THE SUPPLY OF COAL IN THE LONG RUN:
THE CASE OF EASTERN DEEP COAL

by

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CHAPTER I: UNDERGROUND MINING COSTS
COSTS

An estimation of cost functions serves several purposes. In the first place, it allows direct examination of depletion in the coal industry. A method is devised for estimating costs based on the geological characteristics of the coal seam. This function used in conjunction with coal reserve data yields estimates of the future course of depletion. Secondly, the examination of costs provides insight into market structure, by allowing calculation of the minimum efficient mine size. Finally, comparing costs to prices offers a direct measure of industry performance.

The focus is on underground mining costs. The data on eastern strip reserves, as we see below, does not allow construction of a supply function. In the East of the United States, the cost of coal will, at the margin, equal the cost of a large new underground mine. By examining deep mining costs under a variety of assumptions about strip mining output, we are able to indicate the sensitivity of depletion in the industry as a whole to stripping output, as well as indicate a range of likely outcomes.

Previous Attempts at Cost Estimation

Previous estimates of mining costs have been based on engineering data collected by the Bureau of Mines and this effort is no exception. This data is based on an engineer's estimates for mines under assumed conditions and not the experience of actual mines. It is the engineer's best estimate of what it would cost to mine coal under the assumed conditions.

A key element in the engineer's estimate is the assumed role of productivity. The assumed output per man is the same regardless of the size of the mine. Because of the presence of fixed overhead costs, average cost then declines as output expands. Another implicit assumption is the relatively small impact of seam thickness. Productivity per unit shift

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changes from 343 to 312 tons as we move from a 72" seam to a 48" seam, an elasticity of .20. These productivity assumptions are not tested, nor even discussed, either in the Bureau studies or in subsequent work.

There has been one previous attempt to build a cost function from this data. The method used by Charles River Associates utilized the cost estimates of the Bureau of Mines, accepting the assumptions behind the estimates. The impact of physical factors on costs was determined by examining how costs differed between various mines. Thus, mines differing only in seam thickness were compared and the cost difference was attributed to the difference in thickness. A linear relation was assumed so that the difference in cost per ton was simply divided by the difference in thickness to estimate the incremental cost of a thinner seam.

The approach here is, of necessity, based on Bureau of Mines data. However, we test their assumptions of a small impact of seam thickness on costs as well as their assumption that productivity is independent of mine size. This latter assumption leads the Bureau of Mines to conclude that average costs decline up to an output of 5 million tons per year in an underground mine. This, as we see below, greatly overestimates economies of scale in deep mining.

We begin with a discussion of coal mining technology. We then estimate a cost function for mining based on this description of the production process.

Deep Mining Technology

Coal lying under heavy overburden must be mined by deep mining methods. This involves construction of entries to the seam and then mining the seam underground.

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The first distinction of importance relates to the type of opening to the seam. The simplest (and least expensive) is a drift opening. Here the coal outcrops on a hill and the opening to the seam is at the level of the coal. When the seam lies below the surface, either slopes or shafts are constructed to reach the coal. These slopes and shafts provide the access to the seam for men and supplies as well as providing a means for the mined coal to reach the surface. In the past, slopes have been used to depths of up to 400-700 feet. At greater depths shafts are used, although that is now changing and slopes are being driven further down to reach coal seams. These different mine types are illustrated in Figure 1.

The actual technique used to mine the coal is independent of the type of opening. The underground techniques can be classified broadly into cyclic and acyclic techniques. The cyclic technique involves cutting the coal, placing explosives, blasting the coal, and loading. This is called the conventional method, although large new mines today do not generally use this technique. The great increases in productivity in coal mining during the 1950s and 1960s were due to the increasing use of continuous (acyclic) mining systems.

Continuous mining involves the use of large machines that essentially rip or bore the coal from the seam and load it into cars in a continuous action. This eliminates the use of explosives and produces a greater amount of coal per shift.

The most common technique in continuous mining is called the room and pillar method. Using mining machines, haulageways and entries are carved out of the coal seam. As the process continues, pillars of coal are left to support the roof. These pillars form the "rooms." Figure 2 illustrates this method. In the next phase, retreat, the pillars are extracted and the roof is allowed to collapse behind the miners.
Principal Types of Coal Mines

Figure 1

From "Elements of Practical Coal Mining"
A Continuous Mining Unit

Figure 2
A continuous technique used widely in Europe and now being introduced in the United States is longwall mining. In this process, long panels of coal are mined by large ripping machines that move along the face of the panel. The roof is held up by hydraulic jacks that advance as the panel is mined out. This technique is not widely used and our cost functions will be based on the more prevalent room and pillar method.

The actual mining process is complicated and consists of many steps. We need a characterization of cost functions that will not be too simple so as to be unrepresentative of the mining process. It proves convenient to characterize the underground mine as a group of mining units consisting of mining machines and men that mine coal independently of other units. These units use common equipment such as haulageways, transportation systems, and ventilation systems. The total output of the mine is the sum of the outputs of these individual units. The mine itself is the sum of these units plus common equipment.

The individual units are, as mentioned above, composed of both mining machines and men. In continuous mining, the techniques used by the new, large mine, the combination of labor and capital is fixed within a narrow range. We can, without a loss of realism, assume a unit is comprised of machines and miners manning them in a constant proportion.

Physical Characteristics and Costs

The two physical variables upon which the USGS reserve classification is based are the thickness of the seam and the depth at which it lies. The Bureau of Mines, in considering strip coal reserves, considers an additional factor - the ratio of feet of overburden to feet of coal, called the overburden ratio.

A unit, for cost estimation purposes, is defined as a mining machine and two shuttle cars. Other equipment is included with total investment expenditures. Before the Safety Law of 1969, the men in a unit ranged from 7 to 9 according to the American Institute of Mining Engineers, Mining Engineering Handbook, summer 1973, pp. 12-71. Presently, the Bureau of Mines assumes 10 men per unit.
It is clear that these factors correlate with cost. In underground mining, as depth increases, holding coal thickness constant, more must be expended per unit of coal produced. Shafts must be dug, the haulage of coal and mine ventilation become more expensive. The thickness of seam is also a crucial variable. In thin seams the low height of the working area forces miners to work in a crouched position. Movement of equipment is difficult and operations are slower. Most important, though, is that a thick seam allows removal of more coal per area worked. This means quicker operations and hence, cheaper production.

One is impressed in reading a coal mining manual or reports in the trade literature by the complexity of factors in addition to depth and thickness that influence mining costs.

"Natural conditions involve roof, floor, grades, water, methane, and the height of the seam...In addition to these normal conditions, there are, in some mines, rolls in the roof or floor, and clay veins of generally short horizontal distance that intersect the coal seam. All these must be taken into account.

It is possible for an experienced engineer to examine previous conditions of the sections and the immediate area of the section and assess proper penalties. As an example, if the roof is poor, production is reduced by as much as 15% of the available face working time. If the floor is soft, fine clay and water is present, the production handicap could be as much as 15%. If a great deal of methane is being liberated, so that it is necessary to stop the equipment until the gas has been bled off, this delay could run as high as 10%. Fortunately, only a few mines in the United States have such severe conditions. The same remarks apply to all the other natural conditions."

The multiplicity of factors led a Resources for the Future study to conclude that:

"the detailed data in the U.S. Geological Survey report could not have been the basis of costing, for the

American Institute of Mining Engineers, Mining Engineering Handbook, summer 1973, pp. 12-33."
physical criteria used by the survey must be translated into cost equivalents. Yet, there is no systematic means of obtaining such equivalents; there is only the most general relationship - so general as to be useless - between seam thickness and cost, depth and cost, and so forth...a costing of coal resources requires more than the physical factors considered by the survey."  

Clearly, many factors affect costs. We must attempt, if possible, more than a qualitative listing of factors ignored in reserve estimates. No one disputes that thickness and depth are important in determining underground costs, but to respond to the RFF study we must ask, what is the variance about the expected cost given seam thickness? Unfortunately, as with everything in the coal industry, the data are very poor. We present in the following pages an indirect method for examining this variance. [In the next chapter we provide another measure of the variance and use it to develop an economic interpretation of reserves.]

To see what is involved, we specify the following function:

\[ C = \phi (W, Th, WT, R, F, G, N_i) \]

where

- \( W \) = width of seam
- \( Th \) = thickness
- \( WT \) = water conditions
- \( R \) = roof conditions
- \( F \) = floor conditions
- \( G \) = gas conditions
- \( N_i \) = all other factors
- \( C \) = cost per ton

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Further evidence of the importance of these other factors comes in a Law Review article discussing property valuation in West Virginia. The authors are particularly concerned with the valuation of coal lands. They list the following factors as determining costs of mining and therefore the value of the lands. Location, surface facilities limiting the quality of the coal, the dip of the seam, the regularity of the seam, water conditions, floor conditions...Rolla D. Campbell, Lynn C. Johnson, Ernest F. Hays, "Ad Valorem Taxation of Coal Bearing Lands in West Virginia - Assessment and Valuation - A Viewpoint of the Coal Industry," West Virginia Law Review, forthcoming.
This represents the function we would like to estimate. We in fact estimate:

\[ C = \phi (W, Th) + \varepsilon \]

where \( \varepsilon \) represents the effects of the unobserved factors. Assuming these other factors are independent of seam thickness, the more variable and important they are, the larger will be the standard error.

Cost Estimation

The above discussion suggests a method of using engineering data that allows for variation about average conditions. The major impact of the non-observable factors will be reflected in productivity data. The bad roof, etc., leads to a lower rate of productivity since it reduces the productive time per shift. The Bureau of Mines estimates assume implicitly a set of these unobserved factors. With that assumed set, they determine the necessary equipment and labor to reach a given output rate.

We first examine the prior step - arriving at a productivity figure. The effects of seam thickness and mine size on productivity are measured. Given this productivity relationship, we determine the number of units necessary for any level of production conditional upon the seam thickness. We then use engineering data to determine the relationship between mine size and the necessary common equipment and labor. The unobserved factors are reflected in the variance about the expected level of productivity per mining unit.

Productivity

As outlined above, it is the productivity relationship that drives the cost estimate. The importance of this variable is attested to by the citation above from the Mining, Engineering Handbook. Constant industry concern with declining productivity levels, as evidenced in numerous recent articles in the trade literature, further supports the central role of productivity.
There are several ways of capturing the relationship between productivity and seam thickness. A simple engineering analysis suggests productivity per unit shift, all other things being equal, should be proportional to the thickness of the seam. Robinson suggests that a machine cycle, that is the operation of mining in the seam, will cut 18 feet in length and 18 feet in width so that the cubic feet of coal mined is $18 \times 18 \times \text{seam thickness}$. He then calculates the number of cycles per shift as the total amount of productive time divided by the time per cycle.

The difficulty with this approximation is that we have no idea how variable the productivity is and therefore no idea how good the approximation is. Furthermore, it assumes that delays, difficulties, and cycle time are independent of the seam thickness, when in fact an important element of thinner seams is an inability to move men and machines as quickly. Some test of this approximation is clearly needed.

Data

To test these contentions, use is made of information on a set of deep and drift mines in Ohio, Illinois, Pennsylvania, Kentucky and West Virginia. The sample contains all mines producing more than

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7 The reports are the following:

(a) Ohio: Division of Mines Report, Department of Industrial Relations, 1973.

(b) Illinois: Department of Mines and Minerals, 1973 Annual Coal, Oil and Gas Report.


100,000 tons per year by continuous mining methods, and for which the following information was available: seam thickness, the number of mining machines, the number of shifts per day, annual output, days worked.

By limiting the sample to continuous mines, we are focusing on the cost of new mines. In a given supplying region, at the margin, the cost of a new deep mine is equated to the cost of a strip mine. The cost of these mines determines the incremental cost of coal.

