AN EXPERIMENTAL STUDY OF THE SAND TRANSPORTING CAPACITY OF FLOWING WATER ON SANDY BED AND ~~THE~~ the Effect of the COMPOSITION OF THE SAND

by

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for the degree of
MASTER OF SCIENCE
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Signature of Author..
Department of Civil Engineering, September 1, 1935

Signature of Professor in Charge of Research..............................

Signature of Chairman of Departmental
Committee on Graduate Students..................................

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Cambridge, Massachusetts

Sept. 1, 1935

Prof. G. W. Swett,
Secretary of the Faculty,
Massachusetts Institute of Technology,
Cambridge, Massachusetts.

Dear Sir:

I submit herewith my thesis entitled "An Experimental Study of the Sand Transporting Capacity of Flowing water on Sandy Bed and the Composition of the Sand" in partial fulfilment of the requirement for the degree of Master of Science in Civil Engineering.

Respectfully yours

Shiou-dean Chyn

Aknowledgement

The author wants to express the obligation he owed to Prof. K.C. Reynolds for his valuable hints and directions, which made possible the author's work on this subject; also to Prof. G.Gilboy and Mr. D.W. Taylor for their help and instructions.

Contents

Table of Plottings

Nomenclature and Units Used

b,--channel width

D --Water depth

d --grain diameter

d_g--mean grain diameter

F --modified sand uniformity modulus

G --total bed load transported

g --bed load per unit width of channel

h --gage reading of head

T_w--water temperature

q --rate of discharge per unit width of channel

s --surface slope

Subscripts: m --mean value,
c --critical value,
b --measured at bottom.

Units for g,

1 lb./ft./hr. = 0.248 gr./cm./~~sec~~. min.

Synopsis

Both in laboratory work and in field work regarding river hydraulics, the silting and erosion of a movable channel-bed is a serious and complicated problem. A movable bed becomes stable only when an equilibrium between three factors, namely, the rate of flow, the surface slope grade, and the bed material, is reached. This is the so-called non-silting and non-eroding condition, Much work has been done, both in laboratory experiments and field observation, on the theory of sand transportation, to discover seek for the natural law governing this equilibrium condition, but the solving of the whole problem is still waiting for accumulative knowledge.

It was found that for a given sand or other bed material, it is not only its average fineness, but also its mechanical composition, which governs its behavior when it is transported as "bed load". By mechanical composition, is meant it means the variation of the grain diameter.

The main object of the present thesis is to determine to what extent, and in which way direction the sand composition will affect the its total amount transported along the bed by flowing water. The problem of sorting, or the change of relative percentage of the individual elements during transporting, is also analyzed.

Three synthetic sand mixtures were made first, they were of the same average grain size but different composition. With each mixture as bed material, about eleven test runs were made to determine the relationship between rate of discharge, surface slope

grade, and the capacity of sand transportation. The sand caught at the lower end of the test flume during each run was analyzed by sieve analysis.

The results of first part was plotted and the equations fitting the curves were then formulated. The conclusions thus reached are: --

1) The sand composition enters the capacity formula in a complex way; the coefficient, the exponent of slope factor, and the competent constant are all functions of it.

2) The critical tractive power, or the competent constant, is higher for mixtures having more uniformly sized grains.

The result of the second part is that the mean grain diameter and the sand modulus of the material transported are higher than those of the original material. They decrease as the tractive power increases until limiting values are approached.

Previous Work on the Theory of Sand Transportation

In MacDougall's research report "An Experimental Determination of Bed Sediment Transportation", he has compiled the results by both European and American authorities on this subject. Also a complete bibliography of the subject was published this year by the Bureau of Reclamation (2). In this paper then will be only mentioned several previous works which were studied by the author and not included in MacDougall's list.

A. Regarding transported capacity: --

1) Lechalas. (3) He derived in 1871 a formula making use of Dubuat's experiments and data observed on French rivers,

$$G = m(V_b^2 - 0.06)$$

where G is the bed load transported, m is an empirical constant, V_b is the bottom velocity of the water, and 0.06 is a competent constant in metric units. When water depth is constant, this formula can be approximately represented by --

$$G = K(V_m^2 - k)$$

where V_m is the mean velocity of the flowing water.

2) Gilbert's complete formula.(3) Gilbert in his report of experiments made during 1907-1909, after considering three factors separately, gave a complete formula including all the factors,

$$G = b\ (s - \sigma)^n (Q - \kappa)^o (f - \phi)^p$$

where s is the water surface slope, Q is the rate of discharge, and f is the mean fineness, or the reciprocal of the mean grain diameter of the bed material. The constants σ, κ, and ϕ are competent constants. Each competent constant is related to the other two factors, yet the definite function of such relation is still unknown. The mean value of the three exponents resulted from his experiments are

$n = 1.59$, $o = 1.02$, $p = 0.58$

respectively.

Gilbert also have made experiments on mixed grades and found that,

i) addition of finer debris increases the capacity for coarser bed load; and

ii) capacity of transportation varies as slope in the same power for uniform material but with lower competent value.

3) C.H. MacDougall, M.I.T. (1) During 1932-1933,

MacDougall made experiments on three sand mixtures, the results were, --

i) a capacity formula similar to Schoklitsch's

$$g = k_1 s^{m-1}(sq - s_o q_o)$$

in which s_o and q_o are the critical slope and rate of discharge respectively;

ii) the three constants in the above formula, viz., k_1, m, and $s_o q_o$, can be expressed in terms of the mean grain diameter and the sand modulus of the material transported. All the constants vary approximately with the reciprocal of the sand modulus.

4) Straub's rational formula. (4) For the bed sediment transportation, Prof. Straub of the University of Minesota has suggested from theoretical consideration the following formula, --

$$g = s^{1.4} n^{-1.2} q^{3/5}(q^{3/5} - q_o^{3/5})$$

when slope is constant, and

$$g = q^{1.2} n^{1.2} s^{0.7}(s^{.7} - s_o^{.7})$$

when the discharge is constant. The constant "n" is the same as the roughness factor in Manning's formula for mean velocity. He also showed by comparison that these rational formulas are consistent with Gilbert's experiments and Kennedy's erosion formula.

5) Vicksburg U.S. Waterway Experiment Station.(8) Based on an elaborated series of experiments made with river sands, the Vicksburg Experiment Station has derived a formula for the bed load capacity --

$$g = \frac{1}{n}\left(\frac{sD - S_0 D_0}{k}\right)^m$$

where n is the roughness factor, m is a constant which varies only with the type of sand within narrow limits, namely 1.5 to 1.8. k_1 is a sand property factor.

B. Regarding critical conditions: --

1) Airy's sixth power theory, (5)

$$W = K\,V_{cb}^{6}$$

where W is the weight of a single sand particle transported, V_{cb} is the critical bottom velocity.

2) Ho's empirical formula for critical bottom velocity, (6)

$$V_{cb} = 0.204\,d^{0.35}$$

where V_{cb} is the critical velocity measured right at the bottom in meters per second and d is the grain diameter in millimeters.

3) Vicksburg Station's tractive force formula, (8)

$$T_c = 29\sqrt{\frac{d_g}{M}(\rho_1 - \rho)}$$

where T_c is the critical tractive force in grams per square meter, d_g is the mean grain diameter in millimeters, M the sand modulus, ρ_1 and ρ are densities of sand and water respectively.

This formula is a revision of Kramer's with the data from the Station added.

4) Leighly. (7) Prof. Leighly has developed a theory stating that the imagined water prism acting on a unit area of bed is not "1 square foot x depth" but a distorted prism with bed as the lower base, the line of maximum velocity as the upper base, and the transversal and longitudinal planes perpendicular to velocity contours as boundary surfaces. Thus the tractive force is not simply a function of slope and depth, but also a function of form ratio (the ratio between depth and width) and

the location of maximum velocity.

Theoretical Consideration

The tractive power Quite comparable to the idea of tractive force, MacDougall has proposed the term 'tractive power' of a channel. From a theoretical point of view, he has determined that the tractive power of a stream flow is a product of slope and rate of discharge. In o~~r~~ther words, with a given channel bed, the sediment transported per unit width of channel bed is proportional to the product of slope and discharge, with a competent value to be deducted. Expressed in an equation, it will be

$$g = k\ (sq - s_o q_o)$$

where the value of k and $s_o q_o$ depend upon the bed material.

The competent constant $s_o q_o$ may not be exactly the critical condition in case of a mixture, because the sorting effect occurs s or q converges to small value.

Bed material composed of a mixture It is a definite known fact that a certain size grain needs a certain tractive force to set it in motion, the magnitude of the critical tractive force being proportional to the 1.5 power of the grain diameter. But in a mixture, the different grains do not act according to the above law. In the presence of fine particles, the coarser grains may be set in motion at a much lower stage, because the finer particles pave the bed to reduce the friction, also because the coarser particles, when the finer particles around them become eroded away, project higher and produce more turbulence, thus increasing the tractive force locally. It is therefore necessary to introduce another factor beside the mean grain diameter to evaluate the mutual action between finer and coaser grains. This factor would make it possible to reduce any mixture to an equivalent single grained bed material so far as the transported capacity is concerned.

Void ratio is a property of bed material, which is closely related to the bed load motion. It defines the roughness and the percentage of interstices. The former factor tends to anchor the particles but the latter tends to help to dislodge them. The two factors counteract each other to make the effect of void ratio on sand movement uncertain. This is why Gilbert did not have success on this feature.

D_g and M, these two constants were introduced by Krey and Kramer and later used by M.I.T. and Vicksburg Experiment Station. With the factor M, it is thought possible to reduce any mixture with known mechanical composition to a single grained material with mean diameter D_g.

Expressed mathmatically, the two quantities defined by Kramer are as follows;

$$D_g = \frac{\sum_{p=0\%}^{100\%} D\Delta p}{\sum_{p=0\%}^{100\%} p}$$

where D is the grain diameter as measured by the size of the sieve opennings, p is the percentage by weight of the grains larger than that particular D.

$$M = \frac{\sum_{p=50\%}^{100\%} D\Delta p}{\sum_{p=0\%}^{50\%} D\Delta p}$$

In its physical meaning, M is the ratio of D_g of the finer half over that of the coaser half. The value D_g measures the average fineness of the material, while M measures the relative diversity of fineness relative to D_g.

