Side Payments in Marketing

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Side Payments in Marketing

Abstract

Frequent flyer awards, customer satisfaction ratings, internal sales support ratings, and other marketing systems share the property that side payments can be used to increase ratings and/or usage. We examine side payments in a class of systems where a downstream agent (say a sales person) rates an upstream agent (say sales support). If the upstream agent is rewarded based on that rating, the potential might exist for a side payment to increase the rating. Our analyses suggest that, for continuous upstream and downstream reward systems, (1) side payments will almost always occur and (2) the firm can design reward systems to factor out the side payments without any loss of profits. We discuss the practicality and efficiency of such ratings-based reward systems. We extend our analyses to implicit ratings systems and we discuss whether our analyses apply to situations where the upstream and downstream agents work for different firms.
Motivation

Anyone with field experience in marketing or anyone who has taught a case on ethics in marketing is aware that side payments are a common, if unspoken, practice. Among the examples of which we are aware are:

- **Customer satisfaction incentives** -- One major US auto firm pays a bonus of $25 to an auto salesperson whenever the customer rates that salesperson highly on customer satisfaction. The firm’s consultants are aware that some of the $25 makes its way to customers in the form of side payments such as gifts (candy, flowers, etc.). For more examples, see Mohl (1996).

- **Internal customer rating systems** -- Many organizations ask internal customers (say the sales representatives) to rate internal suppliers (say the advertising and product support managers) on how well the internal supplier serves the internal customer (e.g., Zeithaml, Parasuraman, and Berry 1990). In many situations, e.g., Chester (1995) and Shapira and Globerson (1983), the reward the internal supplier receives is based on the rating the internal customer gives. We have heard informally that internal suppliers often provide side payments to internal customers in return for higher ratings.

- **Frequent flyer awards** -- Travelers are given frequent flyer miles for business travel. These miles can be considered side payments when the travelers exchange these credits for personal travel or other awards. We are aware of examples where business travelers choose more expensive routes in order to earn additional miles.

- **Buyer-seller situations** -- Side payments are common as perquisites from sellers to buyers (Murphy 1995). Borus (1995) suggests that $45 billion in overseas sales were so affected.

Side payments, known politely as gainsharing and pejoratively as bribery, are prevalent in marketing. Indeed, many management schools have added ethics modules to their basic marketing courses to discuss these issues and there is much discussion of side payments in economics and law (e.g., Rose-Ackerman 1996). Rather than address marketing side payments with moral imperatives or philosophy, we seek to formalize one class of marketing side payments. We hope that our formal structure clarifies the issues and suggests how such side payments affect marketing activities.¹

¹Our interest in this topic is scientific. Nothing in this paper should be interpreted as advocating side payments.
We begin by focusing on one example of this class -- salesforce ratings of internal sales support. We derive two general results and discuss how these results generalize to ratings that are implicit rather than explicit. We then illustrate how most frequent flyer programs differ from internal rating systems by one critical feature that modifies the managerial implications. We caution the reader that our structure does not apply to all instances of side payments nor do our results substitute for a moral discussion of side payments. However, we hope that the formal structure for a common marketing situation provides valuable insight.

### Formalization

For our formalization we choose an example where an upstream agent (U), say sales support, provides goods and services to a downstream agent (D), say the salesforce. Both agents work for the same firm. The downstream agent is asked to evaluate the upstream agent and the upstream agent receives a reward based on that rating. The reward to sales support might be a monetary bonus, say $1,000 times the rating, or the reward might simply be recognition and an increased chance of promotion. The reward must have effective monetary value to U.

For this section we assume that the rating is a numerical rating, say a 7-point scale going from "unsatisfactory" to "excellent." However, in a later section we discuss how the theory applies to more implicit ratings. We denote this rating with $r$.

Naturally, the actions and efforts of the upstream and downstream agent add value to the firm. In our formalization we call the actions that the upstream agent takes, $u$, and the actions that the downstream agent takes, $d$. For example, $u$ might be the time, effort, and materials needed to produce advertising, brochures, and other sales materials, while $d$ might be sales efforts such as travel to clients, meetings with clients, and written proposals. If U and D do their jobs well, then the firm makes some incremental profit which we denote with $\pi(u, d)$. For example, the firm might make incremental profit on the sales that result from the actions of U and D. We include in $\pi(u, d)$ any costs that U and D cause the firm to incur (printing and mailing the brochures, production costs, etc.), but we do not include the incremental salaries and bonuses paid to U and D.

Accounting systems are rarely exact; incremental profits are hard for the firm to measure. Thus, we make the realistic assumption that the firm can only estimate incremental profits. The firm's estimate, $\hat{\pi}$, is equal to true incremental profits plus zero-mean noise, $e$. That is, $\hat{\pi}=\pi(u, d)+e$. 
The actions taken by the agents are likely to be perceived by the agents as onerous. For example, the salesperson (D) might prefer chatting with his or her colleagues at the office rather than making a difficult cold call on a new client. We denote these costs by $c_u(u)$ for the upstream agent and by $c_d(d)$ for the downstream agent. (Both $c_u$ and $c_d$ are defined in monetary equivalents.)

We define profits as incremental above and beyond that which would have been obtained in the absence of an incentive system for U and D. Finally, we make some reasonable technical assumptions for the formal proofs.²

**Evaluation and Reward System**

We began studying side payments after working with the sales manager for a $2 billion dollar organization that served both consumer and industrial markets. The Chief Executive Officer had begun a program to increase customer satisfaction five-fold and the sales manager felt that his salespeople needed support to achieve this effort. He was preparing to introduce a system in which salespeople rated the support people on how well sales were supported. The sales manager had many years of experience and was quite savvy. He was concerned that the salespeople would demand a side payment in return for a higher rating.

We introduce some formal notation to model this situation. We denote the reward that the upstream agent receives with $v(r)$, an increasing function of $r$. We denote the reward that the downstream agent receives with $w(r, \tilde{r})$, which depends upon the outcomes, $\tilde{r}$, that the firm observes. (We have chosen incremental profit, but, in principle, $\tilde{r}$ could be any noisy indicator of incremental profit such as incremental sales and/or customer satisfaction [net of costs].) In real systems, $w(r, \tilde{r})$, may or may not depend upon $r$. However, we show later that it makes sense for the firm to penalize the downstream agent for a rating that is too high. For example, if the salesperson says that he or she is getting "excellent" sales support, then he or she might be given a higher sales target than the salesperson who says that he or she is getting "unsatisfactory" support. In such cases $w(r, \tilde{r})$ becomes

²Formally, we assume that profit is concave in both arguments and thrice differentiable and that the cost functions are increasing, convex, and thrice differentiable. All functions and variables are real valued and actions are non-negative.
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a function of \( r \). Technically, we restrict our analyses to \( v \) and \( w \) that are concave in their arguments.³

The reward functions are notable for what they do not assume. The upstream agent is paid only on the rating and the downstream agent is paid only on a measure of incremental profits (and perhaps the rating). The firm does not have to observe U’s actions, \( u \), or D’s actions, \( d \). This also means that the firm does not observe the perceived costs, \( c_u(u) \) and \( c_d(d) \).

