A PRICE RESPONSE MODEL DEVELOPED FROM PERCEPTUAL THEORIES

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ABSTRACT

Theories of perception and judgment suggest a structure for a price and promotion response model that is then calibrated on scanner panel data for coffee. Adaptation level and assimilation-contrast theory imply the existence of a reference price and a possible region of customer insensitivity to price (latitude of price acceptance). Prospect theory suggests an asymmetric response to price changes. Attribution theory would predict a negative effect of promotional frequency on product choice. These concepts are operationalized and imbedded in a multinomial logit model.

Two scanner panel databases for regular ground coffee (SAMI and IRI) provide an opportunity to test the model empirically. The results show an asymmetric price response with customers more sensitive to price increases than decreases. This is as would be predicted by prospect theory. There is evidence of a region of price insensitivity and also for a negative effect of promotional frequency on purchase probability. The results are consistent across the two independent databases.

INTRODUCTION

A hierarchy of useful knowledge for providing assistance to marketing managers might be as follows:

(1) General constructs for thinking about marketing phenomena (e.g. price elasticity),

(2) Rough magnitudes of the effects (e.g. price elasticity is in the range of 2 to 3 for brands in the coffee category),

(3) Measurements as close as possible to the case of practical interest to the decision maker (e.g. Butternut Coffee one pound size has a share-price elasticity of 2.3 in Kansas City).

In this paper we turn to the psychological literature for broad constructs that may be applicable to product pricing and promotion. As a potential contribution to (1), we relate several to the pricing of consumer package goods. For (2) we try to construct models that will permit measurements that give ranges of magnitudes for certain price phenomena. Ultimately this may lead to (3), models for specific products in specific practical situations.

Price response models involve a number of issues. One is whether actual price or a perceptually-based transformation of price should be used in modeling the sales-price and share-price relationships. Most economists and many marketers have used actual price, either absolute or relative. By contrast, psychologists, some other marketers and an increasing number of economists have focused on the perceptual aspects of price and, consequently, have put forward such concepts as price-ending effects (Monroe 1973), price thresholds (Uhl and Brown 1971, Monroe 1971), price-quality relationships (Gabor and Granger 1966, Monroe 1973) and reference price (Emery 1970, Monroe 1973). A presumption in these papers is that perceived price is what matters and that perceived price itself is dependent on various factors such as current price, past price and other marketing stimuli. More recently, steps have been taken to meld the two approaches, especially in the study of reference price (Rinne 1981, Winer 1986, Raman and Bass 1988, Lattin and Bucklin 1987, Kalwani, Rinne, Sugita and Yim 1988).

We start with the psychologist's view in the sense of looking to theories of perception and judgment for constructs from which to develop pricing and promotion response models, although we shall happily borrow from the economic literature as applicable. Four potentially relevant psychological theories are: adaptation level, assimilation-contrast, prospect, and attribution. The first three will motivate a price response model in which each theory provides a construct for a particular aspect: adaptation level theory for reference price, assimilation-contrast theory for latitude of price acceptance and prospect theory for asymmetric customer response. The fourth, attribution theory, motivates a model of the effect of promotional frequency on purchasing. We operationalize these concepts for our particular setting, estimate model parameters from data sets, and compare the results with theoretical predictions.

Much of the empirical work on price has examined relationships at the aggregate level. However, aggregate data can obscure valuable information and behavioral processes that are taking place at the household level. The availability of data from scanner panels provides an opportunity for building and calibrating models based on individual decision-making units. Disaggregate models fit well with psychological theories since these ordinarily deal with the individual.

In building and testing our models we shall not be testing the pyschological theories, since these have already been shown valid through extensive experimentation. We shall be examining whether phenomena suggested by the theories can be found in our marketing data using the models we develop.

Two scanner panel data sets on coffee purchases, one from Selling Areas Marketing, Inc. (SAMI) and the other from Information Resources, Inc. (IRI) provide an empirical base for calibrating and testing the models.

THEORETICAL MOTIVATION

ADAPTATION LEVEL THEORY

We use adaptation level theory to motivate reference price and its change as a customer encounters new information.

Adaptation level theory (Helson, 1964) states that the perceived magnitude and effect of a stimulus at any given time depends on the relation of that stimulus to preceding stimuli. The prior stimuli create an adaptation level and subsequent stimuli are judged in relation to it. The judgement and, consequently, the response to a stimulus depend on the relationship between the physical value of that stimulus and the value of the current adaptation level. According to Helson (1964), a change in a stimulus causes "an initial rapid change in activity and sensitivity .... followed by a constant level of activity if the stimulus is continued at a constant intensity."

The adaptation level is the stimulus value at which the scale of judgment is centered or anchored. It is usually taken to be a weighted logarithmic mean of the various physical values of the stimuli. Each of the judgments is in turn determined by the ratio of the stimulus to the adaptation level. In psychophysics research, the test stimuli often increase geometrically and the logarithmic mean fits well. However, in our case, the changes of stimulus are relatively small (from 0 to about 20%) and so we shall use linear models, which are much simpler and are reasonably equivalent over the ranges being considered. This is because for small x, $\log(1+x) \simeq x$.

According to the theory, the past and present context of experience defines an adaptation level, or reference point, relative to which new stimuli are perceived and compared. A simple illustration of this would be as follows. If a person repeatedly lifts a weight of 100 grams, he or she then becomes accustomed to this weight, i.e., adapts to it, and the adaptation level becomes 100 grams.

Application to pricing. For pricing, the theory suggests an adaptation level that depends on previous price experience. The adaptation level will be called the reference price. In most cases, this will not be a price that physically appears on any goods but a price that customers are assumed to form in their minds as a result of experience.

Various operationalizations appear in the literature. Rinne (1981) compares three approaches: (1) setting reference price equal to previous price (2) taking reference price as the weighted mean of the logs of past observations as suggested by Helson (1964), and (3) using an exponential smoothing model motivated by Friedman's (1979) general theory of adaptive expectations. Rinne the imbeds each reference price model in a share-price response equation whose parameters are estimated from a dataset. He finds that models (2) and (3) provide superior fits compared to (1) but neither (2) nor (3) dominates the other.

Raman and Bass (1988) start from an adaptive expectations view of reference price but seek to incorporate the additional phenomenon of price thresholds (Monroe 1973). The basic idea is that consumers have a region of relative price insensitivity about reference price but that at some threshold on either side price response changes and becomes much more pronounced. Raman and Bass, working with aggregate data, determine reference price using a Box-Jenkins auto-regressive model on historical prices. The reference price expression is then imbedded in a model of market share response to price and a switching regression methodology used to identify thresholds.

Winer (1986) tests two different reference price formulation, one based on an extrapolative expectations hypothesis and the other a rational expectations hypothesis. After estimating the parameters of each formulation on his data set, he inserts the reference price into a model relating purchase probability to marketing maix variables. His results do not show a marked difference between the formulations.

Two recent papers build even richer models for the formation of reference price. In Lattin and Bucklin (1987) customers form a reference discount if they are exposed to in-store price-cuts to such a degree that they cross a prescribed threshold level. Thereupon, they tend to expect a discount and so require bigger discounts to achieve the same level of purchase probability that would have occurred had they not crossed the threshold. Kalwani, Rinne, Sugita and Yim (1988) have added other variables besides past prices to their model of reference price. Such variables include promotional frequency and the proneness of the household to buy on promotion.