We cannot make any substantial change in the mechanical composition of a bed material without altering the values D_g and M of it. Therefore these quantities are a unique function of the material. The only defect is t hat if we make the change carefully without changing the 'D_g's of both the upper 50% and the lower 50%, they will not be affected. This means, we still have the freedom to have any kind of sand distribution within within the finer 50% and coaser 50% when D_g and M are both fixed. In order to have a more representative index to the size variation, a modified modulus is suggested:

$$F = \sqrt[4]{\frac{\sum_{p=75\%}^{100\%} D\Delta p}{\sum_{p=0\%}^{25\%} D\Delta p}} \cdot \sqrt{M}$$

Thus the F value takes care of extreme diameters and limits the freedom of change of distribution to within each 25%. In addition, it is reasonable to use the $\frac{1}{2}$ and $\frac{1}{4}$ power. A slight change of D_g in any 50% changes the value of F approximately one half as much, and slight change of D_g in any 25% changes F one fourth as much. It is also not laborious to determine F from a mechanical analysis curve.

Complete movement of a mixture. Schaffernak has deduced from his experiments the following conclusion: "According to size of grain, each mixture corresponds to a certain type of curve of material movement which consists of a flat portion and a steeper portion of curve common to all types of mixtures. The point of flexure corresponds to the critical bottom velocity which is just able to set in motion the entire mixture."

Obviously there are two limiting "critical consitions" for a given mixture. The upper limiting condition is the critical for the coaser grains, or rather the coarsest, above which all particles can be moved with ease. The lower limiting condition is the critical for the finest grains, below which no particle can be moved. Between these limits, there must be a critical bottom velocity which is just able to move the coarsest grains in the presence of the finer elements, and therefore is able to move the mixture as a whole.

Procedure of Experimental Work

The problem To study how the uniformity of a sand mixture affects the capacity of bed-load transportation.

By limiting the problem to bed-load, that part of sand transported in the mode of suspension is entirely neglected. This assumption is permissible when the mixture used does not contain very fine grains.

Method of attack. Because the present objective is to relate the sand transporting capacity to those properties of sand apart from its fineness, it will be advantageous to eliminate D_g first. To accomplish this, three artificial mixtures were made from three sieved sands, each sand having a different mean grain diameter from the others. The three kinds of sand were obtained from natural sand, -- the Plum Island sand -- by successive sieving. By mixing them in proper proportions, the three mixtures were made with approximately equal mean grain diameters but wide varying uniformity, or sand modulus.

With each one of the artificial mixtures as bed material, 2 sets of test runs were made to determine the sand transporting capacity under different hydraulic condition. The hydraulic quantities directly measured were the slope and the rate of discharge. Using these as variables, the two sets were so controlled that the slope was held constant and the discharge varied in the first set and vice versa in the second set.

The reason for using q instead of depth as an independent variable is that it can be precisely measured by a sharp-crested trapazoidal weir. On the other hand the quantity d is very hard to measure when the bottom becomes rough and the water surface is undulating.

Throughout the experiment, all test runs are made with a given slope and discharge and then adjust the sand feeding to keep the bottom at the upper end at a definite elevation. This method was claimed by MacDougall to be preferable to Gilbert's method which is to fix the discharge and rate of sand feeding first and then measure the surface slope after both the bottom and surface of water reached an equilibrium condition. However, one of the test runs, No. 12, was made with the second method with comparable result, which indicates the possibility of the equilibrium. It was thought that this equilibrium takes a long time and waste more sand, therefore in all runs except 12, the procedure is to start with a uniform bed and parallel surface.

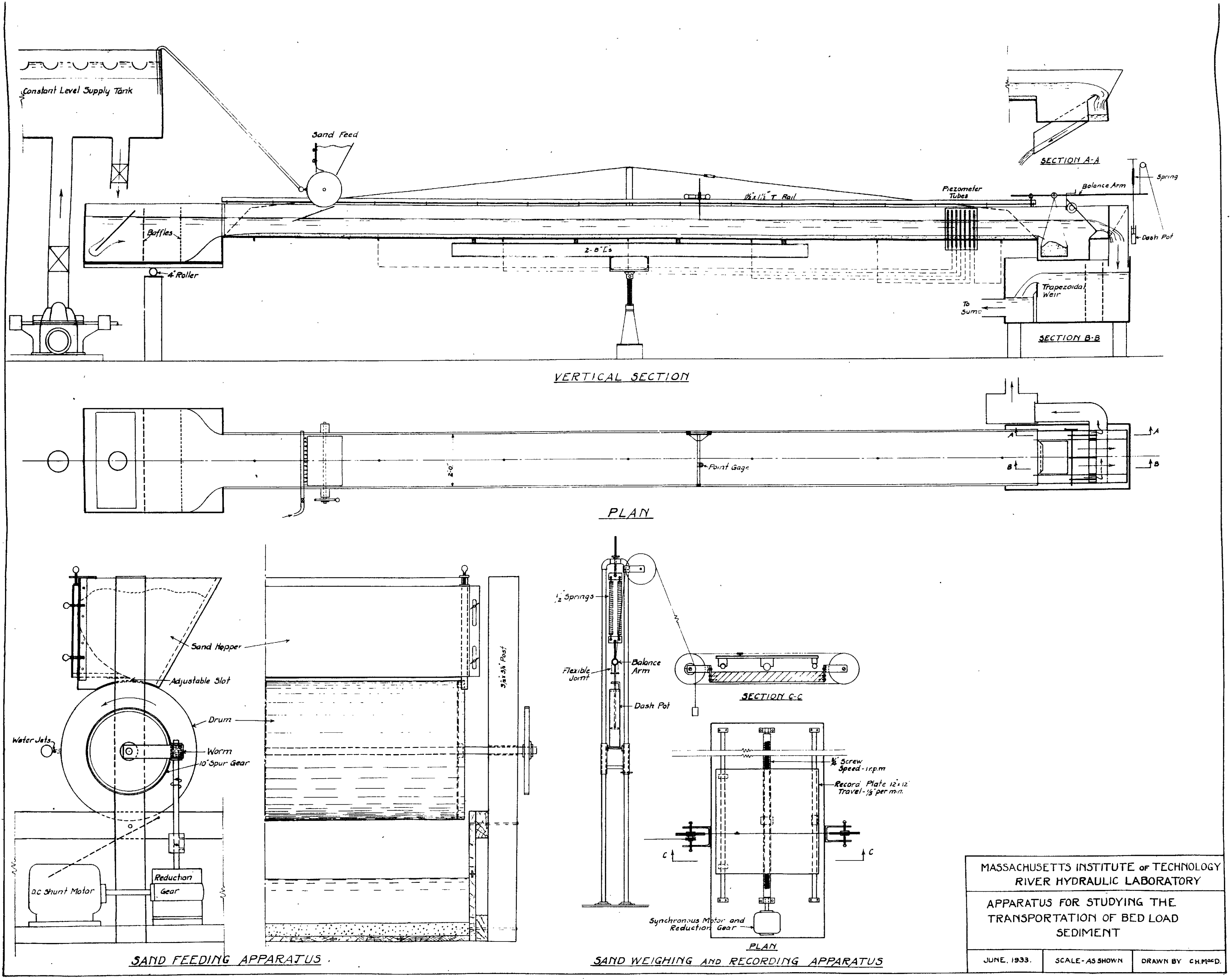
The two sets of readings, if properly taken, serve to determine the capacity formula for each mixture. The initial or critical condition can be computed from each set of data and then checked with each other. Some visual readings of the condition for start of motion were also taken, but due to the difficulty of the criterion, and the diversity between upper and lower limits of initial condition, the visual readings are only used as a check.

In order to get detailed information of the sand composition, each mixture was analyzed with closely spaced series of standard sieves before the run and the results from three samples were averaged. After each run, the sand caught at the lower end of the channel was dried in air and analyzed with mechanical, shaked sieves.

Use of other methods than the sieve analysis may get more accurate result as to the mechanical composition, but due to the character of the sands used, which are comparatively coarse, the sieve method is more convienent.

Range of the experiments. The slope varies in the range from 1:300 to 1:900. The upper limit was fixed by the channel outlet construction, because the tilting gate, when swung to its lowest position, can only allow a slope slightly flatter than 1:300. The lower limit was fixed by the accuracy of tubes and gages.

The lower limit of rate of discharge q was determined by the minimum discharge required to start the general motion of the bed load. The upper limit for q, as well as that for s, was fixed by the possibility of control. Above the upper limits, the excessive high dunes occuring on the bed would make the slope readings not representative. The rapid change in the roughness of the bed also would make the hydraulic condition deviate further from a uniform flow.



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
RIVER HYDRAULIC LABORATORY

APPARATUS FOR STUDYING THE
TRANSPORTATION OF BED LOAD
SEDIMENT

JUNE, 1933. | SCALE - AS SHOWN | DRAWN BY C.H.McD.

Those grains coarser than 4.699 mm. in diameter were sieved out because they would require higher tractive force to start general motion. As shown by Schoklitsch's formula and data compiled by Kramer, the sand particle of such a diameter requires a slope of 0.00216 at depth of 5 cm. to set it in motion. This condition is very near to the upper physical limit of this equipment. The finer elements were not sieved out because they formed a relatively small percent or the total volume.

The water temperature was also taken during each run. This enables one to compute the Reynolds number of each run. It was intended that all the runs are made with turbulent but not shooting flow, to ensure the type of movement of bed material corresponding to the second and third types described below, which is the condition prevailing in nature.

Types of sand movement The sand movement on bed can be divided into the following 5 types:

1) Only isolated grains moved occasionally;
2) General motion occurs, but no riffles developed;
3) Movement with riffle formation;
4) Riffles flattened by strong current; and
5) Unti-dunes formed, the riffles travel in upstream direction.

Apparatus Used

The main equipment is a wooden flume 32 ft. long, 24" wide and 15" deep, with an inlet tank and an outlet tank. Of the 24" width, only the central portion, about 16" wide, leads to the measuring weir. Therefore only the discharge and bed load flowing down that part is to be measured.

The flume is supported by a jack screw at center, and by a roller at its upper end under the inlet tank, so that it is possible to tilt the flume at any slope.

The inlet flow is controlled by a valve which discharges water for a given valve opening at a constant rate from an elevated constant level supply tank. The pump supplying water has a capacity of 2 c.f.s.

The outlet flow is measured by a trapazoidal weir in the tank under the lower end of the flume. The head over the weir is read by a point gage.

The sand bed is molded by a board which is suspended from a rolling frame, resting on rails mounted on the top edges of the flume. The rails are fixed at 2 ft. intervals by adjustable screws so that the slope of the top of the rails can be accurately adjusted.

A point gage mounted on rollers is used for measuring profiles.

At the lower end of the flume, an overflow tilting gate regulated by worm and gears is used as a controlling section. The surface slope is measured by six piezometer tubes at 1.5-meters intervals along the center line of the channel.

The sand feeding is done by an automatic drum feeding apparatus suggested by Gilbert. The sand transported down the lower end is caught in an enlarged section by a basket 15" by 24", which is suspended from a balance arm. At the end of the arm, a spring balance and a wire, leading to an automatic recording machine, are connected to it.

The sand recorder consists of a metal plate moving by synchronous motor at the rate of 1/5" per minute, and a metal loop wire with sparking point which gives a continuous record of the sand weight.

