THE EFFECT OF INTERRUPTIONS ON AIRLINE SCHEDULE CONTROL

by

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ABSTRACT

The airline's schedule plan provides the framework for its entire operation. All airline functions - flight crews, maintenance, ground facilities, sales, and so on - interact smoothly within the bounds set by the schedule. However, interruptions such as a station closing due to bad weather, or the inability of an aircraft to meet its scheduled departure time do occur fairly frequently. These irregularities force the airline to revise its operating plan so as to minimize the potential adverse effects of the interruption.

This report describes several schedule irregularities that can arise and attempts to isolate some of the decision factors involved in the optimal rerouting of flights. Certain of these factors have been modelled for implementation in a real-time computer information system. Using the computer as a decision making aid, and utilizing the "Out-of-Kilter" algorithm - an optimality method from network flow theory - an example is presented to indicate the potential usefulness of these methods to the airline schedule controller.

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CHAPTER 1

INTRODUCTION

1.1 Profitmaking in the Airline Industry

Like most corporations, the scheduled airline is a profit oriented concern; as such it is constantly working to keep its net revenues ahead of its operating costs. The air carrier, unlike most other companies though, must operate within a climate of strict government regulation. The Civil Aeronautics Board (CAB) rules on which routes each interstate airline may fly (and sometimes on a few which they must fly). Further, the CAB affirms the fare structure on domestic routes, and these rates are adhered to by all carriers. So despite the seemingly chaotic system of specialty fares, no single airline, in theory, has an advantage over its competitors based solely on fares.

Additional regulation comes from the Federal Aviation Agency (FAA). This regulation is principally in the areas of safety and air traffic control. The airlines, in general, have realized the importance that a good safety record plays in their economic well being. In fact company policy on safety proceedures has often gone beyond the FAA imposed minima. Thus while high safety standards place an economic burden on an airline, it is a burden that they might reasonably be expected to assume on their own initiative. So at this time, the primary regulatory presence that the carrier feels is that of the CAB. It is to be noted that the blanket of regulation may be draped even tighter in the future as the FAA looks into the question of an Airborne Collision Avoidance System among others[*].

Operating with a so called "Certificate of Convenience and Necessity" from the CAB, the airline knows the route network over which it may fly. The carrier's primary economic concern, then, is in the allocation of its limited inventory of aircraft over these routes so as to realize the most profit.

The detailed listing of all flights - their origins and destinations, the times involved, and the equipment used - is called the schedule plan. The schedule is so central to the airline's well being that the ability of management to come up with a good one and then to run it efficiently is usually the basis for the success or failure of the entire operation. In effect, once the schedule has been set up, limits have been placed on the airline's operation. It puts an upper bound on the amount of revenue that can be possibly received by the scheduled air carrier. It also serves as the operational guide for the interaction of all the supporting functions of the air transportation system. While crew assignments, maintenance rotations, station capacities, ground equipment and personnel requirements serve as constraints for the construction of the schedule, if low costs are to be realized, these groups, and the many others involved, must be efficiently coordinated to support the schedule that finally becomes established.

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[*] At some later date, the FAA may be asked to step into the controversy surrounding the matter of noise levels in the vicinity of airports. Should such a move be necessary, the economic consequences to the air carriers could be quite severe, especially if operational limitations are involved.

1.2 Airline Revenue and Demand

An airline deals in no real, tangible commodity. Its marketable quantity is the guarantee of space on a particular flight. The air carrier's saleable product - the passenger seat - is a transient commodity, for once the aircraft has taken off, the revenue from unsold seats is gone forever. For this reason, management should have the ability to predict with good accuracy future demand levels for various times of the day among the city-pairs in its route network. An excellent discussion of the decisions involved in the construction of an airline schedule is given in Reference 9.

Since it is the airline that is dependent on the traveller and not the opposite, the carrier must provide the kind of service that will bring in customers. A measure of what the airline is giving to its passengers often is expressed by the so called "level of service" variables (LOS). These are outlined in Table I. Usually an increased level of service results in increased revenues but at a greater cost to the company. However, it is not the actual service that is offered but the way in which the potential passenger judges this service in his mind that is the crucial consideration. Thus a history of unsafe operations, strikes or discourteous personnel may still haunt a carrier that has since improved its service. Also assumptions on such things as the elasticity of demand with respect to certain changes in service and the impact of advertising on demand can be of great importance. So while air transportation is becoming increasingly acceptable to the general public, and practically a necessity for some businesses, the air traveller still remains a fickle sort. Delays, lost baggage, surly employees, oversales or cancellations on the part of an airline can quickly drive passengers to the competition. A general

TABLE I

LEVEL OF SERVICE VARIABLES

COST TO USER:

- Ticket Price
- Other Direct Costs
    - Documents
    - Meals
    - Transportation to Airport
    - Gratuities
- Indirect Costs
    - Insurance

TIME:

- Total Trip Time (Door to Door)
- Time spent in Transfer
    - Connections
    - Mode Changes
- Frequency of Service
- Scheduled Departure Times
- Reliability
    - Possibility of a delay or cancellation
    - Possibility of oversales on Flight

SAFETY:

- Probability of a Fatality
- Probability of various Accident Types

COMFORT AND CONVENIENCE:

- Physical Comfort
    - Terminal Facilities
    - Vehicle Type
    - Boarding Methods
- Psychological Factors
    - Privacy
    - Status
- Possibility of Vehicle Changes at Intermediate Points
- Aesthetic or Educational Factors
    - Inflight Entertainment
    - Food and Drinks
- Other Amenities
    - Efficient Ticketing
    - Fast and Careful Baggage Handling
    - Courteous Personnel

laxidasical attitude on the part of the industry or a series of air crashes or a prolonged strike generally manages to send a significant number of would-be travellers to other modes. It usually takes more effort to bring them back than it did to lose them.

A fundamental problem associated with the prediction of demand levels is that the very service that is offered is itself a factor in the generation of this demand. Predictions, then, must be of demand given a stated level of service.

Besides the demand information, the scheduler has to blend the requirements of the crews, the maintenance system, the terminal capacities, and many others, all within the overriding framework of company policy and governmental regulation. The schedule evolves from conflict to compromise, and finally represents management's best estimate of how to allocate its fleet of aircraft so as to most profitably satisfy what it sees to be the time of day demand for air travel between the city-pairs in its network.

1.3 Interruptions in the Schedule

Since both the passenger and the airline depend so heavily on the accuracy of the schedule, it is important to the carrier's profit picture that the schedule be run with as few alterations as possible. Yet some interruptions in service will inevetably occur. The minor ones are planned for by some schedule slackness, but the big ones require some fast rerouting so that the effects are not catastrophic. Delays and cancellations are the third and fourth most important causes of complaints to the CAB (after fares and reservations). Cancellations usually account for around 2% to 3% of an airline's scheduled yearly operations, but in particularly foul weather, monthly cancellations have gone above the 12% to 15% level. Figure I-1 presents cancellation rates by month from 8/63 to 8/64 for the "Big Four" domestic carriers (United, American, Trans World, and Eastern). From a summer level of 1% to 2%, the rate rises to about 5% from November to March. On time performance and cancellation rates for these same carriers are given in Table II, below.

TABLE II

ON-TIME PERFORMANCE for the "Big Four" 1965 (1964)

Carrier	On time to 15 min. late	More than 15 min. late	Cancelled
Eastern	82.27	15.74	1.99 (4.83)
Trans World	81.76	17.14	1.10 (2.11)
American	81.23	16.65	2.12 (3.63)
United	78.92	18.65	2.43 (2.13)

Source: "Air Transport World" April 1966 p. 98
"American Aviation" - various issues

These schedule irregularities occur when an aircraft is not where it should be and/or ready to do what it has been assigned. Examples of this are delays in arriving or departing, cancellations or the loss of an aircraft enroute. Most of these occurrences can be classified as non-critical; that is, the only effects are irritation to the passengers or to their friends waiting on the ground, or possibly a terminal bottleneck in peak hours due to gate saturation, or perhaps a missed connection. While non-critical as far as the schedule is concerned, even these events are bad for the "image" and for this reason should be avoided.

An incident does become critical when it results in a schedule change. A cancellation not only inconveniences the persons

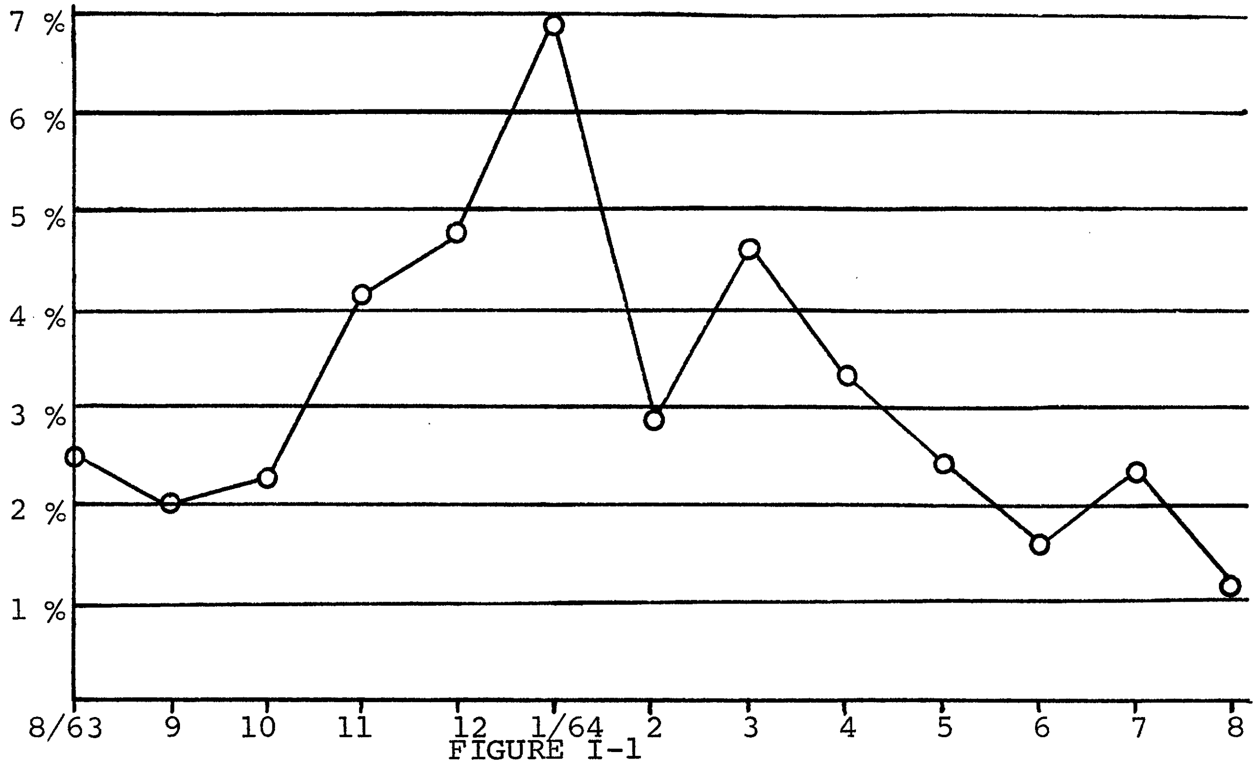

FIGURE I-1

Monthly Cancellation Rate for the "Big Four"

aboard the flight but also causes downline effects since the aircraft will not be available for at least some of its subsequent commitments. Similarly an especially late flight can trigger a chain of late departures if these flights were being held up for connecting passengers.

1.4 Schedule Protection

Because there are so many factors which can inject delays into an air transport operation, schedules are purposely given "reserve coverage" or "schedule protection". Schedule protection is provided by planning fewer hours of flying time than is theoretically possible and thus providing a cushion against unplanned but probable operational difficulties.

Reserve coverage may take any or all of these forms:

- Scheduling longer turnaround times than are necessary. This is schedule "slackness" and it serves to keep departures on time even if connecting arrivals are late.
- Stationing spare aircraft around the system. These aircraft may be able to move in if another aircraft breaks down. Often these spares are used for pilot trainning while "out of service" so that some benefit is derived from them.
- Having aircraft ready to fly long before their planned departure or after their arrival. The aircraft serves as a sort of temporary spare.

Several operational constraints hinder the efficient use of these schedule protection devices. First, maintenance requirements on any aircraft substituted for another must not be neglected. Second, the spare might not be at the right station when there is a breakdown or it might be unavailable for use until after a short delay. Third, substituting an aircraft from a later flight to cover for one on an earlier one may leave the later flight vulnerable. Finally, a spare aircraft might not be of the same type as the one it would replace and especially, might not have the size or the range to fly the route. These measures do afford some degree of flexibility to the system and prevent the occurrence of many potentially dangerous situations.

1.5 The Results of an Interrupted Schedule

If the schedule cannot be maintained, some losses will occur in the process of re-shuffling the remaining flights. Here are a few of the possible areas where losses can creep in, as well as the marginal savings incurred in cancelling a flight:

Tangible Losses -

- Net loss of revenue (from those who cannot be booked on a later flight
- Extra pay to the crews during delays
- Extra passenger handling costs during delays or cancellations (meals, lodging, alternate transportation...)*
- Miscellaneous additional equipment or personnel costs incurred solely by the delayed or cancelled flight (parking)
- Necessity of non-revenue "ferry" flights to balance crew or equipment commitments

Intangible Losses -

- Future revenue loss due to passenger dissatisfaction

Long Range Losses -

- Lower utilization of fleet meaning proportionately higher depreciation and hull insurance per flight hour over the year

Savings on Cancelled Flights -

- Marginal direct operating costs on the flight segment
    - Crew Salaries (Flight and Cabin)
    - Fuel and Oil
    - Landing Fees
    - Food Service
    - Passenger Liability Insurance & Property Damage
    - Misc. Ticketing and Communications Costs
    - Maintenance Costs put off

1.6 Causes of Schedule Irregularities

There are numerous reasons why some flights become delayed or are forced to cancel out. A few of the common causes for these interruptions will be noted here:

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* After the ticket in the flight coupon has been removed by the airline, the carrier is responsible for delivering the passenger to his destination, and for his care until he gets there.

- Bad weather or low visibility resulting in a station closing
- Mechanical breakdown of Equipment
    - On the ground - a delay or cancellation results
    - In the air - a crash or forced landing may occur
- Winds, storm centers or areas of icing conditions may force enroute detours resulting in delays
- Stacking or holding delays on arrival
- Airport capacity saturation may force delays on departure
- Gate congestion at the terminal
- Misc. delays caused by passenger loading, baggage handling, connecting passenger traffic late in arriving, among others

1.7 Schedule Control

Because of the financial importance that fast and efficient rerouting capabilities can have on its operation, the airline generally has some sort of central schedule control group to monitor the operations. This organization can quickly be in touch with almost any part of the system and has at its disposal much data on sales, equipment location and allocation, crew assignments, maintenance logs, and so on. Working under a few management regulations, and with a number of rules based principally on past experience, this group attempts to find the "optimal" reshuffling of flights in an interrupted network.* The use of the term optimal needs explanation. The schedule controllers really search for as many feasible solutions as they can find. They use

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* A description of the United Air Lines schedule control effort is contained in the January 1967 issue of Shield, the UAL employee's magazine. United has about 75 people in their Operations planning Center whose job is to keep schedule interruptions to a minimum. With approximately 1650 segments scheduled per day, a 3% cancellation rate would generate a lot of irritated customers.

data they can bring to bear on the problem in order to develop what seem to be valid courses of action. From among these feasible solutions, one is chosen as the best. Perhaps it looks as if it is the most profitable, or that it inconveniences the fewest number of passengers, or disrupts the fewest number of subsequent schedules. This becomes the basis for the schedule change.

Sometimes the problem presented to the controllers is so complex in nature, or the schedule so large, and the many interrelationships so subtle that what seems to be the optimal solution later turns out to have unforseen consequences. For this reason, regulations, experience and judgement, no matter how keen it is, can never meet all the problems associated with the controlling of airline schedules. In addition, the controller may not have the final word in the decision, especially if time is critical. Station managers, flight dispatchers, or pilots often will act out of necessity according to the way they read the problem; often their decision may not be the the one that a more thorough analysis might render, and they have required that the controller find a rerouting given that their line of action has occurred. As the network patterns and thus the schedules become even more complex, it is more and more likely that the decision maker will be able to see only what appear to be localized optima because of the inability of the human mind to absorb and effectively analyze the pyramiding results of a great many decision possibilities.

Here is where the computer, if adequately programmed, can find a role. Of course, the computer could never conceivably be set up to make exact, accurate, intelligent decisions using the diverse, irrational, and sometimes innacurate data which the human scheduler must work with, but it could become a valuable extension for the decision maker through its ability to quickly accept piles of data and then analyze a large number of alternatives in a complex but predictable framework. The decision maker, then, could have the benefit of seeing the effects that his solutions would have on the entire schedule, not just locally. His ability to come up with better results, by examining more alternatives, often quicker than he could trace through a single plan, would almost surely be improved.

1.8 Looking Ahead

This report examines the choices that are open to the schedule controller and attempts to find a rationale for implementing a real-time computer system for use in the optimal rerouting of aircraft. Because of the complexity of the problem, the role of a data processing system in the capacity of decision maker has not been terrifically successful, but it would seem that by using the computer for the jobs it does well, the decision maker's task could be beneficially expanded. Later a model of the computer system developed herein will be used on an example of an interrupted schedule. For these purposes, the schedule is set up as a capacitated network. The computer will aid the decision maker in setting up possible alternate paths through the network and then the optimal least cost flow will be obtained by a standard network flow algorithm.

Mention will be made here of the assumptions that are used throughout this report. Possible extensions of the theory to "tighten" these assumptions will be found in the Conclusion:

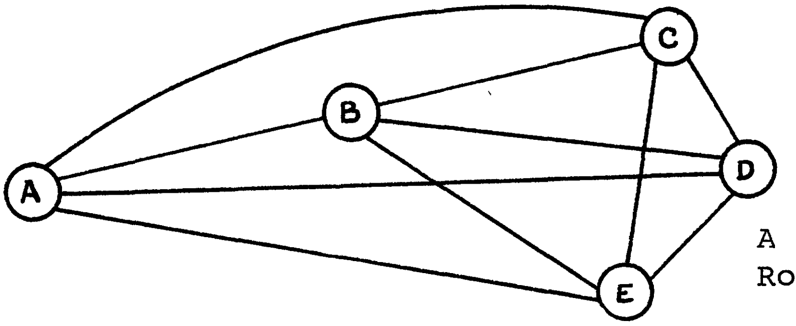
1. The desired route structure has been set up.
2. Passenger operations are dealt with exclusively
3. Data on equipment and locations, crews, union rules, maintenance requirements, etc. are readily available.
4. A single aircraft cost type is used.
5. Whenever possible, route adjustments are to be made so that regular operations may commence the following day.

