A MARKUP INTERPRETATION OF OPTIMAL RULES FOR IRREVERSIBLE INVESTMENT

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WP#3945-97-EFA March 1997
A Markup Interpretation of Optimal Rules for Irreversible Investment*

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This version: February 19, 1997

Abstract: We re-examine the basic investment problem of deciding when to incur a sunk cost to obtain a stochastically fluctuating benefit. The optimal investment rule satisfies a trade-off between a larger versus a later net benefit; we show that this trade-off is closely analogous to the standard trade-off for the pricing decision of a firm that faces a downward sloping demand curve. We reinterpret the optimal investment rule as a markup formula involving an elasticity that has exactly the same form as the formula for a firm's optimal markup of price over marginal cost. This is illustrated with several examples.

JEL Classification Numbers: D92, D81, E22

Keywords: Investment, sunk costs, pricing decisions, optimal markups.

*This research was supported by the National Science Foundation through grants to Dixit and Pindyck, and by M.I.T.'s Center for Energy and Environmental Policy Research. Our thanks to John Leahy for helpful comments.
1. Introduction.

Consider what is probably the most basic irreversible investment problem: a project can be undertaken that requires a sunk cost $C$ and yields a benefit $V$. The cost is known and certain, but the benefit (measured as the present value at the time the cost is incurred) fluctuates as a stationary Markov process $\{V_t\}$ with continuous sample paths.\(^1\) Time is continuous, and at each point the firm must decide whether to invest or to wait and reconsider later. The firm's objective is to maximize the expected present value of net benefits, with a discount rate that is constant and equal to $\rho$.

At time $t$, all of the information about the future evolution of $V$ is summarized in the current value $V_t$. Therefore the optimal decision rule must be of the form: invest now if $V_t$ is in a certain subset of possible values, otherwise wait. Also, because the process is stationary and the discount rate is constant, the optimal rule will be independent of time. So long as the process has positive persistence — i.e., a higher current value $V_t$ shifts the distribution of the random value $V_s$ at any future time $s$ to the right in the sense of first-order stochastic dominance — the rule will be of the form: invest now if $V_t$ is at or above a critical threshold $V^*$, otherwise wait.\(^2\) The problem therefore boils down to determining the optimal choice for the threshold $V^*$.

As first shown by McDonald and Siegel (1986), the optimal $V^*$ exceeds $C$ by a "markup," or premium, that reflects the value of waiting for new information. One can think of the firm as having an option to invest that is akin to a financial call option, and, like the call option, is optimally exercised only when "deep in the money," i.e., when the stock price is at a premium over the exercise price. Thus one can solve the firm's investment problem (and determine the optimal markup) by finding the value of the firm's option to invest and the optimal exercise rule.\(^3\) Indeed, identifying and valuing the firm's option to invest has become the standard approach to solving irreversible investment problems.

\(^1\) $V$ may itself be explained in terms of other more basic economic variables like prices of output and/or inputs; we work simply with the end result.


\(^3\) The option is valued assuming it is exercised optimally, so the valuation of the option yields the optimal exercise rule. See Dixit and Pindyck (1996).
However, as Baily (1995) has pointed out, an alternative way to find the optimal $V^*$ is to examine the trade-off between larger versus later net benefits. Specifically, choosing a larger value for $V^*$ implies that the net benefit, $V^* - C$, will be larger, but will be received at a more distant (but unknown) time in the future, and thus will be discounted more heavily. The optimal choice of $V^*$ is that for which the additional net benefit from making $V^*$ larger just balances the additional cost of discounting.

In this paper, we take this alternative perspective further by developing an intuitively appealing analogy with the trade-off involved in the pricing decision of a firm facing a downward-sloping demand curve — i.e., the trade-off between a higher profit margin and a lower volume of sales. We show that $V$ can be regarded like a price, $(V - C)$ like a profit margin or markup, and the discount factor like a demand curve. The optimal $V^*$ is then given by a markup formula involving the elasticity of the discount factor with respect to $V$, which has exactly the same form as the formula for a firm's optimal markup of price over marginal cost. This suggests extensions of the basic investment problem by analogy with the corresponding extensions of the monopolist's pricing problem. Here we develop one, namely the optimal choice of an ancillary expenditure in advertising or R&D which can speed up the (stochastic) passage to the threshold. The result is analogous to the formula for a monopolist's optimal advertising-to-sales ratio.

