A BEHAVIORAL THEORY OF INTEREST RATE BEHAVIOR

by

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Introduction. In most models of the financial sector, interest rate determination is based upon an assumption that financial markets are in equilibrium in the sense that supply equals demand. However, whether financial markets should be represented as if they are in equilibrium is partially a question of the purpose of the representation and partially an empirical question.\(^1\) Equilibrium representations of the interest rate are appropriate when investigating, for example, issues in general equilibrium theory. On the other hand, investigations focused on aspects of the adjustment of interest rates to equilibrium clearly require a representation of the adjustment processes—that is, a disequilibrium representation. Studies focused on other economic processes should employ an equilibrium representation only if the mechanism that moves interest rates to equilibrium operates fast relative to the processes being studied, otherwise a disequilibrium interest rate formulation is required. In this paper I will present an explicitly disequilibrium theory of the mechanism by which interest rates move. The theory is consistent with empirical tests and consistent with the established partial-equilibrium theories. The theory of interest rate mechanics is appropriate for disequilibrium economic models and provides a starting point for investigating empirical issues such as the speed with which interest rates move to an equilibrium within the financial sector.

The paper begins with a short discussion of the neoclassical economic partial equilibrium\(^2\) theory of interest rates. That discussion concludes with the suggestion that a behaviorally-based theory of interest rate mechanics is desirable at least as a complement to the neoclassical theory. Such a behavioral theory is developed in the following sections and the theory's two parameters are estimated. Finally, the dynamics of the new theory are examined.

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1 The fact that there is a question regarding equilibrium or disequilibrium formulations apparently comes as a surprise to some. Quandt (1985) reports on giving seminars on the question: "On some occasions (mostly in the U.S.) I would be interrupted by someone five minutes into the seminar with the remark, 'What you are trying to do is silly, because everybody knows that prices always clear markets...'. At other times (mostly in Europe) I would be interrupted with the remark, 'What you are trying to do is silly, because everybody knows that prices never clear markets...'."

2 The theories are "partial" in the sense that they are concerned with equilibrium in only one part of the economy, namely the financial sector.
The Interest Rate. Before proceeding it is necessary to be more specific about what is meant by "the interest rate." Figure 1 presents three different rates, the prime rate, the 20-year government bond rate, and the 90-day treasury bill rate. These rates differ with respect to both risk and maturity, and, as figure 1 shows, the different interest rates do not move in perfect lock step. However, the degree of common movement is substantial. The correlation coefficient for each rate with respect to each of the others is above .9. A great deal of the total movement in the rates is accounted for by the common movement. This paper focuses on the common movement by developing a rigorous behavioral theory as if there were only one basic rate.

In a larger macro-economic model other, "non basic" rates could be derived from the basic rate as needed by simple adjustments up or down depending upon whether the other rates are more or less risky than the basic rate. 3 Whether this strategy of using a fundamental rate as a base for other rates is appropriate will depend upon the purpose of the study for which one needs to represent interest rates. The strategy would be inappropriate for a model designed to investigate how the differences between rates differ over time. On the other hand, the strategy of forming interest rates from one basic rate is probably quite appropriate for a model designed to investigate the major economic behavior modes of an industrialized economy: the business cycle, the long wave, inflation. (For discussion of the several major economic behavior modes see Schumpeter 1944; Volker 1978; and Forrester 1976, 1982). It is unlikely that the details of how various rates move against one another is important for an understanding of the major economic behavior modes. Capturing the common movement of interest rates in a basic interest rate is crucial; capturing in a detailed way the fine distinctions between rates is of less importance.

The nominal, short-term, risk-free interest rate is chosen as the basic rate in this paper. This choice is in broad keeping with economic tradition, although many theorists would focus on the real, short-term, risk-free interest rate. In the United States, however, there are no instruments carrying the real rate, and consequently representing demand and supply pressures in a behavioral model might become problematic were the real rate used. The nominal, short-term, risk-free interest rate, on the other hand, corresponds closely to

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3 Translations between short and long rates would also involve expectations regarding changes in the short or long rates and, perhaps, risk premia for different maturity habitats (Modigliani and Shiller 1973, Modigliani and Sutch 1966).
the interest rate on a federal government note. The note is virtually risk-free in terms of default risk because the government has the ability to print money to pay its debt.4

1. Equilibrium Approach: Simultaneous Equations

The theory of interest rate formation developed in this essay is fundamentally a supply and demand formulation. The formulation, however, does not impose the condition that desired supply always equals desired demand, and is therefore distinguishable from supply-demand formulations in most other econometric work. In such work interest rates are chosen to equate desired supply and desired demand (see for example, Friedman 1980a, 1980b, 1977; Friedman and Roley 1980, Modigliani, Rasche, and Cooper 1970, Hendershott 1977, Bosworth and Duesenberry 1973). More specifically, the desired (primary) purchases of a security by lenders and the desired issues of that security by borrowers are specified as functions of the interest rate on that security. The condition that desired purchases (demand) equal desired issues (supply) is then imposed. With this condition the desired amount of securities and the interest rate can be determined.5

For example, if \( g(i,z) \) is a function determining desired loans \( DL \) (or securities purchases) and \( f(i,z) \) a function determining desired borrowing \( DB \) (or issues), where "i" is the interest rate and "z" represents exogenous factors, a system of three simultaneous equations may be written:

\[
\begin{align*}
DL &= g(i,z) \\
DB &= f(i,z) \\
DL &= DB
\end{align*}
\]

4 There have been instances in which governments have defaulted on debt denominated in local currency, though such instances are not common. The default risk on the debt of the industrialized nations is extremely small.

5 Modigliani, Rasche and Cooper (1970) equate the public's demand for demand deposits to the supply offered by banks. The IS-LM approach makes use of simultaneous equalities between goods supplied and goods demanded (IS) and between money demanded and money supplied (LM) to obtain equilibrium figures for GNP and interest rates (Dornbusch and Fischer 1981, Ch. 4, eq. atonement and 12a). Bosworth and Duesenberry (1973) use the equilibrium condition for most, but not all, rates. Hendershott (1977) uses a simultaneous equations approach that permits credit rationing via a "rationing term" in the demand equation.
Equations (1) and (2) may be substituted into (3) to yield:

\[ g(i,z) = f(i,z) \] (4)

which may be solved for a unique interest rate \( i \) under suitable restrictions on \( g \) and \( f \). The interest rate so discovered may then be substituted into either (1) or (2) to uncover the amount demanded and supplied.

The simultaneous equations approach is an equilibrium approach. In equilibrium desired quantities equal actual quantities. The desires of sellers and buyers can both be fulfilled only if the quantities they each have in mind are the same, otherwise one of the groups will find itself short of transacting partners. The simultaneous equations approach requires that sellers' desired quantities always equal buyers' desired quantities (equation (3)). Hence, the simultaneous equations approach carries the assumption that financial markets are always in equilibrium.

Theorists who use simultaneous equations do not argue that the method literally describes the way rates are set. \( F \) and \( g \) are functions that describe the preferences of suppliers and demanders of credit. In an industrial economy, the system of equations describing the preferences of everyone in an industrial economy is too large to be solved, and in any event the necessary information is unavailable. The equilibrium approach is elegant, however, even if it does not describe exactly how rates are determined. The question "What is the interest rate?" can be answered without having to deal with the potentially messy process by which interest rates adjust to the equilibrium.

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6 More precisely, desired quantities equal actual quantities in a stress-free equilibrium. Most generally, an equilibrium might be considered as a condition in which all states (integrations) of a system are constant or are growing at a constant rate. Such a condition can result when desires are not fulfilled but where opposing forces in the system conspire to prevent further movement of actual quantities toward desired. This is termed a "stressed" equilibrium.

7 The key feature here is that markets are cleared by interest rates. Actual stocks do not have to equal the stocks that are (ultimately) desired, but desired flows have to equal actual flows. Partial adjustment models, where sectors are adjusting to desired stocks over some adjustment time, are considered equilibrium models here as long as markets are cleared by interest rates. Some simultaneous equations models are arguably disequilibrium models. For example, Hendershott (1977) includes a "credit rationing" term in some equations for desired issues of securities. This term reduces "issues relative to desired issues, [so that] ... out-of-equilibrium rates are determined" when issues are set equal to purchases (p.66).
The value of the answer produced by the equilibrium simultaneous equations approach depends upon whether the mechanisms adjusting interest rates are fast relative to the economic behavior of interest in a particular study (Cf. Eberlein 1984, p. 183 ff.). If the adjustment process operates relatively quickly, an equilibrium approach will be sound because interest rates will always be almost in equilibrium. For example, if one is interested in the long wave—a process that takes forty to sixty years to unfold—the equilibrium assumption is probably justifiable. Interest rate adjustments probably take far less than fifty years, although the adjustment speed is an empirical question which requires a disequilibrium model to investigate.

The equilibrium approach may be less justifiable if one is interested in the business cycle—a process with a three to seven year period. There is some evidence that a disequilibrium condition in the form of a credit crunch develops in the late stages of most business cycle expansions (Eckstein 1983; Sinai 1976; Wojnilower 1980, 1985; Dubofsky 1985; for more theoretical discussions see Jaffee and Modigliani 1969, Jaffee and Russel 1976, Chiang et al 1984). A credit crunch arises when borrowers would like to borrow at the going rate, but lenders refuse to lend. During a credit crunch, financial markets are not in equilibrium because demand exceeds supply. What evidence we have on the timing of credit crunches suggests that these disequilibria in the financial markets are tied to the business cycle. Consequently, if one is interested in the business cycle or in other economic processes occurring over a period equal to or shorter than the business cycle (say, four years), the equilibrium approach of simultaneous equations may be inappropriate.

2. Disequilibrium Adjustment Processes

The simultaneous equations approach, by assuming that interest rates are in equilibrium, obviates the need for an explicit consideration of the mechanism that moves interest rates to the equilibrium. When the equilibrium assumption is inappropriate however, an interest rate formulation must focus closely on the adjustment process. Walras (1954) appropriately called this process a "groping" ("tatonnement") toward the equilibrium.

