Modeling Conventional and Pumped Hydro-Electric Energy Using Booth-Baleriaux Probabilistic Simulation

by

S. Finger

Working Paper No. MIT-EL 75-009WP

August, 1975

ACKNOWLEDGEMENT:

The work in this paper was supported by Northeast Utilities Service Company.

TABLE OF CONTENTS

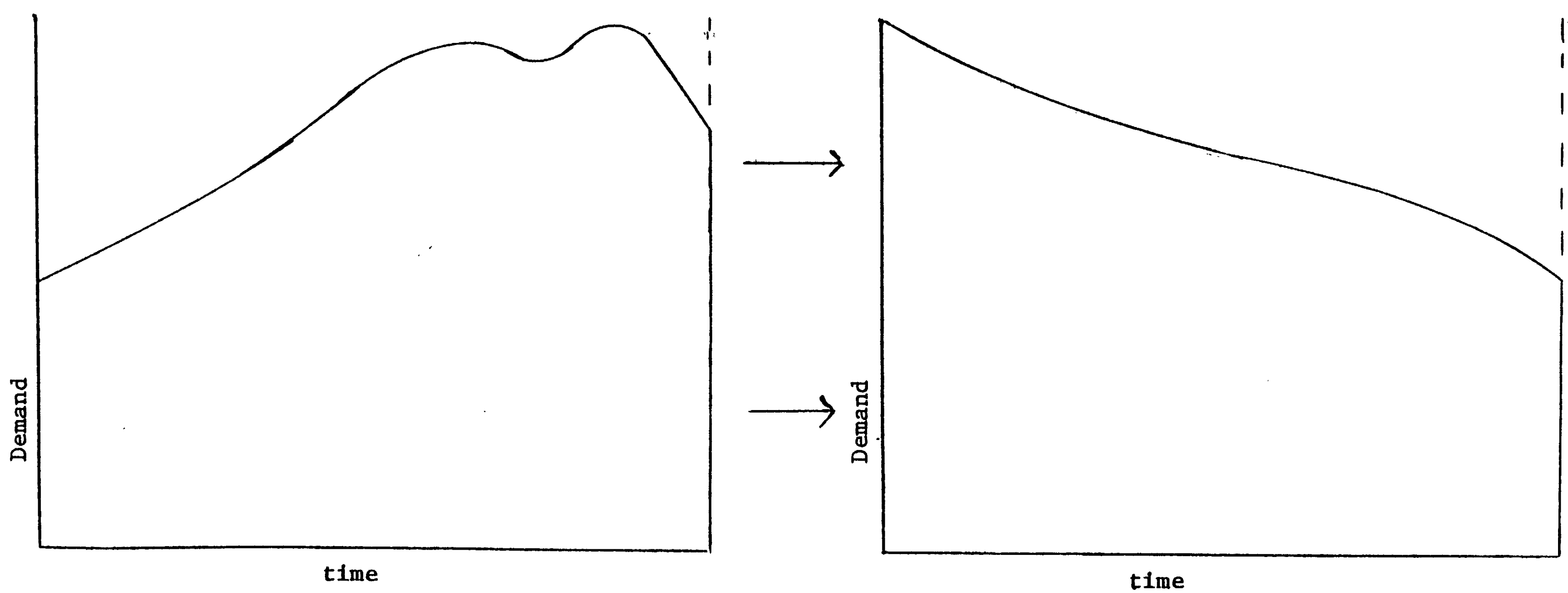
I. Introduction

Booth-Baleriaux probabilistic simulation models the operation of an electrical power system. The two major outputs are the expected energy generated by each unit and the loss of load probability for a given time period. Prior to the introduction of probabilistic simulation, a deterministic model was commonly in use. The deterministic model is easier to implement, but it does not give accurate results because the forced outage rates of units are not accounted for in a realistic manner. Studies using this deterministic method tend to underestimate the use of peaking units. However, to understand how conventional and pumped hydro are modeled using Booth-Baleriaux simulation, it is instructive to begin with the deterministic model.

II. Deterministic Model

A. Thermal and Nuclear Energy

The objective of the planning study is to find the operation schedule with minimum cost which meets the customer demand. The customer demand is a time-dependent stochastic variable. A typical curve is given in Figure 1a. For most long range planning studies, the time-dependent load curve is simplified by converting it into a load duration curve as shown in Figure 1b. The horizontal axis gives the total amount of time that a given demand level occurs. Although some detail is lost, the new load duration curve is easier to work with than the time-dependent curve, particularly for time periods longer than one day. Frequently, the load duration curve is normalized to give the percent of time that a load occurs.



1a. Time-Dependent Load Curve for a Typical Day

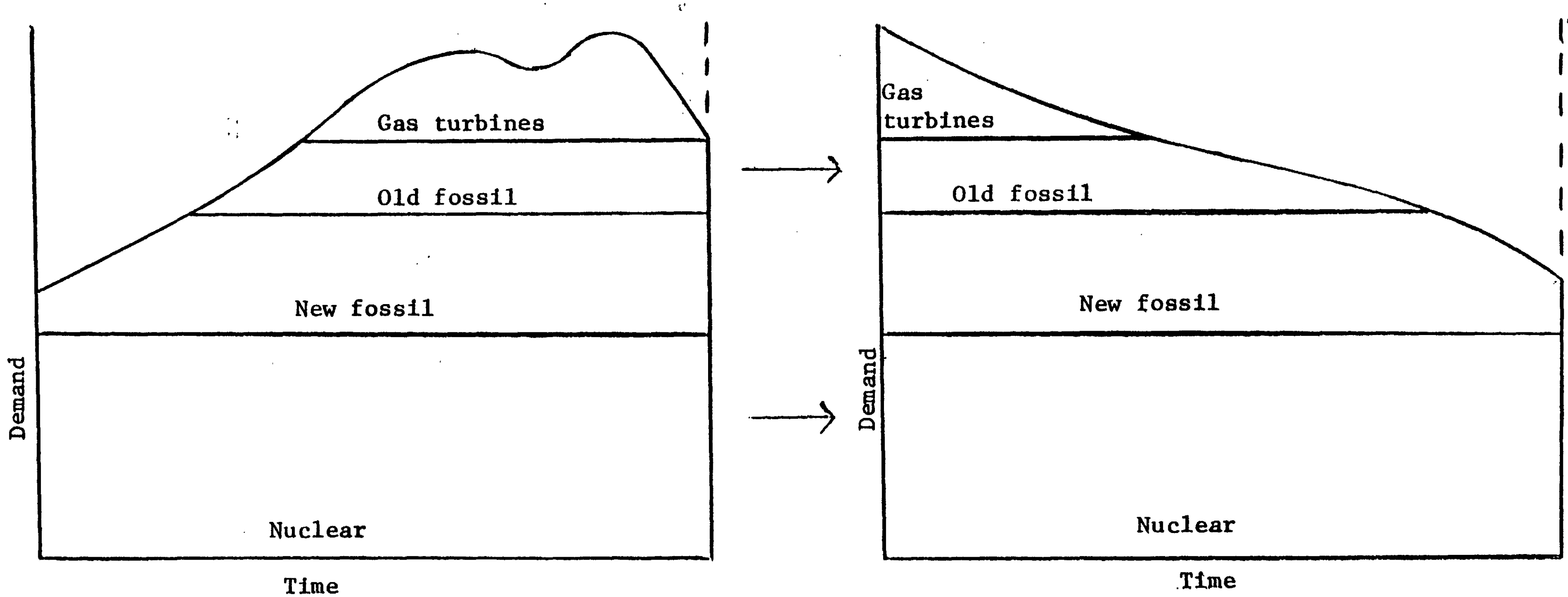
1b. Load Duration Curve of 1a.

Figure 1. Conversion of Time Dependent Curve to Load Duration Curve

An operating schedule for the system is determined by filling in the area under the time-dependent load curve that corresponds to the energy supplied by each unit, as shown in Figure 2a. The height of the area is constrained by the capacity of the unit. From Figure 2a, it is clear that if an on-line unit is always run at the maximum possible capacity, such that the load is just met, then the unit is always loaded and off loaded at the same demand level. This demand level is called the loading point of the unit. As before, the time-dependent load curve can be converted into the load duration curve. The loading point remains unchanged and the total energy supplied by a unit can be calculated by finding the area between the loading point of the unit and the loading point of the next, on the load duration curve as shown in Figure 2b.

In the deterministic method, a unit's capacity on the load duration curve is usually reduced to reflect the random outages of the unit during the operating period. This assumes that the unit is always available at its derated capacity, or equivalently that it has a forced outage rate of zero at its derated capacity. In fact, the unit is not always available. When a unit does fail, more expensive generation must be brought on line to replace it. Since the deterministic model assumes that units never fail, it tends to underestimate the energy supplied by more expensive units.

The minimum cost operating schedule can be found by bringing up the units (or incremental blocks) in order of increasing operating cost. In this way, the least expensive units run for the greatest length of time. Most costly plants are used only for short periods when the demand is high. The final loading sequence of plants, from least to most expensive, is called the economic loading order. The complete schedule is found by bringing the first unit up to its derated capacity and running it 100% of the time as a base loaded unit. Since there is still unmet demand, the unit



2a. Typical Operating Schedule

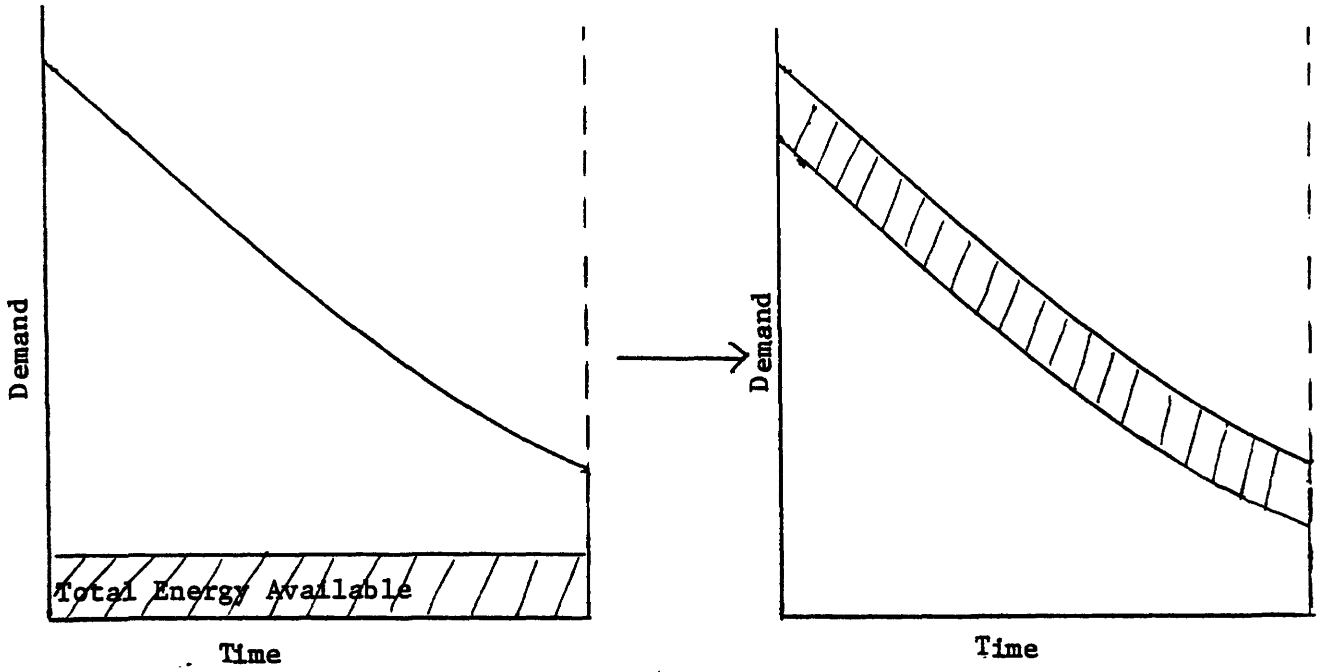
2b. Equivalent Schedule on a Load Duration Curve

Figure 2. Deterministic Operating Schedule

with the next lowest cost is brought on line. This process is continued until all the area under the load duration curve has been filled in. The total cost of the system operation can be computed by multiplying each unit's total megawatt-hours by the cost per megawatt-hour for that unit and then summing the costs over all units.

