THE MEASUREMENT OF MONOPOLY POWER IN DYNAMIC MARKETS*

by

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ABSTRACT

In markets in which price and output are determined intertemporally, the standard Lerner index is a biased and sometimes misleading measure of actual or potential monopoly power. This paper shows how the Lerner index can be modified to provide a meaningful instantaneous measure of monopoly power applicable to dynamic markets, and discusses the aggregation of that instantaneous measure across time. The importance of accounting for intertemporal constraints in antitrust and related applications is illustrated by the analysis of four examples: an exhaustible resource, the "learning curve," costs of adjustment, and dynamic adjustment of demand. An analogous index of monopsony power applicable to dynamic markets is also suggested.

1. Introduction

The measurement and analysis of monopoly and monopsony power have been a prime concern of industrial organization, and are of obvious importance in the design and application of antitrust policy. Much of the recent literature has focused on the structural and behavioral determinants of monopoly and monopsony power, including the characteristics of costs and demand, and the ways in which firms in the market interact with each other. There has been less concern, however, with the development of a measure of monopoly (or monopsony) power. Instead, the Lerner index, first introduced in 1934, has for years been accepted as the standard measure of monopoly power, and is often used as a summary statistic in antitrust applications.[1]

The Lerner index is just the margin between price and marginal cost, i.e. $L = (P - MC)/P$. In a static market, $D = 1/\eta_f$, where η_f is the elasticity of demand facing the firm, so that the firm's elasticity of demand completely determines its monopoly power.[2] However, this need not be the case in a dynamic market. When price and output are determined intertemporally, the Lerner index need not equal the inverse of the firm's elasticity of demand, and neither L nor $1/\eta_f$ will necessarily provide a meaningful measure of monopoly power.

By a dynamic market, I mean one in which price and production are intertemporally determined. Examples include markets for exhaustible or renewable resources, markets in which supply is affected by a learning curve or by the presence of adjustment costs for quasi-fixed factor inputs, and markets in which firms' demands respond over time (as opposed to instantaneously) to changes in price. (Almost all real-world markets are dynamic.) In all of these examples, the Lerner index is insufficient (if not misleading) as a measure of monopoly power. In some cases (e.g. an exhaustible resource) it overstates the extent of monopoly power, and in other cases (e.g. when there are learning curve effects) it understates it. In every case it applies only to an instant of time, while the impact of monopoly power always applies to some interval of time.

As an extreme example of the inapplicability of the Lerner index, even as an instantaneous measure of monopoly power, consider the case of a monopolist producer of an exhaustible resource who faces an isoelastic demand curve and has zero extraction costs. It is well known that the price and production trajectories in this case are identical to those for a perfectly competitive market, i.e. the monopolist has absolutely no monopoly power.[3] The Lerner index, however, will equal 1 at every instant of time. As a second example, consider a monopolist who is starting up production using a technology in which the learning curve is important. Because current production reduces future production costs, current price can be below current marginal cost. Thus even though output will be less than that in a competitive market, the Lerner index can turn out to be negative.

Even if the Lerner index sufficed as an instantaneous measure, it would have little to tell us about the potential impact of monopoly power unless it were properly aggregated across time. Simply calculating "short-run" and "long-run" degrees of monopoly power (based on short-run and long-run demand curves) may not be very informative for two reasons. First, the short-run monopoly price will depend on more than the firm's short-run demand curve, so that the use of that demand curve to calculate a "short-run" degree of monopoly power is misleading. (Consider a firm whose demand is inelastic in the short run but more elastic in the long run as consumers' "habits" adjust or as new firms enter the market. If the firm optimizes, it will initially produce above the point at which marginal cost equals short-run marginal revenue, so that the short-run demand elasticity will overstate the firm's short-run monopoly power.) Second, the firm's gain and consumers' loss from monopoly power depends on the rate at which demand adjusts, which in turn depends on the firm's entire (optimal) price trajectory. Given that the objective is a prescriptive statistic that can be applied to antitrust and related problems, one therefore needs a measure that reflects the trajectory of monopoly power over time, weighted by the firm's revenues (and consumers' expenditures).[4]

In this paper I suggest a simple generalization of the Lerner Index that provides a meaningful instantaneous measure of monopoly power for markets in which price and production are intertemporally determined, and I discuss the aggregation of that measure across time. This generalization is quite straightforward, and simply involves incorporating any relevant "user costs" -- i.e. the sum of (discounted) future costs or benefits that result from current production decisions -- in marginal cost. Although the generalization is straightforward, its application to specific examples of dynamic markets -- natural resources, learning curve effects, adjustment costs, and dynamic demand functions -- provides useful insights into the ways in which intertemporal constraints on production and price affect monopoly power, and why quasi-static analyses (e.g. comparisons of short-run and long-run demand conditions) can be misleading. Much of this paper is therefore devoted to the series of examples listed above.

This paper ignores the general question of how oligopolistic firms interact with each other, even though that will often be a major determinant of the actual monopoly power that prevails in a market. In effect, I am concerned with the measurement of potential monopoly power, which may be much larger than actual monopoly power if firms compete aggressively. If that makes the scope of this paper seem limited, remember that measures of potential monopoly power often play an important role in the evaluation of mergers and acquisitions, the assessment of demages resulting from actual or potential collusive activity, and other aspects of antitrust.[5]

The next section of this paper shows how the Lerner index can be generalized to provide an instantaneous measure of monopoly power that properly accounts for intertemporal constraints on production and price, and discusses the aggregation of that instantaneous measure across time. The following four sections use that measure to examine how monopoly power is affected by resource depletion, the learning curve, costs of adjusting factor inputs, and dynamic adjustment of demand. Finally, I present and discuss a measure of monopsony power applicable to dynamic

markets. As one would expect, this measure is closely related to our measure of monopoly power.

2. A Measure of Monopoly Power

In this section we begin by putting aside the problem that monopoly power -- i.e. a firm's ability to earn "excess profits" -- only has operational meaning over a time interval, and we compare the price, output, and profit flows for a monopolist[6] to those for a competitive market at a particular time t, but recognizing that these variables are intertemporally determined. After adapting the Lerner Index so that it provides a meaningful instantaneous measure of monopoly power, we discuss the problem of aggregating across time.

A. An Instantaneous Measure

The Lerner index assumes static profit maximization, so that the firm produces where marginal revenue equals marginal cost. In a competitive market price will equal marginal cost, and the resulting output is "welfare maximizing," i.e. the sum of consumer plus producer surplus is a maximum. That is the basis for measuring monopoly power in terms of the extent to which price deviates from marginal cost.

In a dynamic setting, risk-neutral firms maximize the sum of their (expected) discounted profits.[7] In a competitive market, this results in the same output path as that which maximizes the sum of (expected) discounted consumer plus producer surplus. This, of course, does not imply that marginal revenue equals marginal cost at every instant, nor that price equals marginal cost under competition. An exhaustible resource market is the obvious textbook example, and as explained earlier, it illustrates the failure of the Lerner index as a measure of monopoly power,

However, the Lerner index can be rescued as an instantaneous measure of monopoly power by altering it as follows:

$$L^*(t) = (P_t - FMC_t)/P_t = 1 - (FMC_t/P_t) \qquad (1)$$

where FMC_t is the full marginal social cost at time t, evaluated at the monopoly output level. By this I mean that first, FMC_t includes any (positive or negative) "user costs" that result from the intertemporal nature of the firm's optimization problem. Second, those user costs are to be calculated under the assumption that the firm is a price taker (i.e. competitive), so that they reflect the change in the present value of future discounted consumer plus producer surplus resulting from an increment in current production. Third, as with simple marginal cost in the standard Lerner index, those user costs are to be evaluated at the monopoly output level. With FMC_t calculated in this way, $0 \leq L^*(t) < 1$ for all t, and $L^*(t) = 0$ in a perfectly competitive market.

The rationale for this measure can be made clear by going back to the example of an exhaustible resource market. We again assume that demand is fixed and iso-elastic, and marginal cost MC = 0, so that the competitive and monopoly output paths are identical. Figure 1 shows the average and marginal revenue curves, and output and price at a particular time t. Note that the monopolist output level is such that

$$MR = MC + \lambda_m = \lambda_m \tag{2}$$

where λ_m is the user cost to the monopolist of one extra unit of cumulative production, or equivalently, the value to the monopolist of the marginal in situ unit of reserves. In a competitive industry, on the other hand, the output level is such that

$$P = MC + \lambda_c = \lambda_c \tag{3}$$

where λ_c is competitive user cost (often referred to as "Hotelling rent").[8] In this case the competitive and monopoly output levels are the same, so that $\lambda_c = \lambda_m + MR$.

It is important to distinguish between the monopoly and competitive user costs, as they are derived from two different objective functions. The monopoly output path is that which maximizes the sum of discounted profits, and monopoly

user cost is the reduction in that sum resulting from one extra unit of cumulative production (i.e. one less unit of in situ reserves). The competitive output path is that which maximizes the sum of discounted consumer plus producer surplus (the latter is just aggregate profit), with competitive user cost the reduction in that sum from one extra unit of cumulative production. Competitive user cost exceeds monopoly user cost because consumer plus producer surplus exceeds the monopolist's profit.[9] In the case shown in Figure 1, the difference between the two user costs is just equal to the difference between average and marginal revenue, so that the output levels are the same.

Since a measure of monopoly power should make a comparison with competitive conditions, or equivalently (in the absence of externalities), with the social welfare maximum, it is competitive user cost that should be added to marginal cost in calculating FMC. In the case shown in Figure 1, price is then equal to FMC, so that $L^*(t) = 0$ for all t, as we would expect.

In reality extraction cost is rarely if ever zero, so that in general a monopolist would produce less than a competitive industry initially, but more later (i.e. the monopolist "overconserves").[10] We must then be careful when calculating competitive user cost. The value of competitive user cost at time t will depend on output at t, as well as the path of (expected) future output. When measuring monopoly power, competitive user cost must be calculated using the monopolist's output path, just as marginal cost is measured at the monopoly output level when calculating the Lerner index in a static model. In this way we obtain the full marginal social cost of producing the monopoly output.

This is illustrated in Figure 2. $\lambda_{c,m}$ is the value of competitive user cost at time t evaluated for the monopoly output path $\{Q^m(t)\}$, where time t is some point after production has commenced. In the Figure, $\lambda_{c,m}$ is shown smaller than $\lambda_{c,c}$, competitive user cost evaluated over $\{Q^c(t)\}$, because monopoly output is (initially) smaller, so that monopoly reserves at time t are larger. Monopoly output at each instant is at the point where marginal revenue equals marginal

cost plus monopoly user cost, λ_m. At that output the "excess" average profit attributable to monopoly power, EP, is price minus full marginal cost, FMC, where FMC = MC + $\lambda_{c,m}$. The instantaneous degree of monopoly power at time t is then given by:

$$L^*(t) = 1 - [MC + \lambda_{c,m}(t)]/P^m(t) \tag{4}$$

Observe from eqn. (4) and Figure 2 that the presence of a positive user cost reduces monopoly power. In other words, given a demand curve and some marginal extraction cost function, the firm's monopoly power will be lower than it would be if its reserves were limited so that its user cost were zero. (Because its total volume of sales is fixed, the monopolist is limited to choosing the allocation of those sales across time.) Use of the standard Lerner index would clearly overstate the firm's true degree of monopoly power. Of course one might ask how important user cost is, and to what extent one would be misled by applying the standard Lerner index. This question is explored in Section 3 of the paper.

