AN OBJECTIVE METHOD FOR FORECASTING THE PROBABILITY OF SNOW IN EASTERN NEW ENGLAND

by

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(1951)

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

(1952)

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Department of Meteorology
August 22, 1952

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ABSTRACT

An Objective Method For Forecasting The Probability Of Snow In Eastern New England

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Submitted to the Department of Meteorology on August 22, 1952 in partial fulfillment of the requirements for the degree of Master of Science.

The value of a probability forecast of precipitation type and the suitability of objective methods for obtaining such a forecast are discussed.

Climatic summaries are prepared in the form of probability diagrams relating the surface temperature at Syracuse and Quebec to the type of precipitation occurring twelve hours later at Portland, Boston, and Nantucket for cases in which the six-hour accumulation is .10 inch or more.

With the aid of these diagrams, forecasts are made on independent data. The forecasts are used in an application to a typical operational problem.

Methods of verification of the forecasts are discussed. The concept of an "equivalent forecaster" is introduced. This concept is discussed in detail in an appendix. The "equivalent forecaster" is found to be useful in comparing directly forecasts which are stated in terms of the probability that an event will occur in any of several classes of events with forecasts which are made without probability statements.

The physical basis for the diagrams is considered. Additional parameters which might be considered in order to increase the accuracy of the forecasts are mentioned.

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Acknowledgment

I would like to express my appreciation for the many constructive suggestions offered by Professor Thomas F. Malone during the preparation of this thesis.

Contents

1. Introduction

The important and difficult problem of forecasting accurately the type and amount of precipitation from winter storms in the New England area a reasonable time in advance has not yet been satisfactorily solved despite the efforts of practising forecasters and a considerable number of research studies. The problem has been made more tractable by forecasting the type and amount of precipitation separately.

Penn (1) has developed a largely objective method for forecasting precipitation amounts from winter coastal storms in the Boston area. These storms account for 80 percent of the total winter precipitation and nearly all precipitation in excess of .35 inch.

Penn (2) and Bristor (3) have developed methods for predicting the type of precipitation at New England stations. These methods are based on forecasting the surface, 850 millibar, or 700 millibar temperatures for the time of the start of precipitation. The forecasts are prepared twelve hours in advance of the onset of precipitation. The incomplete success of the methods developed by Penn and Bristor has been due mainly to the difficulty in forecasting these temperatures. The most critical criterion for distinguishing between snow and rain was found by several investigators to be the temperature in the first 5,000 feet of the atmosphere. Penn has shown that the 850 millibar temperature is most useful at Albany and Portland while the surface temperature serves equally well at Nantucket.

In this study, the type of precipitation at Portland, Boston, and Nantucket is related to the temperature twelve hours in advance at Syracuse and Quebec for cases in which the six-hour accumulation is at least .10 inch at the forecast station. The minimum of .10 inch was used by Penn also and roughly eliminates those cases of snow which have little disruptive effect on public activities.

The choice of Syracuse and Quebec was based on an investigation of five stations in the neighborhood of the forecast stations. The other three stations were eliminated because they appeared to be of no additional forecast value. A cursory examination of many features of the synoptic pattern suggested that the temperature field was of most significant forecast value.

With the aid of forecast diagrams, a forecast of the type of precipitation is made 12 hours in advance. This forecast is made in terms of the probability that the accumulation will be in the form of snow. The forecast method is not restricted to any particular time of day.

Individual diagrams have been constructed for each month at Portland, Boston, and Nantucket. The diagrams are essentially climatic summaries based on data from the period 1940 to 1952. The majority of the initial data is from the first half of the period. Independent data have been collected from the years 1947 to 1952. The accuracy of the forecast method is tested on the independent data.

The physical basis for the diagrams is discussed. The sources of error and the possibility of improving the forecasts are suggested.

An illustration of the application of the probability forecast obtained from the diagrams to a specific operation is presented.

Methods of verification of probability forecasts are discussed in the Appendix.

2. Forecast Diagrams

The terms "snow" and "not snow" are defined in Table 1.

TABLE 1

Present Precipitation Type	Precipitation Type 3 Hours Ago
"snow"	
snow	all types
snow with fog	all types
fog	snow or snow with fog
none	snow or snow with fog
"not snow"	
all types except snow, fog	all types
fog	all types except snow or snow with fog
none	all types except snow or snow with fog

Fig. 1-5, 6-10, 11-14 are the forecast diagrams for Portland, Boston, and Nantucket, respectively, for individual months from November to March. No November diagram is presented for Nantucket since no "snow" cases occur in the data for November at Nantucket.

In each diagram the type of precipitation occurring at the forecast station, whenever the six-hour accumulation is at least .10 inch, is plotted at the intersection of the lines representing the surface temperatures at Syracuse and Quebec twelve hours previously.

Lines are then drawn joining points of equal frequency of occurrence of "snow" cases relative to "not snow" cases. These lines are assigned values from 0 to 1 based upon the number of "snow" cases occurring between two adjacent frequency lines relative to the total number of cases between those lines.

Two sets of parallel straight lines were adopted for the diagrams. In some cases one set consists of a single line. This type of frequency line seemed to be best suited to the data.

Any point which lies outside the .05 frequency line is considered to have less than a .05 probability of being a "snow" point. Similarly any point within the .90 frequency line is considered to have a probability of greater than .90 of being a "snow" point. The probability of"snow" at all other points is determined by a linear interpolation between the values of the frequency lines which bracket the point.

Points which lie between the same two frequency lines are said to belong to the same probability class. The value of a probability class is the average of the values of the frequency lines which define the class. All points for which the probability of "snow" is greater than .90 are said to belong to the probability class .95, which is the average of .90 and 1. The probability class .025 is similarly defined.

The distribution of "snow" and "not snow" points in each probability class for each month is summarized in Tables 2-4 for Portland, Boston, and Nantucket, respectively. The independent data and the initial data are presented separately. The distributions are very similar.

The climatological expectancy of "snow", as determined from the initial data, is also presented in Tables 2-4.

The .50 frequency line is used to make forecasts for the independent data. The results are summarized for individual months in Tables 5-7. At Portland and Nantucket the results are superior for the independent data. For most months the percentage of correct forecasts is nearly the same for the initial data and the independent data.

The fact that more than 50 percent of the forecasts are correct when the .50 frequency line is used as a basis for the forecast is a consequence of the definition of the .50 frequency line. The .50 frequency line is the line along which 50 percent of the cases are "snow" and 50 percent are "not snow". It would be theoretically possible for all the "snow" cases to fall on one side of the line with all the "not snow" cases falling on the opposite side of the line. In this ideal case, the forecasts would be correct 100 percent of the time.

Fig. 15 and 16 illustrate how the diagrams may be used to obtain a forecast. Data for the storm of February 5-7, 1942 are summarized in Table 8. Fig. 15 is an outline of the synoptic pattern at 1830Z February 5, 1942 and Fig. 16 is the pattern twelve hours later. The forecasts are made in terms of the probability that the accumulation in the last six hours of the subsequent twelve hour period will be in the form of "snow". Where the accumulation during the forecast period was less than .10 inch, the forecasts are given in parenthesis. The forecast probabilities are determined by a linear interpolation between the frequency lines which bracket each point. The forecast probabilities have been calculated to the nearest one percent. This is probably greater accuracy than is justified by the amount of data used in constructing the frequency lines or by the accuracy of the method as a whole. In practice, a forecast of the probability class in which a point falls would probably be adequate.

This storm was selected as an example of a typically borderline case. At Boston and Nantucket the precipitation is initially "snow" but later changes to "not snow". The timing of this change is usually very difficult to forecast. Forecasters at Boston know only too well that it may be snowing at Portland while it is raining at Boston, or snowing at Boston while rain is falling at Nantucket. The fact that the forecast diagrams can handle these two problems is demonstrated in this example.

On a .50 probability basis, the diagrams (Fig. 4, 9, and 13) provide 9 correct forecasts and no incorrect forecasts. Since the percentage of correct forecasts for all the independent data is about 85 percent for February, this may be considered a fortunate result.

TABLE 2

Portland												
Probability Class	Nov. S	Nov. NS	Dec. S	Dec. NS	Jan. S	Jan. NS	Feb. S	Feb. NS	Mar. S	Mar. NS	Total S	Total NS
Initial Data												
.025	0	42	0	19	0	7	0	16	1	20	1	104
.15	2	8	0	5	0	4	0	1	1	7	3	25
.375	2	6	4	5	3	9	4	8	4	4	17	32
.55	1	0	1	2	3	2	1	1	5	5	11	10
.70	4	0	2	0	3	1	6	4	1	2	16	7
.85	1	1	2	1	12	2	8	1	6	1	30	6
.95	3	0	4	0	4	0	9	0	1	0	21	0
Total	13	57	13	32	25	25	28	31	19	39	98	184
Total Cases	70		45		50		59		58		282	
Climatological Expectancy of "Snow" $\frac{98}{282} = .35$												
Independent Data												
.025	0	8	0	8	0	15	0	5	0	8	0	44
.15	0	1	0	0	3	7	3	5	2	3	8	16
.375	0	2	1	2	2	8	2	4	1	4	6	20
.55	0	1	3	1	4	1	3	0	3	1	13	4
.70	0	0	0	1	3	0	5	0	0	0	8	1
.85	0	0	4	1	8	0	2	0	2	0	16	1
.95	1	0	3	0	4	0	3	0	1	0	12	0
Total	1	12	11	13	24	31	18	14	9	16	63	86
Total Cases	13		24		55		32		25		149	

TABLE 3

Boston												
Probability Class	Nov.		Dec.		Jan.		Feb.		Mar.		Total	
	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS
Initial Data												
.025	0	56	0	17	0	17	1	14	0	26	1	130
.15	1	9	3	8	0	1	2	9	3	21	9	48
.375	1	8	5	17	4	7	3	9	4	11	17	52
.55	2	0	0	1	4	1	5	4	2	0	13	6
.70	0	0	3	0	21	3	4	1	5	1	33	5
.85	0	0	0	0	1	1	3	2	0	0	4	3
.95	0	0	5	0	2	0	2	0	3	0	12	0
Total	4	73	16	43	32	30	20	39	17	59	89	244
Total Cases	77		59		62		59		76		333	
Climatological Expectancy of "Snow" $\frac{89}{333}$ = .27												
Independent Data												
.025	1	9	0	5	0	18	0	6	0	9	1	47
.15	0	1	0	4	0	1	1	2	2	10	3	18
.375	1	0	3	3	2	2	1	6	1	1	8	12
.55	0	1	3	1	2	1	4	1	1	0	10	4
.70	0	0	0	0	8	3	3	3	3	0	14	6
.85	0	0	0	0	5	0	0	0	1	0	6	0
.95	0	0	1	0	1	0	2	0	0	0	4	0
Total	2	11	7	13	18	25	11	18	8	20	46	87
Total Cases	13		20		43		29		28		133	

TABLE 4

Nantucket										
Probability Class	Dec. S	Dec. NS	Jan. S	Jan. NS	Feb. S	Feb. NS	Mar. S	Mar. NS	Total S	Total NS
Initial Data										
.025	1	33	1	34	0	33	2	50	4	150
.15	1	1	1	8	4	11	0	3	6	23
.375	0	1	6	17	2	4	2	8	10	30
.70	2	1	4	1	7	6	4	2	17	10
.95	7	0	2	0	16	0	0	0	25	0
Total	11	36	14	60	29	54	8	63	62	213
Total Cases	47		74		83		71		275	
Climatological Expectancy of "snow" $\frac{62}{275}$ = .23										
Independent Data										
.025	0	17	0	22	0	10	0	21	0	70
.15	0	2	0	10	1	2	0	3	1	17
.375	1	3	1	8	1	3	1	0	4	14
.70	0	2	0	0	1	1	1	1	2	4
.95	1	0	0	0	2	0	1	0	4	0
Total	2	24	1	40	5	16	3	25	11	105
Total Cases	26		41		21		28		116	

TABLE 5

Portland												
	Nov.		Dec.		Jan.		Feb.		Mar.		Total	
	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS
OBSERVED	FORECAST											
Initial Data												
S	9	4	9	4	22	3	24	4	13	6	77	21
NS	1	56	3	29	5	20	6	25	8	31	23	161
Percent Correct	93		84		84		83		76		84	
Independent Data												
S	1	0	10	1	19	5	13	5	6	3	49	14
NS	1	11	3	10	1	30	0	14	1	15	6	80
Percent Correct	93		83		89		84		84		87	

TABLE 6

Boston												
	Nov.		Dec.		Jan.		Feb.		Mar.		Total	
	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS
OBSERVED	FORECAST											
Initial Data												
S	2	2	8	8	28	4	14	6	10	7	62	27
NS	0	73	1	42	5	25	7	37	1	53	14	230
Percent Correct	97		85		85		78		89		88	
Independent Data												
S	0	2	4	3	16	2	9	2	5	3	34	12
NS	1	10	1	12	4	21	4	14	0	20	10	77
Percent Correct	77		80		86		85		89		87	

TABLE 7

Nantucket	Dec. S	Dec. NS	Jan. S	Jan. NS	Feb. S	Feb. NS	Mar. S	Mar. NS	Total S	Total NS
OBSERVED	FORECAST									
Initial Data										
S	9	2	6	8	23	6	4	4	42	20
NS	1	35	1	59	6	48	2	61	10	203
Percent Correct	94		89		86		91		89	
Independent Data										
S	1	1	0	1	3	2	2	1	6	5
NS	2	22	0	40	1	15	1	24	4	101
Percent Correct	88		98		86		93		92	

TABLE 8

Day	Hour	Temperature		Precipitation Type			Forecast		
		Syr.	Que.	Por.	Bos.	Nan.	Por.	Bos.	Nan.
5	12Z	22	0	None	None	Snow Trace	(.85)	.62	.90
5	18Z	26	19	Snow Trace	Snow .06"	Snow Trace	.82	.54	.34
6	00Z	28	21	Snow .05"	Snow	Snow	.76	.49	.29
6	06Z	30	21	Snow	Snow	Rain	.70	.40	(.56)
6	12Z	30	19	Snow	Rain	Fog None	(.66)	(.45)	(.29)
6	18Z	34	22	Snow	Rain	Fog None	(.42)	(.23)	(.04)

3. Physical Basis for the Diagrams

In a general way one would expect that as the temperature decreased in the neighborhood of the forecast stations, the probability of the precipitation type being "snow" at the forecast stations would increase. To conclude that the surface temperature at Syracuse and at Quebec causes the type of precipitation to be "snow" or "not snow" twelve hours later at the forecast stations would be absurd. However, the factors which determine the surface temperature at Syracuse and Quebec also tend to determine the temperature at Portland, Boston, and Nantucket twelve hours later, and it is these temperatures which, as other investigators have shown, determine the type of precipitation occurring at these stations. The diagrams should be successful to the extent that the factors responsible for the temperature at Syracuse and Quebec are the same factors which determine the subsequent temperature at the forecast stations.