We wish a relatively simple operationalization of reference price that nevertheless captures the basic idea that a customer modifies his or her view of price as new information is encountered. A parsimonious way to do this is with an exponential smoothing process, and the literature cited suggests that it should work well. In the terminology of several of the above papers, we are using an adaptive expectations approach. For each household and product:

current reference price = (carry-over constant)x(previous reference price)
+ (1 - carry-over constant)x(previous actual price).

Further details will be developed below.

PROSPECT THEORY

A number of marketing writers have suggested that customers may have different responses to price increases and decreases (Pessemier 1960, Uhl and Brown 1971, Monroe 1976, Rinne 1981, Raman and Bass 1988). Usually it is suggested that customers respond more negatively to price increases than they do positively to decreases. The empirical work of Pessemier (1960), Uhl and Brown (1971) and Raman and Bass (1988) supports this view, although Monroe (1976) in laboratory experiments reports the opposite and Rinne's empirical results are mixed. In any case it would be desirable to have a theory more general than pricing itself to motivate a model of the phenomenon.

Prospect theory (Kahneman and Tversky 1979) is a candidate. It was developed as an alternative to classical expected utility theory for the purpose of describing and predicting individual choice behavior under risk. As a result of many experiments, the authors and others have shown, first of all, that choice often depends on the way a problem is posed as well as on the characteristics considered in classical theories. Prospect theory, therefore, introduces the concept of a 'frame' or 'reference' and outcomes are expressed as gains or losses from a reference point.

Secondly, the experiments reveal an asymmetric relationship between the subjective values of the outcomes and objectively calculated expected values. The loss of a given amount is viewed more severely than the gain of the same amount. The curve is also concave above the reference point and convex below it. Figure 1, after Kahneman and Tversky (1979), sketches a value function with these properties.

In marketing Thaler (1985) has used prospect theory to good advantage, supplementing it with constructs of his own, to explain a wide variety of consumer choice phenomena. Our use of prospect theory corresponds to his concept of transaction utility.

Application to pricing. For the case of pricing, an increase is a loss in value for the customer and a decrease a gain. The price response function suggested by prospect theory is a reflection of Figure 1 across the vertical axis and therefore appears as in Figure 2. Notice that we have taken the reference price as the reference point for gains and losses and that we indicate a steeper slope for price increases (losses) than price decreases (gains). We shall not attempt to look for the concavity-convexity property of the curve and so represent it with two linear segments.

ASSIMILATION-CONTRAST THEORY

Assimilation-contrast theory suggests that, in the neighborhood of a reference price, customers are likely to perceive prices as different from actual values, tending to downplay small differences from the reference and exaggerating large ones. This leads to a concept of 'latitude of acceptance', a range of relative

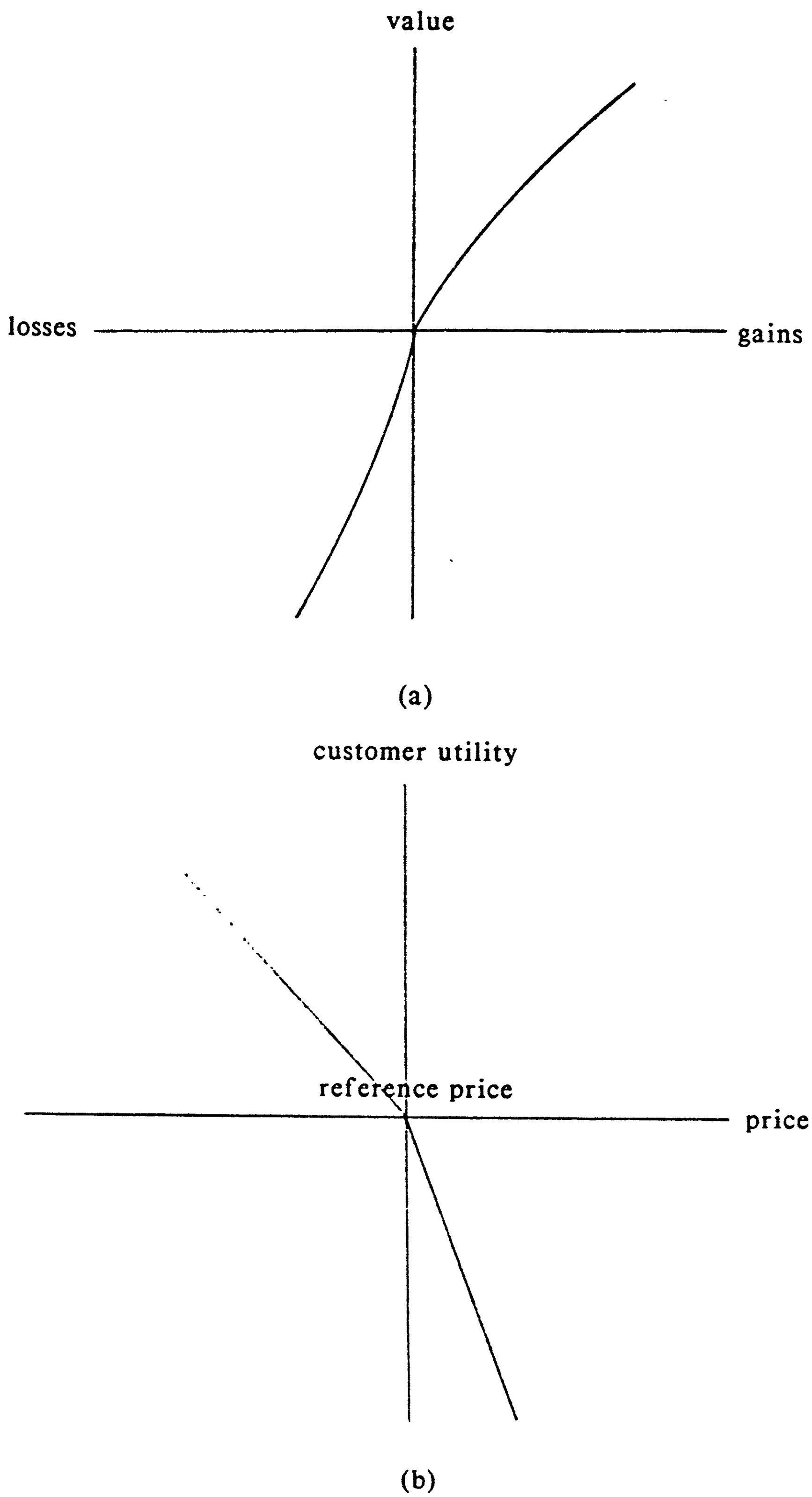

Figure 1. (a) Value function hypothesized in prospect theory shows asymmetry for losses and gains. Curve after Kahneman and Tversky (1979). (b) For application to pricing, price increases are customer losses, decreases are gains. In a linearized version, price increases are expected to have a steeper slope than price decreases.

price insensitivity. Related to this is the notion of a price threshold used by Monroe (1979) and by Raman and Bass (1988).

A number of marketing writers have applied assimilation theory to price perceptions (Emery 1970, Monroe 1971, 1973, Sawyer and Dickson 1984). However, two rather different ideas are discussed. Emery (1970) speaks of "an amount of price variation that has no effect on sales". Sawyer and Dickson (1984) draw a flat place on a curve of perceived price vs. actual price. Raman and Bass (1988) speak of a "region of indifference about a reference price such that changes in price within this region produce no change in perception." It is in this sense that we use the term.