It was thought that the total energy lost in the channel may be measured by the difference between end levels, so that two automatic floats were added to the piezometer tubes No.1 and No.6 and two adjustable screws were attached on the bottom to measure the surface and bed elevations at the ends.

The rate of sand feeding can be adjusted both by the rheostat on the shunt motor and the feeding slit under the sand box. In the first ten runs the feeding was too high, even using the lowest rate, when the tractive power is low. It was necessary to stop the feeding at times. This was improved later by adding resistence to the motor.

In making the sand mixtures, the sands were sieved by screens made with commercial screen cloth, but all the following analyses were made with Tyler's standard sieves. The shaking machine used is a Rotap made by Tyler Co.

The specific gravity of the mixtures was determined by a pychnometer.

Procedure of Making the Tests

The channel bottom was tilted first to a medium slope of 1:600 and kept thus through all the runs. After the flume was properly tilted, wedges were are put under the flume to make it supported on several wooden frmmes and reliefed from strain.

Zero point of measuring gage. The z Zero reading point of the measuring trapazoidal weir was determined by means of a hook gage and level bubble. Three sets of readings at three points on the weir edge were taken and averaged.

Making the sand bed. The flume was then filled at the bottom with about a 2" thickness of the sand mixture to be used. The sand was loosely dumped but later compacted slightly and covered overnight with water 10" deep.

Leveling the top rails. For each particular slope used, the top rails were adjusted as follows: Raise the tailgate to its maximum height, fill the leaks. Open inlet valve until overflow occurs. Wait for half an hour till the surface is quiet. Keep the adjusting screws to an amount required by the computed grade. After that, check the elevation at all points again.

All the elevations were read by the travelling point gage. The maximum erreo allowed in adjusting the rails was 0.04 cm.

Preparing the bed. The bed was then scraped to the desired height and slope by a board, the bottom of which was adjusted by means of a level bubble. After perfectly smoothed, two readings of the bed elevation at the ends were taken. After each run, the bed material on the top layer was turned over and mixed with the lower layer; after each three runs, the sand in each third of the one third stratch of the channel was thoroughly mixed again. The material at the upper end was replaced each run with new sand, because the sorting effect was prominent there.

Starting the run. The tailgate was then raised to maximum top level, the inlet valve was opened gradually until the desired discharge was indicated by the measuring gage. The gate was lowered until the peezometer readings conformed with the desired slope. This condition was maintained at least for half an hour. During the riffle formation, the slope and depth varied quite an amount, it was therefore necessary to wait until this period was over. Nevertheless, in some of the runs, the change took place very slowly so that it was completed in the middle of the runs.

As soon as the equilibrium condition was reached, which was shown by a uniform flow condition and a permanent slope under the fixed rate of discharge and sand feeding, the sand recording machine was switched on, and piezometer readings and gage readings were read. All the readings were repeated at about 5-minute intervals.

Fixing the sand feeding rate. The sand feeding rate was fixed at first by comparison with that of previous runs. Because the discharge or slope varied gradually from the previous one, it was possible to adjudge the feeding rate fairly closely. Then by watching continuously the bed elevation , the sand feeding was adjusted to keep a constant bed at the upper end of the flume.

Profile reading. About 30 minutes after the starting, when the sand curve obtained was enough to show a straight line or steady tendency, the tailgate was then swung up and the run ended. The bed elevation after the run was taken by a round plate gage of 1 inch diameter at 13 sections, 3 points were measured at each section.

Getting sand sample. The sand caught in the weighing basket was then thoroughly mixed and a representative sample, about 600 gr. when dried, was taken.

Method Used for Computing and Plotting

Averaging the slope. The following least squares formula was used for computing surface slope from a set of piezometer reading,

$$s = \frac{1}{150}\,\frac{(h_6 - h_1)\cdot 5 + (h_5 - h_2)\cdot 3 + (h_4 - h_3)}{5^2 + 3^2 + 1}$$

$$= \frac{1}{5250}\left[(h_6 - h_1)\cdot 5 + (h_5 - h_2)\cdot 3 + (h_4 - h_3)\right]$$

where the 'h' s are piezometer readings in centimeter, with the subscripts indicating the number of tubes. The constant 150 is the length of each interval between successive tubes.

Afterwards the average slope of the run was computed by taking an arithmetical mean of the several slope readings.

Averaging the depth. The bed elevations at the 13 sections were plotted as a profile. By adjusting the position of a straight edge, the average depth of water at the end of each run could be read directily from the profile. The mean depths at the beginning and the end of each run were then averaged to obtain the mean depth of the whole run.

Averaging the discharge. The several measuring gage readings were first averaged with weighting ted factors according to the length of interval in which they were taken. Using the average head over the weir, the discharge could be directly read from a calibrating chart.

Rate of sand discharge. To obtain get the real weight of sand transported, the apparent weight recorded was divided by the specific gravity of the particular sand mixture used.

Roughness "n". Because the measured discharge is only that part from the central portion of $16\frac{1}{4}$" wide, the mean hydraulic radius R is neither the mean depth of water nor the wet area divided by $2d + b$. It lies somewhere between these values. Accordingly, the computed n varies between two extremities, but it would be closer to that one computed from the mean depth.

Plotting the capacity curves. The data was first plotted with the straight line formula,* (Curves B3, B4) but it was found later that the straight line formula does not apply to this particular set of data.

Footnote

* The formula referred to on p.6 has two special forms, namely,

$$g = k_1 s(q - k_2/s) = k_1(q - q_o)s, \text{when } s \text{ is constant}, \quad k_2 = sq_o$$

$$g = k_1 q(s - k_2/q) = k_1(s - s_o)q, \text{when } q \text{ is constant}. \quad k_2 = qs_o$$

Both the forms represent a straight line. If this formula applies, the value of $q_o s$ must be identical with $s_o q$, but the values obtained from the plottings are:

Mixture No. 1,	$sq_o = 0.18$,	$s_o q = 0.226$;
Mixture No. 2,	$sq_o = 0.213$,	$s_o q = 0.156$;
Mixture No. 3,	$sq_o = 0.170$,	$s_o q = 0.237$.

The final formula adopted for plotting is one similar to MacDougall's. Its general form is

$$g = k_1 (s^m q - k_2) \qquad (I)$$

where m, k_1 and k_2 are constants depending on the nature of the sand, or more exactly, on the uniformity and density of the sand. Because in each constant slope group, only five test runs were made, it was intended to corelate the data with MacDougall's and his result of using unit power for q was assumed first.

The six groups of data were plotted separately on a sheet of logarithmic paper, using the two special forms of formula (I), namely,

$$g_s = k_1 s^m(q - k_2/s^m) = k_1 s^m(q - q_o),$$

or, $$g_s + k_1 k_2 = (k_1 s^m)\ q \quad ; \text{ and} \qquad \text{(II)}$$

$$g_q = k_1\ q\ (s^m - k_2/q) = k_1 q\ (s^m - s_o^m),$$

or, $$g_q + k_1 k_2 = (k_1 q)\ s^m\ . \qquad \text{(III)}$$

Formula (II0 was plotted with q varying, and (III) plotted with s varying. The conditions which fix the three constants in (I) are therefore:

i) the slope of q curve must be unity;

ii) $k_1 k_2$ in the two cases must be equal; and

iii) $g_s + k_1 k_2 = g_q + k_1 k_2$ when both s and q are equal.

This method of plotting is shown on plate A-7a that the two vertical lines meet the two curves on the same horizontal. meaning?

After the two logarithmic curves had been determined, the constants in formula (I) were determined at once. Thus,

$$k_1 = (k_1 s^m)/s^m = (k_1 q)/q, \text{ and}$$

$$k_2 = (k_1 k_2)/k_1\ .$$

The resulted formula was then plotted on linear plotting paper. (Plates A-7 to A-9)

In making the constant slope or constant discharge plotting, an adjustment was made when the actual slope or discharge deviates too much from the normal slope or the normal discharge of the group. The correction was computed approximately by assuming that $g = k_1(sq - s_o q_o)$ was valid.

The last step was to plot k_1, k_2, and m against 'F' of the three mixtures.

Plotting the sorting curves. The result from the mechanical analysis of sand transported was plotted in constant slope and constant discharge groups. (Plates A-11 to A-16) The fine curve represents the original composition before the run, to be used as a comparison basis. From careful study of the result, a modification was made on composition curves of mixtures No. 1 and No. 3 to eliminate the effect of suspension. Of the original percentages, ½ of those passing through 48-mesh sieve, and 1/8 of those passing through 100-mesh sieve were assumed to be brought into suspension in every run, hence were not caught in the weighing basket. This assumption only made the upper part of the curves round-ended, but did not alter much the general shape.

From the curves, the values d_g and M were determined and plotted against slope and discharge respectively.

Observation, Source of error, and Probable Accuracy

Controlling factors, their steadiness. Of the three factors, the slope is the most difficult one to handle. Not only it is very sensitive to discharge change or bed change, but also it is hard to decide whether to raise or to lower the tailgate when maintaining the slope. The accuracy of piezometers reads to 0.5 mm. which means 4% of 1:600 slope and higher percentage for gentle slopes, but the dune height at tha bottom may vary sometimes from -1.5 cm. to +1.5 cm. This of course causes a local change of depth and hence the grade line. Even a reverse slope may occur in a short stretch.

In most of the test runs, the surface slope consisted of a steeper upper half and a gentler lower half. This may be seen from the profile plottings. The causes of such deviation are:

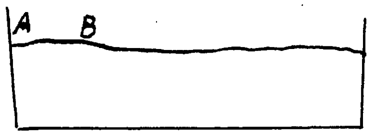
i) The channel flow was not strictly a uniform one, its upper part was somewhat like an approach, hence excessive slope was required to accelerate the flow.

ii) The enlerged section had a backwater effect.

Second to the slope was the bed material factor. Making series of tests of constant slope without disturbing the bed would be a quicker way, but much care would have to be taken to keep the composition of bed material as constant as possible. Therefore , the bed was remolded each time and allowed to develop riffles again. In this way the sorting effect was avoided to a large extent. At the upper end, or in case of very weak flow, the bed seems to be paved with coarse particles which remain untouched.

At very high stage, the discharge measuring gage does not read accurately, because the level in the measuring tank vebrates rhythmically, partly due to the lack of approach, partly due to the unsteady flow over the weir crest.

<u>Transversal condition.</u> The bed velocity along center line was always higher than that along side walls, therefore the sediments carries along the middle third was heavier and moved quicker. When snd dunes were formed, they usually assumes a tongue shape with its point directed downstream.



Another transverse difference was that the finer grains along side wall A were usually eroded off and brought to a stripe B, about 4" apart from A. The accumulation of fine particles at B sometimes produced sub-dunes of small height and closer spacing.

