"Hierarchical Production Planning"

Gabriel R. Bitran*†
Devanath Tirupati**

MIT Sloan School Working Paper #3017-89-MS

May 1989

*Sloan School of Management, Massachusetts Institute of Technology
Cambridge, MA 02139

**Department of Management, The University of Texas at Austin

†This research has been partially supported by the
Leaders for Manufacturing Program.

Hierarchical Production Planning (HPP)

1.0 Introduction

In general terms, production may be defined as the process of converting raw materials into finished products. Manufacturing systems are typically composed of large number of components which have to be managed effectively in order to deliver the final products in right quantities, on time and at an appropriate cost. In systems characterized by multiple products, several plants and warehouses, a wide variety of equipment and operations, production management encompasses a large number of decisions that affect several organizational echelons. To understand the role of Management Science models in supporting those decisions, it is useful to classify them according to the taxonomy proposed by Anthony (1965). He classifies decisions into three categories: strategic planning, tactical planning and operations control.

Strategic planning decisions are mostly concerned with the establishment of managerial policies and the development of resources to satisfy external requirements in a manner that is consistent with the organizational goals. In the area of production management these decisions relate to the design of production facilities and include the following: (i) location and sizing of new plants, (ii) acquisition of new equipment (iii) selection of new product lines, and (iv) design of logistic systems.

These decisions are very important because, to a great extent, they define the competitive position of the firm, its growth rate, and eventually, determine its success or failure. Also these decisions, which are made at fairly high managerial levels, involve large investments, have long term implications and are affected by both external and internal information. Thus, any model-based system to support these decisions should have a broad scope, long planning horizon, and recognize the impact of uncertainties and risk attitudes.

Tactical planning decisions focus on the resource utilization process. At this stage, after decisions have been made regarding physical facilities, the basic problem to be resolved is the allocation of resources such as capacity, work force availability, storage and distribution resources. Typical decisions in this category include utilization of regular and overtime labor, allocation of capacity to product families, accumulation of seasonal inventories, definition of distribution channels, and selection of transportation alternatives. These decisions involve a medium range planning horizon, and the aggregation of items into product families. In the literature, models addressing these issues are classified as aggregate planning models.

Operations Control: Decisions in this category deal with day to day operational and scheduling problems which require complete disaggregation of the information generated at higher levels. Typical decisions at this level include the following: (i) production sequencing and lot sizing at the item level, (ii) assignment of customer orders to individual machines, (iii) inventory accounting and inventory control activities, (iv) dispatching, expediting and processing orders, and (v) vehicle scheduling.

The three types of decisions identified in Anthony's Framework - strategic planning, tactical planning and operational control - differ markedly on several dimensions which have important implications in developing a solution approach to address production planning and scheduling problems. Table 1.1, reproduced from Hax and Candea (1984), summarizes these differences and contrasts the characteristics of the decisions in these three classes. The interdependence among these classes of decisions is very strong and therefore an integrated approach is required to minimize suboptimization. The development of integrated decision models that deal with all the decisions simultaneously, while attractive in principle, has severe drawbacks. First, these models tend to be very large, and in most practical situations, it would be very difficult, if not impossible, to obtain optimal solutions with reasonable effort. Second, even if computational power and methodological capabilities would permit solution of a large detailed model, the approach is inappropriate because it would not be responsive to the management needs at each level of the organization, and would prevent interaction between models and managers at each organization echelon.

The hierarchical approach to production planning and scheduling recognizes these differences. In this framework, the decisions are decomposed into subproblems, which in some way, within the context of the organizational hierarchy, link the higher level decisions with those of lower level in an effective manner. Decisions that are made at higher level impose constraints on the lower level decisions. In turn, detailed decisions provide the necessary feedback to evaluate the quality of aggregate decision making.

Hierarchical planning provides a framework that has application beyond the areas of production planning and operations management. For example, Winkofsky, Baker and Sweeny (1981) consider this approach in the management of research and development resources. Ruefli and Storbeck (1984) examine hierarchical decision processes in a non-production context. In a recent paper, Geoffrion (1987) suggests a hierarchical approach to structured modeling. The aim of structured modeling is to provide a formal mathematical framework and computer based environment for conceiving, representing and manipulating a wide variety of models. The framework of structured modeling uses a hierarchically organized, partitioned, and attributed acyclic graph to represent the structure of a model.

The hierarchical approach to production planning is not a new concept. Early motivation for this approach can be found in Holt et al (1960) and Winters (1962). However, in this chapter, we focus on recent developments and describe applications of this approach to resolve production planning and scheduling problems. It should also be recognized that a number of other approaches have been proposed to address these problems and are described elsewhere in this handbook. For example, Chapter 8 provides an introduction to production planning, and Chapter 10 is devoted to scheduling. Mathematical programming models and methods are discussed in Chapter 9, Materials Requirement Planning is described in Chapter 12, and Just in Time philosophy and Kanban systems are described in Chapter 14.

This chapter is organized as follows. In the following section we provide the basic ingredients of Hierarchical Production Planning (HPP) systems and describe, in detail, models for single and multi-stage systems. This section (Section 2.2) also contains a discussion of some important issues related to aggregation and disaggregation in hierarchical systems. In Section 3 we describe the role of feedback mechanisms in HPP and discuss two different interpretations of this term. Section 4 is devoted to issues related to uncertainties and the role of stochastic programming models in HPP systems. Finally, we provide some concluding remarks in Section 5.

2.0 HPP Systems:

Production planning and scheduling in multiproduct systems has received considerable attention in the operations research literature. The focus of most of this work has been on the analysis of individual components of the overall problem - facilities planning, aggregate capacity planning, inventory control, and detailed scheduling. There are few notable exceptions that provide an integrated solution to these problems. The works of Manne (1958), Dzielinski and Gomory (1965), Lasdon and Terjung (1971) and Zangwill (1966) can be interpreted as efforts to integrate decisions in production planning and scheduling. In this approach, a single detailed model (monolithic formulation) is formulated to determine optimal planning and scheduling decisions. For a detailed discussion of these methods, the reader is referred to Chapter 9 of this handbook.

In contrast, in the hierarchical approach to production planning and control the detailed monolithic formulation is replaced by a sequence of models that are consistent with a hierarchy of decisions that have to be made. Aggregate (strategic and tactical) decisions are made first and impose constraints within which more detailed (operational) decisions are made. In turn, the detailed decisions provide the feedback to evaluate the quality of the aggregate decisions. Figure 2.1, from Meal (1984) illustrates the decision hierarchy in the context of production planning and scheduling. Decisions at the higher levels of the hierarchy are invariably based on aggregate models. The success of the hierarchical approach depends, to a large extent, on the consistency between the aggregation and disaggregation procedures, and on the interaction between the models at the different levels. Each hierarchical level has its own characteristics and aggregation methods are typically influenced by a number of factors that include the following:

(i) length of the planning horizon,

(ii) level of detail of the required information and forecasts,

(iii) scope of the planning activity and

(iv) the authority and responsibility of the manager in charge of executing the plan.

Early work using the hierarchical approach was motivated by planning and scheduling problems in discrete parts, batch manufacturing systems. (See Hax and Meal (1975), Bitran and Hax (1977)). In these applications, end products were aggregated into families and product families were grouped into product types. The upper level models were typically linear and mixed linear integer programs while the lower level models were convex knapsack problems. This approach is described, in detail, later in this section.

The HPP approach, however, is quite general and has been adapted to a wide variety of systems by suitable choice of aggregation and disaggregation schemes and submodels. For example, applications of the approach to a continuous manufacturing process and a job shop can be found in Bradley, Hax and Magnanti (1977, Chapters 6 and 10 respectively).

Axsater and Johnson (1984) have used this approach to provide aggregate models for supporting capacity planning decisions in MRP systems. Their model is based on product and machine groups and is designed to provide consistency between machine capacities and the requirements imposed by the detailed schedules derived by the MRP procedure. This paper is discussed in Section 2.4.

Bitran and Tirupati (1988a,b) describe a single stage, parallel machine scheduling application in which resource allocations are determined by an aggregate model. Aggregation in this application is achieved by classifying jobs into product families. The upper level model in this application is a mixed integer, quadratic program that can be interpreted as a machine grouping and aggregate loading problem. As a result, the detailed scheduling problems at the lower level are considerably simplified. Kusiak and Finke (1987) present a hierarchical approach to address the process planning problem in flexible manufacturing systems.

The foregoing discussion indicates that hierarchical planning represents a philosophy to address complex problems, rather than a specific solution technique. In the next section we illustrate this approach by describing, in detail, the method proposed by Hax and Meal (1975), Bitran and Hax (1977) and related work on production planning and scheduling for single stage batch manufacturing systems. This is followed by a discussion of aggregation and disaggregation methods. We note that the single stage model is a simplification of the manufacturing process. In this model the details of the production process are ignored and the system is modeled as a black box with critical resource(s) that limit its capacity. However, the hierarchical approach is amenable for adaptation to more detailed models. As described in Section 2.2, in more detailed multistage models, we distinguish between different stages of production (such as part production, assembly, etc.) and incorporate resource constraints at each stage.

2.1 Hierarchical Planning in Single Stage Systems:

Hax and Meal (1975) introduced the concept of hierarchical planning by recognizing the differences between tactical and operational decisions. Tactical decisions are associated with aggregate production planning while operational decisions are an outcome of the disaggregation process. The hierarchical structure proposed by Hax and Meal and subsequently used by Bitran and Hax (1977, 1981) and Bitran et al (1981) is based on three levels of product aggregation described below:

Items are the final products delivered to the customer and represent the highest degree of specificity regarding manufactured products. A given product may generate a large number of items differing in characteristics such as color, packaging, labels, size, accessories etc.

Product types are groups of items that have similar unit costs, direct costs, holding costs per unit period, productivities (labor hours per unit of product) and seasonalities.

Families are groups of items that belong to the same product type and share similar setups. That is, whenever a machine is prepared to produce an item in a family, all other items in the same family can be produced with minor change in setups.

The classification described above can be illustrated by considering the product line of a luggage manufacturer. Products of the same size can be aggregated to define product type. Within a product type, items with the same frame can be produced with a common setup and constitute a product family. Items within a product family are distinguished by characteristics such as color, minor variations in material, etc.

An overview of the planning process is described in Figure 2.1.1 and essentially consists of three steps, indicated in the figure by boxes 1,2, and 3. In the first step (box 1) aggregate plans for product types are determined. The planning horizon of this model normally covers a full year to take into consideration the fluctuations of demand requirements for the products. The second step in the process (box 2) results in the disaggregation of the aggregate plan for each type to obtain production quantities for each family. Further disaggregation of the family production lots to determine item quantities is performed in the third step (box 3).

It is important to note two features of the process above. First, while the aggregate plan is run every period, only the results for the first period are implemented. Thus the aggregate plan can be viewed as a "rolling horizon" plan. Second, disaggregation of the aggregate plan (steps 2 and 3) is required only for the first period of the planning horizon. As a consequence, the data collection and data processing is reduced substantially compared to the detailed formulations of the production planning problem.

Hax and Meal (1975) proposed a heuristic to perform the three levels of computations. Bitran and Hax (1977) formalized the hierarchical planning heuristic by suggesting the use of convex knapsack problems to disaggregate the product type and family run quantities into family and item run quantities respectively. This method, referred to as the <u>regular knapsack method</u> is described below. To simplify the presentation of these models, in many instances, we assume that the production lead times are zero. This restriction is not necessary. The model formulations and the corresponding results can be modified easily and extended to cases with constant lead times.

<u>Aggregate Production Planning for Product Types:</u>

The following linear program provides a simple representation of the planning problem at the product type level.

<u>Decision Variables:</u>

X_{it} :the number of units of product type i to be produced in period t,

I_{it} :the number of units of inventory of type i carried from period t to t+1,

R_t, O_t :the regular and overtime hours used during period t respectively.

<u>Parameters;</u>

I: number of product types,

T: the length of the planning horizon,

c_{it}: unit production cost (excluding labor),

h_{it}: inventory carrying cost per unit per period,

r_t: regular time cost per manhour,

o_t: overtime cost per manhour,

rm_t, om_t: total availability of regular and overtime hours in period t respectively,

m_i: hours required to produce one unit of product i, and

d_{it} : effective demand for type i in period t. (A definition of effective demand will be given later in this section.)

Problem (P) Min $\sum_{i=1}^{I} \sum_{t=1}^{T} (c_{it} X_{it} + h_{it} I_{it}) + \sum_{t=1}^{T} (r_t R_t + o_t O_t)$

$$\text{s.t} \quad I_{it-1} + X_{it} - I_{it} = d_{it}, \quad i=1,2,\ldots I, \quad t=1,2,\ldots T$$

$$\sum_{i=1}^{I} m_i X_{it} \le R_t + O_t, \qquad t=1,2,\ldots T$$

$$R_t \le rm_t, \qquad t=1,2,\ldots T$$

$$O_t \le om_t, \qquad t=1,2,\ldots T$$

$$X_{it}, I_{it}, R_t, O_t \ge 0.$$

Note that in this model, X and I represent respectively production and inventory variables at the aggregate or product type level. Since the cost (c_{it}, h_{it}) and productivity (m_i) parameters are required to be the same for all items within a family, it may be necessary to scale the item quantities in accordance to their resource consumption. In that case X and I represent the corresponding weighted average quantities of items within each family. It is worth to note that this procedure will be exact with a single resource constraint. However, with more than one resource, it would be an approximation.

The formulation above can be modified easily to incorporate many features such as hiring and firing, constant production lead time, back orders, subcontracting, lost sales etc. It is important that, whenever seasonal variations are present in the demand pattern of product types, the planning horizon of (P) covers a full seasonal cycle. It is not necessary to formulate the aggregate problem (P) as a linear program. Any model which adequately represents the practical setting under consideration would suffice. Linear programming is a convenient type of model at this level because of its computational efficiency and the wide availability of computer codes. The shadow price information and sensitivity analysis make such models very flexible and can help identify opportunities for capacity expansion, market penetration, introduction of new products etc.

Manufacturing set up costs are ignored in the aggregate model (P). This is motivated by the fact that often set up costs have a secondary impact on total production costs and need to be considered at the detailed or operational level. When this is not the case the hierarchical approach can be modified as described in Bitran, Haas and Hax (1982) or the highest level problem can be formulated as in Graves (1982).

Advantages of Aggregate Planning:

We now describe in detail some of the advantages of the aggregate approach compared to the detailed monolithic model. These can be broadly classified into three categories.

(i) A major benefit of aggregate planning is the substantial savings in the costs of data collection to support the planning model as well as the reduced computational requirements. In a detailed model a major information system may be needed to collect the demand, productivity parameters, and cost data as well as prepare forecasts for thousands of individual items.

Aggregation of items can significantly reduce the cost and effort in demand forecasting and data preparation in addition to reducing the computational costs.

(ii) Another important aspect relates to the accuracy of the data. Unless all items are perfectly correlated, an aggregate forecast of demand will have reduced variance. Given the small number of forecasts required, it is possible to employ more sophisticated techniques such as econometric or time series models, and to obtain judgmental input from the concerned managers. Since the decisions considered in the aggregate model are based on total production quantity rather than item level details, increased forecast accuracy of total demand should improve the decision making process.

(iii) The major advantage of the aggregate approach is in the context of implementation. In a detailed formulation with thousands of items, managers may have difficulties in interacting with the model and comprehending the outputs and may get lost in the details. The aggregate formulation facilitates the managers' understanding of the key tradeoffs involved in the production decisions. At this level of planning, most marketing forecasts are made by product types and manpower decisions are made by broad classes of labor.

A Family Disaggregation Model:

The disaggregation model attempts to allocate production quantities of each product type to the families belonging to that type. Coherent disaggregation requires consistency between the allocations among the families and the product type production determined by the aggregate model. In the disaggregation model (Pi) presented below the objective is to determine run quantities for each family so as to minimize the total set up costs.

Let s_j: set up cost for family j,

Y_{j1}: the number of units of family j to be produced in period 1,

d_j: forecast demand for family j,

lb_j, ub_j: lower and upper bounds for Y_{j1},

X_{i1}: production of product type i in period 1 to be allocated among the families, (note that X_{i1} is an input parameter for the model and is derived from the aggregate plan) and

J(i): set of families in product type i that will runout in period 1.

Problem (Pi) $\min \sum_{j \epsilon J(i)} s_j \, d_j/Y_{j1}$

$$\text{s.t.} \quad \sum_{j \epsilon J(i)} Y_{j1} = X_{i1}$$

$$lb_{j1} \leq Y_{j1} \leq ub_{j1}, j \epsilon J(i)$$

The objective function of Problem (Pi) assumes that the family run quantities are proportional to the set up cost and the annual demand for the family. This assumption which is the basis of the economic order quantity formulation, tends to minimize the average annual set up cost. Observe that the total inventory costs have already been established in the aggregate model and do not appear in problem (Pi).

