usual statistical methods will have difficulty detecting it.

In examining alternative models in the next section we can determine their impulse responses and compare them to what we find in practice.

2.7 Infrequent Purchases

Figure 2.10 is especially interesting because it deals with a consumer durable whose normal time between purchases is measured in years. Some people have argued that the fast advertising response discussed earlier will not apply to infrequently purchased goods. Figure 2.10 refutes this. The reason such goods can respond quickly is simple enough. At any given point of time some group of people is in the market, ready to act. Indeed, potential customers are often seeking information and take a special interest in the advertising for the product class.

However, for infrequent or one-time purchases like houses, refrigerators, books, college educations, or enlistments in the armed services, a new phenomenon is likely to come in: market depletion. Figure 2.12, taken from data of Benjamin, Jolly and Maitland [1960], displays the effect. Successive advertisements in a periodical draw fewer and fewer customers, tending toward an asymptotic value where market depletion is balanced by new entry, or zero if there is none. The authors also observed that when an advertisement was omitted the next one met increased response, indicating a degree of market rejuvenation.

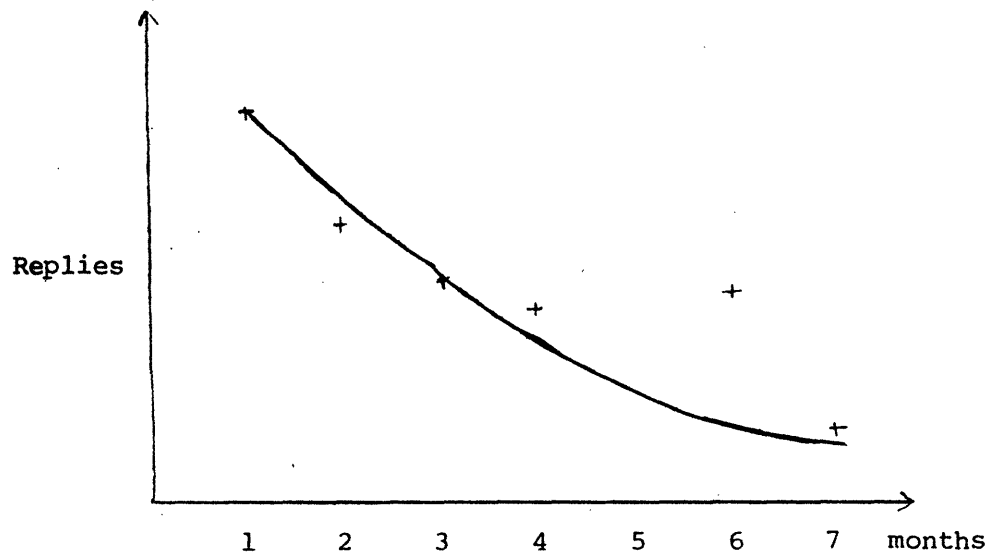


Fig. 2.12 Replies to a series of advertisements in a periodical. Evidence of market depletion and temporary rejuvenation. Benjamin, Jolly, and Maitland [1960].

Although statistical significance is not there, the impulse response curves of Figures 2.10 and 2.11 hint at a negative sales reaction about four months after the advertising pulse. Such borrowing of future sales is a type of temporary market depletion often found in consumer promotions and undoubtedly sometimes occurs with advertising.

2.8 Competition

Companies worry about competition. Surely, if one brand can increase its sales and share by advertising, so can another. Therefore, one brand's advertising will often reduce another brand's sales. Some researchers have studied this phenomenon, for example, Lambin, Naert and Bultez [1975] and Horsky [1978]. Figure 2.13 shows curves derived from data of the former. We argue that an understanding of advertising phenomenon in consumer markets requires competitive models.

2.9 Issues Outstanding

For a number of questions raised earlier, straightforward evidence is scanty.

(a) Advertise where sales are strong or weak? Undoubtedly this question is too simplistic and the right answer depends on conditions. One might expect, for instance, that advertising response would be poor where distribution is weak. On the other hand, a concerted marketing program that includes substantial advertising may be required to gain distribution and the benefits beyond.

The influencing conditions are likely to vary from case to case. Haley [1978] produces evidence for better response where sales are already increasing. Rao and Miller [1975] report a product for which advertising response is greater where share is greater. Competitive

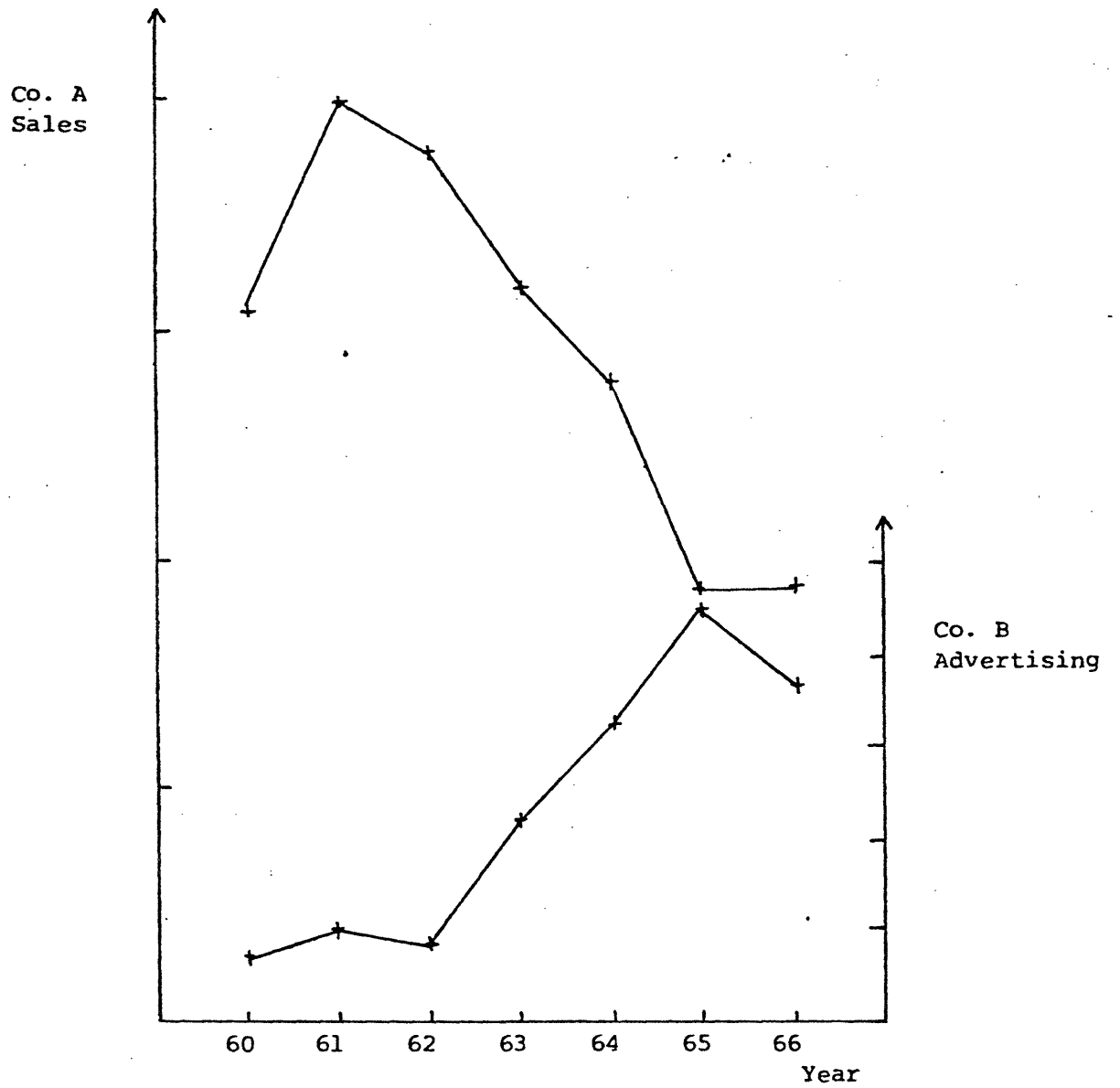


Fig. 2.13 Competitive Effects. Company B conducted an aggressive marketing program including heavy advertising. Data derived from Lambin, Naert, and Bultez [1975].

advertising can affect response. The various conditions need sorting out.

(b) Hysteresis. Are there situations for established products where advertising can carry sales up to new levels to stay there after advertising is reduced? Parsons [1976] explores what appears to be such a case, but good examples are not generally available.

(c) Interactions. How does advertising interact with other marketing variables? Some models assume additive effects, some multiplicative, and others more complicated relationships. They cannot all be right. Interactions are usually much harder to measure than main effects. Some studies have found that advertising response for a product differs from market area to market area. This may result from different product class strength, demographic segmentation, or distribution levels. Much unraveling needs to be done.

2.10 Conclusions

The empirical evidence suggests that at least the following phenomena should be considered in building dynamic models of advertising response:

P1. Sales respond dynamically upward and downward to increases and decreases of advertising and frequently do so at different rates.

P2. Steady-state response can be concave or S-shaped and will often have positive sales at zero advertising.

P3. Competitive advertising affects sales.

P4. The dollar effectiveness of advertising can change over time as the result of changes in media, copy, and other factors.

P5. Products sometimes respond to increased advertising with a

sales increase that falls off even as advertising is held constant.