However, 'latitude of acceptance' is also used to refer to a broad range of prices acceptable for a product class without implying indifference among the prices of individual items. Sherif (1963) uses the term this way in a pioneering psychological study involving price perceptions, as does Monroe (1971) who also uses the term threshold to indicate the boundaries of the latitude of price acceptance. The two uses of latitude of acceptance and price threshold are confusing but do not appear to be conflicting. Rather they are two different applications of the theory: one to different prices for the same product and the other to prices of different products in the same category.

To stay close to the basic pyschophysical phenomenon as it might apply in our case, we describe the classical work of Sherif, Taub and Hovland (1958). The authors conducted experiments to determine the effect of an anchoring weight on people's judgments of a series of other weights. Subjects lifted an anchor weight several times and then judged each of the "series weights" on a 1 to 6 scale. Each anchor was a different amount heavier than the range of the series weights. The authors found that, when the anchor weights were close to the series, subjects assigned higher values to the series weights than when the anchor was absent. This is called the assimilation effect: the judgment is drawn toward the anchor. On the other hand, as the anchor was moved far above the top value of the series, subjects assigned lower values to the series weights than when the anchor was absent. This is called the contrast effect: the judgment is driven away from the anchor. The same assimilation and contrast phenomena occurred for anchors lighter than the series.

The assimilation effect is particularly important, since it provides the theoretical basis for a latitude of acceptance. Consequently we have rescaled and replotted the data of Sherif, Taub and Hovland so that it is in the form of response to deviations from a fixed reference point (anchor). The results appear in Figure 2 with backup details in Appendix 1. Figure 2 clearly shows a depressed place under the 45° line representing assimilation near the origin, although lack of points makes it difficult to determine the exact shape. The contrast effect, which comes out vividly in the histograms of the original article, is muted here by rescaling. Nevertheless, response is higher than the 45° line for large deviations, as would be predicted by the contrast effect. After reflection across the horizontal axis, Figure 2 provides insight on how a price response curve might look for positive deviations from reference.

Application to pricing. As discussed, the assimilation effect suggests that within a certain range about an anchor price, prices deviations may be perceived

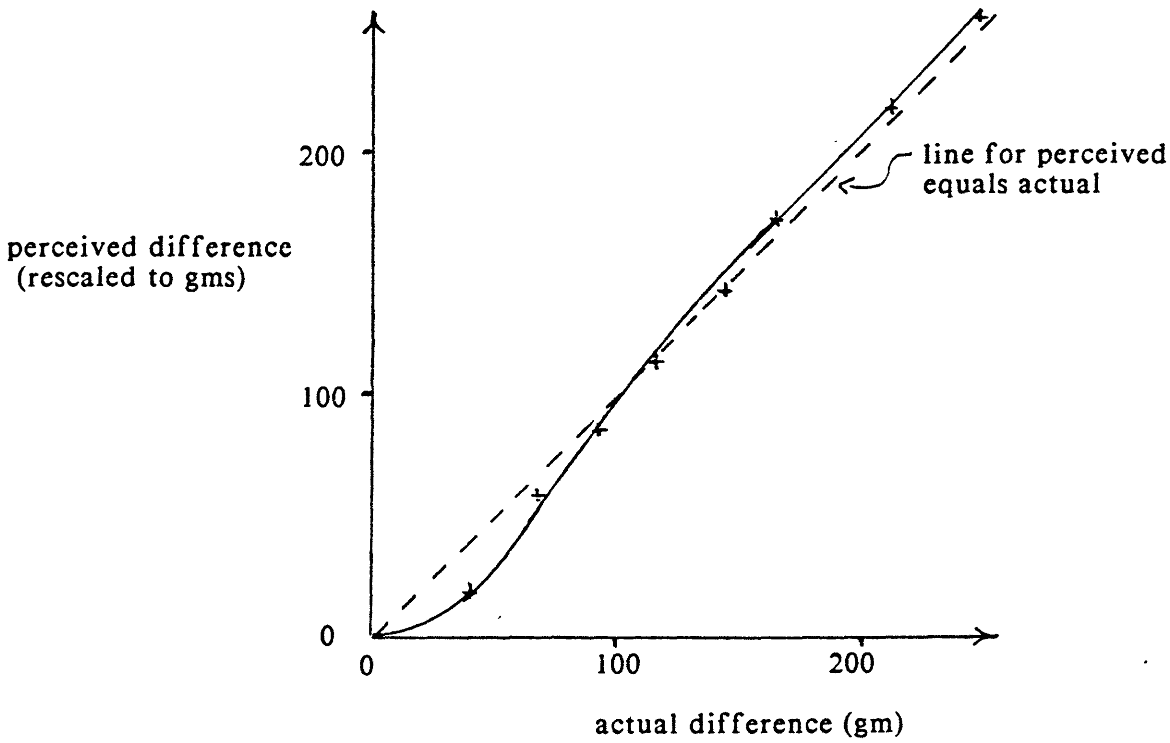

Figure 2. Classical experiments with weights show that, near a reference point, people's perception of a weight changes systematically, with small differences from reference underestimated (assimilation) and large differences overestimated (contrast). Data rescaled and replotted from Sherif, Taub and Hovland (1958).

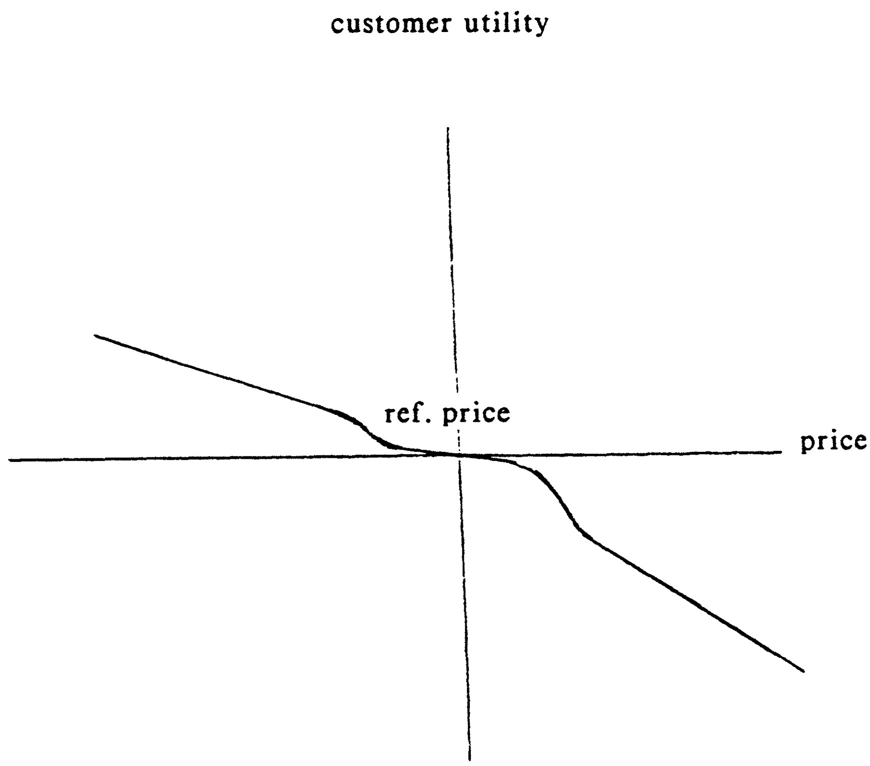

Figure 3. The flattening of response near the reference price as suggested by assimilation theory is superimposed on an asymmetric price curve.

as smaller than they actually are. Outside this range, or "latitude of acceptance," the contrast effect implies that a person will not only take notice of a price difference but may exaggerate it. In Figure 3 we sketch this phenomenon superimposed on an asymmetric price response curve.