The symbols T_S and T_Q will be used as abbreviations for the surface temperature at Syracuse and Quebec, respectively, twelve hours before the time of observation at the forecast stations.

From an informal analysis of the weather patterns that are associated with winter precipitation along the New England coast, three fairly common types may be recognized. Approximately sixty percent of the data may be classified as one or another of these types. The types will be referred to as "normal" (N), "cold air west" (CAW), and "stagnant low east" (SLE).

In the N case, illustrated in Fig. 15 and 16, a cold front advances eastward with the coldest air centered to the north of the forecast stations and cool air behind the cold front, with warm air in between. There is customarily a secondary development and the precipitation is largely pre-warm front. In this case T_S and T_Q are representative of the air which will be over the forecast stations twelve hours later. In the N case the diagrams are most successful.

The CAW pattern is usually associated with post-cold front precipitation at the forecast stations. This type is illustrated in Fig. 20. In this case T_Q is relatively unrelated to the future temperature at Portland, Boston, and Nantucket, being generally too high. T_S is also likely to be too high. The CAW cases account for most of the errors made by the diagrams where T_Q is at least $22^{o}F$ and T_S is at least $26^{o}F$.

The SLE cases are associated with post-frontal precipitation where the air trajectory is from the northeast. Thus T_S is practically useless as an indication of the subsequent temperature at the forecast stations and T_Q tends to be too high. The SLE cases usually fall in the lower right portions of the diagrams and account for most of the "snow" cases where T_S is lower than T_Q. The SLE pattern is illustrated in Fig. 21.

Although the monthly distribution of CAW and SLE cases has not been evaluated, it was noted that these cases occur most frequently in November and March.

From these considerations it would seem advisable to conduct a more rigorous investigation to determine the additional forecast value of a parameter which would distinguish between these three patterns.

4. Application of Probability Forecasts

Probability forecasts have many applications to public activities which ordinary forecasts do not. This fact is established in the Appendix where several types of forecasts are compared. One example will be illustrated here. Although all figures pertaining to the statement of the problem are quite arbitrary, the problem itself is fairly typical of real situations in which probability forecasts are extremely useful.

Let us assume that a department store has found that by heating the sidewalks along its front, the risk of accidents occurring as a result of an accumulation of .10 inch or more of snow may be substantially reduced. The figure .10 inch is taken as the minimum accumulation of snow for which heating the sidewalks is feasible. Approximately six hours is required to heat the sidewalks to the point where snow will melt on contact with the sidewalk under average conditions. The cost of heating the sidewalks for a twelve hour period is $250 and the loss which would be incurred by not heating the sidewalks is, on the average, $1,000. Thus the ratio of cost to contingent loss is $\frac{\$250}{\$1,000}$ or 25 percent. Therefore the sidewalks should be heated whenever the probability of an accumulation of snow of at least .10 inch during a six hour period is .25 or greater.

To determine the value of the forecast diagrams with regard to this operation, Table 9 has been prepared from the independent data summarized in Tables 2-4.

The total cost plus loss which the department store would incur during the period for which the independent data have been collected (1947-1952) has been computed for the following alternative procedures. It is assumed in each case that it is known whether, during any six hour period, there will be an accumulation of precipitation of .10 inch or more.

I. The forecast diagrams are used. The sidewalks are heated whenever the probability of "snow" is at least .25.

II. The sidewalks are heated whenever the accumulation of either "snow" or "not snow" is .10 inch or more.

III. The sidewalks are not heated.

IV. An "ordinary" forecast is used to determine when the sidewalks should be heated. This forecast may be made by any method whatever. It will be assumed that 80 percent of all "ordinary" forecasts are correct. This is probably the best performance which can be obtained by any of the forecast procedures currently in use. The degree to which this assumption approximates a set of actual forecasts, such as is generally available to the public, is discussed in the Appendix.

In I, II, and IV, the heating is begun six hours before the start of the six hour period during which the accumulation takes place and is continued for twelve hours.

Similar computations of the total cost plus loss for the methods I - IV may be made for other ratios of cost to loss. In Table 10, the total cost plus loss for several values of this ratio is presented for each of the four methods. The significance of these totals is investigated in the Appendix.

TABLE 9

Station	Total Cost Plus Loss In Dollars			
	I	II	III	IV
Portland	28,250	37,250	63,000	29,500
Boston	20,000	33,250	46,000	22,750
Nantucket	8,000	29,000	11,000	9,650

TABLE 10

Probability Of "Snow"	Total Cost Plus Loss In Dollars			
	I	II	III	IV
PORTLAND				
.05	5,250	7,450	63,000	15,980
.25	28,250	37,250	63,000	29,500
.50	41,500	74,500	63,000	46,400
.60	49,800	89,400	63,000	53,160
.80	58,200	119,200	63,000	66,680
.90	61,800	134,100	63,000	73,440
BOSTON				
.05	5,250	6,650	46,000	11,910
.25	20,000	33,250	46,000	22,750
.50	34,000	66,500	46,000	36,300
.60	40,000	79,800	46,000	41,720
.80	44,000	106,400	46,000	52,560
.90	45,600	119,700	46,000	57,980
NANTUCKET				
.05	2,300	5,800	11,000	3,690
.25	8,000	29,000	11,000	9,650
.50	10,000	58,000	11,000	17,100
.90	10,600	104,400	11,000	29,020

Appendix

Probability forecasts have, in general, two advantages over forecasts where no probability statement is made. First, probability forecasts express the forecaster's degree of confidence that the forecast event will occur, and second, probability forecasts are better suited to a wide range of applications to specific operations.

To demonstrate these advantages, methods of verification must be derived in which both types of forecasts may be compared quantitatively. Verification systems currently in use are not entirely adequate for this purpose. In this section a method will be developed which will show clearly the improvement which may be obtained when forecasts are made with a statement of the probability that the forecast event will occur in any of several classes of events.

The method of Brier and Allen (5) may be used to verify probability forecasts. The verification of a set of probability forecasts is in terms of a score P which is defined by:

$$P = \frac{1}{N}\sum_{j=1}^{r}\sum_{i=1}^{N}(f_{ij} - E_{ij})^2$$

where

E	=	1	for occurrence of forecast event
E	=	0	for non-occurrence of forecast event
N	=		number of events
r	=		number of classes of events

f_j = probability of each class j

f_{ij} = probability of each class j for the event i

In general, P can have any value from 0 to 2, 0 being the score corresponding to a correct forecast of every event with a probability of 1 in the class in which it occurs.

If the climatological expectancy, or past distribution in each class, of the events to be forecast is known, it is possible to determine the value of P which would be obtained by forecasting each event with a probability equal to the climatological expectancy of its occurrence in any class. This value of P will be designated by P_{CE}. Thus any forecast method may be compared with climatological expectancy. It is shown by Brier and Allen that the best score (i.e., the minimum value of P), which may be achieved, by forecasting the same probability in a given class for every event, is P_{CE}. Since the method of climatological expectancy requires no forecasting skill, the difference, $P_{CE} - P$, is a qualitative indication of the skill of any alternate forecasting procedure.

The following development will provide a quantitative comparison of several practical forecasting methods of general application.

TABLE 11

Symbol	Portland	Boston	Nantucket
P	.23	.22	.12
P_{CE}	.50	.46	.21
P'	.88	.89	.94
P'_{CE}	.75	.77	.90

Sample Computation of P for Portland

	Probability Class							
Event	.025	.15	.375	.55	.70	.85	.95	Total
"Snow"	0	8	6	13	8	16	12	63
"Not Snow"	44	16	20	4	1	1	0	86
			Total Cases = 149					

$$P = \frac{1}{N}\sum_{j=1}^{r}\sum_{i=1}^{N}(f_{ij} - E_{ij})^2$$

$$\begin{aligned}
= \frac{1}{149} \Big[& 0(.025 - 1)^2 + 0(.975 - 0)^2 + 8(.15 - 1)^2 \\
& + 8(.85 - 0)^2 + 6(.375 - 1)^2 + 6(.625 - 0)^2 \\
& + 13(.55 - 1)^2 + 13(.45 - 0)^2 + 8(.70 - 1)^2 \\
& + 8(.30 - 0)^2 + 16(.85 - 1)^2 + 16(.15 - 0)^2 \\
& + 12(.95 - 1)^2 + 12(.05 - 0)^2 + 44(.025 - 0)^2 \\
& + 44(.975 - 1)^2 + 16(.15 - 0)^2 + 16(.85 - 1)^2 \\
& + 20(.375 - 0)^2 + 20(.625 - 1)^2 + 4(.55 - 0)^2 \\
& + 4(.45 - 1)^2 + 1(.70 - 0)^2 + 1(.30 - 1)^2 \\
& + 1(.85 - 0)^2 + 1(.15 - 1)^2 + 0(.95 - 0)^2 \\
& + 0(.05 - 1)^2 \Big]
\end{aligned}$$

$$= .23$$

A useful score may be obtained by computing $P' = 1 - \frac{P}{2}$, which has the range of values 0 to 1. For $P = 0$, $P' = 1$ and for $P = 2$, $P' = 0$. If all forecasts are stated with a probability of either 1 or 0 that an event will occur in a given class, the percentage of correct forecasts will be P'. This is true because, for a correct forecast $(f_{ij} - E_{ij})^2$ will be 0 and for an incorrect forecast $(f_{ij} - E_{ij})^2$ will be 1. Thus any forecasting method may be readily compared with any method for which the stated probability is always 1 or 0.

Values of P, P_{CE}, P', and P'_{CE} have been computed for the independent data summarized in Tables 2-4, where the forecast diagrams are used to forecast the probability of "snow". In this example there are two classes of events, "snow" and "not snow", and $r = 2$. These scores are entered in Table 11.

It would be desirable to compare these scores with corresponding scores derived from a set of actual forecasts of the same events. Since such forecasts are not available, a hypothetical set of forecasts will be introduced as an approximation to the actual forecasts. The following assumptions are made:

(1) Actual forecasts normally available to the public do not include probability statements.

(2) The percentage of correct forecasts is the same for "snow" and for "not snow". Thus if 80 percent of the total number of forecasts are correct, then 80 percent of the forecasts of "snow" are correct and 80 percent of the forecasts of "not snow" are correct.

These assumptions are in general very realistic.

With these assumptions an "equivalent forecaster" may be defined as a set of forecasts of which q percent are correct in each class, and for which the forecast probability is q, and such that the P score of the "equivalent forecaster" is equal to the P score of the forecasting method to which the "equivalent forecaster" is to be compared. The relationship between q and P will now be derived.

The P score achieved by the "equivalent forecaster" is given by:

$$P = \frac{1}{N} N \Big[qC(q-1)^2 + qC(1-q-0)^2 + (1-q)C(q-0)^2 + (1-q)C(1-q-1)^2 + q(1-C)(q-1)^2 + q(1-C)(1-q-0)^2 + (1-q)(1-C)(q-0)^2 + (1-q)(1-C)(1-q-1)^2 \Big]$$

where N is the total number of events
C is the percentage of events in one class

This equation reduces to:

$P = 2q - 2q^2$ which may be expressed as

$$q = \frac{1 + \sqrt{1 - 2P}}{2}$$

It may be shown that the choice of the negative sign for the radical would not alter the interpretation of this relation.

Although this relation has been derived for the case $r = 2$, it is valid for all values of r.

It may be noted that this relation does not involve N or C.

It will now be shown that a set of forecasts of which q percent are correct in each class, and for which the forecast probability is any single value other than q, will have a P score higher than, and thus inferior to the P score of the "equivalent forecaster".

Let the forecast probability be x. Then,

$$P = q(x - 1)^2 + q(1-x - 0)^2 + (1-q)(x - 0)^2 + (1-q)(1-x - 1)^2$$

$$= 2q - 4qx + 2x^2$$

which has a minimum value at $x = q$.

Thus any forecasting method may be represented by an "equivalent forecaster" through a P score and to the extent that the assumptions (1) and (2) above are justified, the "equivalent forecaster" corresponds to a set of actual forecasts such as is available to the public. In this way a set of forecasts made by any method whatever may be verified by determining the "equivalent forecaster" which corresponds to that set of forecasts.

The "equivalent forecaster" will now be computed for several forecasting methods, using the independent data summarized in Tables 2-4. The methods are described below. The results are given in Table 12.

A. The forecast diagrams are used. The probability of "snow" is equal to the probability class in which the point lies on the appropriate forecast diagram. The probability of "not snow" is equal to the probability of "snow" subtracted from 1.

B. Only the .50 frequency line on the forecast diagrams is used. When a point lies within the .50 frequency line, "snow" is forecast with a probability equal to the percentage of "snow" events in the initial data which lie within the .50 frequency line, and "not snow" is forecast with a probability equal to the percentage of "not snow" events in the initial data which lie within the .50 frequency line. When a point lies outside the .50 frequency line, the forecasts are analogously determined.

C. Only the .50 frequency line on the forecast diagrams is used. When a point lies within this line, "snow" is forecast with a probability of 1 and "not snow" is forecast with a probability of 0.

When a point lies outside the .50 frequency line, "snow" is forecast with a probability of 0 and "not snow" is forecast with a probability of 1.

D. The forecasts are based on climatological expectancy. For all events "snow" and "not snow" are forecast with a probability equal to the percentage frequency with which they occur in the initial data.

E. The most common class of events is forecast with a probability of 1 and the remaining classes of events are forecast with a probability of 0. Since, at each station, "not snow" is more common than "snow", "not snow" is forecast with a probability of 1 and "snow" is forecast with a probability of 0.

TABLE 12

	"EQUIVALENT FORECASTER"		
	q		
Method	Portland	Boston	Nantucket
A	.87	.88	.94
B	.88	.84	.92
C	.85	.81	.90
D	.50	.65	.88
E	.50	.50	.89

From Table 12 it may be concluded that:

1) The forecast diagrams are considerably superior to climatological expectancy and probably superior to forecasts currently available to the public.

2) A probability forecast is generally superior to a forecast where no probability is stated. A variable probability forecast is generally superior to a single probability forecast (compare A, B, and C).

3) The best single probability forecast is obtained when the probability is equal to the percentage of correct forecasts. Compare B and C, D and E. When the climatological expectancy of one class is nearly 1 or 0, this fact may be obscured. (See Nantucket.)

4) The forecast diagrams give the best verification score, relative to climatological expectancy, at all stations. However, at Portland, the .50 frequency line alone gives the best score. This may be a result of an insufficiently large sample of independent data.

5) Since q has a minimum value of .50, method E is useless except at Nantucket where the percentage of "snow" events is 9.5 percent.

