EARNINGS VARIABILITY AND VARIANCE BOUNDS TESTS
FOR THE RATIONALITY OF STOCK MARKET PRICES

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1. Introduction

In a recent *Econometrica* paper, LeRoy and Porter [11] derive a set of upper bounds on the volatility of rational stock market prices relative to the volatility of realized aggregate corporate earnings. Using stock prices and earnings measured in real terms for the period 1955-1973, they find that these bounds "...are dramatically violated empirically." In subsequent and closely-related studies, Shiller [16,17] derives upper bounds on the volatility of rational stock market prices relative to the volatility of realized aggregate corporate dividends. Using data for the period 1871-1979, Shiller finds equally dramatic empirical violations of these upper bounds on the volatility of rational stock prices. Thus, the Shiller findings based on 109 years of data provide additional, if not entirely independent, evidence in support of the LeRoy and Porter findings for the post-war period. These collective findings have been widely interpreted as providing impressive evidence against the rationality of stock market prices, and thereby, as also casting doubt on the empirical validity of positing rational expectations in more general economic analyses. In this Comment, we take strong exception to this interpretation, and claim that the LeRoy and Porter variance bounds are wholly unreliable for the purpose of testing stock market rationality.

The key assumptions underlying the LeRoy and Porter variance bound theorems are summarized as follows:

(A) the real conditional expected rate of return on the stock market is constant over time;
(B) stock prices reflect investor beliefs which are rational expectations of future economic earnings;

(C) corporate accounting earnings measured in real terms follow a regular stationary stochastic process.

In discussing their empirical evidence, LeRoy and Porter [11, p. 559] point out that "It is not clear how to interpret our rejection of the hypothesis we have characterized as 'market efficiency.' It should be recognized that our theorems are actually tests of a joint hypothesis, some elements of which have only tenuous support." They go on to say that the most important elements in their joint hypothesis are what we have called assumptions (A) and (B).\textsuperscript{3} In later empirical investigations, Shiller [18] and Grossman and Shiller [6] have strongly argued that the excessive volatility of stock prices cannot be explained by the failure of assumption A. Thus, in light of the long time series of stock price, dividend, and earnings data employed in the respective tests, it would seem that the empirical violations of the bounds leads to a rejection of assumption B--stock market rationality.

This interpretation is further reinforced by the apparent robustness of the variance bounds tests. That is, these tests involve a direct application of a general property of stationary stochastic processes which was pointed out by Hatanaka [7] and Samuelson [15]: Namely, "...optimal...predictions of the future should be less variable than the actual data are then going to be." LeRoy and Porter [11, p. 557] echo this point in their Introduction where they claim that the dispersion of rational stock prices is necessarily less that of earnings, and once again in the discussion of their results which they contend "...are surprisingly powerful considering the generality with which
the distribution of earnings has been specified." [p. 568] In noting the conflict in conclusions between the earlier regression tests of market efficiency and their own results, LeRoy and Porter [11, pp. 558-559] state that "...an explanation that appears more attractive to us is that dispersion tests have greater power than the autocorrelation tests against the hypothesis of market efficiency, given that alternative hypothesis which actually generated the data." It is perhaps not surprising therefore, that the LeRoy and Porter and subsequent variance bounds tests are widely viewed as providing strong evidence against stock market rationality.

To support our claim that such tests are wholly unreliable for testing stock market rationality, we develop an alternative variance bound test which has the feature that observed prices will, of necessity, be judged rational if they fail the LeRoy and Porter tests. That is, if those same observed stock prices were to satisfy LeRoy and Porter's variance bound test, then they would be deemed irrational by our test. Hence, it would seem that for any set of stock market price data, the hypothesis of market rationality can be rejected by some variance bound test.
2. **Conflicts in Variance Bounds Tests**

To demonstrate the seeming empirical paradox that conclusions concerning stock market rationality can be reversed by using different variance bound tests, we consider an economy in which the LeRoy and Porter assumptions (A) and (B) are known to hold, and in which earnings are measured by accountants in accord with generally accepted accounting principles. Under well-established rules, accounting earnings are determined on an accrual basis when a firm's production, sales, and cash collection cycle extends beyond the quarterly or annual accounting period. Thus, revenues are recognized in anticipation of later cash receipts, and cash outlays are capitalized if they can be "reasonably" associated with future revenue. In applying the accrual concepts, "...the accountant follows a simple rule: the investor's interests call for recognizing revenue just as soon as the value changes it represents can be measured reliably" (Shillinglaw, Gordon, and Ronen [20]).