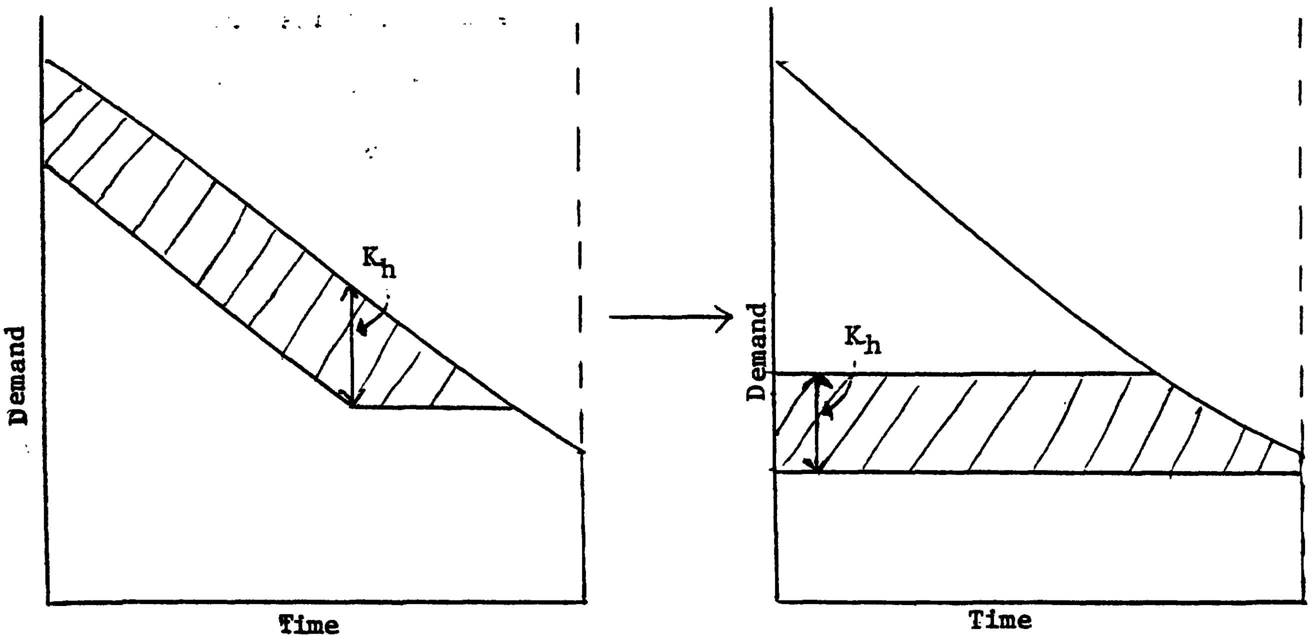
B. Conventional Hydro-electric Energy

The inclusion of conventional hydro-electric power complicates the problem of finding the minimum cost operating plan. The marginal cost of conventional hydro is essentially zero, since there are only operating costs, and no fuel costs. This implies that conventional hydro plants should be first in the economic loading order. However, the total amount of conventional hydro energy available is limited by the river flows and the reservoir size. Usually, the total energy is not sufficient to run the hydro unit 100% of the time at full capacity. There are several possible strategies for discharging all of the hydro energy. One possibility is to load the conventional hydro first, reducing the capacity until the area under the curve is equal to the total energy available. From Figures 3a and 3b, it is clear that this is equivalent to removing the same area from the top of the curve. But because the last units to be loaded are the most expensive to run, the operating cost would be reduced if as much area as possible were removed from the top of the curve. This can be achieved by removing the free hydro energy at full capacity as shown in Figure 3c. This is equivalent to finding the loading point for the hydro-electric unit such that, run at full capacity, the hydro energy is exactly equal to the area under the load duration curve. Figure 3d shows the final result of these manipulations. These manipulations



3a. Hydro Capacity Reduced to Run Unit as Base Loaded

3b. Energy Discharged at Reduced Capacity for Peak Shaving



3c. Equivalent Energy Discharged to Remove Maximum Energy from the Peak K_h = Hydro Capacity

3d. Equivalent Loading Point to Remove Maximum Energy from the Peak

Figure 3. Equivalent Loading Point for Conventional Hydro Units

are necessary because it is not possible to subtract the energy from the top of the curve in the probabilistic model which needs the equivalent loading point. The logic for this point is easier to understand in the deterministic model.

In the process of finding the optimal loading point for the conventional hydro unit, it may be necessary to reduce the running capacity of the previously loaded plant. Because the conventional hydro energy is essentially free, it is always less expensive than the energy it is replacing at its loading point. The remainder of the other unit's capacity can be loaded after the conventional hydro energy has been discharged.

C. Pumped Hydro-electric Energy

The energy stored as pumped hydro is generated by units which are low in the economic loading order, but which are not needed 100% of the time to meet the direct demand. Thus, an artificial demand is placed on these base-loaded units by units which pump water into a reservoir. This stored energy can be released during periods of high demand when more costly units would normally be generating. Since the pumping and generating operations are not completely efficient, the energy available to meet demand using pumped hydro units is less than the energy generated by the base loaded units for pumping. Pumped hydro energy is similar to conventional hydro in that the amount of energy available is limited. However, modeling pumped hydro is complicated by the fact that the energy is not free and that the energy is generated on one part of the curve and discharged on another.

The total energy potentially available from a base loaded unit for pumped hydro can be found by computing the area above the load duration curve for the base loaded unit. Due to the limited capacity of the pumping unit, some of this energy may be unavailable (see Figure 4a). Another limiting factor is the size of the reservoir. When the energy above the curve, subject to the limited pumping capacity and the pumping inefficiency, is equal to the storage capacity of the reservoir, then pumping stops. Taking into account the inefficiencies of generating from pumped storage, the total energy available to meet customer demand will be about two-thirds the energy used for pumping. This results in a marginal generating cost about one and a half times that of the base-loaded unit used for pumping.

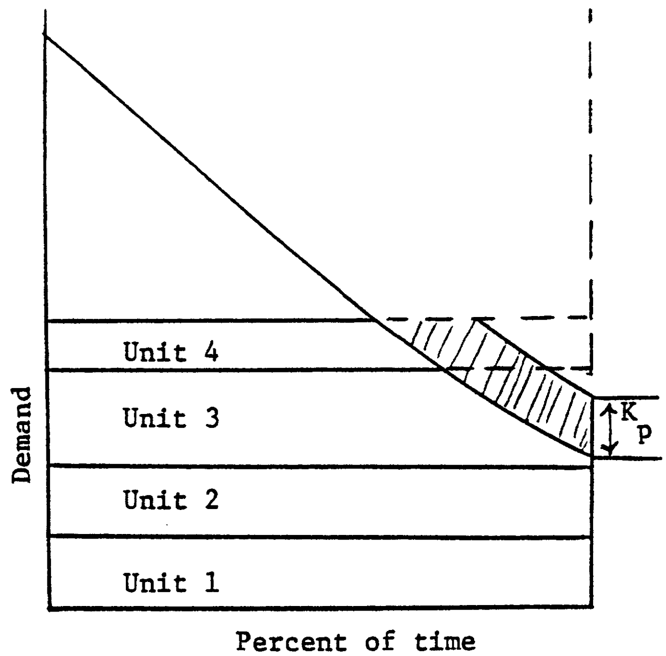
Depending on the system and the shape of the load curve, several base-loaded units may fill a single reservoir, or one base-loaded unit may fill several reservoirs. For the deterministic case, the marginal cost of the pumped hydro will be taken to be the average of the base-loaded costs (with the inefficiencies factored in) weighted by the amount of energy each base-loaded unit provides.

Once the pumped storage increments have been sorted into the economic loading order and the first loading points has been reached, two possibilities can arise. Either the pumped hydro unit has sufficient energy to discharge at full capacity, or it does not. A proof is given in Section G showing that, in the deterministic case, the operating cost of the system is reduced if the pumped hydro is delayed in the loading order until the demand can be met by using the pumped hydro at full capacity. The argument is analogous to the one given for conventional hydro, even though the energy is no longer free. An illustration of the loading of pumped hydro is given in Figure 4.

III. Probabilistic Model

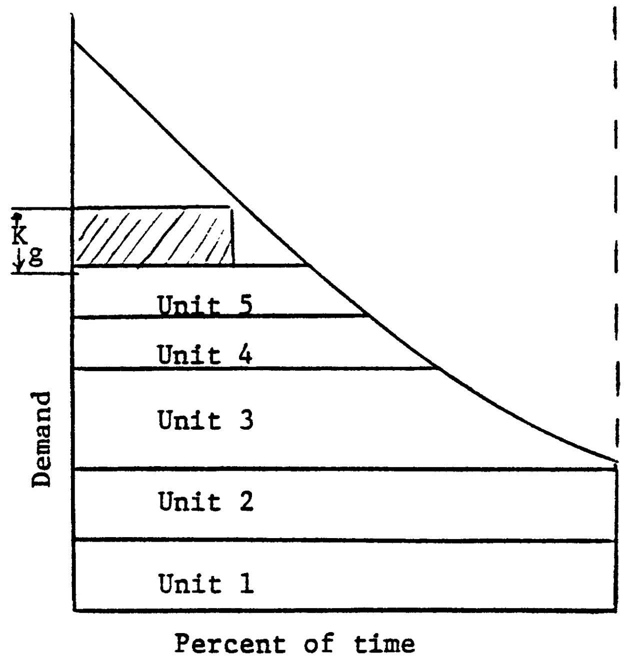
A. Introduction

The probabilistic model presented here was first developed by Baleriaux and Jamoulle in 1967. It was not widely used in the United States until the



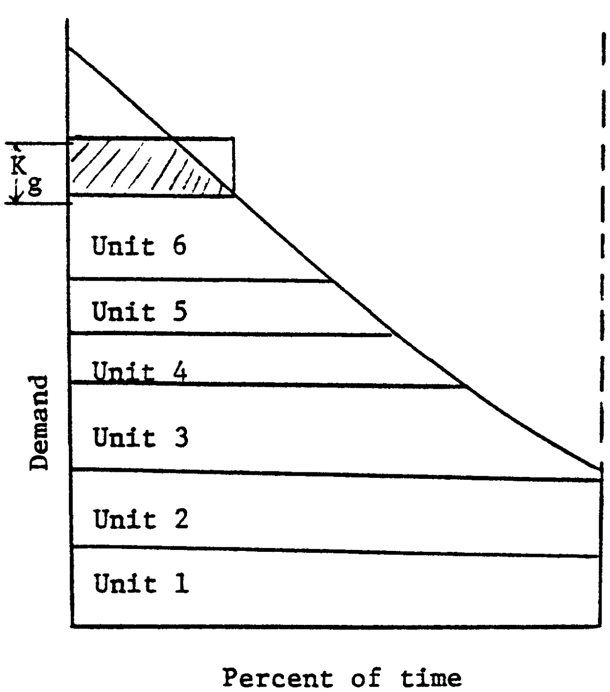
4a. Shaded area is energy available for pumped hydro.

k_p = pumping capacity

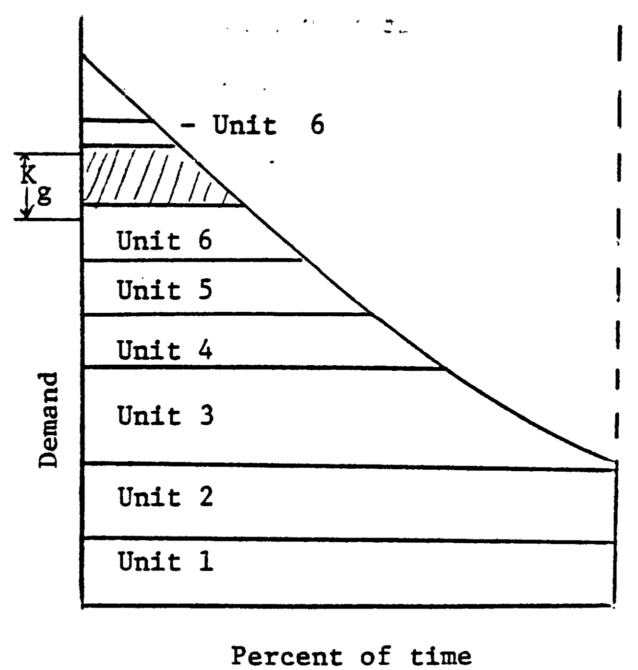


4b. Pumped hydro unit is next in the economic loading order. Energy is reduced by pumping/generating inefficiencies.

K_g = generating capacity



4c. Loading of the pumped hydro is postponed until the energy available exceeds the energy demand



4d. Unit 6 is loaded in increments to allow maximum discharge of pumped hydro.

Figure 4. Loading Sequence for Pumped Hydro

early seventies when Booth wrote several papers clarifying the model. This paper does not follow the original logic used by Baleriaux and Booth, but instead uses a more intuitive approach developed by Deaton at M.I.T. References are given at the end of this paper.

B. Nuclear and Thermal Energy

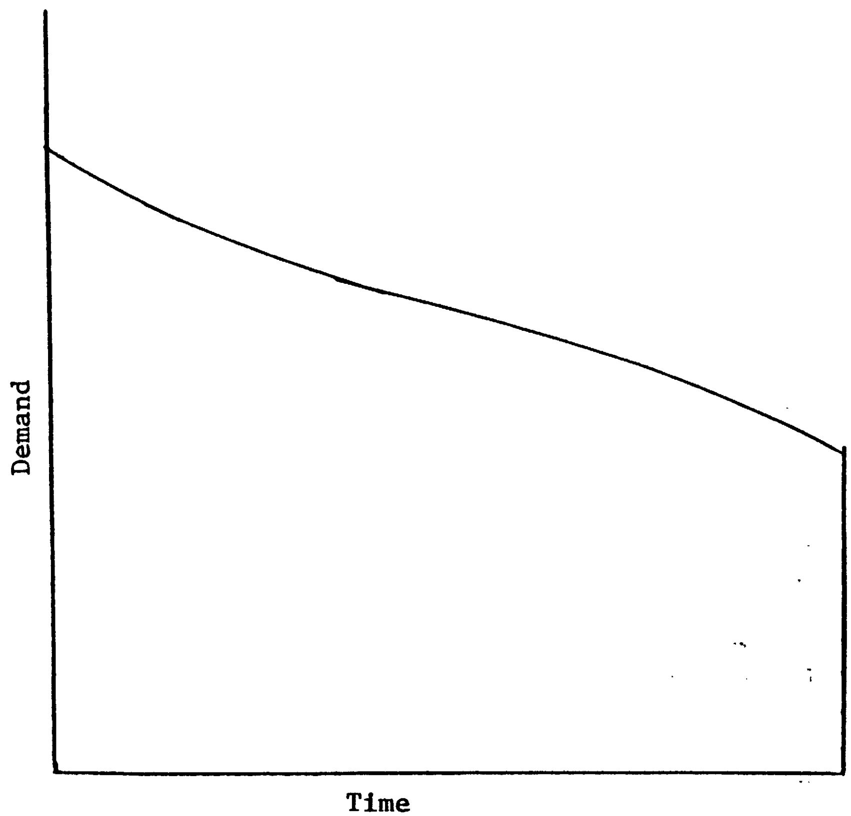
1. Single Increment Logarithm

In Booth-Baleriaux simulation, the power demand on the unit is redefined to be the sum of the direct customer demand and the demand due to outages of previously loaded units. Both of these quantities can be considered as random variables. The load duration curve can be interpreted as a form of the cumulative probability distribution function (CDF) for the customer demand. The probability density function for demand due to forced outage is found from the curve giving the unit's forced outage rate as a function of its running capacity. Using these two curves, the CDF for the equivalent demand can be found using convolution.