2. The Optimal Markup.

Suppose the initial level of the benefit is $V_0$, and consider an arbitrary threshold $V > V_0$. Thus the firm will wait until the first time $T$ at which the benefit $V_T$ has reached $V$, and will then invest. (In technical terms, $T$ is the first-passage time or hitting time from $V_0$ to $V$.) This time $T$ is a random variable, and its distribution can be determined from the known probability law of the evolution of $V_t$. Taking expectations using this distribution, the net present value of this policy is

$$\mathbb{E}[e^{-\sigma T}] (V - C).$$

Note that the expectation of the discount factor in this expression depends on both the initial value $V_0$ and on the threshold value $V$ of our decision rule. We therefore denote this
discount factor as:

\[ D(V_0, V) \equiv \mathcal{E}[e^{-pT}] \quad (1) \]

The optimal threshold, \( V^* \), is the value of \( V \) which maximizes

\[ D(V_0, V) (V - C). \quad (2) \]

The first-order condition for the optimal \( V^* \) is

\[ D(V_0, V') + D_V(V_0, V^*)V^* = D_V(V_0, V^*)C, \quad (3) \]

where \( D_V \) is the partial derivative of the discount factor \( D \) with respect to its second argument, namely the threshold value \( V \), and we are evaluating this at \( V = V^* \). This condition simply says that if the investment opportunity is to be optimally exercised, the expected marginal discounted benefit from the investment should just equal the expected marginal discounted cost.

We can rewrite eqn. (3) in the following equivalent form:

\[ \frac{V^* - C}{V^*} \bigg\{ \frac{D(V_0, V^*)}{D(V_0, V^*)} \bigg\}^{-1} = \frac{1}{\epsilon_D}, \]

where \( \epsilon_D \) denotes the elasticity of the discount factor \( D \) with respect to \( V^* \), i.e., \( \epsilon_D \equiv -V^* D_V / D \). The form of this expression should be very familiar; it is just like the markup pricing rule that follows from equating marginal revenue with marginal cost:

\[ \frac{p - c}{p} = 1/\epsilon_P, \]

where \( p \) is the price, \( c \) is marginal cost, and \( \epsilon_P \) is the magnitude of the price elasticity of the firm's demand.

There is indeed a close connection between equation (4) for the investment markup and the markup pricing rule. To see this, compare the expression for the present value, (2), to that for the firm's profit in the usual pricing problem when marginal cost is constant, namely \( (p - c) q(p) \). A higher \( p \) implies a higher profit margin \( p - c \), but a lower volume of sales \( q(p) \). The trade-off that determines the optimal price is governed by the rate at which \( q(p) \) declines as \( p \) is increased, i.e., by the price elasticity of demand. In our investment problem,
a higher threshold $V^*$ yields a higher margin $(V^* - C)$ of benefits over costs, but a smaller
discount factor $D(V_0, V^*)$ because the process is expected to take longer to reach the higher
threshold. The investment trade-off depends on the elasticity of the discount factor with
respect to the threshold.

We can put this analogy in graphical terms by considering an arbitrary threshold $V$, and
re-writing eqn. (3) as

$$V + D(V_0, V)/D_V(V_0, V) = C. \quad (5)$$

We can think of the first term in this equation, $V = V(D, V_0)$, as the inverse of the discount
factor; it is analogous to the inverse demand, or average revenue function, $p(q)$, for the price-
setting firm. Likewise, the discount factor $D(V, V_0)$ is analogous to quantity for the price-
setting firm, so the left-hand side of eqn. (5) — the marginal benefit from an increase in $D$ —
is analogous to the marginal revenue function.\footnote{To see this, obtain the first-order condition for the investment problem by the discounted net payoff (2) with respect to $D$ instead of $V$, recognizing that $V = V(D, V_0)$:

$$V + D \frac{dV}{dD} - C = 0.$$ 

This can be re-written as eqn. (5) above.} These two functions of the discount factor $D$
are plotted in Figure 1. The optimal threshold $V^*$, and the corresponding optimal discount
factor $D^*(V_0, V^*)$, are found at the point where the marginal benefit $D(V_0, V) + D/D_V$ is
equal to the cost, $C$. Note that $V^* > C$; this is the markup that incorporates the option
premium, or value of waiting. If the firm instead used a simple Net Present Value rule to
decide when to invest, it would invest sooner, when $V = C$, so its discount factor, denoted
by $D^N$, would be larger. (Note that in Figure 1, the current value of the benefit, $V_0$, happens
to be below the cost of the investment, $C$, so the firm would not invest immediately even if
it followed a simple NPV rule, and $D^N < 1$.)