There are other examples of intertemporal pricing and production where user cost is negative, and monopoly power is increased. Consider a market in which firms move down a "learning curve," i.e. as they produce, learning by doing reduces their average and marginal costs. As Spence (1981) has shown, the full marginal cost of current production is then less than current marginal production cost. The reason is that an incremental unit of current production reduces future production costs by moving the firm further down the learning curve, so that production of the unit brings a benefit (a negative user cost) that partly offsets its cost.

This is illustrated in Figure 3, where marginal cost MC_t is constant with respect to the instantaneous rate of output Q_t, but declines (from an initial value of MC_0, asymptotically to MC_∞) as cumulative output increases. Thus if the industry were competitive and there were no externalities (e.g. resulting from the diffusion of experience-based information across firms), price would be less than current marginal cost because of the negative user cost associated

with learning. Price and marginal cost would both fall over time, with price asymptotically approaching marginal cost as the industry matured, learning effects were exhausted, and user cost approached zero.

Similarly, for a monopolist, production will be at the point where $MR = MC_t + \lambda_m$, where λ_m is the (negative) user cost to the monopolist (i.e. the change in the sum of discounted future profits from an extra unit of cumulative production today). However, the user cost relevant to calculating the firm's "excess" average profit EP and its instantaneous degree of monopoly power is $\lambda_{c,m}$, the competitive user cost (i.e. the change in the sum of discounted future surplus from an extra unit of cumulative production), again evaluated for the monopolist's output path. (Figure 3 applies to a point in time after production has commenced; since the monopoly output is less than the competitive output, $\lambda_{c,m}$ is greater in magnitude than $\lambda_{c,c}$.) As Figure 3 shows, because $\lambda_{c,m} < 0$, the effect of learning is to increase the firm's monopoly power, and the use of the standard Lerner index would underestimate that monopoly power. The extent to which the standard Lerner index is biased is examined in Section 4.

In addition to examining resource depletion and the learning curve in detail, later sections of this paper also examine the effects of adjustments costs for a firm's factor inputs, and the effects of dynamic demand adjustment. As those sections show, eqn. (1), with $FMC_t = MC_t + \lambda_{c,m}(t)$, always provides a correct instantaneous measure of monopoly power. We now turn to the aggregation of that instantaneous measure across time.

B. Aggregation Across Time

An instantaneous measure of monopoly power is clearly insufficient as a statistic for antitrust and related analyses. A firm's instantaneous degree of monopoly power might be high initially, and then fall as consumer demand adjusts or as new firms enter the industry. Clearly a complete assessment of a firm's monopoly power requires a method of aggregating the instantaneous index across time.

Remember that neither $L^*(t)$ nor the Lerner index $L(t)$ provide information

about the magnitude of the loss to consumers from monopoly power. ($L^*(t)$ might be higher throughout time in industry A than in industry B, with the damages to consumers higher in B because of its size.) $L^*(t)$ simply measures the excess profit per unit due to monopoly power as a fraction of the price per unit, and so is unit-free. However, the (relative) impact of monopoly power can be approximated by multiplying $L^*(t)$ by expenditure at time t. This suggests the use of expenditure as a weighting variable when aggregating $L^*(t)$ across time.

Alternative weighting variables include quantity and price. Aside from the fact that in practical terms monopoly power is most important when large expenditures are involved, a quantity- or price-weighted index could be a misleading statistic. For example, consider a firm that price discriminates over time, e.g. a book publisher or the manufacturer of "designer" clothing. Initially the firm operates on the inelastic part of its demand curve, with price high and demand low. In this initial period, $L^*(t)$ is relatively large. $L^*(t)$ later falls as the low-elasticity consumers complete their purchases, and price is reduced, with quantity rising. Then a quantity-weighted index will show very little monopoly power (suggesting that a copyright has little value), while a price-weighted index will indicate a permanent ability to maintain price above marginal cost (as though a copyright never loses its value). An expenditure-weighted index, on the other hand, properly reflects the decline in monopoly power and the changing impact on consumers.

Weighting $L^*(t)$ by expenditure yields the following time-aggregated index of monopoly power:

$$I_m(t) = 1 - \frac{\int_t^\infty FMC(\tau)Q(\tau)e^{-r(\tau-t)}d\tau}{\int_t^\infty P(\tau)Q(\tau)e^{-r(\tau-t)}d\tau} \tag{5}$$

Here $FMC(\tau)$ is full marginal social cost at time τ, as discussed above, and future expenditures are discounted at the market interest rate r.[11] This index, of course, is itself time dependent. It describes the monopoly power of a firm looking into the future at a particular point in time, and as conditions change,

so will the firm's monopoly power. This is important in the context of antitrust policy; the time-aggregated degree of monopoly power might be high initially (at $t = 0$), so that antitrust action seems warranted, but such action will be of little value if it takes considerable time to implement, and $I_m(t)$ will fall rapidly over time. Finally, observe that $0 \leq I_m(t) < 1$, and $I_m(t) = 0$ only if $L^*(\tau) = 0$ for all $\tau \geq t$.

The index $I_m(t)$ has value as a summary statistic for antitrust and related applications. Furthermore, it provides a means of identifying those factors that are important determinants of monopoly power, and quantifying their effects. This is best demonstrated through some examples of markets in which the intertemporal aspect of production is important.

3. Exhaustible Resources

Let us return to the case of an exhaustible resource market. Besides illustrating how the index of monopoly power given by eqn. (5) is calculated, we wish to examine the way in which actual or potential monopoly power depends on user cost relative to price, and the extent to which the traditional Lerner index is a biased statistic for real-world resource markets.

The approach is to construct a simple model that can be solved analytically for the competitive and monopoly production rates, and that lets one relate monopoly power to user cost in a straightforward way. Such a model is given by the following assumptions: (i) the reserve base is fixed,[12] (ii) market demand is isoelastic, i.e. $q(p) = bp^{-\eta}$, with $\eta > 1$, and (iii) marginal and average extracion cost are constant with respect to the rate of extraction, but an isoelastic function of reserves, with an elasticity equal to the inverse of the demand elasticity, i.e. $MC = AC = cR^{-1/\eta}$.[13] To deal with problems of common access and aggregation in the competitive case, we will assume that individual firms own shares of a unitized resource stock, or equivalently that firms own their own reserves, but with identical costs, so that competitive production decisions lead to identical output paths for individual firms, and the competi-

tive equilibrium is socially optimal.[14]

Aside from analytical tractability, an important advantage of this model is that it helps to clarify the connection between monopoly power and user cost. In particular, as shown below, in this model L^* and L are functions only of η and ρ, where ρ is the ratio of competitive user cost to competitive price. Also, observe that price and cost rise asymptotically in this model as the reserve base dwindles, but reserves are never exhausted. When c = 0 we have the special case in which the competitive and monopoly output paths are identical, but as c increases, the two will diverge.

The competitive price and production paths are given by the solution of:[15]

$$\max_q \int_0^\infty [\int_0^q p(\nu)d\nu - C(q,R)]\, e^{-rt}dt \tag{6}$$

subject to $$dR/dt = -q, \quad R(0) = R_0 \tag{7}$$

For a monopolist, price and production are given by the solution of:

$$\max_q \int_0^\infty [\, p(q)q - C(q,R)\,]\, e^{-rt}dt \quad , \tag{8}$$

also subject to (7). I have shown elsewhere (1983) that with $p(q) = (q/b)^{-1/\eta}$ and $C(q,R) = cR^{-1/\eta}q$, the <u>competitive</u> rate of production is given by the rule:

$$q_c(R) = b\,[(\tfrac{\eta-1}{\eta})\,A_c + c\,]^{-\eta}R = \phi_c R \tag{9}$$

where $A_c = A_c(\eta,c,r,b)$ is the solution to:

$$(\tfrac{\eta-1}{\eta})\,A_c^{\eta/(\eta-1)} + cA_c^{1/(\eta-1)} = [\,b/(\eta-1)r]^{1/(\eta-1)} \tag{10}$$

Also, the <u>monopoly</u> rate of production is given by:

$$q_m(R) = b\,(\tfrac{\eta-1}{\eta})^{\eta}[(\tfrac{\eta-1}{\eta})\,A_m + c\,]^{-\eta}R = \phi_m R \tag{11}$$

where $A_m = A_m(\eta,c,r,b)$ is the solution to:[16]

$$\left(\frac{\eta-1}{\eta}\right)A_m^{\eta/(\eta-1)} + cA_m^{1/(\eta-1)} = (\eta-1)\left[b/\eta^{\eta}r\right]^{1/(\eta-1)} \tag{12}$$

the reader can check that as $c \to 0$,

$$q_c(R)\,,\ q_m(R) \to \eta r R \tag{13}$$

and that for any given R, $q_c(R) > q_m(R)$ for $c > 0$.

Calculating the degree of monopoly power requires an expression for competitive user cost. As shown in my earlier paper, that user cost is given by:[17]

$$\lambda_c = \left(\frac{\eta-1}{\eta}\right) A_c R^{-1/\eta} \tag{14}$$

Recall that this user cost is to be evaluated over the monopoly output path, and therefore the monopoly path for reserves. The latter is given by $R_m(t) = R_0 e^{-\phi_m t}$, so that $\lambda_{c,m}(t)$ is obtained by substituting this into (14).

We can now compute the instantaneous degree of monopoly power. Substituting (14) into eqn. (4) and noting that the monopoly price is $p_m = (q_m/b)^{-1/\eta} = (\phi_m R/b)^{-1/\eta}$, we have:

$$\begin{aligned} L^*(t) &= 1 - (\phi_m/b)^{1/\eta}\left[c + A_c(\eta-1)/\eta\right] \\ &= 1 - \left(\frac{\eta-1}{\eta}\right)\left[\frac{c + A_c(\eta-1)/\eta}{c + A_m(\eta-1)/\eta}\right] \end{aligned} \tag{15}$$

Observe that in this model, $L^*(t)$ is constant over time, so that $I_m = L^*$. Also, the reader can check first, that $L^* = 0$ when $c = 0$, and second, that $A_c\,, A_m \to 0$ when $r \to \infty$, so that $\lambda_c \to 0$ and $L^* \to L$, where L is the standard Lerner index, and is given by $L = 1 - c(\phi_m/b)^{1/\eta}$. Observe that the standard index is biased by an amount

$$\Delta = L - L^* = \frac{A_c(\eta-1)^2/\eta^2}{c + A_m(\eta-1)/\eta} \tag{16}$$

Note that $\Delta \to 1$ as $c \to 0$.