Problem of suspension. In making two of the three mixtures, the sand C used contained 5.5% of grains passing through 100-mesh sieve. This caused part of the bed material to go into suspension, which could not be caught in the sand basket and therefore formed a lost. It did not, however, interfered with the bed load capacity assuming that the small percentage of suspension only changed the flow velocity in smaller order than the bed load did.

Sorting effect occured even at highest stage. There was no such chance that the bed material moved as a whole unit. A section from one dune to next dube formed a complete cycle of sorting. The grains paved layer over layer according to their sizes with the coarsest settling first. On the top of dunes, there was a layer of finest particles, this made a smooth bed for coarser ones to roll upon. The coarser particles moved with higher velocity but severally, and the finer sand grains moved slower but collectively. The more the uniformly sized fine elements are contained in the mixture, the easier the formation of sand dunes. Because most of the graines would move and settle simultaneously at a small change of bed velocity. The paving action of coarser grains seemed to counteract the dune formation.

Accuracy of the measuring apparatus. The measuring gage for outflow can measures to 0.05 mm. of head except at highest stage, which approximately indicates 0.012 liter per second of discharge. The piezometers can be read to 0.5 mm., but sometimes the parallax effect comes in, which may cause an error of 0.5mm. In most of runs, the level in the piezometer tubes represented a fair average of the oscillating water surface in flune. It usually fluctuated between limits 2mm. apart periodically.

The sand weighing devices worked pretty dependable except that the basket often touched the tank wall due to eccentric loading, it was corrected as soon as perceived.

The accuracy of the spring balance canbe kept within ¼ of a pound of sand weight in water, or 0.4 pound weight of sand in air, which is about 7% of the smallest reading.

When water is clear, the method of using bottom screws for measuring depth directly gives elevation to nearest millimeter, but it seems impossible to get the whole profile reading during the run, the isolated readings can only serve as a check.

Tabulation of Results

I. Mechanical Analysis of the sand mixtures.

Mixture No. 1

Sieve No.	Size of opening	Accumulative percentages		
		1st S.	2nd S.	Average
4	4.699	0.1	0.0	0.1
8	2.362	1.8	0.9	1.4
14	1.168	14.15	13.75	13.95
20	0.833	42.8	43.1	42.95
24	0.701	63.8	63.4	63.1
28	0.589	76.0	73.1	74.5
32	0.495	87.0	81.7	84.35
48	0.295	97.5	92.0	94.75
100	0.147	99.8	98.1	98.95
pan	--	100.0	100.0	100.0

Mixture No. 2

Sieve No.	Size of opening	Accumulative percentage		
		1st S.	2nd S.	Average
14	1.168	4.46	3.62	4.04
20	0.833	51.72	47.89	49.81
24	0.701	86.6	82.15	84.35
28	0.589	95.62	93.45	94.54
48	0.295	99.89	99.35	99.62
pan	--	100.0	100.0	100.0

Mixture No. 3

Sieve No.	Size of openting	Accumulative percentage		
		1st S.	2nd S.	Average
4		0.1	0.04	0.07
8		3.73	4.0	3.88
14		24.55	29.7	27.13
20		26.08	31.9	28.99
24		29.05	34.9	31.98
28		45.75	51.25	48.50
32		50.99	67.40	63.70
48		82.80	87.40	85.10
100		96.20	97.10	96.65
pan		100.0	100.0	100.0

II.Test Runs.

Table I

Run No.	Assum.S	Actu. S.	q cm^3/s/ cm	q c.f.s/f.	D_m cm.
1	1:600	0.00152	202	0.217	5.38
2	1:600	0.00157	224	0.241	5.64
3	1:600	0.00161	252	0.275	6.09
4	1:600	0.00157	308	0.332	6.98
5	1:600	0.00200	590	0.636	10.07
6	1:500	0.00185	246	0.265	5.67
7	1:400	0.00247	247	0.267	5.21
9	1:300	0.00300	243	0.261	4.68
10	1:700	0.00146	258	0.277	5.90
11	1:800	0.00120	255	0.274	6.32
12	1:500	0.00200	252	0.271	5.49
13	1:550	0.00181	251	0.270	5.51
14	1:400	0.00238	317	0.341	6.29
15	1:500	0.00206	307	0.330	6.34
16	1:700	0.00150	309	0.333	7.00
17	1:800	0.00133	312	0.336	7.27
18	1:900	0.00110	310	0.334	7.35
19	1:600	0.00159	206	0.221	5.06
20	1:600	0.00163	255	0.274	5,83
21	1:600	0.00169	310	0.335	6.80
22	1:600	0.00168	367	0.395	7.80
23	1:600	0.00178	490	0.528	8.91
24	1:400	0.00247	313	0.337	6.08
25	1:500	0.00191	314	0.338	6.51
26	1:700	0.00145	312	0.335	6.90
27	1:800	0.00129	309	0.332	7.13
28	1:900	0.00111	307	0.331	7.24
29	1:600	0.00160	255	0.274	5.78
30	1:600	0.00163	208	0.224	5.04
32	1:600	0.00161	313	0.337	6.65
33	1:600	0.00163	367	0.394	7.52
34	1:600	0.00166	503	0.542	9.09

Table II

Run No.	V_m m/s	V_m ft/s	V_c m/s	T_w	R
1	0.375	1.23	0.725	22.5 °C	21200
2	0.398	1.31	0.742	21.2	22800
3	0.413	1.36	0.777	20.2	25000
4	0.443	1.46	0.827	18.9	29800
5	0.586	1.93	0.992	18.5	56250
6	0.434	1.43	0.744	18.4	23400
7	0.476	1.56	0.714	21.0	25130
9	0.518	1.70	0.677	22.2	25320
10	0.437	1.44	0.760	22.2	26950
11	0.404	1.33	0.787	21.8	26330
12	0.459	1.51	0.733	21.8	26100
13	0.456	1.50	0.734	22.4	26400
14	0.504	1.68	0.784	22.4	33900
15	0.484	1.59	0.788	24.8	34370
16	0.441	1.44	0.832	23.9	33560
17	0.429	1.41	0.843	25.0	34760
18	0.422	1.38	0.848	23.0	32970
19	0.407	1.34	0.703	24.7	22780
20	0.436	1.43	0.755	27.0	29660
21	0.456	1.48	0.815	26.0	36150
22	0.470	1.54	0.873	26.6	38750
23	0.550	1.81	0.933	26.5	56500
24	0.515	1.69	0.772	23.6	33760
25	0.482	1.58	0.799	23.6	33900
26	0.452	1.48	0.822	27.0	36400
27	0.433	1.42	0.835	26.9	35900
28	0.424	1.39	0.841	25.5	34590
29	0.441	1.45	0.752	24.5	28080
30	0.412	1.35	0.702	25.0	23200
32	0.470	1.54	0.807	24.0	34070
33	0.488	1.60	0.857	24.6	40500
34	0.553	1.82	0.942	24.8	56140

Table III

Run No.	g #/ft/hr	g gr/cm/~~s~~min.	g adjusted	s^m *	$s^m q$ *
1	6.76	1.68	2.05	0.0000173	0.00349
2	9.18	2.31	2.59	182	408
3	11.72	2.91	3.09	190	480
4	15.82	3.93	4.31	182	562
5	49.20	12.20	9.82	275	1620
6	17.40	4.31	4.45	0.0000241	0.00592
7	34.50	8.55	8.74	392	970
9	44.00	10.90	11.30	545	1324
10	15.15	3.76	3.58	163	421
11	10.78	2.68	2.34	116	295
12	29.00	7.20		275	692
13	22.60	5.61		232	582
14	26.80	6.64		0.00475	1.505
15	24.30	6.03		421	1.293
16	13.75	3.41		317	0.980
17	12.52	3.11		287	0.895
18	9.23	2.29		241	0.745
19	6.76	1.68	1.84	335	0.688
20	13.33	3.31	3.43	321	0.819
21	18.70	4.63	4.54	353	1.093
22	~~28.40~~ 28.10	6.97	6.38	351	1.288
23	36.00	8.92	8.22	369	1.810
24	54.40	13.50		0.000224	0.0701
25	39.40	9.76		156	490
26	18.80	4.67		106	331
27	13.95	3.46		90	278
28	7.37	1.83		73	224
29	17.10	4.24		122	311
30	~~8.18~~ 8.47	2.10		125	260
32	22.00	5.46		123	385
33	29.00	7.19		125	459
34	37.50	9.30		128	645

* The exponent m = 1.69 for No. 1-13
0.807 14-23
1.40 24-34

Table IV

Run No.	Mixture No.	*Description of dunes	n use D_m	Material transp. d_g	Material transp. M
1	3	Not formed	0.0149	0.0948	0.366
2	3	Neglegible	0.0146	0.0948	0.336
3	3	"	0.0154	0.0908	0.335
4	3	Moderate	0.0152	0.0800	0.307
5	3	Excessively high	0.0165	0.0820	0.323
6	3	Neglegible	0.0146	0.948	0.339
7	3	"	0.0146	0.898	0.340
9	3	Moderate	0.0137	0.864	0.301
10	3	Neglegible	0.0134	0.918	0.310
11	3	Not formed	0.0136	0.950	0.354
12	3	Moderate	0.0141	--	--
13	3	"	0.0135	0.934	0.346
14	2	High dunes	0.0151	0.858	0.675
15	2	Moderate	0.0149	0.842	0.644
16	2	High	0.0149	--	--
17	2	High	0.0148	0.890	0.698
18	2	Moderate	0.0138	0.850	0.743
19	2	Moderate	0.0134	0.882	0.777
20	2	High	0.0139	0.846	0.740
21	2	Excessive	0.0151	0.846	0.733
22	2	Excessive	0.0159	0.838	0.738
23	2	"	0.0153	0.856	0.712
24	1	High	0.0149	0.850	0.512
25	1	Moderate	0.0147	0.878	0.514
26	1	"	0.0141	0.878	0.557
27	1	"	0.0142	0.910	0.521
28	1	"	0.0136	0.963	0.541
29	1	Moderate	0.0136	0.884	0.542
30	1	"	0.0134	0.900	0.630
32	1	High	0.0140	0.818	0.542
33	1	"	0.0147	0.850	0.557
34	1	Excessive	0.0149	0.806	0.580

* Neglegible, maximum dune height less than 0.5cm.
Moderate, " " " " " 1.0cm.
High, " " " " " 2.0cm.
Excessively high, " " over 2.0cm.