6) The results are similar at all three forecast stations. Since 116 events is the

minimum number of cases of independent data, the superiority of method A over methods C, D, E, and probably B is significant.

* * * * *

In applications to specific operations, a probability forecast based on a single level of probability is often more useful than any other type of forecast. Frequently the damage caused by some weather phenomenon such as a snow or rain storm, frost, or a flood, for example, may be lessened by some timely protective measures. The ratio of the cost of protection to the contingent loss determines the probability level at which the protective measures are economical. In this situation, the ideal type of forecast is one which is based on this probability level. One example has been presented. Objective methods, such as the forecast diagrams, are particularly well suited to this type of forecast because a highly accurate statement of probability may be made with a minimum of skill. The forecast diagrams require no forecasting skill at all in their application. The time required to prepare the forecast is also important. For the forecast diagrams five minutes is sufficient.

Again the problem of verification of forecast methods in terms of their usefulness for specific operations has been only partially solved. The method of J.C. Thompson (4) of computing the total cost plus loss for several forecasting methods is an important step toward a final solution of the verification problem. It gives a quantitative comparison. This method is used in the construction of Tables 9 and 10.

The relative value of the methods I - IV at different ratios of cost to loss is suggested in Table 10. However, the reasons for the variation in their relative value for different values of this ratio is not entirely clear from the table. Again the concept of an "equivalent forecaster" may be introduced as a forecasting method by which any other method may be measured.

The "equivalent forecaster" is defined in a manner similar to the previous definition. The percentage of correct forecasts is again denoted by q. The"equivalent forecaster" is a set of forecasts, of which q percent are correct in each class, such that the total cost plus loss for the "equivalent forecaster" is the same as the total cost plus loss for the method to which the "equivalent forecaster" is to be compared. If the same assumptions are made as before with regard to forecasts which are currently available to the public, it will be seen that the "equivalent forecaster" is a close approximation to such forecasts.

The relation between q and y_i, where y_i is the percentage of correct forecasts for each class, i, of events for any forecasting method, will be derived.

For a given forecasting method, the total cost plus loss will be:

$$TN\left[y_1SP + (1 - y_1)S + \sum_{i=2}^{r}(1 - y_i)PS_i\right]$$

where
- T is the contingent loss for one event
- N is the number of events
- P is the ratio of cost to contingent loss
- y_1 is the percentage of correct forecasts of the class of events for which protective measures are taken
- y_i is the percentage of correct forecasts in the remaining classes
- r is the number of classes of events
- S is the percentage frequency of occurrence of the event against which protective measures are taken
- S_i is the percentage frequency of occurrence of the remaining classes of events

This may be simplified to

$$TN\left[-y_1S(1 - P) - P\sum_{i=2}^{r} y_iS_i + S + P - PS\right]$$

For the "equivalent forecaster" this becomes

$$TN\left[-qS(1 - P) - qP(1 - S) + S + P - PS\right]$$

Equating these expressions and solving for q:

$$q = \frac{y_1S(1 - P) - P\sum_{i=2}^{r} y_iS_i}{S(1 - P) + P(1 - S)}$$

For method II, in which protective measures are taken for all events, the total cost is TNP

and $$q = q_a = \frac{S(1 - P)}{S(1 - P) + P(1 - S)}$$

For method III, in which protective measures are never taken, the total loss is TNS and

$$q = q_n = \frac{P(1 - S)}{S(1 - P) + P\,(1 - S)}$$

From this:

$$q_n = 1 - q_a$$

In general, $q = y_1 q_a + \dfrac{P \sum_{i=2}^{r} y_i S_i}{S(1 - P) + P(1 - S)}$

For the case $r = 2$,

$$q = y_1 q_a + y_2 q_n$$

$$= y_1 q_a + y_2(1 - q_a)$$

Since q_a decreases as P increases, this equation demonstrates the superiority of a forecasting method in which the percentage of correct forecasts in each class is a function of the ratio of cost to contingent loss relative to a method in which the percentage of correct forecasts in each class is independent of this ratio. Thus a method such as the forecast diagrams is better suited to many specific operational applications than is a method such as the "equivalent forecaster". Since the "equivalent forecaster is probably a close approximation to the type of forecasts now available to the public, it may be inferred that objective methods which give a probability forecast are probably better suited to specific operations than the methods currently being used. This point is emphasized in Fig. 17-19 where methods I - IV are compared for the independent data from Tables 2-4 and Table 10.

In order to be useful at all values of P, a forecasting method must at least be better than methods II and III which require no forecasting techniques. Method I satisfies this requirement. Method IV, which is represented by a horizontal straight line at the appropriate value of q (for Table 10, q = .80) always fails at the extreme values of P because, for these values a very high percentage of correct forecasts of one class is necessary.

Several characteristics of the curves I, II, and III in Fig. 17-19, which are independent of the particular example, may be mentioned.

At P = S,

$$q_a = q_n = .50 \quad \text{and} \quad q = \frac{y_1 + y_2}{2}$$

At P = .50,

$$q_a = S; \quad q_n = 1 - S; \quad q = y_1 S + y_2(1 - S)$$

which is simply the percentage of correct forecasts of all the events.

Where $y_1 = y_2 = q$, a method such as I will normally have a minimum value of q.

References

1 Samuel Penn, "An Objective Method for Forecasting Precipitation Amounts from Winter Coastal Storms for Boston," Monthly Weather Review, vol. 76, No. 8, August 1948, p. 149-161.

2 Samuel Penn, Unpublished Notes

3 Bristor, C.L., "The Critical Temperature Range for the Forecast of Rain or Snow," Thesis for Master of Science Degree, M.I.T., Cambridge, Mass., 1947.

4 Thompson, J.C., "A Numerical Method for Forecasting Rainfall in the Los Angeles Area," Monthly Weather Review, vol. 78, No. 7, July 1950, p. 113-124.

5 Brier, G.W. and Allen, R.W., "Verification of Weather Forecasts," Compendium of Meteorology, Boston, 1951, p. 841-848.

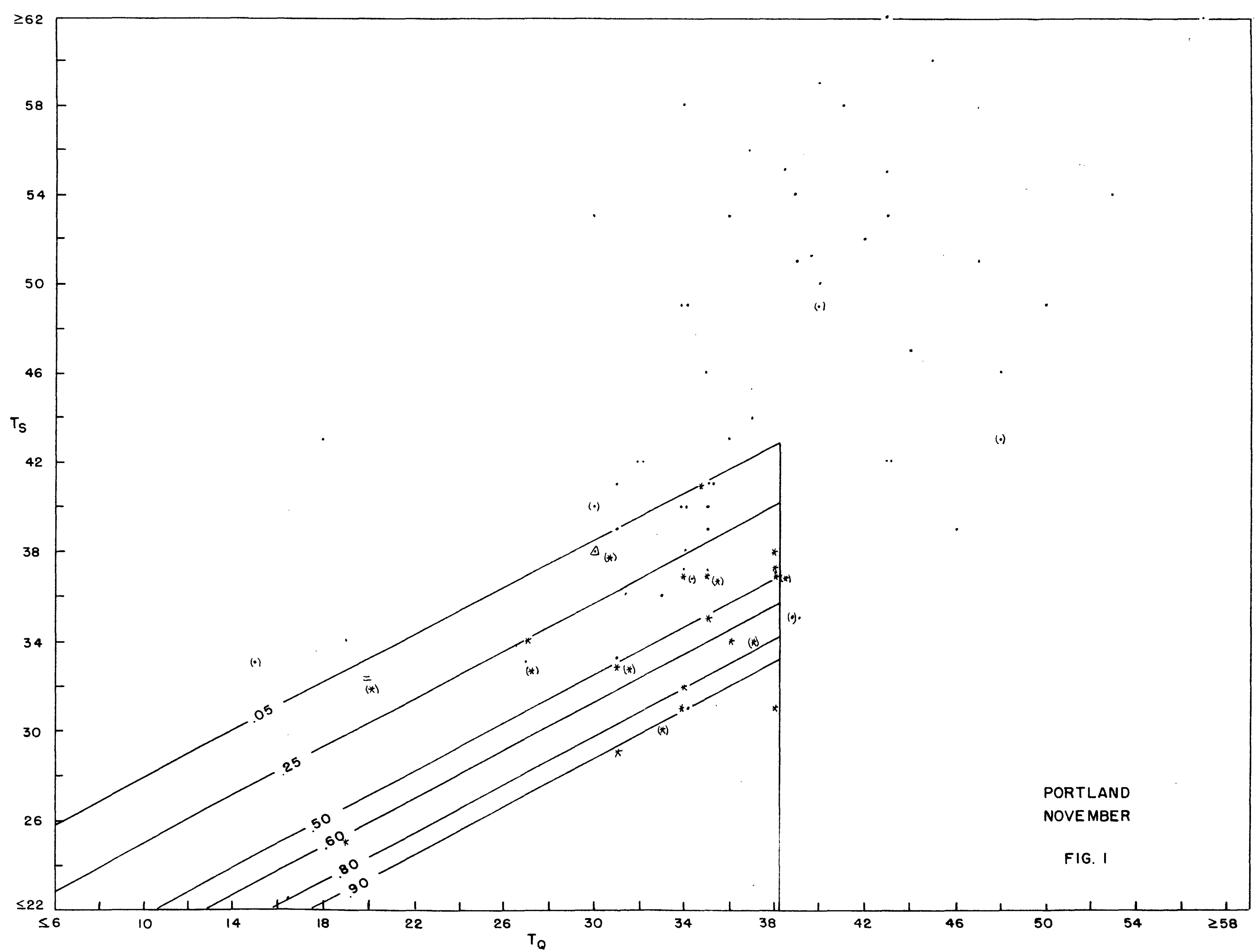
Explanation of Symbols Used in Fig. 1 - 21.

The following precipitation symbols are used:

Symbol	Definition
*	Snow
•	Rain
*̣	Rain and Snow mixed
≡	Fog
=	Fog
= *	Fog and Snow
△	Sleet
△ ▽	Hail
~	Freezing Rain
( )	Precipitation of type indicated occurred 3 hours prior to time of observation
]	Precipitation occurred within past hour but not at time of observation

In Fig. 15, 16, 20, and 21 fronts are designated by the usual symbols. Temperatures are plotted and isobars drawn for these figures.

In Fig. 17-19, I, II, and III refer to the forecasting methods defined on page 17.



T_S
≥62
58
54
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42
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34
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≤22
≤6
10
14
18
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26
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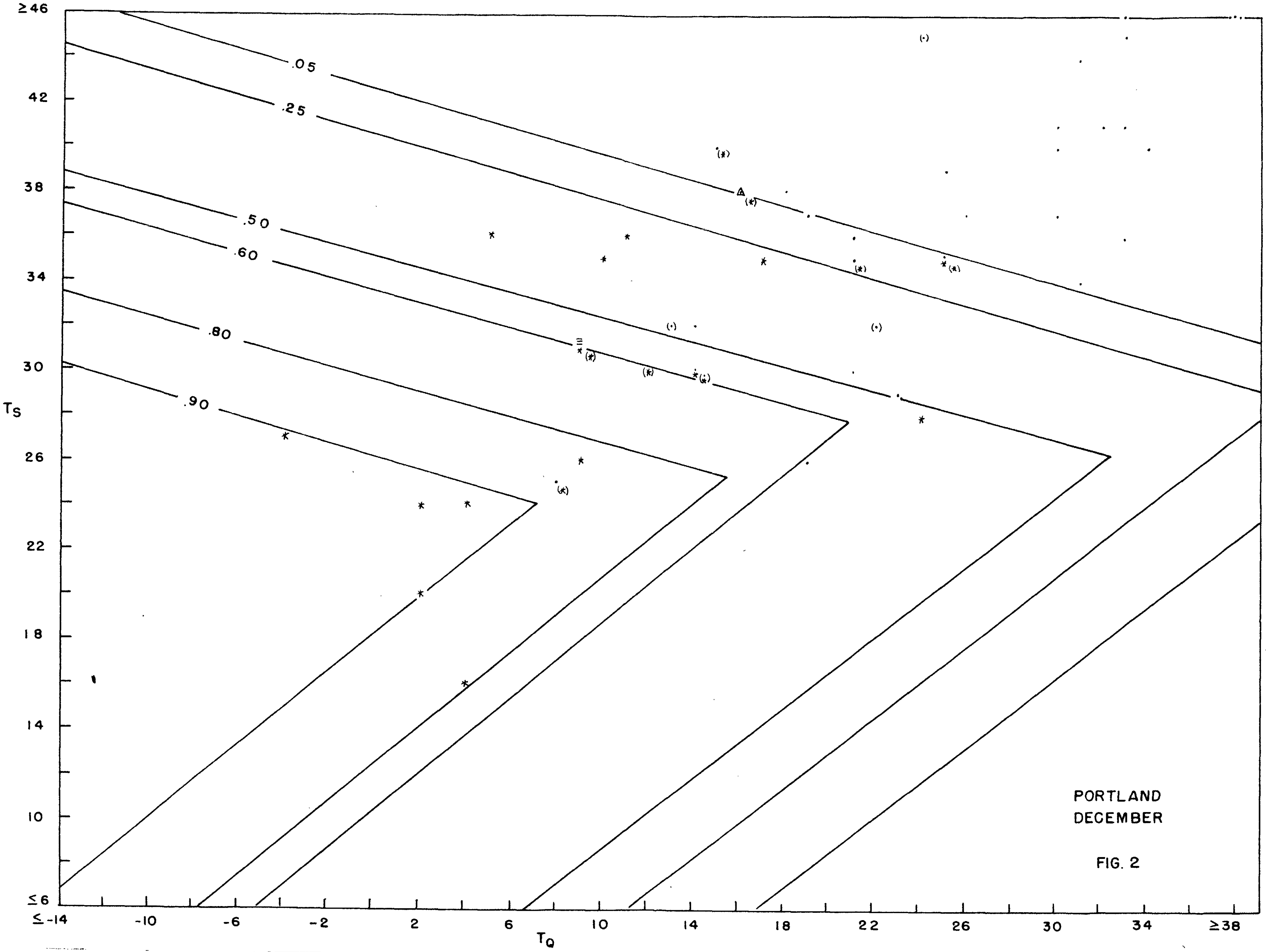
38
42
46
50
54
≥58
T_Q
.05
.25
.50
.60
.80
.90
PORTLAND
NOVEMBER