Clearly the bias in the standard Lerner index depends on the relative importance of user cost. To explore this dependence, it is useful to define the parameter $\rho = |\lambda_{c,c}|/p_c$, i.e. the ratio of competitive user cost to competitive price. Then $0 < \rho < 1$, with $\rho = 0$ if reserves are infinite (i.e. no resource constraint), and $\rho = 1$ when $c = 0$. Observe that in this model p and $\lambda_{c,c}$ are both proportional to $R^{-1/\eta}$, so that ρ is constant and independent of R. Substituting for $\lambda_{c,c}$ and p_c,

$$\rho = \frac{A_c(\eta-1)/\eta}{c + A_c(\eta-1)/\eta} \tag{17}$$

Numerical solutions of the model can now be utilized to examine the dependence of monopoly power on user cost, and the extent of the bias in the standard Lerner index. The model is particularly convenient for this because L^*, L, and therefore Δ <u>are functions only of ρ and η</u>.[18] In other words, once ρ and η have been specified, it is unnecessary to specify c, b, or r. (Given ρ and η, c is determined given b/r.) We therefore want to examine how L^*, L, and Δ vary as ρ increases from zero to one. This dependence is shown in Table 1 and Figures 4A-4E for η = 1.5, 3, 5, and 10.[19]

For many exhaustible resource markets, values of ρ in the vicinity of .1 to .4 are not unreasonable.[20] Observe that for this range of values, the percentage bias in the standard Lerner index becomes quite pronounced, especially when demand is elastic. For example, when $\eta = 5$, L is in the range .23 to .44, but L^* is in the range of only .15 to .06. This suggests that the intertemporal constraints imposed by resource depletion can be quantitatively important determinants of actual or potential monopoly power.

TABLE 1 - EXHAUSTIBLE RESOURCE

A. $\eta = 1.5$

rho	L*	L	delta
0.01	0.6640	0.6673	0.0034
0.05	0.6527	0.6701	0.0174
0.10	0.6377	0.6739	0.0362
0.15	0.6215	0.6783	0.0568
0.20	0.6041	0.6833	0.0792
0.25	0.5852	0.6889	0.1037
0.30	0.5648	0.6954	0.1306
0.35	0.5427	0.7027	0.1601
0.40	0.5187	0.7112	0.1925
0.45	0.4925	0.7209	0.2284
0.50	0.4641	0.7321	0.2679
0.55	0.4331	0.7449	0.3118
0.60	0.3994	0.7598	0.3604
0.65	0.3626	0.7769	0.4143
0.70	0.3226	0.7968	0.4742
0.75	0.2789	0.8197	0.5408
0.80	0.2315	0.8463	0.6148
0.85	0.1801	0.8770	0.6969
0.90	0.1244	0.9124	0.7880
0.95	0.0644	0.9532	0.8888
1.00	0.0000	1.0000	1.0000

B. $\eta = 3.0$

rho	L*	L	delta
0.01	0.3286	0.3354	0.0067
0.05	0.3098	0.3443	0.0345
0.10	0.2862	0.3576	0.0714
0.15	0.2628	0.3734	0.1106
0.20	0.2399	0.3919	0.1520
0.25	0.2176	0.4132	0.1956
0.30	0.1961	0.4372	0.2412
0.35	0.1755	0.4641	0.2886
0.40	0.1560	0.4936	0.3376
0.45	0.1377	0.5257	0.3881
0.50	0.1204	0.5602	0.4398
0.55	0.1042	0.5969	0.4927
0.60	0.0891	0.6356	0.5465
0.65	0.0750	0.6763	0.6012
0.70	0.0619	0.7186	0.6567
0.75	0.0497	0.7624	0.7127
0.80	0.0383	0.8077	0.7693
0.85	0.0277	0.8542	0.8264
0.90	0.0178	0.9018	0.8839
0.95	0.0086	0.9504	0.9418
1.00	-.0000	1.0000	1.0000

C. $\eta = 5.0$

rho	L*	L	delta
0.01	0.1947	0.2027	0.0081
0.05	0.1741	0.2154	0.0413
0.10	0.1504	0.2354	0.0850
0.15	0.1294	0.2600	0.1306
0.20	0.1110	0.2888	0.1778
0.25	0.0951	0.3213	0.2262
0.30	0.0814	0.3570	0.2756
0.35	0.0696	0.3952	0.3256
0.40	0.0594	0.4356	0.3763
0.45	0.0505	0.4778	0.4273
0.50	0.0428	0.5214	0.4786
0.55	0.0360	0.5662	0.5302
0.60	0.0301	0.6120	0.5820
0.65	0.0248	0.6587	0.6339
0.70	0.0201	0.7060	0.6859
0.75	0.0158	0.7540	0.7381
0.80	0.0120	0.8024	0.7904
0.85	0.0086	0.8513	0.8427
0.90	0.0055	0.9005	0.8951
0.95	0.0026	0.9501	0.9475
1.00	0.0000	1.0000	1.0000

D. $\eta = 10.0$

rho	L*	L	delta
0.01	0.0943	0.1033	0.0091
0.05	0.0745	0.1207	0.0463
0.10	0.0561	0.1505	0.0944
0.15	0.0433	0.1868	0.1435
0.20	0.0341	0.2273	0.1932
0.25	0.0274	0.2705	0.2432
0.30	0.0222	0.3156	0.2933
0.35	0.0183	0.3619	0.3436
0.40	0.0151	0.4091	0.3939
0.45	0.0126	0.4569	0.4444
0.50	0.0104	0.5052	0.4948
0.55	0.0086	0.5539	0.5452
0.60	0.0071	0.6028	0.5957
0.65	0.0058	0.6520	0.6462
0.70	0.0046	0.7014	0.6968
0.75	0.0036	0.7509	0.7473
0.80	0.0027	0.8005	0.7978
0.85	0.0019	0.8503	0.8484
0.90	0.0012	0.9001	0.8989
0.95	0.0006	0.9500	0.9495
1.00	-.0000	1.0000	1.0000

4. The Learning Curve

The presence of learning by doing will also cause the standard Lerner index to be a biased measure of monopoly power, but now the bias is in the opposite direction. As explained earlier, user cost is negative, so that a monopolist produces at a point where marginal revenue is below marginal cost.

In specifying a model of learning by doing, assumptions about the diffusion of learning across firms and the strategic interaction of firms are particularly important. For example, Fudenberg and Tirole (1983) have shown that both the level and rate of change of output for oligopolistic firms will depend critically on the way in which they operate strategically. In particular, output is higher if firms follow "closed-loop" strategies (i.e. change their output patterns over time in response to unanticipated changes in their competitors' outputs) than will be the case if firms assume their rivals are pre-committed to fixed output paths. Also, in the first case the diffusion of learning across firms will decrease output, but can increase it in the second case.

At issue, then, is what to compare the monopolist's output to. I choose to compare the monopoly output path to that of a social planner, and it is the latter that I will refer to as "competitive." The reasons for this choice are as follows. First, as Fudenberg and Tirole have shown, if cost is not a sufficiently convex function of instantaneous output, a competitive (i.e. price-taking) equilibrium will not exist. Second, in an oligopolistic context, the strategic interactions of firms can be described by a number of reasonable alternative behavioral modes, and it is not clear how much rationality and sophistication to assign to firms in real world markets. (Economists find closed-loop strategies difficult if not impossible to calculate, and there is little reason to expect that firms are any better at calculating them.) Finally, my interest is a comparison of the monopoly output path with the socially optimal path. Depending on the nature of firms' strategic interactions, the monopoly equilibrium may be better than the "second-best" oligopoly equilibrium,[21] but the issue here is how

monopoly compares to a first-best equilibrium, and most important, how that comparison depends on the intertemporal constraints imposed by the learning curve.

I therefore specify a model that is exactly analogous to the exhaustible resource model developed in the last section. Demand is isoelastic, and marginal cost is an isoelastic function of cumulative production, so that the competitive and monopoly production rates are linear functions of cumulative production, and L^* and L are both functions of only η and ρ. This helps to illustrate the dependence of monopoly power on user cost, and the close relationship between the effects of resource depletion and learning by doing.

As before, demand is given by $q = bp^{-\eta}$. Marginal and average cost are equal and given by $MC = AC = c(a + x)^{-1/\eta}$, where $a > 0$ and x is cumulative production:

$$x(t) = \int_0^t q(\tau)d\tau \tag{18}$$

For simplicity, we write $MC = cx^{-1/\eta}$, and set $x(0) = 1$. As shown in the Appendix, this model has a solution only if c, b/r, and η satisfy the following constraint:

$$c \geqslant \left(\frac{\eta}{\eta - 1}\right)^{(\eta-1)/\eta}\left(\frac{b}{r}\right)^{1/\eta} \tag{19}$$

This constraint implies an upper limit on the magnitude of user cost relative to price. If it is not met, the present discounted value of the flow of net surplus is infinite.

In the Appendix it is shown that the competitive rate of production is given by the rule:

$$q_c(x) = b\left[-\left(\frac{\eta-1}{\eta}\right)B_c + c\right]^{-\eta}x = \theta_c x \tag{20}$$

where $B_c = B_c(\eta,c,r,b)$ is the solution to:

$$-\left(\frac{\eta-1}{\eta}\right)B_c^{\eta/(\eta-1)} + cB_c^{1/(\eta-1)} = [b/(\eta-1)r]^{1/(\eta-1)} \tag{21}$$

and competitive user cost is given by:

$$\lambda_c = -\left(\frac{\eta-1}{\eta}\right)B_c x^{-1/\eta} \tag{22}$$

Also, the monopoly rate of production is given by:

$$q_m(x) = b\ \left(\frac{\eta-1}{\eta}\right)^{\eta}\left[-\left(\frac{\eta-1}{\eta}\right)B_m + c\right]^{-\eta} x = \theta_m x \tag{23}$$

where $B_m = B_m(\eta,c,r,b)$ is the solution to:

$$-\left(\frac{\eta-1}{\eta}\right)B_m^{\ \eta/(\eta-1)} + cB_m^{\ 1/(\eta-1)} = (\eta-1)[\,b/\eta^{\eta}r\,]^{1/(\eta-1)} \tag{24}$$

The close connection between this model and the previous one for an exhaustible resource should be clear. The monopoly and competitive prices of the exhaustible resource rise exponentially as both marginal extraction cost and user cost rise, and production asymptotically approaches zero, while in this example monopoly and competitive production rise exponentially as production cost, user cost, and price all approach zero.

Using the solutions above for production and user cost, our instantaneous measure of monopoly power can be written as:

$$L^*(t) = 1 - \left(\frac{\eta-1}{\eta}\right)\left[\frac{c - B_c(\eta-1)/\eta}{c - B_m(\eta-1)/\eta}\right] \tag{25}$$

Again, $L^*(t)$ is constant over time, so that $I_m = L^*$. Also, $L = 1 - (\theta_m/b)^{1/\eta}c$, so the bias in the standard Lerner index is

$$\Delta = L - L^* = \frac{-B_c(\eta-1)^2/\eta^2}{c - B_m(\eta-1)/\eta} \tag{26}$$

i.e. the standard Lerner index <u>underestimates</u> the true degree of monopoly power. As with an exhaustible resource, as $r \to \infty$, $\lambda_c \to 0$ and $L^* \to L$.