All of these effects hold implications for managerial action. Obviously other important phenomena also exist, some of which have been discussed and others of which remain to be discovered. However, parsimony prompts us to keep the list short.

We now look for models that embrace these basic elements. This list does not seem very demanding, and indeed, where there are competing ways to represent the same phenomenon, we shall not be well equipped to distinguish among them. However, even our simple requirements of face validity will find many models wanting.

3. Models

For twenty years researchers have been adding marketing models to the literature like grains of sand to the beach. By now the pile, if not a dune, is at least a sand castle. Two rather dramatically different model building traditions coexist uneasily in the literature. One, which we shall call apriori, draws heavily on intuition and, although its practitioners are not oblivious to data, the model building goal is to postulate a general structure, not describe a specific application. In this category we place Vidale and Wolfe [1975], Nerlove and Arrow [1962], Little [1966, 1975]. The other tradition is statistical or econometric and usually starts from a specific data base, e.g., time series of sales or share and advertising. In this category are Bass [1969], Bass and Clarke [1972], Montgomery and Silk [1972], and Lambin [1976] to name a few. In addition some older work and an increasing amount of new work is mixed in that it starts with rather more complicated apriori models and endeavors by statistical methods to fit and evaluate them. Examples are Kuehn, McGuire, and Weiss [1966], Sexton [1970] and Horsky [1977].

3.1 Apriori Models

3.1.1 Vidale-Wolfe. Vidale and Wolfe [1957] published one of the earliest and most interesting of advertising response models. They used three basic ideas: (1) sales rate increases with advertising rate, (2) this effect decreases as sales rate approaches a value called saturation and (3) sales constantly erode spontaneously. The authors give empirical illustration of these phenomena. Let

s = sales rate (sales units/period)

\dot{s} = ds/dt

x = advertising rate (dol/period)

ρ = response constant (sales units/dol /period)

λ = decay constant (period⁻¹)

m = saturation sales rate (sales units/period)

Sales might be measured in kilograms, liters, pounds, cases, etc., periods in weeks, months, years, etc.

The Vidale-Wolfe structure is

$$\dot{s} = \rho x [1 - (s/m)] - \lambda s. \quad (3-1)$$

The model contains only three constants, yet displays many of the characteristics one would intuitively attribute to advertising response. Since (3-1) is a first order ordinary differential equation, it has an explicit solution for arbitrary $x(t)$. We report it for completeness, but for more intuitive understanding, we shall display

(a) Sales response to a rectangular pulse.

(b) Impulse response.

(c) Steady state response.

Suppose that at $t = 0$, $s = s(0)$, and a constant rate of advertising $x(t) = x$

is started which lasts until $t = T$ when it drops to zero. Solving (3-1) for such a rectangular pulse yields

$$s(t) = \begin{cases} r(x) + [s(0) - r(x)]e^{-[1+(\rho x/\lambda m)]\lambda t} & 0 \leq t \leq T \\ s(T)e^{-\lambda(t-T)} & T < t \end{cases} \quad (3-2)$$

where

$$r(x) = m(\rho x/\lambda m) / [1+(\rho x/\lambda m)]. \quad (3-3)$$

Equation (3-2) is sketched in Figure 3.1a. Notice that the rise time is primarily affected by the constant ρ and decay time by λ .

The impulse response, expressed as the incremental sales generated by an amount, X , of dollars spent in a very short time at $t = 0$, is

$$\begin{aligned} \Delta s(t) &= s(t) - s(0)e^{-\lambda t} \\ &= [m-s(0)] [1-e^{-\rho X/m}] e^{-\lambda t}, \quad 0 < t \end{aligned} \quad (3-4)$$

and is sketched in Figure 3.1b. Impulse response is exponential with decay constant λ .

The steady state response to a constant advertising rate x , is

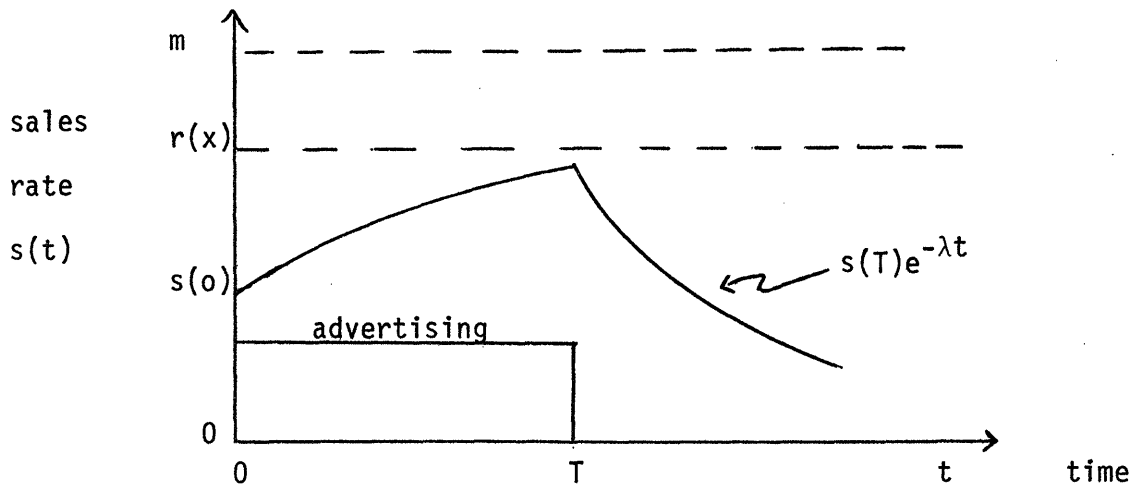
$$s(\infty) = r(x) \quad (3.5)$$

with $r(x)$ given by (3-3) and sketched in Figure 3.1c.

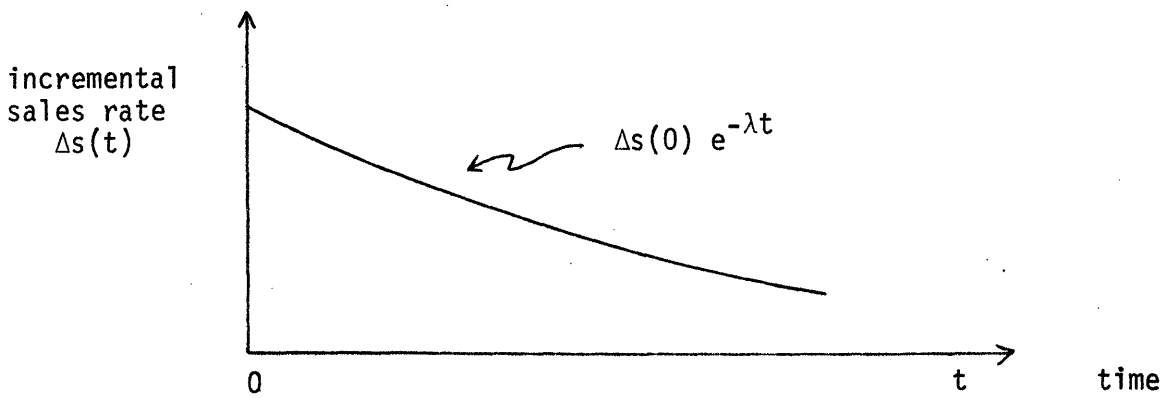
The general solution to the Vidale-Wolfe differential equation for arbitrary $x(t)$ is:

$$s(t) = \left\{ \int_0^t \left[\exp \lambda \int_0^u (1 + \rho x(v)/m\lambda) dv \right] \rho x(u) du + s(0) \right\} \exp \left\{ - \lambda \int_0^t (1 + \rho x(u)/m\lambda) du \right\} .$$

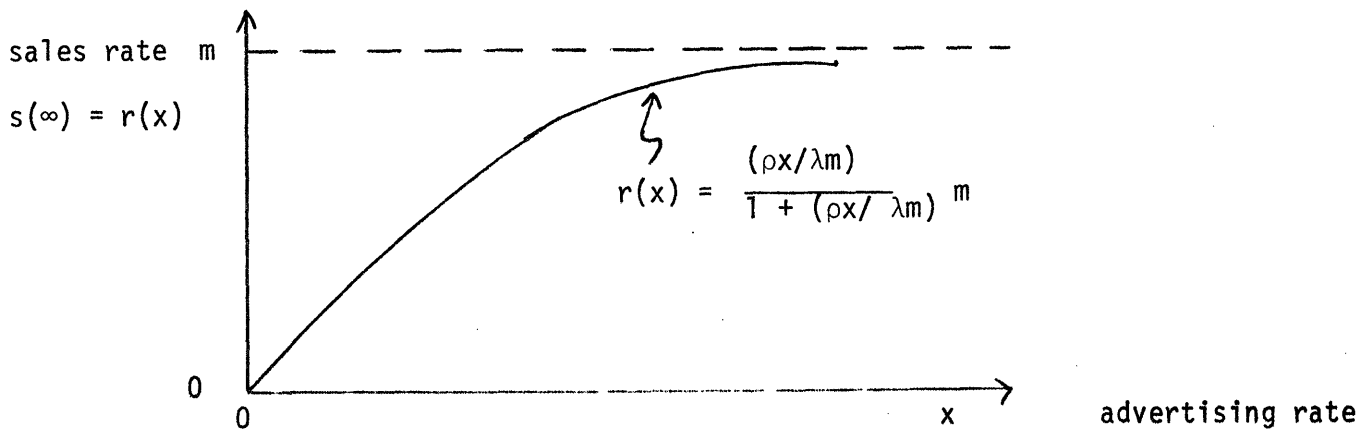
In comparing the Vidale-Wolfe model with our catalog of phenomena, we find that it has different rise and decay times in good agreement with P1. Steady state response, however, is concave, cannot be S-shaped, and has zero sales at zero advertising. This is not the flexibility



(a) Response to rectangular pulse of advertising.