To develop our variance bound test, consider first the polar case where all factors causing changes in firm value are fully and immediately accounted for in the determination of earnings, and where they can be measured "reliably" as soon as they occur, so that accounting earnings are proportional to the firm's value (and therefore, given (A) and (B), to its stock price). Following the notation of LeRoy and Porter, let $y_t$ denote the stock market price at time $t$, $t = 1, \ldots, T - 1$, and $y^*_t$ denote the corresponding "ex post rational price" which equals the discounted value of corporate earnings actually realized subsequent to $t$. Given an economy in which (A) and (B) hold, then the theorem proved in Marsh and Merton [13, pp. 54-57] can be applied *mutatis mutandis* here as follows:


Theorem: If accounting earnings are proportional to firm value in each period \( t \), and if stock prices are set rationally, then for each and every sample path of stock price realizations, \( \text{Var}(y^*) \leq \text{Var}(y) \), with equality holding if and only if all realized prices are identical in the sample \( t = 1, \ldots, T - 1 \).

By inspection, the variance inequality in our theorem is the exact opposite of the one in LeRoy and Porter's Theorem 2 which holds that, appropriately standardized, \( \text{Var}(y^*) \geq \text{Var}(y) \). Hence, if their empirical evidence had not conflicted with market rationality according to their variance bound test, then it would have been inconsistent with market rationality in our test. Moreover, the variance inequality in our theorem is an analytic or "in-sample" inequality. Thus, it must hold for the sample moments \( \text{Var}(y^*) \) and \( \text{Var}(y) \) which can be computed for any finite series of realized stock prices including series of \( \{y^*_t\} \) and \( \{y_t\} \) which are not covariance stationary.

In observing that "although [many] accountants have not formally recognized [it], they have done a remarkably good job of achieving ... the objective of ... [using] a set of rules [in determining earnings] that makes the price-earnings ratio as constant as possible." Black [1, p. 8] provides support for the strict proportionality of accounting earnings to firm value hypothesized in our theorem. The basic inequality in our variance bound test does not, however, turn on the validity of this strict proportionality in the measurement of accounting earnings. Indeed, if, as others believe, changes in firm values (or economic earnings), which are immediately and fully impounded in rational stock market prices, are only slowly reflected in accounting earnings, then our theorem holds a fortiori. An intuitive explanation is as follows: If, as Black suggests, current accounting earnings
are proportional to current price, then the $y_t^*$ is a weighted average of the in-sample realized prices $\{y_t\}$, and hence $y_t^*$ is less volatile than the prices themselves. If accounting earnings are slowly adjusted so that current accounting earnings are a weighted average of current and past prices, then $y_t^*$ will be a weighted average of a weighted average of realized prices which makes it even less volatile relative to the volatility of the realized prices.
3. Resolution of the Paradox

Our theorem and LeRoy and Porter's theorem have in common the hypotheses that (A) the discount rate is constant and that (B) stock prices are rational. Hence, these assumptions are not the source of the "reversal" of the variance inequality. It follows therefore that our assumption about the accounting earnings model is inconsistent with the LeRoy and Porter assumption (C) of a regular stationary process for accounting earnings. Our model does indeed imply that earnings must follow a nonstationary process.

We are not, of course, the first to recognize that the stationarity (or the lack thereof) of the accounting earnings process is important to the validity of the LeRoy and Porter tests. There is, for example, the extensive work by Kleidon [8,9] who uses regression and other time series methods to show that the hypothesis of stationarity for the aggregate Standard and Poor's 500 earnings series can be rejected. Indeed, LeRoy and Porter [11, p. 569] discuss the stationarity issue in a paragraph which concludes "...the dependence of our results on the assumption of stationarity is probably their single most severe limitation." Despite this expressed concern, nowhere else in the paper do they give any indication that this "severe limitation" is the most probable explanation for the substantial empirical rejection of their variance bound tests. Moreover, they present no theoretical or independent empirical foundation for their assumption of stationarity even though there is a long-standing and almost uniform agreement in the accounting literature that accounting earnings (either real or nominal) can be best described by a nonstationary process. In contrast, the earnings and stock price model underlying our variance bound test is consistent with the theoretical and empirical literature in both financial economics and accounting.
Contrary to the apparent belief expressed by LeRoy and Porter [11, p. 569] and others, the stationarity issue is not centered on whether or not "...stockholders can be assumed to have foreseen the assumed trend in earnings." As we have shown elsewhere, if earnings and rational stock prices are best described by nonstationary diffusion processes, then there is no choice of trend which can be applied to the levels of earnings and prices so that the resulting detrended series of earnings and prices are stationary.