We will again use the index $\rho = |\lambda_{c,c}|/p_c$ to measure the relative importance of user cost, and thus the relative importance of the learning curve effect. In this model, ρ is given by:

$$\rho = \frac{B_c(\eta-1)/\eta}{c - B_c(\eta-1)/\eta} \tag{27}$$

The constraint of eqn. (19) implies a corresponding constraint on ρ; the model has a solution only if $\rho \leq 1/(\eta-1)$.

Once again we would like to examine how the degree of monopoly power L^* and the bias in the standard Lerner index vary with η and ρ. This dependence is illustrated numerically in Table 2 and Figure 5. As shown, the standard Lerner index underestimates the true degree of monopoly power, and the bias can be quite significant, especially for larger values of η. For example, when η is 3 or 5 and ρ is .20, the bias exceeds .10 in magnitude.[22]

Just as a positive user cost (an exhaustible resource) reduces a firm's monopoly power, a negative user cost (a learning curve) increases it. The reason is that $|\lambda_c| > |\lambda_m|$, so the incentives associated with learning push a competitive industry further down the market demand curve than they push a monopolist down its marginal revenue curve. Put another way, since the monopolist produces less anyway over the long run, he receives less benefit (in terms of reduced future costs) from increasing current output. Learning therefore leads a monopolist to initially increase output less than it does a competitive industry, and increases the spread between the monopoly and competitive prices. This is just the opposite of what happens with an exhaustible resource. There a monopolist sees a lower cost from the depletion of his reserves, and so has less incentive to conserve relative to a competitive industry; this reduces the spread between the monopoly and competitive prices.

TABLE 2 - LEARNING CURVE

A. $\eta = 1.5$

rho	L*	L	delta
0.00	0.6669	0.6666	-.0003
0.05	0.6797	0.6636	-.0160
0.10	0.6918	0.6610	-.0308
0.15	0.7031	0.6586	-.0445
0.20	0.7137	0.6565	-.0573
0.25	0.7236	0.6546	-.0691
0.30	0.7330	0.6529	-.0801
0.35	0.7417	0.6514	-.0904
0.40	0.7500	0.6500	-.1000
0.45	0.7578	0.6488	-.1090
0.50	0.7651	0.6477	-.1174
0.55	0.7721	0.6467	-.1254
0.60	0.7786	0.6458	-.1328
0.65	0.7849	0.6450	-.1398
0.70	0.7908	0.6443	-.1464
0.75	0.7964	0.6437	-.1527
0.80	0.8017	0.6431	-.1586
0.85	0.8068	0.6426	-.1642
0.90	0.8117	0.6421	-.1695
0.95	0.8163	0.6417	-.1745
1.00	0.8207	0.6414	-.1793
1.05	0.8249	0.6410	-.1839
1.10	0.8289	0.6407	-.1882
1.15	0.8328	0.6405	-.1923
1.20	0.8365	0.6402	-.1962
1.25	0.8400	0.6400	-.2000
1.30	0.8434	0.6398	-.2036
1.35	0.8467	0.6396	-.2070
1.40	0.8498	0.6395	-.2103
1.45	0.8528	0.6394	-.2134
1.50	0.8557	0.6393	-.2164
1.55	0.8585	0.6392	-.2193
1.60	0.8612	0.6391	-.2221
1.65	0.8638	0.6390	-.2248
1.70	0.8663	0.6390	-.2273
1.75	0.8687	0.6389	-.2298
1.80	0.8710	0.6389	-.2322
1.85	0.8733	0.6388	-.2344
1.90	0.8755	0.6388	-.2366
1.95	0.8776	0.6388	-.2387
2.00	0.8796	0.6388	-.2408

B. $\eta = 3.0$

rho	L*	L	delta
0.00	0.3338	0.3331	-.0007
0.05	0.3566	0.3245	-.0322
0.10	0.3795	0.3174	-.0621
0.15	0.4017	0.3119	-.0898
0.20	0.4231	0.3077	-.1154
0.25	0.4437	0.3046	-.1391
0.30	0.4634	0.3024	-.1610
0.35	0.4822	0.3009	-.1812
0.40	0.5000	0.3000	-.2000
0.45	0.5169	0.2995	-.2174
0.50	0.5329	0.2994	-.2335

C. $\eta = 5.0$

rho	L*	L	delta
0.00	0.2005	0.1997	-.0008
0.05	0.2276	0.1890	-.0386
0.10	0.2562	0.1818	-.0744
0.15	0.2848	0.1775	-.1073
0.20	0.3129	0.1755	-.1374
0.25	0.3400	0.1749	-.1650

5. Costs of Adjustment

A firm's long-run cost structure differs from that in the short run because it takes time to alter the capital stock and change the firm's production capacity. As Lucas (1967) and Gould (1968) have shown, one way to capture this is by assuming there are convex costs of adjustment associated with changes in the capital stock (and/or changes in other factor input levels). When adjustment costs are present, a firm will experience an (internal) capital gain or loss when it adjusts to a new long-run equilibrium position, and those capital gains (which occur as capacity is reduced) or losses (occurring as capacity is increased) are part of full marginal cost. As shown below, this reduces monopoly power during periods of industry expansion, and increases it during periods of contraction.

For simplicity, assume output is a function only of capital K, i.e. $q = F(K)$, with $F'(K) > 0$ and $F''(K) < 0$, so that there are diseconomies of scale. The capital stock is assumed to be "quasi-fixed," so that the purchase and installation of "usable" capital at a rate I involves a cost $vI + C(I)$, where v is the purchase price of a unit of capital, and $C(I)$ is the full adjustment cost, with $C(0) = 0$, $C'(I) >(<) 0$ for $I >(<) 0$, and $C''(I) > 0$. Here $C(I)$ includes the cost of installing the capital, training workers to use it, etc. Since this takes time, $C''(I) > 0$, i.e. it is more costly to increase capacity quickly than slowly.[23] Firms are assumed to maximize:

$$\max_{I(t)} \int_0^\infty [\, p(q)q - vI - C(I)\,] e^{-rt} dt \tag{28}$$

subject to

$$\dot{K} = I - \delta K \tag{29}$$

where δ is the depreciation rate, and a dot denotes a time derivative, i.e. $\dot{K} = dK/dt$. Note that competitive firms perform this maximization with $p(q) = \bar{p}$ taken as given.

It is easily shown that the optimal level of investment satisfies:

$$\dot{I} = \frac{1}{C''(I)}\{(r + \delta)[\, v + C'(I)\,] - MR \cdot F_K\} \tag{30}$$

where MR is marginal revenue. (In a competitive market, MR = AR = p.) The

behavior of investment and the capital stock in competitive and monopolistic markets is characterized by the phase diagram of Figure 6. In that figure, $\bar{K}_m^*$ and $\bar{K}_c^*$ are the steady-state equilibrium capital stocks in monopolistic and competitive markets. Because there are diseconomies of scale, in both markets $\dot{I} <(>) 0$ if $K(t) <(>) \bar{K}^*$.

In steady-state equilibrium,

$$MR \cdot F_K = (r+\delta)[v + C'(I)] \tag{31}$$

where the right-hand side of (31) is the cost of a marginal unit of capital. The marginal cost of an additional unit of output is then

$$MC = (r + \delta)[v + C'(I)]/F_K \tag{32}$$

In equilibrium, the use of this "direct" marginal cost in the standard Lerner index would give an unbiased measure of the degree of monopoly power. Furthermore, this "direct" marginal cost can itself be measured -- it is simply the <u>amortized capital outlay</u> required to increase production capacity by one unit.

In disequilibrium, however, <u>full</u> marginal cost is not equal to direct marginal cost. Equating full marginal cost with marginal revenue, observe from eqn. (30) that

$$FMC = MC - C''(I)\dot{I}/F_K \tag{33}$$

The value to the firm of the marginal unit of capital is its total purchase and installation cost, $v + C'(I)$, so that $C''(I)\dot{I}$ is the <u>rate of capital gain</u> on the unit, and $C''(I)\dot{I}/F_K$ is the corresponding capital gain in terms of a marginal unit of production capacity. If a firm is growing so that $K(t) < \bar{K}^*$, $\dot{I} < 0$ and $FMC > MC$. The reason is that as $K(t) \to \bar{K}^*$, the marginal profit rate is falling, so that the value to the firm of a marginal unit of capital is falling. This capital loss <u>raises</u> the full marginal cost of additional production capacity. Conversely, suppose the firms in a competitive market cartelize, agreeing to reduce their aggregate production capacity to a monopoly level (path ABC in Figure 6). Each incremental reduction in capacity raises the marginal profit rate and bestows a

capital gain; $\dot{I} > 0$ and FMC < MC.

In periods of adjustment, the standard Lerner index is therefore a biased measure of monopoly power. The actual degree of monopoly power will depend on whether output is growing or contracting, and cannot be determined simply from an elasticity of demand. The bias is positive during periods of industry expansion, and negative when industry output is contracting. The latter case is important, and the effect is often ignored in analyses of the potential monopoly power from collusion. If a cartel forms in an industry that had been competitive, its monopoly power will exceed the value that would be inferred from the market demand curve (even if that demand curve is static). The bias is largest in the early periods ($\dot{I} \to 0$ as $K \to \bar{K}_m^*$), but depending on the discount rate and the size of marginal adjustment costs $C'(I)$ relative to v, it may be sufficient to significantly affect the time-aggregated index of monopoly power, I_m.[24]

6. Dynamic Demand Functions

In most markets demand responds dynamically to price changes. From the point of view of a firm or group of firms with monopoly power, the response of demand can occur as consumers adjust their spending patterns, or as other (competitive) firms expand their production capacity. If monopoly power is to be exercised rationally, that dynamic response of demand must be taken into account.

The fact that demand is dynamic will not by itself cause the standard Lerner Index to be biased as an instantaneous measure of monopoly power. The reason is that the full marginal social cost of production is equal to direct marginal cost, i.e. competitive producers will produce so that price is equal to marginal cost at every instant (although price will be changing over time). As long as the optimal monopoly price is used to compute the Lerner Index (at each instant), it will not be biased.

A problem arises, however, when the short-run demand curve (or a short-run elasticity) is used to infer a short-run degree of monopoly power. Aside from the obvious problem of time aggregation, the short-run demand curve (taken by

itself) can be misleading as an indicator of a monopolist's short-run monopoly power. To see why, suppose the monopolist's demand curve is more elastic in the long run than the short run. Then it is optimal for the monopolist to initially set output above the point where marginal cost equals short-run marginal revenue; doing so creates a benefit by retarding the response of demand and the adjustment to long-run equilibrium. Just the opposite is the case if the monopolist's demand curve is less elastic in the long run.

The following simple model illustrates how the short-run demand curve can overstate a monopolist's short-run degree of monopoly power. Suppose a group of firms cartelize and gain monopoly power, but if they increase price, the output of a set of "competitive fringe" firms will gradually increase, so that cartel demand is more elastic in the long run that the short. Then the cartel's demand can be written as:

$$q(t) = a_0 - a_1 p(t) - u(t) \tag{34}$$

where u(t) is competitive supply, and itself depends on price as follows:[25]

$$\dot{u} = b_1 p(t) - b_2 u(t) \tag{35}$$

Here b_2 determines the speed at which competitive production responds to price, and the cartel's demand adjusts over time.