CHAPTER 2

SCHEDULING AS A NETWORK FLOW PROBLEM

2.1 Schedule Representation

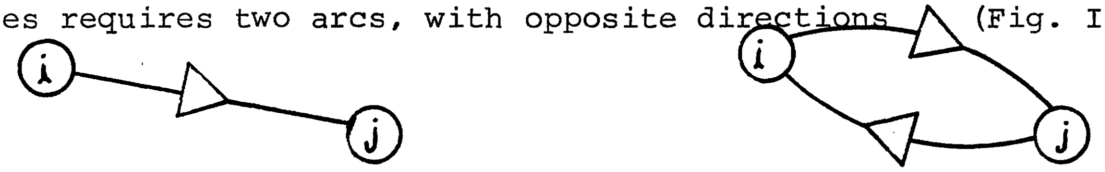
The route structure for an air transportation system can be represented as a two-dimensional network of fixed terminal locations and the airways linking these terminals (Fig. II-1).



A Two-dimensional Route Network

FIGURE II-1

Using the nomenclature of network flow theory, the terminals become "nodes" and the routes between them, "arcs". The flow in this case is aircraft. The flow of aircraft through the arcs is a directed quantity, always from some node i to some other node j. The possibility of flow both ways between two nodes requires two arcs, with opposite directions (Fig. II-2).



From i to j only

From i to j or from j to i

FIGURE II-2 - Directed Arcs

For the purposes of scheduling, a slightly different representation of the network is necessary. What were formerly station nodes are extended in time to cover the entire operational day. The nodes, then, are named by a station location and a time of day and represent the end points of any aircraft movement, such as an arrival, a departure, a retirement from or a return to service, and so on. The arcs between these nodes are the various routes that the aircraft may take. For example, the arc from node A0630 to node B0848 is a flight arc, which, if occupied, means that an aircraft leaves station A at 6:00 A.M.* and is scheduled to arrive at station B at 8:48A.M. Each node at a station is joined to the node immediately following it by what is called a ground arc. The flow on this arc gives the number of aircraft on the ground at any time. The ground arc which joins the last arrival at night to the first departure the next morning is often referred to as the overnight arc.** Of course, the number of aircraft on this arc are those which are parked and generally maintained during the night at that station.

Other possibilities present themselves. A spare aircraft would occupy an arc (a SPARC, perhaps) which has as its origin the first node of the day and its destination, the last one. Whenever this aircraft might be needed, the network would have to

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* Times are given on a 24 hour basis with 1200 as noon and midnight being simultaneously 0000 and 2400. Also all times are local station times.

** On some real networks, there might not be any period of total inactivity over the whole network due to all night flights. Similiarly the technical overnight arc may be just a few minutes depending on when the first and last flights were defined. For the purposes of this report, the system is assumed to be well behaved with no scheduled movements between 0030 and 0630 local times.

have a new arc formed which led from a newly formed node along the "sparc" to another newly formed node at the station. This will be discussed in more detail in the next section. When an aircraft breaks down, is in maintenance, or is otherwise unavailable, an out of service arc, or osarc, would span the time length of the down time period. A very simplified network with no interruptions is shown in Fig. II-3 for the purpose of depicting the arcs and nodes which are described above. Other opportunities for node and arc additions to the basic network will be discussed shortly when the subject of network alterations is examined.

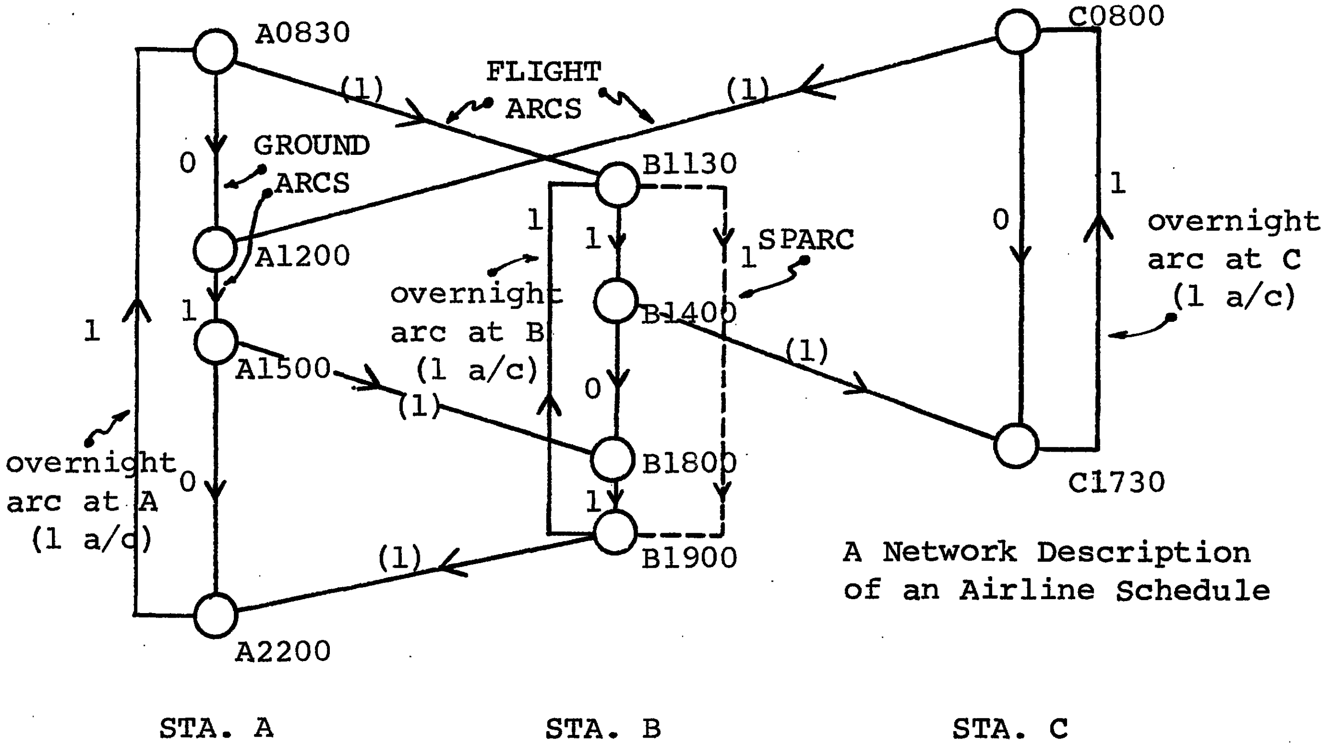


FIGURE II-3

In the above figure, the arrows denote the directionality of the arcs. The numbers in parentheses indicate the number of aircraft on each flight arc. Since the schedule is uninterrupted, there are no unoccupied flight arcs. The numbers beside the ground arcs, the overnight arcs, and the sparc indicate the number of aircraft on each of these arcs. Three aircraft are needed to maintain this simple routing: two to fly, and one to act as spare. One plane overnights at each station. Note that if any of the three stations were a lone maintenance base it would be possible to rotate any one of the aircraft to that base by substituting on to existing routings.

2.2 Flows in Networks

Associated with each arc $\overset{\frown}{ij}$ between nodes i and j, is the value x_{ij}, which denotes the number of aircraft on that arc. Each arc also has two capacity indicators. There is an upper bound on the flow, u_{ij}, and a lower bound, l_{ij}. For a feasible flow situation, it must be true that $l_{ij} \leq x_{ij} \leq u_{ij}$ on every arc. Finally, c_{ij} is the cost of putting one unit of flow through arc $\overset{\frown}{ij}$. Since aircraft are the flowing commodity, x_{ij}, u_{ij}, and l_{ij}, are all integers and all greater than or equal to zero.

A final constraint on the network is that there be a conservation of flow at every node:

$$\sum_{i,k} (x_{ij} - x_{jk}) = 0 \tag{2.1}$$

The network flow problem considered here is that of finding the maximum flow through the network at the minimum cost. That is, the problem is one of finding a feasible flow for which the following sum, taken over all the arcs is a minimum:

$$\sum_{ij} c_{ij}x_{ij} = \text{minimum} \tag{2.2}$$

The problem is to set up the right network with the right costs.

CHAPTER 3

MODIFICATIONS IN THE BASIC NETWORK

3.1 The Basic Network

The network configuration for the unaltered schedule is the basis on which any subsequent modifications will be formed. When the network of flight and ground arcs has been developed, the unaltered routing is obtained by "locking in" all the scheduled flights. This is accomplished by setting $l_{ij}=u_{ij}=1$, on all the flight arcs. The costs for these arcs are obtained by subtracting the revenue for the flight (which is being constantly updated until departure time), from the actual direct operating cost of flying the segment: Cost = -Revenue + Marginal DOC's. Note that this cost does not include the "fixed" DOC charges of hull insurance and depreciation. Since the revenue is usually much higher than these marginal DOC's, c_{ij} is generally negative.

The ground arcs have a lower bound, $l_{ij}=0$, and a high upper bound, say $u_{ij}=9$ (any number will suffice as long as it is higher than the maximum number of aircraft that will be on the ground at any station at one time). The costs for ground arcs is $c_{ij}=0$. More will be said about the relationships between the costs in this model a little later.

It should be noted that, were the costs on the flight arcs generally negative or small and positive, then the max-flow min-cost flow routine would probably return the same solution in the case in which the flights were freely chosen ($l_{ij}=0$; $u_{ij}=1$), as it would for the constrained flight case mentioned above. In either event, the marginal profit found would be the maximum, and would have the value:

$$-\sum_{ij} c_{ij}x_{ij} \tag{3.1}$$

3.2 The Interrupted Network

When an interruption does occur, certain changes are immediately obvious. The nature of the situation tells which arcs must be locked in or locked out of the solution. Take as an example a flight that is delayed 35 minutes enroute to its destination. Here, the node at the destination would have to be moved forward in time by 35 minutes (Fig. III-1).

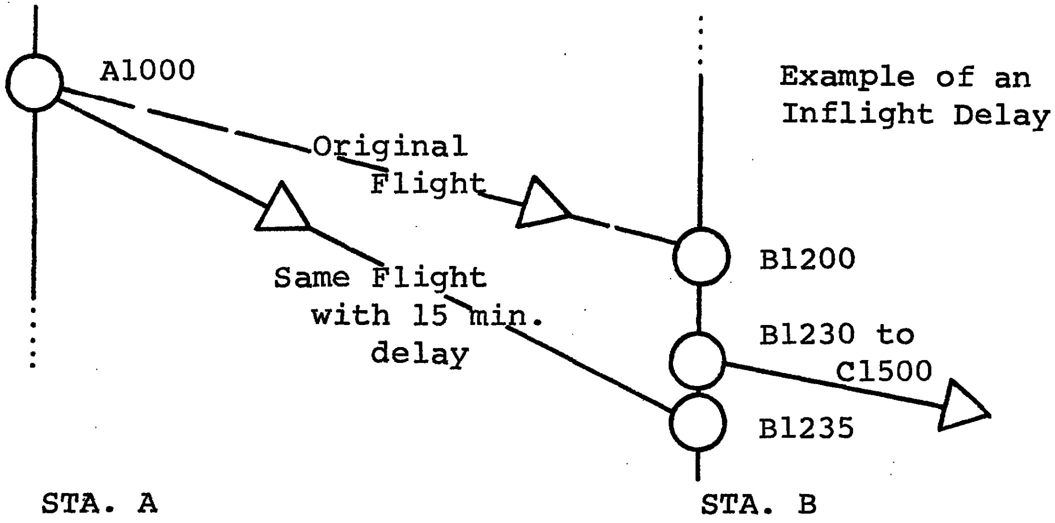


FIGURE III-1

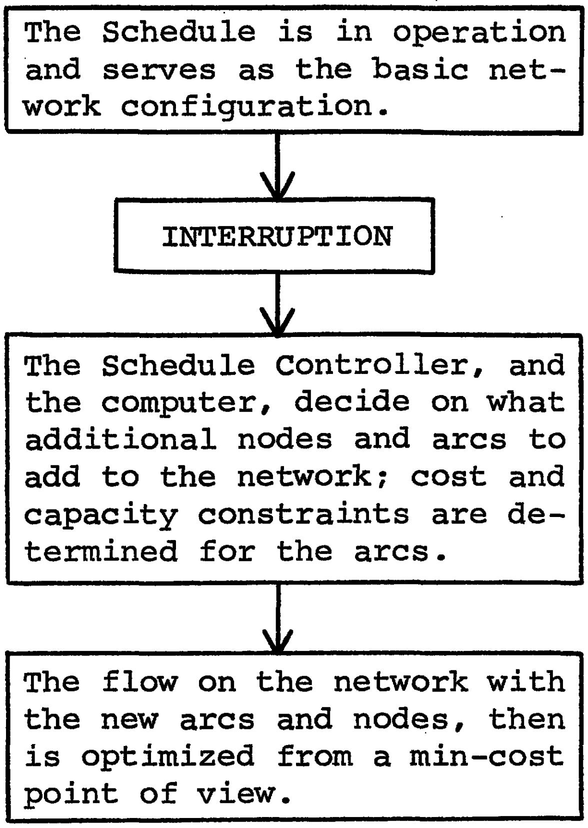
By removing flight arc A1000-B1200, by setting $l_{AB}=u_{AB}=0$, and adding arc A1000-B1235, the new situation has been modelled. If the network were now sent to the optimality routine, some solution would be found. However this solution might be highly inefficient. This model has made no attempt to look at other effects of the interruption. Suppose that there were 20 passengers on the delayed flight who held connecting tickets on flight B1230-C1500. If this flight is sent out on time, these connections would be missed, and the carrier would be liable for assuring that these persons got to their destination. By holding the connecting flight for 5 minutes plus the time necessary to transfer the passengers and their baggage, all would be well except that this flight would be a few minutes late in arriving at C. The various cost advantages to sending the flight out and delaying it should be determined and then placed on alternate policy arcs so that the optimality program can choose whether it is better to hold up the flight or not. Thus it might be found that the connecting flight should go off on time if there are fewer than 5 connecting passengers, but should wait if there are more.

What is being said here is that there are many possible reroutings to adapt for an interrupted schedule. The best one can be found only if the network is set up with all the different alternatives available, with accurate costs, and then having this augmented network "solved" by the minimum cost algorithm. In its most crude form, this functional flow is shown in the block diagram in Figure III-2.

An attempt will now be made to isolate the chacteristics of the various types of schedule irregularities that may occur, and then to develop network modification techniques that can be adapted to these situations.

3.3 Details of Interruption Types

The discussion of the types of interruptions that can come



The Schedule is in operation and serves as the basic network configuration.

INTERRUPTION

The Schedule Controller, and the computer, decide on what additional nodes and arcs to add to the network; cost and capacity constraints are determined for the arcs.

The flow on the network with the new arcs and nodes, then is optimized from a min-cost point of view.

FIGURE III-2 - Schedule Revision

up will follow the outline presented below. Some of the cases proposed are quite infrequent, and so will not be stressed in the following discussion.

I. Permanent Loss of Aircraft
  A. Air Crash
  B. Destroyed on Ground (Fire, Accident,...)
  C. Miscellaneous (Hijacked, Military Emergency,...)

II. Delays
  A. Ground Delays
    1. Equipment Malfunction
    2. Holds on Takeoff due to Traffic Congestion
    3. Terminal Gate Saturation
    4. Loading, Unloading, Baggage Handling, Turnaround Delays
    5. Delays for Late Connections, VIP's, Weather
    6. Emergencies, and Miscellaneous

- B. Inflight Delays
  - 1. Flight Plan Exceeded (Adverse Winds Aloft, Detours to Avoid Storm Centers or Areas of Icing, Altitude Changes in Rough Air, ...)
  - 2. Stacking and Holding at Destination
  - 3. Forced Landing or Return to Origin due to Mechanical or Emergency Reasons (Flight then continues later)

III. Cancellations

- A. Equipment Breakdown
- B. Aircraft did not Arrive from Previous Station
- C. Intended Destination is Closed
- D. Forced Landing (Flight does not continue)
- E. Miscellaneous (Lack of Crew,...)

IV. Station Closings (Weather, Visibility, ATC malfunction)

- A. Station Closed to Inbound Flights Only
- B. Station Closed to Outbound Flights Only
- C. Station Closed Down Completely

These occurrences will now be examined in more detail. The consequences of the interruption, some of the possible factors that may influence the solution, and the network alterations that are necessary, will all be presented whenever possible.

I. Permanent Loss of Aircraft

A. Air Crash (Figure III-3)

Consequences -

As far as the mechanics of the schedule is concerned, the loss of an aircraft, and possibly the crew, means that the routings must be adapted so that they may be serviced with one fewer vechile, and one fewer crew in the rotation.

Possible Solutions -

Of course, nothing can be done for the flight in question. However, the later segments that were to be flown by this aircraft need not necessarily be cancelled if it can be shown that a spare aircraft, or an aircraft on another route can replace the lost vehicle.

Network Alterations -

An out of service arc, "osarc", with $u_{ij} = l_{ij} = 1$ is added from the flight origin to the end of the day. At the end of the day, the overnight arc flow is reduced by one so that the next day's schedule can be adapted to handle the loss.

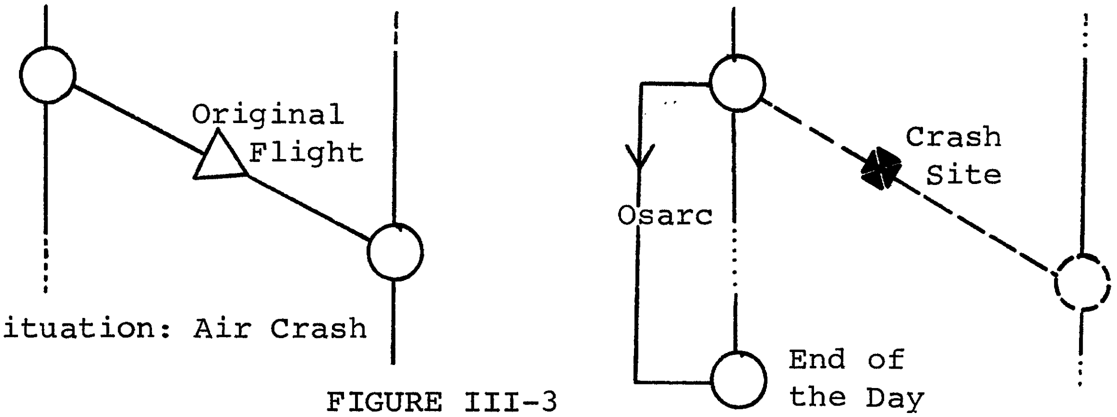


FIGURE III-3

The cases of the aircraft lost on the ground, and the hijacked aircraft will not be looked at due to the infrequent occurrence of this type of situation.