The first constraint of (Pi) assures consistency between the aggregate and disaggregate models. The upper and lower bounds on Y in the second constraint are computed as follows:

$ub_{j1} = \max \{0, os_{j1} - ai_{j1}\}$, and

$lb_{j1} = \max \{0, d_{j1} - ai_{j1} + ss_{j1}\}$.

where os_{j1}, d_{j1}, ai_{j1}, and ss_{j1} denote respectively the overstock limit, the demand, the available inventory, and the safety stock of family j in period 1. J(i) is initially the set of families in product type i that trigger in period 1, i.e., it is the set of indices j such that $d_{j1} + ss_{j1} - ai_{j1} > 0$. Equivalently J(i) can be defined as the set of families whose runout time is less than one time period. All other families are included in a secondary list and are scheduled only if problem (Pi) is infeasible and excess capacity is available for product type i. Bitran and Hax (1981) show that the first constraint of (Pi) can be replaced by $\Sigma_{j \epsilon J(i)} Y_{j1} \leq X_{i1}$ without changing the optimal solution. They also provide an efficient algorithm to solve Problem (Pi).

The above disaggregation approach is motivated by the desire to minimize the set up costs by scheduling only those families that are required to be produced in the period. Hax and Golovin (1978) describe other disaggregation approaches and Bitran et al (1981) provide a comparison of alternate disaggregation procedures. Gabbay (1975) points out that this type of myopic disaggregation could lead to infeasibilities. (This issue is discussed, in detail, later in this section.) Bitran et al (1981) modified the algorithm by introducing a "Look ahead feasibility" rule to counter the problems of infeasibilities.

<u>An Item Disaggregation Model:</u>

Once the quantities Y_{j1} have been determined, it is necessary to allocate this production among items belonging to each family j. For the current planning period all relevant costs have been determined by the previous two stages in the hierarchical process. For example, the inventory holding costs are set by the aggregate plan, while setup costs are determined by the family disaggregation plan. However, the feasible solution chosen will establish the initial conditions for the next period and affect the future costs. It may be observed that the next setup for a family occurs whenever an item in that family is depleted. In order to save setups in future periods, it seems reasonable to distribute the family run quantity among its items in such a way that each item's runout time coincides with the runout time of the family. A direct consequence is that all items will tend to trigger simultaneously. This can be accomplished by solving the following continuous knapsack problem.

Problem (Qj)

$$\text{Min} \sum_{k \epsilon K(j)} \{[(Y_{j1} + \sum_{k \epsilon K(j)} (ai_{k1} - ss_{k1}))/\Sigma_{k \epsilon K(j)} d_{k1}] - [Z_{k1} + ai_{k1} - ss_{k1})/d_{k1}]\}^2$$

$$\text{s.t} \sum_{k \epsilon K(j)} Z_{k1} = Y_{j1}$$

$$lb_{k1} \leq Z_{k1} \leq ub_{k1}, \; k \epsilon K(j).$$

where Z_{k1} is the number of units of item k to be produced in period 1, K(j) is the set of items in family j, d_{k1}, ai_{k1}, ss_{k1}, lb_{k1}, and ub_{k1} represent for item k the same quantities that were discussed for family j in problem (Pi).

The constraints of problem (Qj) are similar to those of problem (Pi) and assure feasibility and consistency during the disaggregation. The two terms inside the square bracket of the objective function represent, respectively, the runout time for family j and the runout time for item k (assuming a perfect forecast). The minimization of the square of the difference will make those quantities as close as possible.

An efficient algorithm to solve (Qj) is presented in Bitran and Hax (1981). It should be noted that the above formulation does not provide for the presence of minor set ups between item changes within a family production run. In such cases the objective function of (Qj) and the solution procedure should be modified to reflect this fact.

In summary, the hierarchical planning system operates as follows:

1. An aggregate forecast is generated for each product type for each period in the planning horizon. Since the number of product types is usually

small, these forecasts can be produced by using fairly sophisticated models (such as regression analysis) that could be prohibitive at the item level. In addition, these forecasts can be reviewed by experienced managers in order to introduce judgmental inputs which the models cannot capture.

2. The product type forecasts are disaggregated into item forecasts by estimating the proportion of total type demand corresponding to each item. These proportions can be updated by using exponential smoothing techniques which are appropriate at the detailed level. Item and family forecasts are required for the first period of the planning horizon.

3. The available inventory for each item is updated. The effective demand for items, families and product types is then computed. (The notion of effective demand is described later in this section.)

4. The production schedule is then determined by solving the aggregate and disaggregation models described earlier. Computer programs to perform these calculations are described in Hax et al (1976).

Issues of Infeasibility and Effective Demand:

The rolling horizon procedure combined with disaggregation may lead to infeasibilities. This may be illustrated by means of a simple example from Bitran and Hax (1977). Consider a 3 period problem with one product type and two items. The demand forecasts are assumed to be perfect and are presented in Table 2.1. The table also provides the initial inventory for each item. The aggregate constraints for the problem are

$$I_0 + X_1 - I_1 = d_1$$

$$I_1 + X_2 - I_2 = d_2$$

$$I_2 + X_3 - I_3 = d_3$$

$$X_1,\ X_2,\ X_3,\ I_0,\ I_1,\ I_2,\ I_3 \geq 0$$

The detailed constraints are

$I_{k0} + Z_{k1} - I_{k1} = d_{k1},\ k = 1,2$

$I_{k1} + Z_{k2} - I_{k2} = d_{k2},\ k = 1,2$

$I_{k2} + Z_{k3} - I_{k3} = d_{k3},\ k = 1,2$

$Z_{kt},\ I_{kt} \geq 0,\ k=1,2,\ t=1,2,3$

Feasibility conditions require that these two constraints are satisfied and that $Z_{1t} + Z_{2t} = X_t,\ t=1,2,3$

For the data in Table 2.1, the reader can verify that although

$X_1 = 8,\ X_2 = 0,\ X_3 = 60,\ I_1 = 29,\ I_2 = 0,\ I_3 = 0$

is a feasible solution to the aggregate problem, it does not have a corresponding disaggregation. The reason for this infeasibility is that the aggregate model ignores the fact that inventory for item 2 cannot be used to satisfy the demand for item 1.

This type of infeasibility can be avoided by working with <u>effective demands</u>. If the initial inventory of an item is not zero, the effective demand for the first period is obtained by subtracting the initial inventory from the demand. If the initial inventory is greater than the demand of the first period then the effective demand for that period is zero. The adjustment process is continued until all inventory is used up. The effective demands for the illustrative example are presented in Table 2.2.

In general, if d_{kt} is the forecast demand for item k in period t, ai_k is its corresponding initial inventory and ss_k its safety stock, the effective demand $\bar{d}_{kt}$ of item k for period t is given by

$$\bar{d}_{kt} = \max\ [0, \sum_{m=1}^{t} (d_{k,m}) - ai_k + ss_k],\ \text{if } \bar{d}_{kt-1}=0 \ (\text{define } \bar{d}_{k0}=0)$$

$$= d_{k,t} \quad \text{otherwise}$$

The effective demand for product type i is then given by the sum of the effective demands of all items belonging to that type, i.e.,

$$\bar{d}_{it} = \sum_{k \epsilon K(i)} \bar{d}_{kt}$$

Even with the use of effective demands, the myopic nature of disaggregation procedure of Bitran and Hax described earlier can give rise to infeasibilities. Their look ahead procedure addresses this issue, but still does not guarantee feasibility. Gabbay (1975) provides a set of feasibility conditions and shows that, if effective demands are used along with these conditions, any feasible solution to the aggregate model generates a feasible solution to the disaggregate model as well. This approach, however, has two drawbacks. First, this approach requires detailed data at the item level for the entire planning horizon which defeats one of the major advantages of the hierarchical approach. Second, the feasibility conditions destroy the knapsack structure of the disaggregation problems and increase the computational complexity. Erschler, Fontan and Merce (1986) address the latter issue and present an equivalent set of feasibility conditions that preserve the knapsack structure of the disaggregation problems. They also interpret the look ahead procedure of Bitran and Hax and show that this procedure is equivalent to imposing the feasibility conditions for two periods - the current period for which disaggregation is required and the following period.

2.2 Aggregation and Disaggregation:

The hierarchical approach described in the previous section should make it clear that aggregation and disaggregation procedures play a crucial role in the success of these methods. This problem is difficult because of the number of factors involved, some of which are not easily quantifiable. In fact, Hax and Meal (1975) provide only guidelines and not a specific procedure for characterizing the product structure of batch production facilities. As described in the previous section, in their framework, items are aggregated to form families and product types. An important consideration in the choice of these procedures is the ability of the disaggregation procedures to obtain feasible solutions at the detailed level. In the previous section we described some of these issues in the context of the Hax and Meal, and Bitran and Hax procedures. In this section we describe other methods that have been reported in the literature for aggregating end items and machines for reducing the size of the planning problem.

The theory of aggregation has been extensively studied in the economic literature. For example, variables like individual prices and incomes are often aggregated into price indices and total incomes. A general overview of this development can be found in Theil (1965), Fisher (1969) and Chipman (1975, 1976). These aggregation problems are similar to, but not identical to those found in production planning problems.

In the operations research literature considerable amount of work has been published on the subject of aggregation of linear and mixed integer linear programs. The primary focus of this work has been on the development of bounds, relative to the original large problem, based on the solution of a smaller problem obtained by aggregation of variables and/or constraints. Since linear and mixed integer linear programs are commonly used to model decisions at the upper levels of the hierarchy, these results are of interest. However, we do not provide a review of the related results in this chapter. Instead, we refer the reader to Geoffrion (1977), and Zipkin (1980a,b) and references therein. The problem of disaggregation has also been considered in the context of aggregate planning (for details see Chapter 8 of this Handbook). Examples of research in this area can be found in Winters (1962) and Zoller (1971). Ritzman et al (1979) present an extensive collection of papers on aggregation and disaggregation in manufacturing and service systems.

Formal approaches to the aggregation problem in the context of hierarchical planning have been considered, among others, by Zipkin (1982) and Axsater (1981). We present some key results from these papers to indicate the flavor of the problems and the difficulties involved. In both approaches the aggregation schemes are derived by focusing on the corresponding disaggregation problems.

Zipkin (1982) considers a multi-item problem in which the cost function for item i, $q_i(y_i)$ is defined on (m',∞) for some $m'<0$ (possibly $-\infty$). The q_i are assumed to have the following form:

$$q_i(y_i) = -c_i y_i + h_i \int_m^{y_i} H(r/\mu_i)dr \qquad (2.2.1)$$

where y_i= inventory of item i, c_i , h_i , and μ_i are constants, $h_i,\mu_i > 0$; H is the same for all i and continuous and increasing on (m',∞); and $m > m'$. The functions q_i are continuously differentiable and strictly convex. The objective is to replace the multiple items by a single aggregate product and obtain a closed form expression for the aggregate cost function, Q(Y). The corresponding disaggregation problem is given by

$$Q(Y) = \min_y q(y), \text{ s.t. } \Sigma y_i = Y$$

where y is an I-vector of y_i, i=1,2,...I,

and $q(y) = \sum_{i=1}^{I} q_i(y_i)$.

Zipkin proposes an approximation for Q(Y), $Q_A(Y)$ which has the same form as (2.2.1) and is given by

$$Q_A(Y) = -c^0 y + h^0 \int_m^Y H(r/\mu)\, dr, \text{ where } \mu = \Sigma_{i=1}^{I} \mu_i$$

The approximation $Q_A(Y)$ is motivated by the desire to define an aggregate problem that is small in size and has the same structure as the detailed problem. It is shown that $Q_A(Y)$ is exact when $c_i = c$ for all i, and

H is monomial, i.e., $H(r) = a\, r^p$ where the constants a and p are either both positive or both negative. The paper provides three methods for determining c^0 and h^0. While the model above has been formulated to describe inventory costs, it can also be used to aggregate production costs. In that case, the variables Y_i represent production quantities and q_i represent production costs.

Zipkin also describes an application of this aggregation scheme to the production smoothing problem of Holt et al (1960) described in Chapter 9 of this handbook and extends it to a broader class of polynomial cost functions. It is shown that the model can also be applied to a two stage facility with J components producing I end items. The aggregate problem, in which the I products are partitioned into N groups, is shown to have the same structure as the detailed problem and is formulated in terms of the product groups. For this approach to be effective it is necessary that J and N should be much smaller than I. Furthermore, the items within a group require the same number of each component which appears to be an unduly restrictive condition.

In a more general context of hierarchical planning, Axsater (1981) considers aggregation procedures in a K item, N machine facility with the following parameters:

I_{kt}: inventory level of item k at end of period t

I_t: $(I_{1t}, I_{2t}, \ldots, I_{Kt})$

Z_{kt}: production of item k in period t

Z_t: $(Z_{1t}, Z_{2t}, \ldots, Z_{Kt})$

a_{kn}: number of units of item k required to produce one unit of item n

A: (a_{kn}), $k,n = 1,2,\ldots K$

d_{kt}: demand for item k in period t

d_t: $(d_{1t}, d_{2t}, \ldots, d_{Kt})$

m_{kn}: production resources on machine n required to produce one unit of k

$M : (m_{kn}),\ k = 1,2,\ldots K;\ n = 1,2,\ldots, N$

The matrices A and M are nonnegative. Constraints defining the item inventories are given by

$$I_t = I_{t-1} + Z_t - A\, Z_t - d_t$$

Axsater considers two types of aggregation procedures to form K′ product groups and N′ machine groups to reduce the size of the planning problem. These are referred to as grouping matrices and general linear aggregation. In the first method grouping matrices $R = (r_{ik})$ and $S = (s_{ik})$ in which all columns are unit vectors, are used to define the aggregation in the following manner:

$r_{ik} = 1$ if item k is included in product group i and 0 otherwise,

$s_{ik} = 1$ if machine k is included in machine group i and 0 otherwise.

Aggregate variables are denoted by $\hat{I}_t$, $\hat{Z}_t$, $\hat{d}_t$. $\hat{A}$ and $\hat{M}$ represent matrices (that correspond to A and M) at the aggregate level and need to be determined. The inventory constraint thus becomes is

$$\hat{I}_{t-1} + \hat{Z}_t - \hat{A}\, \hat{Z}_t - \hat{d}_t = \hat{I}_t$$

For a given production vector Z_t, true component and capacity requirements are given by RAZ_t and SMZ_t and for consistency between the aggregate and the detailed models we require

$$\hat{A}RZ_t = RAZ_t$$

$$\hat{M}SZ_t = SMZ_t$$

"Perfect aggregation" refers to an aggregation scheme that ensures consistency between the aggregate and the detailed models for all possible production vectors Z_t. The necessary and sufficient condition for perfect aggregation is given by finding matrices $\hat{A}$ and $\hat{M}$ that satisfy:

$$\hat{A}R = RA$$

$$\hat{M}S = SM$$

Axsater shows that in general, it is not possible to find perfect aggregation with group matrices and proposes an approximation scheme. This method is motivated by the observation that, while the grouping is static and fixed in the short/medium term, the production vectors are dynamic and vary from period to period. Thus the production vectors can be considered to be stochastic rather than deterministic. Since no single aggregation scheme can be perfect for all realizations of the production vector, a scheme which is perfect in an expected sense may be reasonable. Axsater proposes solution of the following optimization problem (AP) to determine $\hat{A}$ and $\hat{M}$:

$$\text{(AP)} \quad \min_{\hat{A}} E\{ \,||(\hat{A}R-RA)\, Z\, ||^2\}$$

$$\min_{\hat{M}} E\{||(\hat{M}S - SM)\, Z\, ||^2\}$$

$$\text{s.t.} \quad \hat{A}RZ^0 = RAZ^0 \text{ and } \hat{M}SZ^0 = SMZ^0$$

where $Z^0 = E\{Z\}$, and $E\{\cdot\}$ is the expectation operator.