At first glance the specifications of \( v \) and \( w \) seem to imply that the firm does not have to know how the agents’ actions affect profit. But this is not the case. In the formal analysis the firm has to set the base salary for U and for D and the firm has to select the functions, \( v \) and \( w \). In principle, these base salaries and reward functions can depend upon \( \pi(u,d) \), \( c_u(u) \), \( c_d(d) \), and their derivatives. However, such knowledge is a formal requirement for almost any reward system. If the firm wants to maximize its own profits, then it needs to know the marginal productivity of its employees. If it pays them too much, then it has lost the opportunity for profits; if it pays them too little, then they will leave the firm. The key managerial question is whether the ratings-based system makes it more or less practical to select a reward system.

Our perspective is that of the sales manager to whom we spoke. His firm already had a system for paying the salesforce and sales support. Presumably, the firm was able to determine enough about the marginal productivity and perceived costs of its employees and was able to set their salaries accordingly. The sales manager wanted to do better by introducing a rating system to coordinate U and D. He needed to know whether side payments would undermine any potential gain in profit from the rating system. He also wanted to know whether he should invest in monitoring and punishment procedures to preclude side payments. Alternatively, he wanted to know whether he could design a rating system under which there would be no incentives for side payments. Our analyses seek to address his questions:

- What conditions encourage side payments for higher ratings?
- Do side payments decrease profits? Can the firm do as well under a system that allows side payments as it could do under a system that precludes side payments?
- Do the special characteristics of an internal rating system require more detailed knowledge

³A higher rating indicates more effort, thus \( v(r) \) is increasing in \( r \). We also want \( r \) to be an indicator of \( u \)'s effect on \( \pi \), thus we examine \( w \)'s such that \( \frac{\partial w}{\partial u} r > 0 \). For the formal proof we make a technical assumption that \( \partial^2 w/\partial u \partial r \) is bounded away from zero.
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about \( \pi(u,d) \), \( c_u(u) \), and \( c_d(d) \) than the existing incentive system?

By addressing his questions, we hope our analyses provide useful insight for this class of marketing problems. We begin by stating the actions and decisions of the firm, U, and D as a formal game.

The Formal Game

We formalize the order of actions as follows: (1) The firm acts first and announces a reward system, \( v \) and \( w \). Based on this reward system, (2) the upstream agent acts next to select its actions, \( u \), if by doing so it can do better than not acting. (3) The downstream agent observes these actions, but the firm does not. (4) Next, U and D agree on a contract for a side payment, \( s \), and a rating, \( r \). Both do so anticipating what this will imply for D's actions, \( d \), and the resulting expected profit, \( \pi \). If they cannot agree on a contract, D takes no actions. (5) D announces the rating, \( r \). (6) The upstream agent receives its reward, \( v(r) \), based on this rating. (7) The downstream agent, D, acts in its own best interests to choose its actions, \( d \). (8) The firm observes its objective, \( \pi \), and (9) pays D its reward, \( w(r, \pi) \).

Naturally, we assume that the firm will announce a reward system only if it can do better with the actions and profits implied by the reward system than it could do in the absence of a reward system. (Without a reward system, the agents set \( u \) and \( d \) to zero.)

Firm's Goals, Agents’ Goals

We assume that the firm is risk neutral and profit maximizing. Thus, the firm will seek to maximize the expected value of profits minus wages:

\[
expected \text{ net profit} = E[\pi(u,d) - v(r) - w(r,\pi)]
\]

We assume that both the upstream and downstream agents are risk averse and will act in their own best interests to maximize their expected utilities, \( EU_u \) and \( EU_d \).\(^4\) We scale perceived costs so that

\(^4\)Technically, the utility functions are integrable, thrice differentiable, increasing, and concave. Because of our previous assumptions, the expected utility function is well-behaved.
they are measured on the same scale as profits and wages. (Please note: our mnemonic notation for the upstream agent, $U$, is distinguished from the notation for expected utility, $EU$, by the use of italics.) In the absence of a side payment each agent acts in his or her own best interests to maximize the expected utility of wages minus perceived costs.

\[(2)\]

- **upstream agent maximizes** $EU_u[v(r) - c_u(u)]$
- **downstream agent maximizes** $EU_d[w(r,\pi) - c_d(\hat{d})]$

The upstream agent acts first by choosing an action, $u$, that the downstream agent (but not the firm) can observe. In the absence of a side payment, the downstream agent will take $u$ as given and act to maximize the second expression in Equation 2. This means that the downstream agent’s reaction to $u$ implies three continuously differentiable functions, $\hat{r}(u)$, $\hat{d}(u)$, and $\hat{\pi}(u)$, which tell us how $D$ would react to $U$’s choice of actions and how expected profit would be affected if there were no side payment.

Now consider a situation where $U$ offers a side payment, $s$, to $D$ in return for a higher rating, $\hat{r}$. This side payment need not be monetary but it must be valued by $D$ and be costly for $U$ to provide. It might be extranormal service that is valued by $D$ but does not affect $\pi$ directly. Such services might include fancy brochures with $D$’s picture on the cover, sales “training” for $D$ in Aruba, or “warm-up” jackets with the company’s logo that can be given to friends and relatives.

We model this offer as a take it or leave it offer, but we could obtain similar results for other assumptions on how $U$ and $D$ share surplus, if any, from the $(s,\hat{r})$ contract. It will be in $D$’s interest to accept this contract if $D$ can do at least as well with the contract as without. Thus, $D$’s expected utility from the contract must be at least as large as that which $D$ can obtain in the absence of a side payment. This constraint defines critical values of the side payment, $s(u,\hat{r})$, for every action, $u$, and potential rating, $\hat{r}$. After accepting the contract, $D$ will choose its actions in its own best interests. We write $\hat{d} = \hat{d}(u,\hat{r})$ for the action that $D$ chooses in order to maximize its own expected utility. That is, $D$’s actions depend on $U$’s actions and the potential rating. We define $\hat{\pi} = \hat{\pi}(u,\hat{r})$ for the resulting profits. Formally, the implied constraint is:

\[(3)\]

$$EU_d[w(\hat{r},\hat{\pi}) - c_d(\hat{d}) + s(u,\hat{r})] \geq EU_d[w(\hat{r},\hat{\pi}) - c_d(\hat{d})]$$

For a given $v$ and $w$, the upstream agent will select $u$ and $\hat{r}$ to maximize its expected utility. (U’s
choices imply D's choices by Equation 3. Because U is maximizing its own well being, U will select a side payment such that the inequality in Equation 3 is binding.) Thus, if side payments are allowed, we modify the first line of Equation 2 to obtain:

\[ EU_u[\bar{r}, u] = EU_u[v(\bar{r}) - c_u(u) - s(u, \bar{r})] \]

Equations 2, 3, and 4 tell us how U and D will react to any functions, \( v \) and \( w \), chosen by the firm. Subject to these constraints, the firm chooses \( v \) and \( w \) to maximize the profits in Equation 1. In other words, these equations tell us how the firm, U, and D will each act, subject to the formal game, to maximize their own objectives (profit for the firm, expected utility for the agents).