The sample was limited to 100,000 tons per year (tpy) since smaller mines are likely to be on a different production function. Many were closing in 1973-1974 due to their inability to adjust to the Health and Safety Act of 1969.

The seam thickness and days worked come from the annual mining reports of the respective states. The number of mining machines comes from the *Keystone Coal Industry Manual* for 1973. In the case of Illinois, the state report furnished this data. The remaining data also come from *Keystone*. An industry convention is to allow for 20% spare machine capacity. The number of machines therefore is 1.2 times the number of working sections. Productivity per section is derived by dividing total output by the product of days worked, shifts per day, and mining machines/1.2.

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10 This is used in the Bureau of Mines' estimates. Also, see Robinson, op. cit. Of course, this is not a strict rule and represents only an approximation.
The Productivity Relationship

The productivity of a unit in isolation we write as:

\[ q = A(Th)^\gamma \varepsilon \]  \hspace{1cm} (1)

where

- \( q = \text{output per unit shift} \)
- \( Th = \text{seam thickness} \)
- \( \gamma, A = \text{constant} \)
- \( \varepsilon = \text{disturbance reflecting unobserved natural conditions} \).

We never observe \( q \) since we never observe units in isolation. Rather, we derive \( q \) by dividing the mine output by the number of unit shifts. A mine is comprised of many working units. Since these units share common equipment in haulage, etc., their productivity is not independent of each other. As mine size increases, the logistics of haulage of men and supplies become more complicated. A problem with coal haulage equipment will shut down production in several units. The larger the mine, the longer is the travel time to the working face and consequently, the less is the productive time. To capture these scale effects, we rewrite equation (1) as:

\[ Q = A(Th)^\gamma S^\beta \varepsilon \]  \hspace{1cm} (2)

where

- \( Q = \text{mine output} \)
- \( S = \text{producing sections} \)

or

\[ \frac{Q}{S} = A(Th)^\gamma S^{\beta-1} \varepsilon \]  \hspace{1cm} (3)

Taking logarithms, we have:

\[ \log \frac{Q}{S} = \log A + \gamma \log Th + (\beta - 1) \log S + \log \varepsilon \]  \hspace{1cm} (4)

We assume \( \log \varepsilon \) has expected value equal to zero.
The number of continuous mining machines, $M$, serves as a proxy for $S$. Productivity per section, $Q/S$, is calculated as described on page 11.

The results are the following:

$$\log \frac{Q}{S} = 0.429998 + 1.21975 \log \text{Th} - 0.0789645 \log M$$

s.e. (.687018) (.174630) (.106545) (5)

$t$-stat. (.625890) (6.98475) (-.74135)

$R^2 = .5301$ $F(2,45) = 25.3829$ s.e.r. = .381848

The numbers in parentheses are the standard error and the $t$-statistic respectively. The coefficient of $\log \text{Th}$ indicates a greater than proportional effect of seam thickness on productivity. There is also weak evidence of diminishing returns to scale for new mines.

In our sample, we have no observations on the amount of haulage equipment nor on the number of shafts in a given mine. Engineering descriptions suggest that a set of shafts will service from 7-9 producing sections. For larger mines, more shafts will be provided. This has an effect on productivity. We expect to observe output per section declining with the number of sections as congestion effects take their toll. This congestion though reaches a limit when new shafts are sunk to service the next group of producing sections. In other words, we expect to see duplication of units. The effect on observed productivity will be to stem the productivity decline, as measured by $B$. To capture this effect, we introduce a dummy variable for mines with more than 7 producing sections. The equation to be estimated thus is:

$$\log \frac{Q}{S} = \log A + \log \text{Th} + (B-1) \log S + d \log S + \log \epsilon$$

where $d$ is a variable whose value is zero if $S \leq 7$, and $d=1$ if $S > 7$.

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11 See description of Wabash mine of Amax Coal. Coal Age, September 1974, p. 102. Here, a new set of shafts services each set of 9 producing sections.
The results are:

\[
\log \frac{Q}{S} = 0.558208 + 1.25429 \log Th - 0.258036 \log M
\]

\[
s.e. (.689181) \quad (.175405) \quad (.174441)
\]

\[
t \quad (.809959) \quad (7.15084) \quad (-1.47922)
\]

\[+ 0.10144(d)(\log M)
\]

\[
s.e. (.0785854)
\]

\[
t \quad (1.29087)
\]

\[R^2 = .5473
\]

\[s.e.r = .379051
\]

\[F(3,44) = 17.7280
\]

These results support the contention that as the size of the mine increases, productivity declines. After 7 sections this decline is reduced by the addition of more capital, although this capital is not observed by our data.

An examination of the residuals reveals a pattern of heteroscedasticity. It is reasonable to expect the disturbance term to be heteroscedastic. A breakdown in a small mine will, in many instances, idle the entire mine. Productivity per section will be reduced to zero. A larger mine with alternative shafts and more producing sections will, in most instances, be able to continue production in a portion of the mine. The average productivity in the large mine will decline, but not as drastically as in the small mine, since the decline is spread over a larger number of units. The variance is therefore inversely related to the size of mine. We assume the variance of (loge) is inversely proportional to M, the number of mining machines:

\[V(\log e) = \frac{\sigma^2}{M} \tag{8}\]

We weight all observations by \(\sqrt{M}\) so that:

\[V(\sqrt{M} \log e) = E(\sqrt{M} \log e)^2 = \sigma^2 \tag{9}\]
The results after multiplying each observation by the weight $\sqrt{M}$ are as follows:

$$\sqrt{M} \log \frac{Q}{S} = -0.00618340 \sqrt{M} + 1.42174 (\log Th) \sqrt{M}$$

s. error (0.664717) (0.152420)
t. statistic (-0.0093023) (9.32743)

$$-0.332498 (\log M) \sqrt{M} + 0.126954 (\log M)(d)(\sqrt{M}) \quad (10)$$

s. error (0.177801) (0.068666)
t. statistic (-1.87005) (1.84885)

$$F(3,44) = 344.410$$

s.e.r. = .909540

Our expectations are confirmed. Productivity per section declines up to 7 sections. After 7 sections, the rate of decline diminishes. We cannot reject the hypothesis of constant returns after 7 sections at a 95% level of significance.

The effect of seam thickness is again seen to be important. We can reject the hypothesis that $\gamma = 0$. We can also reject the hypothesis that $\gamma = 1$, or the effect of seam thickness is proportional, at a .999 level of confidence.

Another interesting result is the estimate of the variance. The variance reflects the effects of unobservable natural conditions. We have hypothesized that the variance is inversely proportional to mine size. Weighting by mine size, we estimate $\sigma^2$. This is the variance of $\log e$ when $M = 1$. This is the variance of unit in isolation and reflects natural conditions other than seam thickness, as well as observational error and differences in management.
We are interested in the variance of $c$, not the variance of $\log c$, since it is the former that appears directly in the productivity equation:

$$V(c) = e^{\frac{2\mu + \sigma^2}{M}} \left( e^{\frac{\sigma^2}{M}} - 1 \right)$$

(12)

In this case $\mu$, the mean of the distribution of $\log c$ is 0 by assumption. For the case of a single unit,

$$V(c) = e^{\sigma^2} \left( e^{\sigma^2} - 1 \right)$$

(13)

The estimate of $\sigma$, from equation (1), is .909540. $V(c)$ for a single unit is 2.94. This represents a very substantial dispersion. Part of this is due to natural conditions. Part must also be due to management differences and observational error. In the next section we use data that minimizes the effect of these factors in order to see how much of this dispersion can be attributed to natural conditions. Unfortunately, this data is old and the absolute level of productivity has surely changed. Nevertheless, the data do offer qualitative confirmation of this analysis, and suggests that natural conditions other than thickness are important.

The number of sections is related to $M$ by $S = (M/1.2)$. Therefore, the productivity equation is:

$$\frac{Q}{S} = .93 \ Th^{1.42174} S^{-.332498} e \quad \text{if } S \leq 7$$

$$= .95 \ Th^{1.42174} S^{-.205544} e \quad \text{if } S > 7$$

(11)

Influence of Mine Depth

Among the factors leading to the dispersion in productivity is the depth of the mine. The depth would affect productivity by increasing worker travel time to the producing face and thus reducing productive time. To test this we use 20-year-old data on mines that include mine depth.
This data was collected through detailed observation of the individual mines by Bureau of Mines personnel. The productivity relationship should have changed since 1954. However, if depth is important in productivity, it would have been important then. The estimated equation is the same as (6), except that $S$ is observed directly and depth has been added as an explanatory variable.

$$\sqrt{S} \log \frac{Q}{S} = 2.27528 \sqrt{S} + .999683 \log(Th) \sqrt{S} - .360201 \log(S) \sqrt{S}$$

s. error (1.09747) (.212636) (.271249)
t. statistic (2.07321) (4.70138) (-1.32794)

$$+ .218455 \log(S) d (\sqrt{S}) - .110934 \log(DTh) \sqrt{S}$$

s. error (.130035) (.165107)
t. statistic (1.67998) (-.671893)

$R^2 = .9798$

S.E.R = .830768

where $DTh =$ depth of mine.

The results indicate, if anything, a negative effect of depth on productivity. However, the high standard error of the estimate means that the hypothesis that depth is neutral cannot be rejected.

The entire variability is not due to other natural conditions. There will be variability due to errors in the measurement and reporting of productivity per section, differences in management, etc. In this early data, the measurement and reporting errors were much smaller, since the data came from a detailed survey of the 22 mines. It is interesting that the estimates of $\sigma$, the standard error of regressions for both the early and later data are quite close. This indicates that the contribution of measurement errors to the variance is small. The earlier data are also free of the wildcat strikes now affecting the industry.

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12 This data is from the U.S. Bureau of Mines, Information Circular 7696, September 1954. This data is old, but has a use. This data was collected under survey conditions, and errors in observation are likely to be much smaller.
There is the possibility of downward bias in the estimate of the coefficient of thickness. This comes about because in reality we do not have a random sample. The error term reflects, in part, the impact of poor floor, poor roof conditions, etc. If these were distributed randomly in our sample there would be no problem. However, these factors are taken into account in opening mines, so that the thinner seams will be compensated for by better conditions otherwise. At any given level of cost we would expect to see other conditions deteriorating while thickness increases. This bias is mitigated by the fact that these mines are in various regions, were opened at different periods, and represent different coal qualities. All this means that the inverse relation between thickness and other factors is swamped by these developments. For example, at a given level of sulfur content we would expect the negative correlation to hold. When we allow for changes in sulfur, we can observe simultaneously thinner seams and worse conditions.

It is interesting to note that the cost estimates of the Bureau of Mines that lie behind much of the recent Project Independence Blueprint assume a much smaller effect of seam thickness on productivity. This has the effect of underestimating the depletion effect of moving to thinner seams. They also assume a purely deterministic relation between thickness and cost, the implications of which are discussed in the next chapter.

Using eq (11) we can determine the number of units necessary to maintain a given rate of annual production in a given seam thickness.

\[ s = \frac{\check{Q}}{(\check{Q} / S) \times 3 \times 245} \]  

(15)

where \( \check{Q} \) = Annual Output.
We assume a work year of 245 days, three working shifts per day, and solve (15) for $S$ by substituting equation (11) in equation (15):

$$S = \left( \frac{Q}{(735)(0.93)h^{1.42174} \varepsilon} \right)^{1.498} \quad \text{if } S \leq 7$$

(16a)

$$S = \left( \frac{Q}{(735)(0.95)h^{1.42174} \varepsilon} \right)^{1.259} \quad \text{if } S > 7$$

(16b)

**Equipment Expenditures**

Once the number of productive units is known, the necessary auxiliary or common equipment and expenditures can be determined. This is the equipment that provides ventilation, capacity to haul the coal to the surface, and provides transport for men and machines. Since the mine is a collection of producing units, we expect this common equipment to be a function of the number of units. The number of units determines the extent of the workings underground and should therefore determine the need for haulage and ventilating equipment. Included also will be support material such as rescue equipment that will closely correlate with the number of producing units.