FIG. 1


≥46
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≤6
T_S
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-6
-2
2
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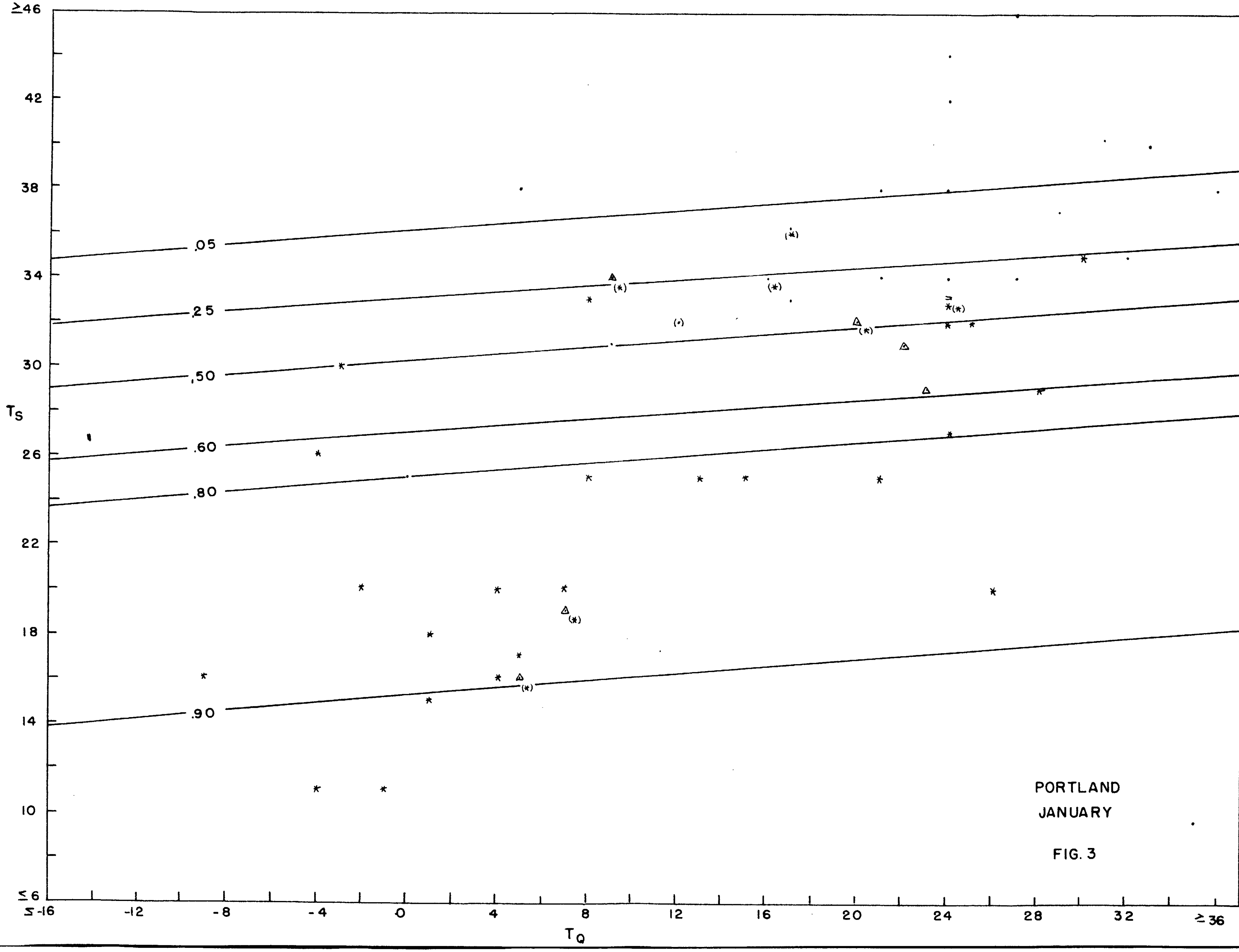
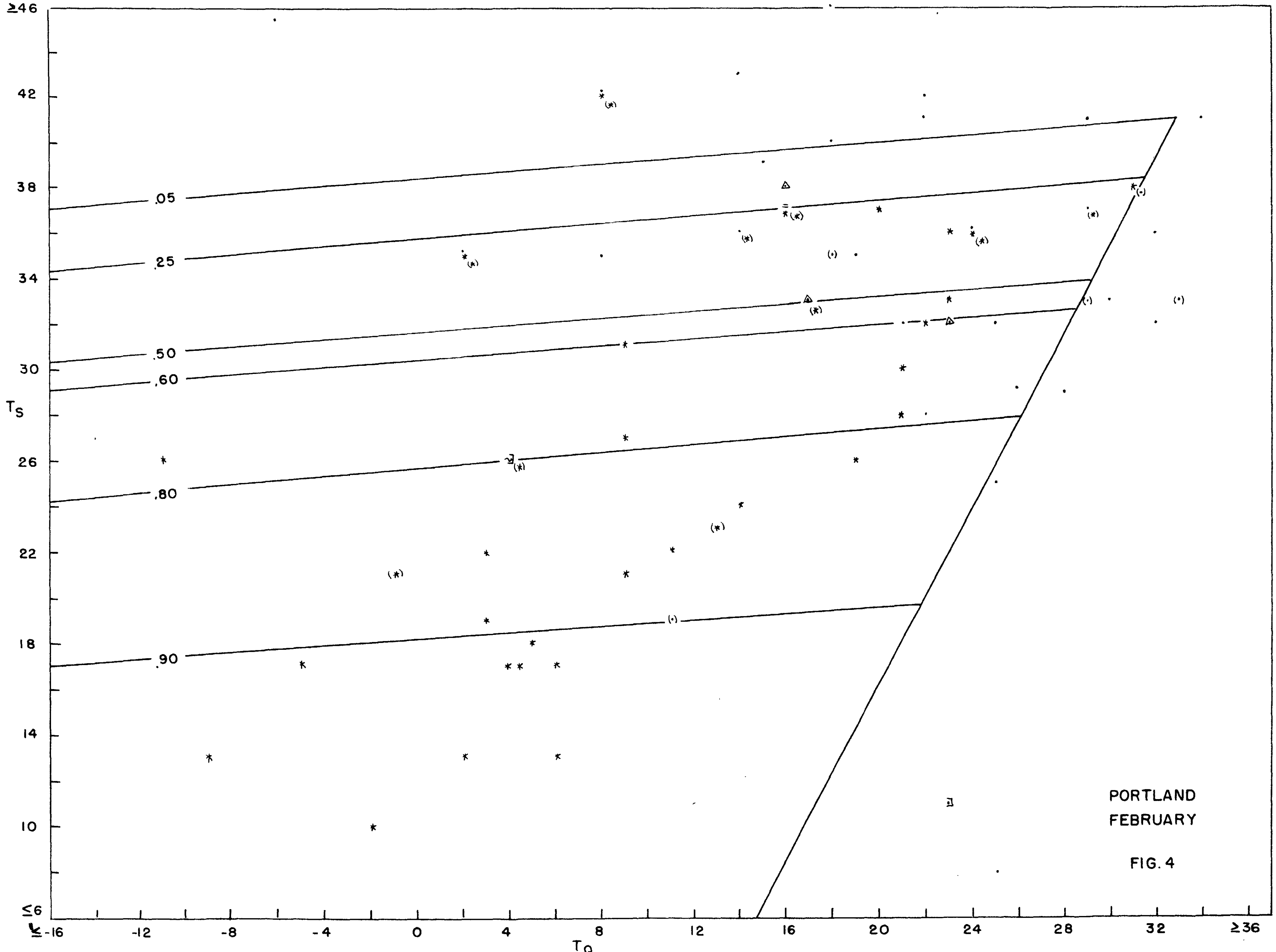
≥38
T_Q
.05
.25
.50
.60
.80
.90
PORTLAND
DECEMBER