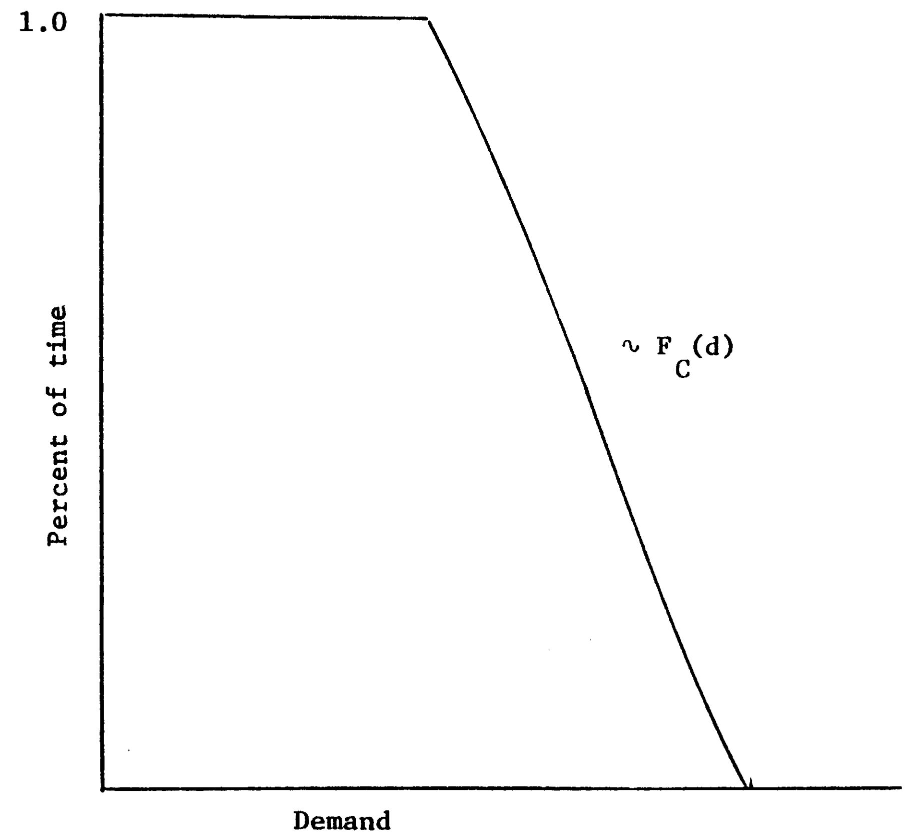
To obtain the CDF for customer demand from the load duration curve, the axes are rotated and the time period is normalized to give percent of time. These operations are shown in Figure 5. The percent of time that a given load level occurs can be interpreted as a probability. That is, there is a probability of one that the load will be greater than the minimum load at any given time and a probability of zero that it will be greater than the maximum load at any given time. The resulting curve is not the cumulative probability distribution function, but one minus the cumulative. The following notation will be used:

$$G_C(d) = Pr(D_C \leq d) = \int_{-\infty}^{d} f_C(x)dx$$

$$F_C(d) = 1 - G_C(d) = Pr(D_C > d) = \int_{d}^{\infty} f_C(x)dx \qquad (1)$$



5a. Original load duration curve



5b. Inverted load duration curve.

Figure 5. Conversion of Load Duration Curve to Cumulative Probability Curve

$G_C(d)$ is the CDF of the customer demand. $F_C(d)$ is the normalized customer load duration curve, and $f_C(d)$ is the probability density function for the customer demand. The subscript of the function identifies the variable which it describes.

By definition, the equivalent load, D_E, is the sum of two random variables:

$$D_E = D_C + D_F \tag{2}$$

where D_C is the direct customer demand and D_F is the demand due to forced outages of units already dispatched. From probability theory, the CDF for the sum of two random variables is given by:

$$G_E(D_E) = \int_{-\infty}^{\infty} \int_{-\infty}^{D_E-D_F} f_{C,F}(D_C,D_F)\,dD_C\,dD_F. \tag{3}$$

The function $f_{C,F}(D_C,D_f)$ is the joint probability density function of the customer demand, D_E, and the forced outage demand, D_F. We can assume these two random variables are independent, which implies that:

$$f_{C,F}(D_C,D_F) = f_C(D_C)f_F(D_F). \tag{4}$$

Using Equation (4), equation (3) can be simplified to:

$$G_E(D_E) = \int_{-\infty}^{\infty} f_F(D_F)\,dD_F \int_{-\infty}^{D_E-D_F} f_C(D_C)\,dD_C. \tag{5}$$

Using the definition of the cumulative distribution function given in Equation (1), Equation (5) becomes:

$$G_E(D_E) = \int_{-\infty}^{\infty} f_F(D_F)G_C(D_E - D_F)\,dD_F. \tag{6}$$

For the case in which the forced outage rate is a discrete random variable, the integral over the probability density function $f_F(D_F)$, can be replaced by the sum over the probability mass function. For a plant with forced outage rate, q, and capacity K, this probability mass function is given by:

$$p_F(D_F) = \begin{cases} p & \text{if } D_F = 0 \\ q & \text{if } D_F = K \end{cases} \tag{7}$$

where p + q = 1. That is, there is a probability, q, that the plant will not perform and the demand due to forced outage will be the capacity of the plant. There is a probability, p, that the plant will perform and the demand due to forced outage will be zero.

Replacing the integral with the sum, Equation (6) becomes:

$$\begin{aligned} & G_E(D_E) = pG_C(D_E) + qG_C(D_E-K) \\ \text{or} \quad & F_E(D_E) = pF_C(D_E) + qF_C(D_E-K). \end{aligned} \tag{8}$$

This final equation gives the new equivalent load duration curve. Figure 6 illustrates how this curve is found using convolution. This curve can be used in much the same way the original load duration curve was used in the deterministic model, except that a new curve must be computed each time another unit is brought on-line.

Units are loaded starting from the left of the equivalent load duration curve. The demand on the first base loaded unit to be brought up is the entire customer demand. There are no outages from previous units, so

$$D_{E1} = D_C \tag{9}$$

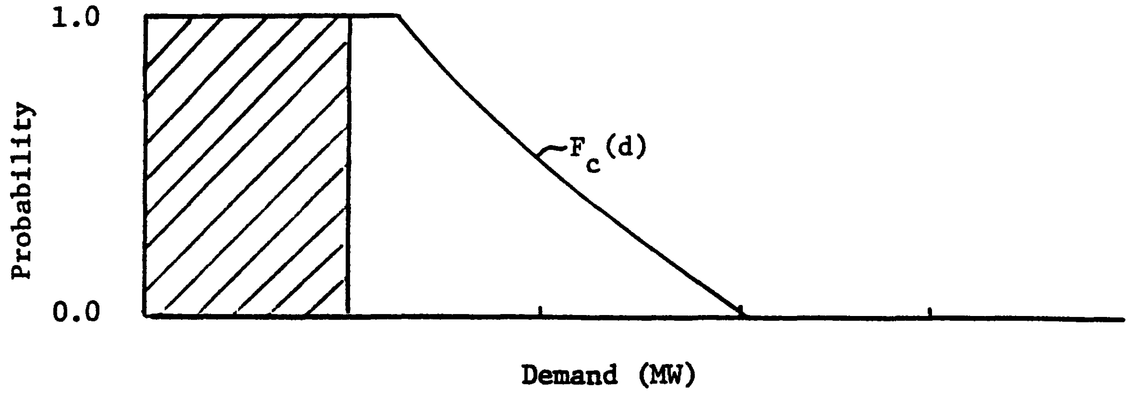


Figure 6a. Loading of a 500 MW plant with 90% availability. Expected Capacity (shaded area) = 500 MW. Expected Energy = Expected Capacity x Availability x Hours = 75600 MWH/week.

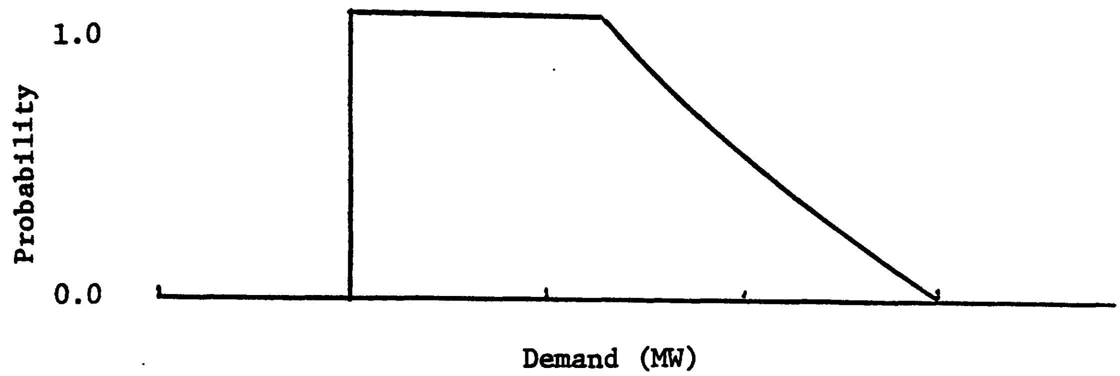


Figure 6b. If the first plant fails, the second plant "sees" the original customer curve. This event has a probability of .10.

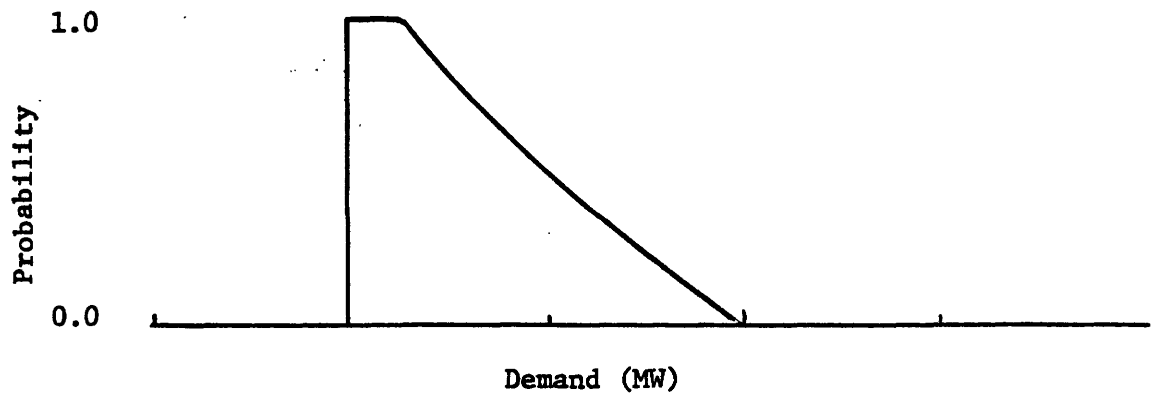


Figure 6c. If the first plant operates, the second plant does not "see" the first 500 MW of customer demand. This event has a probability of .90.

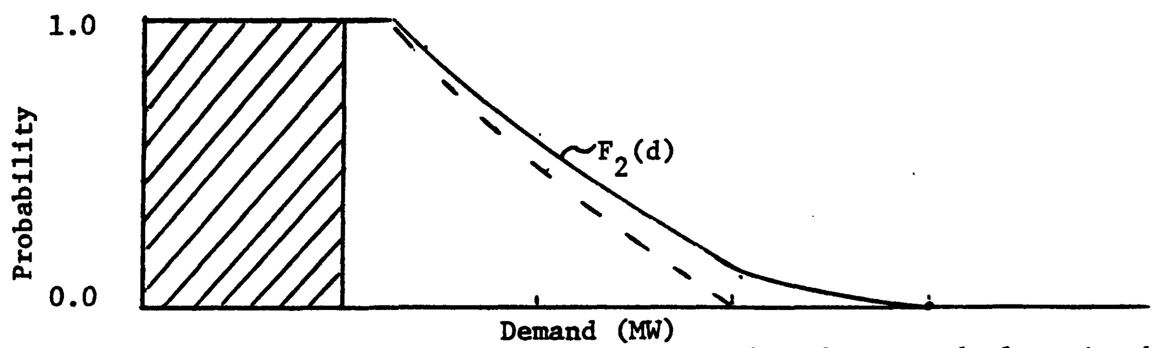


Figure 6d. The equivalent load curve for the second plant is the sum of the two curves weighted by their respective probabilities.

where D_{E1} = equivalent demand on the first unit

D_C = total customer demand.