However, to earn non-zero profits, the firm must retain its employees. Let \( EU_u \) and \( EU_d \) be the minimum expected utilities that U and D require to participate. We assume that these expected utilities represent that which U and D could obtain from other options that are available, such as working for another firm or simply coasting in their current jobs (e.g., Adams 1995). That is, we assume that \( EU_u \) and \( EU_d \) are set by the market. To induce U and D to participate, the firm must set \( v \) and \( w \) such that:

\[ EU_u[v(\bar{r}) - c_u(\bar{u}) - s(\bar{u}, \bar{r})] \geq EU_u \]
\[ EU_d[w(\bar{r}, \bar{u}) - c_d(\bar{d}) + s(u, \bar{r})] \geq EU_d \]

Because the firm is maximizing expected profits it will try to keep wages as low as is feasible. Formally this means that the firm will attempt to select \( v \) and \( w \) such that the constraints implied by Equation 5 are binding.

We now address the sales manager's first question.

**Side Payments Almost Always Occurs**

The upstream/downstream structure almost always guarantees side payments. By this we mean that there are economic incentives for side payments. To preclude side payments, the sales manager would likely have to impose an exogenous system such as a reprimand (or worse) for any perceived impropriety. Such a monitoring and policing system could be costly to the firm.
Interior Solutions

The intuition is best seen in the case where \( v \) and \( w \) imply interior solutions. In this case, in the absence of a side payment, \( D \) will maximize the right hand side of equation 3 by setting the derivatives to zero. That is,

\[
\frac{\partial EU_d[w(\hat{r}, \hat{d}) - c_d(\hat{d})]}{\partial \hat{r}} = 0
\]

Thus, the marginal loss to \( D \) of a very small increase in \( r \) is zero. On the other hand, because \( U \)'s utility is increasing in \( v \) and \( v \) is increasing in \( r \), \( U \) gains by having \( D \) increase \( r \). That is,

\[
\frac{\partial EU_u[v(\hat{r}) - c_u(u)]}{\partial \hat{r}} > 0
\]

Thus, intuitively there appear to be gains to trade at \( \hat{r} \) -- \( U \) gains more than \( D \) loses.

The actual proof is more complex because we have to account for the integration implied by the expected utility operators, but the basic intuition does not change. (A formal proof is given in an appendix that is available from the authors.) For any \( v \) and \( w \) chosen by the firm such that (1) the firm makes positive profits, (2) \( U \) and \( D \) find it better to take some actions than to take no actions, and (3) the \( v \) and \( w \) imply interior solutions, then there are economic incentives to use side payments.

Notice that if \( D \) is not penalized for a higher rating, then \( \partial w/\partial r = 0 \) and equation 6 will hold for all \( \hat{r} \). There will always be gains to trade. In this case even a minimal side payment from \( U \) will persuade \( D \) to rate \( U \) as high as is feasible. When \( w \) does not depend on \( r \), \( U \) need not put in any effort beyond this minimal side payment. Thus, if the firm wants to use a rating system to entice \( U \) to put forth sufficient effort, \( w \) must be a function of \( r \).

Constrained Ratings

The sales manager may ask \( D \) to rate \( U \) on a 7-point scale. For example, the highest rating possible might be "excellent." For some \( v \) and \( w \), this constraint may mean that the optimal solution to Equation 3 (RHS) or Equation 4 is not an interior solution. In general, if \( r \) is constrained to be...
less than some upper bound, \( \bar{r} \), and this upper bound is less than that which D would otherwise choose, then D might find it to be in its best interest to set \( r = \bar{r} \). Formally, such a constraint might replace the equality in equation 6 with an inequality and there may be no gains to trade.

However, constraints on \( r \) do not rule out side payments. For example, Hauser, Simester, and Wernerfelt (1996) demonstrate a linear system in which the firm’s choice of \( v \) and \( w \) causes \( U \) to provide D with a side payment in return for reporting \( r = \bar{r} \). The side payment is necessary in that system because D can not achieve \( EU_d \) without the side payment but can do so with a side payment.

No Room to Trade

There is a final situation we must consider. Suppose \( v \) increases at a slower rate than \( w \) decreases. If this happens over the entire range, there will never be any \( r \) where there are gains to trade. However, such a situation will not occur for a rational firm. If \( v \) increases at a slower rate than \( w \) decreases, then the optimal response for D is to set \( r = 0 \). However, this means that \( U \) will set \( u = 0 \) because any actions incur perceived costs without rewards. This will, in turn, cause D to set \( d = 0 \) and the firm will earn only as much with the reward system in place as it did without the reward system in place. This violates one of our assumptions.

This covers all the cases for continuous \( v \) and \( w \). In an appendix we prove formally that:

**RESULT 1.** For incentive systems in which the rewards to the upstream agent are increasing in the downstream agent’s rating, the upstream agent will provide a positive side payment to the downstream agent unless the firm sets a binding upper bound on the rating (or otherwise precludes side payments). Even with a binding upper bound, there may be side payments.

Side Payments Need Not Hurt the Firm’s Profits

Because most ratings-based reward systems encourage side payments and because it might be expensive to use monitoring and punishment to preclude side payments, we now address the sales manager’s second question. Do the side payments necessarily decrease the firm’s profits? We want to compare the profits from a system that precludes side payments to the profits from a system that allows side payments. We first address whether a ratings-based system with side payments exists that
does not decrease profits and then whether it is practical.

Let's consider the sales manager's firm. Many sales people join and leave the firm annually. Furthermore, the sales skills are not entirely unique to the firm. In theory, in order to retain employees and maximize profits, the firm would have needed to know the details of \( \pi(u,d) \), \( c_u(u) \), and \( c_d(d) \) for each and every employee. However, it appears to have been practical for this large firm to set base salaries and to adjust variable rewards (e.g., sales commissions) by trial and error.

Now let's assume that the firm introduces a rating system which encourages side payments. If the previous system was inefficient, the new system might induce higher \( u \) and \( d \) and the firm will have to reimburse the agents for their increased perceived costs. But how about the side payment? The upstream agent will have to pay \( s \) and the downstream agent will receive \( s \). The firm may be aware of these side payments, but it certainly does not want to set up an accounting system to measure the side payments and adjust salaries accordingly.