We test this hypothesis with the use of the engineering estimates of capita expenditures on hypothetical mines. The initial capital expenditure on other than face equipment is estimated as a function of $S$, the number of working sections. The results are as follows:
\[ I = 3,316,340 + 1,514,080 S \]

s. error \( (754,421) \) \( (72,371) \) \( (17) \)

t. statistic \( (4.39587) \) \( (20.92110) \)

\[ R^2 = .98468 \quad F(1/4) = 437.692 \quad S.E.R. = 69200 \]

Even with the small number of observations, the results yield a good predictive tool. In effect, this aggregates a series of engineering rules used either implicitly or explicitly by the engineers producing the initial cost estimates. It is interesting that engineers often claim that these estimates of required material were done on a case by case basis. However, the high \( R^2 \) indicates an implicit relationship used by them.

Face equipment includes a continuous mining machine and two shuttle cars. Present costs would be $360,000. Deflating by the mining machine price index to get this into 1973 dollars yields $258,387. There is another class of initial capital expenditures. These are not direct expenditures for equipment, but rather include engineering, overhead, and various small construction tasks. The Bureau of Mines deals with these as fixed percentages of initial direct capital expenditure. This is also the procedure of the Coal Task Force of the National Petroleum Council. There is one further source of such estimates, a recent article on developing a new deep mine in Appalachia. Clearly, these expenses should increase with the size of the project.

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13 The wholesale price index for continuous mining machines jumped from 118.9 in 1973 to 160.8 in January 1975. Similarly, the index for shuttle cars jumped to 165.2 from 111.9. The present price is from correspondence with mining machinery company executives. The prices reflect list price and 10% allowance for optional expenditure.


Unfortunately, we have no data to check the assumption of proportionality that lies behind the Bureau of Mines and NPC estimates. We adopt the proportionality factor of the NPC which represents the mid-point of the range, and is 14.4%.\(^\text{16}\)

There is one final class of capital expenditures. Over the life of the mine, new investment will be necessary to replace worn out machinery. We use the machinery lifetimes estimated by the Bureau of Mines in their estimates, and calculate the present discounted value of the spending stream. Then, following the above procedures, we regress the present discounted value of this stream, PDVI, on the number of sections.

The resulting equation is:

\[
\text{PDVI} = 831,764 + 364,876 S \\
\text{S.E.} \quad (274,250) \quad (21528.2) \\
\text{t-Stat.} \quad (3.03286) \quad (16.9487) \\
\] (18)

\[ R^2 = .9829 \]
\[ SE_R = .320556 \]

Operating Supplies

Annual operating supplies should also relate to the number of sections. Operating supplies consist of roof bolts, maintenance of machines, cables, etc., all of which will vary with the number of sections.\(^\text{17}\)

\(^{16}\) In this estimation we assume expenditures occur in the initial year. The range was 10.6 to 18.4 and includes engineering contingency and other indirect expenditures.

\(^{17}\) By assuming a small depletion effect, the Bureau of Mines gets constant operating costs per ton. Since operating costs vary with the number of sections (the constant term is insignificant) and output per section is roughly constant, constant costs per ton result. We expect operating costs to be proportional to the number of sections. However, we allow for changing productivities as thickness changes.
We therefore regress annual expenditures on operating supplies, OC, on the number of sections:

\[ OC = -181,035 + 361,221 S \]

\[ \text{s. error} \ ( -221,708) \ (17,403.6) \]
\[ \text{t. statistic} \ (-.816547) \ (20.7554) \]

\[ R^2 = .9885 \quad F(1/5) = 430.791 \quad \text{S.E.R.} = 259142 \]  

**Labor Costs**

Labor costs are calculated by the Bureau of Mines, based on the union wage agreement. In addition, 35% is added to account for overhead (fringe benefits, etc.). Calculating annual labor costs, LC, as a function of sections, and allowing for the 35% yields:

\[ LC = [377,542 + 452,081(S)] 1.35 \]

\[ \text{s. error} \ (63798.6) \ (5008.08) \]
\[ \text{t. statistic} \ (5.71772) \ (90.2704) \]

\[ R^2 = .9994 \quad F(1/5) = 8148.81 \quad \text{S.E.R.} = 74,520.7 \]

**Other Costs**

Allowance must be made for the union welfare charge per ton ($ .75 in 1973, $.80 in 1974), as well as indirect costs. The Bureau of Mines places indirect operating costs at 15% of total operating costs. These percentage items are troubling, but at present there is no way to be more accurate. Working capital, funds necessary to begin operation, is taken by the Bureau of Mines at 25% of annual labor and operating costs.

The Bureau of Mines estimates exclude the cost of cleaning the coal and loading it into unit-trains. Capital expenditure for loading equipment can be treated as an overhead expenditure. These expenditures are invariant over a wide range of output. The cost of a unit-train facility that can handle an annual output of up to 5 million tons per year
is about $750,000. Annual labor costs are $57,160. These costs can be added to equations 17 and 20 respectively.

Cleaning costs should not be added to the mining cost. A cleaning plant can, and does often, service several mines. There are several sources of cleaning cost estimates. A recent study puts the capital cost of a cleaning plant with a 3 mn tpy capacity at $10,600,000. This probably includes the loading facilities, so the net addition to capital expenditures is $9,850,000. Annual labor cost is $78,180. This yields a cleaning cost of 54.6¢ per ton.

Normally, adjustment must be made for coal lost in cleaning. In other words, the raw coal produced in mining is reduced by 25% in the cleaning process. However, the productivity estimates of this chapter are based on the reported production, which is clean coal. Therefore, no further adjustment is necessary.

An efficient cleaning plant can service several mines of minimum efficient size. The average distance from mine to plant is likely to increase if several mines supply one plant. This additional haulage cost implies that minimum efficient scale for a mine could be larger than estimated here. However, cleaning is a small proportion of total cost. Furthermore, the incremental haulage costs are not likely to be large.

In summary, we have conceptualized the underground drift mine as a conglomerate of individual producing units. We have related the productivity of these units to seam thickness, and in turn related other capital expenditures to the number of underground units. While the data are scarce, the results here suggest that this is the proper way to model the mine. It also indicates that the rewards, in terms of producing more confident results, to a survey of mines that produced more reliable data would be great.

---

18 These costs are from U.S. Bureau of Mines, IC 8535, Cost Analyses of Model Mines for Strip Mining of Coal in the U.S., 1972.

19 TRW, Inc., Coal Program Support Report (prepared for the Federal Energy Administration), June 28, 1974, Figure 3-5A.
Shaft and Slope Mines

So far we have dealt with only drift mines. This is the least expensive of the underground methods. We must, however, allow for the cost of deeper mines. The productivity analysis shows it is realistic to conceive of the underground mine as simply a drift mine with a set of shafts/slopes that provide access to the seam. A unit of minimum efficient scale will have one set of entries. Therefore, we must add the capital cost of constructing these passages as well as an increment to operating costs.

Often mines have more than one set of shafts. However, they are in reality more than one mine with separate access to the seam for men and supplies as well as separate management. The underground portion of the main line haulage system has been included in the drift mine estimations and any economies of scale in that operation show up in the earlier estimation.

For our purposes it is enough to calculate the incremental costs for a mine 1000 feet deep. We need only do this since the reserve data examined in the next chapter distinguish only between coal lying deeper and shallower than this depth. As the next chapter indicates, this allows us to establish limits on the expected increase in coal costs.

The cost of each shaft and slope will be independent of the size of mine. The main determinant of the shaft and slope cost

---

20 An example of this is the Wabash mine of Amax Coal. "We're going to run it essentially as two independent mining operations, each using different portals, but sharing common track and belt haulage systems, reports R.E. Samples, Senior Vice President...", Coal Age, September 1974, p. 102.
is the depth of the seam. The costs of slopes and shafts we take from the experience of a recently developed West Virginia deep mine. This mine is 796 feet deep and involved construction of both a shaft and slope. The cost of the entries, including the slope conveyor and shaft hoist was $6,217,500. The incremental cost to 1000 feet will be very small, since fixed cost is large and average cost per cu-foot of shaft declines rapidly with cu-ft.

To this we must add the greater operating costs that will be incurred. Greater depth will increase the power costs necessary for hauling coal to the surface. It is also likely to increase ventilation power costs as the air must travel a greater distance against greater resistance. These costs, however, are trivial. The additional cost to haul a ton of coal up 1000 feet is less than 2¢ per ton. The additional ventilation cost is $5,540 per year, again a trivial amount.

---


22 To haul a ton of coal up 1,000 feet requires 1.3276 kwhr. If the efficiency of the engine is 80%, the effective power needed is 1.66 kwhr. At 1¢ per kwhr this is a trivial amount. Ventilation costs increase according to the following formula:

$$ HP = \frac{KOV^3(1)}{33,000} $$

where HP = increase in horsepower necessary
O = area of shaft (24)
V = velocity of air (774 cu. ft. per minute)
I = incremental length of airway (2,000 feet)
K = coefficient of friction ($2 \times 10^{-8}$)

This yields an additional power cost of $5,540 per year, again a trivial amount.
Depletion and Costs in Coal Mining

Adding all these costs produces an expression for total cost as a function of the number of producing sections.

The present value of the entire investment for a drift mine is given by summing equations 17 and 18, adding the cost of face equipment (p. 146), loading equipment (p. 149), working capital (p. 148) and allowing for the 14.4% overhead expenditure (pp. 146-147):

Drift mine: Total Investment = 5,578,143 + 2,671,893 (S) (21)
Deep mine: Total Investment = 11,797,643 + 2,671,893 (S) (22)

This can be converted to a per annual ton capital charge in the following manner. Depletion laws allow a deduction of 50% of gross profits or 10% of price, whichever is less. We allow for 50% corporate income tax and a 12% rate of return after tax. We assume the mine is equity financed as is usually the case, and depreciation is by straight-line method. Depreciation is therefore $\frac{1}{20}$ of the present discounted value of total investment.\(^{23}\)

\(^{23}\) A 12% after tax rate of return is used by the Bureau of Mines in their calculation.

A twenty year life of mine is assumed. Straight-line depreciation is used, depreciating $\frac{1}{20}$ of the present discounted value of the entire mine investment over the life of the mine.

The assumption of a twenty-year life is not trivial. For a shorter life, a good deal of the capital equipment investment in slopes and shafts would have a useful life longer than the life of the mine. Thus, capital costs rise as the life of mine declines. This is most acute in a deep mine. Calculations based on Bureau of Mines engineering data indicate that the average cost for a drift mine with 10-years life would increase 50¢ per ton for a 1,030,000 ton per year mine. The data comes from IC-8641, Basic Estimated Capital Investment and Operating Costs for Underground Bituminous Coal Mines, 1974. Rough calculation indicates that a $3.5 million would have to be amortized in 10 years rather than 20. Adding the $6.2 million for shafts and slopes, yields approximately $10 million. A life longer than 20 years would require large costs in maintaining the shafts, etc. The above cost calculations assume therefore that the reserve stock is large enough so that new mines have 20 years assigned reserves.