Given some initial (equilibrium) price p_0 and quantity q_0, and letting C(q) be the cartel's (aggregate) cost function, the cartel sets output to maximize:

$$\max_q \int_0^\infty [\, p(t)q(t) - C(q)\,] e^{-rt} dt \tag{36}$$

subject to (35). It is straightforward to show that the cartel's optimal output trajectory must satisfy:

$$[\, 2\alpha + C''(q)\,]\dot{q} = -(a_0 b_1 \alpha^2 + a_0 \alpha\beta) + 2\alpha\beta q + \beta C'(q) + \alpha(2\beta - r)u \tag{37}$$

where $\alpha = 1/a_1$ and $\beta = b_2 + r + b_1/a_1$. Then $\dot{q} <(>) 0$ if $q(t) >(<) \bar{q}^*$, where $\bar{q}^*$ is the steady-state equilibrium output. Thus if a cartel forms and cuts output, it does not "overshoot" by initially reducing output below the equilibrium level,

but instead reduces it only part of the way, and then slowly reduces it the rest of the way. The reason is that for the cartel there is a negative user cost associated with current production; an incremental unit of production retards expansion by the competitive fringe, and thereby increases future potential profits.[26]

This is illustrated by a phase diagram in Figure 7, and supply and demand curves in Figure 8, for the case $C(q) = cq$. In those figures, the market is initially competitive, and q is the total output of the colluding firms. Once the cartel forms, its optimal output drops to q_1, and then gradually falls to the long-run equilibrium level $\bar{q}^*$ as competitive output expands, and price falls from p_1 to $\bar{p}^*_m$ (path ABC in Figures 7 and 8).[27] Until long-run equilibrium is reached, the cartel's output is <u>above</u> the point where marginal cost equals short-run marginal revenue (again, because of the negative user cost associated with current production). Clearly the cartel's short-run average revenue curve taken by itself overstates the cartel's short-run degree of monopoly power.

The size of the cartel's degree of monopoly power, I_m, depends on how fast the instantaneous index $L^*(t)$ declines over time (as $q(t) \rightarrow \bar{q}^*$), and the time pattern of expenditure. An estimate of I_m therefore requires the calculation of the cartel's entire optimal production trajectory. Even if such a calculation is not feasible, an estimate of I_m based on rough "guessestimate" of q(t) may still be preferable to use of short-run and long-run demand curves.

In some markets, short-run demand is more elastic than long-run demand. This is the case when there is a "stock adjustment effect" -- for example, the product in question is a durable good (say copper or aluminum) with a source of secondary (or "scrap") supply. In this case a monopolist's short-run demand curve <u>understates</u> its true short-run degree of monopoly power. Now the monopolist's short-run output is below the point where marginal cost is equal to short-run marginal revenue, because there is a <u>positive</u> user cost associated with current production (an incremental unit of production retards the reduction of an initially large source of secondary supply from competitive fringe firms). The

implications of this might be easiest to see in the context of the 1945 Alcoa case; a defense of Alcoa based on the argument that its monopoly power was limited in at least the short run is flawed because it ignores the fact that the dynamic adjustment of demand gives the firm an incentive to reduce short-run production.[28]

7. A Measure of Monopsony Power

Monopsony power refers to the ability to purchase at a price below marginal value, or equivalently (since utility maximization implies purchasing up to the point that marginal value is equal to marginal expenditure), at a price below marginal expenditure. A static index of monopsony power exactly analogous to the Lerner index is therefore:

$$M = 1 - (P/ME) = 1 - (P/MV) \tag{38}$$

M is simply the percentage difference between marginal value and price, and in a static market is bounded by zero and one. Just as the Lerner Index compares the monopoly price to the marginal social cost of producing the monopoly output level, this index compares the marginal social value of the monopsony level of sales with the monopsony price.

In a dynamic market, however, M suffers from the same deficiences as the Lerner index. First, when price and quantity are determined intertemporally, a buyer will not necessarily equate current marginal value with current marginal expenditure, so that M may be biased evan as an instantaneous measure. For example, if a monopsonist's current consumption of a good results in a sufficiently large future flow of value (in addition to current value), the current price can exceed current marginal value, and even though less is purchased then in a competitive market, M will be negative. Second, the value of M at an instant of time says little about the potential impact of monopsony. As with monopoly power, a time-aggregated index is needed if it is to be useful as a prescriptive statistic for antitrust and regulatory policy.

The index M can be generalized in the same way that the Lerner index was. Consdier the following instantaneous measure of monopsony power:

$$M^*(t) = 1 - (P_t/FMV_t) \tag{39}$$

where FMV_t is the full marginal social value at time t, evaluated at the monopsony consumption level. As with full marginal cost, FMV_t includes any (positive or negative) "shadow values" that result from the intertemporal nature of the buyer's optimization problem. Also, those shadow values are to be calculated under the assumption that the buyer is a price taker, so that they reflect the change in the present value of future discounted net surplus resulting from an increment in current sales. Finally, the shadow values should be calculated at the monopsony level of sales. Calculated in this way, $M^*(t)$ is an unbiased instantaneous measure, and $0 \leq M^*(t) < 1$, with $M^*(t) = 0$ in a perfectly competitive market.

The following examples should help clarify the rationale for this measure. First, consider a mining firm that uses labor to produce an exhaustible resource. If the firm behaved as a competitive buyer of labor, and if its reserves of the resource were infinite, it would hire labor up to the point where the wage (its average expenditure) was equal to the net marginal revenue product of labor (its marginal value). If reserves are finite, however, each additional unit of labor hired has a negative shadow value associated with the resulting reduction in the future flow of net value because of depletion. As shown in Figure 9, the firm buys a quantity of labor $L_c(t)$ at the point where $AE = FMV_t = MV_t + \lambda_{c,c}(t)$, with $\lambda_{c,c}(t) < 0$, i.e. the effect of future depletion is to reduce the current quantity of labor that is hired. If the firm is a monopsonist in the labor market, it hires labor up to the point where $ME = FMV_t = MV_t + \lambda_m(t)$, $\lambda_m(t) < 0$.

As shown in Figure 9, $|\lambda_m| < |\lambda_{c,c}|$. Again, one must distinguish between the monopsony and competitive shadow values. The monopsony sales path is that which maximizes the sum of the discounted flow of net value to the buyer (in a static market, the area under the demand curve less the expenditure PQ), and the monopsony

shadow value is the increase in that sum resulting from one extra unit of cumulative sales. The competitive sales path is that which maximizes the sum of the discounted flow of total surplus (the area between the demand and supply curves in a static market), and the competitive shadow value is the increase in that sum from an extra unit of cumulative sales. The competitive shadow value is larger in magnitude than the monopsony shadow value because total surplus exceeds the monopsonist's net value.

Since a measure of monopsony power should make a comparison with competitive conditions, it is the competitive shadow value that should be used in calculating full marginal value. Furthermore, that shadow value must be calculated for the monopsony sales path, just as marginal value is measured at the monopsony sales level when calculating the index M in a static model. This yields the full marginal social value of sales at the monopsony level, i.e. $FMV_t = MV_t + \lambda_{c,m}(t)$, and the instantaneous degree of monopsony power is:

$$M^*(t) = 1 - P_m(t) / [MV_t + \lambda_{c,m}(t)] \tag{40}$$

In the exhaustible resource example illustrated in Figure 9, $\lambda_{c,m}(t) < 0$, so that the static index M would overestimate the degree of monopsony power. The exhaustibility of the firm's reserves reduces its monopsony power in the labor market, just as it reduces any monopoly power in the output market.

As a second example, consider a firm that buys a raw material ("steel"), and whose costs fall as it moves down a learning curve. In this case, each unit of steel the firm buys provides current marginal value, but also increases the future flow of value to the firm by accelerating its movement down the learning curve. Now there is a positive shadow value $\lambda_{c,m}(t)$ that must be included in calculating the firm's instantaneous degree of monopsony power in the steel market; the static index M will underestimate that monopsony power.

Once $M^*(t)$ has been calculated, it can be aggregated over time in a manner directly analogous to eqn. (5) for monopoly power. The expenditures P(t)Q(t) used as the weights in eqn. (5) represent the gross income flow to the monopolist. The

analogous gross value flow to the monopsonist is $FMV(t)Q(t)$. It is this, and not expenditures $P(t)Q(t)$ that should be used to weight $M^*(t)$ when aggregating over time. To see this, observe that if expenditures were used, the weights would be the lowest in periods when the degree of monopsony power $M^*(t)$ is the highest (and would approach zero if $M^*(t)$ approached one).[29] Then weighting $M^*(t)$ by $FMV(t)Q(t)$ and discounting, we obtain the following index of monopsony power:

$$I_s(t) = 1 - \frac{\int_t^\infty P(\tau)Q(\tau)e^{-r(\tau-t)}d\tau}{\int_t^\infty FMV(\tau)Q(\tau)e^{-r(\tau-t)}d\tau} \tag{41}$$

Observe that $0 \leqslant I_s(t) < 1$, and $I_s(t) = 0$ only if $M^*(\tau) = 0$ for all $\tau \geqslant t$. $I_s(t)$ is a summary statistics that measures the monopsony power of a buyer locking into the future at a particular point in time.

8. Concluding Remarks

This paper has shown how static measures of monopoly and monopsony power can be inadequate and possibly misleading when applied to dynamic markets. Such static measures include the standard Lerner index, but also short-run and long-run elasticities of demand. We have suggested alternative measures that properly account for intertemporal constraints, and would be applicable as summary statistics in antitrust and related regulatory problems. Also, the application of these measures to a number of examples has shown how a firm's (or cartel's) degree of monopoly power is affected by various forms of intertemporal constraints, including those associated with resource depletion, learning by doing, costs of adjustment, and dynamic adjustment of demand.

We have not attempted to suggest a statistical methodology for estimating the components of these measures of monopoly and monopsony power, and the estimation problems can indeed be formidable. For example, the estimation of user costs in a competitive market can be difficult enough, and here one needs competitive user

cost evaluated over the monopolist's output path, a quantity that is never directly observable.

On the other hand, even though user costs and shadow values may be difficult to estimate, we have seen that ignoring them altogether and simply inferring the degree of monopoly power from a short- or long-run demand curve can be seriously misleading. In some cases it may be preferable to assess monopoly power -- and the resulting damages to consumers -- by constructing dynamic market models to simulate competitive price and output trajectories. Alternatively, some methods do exist for at least roughly estimating the user costs and shadow values discussed in this paper and needed to calculate I_m and I_s.[30] The use of rough estimates may be better than no estimates, and is certainly better than simply ignoring the bias in a static index.