It remains to sort out one potential difficulty. It would be unfortunate if the elasticity
$\epsilon_D$ depended on the initial value $V_0$, as that would imply that if we reconsidered the choice
after some intermediate value $V_1$ had been reached, we would get a different answer for the
optimal $V^*$. To examine this, consider any three values $V_0 < V_1 < V$. Suppose that, along
Figure 1: The Optimal Investment Markup.
any path of the process \( \{V_t\} \), starting at \( V_0 \) the first time the value reaches \( V_1 \) is \( T_1 \), and starting at \( V_t \) the first time it reaches \( V^* \) is \( T_2 \). Then the first time the value reaches \( V^* \) starting at \( V_0 \) is just \( T = T_1 + T_2 \). (In the second interval of time \( T_2 \) we have already supposed that the process does not reach \( V^* \), and in the first interval of time \( T_1 \) the process could not have reached \( V^* \) without hitting \( V_1 \) earlier, which would contradict our definition of \( T_1 \) as the first time to \( V_1 \).) Now

\[ e^{-\rho T} = e^{-\rho T_1} e^{-\rho T_2}, \]

and because of the Markov property of the process \( \{V_t\} \), the random variables \( T_1 \) and \( T_2 \) are independent. Therefore we can take expectations of the two factors on the right-hand side to get

\[ D(V_0, V) = D(V_0, V_1) D(V_1, V). \]

Then

\[ D_V(V_0, V) = D(V_0, V_1) D_V(V_1, V), \]

and

\[ \frac{V D_V(V_0, V)}{D(V_0, V)} = \frac{V D_V(V_1, V)}{D(V_1, V)}. \]  \( \text{(6)} \)

A similar argument can be constructed for \( V_2 < V_0 \), by considering paths where the process starting at \( V_0 \) first falls to \( V_2 \) before rising again and eventually reaching \( V^* \).

This proves that the elasticity is independent of the starting value. In particular, using eqn. (6) we can write the elasticity as

\[ \epsilon_D = \frac{-V^* D_V(V^*, V^*)}{D(V^*, V^*)} = -V^* D_V(V^*, V^*), \]  \( \text{(7)} \)

since \( D(V^*, V^*) = 1 \). Hence the optimal markup rule given by eqn. (4) is independent of the starting value \( V_0 \). This can also be seen in Figure 1; although the discount factors \( D^* \) and \( D^N \) depend on \( V_0 \), the optimal markup \( V^* - C \) does not.

Finally, note that the elasticity of the discount factor, \( \epsilon_D \), can be equivalently expressed in terms of the value of the firm’s option to invest. Let \( F(V) \) denote the value of the firm’s investment option. At the optimal exercise point, \( F(V^*) \) must satisfy the value matching
condition,

\[ F(V^*) = V^* - C, \]

and the smooth pasting condition,

\[ F_V(V^*) = 1. \]

Combining these two conditions, we have:

\[ \frac{V^*}{V^* - C} = \frac{V^* F_V(V^*)}{F(V^*)} = \epsilon_F. \]  \hfill (8)

The right-hand side of (8), denoted by \( \epsilon_F \), is the elasticity of the value of the investment option with respect to the value of the underlying project. Since \( V^*/(V^* - C) = \epsilon_D \), at an optimum the elasticity of the discount factor coincides with the elasticity of the value of the investment option.

3. Examples.

To use this approach to finding optimal investment rules, one must find the discount factor \( D \), given the stochastic process for \( V_t \). This can be done as follows.

Suppose that \( V_t \) follows a general Ito process of the form

\[ dV = f(V)dt + g(V)dz. \]  \hfill (9)

We want to find \( D(V, V^*) = \mathbb{E}[e^{-\rho T}] \), where \( T \) is the hitting time to \( V^* \), starting at \( V < V^* \). Over a small time interval \( dt \), \( V \) will change by a small random amount, \( dV \). Hence (suppressing \( V^* \) for simplicity):

\[ D(V) = e^{-\rho dt} \mathbb{E}[D(V + dV)]. \]

Expanding \( D(V + dV) \) using Ito’s Lemma, noting that \( e^{-\rho dt} = 1 - \rho dt \) for small \( dt \), and substituting eqn. (9) for \( dV \), we obtain the following differential equation for the discount factor:

\[ \frac{1}{2} g^2(V) D_{VV} + f(V) D_V - \rho D = 0. \]  \hfill (10)

This equation must be solved subject to two boundary conditions: (1) \( D(V^*, V^*) = 1 \), and (2) \( D(V, V^*) \to 0 \) as \( V^* - V \) becomes large.
To illustrate, we will obtain solutions using this approach for several different stochastic processes, and draw further analogies to the profit-maximizing decisions of a price-setting firm.