(footnote continued over)
The annual capital charge per ton is given by the following formula:

\[
\text{Return on Equity} = \text{Annual Capital Charge} - \text{Tax}
\]
\[
\text{Tax} = \frac{1}{2} (\text{Annual Charge} - \text{Depreciation} - \text{Depletion})
\]
\[
\text{Depletion} = \frac{1}{2} (\text{Annual Charge} - \text{Depreciation})
\]
\[
\text{Return on Equity} = \text{Annual Charge} - \left[ \left( \text{Annual Charge} - \text{Depreciation} \right) - \frac{1}{2} (\text{Annual Charge} - \text{Depreciation}) \right]
\]
\[
= \frac{3}{4} \text{Annual Charge} + \frac{1}{4} \text{Depreciation}
\]
\[
\text{Annual Charge} = \frac{4}{3} \text{Return on Equity} - \frac{1}{3} \text{Depreciation} \quad (23)
\]

Total capital cost is:

Drift: Total annual capital charge = $888,578 + 425,622 \quad (S) \quad (24)

Deep: Total annual capital charge = 1,879,321 + 425,622 \quad (S)

The Bureau of Mines has estimated expenditures for a working year of 220 days. An examination of our sample indicates an average working year of 236 days in 1973. The industry in that year was plagued by wildcat strikes. Assuming the recently signed contract between the Coal Operators and United Mine Workers will reduce wildcat strikes, we increase the days worked to 245. The estimated annual operating costs must therefore be multiplied by \(\frac{245}{220} = 1.11\). Summing equations (19) and (20), adding indirect operating costs and adjusting for 245-days per year yields:

\[
\text{Total operating costs} = 476,678 + 1,240,159 \quad (S) \quad (25)
\]

(cont.)

There is good empirical support for this number. A recent study by the Bureau of Mines shows that about 5% of production each year is lost because of "working out" of the mines. See Bituminous Coal and Lignite Mine Openings and Closings in the Continental United States, Mineral Industry Surveys, U.S. Department of the Interior, November 1973.
Total annual cost is simply (24) + (25). Summing these and substituting (16) for $S$, yields cost as a function of output and seam thickness. The important estimate is for a single set of sections, shafts, and slopes, since larger mines replicate this unit.

Drift: Total annual cost = $1,365,256 + 1,665,781 \left( \frac{q}{684 \text{ Th}^{1.42174}} \right)$

Deep: Total annual cost = $2,355,999 + 1,665,781 \left( \frac{q}{684 \text{ Th}^{1.42174}} \right)$

There is a cost curve for each seam thickness. An expression for the minimum average cost for any thickness is derived by dividing equation (26) by output, differentiating the resulting expression with respect to $Q$ and solving for $Q^*$, the output that minimizes average cost:

Deep $Q^* = 1.498 \left( \frac{684 \text{ Th}^{1.42174}}{1.42184} \right)$

Table 1 presents these minimum average cost outputs, for an assumed value of $\epsilon$. The minimum average cost output increases with seam thickness. Depletion, that is, the movement to thinner seams, should be accompanied by a diminishing of scale barriers to entry in the industry as the minimum efficient size declines. The minimum efficient size of mines in any seam thickness lies short of the size of the largest underground mines. This implies that the largest of these mines are, in fact, replicating efficient scale. Engineering descriptions of operations of these mines support this contention. Finally, we calculate the average cost of production at the minimum efficient scale for mines of any seam thickness. This is simply:
Drift: \[ AC^* = \frac{\text{TC}(Q^*)}{\text{Q}^*} = \frac{\text{TC}(Q^*)}{954 \text{ Th}^{1.42174} e} = \frac{4305}{\text{Th}^{1.42174} e} \] (28)

Deep: \[ AC^* = \frac{\text{TC}(Q^*)}{1373 \text{ Th}^{1.42174} e} = \frac{5161}{\text{Th}^{1.42174} e} \]

Minimum average cost also depends upon thickness and the natural geological conditions represented by \( e \). We do not know costs unless we know \( e \). We have assumed \( e \) is distributed lognormally. In the estimation it was further assumed that the expected value of log \( e \) is zero.

Table 1 presents minimum average cost and corresponding output for various seam thicknesses and an assumed value of one for \( e \). While costs will increase as seam thickness decreases, the mine output will be adjusted to mitigate the effects of depletion. We show this in Figure 3. The comparison often made is between points A and B. The correct comparison is between A and C. The locus of minimum average cost for any thickness is shown as the dotted line in figure 3.

In summary, we have seen that seam thickness is an important determinant of cost. The effect of other factors leads to a large dispersion about the expected cost. There are important scale effects in mining that make the size of mine not an arbitrary choice.

There is an important economic distinction not covered by this data and estimation. The analysis here is of ex post variation in productivity. Decisions to open mines are based on ex ante expectations. A great deal of the natural conditions in a mine can be anticipated by knowledge of conditions in neighboring mines, information from drill logs, and core samples. Yet we would expect ex post variation to be greater. A test of ex ante expectations would be provided by the data on new mine openings and their seam thickness.

In the next chapter we turn to this data. We use new mine and the dispersion to interpret reserve concepts.
TABLE I
Minimum Average Cost and Corresponding Output
\( (\varepsilon = 1, \text{ excludes welfare fund contribution and coal cleaning cost}) \)

A. Drift Mine

<table>
<thead>
<tr>
<th>Seam Thickness</th>
<th>( \tilde{Q}^* )</th>
<th>( AC^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>28&quot;</td>
<td>108,900</td>
<td>$37.71 per ton</td>
</tr>
<tr>
<td>36&quot;</td>
<td>155,670</td>
<td>26.38</td>
</tr>
<tr>
<td>42&quot;</td>
<td>193,815</td>
<td>21.19</td>
</tr>
<tr>
<td>48&quot;</td>
<td>234,335</td>
<td>17.53</td>
</tr>
<tr>
<td>60&quot;</td>
<td>321,823</td>
<td>12.76</td>
</tr>
<tr>
<td>72&quot;</td>
<td>417,054</td>
<td>9.85</td>
</tr>
</tbody>
</table>

B. Deep Mine

<table>
<thead>
<tr>
<th>Seam Thickness</th>
<th>( \tilde{Q}^* )</th>
<th>( AC^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>28&quot;</td>
<td>156,730</td>
<td>$45.21 per ton</td>
</tr>
<tr>
<td>36&quot;</td>
<td>224,042</td>
<td>31.63</td>
</tr>
<tr>
<td>42&quot;</td>
<td>278,939</td>
<td>25.40</td>
</tr>
<tr>
<td>48&quot;</td>
<td>337,255</td>
<td>21.01</td>
</tr>
<tr>
<td>60&quot;</td>
<td>463,169</td>
<td>15.30</td>
</tr>
<tr>
<td>72&quot;</td>
<td>600,226</td>
<td>11.81</td>
</tr>
</tbody>
</table>

Source: Equations (27) and (28)

Note: Costs are in 1973 dollars
Assumed value of \( \varepsilon \) is only illustrative
Average Cost and Locus of $AC^*$

Figure 3
CHAPTER II: AN ECONOMIC INTERPRETATION OF COAL RESERVES
RESERVES: AN ECONOMIC INTERPRETATION

Introduction

Several recent studies have concluded that the long-run supply curve of coal, over a very wide range, is perfectly elastic.\(^1\) This conclusion emerges in each of these studies from an examination of coal reserve statistics. The conclusion is often stated as "there is enough coal at current rates of output, to last for 2,000 years." Others are even more specific, claiming, for example, that with current technology, at current rates of output, there is enough coal for 500 years at current prices.\(^2\) The most recent report on Project Independence also makes similar claims.\(^3\)

On the other hand, R. L. Gordon has pointed out that we have no idea of the economic relevance of the so-called "reserve" statistics. He suggests that our ignorance about coal reserves might be comparable to that about oil in place.\(^4\)

It is the goal of this chapter to critically examine coal reserve statistics. We have already examined the influence of geology on costs. We use these cost estimates to interpret reserve data. We use data on new mines to check the cost estimates as well as provide a market-determined interpretation of reserves.


\(^2\)Newsweek, January 22, 1973, p. 53.

\(^3\)See The FEA Project Independence Report: An Analytical Assessment and Evaluation, Policy Studies Group, MIT Energy Laboratory, Draft of March 11, 1975, section 2.3.

Reserves and Supply Functions

There are good reasons for attempting to base supply estimates on reserve data. The high prices we are now observing in energy markets are far outside the range of historical prices. This means that predictions based on econometric evidence alone will involve large prediction errors.

More importantly, the supply function for a mineral is likely to involve important nonlinearities. Often, as the quality of the mineral deposit decreases, the quantity available increases. This change in the supply function, unless it occurs in some systematic and previously observed manner, cannot be captured by econometric technique.

Engineers, aware of the perils of purely econometric estimation, have attempted to construct supply curves based on detailed knowledge of the reserve stock. However, at times, the underlying economic relationships have been ignored; and rarely are these engineering functions systematically derived so that the results are reproducible.

This chapter attempts to provide an economic interpretation of coal reserves. The object is to move conceptually from a stock to a flow of output at a given price. In the course of this chapter we hope to illuminate the process of depletion in a natural resource industry.

Process of Supply

In order to understand the various terms used to describe the stock of any mineral, it is helpful to examine the supply process. We can break the process down to three stages--exploration, development, and extraction. The importance and cost of each of these steps varies with the mineral in question. In supplying oil, new discoveries are an important element. Knowledge of the stock in the ground is much greater for coal and consequently exploration is not as central an activity to its supply.
<table>
<thead>
<tr>
<th>Overburden</th>
<th>Measured</th>
<th>Indicated</th>
<th>Inferred</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than 1000 ft.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thin 1)</td>
<td>1.0</td>
<td>6.0</td>
<td>34.0</td>
</tr>
<tr>
<td>Intermediate 2)</td>
<td>3.0</td>
<td>9.0</td>
<td>11.0</td>
</tr>
<tr>
<td>Thick 3)</td>
<td>4.0</td>
<td>8.0</td>
<td>13.0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>8.0</td>
<td>23.0</td>
<td>58.0</td>
</tr>
<tr>
<td>from 1000-2000 ft.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thin</td>
<td>Neg.</td>
<td>0.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Intermediate</td>
<td>Neg.</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Thick</td>
<td>Neg.</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.0</td>
<td>3.5</td>
<td>6.0</td>
</tr>
<tr>
<td>from 2000-3000 ft.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thin</td>
<td>0.0</td>
<td>Neg.</td>
<td>Neg.</td>
</tr>
<tr>
<td>Intermediate</td>
<td>0.0</td>
<td>Neg.</td>
<td>Neg.</td>
</tr>
<tr>
<td>Thick</td>
<td>0.0</td>
<td>0.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>

1) A bituminous coalbed is classified as thin if it is thicker than 14 inches but thinner than 28 inches. Beds of lignite and sub-bituminous coal fall into this category if they are between 2.5 and 5 feet thick.

2) A bituminous coalbed is classified as intermediate in thickness if it is between 28 and 42 inches thick. Beds of lignite and sub-bituminous coal are placed in this category if they are between 5 and 10 feet thick.

3) Only bituminous coalbeds more than 42 inches wide are classified as being thick. Beds of lignite and sub-bituminous coal must be more than 10 feet thick to be placed in this category.

We can conveniently summarize the process with the following diagram:

```
mineral-in-place ↓ Exploration and Discovery

proved reserve ↓ Development

output flow ↓ Extraction
```

At each step, investment is applied to produce an output. The initial step is the exploration activity which produces mineral-in-place. This, in essence, locates the deposit. The next step involves all the preparations necessary in order to mine. In the case of coal, this includes getting to know the coal seam better, its peculiarities, fault lines, quality, etc. It involves tangible site preparation, shaft-sinking, installing haulage, surface facilities, etc. The final stage, once all equipment is in place, is the extraction of coal. Each stage of the process involves an output, and the outputs of the first two stages are often called reserves.