APPENDIX

Here we derive eqns. (20)-(24). The competitive production rule is that which maximizes:

$$\max_{q} \int_0^{\infty} \left[\int_0^{q} p(\nu)d\nu - C(q,x) \right] e^{-rt} dt \tag{A.1}$$

subject to

$$\dot{x} = q \ , \ x(0) = 1 \tag{A.2}$$

with $p(\nu) = b^{1/\eta}\nu^{-1/\eta}$ and $C(q,x) = cqx^{-1/\eta}$. The solution is obtained via dynamic programming. The value function $V(x)$ satisfies the following fundamental equation of optimality:

$$rV = \max_{q} \{\int_0^{q} p(\nu)d\nu - C(q,x) + qV_x\} \tag{A.3}$$

Substituting for $C(q,x)$ and maximizing:

$$- V_x = p(q) - cx^{-1/\eta} \tag{A.4}$$

so that production satisfies:

$$q^*(x) = p^{-1}[-V_x + cx^{-1/\eta}] = b[-V_x + cx^{-1/\eta}]^{-\eta} \tag{A.5}$$

Now substitute (A.5) for q in eqn. (A.3) and rearrange:

$$rV = \frac{b}{(\eta-1)} [- V_x + cx^{-1/\eta}]^{(1-\eta)} \tag{A.6}$$

It is easily seen that this equation has the solution

$$V(x) = B_c x^{(\eta-1)/\eta} \tag{A.7}$$

where B_c is a constant satisfying eqn. (21). Also $V_x = [(\eta-1)/\eta]B_c x^{-1/\eta}$ is the marginal value of an incremental unit of cumulative production, so that competitive user cost $\lambda_c = - V_x$, as in eqn. (22). Finally, substitute V_x into (A.5) to obtain eqn. (2) for $q_c(x)$.

Since $q^*(x) = \theta_c x$, where θ_c is a constant, $x(t) = e^{\theta_c t}$. Then (A.1) can be written as:

$$\max_q \int_0^\infty [\,(\frac{\eta}{\eta - 1})b^{1/\eta}\theta_c^{(\eta-1)/\eta} - c\theta_c\,]\,x^{(\eta-1)/\eta}e^{-rt}dt$$

$$= \int_0^\infty [\,(\frac{\eta}{\eta - 1})b^{1/\eta}\theta_c^{(\eta-1)/\eta} - c\theta_c\,]\,e^{[\gamma(\eta-1)/\eta - r]t}dt \qquad (A.8)$$

This integral becomes infinite if $c < (b/\theta_c)^{1/\eta}\eta/(\eta-1)$ and $\theta_c > \eta r/(\eta-1)$, so that eqn. (19) must hold for convergence.

The monopolist chooses $\{q_m(t)\}$ to maximize

$$\max_q \int_0^\infty [\,p(q)q - C(q,x)\,]\,e^{-rt}dt \qquad (A.9)$$

again subject to (A.2). Equations (23) and (24) are obtained by going through the same steps as in the competitive solution above.

FOOTNOTES

1. For example, Landes and Posner (1981) argue that "the Lerner index provides a precise economic definition of market power." Landes and Posner, and others, refer to "marker power," with "monopoly power" meaning "a high degree of market power." In this paper I use the words "monopoly power" to refer to the ability to profitably raise price above marginal cost, "monopsony power" to refer to the ability to buy at a price below marginal value, and "market power" to mean either monopoly or monopsony power.
2. Determining the elasticity of demand for an individual firm in an oligopolistic market is no easy matter, even in a static context. Under Cournot assumptions, that elasticity can be related to indices of concentration. See, for example, Encaoua and Jacquemin (1980) and Schmalensee (1982b).
3. See Stiglitz (1976).
4. Schmalensee (1982a) discusses the problems associated with aggregating the flow of social cost of monopoly power across time in dynamic markets. This social cost is usually taken to be the traditional "deadweight loss" triangle (see, e.g., Harberger (1954) and Landes and Posner (1981)), and perhaps in addition the resources spent to obtain the monopoly power (see Posner (1975)). In most antitrust applications, the total loss to consumers is the more relevant flow.
5. As Landes and Posner (1981) point out, a finding of successful or attempted monopolization in violation of the Sherman Act requires a demonstration that the defendant has significant monopoly power.
6. In what follows I take the firm's demand function as given. For simplicity, the firm can be viewed as a simple monopolist, and I will refer to its output as the "monopoly output," as opposed to the output that would prevail in a competitive market or socially planned economy.
7. This is what they do, that is, if managers serve the stockholders' interests.
8. In Figure 1, $\lambda_{c,c}$ refers to the competitive user cost evaluated at the competitive output level, and $\lambda_{c,m}$ refers to the competitive user cost evaluated at the monopoly output level. The two output levels are the same, so $\lambda_{c,c} = \lambda_{c,m} = \lambda_c$.

9. In this particular case competitive profit (i.e. producer surplus) and the monopolist's profit are both equal to competitive user cost (i.e. the value of the marginal in situ unit in a competitive market) times the production level.

10. See Stiglitz (1976). This need not always be the case. As Lewis, Matthews, and Burness (1979) have shown, if the elasticity of demand increases with consumption, or if there are quasi-fixed production costs (e.g. leasing fees), the monopoly production rate can initially exceed that for a competitive market.

11. Note that this can be written as

$$I_m(t) = [\int_t^\infty L^*(\tau)PQe^{-r(\tau-t)}d\tau]/[\int_t^\infty PQe^{-r(\tau-t)}d\tau] \quad ,$$

i.e. it is a weighted discounted sum over time of the instantaneous index L^*, where the weights are expenditures, PQ.

12. That is, we ignore the process of exploration and reserve accumulation. This is actually not a very restrictive assumption, if by "reserves" we mean the potential resource base (proved reserves plus potentially discoverable reserves), and if we include the cost of exploration and discovery as part of production cost.

13. The assumption of an isoelastic marginal cost function is widely accepted as a first approximation in petroleum engineering, and leads to the well-known "exponential decline curve." Constraining the elasticity to be $1/\eta$ may seem artificial, but it is necessary for an analytical solution, and it does not compromise the qualitative realism of the model. Also, note that as $\eta \to \infty$, $MC \to c$, a constant.

14. In some resource markets property rights are poorly defined or maintained, so that the common access problem arises. In such cases the competitive equilibrium discussed below should be viewed as an "ideal" and guideline for regulatory policy, and as a comparison to the behavior of a monopolist.

15. Since our assumptions allow us to ignore the common access problems, this is equivalent to each individual producer choosing his production rate $q_i(t)$ to maximize $\int_0^\infty [pq_i - C(q_i,R)]e^{-rt}dt$, where p is taken as exogenous in the maximization.

16. Equations (10) and (12) are of the form $F(z) = z^{k+1} + a_1 z^k - a_2 = 0$, a_1, a_2, $k > 0$, which has one positive solution.

17. Competitive user cost is the marginal social value of an incremental unit of reserves, i.e. $\lambda_c(t) = \partial V_s(t)/\partial R(t)$, where $V_s(t)$ is the present value of the flow of consumer plus producer surplus (from time t on) accruing from the extraction and consumption of reserves $R(t)$. In my earlier paper (1983) I derived q_c using dynamic programming, so that $V_s(t)$ is obtained explicitly.

18. To see this, first re-write eqn. (17) as:

$$c = \left(\frac{\eta - 1}{\eta}\right)\left(\frac{1 - \rho}{\rho}\right) A_c \qquad \text{(i)}$$

and substitute this for c in eqn. (15) for L^*:

$$L^* = 1 - \left(\frac{\eta - 1}{\eta}\right)\left[\frac{1}{(1 - \rho) + \rho A_m/A_c}\right] \qquad \text{(ii)}$$

Thus $L^* = L^*(\eta,\rho,A_m/A_c)$. Now, to show that A_m/A_c is a function only of η and ρ, divide eqn. (10) by eqn. (12), substitute (i) for c, and re-arrange:

$$\rho\left(\frac{A_m}{A_c}\right)^{\eta/(\eta-1)} + (1 - \rho)\left(\frac{A_m}{A_c}\right)^{1/(\eta-1)} = (\eta - 1)^{\eta/(\eta-1)} \qquad \text{(iii)}$$

This equation has a single positive solution $(A_m/A_c)^* = \psi(\eta,\rho)$. The reader can show that $L = L(\eta,\rho)$ as well.

19. In these calculations, $r = .05$ and $b = 1$, and c varies accordingly.

20. Pindyck (1978) has shown that if the world oil market were competitive, the ratio of user cost to price might exceed .4 if the real discount rate were .05, while for the world copper market this ratio is in the range of .2 to .3. In his study of the nickel market, Stollery (1983) finds this ratio to be about .1 to .2.

21. Fudenberg and Tirole shows that the oligopolistic equilibrium can be improved by taxing output in early periods and subsidizing it in later ones.

22. There have been a number of studies that estimate the rate at which cost declines as cumulative production increases; one of the earliest is by Hirsch (1952), and Sahal (1981) provides a more recent survey. Those studies do not, however,

provide estimates of user cost. To determine user cost one would also need information about the demand curves for the products in question.

23. For a general discussion of adjustment costs and their effects, see Nickell (1978). As in Nickell, I assume adjustment costs are a function of gross investment, so that the firm has some positive cost even when it is not expanding its capital stock.

24. For empirical evidence on the size of marginal adjustment costs for U.S. manufacturing as a whole, see Pindyck and Rotemberg (1983).

25. Alternatively, if demand adjusts over time because of consumers' "habit formation," we would replace eqns. (34) and (35) with:

$$q(t) = a_0 - a_1 p(t) + u(t) \qquad (34')$$

and

$$\dot{u} = b_0 - b_1 p(t) - b_2 u(t) \qquad (34')$$

Here u(t) is the "habit" component of demand (or alternatively the component of demand dependent on a stock of durables that can be adjusted only slowly). Note that the slope of the short-run demand curve is $-a_1$, and the slope of the long-run curve is $-(a_1 + b_1/b_2)$.

26. No such user cost would exist in a competitive market, however, because no individual firm can affect the rate of capacity expansion of other firms. Therefore the Lerner index is not biased as an instantaneous measure of monopoly power -- if the correct monopoly price is used in its calculation.

27. To see that $\dot{p} < 0$ as $q(t) \to \bar{q}^*$, combine eqns. (34) and (35):

$$p(t) = [\dot{u} + a_0 b_2 - b_2 q] / (b_1 + a_1 b_2)$$

Observe that $\dot{u} > 0$, so that $p_1 > \bar{p}^*$.

28. Economic analyses of the Alcoa case tend to focus on long-run monopoly power. See Gaskins (1974) and Swan (1980).

29. Alternatively, if the net value flow (FMV - P)Q were used, the weights would be lowest (zero) in periods where $M^*(t)$ was lowest (zero).