Because the two random variables, D_{E1} and D_C, are equivalent, their distribution functions are the same:

$$F_{E1}(d) = F_C(d)$$

$$\text{and} \qquad 0 \le D_{E1} \le D_{max} \tag{10}$$

where D_{max} is the peak customer demand. $F_C(d)$ is just the original load duration curve.

In the deterministic model, a unit was loaded onto the system by filling in the area under the load duration curve. The area gave the KWhrs generated per hour. The total energy was found by multiplying by the length of the time period. To load a unit in the probabilistic model, the area is again filled in. The vertical axis, instead of being the percent of time that a unit is operating at a given capacity, is now the probability that a unit is on-line and operating at that capacity at any given time. Taking the integral over the capacity gives the expectation of the running capacity for the unit at any given time. (A proof is given in section F.) This expected capacity for the first unit is:

$$E(C_1) = \int_0^{K_1} F_C(x)dx \tag{11}$$

where K_1 = nameplate capacity of the first unit

C_1 = random variable describing the running capacity of the first unit.

This is the expected capacity required to meet the equivalent load, without considering the availability of the unit. The total expected energy from the first unit, taking outages into account, is:

$$MWH_1 = p_1 \cdot T \cdot E(C_1) \tag{12}$$

where p_1 = total availability of unit one including forced and planned outages

T = total length of the time period in hours.

The capacity factor, the ratio of running capacity to nameplate capacity, is given by:

$$CF_1 = p_1 \cdot E(C_1)/K_1. \tag{13}$$

The equivalent demand on the second unit to be brought up is the customer demand plus the demand due to the outages of the first unit:

$$\begin{aligned} D_{E2} &= D_C + D_{F1} \\ K_1 &\leq D_{E2} \leq D_{max} + K_1. \end{aligned} \tag{14}$$

Because of the way the equivalent load is defined, the loading point of the second unit on the equivalent load duration curve is the same whether or not the first unit has failed. If the first unit fails, it creates a demand, K_1, so the second unit is loaded when the equivalent demand is K_1. If the first unit had not failed, there would have been no demand due to outage. The first unit would have supplied the demand until the demand exceeded K_1, at which point the second unit would have been loaded.

Equation (8) gives the equivalent load curve for D_{E2}:

$$F_{E2}(d) = p_1 F_C(d) - q_1 F_C(d-K_1). \tag{15}$$

Having used the load duration curve to find the equivalent load curve for the second unit, the expected capacity, the capacity factor, and total energy can be obtained:

$$E(C_2) = \int_{K_1}^{K_1+K_2} F_{E2}(x)dx$$

$$CF_2 = p_2 \cdot E(C)_2/K_2$$

$$MWH_2 = p_2 \cdot T \cdot E(C_2). \tag{16}$$

The loading of the second unit is shown in Figure 6.

Moving on to the third unit, the equivalent load is given by:

$$D_{E3} = D_C + D_{F1} + D_{F2}$$

$$K_1 + K_2 \leq D_{E3} \leq D_{max} + K_1 + K_2. \tag{17}$$

Using the definition of D_{E2} in Equation (14):

$$D_{E3} = D_{E2} + D_{F2}.$$

The distribution for D_{E2} has already been found, so convolution can be used again to find the distribution of D_{E3}:

$$F_{E3}(d) = p_2F_{E2}(d) + q_2F_{E2}(d - K_2)$$

$$E(C_3) = \int_{K_1+K_2}^{K_1+K_2+K_3} F_{E3}(x)dx \tag{18}$$

$$CF_3 = p_3 \cdot E(C_3)/K_3$$

$$MWH_3 = p_3 \cdot T \cdot E(C_3).$$

In general,

$$D_{Er} = D_C + \sum_{i=1}^{r-1} D_{F,i}$$

or

$$D_{Er} = D_{E,r-1} + D_{F,r-1}$$

$$F_{Er}(d) = p_{r-1}F_{E,r-1}(d) + q_{r-1}F_{E,r-1}(d-K_{r-1})$$

$$E(C_r) = \int_{TK_r}^{TK_r + K_r} F_{Er}(x)\, dx \tag{19}$$

$$CF_r = p_r \cdot E(C_r)/K_r$$

$$MWH_r = p_r \cdot T \cdot E(C_r).$$

where r = loading order of the plant.

TK_r = sum of the capacities of the r-1 plants loaded before r.

2. Multiple Increment Algorithm

In the derivation of the equation for the equivalent load, it was assumed that the unit would always be brought to full capacity. In reality, units are frequently brought up to full load in increments. If each increment has a discrete probability of failing, then the probability mass function looks like:

$$p_F(D_F) = \begin{cases} p_r & \text{if } D_F = 0 \\ q_{ri} & \text{if } D_F = K_{ij} \quad j=0,1\ldots J \end{cases}$$

and

$$p_r + \sum_{j=0}^{J} q_{rj} = 1 \tag{20}$$

$$\sum_{j=0}^{J} K_{rj} = K_r$$

where J is the total number of increments of the unit under study.

Before performing the convolution using the new probability mass function for outages, it is necessary to examine the definition of equivalent load:

$$D_{Er} = D_C + \sum_{i=1}^{r-1} D_{Fi}. \tag{21}$$

Included in the demand due to forced outages are outages of increments of plant r that were lower in the loading order. But if a lower increment of a unit failed, then a higher increment will not be available. A lower increment cannot place an outage demand on a higher increment. To account for this, the demand due to forced outage of any earlier increments is removed from the system before the increment is added. The equivalent demand on the k^{th} increment of the r^{th} unit is given by:

$$D_{Erk} = D_{E,r-1} + D_{F,r-1} - \sum_{j=1}^{k-1} D_{Frj}. \qquad (22)$$

To compute the distribution of this random variable, it is easier to consider the system with all of the increments of unit r convolved into the equivalent load. Because the order in which random variables are convolved does not affect the final distribution, to include the k^{th} increment of a unit, one can assume that the k-1 increments were the last ones added to the system. Combining equations (6) and (20), the equivalent load duration curve for the k^{th} increment is:

$$F_{Erk}(d) = p_r F_{E,r-1}(d) + \sum_{j=0}^{k-1} q_{rj} F_{E,r-1}(d-K_{r,k-1}-K_{rj}). \qquad (23)$$

Now suppose we have the equivalent load duration curve for the k^{th} increment. In order to load the k^{th} increment, the outages of the k-1 previous increments have to be deconvolved. Rearranging Equation (23) gives:

$$F_{E,r-1}(d) = \frac{1}{p_r} \left[F_{Erk}(d) - \sum_{j=0}^{k-1} q_{rj} F_{E,r-1}(d-K_{r,k-1}-K_{rj})\right]. \qquad (24)$$

$F_{Erk}(d)$ is the equivalent load curve for the k^{th} increment to be loaded. Equation (24) is used to remove all of the outages of plant r from this curve. Points of the curve can be evaluated even though $F_{E,r-1}(d)$ appears on both sides of the equation. The curve is evaluated starting at $d = 0$. Since $F_E(d)$ always has a value of one for a negative number (i.e., the load is always greater than zero), the right hand side can be evaluated. Through an iterative process, the entire curve can be constructed from left to right.

The k^{th} increment of the r^{th} unit is then loaded back onto the system under the equivalent load curve $F_{E,r-1}$. After the expected energy and capacity factors have been calculated, all k increments are convolved back into the system and the next plant is loaded.

It should be noted that the order of convolution does not change the distribution of the sum of random variables. The increments are considered individually rather than in a group in order to find the proper loading points and expected energies.

C. Conventional Hydro-electric Energy

The treatment of conventional hydro using probabilistic simulation will not be that different from the treatment in the deterministic case. It will be more difficult to reduce the capacity of earlier plants and to compute the area under the curve. At each successive loading point, a test is performed on the feasibility of bringing up the hydro unit. The total energy demand on a unit with a capacity K_h can be found using the current equivalent load duration curve. The total energy demand on the hydro unit is given by:

$$DMWH_h = T \int_{TK}^{TK + K_h} F_{Er}(x)dx. \qquad (25)$$

$F_{Er}(d)$ is the equivalent demand on the next unit to be brought up. Equation (25) is used to find the energy demand on that unit if it were the conventional hydro unit. If $DMWH_h$ is greater than the available hydro energy, then the unit is not loaded. The r^{th} plant in the economic order is loaded instead. If the total energy demand is less than or equal to the available hydro energy, then the conventional hydro unit is loaded.

If one assumes that thermal plants are run only at valve points, then the process is simplified since one has only to find the first loading point at which the available energy is greater than the energy demand. If, however, one wanted to find a lower bound on operating costs, then a procedure would have to be followed which allowed loading or off-loading at any MW level.

To make the most efficient use of the free hydro energy, the previously loaded unit should be off-loaded until the total energy demand balances the hydro energy available. However, changing the capacity of the $r-1^{st}$ unit changes the shape of the equivalent load demand curve for the r^{th} unit. Equation (25) can be rewritten using Equation (19) and changing the capacity of the last unit to K'_{r-1}:

$$DMWH_h = T \cdot p_{r-1} \int_{TK'}^{TK'+K_h} F_{E,r-1}(x)dx + T_{q,r-1} \int_{TK'}^{TK'+K_h} F_{E,r-1}(x-K'_{r-1})dx \qquad (26)$$

where

$$TK' = \sum_{i=1}^{r-2} K_i + k'_{r-1}.$$

If MWH_h is the actual energy available from conventional hydro, we wish to find K'_{r-1} such that $DMWH_h$ is identically equal to MWH_h. This involves solving Equation (26) for K'_{r-1} which is an argument in the limits of the integrals and in the integrand of the second integral. Rather than solving this analytically, we will simply remove the last unit and start adding it in small steps until the energies are balanced to within a set tolerance.

Given the equivalent load duration curve for $F_{Ek}(d)$, the curve for $F_{E,r-1}(d)$ can be found using deconvolution:

$$F_{E,r-1}(d) = \frac{1}{p_{r-1}} [F_{Eh}(d) - q_{r-1}F_{E,r-1}(d-K_{r-1})]. \tag{27}$$

Once $F_{E,r-1}(d)$ is computed, the capacity of $r-1^{st}$ plant can be added in steps. Let K_{r-1} be the size of the k^{th} step. Each time an increment is added, a new load curve is computed and the right hand side of Equation (26) is evaluated. As soon as the value is greater than MWH_h, the current load curve which includes the capacity up to and including the i^{th} step is saved. The conventional hydro unit is loaded under this curve. Its expected energy and capacity factor are computed. It is then convolved into the system to give the equivalent demand on the remaining capacity of the off-loaded unit. In order to load the remaining capacity the outages of the earlier portion have to be removed using the multiple increment algorithm. The final curve is the same as if both units had just been convolved in. The intervening steps were needed to find the proper loading point for the hydro unit.

D. Pumped Hydro-electric Energy

Once the energy available to pumped hydro has been computed, its treatment is very similar to conventional hydro. However, there are additional problems with pumped hydro in that one has to; (1) compute the expected excess energy available from base loaded units, given that each such unit has a probability of failure; (2) compute the probability that a pumped hydro unit generates a given amount of energy, that is, that it has sufficient water available and that the generator does not fail; and (3) compute the expected cost of the pumped hydro energy.

From the equivalent load curve (Figure 6), one can see that the base loaded plants will frequently have excess capacity available for pumped hydro. (Recall that the area under the curve is the expected capacity, not energy, required to meet the load demand). This excess capacity is available with a probability, p, the availability of the base loaded unit.

Thus, the expected supply for pumped hydro units could be expressed as the convolution of random variables corresponding to the excess capacity available from each base loaded unit. However, due to the time dependent nature of the original load curve this excess capacity is not available to pumped hydro all the time. In fact, during certain parts of the day, the entire capacity will be used to meet customer demand, while during others, most or all of the capacity will be used for pumped hydro. This means that the capacity available for pumped hydro will be greater than the expected excess capacity (the area above the equivalent load duration curve), but it will be available for less than the availability of the base loaded unit. The new availability includes the probability that the unit is not forced out as well as the time-dependent availability. This is unfortunate because it is difficult to justify interpreting the percent of time available as the probability of being available. Indeed, to do so would violate the assumption that availabilities are independent because if one base loaded unit has excess capacity available for pumped hydro then so do the rest of the units higher in the loading order.

An alternative method is to include the time-dependency by computing the expected energy available for pumped hydro and assuming that the energy is available with probability, p. This method avoids the problems mentioned above, although it obscures some detail. In particular, it does not allow for the inclusion of the limited pumping capacity. However, it is conceptually clearer and considerably easier to implement. For these reasons, the rest of this paper will focus on the second approach. Reference can be made to Section H where an example is worked through.

1. Base Loaded Energy Supply

The base loaded energy available for pumped hydro is the area above the load curve multiplied by the length of the time period. This energy is available with the same availability as the base loaded plant. Using this fact one can create a cumulative supply curve of base loaded energy, $F_{S,B}(s)$, by convolving random variables of the form:

$$P(s_B = x) = \begin{cases} p & \text{if } x = Ex \\ q & \text{if } x = 0 \\ o & \text{otherwise} \end{cases} \tag{27}$$

p = availability of the plant

Ex = energy above the curve

Then:

$$F_{S,B}(s) = q_{B-1}F_{S,B-1}(s) + p_{B-1}F_{S,B-1}(s-Ex) \tag{28}$$

where

$F_{S,B}(s)$ = cumulative supply curve before Bth plant is loaded

B = loading order of the base plant

s = given supply level

N.B. (1) The cumulative supply distribution is a series of step functions formed from discrete random variables; (2) the p's and q's are reversed from the forced outage demand random variable, because the supply is being created, not consumed; (3) The original supply curve is one for all MWH levels less than or equal to zero and zero for all MWH levels greater than zero.