This is a basic confusion. The term is not important, but it is important to realize that mineral-in-place is the output of exploration, and proved reserves the output of development investment. This is the usage in the oil industry, but the situation in other minerals is confused because of the indiscriminate use of the term reserves.

In the coal industry the published "reserve" statistics refer not to the output of development expenditure, but rather to mineral-in-place. Because of the nearness to the surface of many coal seams, previous oil and gas drilling, and the relative homogeneity of coal seams,

---

information as to where coal is located is good. Projection over fairly wide areas is feasible in coal geology, whereas this is not the case with most minerals. We therefore have a relatively good knowledge of where and how much coal there is. The important question is the cost of development and extraction from this "mineral-in-place."

Existing Estimates

The basic information on coal "reserves" was developed by the United States Geological Survey (USGS). The reserves are broken down into categories according to dimensions of the deposit and the certainty with which the deposits are known to exist. The physical aspects are defined according to the rank of coal. Thus, the thick classification refers to different dimensions for bituminous coal than it does for lignite. The definitions are presented below in Table 1.

The second class of distinctions between deposits relates to certainty. The terms measured, indicated and inferred are defined by the U.S.G.S. as follows:

**Measured:** Measured resources are resources for which tonnage is computed from dimensions revealed in outcrops, trenches, mine workings, and drill holes. The points of observation and measurement are so closely spaced, and the thickness and extent of the coal are so well defined, that the computed tonnage is judged to be accurate within 20 percent of the true tonnage. Although the spacing of the points of observation necessary to demonstrate continuity of coal differs from region to region according to the character of the coal beds, the points of observation are, in general, about half a mile apart.

---

6 Paul Averitt, Coal Resources of the United States, USGS Bulletin 1275, 1970. (Data refers to January 1, 1967.)
**Indicated:** Indicated resources are resources for which tonnage is computed partly from specific measurements and partly from projection of visible data for a reasonable distance on the basis of geologic evidence. In general, the points of observation are about 1 mile apart from beds of known continuity. In several states, particularly Alabama, Colorado, Iowa, Montana, and Washington, where the amount of measured resources is very small, the measured and indicated categories have been combined.

**Inferred:** Inferred resources are resources for which quantitative estimates are based largely on broad knowledge of the geologic character of the bed or region and for which few measurements of bed thickness are available. The estimates are based primarily on an assumed continuity in areas remote from outcrops of beds, which in areas near outcrops were used to calculate tonnage classes as measured or indicated. In the interest of conservatism, the areas in which the coal is classed as inferred are restricted as described under the heading "Areal Extent of Beds." In general, inferred coal lies more than 2 miles from the outcrop or from points for which mining or drilling information is available.

**Unclassified:** For a few states, particularly Georgia, Maryland, Pennsylvania, Utah and West Virginia, the calculated resources have not been divided into the measured, indicated and inferred categories.

The percentage figures that appear in Table 1 are based on the distribution in those states for which information is available. It is assumed that the same distribution exists in all states, including those for which data are not available.  

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7 The states for which a breakdown was not available are Georgia, Maryland, Pennsylvania, Utah and West Virginia. It was assumed by the USGS that they would reflect the distribution in other states. Averitt, op. cit., p. 31.
An Economic Interpretation of Reserves

Table 1 presents the classification that must be worked with. The questions that must be answered are: (1) How accurately do the characteristics define a cost function, (2) Is the measure of "reserves" used by the Bureau of Mines that portion of the stock that is available at constant costs, and if it is not, (3) How will prices behave as output increases? We have already partially answered (1). In this section we provide another test of (1), and deal with the uncertainty of the deposits. We then turn to questions (2) and (3).

Economic Distinctions: Certainty

Measured reserves represent reserves that have been drilled before commencing development. Indicated reserves have been drilled, but to a lesser degree. These reserves are known with less certainty. The uncertainty includes the possibility that there is a fault, making mining difficult, or that the seam thickness diminishes making mining more costly. The extreme is, of course, that the seam thins to zero, meaning the deposit is very small.

The miner has the alternative of mining a measured seam that is thinner or drilling an indicated seam with a greater expected thickness. At the margin, he equates these two alternatives. Therefore, the maximum he is willing to pay for drilling is given by the increment in costs in the measured portion. Similarly, if it is cheap to drill and the probability of finding a thicker seam is great, he will not move to much thinner seams.
In fact, drilling is cheap. The indicated reserves are estimated from observations more widely spaced than is the case for measured reserves. Measured reserves are said by the USGS to be accurate within 20%. No estimate of accuracy is given for indicated or inferred reserves. The difference in knowledge is represented by five core-holes per square mile. Observations in the measured stock are one-half mile apart and one mile apart in the indicated portion. The individual miner faces a greater risk when dealing with an individual parcel of indicated reserves. However, this risk is quite limited. After drilling, the miner can choose not to develop if the results indicate a non-economic parcel. The most that is at risk is the cost of the drilling.

When we consider industry-wide behavior, even this risk disappears. This risk can be "diversified away" by drilling in enough separate parcels. If all the indicated reserves were drilled, the individual parcel results would vary, but unless the estimation process were biased, we would find the estimated "expected" amount of coal. The large number of individual parcels and the law of large numbers assure this.

Assuming that all the holes are drilled, we can ask what cost this adds to the cost of development. The cost of drilling diamond cores is about $15 per vertical foot down to a depth of 1000 feet. Therefore, the cost of drilling 5 holes in a one-mile square parcel is $75,000.

---

8 See Appendix 1 for a measure of the relative risk. The 20% error itself is hard to interpret. Since the total is the sum of many 1/2 mile square parcels, we would expect an unbiased procedure to give a good expected total, although our individual estimate would be subject to error. Furthermore, literally, a 20% error means that the error process is not independent of the size of the seam. Clearly, this complicates interpretation. More work by geologists is needed on this subject.

9 The limited exploration risk can be diversified away by a large enough company exploring over a large enough number of parcels. However, since the cost of exploring is small, this is a relatively unimportant scale economy.

10 Peter T. Flawn, Mineral Resources, Rand McNally & Co., 1966, p. 27. The figure there is $10/foot down to 1000 feet. This was adjusted for price increases since 1966 of 50% in the implicit price deflation for
This is a capital expenditure that must be amortized over the life of the investment. The life of the investment depends upon total reserves and annual output:

\[ c = \frac{\$75,000}{q (1-e^{-rt})} \]

where \( r \) = discount rate
\( q \) = annual output rate
\( t \) = life of investment

The life of the investment is simply \( \frac{R}{q} \), where \( R \) is reserves "proved."

\( R \), in turn, is equal to:

\[ R = 640(1800)(Th) \]

where \( Th \) = thickness of seam in feet. The 640 refers to acres per square mile, and 1800 is the tons of coal per acre-foot. We can use a simple example to see that this cost is trivial. Assume the reserves are in a seam with expected thickness of 42", and output will be 100,000 tons per year.

\[ c = \frac{(75,000)(.12)}{100,000(1-e^{-rt})} \]

\[ = \$0.09 \text{ per ton} \]

The importance of this exercise is that for all intents and purposes we can ignore the difference between indicated and measured reserves. When we inquire as to the portion of the stock being mined today, we need not be concerned whether a new mine was developed from the measured or indicated portion of the stock. More importantly, when we ask what will happen to prices as output expands, we can also aggregate the two categories.
The uncertainty with regard to inferred reserves is much greater since these estimates are based on broad geologic information over wide areas and not core samples. There is not a large number of individual parcels; estimates are based on broad extrapolation. The uncertainty in these estimates is reflected in large adjustments in the estimates as new information becomes available. A good example is the massive re-evaluation of western coal reserves. The original estimates were based on a few observations and extrapolation. Currently drilling is taking place and the estimates are changing.\footnote{Compare for example, the estimates of Montana reserves in the U.S. of Mines, IC8531, Stuppable Reserves of Bituminous Coal and Lignite in the United States, 1971, with Robert E. Matson and John Blumer, Quality and Reserves of Strippable Coal Selected Deposits, Bulletin 91 Montana Bureau of Mines and Geology, Dec. 1973.}

**Economic Interpretation of Reserves: Thickness of Seam and Compensating Factors**

The error term as index

The preceding chapter demonstrated that factors other than seam thickness are important determinants of costs. Their influence is reflected in the error term in each of the equations. The error term can be thought of as an index. The more favorable the combination of roof conditions, gas conditions, etc., the higher the value of the index. At a given level of cost, there is an inverse relation between seam thickness and the index. If the level of cost is to be maintained, a thinner seam can only be worked if other conditions are sufficiently favorable to offset the decline in seam thickness. This relation is depicted in Figure 1.

This diagram is analogous to a production isoquant. The level of cost represented by $C_0$ can be maintained by any combination of thickness and $\varepsilon$ along the curve. Curve $C_1$ represents a higher cost level, since every point represents a smaller thickness for a given $\varepsilon$ than does curve $C_1$. 
Relation Between $Th$ and $\varepsilon$ for a Given Level of Cost

Figure 1
In the previous chapter the measure $\varepsilon$ was limited to factors directly affecting productivity. We attempted to isolate, as much as possible, natural geological factors. This, however, is not necessary. An important factor not reflected in productivity data is locational advantage. A mine favorably located with respect to transport facilities or to a consuming center can afford to mine, all other things equal, in a thinner seam. This advantage then is directly substitutable for greater seam thickness. Reserve data tell us solely about the distribution of seam thicknesses.

Also included as a cost-determining factor is the size of the reserve parcel. In Chapter I, it was pointed out (see footnote 23) that when the life of the mine is less than 20 years, the useful life of some investment exceeds the life of the mine. Larger parcels will reduce the average cost of this class of investments by extending the life of the mine. Dispersion in this variable will also be reflected in the overall dispersion of seam thicknesses of new mines.

The fact that all mines of equal cost have a pair of $(th, \varepsilon)$ that satisfies $K = C$, allows for another test of the variability of $ThY_{\varepsilon}$ $\varepsilon$. And, in this case, transport cost and parcel size differentials can also be included.

If factors other than thickness are important and variable, in a set of equal minimum average cost mines, we would expect to observe a disperse distribution of new seam thicknesses. Since we cannot directly observe costs, we must have some indirect method of approximating a set of equal-cost mines. We restrict the sample to new large mines (100,000 + tons per year) opened between 1968-1973. The sample is subdivided by geographical region and quality. The assumption is that depletion was small in this five year period. Therefore, coal mines in the same region
mining similar quality coal, opened in this limited period, should represent a set approximating equal-cost mines. Since by the time a mine is opened, remaining uncertainty about seam thickness is small, the new mines represent **ex ante** expectations about how costly it will be to mine at the various seam thicknesses of the new mines. Furthermore, this data reflects locational advantage.

The observations come from the *Keystone Coal Industry Manual*, from the years 1970, 1971, 1972, 1973, and, in the case of Illinois, from the Annual Report of the Department of Mines. Illinois lists mines by years of opening. This is done on a less systematic basis in the *Keystone Coal Manual*. For all other states, the lists of mines appearing in the *Keystone manual* in each year were compared. In this way, new mine openings could be ascertained. In some cases, the year of opening was listed directly in *Keystone*. The seam thickness and sulfur contents in most cases come from the *Keystone Coal Manual*. These were supplemented with state mining reports for seam thickness. In some cases, sulfur content was not listed in *Keystone*, but an approximate value came from *Producers of Coal from Appalachian States*, published by the *Keystone Coal Industry Manual*.

The distributions confirm our early estimate that thickness alone cannot describe a cost function. If costs were determined only by the thickness, these distributions would concentrate over a narrow range. There would, in other words, be little dispersion in the distribution of seam thickness. Equal cost mines would all be of the same thickness. Transport differentials would account for some small dispersion within the coal-supplying regions. Figures 2-4 show the distribution of seam thicknesses of new mines.

