Application, Predictive Test, and Strategy
Implications for a Dynamic Model of Consumer
Response to Marketing

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SSM W.P. # 1244-81 June 1981

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ABSTRACT

Marketing actions such as advertising or direct mail promotion affect sales in order to increase profitability or achieve the social goals of management. A model of consumer response helps managers understand and forecast the impact of such marketing actions so that they might evaluate these actions. But, particularly during a new product launch, the dynamics of consumer response are important. For example, a manager wants to evaluate the impact of advertising on consumer awareness of the new product, determine how quickly consumers will try the product once aware, and predict whether they will repeat purchase the product.

This paper describes and evaluates the application to a new product launch of a dynamic stochastic model of consumer response. The model describes, then forecasts, how consumers respond to a new transportation service and to the marketing strategies used during its introduction. The model is estimated on survey data during the first 11 weeks of service. Forecasts over the next 19 weeks are then compared to actual ridership as measured by dispatch records.

The model is simple. At any point in time, consumers are described by a set of 'behavioral states', indicating (1) whether they are aware of the new service (DART) and (2) what mode of transportation was used for their last trip. Behavior is described by movement among behavioral states. E.g., if a car user tries DART, he makes a transition from 'car used for last trip' to 'DART used for last trip'. The transition probabilities and the rate of transition are dependent on marketing strategies (direct mail, publicity), word of mouth, consumer perceptions, availability of a mode, and budget allocation to transportation.

The advantages and disadvantages of the model and the measurements are discussed with respect to predictive ability and managerial utility.
1. RESEARCH GOALS

Our ability to model consumer response advances through the interaction of theory, methodology, and practice. In a recent article, Hauser and Wisniewski [11], we developed a new methodology to integrate diverse mathematical models in stochastic brand choice, diffusion of innovations, test market analysis, and some aspects of information flow. Conceptually, we represent consumer behavior as flows among a series of "behavioral states" which represent either information processing states, e.g., 'aware' or 'unaware', or behavior, e.g. 'last brand purchased was Ivory'. The probabilistic "flows" are modeled as linear functions of marketing mix variables such as relative advertising or information processing variables such as word of mouth. We developed a practical estimation procedure to obtain the parameters of the system. Once the parameters are estimated, statistics of managerial interest such as the mean and variance of sales, penetration of a deal, or cumulative awareness are all obtained with closed form formulae. Furthermore, simulation suggests that good statistical fits can be obtained with reasonable sample sizes (200-500 observations for a 5-state process).

Theoretically, the methodology shows promise as a means to build and test practical models of consumer behavior, but the ability of such a model to explain and predict actual consumer behavior in a managerially relevant environment remains an empirical question.

The purpose of this paper is to address the empirical question. Since the methodology was derived for abstract states and explanatory variables, one research goal is to evaluate the feasibility of using the methodology to implement a simple consumer model in a managerial environment. We critically assess the practicality of measurement, estimation, and use in forecasting. Another research goal is to evaluate the extent to which the specific consumer model and measurements describe consumer behavior over periods of estimation and forecast consumer behavior over future time periods.

Good descriptive and predictive ability imply that the model captures many important phenomena. However, it does not imply that all phenomena are modeled or that other models are not also acceptable. Thus, it is important to evaluate the limits of the model's managerial utility and predictive ability. In this way we learn from our empirical experience so that we might begin to generalize to other situations.
This paper is empirical. A number of tradeoffs (sample size, questionnaire design, data collection strategy, behavioral states, explanatory variables, bias corrections, etc.) were necessary to construct the empirical realization of the theoretical model. We made one set of judgements. Other researchers with different goals and philosophies might make different empirical judgements. Thus, we estimate the model based on survey data and compare predictions to actual sales obtained from unobtrusive observation of consumer behavior. In this way, survey errors and specification errors work against good predictive ability. Finally, so that others can test alternative models we will provide at cost the raw data upon request.

Since a review of the literature and a detailed technical derivation of the theoretical methodology are published in Hauser and Wisniewski [11], we do not repeat them here. Instead Appendix 1 briefly summarizes the key results. We begin with the research context.

2. RESEARCH CONTEXT

The model was developed to evaluate the impact of a marketing campaign used to support the introduction of an innovative transportation service in Schaumburg, Il. The Village of Schaumburg is a northwest suburb of Chicago with a population of approximately 51,000 people (16,000 households). Schaumburg covers a 6 mile x 7 mile area consisting primarily of single family homes but with some newer apartment and condominium buildings. There is no large central business district, but Schaumburg does contain one of the largest shopping malls in the midwest. The existing transportation system consists of commuter rail lines to downtown Chicago and limited conventional bus service (6 vehicles over 6 routes in peak hours, 1 vehicle over 1 route in the off-peak hours) serving approximately 200 roundtrips per day. There are an average of 1.8 automobiles per household in Schaumburg.

The transportation innovation is a demand responsive dial-a-ride service called DART which was funded by the Urban Mass Transportation Administration (U.S. Department of Transportation) in cooperation with the Chicago area Regional Transportation Authority (RTA) and the Village of Schaumburg. The primary mode of operation is for passengers to call a dispatcher who arranges for them to be picked up and brought to their destination with stops along the way to serve other passengers. The service began officially on October 15, 1979, with four 22-passenger white school buses operating a total of 28 vehicle hours per day. Service was available

1 There was a two-week, no-fare, no-promotion service from October 1 to October 12 to familiarize the dispatchers and drivers with the mode of operation and with the Schaumburg street system.
from 9:00 A.M. - 5:30 P.M. Monday through Friday. The fare was 80¢ with half-fare available to the elderly, the handicapped, and students on their way to or from school or school-related activities.

The marketing consisted primarily of newspaper publicity and information brochures (see Appendix 2) distributed at banks, schools, and other locations. On December 1 and March 3, the Village used direct mail promotions based on the information brochures.

There were two levels of managerial goals. On the local level, the transit manager was interested in evaluating and improving his marketing strategy and in determining whether service improvements (more vehicles, longer hours, etc.) were necessary to achieve the ridership goals of the Village. On the national policy level, the DART service was part of a federal program of service and method demonstrations (SMD's). The U. S. Department of Transportation was interested in evaluating the impact of marketing for use in future SMD's. In addition, they shared our goal in testing the predictive accuracy of a simple dynamic model of consumer response. To the extent that the simple model predicts well, an evolution of that model could be used for nationwide evaluation of potential dial-a-ride SMD sites.

The operating plan (vehicle hours, dispatching strategy) was constant throughout the model evaluation although system performance varied. Operating changes to improve service were made after the model evaluation period, partially as the result of the model's implications. Thus our explanatory variables include marketing strategy (direct mail, publicity), diffusion phenomena (word of mouth) and measures of the impact of the variation in system performance.

3. CONSUMER MODEL

Loyal transit usage does not develop overnight. Consumers must first become aware of the new service. Even if they become aware of the service, they will not all try it immediately. Instead ridership (at least trial) is likely to grow over time as those who are aware of the service try it. Marketing theory suggests these awareness and trial processes are not automatic but influenced by marketing strategies and system performance.

Consider a marketing strategy of mailing information brochures to all households in the target market. The impact of this strategy can occur in many ways. First, those who read the brochure will become aware of DART, its hours of operation, and its

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2 New vehicles replaced the school buses during the predictive period but not the estimation period. Such an unmodeled change in operating strategy represents noise in the predictive period and biases the model against good prediction. Thus if we predict well despite not modeling vehicle change, we could expect a model which includes vehicle change to predict as least as well.
fare. In addition to communicating the characteristics of the system, the brochure may also contain persuasive messages to influence a consumer's preference for DART. But no one would expect the marketing strategy to cause 100% awareness, encourage 100% trial, or assure 100% repeat. Some consumers may not receive the brochure, some may not read it, some may read it but not seriously consider the information, and some may seriously consider DART and reject it as not filling their needs. Furthermore, the impact will be greatest in the week of the mailing and decay as time passes.

It is possible to describe this process by a complex model of information processing including "behavioral states" for awareness, full information, intent, trial and repeat. (See Bettman [2]) Instead, we choose a simpler beginning model describing consumers as either unaware of DART or aware of DART. In addition to their level of awareness, at any point in time consumers can also be classified by how they made their last trip. For example, for those aware of DART the last trip was either by DART, by the existing bus system (BUS), or by car. (Models for other communities might also include taxi, walk or bicycle but an earlier survey indicated that the use of these transportation modes in Schaumburg was negligible compared to car and transit.) Since the Village believed that those who were passengers in cars were more likely to switch to transit than those who drove cars, we classify car users as either car drivers or car passengers. Thus we can describe each consumer at each point in time as being in one of the seven "behavioral states" as indicated in figure 1.