The constraints of (AP) assure that the aggregation defined by $\hat{A}$ and $\hat{M}$ is perfect in an expected sense while the objective function is the expected value of the squared Euclidean norm. Axsater shows that a general solution to (AP) is given by

$$\hat{A} = RAR* + G'G$$

$$\hat{M} = SMR* + G''G$$

where G', G'' are arbitrary matrices of dimension $K' \times K'$ and $K' \times N'$ respectively,

$$R* = Z^0Z^{0T}R^T \,/\, Z^{0T}R^TRZ^0 + (I - Z^0Z^{0T}R^TR \,/\, Z^{0T}R^TRZ^0)\; PR^TQ^t,$$

$G = I - RZ^0Z^{0T}R^T / Z^{0T}R^TRZ^0 - QQ^t$,

$Q = (I - RZ^0Z^{0T}R^T / Z^{0T}R^TRZ^0)\ RPR^T\ (I - RZ^0Z^{0T}R^T / Z^{0T}R^TRZ^0)$,

Q^t = pseudo - inverse of Q and

$P = E\ \{ (Z - Z^0)\ (Z - Z^0)^T \}$

The procedure above to determine $\hat{A}$ and $\hat{M}$ is computationally reasonable. However, a major drawback is the data requirement. While it is easy to obtain the expected production vectors at the item (detailed) level, it is extremely difficult to estimate the variance / covariance matrix P at this level of detail. In practice, the issue is further complicated because it is necessary to determine the grouping matrices R and S together with $\hat{A}$ and $\hat{M}$.

Axsater also considers an alternative scheme - general linear aggregation and provides necessary and sufficient conditions to obtain perfect aggregation. However, this procedure does not seem very attractive since it requires assignment of fractions of items to different product groups. For further results on this subject, the reader is referred to Axsater et al (1983), Axsater and Jonsson (1984), and Axsater (1986).

The above discussion illustrates some of the difficulties associated with aggregation of items and machines and suggests that the definition of appropriate hierarchies of products and machines, in practice, is still an imprecise science. It also highlights the need to develop easily implementable aggregation methods.

2.3 Multistage Models:

The single stage models described so far in this chapter capture essential features of the hierarchical approach to production planning. Extension of this approach to systems with multiple stages requires coordination between the different stages which introduces an additional dimension of complexity. In this section we discuss some of these issues by focusing on the extensions by Meal (1978) and Bitran et al (1982) for two stage systems. Related work can also be found in Maxwell et al (1983) and Gelders and Van Wassenhove (1982). Beyond these references, the literature on HPP for multi-stage systems is quite scanty. This area presents potential for further research.

Meal (1978) describes an integrated distribution planning and control system which highlights some of the difficulties encountered in extending the hierarchical approach to multistage systems. A schematic diagram of the system is presented in Figure 2.3.1. The first two stages model the manufacturing system and correspond to parts production and assembly operations, while the third stage represents the distribution system. The major objective of the planning system is to achieve an integrated control of operations at the three stages. There is no attempt to optimize the decisions at either the aggregate or detailed levels. Figure 2.3.2 presents an outline of the planning system and describes the data requirements and the flow of control information. It can be observed from this diagram that a two level hierarchical system is used to control the operations in the production stages. The aggregate plan in this system is essentially a manpower plan for a horizon of 9 to 18 months. Unlike the models described in the previous section, in this system there is no mechanism to ensure consistency between the aggregation and disaggregation decisions, except for the imposition of capacity constraints. In this respect, the link between the two hierarchical levels can be considered relatively weak, compared to the previous models. Consistent with the system objectives, detailed schedules (disaggregation decisions) are based on tight coupling between the various stages of the production and distribution system, which is achieved by adopting a base stock control procedure. The inventory levels of the system, however, are determined by the aggregate model and are based on long-term forecasts. Figure 2.3.3 presents an outline of the two level hierarchical control system for the two production stages.

In contrast to the manufacturing stages, no aggregate planning is done at the distribution stage. This was considered unnecessary in view of the excess capacity available in order processing and shipping activities. Detailed shipping schedules are prepared daily using a 1 to 3 week horizon and are based on a base stock policy. The outline of the control system for this stage is presented in Figure 2.3.4. Meal (1978) describes several heuristic rules employed in the system to develop aggregate plans and detailed schedules.

Bitran et al (1982) present an extension of the model described in Section 2.1 to a two stage system. The two stages represent respectively, parts production and assembly. Figure 2.3.5 provides a schematic overview of their approach which may be summarized as follows:

1. Aggregation of products and parts.
2. Aggregate planning for the two stages using an integrated model to guarantee appropriate coordination between stages.
3. Aggregate plans for parts and finished products are disaggregated to determine detailed schedules.
4. Reconcile possible differences at the detailed level via part inventories.

The hierarchy for the assembly stage (end products) is the same as that described in Section 2.1 for single stage systems. At the parts production stage only one level of aggregation is employed and parts are classified into part types. Thus, part items are individual parts required as a component to a product item or having an independent demand as a service or spare part. Part types are groups of part items having similar direct production costs, holding costs per part period, and consume the same amount of resources per part. This particular classification was motivated by the fact that, in the application considered, there was no significant shared setups among the parts. The approach, however, is quite general and can be extended to other two stage systems with different levels of aggregation at each stage.

We introduce the following additional notation to describe the aggregation and disaggregation models (This notation is similar to the one used in Section 2.1 with "^" denoting the corresponding variables at the part production stage.):

L: fabrication lead time for parts

J(i): the set of indices of product families in product type i

N(k): the set of indices of parts in part type k

$\hat{h}_{kt}$: holding cost per unit of inventory of part type k from period t to t+1

$\hat{ss}_{kt}$: safety stock for part type k in period t

$\hat{os}_{kt}$: over stock limit for part type k in period t

$\hat{r}_{kt}$: cost of one hour of regular time at the part production stage

$\hat{o}_{kt}$: cost of one hour of overtime at the part production stage

$\hat{m}_{k}$: units of labor consumed to produce one unit of part type k

$(\hat{rm})_{t}$: availability of regular time in period t at the part production stage

$(\hat{om})_{t}$: availability of overtime in period t at the part production stage

$\hat{D}_{kl}$: demand of part type k over the run out time

f_{ijkn}: number of units of part n required by each unit of product family j, $n \epsilon N(k)$, $j \epsilon J(i)$

f_{ik}: average number of parts of type k required to produce one unit of product type i

$\hat{s}_k$: setup cost for parts in part type k

$\hat{lb}_{n1}$ and $\hat{ub}_{n1}$: lower and upper bounds on production quantity of part n in period 1

$\hat{R}_{kt}$: regular time hours for part type k in period t

$\hat{O}_{kt}$: overtime hours for part type k in period t

$\hat{I}_{kt}$: inventory of part type k at the end of period t

$\hat{Q}_{n1}$: quantity of part n scheduled for production in period 1

$\hat{ai}_{nt}$: inventory position of part n at time period t (includes the number of parts on order or being fabricated that will become available in period t.)

Aggregate Production Planning for Part Types and Product Types

The aggregate model is a linear program similar to the single stage model and is formulated as follows:

$$[TP] \min \Sigma_{t=1}^{T} \Sigma_{i=1}^{I} (h_{it}\, I_{it} + r_t\, R_{it} + O_t\, O_{it}) +$$

$$\Sigma_{t=1}^{T-L} \Sigma_{k=1}^{K} (\hat{h}_{kt}\, \hat{I}_{kt} + \hat{r}_t\, \hat{R}_{kt} + \hat{o}_t\, \hat{O}_{kt})$$

$$\text{s.t.} \quad I_{it-1} + m_i(R_{it} + O_{it}) - I_{it} = d_{it},\ i=1,2,\ldots I,\ t=1,2,\ldots T \qquad (2.3.1)$$

$$\sum_{i=1}^{I} R_{it} \le (rm)_t,\ t = 1,2,\ldots T \qquad (2.3.2)$$

$$\sum_{i=1}^{I} O_{it} \le (om)_t,\ t = 1,2,\ldots T \qquad (2.3.3)$$

$$ss_{it} \le I_{it} \le os_{it},\ i = 1,2,\ldots I,\ t = 1,2,\ldots T \qquad (2.3.4)$$

$$\sum_{k=1}^{K} \hat{R}_{kt} \le (\hat{rm})_t,\ t=1,2,\ldots T-L \qquad (2.3.5)$$

$$\sum_{k=1}^{K} \hat{O}_{kt} \leq (\hat{om})_t, \quad t=1,2\ldots T\text{-}L \qquad (2.3.6)$$

$$\hat{ss}_{kt} \leq \hat{I}_{kt} \leq \hat{os}_{kt}, \quad k = 1,2,\ldots K, \quad t = 1,2,\ldots T\text{-}L \qquad (2.3.7)$$

$$\hat{I}_{kt-1} + \hat{m}_k (\hat{R}_{kt} + \hat{O}_{kt}) - \hat{I}_{kt} = \sum_{i=1}^{I} f_{ik}\, m_i (R_{it+L} + O_{it+L}) \qquad (2.3.8)$$

$$k=1,2,\ldots K, \quad t=1,2\ldots T\text{-}L$$

$$R_{it}, O_{it}, \hat{R}_{kt}, \hat{O}_{kt}, \hat{I}_{kt} \geq 0 \qquad (2.4.9)$$

Constraints (2.3.1) and (2.3.8) represent respectively, the inventory balance for the product and part types. The product type demand as explained earlier is the net effective demand and the initial inventory I_{k1} is equal to the safety stock ss_{k1}. Constraints (2.3.8) couple part type requirements and product type production and represent the link between the two stages. The fabrication lead time is modeled as a time lag between initiation of part production and the availability of parts at the assembly stage. Thus part production in period t ($\hat{m}_k$ ($\hat{R}_{kt}$ + $\hat{O}_{kt}$)) is available for assembly in period t+L. The right hand side of (2.3.8) represents the demand for part type k in the assembly stage in period t+L and defines the demand at the part production stage in period t. The other constraints involve either part types or product types but not both. (2.3.2) and (2.3.3) are the regular time and overtime constraints at the assembly stage while (2.3.5) and (2.3.6) are the corresponding constraints at the parts production stage. (2.3.4) and (2.3.7) reflect upper and lower limits on inventories of product and part types respectively. These limits are defined in a manner similar to those in the single stage model. It may be noted that the parts required for the first L periods of production at the assembly stage (constraint 2.3.1) are already being manufactured, or have already been ordered. Still, these constraints are included to make the system responsive to forecast changes in each period.

This is motivated by the fact that at the parts production stage minor variations can be absorbed by either expediting or by having a supplier make a special delivery.

We note that problem TP is similar to the aggregate problem (P) of Section 2.1 and all remarks pertaining to the advantages and disadvantages of the linear programming formulation apply to (TP) as well. Also, TP can be modified to incorporate features such as planned back orders, hiring and layoffs, lost sales and subcontracting. In the same vein, TP is also solved with a rolling horizon of length T. At the end of each period, new information becomes available and is used to update the model. Only the results pertaining to the first L+1 periods for product types, and the first period for part types are implemented.

A critical parameter in problem TP is the definition of f_{ik}. It is the weighted average of the f_{ijkn} and is defined as

$$f_{ik} = \sum_{j \epsilon J(i)} \sum_{n \epsilon N(k)} \bar{d}_j \; f_{ijkn} \,/ \sum_{j \epsilon J(i)} \bar{d}_j$$

where J(i) is the set of indices of the product families in product type i, N(k) is the set of indices in part type k, $\bar{d}_j$ is the annual demand of family j. It is important to realize that f_{ik} is a weighted average of the parts required by individual items in the family. Thus the solution of the aggregate problem does not assure the existence of a feasible disaggregation, even with perfect forecasts. The authors observe that in practice this is not a critical issue and can be taken care of by using safety stocks to provide protection against variations in the bills of materials. These observations are partly justified by a result which guarantees that, under the following conditions:

(i) perfect forecasts are available,

(ii) initial inventory of every family is zero,

(iii) Problem TP is solved just once (it is not solved on a rolling horizon basis), and

(iv) the first L constraints of (2.3.1) are deleted;

the initial inventory of part type k together with the production scheduled by TP up to period t is sufficient to satisfy the sum of the demands, corresponding to the interval [1,t], of all parts in part type k for every t, such that $1 \leq t \leq T - L$. This result is easier to understand after reading the disaggregation scheme described below.

Disaggregation Procedure:

The disaggregation of the aggregate solution to TP is achieved in two steps. In the first step product family requirements for the first L+1 periods and part requirements for the first period are determined jointly to assure consistency between the two stages, while in the second step the detailed item schedule is developed for the assembly (second) stage.

Step 1: Product Family and Part Requirements:

Let X_{it} and $\hat{X}_{kt}$ denote the production of product type i and part type k respectively in period t, i.e., $X_{it} = m_i (R_{it} + O_{it})$, $\hat{X}_{kt} = \hat{m}_k (\hat{R}_{kt} + \hat{O}_{kt})$.

The disaggregation model to determine the production quantities and for product families parts is described as follows:

Problem (TD):

$$\min \sum_{t=1}^{L+1} \sum_{i=1}^{I} \sum_{j \epsilon J(i,t)} s_j D_{jt} / Q_{jt} + \sum_{k=1}^{K} \sum_{n \epsilon N(k,1)} \hat{s}_k \hat{D}_{k1} / \hat{Q}_{n1}$$

$$\text{s.t} \quad \sum_{i=1}^{I} \sum_{j \epsilon J(i,t)} f_{ijkn} Q_{jt} \leq \hat{a}i_{nt} - \hat{s}s_{nt}, \quad k=1,2,\ldots k, \; n \epsilon N(k,t), \; t=1,2,\ldots L+1 \qquad (2.3.11)$$

$$\sum_{i=1}^{I} \sum_{j\epsilon J(i,L+1)} f_{ijkn} Q_{jL+1} \leq \hat{a}i_{nL+1} - \hat{s}s_{nL+1} + \hat{Q}_{n1}, \quad k=1,2,..K, \; n\epsilon N(k,1) \tag{2.3.12}$$

$$\sum_{j\epsilon J(i,t)} Q_{jt} = X_{it} \quad i=1,2,\ldots I, \; t=1,2,\ldots L+1 \tag{2.3.13}$$

$$\sum_{n\epsilon N(k,1)} \hat{Q}_{n1} = \hat{X}_{k1}, \; k=1,2,\ldots K \tag{2.3.14}$$

$$lb_{jt} \leq Q_{jt} \leq ub_{jt}, \; j\epsilon J(i,t), \; i=1,2,..I, \; t=1,2,\ldots L+1 \tag{2.3.15}$$

$$\hat{l}b_{n1} \leq \hat{Q}_{n1} \leq \hat{u}b_{n1}, \; n\epsilon N(k,1), \; k=1,2,\ldots K \tag{2.3.16}$$

where J(i,t) and N(k,1) are the set of indices of families in product type i and parts in part type k that are triggered (will run out) in period t and 1 respectively. Similarly to the single stage models of Section 2.1, the objective of the disaggregation procedure is to minimize the set up costs at both stages. The objective function of (TD) assumes that while a set up is required for each part, the set up cost is equal for parts within a part type. Thus $\hat{s}_k$ can be interpreted as the average set up cost for parts within part type k. Constraints 2.3.11 could have been omitted and the production within the lead time "frozen". However, this was not done in problem TD since some corrections can be accommodated in practice either by expediting production or by having special deliveries made by suppliers.

Constraints (2.3.12) together with (2.3.11) assure that the part production lots are sufficient to meet part requirements for L+1 periods, (2.3.13) and (2.3.14) ensure consistency between the aggregate model and the scheduled lot sizes. (2.3.15) and (2.3.16) impose upper and lower bounds on the production lot sizes. These bounds are defined in the same manner as in single stage models.

Problem TD has a convex objective function and linear constraints and is similar to the disaggregation problem (Pi) described in Section 2.1. It can be shown that (TD) can be decomposed into continuous convex knapsack subproblems and each can be solved through procedures similar to those used to solve problem (Pi).

Step 2: Disaggregation to determine product item requirements:

Once the product family run quantities are determined, the item run quantities are computed by solving a disaggregation problem similar to that encountered in single stage problems (problem Qj of Section 2.1). The product item run quantities are determined by equalizing the run out times for items within a family.

2.4 Materials Requirement Planning (MRP) and HPP:

MRP is perhaps the most commonly used approach to deal with multi-stage production planning problems and has become a benchmark for evaluation of HPP systems. (For a detailed description of this approach, the reader is referred to Chapter 12 of this handbook and references therein.) The basic idea in MRP is to start with a master schedule for the final products, which is then exploded to compute the requirements for all parts. When all the requirements for a given part have been consolidated, an individual production schedule is developed for each item, based on appropriate lot sizing procedures. Typically, MRP systems offer a number of alternative methods for deriving item and part schedules. In most MRP systems, the master schedule is an external input. Thus MRP can be viewed as an information system and a simulation tool that generates proposals for production schedules which managers can evaluate in terms of their feasibility and cost effectiveness. In its present structure, MRP does not deal directly with optimization criteria associated with multilevel production problems. The lack of appropriate support for managers to generate good master schedules usually leads to infeasibilities of schedules due to capacity constraints. This fact is cited as one of the major weaknesses of MRP.