The Financial Intermediary. A disequilibrium theory is a theory of how rates are set. The answer to how may be approached by first answering who and why. In
Walras' formulation, an "auctioneer"\(^8\) acts to clear a market represented by a group of people gathered (or linked) together to buy and sell securities. The auctioneer calls out an interest rate, participants make offers to lend and borrow at this interest rate. If desired loans do not match desired borrowing, the auctioneer calls out a new interest rate. He continues in this fashion until offers to borrow equal offers to lend, at which point he permits the participants to transact.

Walras' notion of an auctioneer is considered even by those who use it as "a fairly extreme idealization of the mechanism by which prices are determined" (Malinvaud 1972, 140). Indeed, the auction is actually only a bit less unwieldy than the solution of simultaneous equations. Instead of requiring massive amounts of information and computational power, this process requires the massive and patient cooperation of institutions and the public at large. Walras' auction is little more than a hypothetical computational technique for the solution of a set of simultaneous equations (Goodwin 1951).

The lack of realism in Walras' adjustment process may be identified in at least two ways. First, transactions cannot take place before an equilibrium price is determined, whereas on real-world commodity and stock exchanges "without exception contracts are made at each of the prices called." (Malinvaud p. 139). Second, the auctioneer, although a kind of intermediary, is not permitted to take a position in the market, that is, he is not permitted to buy and sell for his own account, unlike his real-life counterparts. Realism may be improved, therefore, by permitting transactions out of equilibrium and by replacing Walras' auctioneer with an aggregated financial intermediary. The simplification involved in taking this step is quite the reverse of that made by Walras: The intermediary, rather than being a party to no transactions, can be assumed to be a party to every transaction.

The assumption that an aggregated financial intermediary is a party to every trade of financial securities closely mirrors the real world. Granted, in some transactions the price on a security is negotiated by both parties as, for example, in the case of direct placement of commercial paper by finance companies. But, in a modern economy, most financial transactions go through a dealer or a financial intermediary who stands ready to buy and sell at a price it quotes. For example, publicly traded commercial paper does go through an intermediary. Similarly, banks set the rates in the commercial loan market. It is true

\(^8\) The term "auctioneer" is common; "Master of Ceremonies" would better describe the intended function.
enough that some loan officers, more adventurous perhaps than others, will argue on behalf of their customers for special pricing. But most officers for most loans will apply standard pricing. The case is even clearer for a bank's liabilities: Banks stand ready to accept deposits at posted rates. Finally, in the largest financial market, the market for U.S. treasury securities, dealers are prepared to buy and sell at prices they quote. Financial intermediaries set prices for the bulk of financial transactions in the U.S. In other financial centers the story is much the same. Great Britain's recent "big bang" has moved virtually all financial market transactions into the hands of dealers. To a good approximation intermediaries set interest rates.

Adjusting Interest Rates to Manage "Inventories." Before moving on to discuss how intermediaries set the interest rate, it is useful to discuss why intermediaries set interest rates. It is important to realize that intermediaries make their profits primarily on the spread between rates and not on the level of rates. A dealer makes a profit on the spread between the bid and ask prices; A bank makes its profits on the difference between the cost of funds (e.g. deposits) and the rate of interest on loans. I shall argue below that the ultimate reasons that intermediaries increase or lower the interest rate also tend to keep the spreads constant. Consequently, the assumption of a constant spread is not a bad approximation. Given a constant spread, changes in the level of interest rates do not affect an intermediary's profits. The most important reason intermediaries alter the level of interest rates is not to glean directly a higher profit, but rather to adjust inventories of securities and reserves to more appropriate levels.\(^9\)

Inventories of reserves and securities are necessary in order for intermediaries to permit disequilibrium trading. If the number of securities offered for sale to the intermediary at the going interest rate does not equal the number of securities desired to be bought, the intermediary can absorb the difference if it holds inventories. The intermediary must have an inventory of money or "reserves," and, depending upon the type of intermediary, it may have an inventory of securities as well. A primary market intermediary, such as a bank, only needs an inventory of reserves. When deposit inflows exceed loan outflows, reserves will increase. Conversely, when deposit inflows are less than loan outflows, reserves will decrease. The reserves are an inventory which at times accumulates excess cash inflow and which at other times permits outflows to exceed

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\(^9\) Adjusting inventories to more appropriate levels may, of course, increase profits indirectly by balancing the costs of holding too much and too little inventory.
inflows. An intermediary working in the secondary markets, such as a government bond dealer, needs both an inventory of money and an inventory of securities. When demand from buyers for securities exceeds the supply from sellers, the dealer will dip into his inventory of securities to meet the demand while he will accumulate the excess inflow of money in his inventory of "reserves." In contrast, when the demand of security buyers is less than the supply of security sellers, the dealer will dip into his reserves to meet the excess money outflow while accumulating in his inventory of securities the excess inflow of securities. Inventories for financial intermediaries are as necessary as the inventories of manufacturers and for the same reason: The inventories decouple supply and demand.  

Intermediaries must manage their inventory positions. Inventories that are too high entail excessive carrying costs while inventories that are too low reduce the intermediaries' capacity to handle trades, thereby limiting the intermediaries' profits. Somewhere between inventories that are too low and inventories that are too high is a desired inventory level. The intermediary must control the in- and out-flows from the inventories in order to bring his actual inventories into line with his desired inventories. The intermediary can control its inventories through price and/or through rationing. When a bank's inventory of reserves is low (i.e. it is illiquid) it can reduce the amount of loans it makes below the amount demanded—that is, the bank can ration credit. On the other hand, the bank can raise the interest rate it charges on loans and the rate it offers on deposits. The result will be to encourage depositors and to discourage borrowers, thereby tending to increase

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10 When inventories become low, intermediaries may refuse or be unable to accept certain transactions. Such a situation when economy-wide is a credit crunch. (For discussion see Hines, 1987.)

11 The desired inventory levels might be seen as the solution of a constrained optimization problem. Whether intermediaries actually approach the problem through the formal calculus of optimization is not known. Certainly they make trade-offs between the costs of high inventories and forgone profits of low trading volumes.
reserves. The spread between the cost of funds and the interest rate on loans will tend to remain constant. Conversely, if reserves are too high, that is, if the bank is excessively liquid, the bank can decrease the rate it charges on loans and decrease the rate it offers on deposits. The result will be to encourage borrowers and discourage depositors, thereby tending to decrease its reserves, and, again, tending to maintain the spread. In this case of excessive liquidity, the bank could also ration deposits, although deposit rationing seems to be rarer than credit rationing. The bank can thus use credit and deposit rationing and interest rate adjustments to control its inventories.

The methods of inventory control available to a dealer-type financial intermediary are the same as those available to a commercial bank. Figure 2 is a highly simplified sketch of a bond dealer's two inventories: bonds and money. Assume for a moment that the bond dealer begins in an equilibrium in which his two inventories are at their desired levels and that people begin selling bonds to the dealer faster than they buy bonds from him. The dealer's inventory of bonds will increase above the desired level while the dealer's inventory of money will decrease below the desired level. The dealer will be illiquid. The dealer can do two things: He can restrict his purchases of bonds and/or he can increase the

12 Naturally, reserves in the overall federal reserve system are fixed by the Fed. An individual bank will try to get a larger share of the reserves by raising the rate it offers on deposits.

13 This implies that financial intermediaries believe their customers' demand schedules for securities are negatively sloped and supply schedules are positively sloped. Zannetos (1966) shows how demand schedules can have positively sloped segments, and supply schedules negatively sloped segments if price expectations are elastic (i.e., if people expect future prices to change more than in proportion to the change in current prices). The beliefs of intermediaries are not the same as what the supply and demand schedules of their customers actually are. Consequently, the assumption that intermediaries believe their customer's have normally shaped demand and supply functions, says nothing about the actual shape of the supply and demand functions. If the world is characterized by oddly shaped supply and demand functions one might anticipate wide swings in interest rates and in prices of financial securities. It is possible, though controversial, that such wide swings may in fact be present (Shiller 1981, Marsh and Merton 1983). Although, the investigation of this possibility is beyond the scope of this paper, it seems that the interest rate formulation presented herein combined with
interest rate (i.e. lower the price) at which he offers to buy and sell. Raising the interest rate, lowering price, will discourage people from selling their bonds to that particular dealer and will encourage them to buy bonds from him. Hence his inventory of bonds will decrease and his inventory of money will increase relative to leaving the price unchanged—the dealer's illiquidity will be eased. If the dealer faces excessive liquidity (i.e. too much money and too few bonds) he can restrict his sales of bonds and/or he can lower the interest rate (raise the price). In sum, the dealer, like the commercial bank, can use restrictions on purchases and sales of bonds and/or interest rate adjustments to control his inventory positions.

The intermediary's desire to balance inventories operates to equate supply and demand. When supply and demand are in balance, the intermediary's inventories are unchanging. If inventories are at their desired levels and supply equals demand, the intermediary will be unmotivated to change interest rates. If supply and demand are not equal, inventories will increase above or below desired levels and the intermediary will be motivated to alter the interest rate in order to influence supply and demand to move his inventories back into line.

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An increase in the interest rate is equivalent to a decrease in the price. This can be easily seen in the case of a consol such as the British government issued. A consol is a coupon bond with an infinite maturity; the issuer pays a coupon forever, but never repays the principle. The interest rate in the secondary market on a consol is: $r = c/P$ where $r$ is the interest rate, $c$ is the fixed annual payment or coupon and $P$ is the price as determined by a dealer. Since $c$ is fixed, an increase in $r$ means that $P$ must decline.

One might argue that while intermediaries do try to balance their inventories, they also try to balance supply and demand directly to prevent their inventories from moving too far out of line. They might, for example, take (short) moving averages of their customers' purchases and sales. When the moving averages of purchases and sales are out of kilter, the intermediary might react by altering the interest rate. This question should be resolved through observations of and discussions with intermediaries. Even in the absence of this fieldwork, however, one may examine the nature of the difference more carefully by writing the integral equations for the inventory of securities and the purchases and sales average.