Our sales manager would like to use \( v \) and \( w \) to adjust the compensation to \( U \) and \( D \). In particular, he must change \( v \) so that \( U \) gets enough additional compensation to pay \( s \) (otherwise \( U \) will quit) and he must change \( w \) so that \( D \)'s compensation is reduced by \( s \) (otherwise he is paying \( D \) more than the market wage). However, in adjusting \( v \) and \( w \) the sales manager wants to be able to achieve the desired \( u \) and \( d \), that is, he wants as much profit from the new system as he earned with the old. All of this must happen with the agents acting in their own best interests. The following result says simply that the sales manager can choose the appropriate \( v \) and \( w \). The result is reasonably general. It holds for fairly general concave profit, cost, and reward functions. The non-side-payment reward systems can be profit maximizing, but they need not be.

**RESULT 2.** If the firm can preclude side payments and choose a reward system, \( v \) and \( w \), such that the upstream and downstream agents, acting in their own best interests, choose actions \( u' \) and \( d' \), then there exists a reward system such that (1) there is no loss of profits to the firm and (2) the upstream and downstream agents, still acting in their own best interests, choose \( u' \) and \( d' \) even though they are free to make side payments in return for higher ratings.

The basic proof follows the intuition of the sales force example. The modified reward system changes the slopes of \( v \) and \( w \) with respect to \( r \) to achieve the new equilibrium implied by side payments. The change in the slope of \( v \) offsets the cost of the side payment to \( U \) and the change in
the slope of $w$ reduces D's rewards accordingly. Together these changes do not affect the first-order conditions for $\tilde{u}$ and $\tilde{d}$ nor do they add any risk. (Recall that the rating is given before the noisy outcome, $\tilde{\tau}$, is observed. Thus, the change in the rating imposes no new risks to either U or D.)

**Practical Internal Ratings Systems (which allow Side Payments)**

Result 2 answers the sales manager's second question, but not his third question. Result 2 guarantees that a rating system can be found, not that it is practical. Assuming the sales manager already knows how to set base salaries and sales commissions, we want to know whether the slopes of $v$ and $w$ can be set by practical methods.

Ratings systems do exist (e.g., Zeithaml, Parasuraman, and Berry 1990, Chester 1995, and Shapira and Globerson 1983, and our own experience) so some must be practical. They have survived the market test, so they must provide reasonable profits. More formally, Hauser, Simester, and Wernerfelt (1996) provide two examples of practical internal customer ratings systems -- a linear variable outcome-based system and a quadratic target-value system. In both systems the firm's profits are robust with respect to the slopes of $v$ and $w$. These systems are practical because the slopes can be set by judgment or by trial and error with little loss of profit (versus optimal slopes). Thus, the challenge of setting the slopes of $v$ and $w$ seems to be comparable to the challenge of setting base salaries and sales commissions.

However, the simple rating systems do not always handle risk optimally. Result 2 gives the sales manager confidence to begin with easy-to-implement systems and then tinker with more complex systems in the pursuit of higher overall profits. He knows that the side payments introduced by more complex systems need not decrease profits.

**Implicit Ratings**

To illustrate implicit ratings, we use another (hypothetical) example. Suppose that a large firm uses a wholly owned internal travel service. This service might include a fleet of private planes and vans for executives who must travel site to site. Suppose that this firm tracks the number of executive trips (perhaps weighted by distance and type of vehicle) and the total cost of providing the travel service. It evaluates and rewards the internal travel service based on the number of trips. If
the internal travel service is the upstream agent (U) and the executives are the downstream agent (D), then the number of trips acts like an implicit rating, \( r \). (For the purposes of this illustration ignore any free-riding within divisions.) Specifically, the rewards to the internal travel service (U) are increasing in the number of trips \( (r) \) and the rewards to the executives are decreasing in the number of trips. (Presumably more trips are more onerous to the executives.) Mathematically we now have the same structure as that for the explicit ratings except that profits may depend upon the number of trips above and beyond the efforts of U and D. Even when the profit function depends explicitly on the implicit rating, e.g., \( \pi(u,d,r) \), the analogs of Results 1 and 2 hold. (See appendix.) The internal travel service will (most likely) offer side payments for increased business travel and the firm can choose a reward system so that the side payments do not lower profits.

**Are Side Payments Ever Efficient?**

Side payments within a firm might be attractive to the firm if the upstream agent can compensate the downstream agent more efficiently than the firm. For example, if the internal travel service faces a peak-loading problem, it will have excess capacity during off-peak hours. The marginal cost of providing transportation during these evening, weekend, or vacation times might be relatively low. If the travel service is efficient, then the cost of providing this travel might be less than the traveler could otherwise obtain. The firm might find it attractive to allow the internal travel service to compensate employees with an internal frequent flyer program. Further efficiencies result when this benefit is not taxed (e.g., Peterson 1996).

We can get an idea of the magnitude of internal travel costs by examining data on external travel services. TWA estimates that the average cost of a frequent flyer trip is $28, considerably less than the traveler would pay for a comparable flight (Peterson 1996). To estimate the value to the traveler, we examine firms which reimburse employees who use frequent flyer tickets for business travel. Presumably the value to the flyer is at least this large. For example, one large real estate development firm reimburses its employees when they use (personal) frequent flyer miles for business travel. The reimbursement is the price of the least expensive commercial ticket. This is clearly above $28.\(^5\) If these numbers are representative, the value to D appears to exceed the cost to U.

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\(^5\)Personal discussions with an employee from that firm. His most recent transaction was a $535 reimbursement.
Side Payments Across Firm Boundaries

It is tempting to apply our results to commercial frequent flyer programs. Not only are they a pervasive marketing program in the airline industry (Boston Globe, March 13, 1994), but credit card companies, long distance telephone companies, and electric utilities now offer frequent flyer miles. (The market price that airlines charge these firms is currently 2¢ per mile.) By defining the number of trips (appropriately weighted) as an implicit rating, we get a related formal structure -- but with one critical difference. The flyer's firm no longer can announce $v(r)$ because the price that an airline receives for a trip is set by the market.\(^6\) Result 1 still holds because it applies for any $v(r)$ under which the firm chooses to operate. However, Result 2 no longer guarantees that the firm can choose a reward system that allows side payments and does not hurt profits. Technically, Result 2 still says that $v$ and $w$ exist such that profits are not decreased. However, because a single firm no longer controls $v$ and $w$, there is no easy mechanism with which a single firm can set $v$ and $w$ such that profits do not decrease. The efficiency gains of frequent flyer programs may or may not offset their costs to the airline or to the flyer's firm. Similar comments apply to sales perquisites and related marketing side payments across firm boundaries. Reward functions exist, but the firm(s) may be unable to select them.