![Figure 1: Conceptual Representation of Consumer Model](image)

**Key:**
- BUS = Conventional Bus Service
- DART = Dial-a-Ride Transportation
- CD = Car as a driver
- CP = Car as a passenger
Consumer behavior is movement from one behavioral state to another including self-flows. For example, before a consumer becomes aware of DART he "flows" among the three modes of transportation in the box in the left of figure 1. Each "flow" is an act of taking a trip by one mode. Over time as the result of advertising, word of mouth, and other variables consumers become aware of DART as represented by the arrow marked 'A' in figure 1. Once a consumer becomes aware of DART, his behavior is modeled by the four behavioral states in the box in the right of figure 1. The whole process is continuous in time with flows probabilistically dependent on the explanatory variables.

Analytically we model figure 1 as a semi-Markov process defined by (1) $f_i(t)$, the probability distribution of times, t, until a consumer who just entered state $S_i$ will leave it, and (2) $q_{ij}$, the probability that the next state he flows to (from $S_i$) is state $S_j$. In marketing terms, $f_i(t)$ is the distribution of interpurchase times and $q_{ij}$ are the switching probabilities. The methodology can handle either Erlang or negative exponential distributions for $f_i(t)$, but previous work by Leman [15] suggests that transportation mode choice is best modeled by a negative exponential distribution, $f_i(t) = \mu_i \exp(-\mu_i t)$, where $\mu_i$ is the "flow rate" from state $S_i$.

When $f_i(t)$ is negative exponential, the semi-Markov process becomes a continuous time Markov process and can be summarized by a set of flow rates, $a_{ij} = \mu_i q_{ij}$, where $a_{ij}\Delta t$ is the probability that a consumer flows from $S_i$ to $S_j$ in time $\Delta t$. Since $\mu_i$ and $q_{ij}$ can be recovered from the $a_{ij}$, i.e., $q_{ij} = a_{ij}/\sum_k a_{ik}$ and $\mu_i = \sum_k a_{ik}$, we deal directly with the flow rates, $a_{ij}$. Derivations are given in Hauser and Wisniewski (1981). (The notation, $\sum^0$ implies special consideration for self-flows and is described in the appendix.)

We model the flow rate from state to state, $a_{ij}$, as a linear function of the explanatory variables. For example, the flow from "BUS/Unaware" to "DART/Aware" might depend on the number of direct mail pieces sent out, the amount of newspaper coverage, the availability of DART, and the probability that a consumer prefers DART to BUS. This preference probability is in turn dependent upon consumers' relative perceptions of DART and BUS. Note that in any time period, a consumer can make a number of transitions. E.G., on a given day he may ride the BUS to work, get a ride home, and in the evening read his mail, become aware of DART, and use it for a trip to the shopping mall. Thus during that day he would (1) flow into "BUS/Unaware", etc.

Application of the model to frequently purchased consumer products might require Erlang rather than negative exponential interpurchase times. See Jeuland, Bass and Wright [13], Massy, Montgomery, and Morrison [16], and Zufryden [28]. The methodology can handle Erlang distributions with a modification in state definitions. See Hauser and Wisniewski for details [11].
(2) flow from "BUS/Unaware" to "Car Passenger/Unaware", (3) flow from "Car Passenger/Unaware" to "Car Passenger/Aware" and (4) flow from "Car Passenger/Aware" to "DART/Aware."

Analytically, let $x_{ijn}$ be the value of the $i$th explanatory variable, say the percentage of people receiving a brochure, affecting the i to j flow in period n, and let $w_i$ be the "importance" of the $i$th variable. Then we model the impact of the variables as

$$a_{ijn} = \sum w_i x_{ijn} \quad (1)$$

For example, $w_1$ might be the "importance" of direct mail. Once we know the $w_i$ parameters we can describe the system.

We postulate that flows from "unaware" to "aware" are a function of direct mail, publicity and word of mouth. Flows among usage states (e.g., BUS to DART) depend on the relative availability of the modes, on the consumers' budget for transportation, and on consumers' perceptions of the various characteristics of the modes. Perceptions are in turn dependent on marketing and system performance. As described in section 5, these hypotheses are based on Brunswik's model [4] of consumer information processing but will be tested empirically.

**Estimation of Model Parameters**

So far we have described a system that is continuous in time, but it is not feasible empirically to observe each and every transition, i.e., every flow. We can observe a series of discrete snapshots at $t = T_0, T_1, T_2, ..., T_n$. (These do not need to be equal time intervals.) In each time period, $T_{n-1}$ to $T_n$, we observe the number of consumers, $C_{in}$, in each state, $S_i$, at the start of the period, $T_{n-1}$, and the number of these, $C_{ijn}$, who end up in each state, $S_j$, at the end of the time period. $C_{in}$ and $C_{ijn}$ are readily obtained from panel data or, with some recall bias, from periodic surveys. For example, $C_{45n}$ might be the number of consumers who were BUS users on November 1, and who are DART users on November 15.

Let $\tilde{p}_{ijn} = C_{ijn}/C_{in}$, i.e., $\tilde{p}_{ijn}$ is an estimate of the probability that a consumer is in $S_i$ at $T_{n-1}$ and in $S_j$ at $T_n$. Let $\tilde{P}_n = \{\tilde{p}_{ijn}\}$ be the matrix of the $\tilde{p}_{ijn}$'s. Let $a_{ijn}$ be "estimated" flow rates and let $\tilde{A}_n = \{a_{ijn}\}$ be the matrix of the $a_{ijn}$'s. Then Hauser and Wisniewski [11] show that the maximum likelihood estimators of the $w_i$ are approximated by the following regression equation:

$$\tilde{a}_{ijn} = \sum w_i x_{ijn} + \text{error} \quad (2)$$

where $\tilde{A}_n = E_n^{-1} [\log \tilde{A}_n] E_n$, $E_n$ is the matrix of eigenvectors of $\tilde{P}_n$, and $[\log \tilde{A}_n]$ is a matrix with the logarithms of the eigenvalues of $\tilde{P}_n$ on the diagonal and zeros elsewhere.
Equation 2 allows us to estimate the "importance" weights, \(w_k\), by first using an eigenstructure computer package to transform the frequency data, \(C_{in}\) and \(C_{ijn}\), and then using ordinary regression to obtain the estimates, \(\hat{w}^k\). Simulation suggests that good estimates can be obtained if (1) the average \(C_{ijn}\) is greater than 20 and (2) \(t_n = T_n - T_{n-1}\) is short relative to the time it takes the process to reach equilibrium.

**Statistics of Managerial Interest**

To forecast we use the estimated importance weights and the observed explanatory variables to estimate the flow rates, i.e., \(\hat{a}_{ijn} = \sum_k \hat{w}^k x_{ijn}\) where \(\hat{\cdot}\) indicates estimate. The forecast flow probabilities, \(\hat{p}_{ijn}(t_n)\), are given by an eigenstructure formulae given in the appendix.

Although the forecast flow probabilities can describe the system, a manager wants descriptions of consumer response that are more similar to the types of statistics with which he normally deals. Two important summary statistics that the model can provide are (1) cumulative awareness and (2) rides per period. Cumulative awareness is the total number of consumers aware of DART by period \(n\). We compute cumulative awareness by calculating the number of consumers who flow out of the "Unaware" states. We compute rides per period by calculating the number of times any consumer flows into "DART/Aware", including flows from "DART/Aware" back into "DART/Aware". While the mathematics of computing these statistics is complex, algebraic formulae have been developed which are readily adaptable to computer analysis. See Appendix 1.

A final managerial question that is addressed by the model is what will happen in the long run if a strategy is continued. We call the long run ridership per period resulting from a stable strategy the "equilibrium" ridership. To compute equilibrium ridership we assume that the same managerial strategy is applied in every period and use the sales formulae to compute what happens as the number of periods becomes very large. The equilibrium ridership also can be calculated with a simple formulae based on the estimated flow rates, \(\hat{a}_{ijn}\).

This completes our brief description of the model. Details will become more clear as we proceed to the empirical model development.