Let

\[ \text{Securities inventory}_t = \int_0^t (\text{Sales}_s - \text{Purchases}_s) \, ds \quad (i) \]

\[ \text{Sales-Purchases Average} = \frac{1}{t} \int_{t-t}^t (\text{Sales}_s - \text{Purchases}_s) \, ds \quad (ii) \]

An exploration of the consequences of assuming intermediaries consider both (i) and (ii) (rather than only (i)) when they set rates might be worthwhile. However, adding (ii) to the intermediary's concerns would probably have only a second order effect because the information in (ii) is contained in (i). Adding (ii) would likely result in a less oscillatory system, however, the models considered in this paper have little tendency to oscillate even without the introduction of (ii). Consequently, in this paper I
Anchoring and Adjustment. Having discussed who sets interest rates and why, it is now time to consider how interest rates are set. The approach taken here relies on behavioral decision theory (see Hogarth 1980 and Einhorn and Hogarth 1981 for surveys of the field). Behavioral decision theory is a young field. Certainly, more observation and more theory are required. Nonetheless, the body of available results is already sufficient to permit experimenting with corporate and economic models that are justified primarily by behavioral observations and generalizations. 17

A large and growing segment of the work in behavioral decision theory suggests that the process termed "adjustment and anchoring" is a common strategy for arriving at judgments like determining interest rates, where a lack of sufficient understanding, information, or time precludes solving a problem from first principles. In the adjustment and anchoring process people come to a judgment by adjusting a preexisting quantity (the anchor) by taking account of currently available information (Tversky and Kahneman 1974, Hogarth 1980 p.47, Slovic et al 1982, and Kleinmuntz 1985).

As an example consider an experiment reported by Lichtenstein et al (1978). Seventy four people were divided into two groups. One group was told that one thousand people die from electrocution every year in the United States. The people in this group were then asked to estimate how many people die each year from each of 40 other lethal events (e.g. train collisions, venomous bites, small pox, murder). The second group was told that fifty thousand people die each year from motor vehicle accidents. They, too, were given the list of other lethal events and asked to estimate how many people die each year from each cause.

The average estimate from the "1,000 electrocutions group" was lower than the average estimate from the "50,000 motor vehicle accidents group" for ninety percent of the

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17 The use of behavioral decision theory in the present context involves an aggregative leap. Behavioral decision theory deals with how individuals make decisions. This paper is concerned with representing an aggregation of decisions made by many people. The leap from individual to group is well sanctioned in macro-economics. For example, the savings/consumption formulation of the life-cycle theory is developed in terms of an individual and then extended to an aggregation of individuals in an economy. The leap requires checking results at the macro-level. One way of checking is to test the resulting formulation statistically using macro-economic time series data. Initial statistical tests are reported in this paper.
other lethal events. This difference between groups seems to result from each of the two groups using a different base, a different anchor. People in the first group used 1,000 deaths per year as an anchor; people in the other group used 50,000 deaths per year as an anchor. Beginning from a lower base or anchor, people in the "electrocutions group" ended up with a lower estimate than the people in the "motor vehicle accidents group." The wide difference in the anchors allowed the results of the judgmental strategy to be observed. In both groups people formed their judgments by anchoring and adjustment.

Adjustment and Anchoring in an Interest Rate Formulation. A central hypothesis of this paper is that the adjustment and anchoring strategy undergirds the interest rate setting process. More particularly, it is hypothesized that in the presence of a supply and demand imbalance, summarized by the inventory positions of intermediaries, intermediaries choose a current interest rate by adjusting up or down an anchor.

The suggestion that interest rates are set by anchoring and adjustment is not necessarily a new one. In the "modern" interpretation of Walras (Samuelson 1947 p. 270, Goodwin 1951, Negishi 1962), used frequently in disequilibrium models (Quandt 1985, particularly eq 2-4), the movement in interest rate is proportional to the difference between supply and demand of securities:18

\[
\frac{d}{dt}\text{Interest rate}_t = k(\text{Supply}_t - \text{Demand}_t)
\] (5)
or using the infinitesimal \( \delta \)

\[
\text{Interest rate}_t = \text{Interest rate}_{t-\delta} + \delta k(\text{Supply}_{t-\delta} - \text{Demand}_{t-\delta})
\] (6)

Equation (6) makes clear that the "modern" interpretation of Walras is itself an anchoring and adjustment process. The anchor is the interest rate a moment ago and the adjustment is proportional to the difference between supply and demand.

The Walrasian formulation possesses the virtue of simplicity and may be well suited to the analytical needs of stability theory, but the formulation suffers from several flaws. First the state of supply and demand are incorrectly assumed to be known in the Walrasian approach. Further, the motivation of participants is not made clear: Why do people wish to equate supply and demand? On a more technical level, the formulation assumes the interest rate is continuous even though it appears that interest rates can jump in response to

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18 The theory is usually represented in terms of price, rather than interest rates. I have converted to interest rates which entails a transposition of the supply and demand terms on the right hand side.
news. Finally, the adjustment to bring supply and demand into line is the same as the movement in the anchor. Hence, the aggressiveness with which the interest rate is raised above or below the anchor is not separated from the speed with which financial-market participants move their anchor. The Walrasian formulation precludes, for example, the possibility that dealers would adjust the interest rate with respect to the anchor and then wait for the response from suppliers and demanders before adjusting the anchor.

As an alternative to the Walrasian formulation, consider the possibility that pressures causing interest rates to move are experienced by intermediaries as discrepancies between their actual and their desired inventories of securities or reserves. In response to such pressure, intermediaries immediately move their posted rates above or below their assessment of "where rates are," abstracting from transitory pressures. "Where rates are" is an anchor. The anchor is a characterization of the current rate environment and will be termed the "underlying interest rate" in this paper. A mathematical formulation suitable for use in a simulation model involves clarifying three issues. First, the manner in which the current interest rate is adjusted above or below the anchor must be considered; second, the way people form the anchor itself must be dealt with; and, finally, the manner in which inventories are translated into an adjustment term must be described.

**Multiplicative Adjustment.** Two simple basic formulations for the adjustment around the anchor suggest themselves. One is additive, the other multiplicative.

\[
\begin{align*}
RFIR_t &= UIR_t + ALR_t \\
RFIR_t &= UIR_t \times ALR_t \\
UIR_t &= f(\text{past interest rates}) \\
ALR_t &= g(\text{intermediaries' liquidity})
\end{align*}
\]

Where RFIR is the risk free interest rate, UIR is the underlying interest rate (the anchor), ALR is the adjustment from liquidity to the risk free rate, and f and g are functions discussed below.

Kahneman and Tversky (1982, p.168) report an experiment which bears on the question of additive versus multiplicative forms. Kahneman and Tversky asked one set of respondents to imagine they were buying a calculator for fifteen dollars. The salesman, however, says the identical calculator is on sale for ten dollars at the other branch, a twenty-minute drive away. A majority of respondents presented with this situation said they would make the drive. A second set of respondents were presented with an identical situation except they were told the calculator's price was one hundred and twenty five
dollars; like those in the first group, they were informed they could save five dollars by driving across town. The majority of respondents presented with the second version reported they would not make the trip to the branch store.\textsuperscript{19} The experimental results suggest that a given difference in price becomes psychologically less important as the price increases.

Kahneman and Tversky’s experiment suggests that for a given state of liquidity the difference between the anchor and the risk-free rate should increase as rates increase in order for the psychological impact of the difference to remain the same. A functional form in which the effect of liquidity enters multiplicatively possesses this characteristic. Equation (8) can be rearranged in terms of the gap between the risk free interest rate (RFIR) and the anchor (UIR) to show this explicitly.

\[ RFIR_t - UIR_t = UIR_t \cdot (ALR_t - 1) \]  

Clearly, for a given adjustment from liquidity, the difference between the risk free rate and the anchor increases as interest rates (represented by the anchor) increase. Hence, during periods of high interest rates, indicated by a high value of UIR, the gap between UIR and RFIR will be greater than during periods of low rates for identical values of ALR.

**The Underlying Interest Rate.** The underlying interest rate is that rate which people feel generally holds at the present time, abstracting from the effects of transitory pressures on the actual interest rate. The underlying interest rate is the reference condition to which people have become accustomed; it continually adjusts as interest rates change and people become accustomed to the changed environment. A weighted average with recent experience weighted most heavily would seem to be an appropriate mathematical representation of this concept. An exponential average possesses this property and is easy to represent in a simulation model (Forrester 1961, p. 406-411). While a small modification will be considered in the section of this essay dealing with behavior and in appendix 2, for most purposes the underlying rate may be defined as:

\[ \frac{d}{dt} UIR_t = (RFIR_t - UIR_t) / \text{TAUR} \]  

\textsuperscript{19} Kahneman and Tversky kept the total dollar purchase constant by including in the question another item (a jacket) that was not on sale.
where:

- UIR - Underlying Interest Rate
- RFIR - Risk Free Interest Rate
- TAUIR - Time to Adjust Underlying Interest Rate

Equation (11) represents a translation of an inherently psychological phenomenon. The underlying interest rate is not an expectation in the ordinary use of the term as meaning a forecast of what something will be. Rather, (8) states that the underlying rate is the assessment of what the short-term, risk-free interest rate (RFIR) would be in the absence of liquidity pressures, that is, if the term representing liquidity pressures (ALR) took on its neutral value of 1 (the neutral value is 0 in alternative equation (7)). Equation (11) suggests that people making this assessment form a weighted average in which recent history is weighted most heavily. It seems likely that this is, by and large, a reasonable assumption. Subsequent research, however, may suggest refinements. For example, work on the term structure (Modigliani and Sutch (1966), Modigliani and Shiller (1973)) and work on inflation and oil-price forecasts (Sterman 1986a, 1986b) suggest that trend-extrapolation may be important. Zannetos (1966) raises the further intriguing possibility that price extrapolations may be biased so that the expected relative change in prices may exceed the current perceived relative change in prices (i.e. the price elasticity of expectations may be greater than 1). Although these studies considered expectations of future quantities and are not directly applicable to the formation of the underlying rate, extrapolations, biased or not, also may be present when forming assessments of current underlying conditions. An investigation designed to explore the underlying rate more carefully would be worthwhile. Until more information is available, it is reasonable to proceed under the simpler assumption that the underlying rate is an exponential average of past rates.