**Geometric Brownian Motion.**

First, suppose that $V_t$ follows the geometric Brownian motion
\[
dV = \alpha V dV + \sigma V dz, \tag{11}
\]
with $\alpha < \rho$. Then $f(V) = \alpha V$ and $g(V) = \sigma V$, and it is easily seen that the solution to eqn. (10) is
\[
D(V_0, V) = \left(\frac{V}{V_0}\right)^{\beta_1}, \tag{12}
\]
where $\beta_1$ is the positive root (exceeding unity) of the following quadratic equation in $\beta$:
\[
\frac{1}{2} \sigma^2 \beta (\beta - 1) + \alpha \beta - \rho = 0; \tag{13}
\]
see Dixit and Pindyck (1996, p. 316).

In this case the elasticity of the discount factor is constant and equal to $\beta_1$. The markup formula (4) thus implies a constant proportional mark-up,
\[
\frac{(V^* - C)}{V^*} = 1/\beta_1,
\]
or
\[
V^* = \frac{\beta_1}{\beta_1 - 1} C.
\]
This well-known result is analogous to the price-cost markup rule for a firm facing an isoelastic demand curve. A geometric Brownian motion for $V_t$ implies an isoelastic discount factor because the probability distribution for percentage changes in $V$ is independent of $V$, so changes in the discount factor resulting from a percentage change in $V$ will also be independent of $V$.

**Arithmetic Brownian Motion.**

Next, suppose $V$ follows the arithmetic Brownian motion
\[
dV = \alpha dt + \sigma dz. \tag{14}
\]
Then the solution to eqn. (10) is

\[ D(V_0, V) = \exp[-\gamma_1 (V - V_0)], \]

where \( \gamma_1 \) is the positive root of the quadratic

\[ \frac{1}{2} \sigma^2 \gamma^2 + \alpha \gamma - \rho = 0; \]

see Harrison (1985, p. 42). In this case, the elasticity of the discount factor is \( \gamma_1 V \). Hence \((V^* - C)/V^* = 1/\gamma_1 V^*\), and we get a constant additive mark-up:

\[ V^* = C + (1/\gamma_1). \]

This is analogous to the markup formula for a firm facing an exponential demand curve. (For the demand curve \( q(p) = a \exp[-bp]\), the elasticity of demand is \( bp\), and the profit-maximizing price is \( p^* = c + 1/b\).)

**Mean-Reverting Process.**

Finally, suppose that \( V_t \) follows the mean-reverting process:

\[ dV = r(V - V)V dt + \sigma V dz. \]

The value, \( V \), might follow such a process if, for example, the firm’s output was a durable good so that its demand was subject to a stock adjustment process.

Then eqn. (10) for \( D(V, V_1) \) becomes:

\[ \frac{1}{2} \sigma^2 V^2 D_{VV}(V, V_1) + \eta(V - V)V D_V(V, V_1) - \rho D(V, V_1) = 0. \]

This equation has the following solution (see Dixit and Pindyck, pp. 162–163):

\[ F(V, V_1) = AV^\theta H \left( \frac{2\eta}{\sigma^2} V, \theta, b \right), \]

where \( A \) depends on \( V_1, \theta \) is the positive solution to the quadratic equation

\[ \frac{1}{2} \sigma^2 \theta (\theta - 1) + \eta V \theta - \rho = 0, \]
and

\[ b = 2 \left( \theta + \frac{\eta V}{\sigma^2} \right). \]

Here \( H(x, \theta, b) \) is the confluent hypergeometric function, which has the following series representation:

\[
H(x, \theta, b) = 1 + \frac{\theta}{b} x + \frac{\theta(\theta + 1)x^2}{b(b + 1)2!} + \frac{\theta(\theta + 1)(\theta + 2)x^3}{b(b + 1)(b + 2)3!} + \ldots.
\]

The limiting behavior of the solution is used to determine \( A \). When \( V \) approaches \( V_1 \), the first hitting time \( T \) must approach zero, which means that \( F(V, V_1) \to 1 \). Thus,

\[
A = \frac{1}{V_1^\theta H \left( \frac{2\eta V_1}{\sigma^2}, \theta, b \right)}.
\]