(b) Impulse response to sharp pulse at $t=0$.



(c) Steady state response.

Figure 3.1 Vidale-Wolfe model: sales response to advertising.

called for by P2. The model does not consider competitive advertising in disagreement with P3. No explicit provision is made for changes in copy or media effectiveness as required by P4, although ρ could be made to perform that role. The temporary sales increases of P5 are not handled. In addition the exponential impulse response corresponds only weakly to Figure 2.10 and 2.11.

3.1.2 Nerlove-Arrow. In a study of advertising dynamics Nerlove and Arrow (1962) employ the term "good will", which "summarizes the effects of current and past advertising outlays on demand." Let

A = stock of goodwill (dollars)

x = advertising rate (dol/period)

\dot{A} = dA/dt (dol/period)

δ = goodwill depreciation rate (period⁻¹)

They postulate that growth and decay of goodwill behave according to

$$\dot{A} = x - \delta A \quad (3-6)$$

Goodwill, price and other variables affect sales. Let

p = price (dol/unit)

z = variables uncontrolled by the firm.

$s = s(p, A, z)$ = sales rate (units/period)

The authors' stated purpose is to investigate mathematical conditions required of optimal policies under various circumstances.

Our interest is in sales response. Since sales is presumably a monotone transformation of goodwill, the shape of rectangular, impulse and steady-state response for sales will closely depend on that for goodwill. Response to a rectangular advertising input, $x(t) = x$ for $0 \leq t \leq T$ and $x(t) = 0$ for $t > T$ is

$$A(t) = \begin{cases} A(0)e^{-\delta t} + (x/\delta) [1 - e^{-\delta t}] & 0 \leq t \leq T \\ A(T)e^{-\delta(t-T)} & T \leq t \end{cases} \quad (3-7)$$

Incremental response to an impulse of X dollars administered at $t = 0$ is

$$\Delta A(t) = A(t) - A(0) e^{-\delta t} = \chi e^{-\delta t} \quad 0 < t \quad (3-8)$$

Steady state response to constant $x(t) = x$ is linear.

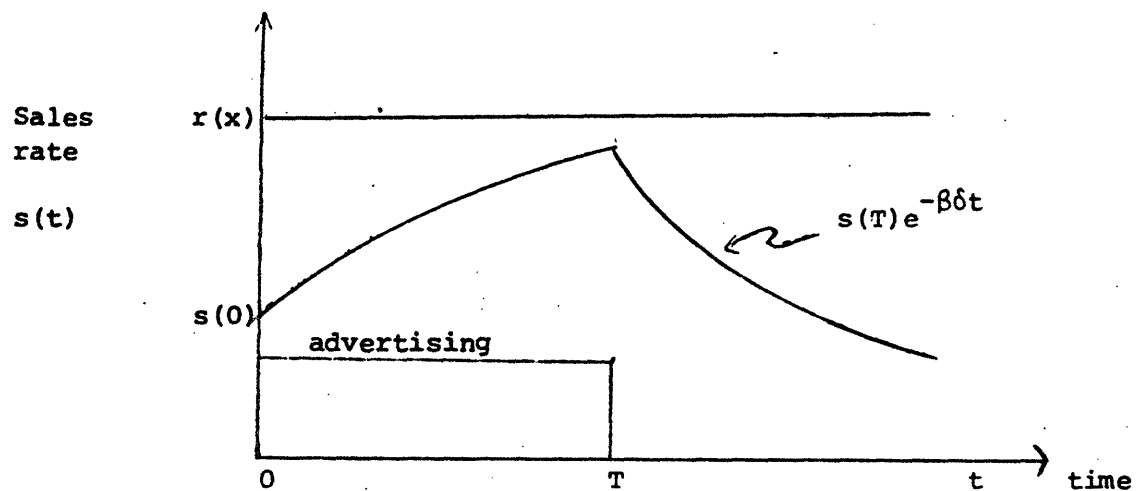
$$A(\infty) = x/\delta \quad (3-9)$$

At a later stage of their paper, Nerlove and Arrow investigate the constant elasticity response function $s = k p^{-\eta} A^\beta z^\xi$, which, for present purposes, can be written

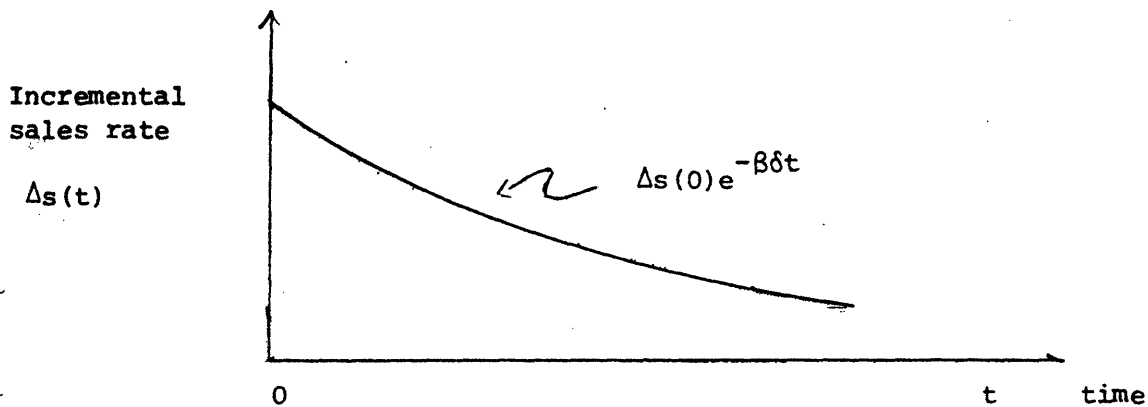
$$s(t) = k A(t)^\beta \quad (3-10)$$

with $\beta < 1$ for meaningful functions. Figure 3.2 sketches rectangular, impulse, and steady state sales responses.

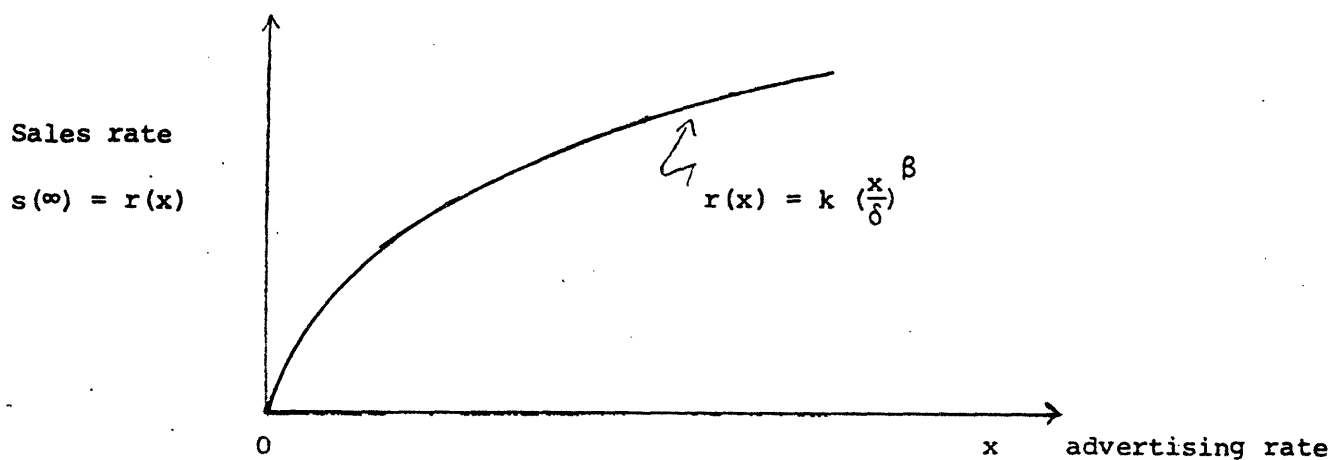
The Nerlove-Arrow model views advertising as piling up "good will," which continuously leaks away. The current stock of "good will" drives a steady state response function, exemplified as a constant elasticity model. The process is somewhat similar to the Vidale-Wolfe model but the latter differentiates between rise and decay as required by P1 whereas Nerlove-Arrow does not. The steady-state of the Nerlove-Arrow constant elasticity model has the problem of zero sales at zero advertising and lacks the possibility of an S-shape, thereby lacking the flexibility of P2. There is no consideration of competition (P3), changing effectiveness (P4), or temporary sales increases (P5). The authors give no empirical evidence for their model.



(a) Response to rectangular pulse of advertising.



(b) Impulse response to sharp pulse at $t=0$.



(c) Steady state response.