FIG. 2



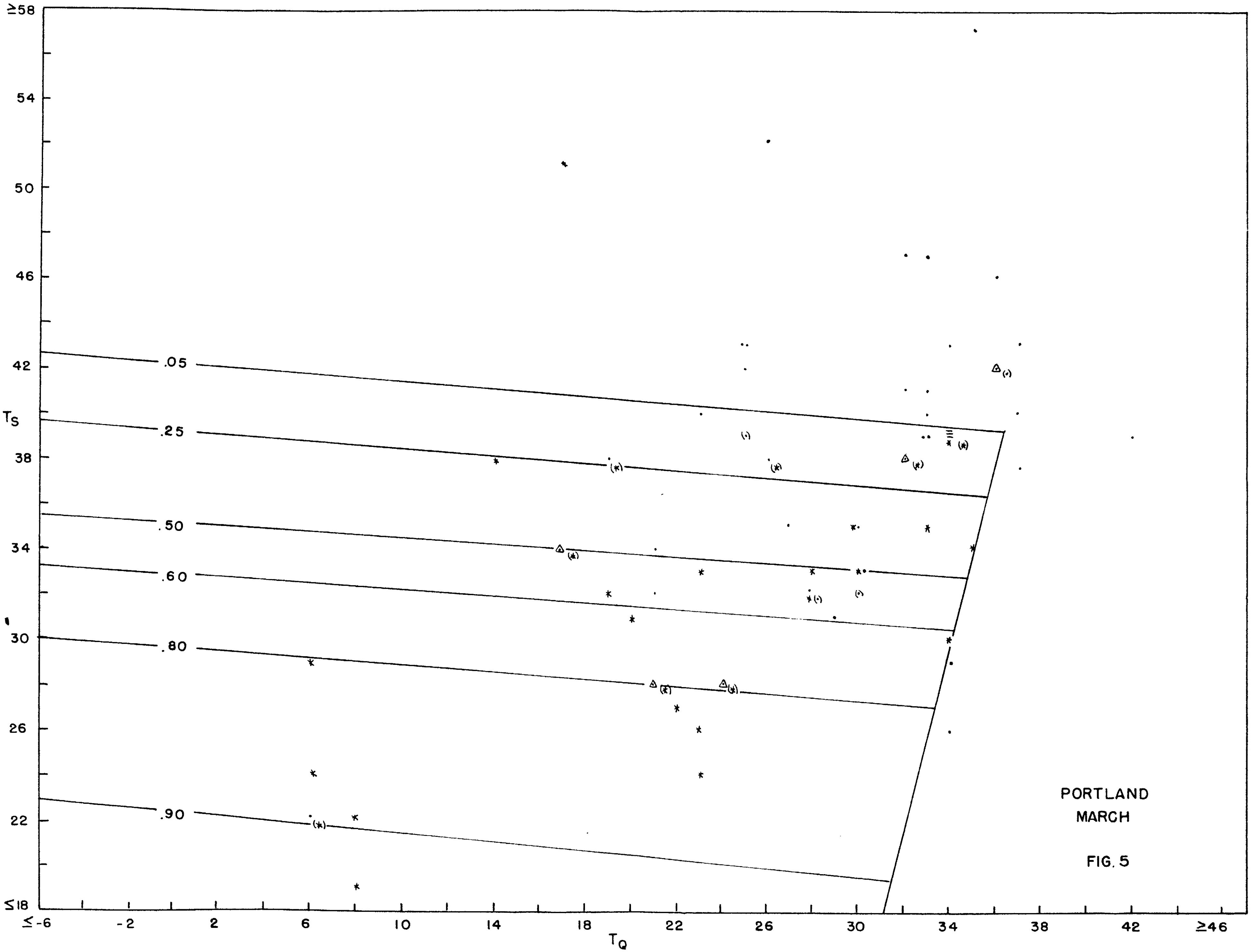
PORTLAND
JANUARY

FIG. 3



PORTLAND
FEBRUARY

FIG. 4


≥58
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T_S
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≤18
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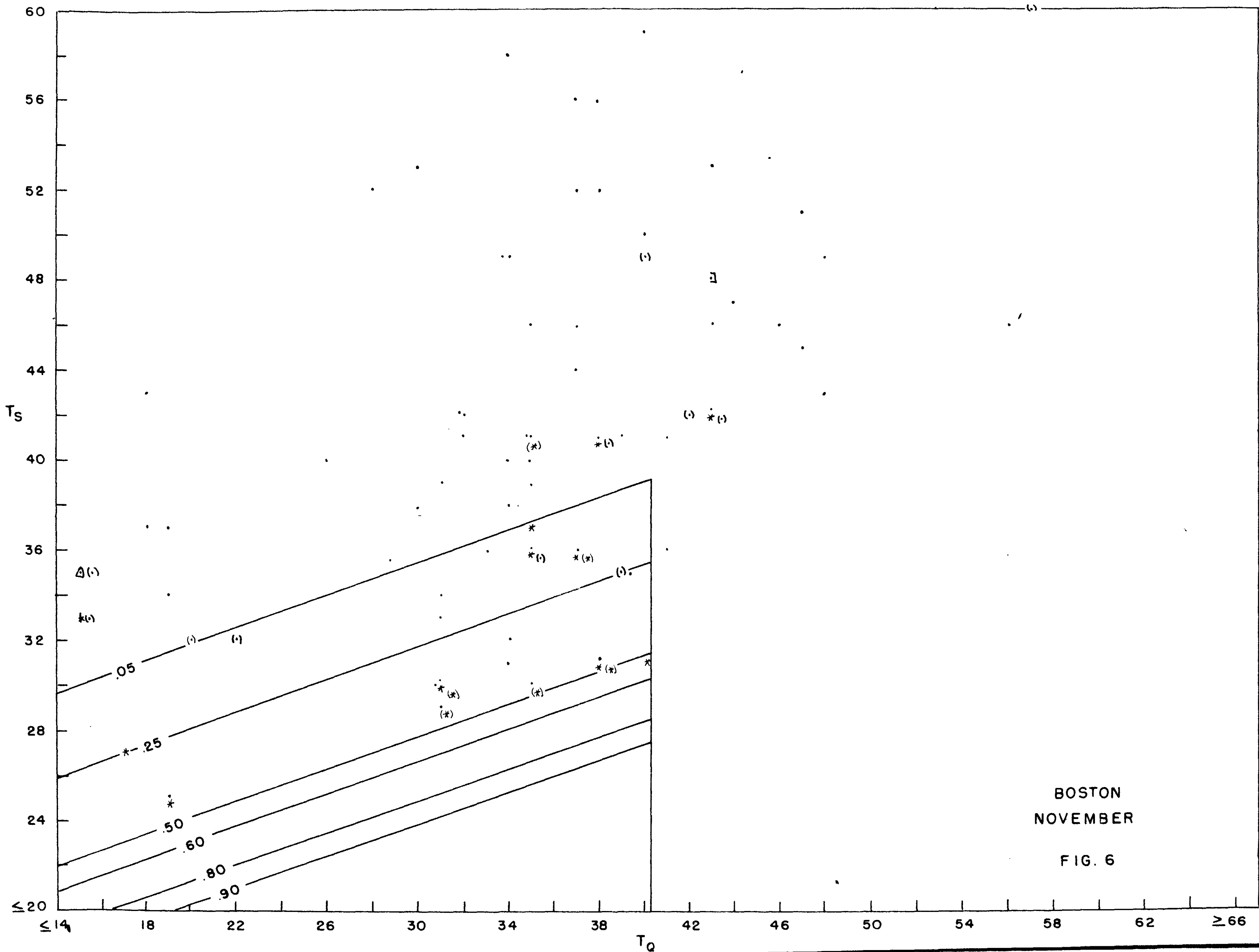
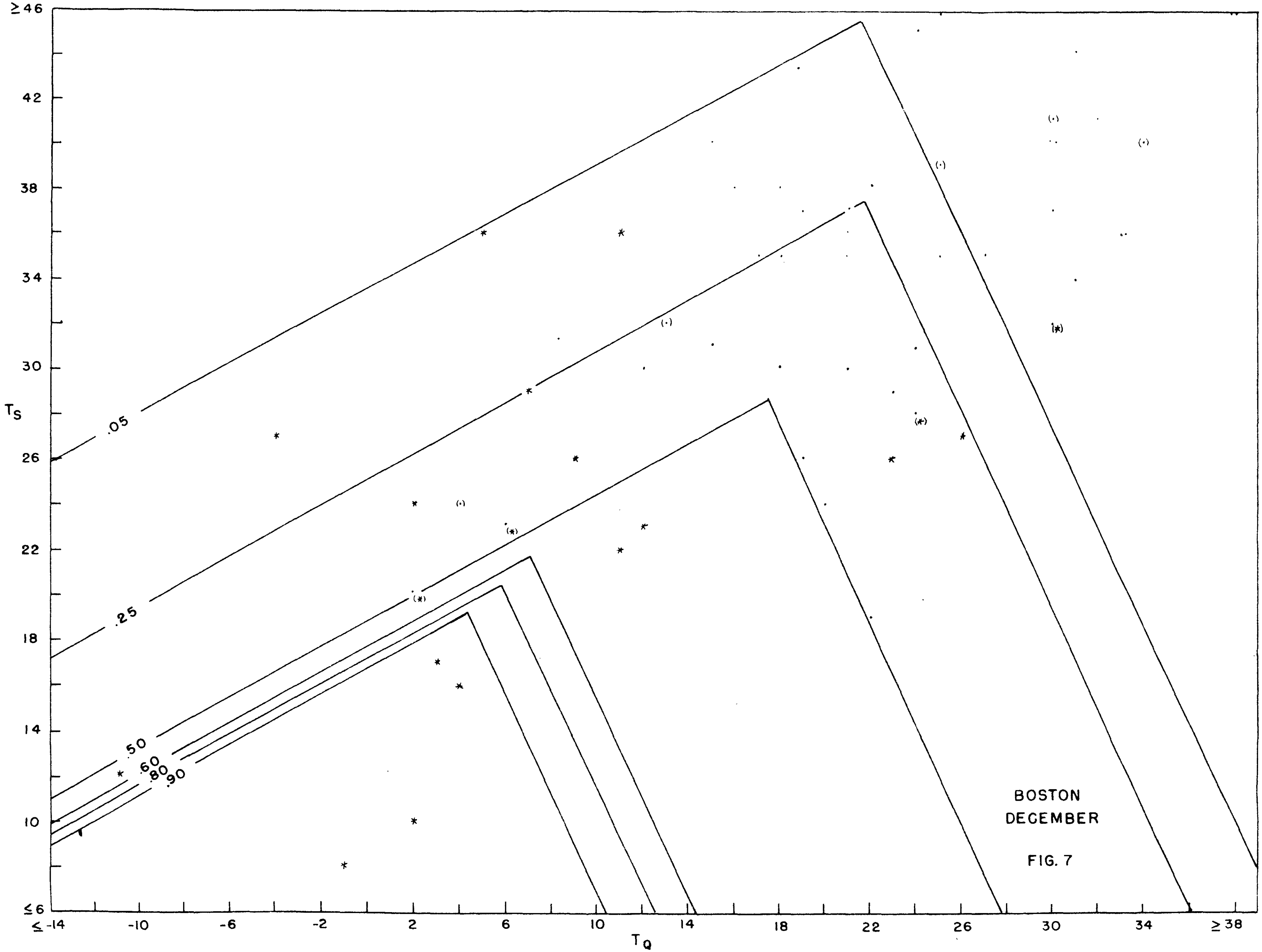
≥46
T_Q
.05
.25
.50
.60
.80
.90
PORTLAND
MARCH

FIG. 5



BOSTON
NOVEMBER

FIG. 6


≥46
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≤6
T_S
≤-14
-10
-6
-2
2
6
10
14
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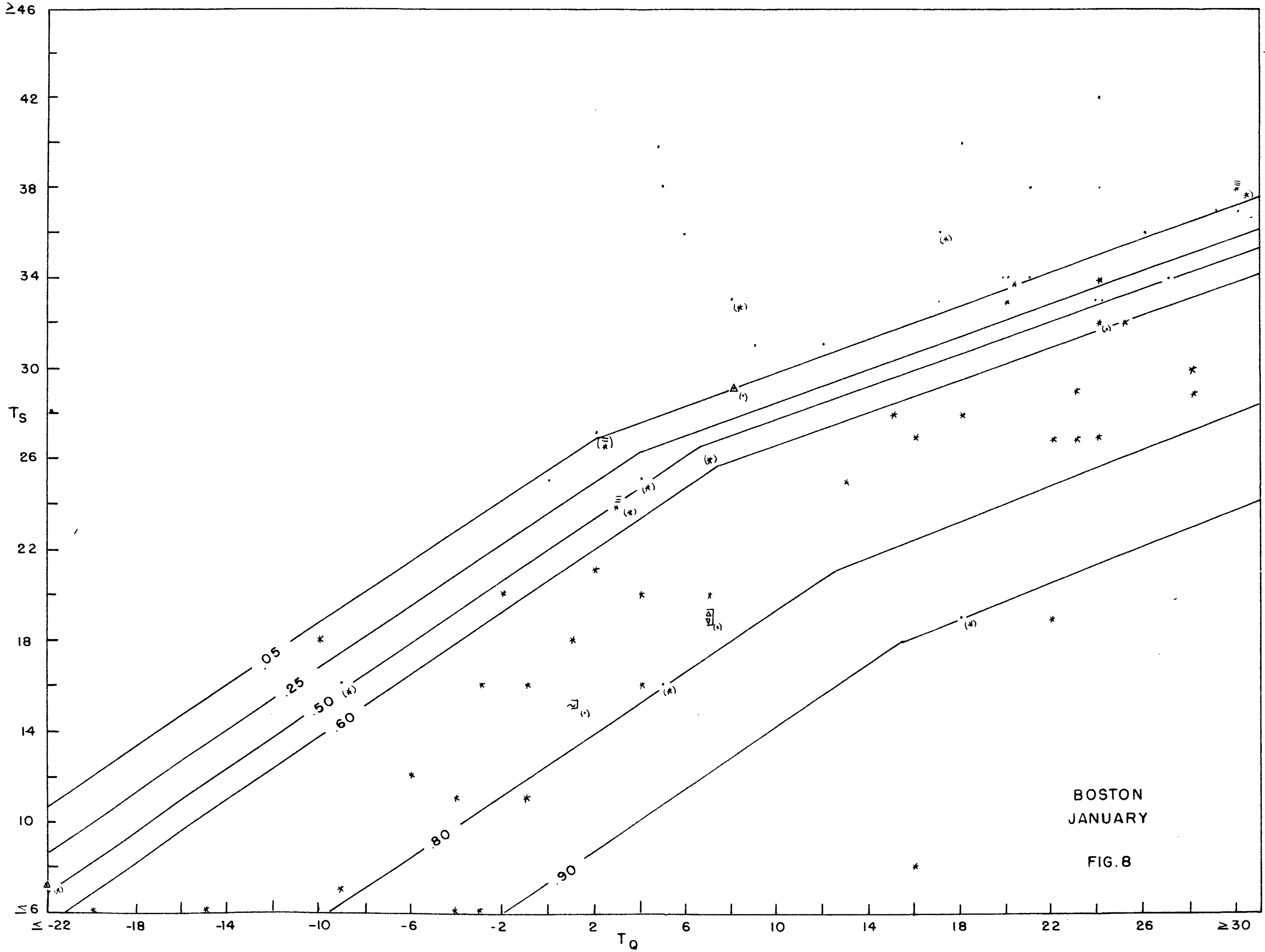
30
34
≥38
T_Q
.05
.25
.50
.60
.80
.90
BOSTON
DECEMBER

FIG. 7


≥46
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T_S
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≤6
≤ -22
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-14
-10
-6
-2
2
6
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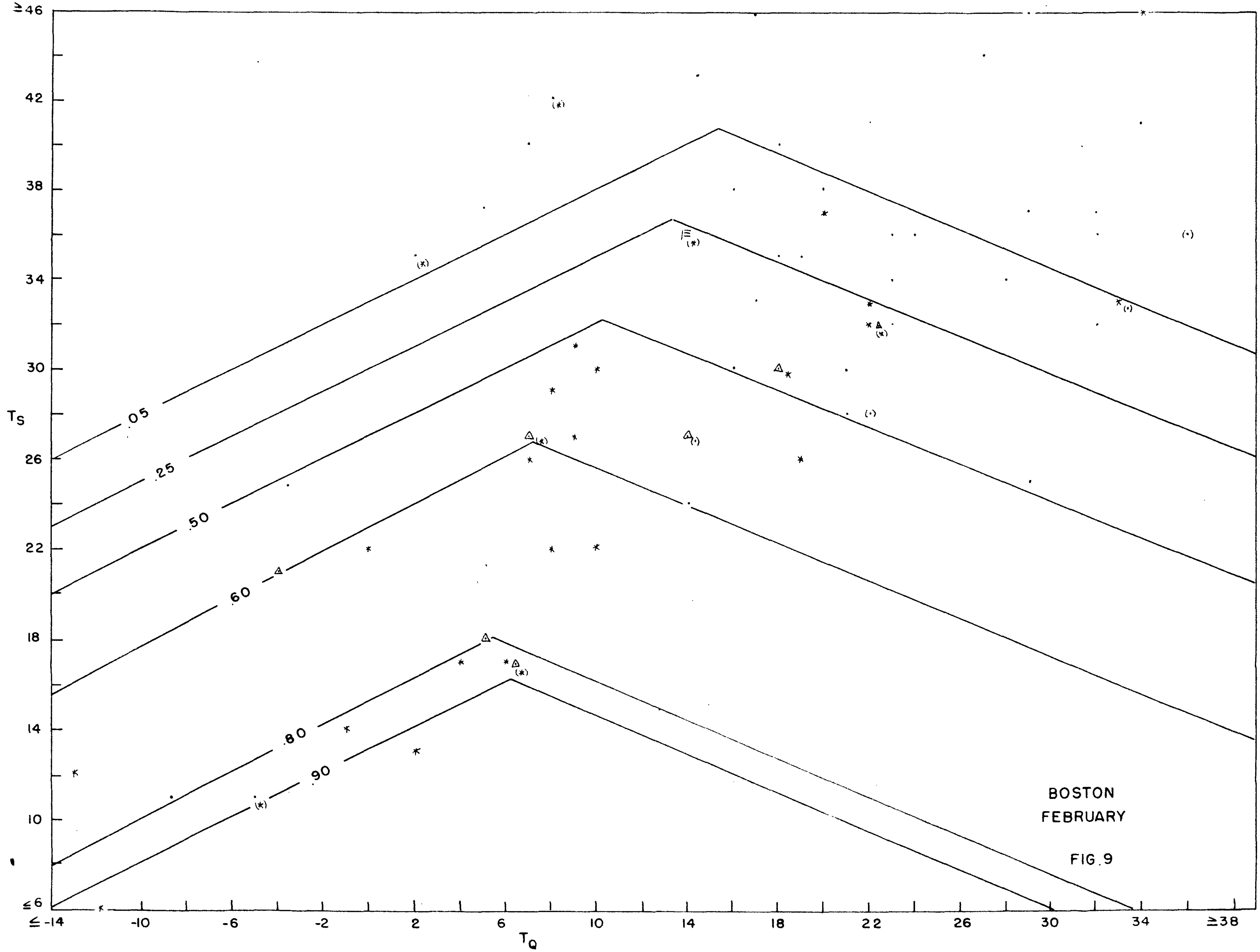
22
26
≥30
T_Q
.05
.25
.50
.60
.80
.90
BOSTON
JANUARY
FIG. 8



≥46
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≤6
T_S
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-2
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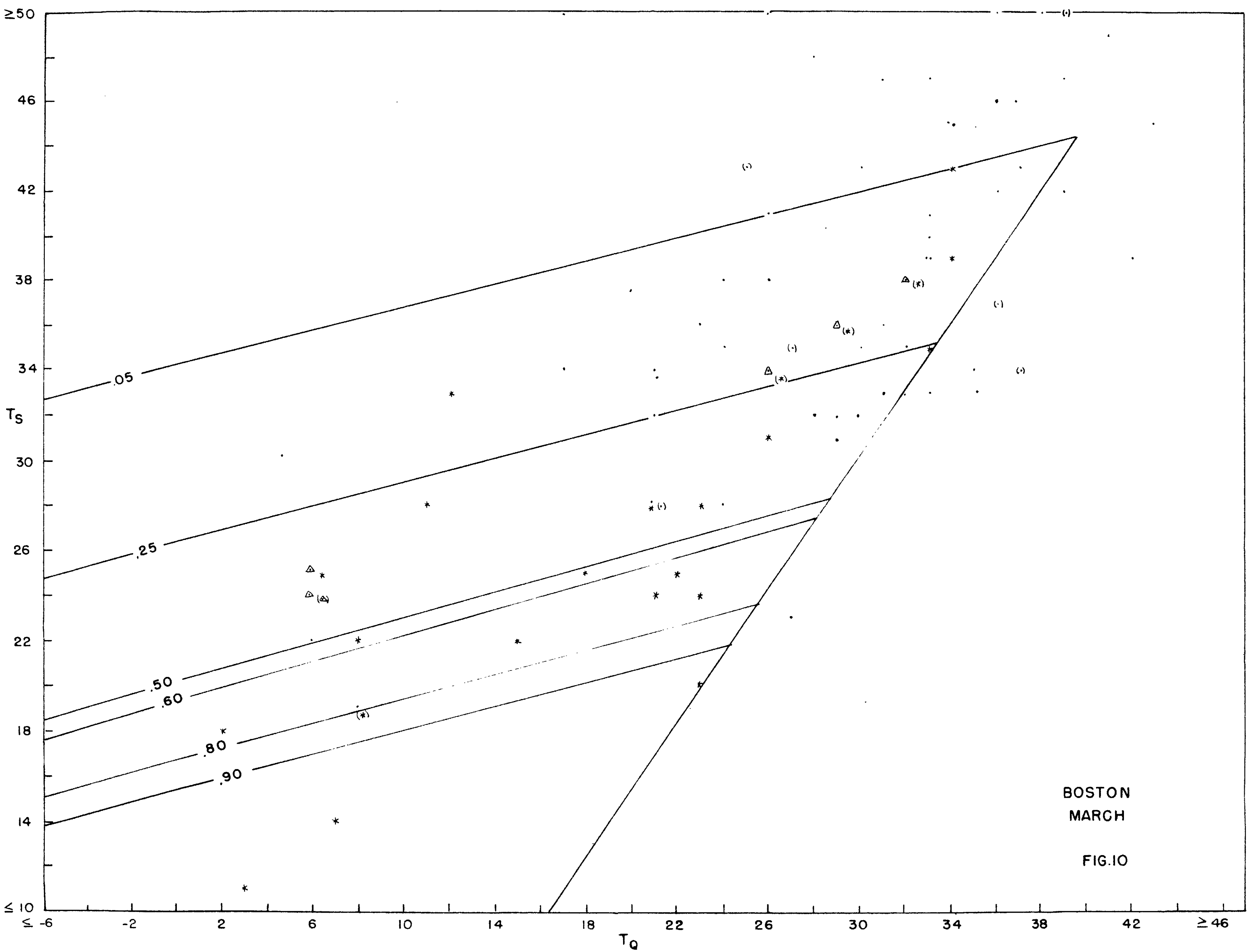
≥38
T_Q
.05
.25
.50
.60
.80
.90
BOSTON
FEBRUARY