2. Energy Consumed by Pumped Hydro

After all the base loaded plants have been convolved, the curve of $F_{S,B}(s)$ is the cumulative supply curve of energy available for pumped hydro. We now define a new random variable, S_E, the equivalent energy supply:

$$S_E = S_B + S_F \tag{29}$$

where

S_B = original supply available

S_F = supply available due to outages of earlier pumped hydro units.

When a pumped hydro unit creates demand on the base loaded units, the energy available is a function of the original base loaded supply plus a supply due to the failure of earlier pumped hydro units.

Because of the inefficiencies in pumping, the demand created by a pumped hydro unit is greater than the MWH size of the reservoir. The pumped hydro demand, H, is given by:

$$H = z/e \tag{30}$$

where

z = size of reservoir (MWH)

e = pumping efficiency.

The distribution of the random variable, S_F, corresponding to the supply created by the failure of previous pumped hydro units, is formed by the convolution of random variables having the following distribution:

$$Pr(S_F = x) = \begin{cases} q & \text{if } x = H \\ p & \text{if } x = 0 \\ o & \text{otherwise} \end{cases} \qquad (31)$$

The distribution of S_F has the same form as the distribution of demand due to forced outages in the load convolution, except that now the random variable is energy supply rather than power demand.

The procedure for finding the expected energy for a pumped hydro unit is completely analogous to finding the expected capacity for a non-hydro unit. Each pumped hydro unit is loaded under the supply curve at its demand, H, just as the units were loaded under the equivalent demand curve at their capacity. The expected energy demand is found by computing the area under the curve, between the loading point and the demand level, just as the expected capacity was computed before. The outages of the pumped hydro unit are convolved into the equivalent supply curve to account for the fact that failure of pumped hydro units makes more energy available to other units. At any point, the distribution of the equivalent supply, S_E, can be written as:

$$F_{s,u}(S) = p_{u-1} F_{s,u-1}(S) + q_{u-1} F_{s,u-1}(S-H_{u-1}) \qquad (32)$$

$$E(MWHC_u) = p_u \int_L^{L+H} F_{s,u-1}(x)dx. \qquad (33)$$

where

$F_{S,u}$ = equivalent supply curve before the u^{th} plant is loaded.

$F_{S,1} = F_{S,B}$, the final base loaded supply curve

u = loading order of the pumped hydro unit

p_u = probability that the pump does not fail

L = loading point on the supply curve

$MWHC_u$ = megawatt hours consumed.

Finally, the expected energy available is:

$$E(MWH_u) = E(MWHC_u)\cdot e_u \tag{34}$$

Comparing this set of equations to those for the demand curve, the only difference is that time is not included in any of the equations. Time is included in the original calculation of the expected energy available.

The expected cost of the energy can be found by assuming that the base loaded plants are convolved into the supply curve in the same order in which they loaded into the system load demand curve. This is the same as saying that the first pumped hydro unit is probably supplied by the first available and therefore cheapest base loaded unit.

The expected cost can be written as:

$$E[c(s)] = \sum_{i=1}^{n} c_i \ \Pr\{\text{unit } i \text{ supplies the energy at } S\} \tag{35}$$

where

$c(s)$ = cost of energy at point d on the supply curve

c_i = cost of unit i

n = total number of base loaded plants.

From Figure 7 in Section H it is clear that the probability that a certain unit supplies the energy at any one point can be found during the convolution process given that we convolve the plants in the proper order. If the units are convolved in a different order, then the final supply curve is the same but the internal blocks are different resulting in different costs.

Equation (35) can be rewritten as;

$$\bar{c}_B(s) = \bar{c}_{B-1}(s) + c_B[F_{S,B}(s) - F_{S,B-1}(s)] \tag{36}$$

where

$\bar{c}_B(s)$ = expected cost of the supply at point s after B is loaded

c_B = cost per MWH of plant B.

The last term in Equation (36) contains the expected energy to be supplied by any base loaded plant at any point s. This value:

$$\Delta MWH_B = [F_{S,B}(s) - F_{S,B-1}(s)] \tag{37}$$

can be accumulated for each plant to give the total energy expected from each.

The expected cost for the pumped hydro energy can be found by summing the expected cost of the original supply plus the expected cost of the supply due to outages. Since the first pumped hydro unit has no supply due to outages:

$$\bar{c}_1(s) = \bar{c}_N(s)$$

and $$E(c_1) = \sum_{s=0}^{H_1} \bar{c}_1(s)$$

where $c_N(s)$ = cost curve after all base loaded plants are convolved. (38)

In general for base loaded units:

$$F_{S,B}(s) = q_{B-1}F_{S,B-1}(s) + p_{B-1}F_{S,B-1}(s-Ex)$$

$$E(MWH_B) = \int_L^{L + EX_B} [F_{S,B}(x) - F_{S,B-1}(x)]dx \tag{39}$$

$$\bar{c}_B\,(s) = \bar{c}_{B-1}(s) + c_B \cdot [F_{S,B}(s) - F_{S,B-1}(s)]$$

For pumped hydro units:

$$F_{s,u}(s) = p_{u-1}F_{s,u-1}(s) + q_{u-1}F_{s,u-1}(s-H_{u-1})$$

$$E(MWH_u) = P_u{*}e \cdot \int_{L}^{L+H} F_{s,u-1}(x)dx \tag{40}$$

$$\overline{c}_u(s) = \overline{c}_{u-1}(s) + c_{u-1}[F_{s,u}(s) - F_{s,u-1}(s)]$$

$$c_u = \int_{L}^{L+H} \overline{c}_u(x)dx$$

E. Loss of Load Probability

After the last unit has been loaded, the final curve is the equivalent load curve for the entire system. Since the loss of load probability is defined to be the percent of time that the customer demand cannot be met, its value can be read directly from the final curve. The energy demand that cannot be supplied is given by:

$$\overline{E}_N = T \int_{TK_N}^{\infty} F_{EN}(x)dx \tag{41}$$

where TK_N is the total installed capacity of the system. Figure 6 shows the final system configuration.

F. Expected Value of the Running Capacity

In the deterministic model, the energy can be found from the area under the load duration curve. In the probabilistic model, this integral must be reinterpreted because the vertical axis does not have the dimension of time. To make the reinterpretation, a new random variable, the running capacity of unit r, is defined:

$$C_r = D_{Er} - TK_r \tag{42}$$

where TK_r is the loading point of unit r. The cumulative of C_r is:

$$G_{cr}(S) = Pr(0 \leq C_r \leq S) \tag{43}$$

or $$F_{cr}(S) = Pr(C_r > S)$$

By definition of C_r, $F_{cr}(S)$ is equivalent to:

$$F_{cr}(S) = Pr(C_r > S) = Pr(D_{Er} > S + TK_r) = F_{Er}(S+TK_r). \tag{44}$$

From Equation (1):

$$F_{cr}(S) = \int_{s}^{\infty} f_{cr}(x)dx$$

$$= \int_{S+TK_r}^{\infty} f_{Er}(x)dx$$

$$= F_{Er}(S+TK_r). \tag{45}$$

The expected value of a random variable is defined to be:

$$E(x) = \int_{x_{min}}^{x_{max}} x_o F_x(x_o)dx_o$$

$$x_{min} \leq x \leq x_{max}. \tag{46}$$

Integrating Equation (46) by parts gives:

$$E(x) = x_o G_x(x_o)]_{x_{min}}^{x_{max}} - \int_{x_{min}}^{x_{max}} G_x(x_o)dx_o$$

$$= x_{max} - \int_{x_{min}}^{x_{max}} G_c(x_o)dx_o$$

$$= x_{max} - \int_{x_{min}}^{x_{max}} [1-F_x(X_o)]dx_o$$

$$= x_{min} + \int_{x_{min}}^{x_{max}} F_x(x_o)dx_o. \tag{47}$$

Using this definition and Equation (45), the expectation of C_r is:

$$E(C_r) = \int_o^{K_r} F_{cr}(t)dt$$

$$= \int_o^{K_r} F_{Er}(t+TK_r)dt$$

$$= \int_{TK_r}^{TK_r+K_r} F_{Er}(x)dx. \tag{48}$$

This is just the area under the equivalent load duration curve. Therefore, even though the calculations for the energy produced by a unit appear to be the same, the interpretation of the variables is quite different. In the probabilistic model, the expected running capacity at any given time is computed and then multiplied by the length of the time period.

G. Pumped Hydro Dispatch Strategy

Suppose a system in which plants are loaded in order of increasing operating cost. All pumped hydro units are loaded at the point where their costs become competitive. The capacity of the pumped hydro units is reduced so that a unit can generate for the required length of time at the point where it is first competitive.

Now suppose that the running capacity of the pumped hydro unit is increased. Because the energy remains constant, the hours of operation must be reduced. This means that the pumped hydro unit must be moved up in the loading order (see Figure 4). Moving the pumped hydro unit creates two effects. One is that the plant directly above the pumped hydro unit must generate longer to make up for the hours that the pumped hydro unit is no longer supplying. The other is to decrease the capacity requirements on units higher in the loading order.

The extra cost required to make up for the loss in pumped hydro generation time is:

$$\Delta c' = \Delta H * MW * c_r \tag{49}$$

And the savings from decreasing the capacity requirement is:

$$\Delta c = \Delta MW \ (H-\Delta H) * c_k \tag{50}$$

where

MW = original pumped hydro running capacity

ΔMW = increase in pumped hydro running capacity

H = original generation hours

ΔH = decrease in generation hours

c_r = cost of replacement generation

c_k = cost of unit(s) displaced.

Since the plants were loaded in order of increasing cost:

$$c_k \geq c_r \qquad \forall \ k \tag{51}$$

where k indexes all plants above the pumped hydro unit. In some cases the replacement generation and the displaced capacity may be for the same unit, however, Equation (51) still holds.

The pumped hydro energy remains constant:

$$E = H * MW = (H-\Delta H) * (MW+\Delta MW) \tag{52}$$

or

$$(H-\Delta H) * \Delta MW = \Delta H * MW \tag{53}$$

This implies:

$$\begin{aligned} \Delta c - \Delta c' &= \Delta MW*(H- H)*c_k - \Delta H*MW*c_r \\ &= \Delta H*MW(c_k-c_r) \\ &\geq 0 \end{aligned} \tag{54}$$

by Equations (51) and (53). Therefore, the savings are always greater than or equal to the additional cost of delaying the pumped hydro.

Therefore, it is always advantageous to increase the running capacity of the pumped hydro as far as possible. Equation (54) is equivalent to stating that the same amount of energy must be generated no matter what, and that it is always cheaper to generate the energy toward the bottom of the load duration curve.

The argument given above does not carry through to the probabilistic model since there is no upper limit on the capacity required by the system to meet the peak. That is, each additional MW at the top of the curve reduces the loss of load probability, but, the loss of load probability never reaches zero. (There is always a finite chance that all plants will fail).

In the probabilistic case, increasing the capacity of the pumped hydro may actually increase the costs since the additional capacity places an outage demand on future units. However, the additional costs would be relatively small, and the overall effect would be to increase the reliability of the system

H. Numerical Example of Simulation

Suppose one is given a system of five thermal plants, one conventional hydro plant and two pumped hydro plants with the characteristics given in Table 1. Also given is the customer load duration curve for a summer week shown in Figure 7a. With this data, the expected energy and expected cost for each unit can be computed as well as the loss-of-load probability for the system.