Discussion of Results

Under fixed hydraulic conditions, using sand mixture of same mean grain diameter, the sand transporting capacity may be expected to be simply a function of the sand composition, thus,

$$g = \phi(F) \cdot f(s, q, d_g)$$
$$= K\,\phi(F)$$

If the experiments were so arranged that the factor F was chosed as the only variable, the result came out would determine (F) directly. The nature of the experiment, however, prohibits such doing, because it would require a huge amount of sand to pave the channel bed each time. Also since the result of the present experiments shows clearly that the sand transportation cannot be expressed in any simple form of explicit function of 'F',* the method adopted seems to be superior to the method of varying F.

* (This relation may be seen from the plottings B3 and B4, that for same s and q, the capacity g for the mixtures is in the following order according to the magnitude,

Mixture No. 1, No. 2, No.3

in one case, and in the order

Mixture No. 1, No. 3, No. 2

in the other, also that below a certain point, where the straight lines cross each other, the above orders are reversed.

The plotting of the capacity curve for mixture No. 3 has been done with great difficulty, because the data from the two separate groups did not consistent in any way. The systematic error was not yet found definitely. One of the reasons which may be mentioned is the change made on the lower end sill or the channel bed. Its top was flushed with the bed during the first 7 runs, but was depressed later to about one centimeter lower than the bed elevation. The final curve was plotted by considering the consistent points only.

The result compares not favorably with that of Gilbert's experiments in one phase, i.e., the exponent "o" in the formula

$$g = b_2(q - \tau)^o$$

is in general too low. (Gilbert's values vary from 0.79 to 1.24, while the results from this experiment are 0.55, 0.66, and 0.95.) On the other hand, the three exponents for s are nearly equal in spite of the variation of the sand uniformity. (Plate B2) This agree s with Gilbert's result on mixtures.

It is stated in the Vicksburg Station's report that, when the tractive force is low, the mean grain diameter of the transported material is higher than that of the original, though the usual idea believes that it should be lower. (p. $\overset{33}{32}$, ref. 8)

Another matter of interest~~ing~~ is that when the capacity g is plotted against the mean velocity V_m, the resulting curve resulted consists of two parts: the first part assymptotes a straight line but with increasing slope; the second part starts with a very steep slope and then bends downward. Quite accidentally, the velocity at the point of intersection of these two parts is just the velocity at which the riffles change from moderate or neglegible height to high dunes. The same curve for mixture No.2 however, does not show the diffirence before and after that point. It may be interpreted as this: The sorting effect becomes prominent after the riffles have been completely developed.

When the completely sorted material advances in the form of sand dunes, the capacity g is materially changed and does not follow the same law as before. The higher the uniformity of the sand, the less would be the effect of sorting on capacity.

From table II, it may be concluded that all the runs were made with streaming turbulent flow, as all the 'V_m's were less than V critical, and all the Reynolds numbers were higher than 2000.

Conclusions

First part, -- capacity of sand transportation. The capacity of the transportation of a channel flow can be expressed in the form

$$g = k_1(s^m q - k_2).$$

With the mean grain diameter of the sand held fast, the constants k_1, k_2, and m are functions of the uniformity modulus F (or M) of the sand.

i) k_1 decreases with increasing F, in an order higher than the straight line relation.

ii) k_2 increases with increasing F, in an order higher than the straight line relation.

iii) m decreases with increasing F, in an order higher than the straight line relation. When the curve is extrapolated to uniform material, that is, $F = 1$, the exponent m will be less than unity.

Second part, hydraulic condition and sorting effect.

i) Both d_g and F of the transported material, when the tractive force is low, are higher than those of the original material. As the tractive force or tractive power is increased, d_g and F decrease gradually until a point is reached, after which they remain almost constant. This point corresponds to the state of excessive dune formation.

ii) As the tractive power increases, the percentage of coarser elements decreases and that of the finer elements increases.

iii) The more the uniformly sized fine elements the mixture contains, the easier the developement of sand dunes. The word'fine' has a relative meaning to the tractive power.)

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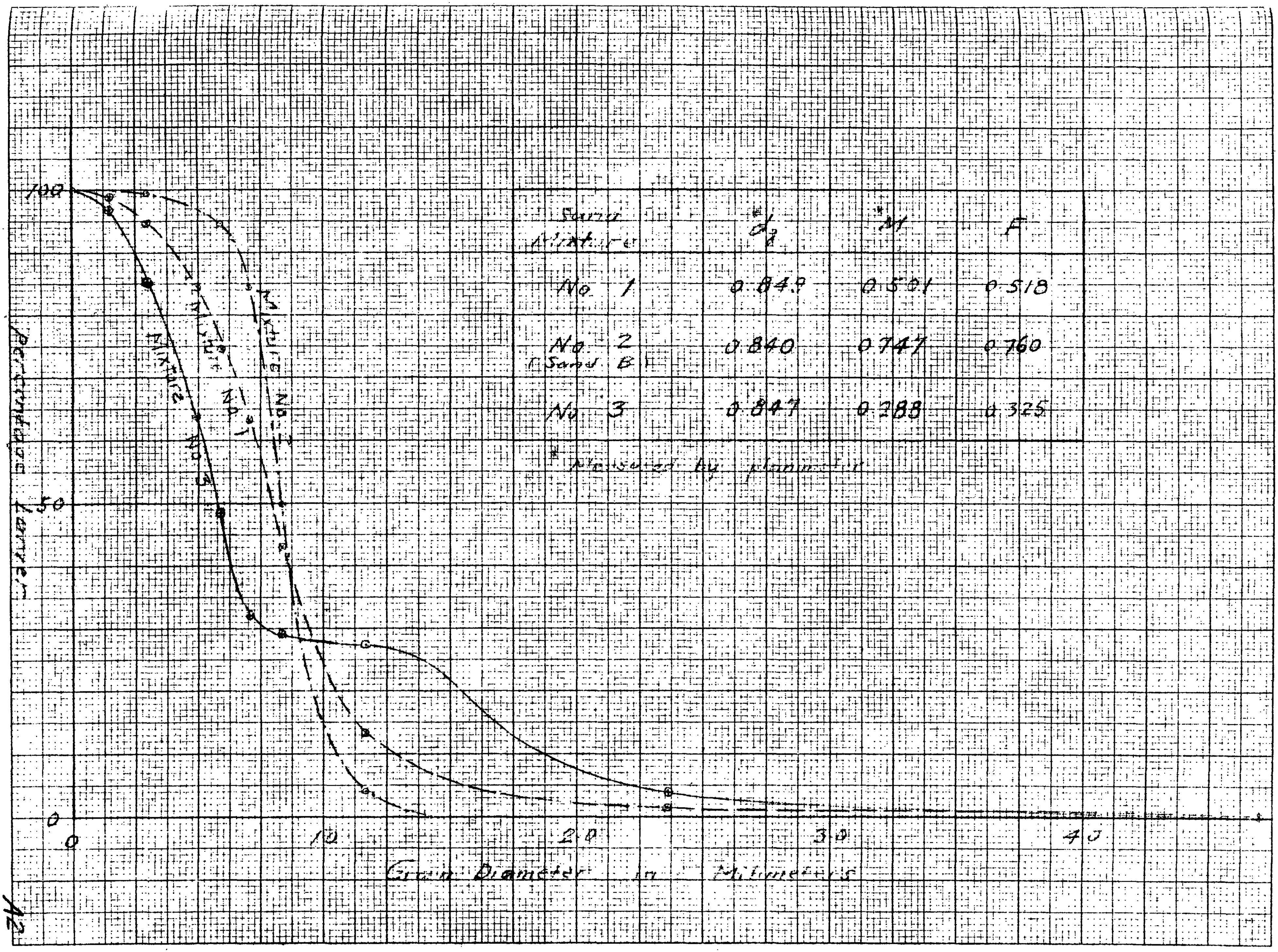
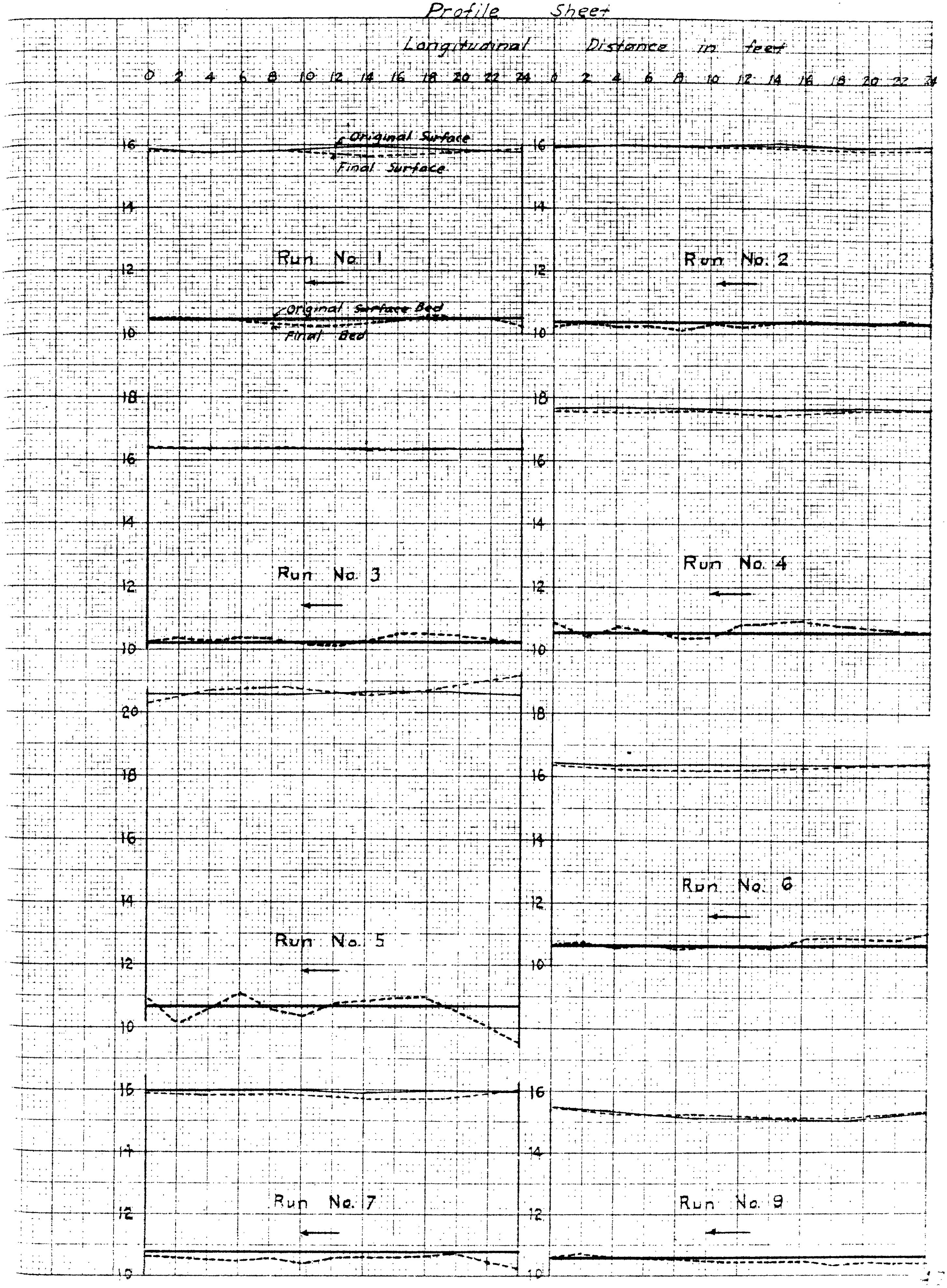