### 4. DATA COLLECTION

The primary data on which our analyses are based is a series of sixteen identical twelve-page mail surveys sent periodically and randomly to Schaumburg residents and to residents of a neighboring community (Hoffman Estates) who are in the DART service area. The early surveys were mailed out at relatively
short intervals (1 week) to be sensitive to rapid behavior changes likely to occur once service began. Later surveys were mailed at longer intervals (2 weeks, then 4 weeks) to enable us to track the behavior changes over the length of the demonstration project and to do so at a reasonable cost of data collection. Archival records of ridership, publicity and mailings were collected to establish some of the explanatory variables and to provide a non-survey test of our model's predictions of behavior. We describe each in turn.

Pre-Analysis

Prior to implementation of service, we performed a pre-analysis to help the Village establish the image they wished to portray for DART. The name of the service and its core benefit proposition were developed with this analysis. The pre-analysis was based on focus groups, a telephone survey to establish ridership patterns and a mail survey sent to 1500 residents. See Wisniewski [27] for details.

For our purposes, this pre-analysis, combined with an earlier analysis in Evanston, Il. (Hauser, Tybout, and Koppelman [9]) provided a measurement instrument with validated scales to measure usage and perceptions of transportation modes. Tybout and Hauser [25] use archival ridership data to evaluate the Evanston forecasts which were made with a static version of the model in the left box of figure 1. Their analyses provided further input to the development of the measurement instrument.

Periodic Surveys

In each observation period between September 20, 1979 and April 17, 1980 periodic surveys provided measures of perception and preference, provided the data necessary for the dependent measures used to estimate in the dynamic model, and provided self-reported descriptions of media, mail promotion and word of mouth. The surveys were developed based on the pre-analysis, focus groups and extensive pretests. In fact, an abridged version of the survey was used to monitor a June 1979 strategy modification in the conventional bus system. Although that sample was small, the results suggest that the survey questions were sufficiently sensitive to identify changes in behavioral states. We describe the specific survey measures in section 5.

Table 1 indicates the mailing dates, the sample sizes, and the response rates of the periodic surveys. The overall response rate of 30.4 percent (of which 91.2 percent were complete and usable) is a moderate response rate for mail surveys

DART (Dial-A-Ride Transportation) was chosen to connote a service that was everywhere (convenient) and provided speedy service. Prior to the analysis, the Village was considering STEP (Schaumburg Transportation Energy Conservation Program). The logo was to be a drawing of people riding in a shoe. Tests indicated that DART communicated the core benefit proposition better than STEP and other potential names.
(Hauser, Tybout, and Koppelman achieved a 41.2 percent return). Comparisons with a 1978 census in Schaumburg indicate a slight bias toward males (55%) and a slight undersampling of the elderly and students but no other significant differences at the .01 level. A further (but untestable) hypothesis is that there might be a slight bias in return toward those more interested in public transportation. In either case, such biases work against successful prediction of archival ridership and thus make the predictive test a more stringent test of the model.

Table 1 also indicates which surveys provided data for the estimation of the model and which surveys provided data for the predictive tests of the model. Period 1 was prior to implementation. Periods 2 through 9 provided the data to estimate the parameters of the model, the \(w_j\)'s. Periods 10 through 16 provided explanatory variables only. Note that the model is developed based on the first 11 weeks of service and is used to predict ridership for the next 19 weeks.

Archival Data

The explanatory variables for media and promotion and the ridership counts to test the model were obtained by unobtrusive observation. We kept records on when articles on DART appeared in each newspaper, how long they were, and what percentage of Schaumburg residents subscribed to each newspaper. We also noted the dates and the coverage of both direct mail campaigns. Ridership was obtained from dispatch records. For each service call, the dispatcher recorded who rode the system, when they rode it, where they were picked up, and where they were dropped off. Finally it is important to note that there was no service on holidays (Nov. 22, Dec. 25, Jan. 1).

Table 1: Mailing Dates and Response Rates of Periodic Surveys

<table>
<thead>
<tr>
<th>WAVE NUMBER</th>
<th>DATE MAILED</th>
<th>SAMPLE SIZE</th>
<th>PERCENT RETURN</th>
<th>ESTIMATION</th>
<th>PREDICTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sept. 20</td>
<td>320</td>
<td>26.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Oct. 4</td>
<td>400</td>
<td>29.8</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Oct. 11</td>
<td>725</td>
<td>27.2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Oct. 18</td>
<td>700</td>
<td>30.0</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Oct. 25</td>
<td>600</td>
<td>33.8</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Nov. 1</td>
<td>750</td>
<td>36.8</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Nov. 15</td>
<td>750</td>
<td>17.1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Nov. 29</td>
<td>750</td>
<td>31.9</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Dec. 13</td>
<td>750</td>
<td>22.7</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Dec. 27</td>
<td>750</td>
<td>30.9</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Jan. 10</td>
<td>600</td>
<td>33.7</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Jan. 24</td>
<td>600</td>
<td>33.7</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Feb. 7</td>
<td>600</td>
<td>34.7</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Feb. 21</td>
<td>600</td>
<td>35.3</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Mar. 21</td>
<td>600</td>
<td>34.3</td>
<td>✓</td>
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</tr>
<tr>
<td>16</td>
<td>Apr. 17</td>
<td>500</td>
<td>30.2</td>
<td>✓</td>
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</table>
5. OPERATIONALIZATION OF THE CONSUMER MODEL

To apply to the consumer model, we must select measures to operationalize the dependent and the explanatory measures. Any operationalizations require tradeoffs. In the Schaumburg analysis, we make a number of tradeoffs to develop a feasible model. In making our decisions we were guided by the goals of parsimony, paramorphism, and actionability. Other research goals might lead to different empirical decisions.

**Parsimony.** We are interested in an evolutionary model that can explain behavior with a small set of explanatory variables. We favor the less complex model if it can provide the necessary managerial insights.

**Paramorphism.** A model is a representation of the essential elements of consumer behavior. Instead of requiring a complete description of information processing, we require a model that says consumers behave "as if" this were the way they process information. We develop the "best" model the data allows, but we are willing to make practical decisions to make the model feasible. We require only that all "fitting" parameters necessary to implement the practical decisions be determined from the estimation data. This requirement tends to bias against good prediction. If the model predicts well under these conditions, it would predict at least as well were the model improved.

**Actionable.** Whenever possible we select explanatory variables over which a manager can exercise control. We are interested in how well these variables explain behavior recognizing there may be other, unobserved, causes of behavior. In our case we achieve actionability for the marketing variables, but because there is no variation in the operating decisions within the data period, we use surrogates (perceptions) to observe the variation in system performance (which was not directly measured). Forecasting under new operating decisions, as opposed to marketing decisions, would require an external model linking operating decisions to perceptions.

**Behavioral States**

We prefer the model in figure 1, but our data does not contain consumer perceptions of BUS, car driver and car passenger for those consumers who are unaware of DART. Managerially, we are most interested in strategies that affect ridership on DART. Marketing strategies cause awareness and influence consumers to modify their ridership patterns. System performance has its greatest impact on consumers' ridership patterns. Based on discussions with representatives of the Village of Schaumburg and the RTA we selected the five-state model shown in figure 2 as sufficient for the identification of managerial strategies affecting DART ridership. This model requires 16 independent flows (excluding self-flows). Since our data averages 200 consumers each making five trips per week, this is well within the simulation limits suggested by the
analysis (Hauser and Wisniewski [11]).

<table>
<thead>
<tr>
<th>Behavioral States</th>
<th>UD</th>
<th>CD/A</th>
<th>CP/A</th>
<th>B/A</th>
<th>D/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaware of DART</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Car Driver/Aware</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Car Passenger/Aware</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>BUS/Aware</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>DART/Aware</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Figure 2: Behavioral States Used in the Model of Consumer Response to DART (√ indicates potential flows)

**Dependent Variables for Estimation Purposes**

To describe a consumer, we must identify whether a consumer is aware of DART and what mode of transportation he has used last. To identify the percent of consumers, \( \bar{p}_{ijn}(t_n) \), that flow from state to state in period \( n \) we must identify which behavioral state describes a consumer at the beginning of an observation period, time \( T_{n-1} \), and at the end of an observation period, time \( T_n \). (\( t_n = T_n - T_{n-1} \)).

*Awareness*. Awareness of DART at the end of the period (time \( T_n \)) was measured in survey \( n \) by a direct awareness question placed among awareness and familiarity questions about four existing services. Awareness of DART at the start of the period (time \( T_{n-1} \)) was measured in survey \( n \) by a recall question asking consumers when they first learned of DART. See table 2. Each question is potentially biased. However, we are only measuring changes in awareness and are thus mainly concerned with the relative bias, i.e., the difference in bias between the two measures.