Hence the discount factor becomes:

\[
D(V_0, V) = \left( \frac{V_0}{V} \right)^\theta \frac{H \left( \frac{2\eta V_0}{\sigma^2}, \theta, b \right)}{H \left( \frac{2\eta V}{\sigma^2}, \theta, b \right)}
\]  \hspace{1cm} (16)

From the series representation, we obtain the following relationship between \( H \) and its derivative with respect to the first argument:

\[
H_x(x, \theta, b) = \frac{\theta}{b} H(x, \theta + 1, b + 1).
\]

Using this, we can determine that the elasticity of the discount factor at the optimum \( V^* \) is:

\[
\epsilon_D = \theta \left[ 1 + \frac{2\eta V^*}{\sigma^2 b} H \left( \frac{2\eta V^*}{\sigma^2}, \theta + 1, b + 1 \right) \right].
\]  \hspace{1cm} (17)

Thus, \( \epsilon_D \) is equal to a constant \( \theta \) — which represents pure geometric growth — plus a term which corrects for the mean reversion effect. As the mean reversion speed \( \eta \) approaches zero, the second term also goes to zero, and \( \theta \) approaches \( \beta_1 \), as in the case of geometric Brownian motion. As \( \eta \) increases, mean reversion dominates.

The implications of mean reversion are easiest to see from some numerical calculations. Mean reversion implies that \( V \) is expected to stay close to \( \overline{V} \). Hence when \( V - \overline{V} \) is small, the discount factor must be larger for the mean-reverting process than for the corresponding geometric Brownian motion. Likewise if \( V - \overline{V} \) is large, it can be expected to decline, so that the discount factor will be relatively small. Figure 2 illustrates this; it shows the discount
Figure 2: Discount Factor for Mean-Reverting Process and Geometric Brownian Motion 
($\rho = .05, \sigma = .2, V = 1, V_0 = 1$).

Figure 3: Elasticity of Discount Factor as a Function of the Speed of Mean Reversion.

Figure 3: Elasticity of Discount Factor as a Function of the Speed of Mean Reversion.
factor as a function of $V$ for a mean-reverting process ($\eta = 0.2$) and a geometric Brownian motion ($\eta = 0$). (In both cases, $p = .05$, $\sigma = 0.2$, $V = 1$, and $V_0 = 1$.) This effect of mean reversion is also reflected in the elasticity of the discount factor, which is increasing in $V$. For example, $\epsilon_D(V = 1) = 1.4$ and $\epsilon_D(V = 2) = 8.54$; while the corresponding constant elasticity for the geometric Brownian motion ($\eta = 0$) is $\beta_1 = 2.16$. Figure 3 shows how the elasticity depends on the speed of mean reversion, $\eta$. When $V - \overline{V}$ is small ($V = 1.0$), $\epsilon_D$ decreases with $\eta$, but when it is large ($V = 2.0$), it increases with $\eta$.

4. **Ancillary Investments in Advertising or R&D.**

The close connection between investment decisions and pricing decisions has pedagogical value, but also provides insight into investment-related decisions more broadly. As an example, consider a price-setting firm that must also decide how much money, $A$, to spend on advertising, given its demand $q = q(p, A)$, with $\partial q / \partial A > 0$. As students are taught in intermediate microeconomics courses, the profit-maximizing advertising-to-sales ratio is given by:

$$\frac{A}{pq} = \frac{\epsilon_A}{\epsilon_P}, \quad (18)$$

where $\epsilon_A = (A/q)\partial q / \partial A$ is the firm's advertising elasticity of demand, and $\epsilon_P$ is the price elasticity of demand.\(^5\)

Now let us return to our investment problem. Suppose that the firm, prior to making the sunk expenditure $C$ in return for the benefit $V$, can make an ancillary investment, costing $A$, in advertising, marketing, or R&D activities. The exact nature of these activities is unimportant; what matters is that they lead to more rapid increases in $V$, and hence to an increase in the discount factor $D(V_0, V)$. We can then re-state our investment problem as:

$$\max_{V, A} [(V - C)D(V_0, V, A) - A]. \quad (19)$$

The two first-order conditions for this problem are

$$D(V_0, V, A) + (V - C)D_V(V_0, V, A) = 0, \quad (20)$$

\(^5\)Eqn. (18) follows from maximizing profit with respect to $p$ and $A$, and is sometimes referred to as the Dorfman-Steiner (1954) theorem.
and
\[(V - C)D_A(V, V, A) = 1 = 0. \quad (21)\]