Summary

Side payments are common in marketing. Our analysis suggests that this should not be surprising because (1) the structure of intrafirm upstream-downstream ratings systems often guarantees that there are economic incentives for side payments and (2) if the firm can control the reward functions it can always factor the side payments into the compensation system with no loss of profits. These results hold even if the rating is implicit. Side payments may not occur if the rating is constrained appropriately (Result 1) or if the firm or society uses peer pressure, cultural norms, or punishment to prevent side payments. When side payments are made across firm boundaries, a single firm may not have sufficient control of the reward functions to factor out side payments.

\(^6\)If the firm is sufficiently large relative to the market it might approach a monopsony and, in some formal games, control the price. For example, the US military receives special (lower) fares from the airlines.
We hope that the ideas of this paper apply to other marketing situations and generalize beyond marketing. For example, in Wernerfelt, Simester, and Hauser (1996) we use related concepts to analyze side payments between supervisors and those who are supervised.
REFERENCES


Appendix: Formal Proofs (To be made available from the authors, September 1996.)

RESULT 1. For incentive systems in which the rewards to the upstream agent are increasing in the downstream agent’s rating, the upstream agent will provide a positive side payment to the downstream agent unless the firm sets a binding upper bound on the rating (or otherwise precludes side payments). Even with a binding upper bound, there may be side payments.

Proof. Consider a given \( u \). Let \( f(e) \) be the density function for \( e \). Consider first the case where there is no upper bound on \( r \) or it is not binding. \( u \) will seek to maximize the expression in Equation 4 subject to the conditions imposed by \( D \)'s maximization problems in Equation 3. Let \( \Gamma_u = v - c_u - s \). Differentiating Equation 4 we obtain:

\[
\int \frac{\partial U_u}{\partial \Gamma_u} \left[ \frac{\partial v(\bar{r}) - c_u(u) - s(u, \bar{r})}{\partial \bar{r}} \right] f(e) \text{d}e = 0
\]

By assumption, \( \partial U_u/\partial \Gamma_u > 0 \). The error, \( e \), appears in \( \bar{r} \), but it does not appear in \( v \), \( c_u \) or \( s \). Thus, this integral can be zero if and only if \( \partial U_u/v - c_u - s)/\partial \bar{r} = 0 \). Thus, this first order condition holds if only if:

\[
(A1) \quad \frac{\partial v(\bar{r})}{\partial \bar{r}} - \frac{\partial s(u, \bar{r})}{\partial \bar{r}} = 0
\]

Let \( \Gamma_u = w - c_u + s \). We now use implicit differentiation on Equation 3 recognizing that the right-hand side (RHS) does not depend on \( \bar{r} \).

\[
(A2) \quad \int \frac{\partial U_d}{\partial \Gamma_d} \left[ \frac{\partial w(\bar{r}, \pi)}{\partial \bar{r}} + \frac{\partial s(u, \bar{r})}{\partial \bar{r}} \right] f(e) \text{d}e = 0
\]

By assumption \( \partial U_d/\partial \Gamma_d \), which depends on \( e \), is positive. Furthermore, \( s(u, \bar{r}) \) does not depend upon \( \bar{r} \), and \( \partial \Gamma_d/\partial w = \partial \Gamma_d/\partial s = 1 \). Hence, Equation A2 becomes:

\[
(A3) \quad \int \frac{\partial U_d}{\partial \Gamma_d} \frac{\partial w(\bar{r}, \pi)}{\partial \bar{r}} f(e) \text{d}e = - \frac{\partial s(u, \bar{r})}{\partial \bar{r}} \int \frac{\partial U_d}{\partial \Gamma_d} f(e) \text{d}e
\]

From A1 we have \( \partial s(\bar{r}, u)/\partial \bar{r} = \partial v(\bar{r})/\partial \bar{r} \). But \( \partial v(\bar{r})/\partial \bar{r} > 0 \) by assumption. Thus, the RHS of A3 is negative:

\[
(A4) \quad \int \frac{\partial U_d}{\partial \Gamma_d} \frac{\partial w(\bar{r}, \pi)}{\partial \bar{r}} f(e) \text{d}e < 0
\]

By similar arguments we use implicit differentiation on Equation 3 to obtain.

\[
(A5) \quad \int \frac{\partial U_d}{\partial \Gamma_d} \frac{\partial w(\bar{r}, \pi)}{\partial \bar{r}} f(e) \text{d}e = 0
\]

Finally, we differentiate the left-hand side (LHS) of A4 or A5 to demonstrate that the second derivative with respect to \( \bar{r} \) is negative (concave) because \( \partial U_d/\partial \Gamma_d > 0 \) and \( \partial^2 w/\partial \bar{r}^2 < 0 \). Thus, we have shown that the first derivative of a concave function is negative at \( \bar{r} \) and zero at \( \hat{r} \). Hence, \( \bar{r} > \hat{r} \). The side payment is
positive by Equation 3. In the case where \( r \) is constrained, we add a Lagrange multiplier, \(-\lambda(r-r)\), to \( U \)'s optimization problem. This might allow a solution of the form \( \partial v/\partial r = \lambda \) and \( \partial s/\partial r = 0 \). If \( u \) were limited to a finite set, we obtain a result similar to that for Lagrange multipliers by using a series of piecewise functions for \( w \) such that each piece corresponds to a different action by \( U \). Finally, the reader can verify side payments for \( v=v_o+v,r \) and \( w=w_o+w_o(1-r)+w_o,\hat{r} \) for \( s \in [0,1] \) and sufficiently large \( v \) and \( w \).

**RESULT 2.** If the firm can preclude side payments and choose a reward system, \( v \) and \( w \), such that the upstream and downstream agents, acting in their own best interests, choose actions \( u^* \) and \( d^* \), then there exists a reward system such that (1) there is no loss of profits to the firm and (2) the upstream and downstream agents, still acting in their own best interests, choose \( u^* \) and \( d^* \) even though they are free to make side payments in return for higher ratings.

**Proof.** We prove the theorem for any implementable actions, \( u^* \) and \( d^* \). We begin with interior solutions for \( r \). Let \( v^*(r) \) and \( w^*(r,\hat{r}) \) implement \( u^* \) and \( d^* \) without side payments. \((\pi^o, c^o_u, c^o_d) \) are shorthand for \( \pi(u^*, d^*) \) and \( c^o_d(d^*) \), respectively. Define \( \Gamma^o_d = w^*(r,\hat{r})-c^o_d(d^*) \) and \( \Gamma^o_u = v^*(r)-c^o_u(u^*) \). Then \( r^o, u^o, \) and \( d^o \) satisfy the following first order conditions.