FIG. 9


≥50
46
42
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34
30
26
22
18
14
≤ 10
T_S
≤ -6
-2
2
6
10
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26
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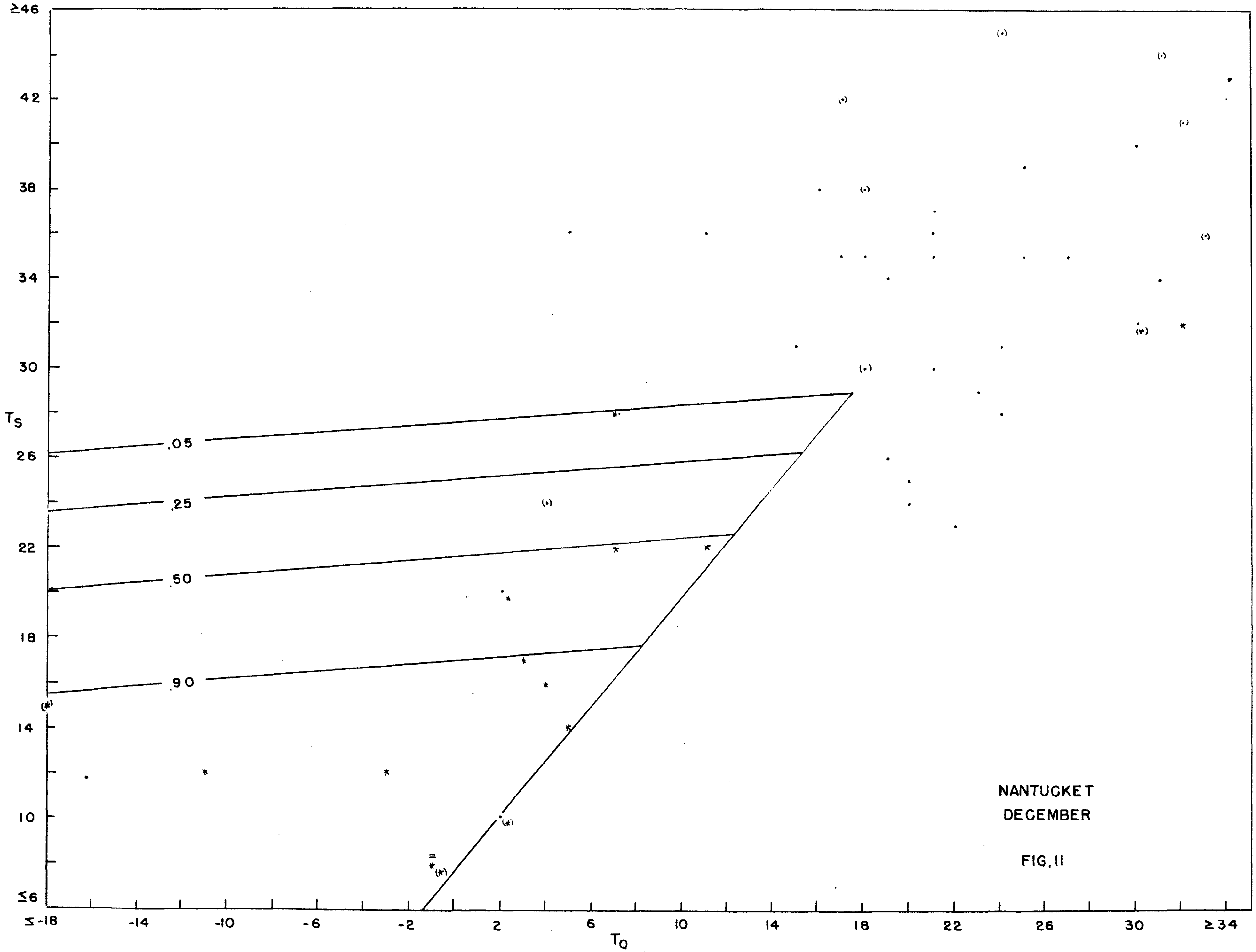
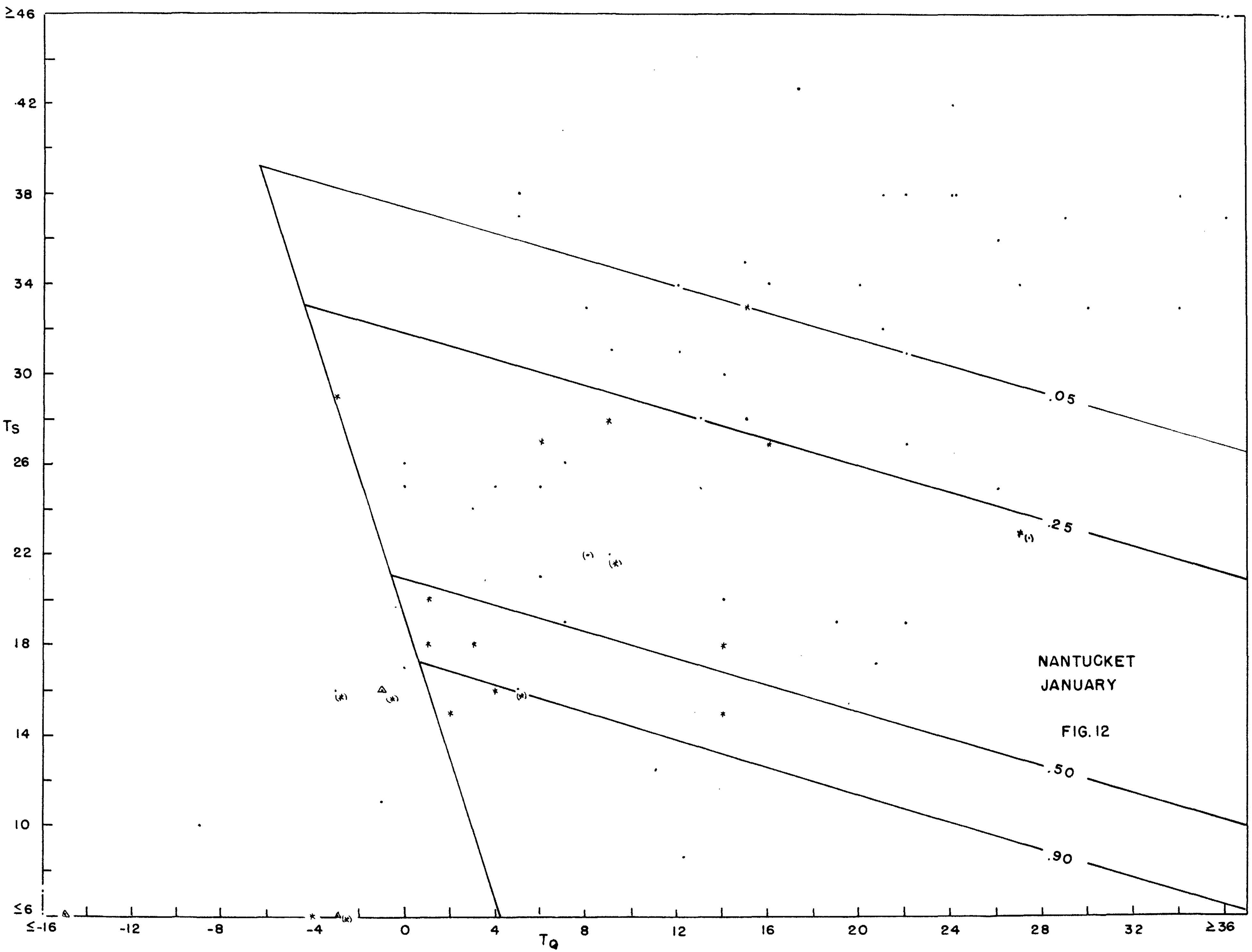
38
42
≥46
T_Q
.05
.25
.50
.60
.80
.90
BOSTON
MARCH

FIG.10



NANTUCKET
DECEMBER

FIG. 11



NANTUCKET
JANUARY
.05
.25
.50
.90
T_S
T_Q
≥46
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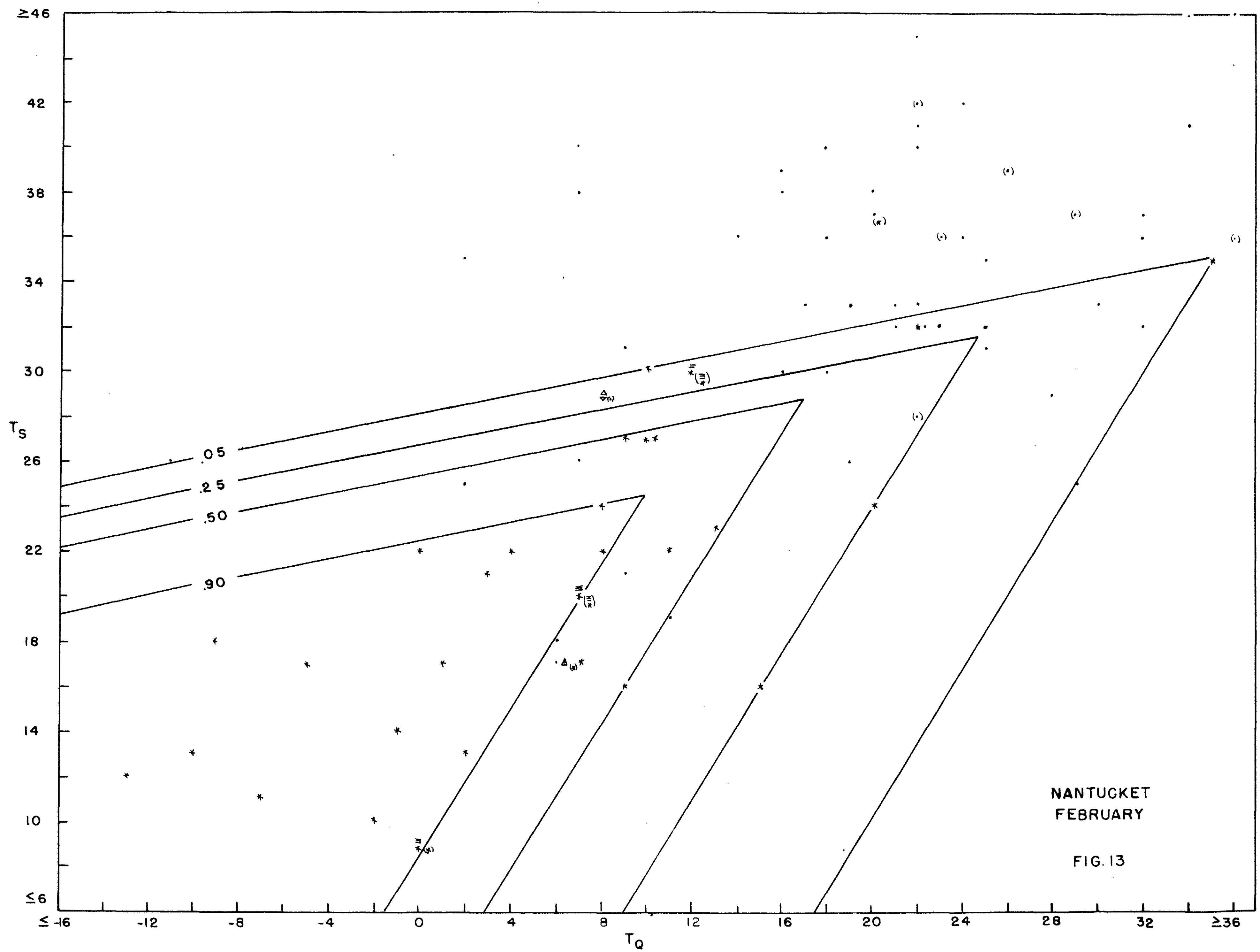
≤6
≤-16
-12
-8
-4
0
4
8
12
16
20
24
28
32
≥36