30. See footnotes 20 and 22.

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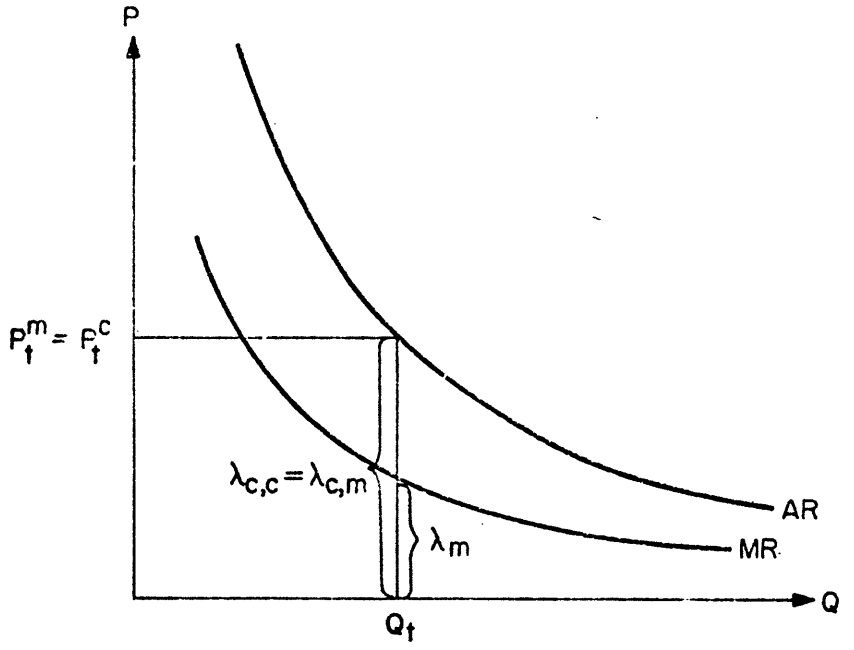