Sand Mixture	*d/d	*M	F
No 1	0.849	0.501	0.518
No 2 (Sand B)	0.840	0.747	0.760
No 3	0.847	0.288	0.325

* Measured by planimeter

Profile Sheet



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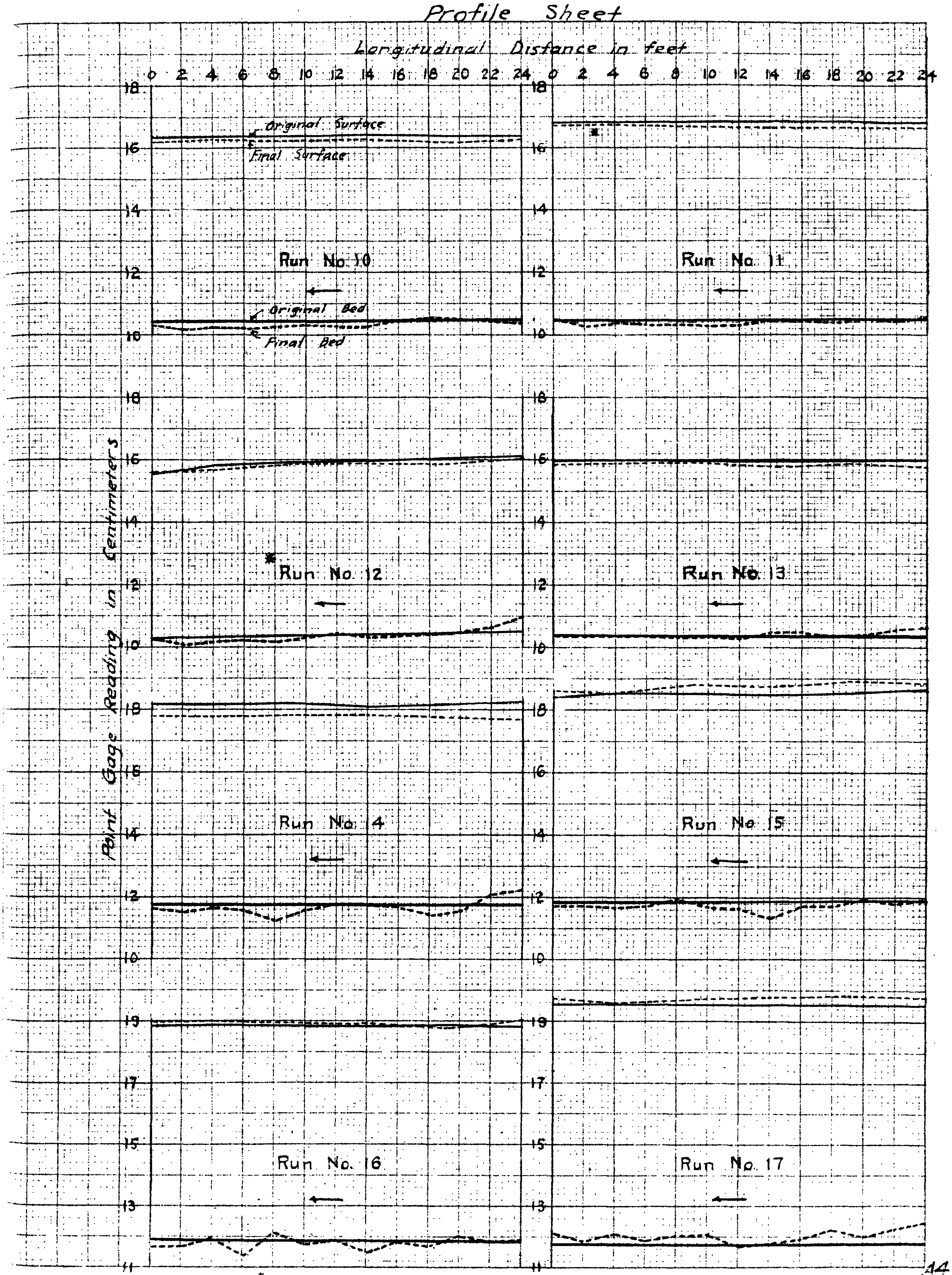
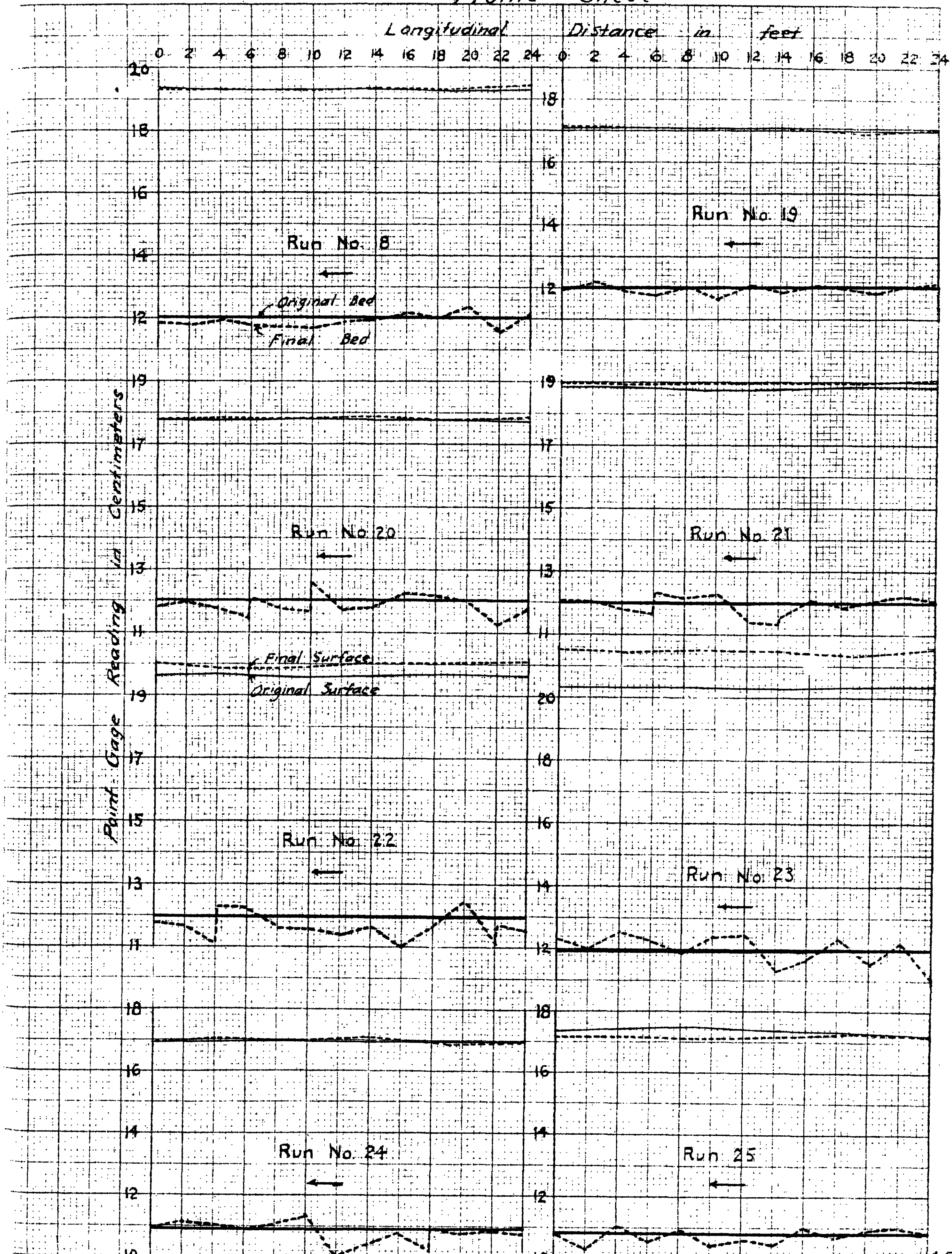