Fig. 3.2 Nerlove-Arrow model: sales response to advertising

3.1.3 Lanchester Models. We shall give the name Lanchester, to a flexible class of competitive marketing models that have a strong resemblance to Lanchester's models of warfare. The basic idea was introduced by Kimball [1957]. A model of this form has also been considered by Deal and Zions [1973]. We concentrate on a basic two-competitor case and later point out certain generalizations. Let

s_1 = sales rate of brand 1 (units/period)

s_2 = sales rate of brand 2 (units/period)

x_1 = advertising rate of brand 1 (dol/period)

x_2 = advertising rate of brand 2 (dol/period)

ρ_1 = advertising effectiveness constant of brand 1 (dol⁻¹)

ρ_2 = advertising effectiveness constant of brand 2 (dol⁻¹)

m = total market sales rate (units/period)

$$s_1 + s_2 = m \tag{3-11}$$

The basic Lanchester model is

$$\dot{s}_1 = \rho_1 x_1 s_2 - \rho_2 x_2 s_1 \tag{3-12a}$$

$$\dot{s}_2 = \rho_2 x_2 s_1 - \rho_1 x_1 s_2 \tag{3-12b}$$

Thus, company 1 wins sales proportional to its advertising and to company 2's sales. At the same time company 1 is losing sales proportional to its own sales and company 2's advertising. The situation is entirely symmetric for company 2. The coefficients ρ_1 and ρ_2 permit different advertising dollar efficiencies due to copy, media buying, and other product and market characteristics.

A number of interesting properties of the model emerge from simple analyses. First, we make the substitutions:

$$s_2 = m - s_1 \quad (3-13a)$$

$$\rho = \rho_1 m \quad (3-13b)$$

$$\lambda = \rho_2 x_2 \quad (3-13c)$$

Dropping the now redundant subscript 1, we obtain

$$\dot{s} = \rho x [1 - (s/m)] - \lambda s,$$

which is just the Vidale-Wolfe model. Thus, the Lanchester equations (3-12) form a competitive generalization of Vidale-Wolfe. Note that the decay constant of the Vidale-Wolfe model is now expressed in terms of the competitor's advertising rate.

It follows that, for the case of fixed competitive advertising, appropriate substitutions into (3-2) to (3-5) give the rectangular pulse, impulse, and steady state responses and Figure 3.1 portrays their shapes. The case of time-varying advertising and/or time-varying competitive advertising converts into a first order differential equation which can be solved explicitly if desired.

The steady state response functions help build intuition about the competitive affects of advertising. Solving (3-13) yields

$$s_1 = m(\rho_1 x_1) / (\rho_1 x_1 + \rho_2 x_2) \quad (3-14a)$$

$$s_2 = m(\rho_2 x_2) / (\rho_1 x_1 + \rho_2 x_2) \quad (3-14b)$$

Of great interest is the property that one company's response function depends on another company's advertising rate. This is sketched in Figure 3.3.

Response models of the general type $us/(us + them)$ are well known. In particular Friedman [1959], Mills [1960], and Bell, Keeney and Little

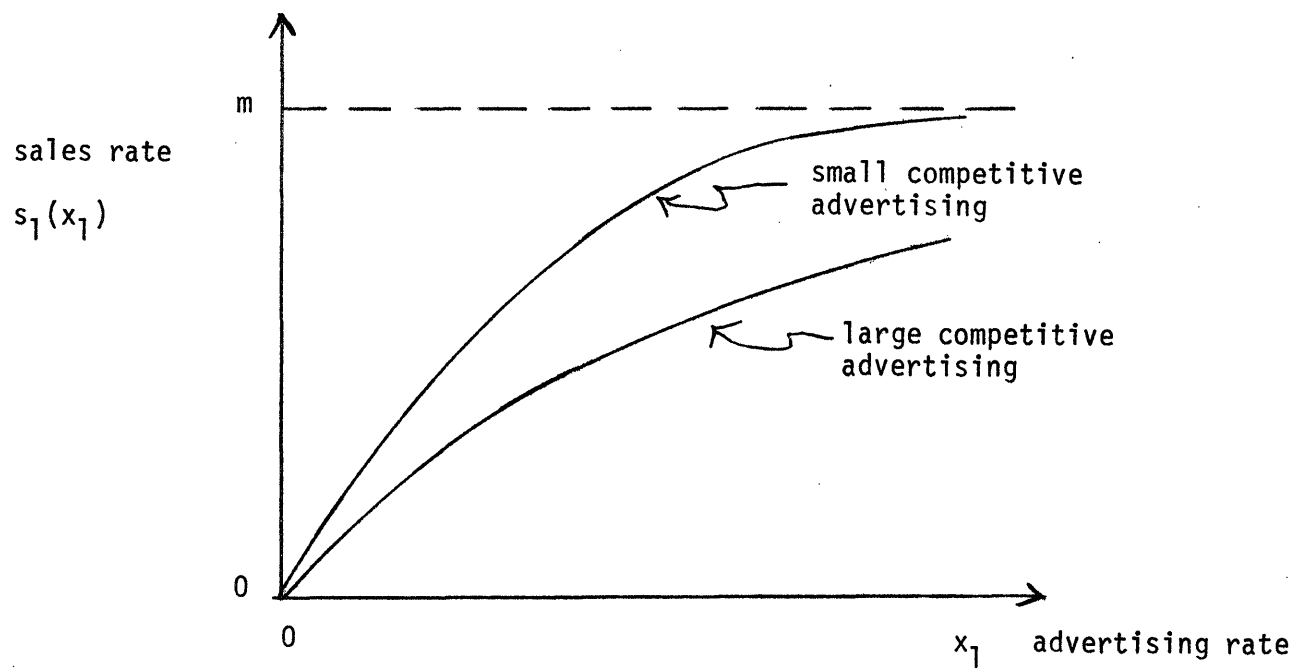


Figure 3.3 Steady state response of two competitor Lanchester models.

[1975] study them. These papers refer to generalizations to N competitors, other functions of advertising, various game theoretic issues, and generalizations beyond advertising. A straightforward expansion of (3-12) to N competitors with x_j generalized to $x_j^{\epsilon_j}$ produces a model with many of the requested phenomena:

$$\dot{s}_i = \rho_i x_i^{\epsilon_i} \sum_{j \neq i} s_j - \left(\sum_{j \neq i} \rho_j x_j^{\epsilon_j} \right) s_i \quad i=1, \dots, N \quad (3-15)$$

$$\sum_{j=1}^N s_j = m \quad (3-16)$$

In steady state

$$s_i = m \rho_i x_i^{\epsilon_i} / \sum_{j=1}^N \rho_j x_j^{\epsilon_j} \quad i=1, \dots, N \quad (3-17)$$

The response function (3-17) is quite versatile, being S-shaped in x_i for $\epsilon_i > 1$ and concave for $0 \leq \epsilon_i \leq 1$. Thus, if we think of the ρ_i as carrying media and copy effectiveness, the Lanchester model (3-15) displays phenomena P1 - P4 except for non-zero sales at zero advertising. The model does not display P5, erosion of incremental sales under constant advertising.

A further generalization would be to make each brand's advertising differentially effective against each other brand, e.g., change $\rho_i x_i^{\epsilon_i} s_j$ to $\rho_{ij} x_i^{\epsilon_i} s_j$. Another feature would be to let the total market size m depend on total industry advertising.

3.1.4 Brandaid. Little [1975] presents a general, flexible structure for modeling the effect of the marketing-mix on company sales.

The advertising submodel works as follows. Let

t = time in discrete units (periods)

$s(t)$ = brand sales rate (units/period)

$a(t)$ = brand advertising rate (index)

$r(a)$ = long run (steady-state) advertising response (units/period)

$\alpha(a)$ = carry-over constant (period⁻¹)

Customer purchases are presumed to have persistence so that current sales

are a weighted combination of previous sales and long run response.

$$s(t) = \alpha s(t-1) + (1-\alpha)r(a(t)) \quad (3-18)$$

Steady state response is arbitrary; in particular, it can be S-shaped and have a non-zero origin as sketched in Figure 3.4. The burden of calibration is placed on the user. In applications to date some companies have made empirical measurements that guide the setting of $r(a)$ and some have used managerial judgement or a mix of the two.

The model anticipates that media and copy effectiveness may vary over time. Advertising consists of messages delivered to individuals by exposures in media paid for by dollars. These ideas are modeled by

$$\begin{aligned} \text{advertising rate} &= (\text{copy effectiveness}) \times (\text{media efficiency}) \\ &\quad \times (\text{spending rate}) \end{aligned}$$

Let $h(t)$ be copy effectiveness, $k(t)$ media efficiency, $x(t)$ spending rate, and h_0 , k_0 and x_0 normalizing constants for these quantities. Then the advertising rate, $a(t)$, is given by

$$a(t) = h(t)k(t)x(t)/h_0k_0x_0. \quad (3-19)$$

This quantity can drive the response function, or, as a further embellishment, a weighted combination of current and past advertising can be used. A simple exponential smoothing model is

$$\hat{a}(t) = \beta \hat{a}(t-1) + (1-\beta)a(t) \quad (3-20)$$

where $\hat{a}(t)$ is the effective advertising at t and β is a carry-over constant for advertising exposure (units of fraction/period, $0 < \beta < 1$).

The Brandaid advertising model meets the criteria of flexibility in dynamic and steady-state response (P1, P2) and treats changing effectiveness (P4). The Brandaid paper also presents a way to model competition that lends itself well to calibration by managerial judgement in decision calculus style but seems less suited to our purposes here. The model has no mechanism for handling the temporary sales increase phenomenon P5.