An exponential average adjusts toward current conditions. If, for example, the interest rate had been at one rate for a long time and jumped suddenly to a higher rate, the underlying rate would gradually move to the higher rate. The speed of adjustment of UIR toward RFIR depends on the parameter TAUIR. Figure 3 charts the adjustment path of UIR for a step increase in RFIR for several values of TAUIR. Statistical estimation of the parameters, including TAUIR, appears below.

**Effect of Liquidity.** While the terminology of "liquidity" tends to restrict attention to money balances, it is well to note that the effect with which we are concerned can be viewed as a function of either the inventories of securities or the inventory of money. One can work with either inventory because an increase in one implies an equal
Because the concern here is with movements common to rates on all securities, a potentially troublesome aggregation issue can be largely avoided by considering the inventory of money, since instruments used as money will be quite similar. Consequently, money inventories will be explicitly considered and the "liquidity" terminology is descriptive.

The simplest variable measuring the state of liquidity is the difference between actual and desired money holdings. However, this difference is measured in dollars and is sensitive to the price level and institutional size, which one might expect to be unimportant in the determination of interest rates. Inventory discrepancies, therefore, should be measured relative to some base. Either desired inventories or actual inventories are prime candidates. Choosing desired inventories as the base yields the following expression where intermediaries' money inventories are termed "reserves," meaning unborrowed reserves of the intermediaries:

\[ ALR = f(RAR) \] (12)
\[ RAR = (R - DR) / DR \] (13)

where:

- **ALR** - Adjustment from Liquidity to the risk free Rate
- **RAR** - Relative Available Reserves
- **R** - Reserves
- **DR** - Desired Reserves
- **f** - a function

The function \( f \) is not completely arbitrary. It must take the neutral value of 1 when \( RAR \) takes the value of 0, indicating reserves are equal to desired. It must be negatively sloped indicating that the interest rate will increase as intermediaries become less liquid. The function cannot drop below zero, because nominal rates are never negative. A flexible function which satisfies these properties and which will prove quite convenient later is the exponential function:

\[ ALR = f(RAR) = e^{-\alpha \cdot RAR} \] (14)

This function, for several values of \( \alpha \), appears as figure 4.

3. **Estimation**

This section and the next are intended to extend the understanding of and confidence in the behavioral theory of interest rates presented above. In this section the
parameters of the interest rate theory will be estimated using single equation techniques. The parameters estimated are of reasonable magnitude and of the predicted sign. These statistical results, while not conclusive, increase confidence in using the behavioral theory in a larger macro-economic model.

**Estimation.** Equations (8), (11), (13), and (14) constitute a theory of interest rate formation. The equations are reproduced below for convenience.

\[
\begin{align*}
RFIR_t &= UIR_t \times ALR_t \\
\frac{d}{dt}UIR_t &= \frac{(RFIR_t - UIR_t)}{TAUIR} \\
ALR_t &= f(RAR_t) = e^{-\alpha RAR_t} \\
RAR_t &= \frac{(R_t - DR_t)}{DR_t}
\end{align*}
\]

where:

- RFIR - Risk Free Interest Rate
- UIR - Underlying Interest Rate
- ALR - Adjustment from Liquidity to the Risk Free Rate
- TAUIR - Time to Adjust Underlying Interest Rate
- R - Reserves
- DR - Desired Reserves

The stock and flow diagram\(^2\) of the structure may be drawn as in figure 5.

The structure developed above, although based upon empirical observation at the individual level, is intended to represent macro behavior. It is necessary now to present evidence that the formulation is consistent with macro-economic observation. Toward this end, the structure will be estimated statistically, significant parameter estimates of reasonable magnitude and predicted sign will constitute evidence that the formulation is consistent with macro-economic data on interest rates and reserves.

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\(^2\) Stock and flow diagrams are in common use in system dynamics. They provide a close pictorial representation of a system of integral equations. In the diagram, the box represents an integration. The heavy arrow into the box represents a rate of flow which is controlled by the "valve" symbol termed CUIR (Change in UIR). CUIR, mathematically, is the time derivative of UIR. Entities not in boxes represent constants or functions. Single-line arrows represent information connections and reveal the arguments of each function. For a good discussion of diagramming conventions in system dynamics see Richardson and Pugh (1981, chapters 1-2) and Morecroft (1982).
The first step is to convert equations (15)-(18) into a form which can be estimated. A convenient way to do this is to solve the differential equation. Substituting (15) into (16) and solving for UIR yields:

\[ \ln(\text{UIR}_t) = \ln(\text{UIR}_t) + \frac{1}{\text{TAUIR}} \int_{t_0}^{t} (\text{ALR}_s - 1) \, ds \]  

From (15) we know that:

\[ \text{UIR}_t = \frac{\text{RFIR}_t}{\text{ALR}_t} \]  

Substituting (20) into (19) and rearranging yields:

\[ \ln(\text{RFIR}_t) = \ln(\text{UIR}_t) + \ln(\text{ALR}_t) + \frac{1}{\text{TAUIR}} \int_{t_0}^{t} (\text{ALR}_s - 1) \, ds \]  

Substituting for ALR using (17) and reexpressing the integral produces:

\[ \ln(\text{RFIR}_t) = \ln(\text{UIR}_t) - \alpha \text{RAR}_t + \frac{1}{\text{TAUIR}} \left[ e^{-\alpha \text{RAR}_t} - (t-t_0) \right] \]  

or for estimating purposes:

\[ \ln(\text{RFIR}_t) = k + z \alpha \text{RAR}_t + h \left[ \text{CELER}_t - (t-t_0) \right] \]  

where:

- \( k \) is constant for any particular regression. It is the logarithm of the underlying rate at time \( 0 \), i.e. at the beginning of the data used in the regression.
- \( z \) is \((-\alpha)\)
- \( h \) is \((1/\text{TAUIR})\)
- \( \text{CELER} \) is a month-by-month accumulation.

It would also be possible to approximate the differential equation with a difference equation. As will be seen, the solution of the differential equation results in a "quasi" linear equation. The difference equation would be non-linear. Further, there is some evidence that the use of difference equations as an approximation to an underlying continuous-time model can cause problems (Richardson, 1981).
The integration in equation (22) may be approximated by a month-by-month accumulation CELR. The approximation would be exact if RAR were constant during the month, changing only in the instantaneous transition from one month to the next.

The entire equation may be estimated by the following method: Start with an estimate of $\alpha$ and form the month-by-month accumulation of the exponential CELR. Next, estimate $h$ and the coefficient $z$ using ordinary least squares. Form a new month-by-month accumulation CELR by adjusting the old estimate of $\alpha$ toward $(-z)$. Continue in this manner until the $\alpha$ used to form the accumulation CELR and $(-z)$ converge.

The question of what data to use in estimating (23) remains to be discussed. Data on the risk-free nominal interest rate poses no significant problem since time series on treasury bills are readily available (see appendix 1). Observation of RAR poses a greater problem. Data relating to depository institutions can be used with the assumption that the reserve positions of depository institutions are highly correlated with the reserve positions of other intermediaries. This assumption is discussed at greater length below. In forming RAR for depository institutions, a problem exists. While information concerning the reserves of depository institutions is available (see appendix 1), data on desired reserves is not. Here, I will use required reserves (see appendix 1) as a proxy for desired reserves. This seems conscionable since required reserves are likely to be the major determinant of desired reserves. Other factors, such as changing interest rate spreads (Cf. Modigliani, Rasche and Cooper 1970, especially eq. 3.11), fluctuations in the degree of Federal Reserve displeasure at borrowing from the discount window, the covariance between deposits and withdrawals, and the degree of risk in the lending portfolio, will result in variations in desired reserves away from the level of required reserves.

It is possible to consider the distortion introduced by using RAR of depository institutions instead of RAR of all intermediaries. It seems likely that the relative inventory positions of all intermediaries are highly correlated because financial instruments may be quickly traded between intermediaries. It is difficult to imagine a pool of excess liquidity obtaining for a significant period in one set of intermediaries while another class is illiquid. It must be true that the relative available reserves of different intermediaries are correlated.

---

22 The difficulty in deriving an estimable expression for free reserves is noted by Modigliani, Rasche, and Cooper (1970).
This means that:

$$RAR_t = j^*RAR_d + e_t$$  \hspace{1cm} (24)

where:

- $RAR_d$ is relative available reserves for depository institutions
- $e$ is a disturbance term
- $j > 0$

This means that:

$$ALR_t = e^{-\alpha}RAR_t = e^{-\alpha}j^*RAR_d$$  \hspace{1cm} (25)

and the estimate ($-z$), obtained from (23) is actually an estimate of $\alpha^*j$. Because theory calls for positive $j$ and $\alpha$, a positive estimate ($-z$) would constitute evidence in favor of the theory.  

OLS estimation of equation (19), using data from January of 1959 to October of 1980, produces the results summarized on line 1 in figure 6. The low Durbin-Watson statistic in line 1 suggests the presence of an autocorrelated disturbance term. Figure 7 presents the autocorrelation and partial autocorrelation function for the errors. The gradual, though uneven, fall in the auto-correlation function and the initial spike in the partial autocorrelation function suggests a first order autocorrelated process (Box and Jenkins 1976).

---

23 No intercept constant appears in 24 because such a constant would imply that balanced liquidity among depository institutions implies imbalanced liquidity elsewhere in the economy. This seems unreasonable.

24 Bias is also introduced by the error term in (24). In the absence of an obvious instrument no correction has been made for this factor.