\[
\begin{align*}
\frac{\partial EU_d}{\partial r} &= \int_{-\Gamma^o_d}^{\Gamma^o_d} \left[ \frac{\partial \pi^o}{\partial r} \frac{\partial w^o(r^o,\pi^o)}{\partial r} + \frac{\partial \pi^o}{\partial d} \frac{\partial c^o_d(d^o)}{\partial d} \right] f(e)de = 0 \\
\frac{\partial EU_d}{\partial d} &= \int_{-\Gamma^o_d}^{\Gamma^o_d} \left[ \frac{\partial \pi^o}{\partial w^o} \frac{\partial v^o(r^o,\pi^o)}{\partial r} \frac{\partial \pi^o}{\partial d} \frac{\partial c^o_d(d^o)}{\partial d} \right] f(e)de = 0 \\
\frac{\partial EU_u}{\partial u} &= \int_{-\Gamma^o_u}^{\Gamma^o_u} \left[ \frac{\partial \pi^o}{\partial u} \frac{\partial c^o_u(u^o)}{\partial u} + \frac{\partial \pi^o}{\partial d} \frac{\partial c^o_d(d^o)}{\partial d} \right] f(e)de = 0
\end{align*}
\]

At this point we could continue to track through all the integral expectations as we did in the proof to Result 1. However, this notational nightmare adds no new insight to the basic proof. (Our proof demonstrates that two sets of first-order conditions lead to the same \( u^o \) and \( d^o \). With the integrals, these first-order conditions are still equal. Thus we work with the terms in the brackets. This would be exact if the error, \( e \), were additive to \( w \) rather than to \( \pi \). For the more sophisticated reader, we provide a supplemental appendix that contains an alternative proof with the integrals. In addition, that proof allows \( \pi \) to be a function of \( r \). This generalization is described in the text. Since \( \partial \Gamma^o_d/\partial w=-\partial \Gamma^o_u/\partial c^o_u=\partial \Gamma^o_u/\partial v=-\partial \Gamma^o_u/\partial c^o_u=1 \), the simplified first-order conditions reduce to:

\[
\begin{align*}
\frac{\partial w^o(r^o,\pi^o)}{\partial r} &= 0 \\
\frac{\partial w^o(r^o,\pi^o)}{\partial \pi} \frac{\partial \pi^o}{\partial d} \frac{\partial c^o_d(d^o)}{\partial d} - \frac{\partial c^o_d(d^o)}{\partial d} &= 0 \\
\frac{\partial v^o(r^o)}{\partial r} - \frac{\partial c^o_u(u^o)}{\partial u} &= 0
\end{align*}
\]
Now allow side payments and select new reward functions, \( w(r, \pi) = w'(r, \pi) - \alpha r + v'(r') \) and \( v(r) = \alpha r \). Recall that the firm must reimburse U and D for their risk costs because \( \pi \) contains noise and both U and D are risk averse. Under the specified reward system (for no side payments) only D incurs risk due to \( w'(r, \bar{\pi}) \). Because the noise does not affect \( r \), the only risk that D will incur under the new reward system is still due to \( w'(r, \bar{\pi}) \). Thus, if the new reward system implements \( v', d', r' \), then the cost of risk will be the same for D and hence for the firm which must reimburse D for that risk. Now, we must only prove that an \( \alpha \) can be chosen such that the new reward functions implement \( u', d', \) and \( r' \) when side payments are allowed.

For a given \( u' \), without a side payment, D would maximize the RHS of Equation 3. After simplification similar to that used to derive Equations A6-A8, \( d(u') \) and \( r(u') \) are defined by the following. (\( \bar{\pi} \) is shorthand for \( \pi(u', d) \).)

\[
\frac{\partial w'(r, \bar{\pi})}{\partial r} - \alpha = 0 \tag{A9}
\]

\[
\frac{\partial w'(r, \bar{\pi})}{\partial \pi} \frac{\partial \pi(u', d)}{\partial d} - \frac{\partial c_d(d)}{\partial d} = 0 \tag{A10}
\]

For a given \( u' \), with a side payment contract \( (s, r) \), D will choose \( \tilde{d} \) to maximize the LHS of Equation 3. After simplifying we obtain (\( \bar{\pi} \) is shorthand for \( \pi(u', \tilde{d}) \)).

\[
\frac{\partial w'(r, \bar{\pi})}{\partial \pi} \frac{\partial \pi(u', \tilde{d})}{\partial \tilde{d}} - \frac{\partial c_d(\tilde{d})}{\partial \tilde{d}} = 0 \tag{A11}
\]

If \( u = u' \) and \( r = r' \), then Equation A11 is the same as Equation A7 implying \( \tilde{d} = d' \).

Now U chooses \( \bar{r}, \tilde{u}, \) and \( s \) to maximize Equation 4 subject to Equation 3. Use the definition of \( \Gamma_d \) and \( \Gamma_u \) from the proof to Result 1. We first differentiate Equation 4 and simplify (review A1).

\[
\alpha - \frac{\partial s}{\partial r} = 0 \tag{A12}
\]

\[
- \frac{\partial c_u(\tilde{u})}{\partial \tilde{u}} - \frac{\partial s}{\partial \tilde{u}} = 0 \tag{A13}
\]

We use implicit differentiation on Equation 3. (U will choose \( s \) such that Equation 3 is binding. The RHS is not a function of \( \bar{r} \).) After simplification (review A2→A3):

\[
\frac{\partial w'(r, \bar{\pi})}{\partial r} - \alpha + \frac{\partial s}{\partial \tilde{r}} = 0 \tag{A14}
\]

Substituting Equation A12 into Equation A14 yields Equation A4 thus if \( u' = u' \), then \( r = r' \) whenever \( u' = u' \) and \( r = r' \). We must now show that we can choose \( \alpha \) such that \( u' = u' \).

We begin by implicitly differentiating Equation 3 with respect to \( u \). After extensive simplification this becomes Equation A15. (We use the first-order conditions in Equations A7, A9, and A14 to eliminate many terms by the envelop theorem and we use Equation A13 to substitute \( \partial s/\partial \tilde{u} = -\partial c_u(\tilde{u})/\partial \tilde{u} \).)
We must now demonstrate that we can choose an \( \alpha \) such Equation A15 holds when the participation constraints hold. (We first replace \( v'(s') \) with \( K(\alpha) \) such that the participation constraints hold.) Let us fix \( \tilde{u}=u' \) and \( \tilde{r}=r' \), then only the RHS of equation A15 varies as \( \alpha \) varies. First consider \( \alpha=0 \). When \( \alpha=0 \), \( \tilde{u}=u' \), and \( \tilde{r}=r' \), then \( \tilde{r}=r'' \) and \( \tilde{d}=d'' \) by Equations A9 and A10 which become the same as A6 and A7. Because the participation constraints hold, \( \tilde{\Gamma}_d=\Gamma_d \), thus the term on the RHS of equation A15 is the same as the first term on the LHS of Equation A15. Since \( \partial U_f/\partial \Gamma_d>0 \) and \( \partial c_d/\partial u>0 \), this implies that for Equation A15 we have LHS < RHS.