If there were no relative bias, the overall percent of consumers aware at the end of the \( (n-1) \)th period, time \( T_{n-1} \), should equal the overall percent aware at the beginning of the \( n \)th period, also time \( T_{n-1} \). Examination of overall percentages indicates that relative to direct measurement the recall question underestimates awareness at time \( T_{n-1} \).
Table 2: Survey Measures of Awareness

(A) Direct Measure (time $T_n$, survey $n$)

3. Are you aware of the following bus services in Schaumburg? (please check only one box for each service)

<table>
<thead>
<tr>
<th>Service</th>
<th>No, I am not aware of this service</th>
<th>Yes, but I am not sure of the details</th>
<th>Yes, I am thoroughly familiar with the details</th>
<th>Yes, and I have used this service</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior Citizen Bus</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Rush-hour commuter bus to Roselle train station</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Dial-A-Ride bus service (DART)</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Regular RTA bus service</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

(B) Recall Measure (time $T_{n-1}$, survey $n$)

6. When did you first learn of Dial-A-Ride bus service?

[ ] this is the first time I've heard of the service (GO TO QUESTION 11)
[ ] within the last 2 weeks
[ ] over 2 weeks ago

In an attempt to adjust for the underestimated recall, we developed a regression model (periods 2 through 16) to modify the recall measure to match the direct measurement. See table 3. Basically, the model in table 3 estimates directly measured awareness (time $T_{n-1}$, survey $n$) as 74% of recall awareness (time $T_n$, survey $n$) modified by a series of (0,1) dummy variables that correct for the environment of measurement. "Period length" accounts for the fact that $t_n$ varies from 1 week to 2 weeks to 4 weeks. "Target Sample" accounts for postal error resulting in partial non-delivery of surveys to a neighboring community in the first five periods. "Daily Herald" and "Voice of Schaumburg" account for media publicity which seem to affect the measurement bias as well as awareness. Because of the small sample size, we retained all variables even though the significance goes as high as the .15 level. This adjustment is necessary due to a relative bias in recall measurement of membership in a behavioral state. It would not be necessary if panel data could be used to obtain the dependent measures of awareness.

Table 3: Regression Equation Used to Adjust Recall Measure of Awareness to Direct Measure of Awareness (Dependent Measure is directly measured awareness, time $T_{n-1}$, survey $n$)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall Awareness</td>
<td>.74</td>
</tr>
<tr>
<td>Period Length - 1 week</td>
<td>-</td>
</tr>
<tr>
<td>Period Length - 2 weeks</td>
<td>.12</td>
</tr>
<tr>
<td>Period Length - 4 weeks</td>
<td>.17</td>
</tr>
<tr>
<td>Target Sample</td>
<td>.11</td>
</tr>
<tr>
<td>Daily Herald</td>
<td>-.08</td>
</tr>
<tr>
<td>Voice of Schaumburg</td>
<td>.09</td>
</tr>
<tr>
<td>Constant</td>
<td>.15</td>
</tr>
<tr>
<td>$R^2$ (adjusted)</td>
<td>.92</td>
</tr>
</tbody>
</table>
Behavior. Simulation and analytic arguments suggest that the time period of observation, \( t_n \), be short compared to the dynamics of interest. This is no problem for the typical consumer product with interpurchase times on the order of weeks. For transportation service interpurchase times are fractions of days, short compared to the weekly observation periods. We overcome this potential problem by recognizing that the pattern of behavior (choice of mode portfolio) is likely to change at a much slower rate. This is particularly true with respect to DART which for most consumers, is a mode that is used for occasional trips.

The portfolio of usage of transportation modes was measured by self-reported frequency of use at \( T_n \) and at \( T_{n-1} \). Unlike awareness, there was no overall relative bias identified for self-reported frequency of use. See Table 4. Flow percentages, \( P_{ijn}(t_n) \), were estimated as a weighted average of individual flow percentages. For example, suppose consumer 1 made 6 BUS trips per week at time \( T_{n-1} \) and he made 4 BUS trips and 2 DART trips at time \( T_n \). Then, we count his contribution to flows as 4 BUS-to-BUS flows and 2 BUS-to-DART flows. By summing flows across consumers and dividing by total observed trips we obtain the flow percentages. Note that consumers are implicitly weighted by the number of trips they make per week. The reasonableness of this interpretation of figure 2 remains an empirical question.

Explanatory Variables - Marketing Strategy

The primary marketing strategies used by Schaumburg were direct mail and media publicity.

Archival Measures. We recorded when publicity appeared in each of the five Schaumburg newspapers (Schaumburg Daily Herald, Voice of Schaumburg, Record Newspaper, Chicago Tribune, and Chicago Sun Times) and we know the circulation of each newspaper within Schaumburg. Thus one archival measure is a series of dummy variables which take on \((0,1)\) values depending upon whether or not publicity appeared in that newspaper during the observation period. A more accurate measure would be to weight the variables by their percent circulation. However, such a series of variables does not take into consideration the overlap in readership among the five newspapers.

To account for overlap in readership we created a composite variable to estimate the "reach" of the publicity, i.e., the percent of consumers who read at least one newspaper containing an article on DART. This variable was computed by standard rules of probability (assuming independent events). For example, if two articles appeared between time \( T_{n-1} \) and \( T_n \), one in a newspaper with 60% circulation and one in a newspaper with 20% circulation, then the reach variable takes on a value of .68 representing .60 plus .20 minus the overlap (\(.60)(.20)\).
Table 4: Survey Measures of Behavior

(A) Direct Measure (time $T_n$, survey $n$)

12. Within the last 7 days, how many round trips did you make within Schaumburg Township by each of the following means of transportation for a purpose other than going to a full-time place of employment. (A round trip starts and ends in the same place; for example, home + destination 1 + destination 2 + home would be 1 round trip.)

<table>
<thead>
<tr>
<th></th>
<th>None</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5-6</th>
<th>7 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car as a driver</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Car as a passenger</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Dial-A-Ride</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Regular RTA buses</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Other (please specify)</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

(B) Recall Measure (time $T_{n-1}$, survey $n$)

13. In the week just before this last week (i.e. 8-14 days ago), please estimate how many round trips you made within the Township for a purpose other than going to a full-time job by:

<table>
<thead>
<tr>
<th></th>
<th>None</th>
<th>1</th>
<th>2</th>
<th>3-4</th>
<th>5-6</th>
<th>7 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car as a driver</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Car as a passenger</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Dial-A-Ride</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Regular RTA buses</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Other (please specify)</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

Direct mail was measured as the percent of consumers to whom the brochure was mailed. Finally we created an alternative "marketing" variable which included the direct mail variable in the calculation of reach.

Survey Measures. The archival variables are actionable but may be less accurate than survey measures which can measure whether the consumer actually received and noted the direct mail brochure or the newspaper article. To test this hypothesis we used the alternative operationalizations of the marketing variables appearing in lines 1, 3, 5, and 8 of table 5.
Table 5: Alternative (Survey) Operationalizations of Marketing and Word of Mouth Variables

1. Consider only information and events of the last 7 days. How did the following effect your decision to try or use Dial-A-Ride in the last 7 days?

<table>
<thead>
<tr>
<th>This occurred in the last 7 days, and had:</th>
<th>This did not happen in the last 7 days</th>
<th>A negative effect on my decision to try/use</th>
<th>No effect on my decision to try/use</th>
<th>A positive effect on my decision to try/use</th>
</tr>
</thead>
<tbody>
<tr>
<td>mailings to my residence on Dial-A-Ride</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>receipt of free-ride or multi-ride coupon(s)</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>receipt of brochure/ fact sheet</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>calls to the Village Transit Manager</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>reading newspaper articles on Dial-A-Ride</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>saw the Dial-A-Ride buses in operation</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>word of mouth from friends/associates</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>DTA advertising</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>gas price increases/ shortages</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>no other means of transportation available</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>other</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

Explanatory Variables - Word of Mouth

The literature on the diffusion of innovations (Rogers and Shoemaker [17]) suggests that word of mouth can have a major influence on awareness and on the adoption of innovations.

Archival Measures: Bass [1] has developed a parsimonious model to account for diffusion phenomena in consumer durable goods. This model has been applied in a variety of product categories (Dodds [5], Nevers [20]) and has predicted well in many of those categories. In that model, Bass operationalizes word of mouth as the number of consumers who have already adopted the innovation. For DART, this operationalization corresponds to the total number of consumers (as measured from the archives) who have tried the system by time $T_{n-1}$. For forecasting purposes, this variable is endogenous to the model.