II. Delays

A. Ground Delays (Figure III-4)

Consequences -

When an aircraft is delayed for some time, revenue begins to be lost as passengers decide not to wait (this is especially true if there are competing flights available). Also, a late departure means a late arrival at the destination. This can cause problems with connecting passengers or in the ability to turn the aircraft around for the next departure.

Possible Solutions -

A spare aircraft may be able to take over. An aircraft from a later flight might be put on the ealier route in the hope that the down aircraft will be able to take over on the later segment. (Note that maintenance requirements must not be endangered). At the destination, connectinf flights may be delayed, and a spare vehicle may be used to fly the continuing routing.

Network Alteration -

An osarc is set up for the duration of the delay and the aircraft is locked on it. If there is a spare available, an arc must be added in order to bring it into service. Non-revenue "ferry" flights may be set up from other nearby stations. Downline connecting flights would have the option of either leaving on time or waiting for connections (this option will be explained shortly).

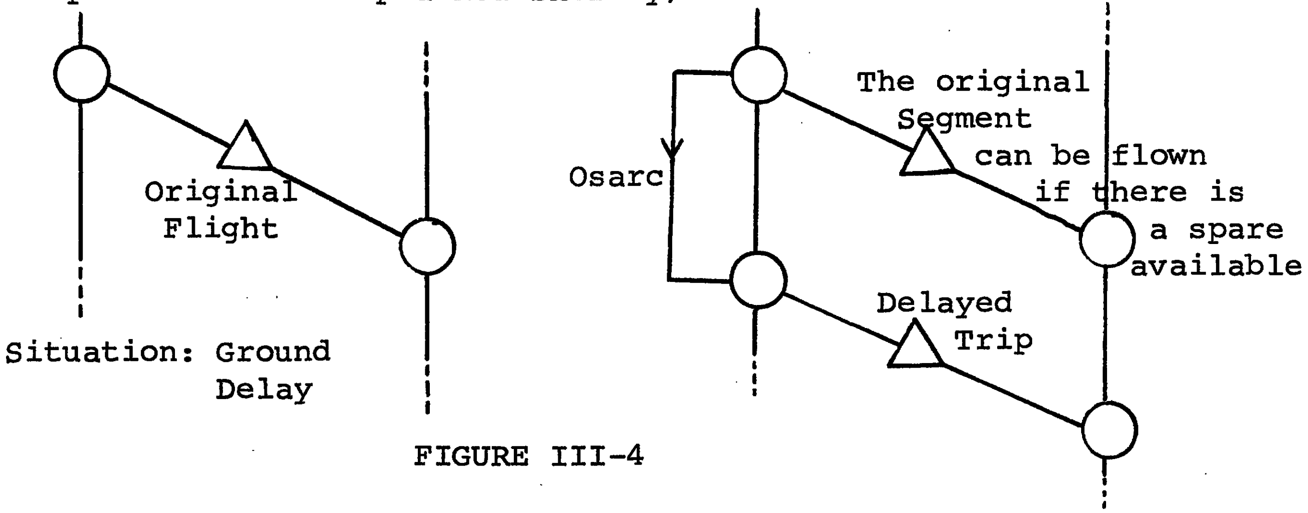


FIGURE III-4

B. Inflight Delays (Figure III-5)

Consequences -

Inflight dealys mean late arrivals at the destination, and usually incur increased trip costs for the extra fuel used, the longer crew period, etc. The late arrival would have much the same effect on later flights as the delayed flights mentioned above.

Possible Solutions -

At the destination, an analysis similar to the ground delay case would be needed.

Network Alterations -

The normal flight would be removed and replaced with one from the same origin but with a later destination time. Further alterations would follow the previous case.

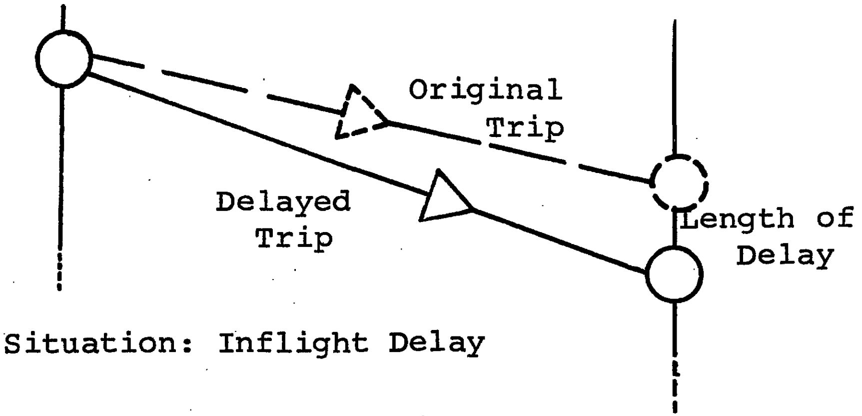


FIGURE III-5

IV. Cancellations

Consequences -

There is a loss of revenue involved in a cancelled flight, if the passengers can not be accommodated on a later segment. Sometimes passenger handling costs, such as meals, lodging or alternate transportation will be incurred. When a flight is cancelled, the aircraft stays where it should no longer be, and does not show up where it should be. Note the special case of a forced landing where the aircraft might even end up at a station that is not in the carrier's route structure.

Possible Solutions -

1. In the case of the intended station being closed, the aircraft may either not fly at all or purposely fly to a base near to the destination so that the passengers can be sent in quickly by surface mode. (Figure III-6)
2. If the equipment fails, the situation is similar to the case of the delay, and spares or substitutions can be sought.

Network Alterations -

1. If the vehicle is ready to fly but the destination is closed, set up possible alternate destinations.
2. For breakdown in equipment, use osarc as before.

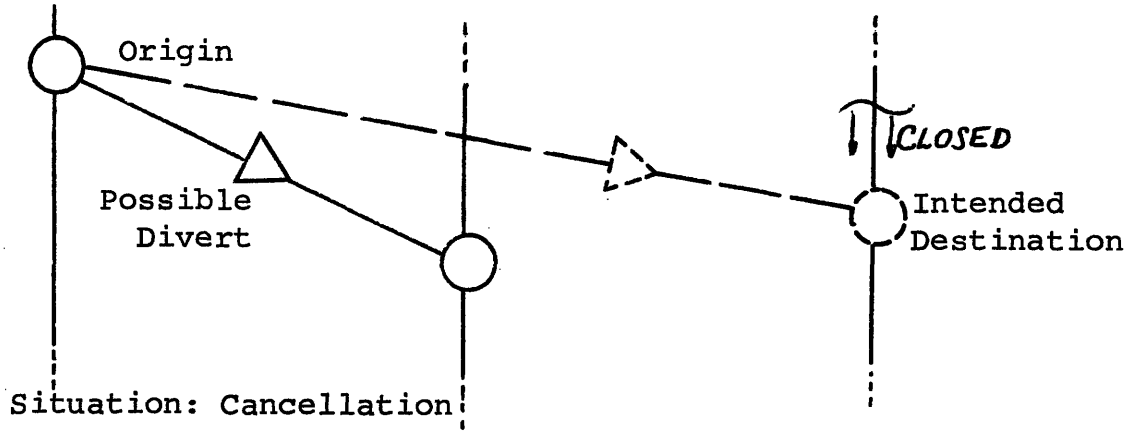


FIGURE III-6

IV. Station Closing

A. Closed to Inbound Only (Fig. III-7)

Consequences -

All inbound flights are locked out but aircraft on the ground may be flown out if desired. There is a possible loss of revenue if upstream stations must cancel out some flights. The flights on the ground can be flown or not depending on the advantage of this move.

Possible Solutions -

The aircraft inflight have three possible options:

1. They may return to the originating station, or
2. If the closed station is an intermediate stop on a through segment, the aircraft may overfly to its next scheduled destination, or
3. The flight may divert to a nearby open station where the passengers either will be flown to their destination later or sent to it by surface means.

Flights to the closed station which have not departed may be purposely sent out with the intention of diverting. Also, a flight might leave for the closed station anticipating that the station will open during the duration of the flight.

Network Alterations -

The arc additions for the three options listed above will be described in the next section. Alterations involve locking out all flights to the closed station, and adding arcs for return, overfly, and divert possibilities.

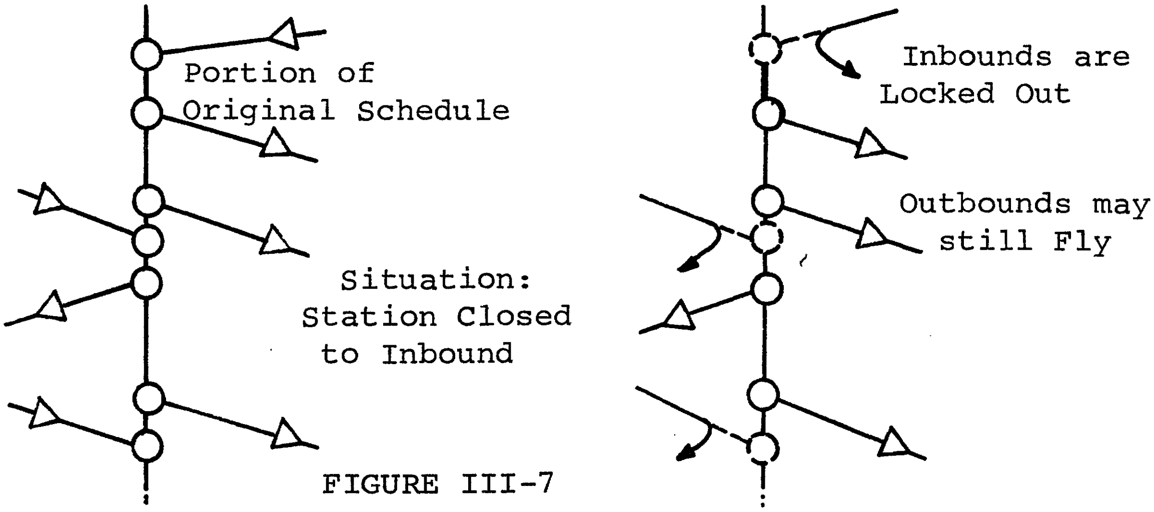


FIGURE III-7

B. <u>Closed to Outbound Only</u> (Fig. III-8)

Consequences -

Flights may come in to the station, but once they are in, they must remain there for the duration. There may be capacity limitations at the base which must be observed. Further, the airline might choose to overfly this base in order to keep as much of its fleet available as possible. All flights out of the station must be cancelled.

Possible Solutions -

Divert flights to a nearby station and bring in the passengers by a later flight or a different mode; this keeps the aircraft in service.

Network Alterations -

Lock in all aircraft on the ground during the period of the station closing. Add the possibility of inflight diverts for the inbound segments.

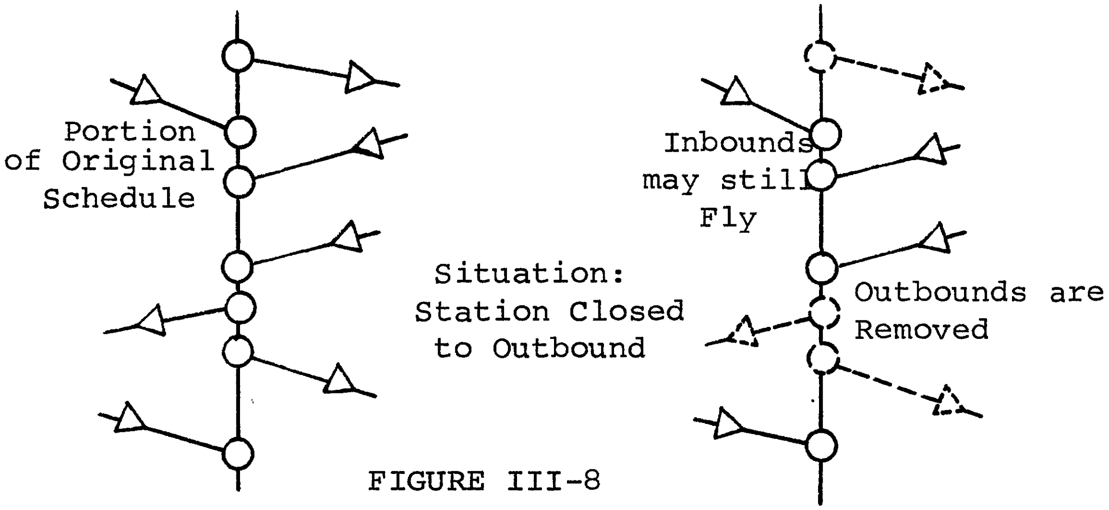


FIGURE III-8

C. <u>Station Closed Down Completely</u> (Fig. III-9)

Consequences -

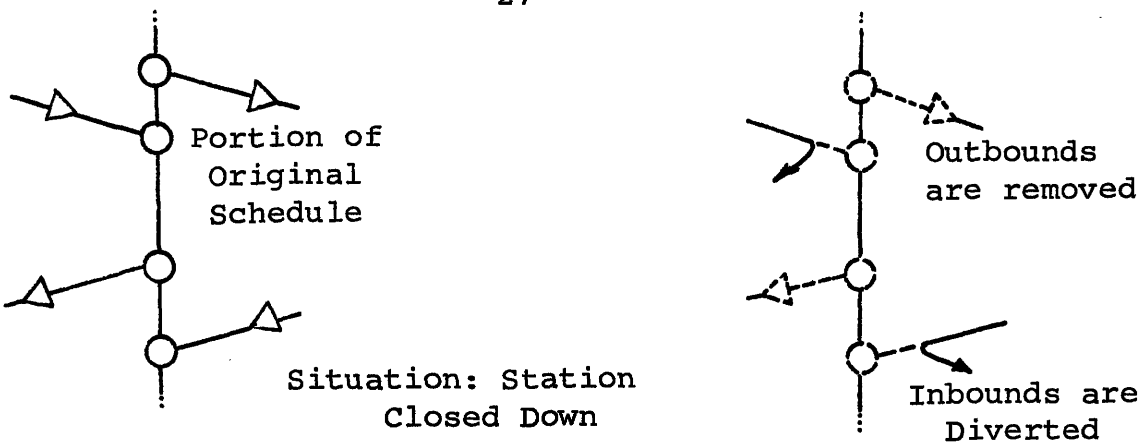
There are no movements in or out of the station. Upstream and downstream adjustments will be necessary due to the shortage of equipment. Divert inbound flights as before.

Possible Solutions -

If the time that the station will be down is known fairly certainly, then by locking out the inbound flights and putting all the grounded aircraft on "osarcs" for the time of closing, and resolving the schedule, the network can be run, and the aircraft are in their most beneficial locations for a smooth return to service when the station opens.

Network Alterations -

Cancel out the outbound flights, and use alternate divert possibilities as before. If the situation is as described previously, then osarcs would be added to simulate the closing by taking all the grounded vehicles out of service.



Situation: Station Closed Down

FIGURE III-9

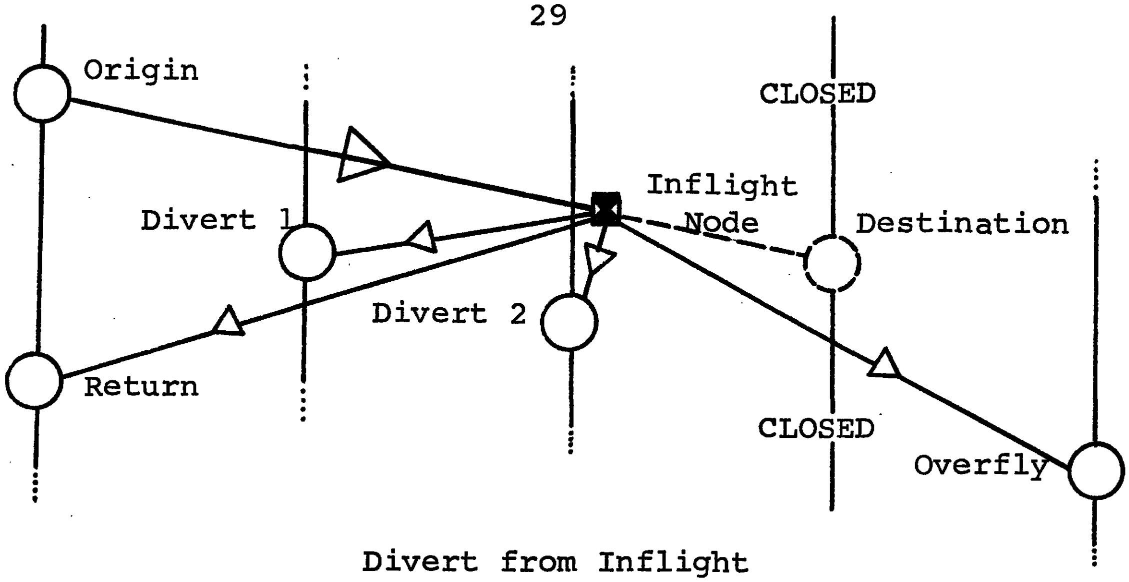
These, then, are the basic types of schedule interruptions that the airline faces. It is an easy matter to adapt the network to take the interruption itself into account. This usually means just locking in or locking out some arcs. The real job for the decision maker is in deciding how the rest of the schedule should adapt to compensate for the irregularity. As has been shown, this involves the careful addition of alternate decision possibilities to the basic network so that the optimality program can do the work of choosing the best solution.

3.4 Alternate Network Routings

Two different network alteration schemes will be presented here. The first of these is called the "Divert Option", and the other is the "Alternate Delay Option". These are basic network packages which can be added when necessary to simulate the choices open to the controller in the process of reallocation of flights.

The "Divert Option" is used when there is a station closing type of interruption. As was mentioned previously, there are three choices open to the flight which is enroute to this base. These are, return to the origin, overfly to the next destination, and divert to a nearby open station. There are numerous economic consequences related to any of these alternatives, depending on many factors including the length of the flight, how far it has progressed, the number of passengers aboard, and so forth. To assure that the rerouting will be a good one, accurate cost estimates for each of these choices must be assured. Two representations of the network modifications required by the inflight divert are given in Figures III-10 and III-11. In the first of these, an inflight node is added at the time-position of the flight when the station closes down. The flight arc from the origin to the inflight node is locked in, and the arc from the inflight node to the destination is removed. The return to origin case requires the addition of an arc from the inflight node back to the origin. Costs on this arc depend on the details of the problem. Marginal DOC's are available, but the revenue that is lost depends on how many of the passengers were able to be booked on later flights. Also passenger handling costs for meals and/or lodging might be involved. An arc is added to indicate an overflight to the next destination. Revenue loss here involves the return of those passengers destined for the closed station by other means, and the loss from those customers at the closed station who would have taken the flight. The possibility of a diverted flight to the two nearest open stations is presented. Similar to the overflight, costs of transshipping passengers, lost revenue at the closed base, and extra operating costs for the diversion are incurred.