25 Data on bank reserves is conveniently available from January of 1959 through the present. However, beginning in October of 1980, the definition of what kinds of depository institutions maintained reserves began to change as the requirements of the Monetary Control Act of 1980 were phased in. Consequently, data from January of 1959 to October of 1980 is used in the regressions below as a consistent set of observations.
The structure of the errors prompts a reestimation using the Cochrane-Orcutt procedure to make a first order autocorrelation correction (Johnston 1972, pp. 261-263). Results are summarized in line 2 in figure 6. The time constant (TAUIR) implied by the estimate for h is a reasonable 17 months or 1.4 years. The estimate (−z) of α*j is 4. If α is between 0.5 and 2, j is between 8 and 2—a reasonable, if somewhat wide, range. The autocorrelation function and partial autocorrelation functions for the errors from this regression appear as figure 8. The process is indistinguishable from white noise. As a measure of fit, $R^2$ is a bit misleading here, representing as it does the fraction of explained variance in RFIR, including the variance explained by the lagged term RFIR\(_{t-1}\) necessitated by the autocorrelation correction. A better measure is the explained variance of the generalized difference (GD) of RFIR:

$$GD = RFIR_t - \rho RFIR_{t-1}$$

The $R^2$ of the generalized difference, appearing under the heading "GD $R^2$" in figure 6, removes the effect of the autocorrelation correction from the measure of fit. Using this measure, about 19% of the variation in the risk free rate is accounted for by the regression.

In order to test the stability of these results the data has been divided in half and regressions performed on each half. The third line of figure 6 contains estimates using the first half of the data, while the fourth line contains estimates using the second half of the data. It is clear that decreasing the amount of data increases the variance of the estimates of both h (1/TAUIR) and z (−α*j). Although the increased variance of the estimates makes it difficult to observe changes in the parameters, there is no indication that the relationship tested by the regression has changed significantly over the period from the beginning of 1959 through October of 1980. It is of interest to point out that the regression for the more recent period is better, having smaller variance about the parameter estimates and having a larger "GD $R^2$" than the regression on the earlier data. Although an explanation for the shift in fit is lacking, the fact that the fit is improving should be reassuring to those who wish to use the theory presented in his paper in practical applications. Several other regressions are reported in appendix 2.
4. Simulation

Two simulations will now be considered in order to explore the dynamics and to relate the structure to partial equilibrium theories of interest rates. The first simulation will be concerned with the isolated interest rate structure derived and estimated above. The second simulation, incorporating the demand for money, will show the structure is compatible with a partial equilibrium determined by the simultaneous equations approach, and will provide an indication of how quickly interest rates adjust to a new equilibrium.

The Isolated Structure. A simulation of the structure appears in figure 9. In this simulation TAUIR is set to its estimated value of 1.4 years. \( \alpha \) is set to 4 which makes the implicit assumption that \( j=1 \). The pattern of behavior, of course, is not sensitive to the particular parameters chosen. We will assume that initially UIR is 2% per year, and reserves are equal to desired reserves, so RAR begins at its equilibrium value of zero. The structure is disturbed from equilibrium by a 10% step decrease in reserves; after some years reserves step up to their original (equilibrium) level. Plotted in the diagram of figure 9 are the risk free interest rate (RFIR), the underlying interest rate (UIR), and relative available reserves (RAR).

Initially the structure is in equilibrium, with the risk free rate and the underlying rate equal to 2%. Because desired reserves remain unchanged, the 10% decrease in reserves translates into a step down in RAR from 0 to \(-1\), as shown in the diagram. ALR immediately rises to about 1.5 and, as seen in the diagram, the risk free rate instantaneously rises to about 150% of the underlying interest rate. The underlying rate now begins adapting to the new rate environment, that is, it begins moving toward the risk free rate. As long as RAR remains at \(-1\), however, the adjustment to the risk free rate remains at 150% of the underlying rate. Consequently, the underlying rate moves toward an advancing target and, as long as reserves remain low, both the risk free rate and the underlying rate grow exponentially.

Finally, reserves instantaneously step back up to their original level, returning RAR to its neutral value of 0. Pressures in the financial system are instantaneously relieved and the risk-free rate immediately returns to the underlying interest rate. With balance in supply and demand restored, there are no further pressures to generate movement in the rates.

A less technical explanation for the behavior may also be made: Because desired reserves remain unchanged, the decrease in reserves translates into a liquidity squeeze for
the intermediary. The intermediary boosts its interest rate above the prevailing rate environment in order to discourage borrowers who (it believes) would deplete its reserves and in order to attract depositors who (it believes) will increase its reserves. The assumptions are cruel though, and the increase in the interest rate does not relieve the liquidity strain. The intermediary begins to believe that the current interest rate environment is higher than it thought, and boosts its interest rate again in order to keep the pressure on lenders and to maintain its pull on deposits. This process continues until, just as suddenly as it began, the liquidity crisis ends. The intermediary, no longer wishing to pressure its borrowers nor to attract more than the usual amount of deposits, drops its rate to a level consistent with its understanding of the prevailing interest rates in the market place.

The dynamics of the isolated structure appear to be reasonable. The reverse experiment also could have been carried out: A step increase in reserves will cause rates to decay exponentially. There is a potential problem in this case, however. Equation (15) indicates that once the underlying rate reaches zero, the risk free rate will also equal zero. Equation (16) indicates that if the two rates are identical, no further movement occurs in the structure. Consequently this structure carries the implication that rates will get "stuck" if the underlying rate reaches zero. The appendix to this essay contains a small modification based on intermediation costs which corrects this problem. Because evidence of the small modification is difficult to pick up in the available time-series data, the estimation results for "normal" operating ranges still hold and we will proceed directly to consider a more important extension to the model: feedback from interest rates to intermediaries' liquidity.

The Extended Model. The above simulations explored the dynamic implications of the interest rate formulation in isolation. As discussed extensively above, financial intermediaries adjust the interest rate in order to adjust their own liquidity. Considering the interest rate formulation in isolation ignores the feedback from interest rates to the intermediary's own liquidity. Incorporating a demand for money provides a major part of the missing link. In this and the immediately following sections the simple isolated interest rate structure is extended to incorporate the demand for money. In addition to incorporating the "missing link", the extended model also demonstrates that the

26 In a truly continuous-time simulation, UIR will never reach zero. However, since actual simulations must proceed by finite steps, UIR can hit zero in an actual simulation. Furthermore, the problem of getting "stuck" does not materialize suddenly; a UIR "very close" to zero will be "very close" to being stuck at that value.
formulation for interest rates proposed in this paper is compatible with the more traditional, simultaneous equations representation of interest rates in two ways: First, the extended model has an equilibrium which can be found via simultaneous equations, and second, under reasonable parameter combinations, the dynamics of the extended model can closely approximate a "jump" to a new equilibrium.

Figure 10 presents a model in which money demand is endogenous. In the extended model the banking system's liquidity depends upon deposits held by the public and the public's deposits are a function of interest rates. The loop composed of heavy arrows is an equilibrating, negative loop as may be seen by tracing around the effects of a liquidity squeeze. Low liquidity (measured by low RAR) will cause banks to raise the interest rate. A higher interest rate implies a higher opportunity cost to holding deposits rather than interest bearing securities. Consequently, a higher interest rate will lead businesses and individuals to decrease their desired deposits. Lower desired deposits will lead to lower actual deposits. Lower deposits will relieve the liquidity squeeze. The negative loop will tend to restore the system to its desired state of liquidity.

More specifically, the banking system's relative available reserves (RAR) is defined as above (equation (18)) as:

$$ RAR_t = \frac{R_t - DR_t}{DR_t} $$  \hspace{1cm} (26)

where:

RAR - Relative Available Reserves  
R - Reserves  
DR - Desired Reserves

The desired reserves of the banking system may be written as:

$$ DR_t = D_t \times RR $$  \hspace{1cm} (27)

where:

D - Deposits  
RR - Reserve Requirements

Hence, as deposits held by the public go down the banking system's liquidity, as measured by RAR, goes up. Deposits are a level whose rate of change depends upon interest rates. As rates go up, desired demand deposits, and hence deposits, go down because it is more expensive in terms of interest foregone to hold deposits. There is extensive empirical literature on the demand for money which can be brought to bear on the
formulation of desired deposits. Commonly, empirical studies of the demand for money are based on a relatively simple money demand equation (Goldfeld 1973, 1976; Judd and Scadding 1982) which, in turn, is based on an inventory theory of money holdings (Baumol 1952; Tobin 1956; Miller and Orr 1966, 1968). The formulations used in econometric studies, intended for econometric models, may be manipulated to yield equivalent expressions well suited to use in system dynamics models.