In contrast, the objective in HPP is to develop, at an aggregate level, a joint product type-part type schedule that is consistent and recognizes resource (capacity) limits. The schedule also attempts to minimize the primary costs. Moreover, the aggregate plan is concise and facilitates the understanding of its implications. The disaggregation procedures in HPP focus only on the time periods that cover a lead time, which avoids excessive amount of data and computational work. Bitran, Haas and Hax (1982) contrast the two approaches for a two-stage system and provide a numerical example to illustrate these differences. In this illustration, the master schedule for MRP was determined using the hierarchical procedure of Section 2.1 for single stage systems (end products). The Silver-Meal heuristic was used in developing detailed schedules for parts and items. The two stage model described in Section 2.3 was used to derive the schedules based on the HPP approach. The computational results of that paper suggest that considering explicitly the cost criteria and capacity constraints significantly improves the quality of the production plans.

Axsater and Jonsson (1984) likewise note the limitations of MRP and suggest that the HPP philosophy can be used to develop systems to support MRP and overcome some of its limitations. They observe that MRP can be considered as a particular form of hierarchical structure, with end products at the highest level and parts and components at the lower levels. They remark that this hierarchy is not suitable in the presence of capacity limitations and suggest that the master schedule for MRP should be developed on the basis of other aggregation methods. Their paper reports simulation results of such a model developed for a Swedish company manufacturing rock drilling equipment. In the aggregate model items and parts are aggregated into three product groups based on number of operations; purchased items,

items with at most five operations and the rest with more than five operations. Axsater and Jonsson examine two alternative methods for aggregation of machine centers into machine groups. In the first method machine groups are based on production load; a utilization of 75% was used as a cut-off to form two machine groups. In the second method, aggregation is based on similarity of product flows. In this method a similarity coefficient sij between two machines i and j is defined as

$$s_{ij} = \frac{\text{number of items processed on both machines i and j}}{\text{number of items processed on machine i and/or machine j}}$$

This measure developed by McAuley (1972) is commonly used in Group Technology and FMS applications in clustering parts and machines. For machine groups I and J containing n_I and n_J machines respectively, the similarity coefficient is defined in an analogous manner as

$$s_{IJ} = \frac{1}{n_I n_j} \sum_{i \epsilon I} \sum_{j \epsilon J} s_{ij}$$

Axsater and Jonsson suggest the use of models similar to those described in Section 2.2 for determining aggregation matrices specifying part and capacity requirements for a given grouping scheme. Aggregate production plans are determined sequentially using a hierarchy of objectives. First, aggregate plans for order releases are found by minimizing the sum of echelon stock and undelivered orders. In the second step production of raw materials and parts are determined. The priority of objectives in this step are as follows:

(i) minimize the deviation of machine load from capacity,

(ii) minimize the echelon stock at each stage, and

(iii) minimize the net production.

The objective of disaggregation procedures is to obtain order releases for end items and parts that are consistent with the aggregate plans. The authors suggest that the MRP logic may be used in deriving the detailed schedule by modifying the order quantities to maintain consistency between the two levels. However, they adopt a procedure in which order release times are altered at each stage and the MRP logic is not strictly observed. The detailed schedules are derived based on the following priority scheme for order releases.

(i) Items needed to replenish safety stocks of final products.

(ii) Final products with negative slack.

(iii) Orders with earlier starting date according to the MRP system.

Simulation experiments with the two systems indicate that the hierarchical planning approach results in lower costs (statistically significant at 3% level with t-test and 7% with Wilcoxon test) compared to the stand alone MRP system. These experiments also examine the impact of alternate methods for aggregation and disaggregation procedures on the quality of the schedules.

The discussion above demonstrates the fact that MRP and HPP concepts are complementary rather than competitive. (Meal et al (1987) illustrate this complementarity with an application from the computer industry.) For example, MRP systems can be improved by use of optimization models at the aggregate level to derive good master schedules. Also, HPP methods have been developed for single and two stage systems only and extensions to multistage plants are not easy. The literature on this subject is quite scanty and there is considerable research potential to develop systems that integrate features of both systems.

3.0 Feedback Mechanisms in HPP

An important component of hierarchical systems is the interaction between models at different levels to ensure consistency between the planning and scheduling decisions. "Feedback" refers to this interaction and represents a critical link between the aggregate and the detailed decisions. This term, however, has been used with different meanings in the context of HPP. For some, feedback refers to the flow of information from the disaggregation problem to the aggregate level at the end of each period. To others it has meant a mechanism like the pricing procedure in generalized programming or like the obtainment of a sequence of convergent dual variables in subgradient algorithms. In this section we review both interpretations and the results presented in the literature. This discussion is primarily based on the models of single stage systems presented in Section 2.1.

In its simplest form the feedback from the detailed level includes the actual realization of the production and demands for each item. The information to the aggregate problem is the inventory levels of the product type. Another component of the feedback, not necessarily from the detailed level, includes revised forecasts of the demand at the product type level. A consequence of this information is the modification of the aggregate plan by resolving problem (P) in each time period. The solution of the aggregate problem on a rolling horizon basis can thus be interpreted as the manifestation of the feedback mechanism in the Bitran and Hax procedure.

This information flow has other uses as well. For example, the myopic procedure of Section 2.1, for disaggregation of product type quantities into family lot sizes, could lead to infeasibilities. Bitran et al (1981) propose a "look ahead feasibility rule" to overcome this problem. In this modification the families scheduled for production in a given period are based on (i) the revised aggregate plan, and (ii) the revised demand forecasts for the first two periods. This procedure is still based on knapsack problems and is computationally efficient.

This type of feedback mechanism has cost implications also. It may be recalled that the hierarchical structure of the models presented in section 2.1 is suitable for systems in which the set up costs are not very significant. In such cases ignoring such costs at the aggregate level is not very critical. Bitran, Haas and Hax (1982) observe that these procedures are effective as long as the set up costs do not exceed 15% of the total production costs. They propose a modification of the regular knapsack method for cases with high set up cost. This heuristic modification adjusts the family run quantities (determined by the solution to problems P) to a value as close as possible to the "ideal lots" of the corresponding dynamic lot sizing problem. Motivated by computational considerations, the authors propose the use of the Silver-Meal heuristic to determine these ideal lot sizes.

Graves (1982) presents an alternative method to address the feedback question. In this approach "feedback" between levels corresponds to the pricing procedure of generalized programming methods. Graves first formulates a mixed integer programming model combining the product type planning and family disaggregation decisions. He proposes the use of Lagrangean relaxation to solve the problem. The mixed integer programming model is described as follows:

$$\text{(MM)} \min \sum_{t=1}^{T} (o_t O_t + h_{it} I_{it}) + \sum_{j \epsilon J(i)} \sum_{t=1}^{T} s_t \hat{Y}_{jt}$$

$$\text{s.t} \quad I_{it-1} + X_{it} - I_{it} = d_{it} \qquad i=1,2,\ldots I \quad t=1,2,\ldots T \qquad (3.1)$$

$$\sum_{i=1}^{I} m_i X_{it} - O_t \leq (rm)_t \, , \quad t=1,2,\ldots T \qquad (3.2)$$

$$\sum_{j \epsilon J(i)} \hat{I}_{jt} - I_{it} = 0 \qquad , \; i=1,2,\ldots I \quad t=1,2,\ldots T \qquad (3.3)$$

$$Y_{jt} + I_{jt-1} - I_{jt} = 0 \qquad j \epsilon J(i), \; i=1,2,\ldots I \qquad (3.4)$$

$$Y_{jt} - \hat{m}_j \hat{Y}_{jt} \leq 0 \qquad t=1,2,\ldots T \qquad (3.5)$$

$$\hat{Y}_{jt} \in \{0,1\} \tag{3.6}$$

$$O_t,\ X_{it},\ Y_{jt},\ I_{it},\ \hat{I}_{jt} \geq 0 \tag{3.7}$$

where $\hat{I}_{jt}$ and $\hat{Y}_{jt}$ are the additional variables introduced at the family level. $\hat{I}_{jt}$ is the inventory of family j at the end of period t, while $\hat{Y}_{jt}$ is a binary variable associated with the set up of family j in period t.

The objective function of model (MM) minimizes the inventory holding costs, overtime costs and the set up costs. This model assumes that the regular time and other production costs are fixed and hence excluded from the objective function. Constraints (3.1) and (3.2) correspond to the aggregate decisions, while (3.4), (3.5) and (3.6) correspond to the family disaggregation problem. (3.3) represent the linking constraints between the two models. Relaxation of constraints (3.3) give rise to the dual problem

(MMD) max $L(\lambda)$

where $L(\lambda) = \min \left[Z + \Sigma_{i,t}\, \lambda_{it}\, ({}_j\Sigma\, \hat{I}_{jt} - I_{it})\right]$

s.t $Z = \Sigma_t\, (o_t\, O_t + {}_i\Sigma\, h_{it}\, I_{it}) + {}_j\Sigma\, {}_t\Sigma\, \hat{s}_j\, \hat{Y}_{jt}$

and (3.1), (3.2), (3.4), (3.5), (3.6) and (3.7)

The dual problem (MMD) decomposes into subproblems P' and P_i' described below.

P' $\min \Sigma \{o_t\, O_t + {}_i\Sigma\, I_{it}(h_{it} - \lambda_{it})\}$

s.t (3.1), (3.2) and (3.7)

P_i': $\min \sum_{j \in J(i)} \sum_{t=1}^{T} s_j\, \hat{Y}_{jt} + \lambda_{it}\, \hat{I}_{jt}$

s.t (3.4), (3.5), (3.6) and (3.7)

Problems P and P' differ in the definition of the inventory carrying costs. In P' the parameter h_{it} is modified to adjust for the dual multiplier of constraint (3.3). Problems P_i' address the same issue as P_i , i.e., disaggregation of product types into family lots, but has a different structure. P_i' decomposes into a set of uncapacitated Wagner-Whitin type lot sizing problems that can be solved efficiently using dynamic programming algorithms. Graves presents an iterative procedure to solve (MMD) and suggests that a heuristic or a branch and bound procedure may be followed to obtain a good feasible solution.

A comparison of the two approaches suggests that the Lagrangean relaxation procedure is likely to provide more cost effective schedules that are likely to be significantly better in cases with high set up costs. However, this should be balanced against the computational requirements and the complexity of the algorithm.

4.0 HPP and Stochastic Programming

The hierarchical models described so far in this chapter are primarily deterministic in nature. However, in many real life production situations uncertainties cannot be ignored. In this section we describe three applications of the hierarchical approach to provide an overview of the research in this area. In Section 4.1 we present a job shop design / scheduling problem. This is based on the work by Dempster et al (1980,1981) and provides a framework for evaluating the hierarchical approach. Section 4.2 is based on the work by Bitran, Haas and Matsuo (1986) and deals with the production planning and scheduling problem in the manufacture of style goods. In Section 4.3 we describe Gershwin's (1987) framework for addressing scheduling and control problems in dynamic manufacturing systems with machine failures, setups and demand changes.

4.1 A Job Shop Design / Scheduling Problem

Dempster at al (1981) argue that hierarchical models represent a stochastic, multi-level decision process in which decisions at higher levels are often based on aggregate imperfect information. They suggest that such decisions should be based on accurate models of lower level activities that incorporate stochastic parameters to capture the uncertainties in the detailed decisions. They suggest that the objective at each level be the minimization of the current costs plus the expected value of the lower level decisions. The combination of stochastic optimization and scheduling problems makes the resulting formulations very hard to solve. The authors interpret the hierarchical approach as heuristics to solve the global problem. They suggest that the multistage stochastic formulation provides a useful framework to evaluate alternative approaches to solve the problem addressed by hierarchical production planning. Their results focus primarily on the analysis and on the development of bounds and heuristics to solve approximately their stochastic programming formulation. In what follows, we illustrate their approach by means of a simple two-level problem described in Dempster et al (1980).

In this simplified example, it is assumed that the number of jobs to be processed is known. The design (higher level) decision consists in determining the number of identical parallel machines to be purchased. At this stage, the job processing times are unknown and are assumed to be random variables with independent distributions. At the detailed level, the number of machines is considered fixed since it is an output of the first stage decisions, and the job processing times are known exactly. The resulting problem is to determine a schedule which minimizes the makespan. (Makespan of a schedule denotes the time required to complete all the jobs.) We use the following notation to describe the model formulations:

n: number of jobs

c: cost of a single machine

m: number of machines

p_j: processing time of job j

$p = (p_1, p_2, \ldots, p_n)$

$C^*(m,\tilde{p})$: minimum makespan to complete n jobs with m machines and known $\tilde{p}$. A tilde (~) above a variable indicates that it is a random variable and E denotes its expected value.

Dempster et al propose the following two stage stochastic problem to model this situation:

(MSP) $Z^* = \min \{cm + E\, C^*(m, \tilde{p})\}$

The deterministic, parallel machine scheduling problem (with fixed number of machines) to minimize the makespan represents the detailed (second) level optimization problem. This problem, by itself is NP-hard which makes MSP also very difficult to solve. The authors propose a two level hierarchical procedure to solve the problem approximately. At the detailed level, schedules are obtained by a list processing heuristic. In this method, jobs are chosen in an arbitrary manner and assigned to machines by a single pass heuristic. Each job is placed on the machine that has the least processing load already assigned. The makespan corresponding to this schedule is denoted by $C^{Ls}(m,p)$. At the first level an approximate solution to MSP is obtained by solving a related problem MSP' given below.

(MSP') $\min_m \{cm + E\, \tilde{P}/m\}$ where $P = \sum_{j=1}^{n} p_j$

The above approximation is motivated by the facts that P/m represents a lower bound on the make span and it is asymptotically optimal in the number of jobs. The optimal solution to MSP' is given by m^H such that $m^H \epsilon \{\lfloor E\tilde{P}/c \rfloor, \lceil E\tilde{P}/c \rceil\}$, subject to $m^H \geq 1$, where $\lceil a \rceil$ denotes the smallest integer not less than a and $\lfloor a \rfloor$ denotes the largest integer not greater than a. The overall value realized by this hierarchical approach is then given by

$$Z^H = cm^H + E\ C^{Ls}\ (m^H,\ \tilde{P})$$

It is easy to show that $Z_H/Z^* \leq 1 + E\ p_{max}/\ (2\sqrt{cE\tilde{P}})$ where $p_{max} = \max\ \{p_j\}$. This result provides bounds on the performance of the hierarchical approach. The bound is reasonable as long as p_{max} is sufficiently small. The authors also show that if the p_j s have independent and identical distributions with finite second moments, then the hierarchical system is asymptotically optimal in the sense that

$$\lim_{n\to\infty} (Ep_{max}/\sqrt{E\tilde{P}}) = 0 \text{ and hence } \lim_{n\to\infty} (Z^H/Z^*) = 1.$$

It is interesting to note that the hierarchical structure proposed for this problem is consistent with the Hax-Meal framework described earlier. At the first level all jobs are replaced by the aggregate processing requirements and complicating details are omitted. The authors conjecture that similar approaches would work well for more complicated systems because the instances for which the higher level assumptions are severely violated occur with decreasingly small probability as the problem grows larger.

A detailed discussion of this approach and extensions to more elaborate models can be found in Dempster et al (1981), Dempster (1982) and Lenstra et al (1984). Their research focuses on (i) the development of heuristics to solve the multistage stochastic program and (ii) the derivation of relations between performance measures in related model formulations. A summary of the results relating the performance measures for a two stage decision model is presented in Figure 4.1.1. In developing these relations, the authors consider, in addition to the exact and approximate formulation based on the hierarchical approach, a third model based on perfect information. This "omniscient" model represents the best scenario in which all information is known with certainty before the first stage decisions are made and provides a lower bound on the multistage decision model. The following notation would be useful in interpreting the results of Figure 4.1.1.

x: first stage decisions,

X: set of feasible decisions at the first stage,

$\tilde{w}$: vector of resource requirements at the second stage,

F: distribution function of w

W: sample space for w

$g^*(x,w)$: cost of optimal decision at the second stage, given the first stage decision x and the realization w of resource requirements,

$f(x)$: cost of acquisition of x at the first stage, and

$Z^*(x,w)$: $f(x) + g^*(x,w)$.