Several systematic facts emerge from this distribution data. It is clear that there is overlap in the range of seam thicknesses within

---


Histogram - Seam Thicknesses of New Mines

SEAM THICKNESS (INCHES)

FREQUENCY (NO. OF MINES)

SOUTHERN APPALACHIA
LOW SULFUR SHAFT/SLOPE MINES

Figure 2
Histogram - Seam Thicknesses of New Mines

ILLINOIS-WEST KENTUCKY
HIGH SULFUR-SHAFT MINES

Figure 3
Histogram - Seam Thicknesses of New Mines

NORTHERN APPALACHIA
HIGH SULFUR SHAFT MINES

SEAM THICKNESS (INCHES)

NORTHERN APPALACHIA
HIGH SULFUR DRIFT MINES

SEAM THICKNESS (INCHES)

Figure 4
a region for high sulfur and low sulfur and deep and drift mines. Expectations formed in earlier chapters are confirmed. The lower cost of drift mining relative to deep is reflected by a downward shift in the distribution. This reflects the cost of sinking shafts. Similarly, the downward shift in the low-sulfur distribution relative to the high-sulfur mines reflects the premium paid for low-sulfur coal.

The Incremental Cost of Coal

The unanswered question about reserve estimation is what portion of the stock is being mined today. This portion establishes today's incremental cost of coal. Furthermore, it is confusion about this portion that has led to the assertion of "there is enough coal at current costs for 500 years."

Economic theory predicts that the least-cost deposits will be exploited first. The incremental mine represents the highest cost mine that must be opened to satisfy demand. The initial temptation is to use the lower extreme of seam thickness for new mines as the estimate. This procedure, though, is surely biased. The above discussion indicates that both a thickness and a value of $c$ must be specified. The lower extreme seam thickness represents the highest value of $c$. Any mine lying on the $Th-c$ curve of Diagram 1 is suitable since they represent mines of constant cost. The entire distribution represents "incremental" mines. The correct question is, therefore, what cost of coal is implied by the distribution of new mines? In Chapter I, minimum average cost was estimated as:

$$C = \frac{K}{Th^\gamma c}$$

(1)

where $K = 5161$ for deep mines

$K = 4305$ for drift mines

$\gamma = 1.42174$

A more precise reckoning would consider metallurgical and steam coal. However, as pointed out above, the markets, except for the premium metallurgical costs, are merging.
It will be recalled that \( f(c) \) was assumed to be lognormal with parameters \((0, \sigma^2)\). The problem is to estimate cost, given the observations on \( T_h \). This involves selecting a value of \( T_h \) to characterize the distribution. The cost is then given by \((K/\overline{T_h})\) where \( T_h \) is the chosen thickness. Because of the variation of the other factors, the thickness chosen should be a central value of the observed distribution. Given the lognormality of the distribution, the cost estimate that maximizes the likelihood of observing the distribution of thicknesses is obtained by using the geometric mean of the observed thicknesses as \( T_h \). These thicknesses are reproduced in Column 4 of Table 2.

In Table 2 the cost estimates are presented for the various geographical regions and quality characteristics. The closeness of the estimate derived from either the deep or drift mines within each category indicates the method is sound and internally consistent. In Northern Appalachia, the drift mine estimate yields a higher cost than the estimate for deep mines. This is due to the higher average sulfur content of the deep mines in the sample. The deep mines all were above 2%, while the drift mines were below 2% sulfur content.

The probability of observing the given distribution of thicknesses is:

\[
P(\varepsilon_1 = \frac{K}{T_h Y_c}, \varepsilon_2 = \frac{K}{T_h Y_c} \ldots) = \prod \pi p_i(\frac{K}{T_h Y_c})
\]

Expression (3) results from substituting the expression for the normal distribution into (2). This is maximized when:

\[
\sum (\log K - \gamma \log T_h - \log \overline{c})^2 = 0
\]

or

\[
n \log \overline{c} = \frac{\sum (\log K - \gamma \log T_h)}{n}
\]

\[
\overline{c} = K / (\overline{T_h})^{1/n}
\]
### TABLE 2: INCREMENTAL COST OF COAL

#### A. Shaft and Slope Mines

<table>
<thead>
<tr>
<th>Area</th>
<th>Sulfur Content</th>
<th>No. of Observations</th>
<th>&quot;Incremental mine seam thickness&quot;</th>
<th>Minimum Average Cost for Incremental Mine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Appalachia</td>
<td>High Sulfur (&gt;1%)</td>
<td>7</td>
<td>63.8&quot;</td>
<td>$14.02</td>
</tr>
<tr>
<td>Southern Appalachia</td>
<td>Low Sulfur (&lt;1%)</td>
<td>6</td>
<td>48&quot;</td>
<td>21.01</td>
</tr>
<tr>
<td>Illinois Basin</td>
<td>Mostly High Sulfur</td>
<td>14</td>
<td>69&quot;</td>
<td>12.54</td>
</tr>
</tbody>
</table>

#### B. Drift Mines

<table>
<thead>
<tr>
<th>Area</th>
<th>Sulfur Content</th>
<th>No. of Observations</th>
<th>&quot;Incremental mine seam thickness&quot;</th>
<th>Minimum Average Cost for Incremental Mine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Appalachia</td>
<td>High Sulfur (&gt;1%)</td>
<td>7</td>
<td>52.3&quot;</td>
<td>15.51</td>
</tr>
<tr>
<td>Southern Appalachia</td>
<td>Low Sulfur (&lt;1%)</td>
<td>24</td>
<td>41&quot;</td>
<td>21.93</td>
</tr>
</tbody>
</table>

Source: Text and Equation (1) Chapter VI
The costs estimated, here, are higher than the long-term contract prices in that period. This upward bias can be attributed to several factors. There is the possibility that the Bureau of Mines data is biased. Equipment prices reflect list prices and actual sales might be below list. Furthermore, there is a general tendency in engineering estimate toward allowance for special contingencies, biasing the cost estimates upward. These contingencies are properly considered as variance about the expected cost and should not be calculated in the estimate of the average.

A second explanation is that the productivity equation reflects the recent turmoil in the industry. Industry expectations could be that over the long run, productivity will return to previous levels and the productivity relation estimated with 1973 data does not reflect those expectations. The National Petroleum Council report on coal availability supports this hypothesis. That study, reflecting industry opinion, expected average productivity to stop its decline and return to pre-1969 values. However, since the completion of that report in 1972, productivity has not increased, and it is hard to assess whether this experience has changed the long-run expectations.

The bias notwithstanding, this way of dealing with the overall problem has great advantages. Explicit consideration is given to non-observed cost determining factors. It also suggests the type of data collection that, done on a systematic basis, would provide meaningful economic information about coal costs and reserves. Furthermore, the indication of bias in the engineering estimates suggests more careful engineering calculations are in order.

A Re-Interpretation of Reserve Categories

The correct use of the derived cost function is with the geometric mean thickness of new mines. We call this the "incremental seam thickness," with the understanding that it represents an entire distribution of new mines. What then is the meaning of the established reserve categories?

Comparison Between Alternative Interpretations of "Reserves"

Figure 5
The most recently developed reserve concept is the "Demonstrated Reserve Base." Adding together all measured and indicated reserves greater than 28", the Bureau of Mines estimates the "Demonstrated Reserve Base" of deep coal. This base of 161 million tons plus estimates of strip coal in the Eastern United States is used to justify the assumption of little depletion from 1975-1990 in the Federal Energy Administration Report (see footnote 3).

Clearly, aggregation of all seams 28" or thicker is extreme. It is certainly true that some coal in 28" seams is mined today. However, this is only low-sulfur coal or coal with other offsetting factors. The thickness of point A in Figure 1 is observed, yet the value of ε is not. All coal in 28" seams is not available at today's level of cost. The error is to infer a general proposition from special circumstances. Coal in 28" seams is exploited, but only very high quality coal or coal seams with important compensating factors.

Even all the coal in 42" or thicker seams is an overestimate of coal available at today's costs. The cost today of high sulfur coal is represented by a distribution with an "average thickness" of about 64 inches. If mining were to proceed such that the "average" thickness of new mines were 42", this would represent a 66% increase in costs.

The error is to assume the supply curve is as FEA in the diagram below, when, in fact, a more correct interpretation of reserve data is given by supply curve S. Supply curve FEA is drawn horizontally at the level of cost obtaining when the incremental mine is in a 28 inch seam. The true cost is below that, and the true supply curve slopes upward as in supply curve S. Table 3 shows the difference between incremental cost today and the standard used by the Bureau of Mines.
TABLE 3
COMPARISON OF "INCREMENTAL MINE" AND RESERVE BASE CRITERIA

<table>
<thead>
<tr>
<th>Area</th>
<th>Incremental Deep Mine</th>
<th>&quot;Demonstrated Reserve Base&quot; Criteria</th>
<th>% Increase in Cost from Incremental mine to B of M Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>High Sulfur Coal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northern Appalachia</td>
<td>63.8&quot;</td>
<td>28&quot;</td>
<td>222%</td>
</tr>
<tr>
<td>Southern Appalachia</td>
<td>NA (see note)</td>
<td>28&quot;</td>
<td></td>
</tr>
<tr>
<td>Illinois Basin</td>
<td>69&quot;</td>
<td>28&quot;</td>
<td>262%</td>
</tr>
<tr>
<td></td>
<td>Low Sulfur Coal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northern Appalachia</td>
<td>NA</td>
<td>28&quot;</td>
<td></td>
</tr>
<tr>
<td>Southern Appalachia</td>
<td>48&quot;</td>
<td>28&quot;</td>
<td>115%</td>
</tr>
<tr>
<td>Illinois Basin</td>
<td>NA</td>
<td>28&quot;</td>
<td></td>
</tr>
</tbody>
</table>

Source for Table 3:

Col. 2: Text and Table 2, Chapter II.


Col. 4: Costs were calculated using equation (1) Chapter II, for the incremental thickness and the thickness represented by the reserve criterion.

NA indicates that there were too few observations for reliable estimation. The best guess for the incremental mine in these cases is the mine of comparable quality coal in the neighboring area.
The Incremental Strip Mine

For strip mining, the only available measure of the economic validity of the criteria is in Illinois where we have some information on stripping ratios of new mines. The 18:1 cut-off appears to be too stringent, however the sample is quite small. The three greatest ratios are for the Consolidation Coal Company Norris mine which ranges from 16.8-25, the Midland Company mine at 18.36 and the Eads Company mine which reaches 16.7. However, in each of these there are special compensatory circumstances. The Midland mine is in Peoria County, which gives it a transport advantage of 200 miles over mines in southern Illinois. This would lead to a difference of at least $1 per ton. Similarly, for the Norris mine located in Fulton, Co. Finally, the Eads mine is in the so-called "quality triangle" in southern Illinois, and mines from a low-sulfur seam. This, as we have seen, could account for a large difference in coal price. Most of the other mines are quite small.

The large published reserve statistics are at variance with industry opinion about the supply of strip reserves in the Illinois Basin:

Although mining conditions, overburden ratios and other factors affecting the economical production of coal vary considerably from mine to mine and are subject to changing conditions of a transitory nature, there have been no material changes during the past several years affecting generally the coal reserves being mined by Ayrshire or in mining conditions, and no such changes are presently anticipated. However, as the best strip-ping locations are gradually depleted by the coal industry, the

The incremental cost in 1970 was .004 per ton-mile. This represents 80¢ per ton, which when adjusted for transport cost increases through September 1973, yields $1.05. Estimates from M.B. Zimmerman, Long-Run Mineral Supply: The Case of Coal in the United States, unpublished PhD dissertation, M.I.T., August 1975.
cost of operating will increase as a result of deeper over-
burdens and thinner coal seams. While larger and more
efficient machinery has served in the past to offset these
increased costs, that portion of the coal industry that
produces largely from strip mines, as does Ayshire, will in
time become subject to lower profit margins and to competi-
tive procedures from underground producers, which have his-
torically had higher production costs, and from other fuel
sources and will be forced, to the extent they have reserves,
to open underground mines.