The jumping off point for most econometric studies of the demand for money is an equation like (28) below.

\[ DD_t = a \cdot Y_t^d \cdot RFIR_t^b \]  

where:

- \( DD \) - Desired Deposits
- \( Y \) - Transactions
- \( RFIR \) - Interest Rate
- \( a, b, d \) - Constants

Equation (28) can be recast as:

\[ DD_t = (a \cdot RY_t^d \cdot RI_t^b) \cdot (Y_t/RY)^d \cdot (RFIR_t/RI)^b \]  

Defining the first term in parenthesis as Reference Deposits (RD), the second term in parenthesis as relative income (RY), and the third term in parenthesis as the relative interest rate (RI) equation (29) can be rewritten as:

\[ DD_t = RD \cdot RY_t^d \cdot RI_t^b \]  

Finally, defining the second term on the right hand side as the effect of income on deposits (EYD) and the third term as the effect of interest rates on desired deposits (EID) yields:

\[ DD_t = RD \cdot EYD_t \cdot EID_t \]  

Empirical work on the demand for money usually assumes that actual money balances adjust to desired balances with a lag. Estimation is usually carried out using logarithms (Judd and Scadding 1982, Goldfeld 1973, 1976) as in the equation below:

\[ \ln(D_t) - \ln(D_{t-1}) = c(\ln(DD_t) - \ln(D_{t-1})) \]  

\[ \text{27 In the model here there is no currency, so money and deposits are synonymous. The appropriate estimates, therefore, are those for the demand for money rather than those which deal only with the demand for demand deposits.} \]
To see what this equation implies for the underlying adjustment process begin by taking the anti-logs. This yields:

\[(D_t/D_{t-1}) = (DD_t/DT_{t-1})^c\]  

(33)

Multiplying by \(D_{t-1}\) produces:

\[D_t = (DD_t/D_{t-1})^c*DT_{t-1}\]  

(34)

To find the change in deposits each period, subtract \(D_{t-1}\) from each side:

\[D_t - D_{t-1} = \{(DD_t/D_{t-1})^c - 1\}*D_{t-1}\]  

(35)

or, in continuous time notation:  

\[\left(\frac{d}{dt}\right)D_t = \{(DD_t/D_t)^c - 1\}*D_t\]  

(36)

Defining the term in parenthesis as relative desired deposits (RDD) yields:

\[\left(\frac{d}{dt}\right)D_t = \{(RDD_t)^c - 1\}*D_t\]  

(37)

and defining the term in curly brackets as the fractional change in deposits (FCD) yields:

\[\left(\frac{d}{dt}\right)D_t = FCD_t*D_t\]  

(38)

Equations (29) and (36) together with equations (15)-(18) represent a model of interest rate formation in a banking system with endogenous demand for money. Equations (29) and (36) have been decomposed above to produce an intuitively appealing formulation. For convenience the equations of the model are presented together below.

\[RFIR_t = UIR_t*ALR_t\]  

(39)

\[\left(\frac{d}{dt}\right)UIR_t = (RFIR_t - UIR_t)/TAUIR\]  

(40)

\[ALR_t = f(RAR_t) = e^{-\alpha*RAR_t}\]  

(41)

\[RAR_t = (R_t - DR_t)/R_t\]  

(42)

\[DR_t = D_t*RR\]  

(43)

\[\left(\frac{d}{dt}\right)D_t = FCD_t*D_t\]  

(44)

\[FCD_t = (RDD_t)^c - 1\]  

(45)

\[RDD_t = DD_t/DT_t\]  

(46)

\[DD_t = RD*EYD*EID_t\]  

(47)

\[EID_t = RI_t\]  

(48)

\[RI_t = RFIR_t/RIR\]  

(49)

where:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RFIR</td>
<td>Risk Free Interest Rate</td>
</tr>
<tr>
<td>UIR</td>
<td>Underlying Interest Rate</td>
</tr>
<tr>
<td>ALR</td>
<td>Adjustment from Liquidity to the risk free Rate</td>
</tr>
</tbody>
</table>

28 This is an approximation. The approximation becomes exact as the period of the discrete-time formulation goes to zero.
In the above model, EYD has been held constant and below it is set equal to its neutral value of 1. The effect of income is not considered further here, despite its importance, because such a consideration would take us well beyond the confines of the present partial disequilibrium consideration of interest rates.

**Parameters in the Extended Model.** The previously obtained estimates for TAUIR and α, 1.4 and 4 respectively, will be used initially in what follows, and their estimated standard deviations will be used in subsequent sensitivity tests. Because it is not the purpose here to add to the already voluminous literature on money demand, values for b and c will be based on estimates from the other studies. The stability of money demand represented by equations (28) and (32) has been questioned (Goldfeld 1976, Judd and Scadding 1982) and many modifications to the "conventional" equation have been suggested to correct for instability. Through all of these modifications, however, the estimates for the parameters b and c above have remained in a relatively narrow range.

The post 1973 literature on money demand equations has been reviewed by Judd and Scadding (1982). Figure 11 below shows the estimates of the parameters of concern here from the equations discussed in Judd and Scadding. Because most empirical studies use more than one interest rate, there is a potential problem in using the results from

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29 I have shown parameter estimates rather than coefficient estimates. The translation is as follows:

- Adjustment Speed Parameter = 1 - (coefficient on lagged money)
- Interest Rate Parameter = (Coefficient on Interest rate)/(Adjustment Speed Parameter)

30 Only equations for M1 and containing transactions variables and without imposed constraints are shown in figure 6.
the studies for an estimate of the single interest rate parameter $b$ needed in the present context. A sense for the combined impact of interest rates can be gained, however, by adding together the parameter estimates on each of the separate interest rates as shown in the table. The sum would be precisely correct if one could write

$$\text{Rate}_{1t} = \mu \text{Rate}_{2t}$$

(50)

because in this case

$$(\text{Rate}_{1t})^w * (\text{Rate}_{2t})^q = \mu^q \text{Rate}_{1t}^{(w+q)}$$

so that

$$b = (w+q)$$

(51)

Because equation (50) does not hold exactly, considering the sum of the parameters in estimated money demand equations to be equal to the single parameter needed for equation (48) is only an approximation. It is, however, interesting to note that the sum of the interest rate parameter estimates is less variable between equations than the parameter estimates for the individual interest rates. It is likely that the collinearity between interest rates makes the parameter estimates for a single rate less reliable than the sum of the parameter estimates.

The entries in the column labeled "Sum of Interest Rate Coefficients" are estimates of the value of the parameter $b$ in equation (48). Thirteen of the sixteen entries are between .18 and .31. The validity of each of the three remaining equations is questionable:

Equation (1.2) contains a positive estimate for the coefficient on the interest rate on time deposits and is therefore suspect (indeed Garcia and Pak argue that (1.2) is "unacceptable" (1979 p. 330)). Goldfeld considers equation (A2.4) "clearly a step in the wrong direction" (1976, p. 697). Equation (A2.5) is conceptually quite similar to (A2.6) (both include terms for debits), but (A2.6) is considered by Goldfeld to be superior to (A2.5) (Goldfeld p. 697). In brief the parameter estimates that differ markedly from the .2 to .3 range are considered by the researchers responsible to be inferior money demand equations. The difference in EID for $b=.2$ and for $b=.3$ is not large for most values of RI as can be seen from the following expression.

$$(\text{EID}_{\text{large}} - \text{EID}_{\text{small}})/\text{EID}_{\text{small}} = (RI^{.3} - RI^{.2})/RI^{.2} = RI^{.1} - 1$$

A value of .25 for $b$ will be used initially.

Figure 11 also presents estimates for the "adjustment speed," parameter $c$ in equation (45). Equation (1.2) has already been questioned. All the other parameter estimates fall between about sixteen and 30 per cent per quarter. An estimate of .2 per quarter (.8 per year) will be used initially.
In brief, the parameters in the extended model are:

\[ \alpha = 4 \]
\[ \text{TAVIR} = 1.4 \text{ years} \]
\[ c = .80 \text{ per year} \]
\[ b = .25 \]

These values fall into the mid-range of the estimates available. The sensitivity of the dynamics to the uncertainty in these estimates will be considered later.

**Comparative Statics and Dynamic Adjustment in the Extended Model.**

The extended model is compatible with more traditional simultaneous equations approaches to representing interest rates in the sense that the extended model has a unique non-zero equilibrium and the methods of simultaneous equations can be used to find it. Desired deposits are given by equation (47). After some rearranging and substitution and making use of the equilibrium assumption that \( EYD = 1 \), desired deposits may be written:

\[
DD_t = \frac{RD}{RIR} \cdot RFIR^b_t
\]

(53)

The supply of deposits \( (SD) \) may be found by solving equation (42) for desired reserves, substituting using (43), and realizing that in equilibrium desired reserves will be equal to reserves. Thus:

\[
SD_t = \frac{R}{RR}
\]

(54)

The equilibrium condition is that supply equals demand simultaneously or

\[
SD_{\infty} = DD_{\infty}
\]

(55)

After making the suitable substitutions one finds that in equilibrium

\[
RFIR_{\infty} = RIR \cdot \left( \frac{R}{RR \cdot RD} \right)^{1/b}
\]

(56)

\[
D_{\infty} = \frac{R}{RR}
\]

(57)

The equilibrium impact of a ten per cent reduction in reserves can now be calculated. Using \( b = -0.25 \), \( RIR = 2 \), \( RD = 10 \), \( RR = 0.1 \) as the relevant parameters, and setting \( R \) initially equal to 1, a quick calculation shows that in equilibrium \( RFIR = 2 \) and \( D = 10 \). If \( R \) is dropped to 0.9, the new equilibrium will be where \( D = 9 \) and \( RFIR = 3.05 \). A ten per cent reduction in reserves implies a ten per cent reduction in deposits and a fifty per cent increase in the interest rate in partial equilibrium.  

\[ 31 \]

It is well to stress the discussion concerns a partial equilibrium in the financial markets. The derivation holds if transactions remain fixed. Clearly, transactions are not fixed. Assuming that transactions are fixed is reasonable only if the adjustment through deposits is fast relative to changes in real economic activity. The extended model includes the dynamic mechanisms necessary to gain a grasp of the speed of the response through deposits.
The extended model represented by equations (39) to (49) is a disequilibrium model that has an equilibrium consistent with the simultaneous equations approach. However, the extended model is a more general case than the simultaneous equations representation: It contains the dynamics by which the economy moves to equilibrium.

Consider the dynamic forces at work when the Federal Reserve reduces reserves. Intermediaries will feel a liquidity squeeze. In response, they will increase rates above the underlying interest rate in order to attract deposits and purchasers of securities, while discouraging borrowers and sellers of securities. Then, two dynamic processes will be called into play: The intermediaries will begin to reassess the underlying rate, increasing it as the interest rate remains high. This, in isolation, leads to a growing risk free rate as was seen in the simulations of the isolated structure. In the extended model, however, an additional force is called into play: Businesses and individuals begin reducing their deposits in the presence of a higher interest rate. Reduced deposits means lower desired reserves and, consequently, a lessened liquidity squeeze. In isolation the reaction of businesses and individuals would tend to push the interest rate down.