FIGURE 1 - EXHAUSTIBLE RESOURCE, MC = 0

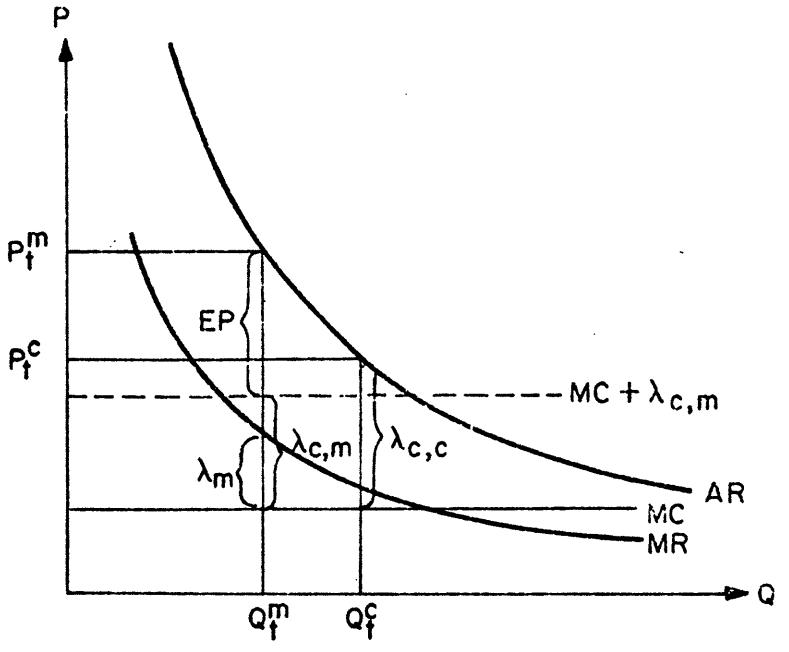

FIGURE 2 - EXHAUSTIBLE RESOURCE, MC > 0

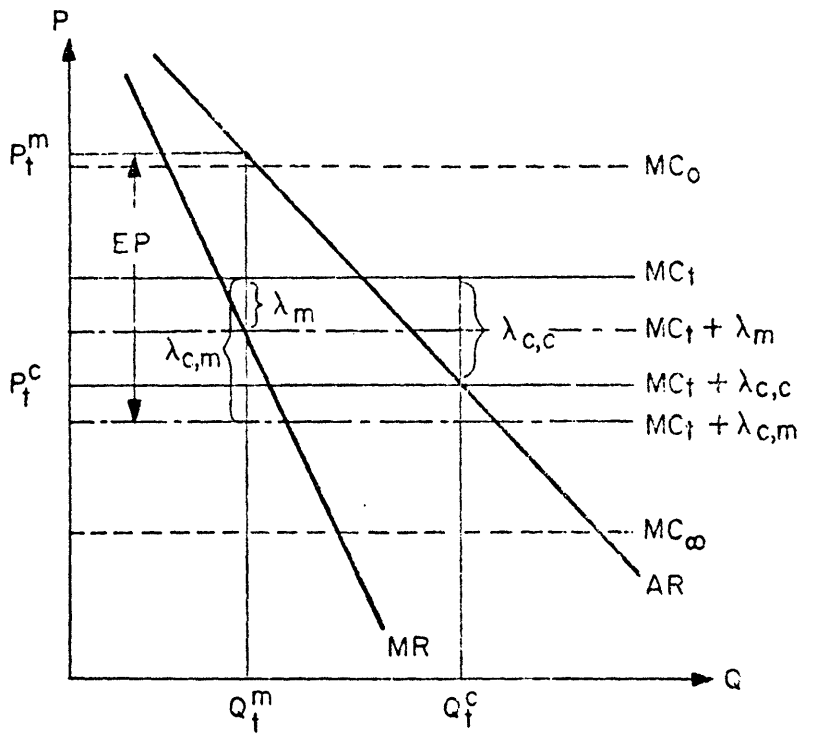

FIGURE 3 - LEARNING CURVE

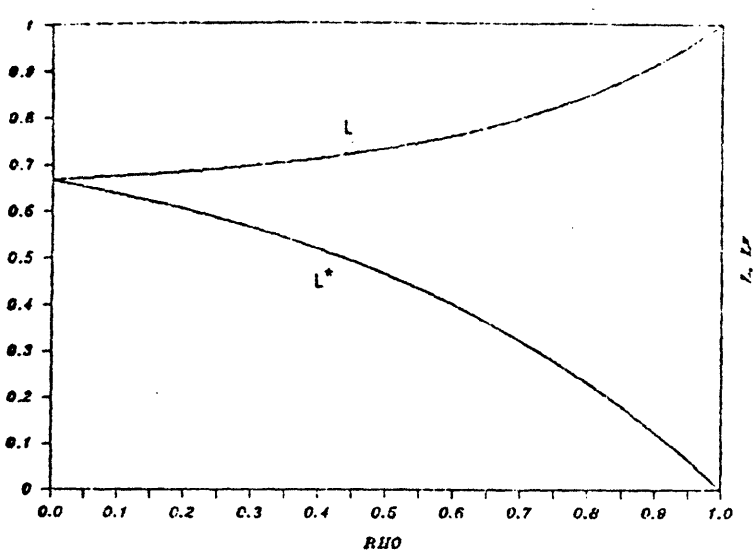

FIGURE 4A - ELASTICITY OF DEMAND = 1.5

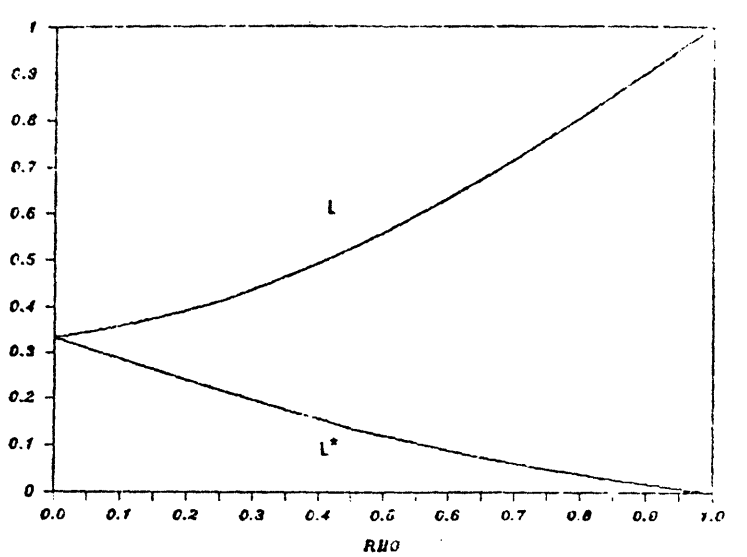

FIGURE 4B - ELASTICITY OF DEMAND = 3.0

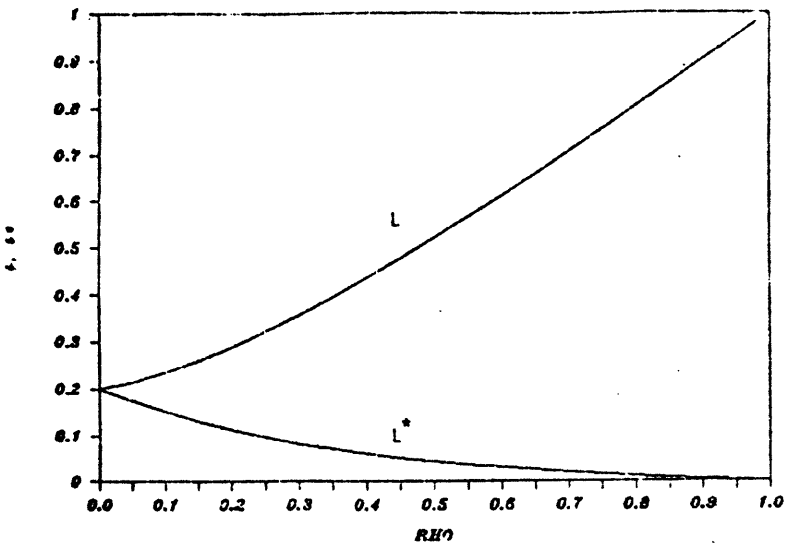

FIGURE 4C - ELASTICITY OF DEMAND = 5.0

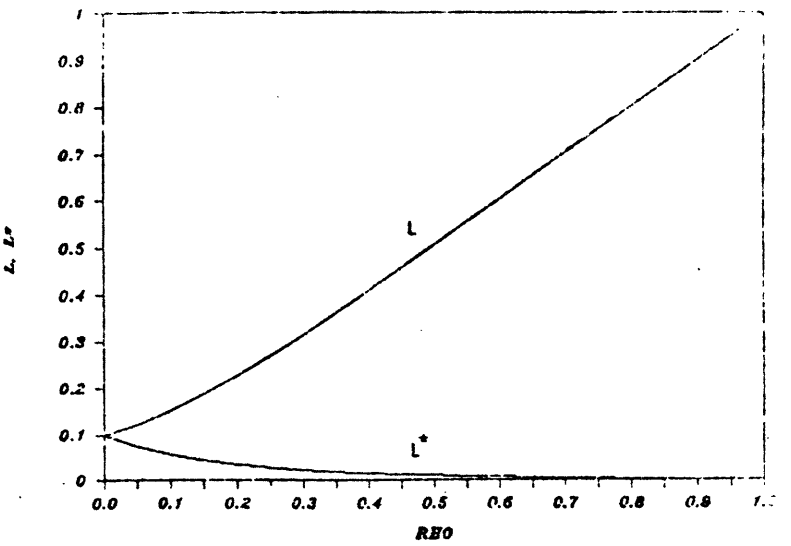

FIGURE 4D - ELASTICITY OF DEMAND = 10.0

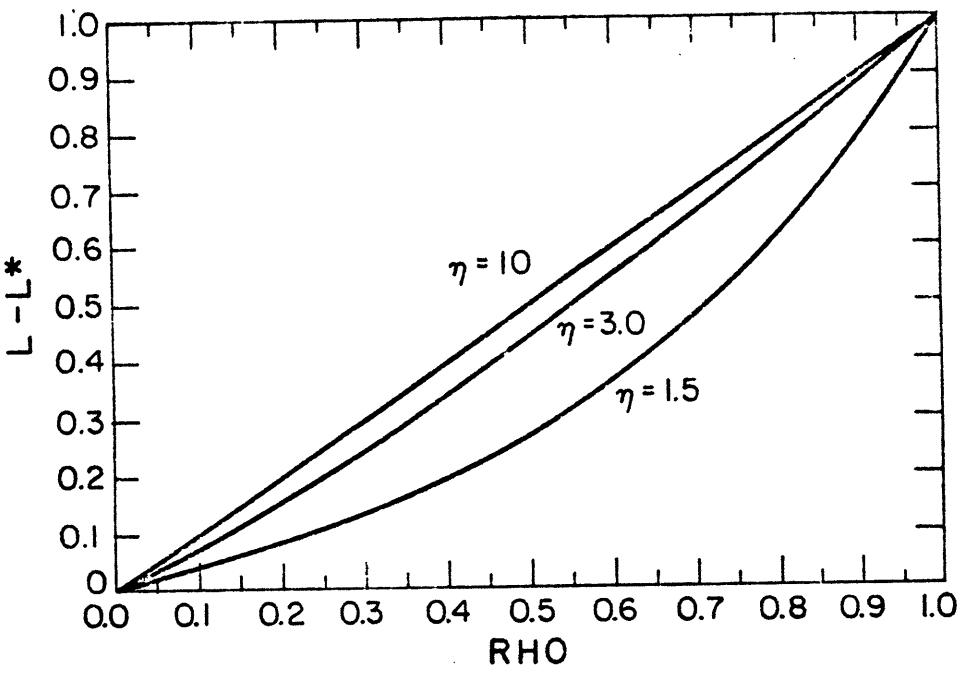

FIGURE 4E - EXHAUSTIBLE RESOURCE: BIAS ($L - L^*$)
AS FUNCTION OF RHO

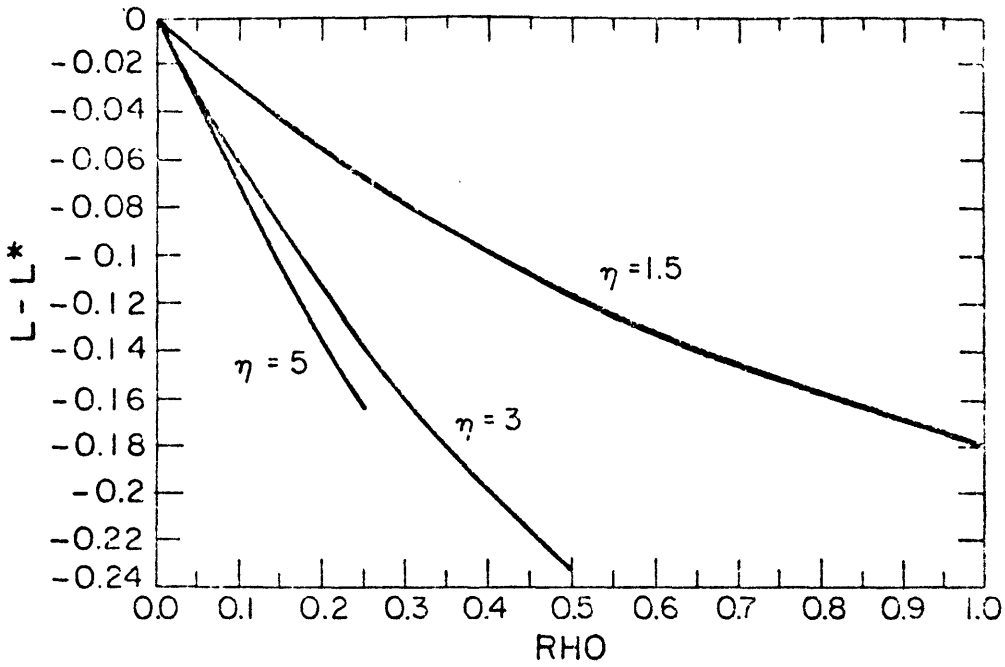

FIGURE 5 - LEARNING CURVE BIAS (L - L*) AS FUNCTION OF RHO

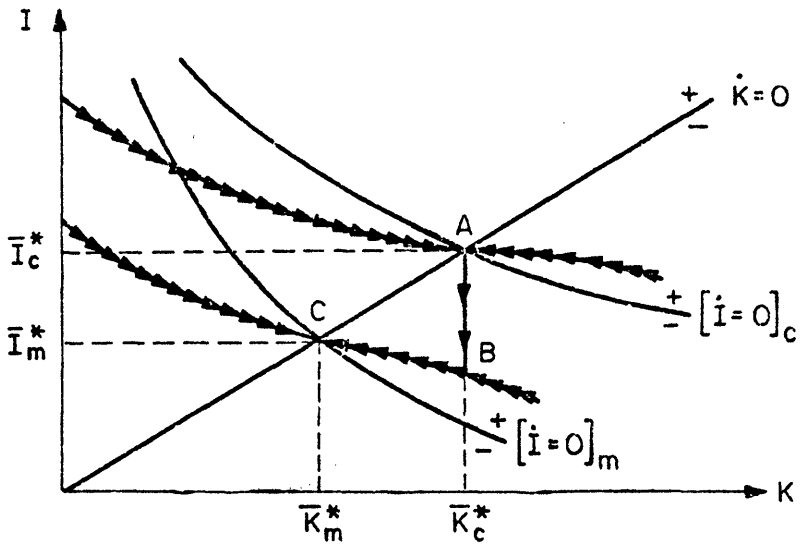

FIGURE 6 - COSTS OF ADJUSTMENT

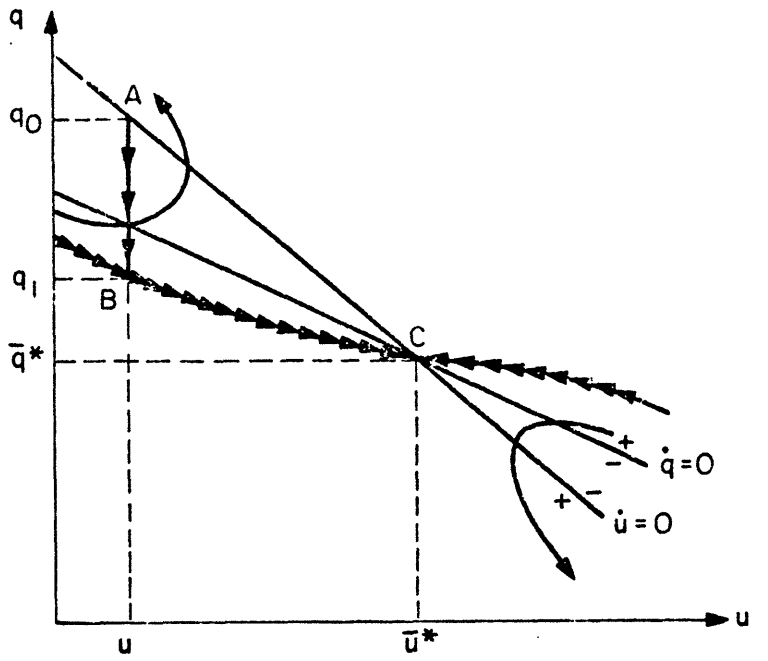

FIGURE 7 - PHASE DIAGRAM FOR CARTEL AND COMPETITIVE FRINGE

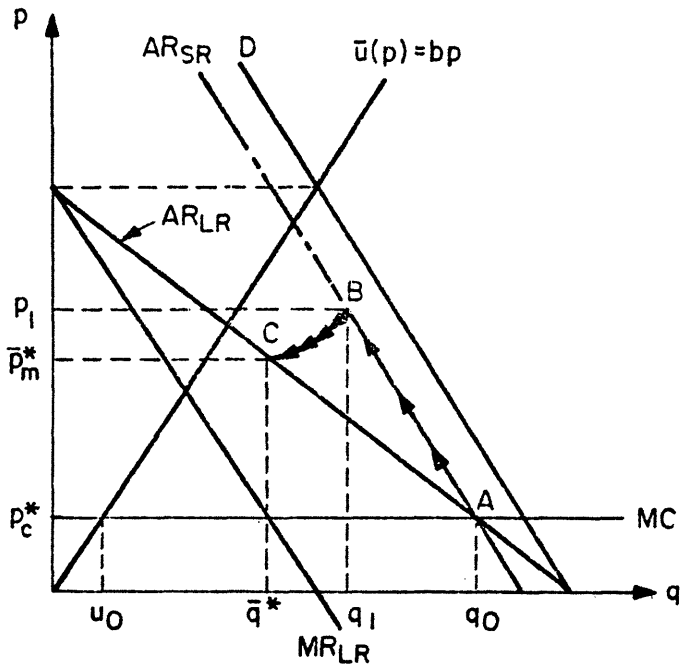

FIGURE 8 - CARTEL AND COMPETITIVE FRINGE

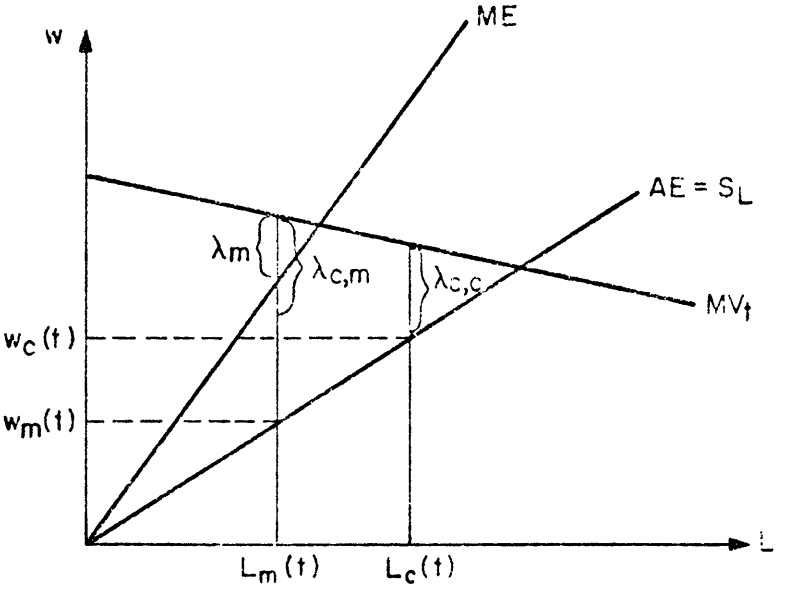

FIGURE 9 - RESOURCE PRODUCER WITH MONOPSONY POWER IN LABOR MARKET