The critical point of this Comment is that misspecification of stationarity in the earnings process does not just weaken the power of the LeRoy and Porter variance bound test by introducing "econometric noise"—instead, it actually reverses the fundamental variance bound inequality used to test market efficiency. Thus, an empirical result that the upper bound of rational stock price volatility is significantly violated in the LeRoy and Porter test immediately implies that stock price volatility is significantly within the range predicted for rational prices in our test.

The class of nonstationary processes for earnings posited in our theorem is hardly pathological. As we have indicated, some would even argue that it is far more plausible than the stationary processes assumed by LeRoy and Porter. Hence, in the spirit of Leamer's [10] discussion of hypothesis testing, the LeRoy and Porter variance bound relations—viewed as tests of market rationality—are extremely fragile insofar as "...there are assumptions within the set under consideration that lead to radically different conclusions."

Our conclusion that such variance bounds relations are wholly unreliable for testing stock market rationality does not imply that the LeRoy and Porter analysis is without merit. Indeed, if one accepts the hypothesis that stock
prices are (at least, approximately) rational, then both their tests and the related Shiller variance bound tests provide impressive evidence for rejecting the hypothesis that earnings or dividends or stock prices follow a stationary process (with or without a trend).
FOOTNOTES

1 Dividends and earnings are highly related to each other. For example, Lintner [12] reported finding a strong interaction between a corporation's earnings and its dividend decision in his interviews with managers, and he subsequently used earnings as the driving variable in his model of corporate dividend policy.

2 By rationality, we mean that a firm's stock price equals the present value of its expected future cash flows.

3 In the discussion of their tests, LeRoy and Porter [11, p. 559] append another assumption to those we have listed as (A), (B), and (C), namely "...the present value relation (or, in the stock market application, the equivalent martingale assumption)." If this added assumption is meant to refer to the martingale assumption, it becomes equivalent to the constant discount rate assumption. On the other hand, if it refers to the assumption that a firm's "intrinsic" value equals the discounted value of its expected cash flows, Brock [2] showed that this follows once the transversality condition is imposed.
4 The proof given in Marsh and Merton [13] assumes that the terminal value of \( y^* \) equals the average of the sample series of stock prices, as in Shiller [17]. LeRoy and Porter do not specify what terminal condition is used in obtaining the results in their tables. However, the Marsh and Merton [13] proof also holds when the terminal \( y^* \) is set equal to the corresponding observed terminal stock price, and we presume that LeRoy and Porter used one of these two "natural" terminal value procedures. Moreover, for long time series sample periods, the choice of a terminal \( y^* \) should have no significant influence on the estimated value of \( \text{Var}(y^*) \) and its relation to \( \text{Var}(y) \).

5 In this Comment, we concentrate on LeRoy and Porter's "ex post rational price" test, which is their Theorem 2. We note that their proof of Theorem 1 follows from their proof of Theorem 2. More generally however, as we discuss in Marsh and Merton [13, pp. 36-37 and pp. 59-61], the same basic issues which we raise here will apply to "innovations tests" like LeRoy and Porter's Theorem 1.

6 Sampling considerations are thus irrelevant to our theorem. For an analysis of the effect of sampling errors on the variance bound tests, see Flavin [4].

7 Because earnings are now a distributed lag of past prices, some of the sample period earnings (and thus, computed \( y_t^* \)) will depend on out-of-sample stock prices. Thus, the inequality in our theorem may not obtain in-sample for every sample size. However, the theorem will be robust with respect to the choice of "start-up" values for lagged prices when the length of the sample period is reasonably long.
8 See, for example, Foster [5, Ch. 4].

9 See Marsh and Merton [13] for further discussion. It should be noted that the constant discount rate or martingale assumption which LeRoy and Porter take to be the standard belief in finance is consistent with the geometric Brownian motion process for economic earnings and firm values often assumed in finance. However, as Fama [3] and Myers and Turnbull [14] have shown, a constant discount rate is inconsistent with a stationary process for earnings when investors are risk averse. That is, the LeRoy and Porter assumptions (A) and (C) are inconsistent.

10 Marsh and Merton [13, pp. 33-35].
REFERENCES