Figure 4.1.1 describes the relations between the following three models:

<u>Stochastic program for the two stage decision process:</u>

$$EZ^* = E(Z^*(x^*,\tilde{w})) = \min_{x \in X} \{E(Z^*(x,\tilde{w}))\}$$

<u>Omniscient Model:</u>

$$Z^0 = Z^*(x^0(w),w) = \min_{x \in X} \{Z^*(x,w)\}$$

Note that Z^0 (like Z^*) is a function of the resource requirements w and is a random variable. The expectation of Z^0 with respect to w, EZ^0 would be the appropriate measure to compare the performance of this model with that of the stochastic program, EZ^*.

<u>Hierarchical Approach</u>:

In this model, in the first stage decision, $E(g^*(x,w))$ is replaced by an estimate $g^{H1}(x)$, and the first stage decision x^H is determined as

$$Z^{H1}(x^H) = \min_{x \in X} \{Z^{H1}(x)\} = \min_{x \in X} \{f(x) + g^{H1}(x)\}$$

In the second stage x^H is given and the requirements w are known. The decisions are made using a detailed model at a cost of $g^{H2}(x^H,w)$. (The authors observe that while $g^{H2}(x^H,w)$ could be the result of solving optimally the detailed model, in most cases, the detailed problems are hard. In these cases, $g^{H2}(x^H,w)$ may be the result of an approximate solution to the second stage model.) Also, the information available at the two decision stages is different, and the functions $g^{H1}(x)$ and $g^{H2}(x^H,w)$ are usually different. The cost of the decisions based on this hierarchical approach is then given by

$$Z^H = f(x^H) + g^{H2}(x^H, w).$$

Z^H, like Z^0, is a function of w, and EZ^H is the appropriate measure for comparison with EZ^*.

Figure 4.1.1 presents a summary of results relating to the ratios of cost functions Z^*, Z^H and Z^0 and their expectations. For example, if $Z^H/Z^0 \to 1$ with probability 1 (wp1), then $Z^H/Z^* \to 1$ wp1. However, the converse is not true, and $Z^H/Z^* \to 1$ wp1 does not imply that $Z^H/Z^0 \to 1$ wp1. Similarly, each of the conditions $Z^H/Z^0 \to 1$ in probability (ip), $E(Z^H/Z^0) \to 1$ and $EZ^H/EZ^0 \to 1$ imply that the others are true. Also these conditions imply that the corresponding results hold for Z^H/Z^*. Again, the converse is not true and $Z^H/Z^* \to 1$ ip does not imply that $Z^H/Z^0 \to 1$ ip.

The approach above is most suitable when the higher level decisions are irreversible as in the case of acquisition of machines. For example, in the production planning problems discussed in sections 2 and 3, the aggregate decisions were flexible in the sense that plans were revised every period. In contrast, the models of Dempster et al assume that the first stage decisions, once made, cannot be altered (as in the case of purchase of machines).

4.2 Production Planning and Scheduling with Stochastic Demand

In the production planning and scheduling models described in Sections 2 and 3 uncertainties occur primarily because of errors in demand estimates. Also, aggregate decisions are somewhat flexible and permit minor changes based on the forecast revisions. The rolling horizon approach to the aggregate problem incorporates this aspect of the problem. A justification for the disaggregation procedure of Section 2 for the case with stochastic demands can be found in Agnihotri et al (1982). The authors show that, with stochastic demands, the Bitran and Hax disaggregation procedure provides a lower bound to the family run out time.

Bitran, Haas and Matsuo (1986) explicitly consider uncertainties in demand estimates and forecast revisions while examining production planning and scheduling issues in the manufacture of style goods. Style goods are characterized by a very short selling season and stochastic demand. Because of capacity limitations, manufacturers of style goods usually build up inventory over the year preparing for demand in the selling season. If the demand exceeds on-hand inventory, a shortage cost is incurred, while if the opposite occurs, an overage cost is incurred. Examples of style goods can be found in a variety of situations ranging from clothing to consumer durables. The problem is similar to the multi-item newsboy problem with capacity constraints with two additional characteristics described below.

First, the products have a hierarchical structure. That is, individual items are categorized into families. A family is defined as a set of items that share a common setup, consume the same amount of resources per unit, and have the same magnitude of forecast errors. Setup costs are so large that all items within a family must be produced in a single setup.

Hence, the production planning decisions consist in determining the sequence in which the product families will be produced and the production lot sizes for items within each family with the objective of minimizing the total cost.

The second feature relates to demand forecasts and revisions during the planning horizon. The mean demand for each family is assumed to be invariant over time. However, demand forecasts for items are revised continuously over the planning horizon. The authors assume that the planners can estimate the improvement in the accuracy of forecasts, perhaps based on historical trends. Forecast accuracy is measured by standard deviation of forecast errors. For example, the volume of a standard line of products can be forecast nearly as accurately in January as in October, while the accuracy of forecasts for new products can be expected to significantly improve over time. Intuition suggests that to take advantage of this characteristic, some standard products should be produced early in the year and the production of families with a potential for large improvement in forecast should be deferred.

Bitran, Haas and Matsuo formulate the problem as a mixed integer stochastic program and propose a two-stage hierarchical approach to solve this difficult problem. The aggregate problem is formulated as a deterministic mixed integer program that provides a lower bound on the optimal solution. The solution to this problem determines the set of product families to be produced in each period. The second level problem may be interpreted as a disaggregation stage where item lot sizes are determined for families scheduled in each period. We now describe the problem in detail.

<u>Notation:</u>

i: index for families, n: number of families

j: index for items, N: number of items

$J(i)$: set of indices of items in family i, n_i: number of items in family i

T: number of time periods in the planning horizon

m_{jt}: time-t forecast of the demand of item j (This parameter represents the demand forecast for item j at time t and is discussed in detail later.)

M_i: mean demand for family i

X_{jt}: production quantity of item j in period t

X_{it}: production quantity of family i in period t

r_{it}: resource consumption for producing one unit of an item in family i in period t

R_t: resource available in period t

s_{it}: set up cost for family i in period t

h_{jt}: inventory holding cost of item j in period t

v_{jt}: material cost of item j in period t

v_{jt}': variable production and inventory holding cost of item j

$$= v_{jt} + \sum_{k=t}^{T} h_{jk}$$

P_j: unit selling price of item j

B_j: loss of goodwill due to shortage of one unit of item j

G_j: salvage value of item j

$\delta(X_{jt}) = 1$ if $X_{jt} > 0$ and 0 otherwise

$d_{.t}$: N-component vector $(d_{1t}, d_{2t}, \ldots, d_{Nt})$

$m_{.t}$: N-component vector $(m_{1t}, m_{2t}, \ldots, m_{Nt})$

Forecasts of item demands and their revisions play an important role in the production planning and scheduling problem examined by Bitran, Haas and Matsuo. The authors make several assumptions in characterizing the demand behavior. These are summarized below:

1. The demand estimates in period t for items within a family follow a joint normal distribution.

2. The mean family demand is known in period 1 for all families and does not change over time, i.e.,

   $\sum_{j \in J(i)} m_{jt} = M_i, \ i=1,2,\ldots n, \ t=1,2,\ldots T$

3. The demand estimates of items in family i have a covariance matrix $\sigma_{it}^2 \Sigma_i$ in period t for i = 1,2,...n, t=1, 2,...T, where Σ_i is an n_i x n_i correlation coefficient matrix.

4. The precision of the forecasts are known in period 1, i.e., the standard deviation of forecast errors of items in family i, σ_{it}, is known for i = 1,2,...n, t = 1,2,...T in period 1.

5. In period t, the demand of items in family i are denoted by random variables $(d_{1t}, d_{2t}, \ldots d_{n_i t})$ with joint normal distribution

   $N\left((m_{1t}, m_{2t}, \ldots m_{n_i t}), \ \sigma_{it}^2 \ \Sigma_i\right)$

6. The forecast accuracy is assumed not to decrease as t increases, i.e.,

   $\sigma_{i1} \geq \sigma_{i2} \geq \ldots \geq \sigma_{iT}, \ i=1,2,\ldots n$

The above assumptions imply that the forecasts in period t for items in family i, $(m_{1t}, m_{2t}, \ldots m_{n_i t})$ follow a joint normal distribution with mean $(m_{11}, m_{21}, \ldots m_{n_i 1})$ and covariance matrix $(\sigma_{i1}^2 - \sigma_{it}^2)\Sigma_i$.

The formulation in Bitran, Haas and Matsuo applies when the setup costs of producing each family are substantial. The authors assume that each family is setup exactly once in the planning horizon and that all production of a given family occurs during one period. They point out that this assumption is unlikely to be critical when the number of families is much larger than the number of time periods. A consequence of these assumptions is that, for each family i, only one of the X_{it}s, t=1, 2,...T is positive. The cost function can then be formulated in a manner similar to that in a newsboy problem. The overage and underage costs of producing item j in period t are then given by $(v'_{jt} - G_j)$ and $(P_j + B_j - v'_{jt})$ respectively. The cost of producing family i in period t can then be written as follows:

$$\sum_{j \epsilon J(i)} f_{jt}(X_{jt}) + s_{it}$$

where $f_{jt}(x_{jt}) = (P_j + B_j - v'_{jt})\,(d_{jt} - x_{jt}) + v'_{jt}\, d_{jt}$, if $d_{jt} \geq x_{jt}$

$= (v'_{jt} - G_j)\,(x_{jt} - d_{jt}) + v'_{jt}\, d_{jt}$, if $d_{jt} < x_{jt}$.

The stochastic mixed integer program (P) presented below models the production planning problem as a cost minimization program.

(P)

$$V_p = \min \sum_{t=1}^{T} E_{m.t|m.1} \min E_{d_{.t}|m_{.t}} \sum_{i=1}^{n} \{ \sum_{j \epsilon J(i)} f_{jt}(X_{jt}) + s_{it} \} Y_{it}$$

$$\text{s.t } \sum_{t=1}^{T} Y_{it} = 1, \quad i=1,2,\ldots n \qquad (4.2.1)$$

$$\sum_{i=1}^{n} \sum_{j \epsilon J(i)} r_{it}\, X_{jt} \leq R_t, \quad t=1,2,\ldots T \qquad (4.2.2)$$

$$X_{jt} \leq M\, Y_{it}, \quad j \epsilon J(i), \quad t=1,2,\ldots T, \quad i=1,2,\ldots n \qquad (4.2.3)$$

$$Y_{it} \ \epsilon \ \{0,1\}, \quad i=1,2,\ldots n, \quad t=1,2,\ldots T \qquad (4.2.4)$$

$$X_{jt} \geq 0, \quad j \epsilon J(i), \quad i=1,2,\ldots n, \quad t=1,2,\ldots T \qquad (4.2.5)$$

$[\min E_{d_{.t}|m_{.t}} [\sum_{j \epsilon J(i)} f_{jt}(X_{jt})] + s_{it}]\, Y_{it}$ represents the optimum cost of scheduling family i in period t. Note that at time t, the forecasts are $m_{.t}$ and demands for items in family i follow a joint normal distribution with mean $(m_{it}, m_{2t}, \ldots, m_{n_i t})$ and covariance matrix $(\sigma^2_{i1} - \sigma^2_{it})\, \Sigma_i$. However, in period 1, $m_{.t}$ are random variables (with mean $m_{.1}$) and the expected cost in period 1 for scheduling family i in period t is given by the expectation of the expression above with respect to $m_{.t}$. Hence the objective function of

(P) can be interpreted as the expected cost of the production plan that is based on information available in period 1. The constraints of problem (P) are rather straight forward. (4.2.1) ensures that each family is produced exactly once, while (4.2.2) impose resource restrictions in each period. (4.2.3) assures consistency between family and item production schedules.

It is obvious that (P) is a hard problem with little hope for obtaining an optimal solution for most practical cases. Bitran, Haas and Matsuo propose an approximate solution based on a hierarchical approach. In period 1 an aggregate problem (MIP) presented below is solved at the product family level.

$$\text{MIP:} \quad V_{mip} = \min \sum_{i=1}^{n} \sum_{t=1}^{T} f_{it} (X_{it}) Y_{it}$$

s.t. (4.2.1), (4.2.2), (4.2.3), (4.2.4) and (4.2.5),

where $f_{it}(X_{it}) = \min E_{d_{.t}|m_{.t}=m_{.1}} \sum_{j \epsilon J(i)} f_{jt}(X_{jt}) + s_{it}$

$$\text{s.t.} \sum_{j \epsilon J(i)} X_{jt} = X_{it}, \; X_{jt} \geq 0$$

The purpose of the aggregate problem MIP is to specify the families that need to be produced in each period. The item lot sizes determined by MIP are ignored. Instead, the authors propose the solution of the disaggregation problem (SP), given below, in each period.

$$\text{SP:} \quad v(s_t, m_{.t}) = \min E_{d_{.t}|m_{.t}} \sum_{i \epsilon S_t} \sum_{j \epsilon J(i)} f_{jt} (X_{jt}) + \sum_{i \epsilon S_t} s_{it}$$

$$\text{s.t.} \quad \sum_{i \epsilon S_t} \sum_{j \epsilon J(i)} r_{it} X_{jt} \leq R_t,$$

$$X_{jt} \geq 0, \; j \epsilon J(i), \; i \epsilon S_t$$

where S_t = set of families scheduled in period t, determined by MIP.

Bitran, Haas and Matsuo provide extensive justification for the approach described above. They show that V_{mip} is a lower bound on V_p. They also demonstrate that if the non-negativity constraints (4.2.5) are relaxed, the optimal objective function values for problems P and MIP (V_p, and V_{mip},) are equal. It is further argued that in most applications the non-negativity constraints are violated with low probability and hence MIP should be a good approximation to P. To obtain detailed schedules (item production quantities), the authors show that problem SP provides a superior solution (a better lower bound) compared to the one obtained by disaggregating the product family lot sizes determined by MIP. The paper also presents approximate solution procedures for MIP and SP and provides bounds on the performance of the heuristics.

One of the limitations of the approach described above is the restriction that the production of one family be started and completed during the same period. This constraint seems rather artificial and may become important when the number of families is not very large. In a recent paper, Matsuo (1987) examines a different formulation of the problem and presents solution procedures that eliminate this restriction. He formulates a stochastic sequencing problem that simultaneously determines product sequence and production volumes for the style goods production planning problem. In his formulation time is treated as a continuous variable. Matsuo's solution procedure is also based on the hierarchical approach. In the first stage family lot sizes and sequence are determined by specifying, for each family, the start and finish times of production. In the second stage the family lot sizes are disaggregated. The analysis in this paper is rather involved, but the sequencing rules that are derived are intuitively appealing, elegant and surprisingly simple.

4.3 Production Control and Scheduling in the Presence of Machine Breakdowns

Gershwin (1987) considers scheduling problems in dynamic manufacturing systems with machine failures, setups, demand changes etc., and proposes a hierarchical structure based on the frequency of occurrence of different types of events. This framework is based on the assumption that events tend to occur in a discrete spectrum which define the hierarchical levels. For example, the frequency of additions of machines is an order of magnitude smaller than setup decisions, which in turn, occur less frequently than item production. A central assumption in this approach is that activities can be grouped into sets J_1, J_2, ...such that for each set J_k, there exists a characteristic frequency f_k satisfying

$$f_1 << f_2 << \ldots << f_k << f_{k+1} << \ldots$$

In this framework the hierarchical levels are defined by the frequency of activities (sets J_k). In modeling the decisions at each level, quantities that vary slowly (variables that correspond to higher levels) are treated as static, or constant, and discrete. Quantities that vary much faster (variables at lower levels) are modeled in a way that ignores the variations, for example, replacing fast moving variables by their averages. These ideas may be illustrated by considering a production planning example. In aggregate planning models, the number of machines (variables that correspond to higher level of hierarchy) are considered fixed. Also, in these models, details such as machine breakdowns (lower level variables) are ignored. However, the effect of breakdowns is usually accounted for by factoring an adjustment (based on expected behavior) in available capacity. An interesting feature in this approach is the treatment of capacity. Gershwin makes a distinction between capacities at different hierarchical levels. Figure 4.3.1, which describes the capacity definitions for an item in a multi-item shop that is modeled with three hierarchical levels, provides an example to clarify this idea. In this example, the terms capacity and production rate are synonymous. In the figure u^1 is the aggregate production rate (at level 1) that is equal to the demand rate and represents the capacity available for this item. u^2, the capacity at the next level, is the relevant capacity when the setup decisions are considered. At the operational level, when breakdowns occur, the production rate u^3 is higher than u^2 and u^1. Gershwin also distinguishes between controllable variables and activities such as breakdowns and repair that are beyond control.