* Surface Slope not parallel to rail

Profile Sheet



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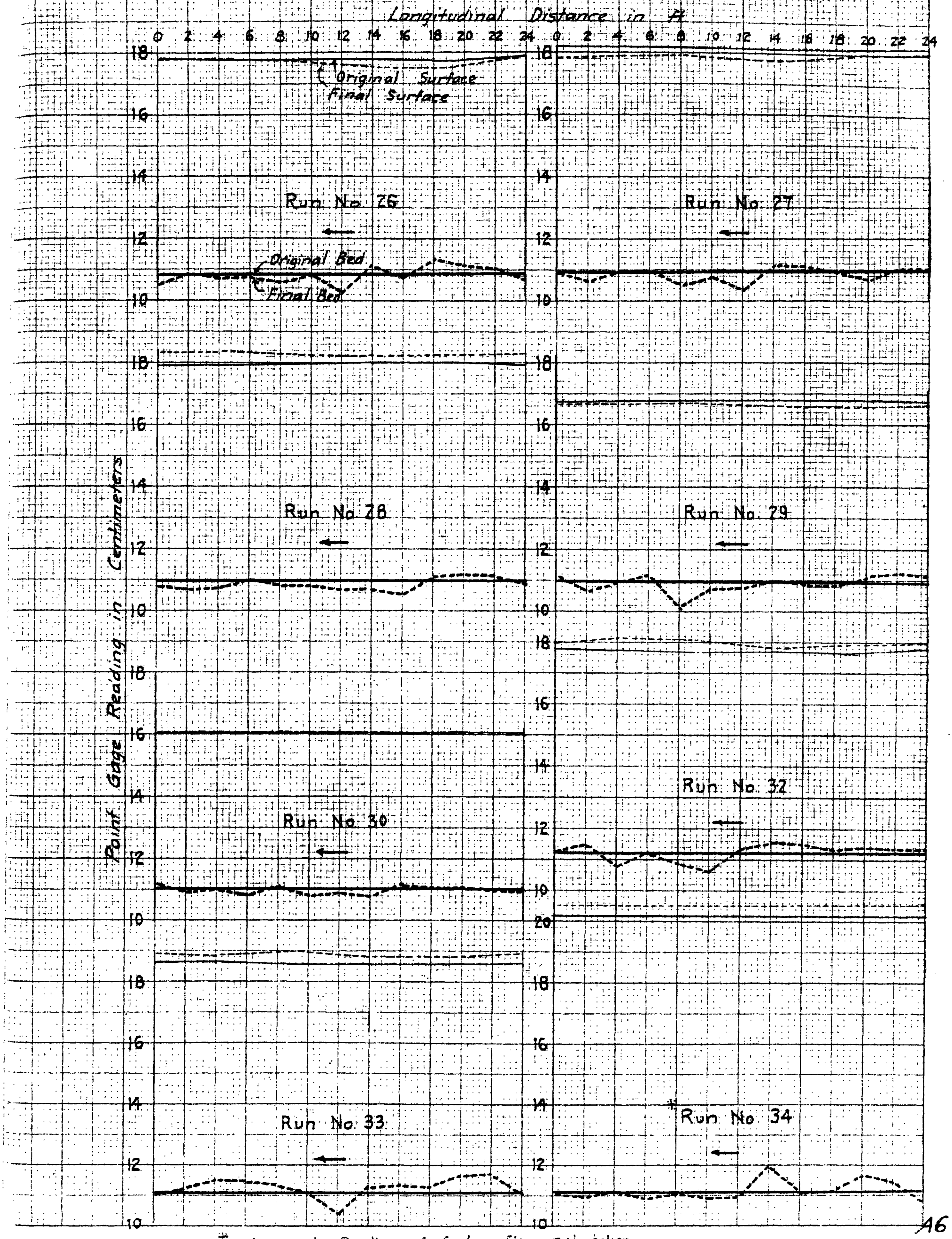
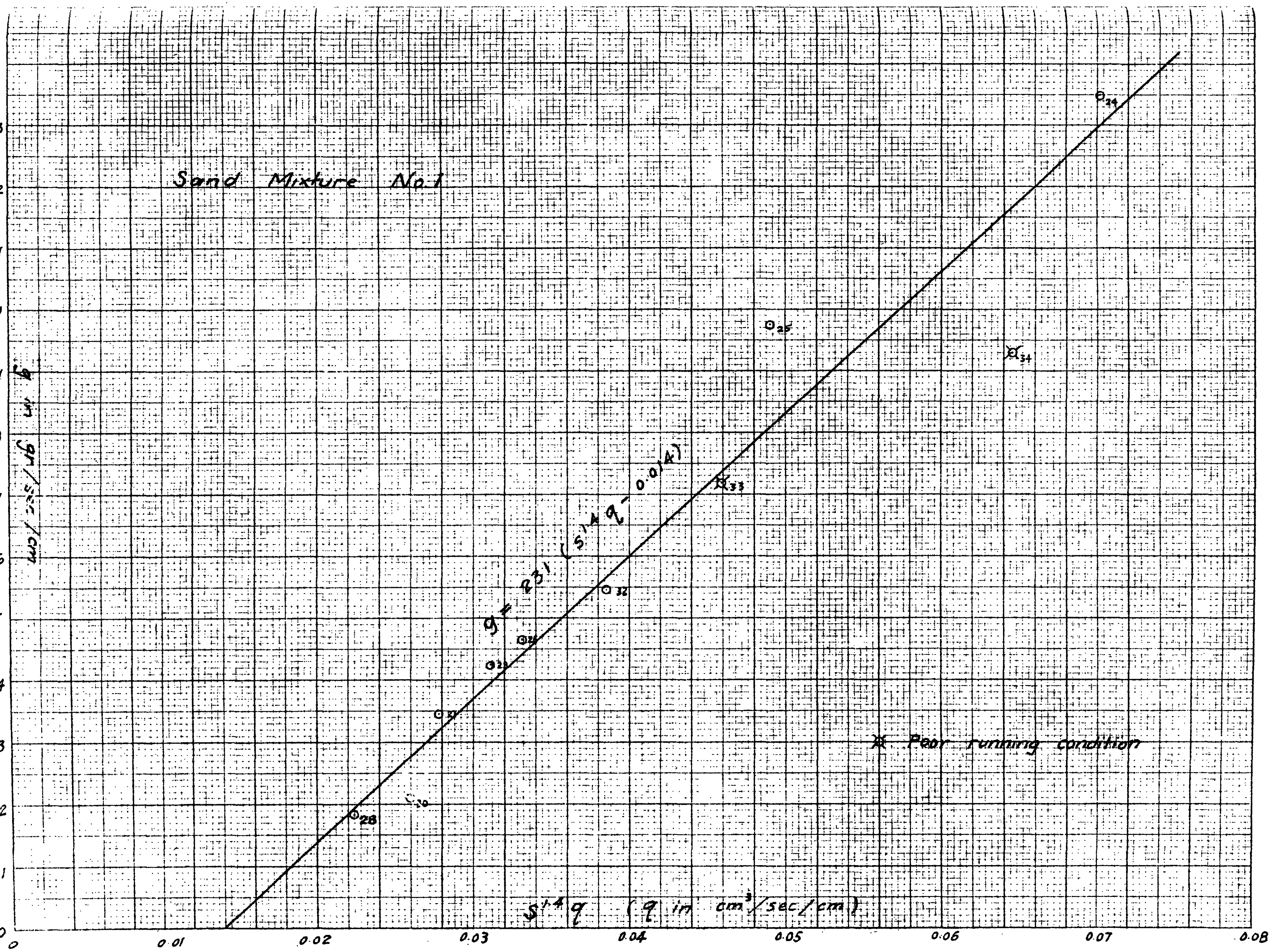
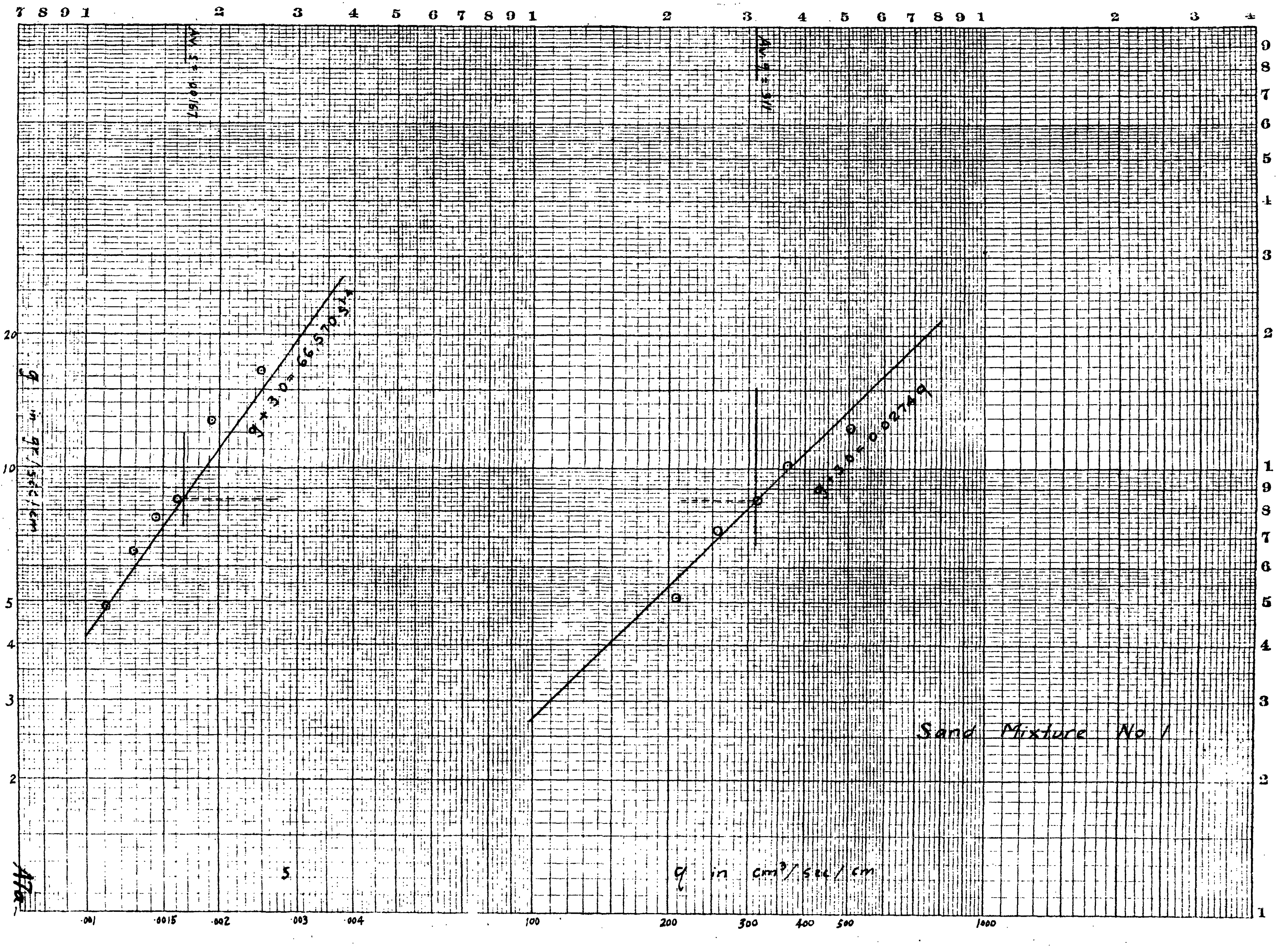
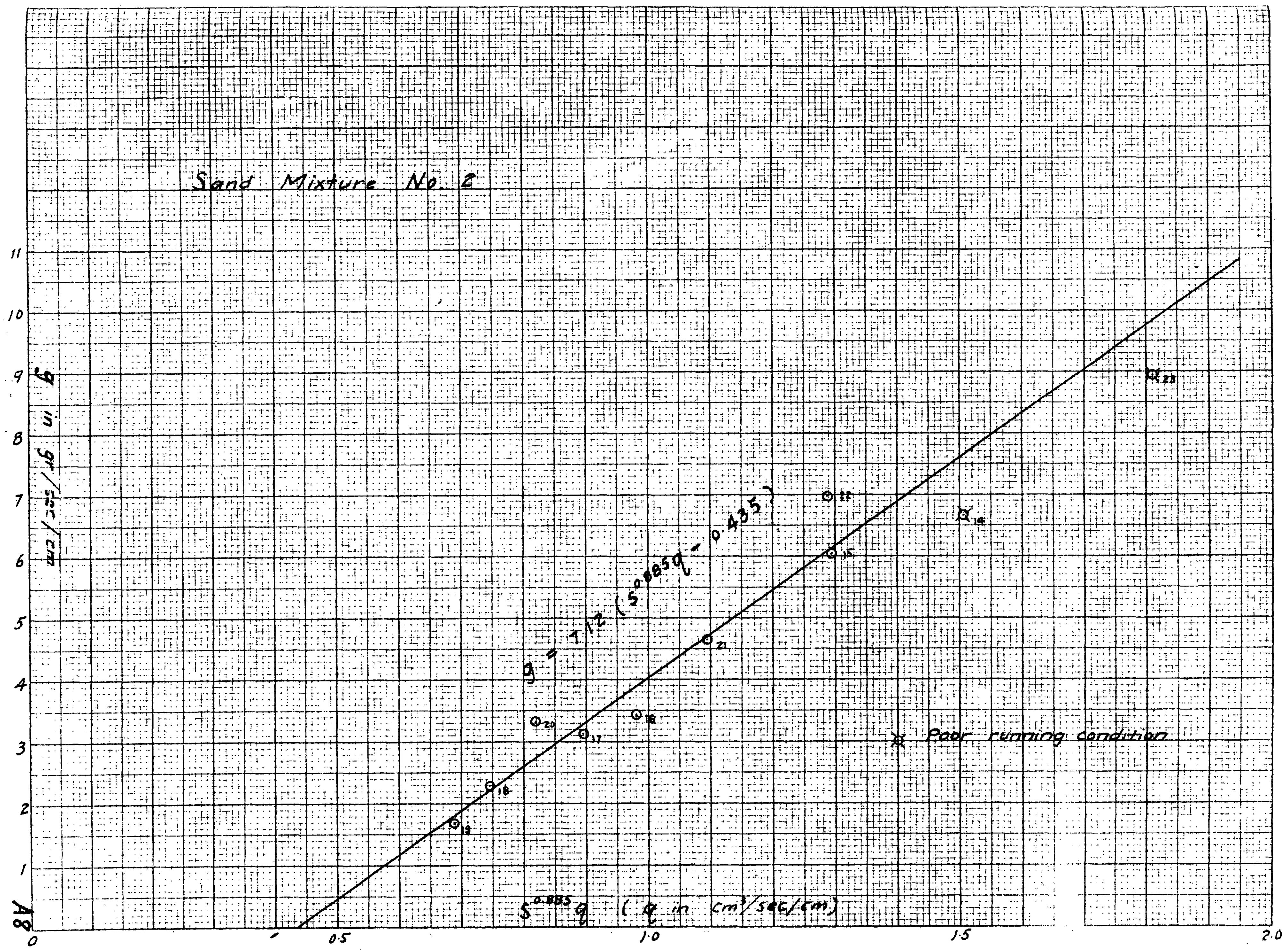