Now define the elasticities of the discount factor with respect to \(V\) and \(A\), respectively, as \(\varepsilon^D_V \equiv -VDV/D\) and \(\varepsilon^D_A \equiv AD_A/D\). Then by combining the first-order conditions (20) and (21), it is easy to see that
\[\frac{A}{DV} = \frac{\varepsilon^D_A}{\varepsilon^D_V}. \quad (22)\]

Eqn. (22) is a condition for the optimal ratio of expenditures on advertising (or marketing, or R\&D) to the expected discounted value of the benefit. (Remember that the actual discounted value of the benefit is unknown because the time until \(V\) reaches the threshold \(V^*\) is stochastic; \(DV\) is the expected discounted value of the threshold \(V^*\).) It is exactly analogous to condition (18) for the advertising-to-sales ratio of a price-setting firm.

As an example, suppose a pharmaceutical firm is deciding whether to invest in a plant to produce a new drug. Suppose the benefit from this investment, \(V_t\), follows the geometric Brownian motion of eqn. (11), and will grow over time (at expected rate \(\alpha\)) even before the plant is built as doctors and patients learn about the drug. However, the expected growth rate \(\alpha\) can be increased via expenditures \(A\) on advertising and marketing.\(^6\)

To determine the optimal level of \(A\) for this example, note that the discount factor is again given by eqn. (12), with \(\beta_1\) again the solution to the quadratic eqn. (13). Hence the elasticity \(\varepsilon^D_V\) is again equal to \(\beta_1\). But now \(\beta_1\) is a function of \(A\), since \(\alpha\) depends on \(A\). Differentiating the quadratic eqn. (13) with respect to \(A\) and rearranging yields the following expression for the elasticity \(\varepsilon^D_A:\)
\[\varepsilon^D_A = \frac{A \log D(\alpha/DA)}{\sigma^2 \beta_1 + \alpha - \frac{1}{2} \sigma^2}. \quad (23)\]

Defining the elasticity \(\varepsilon^* \equiv (A/\alpha)da/DA\), the optimal ratio of \(A\) to the discounted benefit is thus given by:
\[\frac{A}{DV} = \frac{\alpha \varepsilon^* \log(V^*/V_0)}{\sigma^2 \beta_1 + \alpha - \frac{1}{2} \sigma^2}. \quad (24)\]

\(^6\)We treat \(A\) as a lump-sum expenditure. If the advertising and marketing expenses must be spread out over time, then \(A\) is just the present value of those expenses.
This ratio will be larger the larger is $e_A^a$ — the more productive is advertising and marketing, the more that should be done. But note that this ratio is also larger the larger is the threshold $V^*$. A larger $V^*$ implies that the option to invest is more valuable (the expected net payoff $(V^* - C)$ is larger), which increases the expected return from advertising and marketing expenditures. Hence this ratio is larger if there is greater uncertainty over the evolution of $V$; an increase in $\sigma$ increases $V^*$, and (with some algebra) can be shown to reduce the denominator of (24). Finally, note that $A \to 0$ as $V^*/V_0 \to 1$; when $V^* = V_0$ there is no option premium, and thus no benefit to increasing $\alpha$.

5. Conclusions.

Framing the optimal investment decision as a trade-off between larger versus later net benefits has allowed us to interpret the investment rule as a simple markup formula involving an elasticity. We have seen that the markup is exactly analogous to a firm’s optimal markup of price over marginal cost. For economists, this may be more intuitively appealing than the standard approach to irreversible investment problems in which one values the firm’s option to invest and finds the optimal exercise rule.

If the benefit, $V$, follows a geometric Brownian motion — as is typically assumed in applications — then the markup formula is particularly simple, since the elasticity of the discount factor is constant and equal to $\beta_1$, the solution to the fundamental quadratic equation (13). In this case the discount factor is isoelastic with respect to $V$, so the investment problem is analogous to the pricing problem for a firm facing an isoelastic demand curve.

Even if $V$ does not follow a geometric Brownian motion, this markup formulation provides a rule of thumb that can be of value to practitioners. Compared to equating marginal cost with marginal revenue, it can be easier for a manager to think about pricing in terms of a markup based on the elasticity of demand, estimates of which can be based on statistics or on judgment. Likewise, it can be easier to think about investment hurdles as a markup based on the elasticity of the discount factor, “estimates” of which can be found analytically or judgmentally.
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