The width of a latitude of price acceptance will vary. When a customer has become used to a single price for a long time, the latitude is likely to be narrow. An illustration might be a subway or bus fare which has been in effect for several years. Any change, even a small one, is quickly noticed. However, when a person is unsure of the 'customary price' because price has varied considerably in the past, the latitude of acceptance is likely to be great. A current example is airline fares; price wars and the proliferation of special fares have introduced great price uncertainty so that a customer often has difficulty perceiving whether a quoted fare is high or low. With respect to food products, Uhl and Brown (1971) found that customers' ability to perceive price changes was weaker for products with frequent price changes than for products with more stable prices.

Two issues now arise: First, how is the anchor point determined? Our approach is to take reference price as the anchor point. Second, what is an appropriate measure of latitude of price acceptance? Here we assume that latitude varies with the situation and can be related to price variability in ways to be operationalized below.

COMBINING THEORIES INTO A PRICING MODEL

A combined theory would have the following features. Each individual would have a reference price that depends on past price experience and so changes with time. Around the reference price would be a latitude of price acceptance in which the customer is relatively insensitive to price changes. The width of the latitude would depend on the price variation experienced by the customer and so adapts over time. Beyond the end points of the latitude of acceptance, price sensitivity would take a sharp increase because of the contrast effect. Finally, outside the latitude of acceptance, the response to price decreases (gains) should be steeper than to price increases (losses).

These several ideas can be represented in a set of models of increasing complexity, each adding a new phenomenom, as shown in Figure 4. Figure 4a shows a simple linear response to deviations from reference price. Figure 4b introduces asymmetry as expected from prospect theory. Figure 4c further adds a piece-wise linear, potentially flat segment to represent a latitude of acceptance.

ATTRIBUTION THEORY

Attribution theory predicts that, although promotions may increase the probability of immediate purchase, they may also create a negative after-effect on the customer's perceived value of a product. This idea was first introduced and supported empirically by Dodson, Tybout and Sternthal (1978).

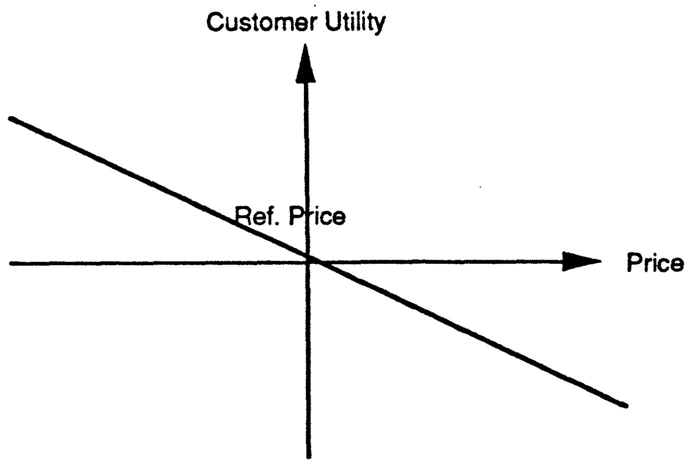

Figure 4 (a)

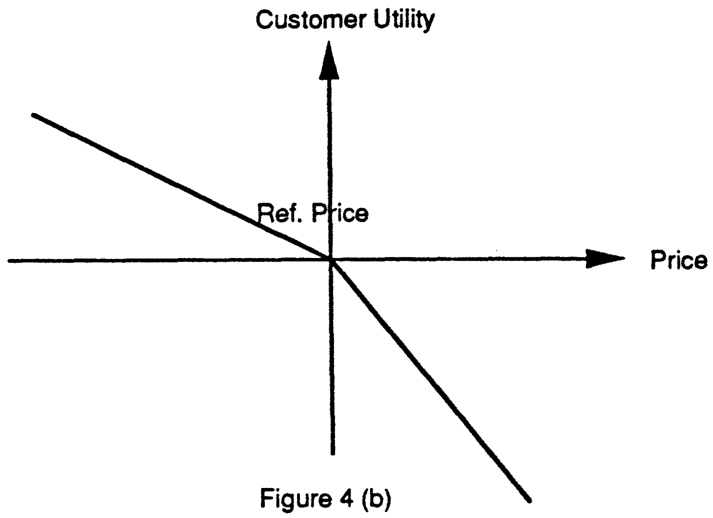

Figure 4 (b)

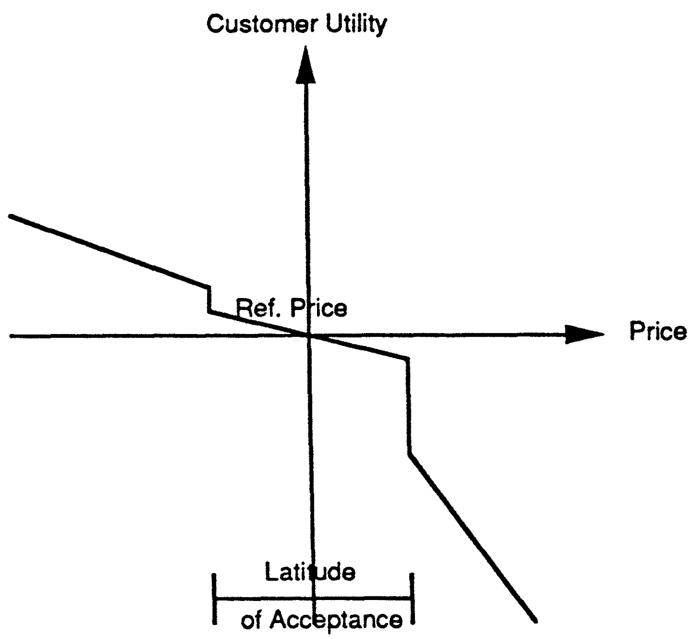

Figure 4 (c)

Figure 4. A family of price response models of increasing complexity. (a) Customer utility is negatively related to price in a simple linear fashion. (b) Prospect theory implies a steeper slope for price increases than decreases. (c) Assimilation theory suggests that small changes from reference will be underestimated.

A central proposition of attribution theory is that individuals determine their attitude toward an object in part by examining their own behavior and the circumstances surrounding it. People tend to ascribe their behavior to either internal or external causes. If an individual attributes her/his behavior to her/his own beliefs then the probability of repeating that behavior is enhanced. However, if there is a plausible external cause, then the behavior will often be ascribed to that cause and the probability of repeating the behavior is diminished.

Application to promotion. Attribution theory would predict that the higher the frequency of promotion, the lower will be the probability of purchase in a subsequent period. The argument is follows: Suppose there is a brand A which is promoted fairly frequently. Let us assume that a customer purchases brand A initially for its intrinsic value. However, since the brand is often on promotion, the customer may start to feel that he/she is purchasing the brand because it is on promotion, i.e., the customer may begin to ascribe the cause of purchase to promotion (an external cause) rather than to his/her own preference (an internal cause). To the extent that the purchases are attributed to promotion, a diminishing in perceived value will take place. For further discussion, see Dodson, Tybout and Sternthal (1978).

One may note that the negative attribution is especially likely to occur if the brand is promoted frequently. Therefore, a negative relation is expected between the frequency of promotion and probability of purchase. Notice, however, that one would still expect a positive impact of promotion on the current purchase relative to the probability of making that purchase without the promotion.