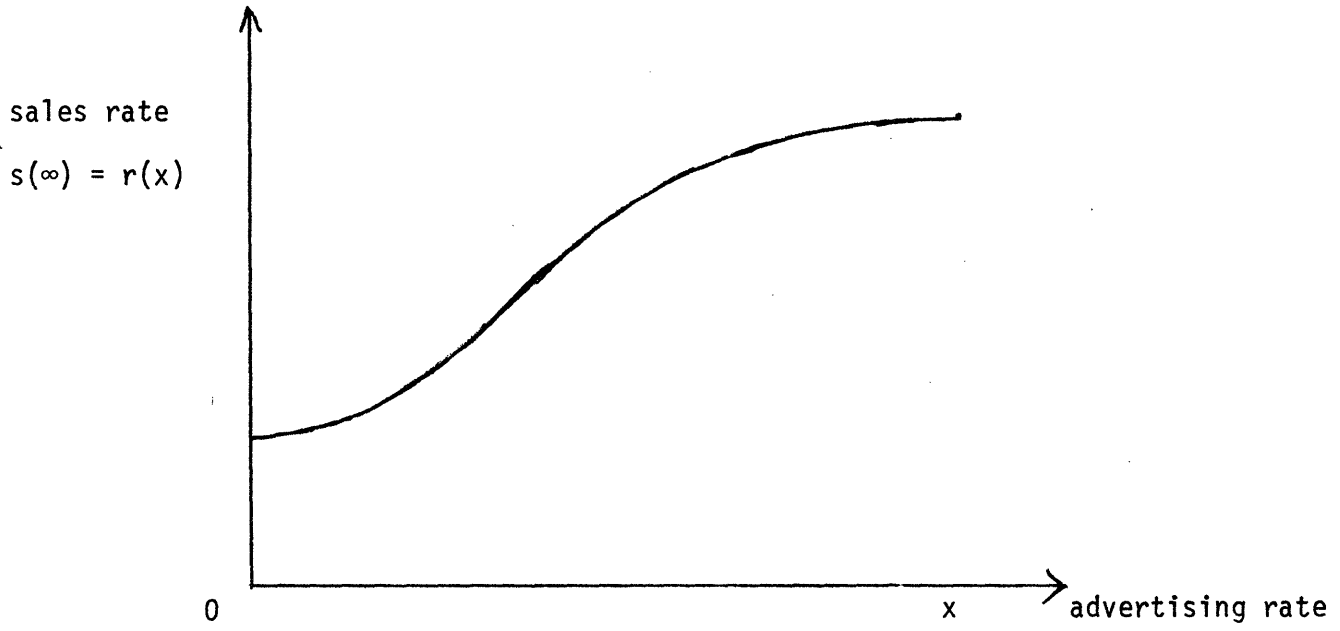


Figure 3.4 S-shaped response with nonzero origin accomodated by Brandaid.

We now show that the previous three models are special cases of Brandaid; or, if you prefer, the previous models have gone out on a limb with specific postulates where Brandaid has refused to make commitments.

Consider first the Vidale-Wolfe model. We convert it to discrete time by the approximation

$$\dot{s} \approx [s(t) - s(t-h)]/h \quad (3-21)$$

where h is a small interval of time. Taking the time unit equal to h (i.e. setting $h=1$) and defining

$$\alpha(x) = 1/[1 + \lambda + (\rho x/m)] \quad (3-22a)$$

$$r(x) = (\rho x/\lambda m)/[1 + (\rho x/\lambda m)] \quad (3-22b)$$

we obtain, by substituting (3-21) and (3-22) into (3-1) and rearranging,

$$s(t) = \alpha(x)s(t-1) + [1-\alpha(x)]r(x)$$

This is just the Brandaid advertising model with $a(t) = x(t)$, the spending rate. Notice in (3-22) that $0 < \alpha < 1$ and that $r(x)$ is indeed the steady-state response of the Vidale-Wolfe model.

The implications of the relation between the two models are several. First, by appropriate specification of $\alpha(x)$, Brandaid can have different rise and decay times. Second, the Brandaid advertising model turns into "our brand" of a two-brand discrete time Lanchester model through substitution of (3-13b) and (3-13c) into (3-22). N competitor generalizations are also possible so that in fact the Lanchester model (3-15) can be cast into the Brandaid format.

The Nerlove-Arrow model in discrete time is a special case too. Set $\alpha=0$ in (3-18), suppress $h(t)$ and $k(t)$ in (3-19) and drive the response function in (3-18) with the effective advertising $\hat{a}(t)$ of (3-20). Effective advertising corresponds to Nerlove and Arrow's goodwill.

Finally, we note two straightforward generalizations of the lag structure. For (3-18)

$$\dot{s}(t) = \sum_{i=1}^{\infty} \alpha_i s(t-i) + (1 - \sum_{i=1}^{\infty} \alpha_i) r(a(t)), \quad (3-23)$$

and for (3-20).

$$\dot{\tilde{a}}(t) = \sum_{i=0}^{\infty} \beta_i a(t-i) \text{ where } \sum_{i=0}^{\infty} \beta_i = 1. \quad (3-24)$$

These generalizations are not especially parsimonious as each added parameter puts more burden on calibration. A situation in which additional sales lags might be desired is when sales are measured by factory shipments so that the distribution pipelines put lags between customer purchase and point of measurement.

3.1.5 Other Models. The literature contains a variety of other a priori models, a number of which we report here.

Saseini [1971] postulates sales dynamics in the form

$$\dot{s} = g(s, x, t) \quad (3-25)$$

where g is a known function that increases with advertising, x , and decreases with sales, s , ($\partial g / \partial x \geq 0$, $\partial g / \partial s \leq 0$). Vidale-Wolfe (3-1) is a special case. Schmalensee [1973] goes a step further by postulating that, at every moment, there is an equilibrium demand toward which actual sales are moving. Equilibrium demand corresponds to our steady state sales rate with the addition that, in principle, the equilibrium point can change with time. In our notation, let $r = r(x, p, t)$ be the steady state sales rate as a function of advertising, x , price, p , and possibly t . Schmalensee postulates

$$\dot{s} = F[r(x, p, t), s(t)] \quad (3-26)$$

and assumes $\partial F / \partial r > 0$ and $\partial F / \partial s < 0$.

Again, Vidale-Wolfe can be cast in this form, using (3-1) and (3-3):

$$\dot{s} = [\lambda/(1-r/m)](r-s). \quad (3-27)$$

The Brandaid advertising model fits into Sasieni's form but not quite into Schmalensee's. In continuous time Brandaid becomes

$$\dot{s} = \gamma(x)[r(x)-s] \quad (3-28)$$

where $\gamma(x) = \lim_{h \rightarrow 0} [1 - \alpha(x)]/h$ is the carry-over function converted to a decay factor. The existence of $\gamma(x)$ keeps (3-28) from being in the form (3-26).

Sasieni and Schmalensee each have as a goal the characterization of optimal policies and so make as few assumptions as possible about response. This leads to very general formulations. Both are quite flexible on response upward and downward and on the shape of steady response. At the same time this means they specify relatively little about the mechanisms of advertising. Sasieni does not explicitly consider competition. Schmalensee introduces it only to the extent of formally indicating a competitive advertising variable in the equilibrium demand function.

A variety of generalizations and modifications of the Vidale-Wolfe and Nerlove-Arrow models have been proposed. Mann [1975] generalizes the Nerlove-Arrow exponential weighting of past advertising for determining goodwill to more arbitrary weightings. Sethi, Turner and Newman [1973] do approximately the same thing to Vidale-Wolfe. They introduce a variable termed market attitude determined by present and past advertising. Current advertising is thereby replaced in the model by a linearly weighted combination of present and past advertising.

Sethi [1975] proposes a model

$$\dot{s} = \rho \log x = \lambda s$$

which exchanges the Vidale-Wolfe sales saturation process in (3-1) for a log function. Steady-state response now becomes the strictly concave function $s = (\rho/\lambda)\log x$. From the point of view of our catalog of phenomena this has about the same advantages and disadvantages as Vidale-Wolfe except for the added drawback that the log model makes no sense at zero advertising. Burdet and Sethi [1976] also present a discrete time model of Brandaid form with linear steady state response, an undesirable feature.

In the early and mid 1960's researchers created quite a few of speculative and often interesting models. Kuehn [1961] presents a general marketing mix model motivated by the linear learning description of brand switching. Viewed as an advertising response model, sales consist of a retained fraction of past sales plus new input. The new input is linear in the brand's share of total advertising and in the brand's share of various interaction functions between advertising and other marketing variables. Shakun [1966] gives a competitive model in which a firm's market share is share of total advertising but each firm's expenditure is weighted by its market share from the previous period. Industry sales of the product category are a saturating function of effective industry advertising. This in turn is a weighted combination of past effective advertising and new spending, diminished possibly by the spending on competing categories of products. Gupta and Krishnan [1967] define effective advertising as a linear weighting of past advertising. Then, in a competitive model, market share equals company share of total effective advertising.

These models are all competitive and so satisfy our phenomenon P3. However, from the vantage point of today, they lack flexibility in rise and decay rate (P1) and have rather inflexible concave steady state response functions (P2).