FIG. 12



≥46
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≤6
T_S
≤-16
-12
-8
-4
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≥36
T_Q
.05
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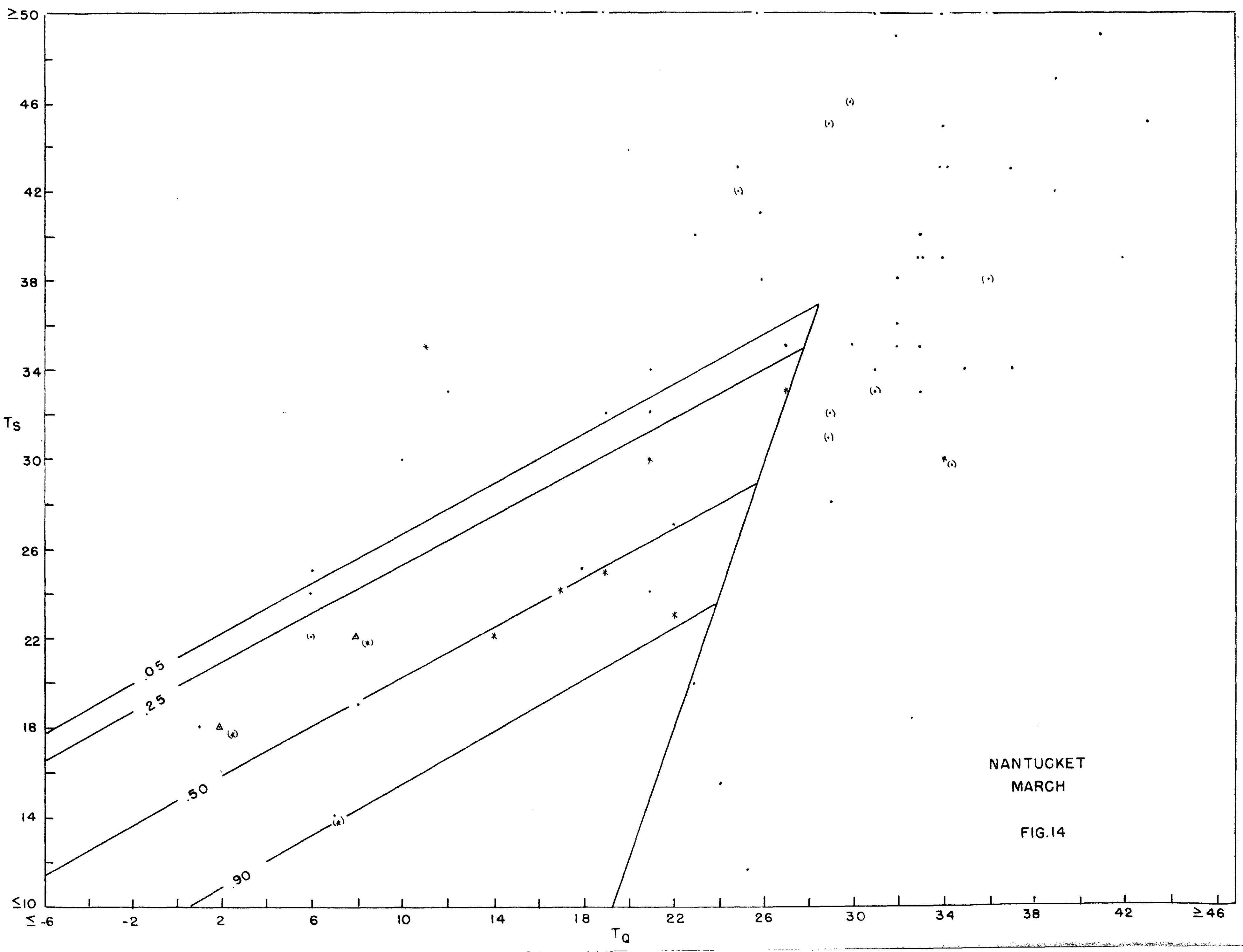
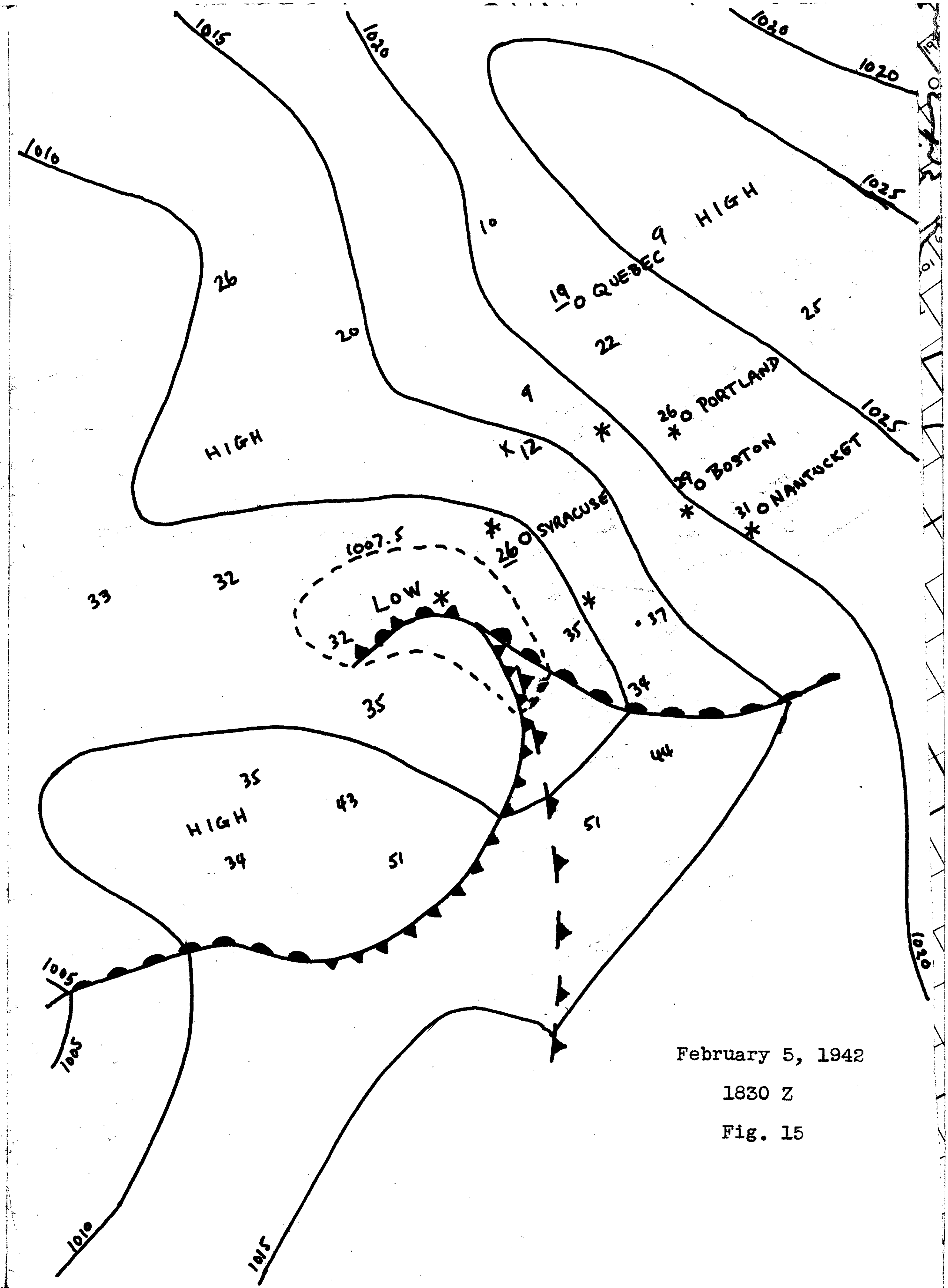
.50
.90
NANTUCKET
FEBRUARY