Survey Measures. We also tested an alternative operationalization which asked consumers to self-report whether they had received information about DART by "word of mouth from friends or associates". See table 5, line 7.

Explanatory Variables - Imbedded 'Lens' Model

Marketing strategies and word of mouth should impact awareness flows directly. Changes in usage patterns are more complex. Operating strategies and marketing strategies affect behavior but psychological theory (e.g., Brunswik [4]) suggests that their impact on behavior is moderated by a series of intervening variables. This conceptual representation of the impact of marketing strategy is called the 'Lens' model. It is similar to models developed by Shocker and Srinivasan [23], Hauser and Urban [10], and Sternthal and Craig [24].
See figure 3. In the 'Lens' model, operating strategies and marketing strategies affect objective reality (e.g., physical characteristics of the transportation system such as travel time) and psycho-social cues (e.g., advertising). These influence subjective reality as represented by consumers' perceptions of the various means of transportation. Based on these perceptions, consumers form their preferences. Behavior is then based on preference but moderated by situational constraints such as availability or budgets. Feedback loops such as the dotted line in figure 3 are also possible.

In situations where operating strategies vary, we could measure and estimate the full 'Lens' model for diagnostic purposes or the reduced form model (strategies → behavior) for predictive purposes. In our situation operating strategies are constant throughout the estimation period. Physical characteristics such as travel time vary but were not directly measured due to cost considerations. Instead we used direct survey measures of perceptions and constraints to reflect the variation in system performance. In this way, we develop a model which can evolve through the addition of an external model linking operating and marketing strategies to perceptions. Such models are feasible and have been developed in other contexts. See Green, et al [6], Green and DeSarbo [7], Hauser and Simmie [8], Neslin [19], and Urban and Hauser [26].

OPERATING STRATEGIES → PHYSICAL CHARACTERISTICS → PERCEPTIONS → PREFERENCE
MARKETING STRATEGIES → PSYCHO-SOCIAL CUES → CONSTRAINTS → BEHAVIOR

Figure 3: "Lens" Model Relating Operating and Marketing Strategies to Behavior Through Preference and Availability Constraints

Perceptions. Perceptions are measured by having consumers evaluate car as a driver, car as a passenger, RTA bus, and DART on the series of eighteen agree/disagree scales developed to measure the constructs identified in the preanalysis. Table 6 shows the first of these eighteen scales as it appeared in the periodic surveys.

5 The direct measurement of physical characteristics for all four modes of transports is a non-trivial, labor intensive, expensive data collection process.
Since the eighteen scales are redundant measures of the perceptual constructs we use factor analysis\(^6\) to reduce the eighteen scales to four perceptual constructs: 'convenience', 'ease-of-use', 'safety', and 'general opinion'. 'Convenience' reflects the ability to "come and go as I wish", on time performance, the hassle in arranging for use, and the fit to the consumer's schedule. 'Ease-of-use' reflects whether the mode is not tiring, enjoyable, and easy to use in bad weather. 'Safety' includes the fear of crime and accidents. 'General opinion' is a series of scales to measure beliefs about personal and social norms. Factor scores based on the eighteen scales are the explanatory variables used to represent the four perceptual constructs.

Table 6: Example Scale to Measure Perceptions

<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree Nor Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>It would require a lot of effort to travel around Schaumburg by:</td>
<td>Car as a driver</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td></td>
<td>Car as a passenger</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td></td>
<td>Dial-A-Ride</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td></td>
<td>Regular RTA bus</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

Preference. Preference is measured in the surveys by having consumers rank order the four modes of transportation in terms of preference. To use this construct in the model we want to be able to predict preference as a function of the measured perceptions. To accomplish this we use the multinomial logit model (McFadden [13]) which predicts the probability that a consumer with a given set of perceptions prefers a given mode. As shown in table 7, the estimated model does quite well in predicting preference based on the measured perceptions (82% of the consumers correctly predicted, 63% of the uncertainty explained). This model is estimated based on survey periods 2 through 9. Alternative specific constants are included to insure consistent estimates.

To create the explanatory variable that is used in the dynamic consumer model we use the logit equation to compute the estimated probability, \( L_{cj} \), that consumer \( c \) prefers mode \( j \). That is,

\[ \text{Preference is measured in the surveys by having consumers rank order the four modes of transportation in terms of preference. To use this construct in the model we want to be able to predict preference as a function of the measured perceptions. To accomplish this we use the multinomial logit model (McFadden [13]) which predicts the probability that a consumer with a given set of perceptions prefers a given mode. As shown in table 7, the estimated model does quite well in predicting preference based on the measured perceptions (82% of the consumers correctly predicted, 63% of the uncertainty explained). This model is estimated based on survey periods 2 through 9. Alternative specific constants are included to insure consistent estimates.}

\[ \text{To create the explanatory variable that is used in the dynamic consumer model we use the logit equation to compute the estimated probability, } L_{cj}, \text{ that consumer } c \text{ prefers mode } j. \text{ That is,} \]

\[ L_{cj} \]

\( \text{The factor analysis is run across stimuli and subjects. The four factor solution was selected based on eigenvalue and scree rules (Rummel [22]) and ease of interpretation. It explains 63.3\% of the total variance.} \)
\[ L_{cj} = \exp(\sum_k \beta_k y_{ckj} + \delta_j) / \sum \exp(\sum_k \beta_k y_{ckj} + \delta_j) \]  

where \( \beta_k \) are the estimated logit coefficient (table 7) and \( y_{ckj} \) is the factor score estimating consumer c's perceptions of mode j for perceptual dimension k. \( \delta_j \) is the alternative specific constant for mode j. The coefficients are estimated based on data in periods 2-9. The same coefficients are used to forecast for periods 10-16.

Table 7: Preference Estimation Via Logit Analysis  
(Dependent measure is first preference)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convenience</td>
<td>.86</td>
<td>5.7</td>
</tr>
<tr>
<td>Ease of use</td>
<td>.64</td>
<td>5.8</td>
</tr>
<tr>
<td>Safety</td>
<td>.55</td>
<td>3.5</td>
</tr>
<tr>
<td>General Opinion</td>
<td>1.28</td>
<td>10.1</td>
</tr>
<tr>
<td>Constants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car Driver</td>
<td>2.07</td>
<td>10.2</td>
</tr>
<tr>
<td>Car Passenger</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BUS</td>
<td>.68</td>
<td>2.4</td>
</tr>
<tr>
<td>DART</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Percent Correctly Predicted</td>
<td>82.2</td>
<td>-</td>
</tr>
<tr>
<td>Percent Uncertainty Explained</td>
<td>62.5</td>
<td>-</td>
</tr>
</tbody>
</table>

*Insignificant at the .05 level and dropped from the final model. The car passenger constant is arbitrarily set to zero since the constants measure relative effects.

Constraints. The final variable we need based on the 'Lens' model is a measure of the constraints faced by the consumer. Previous transportation research has identified availability as a major constraint on choice of mode. We operationalized availability with the scale in table 8a. Economic theory suggests that a consumer's budget allocation to a product category is a major constraint on choice (Blackorby, Primont and Russell [3]). We operationalize budget allocation with the categorical scale in table 8b.

Preference Inertia. Neslin [18] showed empirically that consumers exhibit an inertia factor when considering innovative services, i.e., their delay in trying a new service is more than could be explained by their relative preferences among the new and existing services. We operationalize inertia by a (0,1) variable affecting flows into DART. If this variable is significant and negative then it could represent inertia. If it is significant and positive it could represent a bias towards innovations.
Table 8: Constraints on Choice

(A) Availability Constraints

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Usually readily available</th>
<th>Usually available, but with great difficulty</th>
<th>Usually not available</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car as a driver</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Car as a passenger</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Dial-a-Ride</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Regular RTA buses</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Other (please specify)</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

(B) Budget Allocation to Transportation

10. Do you consider transportation costs to be:

[ ] a large part of your yearly household expenses
[ ] a moderate part of your yearly household expenses
[ ] a small part of your yearly household expenses

6. MODEL ESTIMATION

Section 5 provides measures of the dependent variables, \( \tilde{p}_{ijn}(t_n) \), and the explanatory variables. (To review, \( \tilde{p}_{ijn}(t_n) \) is the percentage of consumers who are in \( S_i \) at time \( T_{n-1} \) and in \( S_j \) at time \( T_n \) where \( t_n = T_n - T_{n-1} \); \( x_{ijn} \) is the value of the \( s \)th explanatory variable takes on for the \( i \) to \( j \) flow in period \( n \) for consumer \( c \). We choose the macro-flow model described in Hauser and Wisniewski [11] and hence use representative variables, \( r_{ijn} = (1/C_{in}) \sum\Sigma x_{ijn} \) where \( C_{in} \) is the number of consumers in state \( S_i \) at the beginning of period \( n \).) We use the regression approximation to obtain the estimates, \( \hat{w}_z \), of the parameters of the system. (Review equation 2.) The dependent variables in the regression are the \( \tilde{a}_{ijn} \)'s obtained from the \( C_{ijn} \)'s and \( C_{in} \)'s. The independent variables are the representative variables, \( r_{ijn} \), which measure the average effect of marketing strategy, word of mouth, preference, constraints, and preference inertia. These variables take on different values for different \( i \)-\( j \) combinations. E.G., for preference, \( x_{ijn} = \frac{\tilde{p}_{ijn}}{\tilde{p}_{ijn}} \), where \( \tilde{p}_{ijn} = \frac{1}{C_{in}} \sum\Sigma C_{ijn} \).