To begin the simulation, the plants are ordered by their operating costs to get the economic loading order. The conventional hydro unit, which has no operating cost, is first in the loading order. The load curve is for a winter month, so there is conventional hydro energy available. Since the hydro unit is the first to be loaded, there is no demand due to forced outages and the plant curve is the customer load duration curve. If the

Table I. Plant Characteristics

Plant Name	Plant Type	Capacity (MW)	Forced Outage Rate	Operating Cost ($/MWH)	Hydro Size (MWH/week)	Pumping/ Generating Efficiency
A	Coal Base	500	20%	$12.00	-	-
B	Oil Inter- mediate	250	10%	$20.00	-	-
C	Nuclear Base	750	10%	$ 7.00	-	-
D	Convention- al Hydro	30	1%	0.00	5040-winter 0-summer	-
E	Oil Base	100	15%	$15.00	-	-
F	Pumped Hydro 100	100	5%	-	1050	70%
G	Pumped Hydro 50	50	5%	-	700	70%
H	Gas Turbine	100	10%	$25.00	-	-

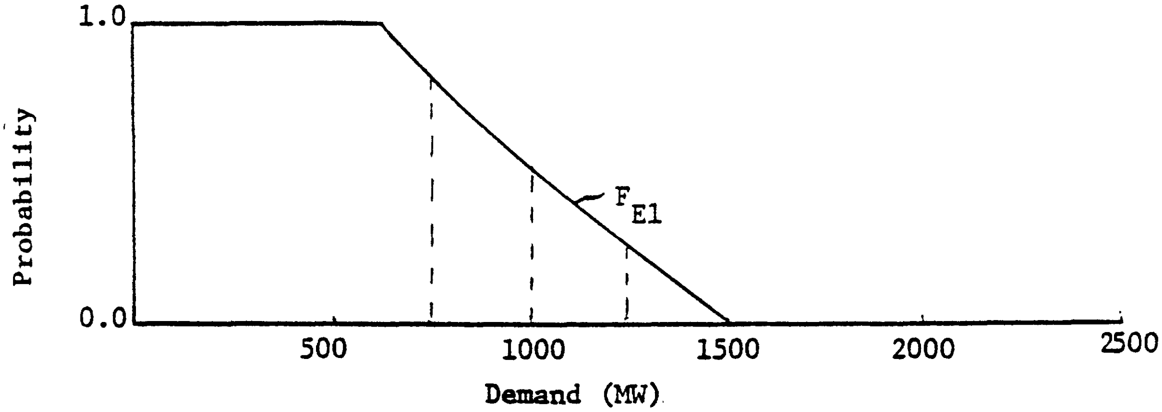


Figure 7a. Original Customer Demand Curve

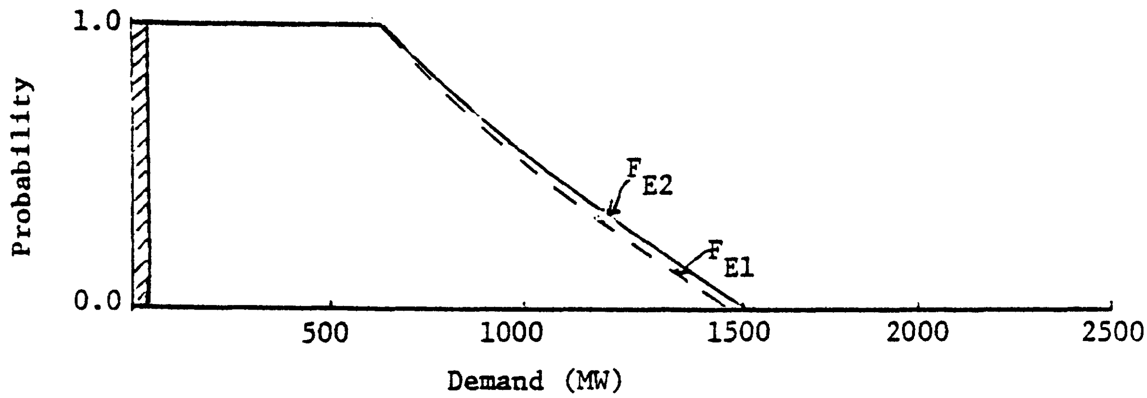


Figure 7b. Equivalent Load Curve after Loading Conventional Hydro, Plant D

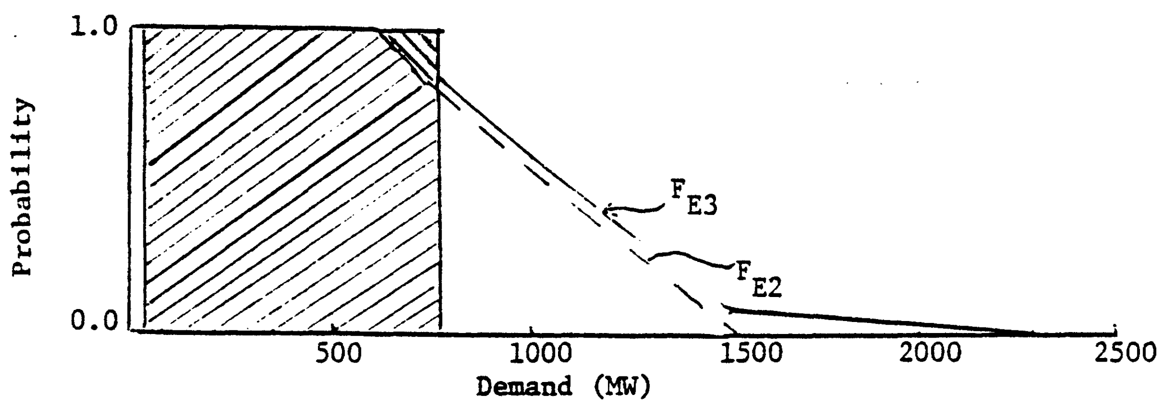


Figure 7c. Equivalent Load Curve after Loading Nuclear Base, Plant C

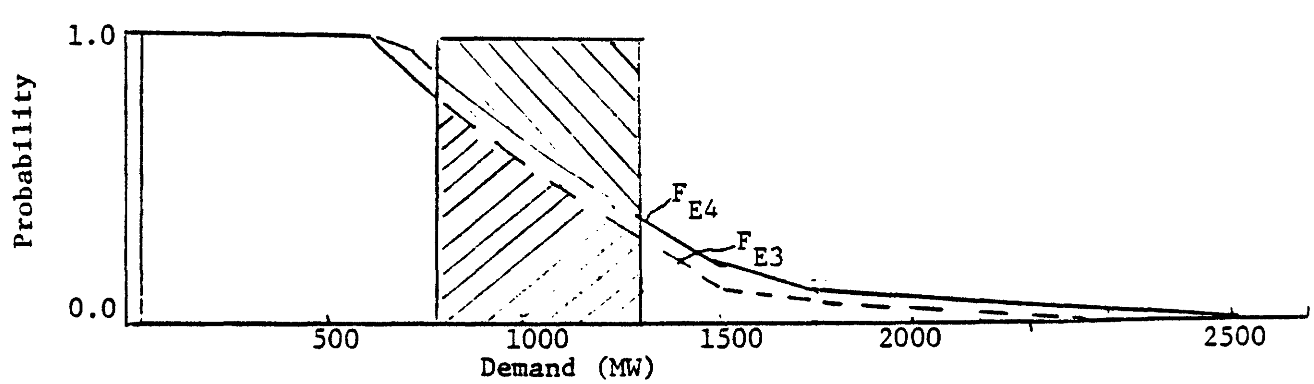


Figure 7d. Equivalent Load Curve after Loading Coal Base, Plant A

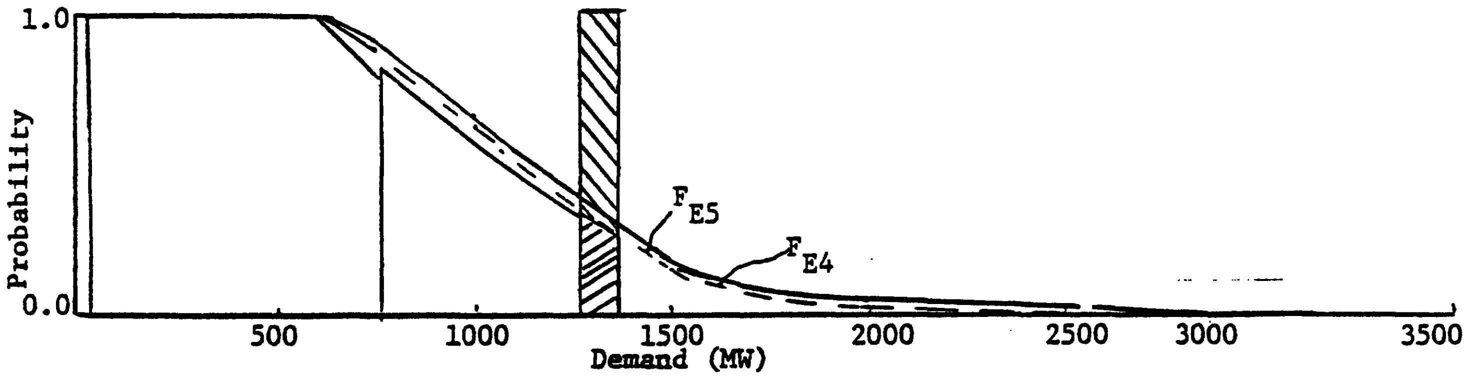


Figure 7e. Equivalent Load Curve after Loading Oil Base, Plant E.

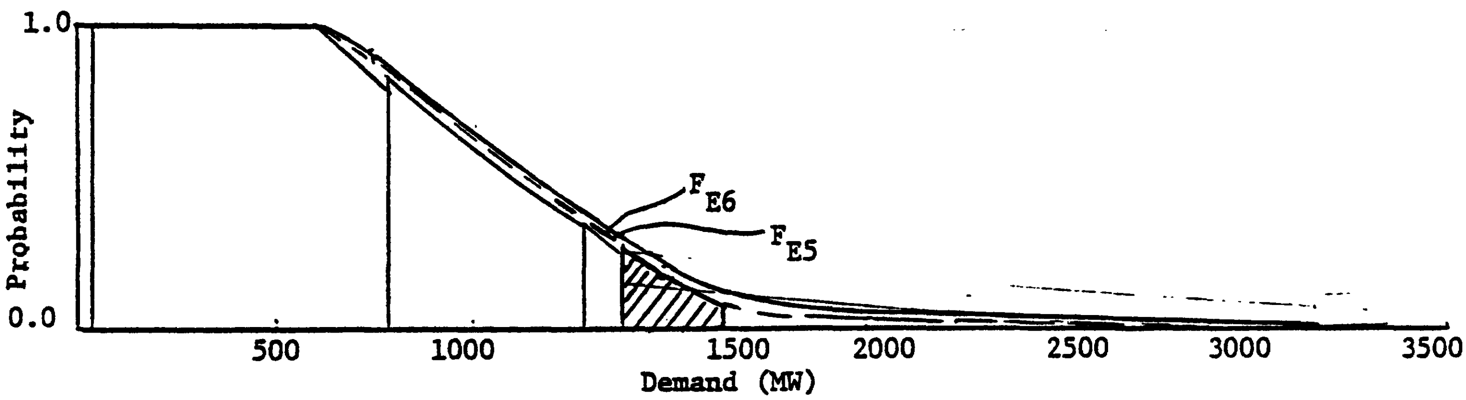


Figure 7f. Equivalent Load Curve after Loading Oil Intermediate, Plant B.

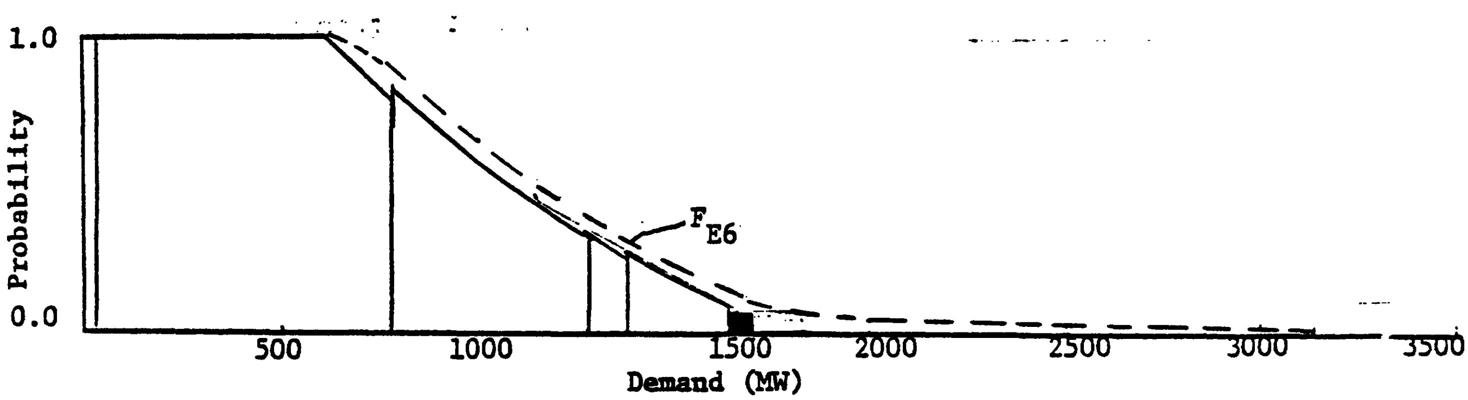


Figure 7g. Test on Pumped Hydro Energy, Unit not Loaded, Insufficient Energy.