...It is evident that reserves of strippable coal are limited
in relation to the present level of production. For this
reason we believe that peak production of strip coal will be
reached within a very short time and a gradual decline will
ensue thereafter. Meanwhile there will be substantial growth
in underground production... 19

...For half a century the percentage of coal production pro-
vided through strip mining has steadily increased. Recent
trends indicate a peak in percentage, if not in tonnage, is
being approached. Important among the reasons for this is the
decline in economic advantage enjoyed by strip mining compared
to underground mining... 20

18 American Metal Climax, Inc., Listing Application to New York Stock
19 Paul Weir Company, Coal Reserves and Deep Mining Report, General
20 Hubert E. Risser, "Coal Strip Mining - Is it Reaching a Peak?" Paper
presented at Fall Meeting, Society of Mining Engineers,
Minneapolis, Minnesota, September 18, 1968, cited in Paul Weir
Company, op. cit., p. 773.
An explanation lies in the size of the reserve parcel. Since there are economies of scale in strip mining, due largely to greater utilization of capital equipment, large size parcels can produce more efficiently. To support a large operation and justify site development expenditures, a large parcel is needed. Reserve data ignores this distinction. Furthermore, the tendency to go to the extreme exists with the strip mining reserve data as it does with underground reserves. In light of the discussion of deep reserves, it is consistent to observe large "reserves" and an upward sloping supply curve. Unfortunately, the data do not allow a more complete treatment of this question.

There is, however, an exception to this in Western (west of the Mississippi River) reserves. There, variable ratios were chosen. A different economic cut-off was chosen for each coal field in the area. The cut-offs were chosen according to thickness currently being mined. This then reflects both transport and quality differentials and are more reliable estimates of the portion of the curve that is perfectly elastic.

The reserves for the Western states are enormous. All the coal is low in sulfur and relatively low in heat content. It has been pointed out that this combination means that to satisfy sulfur restrictions setting limits on emissions per million Btu, the sulfur content must be lower for this low Btu coal than for eastern coal. The extent of reserves with a low enough sulfur content is not known.

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21 Telephone conversation with Robert E. Matson, Montana State Geologist.

22 M. Reiber has pointed this out in Low Sulfur Coal: A Revision of Reserve and Supply Estimates, CAC Document No. 88, Revised, University of Illinois, November 30, 1973. However, his subsequent estimates of the totals of low-sulfur Western coal are biased downward because of the arbitrary assumption that all coal in the less than .7% sulfur category contains .65% sulfur. By this assumption he eliminates over 14 billion tons of low sulfur coal as not suitable because of low Btu content. Surely some of this 14 billion tons is at a sulfur level less than .65% and acceptable for burning. His procedure, by assumption, eliminates all the coal of the Powder River Basin, the most important area of Montana and Wyoming.
APPENDIX 1

Relative Riskiness of Measured and Indicated "Reserves"

The essence of the problem is that thickness is a random variable with mean $T$ and variance $\sigma^2$. The reserve estimator has 4 observations on thickness. He interpolates between them so as to estimate the volume of a regular object.\(^1\) The diagram below indicates the problem.

The geologist has information on the thickness of the seam at points a, b, c, and e. He estimates the volume of the object shown by solid lines. The true seam has random variation about the solid lines. These are shown by the dotted lines. We assume this random variation is independent of seam thickness. For simplicity, we assume all the variation occurs in the lower boundary. The true volume is given by:

$$V = \int_0^d A(x) \, dx$$

\(^1\)See U.S. Bureau of Mines, Information Circular IC8283, for a description of reserve estimation technique.
where \( d \) is the distance between observations and \( A(x) \) is the area of a typical "slice" of the object.

The true area we call \( g(x) + \varepsilon \) where \( \varepsilon \) represents the random variation in seam thickness. Then:

\[
V = \int_{0}^{d} \int_{0}^{d} g(x)dx = V + d^2 \varepsilon
\]

where \( \tilde{V} \) is the estimated volume. The variance of \( \tilde{V} \) is given by:

\[
E(\tilde{V} - V)^2 = d^4E(\varepsilon^2) = \sigma^2 \varepsilon d^4
\]

Since for measured reserves, \( d = \frac{1}{2} \), and for indicated reserves, \( d = 1 \), the ratio of the standard deviation is

\[
\frac{1}{\left(\frac{1}{2}\right)^2} = 4
\]
CHAPTER III: CONCLUSIONS: DEPLETION AND COSTS IN THE LONG RUN
Conclusions: Depletion and Costs in the Long Run

The analysis of the previous chapters can now be used to deal with the question of depletion in the coal industry. Depletion is the movement to costlier deposits. The behavior of costs as cumulative output increases is the effect we seek to measure.

Chapter II presented an interpretation of the reserve concept known as the demonstrated reserve base. It was shown that this is too broad an aggregate with which to measure depletion. The limits to depletion implied by this concept are far above the current incremental mining cost and are not likely to be reached soon. Even coal in seams 42 inches or thicker is too broad a measure to establish likely depletion effects. Information on the distribution between 42 inches and the maximum thickness mined is needed in order to establish the future course of depletion. Unfortunately, comprehensive information on this distribution is lacking. The estimation procedure developed here can serve as an approximation. Furthermore, the procedure establishes a means for more accurate estimation as more data become available.

The procedure relies on information available on the complete distribution of coal reserves by thickness for a large and important coal producing county--Pike County in East Kentucky.

The Distribution of Coal by Seam Thickness

Information that was made available for this study details the complete distribution of coal by seam thickness in Pike County, Kentucky. The object is to describe this actual distribution by a well-known statistical distribution. The lognormal distribution has been used toward similar purposes in studies of other solid minerals, as well as for oil reservoirs,
Figure 1

The distribution of coal reserves by seam thickness in Pike County, Kentucky.
and proves valuable here.\textsuperscript{2}

In their study of the lognormal distribution Aitchison and Brown outline a procedure that provides an approximate test of lognormality.\textsuperscript{3} The cumulative distribution is plotted on logarithmic probability paper. If the generating process is lognormal, the points should approximate a straight line. In Figure 1 the distribution of tons of coal by seam thickness is plotted. For example, fifty-eight percent of the tonnage in Pike County lies in seams of 41 inches or less. It appears that the lognormal distribution is an adequate representation of the distribution of coal by seam thickness.

It is assumed that such a lognormal distribution applies for the various coal-producing states.\textsuperscript{4} Pike County has, in fact, larger total reserves than several individual coal-producing states.\textsuperscript{5} Finally, it is assumed that the variance of the distribution is equal in all states but that the mean of the distribution changes from state to state. And, of course, the total tonnage varies from state to state. The variance is calculated by a graphical method discussed by Aitchison and Brown.\textsuperscript{6}

\textbf{The Distribution of Costs}

Chapters I and II demonstrated that seam thickness alone is inadequate to derive a cost function. The myriad other unobservable factors were lumped together in the error term of the regressions. It was assumed that this index was lognormally distributed, with parameters $0$ and $\sigma^2\epsilon$ and is independent of seam thickness. A test of this assumption is provided in the Appendix to this chapter.

The distribution of coal by cost is then the distribution of $\frac{\nu}{m\epsilon}$, which reflects the distribution of coal by seam thickness, as well as the distribution of $\epsilon$ or "other" factors.
It is convenient to deal with the distribution of \( \log C \):

\[
\log C = \log K - \gamma \log Th - \log \epsilon. \tag{1}
\]

This is normal with mean \((\log K - \log Th)\) and variance \(\gamma^2 \sigma_{\log Th}^2 + \sigma_{\log \epsilon}^2\)

where \(\log Th\) is the mean of the log of thicknesses.

The distribution observed today, however, is truncated. Economic theory predicts that the least cost deposits will be exploited first. Therefore, the cost of mining the coal remaining in the ground in any area is equal to or greater than the present incremental cost. If the original probability distribution was \((\log C)\), the remaining distribution of coal according to the cost of mining is

\[
\phi(\log C) \int_1^{\log \bar{C}} \phi(\log C) \, dC \tag{2}
\]

This is illustrated in Figure 2.7.

Parameter Estimates

This simple model allows calculation of depletion effects. The necessary parameters are:

\[
\sigma_{\log \epsilon}^2 = \text{variance of other factors}
\]

\[
\sigma_{\log Th}^2 = \text{variance of coal-seam thickness distribution}
\]

\[
\log Th = \text{the mean of the log of seam thickness in each state}
\]

\[
\log \bar{C} = \text{the log of incremental coal cost in each state}
\]

\[
\gamma = \text{the elasticity of cost with respect to seam thickness}
\]

The variance of other factors comes from Equation (10), Chapter I. Since the variance is dependent upon the size of mine, the variance is calculated for the number of units used to produce the minimum average cost output. The variance of seam thickness is, as described above, estimated from Figure 1. The mean seam thickness for each state is calculated in the following manner. Data are available on coal in seams 28"-42", and 42" or
Distribution of Tons of Coal by the Log of the Cost of Exploitation

Figure 2
above for each state. The percentage of the total tonnage in seams 42" or thicker is calculated from the following formula based on the normal distribution:

\[ \log 42 = \log \bar{Th} + \sigma_{Th} U_{42} \]

where \( U_{42} \) is the point on the standard normal distribution corresponding to the cumulative percentage represented by \( \log 42 \). If, for example, 50% of the coal in the state is in seams 42 inches or thicker, \( U_{42} \) is zero. Fifty percent of the standard normal distribution lies above and fifty percent below zero.

The final parameter, \( \bar{C} \), is calculated in the manner described in Chapter II and represents the incremental mine thickness for deep mines. For those states where not enough new mines were observed to provide a good estimate of \( \bar{C} \), the value was assumed equal to the regional average calculated in Chapter II.

In Northern Appalachia and the Illinois Basin, all coal was treated equally. In Southern Appalachia the distinction between high and low-sulfur coal is important. Consequently, there the procedure was applied separately for coal with sulfur content less than 1% and for coal of sulfur content greater than 1%. It is assumed that sulfur is distributed independently of seam thickness. However, estimates of Chapter II show that previous depletion has pushed the truncation point, \( \bar{C} \), lower in the case of low sulfur coal.

Once these parameters are known, the probability distribution is known and values can be obtained from a table of the normal distribution. The total tons of coal in any cost category is obtained by multiplying the relevant tonnage total by the relevant probability. The amount of coal in the ground, available at up to 5% more than current cost in Illinois, for
example, is total state tonnage multiplied by:

\[
\int \frac{\log 1.05c}{\log c} \left[ \frac{\phi(\log c)}{1 - \int_{-\infty}^{\log z} \phi(\log c) dc} \right] dc
\]  

(3)

This function defines implicitly a "cumulative" cost function. It shows how costs rise with cumulative output. For any cumulative output total, the function implicitly gives the upper limit of integration, or the marginal cost of mining resulting from having mined that total volume of coal.

**Depletion in the Long Run**

Table 1 presents the life of the reserve stock at current rates of output. Reserves are defined here in a special way--all the coal in the ground in the measured and indicated categories available at less than a specified cost level. Thus, column one of the table lists the coal available at up to a 5% cost increase above current cost. The total available is expressed as a multiple of current annual output.

The table leads to some interesting conclusions. In the first place it puts earlier "reserve" estimates into perspective. The National Petroleum Council, as well as the Federal Energy Administration, have all based their assumption of perfect elasticity upon the Demonstrated Reserve Base. This was shown to be an overestimate of the elastic portion of the supply curve. Exactly how much of an exaggeration can be seen. High-sulfur coal in Appalachia available at less than 5% above current costs is only 9 times current annual output. Low-sulfur coal available at less than a 5% increase, represents less than 4 years current annual output.