The regression is run across time periods 2-9 and across ten of the sixteen independent, non-zero flows in figure 2. Based on the interpretations of the consumer model in section 5 and the simulation results in Hauser and Wisniewski [11], the sample size and observation period length should be sufficient to obtain reasonable estimates, \( \hat{w}_x \), of the \( w_x \).

In the estimation periods the sample size, \( C_{in} \), for flows out of the low share modes, BUS and DART, were below the simulation limits. Flows into these modes were based on sufficient sample sizes. This step reduces the number of observations but should not bias the estimates.
Model Development

According to the theory of section 5, the marketing variables, publicity and direct mail, and word of mouth should impact flows out of awareness. The 'Lens' model variables, preference, availability, budget allocation, and inertia, should impact usage flows. Note that preference is in turn a composite variable created with the logit model in equation 3 and table 7. Examination of the correlation matrix indicated that publicity and direct mail were collinear, but all other variables were within the simulation limits. Since the collinearity in the marketing variables was structural - whenever the Village mailed out brochures, the newspaper ran a news item - we use marketing reach to replace publicity and direct mail. The complete model is shown in table 9 as model 1 where we have chosen archival variables for marketing reach and word of mouth.

Table 9: Model Development

<table>
<thead>
<tr>
<th></th>
<th>(1) Full Model</th>
<th>(2) Selected Model</th>
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<tr>
<td><strong>Impacts on Awareness Flows</strong></td>
<td></td>
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<td>Marketing Reach</td>
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<tr>
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<td>Budget Allocation</td>
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<tr>
<td>Moderate</td>
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<tr>
<td><strong>Ridership Correlation</strong></td>
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<td>.94</td>
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*Both regressions are significant at the .01 level.
All coefficients except those starred are significant at the .10 level.
Model (1) is not significantly better than Model (2).
F(4,71) = 1.36  (p > .10)

Preference is a composite variable created from the measured perceptions with equation 3 and the parameters in table 7.

While Model 1 fits the data well (adjusted R² = .82), many of the variables are not significant at the .10 level. We delete these variables and reestimate the model. See Model 2 in table 9. All non-constant variables are now significant.
at the .10 level and a comparable fit is obtained (adjusted $R^2 = .82$). Comparison of the two models indicates that Model 1 is not significantly better than Model 2 at the .10 level of statistical significance. \[F(4,71) = 1.36\]

As a further test of descriptive fit, we compared ridership predicted by the model with the ridership observed in the archival data. This is by no means a guaranteed fit. First, the dependent measure in the regression is flows among behavioral states. Ridership is a structural output of the model dependent upon the adequacy of the probabilistic model in representing behavior. Second, the model estimation is based on survey data while the archives are based on observed behavior.

As shown in figure 4, the model fits the archival data reasonably well, successfully predicting the increase during November 1-14 (due to publicity), the downtown during November 15-29 (due to riders who tried the service but did not repeat), and the upturn during November 29-December 12 (due in part to the direct mail campaign late in that period). Dates in figure 4 represent when the survey was mailed out. Correlation between predicted and actual ridership was .94.

Finally, as expected, a model with the same variables but based on the full data including low probability flows (16 flows x 8 periods) does not do significantly better (adjusted $R^2 = .68$, correlation = .95) and yields similar coefficients. Based on these results and the criteria of pasimy we use the coefficients in model 2 to forecast ridership from December 27 through the period beginning April 17.

**Alternative Operationalizations**

We used the actionability criterion to select the archival operationalizations of the explanatory variables. However, if the self-reported variables do significantly better we would have to reassess our position and plan future research to further understand how managerial actions impact the intervening self-reported measures. To investigate this issue, we ran regressions with selected models based on alternative operationalizations of the explanatory variables.

Based on the statistics that measure descriptive fit, these models did not do significantly better than Model 2. See table 10. Model 2 is the same Model 2 reported in table 9.

Model 3 breaks the "marketing reach" variable into its component parts. Although its descriptive statistics are comparable to those of Model 2, the marketing variables are now insignificant at the .10 level - probably due to the collinearity among the explanatory variables. Since collinearity blurs the interpretation of Model 3, we retain Model 2.
weekly ridership

or

UAKI

Table

I

10-30-00

observed ridership
(predicted ridership
(model from questionnaire data)

Figure 4: Comparison of Actual and Estimated Ridership for the Estimation Period

Model 4 replaces the archival marketing and word of mouth variables with self-reported measures. Again, the descriptive statistics are comparable to those of Model 2, but the variables are all insignificant at the .10 level. Since, in this application, the self-reported variables do not do significantly better than the more actionable archival variables we select Model 2 for its managerial relevance. Other researchers with other goals, e.g., an investigator of cognitive processing, might further investigate Model 4 because it is based on measures of consumers' subjective evaluations of the impacts of marketing strategies.

Model 5 replaces preference probabilities (equation 2) with a weighted sum of the perceptual dimensions ($\sum_k \beta_k y_{ckj} + \delta_j$). Model 5 does not do as well as Model 2 suggesting that preference probability may be the better measure. We again select Model 2 based on the theoretical justification that, in the absence of other variables, an explanatory variable should be proportional to a transition probability.

Finally, we tested three models which each deleted one of the non-constant variables in Model 2. In each case, Model 2 was significantly better at the .10 level.
Final Selection

Based on the statistical, managerial and theoretical considerations we select Model 2 for predictive testing.

Table 10: Alternative Operationalizations

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<th>(3)</th>
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<td>.94</td>
<td>.78</td>
</tr>
</tbody>
</table>

*All regressions are significant at the .01 level.
All coefficients except those starred are significant at the .10 level.

7. PREDICTIVE TEST

Model 2 fits the estimation data well. If it is to be useful for managers and planners it must also do reasonably well in predicting consumer behavior under a changing operating environment. We construct the following predictive test:

(1) marketing strategy - Use the archival measures of the marketing strategy variables for periods 10 through 16 (percent mailing in each period, newspaper articles weighted by newspaper circulation).

(2) operating environment - Use the measured perceptions and availability for periods 10 through 16. These measures account for (unobserved) operating changes.

(3) preference - Use the probability equation (equation 3) with the parameters, \( \hat{\beta}_k, \hat{\delta}_j \) estimated in periods 2 through 9.

(4) share forecast - Use the consumer model to forecast share of ridership by DART in periods 10 through 16.

(5) ridership forecast - Assume total weekly trip making behavior is the same (except for holidays) throughout the forecast period. DART ridership is then (share) \( \times \) (total trips).

(6) comparison - Compare the forecast ridership to that measured directly by the archival data for periods 10 through 16.
We feel that this procedure provides a true predictive test because the dynamic consumer model is being used to forecast future behavior as much as 19 weeks in advance - a time period longer than the 11 week period of observation. Not only was the model estimated based on data collected in periods 2 through 9, but its structure and operationalization of explanatory variables were selected based on data collected in periods 2 through 9. Furthermore, as noted above, the predictions that are being tested (ridership in periods 10 through 16) are a structural output of the model, not the dependent measure used in estimation.

The basic output of the predictive test is shown in figure 5. Overall the model predicts well, but more importantly it correctly forecasts the inflection points in the data. (The inflection points are explainable by marketing actions and trial-repeat phenomena.) The correlation of predicted ridership with forecast ridership is .83. Because there are many potential biases\(^8\) in the data, all of which work against successful prediction, we feel the predictions in figure 5 are quite respectable. Finally, Model 2 outperforms Models 1, 4 and 5 which predict with correlations of .42, .81, and .27 respectively, and performs as well as the collinear model, Model 3, which predicts with correlation .85.