Continue to fix \( \tilde{u}=u' \) and \( \tilde{r}=r' \) and let \( \alpha \to \infty \). We use the implicit function theorem on Equations A9 and A10 (differentiating with respect to \( \alpha \)) to obtain:

\[
(A16) \quad \left\{ \frac{\partial \tilde{w}^o}{\partial \alpha} + \frac{\partial \pi}{\partial \alpha} \left( \frac{\partial \tilde{w}^o}{\partial \tilde{d}} \right) \frac{\partial \tilde{d}}{\partial \alpha} \right\}^{\frac{\partial \tilde{w}^o}{\partial \tilde{d}}} \frac{\partial \tilde{\pi}}{\partial \tilde{d}} + \frac{\partial \tilde{w}^o}{\partial \tilde{d}} \frac{\partial \tilde{\pi}}{\partial \tilde{d}} - \frac{\partial \tilde{d}}{\partial \tilde{d}} = 1
\]

By the assumptions of the text, the term in the large brackets, \( \{\} \), is negative, thus \( \tilde{r}/\partial \alpha<0 \). Because the second partials are bounded from zero, this implies that \( \tilde{r} \to -\infty \) as \( \alpha \to -\infty \). Then, because \( \partial(\tilde{w}^o/\partial \tilde{\pi})/\partial r \) is bounded from zero, we have \( \partial v''(\tilde{r}, \tilde{\pi})/\partial \tilde{\pi} \to -\infty \). Hence, for \( \alpha \to -\infty \) in Equation A15 we have LHS > RHS.

Finally, Equations A9 and A10 tell us that the change with respect to \( \alpha \) is continuous, thus there must be an \( \alpha \) between 0 and \( \infty \) such that for Equation A15 LHS = RHS for \( \tilde{u}=u'' \).

To summarize, we have proven that an \( \alpha>0 \) exists such that Equation A15 is satisfied. Thus, \( \tilde{u} \), \( \tilde{d} \), and \( \tilde{r} \) must satisfy Equations A6-A8 hence \( \tilde{u}=u'', \ \tilde{d}=d'', \ \tilde{r}=r'' \) for \( w(r, \pi) = w''(r, \pi) - \alpha r + v''(r') \), and side payments are allowed. The proof for constrained \( r \) requires that we introduce Lagrange multipliers in Equations A6, A8, A9, A12, and A14. This does not affect the arguments for \( \tilde{r}=r'' \) and \( \tilde{d}=d'' \). We then use the new A9 and A14 to simplify for Equation A15 and the rest of the proof follows. \( \square \)
In the appendix to "Side Payments in Marketing" we provided an abridged proof to Result 2. The proof was abridged because we did not carry through all of the integrals associated with expected utilities and we did not make \( \pi \) an explicit function of \( r \). Most readers will find that proof easier to follow. In this supplemental appendix we provide an alternative proof to Result 2. In this proof we carry through all the integrals and we make \( \pi \) an explicit function of \( r \). In addition, for brevity, we make greater use of agency theory vocabulary and we leave a few algebraic details to the reader. Finally, we generalize the result to more general utility functions.

**Result 2.** If the firm can preclude side payments and choose a reward system, \( v \) and \( w \), such that the upstream and downstream agents, acting in their own best interests, choose actions \( u^o \) and \( d' \), then there exists a reward system such that (1) there is no loss of profits to the firm and (2) the upstream and downstream agents, still acting in their own best interests, choose \( u^o \) and \( d' \) even though they are free to make side payments in return for higher ratings.

**Proof.** Let us first denote the game in which the firm precludes side payments by \( G^o \) and the game in which side payments are allowed as \( G' \). Formally, we show that if \((v^o,w^o)\) implement \((u^o,d^p,r^o)\) in \( G^o \), and the participation constraints bind, then there exist real numbers \( \gamma, \Theta_u, \Theta_d \) such that \((v^o,w^o) = (0.5\gamma^2 + \gamma r + \Theta_u, w^o - 0.5\gamma^2 - \gamma r + \Theta_d)\) leads to \( u \) and \( D \) choosing \((u^o,d',r^o)\) in \( G' \) at the same expected cost to the firm.\(^1\)

We first generalize the reward and cost functions for the upstream and downstream agents as: \( U_u = U_u(x,u) \) and \( U_d = U_d(y,d) \) where the first argument in each function represents the money transfers resulting from the side payments and wage payments and the second argument reflects the disutility of effort.\(^2\)

Note that under the new reward system only \( D \) incurs risk and this risk is due to \( w^o \) (as under the old reward system). Hence, the risk premium paid by the firm will be unchanged if we implement \((u^o,d^o,r^o)\) in \( G' \). Moreover, if we can implement \((u^o,d^p,r^o)\) in \( G' \), we can adjust \( \Theta_u, \Theta_d \) until both participation constraints bind. Therefore, we need only show that we can choose \( \gamma \) to implement \((u^o,d^p,r^o)\) in \( G' \) with \((0.5\gamma^2 + \gamma r + \Theta_u, w^o - 0.5\gamma^2 - \gamma r + \Theta_d)\). To do so we first show that if we can implement \( u^o \) and \( d^p \) with \((0.5\gamma^2 + \gamma r + \Theta_u, w^o - 0.5\gamma^2 - \gamma r + \Theta_d)\) then, when the participation constraints are binding, we will also implement \( r^o \). Next we show that if we can implement \( u^o \) and \( r^o \) with \((0.5\gamma^2 + \gamma r + \Theta_u, w^o - 0.5\gamma^2 - \gamma r + \Theta_d)\) then, when the participation constraints are binding, we will also implement \( d^p \). Finally we show that we can choose a \( \gamma \) so that \((0.5\gamma^2 + \gamma r + \Theta_u, w^o - 0.5\gamma^2 - \gamma r + \Theta_d)\) implements \( u^o \) given \( d^p \) and \( r^o \).

Note that \( r^o \) is determined by \( D \) in \( G^o \) from the following first order condition:

\[^1\]Note that we will assume that \( w^o \) satisfies the same assumptions as \( w \) and that utility is strictly increasing in monetary payments (with the slope bounded away from zero). Finally we also require that \( \forall d \pi \) is increasing in \( u \) when \( u = u^o \), and that the integrals over \( f(e) \) are defined correctly. These are all rather weak (and natural) assumptions.

\[^2\]That is: \( x = v - s \) and \( y = w + s \).
(SA1) \[ \int \frac{\partial U_d}{\partial y} \left[ \frac{\partial w^*}{\partial \pi} \frac{\partial \pi}{\partial r} + \frac{\partial w^*}{\partial r} \right] dF = 0 \]

Under \( G' \), \( r \) is determined by \( U \) maximizing \( \int U_d(v^l - s, u) dF \), which yields the following first order condition:

(SA2) \[ \int \frac{\partial U_d}{\partial x} \left( \gamma - \frac{\partial s}{\partial r} \right) dF = 0 \]