A number of writers have noted that the recent purchase of a product on promotion tends to lower the probability of purchasing it on the next occasion, other variables being held constant. See for example, Shoemaker and Shoaf (1977) and Guadagni and Little (1983). The approach here differs in that it chooses a broad measure, promotional frequency, to characterize the overall promotional activity of the product.

The phenomenon is modeled by its own variable and so can be treated as a separable submodel in our overall price response function.

THE MULTINOMIAL LOGIT MODEL OF CHOICE

The multinomial logit model of probability of choice is well suited for modeling at the household level and has been used successfully in a number of marketing studies (Gensch and Recker 1979, Guadagni and Little 1983). In particular we draw heavily on the last. Essentially, we remove the pricing part of Guadagni and Little's model and substitute a new, behaviorally motivated structure. Related modifications are also made to promotion variables.

The multinomial logit, as it will be used here, calculates the probability of choice by an individual as a function of inferred utilities for a set of alternatives. Let

P^i_k = probability that individual i will choose alternative k;

v^i_k = deterministic component of utility of alternative k for i.

Then

$$P^i_k = \exp(v^i_k)/\Sigma_j \exp(v^i_j).$$

We shall express the utilities as linear functions of explanatory variables:

$$v^i_k = \Sigma_r b_{rk} x^i_{rk}$$

where

x^i_{rk} = observed value of explanatory variable r for alternative k for customer i,

b_{rk} = utility weight of explanatory variable r for alternative k.

Utility is not directly observable. However, observations are available on actual choices and the explanatory variables associated with those choices. Maximum likelihood estimation then determines the b's. For a comprehensive discussion see Ben Akiva and Lerman (1985).

An important advantage of the multinomial logit, as used here, is that it provides a fully competitive model. That is, the past history and marketing actions of every brand-size enters into the customer's probability of purchase of any brand-size, and, therefore, into the estimation of all response parameters of the model.

For a number of purposes we shall wish criteria for evaluating the logit calibration. Standard logit output provides asymptotic t-values for estimated coefficients and the final value of the log likelihood. In addition we often use U^2 (Hauser 1978), a measure of uncertainty explained by the calibrated model beyond a chosen reference model. U^2 varies from 0 (nothing further explained) to 1 (perfect explanation). $U^2 = 1-L(X)/L_0$ where L(X) is the log likelihood of the calibrated model with explanatory variables, X, and L_0 is the log likelihood of a reference model. For the latter we use equal probabilties for all alternatives.

DATA AND VARIABLES

DATA

Two scanner panel databases provide the raw data for our empirical work. One of them (SAMI) consists of ground coffee panel and store records from four Kansas City supermarkets for the 78-week period September 14, 1978 to March 12, 1980. This is the same database used by Guadagni and Little (1983). The store sales contain weekly movement for each UPC in the category as well as the shelf price for each item each week. A single panel purchase record contains the household number, the date of purchase, the UPC, and the price paid. Households with reporting gaps and households that join in the middle of the period have been omitted. Light and non-users of ground coffee (less than five purchases of relevant brand-sizes during the period under consideration) have been eliminated. Of the 200 families used by Guadagni and Little, 175 remain. The group made 2966 purchases in the brand-sizes considered during the 32-week period, March 8 to October 17, 1979. This is the calibration period.

In addition to the calibration period, 757 purchases over the previous 25 weeks have been used for initialization. Each purchase is treated as an observation so that we are combining cross-section and time-series data.

To check the results found with the SAMI data, we perform the analysis on a second data base: the IRI academic coffee data base. This consists of continuously reporting households for two years from April 7, 1980 to April 4, 1982, a period of 102 weeks. Of these the first 30 weeks are used for initialization and the remaining 72 constitute the calibration period. After appropriate cleaning of the data, 88 families have been chosen at random. These families made 2267 purchases in the calibration period and 788 purchases in the initialization period.

ALTERNATIVES

The first step in setting up the model is to identify a set of alternatives for the customers to choose from. We model brand-sizes and, to obtain a relatively homogeneous set of alternatives, we restrict consideration to regular ground caffeinated coffee. For a discussion of the rationale, see Guadagni and Little.

In working with the SAMI data base, Guadagni and Little model the eight largest selling brand-sizes. In our case we have dropped brand-sizes with very little price variability i.e. brand-sizes for which the price variability is smaller by an order of magnitude than for the retained products. As a result we keep six brand-sizes: MH.S, MH.L, BUT.S, BUT.L, FOL.S, and FOL.L. These products are Maxwell House, Butternut, and Folgers in one pound (S) and three pound (L) sizes.

In the case of IRI data base, we model 9 brand-sizes. Our cut-off in this case is a market share of 4%. This provides the choice set: FOL.S, CFN.S, MH.S, HILLS.S, CHN.S, MH.L, CHN.L, MART.S and CS.S. These products are Folgers, Chock-full-o'-Nuts, Hills Brothers, private label of a principal chain, Maxwell House, Martinson, and Chase and Sanborn with sizes one pound (S) and two pound (L).

BASIC EXPLANATORY VARIABLES

The model uses several non-price variables taken from Guadagni and Little including brand loyalty, size loyalty, promotion, and the brand-size constants.

Brand loyalty. An individual customer (household) is assumed to have certain brand preferences that can be captured by observing past behavior. We define a variable, which will be called brand loyalty, for each customer. Let

$x^i_{1k}(t)$ = brand loyalty for brand of brand-size k for t^{th} coffee purchase of customer i.

$$x^i_{1k}(t) = (\alpha_b)x^i_{1k}(t-1)+(1-\alpha_b)\begin{cases} 1 & \text{if customer i bought brand of alternative k at purchase occasion t-1,} \\ 0 & \text{otherwise.} \end{cases}$$

The carry-over constant is α_b. To start up brand loyalty, we set $x^i_{1k}(1)$ to α_b if the brand of alternative k was the first purchase for the customer i, otherwise $(1-\alpha_b)$/(number of brands - 1). As a result the loyalties for a customer always add up to one across brands.

Size loyalty. Similarly,

$x^i_{2k}(t)$ = size loyalty for size of brand-size k for t^{th} coffee purchase of customer i.

$$x^i_{2k}(t) = (\alpha_s)x^i_{2k}(t-1)+(1-\alpha_s)\begin{cases} 1 & \text{if customer i bought size of alternative k at purchase occasion t-1,} \\ 0 & \text{otherwise.} \end{cases}$$

Here α_s is the carry-over constant for size loyalty. Initialization is analagous to brand loyalty. Through iterative search the carry-over constants for brand and size loyalties have been set at 0.875 and 0.812 respectively. The log-likelihood is not very sensitive to the smoothing constants in the neighborhood of these values.