In addition to ridership, the model predicts other statistics of managerial interest. For example, figure 6 compares forecast cumulative awareness with cumulative awareness as measured by the surveys. The correlation between predicted and actual is .98. (The predictive test based on cumulative awareness is less stringent since the observed variable is a survey measure. The correlation is guaranteed to be high since cumulative awareness is a monotonically increasing measure.)

Based on these results we are encouraged about the accuracy of the dynamic consumer model, both its theoretical potential as developed in Hauser and Winiweski [11] and its operationalization as presented in this paper. In section 9 we discuss future research that has the potential to improve applications of the model.

8. MANAGERIAL RECOMMENDATIONS

The consumer model was developed as a forecasting tool, but prior to the predictive test it was an unvalidated forecasting tool. Thus, managerial decisions were not based on the model until the 30th week of service.

---

8 Among the potential biases in the data are non-response issues, seasonality not modeled, potentially small sample size, recall measures that require adjustment, measurement error in the survey, collinearity in the archival explanatory variables, and DART having a small share of the market.
Figure 5: Predictive Test for Dynamic Consumer Model Based on Seven Months of DART Service in Schaumburg, IL.

Figure 6: Comparison of Predicted and Observed Awareness of DART in Schaumburg, IL.
Before we discuss strategy, we note two additional bits of information: (1) average perceptions of DART were relatively stable throughout the observation periods and (2) the correlations between average perceptions of DART and the archival marketing variables were all insignificant at the .10 level. Interpreting this information within the context of the model, it appears that the aggregate effect of publicity and direct mail for DART in Schaumburg was to create awareness and communicate the characteristics of DART. Thus the effect of such strategies beyond the 30th week would be at most a 10% increase in ridership due to increasing awareness from 90% to almost 100%. We caution the reader that this does not rule out more active media, such as television advertising, which was not tested in Schaumburg.

Once consumers are aware of DART, the model explains the dynamic growth in ridership with a continuous time Markov interpretation of the static multinomial 'Lens' model. Using this model to predict equilibrium ridership, we found our estimates to be below the targeted goals of Schaumburg. Since the transit manager now accepts our model, he decided to take steps to increase perceived 'convenience', 'ease of use', 'safety', 'general opinion', and/or availability. While our model has not yet evolved to explicitly model the link from operating strategies to perceptions and constraints, we can gain some insight through qualitative diagnostic information.

We examine the perceptual map in figure 7. The points indicate how the average consumer perceives transportation in Schaumburg, the length and direction of the arrows are based on the relative importances of the perceptual dimensions in table 7. In addition, figure 7 indicates the relative perceived availability of the modes of transportation. Figure 7 suggests that improvements in convenience and ease of use and increased availability would have a major impact on ridership. (The goal is to improve the relative position of DART in the direction of the arrows.)

Discussions with representatives of the Village indicated that DART was providing as good a service as was possible within the current operating constraints (number of budgeted vehicle hours). The identified need was for more vehicles per hour or extended hours. Both of these strategies translate into more vehicle hours. Increased vehicle hours plus the improved weather in May through August (which affects 'ease of use') should increase ridership beyond the current levels.

These suggestions were made to the Village. Partially as the result of the analysis, the budgeted vehicle hours were increased in May, 1980 from 28 hours per week to 43 hours per week. Ridership is now running at approximately 1300 rides per week.
Figure 7: Relative Perceptions of the Four Modes of Transportation in Schaumburg, IL. (The arrows indicate relative importance of the dimensions. Availability is modeled as a constraint affecting behavior directly.)

It is interesting to note that had we been willing to accept the model's predictions and diagnostic information prior to predictive testing, many of the managerial recommendations could have been made after the 11th week rather than after the 30th week. Furthermore, the timing and magnitude of publicity and direct mail could have been selected to achieve desired levels of growth in service. (See Horsky [12] for optimization procedures to select a time stream for advertising expenditures.)

Finally, the predictive success of the simple model (Model 2) encourages us to develop expanded models that include additional phenomena of interest. In particular, the dynamic model in figure 1 appears to be a useful framework for incorporating explicit submodels linking operating strategies to objectively measured system performance and linking objectively measured system performance to perceived system performance. Such an expanded model should provide a useful planning tool for future applications in other communities.

9. DISCUSSION

One of our research goals was to critically evaluate the practicality of the semi-Markov methodology and the empirical reasonableness of the consumer model in figures 1, 2, and 3.
Feasibility

No major difficulties were encountered in implementing the semi-Markov methodology. The required sample sizes per period are reasonable for many consumer goods. The estimation and forecasting were straightforward tasks requiring only existing regression and eigenstructure computer software. The dependent variables are measurable. The bias in recall awareness appears correctable.

Paramorphism

The predictive test was constructed to minimize the likelihood that good results could be spurious. Good results are not guaranteed. Models can be constructed which produce ridership forecasts which are not correlated with actual ridership. (Correlations are insignificant for Model 5 in table 10). The selected model and its predictions have good face validity. For example, the rapid rises in awareness and ridership all appear explainable by either direct mail or publicity. The declines appear explainable by consumers trying DART but only some consumers continuing to ride DART.

Managerial Utility

The model helped a manager make a better decision, but its managerial utility could be improved through evolution. In particular, conjoint models linking physical characteristics to perceptions would enable the manager to optimize over marketing strategies and operating decisions rather than simply over direct mail and publicity. We view this evolutionary capability of the model as one of its strengths.

Aggregation Issues

The macro-flow assumption does not appear to greatly impair the model's predictive ability in this application. However, information is lost in aggregation of the explanatory variables. Even though publicity, direct mail and word of mouth do not correlate with average perceptions, they have small, but statistically significant, correlations at the level of the individual consumer. For example, self-reported receipt of newspaper publicity has a .11 correlation with 'convenience' and a .12 correlation with 'ease of use' at the disaggregate level. Both are significant at the .10 level. A fully disaggregate model, which is only possible with full maximum-likelihood computer software, could conceivably increase the diagnostic power of the consumer model. Another benefit would be greater efficiency allowing potentially smaller estimation samples.
Insignificant Variables

Word of mouth, budget allocation, and inertia were not significant in Model 1. There are a number of possible explanations including the following. DART has a small market share and hence the impact of word of mouth may be too small to measure relative to the much larger impacts of the marketing variables. This is not necessarily true for other innovations where word of mouth has proven significant (Bass [1], Rogers and Shoemaker [17]). Budget allocations are small for transportation and were only measured with a three-level categorial variable. This important economic variable may be significant with improved measures. Preference inertia has been identified for health care which requires a major commitment by the consumer (Neslin [14]), it may be insignificant for DART due to the low commitment required to try DART. It is likely that these and other variables could prove significant in other applications of the model.

10. CONCLUSIONS

We are encouraged by the practicality and predictive accuracy of the simple consumer model. Any model requires tradeoffs. We feel that the advantages of the semi-Markov methodology justify its limitations for many marketing science applications. For example, in a recent application Lange [14] used a continuous time Markov model similar to the right box of figure 1 to analyze consumer purchasing behavior for ground coffee. Using Universal Product Code (UPC) technology Lange was able to observe consumer purchases (the dependent variables) at automated supermarket checkout stations and to observe the purchase environment with respect to price, price cuts, store promotions, consumer's inventory, and whether the brand was in or out of stock (the explanatory variables). Lange obtained a lower $R^2$ (.304) and predictive accuracy was not quite as good as figure 5, but he did accurately forecast many of the inflection points in the data. His analyses show promise for future model development with products other than transportation services. We also posit that his application would be improved with the addition of survey measures of perception and preference.
APPENDIX 1

TECHNICAL DETAILS ON THE CONSUMER MODEL

The following equations briefly summarize the main results from Hauser and Wisniewski [11].

**Estimation**

Define $S_i(T_n) = 1$ if the consumer is in state $i$ at time $T_n$ and $S_i(T_n) = 0$ otherwise. The statistic we use to describe the process is the probability, $P_{ij}(t_n)$, that the consumer is in state $S_j$ at time $T_n$ given that he started in state $S_i$ at time $T_{n-1}$. I.e.,

$$P_{ij}(t_n) = \text{Prob} \{ S_j(T_n) = 1 \mid S_i(T_{n-1}) = 1 \} \quad (A-1)$$

where $t_n = T_n - T_{n-1}$. Any data collection procedure that provides observations on $S_j(T_n)$ and $S_i(T_{n-1})$ can be used to implement the model.