FIG. 13



NANTUCKET
MARCH

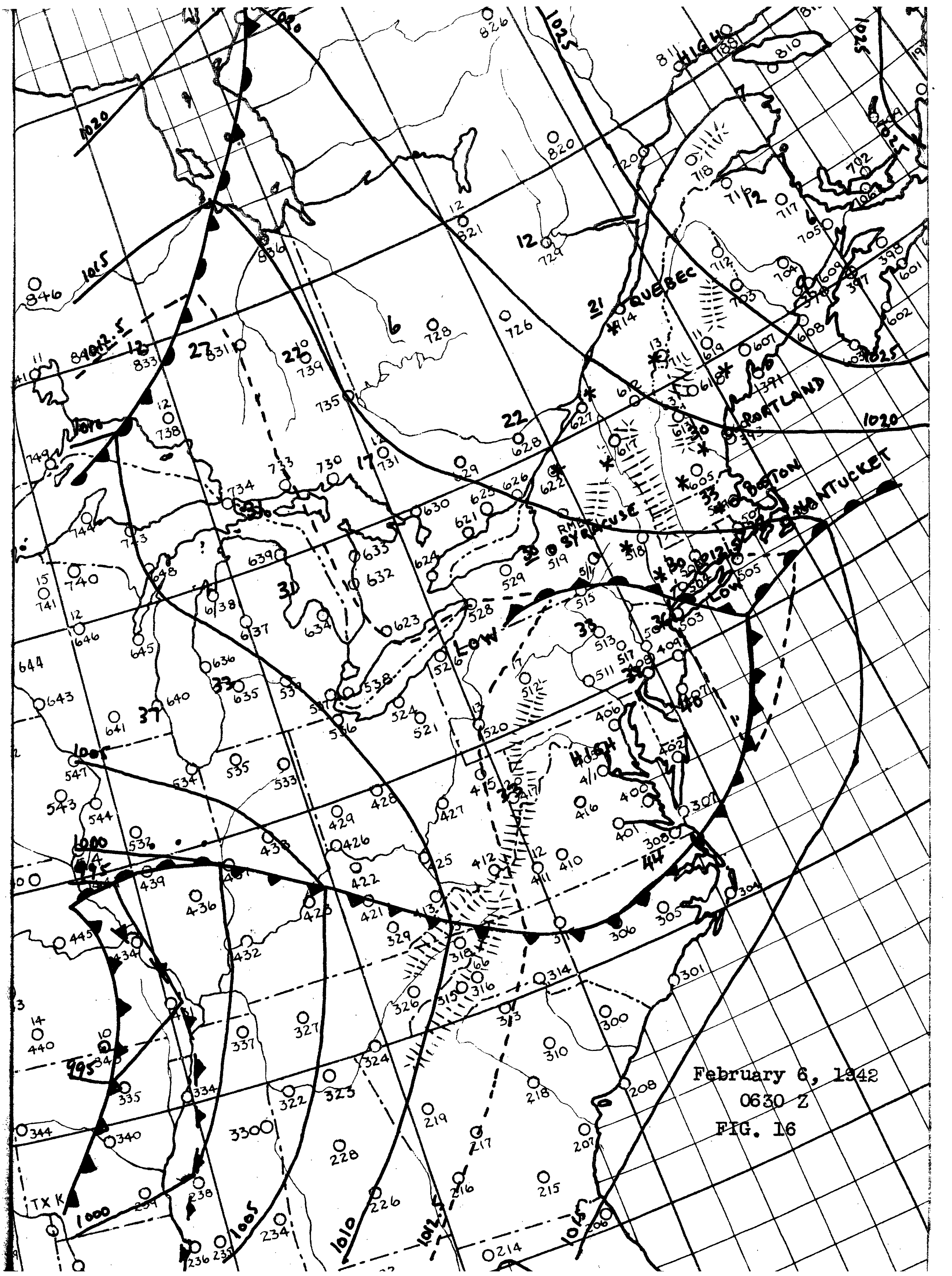
FIG. 14



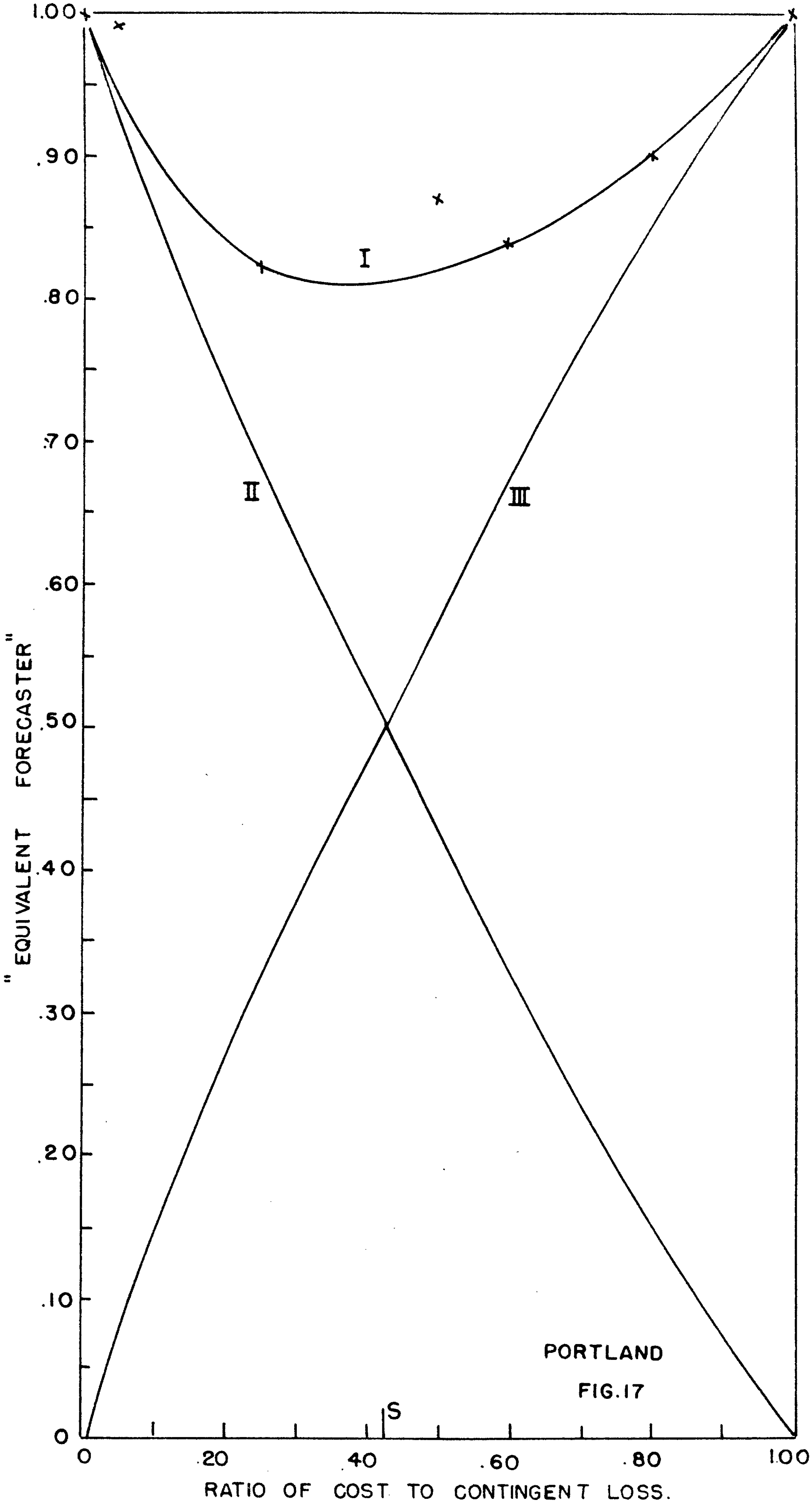
February 5, 1942

1830 Z

Fig. 15

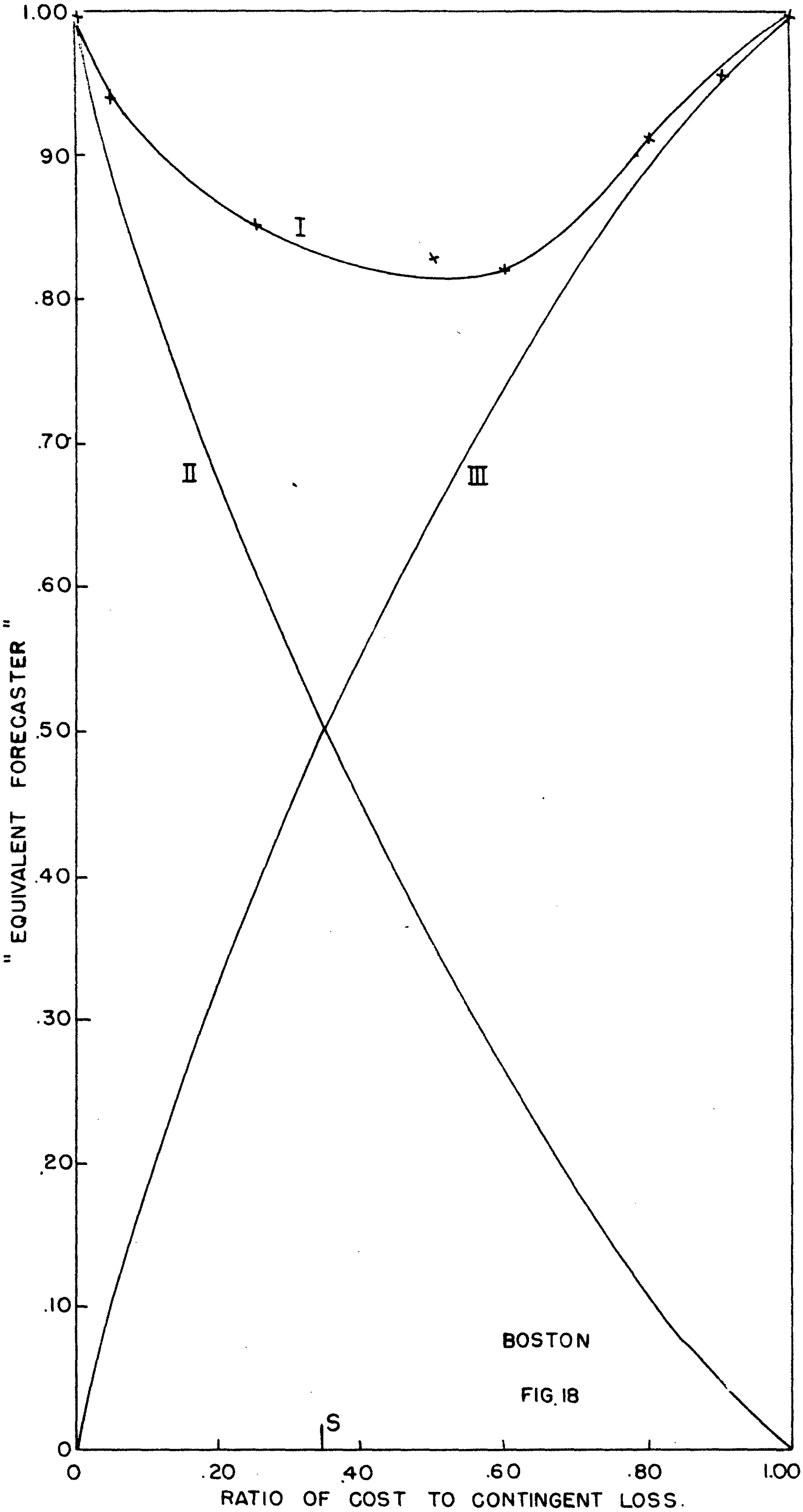


February 6, 1942
0630 Z
FIG. 16



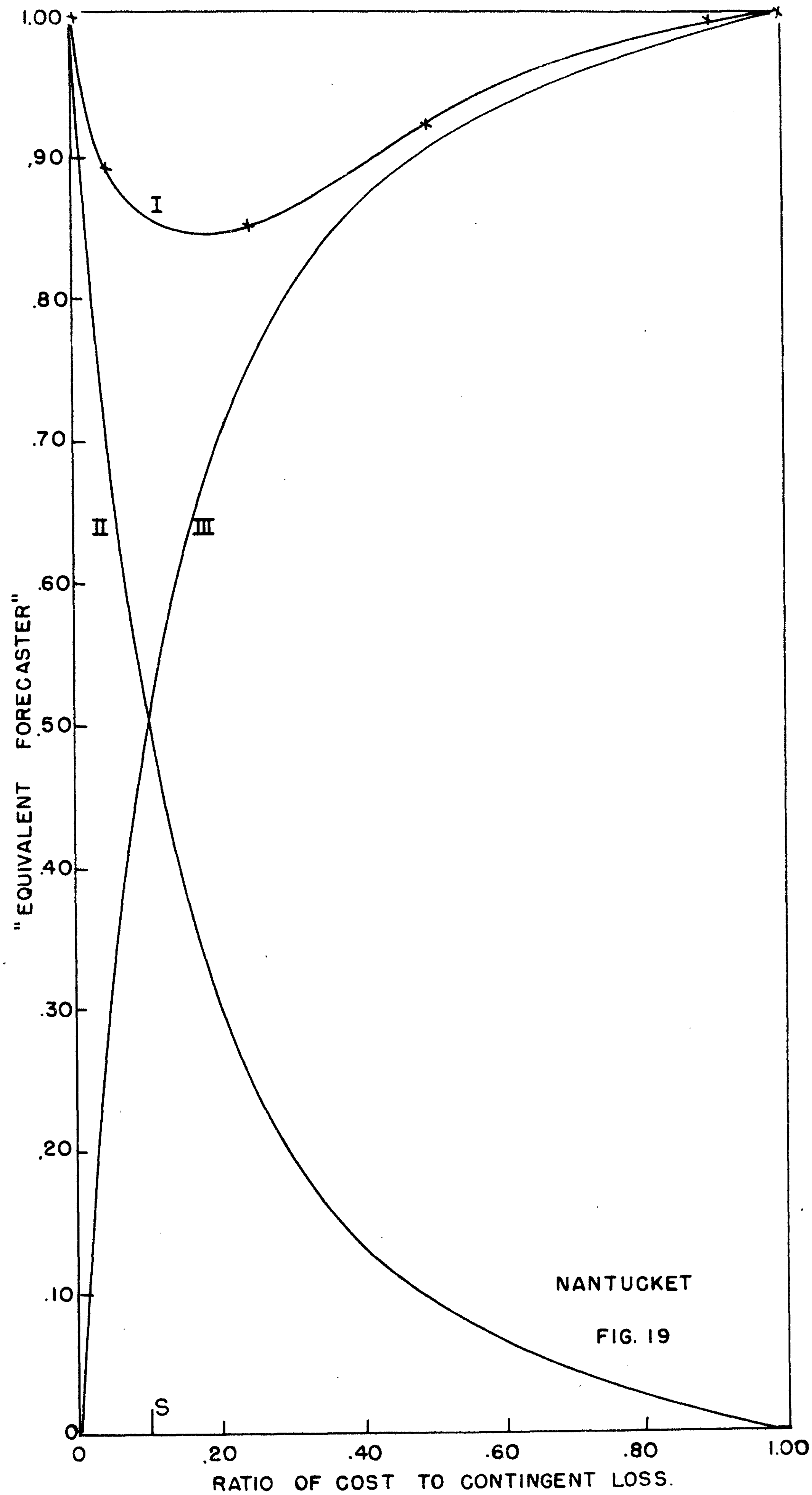
PORTLAND

FIG. 17



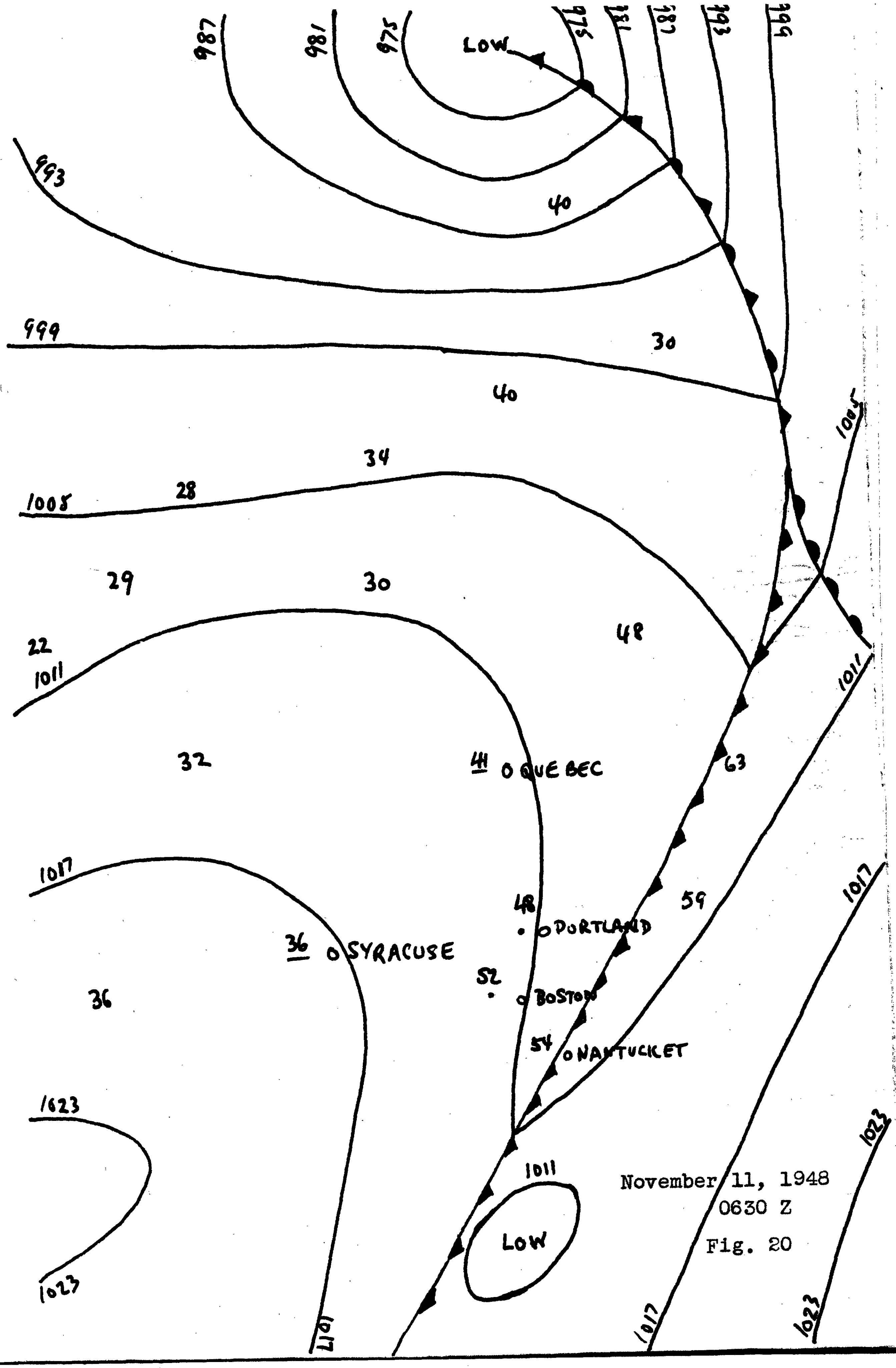
BOSTON

FIG. 18

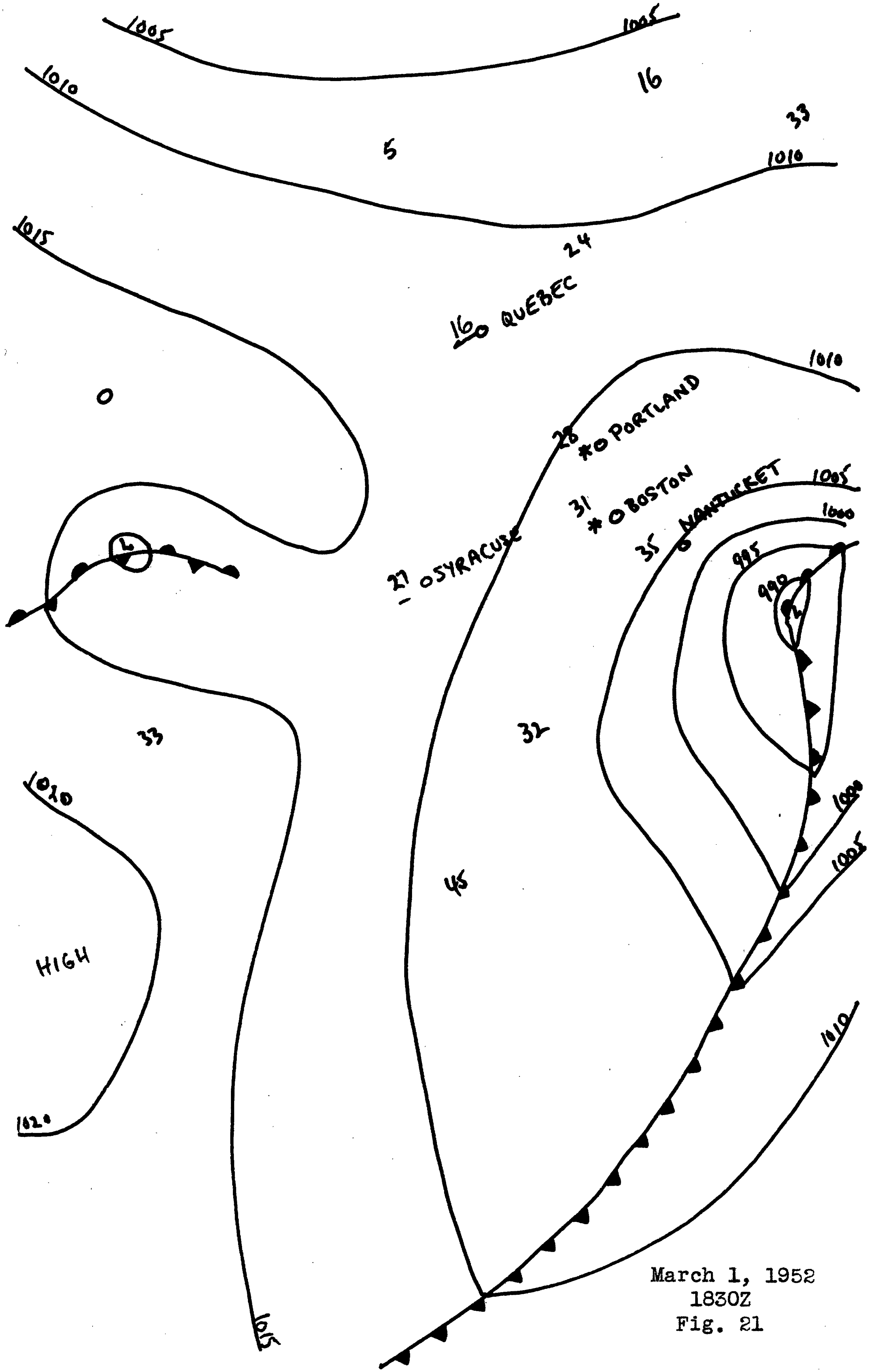


NANTUCKET

FIG. 19



November 11, 1948
0630 Z
Fig. 20



March 1, 1952
1830Z
Fig. 21