The recent report of the Federal Energy Administration Coal Task Force assumes no depletion over a period of 15 years. Furthermore, this assumption is made for output rates almost double current rates. The estimates here indicate that at double current output rates, depletion is, in fact, significant.
Table 1

Life of Stock at Current Rates of Output

(Depletion Limited to 5%, 11%, 22% Above Current Costs)

<table>
<thead>
<tr>
<th>Region</th>
<th>+5%</th>
<th>+11%</th>
<th>+22%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Appalachia</td>
<td>8.9 years</td>
<td>18.3 years</td>
<td>38.1 years</td>
</tr>
<tr>
<td>Souther Appalachia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low Sulfur</td>
<td>2.8</td>
<td>5.6</td>
<td>14.3</td>
</tr>
<tr>
<td>High Sulfur</td>
<td>8.9</td>
<td>18.3</td>
<td>38.2</td>
</tr>
<tr>
<td>Illinois Basin</td>
<td>23.0</td>
<td>47.1</td>
<td>96.6</td>
</tr>
</tbody>
</table>

Source: Text
In nineteen years, depletion would have led to a cost increase of more than 22% in the case of Eastern high-sulfur coal, and a little less than 11% in the Midwest. The Midwestern coal industry could therefore be expected to expand more rapidly than the Eastern industry, expanding the geographical extent of its markets.

Table 1 can be used to shed light on some policy questions now facing the United States. The effects of complete ban on strip mining, for example, can be approximated by the case of doubling present output. Banning strip mining, and maintaining current output levels, would force a doubling of deep mining output. The effect on costs is an increase of 22% rather than 11% over the course of 19 years. If, at the same time, it is attempted to double the total annual output, costs would rise by a total of 22% in ten years. This is, of course, an extreme case. The more gradual the cutback in stripping output, the smaller the cost increase. Given adequate adjustment times, these are the long-run costs of such policies. The short-run impact on cost could, of course, be quite different since strip mines can be brought on stream at a faster rate than underground mines.

The table also sheds light on the costs of the Clean Air Act. Assume strip mining and deep mining continue to account for the same proportions of output as in 1973. Then, increased substitution of low-sulfur coal will cause that already rapidly upward shifting supply curve to shift even further. Costs, over 20 years, would rise by 75% if low-sulfur output doubled. A doubling represents a total increase of 160 million tons. This falls short of recent estimates of the increases needed to satisfy the Clean Air Act. Two factors will serve to limit this rise. The successful development of stack-gas scrubbing devices will limit the cost of low-sulfur coal to the cost of high-sulfur coal plus the cost of scrubbing.
Secondly, Western strip mined coal will begin to penetrate further east. As discussed in the previous chapter, it is uncertain how much of the low-Btu western coal will in fact satisfy standards for low-sulfur coal. One thing, however, is clear. A simultaneous push to decrease strip mining and substitute low-sulfur coal could prove very costly. Society clearly must trade-off between these two valid environmental goals.

User Costs

The discussion of the last three chapters has been limited to the costs of mining coal. Price will reflect, in addition to the mining and processing costs, the rent earned by the stock—the so-called user cost. User cost arises because the stock of coal is depletable. The problem of complete exhaustion of the stock is unimportant, since the amount of coal in the ground is very large and eventual exhaustion, if it even comes at all, is so far in the future as to have an unmeasurable effect upon price. However, the exploitation of a ton of coal in a thick seam today hastens the time when mining will proceed to thinner and, ceteris paribus, more expensive seams. This movement to more expensive seams is reflected in today's price. This reflection is simply the present value of the future increments in cost. The further into the future this movement to high cost sources, the smaller the user cost. The greater the rate of output, the faster the time-rate of increase of costs and the higher the user cost.

Clearly, user cost depends upon demand and upon the substitutability among fuels as well as upon cumulative costs. However, the more gentle the slope of the cumulative cost curve given any demand function, the smaller the user cost.
While far short of a calculation of user cost, we can indicate the user cost compatible with any pattern of output over time using the concept of a backstop technology—an energy source infinitely available at a high cost. Here we provide an illustrative calculation for high sulfur coal at a constant rate of output equal to today's level. It is assumed that the backstop, the breeder reactor, becomes available at 5 times the level of today's coal cost.

The user cost is then the present value of the future cost increments until the cost of coal reaches the level of cost of the backstop. The present value of future cost increments for today's level of output is approximated in the following manner. An exponential rate of growth in cost is estimated for several intervals of cost increase. For example, at current rates of output costs will rise by 5% in a period of 8.9 years in the case of high-sulfur coal. Cost will rise by 11% in 18.3 years and so forth. For each interval an average rate of growth is calculated so that:

\[ c(t) = c(t_0)e^{jt} \]

for \( t \) within the interval. This can be carried out until cost reaches 5 times current cost. In the case of high-sulfur coal this would take 300 years. But the present value of a cost increment 75 years from now at any reasonable interest rate will have a negligible effect on the result.

User cost is then the present value of the increments in cost:

\[
U = \int_{0}^{T} \frac{dc}{dt} e^{-rt} \, dt
\]

\[
= \int_{0}^{t_1} c(0)e^{(j_1-r)t} \, dt + \ldots + \int_{t_{N}}^{T} J_{N}C(0)e^{(j_{N}-r)t} \, dt
\]
where $J_i = \text{average annual rate of growth and } T = \text{the year in which } c(t) \text{ reaches the cost of the backstop. The result depends, of course, upon the interest rate. For an interest rate of 10\%, the user cost is five percent of current costs. If the real interest rate is only 5\%, the user cost rises to 10\% of current cost in the above example.}

Low-sulfur coal, unfortunately, is more difficult to treat. A very large proportion of the low-sulfur coal produced (80\%) is used for the manufacture of coke. As such, it is valued not for its heat value, but for its carbon. There are at present techniques for iron reduction under development which rely on hydrogen and not carbon. These can serve as the back-stop technology for coke manufacture. However, the relevant resource base includes much more than coal since petroleum and even wood are sources of carbon. A calculation of user cost here would take us far afield.

In summary, the derivation of the cumulative cost curve is one element in the calculation of rent. Here we have provided all illustrative calculation for high-sulfur coal, but an actual calculation of user cost calls for more complete specification of the demand functions, how they will shift over time, and more precise specifications of new technologies.

Rising Factor Prices

Costs will be affected by rising factor prices as well as by depletion. Wages can be expected to rise in real terms. Increases will occur in order to attract workers to an expanding industry. Furthermore, the United Mine Workers Union is in a position to raise wages as prices of substitute fuels rise. No one knows how fast wages will rise or how high they will go. However, because of the labor intensity of production, real wage increases can lead to significant increases in cost. The cost function of Chapter I
can be used with any rate of wage increase.

In short, many factors impact upon the costs of coal. Only one is a natural phenomenon—depletion. Policy choices will also affect the price, forcing more rapid movement up the long-run cost curve. Previous analyses, based on a misinterpretation of reserve data, have ignored these policy questions. Technological change has not been treated here, but in the longer term new techniques can be expected to work against depletion and rising costs.

Summary

We have taken a very disaggregated view of long-run mineral supply, in this case, by an examination of costs and reserves of coal. This procedure allows a direct consideration of depletion, something difficult to capture in purely econometric approaches to mineral supply. We have seen that cost functions can be constructed using engineering data. These functions can then be combined with reserve data. Finally, prediction of costs must consider changing factor prices, as well as depletion. And in the longer-run technological change can be an important factor.
Footnotes

1. This information comes from a private engineering consulting firm.


4. The distribution \( \phi(\log Th) \) is the original distribution of seam thicknesses before mining. However, depletion of original resources has been small, and the present distribution is taken as an approximation of the original.

5. The total underground reserves in seams 28" or more in the measured and indicated categories is given by the Bureau of Mines as 2.2 billion tons. This is larger than the reserves of the coal-producing states of Alabama, Virginia, Tennessee and Maryland. It represents 73% of the reserves of Eastern Kentucky. U.S. Bureau of Mines, Information Circular 8655, The Reserve Base of Bituminous Coal and Anthracite for Underground Mining in the Eastern United States, 1974.

6. There is a problem of the proper level of aggregation. Too small a county would clearly not represent a lognormal. Pike County is large enough to represent a lognormal distribution. When, however, individual counties are aggregated the resulting distribution is the sum of lognormals, which is not lognormal. There is therefore some distortion involved in applying the lognormal to the largest of the coal producing states. Future work can divide the larger areas into smaller units for which the lognormal is the proper characterization.

7. Aitchison and Brown, op. cit.

8. In any state, the level of aggregation used here, transport differentials will not be large.

9. The number of sections, \( S^* \), is two. This results from equations 16a and 27, Chapter 5.


11. For the states where at least three new mines were in the sample of Chapter II, separate estimates of \( C \) were calculated. For the others the regional \( C \) was used.

12. The relevant tonnage must account for coal that will be lost in the mining process and for coal that is inaccessible because it lies under streams, towns, in multiple coal beds where underlying or overlying...
seams have been mined, etc. The recovery ratio in underground mines averages 57%. To reflect the other losses, the usual convention is to use a 50% overall recovery ratio. See, Averitt, op. cit., p. 29.

12. The inferred totals have been excluded from the reserve base as estimated by the Bureau of Mines in The Reserve Base of Bituminous Coal and Anthracite for Underground Mining in the Eastern United States, Information Circular 8655. These totals were excluded because of their highly conjectural nature. In the Eastern United States the unexplored areas are likely to be small. Furthermore, those areas not explored are likely to have been left untouched because of the inaccessibility of the area. The net result of this omission is to overstate to some degree the rate of depletion at higher cost levels since for the reason mentioned, these areas are likely to represent higher cost deposits.


14. A common error of other work in this area, particularly the National Petroleum Council study, is to assume regional relationships remain constant. Because of the different supply elasticities in the various regions, these relationships will change over time.

15. Output of low-sulfur coal in 1970, the only year for which such data are available, was 160 million tons. Recent estimates of the "deficit" of low-sulfur coal meeting the EPA standards was 225 million tons. Furthermore, coal with sulfur levels of 1% is above the approximately .7% standard of the EPA. For estimates of this "deficit" see Coal Week, McGraw-Hill, April 28, 1975, p. 5.


Appendix

There is a simple test of the hypothesis of lognormality of this error term. Since the new mines of Chapter II are equal in cost, the distribution of thicknesses should be the following:

\[ Th = \left( \frac{K}{\xi \epsilon} \right)^{-\frac{1}{\xi}} \]

where \( \xi \) is the cost. Since \( \epsilon \) is lognormal, this distribution is lognormal. Plotting the distribution of seam thicknesses on logarithmic-probability paper indicates that lognormality is a good approximation. The percentiles are calculated by listing the thickness of each mine in increasing order. If there are \( N \) mines, each mine is taken as corresponding to the \((1/N+1)\) percentile. The results are seen in the diagrams 3 through 7.
Figure 3
Southern Appalachian Law-Sulfer Drift Mines

PERCENTILE

SEAM THICKNESS (INCHES)
Figure 4

Southern Appalachian Low-Sulfur Shaft Mines

PERCENTILE

SEAM THICKNESS (INCHES)
Figure 6
Northern Appalachian High-Sulfur Drift Mines

PERCENTILE

SEAM THICKNESS (INCHES)
Figure 7
Illinois/West Kentucky High-Sulfur Shaft Mines

PERCENTILE

99.99
89
79
69
59
49
39
29
19
0.01

SEAM THICKNESS (INCHES)

0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100