In a totally different direction of development, Tapiero [1975] studies a diffusion model of sales response to advertising. The model views sales as uncertain and the result of a stochastic process. However, the underlying response dynamics are basically Vidale-Wolfe. In still another approach Gould [1970] describes the advertising process as a diffusion of information among individuals. His resulting differential equation is identical to Vidale-Wolfe.

3.2 Econometric Models

Whereas one group of researchers has proposed and promoted a priori models, another has embraced specific data bases and applied econometric methods to them. The amount of econometric work is large. Clarke [1976] finds more than 70 studies and, at that, restricts himself to those amenable to inferences about the cumulative effect of advertising. Lambin [1976] alone analyzes 107 brands and reports 291 regressions.

Such studies take the historical data as it comes. The data may or may not contain sufficiently clean changes in advertising to draw solid inferences. Notice that most of our earlier examples of advertising phenomena were drawn from field experiments. It is easier to identify specific effects by direct manipulations than by sifting through the historical record with an econometric sieve. The drawback to experiments, of course, is that they require considerable effort to mount.

Most of the econometric studies use models that are linear or linear in the logarithms of the variables, with or without lagging some of the variables. Simultaneous equation models are common. Researchers frequently add exploratory variables as available, such as other marketing activities, economic indicators, and dummy variables for special circumstances. We examine several major classes of econometric work, focussing, however, only on the advertising response models therein.

1. Linear in advertising. Let

s_t = sales in period t (sales units).

x_t = advertising in period t (dollars).

a_i, b_i = constants

A parsimonious linear model used, for example, by Bass and Clarke [1972] is:

$$s_t = a_0 + \sum_{i=0}^L b_i x_{t-i} \quad (3.2.1)$$

The model has a linear steady state response, given by $s = a_0 + (\sum b_i)x$ and an arbitrary impulse response, represented by the coefficients: b_0, b_1, \dots, b_L . See Figure 3.5.

A related model, used by Palda [1964] and others, includes previous sales as well as advertising as explanatory variables.

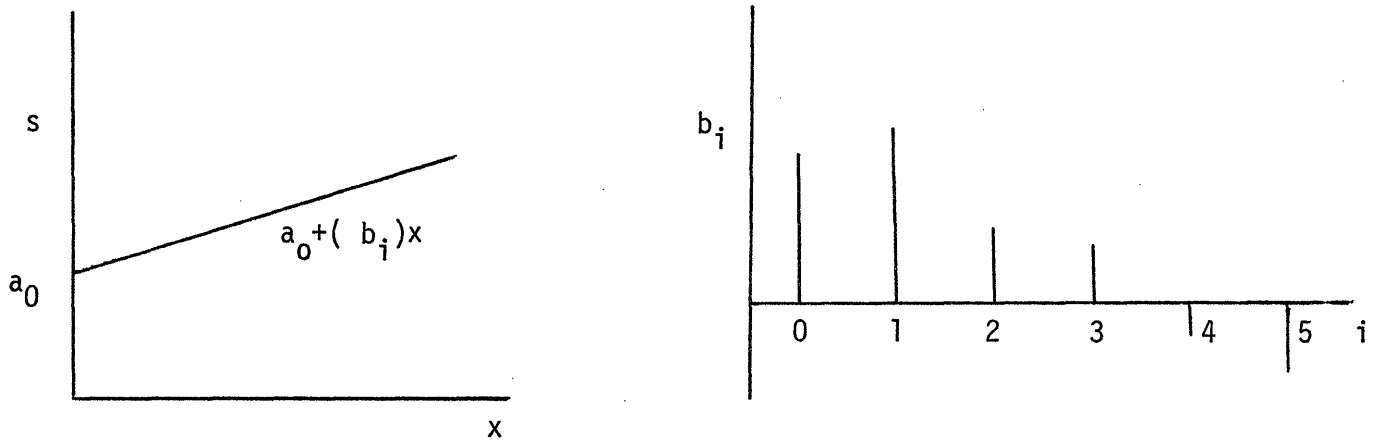
$$s_t = a_0 + a_1 s_{t-1} + b_0 x_t \quad (3.2.2)$$

Meaningful values of a_1 are in (0,1). This model also has a linear steady-state response function, $s = [a_0 / (1-a_1)] + [b_0 / (1-a_1)]x$. and an exponential (geometric) impulse response with n^{th} term $b_0 a_1^n$.

These two models differ considerably in statistical estimation properties, a fact which has generated considerable discussion (Houston and Weiss [1975]), but from our point of view they are similar. Model (3.2.2) can be put into the form of (3.2.1) with $L = \infty$ by successive substitutions. We note that either model can easily be cast into the Brandaid format of (3-18).

These models contain very few of the advertising phenomena described earlier. Linear response is not credible over an indefinite range and obviously fails the requirements of P3.

The impulse response of (3.2.1) is versatile but rise times and



(a) Steady state

(b) Impulse response

Figure 3.5 Steady state and impulse response of linear model (3.2.1).

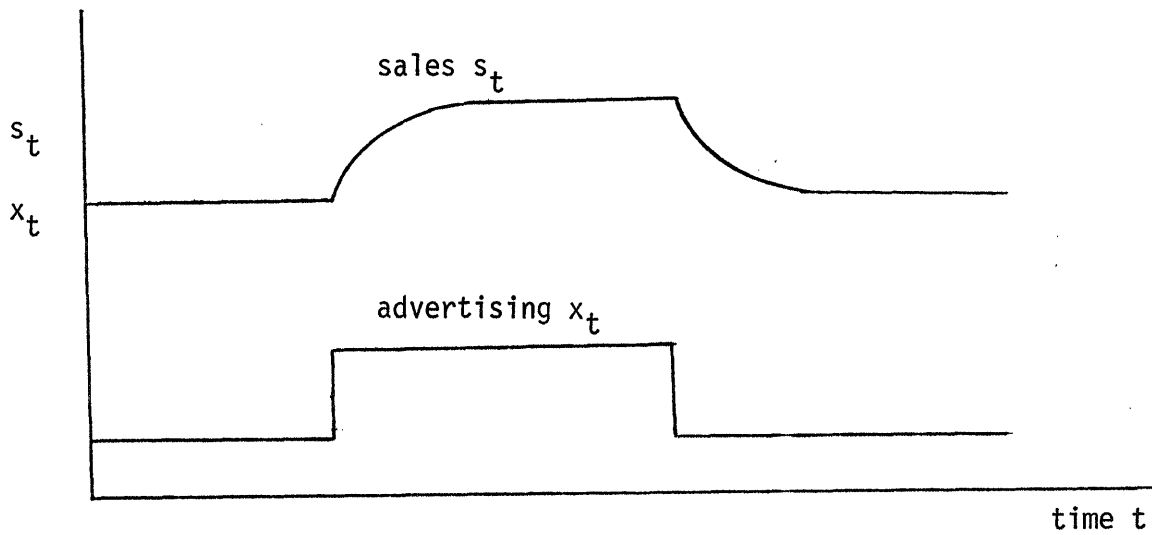


Figure 3.6 In the linear model (3.2.1) and product form model (3.2.3) a rapid rise time also means a rapid decay time.

decay times between steady state levels are essentially the same. To see this, observe that, if sales are in steady state under advertising rate x and we increase the rate by Δ , then n periods later sales will be incremented by $\Delta(\sum_{i=0}^n b_i)$. If after establishing steady state at the new higher advertising, we decrease advertising by Δ back to x , then n periods later sales will be reduced by $\Delta(\sum_{i=0}^n b_i)$, the same amount. This is sketched in Figure 3.6. Thus linear models fail phenomenon P1.

Linear models have been extended to include competitive advertising variables. (See, for example, Picconi and Olson [1978] Model 5.) This is desirable but, of course, does not circumvent the difficulties already discussed.

2. Product form models. Many writers use models of the form

$$s_t = a_0 \prod_{i=0}^L x_{t-i}^{b_i} \quad (3.2.3a)$$

which, after taking logs, becomes linear in the constants:

$$\ln s_t = \ln a_0 + \sum_{i=0}^L b_i \ln x_{t-i} \quad (3.2.3b)$$

A lagged sales term may be added:

$$s_t = a_0 s_{t-1}^{a_1} \prod_{i=0}^L x_{t-i}^{b_i} \quad (3.2.4)$$

and sometimes more than one. Logs again linearize the expression with

respect to the constants and thereby greatly simplify the task of estimating them from data. Models (3.2.3) and (3.2.4) are analogs of the linear (3.2.1) and (3.2.2). The product form is widely used. Examples may be found, for instance, in Montgomery and Silk [1972] and many in Lambin[1976].

Product form models have an obvious defect, namely, zero advertising produces zero sales and, if lagged advertising terms are included, zero advertising in any lagged period produces zero sales in the current period. The situation is particularly acute for applications with short period lengths (e.g., months or weeks), since zero advertising in such intervals is quite common. A constant can be added to the advertising variable but an a priori constant represents a strong assumption about the shape of the response function and letting the calibration pick the constant loses the advantages of linearity for estimation.

Models in product form fail to conform to our required phenomena in other ways. S-shaped response is precluded. Rise and decay from steady-state involve symmetric factors. Thus in (3.2.3), if sales are in steady state with advertising x and a jump of Δ is made in advertising, then n periods later sales will be multiplied by a factor $(1 + (\Delta/x))^k$ where $K = \sum_{i=0}^n b_i$. If, after reaching steady state with advertising of $x + \Delta$, advertising is reduced to x , then n periods later sales will be divided by the same factor.