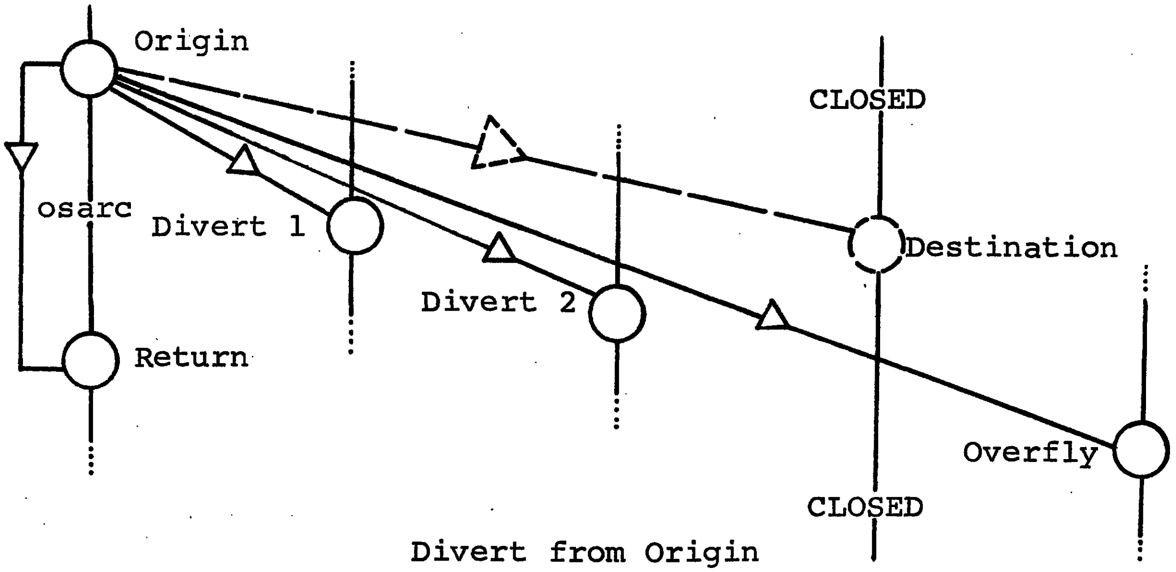
In the second figure, the same situation is presented, but the way in which the arcs are handled is changed. Instead of an inflight node being added, all arcs are added from the origin. This creates the fiction that flights of duration sufficient to model the real situation leave from the origin. The original flight is removed. Costs on the arcs added are as before, but



Divert from Inflight

FIGURE III-10

This is the inflight "Divert Option" in which an inflight node is added, from which the various alternate routes are branched.



Divert from Origin

FIGURE III-11

This is the same situation as above, but instead of an inflight node, all alternate arcs emanate from the origin.

with the additional expense of the marginal DOC's for the part of the flight completed before any changes were made. This representation is not as close to the real situation as the other one, but it is simpler to implement.

Whenever there is a closed station and flights enroute to it, or whenever there are flights due to leave for this station and scheduled to arrive before the base opens again, then this "package" of arc and node additions can be placed in the new network which then goes to the optimality program.

In the "Alternate Delay Option", the choice is given to send a flight out on time, to delay it for a number of specified intervals, or to cancel it all together. The configuration is shown in Figure III-12. Here, flight arcs are provided which correspond to on-time operation, as well as delays of 15 minutes, 30 minutes, and 1 hour. Now, if the flight is taken at all, only one of these arcs can be flown at the exclusion of the others. Therefore, an arc is added from the last movement preceeding the on-time departure, to an artificial "star" node. The upper limit of this arc is 1. Arcs are then added from the star node to the alternate delay arcs. For all these arcs, $c_{ij} = 0$, $l_{ij} = 0$, and $u_{ij} = 1$. The configuration is designed so that there will be the right flow on the intermediate ground arcs no matter which of the delay alternatives is chosen.

The delay arcs group are added to the network at each node where there is a possibility that a delay might be advisable for any reason. Some of these possible placements might not be at all obvious, since they might be required by a chain of events that is complex in nature, thus care must be taken in the use of these methods. As in the previous discussion, it is of prime importance that an accurate cost picture is developed.

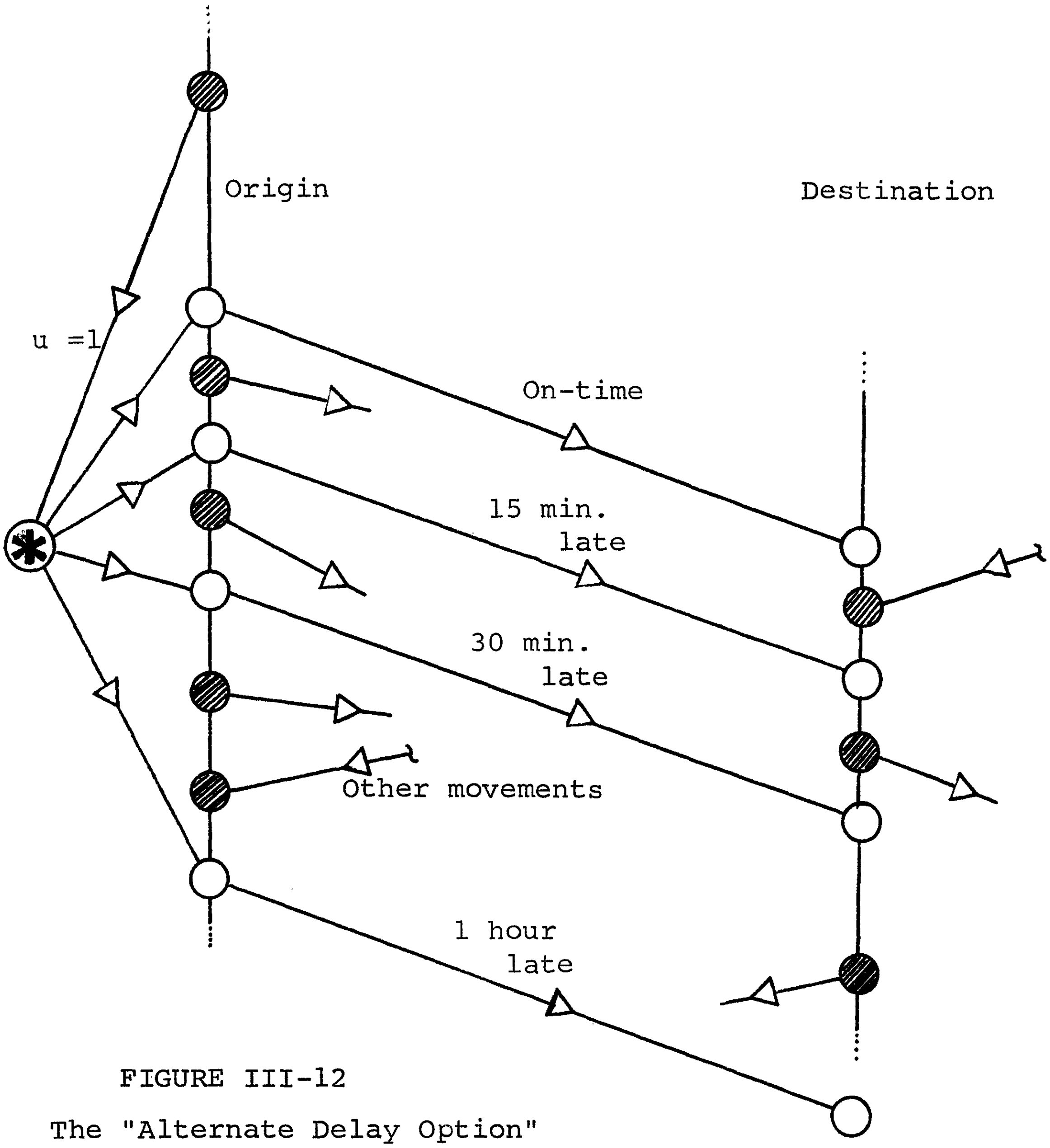


FIGURE III-12

The "Alternate Delay Option"

3.5 Further Alterations

There are other network alteration types which are perhaps not as complicated as the two presented above. Mention has already been made of the use of spare aircraft to give schedule protection. To bring a spare vehicle into service requires that an arc be added between the "sparc" and the station where the spare is located.

A final example will be the use of non-revenue "ferry" flights. In most cases, the use of "ferry" flights is expensive. There is no revenue since the flight is not a regularly scheduled departure. Usually these flights are made necessary when, at the end of the day, the various aircraft in the fleet are not all where they should be either for the start of the next day's service or for their scheduled maintenance. Sometimes, though, it may be more inexpensive to bring in an aircraft from a nearby station to replace an aircraft on a very lucrative route, than it would be to have that vehicle carry out its assigned schedule and cancel out the good route.

Thus, in altering the schedule, the opportunity must be made available for "ferry" flights if the optimality routine should happen to find that this course of action is actually the best available.

A couple of general comments will be used to finish up this section. First, the optimality routine and the network should be such that practically no time is wasted in running through the already completed part of the schedule. The system should be internally "locking in" the flight segments as they go off during the day so that a minimum of effort is expended on this part of the network. This means that as the day goes on, the solution to the problems should be getting faster, since there are fewer later alternatives to evaluate.*

Second, the program should be efficient in choosing those sections of the schedule on which to work and which to ignore as being totally irrelevant to the interruption and to its subsequent solution.

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* Appendix I contains some comments on the possible reduction of the already completed schedule into a "composite node". This might be necessary if there were critical storage problems.

CHAPTER 4

COMPUTER APPLICATIONS

4.1 The Use of Computers in Airline Operations

In some respects the airline industry has been among the leaders in introducing computers into its operations. High speed data processing has given the airlines the ability to keep an accurate, up to the minute record of their complete inventory of seats available, and to have instant access to myriads of stored data. Yet the airlines have lagged considerably in placing any real emphasis on the use of the computer for help in decision making. Since middle and lower managers spend most of their time making decisions on recurring problems, it might be sensible to use the computer in this role and thus free these men for more important tasks. There are several reasons why an intensive effort has not been made in this area.

1. High Cost - It has cost the airlines several tens of millions of dollars just to obtain and program the "bookkeeping" type of systems which they now have. The costs of automating for certain decision making functions would be quite high, and some people think that it is just not worth the cost.

2. Equipment - Only recently has a new generation of computer hardware become available so that detailed programs of the size of most real workd problems of interese be even considered.

3. Programming Difficulty - There is a general feeling that most of the real problems that the decision maker faces are so frought with irrational, conflicting, and sometimes seemingly

illogical inputs that only the human mind could quickly decide what is valid and what weight to give it. Much of the decision making of the schedule controller is based on probabilistic data. For example, there may be a chance of a station closing later in the day, and this might be the basis for some decision that the "logical" capacity of the computer might reject. Tied in with the above reasoning is the idea that any large scale program would be so full of simplifying assumptions that it would end up sending out answers that were a crude approximation to reality.

4. Fear - There is some suspicion among the managers whose whose jobs were to be automated that the computer was going to move in and they were going to move out and not up.

5. Previous Bad Experiences - A general tendency for people who are not familiar with the limits of computers to either expect too much or too little from the results. When too little is expected, the job gets done, but either the result is not recognized or it doesn't seem worth the cost. The serious problem occurs when too much is expected. In awe of the ability of the machine, the data recipient will take the results to be the ultimate in truth. Thus by not looking at the programming assumptions, the person ends up doubting the ability of the computer to do accurate work when the previous results prove slightly inaccurate.

To a certain extent there are some valid arguements in all these views. No computer can duplicate the human thought processes for it is debatible whether even two people react in the same way to a given set of stimuli. But the jobs that computers have learned to do, they do extraordinarily well, and as programming has become more sophistocated, the list of things that they can do has increased rapidly. It would be foolish not to at least explore their possible areas of usefulness.

4.2 The Computer and Schedule Control

The current use of computers in the area of schedule control is practically nil. Their only important function is again in the capacity of a huge, dependable data storage and retrieval device. Airline computers are generally real-time systems. This means that they can be made to give very fast response to random input requests. Input is usually accomplished by a keyboard or by punching a button; output is either typed or flashed on a cathode ray tube. It would be in this type of set up that a computerized "schedule revision program" would be the most useful.

At the time of a schedule interruption or in anticipation of one, a controller would convey the nature of the irregularity to the revision program. He would quickly have the computer's estimate of possible alternate nodes and arcs to add to the network, and the values for these arcs. With these suggestions and his own ideas about the characteristics of the situation, the controller would set up a new network of alternate routings and have the minimum cost flow optimality routine return the changes in flights required by this strategy. A great number of different plans could be tested in a very short time, and the long range effects of all the differing plans would be known. From among these results, one would be implemented and become the basis for the revised aircraft routing.

Thus the purpose of the computerized revision program is not to put the schedule controller out of a job. This program would be merely another addition to his analytical bag of tricks, albeit a powerful one. Hopefully the use of data processing in this capacity would enable him to extend his analysis more deeply and more accurately than is possible at the present time, and perhaps with more speed.

4.3 System Configuration

With an increased emphasis on computer systems, it is logical to assume that the operating structure of management in the future will be built around a fully integrated management information system. A representation of such a system as it might someday evolve for an airline is shown in Figure IV-1. In this set up, the computer has a hand in many decision oriented problems as well as in data storage. In this report, interest is centered on the functional grouping under the area "schedule control". The exact nature of this system must, of course, follow the logical development that the airline feels is right for their operations. Therefore, the scheme that is given here is meant only to depict the various interactions that seem evident. This possible system configuration is presented in Figure IV-2.

As can be seen, the revised schedule control program is tied in with and receives data from other portions of the information system, specifically from reservations update, optimal aircraft scheduling, crew scheduling and maintenance assignment. The following comments should serve to explain the configuration as given in Figure IV-2.

By means of the remote agent sets that the station personnel use to deal with the reservations computer, a constant update on sales is maintained. Using a predetermined set of regulations, the current revenue picture is extrapolated to give a guess on the expected revenue on all later flights in the schedule. Other regulations, along with directives and data on the marginal costs which are incurred in various situations then are fed, along with the revenue estimates, to a program which computes the costs on various arcs which may be added to the network. From this cost data, the expected marginal profit for any schedule configuration can be determined.