The objective of this approach is to determine an optimal control strategy at the detailed level. The control strategy is specified by selecting the time to initiate each controllable event. Gershwin proposes the solution of one or two problems at each level to derive the control strategy. These problems are termed as the hedging point strategy and the staircase strategy. In the hedging point strategy problem at level k, the objective is to determine level-k capacities u_j^k , $j \epsilon J_m$, $m > k$, i.e., determine u_j^k for all activities that occur more frequently than f_k . Constraints are imposed by the total capacity available and the decisions at the higher levels. The staircase strategy problem can be interpreted as the allocation of resources among activities at level k, consistent with production rates u^{k-1} determined at the previous level.

Gershwin, Akella and Choong (1985), Kimemia and Gershwin (1983) describe some applications of this approach and discuss the detailed formulation and solution procedures for the staircase and hedging point strategy problems. The two machine, two product example of Figure 4.3.2 described in Gershwin (1987) can be used to illustrate the basic ideas of this approach. In this example, machine 1 is an unreliable, flexible machine that can produce both parts type 1 and 2. No setups are required to change from product type 1 to 2 or vice versa. Machine 2 is dedicated to production of part type 1 and is totally reliable. The following data is available.

p: failure rate for machine 1

r: repair rate for machine 1

t_{ij}: duration of the operation of part type j on machine i (t_{11}, t_{12}, t_{21})

d_{ij}: demand rates of part type j on machine i (d_{11}, d_{12}, d_{21})

It is assumed that (i)t_{ij} and $1/d_{ij}$ are of the same order of magnitude, and (ii) 1/r and 1/p are of the same order of magnitude which is greater than t_{ij}. The state of the system is specified by α, the repair state of machine 1 and x_{11}, x_{12}, and x_{21}; where x_{ij} is the surplus inventory and is defined as the excess of production capacity over the cumulative requirement. An example of a staircase strategy at level 2 for this example is presented in Figure 4.3.3, which describes the loading decisions as a function of the system state. At level 1, the hedging point strategy problem can be formulated as follows:

$$\min_{u_{ij}} \quad \Sigma \; c_{ij}\,(x, \alpha)\, u_{ij}$$

$$\text{s.t.} \quad t_{11}\, u_{11} + t_{12}\, u_{12} \le \alpha$$

$$t_{21}\, u_{21} \le 1$$

$$u_{11},\ u_{12},\ u_{21} \ge 0$$

where u_{ij} is the production rate of part type j on machine i (decision variable), $c_{ij}(x, \alpha)$ is the cost of maintaining production rate u_{ij} with the system state (x, α).

The approach described above is a fairly recent development in the application of the hierarchical approach to control and scheduling problems in discrete manufacturing systems. There are several outstanding issues that need to be resolved before the methods can be applied widely. These problems include the following:

(i) Development of methods to model systems in which the time scales for various activities are not widely separated,

(ii) formulating and solving hedging point problems with non-Markov events,

(iii) new formulations and solution procedures for the staircase strategy to obtain loading patterns that are very close to a given rate, and

(iv) aggregation issues in modeling higher level activities.

5.0 Conclusions

In this chapter we have described the basic features of the hierarchical approach in addressing planning and scheduling problems. We have also described some of the models and presented results of general interest in this context. These models span a wide variety of manufacturing environments ranging from continuous processes to discrete systems such as batch and job shops. The approach has been successful on two dimensions. First, the hierarchical framework is attractive to practitioners as evidenced by the several applications that have been reported. Second, considerable amount of research has been generated in developing appropriate models and solution methods.

However, in spite of the developments in the last decade, there is a need for good models that support decisions in more complex environments. We note four areas with potential for further research. First, the problem of aggregation and disaggregation has not been resolved satisfactorily. Second, detailed models have been developed for single and two stage systems only. It is not clear how such models can be extended to more complex systems. Third, the issue of feedback in hierarchical systems has not been explored adequately and merits further research. An finally, except in a few cases, good models have not been developed for systems characterized by uncertainties.

REFERENCES

Agnihotri, 1982. Stochastic Allocation Rules. Operations Research 30, 545-555.

Andersson, H., S. Axsater, and H. Jonsson. 1981. Hierarchical Material Requirements Planning. Int. J. Prod. Res. 19, 45-57.

Anthony, R.N. 1965. Planning and Control Systems: A Framework for Analysis. Harvard University, Graduate School of Business Administration, Cambridge, Massachusetts.

Axsater, S. 1981. Aggregation of Product Data For Hierarchical Production Planning. Operations Research 29, 744-756.

Axsater, S. 1986. Feasibility of Aggregate Production Plans. Operations Research 34, 796-800.

Axsater, S. and H. Jonsson. 1984. Aggregation and Disaggregation in Hierarchical Production Planning. Eur. J. Oper. Res. 17, 338-350.

Axsater, S., H. Jonsson, and Thorsterson. 1983. Approximate Aggregation of Product Data. Eng. Cost Prod. Econ. 7, 119-126.

Bitran, G.R. and A.C. Hax. 1977. One the Design of Hierarchical Production Planning Systems. Decision Science 8, 28-55.

Bitran, G.R. and A.C. Hax. 1981. Disaggregation and Resource Allocation Using Convex Knapsack Problems. Management Science 27, 431-441.

Bitran, G.R., E.A. Haas, and A.C. Hax. 1981. Hierarchical Production Planning: A Single Stage System. Operations Research 29, 717-743.

Bitran, G.R., E.A. Haas, and A.C. Hax. 1982. Hierarchical Production Planning: A Two Stage System. Operations Research 30, 232-251.

Bitran, G.R., E.A. Haas, and H. Matsuo. 1986. Production Planning of Style Goods with High Set-up Costs and Forecast Revisions. Operations Research 34, 226-236.

Bitran, G.R. and D. Tirupati. 1988a. Planning and Scheduling for Epitaxial Wafer Production Facilities. Operations Research 36, 1, 34-49.

Bitran, G.R. and D. Tirupati. 1988b. Development and Implementation of a Scheduling System: An Application in a Wafer Fabrication Facility. Operations Research, 36, 3, 377-395.

Bradley, S.P., A.C. Hax, and T.L. Magnanti. 1977. Applied Mathematical Programming, Addison Wesley, Reading, Massachusetts. Chapter 6 based on the technical paper, Integration of Strategic and Tactical Planning in the Aluminum Industry by A.C. Hax, Operations Research Center, M.I.T., Working Paper 026-73, September 1973; Chapter 10 based on the paper by R.J. Armstrong and A.C. Hax, A Hierarchical Approach for a Naval Tender Job-Shop Design, Technical Report No. 101, Operations Research Center, M.I.T., Cambridge, Massachusetts.

Chipman, J.S. 1975. The Aggregation Problem in Economics. Suppl. Adv. Appl. Prob. 7, 72-83.

Chipman, J.S. 1976. Estimation and Aggregation in Econometrics: An Application of the Theory of Generalized Inverses. In Generalized Inverses and Application. 549-570, M.Z. Nashed (ed.). Academic Press, New York.

Dempster, M.A.H., M.L. Fisher, L. Jansen, B.J. Lageweg, J.K. Lenstra, and A.H.G. Rinnooy Kan. 1981. Analytical Evaluations of Hierarchical Planning Systems. Operations Research 9, 707-716.

Dempster, M.A.H. 1982. A Stochastic Approach to Hierarchical Planning and Scheduling. M.A.H. Dempster et al (eds.), Deterministic and Stochastic Scheduling, 271-296.

Doumeingts, G., L. Pun, M. Mondain, and D. Breuil. 1978. Decision-Making Systems for Production Control Planning and Scheduling. Int. J. Prod. Res. 16, 137-152.

Dzielinski, B.P. and R.E. Gomory, 1965. Optimal Programming of Lot Sizes Inventory and Labor Allocations, Management Science, 11, 9, 874-890.

Erschler, J., G. Fontan, and C. Merce. 1986. Consistency of the Disaggregation Process in Hierarchical Planning. Operations Research 34, 464-469.

Fisher, W.D. 1969. Clustering and Aggregation in Economics. The Johns Hopkins Press, Baltimore.

Gabbay, H. 1975. Hierarchical Approach to Production Planning. Technical Report No. 120, Operations Research Center, M.I.T., Cambridge, Massachusetts.

Gabbay, H. 1979. Optimal Aggregation and Disaggregation in Hierarchical Planning. In Disaggregation: Problems in Manufacturing and Service Organizations. L.P. Ritzman, et al (eds.), Martinus Nijhoff Publishing, Boston, Massachusetts.

Gelders, L.F. and L.N. Van Wassenhove, 1982. Hierarchical Integration in Production Planning: Theory and Practice, Journal of Operations Management, 3, 1, 27-35.

Geoffrion, A.M. 1977. Aggregation Theory and its Application to Modeling in Mathematical Programming. Working Paper No. 278, Western Management Science Inst., Univ. of California, Los Angeles.

Geoffrion, A.M. 1987. An Introduction to Structured Modeling. Management Science 33, 5, 547-588.

Gershwin, S.B. July 1987. A Hierarchical Framework for Discrete Event Scheduling in Manufacturing Systems. LIDS-P-1682, MIT, Presented at the IIASA Workshop in Discrete Event Systems: Models and Applications, Soprun, Hungary.

Gershwin, S.B., R. Akella, and Y.F. Choong. 1985. Short-Term Production Scheduling of an Automated Manufacturing Facility. IBM Jour. of Res. and Dev. 29, 4, 392-400.

Golovin, J. J. 1975. Hierarchical Integration of Planning and Control, Technical Report No. 116, Operations Research Center, M.I.T., Cambridge, Massachusetts.

Graves, S. C. 1982. Using Lagrangian Techniques to Solve Hierarchical Production Planning Problems. Management Science 28, 260-275.

Hax, A. C. 1976. The Design of Large Scale Logistics Systems: A Survey and An Approach. In Modern Trends in Logistics Research, W. H. Marlow (ed.), M.I.T. Press, Cambridge, Massachusetts.

Hax, A. C. 1978. Aggregate Production Planning. In Handbook of Operations Research, J. Moder and S. E. Elmaghraby (eds.), Van Nostrand Reinhold, New York.

Hax, A. C. and D. Candea. 1984. Production and Inventory Management. Prentice-Hall, Englewood Cliffs, New Jersey.

Hax, A. C. and J. J. Golovin. 1978. Hierarchical Production Planning Systems. In Studies in Operations Management, A. C. Hax (ed.), North Holland Publishing Company, Amserdam.

Hax, A. C. and J. J. Golovin. 1978. Computer Based Operations Management System (COMS). In Studies in Operations Management, A. C. Hax (ed.), North Holland Publishing Company, Amsterdam.

Hax, A. C., J. J. Golovin, M. Bosyj, and T. Victor. 1976. COMS: A Computer-Based Operations Management System. Technical Report No. 121, Operations Research Center, M.I.T., Cambridge, Massachusetts.

Hax, A. C. and N. S. Majluf. 1984. The Corporate Strategic Planning Process. Interfaces 14, 47-60.

Hax, A. C. and H. C. Meal. 1975. Hierarchical Integration of Production Planning and Scheduling. In Studies in Management Sciences, Vol. 1, Logistics, M. A. Geisler (ed.), North Holland-American Elsevier, New York.

Holt, C. C., F. Modigliani, J. F. Muth, and H. A. Simon. 1960. Planning Production, Inventories and Work Force. Prentice-Hall, Englewood Cliffs, New Jersey.

Jaikumar, R. 1974. An Operational Optimization Procedure for Production Scheduling. J. Compt. Opns. Res. 1, 191-200.

Kimemia, J., and G. B. Gershwin. 1983. An Algorithm for the Computer Control of a Flexible Manufacturing System. IIE Trans. 15, 4, 353-362.

Kusiak, A., and G. Finke. 1987. Hierarchical Approach to the Process Planning Problem. Discrete Applied Mathematics, 18, 175-184.

Lasdon, L.S. and R.C. Terjung, 1971. An Efficient Algorithm for Multi-Item Scheduling. Operations Research, 19, 4, 946-969.

Lenstra, J.K., A.H.G. Rinnooy Kan, and L. Stougie. 1984. A Framework for the Probabilistic Analysis of Hierarchical Planning Systems. Annals of Operations Research 1, 23-42.

Machulak, G.T., C.L. Moodie, and T.J. Williams. 1980. Computerized Hierarchical Production Control in Steel Manufacture. Int. J. Prod Res. 18, 455-465.

Manne, A.S., 1958. Programming Economic Lot Sizes, Management Science, 4, 2, 114-135.

Matsuo, H. 1987. A Stochastic Sequencing Problem for Style Goods with Forecast Revisions and Hierarchical Structure. Dept. of Management, The University of Texas at Austin, forthcoming in Management Science.

Maxwell, W., J.A. Muckstadt, L.J. Thomas, and J. Vandereecken. 1983. A Modeling Framework for Planning and Control of Production in Discrete Parts Manufacturing and Assembly Systems. Interfaces 13, 92-104.

Meal, H.C. 1975. Manufacturing Control and Material Requirements Planning. Presented at the Seventh Annual Meeting of the American Institute for Decision Sciences, Cincinnati, Ohio.

Meal, H.C. 1978. A Study of Multi-Stage Production Planning. Chapter 9 in Studies in Operations Management, A.C. Hax (ed.), North Holland.

Meal, H.C., 1984. Putting Production Decisions Where They Belong, Harvard Business Review, 62, 2, 102-111.

Meal, H.C., M.H. Wachter and D.C. Whybark, 1987. Material Requirements Planning in Hierarchical Production Planning Systems. Int. J. Prod. Res., 25, 7, 947-956.

Mitchell, J.C. 1985. A Hierarchical Production Planning Model to Allocate Scarce Resources. AT&T Bell Laboratories, Holmdel, New Jersey.

Ritzman, L.P., L.J. Krajewski, W.L. Berry, S.H. Goodman, S.T. Hardy and L.D. Vitt (eds.). 1979. Disaggregation Problems in Manufacturing and Service Organizations, Martinus Nijhoff, Boston.

Ruefli, T.W., and J. Storbeck. 1984. NOTE: Removing a Self-Imposed Restriction in Modeling Hierarchical Decision Processes. Management Science 30, 389-392.

Steward, S.M., S.G. Taylor, and S.F. Bolander. 1985. Progress in Integrating and Optimizing Production Plans and Schedules. Int. J. Prod. Res. 23, 609f.

Theil, H. 1965. Linear Aggregation of Economic Relations. North Holland, Amsterdam.

Wijngaard, J. 1983. Aggregation in Manpower Planning. Management Science 29, 1427-35.

Winkofsky, E.P., N.R. Baker and D.J. Sweeny. 1981. A Decision Process Model by R&D Resource Allocation in Hierarchical Organizations, Management Science, 27, 268-283.

Winters, P.R. 1962. Constrained Inventory Rules for Production Smoothing. Management Science 8, 470-481.

Zangwill, W. 1966. A Deterministic Multiproduct, Multifacility Production and Inventory Model, Operations Research, 14, 3,

Zipkin, P. 1980a. Bounds on the Effect of Aggregating Variables in Linear Programs, Operations Research, 28, 2, 403-418.

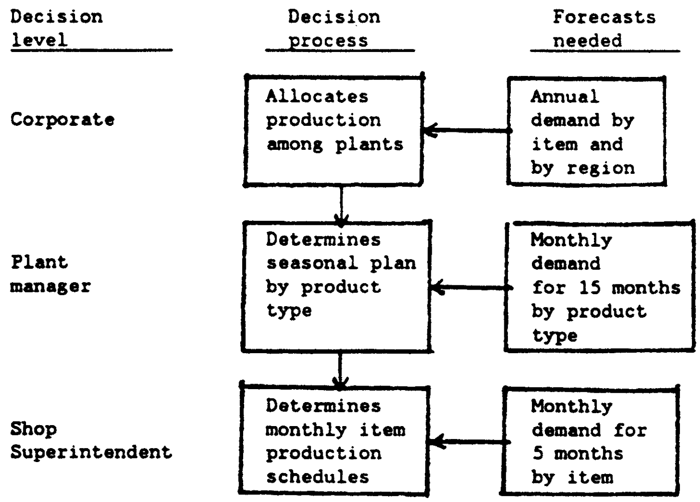
Zipkin, P. 1980b. Bounds for Row-Aggregation in Linear Programming, Operations Research, 28, 4, 903-916.

Zipkin, P. 1982. Exact and Approximate Cost Functions for Product Aggregates. Management Science 28, 1002-1012.