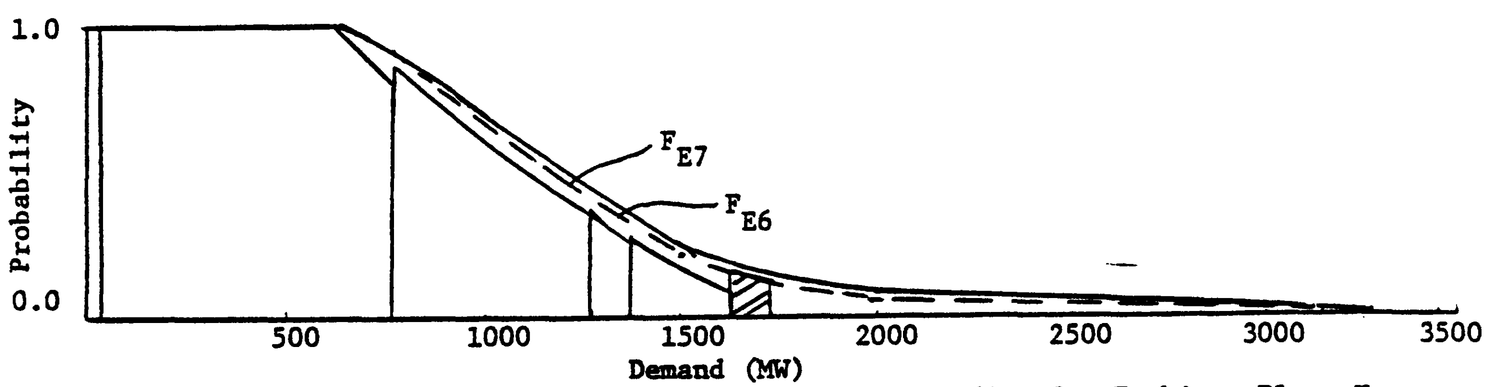


Figure 7h. Equivalent Load Duration Curve after Loading Gas Turbing, Plant H,

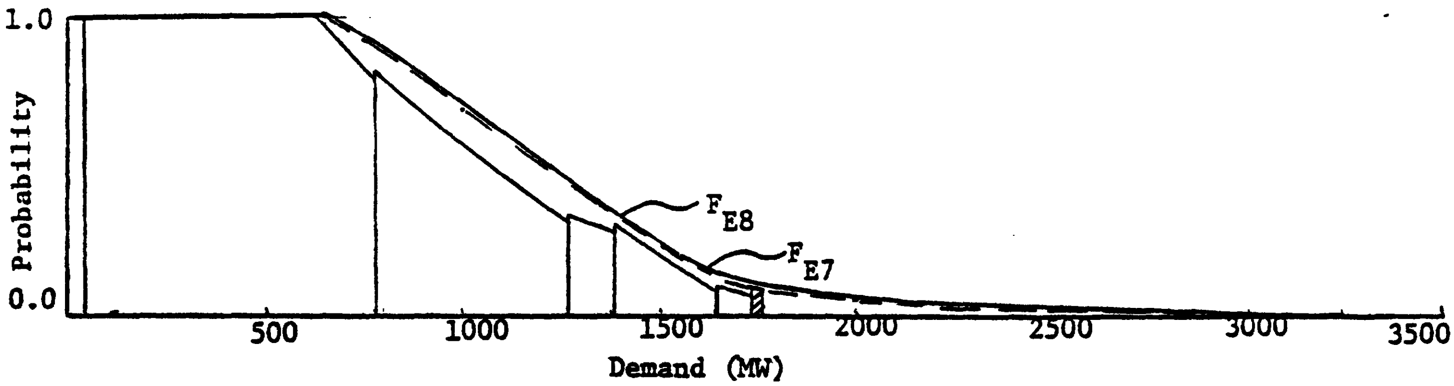


Figure 7i. Equivalent Load Curve after Loading Pumped Hydro 50, Plant G at Reduced Capacity

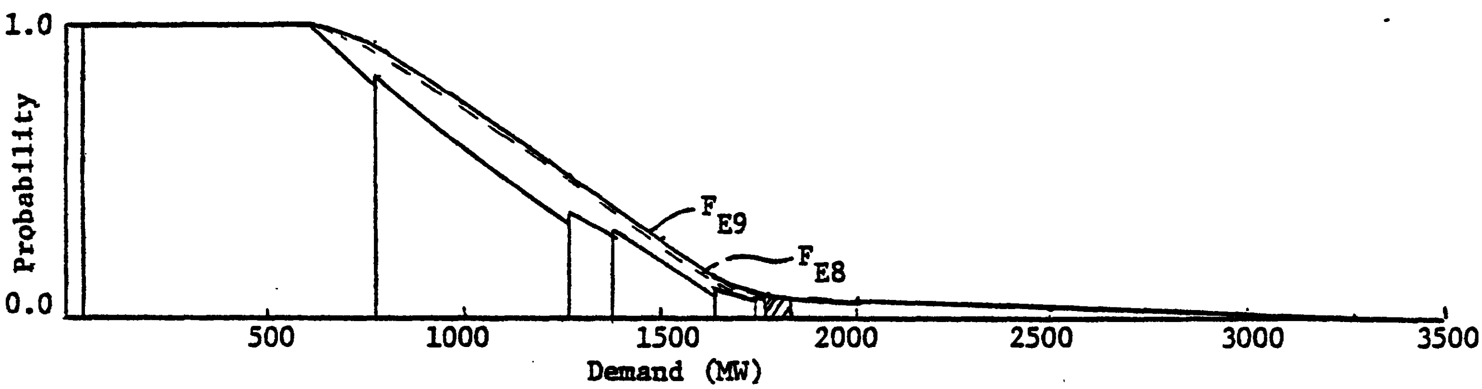


Figure 7j. Equivalent Load Curve after Loading Pumped Hydro 100, Plant F at Reduced Capacity.

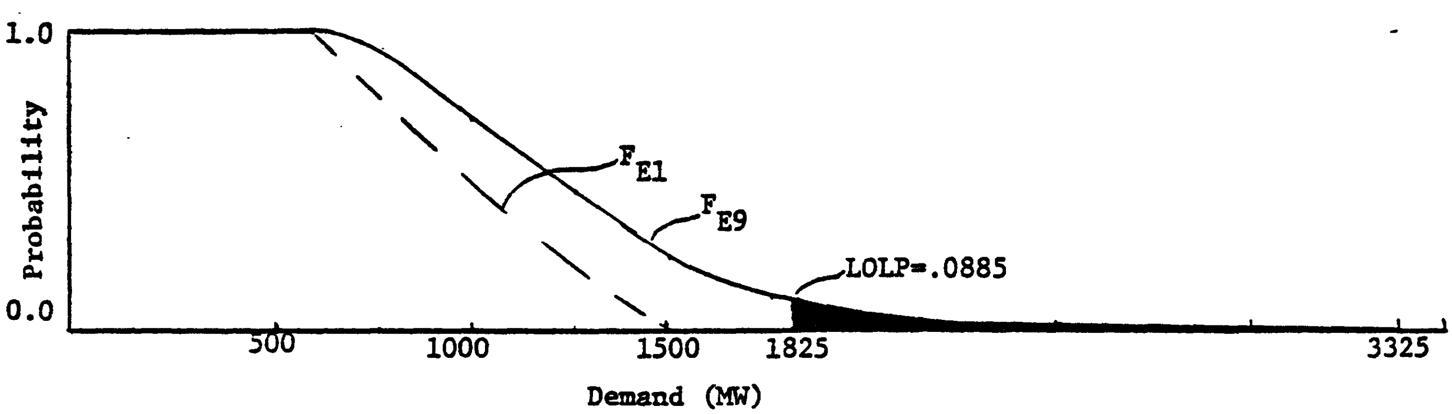


Figure 7k. Final Equivalent Load Curve. Shaded area is the unserved energy.

conventional hydro unit were loaded at 30 MWs at the beginning of the curve, there would be a demand of 5040 MWHrs on it. The demand value is found by computing the area under the load curve (the expected running capacity) and multiplying it by the number of hours in the time period. So for this case the demand would be (30 MWs * 1.0 * 168 hours/week) = 5040 MWHrs/week. (See Figure 7b.) The energy demand is equal to the energy available, so the hydro unit is loaded. The expected energy of the conventional hydro unit is the energy demand times the units' availability. Using the notation of Section IIIb:

$$K_1 = 30 \text{ MWs}$$

$$E(C_1) = 1.0*30\text{MWs} = 30\text{MWs}$$

$$MWH_1 = .99 * 168 * 30\text{MWs} = 4989.6 \text{ MWHs/week}$$

$$CF_1 = .99 * 30\text{MWs}/30\text{MWs} = .99$$

$$COST_1 = 0.0 * \text{MWHs} = \$0.0/\text{week}$$

All final values are given in Table II.

After the unit has been loaded, a new load curve is computed using Equation (15):

$$F_{E2}(d) = p_1\ F_{E1}(d) + q_1\ F_{E1}(d-K_1).$$

The curve F_{E2} is the demand curve that the second unit sees. Sample calculations are worked out here. The complete curve is given in Table III. In the calculations below all values that do not appear explicitly in the table are computed using linear interpolation. Remember that the probability that the load is greater than a negative number is one.

$$\begin{aligned} F_{E2}(625) &= .99\ F_{E1}(625) + 0.1\ F_{E1}(625-30) \\ &= .99 * 1.0 + 0.1 * 1.0 = 1.0 \end{aligned}$$

$$\begin{aligned} F_{E2}(1000) &= .99\ F_{E1}(1000) + 0.1\ F_{E1}(970) \\ &= .99 * .5 + .01 * .5444 \\ &= .5004 \end{aligned}$$

Table II Plant Characteristics After Loading

Plant Loading Order	Plant Name	Capacity (MW)	Loading Point (MW)	Expected Energy (MWHs/week)	Expected Cost ($/week)	Capacity Factor (excluding energy for pumped hydro)	Expected Energy Available for pumped hydro (MWHs/wk)	Expected Energy Used for Pumped Hydro (MWHs/week)
1	D	30	0	4990	0.0	.99	-	
2	C	750	30	111,456	780,191	.88	1944	1944
3	A	500	780	36,268	435,213	.43	30932	444.8
4	E	100	1280	4,000	60,000	.24	10503	88.74
5	B	250	1380	7,778	155,564	.18	-	
6	H	100	1630	2,235	55,885	.13	-	
7	G	50 (35)	1730	654	7,507	.078	-	
8	F	100 (60)	1765	950	12,868	.057	-	

Table III Equivalent Load Duration Curves

MW Level / Curve	625	750	1000	1250	1500	1750	2000	2250	2500	2750	3000	3250	3500
Customer Load Curve = F_{E1}	1.0	.8700	.5000	.1700	0.0								
F_{E2}	1.0	.8736	.5004	.1704	.0002	0.0							
F_{E3}	1.0	.8862	.5504	.2534	.0875	.05	.017	0.0					
F_{E4}	1.0	.9090	.6403	.3800	.1801	.0907	.0311	.01					
F_{E5}	1.0	.9199	.6566	.3956	.1964	.0961	.0347	.0113	.0038	.0002	0.0		
F_{E6}	1.0	.9279	.6829	.4217	.2163	.1061	.0408	.0136	.0046	.0022	.0002	0.0	
F_{E7}	1.0	.9337	.6927	.4322	.2245	.1105	.0438	.0150	.0050	.0023	.0003	0.0	
F_{E8}	1.0	.9514	.6944	.4340	.2260	.1113	.0443	.0152	.0051	.0023	.0003	10^{-7}	0.0
F_{E9}	1.0	.9526	.6975	.4371	.2395	.1071	.0451	.0156	.0052	.0023	.0003	10^{-7}	0.0

1. Entries are probability that the equivalent load exceeds the given MW level.

$$F_{E2}(1500) = .99\ F_{E1}(1500) + 0.1\ F_{E1}(1470)$$
$$= .99 * 0.0 + 0.1 * .0204$$
$$= .0002$$

The new maximum for the equivalent load duration curve is 1530 MWs and the loading point for the next unit will be 30 MWs.

The three base loaded units are loaded in much the same way except that it is not necessary to test on available versus demand energy. In addition, for each base loaded unit, one computes the excess capacity which could be used for pumped hydro. (In Figure 7, this is the shaded area above the curve.) The expected capacity available for pumped hydro is converted to energy by multiplying by the hours in a week. Each of these blocks of energy is available with a given probability, p, and a given cost. This information can be used to create an energy supply curve for pumped hydro.

So, for example, after the first base loaded plant is loaded, one computes that it has an expected energy of 111,456 MWHs and excess energy of 2,160 MWHs which is available 90% of the time at a cost of $7.00 per MWH. Figure 8a shows the beginning of the supply cumulative probability curve and the supply cost curve.

Remembering that for the original supply probability curve the probability for negative numbers is one, one obtains:

$$F_{S2}(s) = q_1 F_{S1}(s) + p_1 F_{S1}(s\text{-}Ex)$$
$$F_{S2}(0) = .1 + F_{S1}(0) + .9 * F_{S1}(-2160)$$
$$= .1 * 1.0 + .9 * 1.0 = 1.0$$
$$F_{S2}(100) = .1 * F_{S1}(10) + .9 * F_{S1}(-2150)$$
$$= .1 * 0.0 + .9 * 1.0 = .9$$
$$F_{S2}(2160) = .1 * F_{S1}(2160) + .9 * F_{S1}(0)$$
$$= .1 * 0.0 + .9 * 1.0 = .9$$