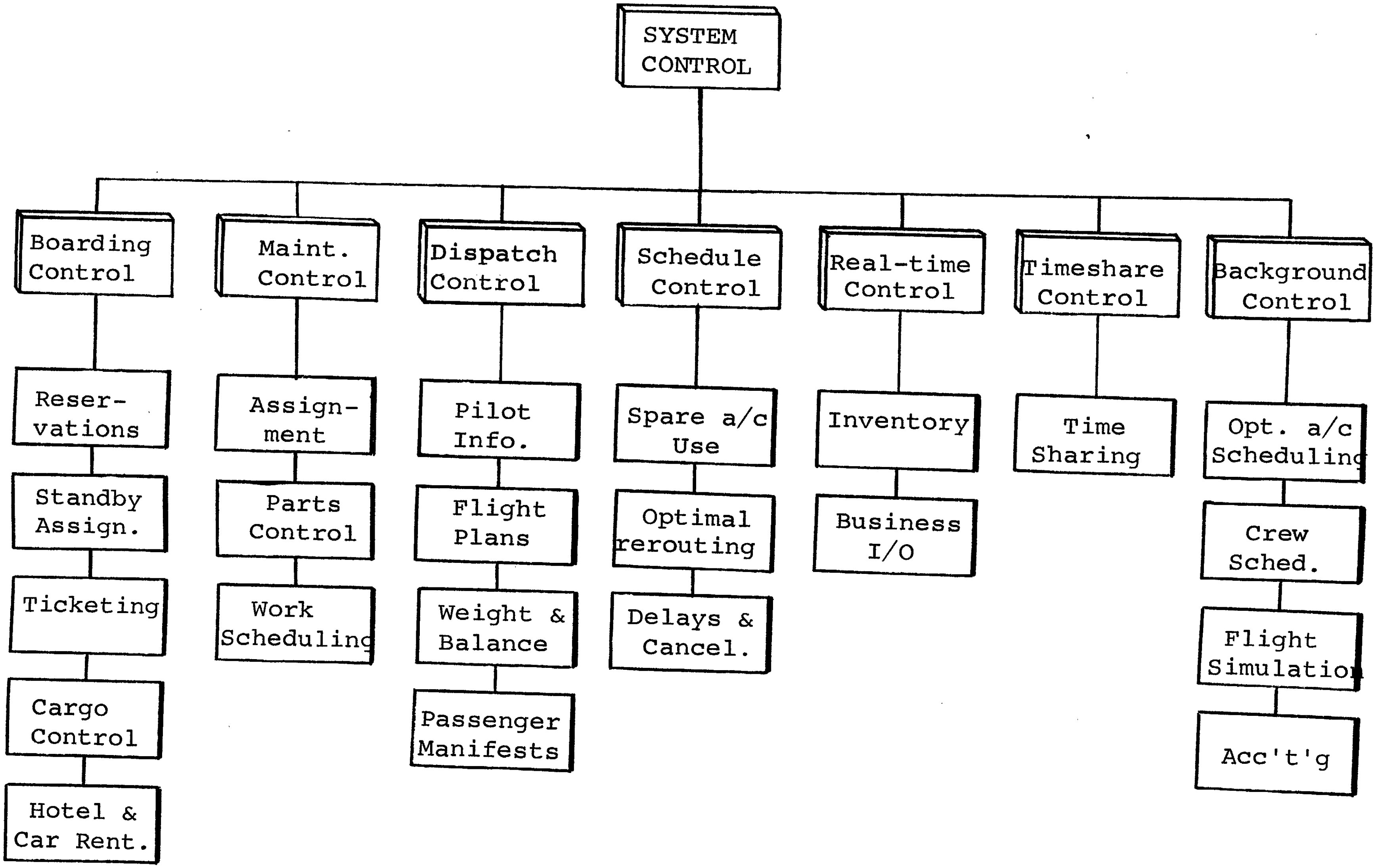

FIGURE IV-1

Computerized Management Information System for Airline Operations

Source: Adapted from Fig. VI-2 of Ref. 8

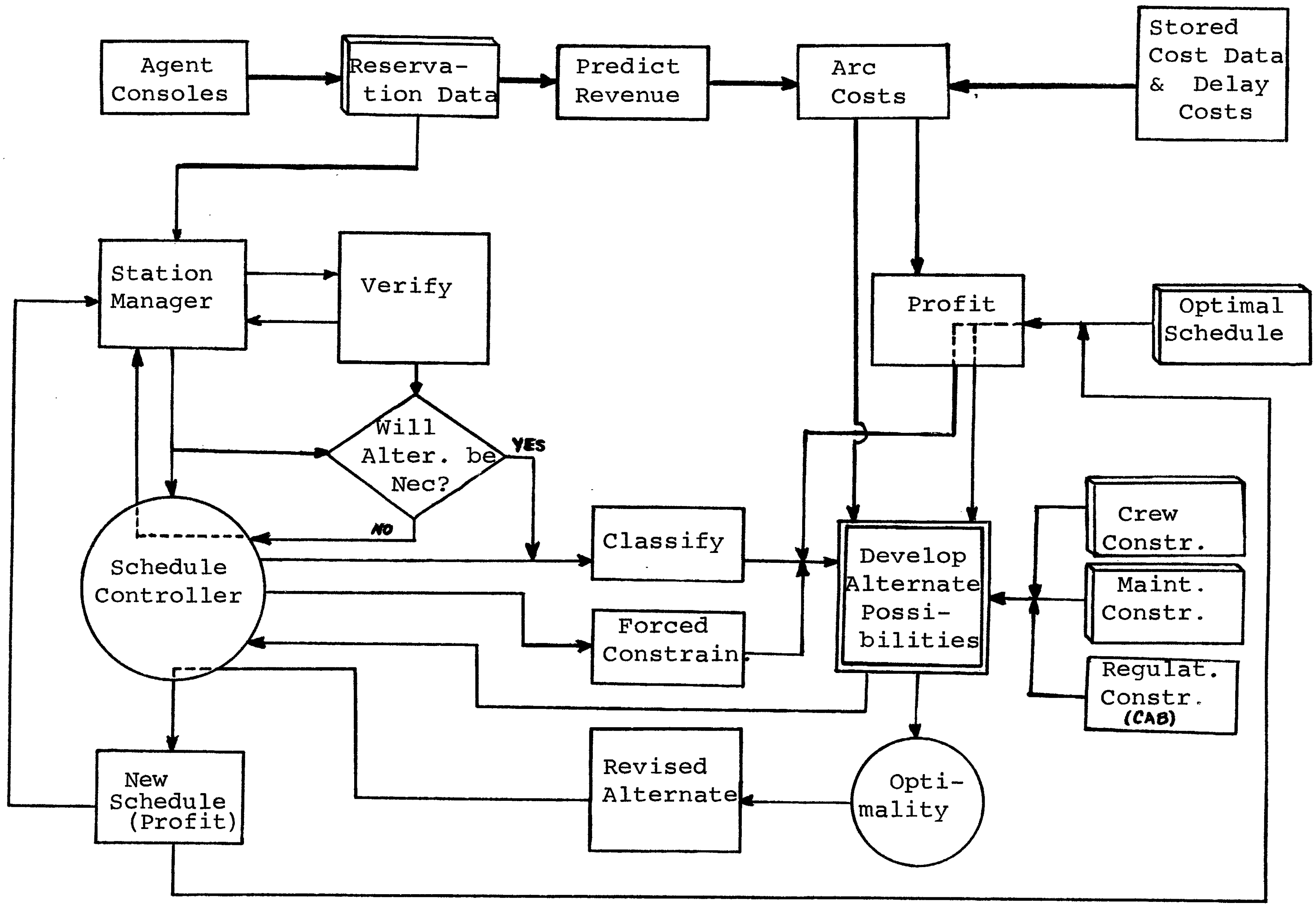

FIGURE IV-2 Schedule Control Functional Flows

Each station has a manager or dispatcher who would verify each aircraft movement that occurred at his station. If a scheduled movement is not verified, the computer would ask the dispatcher for information. In the case of an irregularity, (other that minor perturbations in the arrival or departure times), the dispatcher notifies the schedule controller. If the situation still proves to be non-critical after brief examination, no further action is undertaken, and the schedule remains unchanged even though there may have been a minor delay involved.

If, however, it is found that the interruption is of such a nature that it will disrupt service, the pertainent information is sent on to the schedule revision routine. At this point, the controller is gathering all the data that he normally would, and is checking the availability of spares, etc. He has access to the stored data on crew and maintenance constraints, and CAB directives.* He may desire to force certain constraints on the solution, for example, he might want to find the optimal reallocation of flights given that certain ones are forced to be flown. By changing the flights which are locked into the solution, and rerunning the optimization, a number of different strategies can be evaluated.

With the information that has been given to it about the nature of the interruption, the computer decides which nodes and arcs should be created to allow for any possible alterations. The controller may add or delete arcs as he feels is proper, and can monitor the values that the computer sets on the arcs. Accept-

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* The carrier may have received warnings about too many cancellations on a particular route; management decides that this route should be locked into the solution wherever possible regardless of the cost disadvantage.

able arcs and node additions must be consistant with the many outside constraints on the solution.

It is envisioned that all this schedule altering will be done internally. That is, one part of the program decides what alterations are necessary and then it calls another part of the program which sets up the arc and node additions in a form that is accepted by the optimization routine. The controller's forced constraints are accepted by the program and included when the new network is being set up.

Once the alternatives are decided upon, the result is a new capacitated network with arc flow costs. This network is solved for the max-flow min-cost solution by the optimality routine which returns the solution to the controller. This optimality algorithm is quite efficient in finding new solutions for networks which are basically similar to a previous configuration. So, since subsequent schedule changes build upon themselves, a large number of passes can be made through the routine in little time.

The controller now makes a decision on the best course of action on the basis of the results just obtained, and conveys this information to the stations involved. The newly revised schedule and the profit picture now become the basis for any subsequent alterations.

To implement a system such as this in its entirety would be an ambitious undertaking. Yet hopefully the flexibility that it would give to the operations would make its difficulties worth resolving.

In the next section, the optimality routine used in the model for this report is examined.

CHAPTER 5

THE OPTIMALITY PROGRAM

5.1 Characteristics of the Optimality Routine

Throughout the previous sections, mention has been made of the optimality routine without any real attempt at a definition or elaboration of this part of the program. The optimization routine is right at the heart of the "Revised Airline Schedule Program". As envisioned, it would take any network of nodes and arcs, along with the arc costs and the capacity constraints, and find the minimum cost flow solution through this network.

There has been considerable effort in the past few years on the creation of network flow algorithms which would efficiently solve max-flow min-cost problems. A number of methods are now in existance, and these, as well as work not yet completed, would be used to specifically tailor a network flow routine to the exact nature of the airline's operations. Special emphasis would have to be given to the as yet unsolved problem of multi-commodity flows (Ref. 5), if the possibility of an interchange of aircraft types were to be included in the model.

It might also be possible to use some of the techniques of analog computer mechanization to simulate the capacitated network. Then by patching in predesigned electronic elements, the re-assigned flow of aircraft could be read out as a steady state current between the node pairs of the analog network.

Whatever methods finally have a part in determining the form of the algorithm, there are three tasks which the program must be able to perform:

1. It must find the maximal flow through a capacitated network, while minimizing the sum of the flow costs through each arc.
2. It must be able to deal in integer flows.
3. There must be the ability to quickly find new solutions on the basis of existing ones.

5.2 The "Out-of-Kilter" Method

A very efficient method for solving network flow problems of the type described here is the "Out-of-Kilter" method of D.R. Fulkerson. This algorithm is described in Reference 3.* All of the criteria listed above are satisfied by this method. It works on a network of nodes and directed arcs as was described in Chapter 2. Each arc has the capacity constraints, l_{ij} and u_{ij}, and the flow cost, c_{ij}, given. These are all integers. Recall that $0 \leqslant l_{ij} \leqslant u_{ij}$, but that c_{ij} is unrestricted in sign.

When a non-negative integral vector $\vec{x} = (x_{ij})$, with one component for each arc $\widehat{ij}$, can be found which satisfies the requirement that there be conservation of flow at each of the n nodes in the network,

$$\text{(for } i=1,\ldots,n) \qquad \sum_{j} (x_{ij} - x_{ji}) = 0 \qquad (5.1)$$

then there is said to be a network circulation. If the components of the flow vector, $\vec{x} = (x_{ij})$ also satisfy the capacity limits,

$$l_{ij} \leqslant x_{ij} \leqslant u_{ij} \qquad \text{for all } \widehat{ij} \qquad (5.2)$$

then $\vec{x}$ is called a feasible circulation.

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* It is also found in Reference 2, p. 162-169.

The "Out-of-Kilter" Flow algorithm (OKF) constructs an optimal feasible circulation, that is, one which finds:

$$\sum_{\widehat{ij}} c_{ij}x_{ij} = \text{minimum} \qquad (5.3)$$

If there is no feasible circulation, the algorithm reveals this fact.

Associated with each node i is an integral node price π_i. Then $\vec{\pi} = (\pi_i)$. Fulkerson[3] shows that if $\vec{x} = (x_{ij})$ is a feasible circulation, and there is a pricing vector $\vec{\pi} = (\pi_i)$ such that the following relationships hold, then $\vec{x} = (x_{ij})$ is the optimal feasible solution.

$$c_{ij} + \pi_i - \pi_j > 0 \implies x_{ij} = l_{ij} \qquad (5.4)$$

$$c_{ij} + \pi_i - \pi_j < 0 \implies x_{ij} = u_{ij} \qquad (5.5)$$

For compactness, a marginal cost, $\overline{c_{ij}}$, can be defined:

$$\overline{c_{ij}} = c_{ij} + \pi_i - \pi_j. \qquad (5.6)$$

Each arc in the network is classified as either "in kilter" or "out of kilter" depending on the values of the marginal cost, $\overline{c_{ij}}$, and the flow, x_{ij}. Table III lists the various arc states and the corresponding values of $\overline{c_{ij}}$, and x_{ij}.

To solve the min-cost problem, all the arcs in the network must be brought into kilter.

Two advantages of the "Out-of-Kilter" method will be mentioned here. First, the OKF method can be started with any arbitrary flow, feasible or not, along with any arbitrary pricing vector. However, a good initial guess for $\vec{x}$ and $\vec{\pi}$ will decrease

TABLE III

Arc Kilter States

Notation	Sign of $\overline{c}_{ij}$, Marginal Cost	Value on x_{ij}	State
α	$\overline{c}_{ij} > 0$	$x_{ij} = l_{ij}$	IN KILTER
β	$\overline{c}_{ij} = 0$	$l_{ij} \leqslant x_{ij} \leqslant u_{ij}$	IN KILTER
γ	$\overline{c}_{ij} < 0$	$x_{ij} = u_{ij}$	IN KILTER
α_1	$\overline{c}_{ij} > 0$	$x_{ij} < l_{ij}$	OUT OF KILTER (Infeasible Flow)
β_1	$\overline{c}_{ij} = 0$	$x_{ij} < l_{ij}$	OUT OF KILTER (Infeasible Flow)
γ_1	$\overline{c}_{ij} < 0$	$x_{ij} < u_{ij}$	OUT OF KILTER (Non-optimal Flow)
α_2	$\overline{c}_{ij} > 0$	$x_{ij} > l_{ij}$	OUT OF KILTER (Non-optimal Flow)
β_2	$\overline{c}_{ij} = 0$	$x_{ij} > u_{ij}$	OUT OF KILTER (Infeasible Flow)
γ_2	$\overline{c}_{ij} < 0$	$x_{ij} > u_{ij}$	OUT OF KILTER (Infeasible Flow)

the computation time significantly. If the method is started off with a feasible circulation, then kilter states α_1, β_1, β_2, and γ_2 are empty to begin with and remain that way.

Second, unlike some other optimality routines, the OKF algorithm is a "one step method". That is, once an arc is put "in kilter" it never goes back "out of kilter". Therefore the computation is always advancing toward a solution. Comparison can be made with the two-phase Simplex method, for example, in which there is no guarantee that an arc, once "in kilter", would not go "out of kilter" at a later stage in the computation.

The process by which the OKF algorithm works to put all the arcs into kilter is called labeling. Starting with any $\vec{x}$ and $\vec{\pi}$, the algorithm locates an "out of kilter" arc, defines its origin and terminal nodes, and attempts to put this arc into kilter. The labeling rules are given in Reference 3, and will not be repeated here. The termination of the labeling proceedure will be explained, though. A labeling pass ends either in a "breakthrough", or in a "non-breakthrough". A breakthrough occurs if the terminal node has been labelled; the path from the origin is found by backtracking from the terminal node.

If a non-breakthrough occurs, there may or may not be a feasible solution. Let L denote the set of all labelled nodes, and let $\overline{L}$ be the set of the unlabelled nodes. Define two subsets of arcs:

$$\mathcal{A}_1 = \left\{ \widehat{ij} \,\middle|\, i \in L,\ j \in \overline{L},\ \overline{c}_{ij} > 0,\ x_{ij} \leqslant u_{ij} \right\} \qquad (5.7)$$

$$\mathcal{A}_2 = \left\{ \widehat{ji} \,\middle|\, i \in L,\ j \in \overline{L},\ \overline{c}_{ji} < 0,\ x_{ji} \geqslant l_{ji} \right\} \qquad (5.8)$$

Then find δ , where:

$$\delta_1 = \min\ (\overline{c}_{ij}) \quad \text{FOR all } \widehat{ij} \in \mathcal{A}_1 \qquad (5.9)$$

$$\delta_2 = \min\ (-\overline{c}_{ji}) \quad \text{FOR all } \widehat{ji} \in \mathcal{A}_2 \qquad (5.10)$$

$$\delta = \min\ (\delta_1, \delta_2) \qquad (5.11)$$

If $\mathcal{A}_i$ is non-empty, δ_i is a positive integer; if $\mathcal{A}_i$ is empty, δ_i is infinite.

If, after a labeling pass, the arc is not in kilter, either an alteration is made in the flow in the path from the origin to the destination (if there was a breakthrough), or a change is made in the node prices* (if there was a non-breakthrough), and the labeling process is repeated with the same arc. When the arc is finally put into kilter, another out of kilter arc is sought, and

---

* of the unlabelled nodes, that is those in the subset $\overline{L}$.

the whole business starts again. If the result of the labeling process is ever a non-breakthrough with $\delta = \infty$, then there is no feasible circulation for the network.

Therefore, the algorithm terminates after a finite number of steps, either with all the arcs in kilter (in which case, the feasible circulation is optimal), or with the result that there is no feasible solution.

5.3 The "Out-of-Kilter" Computer Program

A Fortran II and FAP program using the "Out-of-Kilter" method of solving min-cost flow problems has been developed by R.J. Clasen of the RAND Corp. (Ref. 11). The program is quite versitile and has several interesting options built in, including a way of changing the constraints on the arcs in the network and rerunning without having to input the entire data deck again (Alter Option). For use on the IBM 7094, with about 32,000 cells available, the program can handle a network of 1500 nodes and 4500 arcs. This uses 27,002 cells for the various node and arc storage, and about 5000 cells for the program block.* If the number of cells available were fewer than 27000, then the network would also have to be modified. For example, the program that is run on the 7094 at MIT uses a maximum of 1000 nodes and 3000 arcs (accounting for 18,002 cells), because the capacity of the machine is reduced for time-sharing. Indications are that the newer IBM System 360 will be able to handle more than twice the number of node and arc inputs, and thus will afford much larger networks to be worked on.

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* The number of nodes and arcs may be varied with minor program modifications. The constraint is that $4a + 3n + \max(a,3n+2) =$ storage capacity for arcs and nodes.

The program controls are entered by punched card, and the input data is either by cards or tape. The arc data inputs are:

- Origin name (i)
- Destination name (j)
- Lower bound (l_{ij})
- Upper bound (u_{ij})
- Cost (c_{ij})
- Estimate of the Flow (x_{ij}) - optional

The output can be any combination of printed, tape, or punched cards, and contains the input data (without the flow estimate), along with the following:

- The Actual Flow (x_{ij})
- The Product of the Flow and the Cost for Each Arc $(x_{ij}c_{ij})$
- Node Price of Origin (π_i)
- Node Price of Destination (π_j)
- Marginal Cost $(\overline{c}_{ij} = c_{ij} + \pi_i - \pi_j)$

The letter "N" is printed by any arc which could not be brought into kilter, indicating that there is no feasible solution. The output message "Total System Contribution" gives the value of

$$-\sum_{\widehat{ij}} c_{ij}x_{ij} \qquad (5.12)$$

The OKF sequence of operations in a very basic form is shown in the flow chart in Figure V-1.