Promotion. We determine promotional activity in the SAMI data base in the same manner as Guadagni and Little (1983). Let

$$x^i_{3k}(t) = \begin{cases} 1 & \text{if brand-size k was on promotion at time of customer i's } t^{th} \text{ coffee purchase,} \\ 0 & \text{otherwise.} \end{cases}$$

In the IRI data base, we replace this promotion variable with two better ones collected by IRI, advertising feature and display. These are 0-1 variables defined by

$$x^i_{4k}(t) = \begin{cases} 1 & \text{if brand-size k had an advertising feature at time of customer i's } t^{th} \text{ coffee purchase,} \\ 0 & \text{otherwise.} \end{cases}$$

$$x^i_{5k}(t) = \begin{cases} 1 & \text{if brand-size k had a special display at time of customer i's } t^{th} \text{ coffee purchase,} \\ 0 & \text{otherwise.} \end{cases}$$

Brand-size Constants. The utility function for a brand-size will include an additive constant specific to that alternative. This is accomplished by a set of dummy variables, one for each brand-size alternative, K, (K=1,2,...) except that one must be omitted to avoid singularity in the maximum likelihood estimation. The omitted variable has an implicit brand-size constant of zero. Let

$$x^i_{0Kk}(t) = \begin{cases} 1 & \text{if brand-size } k = K, \\ 0 & \text{otherwise.} \end{cases}$$

The brand-size specific constants capture any constant utility attributable to the product and not explained by the other variables of the model.

OPERATIONALIZING PROMOTIONAL FREQUENCY

Attribution theory suggests that frequent promotion of a brand may erode its perceived value to a customer. Therefore we introduce a variable to describe how often a customer has encountered a promotion for a particular brand-size in the past. Let

$x^i_{6k}(t)$ = promotion frequency for brand-size k at time of customer i's t^{th} coffee purchase.

$$x^i_{6k}(t) = \Sigma_{s<t}\, x^i_{3k}(s) \;/\; (\text{Total number of coffee purchases by customer i prior to } t).$$

The above definition applies to the SAMI data base. In the case of IRI an brand-size is considered to be on promotion if it is either featured or displayed or both. Appropriate changes are made in the definition of x_6. Our formulation gives equal weight to all promotions from the start of the data series.

Our hypothesis is that promotional frequency adversely affects probability of purchase. In other words, letting

contribution to customer i's utility = $\beta_F\, x^i_{6k}(t)$,

we expect the coefficient β_F to be negative in the empirical testing.

OPERATIONALIZING THE PRICING MODEL

The contribution of price to a customer's utility is modeled as a linear function of reference price plus further piece-wise linear components as sketched in Figure 4.

Reference price. Reference price for each customer is taken to be an exponential smoothing of past prices encountered by the customer for that brand-size. Let

$p^i_k(t)$ = actual shelf price of brand-size k at time of customer i's t^{th} purchase (dollars/ounce),

$p^i_{Rk}(t)$ = reference price of brand-size k for customer i at time of t^{th} purchase (dollars/ounce).

Then

$$p^i_{Rk}(t) = \beta_R\, p^i_{Rk}(t-1) + (1-\beta_R)\, p^i_k(t-1).$$

This can also be written

$$p^i_{Rk}(t) = p^i_{Rk}(t-1) + (1-\beta_R)(p^i_k(t-1)-p^i_{Rk}(t-1)).$$

Thus each brand-size is assigned its own reference price. The reference prices are constantly updated, with the customer making adjustments depending on the price difference between the previous purchase price and the previous reference. The value of β_R has been picked by grid search to maximize the U^2 criterion (which is equivalent to maximizing likelihood). This gives $\beta_R = 0.85$.

Notice that the shelf price used to form the reference price will often contain promotional price cuts. Thus, if a product is on promotion nearly all the time, the customer will come to have a reference price that is almost the same as the promotional price.

Latitude of price acceptance. Figure 4 has illustrated the latitude of price acceptance. We take its width to be proportional to price variability, which, in turn, is derived from a smoothed function of the deviations between actual price and reference price. Specifically, let

$a^i_k(t)$ = latitude of price acceptance for brand-size k for customer i on purchase t;

$a^i_k(t)$ = $\delta\, s^i_k(t)$

where

$s^i_k(t)$ = variability of price for brand-size k for customer i on purchase t (dollars/ounce),

and

$$[s^i_k(t)]^2 = \gamma[s^i_k(t-1)]^2 + (1-\gamma)[p^i_k(t-1)-p^i_{Rk}(t-1)]^2.$$

Thus the latitude of price acceptance is set proportional to smoothed fluctuations of price around the reference price as measured by a standard deviation-like quantity. We determine γ through a grid search to maximize U^2. This yields $\gamma = 0.80$. The value of δ determines the width of the latitude of acceptance and will be the subject of a subsequent sensitivity analysis.

Figure 4c sketches customer utility vs. price as a three-piece linear model centered on the reference price, p_R. The pieces are: a hypothesized flat place of width equal to the latitude of acceptance around p_R plus two negatively sloping pieces, one on each side. Three mutually exclusive transformations of price cover the three ranges. Suppressing the notation for customer and brand, let

m = mid-range variable for prices within the latitude of price acceptance,

l = customer "loss" variable operating for prices higher than the latitude of acceptance,

g = customer "gain" variable operating for prices lower than the latitude of acceptance.

We take

$$m = \begin{cases} (p-p_R)/s & \text{if } p_R - a/2 < p < p_R + a/2 \ , \\ 0 & \text{otherwise;} \end{cases}$$

$$l = \begin{cases} (p-p_R)/s & \text{if } p > p_R + a/2 \ , \\ 0 & \text{otherwise;} \end{cases}$$

$$g = \begin{cases} (p_R-p)/s & \text{if } p < p_R - a/2 \ , \\ 0 & \text{otherwise.} \end{cases}$$

Notice in these three variables that the difference between purchase price and reference price is divided by s, the measure of price variability. With this scaling, m ranges over $(-\delta/2,+\delta/2)$, while l and g range over $(\delta/2,\text{infinity})$, permitting δ to set the latitude of acceptance.

We now write the contribution of price to customer utility in the multinomial logit model:

$$\begin{array}{ll} \text{Contribution of price to utility} & = \beta_R\, p^i_{Rk}(t) + \beta_G\, g^i_k(t) \\ \text{of customer i for brand-size k} & \\ \text{at purchase t} & \qquad + \beta_M\, m^i_k(t) + \beta_L\, l^i_k(t). \end{array}$$

The theories described earlier provide hypotheses as shown in Table 1.

Table 1. Hypotheses to be tested.

1)	$\beta_R < 0$	Higher reference price decreases customer utility.
2)	$\beta_L < 0$	Positive deviations from reference decrease utility.
3)	$\beta_G > 0$	Negative deviations from reference increase utility.
4)	$\lvert\beta_L\rvert > \lvert\beta_G\rvert$	Slope will be steeper for losses than gains.
5)	$\beta_M = 0$	Changes in prices within the latitude of acceptance have little effect.
6)	$\beta_F < 0$	Increasing promotion frequency has a negative effect

---

CALIBRATION AND TESTING

To test our hypotheses we define a base case which we examine in detail and then do sensitivity analyses around it. The base case sets $\delta = 1.0$ in the latitude of acceptance. This says that the presumed flat portion in price response has a width of about a standard deviation of price. Table 2 shows the logit calibration for the two databases.