Let $a_{ijn}$ for $j \neq i$ be the flow rate, i.e., the probability that a consumer flows from $S_i$ to $S_j$ in time $\Delta t$ is $a_{ijn} \Delta t$. Define $a_{inn} = \sum_j a_{ijn}$. Define the matrices $P_n(t_n) = \{p_{ij}(t_n)\}$ and $A_n = \{a_{ijn}\}$. Then

$$P_n(t_n) = \exp(A_n t_n) = \sum_{r=0}^{\infty} A_n^r t_n^r / r! \quad (A-2)$$

which is a highly non-linear system of equations. If $\Theta_n$ is a matrix with the eigenvalues of $A_n$ on the diagonal and zeros elsewhere and $E_n$ is the matrix of eigenvectors then $P_n(t_n)$ can be obtained from $A_n$ by $P_n(t_n) = E_n[\exp(\Theta_n t_n)]E_n^{-1}$ where the $\exp(\cdot)$ operation applies separately to each eigenvalue.

Let $x_{ijn}$ be the value that the $\ell$th explanatory variable takes on for the $i \to j$ flow in period $n$. Let $w_\ell$ be the (unknown) importance weight of the $\ell$th explanatory variable. Assume

$$a_{ijn} = w_\ell x_{ijn} \quad (A-3)$$

We also define variables $x_{iijn}$ to carry information about flows from $S_i$ to $S_i$. Let $a_{inn} = \sum_\ell w_\ell x_{iijn}$. Define the matrix $X_n = \{x_{ijn}\}$.

In the macro-flow version of the theory we observe the number of consumers, $C_{ijn}$, who are in state $S_i$ at $T_{n-1}$ and in state $S_j$ at $T_n$. The log-likelihood function, $L$, is then:

$$L = \sum_{i,j} \sum_{n} C_{ijn} \log(\exp(\sum_\ell w_\ell x_{ijn} t_n))_{ij} \quad (A-4)$$
where the notation, \( \{M\}_{ij} \), indicates the i-jth element of matrix M.

The main estimation result is that an approximation to the maximum likelihood estimators of the \( w_s \)s can be obtained by solving the following regression equation:

\[
\tilde{E}_n [\log \tilde{\Lambda}_n] \tilde{E}_n^{-1} = \Sigma \tilde{w}_x \tilde{n}^t 
\]

for all \( n \) \hspace{1cm} (A-5)

where \( \tilde{E}_n \) is the matrix of eigenvectors of \( \tilde{P}_n(t_n) \) and \( \tilde{[\log \tilde{\Lambda}_n]} \) is a matrix with logarithms of the eigenvalues on the principal diagonal and zeros elsewhere. 

\( \tilde{P}_n(t_n) \) is the matrix of \( \{C_{ijn}/C_{in}\} \), i.e., the observed percentages of consumers who are in state \( S_j \) at \( T_n \) given they were in state \( S_i \) at \( T_{n-1} \). This is the regression equation described in equation 2. Note that since the diagonal elements of both sides of equation A-5 are functions of the non-diagonal elements, the diagonal equations are deleted from the estimation.

Forecasting Equations

Once we estimate the importance weights, \( \hat{w}_x \), we can use equation A-3 to estimate the flow rates, \( \hat{a}_{ijn} \), for future periods. This assumes of course we can forecast the explanatory variables, \( x_{ijzn} \), for those periods. The estimated flow rates then determine completely the probabilistic system, via equation A-2.

Cumulative statistics. Cumulative awareness, cumulative trial, and other cumulative statistics are simply the total percentage of consumers who flow into a state, say state \( S_j \), by time \( T_n \). We call this statistic penetration.

If we define \( \tilde{l}A_{n} \) such that \( \tilde{l}a_{ink} = a_{ink} \) for \( i \neq j \) and \( \tilde{l}a_{jk} = 0 \) then penetration is given by:

\[
\text{penetration (into state } S_j) = \Sigma \tilde{l}p_{i}(T_{n-1})\tilde{l}p_{ijn}(t_n) 
\]

where 

\[
\tilde{l}p_{i}(t_n) = \exp (\tilde{l}A_{n}t_n)
\]

and \( \tilde{l}p_{i}(T_{n-1}) \) is the probability that the consumer is in state \( S_i \) at time \( T_{n-1} \). Equation A-6 is used recursively when calculating penetration over more than one observation period.

Sales. Expected sales and the variance of sales are computed via moment generating functions. Expected sales are given by:

\[
\text{Expected sales} = \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma (T_{n-1})p_{ikn}(t_n)a_{kijn}t_n
\]
where $\sum_k^D$ means we use $a_{ijn}^0$ in the sum. The equation for the variance of sales is given in Hauser and Wisniewski [11].

**Equilibrium statistics.** We calculate equilibrium statistics by letting $x_{ijn}$ tend to its long run ($t \to \infty$) value, $x_{ij}^L$. We use equation A-3 to calculate the long run flow rates, $a_{ij}$ and equilibrium sales are given by:

$$\text{Expected equilibrium sales rate} = \sum_k^D \pi_k^0 a_{kj}^0 = \pi_j^0 (a_{jj}^0 - a_{jj}) \quad (A-8)$$

where the equilibrium $\pi_j$ are determined by solving the matrix equation $\pi^T A = 0$ subject to $\sum_k \pi_k = 1$.

**Simulation Results**

Simulation analyses determined whether the model could recover known data under cases of varying sample size and time periods. The key results are (1) that the sample size per time period should be greater than $20 \times$ (number of non-zero flows), and (2) that the length of the observation period should be short relative to the time it takes the process to reach equilibrium.
APPENDIX 2

Information Brochure Mailed to Schaumburg Residents

Village of Schaumburg

Regional Transportation Authority

U.S. Department of Transportation

For Schaumburg

Dial-a-Ride Transportation

DART

Call 255-4700
Public Transportation takes a
Major Step Forward in Schaumburg
with
din-a-ride transportation
dart
Sponsored by The Village of Schaumburg
and the Regional Transportation Authority
(RTA), dart offers door to door public
transportation that is as near as your
telephone. Just call 255-4700 and a two-way
radio equipped bus will be routed to pick
you up at or near your door and take you
anywhere in Schaumburg while picking up
and dropping passengers along the way.
Fares are low on dart. For $1.00 you can
call to anywhere in Schaumburg and with
a low cost transfer use RTA buses to many
driving regions of the Northwest Suburbs to
Chicago.

HOW TO USE dart
Call dart at 255-4700 and tell the dispatcher
who answers:
• Where you are
• Where you wish to go
• The time you wish to leave or you wish
to arrive.
• Your name
• Your phone number—in case we must
call you.
• Number in your party

GIVE dart TIME TO SERVE
YOU BETTER
If you miss an appointment give
dart time to serve you better. It will norm-
ally take 30 minutes (sometimes more some-
times less) to route a bus to pick you up
and 30 minutes to make the average trip.
Therefore, give dart at least 60 minutes or
call dart in advance and we will reserve a
spot for you and call you a few minutes be-
fore we pick you up.

BE READY FOR dart
The dart dispatcher will tell you when the
bus will arrive. Please be ready to catch it a
few minutes before, in order to assure speedy
service for you and other passengers. The
bus will signal its arrival by blowing its
horn and will only wait 30 seconds for you
to appear.

RETURN TRIPS ON dart
If you know in advance when you want
to return, just tell the dart dispatcher and
he will schedule it. If not, just call 255-4700
when you are ready.

HOURS OF SERVICE
dart buses will begin picking up passengers
at 9:00 A.M. Monday thru Friday. Calls
will be accepted from 8:30 A.M. The last
passengers will be picked up at 5:30 P.M.
dart will not operate on Saturdays, Sundays
or Holidays.

FARES ON dart

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TRANSFERS
FROM dart TO RTA ROUTES
| Regular Fares | .10 |
| Reduced Fares* | .05 |

dart ACCEPTS RTA TRANSFERS
Upon payment of surcharge

| Regular | .20 |
| Reduced* | .15 |

• PERSONS ELIGIBLE FOR REDUCED
FARES
• Senior Citizens 65 or over with RTA
Special Users Card. (Can be obtained at
many locations)
• Handicapped Persons with RTA Special
Users card. (Applications can be obtained
by calling toll free (800) 572-7000)
• Students through grade 12—
to and from school.
• Children 7-11—Children under 7 with
an adult are free.
REFERENCES