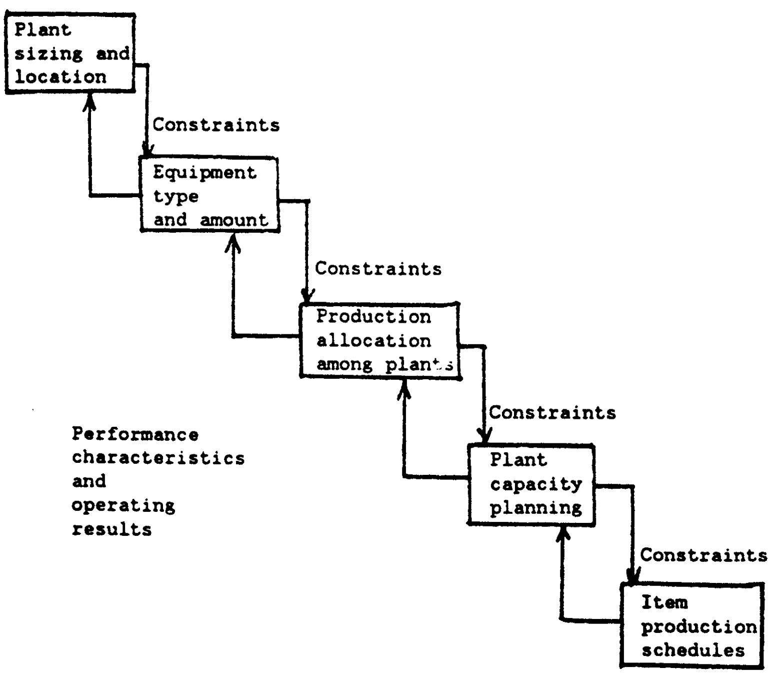
Zoller, K. 1971. Optimal Disaggregation of Aggregate Production Plans, Management Science, 17, 8, B533-B549.

TABLE 1.1 Differentation Factors of the Three Decision Categories

Factor	Strategic Planning	Management Control (Tactical Planning)	Operational Control
Purpose	Management of change, resource acquisition	Resource utilization	Execution, evaluation, and control
Implementation instruments	Policies, objectives, capital investments	Budgets	Procedures, reports
Planning horizon	Long	Medium	Short
Scope	Broad, corporate level	Medium, plant level	Narrow, job shop level
Level of management involvement	Top	Middle	Low
Frequency of replanning	Low	Medium	High
Source of information	Largely external	External and internal	Largely internal
Level of aggregation	Largely aggregated	Moderately aggregated	Largely Detailed
Required accuracy	Low	Medium	High
Degree of uncertainty	High	Medium	Low
Degree of risk	High	Medium	Low



Hierarchical Planning Process



Production Planning Decision Hierarchy

Figure 2.1 An Overview of Hierarchical Planning Approach

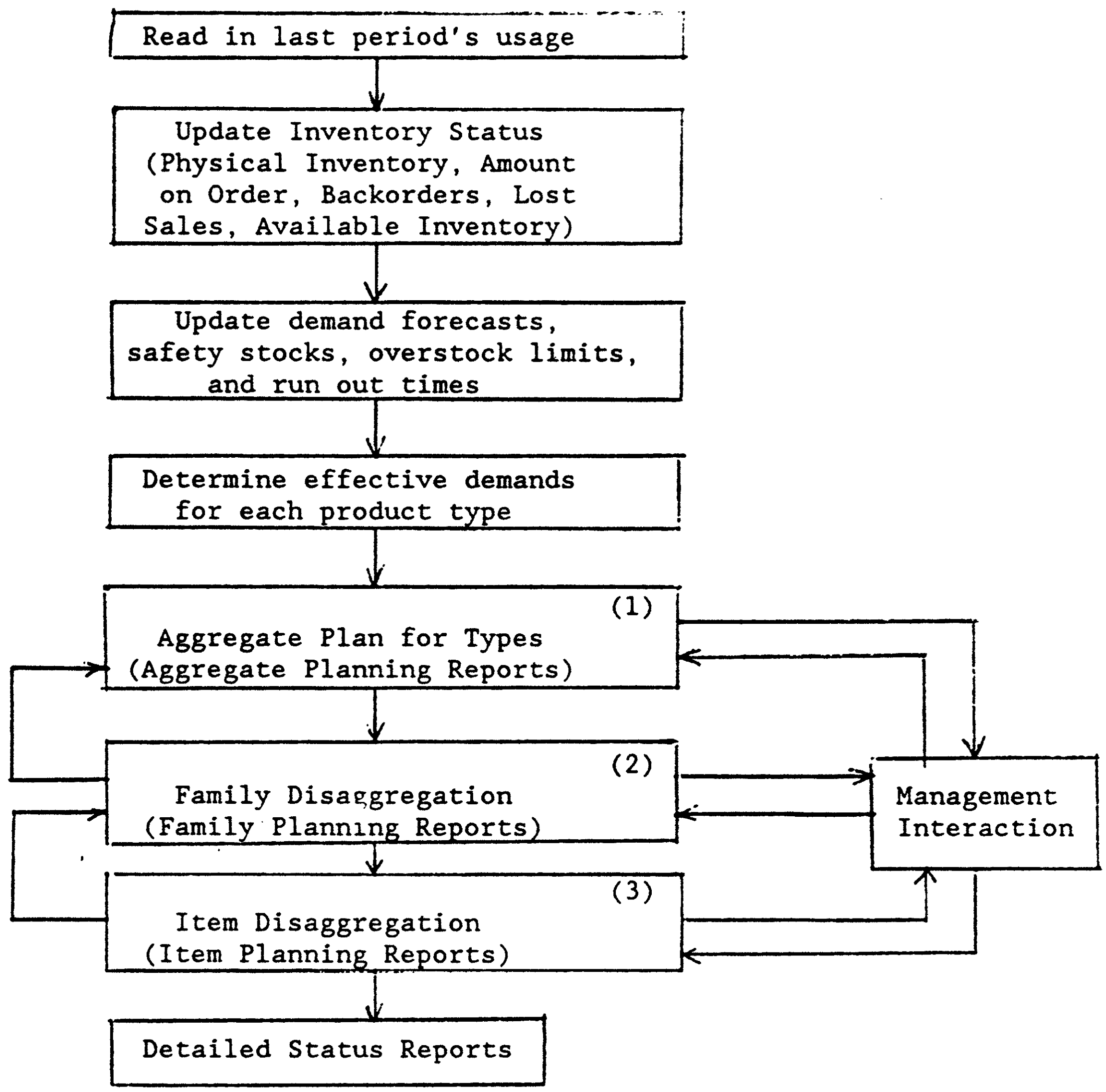


Figure 2.1.1. Conceptual overview of hierarchical planning system.

Table 2.1

DEMAND				
Item	Period t=1	Period t=2	Period t=3	Initial Inventory
k = 1	$d_{11} = 5$	$d_{12} = 17$	$d_{13} = 30$	$I_{10} = 9$
k = 2	$d_{21} = 3$	$d_{22} = 12$	$d_{23} = 30$	$I_{20} = 20$
Total	$d_1 = 8$	$d_2 = 29$	$d_3 = 60$	$I_0 = 29$

TABLE 2.2

	Effective Demand			
Item	Period t=1	Period t=2	Period t=3	Initial Inventory
k = 1	$d_{11} = 0$	$d_{12} = 13$	$d_{13} = 30$	0
k = 2	$d_{21} = 0$	$d_{22} = 0$	$d_{23} = 25$	0
Total	$d_1 = 0$	$d_2 = 13$	$d_3 = 55$	0

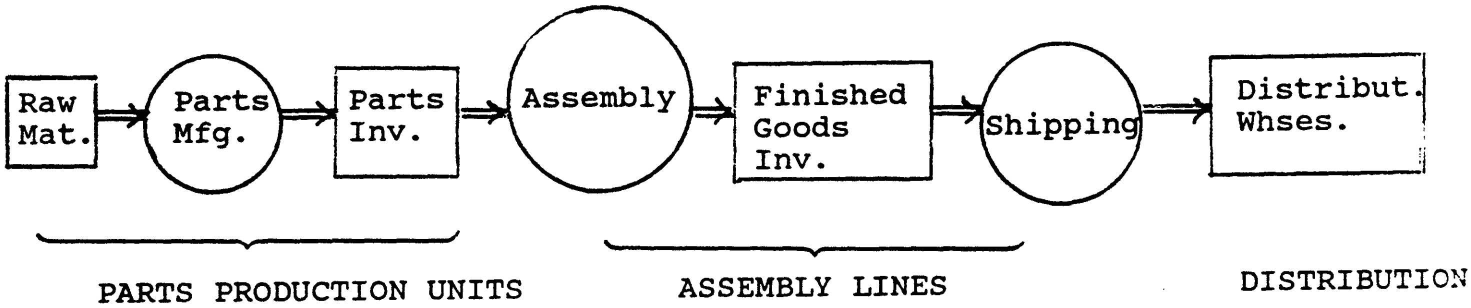


Figure 2.3.1 Major stages in the production and distribution system.

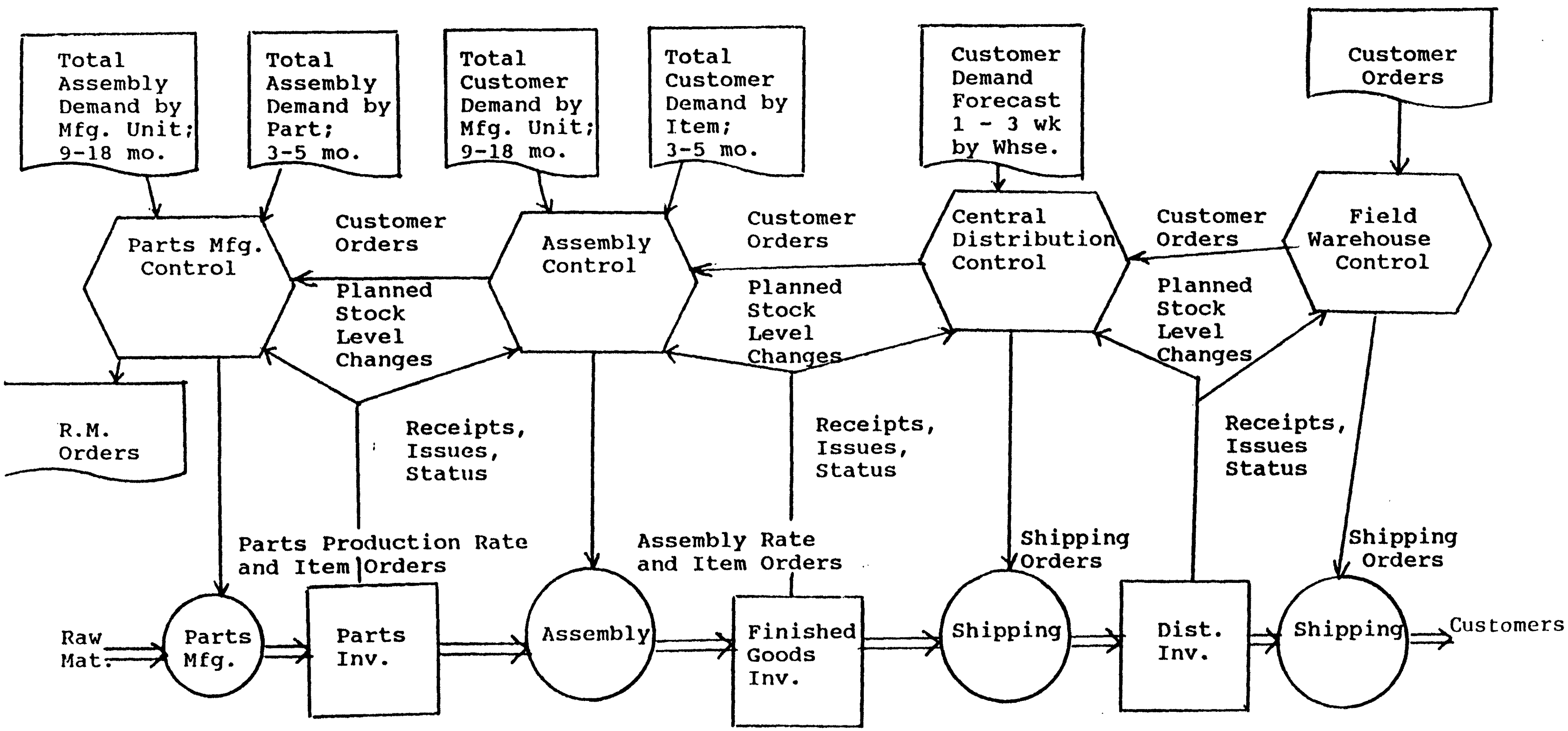


Figure 2.3.2 Major information inputs and flow of control information.

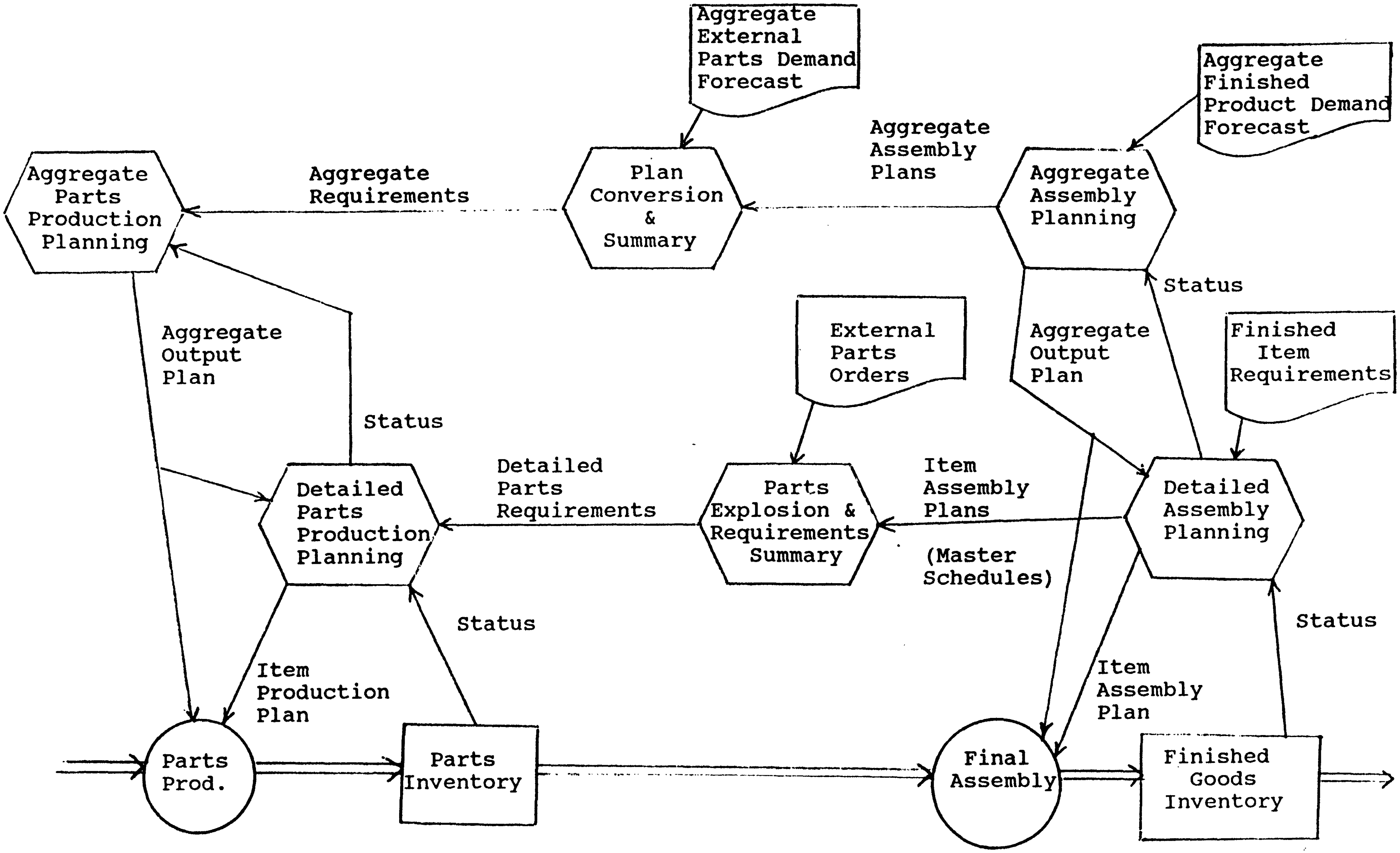


Figure 2.3.3. Overall structure of two-level, two stage planning system.