5.4 Modifications for Internal Monitoring of the OKF Routine

In its normal operation, the OKF algorithm uses punched control cards, and either punched or tape input data. For an internal

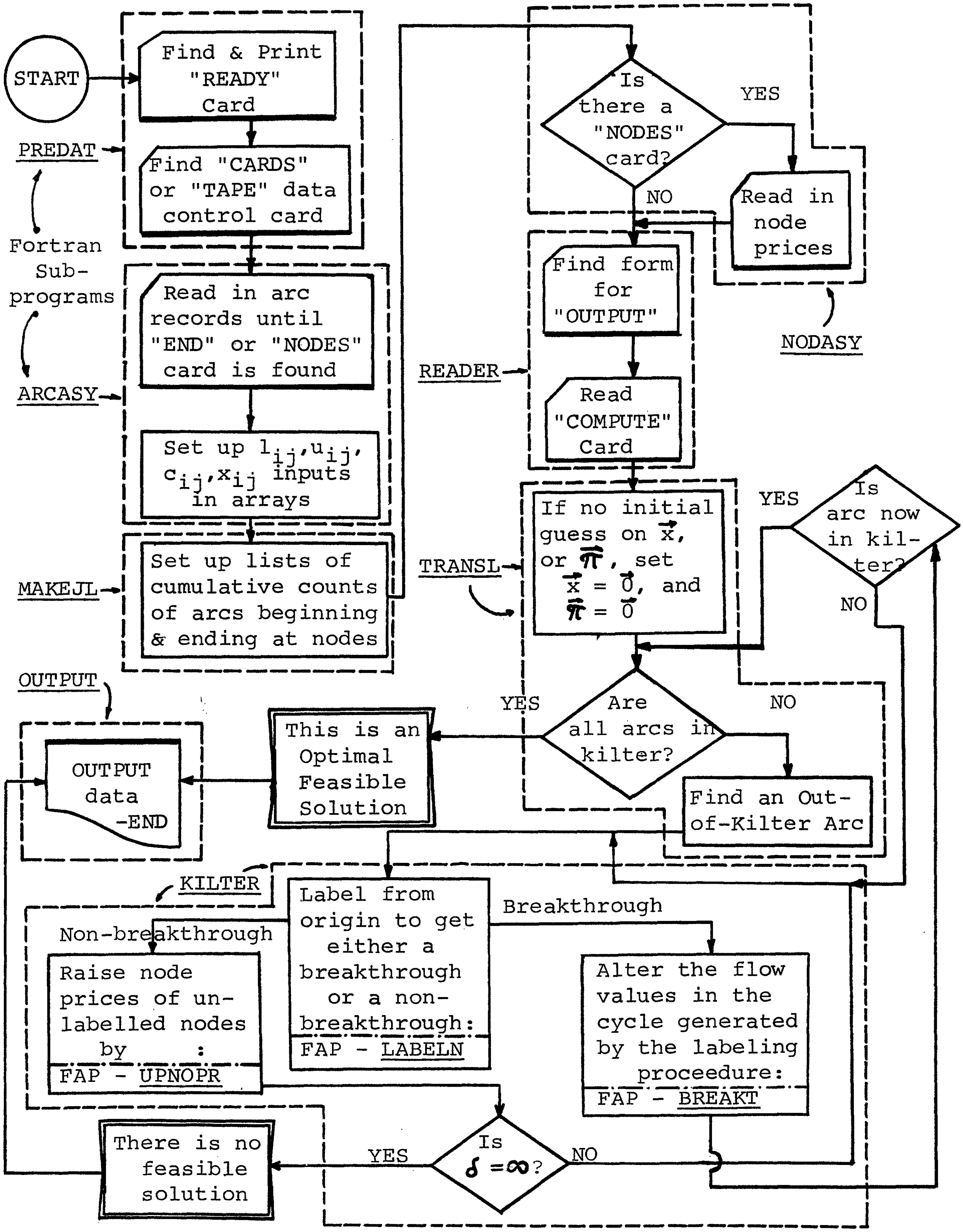


FIGURE V-1

"Out-of-Kilter" algorithm Sequence of Operations or dynamic operation as would be useful in the schedule revision program, the network modification routine would make alterations in the basic network according to pre-programmed decision rules, and depending on the characteristics of the interruption. It would then set up these new nodes and arcs in a data format compatible with the optimality routine, and call this routine to perform the actual optimization.

Thus while the OKF algorithm is a powerful one, and is the one which will be used in the subsequent example, its input-output proceedure does not correspond to the dynamic type of system which has been presented. Specifically, to model an internally called program, the control cards as well as the arc input data would have to be put on tape, and called by the OKF as a subroutine. Certainly nothing of value is to be gained by working through the laborious task that this would entail. Thus, for the purposes of the example which will be presented in the next section, the computations will be performed in two steps. First, a decision will be made as to which arcs and nodes should be added to the basic network (this job would eventially be handled by the machine with supervision of the schedule controller). Then the data will be punched on cards and submitted by batch processing to the OKF routine for solution.

In other words, a two-step batch process will be used in place of the internally integrated system. It is emphasized that there is no loss of touch with the real problem at hand in bowing to this expediency, since it is the results of the methods detailed previously, and not the testing of a possible system configuration that is of importance here.

CHAPTER VI

AN EXAMPLE

6.1 A Brief Outline

In this section a complete example of a computerized solution for an interrupted airline schedule will be presented. The program utilizes the "Out-of-Kilter" algorithm on the IBM 7094 computing facility. The basic difference between this example and that envisioned for a working system is, as was mentioned just previously, that the network additions will be picked by hand, punched, and given to the computer to solve, rather than having the whole thing done within the machine, with human supervision and alteration allowed.

The following list of steps should serve as a rough guide to the construction of the example:

- Station locations (here five) are chosen,
- A vehicle type (may be hypothetical) is found,
- The distance matrix is laid out,
- Interstation block times are computed,
- Origin and destination times (difference) are found if there are time zones in the structure,
- The no-tax fare matrix is found,
- A desired schedule is set up giving the number of vehicles required, their commitments, the location of spares, and so on,
- A hypothetical revenue picture is developed by listing the expected load factors on each flight segment,

- The DOC's v. stage length are assumed,
- Some estimate is made of how much of the DOC and variable IOC is real marginal operating cost for the flight segment,
- The marginal profit for each segment is computed,
- Find the total system contribution to marginal profit either by hand or by means of the OKF program with all flights locked in,
- Define the interruption,
- Decide on what nodes and arcs are to be added to the network to allow the algorithm to be able to find the right solution,
- Place costs and limits on the new arcs,
- Submit the augmented network to the OKF which returns the new routing and the new total marginal costs,
- Other constraints can be placed on the solution by means of the "Alter" option: lock in certain flights and resolve,
- Choose the best alternative.

6.2 The Route Structure

The route model which was chosen consists of five stations; these, by the way, correspond amazingly close to Boston (A), Los Angeles (B), New York (C), Washington (D), and Chicago (E). It was assumed at the outset that station D has no overnight aircraft, if at all possible, and that station B is the main overhaul base, but overnight repairs can be done at the others.

The vehicle is an intermediate to long range jet similar to the Boeing 720.

The distance matrix for the various city-pair combinations is given in Figure VI-1. The shortest hop is between A and C, while the longest haul is between A and B. The block to block times between the cities is shown in Figure VI-2. These are average times between the various real station combinations which were mentioned above. Since stations A, C, and D are in the Eastern time zone, E in the Central, and B in the Pacific, there is a "local time differential" in the origin and destination times whenever a time zone is crossed. Station E is one hour earlier than A, C, and D, and station B is three hours earlier. Figure VI-3 shows the difference in local times for flights between the stations. For example, while the block time for a trip from A to B is 5.7 hours, the local time differential is only 2.7 hours.

A possible schedule plan was constructed for flights between the city-pairs in the network. This schedule is presented as Figure VI-4. It is not meant as a clear working picture, but only to provide an overall view of the operations. It can be seen that there are eleven aircraft represented on this schedule. Two overnight and begin their daily routing from A, two from C, none from D, three from E, and four from B. Note also that one of the aircraft at station E is designated as a spare, and so is always at E in the uninterrupted schedule. It should also be noted that any vehicles which are in maintenence are not shown on this routing chart, where in a complete schedule, they would be, so that the fleet could be effectively rotated for required maintenence and overhaul. The schedule in Figure VI-4 is read in this way: The horizontal line which jogs between D and E, and then again between E and B is a line of constant local station time. A straight line drawn horizontally at any time would give the local time at each station at any given instant. Thus the 0900 line at A, C, and D, intersects E0800 and B0600.

TO / FROM	A	B	C	D	E
A	——	2610	185	380	865
B	2610	——	2450	2290	1750
C	185	2450	——	200	720
D	380	2290	200	——	610
E	865	1750	720	610	——

FIGURE VI-1 Distance Matrix

TO / FROM	A	B	C	D	E
A	——	342	48	66	144
B	288	——	282	270	210
C	48	336	——	54	126
D	66	306	54	——	114
E	114	228	102	90	——

FIGURE VI-2 Block to Block Times

TO / FROM	A	B	C	D	E
A	——	2.7	.8	1.1	1.4
B	7.8	——	7.7	7.5	5.5
C	.8	2.6	——	.9	1.1
D	1.1	2.1	.9	——	.9
E	2.9	1.8	2.7	2.5	——

FIGURE VI-3 "Local Time Differentials"

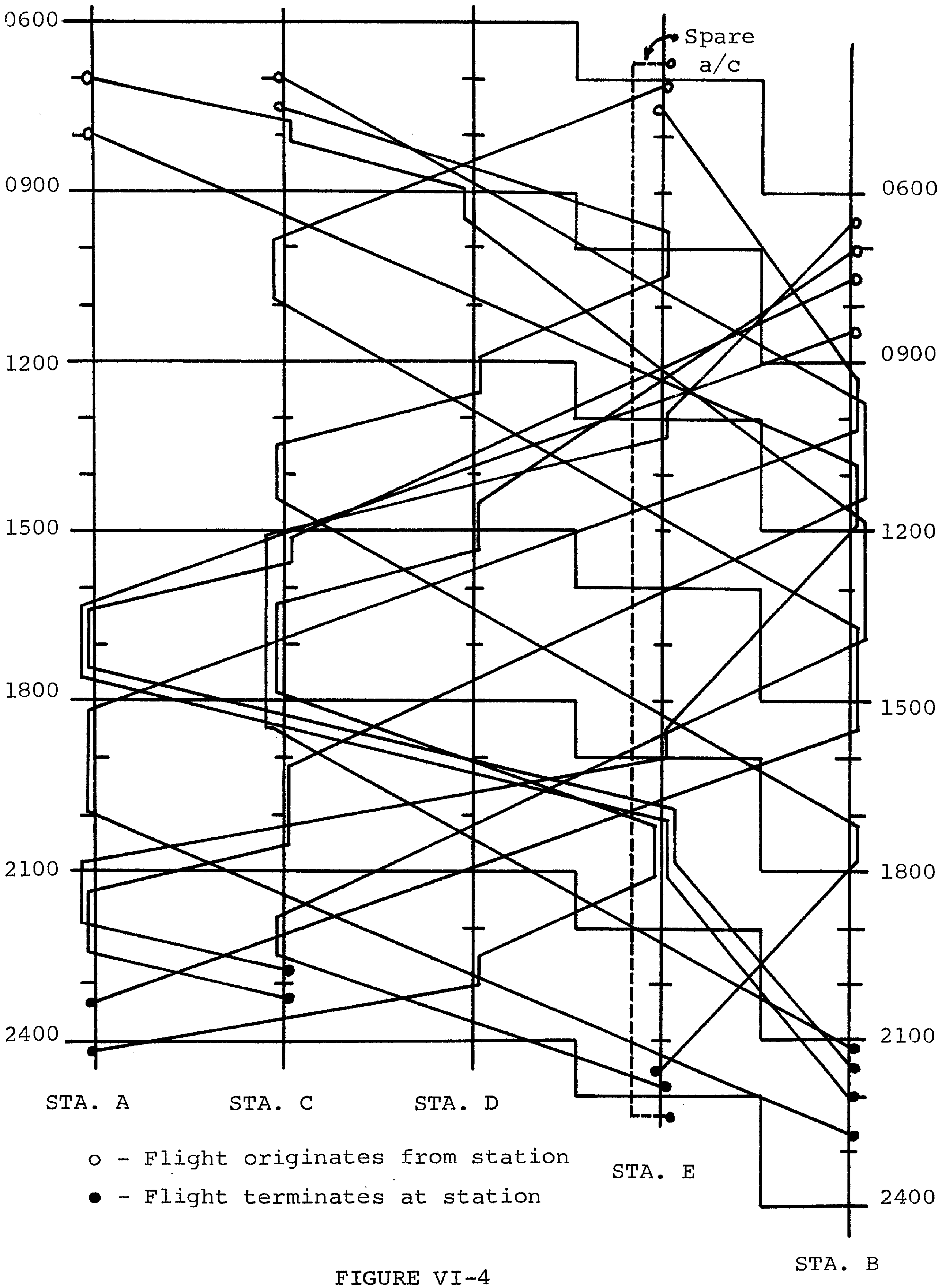


FIGURE VI-4

Schedule for Uninterrupted Service

This schedule shows the daily commitment of the fleet. Figure VI-5 is a representation of the route network showing the stations which are linked by non-stop segments. There are no flights from A to D or from D to E. The number of segments operated per day between the various city-pairs is shown in Figure VI-6.

A more easily readable listing of the allocation of aircraft over the routes is contained in Table **IV** . This is a listing of the daily flight itineraries for the eleven aircraft in the fleet. The total block time turned in by the fleet in normal operations is 121.0 hours per day. This comes to a block utilization of 11 hours per day per aircraft (but if there were one additional vehicle in the shops at all times, this figure would drop to 10.1 hours per day).

In this schedule, no final flight arrives at its intended overnight location after 0006 hours , or departs before 0630. Thus there is a well defined overnight period. For the model, it is assumed to be company policy that if possible, all network changes are to be completed so that the aircraft are in their correct locations for the start of service the next day.

6.3 Arc Costs

The broad categories of Direct and Indirect operating costs do not correspond to fixed and variable costs in the operation. While, in general, DOC's represent the variable costs associated with actually sending out a flight, and IOC's contain mostly fixed overhead and administrative costs, there is a good deal of mixing between the two. For example, some of the flight crew pay is fixed whether there is a flight or

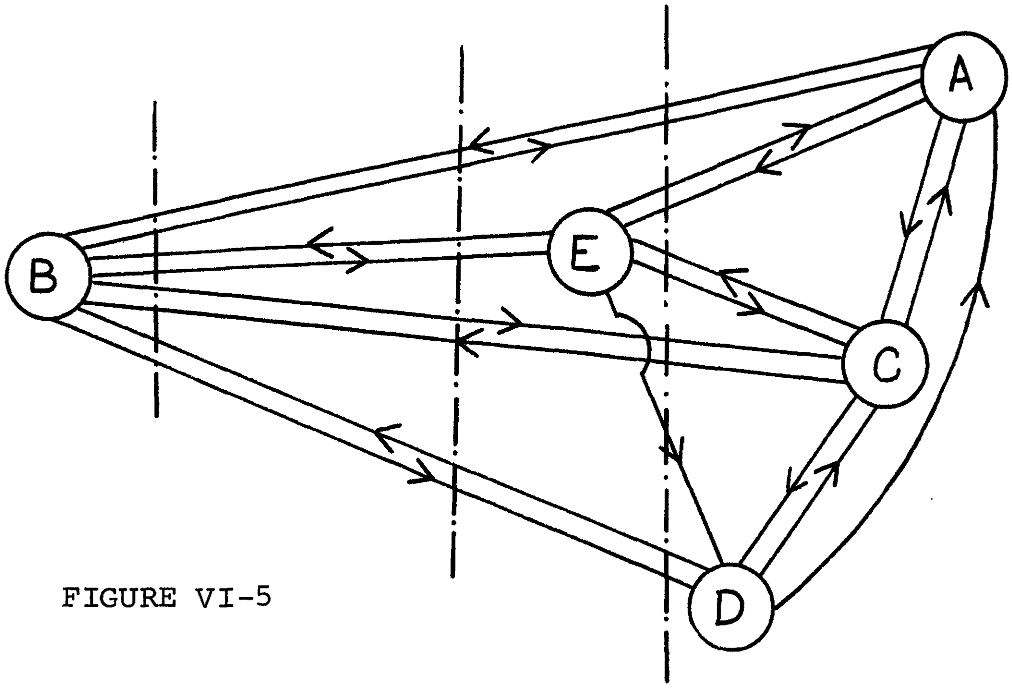


FIGURE VI-5

Directed Arcs Between Stations in the Route Structure

TO FROM	A	B	C	D	E
A	—	2	3	0	2
B	3	—	3	1	3
C	2	4	—	1	3
D	1	1	2	→	0
E	1	3	2	2	—

FIGURE VI-6

Number of Flights Between Various City-pairs

TABLE IV

Flight Itineraries For The Uninterrupted Schedule

a/c	ITINERARY (Block Time in hours)	Total Flight Time - hrs.
1.	o E0700 - E2348 ● (Spare)	0
2.	o A0700 - C0748 (0.8) → C0806 - D0900 (0.9) → D0930 - B1142 (5.1) → B1400 - C2142 (4.7) → C2242 - E2348 (2.1) ●	13.6
3.	o A0800 - B1042 (5.7) → B1200 - E1730 (3.5) → E1800 - A2054 (1.9) → A2200 - C2248 (0.8) ●	11.9
4.	o C0700 - B0936 (5.6) → B1130 - C1912 (4.7) → C2030 - A2118 (0.8) → A2230 - C2318 (0.8) ●	11.9
5.	o C0730 - E0836 (2.1) → E0930 - D1200 (1.5) → D1230 - C1324 (0.9) → C1430 - B1706 (5.6) → B1800 - E2330 (3.5) ●	13.6
6.	o E0706 - C0948 (1.7) → C1100 - B1336 (5.6) → B1530 - A2318 (3.5) ●	12.1
7.	o E0730 - B0918 (3.8) → B1018 - A1806 (4.8) → A2000 - B2242 (5.7) ●	14.3
8.	o B0630 - E1200 (3.5) → E1218 - C1500 (1.7) → C1830 - B2106 (5.6) ●	10.8
9.	o B0700 - D1430 (4.5) → D1518 - C1612 (0.9) → C1800 - E1906 (2.1) → E2000 - D2230 (1.5) → D2300 - A0006 (1.1) ●	10.1
10.	o B0730 - C1512 (4.7) → C1536 - A1624 (0.8) → A1730 - E1854 (2.4) E1930 - B2118 (3.8) ●	11.7
11.	o B0830 - A1618 (4.8) → A1736 - E1900 (2.4) → E2006 - B2200 (3.8) ●	11.0
	TOTAL	121.0

not even though it is all charged to DOC's; similiarly, much of the cabin crew pay depends on the length of a flight, but all of this is charged to IOC.

When the yearly accounting of operations is made, the fleet depreciation and hull insurance is charged off at a fixed rate per flight hour depending on the yearly utilization. Therefore, for all intents, these sizable portions of the direct operating costs can be considered to be fixed costs in the short run.

For the purposes of this model, only real flight costs, and real expected losses for delays and cancellations will be taken into account. Thus the arc costs for regular flights is the actual marginal flight costs involved minus the total revenue on the flight. Of course, this figure is not profit; it is much too high in the long run. But for the immediate picture when the decision is to cancel or not to cancel a particular flight, the relative costs of the decision are the deciding factor.