Thus we conclude that the usual product form models fail to exhibit the key phenomena P1 and P2.

3. Models additive in nonlinear functions of advertising. A number of writers (e.g. Lambin [1972]) have used models like (3.2.1) with the change that x_t is the share of advertising, i.e.

$$x_t = \frac{\text{brand advertising in } t}{\text{sum of advertising of all brands in } t}.$$

Often this is coupled with s_t changed to market share. A model of this type satisfies two important goals: it is nonlinear in brand advertising and contains competitive effects. However, the simple share approach does not permit competitors to have different effectiveness and is rather rigid in its nonlinearity. For example, it cannot be S-shaped and so fails P2.

A variant is to use relative advertising, i.e. the denominator of x_t excludes the brand's own advertising (e.g., Clarke [1973]). Also product forms are sometimes used. However, the drawbacks cited above remain. In other cases (Palda [1964], Picconi and Olson [1978]), equation (3.2.2) is used with x_t equal the log of advertising in t . This produces diminishing returns but cannot be S-shaped, has symmetrical rise and decay times, and becomes meaningless at zero advertising.

4. Simultaneous equation models. A serious problem arises in analyzing historical data because many companies set their advertising budgets, at least in part, on the basis of sales. If the direction of causality between advertising and sales is partly reversed, biased and spurious results can occur (Schmalensee[1972]).

Simultaneous equation models are designed to counter this problem. Bass [1969] and Bass and Parsons [1969], for example, use the technique. However, the advertising response models generally used in the equation systems are

product form. As a result they have the problems already discussed.

What can we conclude? First, most of the commonly used econometric models of sales response to advertising do not have structures that will accommodate the set of the dynamic phenomena identified earlier. These models are particularly weak in flexibility of shape for the response curve and in allowing different rise and decay rates. None of the models consider phenomena, P5, sales increases under increased advertising that decay with constant advertising. However, a researcher would be unlikely to hypothesize this phenomenon without experimental data like that provided by Haley.

To this writer the standard econometric forms (3.2.1 - 3.2.4) are not so much models of advertising as convenient functions fit to the advertising response process in the neighborhood of historical operations. Such a fitting process may be useful. For example a linear model might well be reasonable if the data do not contain a large enough variance in advertising to permit meaningful calibration of a nonlinear model. The coefficient from a linear statistical model might be combined with estimates from other sources about the effects of very large or very small advertising rates to calibrate a decision model. However, the purpose of building the statistical model would then be quite different from our modeling objectives here, which are to find the structure of advertising response that might appropriately be incorporated into the decision model.

The sheer volume of econometric work has led to some empirical generalizations. For example Clarke [1976] makes a convincing case for a short term effect of advertising on the order of a few months. He also demolishes certain empirically based arguments for long run

effects by showing them to be artifacts of the time period used in the econometric work. Lambin [1976] also draws generalizations from his massive study, although some are not entirely persuasive. For example, he says (p. 95) that there is no S-curve because product form and logarithmic models fit better than linear ones. This seems an insufficient argument and, indeed, he seems to contradict himself by later advocating the existence of threshold effects (p. 127).

3.3 Apriori Models with Calibration

A number of researchers have taken the approach of defining advertising models rather independently of standard econometric forms and then devising means to calibrate them on specific historical databases. This is an important direction of research, although, as is always true with non-experimental data, the researcher is dependent on historical variations to make measurement possible. Furthermore, most of the more elaborate models are nonlinear in some of the parameters. This introduces a host of calibration problems, not the least of which is the assessment of the quality of the parameter estimates.

Kuehn, McGuire and Weiss [1966] present an early and ambitious example of an apriori model calibrated on historical data. Let

s_{it} = market share of brand i in time period t .

p_{it} = price of brand i in t .

x_{it} = advertising spending of brand i in t .

a_{it} = effective advertising in t .

Unknown constants are:

α, β = carry-over constants for sales and advertising

b = weighting constant reflecting amount of sales
not affected by advertising.

ϵ = advertising sensitivity exponent.

δ = price sensitivity exponent

e_i = brand i advertising effectiveness coefficient

k_i = brand i effectiveness coefficient due to other marketing activities

$$s_{it} = \alpha s_{i,t-1} + (1-\alpha) \left\{ b k_i p_{it}^{-\delta} / \sum_j k_j p_{jt}^{-\delta} + (1-b) k_i p_{it}^{-\delta} a_{it}^{\epsilon} / \sum_j k_j p_{jt}^{-\delta} a_{jt}^{\epsilon} \right\} \quad (3.3.1)$$

$$a_{it} = \beta a_{i,t-1} + (1-\beta)^{\epsilon} x_{it} \quad (3.3.2)$$

$$\sum s_{jt} = 1 \quad (3.3.3)$$

By means of nonlinear estimation on historical time series the authors determine twelve constants required in their particular case.

The model has several interesting features. Its general form is that of (3-18), the Brandaid advertising submodel, but with price effects imbedded in it. The steady state response function is in the braces {} and is essentially the steady state of a Lanchester model with an additive term representing sales at zero advertising. Response can be either S-shaped or concave. It is interesting to note that the fitted value of ϵ was 2.57 so that response is S-shaped in the specific application. Effective advertising is an exponentially smoothed function of spending (3.3.2). The constraint (3.3.3) forces the market shares to add to one in the model and is an integral part of the estimation. The model contains many, although not all, of the phenomena laid out earlier as desirable.

Horsky [1977] builds an interesting model and calibrates it on cigarette data. He considers a two competitor case, one competitor being the brand of interest and the other the rest of the industry. Let

s_{it} = market share of competitor i in period t .

x_{it} = advertising spending of competitor i in t .

a_{it} = effective advertising or goodwill of competitor i in t .

β_i = carry-over constant for advertising.

ρ_i = effectiveness constant for advertising.

Horsky's model for competitor 1 is

$$s_{it} - s_{1, t-1} = \rho_1 a_{1t} s_{2t} - \rho_2 a_{2t} s_{1t} \quad (3.3.4)$$

with a symmetric equation for competitor 2. Effective advertising is given by

$$a_{it} = \beta_i a_{it} + (1 - \beta_i) x_{it} \quad i=1,2 \quad (3.3.5)$$

In our terminology this is a two-competitor Lanchester model in discrete time driven by exponentially weighted past advertising. It can have different rise and decay rates, thereby satisfying phenomenon P1. The steady state response is somewhat inflexible, being concave and having zero sales at zero advertising. Nevertheless, the model is a considerable step up in complexity from most current econometric models and non-linear estimation is required.

Parsons [1975] tackles the problem of time varying advertising effectiveness. Armed with sales and advertising data for a household cleaner from 1886 to 1905 he adds a time varying coefficient to a standard product-form econometric model and finds the change in advertising effectiveness over the product life cycle. Again, nonlinear estimation is required. Pekelman and Tse [1976] model copy wearout and replacement as a time-varying coefficient in a Lanchester-like competitive model and track the coefficient with Kalman filter techniques. Turner and Wiginton [1976] use non-linear techniques to calibrate the Vidale-Wolfe model on aggregate industry sales and advertising for filter cigarettes.

These examples show that, when researchers abandon the estimation conveniences of standard econometric models, they can build more realistic models and calibrate them using nonlinear methods.

4. Conclusions

We have reviewed a large amount of material on the sales effects of advertising for established products. What can we now say about representing these processes with models?

A first conclusion is that advertising is rich with phenomena. We are dealing with communication and its influence on purchase behavior. Perhaps it is presumptuous to expect any regularity that can be reduced to models with only a few parameters. Yet measurements have brought out many recurrent characteristics: an upward response of sales that takes place soon after increased advertising; a relatively slower sales decay on withdrawal that we attribute to customer satisfaction; sales saturation at high advertising levels; a possible threshold-like effect at low levels; the change of effectiveness over time because of media and copy changes; the loss of sales due to competitive advertising; and the effect presented by Haley that an advertising increase sometimes brings only a temporary sales increase. The magnitude and timing of all these effects are of great practical interest in making advertising decisions.

At the same time, many other effects remain to be uncovered and understood. The S-shaped curve is still on shaky ground. Is pulsing an effective policy and, if so, how long should pulses last? Does the S-shaped curve (essentially a static notion) provide an adequate theory for deriving optimal pulsing policies? What about the reported phenomenon that advertising is more effective when sales are increasing? More measurement and understanding are called for.

A second conclusion is that, although we have an apparent richness of models, many of them are rearrangements of a few key ideas. The Vidale-Wolfe constructs are surviving well, even though generalizations of the original model are very much in order. The competitive Lanchester generalization in which advertising rate is raised to a power looks quite versatile at the moment. It needs a change that will permit positive sales at zero advertising but this could be achieved by defining a component of sales not affected by advertising. The Lanchester model can be used in differential equation form or put in discrete time using the form of the Brandaid advertising submodel.

We have introduced copy and media effectiveness as a multiplier on spending, but this is not the only way to do it and time will tell whether it is the best way. Nowhere have we presented a model for phenomenon, P5, the temporary increase in sales under a permanent increase in advertising. A parsimonious adaptation of a new trier model might help here.