Using the implicit function theorem on (3) we get:

(SA3) \[ \frac{\partial x}{\partial r} = \left( \int \frac{\partial U_d}{\partial y} \gamma dF - \int \frac{\partial U_d}{\partial y} \left[ \frac{\partial w^*}{\partial \pi} \frac{\partial \pi}{\partial r} + \frac{\partial w^*}{\partial r} \right] dF \right) \left( \int \frac{\partial U_d}{\partial y} dF \right)^{-1} = 0 \]

Inserting this into (SA2) yields:

(SA4) \[ \frac{\partial U_d}{\partial x} \left( \gamma - \gamma + \int \frac{\partial U_d}{\partial y} \left[ \frac{\partial w^*}{\partial \pi} \frac{\partial \pi}{\partial r} + \frac{\partial w^*}{\partial r} \right] dF \left( \int \frac{\partial U_d}{\partial y} dF \right)^{-1} \right) = 0 \]

This is zero if (SA1) is zero, since all derivatives are evaluated at their participation constraints. So if we can use \((0.5\gamma^2 + \gamma r + \theta_u, w^p - 0.5\gamma^2 - \gamma r + \theta_d)\) to implement \( u^o \) and \( d^o \) in \( G^i \), we will also implement \( r^o \). Note that this result holds even if there is an upper bound on \( r \) that constrains \( r^o \) in \( G^o \). Although SA1 will not equal zero in those circumstances, because SA4 equals SA1 we know that the constraint will also be binding in \( G^o \).

We turn now to implementation of \( d^o \), and make the analogous argument that if \((0.5\gamma^2 + \gamma r + \theta_u, w^o - 0.5\gamma^2 - \gamma r + \theta_d)\) implement \( u^o \) and \( r^o \) in \( G^i \), we will also implement \( d^o \). Note that \( d^o \) is determined by \( D \) in \( G^o \) from the following first order condition:

(SA5) \[ \int \frac{\partial U_d}{\partial y} \left( \frac{\partial w^*}{\partial \pi} \frac{\partial \pi}{\partial d} \right) dF + \int \frac{\partial U_d}{\partial d} dF = 0 \]

Under \( G' \), \( d \) is determined by \( D \) maximizing \( \int U_d(v^l + s, d) dF \), which yields the following first order condition:
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\[
(SA6) \quad \int \frac{\partial U_d}{\partial y} \left( \frac{\partial w^\rho}{\partial \pi} \frac{\partial \pi}{\partial d} - \frac{\partial w^\rho}{\partial \pi} \frac{\partial \pi}{\partial d} \right) \, dF + \int \frac{\partial U_d}{\partial d} \, dF = 0
\]

This is zero if \((SA5)\) is zero since all derivatives are again evaluated at their participation constraints. So we now need only show that we can find a \(\gamma\) that implements \(u^\rho\) using \((0.5\gamma^2 + \gamma r + \Theta_u, w^\rho - 0.5\gamma^2 - \gamma r + \Theta_d)\). Under \(G'\), \(u\) is determined by \(U\) maximizing \(\int U_d(v^1 - s, u) \, dF\), which (using the envelope theorem) yields the following first order condition:

\[
(SA7) \quad \frac{\partial U_u}{\partial x} \left[ \int \frac{\partial U_d}{\partial y} \frac{\partial w^\rho}{\partial \pi} \frac{\partial \pi}{\partial u} \, dF - \int \frac{\partial U_d}{\partial y} \frac{\partial w^\rho}{\partial \pi} \frac{\partial \pi}{\partial d} \, dF \right] \left[ \frac{\partial U_d}{\partial y} \right]^{-1} + \frac{\partial U_u}{\partial u} = 0
\]

where:

\[
a = (w^1(u^\rho, d^\rho, r^\rho), s, d^\rho) \\
b = (w^1(u^\rho, d^\rho, r^\rho), d^\rho)
\]

Suppose again that we fix \(u^\rho\), \(d^\rho\) and \(r^\rho\) and use \(\Theta_u, \Theta_d\) to ensure that the participation constraints remain binding, does there exist a \(\gamma\) such that \((SA7)\) holds?

Under these constraints, only the second term \(\frac{\partial w^\rho}{\partial \pi} \frac{\partial \pi}{\partial u}\) changes as we change \(\gamma\). On the other hand \((6)\) tells us that the change is continuous. Suppose first that \(\gamma = 0\). In this case, \(s = 0, \rho = r^\rho\) and \(d = d^\rho\) so the left side of \((SA7)\) reduces to \(\frac{\partial U_u}{\partial d} < 0\). Consider next what happens as \(\gamma \to -\infty\). Using the implicit function theorem on \((6)\), we see that \(\frac{d\hat{r}}{d\gamma}\) is negative iff:

\[
(SA8) \quad \int \frac{\partial^2 U_u}{\partial y^2} \left[ \frac{\partial w^\rho}{\partial \pi} \frac{\partial \pi}{\partial r} + \frac{\partial w^\rho}{\partial \pi} \frac{\partial \pi}{\partial d} - \gamma \right] (\gamma + \hat{r}) \, dF + \int \frac{\partial U_u}{\partial y} \, dF > 0
\]

We would like to show that \(\hat{r} \to -\infty\) as \(\gamma \to -\infty\) (we assume that \(D\)'s threat point \(\hat{r}\) has no lower bound). Let us assume, to derive a contradiction that this is not the case. This implies that \((SA8)\) is not satisfied, \((\gamma + \hat{r}) > 0\) and \(\frac{\partial w^\rho}{\partial \pi}\) is increasing as \(\gamma \to -\infty\). Note that because \(w^\rho\) and \(\pi\) are concave in \(r\), \(\frac{\partial w^\rho}{\partial r}\) and \(\frac{\partial \pi}{\partial r}\) are non-increasing as \(\gamma\) increases. Hence for sufficiently large \(\gamma\) we know that
\[
\left[ \frac{\partial w^*}{\partial r} \frac{\partial \pi}{\partial r} + \frac{\partial w^*}{\partial r} - \gamma \right] < 0. \quad \text{Thus we have derived a contradiction because if}
\]
\[
\left[ \frac{\partial w^*}{\partial \pi} \frac{\partial \pi}{\partial r} + \frac{\partial w^*}{\partial r} - \gamma \right] < 0 \quad \text{and } (\gamma + r) > 0 \text{ then both terms in Equation (SA8) are positive as } \gamma \to \infty.
\]
Hence we know that \( r \to -\infty \) as \( \gamma \to \infty \). Since by assumption \( \frac{\partial w^*}{\partial \pi} \to -\infty \) as \( r \to -\infty \), this will eventually make the left side of (SA7) positive. So by continuity there exists a \( \gamma \) such that (SA7) holds and this \( \gamma \) implements \( u^* \). \( \blacksquare \)