The two dynamic responses, upward revisions of the underlying interest rate and reduced deposits, work in opposite directions. The reaction of the risk free rate at any point in time will depend on the relative strengths of the two responses: On the one hand, the risk free rate will depend on how sensitive business and individual desired deposits are to the interest rate. On the other hand, the behavior of the risk free rate will depend on how quickly intermediaries reassess the underlying rate and on how far above the underlying rate they boost the risk free rate. These characteristics of businesses, individuals and intermediaries are captured in the parameters of the extended model:

- $b$: Relationship between desired deposits and the interest rate. Larger values indicate desired deposits are more sensitive to the interest rate.
- $c$: The speed with which deposits adjust to desired deposits. Larger values of $b$ denote a faster adjust speed.
- $\tau$: The speed with which intermediaries adjust the underlying rate toward the risk free rate.
- $\alpha$: The degree to which intermediaries raise the risk free rate above the underlying rate in response to liquidity pressures.

Figure 12 presents a plot of the risk free interest rate and the equilibrium risk free interest rate. Surprisingly, perhaps, the risk free rate jumps immediately to its equilibrium.
The initial increase in the risk free rate (governed by \( \alpha \)) moves the risk free rate to its equilibrium value. Then, the decreasing liquidity squeeze caused by depositors reducing their deposits (parameters \( b,c \)) balances the increasing underlying interest rate (parameter \( \text{TAUIR} \)). This simulation shows that the disequilibrium formulation of the extended model is consistent with the simultaneous equations approach in another important sense: The extended model can jump to a new equilibrium under suitable choices of the parameters.

It is an intriguing result suggesting, perhaps, that there may be a learning process by which financial intermediaries and businesses and individuals come to have a set of compatible responses that favor stability in interest rates. Unfortunately, this conclusion will need more substantiation than that offered by this paper. Although it is true that the jump to equilibrium holds up very well under a variety of test inputs given the parameters, these parameters are subject to uncertainty. \( \alpha \) and \( \text{TAUIR} \) have been estimated in this paper and have a probability distribution about them with the standard deviations shown in figure 6. Parameters \( b \) and \( c \) are typical values from several other studies as discussed above. There is sufficient uncertainty in the parameters to produce behavior different from the jump to equilibrium. For example, figure 13 shows a far more sluggish response to a ten per cent reduction in reserves. The simulation was obtained by assuming intermediaries change rates more cautiously and more slowly (\( \alpha \) reduced by one standard deviation and \( \text{TAUIR} \) increased by one standard deviation) and by assuming businesses and individuals move their deposits more slowly toward a more extreme goal (\( b \) reduced to .18 and \( c \) increased to 1.2 — the end points of their typical ranges). Under these assumptions of how people react, interest rates remain below their equilibrium values for years. The reverse assumptions, Intermediaries more aggressive and faster, depositors less extreme but faster (\( \alpha \) increased and \( \text{TAUIR} \) reduced by one standard deviation; \( b \) increased and \( c \) reduced to the end points of the "typical" ranges) produces a large overshoot as shown in figure 14.

Further clouding the evidence in this paper that rates immediately jump to new equilibria is the absence of an important feedback. Interest rates affect decisions on the

32 The equilibrium is a partial equilibrium, because there is no feedback through income.

33 There is a slight tendency to overshoot for large (20%) reductions in reserves, a tendency to undershoot, then overshoot for reductions in reserves. The response to sinusoidally varying reserves is very good for periods of two years and four years.
"real" side of the economy—decisions to invest and to hire or fire. Those decisions will determine the need for deposits to cover transactions. Movements in the real economy, therefore, will alter the response of deposits to interest rates. An investigation into the impact of the physical responses of the economy moves us beyond the confines of this paper.

5. Summary and Conclusions

This paper has presented a behavioral model of interest rate mechanics. The theory is based upon the observation that most financial transactions go through financial intermediaries at prices set by those intermediaries. Intermediaries raise and lower interest rates in order to adjust their financial inventories (i.e. their inventories of securities and reserves). Intermediaries use an anchoring and adjustment strategy to decide where interest rates should be. In the presence of financial inventory imbalances, the intermediary will raise or lower its posted interest rates above or below an anchor here termed the underlying interest rate. The underlying interest rate represents the interest rate environment, the expectation of what the interest rate would be now if it were not for transitory pressures such as financial inventory imbalances.

The financial inventory positions of financial intermediaries accumulate, and thereby summarize, the entire history of supply and demand. For example, a low money inventory (or a high securities inventory) indicates that people in the non-financial sectors have been selling securities at a rate greater than they have been buying them. An increase in the interest rate is the response which should bring demand and supply back into line. An increase in interest rates is also the response that the illiquid intermediary will be inclined to make to adjust its own inventories. In brief the behavioral theory is a supply and demand theory.

Empirical support for the behavioral theory exists at a micro and at a macro level of aggregation. Research into the nature of human decision making at the individual level indicates that anchoring and adjustment is a common strategy for making decisions like those involved in interest rate determination. Econometric estimation was used to validate the theory at a macro-economic level of aggregation.

An "extended model" in which the interest rate theory was coupled with a representation of money demand showed that the behavioral theory can contain an equilibrium that may be determined by simultaneous equations techniques. However, the
theory developed in this paper also contains the behavioral adjustment processes that bring interest rates into equilibrium. The extended model can behave like more traditional partial equilibrium models of interest rate determination—interest rates in the extended model can jump to new (partial) equilibria and stay there. But, with slightly altered parameters the extended model can exhibit different behavior: Interest rates can dally below the equilibrium rate for an extended period of time, or interest rates can overshoot the mark.

The research reported in this paper can be extended in several directions. Further work in empirical estimation of parameters, theory building, and policy analysis would all be worthwhile. Better data can be collected on the reserves and desired reserves of intermediaries. Alternatively, more sophisticated estimation techniques might be utilized to correct for biases introduced by using surrogate measures. Observation of the way interest rates are actually determined by those who actually determine them would provide an even more valuable opportunity to confirm or disconfirm the formulation suggested in this paper. In addition, such observation might reveal the ways in which the parameters of the formulation are themselves formed, allowing one to deepen the current model by transforming its parameters into variables.

The theory might also be extended through disaggregation. Although this paper dealt at a highly aggregated level, the disequilibrium process described herein could be applied to a more detailed model which included several different securities and several different intermediaries. Such a disaggregation would illuminate the processes by which the different interest rates move relative to one another.

Finally, the current work might be extended by investigating the policy implications of this theory of interest rate formation. What are the causes and consequences of credit crunches. How does the Fed influence the business cycle and other economic behavior modes? What are the impacts of targeting interest rates, reserves, money stock, or unemployment? Are there other targets or other rules for guiding open market operations which are more powerful or more reliable for fostering economic growth? Answering questions such as these will involve embedding this interest rate theory into increasingly more detailed models.
Appendix 1: Data Sources

This appendix provides information on the data, and their transformation, used to estimate equation (19). The risk-free rate is based on CITIBASE's (Citibank 1985) secondary market rate on three month treasury bills (data series FYGM3, see also Federal Reserve Bulletin, table 1.20)). This rate is calculated on a bank-discount basis.

Relative available reserves (RAR) are defined as the difference between reserves and desired reserves, divided by desired reserves. The data series used for reserves was CITIBASE's FZRNB, nonborrowed reserves of depository institutions, and for desired reserves CITIBASE's FZRQ, required reserves of depository institutions. Neither FZRNB nor FZRQ are seasonally adjusted, nor are they adjusted for changes in reserve requirements, in particular the change in institutions required to hold reserves associated with the Monetary Control Act of 1980. To avoid definitional problems introduced with the monetary control act of 1980, the data series used end in October of 1980. Using data adjusted for the Monetary Control Act of 1980 and using the data through 1987 materially affects the estimates. These alternative estimates would imply that the interest moves far more slowly. The estimates using these data, however, have lower $R^2$ and higher standard deviations of the estimates.
Appendix 2: Additional Regressions

Several regressions were performed in addition to those discussed in the text. One theory of the autocorrelated error term is that it arises from the month-by-month accumulation over the entire span of the regression. Lines 5 and 6 represent OLS and GLS regressions respectively in which the extended monthly accumulation is replaced by a twelve-month accumulation. The improved Durbin-Watson statistic of line 5 compared to line 1, indicates that, perhaps, some of the autocorrelation is removed by this Arlying R, although other explanations are also possible. The GLS correction yields estimates comparable to those of line 2, however the regression in terms of $R^2$ and significance of the coefficients is not quite as good. Line 7 tests whether moving to a one-month accumulation is helpful. The high standard error on H suggests that this procedure makes it difficult to obtain a good measure of the adjustment speed of the underlying rate—the underlying interest rate appears to adjust more slowly than one month.

The impact of inflation is briefly explored in lines 8 and 9. In line 8, the current period inflation rate is added to the regression equation; in line 9 the change in the inflation rate is added to the basic regression equation. In both tests, the inflation term is insignificant and carries a sign opposite from what would be expected from the Fischer theory. These results give some preliminary evidence that theory of interest rate mechanics is indeed a more proximate theory of interest rates than Irving Fischer's—that is, the influence of inflation works through liquidity, rather than directly on interest rates.
Appendix 3: Modification to Interest Rate Structure

As discussed in the text, the structure contains a risk that rates will become "stuck" at zero. A small modification removes this risk. The change involves introducing a new variable, the indicated underlying rate (IUR), toward which the underlying rate adjusts. The indicated underlying rate is identical to the risk-free rate, except the indicated underlying rate does not go to zero, but rather to some minimum value.

Behaviorally such a modification implies there is some lowest reference rate which is greater than zero. Economically this minimum rate may be determined by the costs of intermediation. The minimum rate may be interpreted as the average cost of intermediation expressed as a fraction of the amount intermediated.