Table 2. Base Case: $\delta = 1.0$

Variable	SAMI database	IRI database
Brand Loyalty	2.80 (30.8)	4.86 (38.8)
Size Loyalty	2.69 (26.0)	2.87 (13.7)
Promotion	1.79 (22.6)	--
Feature	--	1.26 (9.65)
Display	--	0.78 (5.9)
Reference Price	-8.53 (-2.7)	-64.71 (-16.2)
Loss	-0.56 (-8.4)	-0.81 (-5.7)
Gain	0.41 (7.9)	0.58 (5.5)
Mid-range	-0.13 (-0.7)	1.44 (0.8)
Prom. Frequency	-0.48 (-2.9)	-0.28 (-1.8)
Brand Size Constants:		
MH.S	0.29 (2.8)	0.94 (3.6)
MH.L	0	1.18 (4.2)
BUT.S	0.10 (1.0)	--
BUT.L	-0.20 (-1.8)	--
FOL.S	0.53 (5.13)	0.34 (1.40)
FOL.L	-0.04 (-0.3)	--

CFN.S	--	-0.08 (-0.3)
HILLS.S	--	0.04 (0.2)
CHN.S	--	-0.85 (-4.0)
CHN.M	--	0
MART.S	--	0.11 (0.4)
CS.S	--	0.69 (2.2)
U^2	0.4779	0.6374
Loglikelihood	-2772.1756	-1635.9128

(t-values in parentheses)

Both sets of parameters show certain characteristics typical of this type of model. The t-values for the coefficients of loyalty and promotion (display and feature in the case of IRI) are very large. These variables account for much of the explained uncertainty reported in U^2. The brand-size constants are mostly small, although a few seem to pick up a residual uniqueness not explained by the rest of the variables. The SAMI and IRI calibrations are somewhat different but not surprisingly so, considering that the data come from different households, cities and time periods and involve different selections of brand-sizes.

With respect to our hypotheses, the results are quite supportive and the two databases remarkably consistent. First of all, the basic negative effects of price on utility and thence probability of purchase are supported by the negative coefficient β_R for reference price, negative β_L for substantial price increases (customer "losses"), and positive β_G for substantial price decreases (customer "gains"). All of these are solidly significant from a statistical point of view in both databases, as would be expected.

A more interesting parameter is β_M, the coefficient for price deviations in the mid-range, i.e., within the hypothesized latitude of acceptance. We find that the coefficient is negative in the SAMI database, positive in IRI, but not significantly different from zero in either. Thus the existence of a latitude of price acceptance seems to be supported, a potentially important finding.

Another key hypothesis, borrowed from prospect theory, is that the slope of losses is steeper than that of gains. Let

$$\theta = |\beta_L| - |\beta_G| = -(\beta_L + \beta_G).$$

The question now becomes: is $\theta > 0$? The answer is that in both databases it is, supporting the hypothesis in terms of having the right algebraic sign. Statistical significance is a more delicate issue. An approach to addressing this is to use the above definition of Θ to eliminate one of the β's in the model and re-estimate the parameters. Θ becomes the coefficient of the variable (-g-l) and the model remains algebraically the same. The results are shown in Table 3. We find that $\Theta > 0$ in each database but that the result does not quite reach conventional levels of significance (t=1.74 in SAMI and t=1.35 in IRI).

The last hypothesis is that promotional frequency will have a negative effect, i.e., $\beta_F < 0$. We find that it does in both databases. In SAMI the value is statistically significant; in IRI it is not quite.

Since the SAMI and IRI results are statistically independent, it is natural to ask whether there is some way to combine them for making statistical tests. One simple method (suggested to us by A. Barnett) is to average the coefficients derived from the two databases. Since the variance of the sum of independent random variables is the sum of their variances and we can obtain the latter from the standard errors produced by the logit runs, it is a simple matter to calculate a t-value for the average of the coefficients. This is shown in Table 3. We find that now both Θ and β_F are statistically significant at better than a 95% level in the combined results.

If we try the combined test on the values of β_M, the slope in the proposed latitude of acceptance region, it is still not significantly from zero (t=0.9).

Table 3. Key parameters and their t-values (in parentheses) for base case: δ = 1.0.

variable	coefficient	SAMI	IRI	Combined
Reference price	β_F	-8.53 (-2.7)	-64.71 (-16.2)	*
Loss	β_L	-0.56 (-8.4)	-0.81 (-5.7)	*
Gain	β_G	0.41 (7.9)	0.58 (5.5)	*
Mid-range	β_M	-0.13 (-0.7)	1.44 (0.8)	0.66 (0.9)
Promotional frequency	β_F	-0.48 (-2.9)	-0.28 (-1.8)	-0.38 (-3.3)
-Loss-Gain	Θ	0.15 (1.7)	0.23 (1.4)	0.19 (2.0)

* Must be significant because both SAMI and IRI results are.

<u>Sensitivity analysis of latitude of acceptance</u>. So far we have examined a base case with latitude of acceptance selected by δ = 1.0. By varying delta we can make the width of the hypothesized flat region go from zero to any size we wish. We present two cases.

$\underline{\delta = 0}$. This case eliminates the latitude of acceptance so that the price model reduces to two segments, one for gains and one for losses. The coefficients of interest appear in Table 4 and a complete set in Appendix 2. Excepting the omitted β_M, we see that all hypotheses are supported, if not in both databases separately, then in the combined results.

$\underline{\delta = 2.0}$. Now we have a wide value for the proposed latitude. The relevant coeffients again appear in Table 4 and the complete set in Appendix 2. Setting aside latitude of acceptance for the moment, we see that all the other hypotheses are supported, if not in both databases, then in the combined results, with the exception of $\theta > 0$, which does not quite make statistical significance.

The interesting point, however, is that slope, β_M, of price deviations in the mid-range within the proposed latitude of acceptance is now significantly negative in both databases. This suggests that we are trying to extend the latitude too far: While the construct was supported at a tighter width ($\delta = 1.0$), it is rejected at the larger width ($\delta = 2.0$).

Table 4. Sensitivity analysis on width of latitude of acceptance: Key parameters and their t-values (in parentheses) for $\delta = 0$ and $\delta = 2.0$.

		$\delta = 0$			$\delta = 2.0$		
var.	coef.	SAMI	IRI	ave.	SAMI	IRI	ave.
Ref. price	β_F	-8.19 (-2.6)	-64.57 (-16.2)	*	-8.10 (-2.5)	-64.70 (-16.3)	*
Loss	β_L	-0.58 (-8.3)	-0.82 (-5.7)	*	-0.55 (-8.5)	-0.78 (-5.6)	*
Gain	β_G	0.40 (7.4)	0.56 (5.3)	*	0.42 (8.4)	0.58 (5.6)	*
Mid-range	β_M	-	-	-	-0.51 (-6.4)	-1.34 (-2.8)	*
Prom. freq.	β_F	-0.46 (-2.9)	-0.29 (-1.8)	-0.38 (-3.3)	-0.46 (-2.9)	-0.27 (-1.7)	-0.37 (-3.2)
-Loss -Gain	θ	0.18 (2.1)	0.26 (1.5)	0.22 (2.3)	0.13 (1.7)	0.20 (1.2)	0.17 (1.8)

* Must be significant because both SAMI and IRI results are.

To gain a better understanding of the width of the latitude of acceptance we have performed an exploration of δ beyond the three cases discussed above. We find a price-insensitive region (slope not significant) around the reference price, but the slope depends on how wide we make the interval. As the width, δ, moves away from zero, the region remains insensitive to price for δ up to about 1.0. As the width increases further toward $\delta = 2.0$, price sensitivity becomes evident (magnitude of slope significant).

DISCUSSSION

The most interesting results are:

(1) the asymmetric price response: customers are more sensitive to increases than to decreases as predicted by prospect theory;

(2) the relatively flat spot around reference price: there is a reduced price sensitivity with a width of about a standard deviation;

(3) the negative impact of promotional frequency on probability of purchase.

Although some individual coefficients are not significant at conventional levels, it is striking to find consistent results across the two databases for all three cases, and, in the base case that is our central model, all hypotheses are supported in the analysis that combines both databases.