The direct operating costs for an aircraft type varies from airline to airline depending on the maintenence philosophy, and its efficiency, the crew pay, and other factors. For this example it is assumed that the carrier has accurate cost data from experience. The DOC v. statute mile graph which is assumed is shown in Figure VI-7. Here the vertical lines are drawn at all the intercity distances for the route structure. The DOC's at the various distances are also listed in Figure VI-9. Some estimate must be made on how big the marginal flight costs are. Table V, below, gives the percentage of the DOC that various flight costs involve:

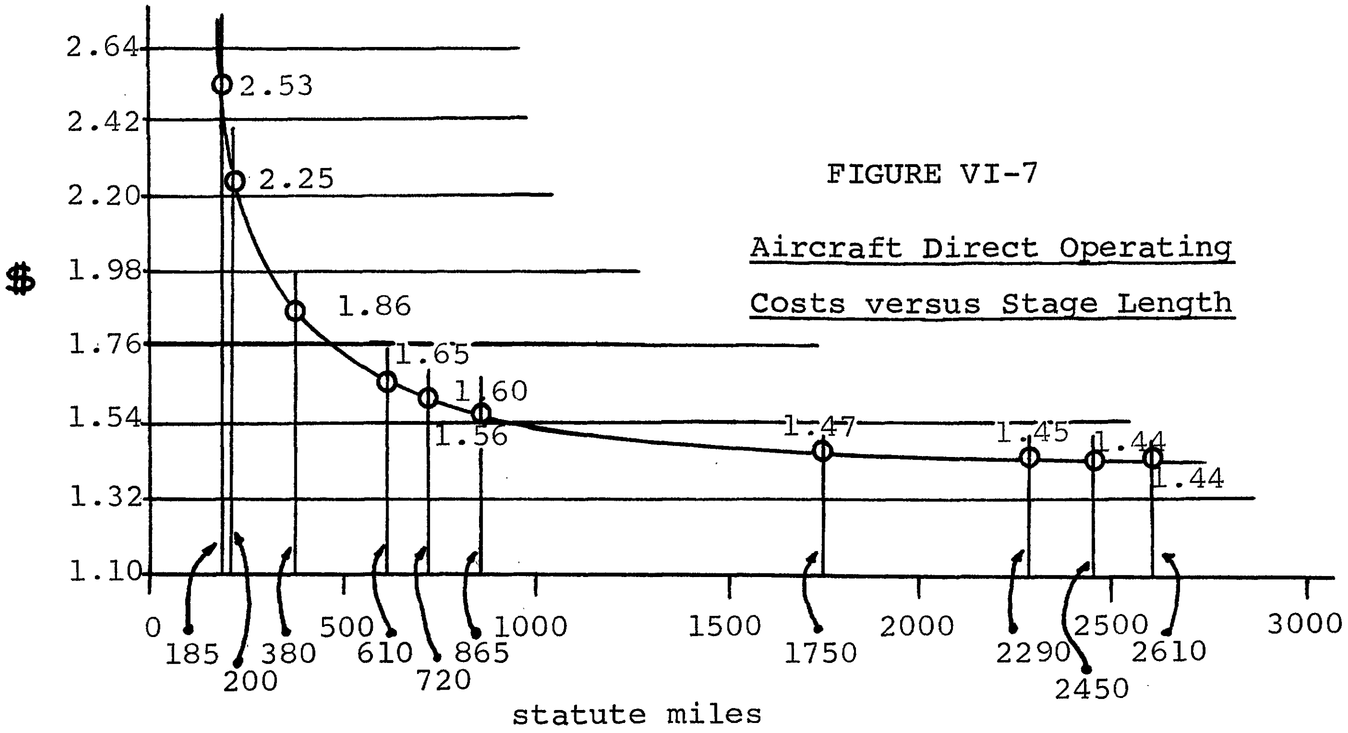


FIGURE VI-7

Aircraft Direct Operating Costs versus Stage Length

TABLE V

Marginal Direct Operating Costs	
Flight Crew	22% of DOC
Fuel & Oil	31% "
Cabin Crew	3% "
Food Service	5.5% "
Liab. Ins. on Px	2% "
Misc.	.5% "
Maintenence	22% "
Landing Fees	$100.00
Total	86% + $100.-

Figure VI-8 gives the fare structure over the routes. These approximate the no-tax rates for the real stations.

The list of the flight costs is contained in Figure VI-10.

TO FROM	A	B	C	D	E
A	——	$158.-	$16.-	$28.-	$54.-
B	$158.-	——	$150.-	$143.-	$108.-
C	$16.-	$150.-	——	$18.-	$46.-
D	$28.-	$143.-	$18.-	——	$41.-
E	$54.-	$108.-	$46.-	$41.-	——

FIGURE VI-8

Fare Matrix - No Tax

TO FROM	A	B	C	D	E
A	——	1.44	2.53	1.86	1.56
B	1.44	——	1.44	1.45	1.47
C	2.53	1.44	——	2.25	1.60
D	1.86	1.45	2.25	——	1.65
E	1.56	1.47	1.60	1.65	——

FIGURE VI-9

Direct Operating Costs - Dollars per Aircraft Mile

TO FROM	A	B	C	D	E
A	——	$3332.-	$503.-	$708.-	$1260.-
B	$3332.-	——	$3134.-	$2856.-	$2313.-
C	$503.-	$3134.-	——	$487.-	$1091.-
D	$708.-	$2856.-	$487.-	——	$966.-
E	$1260.-	$2313.-	$1091.-	$966.-	——

Figure VI-10

Marginal Trip Costs

This list was obtained by multiplying 86% of the DOC by the distance and then adding $100.00. This represents the cost of a non-revenue ferry flight.

An estimate was made on the number of passengers on each flight segment. This assumes 110 passengers maximum. The total revenue for the flight is then found. By subtracting the marginal cost,from Figure VI-10, from the total revenue, the marginal profit for each arc can be found. Table VI is a compilation of this information.

Using these data for the flight costs, the uninterrupted schedule can be run. The total system contribution to marginal profit for the regular case is $ 135,190.00.

Notice that the costs on the ground segments are set at zero.

6.4 The Interrupted Schedule (Example)

The following situation is presented as an example of the modification program being considered:

> The Flight Engineer on flight B0700 - D1430 discovers a breakdown in the backup hydraulic system upon landing at station D at 1430. It is estimated that the aircraft will be out of service from 1430 to 1930 hours.
>
> Schedule Control was notified of the situation at 1433 (Local time at D), and they, in turn, contacted station E to check on the availability of the spare vehicle. They also commenced the reallocation analysis.
>
> Station E advised that the aircraft and crew were both ready to go, and that the spare could leave for any station in the system by 1348 (Local Time at E). A quick turnaround and loading can be done in 30 minutes at the

TABLE VI

Expected Revenue and Marginal Costs for Scheduled Flights

i	j	Load Factor	Num. Px	Fare	Revenue	Marg. Cost	Marg. Profit
A0700	C0748	70%	77	16.-	1232.-	503.-	729.-
A0800	B1042	60%	66	158.-	10428.-	3332.-	7096.-
A1730	E1854	60%	66	54.-	3564.-	1260.-	2304.-
A1736	E1900	35%	39	54.-	2106.-	1260.-	846.-
A2000	B2242	55%	61	158.-	9638.-	3332.-	6306.-
A2200	C2248	55%	61	16.-	976.-	503.-	473.-
A2230	C2318	30%	33	16.-	528.-	503.-	25.-
B0630	E1200	55%	61	108.-	6588.-	2313.-	4275.-
B0700	D1430	80%	88	143.-	12584.-	2856.-	9728.-
B0730	C1512	75%	83	150.-	12450.-	3134.-	9316.-
B0830	A1618	60%	66	158.-	10428.-	3332.-	7096.-
B1018	A1806	35%	39	158.-	6162.-	3332.-	2830.-
B1130	C1912	45%	50	150.-	7500.-	3134.-	4366.-
B1200	E1730	40%	44	108.-	4752.-	2313.-	2439.-
B1400	C2142	50%	55	150.-	8250.-	3134.-	5116.-
B1530	A2318	55%	61	158.-	9638.-	3332.-	6306.-
B1800	E2330	65%	72	108.-	7776.-	2313.-	5463.-
C0700	B0936	70%	77	150.-	11550.-	3134.-	8416.-
C0730	E0836	65%	72	46.-	3312.-	1091.-	2221.-
C0806	D0900	50%	55	18.-	990.-	487.-	503.-
C1100	B1336	40%	44	150.-	6600.-	3134.-	3466.-

TABLE VI - (Cont'd)

Expected Revenue and Marginal Costs for Scheduled Flights

i	j	Load Factor	Num. Px	Fare	Revenue	Marg. Cost	Marg. Profit
C1430	B1706	35%	39	150.-	5850.-	3134.-	2716.-
C1536	A1624	50%	55	16.-	880.-	503.-	377.-
C1800	E1906	75%	83	46.-	3818.-	1091.-	2727.-
C1830	B2106	80%	88	150.-	13200.-	3134.-	10066.-
C2030	A2118	20%	22	16.-	352.-	503.-	- 151.-
C2242	E2348	45%	50	46.-	2300.-	1091.-	1209.-
D0936	B1142	40%	44	143.-	6292.-	2856.-	3436.-
D1230	C1324	45%	50	18.-	900.-	487.-	413.-
D1518	C1612	50%	55	18.-	990.-	487.-	503.-
D2300	A0006	60%	66	28.-	1848.-	708.-	1140.-
E0706	C0948	65%	72	46.-	3312.-	1091.-	2221.-
E0730	B0918	70%	77	108.-	8316.-	2313.-	6003.-
E0930	D1200	55%	61	41.-	2501.-	966.-	1535.-
E1218	C1500	45%	50	46.-	2300.-	1091.-	1209.-
E1800	A2054	95%	105	54.-	5670.-	1260.-	4410.-
E1930	B2118	70%	77	108.-	8316.-	2313.-	6003.-
E2000	D2230	40%	44	41.-	1804.-	966.-	838.-
E2006	B2200	30%	33	108.-	3564.-	2313.-	1251.-

destination. Therefore, the spare could leave D on a delayed segment at D1648. Control also found that the spare could be flown in and reloaded by C1700 or by A1712.

The schedule controller immediately spotted a number of feasible alternatives, but he decided to have the revised schedule control program do a little of his analysis. He set up the rules that the delays caused anywhere in the system must not exceed two hours in any event. He found a way of alloting punitive costs for any delays or cancellations, and asked the computer for a new network.

In passing, here are a few of the feasible solutions that the controller was able to see right off:

1. Fly the spare on E1348 - D1648 and have it take delayed flight D1648 - C1742. The marginal profit for this flight is 350., down from 503. for the on-time segment. The spare then flys C1800 - E1906, and E2000 - D2230 for the down aircraft; then either the repaired vehicle can fly its last route, D2300 - A0006 and the spare is ferried to E, or the spare can fly the last segment, and the repaired aircraft can be sent back to E to act as the next day's spare.
2. Cancel the D1518 - C1612 segment and send the spare straight to C on E1348 - C1700. The spare can then pick up on flight C1800 - E1906 and proceed as in 1.
3. The spare could stay at E. D1518 - C1612 would be cancelled as would one of the flights out of C. If the C1800 - E1906 flight is flown, then the spare could take over or not when the flight got in.

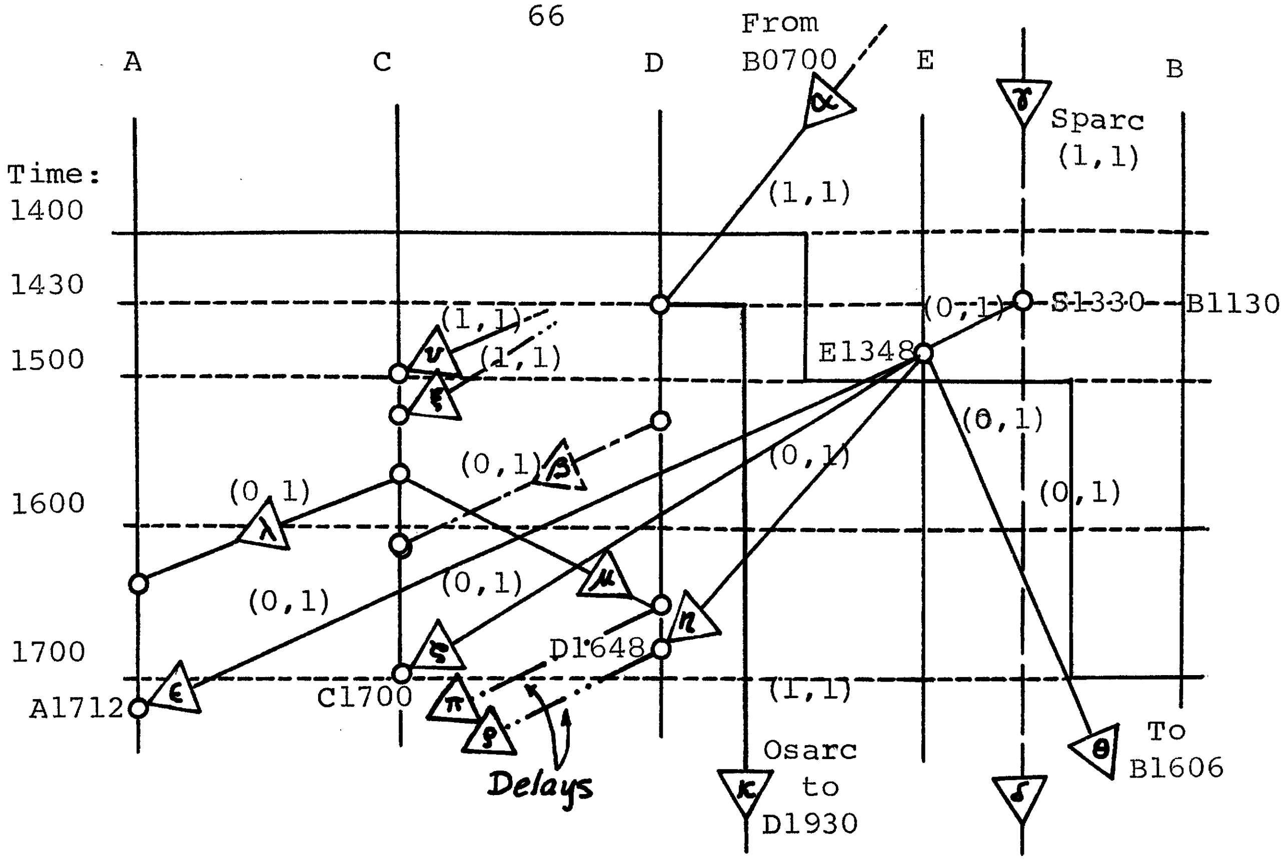
The scheduler asked the algorithm to add the necessary nodes and arcs to assure an alternate solution that was a good one. Since there is an update process going on continually, all the flights which had been completed earlier in the day, or were inflight, were locked in with $l_{ij}=u_{ij}=1$.

The program added an "osarc" from D1430 to D1930 and set $l_{ij}=u_{ij}=1$ on it. This took the down craft out of service. A node, S1330, was added in the "sparc" at station E. The spare was locked on the sparc from E0700 to S1330. An arc from S1330 to E1348 with $l_{ij}=0$, $u_{ij}=1$, gives the spare the path to come into service. The "sparc" continues from S1330 to E2348 (the end of the day), with $l_{ij}=0$, $u_{ij}=1$. The cost on this segment is zero. The cost on S1330 to E1348 is zero also, but it could be set to some trivially small cost such as $1.00 just to have the aircraft stay on the sparc if it is not going to be put into service.

Now, from E1348, ferry flights with no revenue can be set up. The limits are $l_{ij}=0$, and $u_{ij}=1$. None of these arcs has to be flown, but the possibility is there if necessary. Figure VI-11 shows some of the options available at the time of the interruption. The aircraft breaks down at the line 1430, which means S1330 and B1130. The flight into D1430 is marked α in the arrow, and is locked in. Sparc segment γ is also locked in. The osarc is shown by the notation κ , while the four possible ferry flights from E1348 are given by ϵ , ζ , η , and θ . The original flight continuation D1518 - C1612 is labelled β .

The computer also decided that with two flights ν , and ξ , coming in to C, the possibility of a ferry flight from C to D would be worth including in the analysis. The earliest that either of the vehicles could be turned around was 1536. There is a regularly scheduled departure from C to A at 1536, but an alternate arc C1536 - D1630 was provided. The regular run is given by λ in the Figure, and the new arc is μ .

There is now the possibility of two delayed flights from



The notation on the arcs denotes the lower and upper bounds respectively (l_{ij}, u_{ij})

FIGURE VI-11 - Schedule Revision Alternatives

station D.(π, ρ) From an original marginal profit of \$503.00, the delayed flights can produce \$350.00 and \$200.00 respectively.

The delay arcs must be handled in a slightly different way from the "Alternate Delay option" mentioned in an earlier chapter. This is because outside flights are involved in the delays. The consideration again is that if either delayed flight goes, the other can't go. In other words, the costs on the delayed arcs are for the particular delayed segment given that the other one did not go. Figure VI-12 is a graphical view of the modified delay set up. Here the flights come into D1630 and D1648 from C and E as was shown above. The delay arcs then travel to mixing arcs M1724, and M1742 with zero cost, $l_{ij}=0$, and $u_{ij}=1$. Arcs pass between these nodes to node

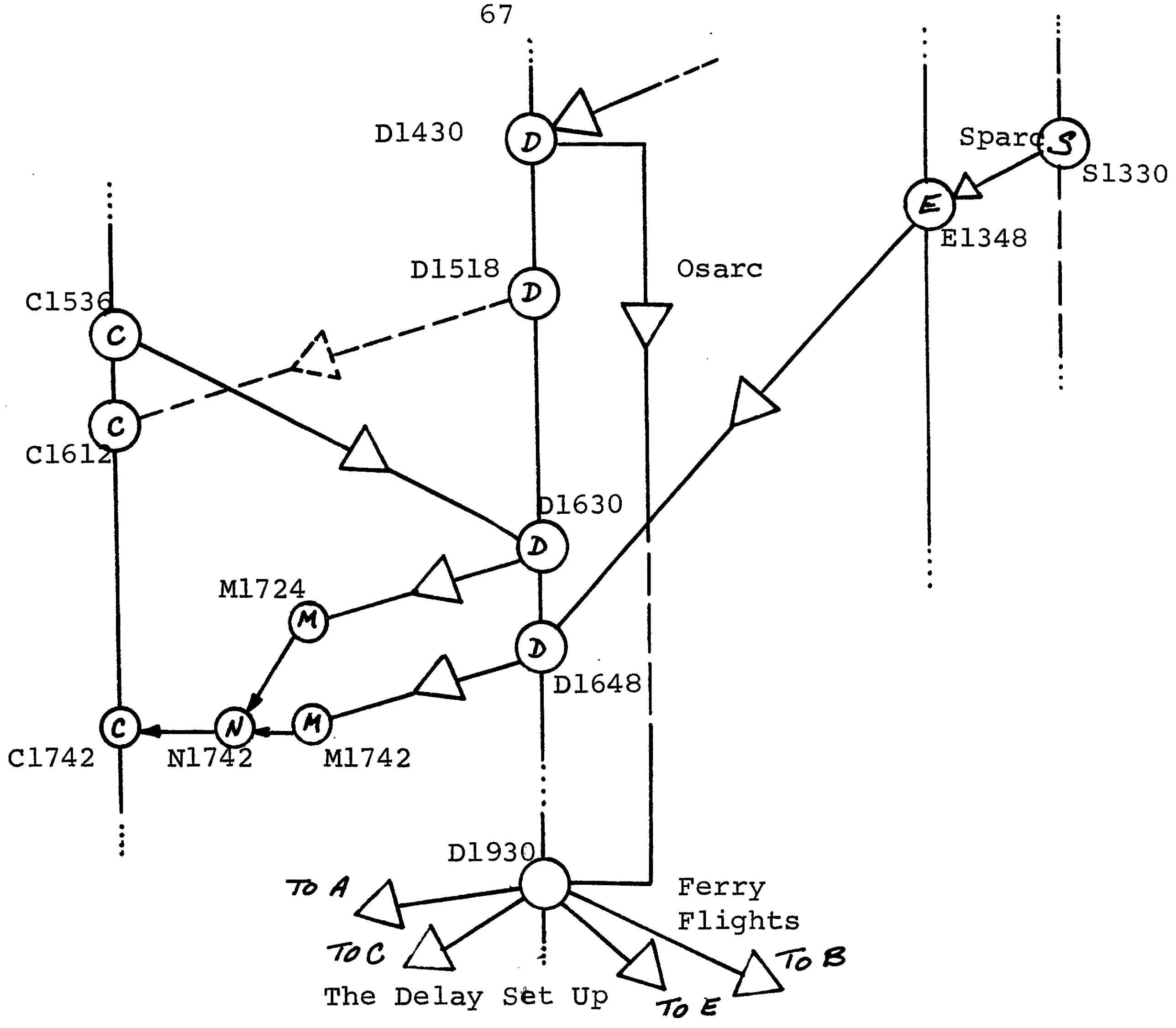


FIGURE VI-12

N1742 with the same limits and cost. A single arc runs from N1742 to C1742 with $l_{ij}=0$, and $u_{ij}=1$. This upper limit allows at most one of the delayed flights to be taken. The one which is taken can be found by the flow in arc D - M.