A restatement of the interest rate formulation using a possible definition of the indicated underlying rate is:\[RFIR_t = UIR_t \times ELR_t\] (A2.1)
\[\frac{d}{dt}UIR_t = (IUR_t - UIR_t)/TAUIR\] (A2.2)
\[IUR_t = \max(RFIR_t, MUIR)\] (A2.3)
\[ALR_t = e^{-\alpha \times RAR_t}\] (A2.4)

where:
- RFIR - Risk Free Interest Rate
- UIR - Underlying Risk Free Interest Rate
- ALR - Affect of Liquidity on the risk free Rate
- IUR - Indicated Underlying Interest Rate
- MUIR - Minimum Underlying Interest Rate, a constant
- TAUIR - Time to Adjust the Underlying Interest Rate
- RAR - Relative Available Reserves
- e - The exponential function
- \(\alpha\) - a constant

The new definition of the underlying interest rate appears as equation (A2.3).

\[\text{A function involving a gradual approach to MUIR, in place of the abrupt change introduced by the max function, would also be a possible assumption. The use of such a function, however, introduces additional complications and further dynamics without materially enriching the current discussion.}\]
Appendix 4: Model Documentation

Figure 9 was generated by a model created using STELLA. Figures 12, 13, and 14 were generated by a model written in DYNAMO. Equation listings for each model follow. Note that "changes files" for the DYNAMO program are given at the end of the listing.
BASIC INTEREST RATE STRUCTURE
(Used in Figure 9)
STELLA Listing

JAMES H. HINES, JR.

□ UIR = UIR + CUIR
   INIT(UIR) = 2
□ a = 4
□ ALR = EXP(-a*RAR)
□ CUIR = (RFIR/4)/TAUIR
□ DR = 1
□ R = 1+(STEP(-1,.5)-STEP(-1,4.5))
□ RAR = (R-DR)/DR
□ RFIR = UIR*ALR
□ TAUIR = 1.4
EXTENDED INTEREST RATE MODEL
(Used in Figures 12, 13, and 14)
DYNAMO listing

JAMES H. HINES, JR.

A RFIR.K = UIR.K * ALR.K
RISK FREE INTEREST RATE (FRACTION/YEAR)
L UIR.K = UIR.J + DT * CUIR.J
N UIR = RFIR*K*(R/(RR*RD))**(1/B))
UNDERLYING INTEREST RATE
A CUIR.K = (RFIR.K - UIR.K) / TAUIR
CHANGE IN THE UNDERLYING INTEREST RATE
(FRACTION/YEAR/YEAR)
C TAUIR = 1.4
TIME TO ADJUST THE UNDERLYING INTEREST RATE (YEARS)
A ALR.K = EXP(-A * RAR.K)
ADJUSTMENT FROM LIQUIDITY FOR THE RISK FREE RATE
(DIMENSIONLESS)
C A = 4
COEFFICIENT ON LIQUIDITY ADJUSTMENT
A RAR.K = (R.K - DR.K) / DR.K
RELATIVE AVAILABLE RESERVES (DIMENSIONLESS)
A R.K = IR * STUP.K * WAVE.K
RESERVES (CURRENCY UNITS)
C IR = 1
INITIAL RESERVES
A STUP.K = 1 + STEP(FSIR, TFSIR)
STEP-UP (DIMENSIONLESS)
C FSIR = 0
FRACTION STEP INCREASE IN RESERVES (DIMENSIONLESS)
C TFSIR = .5
TIME FOR FRACTION STEP INCREASE IN RESERVES (YEAR)
A WAVE.K = 1 + WAVAMP * SIN(6.283 * TIME.K / WAVPER) * STEP(1, TSWAVE)
WAVE (DIMENSIONLESS)
C TSWAVE = .5
TIME TO START WAVE (YEAR)
C WAVAMP = 0
WAVE FRACTIONAL AMPLITUDE
C WAVPER = 2
WAVE PERIOD
A DR.K = D.K * RR
DESIRED RESERVES (CURRENCY UNITS)
C RR = .1
RESERVE REQUIREMENT
L D.K = D.J + DT * CID.J
N D = R/RR
DEPOSITS (CURRENCY UNITS)
A CID.K = FCD.K * D.K
CHANGE IN DEPOSITS (CURRENCY UNITS/YEAR)
A FCD.K = (RDD.K ** C) - 1
FRACTIONAL CHANGE IN DEPOSITS (FRACTION/YEAR)
C C=.8
COEFFICIENT ON FRACTIONAL CHANGE IN DEPOSITS
A RDD.K=DD.K/D.K
RELATIVE DESIRED DEPOSITS (DIMENSIONLESS)
A DD.K=RD*EYD*EID.K
DESIRED DEPOSITS (CURRENCY UNITS)
C RD=10
REFERENCE DEPOSITS (CURRENCY UNITS)
C EYD=1
EFFECT OF TRANSACTIONS ON DESIRED DEPOSITS
A EID.K=RI.K**B
EFFECT OF INTEREST RATES ON DESIRED DEPOSITS
(DIMENSIONLESS)
C B=-.25
COEFFICIENT ON EID
A RI.K=RFIR.K/RIR
RELATIVE INTEREST RATE (DIMENSIONLESS)
C RIR=2
REFERENCE INTEREST RATE (FRACTION/YEAR)

A EQUILRT.K=RIR*((R.K/(RR*RD))**(1/B))
EQUILIBRIUM RISK FREE RATE (FRACTION/YEAR)
A EQUILDP.K=R.K/RR
EQUILIBRIUM DEPOSITS (CURRENCY UNITS)
SPEC LENGTH=5/SAVPER=.0625/DT=.0625REL_ERR=0
Changes file for FIGURE 12: "Behavior of Extended Model - Parameters"

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Changes file for FIGURE 13: "Behavior of Extended Model - Slow Response" Parameters

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Changes file for FIGURE 14: "Behavior of Extended Model - Overshoot Response" Parameters

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REFERENCES


Figure 1: Selected Interest Rates

Selected Interest Rates. Although interest rates do not move in precise lock-step, the degree of common movement is significant.
Figure 2: Simplified Sketch of a Securities Dealer
The Securities Dealer: As people sell their bonds to a dealer, the dealer's money balance declines while his stock of bonds increases. As people buy bonds from a dealer, the dealer's money balance increases while his stock of money decreases.
Figure 3: Exponential Adjustment Paths
Exponential Adjustment Paths: The response of the underlying interest rate to a nominal step increase in the risk free rate depends on the parameter TAUIR.
The adjustment of the risk free rate from liquidity (ALR) depends upon the value of relative available reserves (RAR) and on the parameter $\alpha$.

**Figure 4: The Exponential Function**

The adjustment of the risk free rate from liquidity (ALR) depends upon the value of relative available reserves (RAR) and on the parameter $\alpha$. 
Figure 5: Basic Interest Rate Structure
The risk free interest rate is the underlying interest rate adjusted for liquidity. The underlying interest rate is a weighted average of past values of the risk free rate. The adjustment from liquidity is determined by the availability of reserves relative to desired reserves.
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Figure 6: Estimation Results

| K | Estimated value of the underlying interest at the beginning of the data range for each regression. |
| H | Adjustment speed parameter for the underlying interest rate. 1/H gives the adjustment speed in months. |
| Z | Parameter governing strength of adjustment from liquidity. The greater the magnitude of Z, the greater the response to liquidity pressures. |
| ρ | Autocorrelation coefficient of the errors. |
| I | Coefficient on inflation in line 8, coefficient on change in inflation in line 9. |
| R² | Percentage of variation of the risk free interest rate explained by the regression. |
| GĐR² | Percentage of variation of the generalized difference of the risk free rate explained by the regression. |
| RFIRₜ - RFIRₜ₋₁ | The generalized difference is |
Figure 7: Autocorrelations for OLS regression error term suggesting the presence of first order autocorrelation in the error term of the regression.
Figure 8: Autocorrelations for GLS
Autocorrelagram and partial autocorrelagram of the GLS regression error terms. The GLS procedure has resulted in an uncorrelated error term.
Intermediaries increase the risk free interest rate above the underlying rate when they are illiquid (i.e., when relative available reserves are below 0). They adjust their assessment of the underlying rate toward the risk free rate causing rates to move upward during periods of illiquidity. Intermediaries drop the risk free rate back to the level of the underlying interest rate when liquidity pressures are removed.
Figure 10: The Extended Model
Insufficient liquidity (i.e., reserves lower than desired reserves) will lead banks to increase the interest rate. The increased interest rate will cause people and businesses to reduce their deposits. The reduction in deposits will reduce the desired reserves of banks. A reduction in desired reserves will relieve the illiquidity of banks.
### Interest Rate Coefficients and Adjustment Speed from Alternative Money Demand Equations

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<td>A2.8 Goldfeld (1976)</td>
<td>5.6</td>
<td>-.063</td>
</tr>
<tr>
<td>† * A3.1 Hafer and Hein (1979)</td>
<td>2.2</td>
<td>-.121</td>
</tr>
<tr>
<td>A3.3 Hafer and Hein (1979)</td>
<td>2.4</td>
<td>-.042</td>
</tr>
</tbody>
</table>

* The equation shown here corrects Judd and Scadding's typographical errors on the coefficients of government bonds and dividend price ratio. Judd and Scadding's rounding is preserved in the other equations.

† Equations for Hafer and Hein show the table number before the decimal. Garcia and Pak's equation comes from the third equation of table 1.

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**Figure 11: Interest Rate Coefficients and Adjustment Speed**

Judd and Scadding (1982) reviewed other economists' statistical estimates of coefficients on determinants of the money supply. The first column gives the equation number in Judd and Scadding's review; the second column gives the author and date of the original study; the third column gives the equation number in the original study.
Figure 12: Behavior of Extended Model

With \( \alpha = 4, T \text{AUIR} = 1.4, B = 0.25, \) and \( C = 8, \) the risk free rate jumps to a new equilibrium value and stays there in response to a 10% decrease in reserves.
With $\alpha = 3.4$, $\text{TAUIR} = 2.2$, $B = .18$, and $C = 1.2$, the risk free rate moves slowly to a new equilibrium in response to a 10% decrease in reserves.
Figure 14: Behavior of Extended Model: Overshoot Response

With $\alpha = 4.5$, $TAUIR = 1$, $B = .31$, and $C = .64$, the risk free rate moves and overshoots the new equilibrium in response to a 10% decrease in reserves.