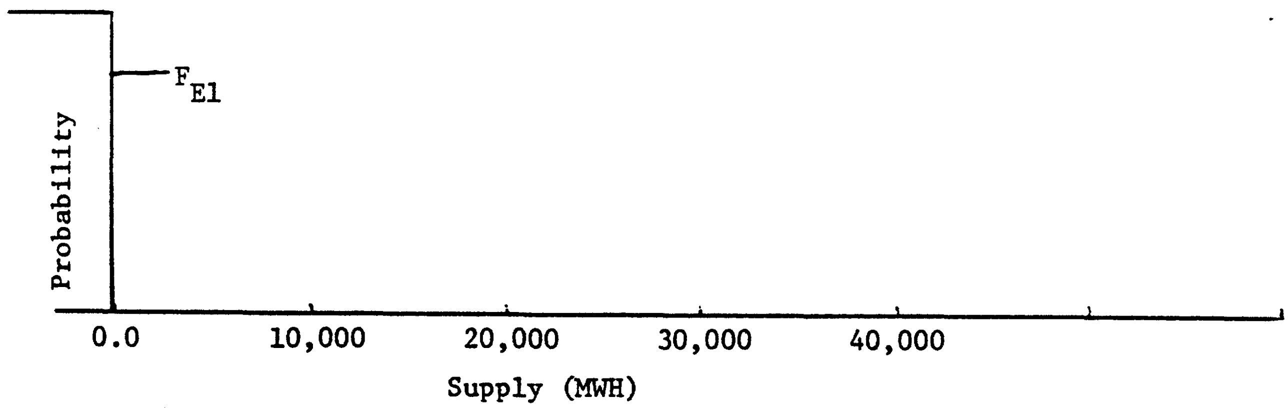
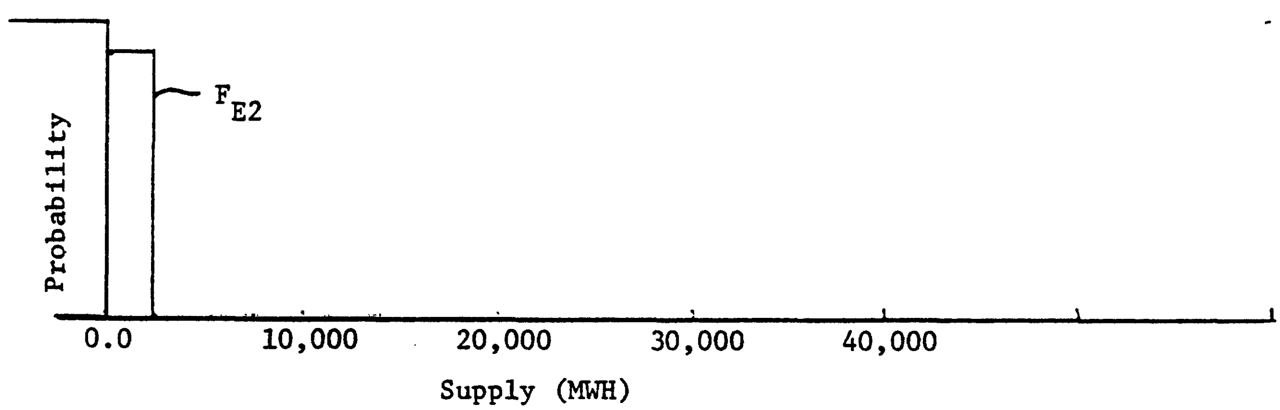


Fig. 8a. Original Pumped Hydro Energy Supply Curve



Dig. 8b. Supply Curve after Energy from Nuclear Base, Plant C

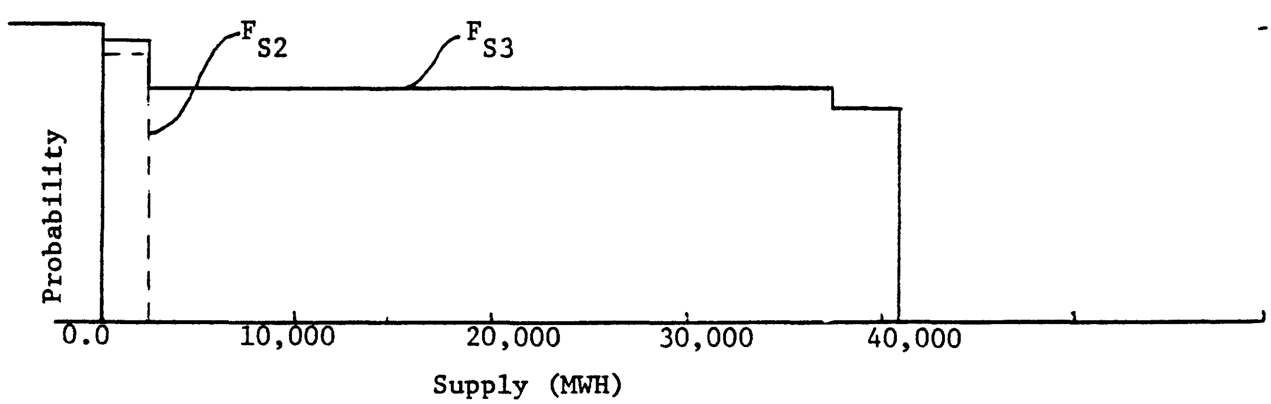


Fig. 8c. Supply Curve after Energy from Coal Base, Plant A

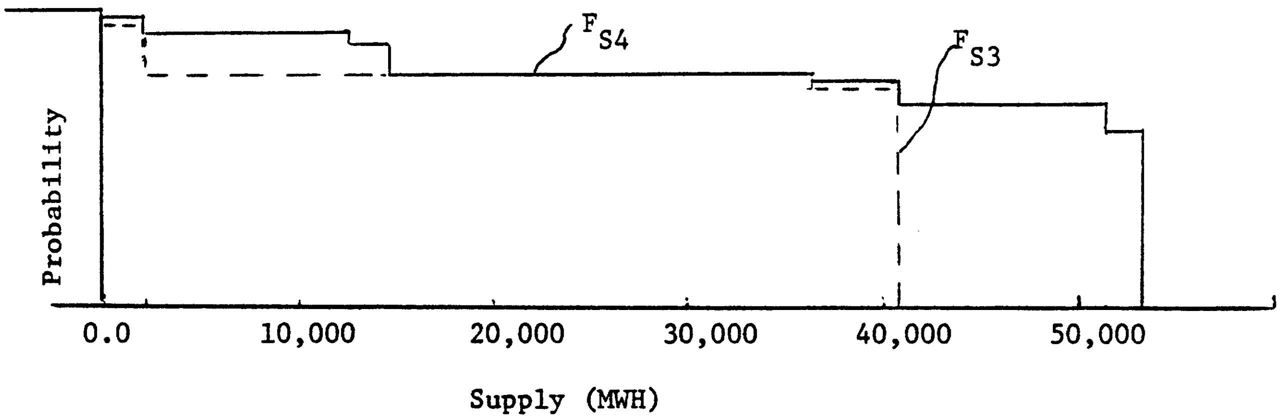


Fig. 8d. Supply Curve after Energy from Oil Base, Plant E

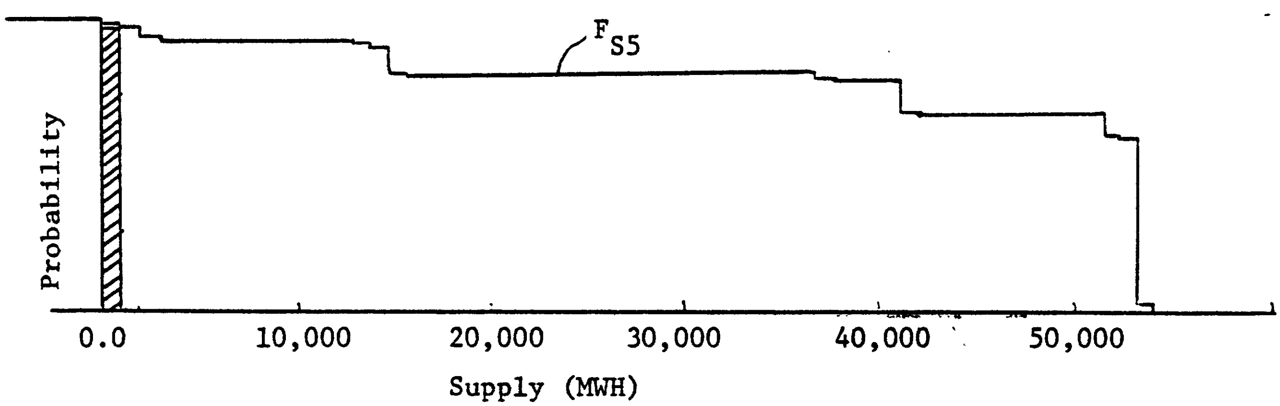


Figure 8e. Supply Curve after Loading Pumped Hydro 50, Plant G

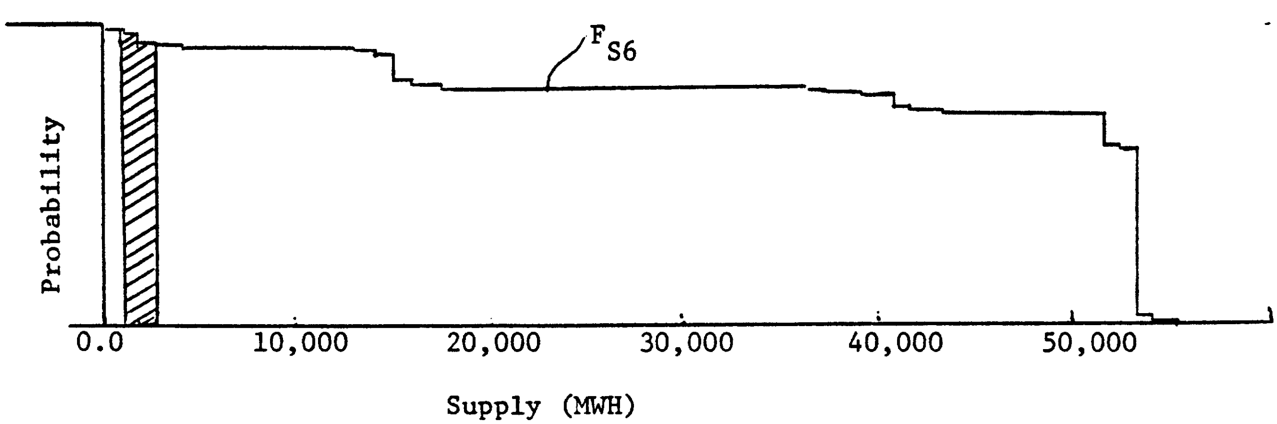


Fig. 8f. Supply Curve after Loading Pumped Hydro 100, Plant F

$$F_{S2}(2170) = .1 * F_{S1}(2170) + .0 * F_{S1}(10)$$
$$= .1 * 0.0 + .9 * 0.0 = 0.0$$

For the cost curve one has:

$$\overline{c}_1(s) = \overline{c}_1(s) + p_1 * c_1 * [F_{S1}(s\text{-}Ex) - F_{S1}(s)]$$
$$\overline{c}_2(0) = \overline{c}_1(0) + .9 * \$7.0 * [F_{S1}(-2160) - F_{S1}(s)]$$
$$= 0.0 + .9 * \$7.0 * [1.0-1.0] = 0.0$$
$$\overline{c}_2(2160) = \overline{c}_1(2160) + .9 * \$7.0 * [F_{S1}(0) - F_{S1}(2160)]$$
$$= 0.0 + .9 * \$7.0 * [1.0-0.0] = \$6.3$$

Once all the base loaded units have been loaded, one can compute the expected energy supply available to each of the pumped hydro units. Since each pumped hydro unit consumes more energy than it produces, the units are loaded under the supply curve at their size divided by their efficiency. The area under the supply curve (see Figure 83) is the expected energy available to the first unit. Dividing this number by the efficiency will give the expected energy the unit can supply to the system. To find the expected cost of the pumped hydro energy, one recomputes the area under the supply curve multiplying each incremental block of energy by its cost as shown below. The total cost is not adjusted by the efficiency since the unit is charged for all the energy it consumes.

The first pumped hydro unit is loaded between 0 and 1000 MWHs. This block of energy is available at a single cost (see Figure 8e). Since a cumulative cost curve was not kept, the cost per MWH of this energy can be found by noting that 90% of the energy was supplied by Plant C, 8% by Plant A, and 1.7% by Plant E. These percentages can be found from Table IV by taking the difference between two supply curves at the desired level. The difference is the incremental supply by the next unit at that level. The expected cost per MWH for the first pumped hydro unit (the seventh plant) is the product of these incremental supplies and their costs from Table I:

Table IV Energy Supply Curves

MWH Level / Curve	≥ 0 2160	>2160 12356	>12357 14487	>14487 38666	>38666 40826	>10826 51023	>51023 53183	>53183 -54183	>54183 -55683	>55683
F_{S1}	0.0									
F_{S2}	.90	0.0								
F_{S3}	.980	.80	.80	.80	.72	0.0				
F_{S4}	.997	.970	.953	.80	.788	.68	.612	0.0		
F_{S5}	.9972	.9714	.9539	.8077	.7886	.6854	.6154	.0306	0.0	
F_{S6}	.9973	.9727	.9548	.8150	.7896	.6906	.6189	.0598	.0015	0.0

1. Entries are probability that energy demand is strictly greater than the given energy level.

2. Curve F_{S4} is the final supply curve, analogous to the customer load curve. Curves F_{S5} and F_{S6} are equivalent supply curves. (All the break points are not given for the last two curves. See Figure 8f.)

$$c_7 = \$7. * .9 + \$12 * .08 + \$15 * .017$$
$$= \$7.52/\text{MWH}$$

After the first pumped hydro unit has been loaded a new equivalent supply curve is computed to account for the fact that the first unit may fail making additional energy available to the second unit. The procedure for computing the new curve is completely analogous to computing the new equivalent demand curve. Results are given in Table IV.

The cost for the second pumped hydro unit is computed in the same way it was for the first. Also included in its cost, though, is the energy available because the first unit was down. This amounts to .02% of its expected energy between the MWH levels of 2160 and 2,500 at a cost of $7.52/MWH. The cost per MWH for the second unit is found by weighting the costs by the energy available in each block. Final results are given in Table II.

IV. References

1. Baleriaux, H., Jamoulle, E., Linard de Guertechin, F., "Simulation de l'exploitation d'un parc de machines thermiques de production d'electricité couplé à des stations de pompage," Revue E, Vol. V, No. 7, 1967, pp. 225-245.

2. Booth, R.R., "Power System Simulation Model Based on Probability Analysis," IEEE Transactions on Power Apparatus and Systems, PAS-91, January/February 1972, pp. 62-69.

3. Deaton, P.F., Utility System Integration and Optimization, PhD Thesis, M.I.T., 1973.