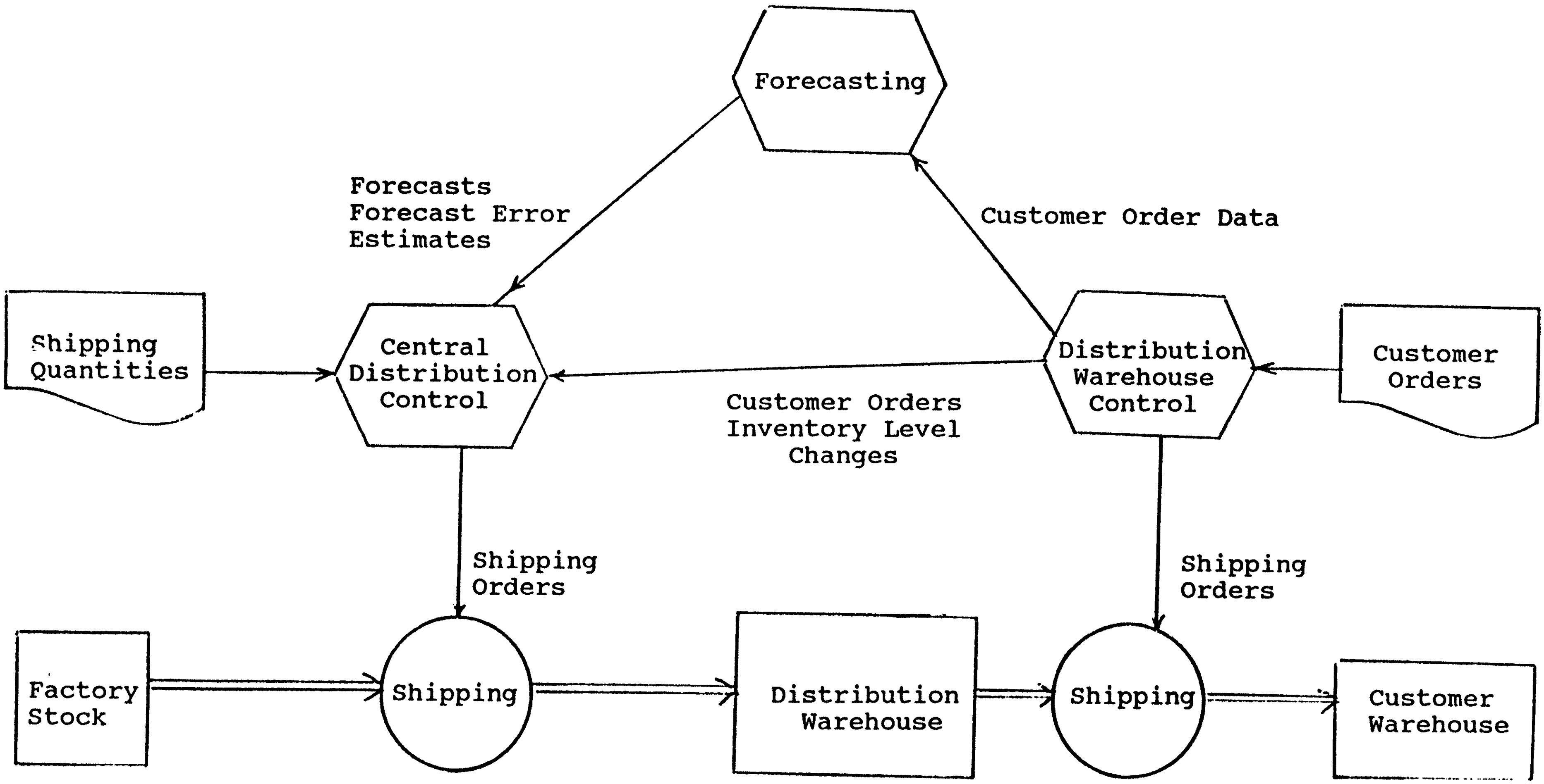


Figure 2.3.4. Distribution control system.

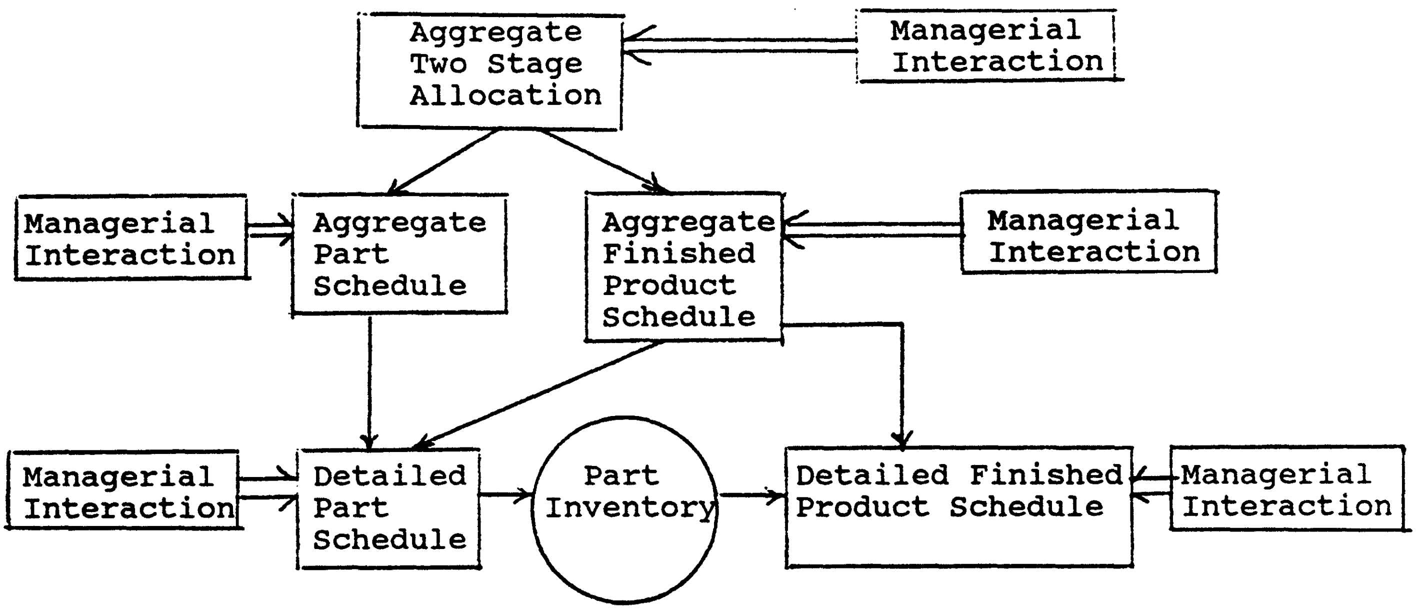


Figure 2.3.5 A conceptual overview of a hierarchical production planning system for a fabrication and assembly process.

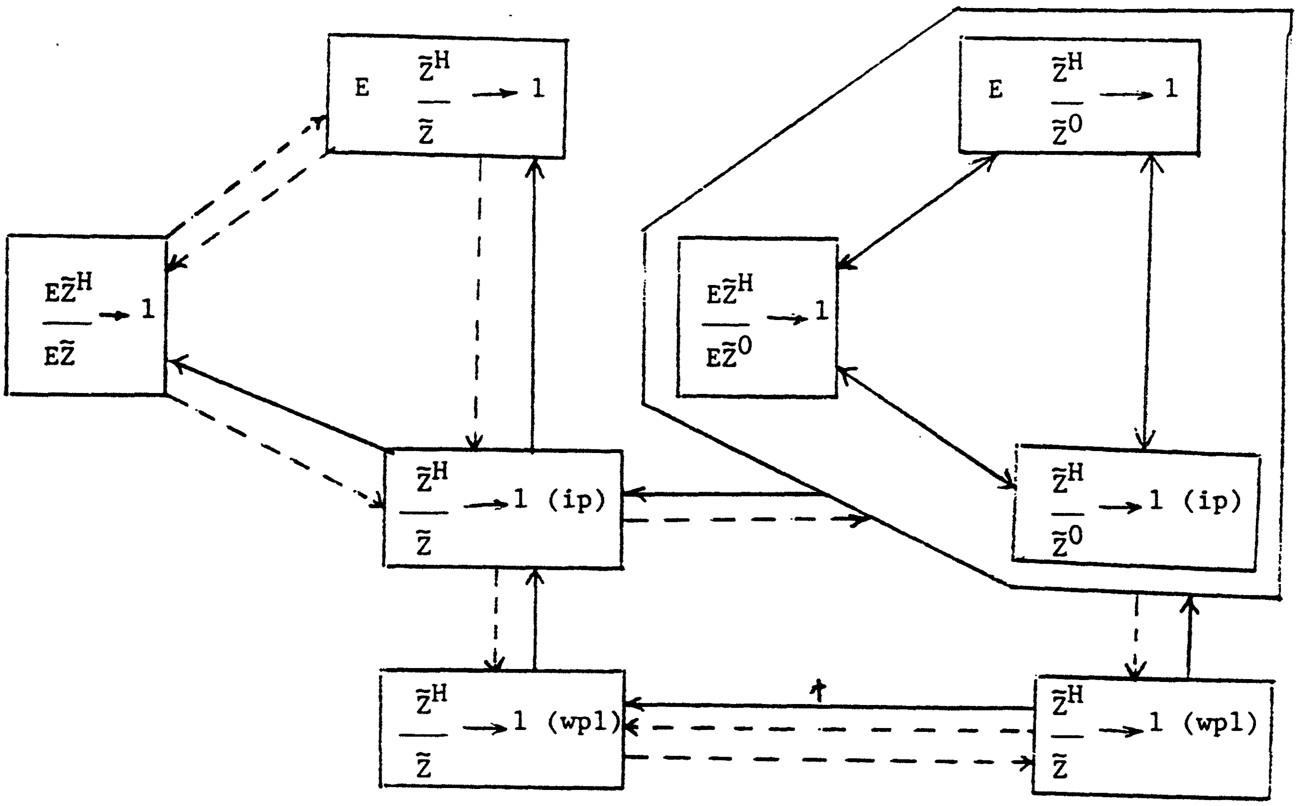


Figure 4.1.1 Relations between performance measures. ——→ : Valid implication; — — — → : invalid implication.

†: if Z^H/Z^* has a finite limit (wp1).
(ip): in probability
(wp1): with probability 1

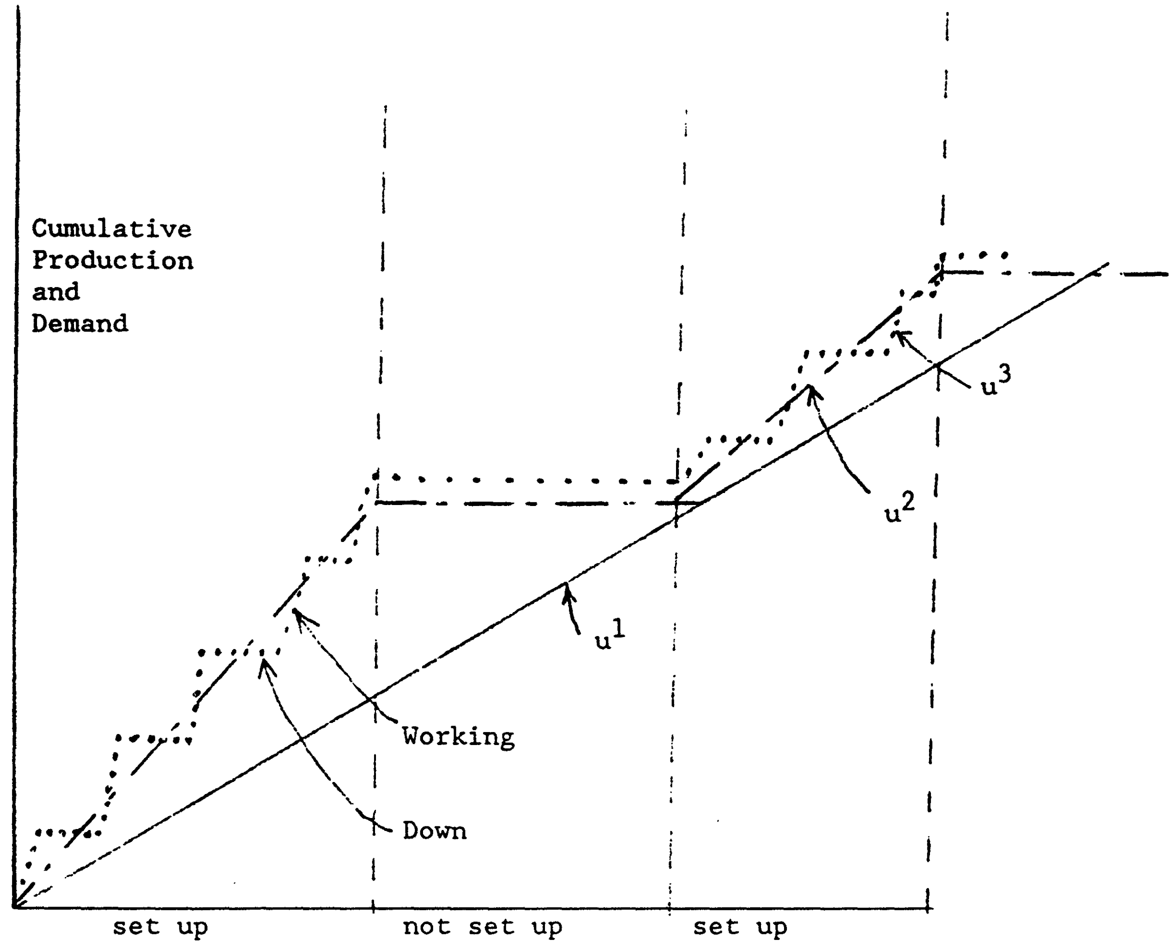
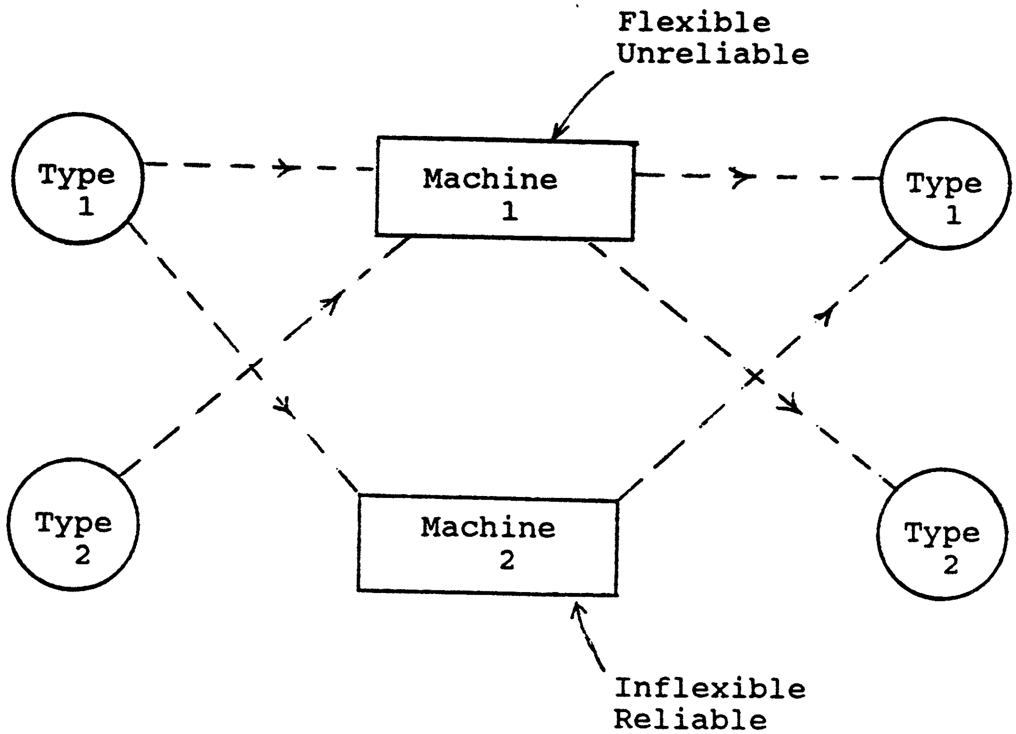

Cumulative Production and Demand
Working
Down
u^1
u^2
u^3
set up
not set up
set up

Figure 4.3.1



Flexible
Unreliable
Type
1
Machine
1
Type
1
Type
2
Machine
2
Type
2
Inflexible
Reliable

Figure 4.3.2

Figure 4.3.3

Level 2: Staircase Strategy

Loading a Type j part into Machine 1 is eligible if:

1. The number of Type j parts made up to time t on Machine 1 is less than

   $\int_0^t u_{1j}(s)ds$, and

2. Machine 1 is now idle.

Loading a Type 1 part into Machine 2 is eligible if:

1. The number of Type 1 parts made up to time t on Machine 2 is less than

   $\int_0^t u_{21}(s)ds$, and

2. Machine 2 is now idle.