Since there may be some interstation shuffling of aircraft, the opportunity for ferry flights must be provided. The first series of these flights is set up from D1930 when the down aircraft comes back into service. Further interstation transfer flights can be provided at the end of the day for each station combination. In this way, ferry flights were set up from A0006, B2242, C2318, D2300, and E2348. Some of these flights, if used, would get into their stations quite far into the morning, but always before the first flight was due to depart. So, to conform to the policy of having the schedule readyto go at the start of the day, the number of aircraft that are supposed to be on each overnight arc (as found from the uninterrupted schedule run), is locked in to the arc from the last possible arrival until the first departure of the day.

As was mentioned earlier, this arc and node addition was done by hand, meaning that the additions which seemed to be those that the computer would be programmed to include, were included. However, it may well be that some possibilities were overlooked. As far as the punching of the data is concerned, the biggest job is in removing the ground arcs when a node is inserted between two existing nodes, and then setting up the two new ground arcs. This is a nice task for the machine because of the basically simple decisions and hard work involved.

6.5. Results

The network, as altered above, was run, and the following results were obtained. The flight D1518 - C1612 was, of course, cancelled because there was no way to get an operating aircraft to the spot in time. At C1536, the regularly scheduled run was chosen over the alternate to D1630. The spare aircraft at station E was brought in and ferried to station D where it went out on a delayed flight making a marginal profit of 200.00. This vehicle then proceeded to fly C1800 - E1906. The normal continuation for the aircraft on this route is E2000 - D2230, but the algorithm decided to remove this segment, leave this aircraft at E as the spare (it was the original spare), and have the vehicle at D, now repaired, take over its last segment of the day from D2300 to A0006.

The algorithm made one other change in the routing. In the itinerary of Table IV, aircraft 4 ends its service with a round trip between C and A. Now the flight from C to A realizes a loss of $151.00 while the return segment makes a profit of $25.00. Therefore, the routine decided that since these flights were not locked in, the best thing to do was to drop the entire round trip, and retire the vehicle early at C. If all of these modifications were adopted, the total system contribution to profit becomes $133,209.00.

If it was decided to lock in the C - A - C round trip, the contribution to profit would drop to $133,083.00.

The routing changes that are made necessary by the down aircraft are shown in Figure VI-13.

An alter run was made to see what the effect would be if flight E2000 - D2230 were forced into the solution. The alteration necessary is exactly what is expected. The spare flys this route and the D2300 - A0006 route and takes over for the other aircraft the next morning; meanwhile, the repaired aircraft is ferried over to E and it becomes the spare. The ferry flight loses $966.00 while the flight makes $838.00. The marginal profit (with the A - C - A segments included) is $132,955.00.

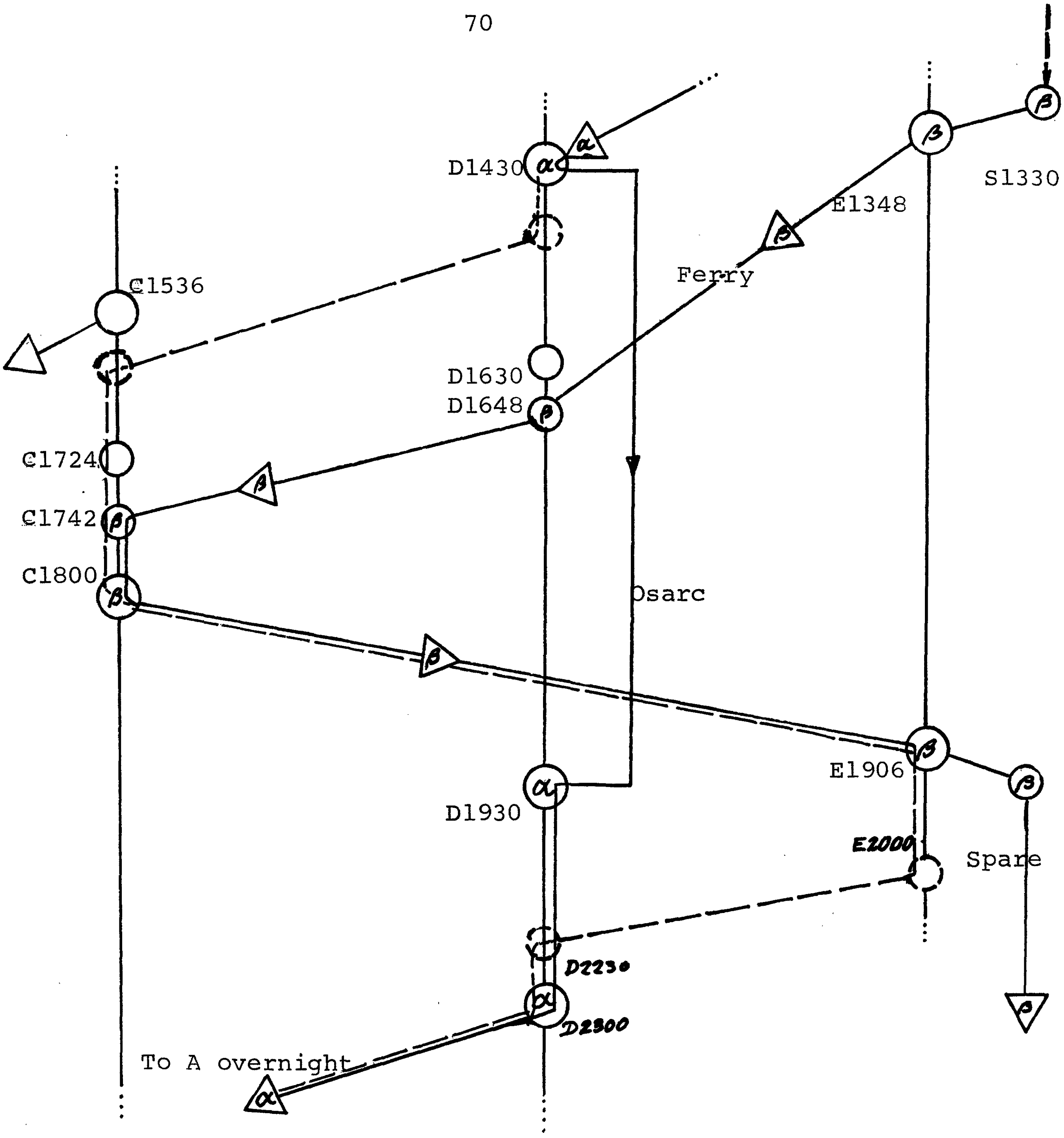


FIGURE VI-13

Network Modifications Chosen by OKF

CHAPTER 7

CONCLUSION

7.1 Some Limitations

While a fairly thorough examination of the problems besetting the schedule controller has been presented, in many areas the situations have been oversimplified. This, certainly, is due primarily to the vast scope of the problem and the difficulty of modelling real world situations simply.

The last few years has seen such a magnificent increase in the development of computer hardware that the "software" - the programs have not been written which can keep pace with their capabilities. Some paths of evolution of the computer's role in industry are becoming evident; with its tremendous versitility, the question is increasingly not whether to utilize data processing, but when. Yet just because the machines are there, and the men might be found who could make them work,is not a valid reason to jump off into problems that for one reason or another shouldn't be undertaken. A real searching analysis of the costs involved versus the benefits derived must proceed the final committment.

A major limitation on the model presented in this report is the assumption of a single aircraft cost type. This restriction is imposed so that the optimality routine can function. The network flow routine can deal only in a homogeneous flowing quantity. Now in the real situation, different aircraft types are freely substituted onto routes to replace down vehicles.

Therefore, for the computer to be of assistance to the controller, it must be able to do what he can do, as well as reaching beyond in other areas. A great deal of work needs to be done in this area since it is so central to a good solution. In Figure VII-1 an unworked upon idea for dual arc networks is shown. For the case of two vehicles, one type would occupy its own arc set, but a flow in either of the parallel arcs would constitute a flow. Constraints are that for the parallel flight segments, only one, at most can be flown, and the flows must depart from and arrive at the correct type of arc. This complicates things somewhat, since the set up used in the example where the flow from two arcs was brought together to assure a maximum of one aircraft in the two arcs, would not be able to send the vehicle on to its correct arc type.

This is just an idea, and, as was mentioned, no work has been done on it.

Again, more work must be done on ways of integrating the information that the airline has on its crew rotations and maintenance schedules into the schedule control decision process.

7.2 Future Work

An immediate follow-on to the work done here would be in the construction of programs for the automatic selection of network arc and node additions given the information concerning an interruption. Following this, the selected additions should be introduced into the network without hand computations. A system in this form would form a compact basis from which a real examination of the problem could follow.

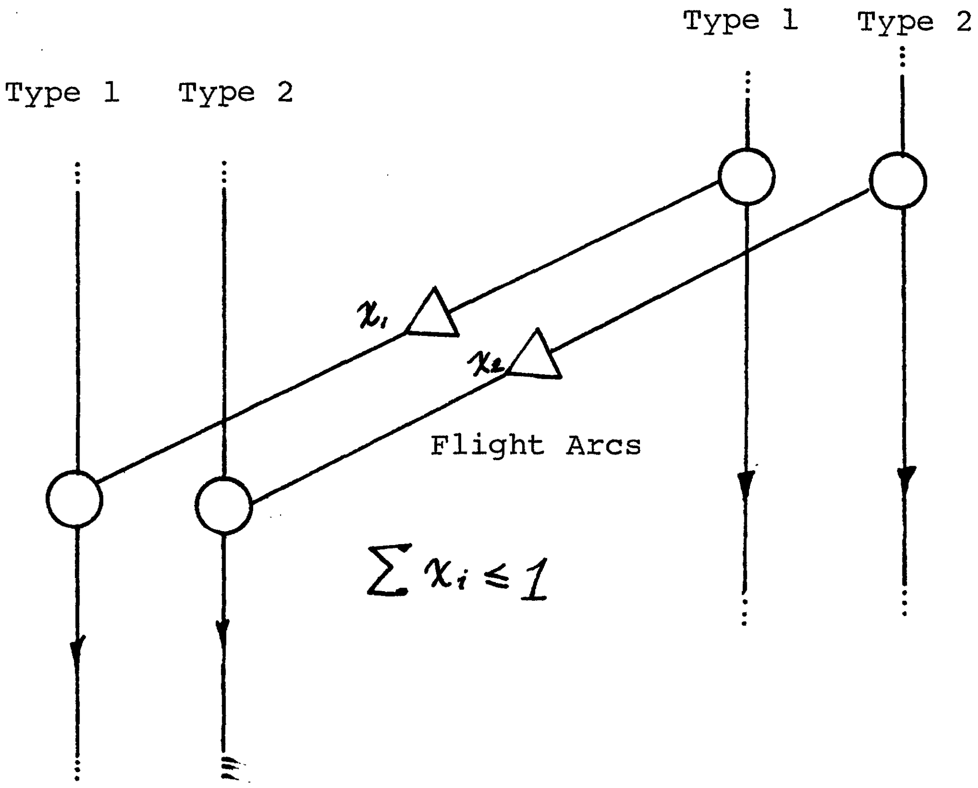


FIGURE VII-1

<u>Multi-commodity Flows</u>

APPENDIX I

REDUCTION IN THE SIZE OF THE COMPLETED PART OF THE SCHEDULE

The network model for an airline's scheduled service is usually extremely large. An operation of 1000 flights per day can easily require 2000 nodes and 5000 arcs in the unmodified case. The addition of even minor alterations can boost this total significantly.

For computerized operations, two important factors are the size of the program and the speed of computation. Existing flow methods, such as the out-of-kilter algorithm, are quite fast, even on very large programs. With an existing solution on hand, modified networks can be solved for optimality even faster.

Often, very large problems can cause overflow conditions in the machine; these programs must then be pared down to become useable.

Since the network is a representation in both time and space, as the day progresses, more and more flights will be completed, and thus removed from consideration in the event of an interruption. In the example presented in the report, the completed flights are locked into the network and thus take almost no time to include in the computation. If storage limits are in jeopardy, another representation is needed.

Presented here is a method of compressing the completed part of the schedule into a single conservation node, and a minimum of arcs to assure the continuity of aircraft at each station. Further the flights in the air at the time of the interruption must be taken into account. Figure A-1 is a possible network configuration for this alteration. In brief, here is a list of the considerations in setting up the compression network:

1. At the time of the interruption, read the number of aircraft on the ground at each station (if a movement occurs at that time, let it occur).
2. Let the overnight arc convey the number of aircraft overnight at each station to a "start of the day" node at each station.
3. Send to the "conservation" node the difference between the number of aircraft overnighting at the station and the number on the ground at the time of the interruption. If this number is negative, send no aircraft to the conservation node.
4. From the conservation node, send one aircraft to the departure node for each inflight vehicle.
5. Send the remaining aircraft to the stations where the difference between overnighting aircraft and those required at the interruption time are negative.

Because of the lack of application to the immediate report, no testing of this section was undertaken.

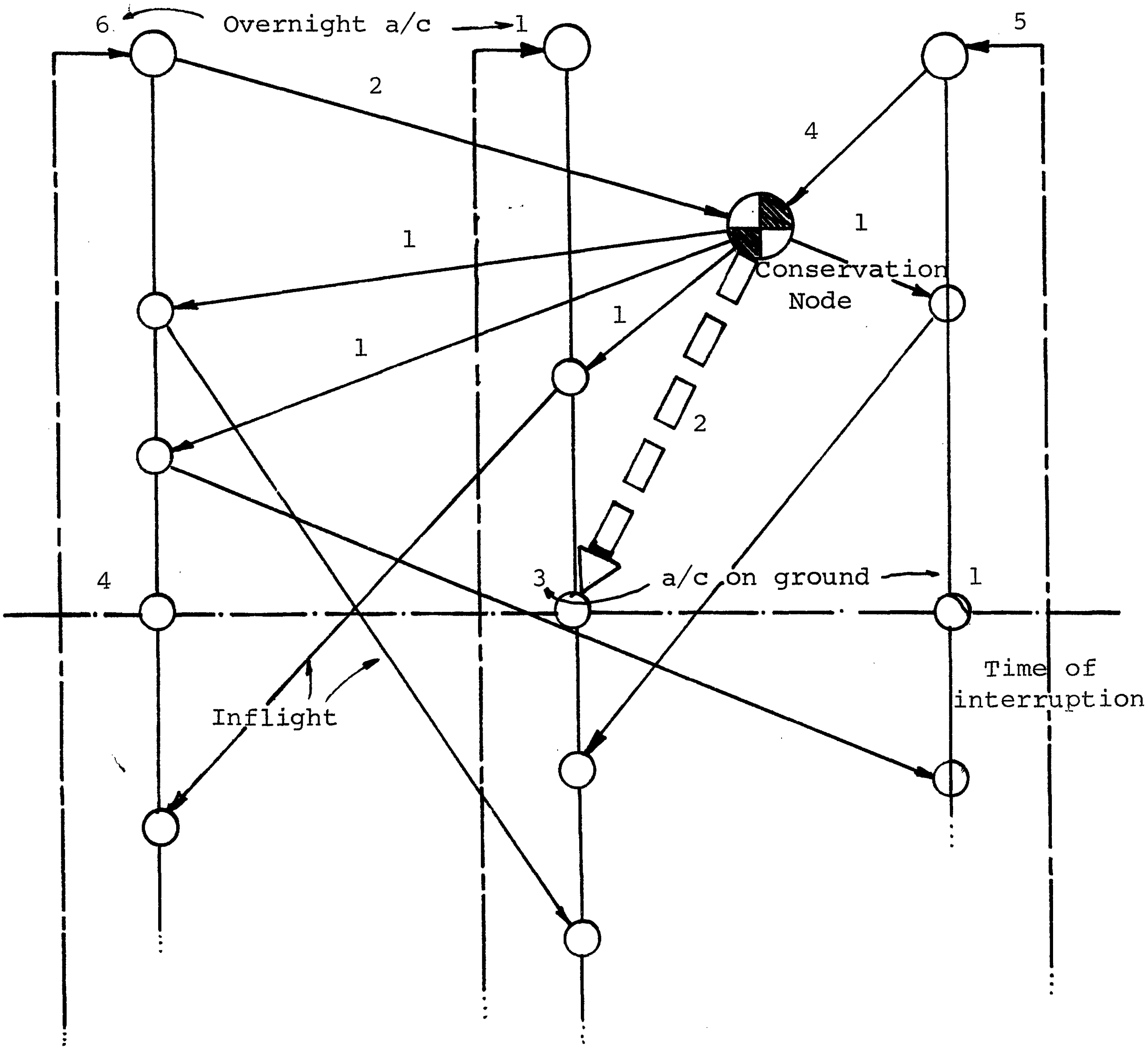


FIGURE A-1

Network Compression

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