A third conclusion, and possibly a controversial one, is that the commonly used econometric models are of limited value in advertising. Their functional forms generally fail to represent advertising processes except possibly over a limited range. Add to this the problems of collinearity, autocorrelation and simultaneity. An approach that initially appeared promising for learning about advertising by applying standard tools to widely available historical data begins to look less inviting.

In any case, a fourth conclusion is that in analyzing historical data, we should specify more realistic a priori models and put the burden on the statisticians and ourselves for developing and using appropriate calibration methods.

Finally, we observe that, at least in the literature, there is an under-use of separate calibrations for different parts of a model. Particularly for decision making, we must include in our models all the phenomena that affect the decision. This will often lead to calibrating the model in several parts from eclectic data sources.

REFERENCES

- R.L. Ackoff and J.R. Emshoff, "Advertising research at Anheuser-Busch, Inc. (1963-68)," Sloan Management Review, 16, 1-16 (Winter 1975).
- F.M. Bass, "A simultaneous equation regression study of advertising and sales of cigarettes," Journal of Marketing Research, 6, 291-300 (August 1969).
- F.M. Bass and D.G. Clarke, "Testing distributed lag models of advertising effect," Journal of Marketing Research, 9, 298-308 (August 1972).
- F. M. Bass and L. J. Parsons, "Simultaneous-equation regression analysis of sales and advertising," Applied Economics, 1, 103-24 (1969).
- D.E. Bell, R.L. Keeney, and J.D.C. Little, "A market share theorem," Journal of Marketing Research, 12, 136-41 (May 1975)
- B. Benjamin, W.P. Jolly, and J. Maitland, "Operational research and advertising: theories of response," Operational Research Quarterly, 11, 205-18 (1960).
- B. Benjamin and J. Maitland, "Operational research and advertising: some experiments in the use of analogies," Operational Research Quarterly, 9, 207-17 (1958).
- D. Bloom, A. Jay, and T. Twyman, "The validity of advertising pretests," Journal of Advertising Research, 17, No. 2, 7-16 (April 1977).
- C. Burdet and S. P. Sethi, "On the maximum principle for a class of discrete dynamical systems with lags," Journal of Optimization Theory and Application, 19, 445-53 (1976).
- D. G. Clarke, "Econometric measurement of the duration of advertising effect on sales," Journal of Marketing Research, 18, 345-57 (November 1976).
- D. G. Clarke, "Sales-advertising cross-elasticities and advertising competition," Journal of Marketing Research, 10, 250-61, (August 1973).
- K.R. Deal and S. Zions, "A differential game solution to the problem of determining the optimal timing of advertising expenditures," Proc. Second Annual Northeast Regional AIDS Conference, American Institute of Decision Sciences, Atlanta, Ga. (1973).
- L. Friedman, "Game theory in the allocation of advertising expenditures," Operations Research, 6, 699-709 (September 1958).
- D.H. Gensch, "A computer simulation model for selecting advertising schedules," Journal of Marketing Research, 6, 203-14 (May 1969).

- S. K. Gupta and K. S. Krishnan, "Differential equation approach to marketing," Operations Research, 15, 1030-39 (November 1967).
- R.I. Haley, "Sales effects of media weight," Journal of Advertising Research, 18, No. 3, 9-18 (June 1978).
- R. M. Helmer and J. K. Johansson, "An exposition of the Box-Jenkins transfer function analysis with an application to the advertising-sales relationship," Journal of Marketing Research, 14, 227-39 (May 1977).
- D. Horsky, "An empirical analysis of the optimal advertising policy," Management Science, 23, 1037-49 (June 1977).
- F.S. Houston and D.L. Weiss, "Cumulative advertising effects: the role of serial correlation," Decision Sciences, 6, 471-81 (July 1975).
- G.E. Kimball, "Some industrial applications of military operations research methods," Operations Research, 5, 201-4 (April 1957).
- A. A. Kuehn, "A model for budgeting advertising," 315-53 in F. M. Bass et al. (eds.), Mathematical Models and Methods in Marketing, Richard D. Irwin, Homewood, IL, (1961).
- A.A. Kuehn, T.W. McGuire, and D.L. Weiss, "Measuring the effectiveness of advertising," 185-94 in R.M. Haas, ed., Science, Technology and Marketing, American Marketing Association, Chicago, 1966.
- J.-J. Lambin, "A computer online marketing mix model," Journal of Marketing Research, 9, 119-126, (May 1972).
- J.-J. Lambin, Advertising, Competition and Market Conduct in Oligopoly over Time North Holland, Amsterdam (1976).
- J.-J. Lambin, P.A. Naert, and A. Bultez, "Optimal marketing behavior in oligopoly," European Economic Review, 6, 105-128 (1975)
- G.L. Lilien, "Model relativism: a situational approach to model building," Interfaces, 5, No. 3, 11-18 (May 1975).
- J.D.C. Little, "Brandaid: a marketing-mix model, parts 1 and 2," Operations Research, 23, 628-73 (July 1975).
- J.D.C. Little, "A model of adaptive control of promotional spending," Operations Research, 14, 1075-97 (November 1966).
- J.D.C. Little and L.M. Lodish, "A media planning calculus," Operations Research, 17, 1-35 (January 1969).
- L.M. Lodish, "Empirical studies on industrial response to exposure patterns," Journal of Marketing Research, 8, 212-18 (May 1971).
- D. H. Mann, "Optimal theoretic stock models; a generalization of delayed response from promotional expenditures," Management Science, 21, 823-32 (March 1975).

- C. McDonald, "What is the short-term effect of advertising?" Proceedings of 1970 ESOMAR Conference 463-85
- H.D. Mills, "A study in promotional competition," in F.M. Bass et al., eds., Mathematical Models and Methods in Marketing, Richard D. Irwin, Homewood, IL (1961).
- D.B. Montgomery and A.J. Silk, "Estimating dynamic effects of marketing communications expenditures," Management Science, 18, B485-501 (June 1972).
- M. Nerlove and K.J. Arrow, "Optimal advertising policy under dynamic conditions," Economica, 29, 129-42 (May 1962).
- K. Palda, The Measurement of Cumulative Advertising Effects, Prentice-Hall, Englewood Cliffs, New Jersey (1964).
- L.J. Parsons, "A rachet model of advertising carryover effects," Journal of Marketing Research, 13, 76-9 (February 1976).
- L. J. Parsons, "The product life cycle and time-varying advertising elasticities," Journal of Marketing Research, 12, 476-80 (November 1975).
- D. Pekelman and E. Tse, "Experimentation and control in advertising, an adaptive control approach," Wharton School Working Paper, 76-04-01, University of Pennsylvania, Philadelphia (March 1976).
- M. J. Picconi and C. L. Olsen, "Advertising decision rules in a multibrand environment: optimal control theory and evidence," Journal of Marketing Research, 15, 82-92 (February 1978).
- A. G. Rao, "Proctivity of the marketing-mix: measuring the impact of advertising and consumer and trade promotions on sales," Paper presented at ANA Advertising Research Workshop, (April 1978).
- A.G. Rao and P.B. Miller, "Advertising/sales response functions," Journal of Advertising Research, 15, 7-15 (April 1975).
- M. W. Sasieni, "Optimal advertising expenditures," Management Science, 18, No. 4 Part II, 64-72 (December 1971).
- R. Schmalensee, The Economics of Advertising, North Holland, Amsterdam, (1972).
- S.P. Sethi, "A comparison between the effect of pulse versus continuous television advertising on buyer behavior," 563-67 in F.C. Allvine, ed., 1971 Combined Proceedings, American Marketing Association, Chicago.
- S.P. Sethi, "Dynamic optimal control models in advertising: a survey," SIAM Review, 19, 685-725 (October 1977).

- S. P. Sethi, "Optimal control of a logarithmic advertising model," Operational Research Quarterly, 26, 317-19 (1975).
- S. P. Sethi, R. E. Turner, and C. P. Neuman, "Policy implications of an intertemporal analysis of advertising budgeting models," Proceedings 4th Annual Midwest AIDS Conference, American Institute of Decision Sciences, Atlanta, GA, A15-18 (1973).
- M. F. Shakun, "A dynamic model for competitive marketing in coupled markets," Management Science, 12, B525-30 (August 1966).
- M.K. Starr, "OR in marketing: a case of failure and renewal" in K.B. Haley, ed., OR'78, North Holland, Amsterdam (1978).
- E.C. Strong, "The use of field experimental observations in estimating advertising recall," Journal of Marketing Research, 11, 369-78 (November 1974).
- C. S. Tapiero, "On-line and adaptive optimum advertising control by a diffusion approximation," Operations Research, 23, 890-907, (September 1975).
- R. E. Turner and J. C. Wiginton, "Advertising expenditure trajectories: an empirical study for filter cigarettes 1953-1965," Decision Sciences, 7, 496-509, (1976).
- G.L. Urban, "Building models for decision-makers," Interfaces, 4, No. 3, 1-11 (May 1974).
- M.L. Vidale and H.B. Wolfe, "An operations research study of sales response to advertising," Operations Research, 5, 370-81 (June 1957).
- R.S. Weinberg, An Analytical Approach to Advertising Expenditure Strategy, Association of National Advertisers, New York (1960).
- F.S. Zufryden, "Media scheduling: a stochastic dynamic model approach," Management Science, 18, B145-54 (August 1973).