Practical implications. The practical implications of our results are several. First, the presence of a region of price insensitivity suggests that marketers wishing to increase prices should nibble not bite. Small price increases are less hazardous if they stay within the latitude of acceptance. One may reasonably ask, however: what is to stop one from using small steps to increase price indefinitely without appreciable loss? the answer lies within the model - this is prevented, or at least penalized, by the reference price, which has a large negative coefficient. As price increases are added, reference price goes up and reduces the probability of purchase.

On the price decrease side, a marketer will usually wish to make changes dramatic enough to exceed the insensitive region and so pick up the contrast effect. In this respect it is interesting to note that most promotional price cuts are fairly substantial (e.g., 10-20%).

Our findings on promotion frequency raise a warning flag on promoting too often. Obviously there is a trade-off between the big short term boost of the promotional price cut and the longer term erosion of probability of purchase from frequent promotions. A good policy may be to stimulate relatively deep promotional price cuts, enough to reach outside the price insensitive region, but not to do this very often. The model presented here provides an approach to appraising such trade-offs. The effects of different policies and scenarios on share and profits can be simulated.

Limitations, extensions. The model proposed and tested here is felt to be desirable for several reasons. First, it has a clear theoretical foundation. Second, it incorporates phenomena of practical importance. Third, it appears to be applicable to other products and categories. At the same time, however, in order to apply the perceptual theories to a marketing setting, it has been necessary to operationalize the psychological constructs in specific ways.

These are not unique and other researchers may find better ones. Our results are generally encouraging, but a single application, even on two databases, cannot be considered definitive. More testing would certainly be desirable, especially on other product categories.

Several elaborations suggest themselves immediately: Perhaps the latitude of acceptance region is asymmetric. It would be interesting to try more segments in the price response function. Quite likely there are interactions with other marketing variables, for example, feature advertising. These and certain other investigations lie beyond the limits of the present data sets but some of them should become feasible in the larger scanner panel databases now coming into existence.

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APPENDIX 1

Replot of data from Sherif, Taub and Hovland (1958)

In the classical experiments of Sherif, Taub and Hovland (1958), subjects judged a series of 6 weights (55, 75, 93, 109, 125, and 141 gm) on a 1 to 6 scale. The median estimate was about 3.13, which, by linear interpolation within the extremes of the series, corresponds to 91.6 gm. Following the initial judgments of the series, anchor weights were introduced and reinforced, one at a time. Each time, the subject then judged the series weights again on the 1 to 6 scale. Thus the anchor was varied and the stimulus held constant, whereas we would like a plot with reference held constant and the stimulus varying. We handle this by plotting the experimental data as differences between stimulus and anchor.

Another feature of the experiment was that the subject was told each time that the anchor represented 6 on the scale. Since the anchors varied from 141 gm to 347 gm and the lightest weight in the original series stayed at 55 gm, the meaning of the 1 to 6 scale changed from one anchor point to the next, unlike the situation in our application. Therefore we have rescaled each measurement to a common basis labeled "perceived weight." This is constructed by linearly converting the range 1 to 6 into a range going from 55 gm to the anchor weight. Taking the difference between 91.6 and these perceived weights, we learn how much the perceived weight was shifted by the presence of each anchor. Negative shifts represent assimilation and positive ones contrast.

Median values for the series weights were measured from Figure 5 in Sherif and Hovland (1961), which reprints the experimental results in an enlarged graph. Our calculations for the plot are shown below. The point for the anchor of 288 gm is anomalous in the original data and, since we are not critiquing the basic paper, we omit this outlier in the plot.

Anchor (gm)	Smallest weight in series (gm)	Median judgment of series wts. (1 to 6 scale)	Median judgment converted to perceived wt. (gm)	Shift= 91.6 - perceived weight (gm)	Differences actual (anchor - 101)	Differences perceived (actual + shift)
none*		3.13	91.6	0	0	0
141	55	4.34	112.5	-20.9	40	19.1
168	55	2.98	99.8	- 8.2	67	58.8
193	55	2.59	98.9	- 7.3	92	84.7
219	55	2.23	95.3	- 3.7	118	114.3
244	55	1.98	92.0	- 0.4	143	142.6
267	55	1.74	86.4	5.2	166	171.2
288	55	2.02	102.5	-10.9	187	176.1
312	55	1.61	86.3	5.3	211	216.3
347	55	1.54	86.5	5.1	246	251.1

Example calculation of perceived weight: $112.4 = 55+(141-55)(4.34-1)/(6-1)$

* However, we need a physical reference point to represent the series weights. We take the median, which is 101 gm.

APPENDIX 2

Table 5. $\delta = 0$

	SAMI	IRI
Brand Loyalty	2.82 (30.81)	4.86 (38.84)
Size Loyalty	2.68 (25.97)	2.87 (13.64)
Promotion	1.77 (22.54)	- -
Feature	- -	1.27 (9.71)
Display	- -	0.78 (5.88)
Reference Price	-8.19 (-2.58)	-64.57 (-16.20)
Loss	-0.58 (-8.27)	-0.82 (-5.70)
Gain	+0.40 (7.39)	+0.56 (5.32)
Prom. Freq.	-0.46 (-2.86)	-0.29 (-1.80)
Brand Size Constants:		
MH.S	0.29 (2.84)	0.94 (3.63)
MH.L	0	1.18 (4.24)
BUT.S	0.10 (0.96)	- -
BUT.L	-0.20 (-1.79)	- -
FOL.S	0.54 (5.23)	0.34 (1.40)
FOL.L	-.03 (-0.27)	- -
CFN.S	- -	-0.07 (-0.31)
HILLS.S	- -	0.05 (0.19)
CHN.S	- -	-0.84 (-3.95)
CHN.M	- -	0
MART.S	- -	0.12 (0.44)
CS.S	- -	0.69 (2.18)
U^2	0.4777	0.6373
Log likelihood	-2773.6596	-1636.5362

(t-values in parentheses)

Table 6. $\delta = 2.0$

	SAMI	IRI
Brand Loyalty	2.82	4.86
	(30.81)	(38.82)
Size Loyalty	2.68	2.87
	(25.95)	(13.67)
Promotion	1.77	- -
	(22.25)	
Feature	- -	1.26
	- -	(9.68)
Display		0.77
		(5.82)
Reference Price	-8.10	-64.70
	(-2.53)	(-16.24)
Loss	-0.55	-0.78
	(-8.52)	(-5.62)
Gain	0.42	0.58
	(8.35)	(5.64)
Mid-range	-0.51	-1.34
	(-6.39)	(-2.76)
Prom. Freq.	-0.46	-0.27
	(-2.86)	(-1.71)
Brand Size Constants:		
MH.S	0.29	0.93
	(2.85)	(3.60)
MH.L	0	1.18
		(4.22)
BUT.S	0.10	- -
	(0.98)	
BUT.L	-0.20	- -
	(-1.91)	
FOL.S	0.54	0.33
	(5.24)	(1.36)
FOL.L	-0.03	- -
	(-0.29)	
CFN.S	- -	-0.08
		(-0.34)
HILLS.S	- -	0.03
		(0.14)
CHN.S	- -	-0.85
		(-4.00)
CHN.M	- -	0
MART.S	- -	0.11
		(0.39)
CS.S	- -	0.69
		(2.16)
U^2	0.4776	0.6374
Log likelihood	-2774.2425	-1635.8877

(t-values in parentheses)