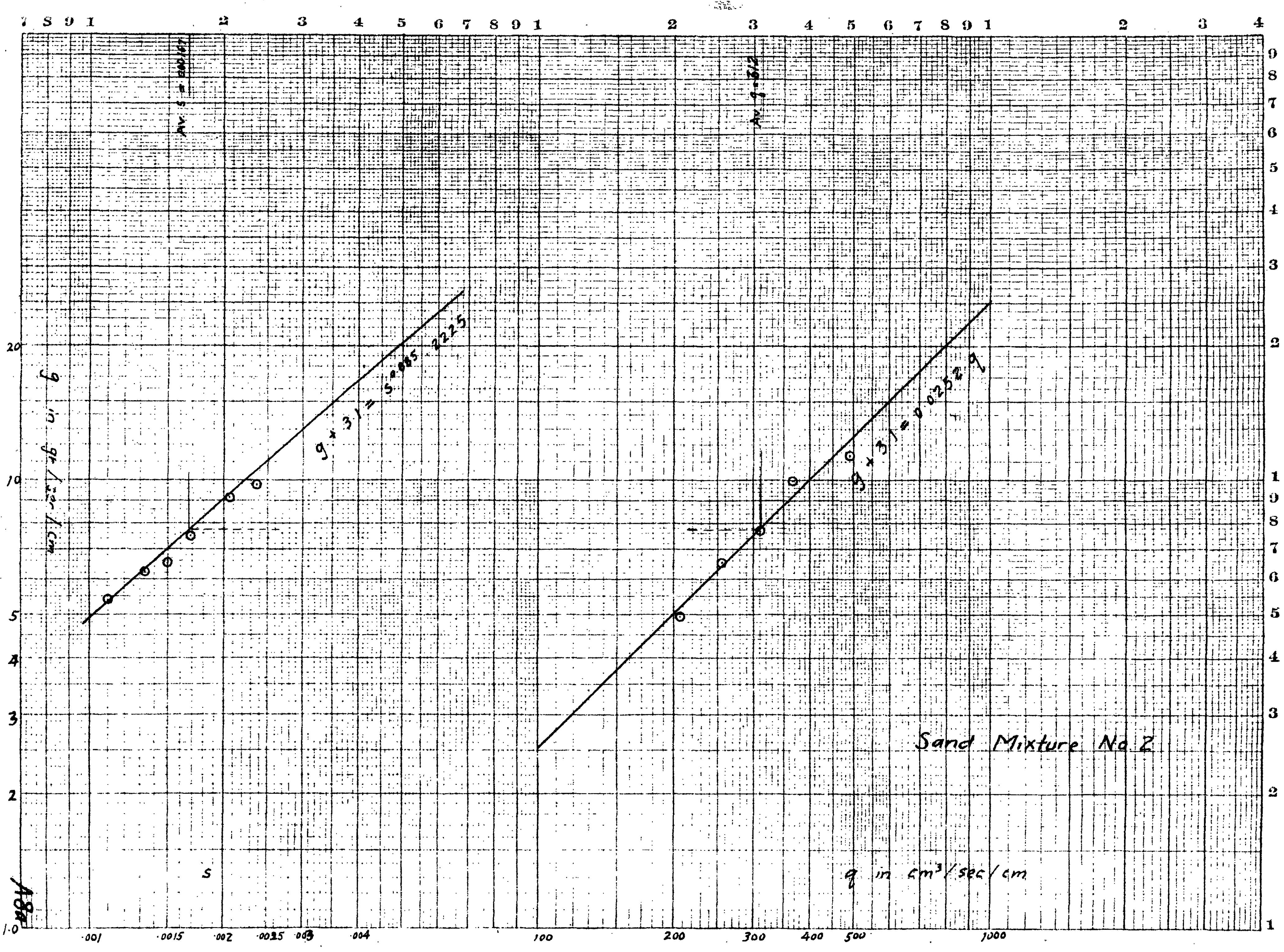
* Intermiate Reading of Surface Elev. not taken


Sand Mixture No.1
g in gr/sec/cm
$s^{1.4} q$ (q in cm^3/sec/cm)
$g = 231 (s^{1.4} q - 0.014)$
Poor running condition
0
1
2
3
4
5
6
7
8
9
10
11
12
13
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
24
25
34
33
32
26
23
27
30
28

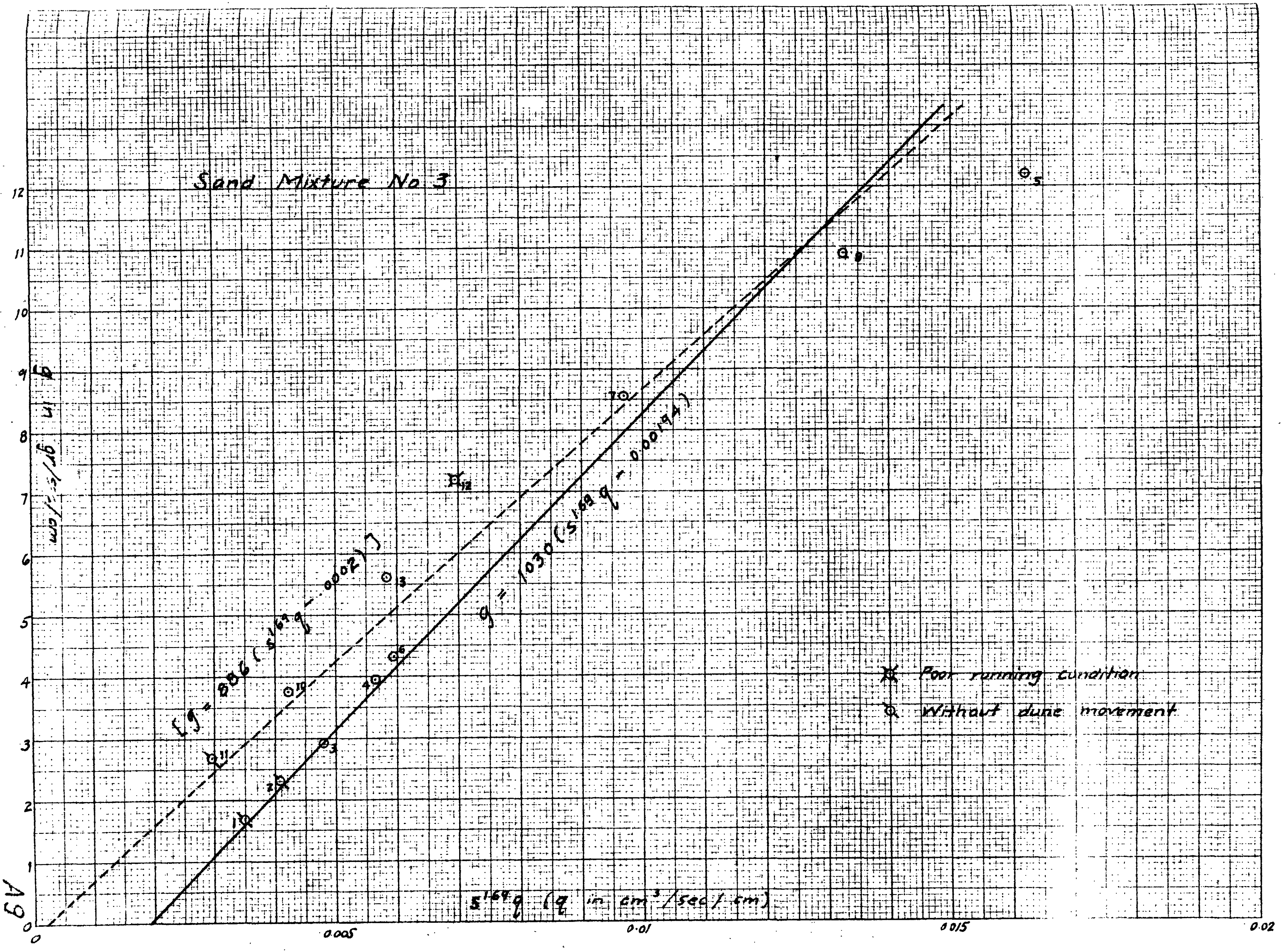
Sand Mixture No 1
g × 3.0 = 66,570 S1.4
g × 3.0 = 0.0274 q
Av. S = .00167
Av. q = 311
g in gr/sec/cm
q in cm³/sec/cm
5
.001
.0015
.002
.003
.004
100
200
300
400
500
1000
1
5
10
20
47a

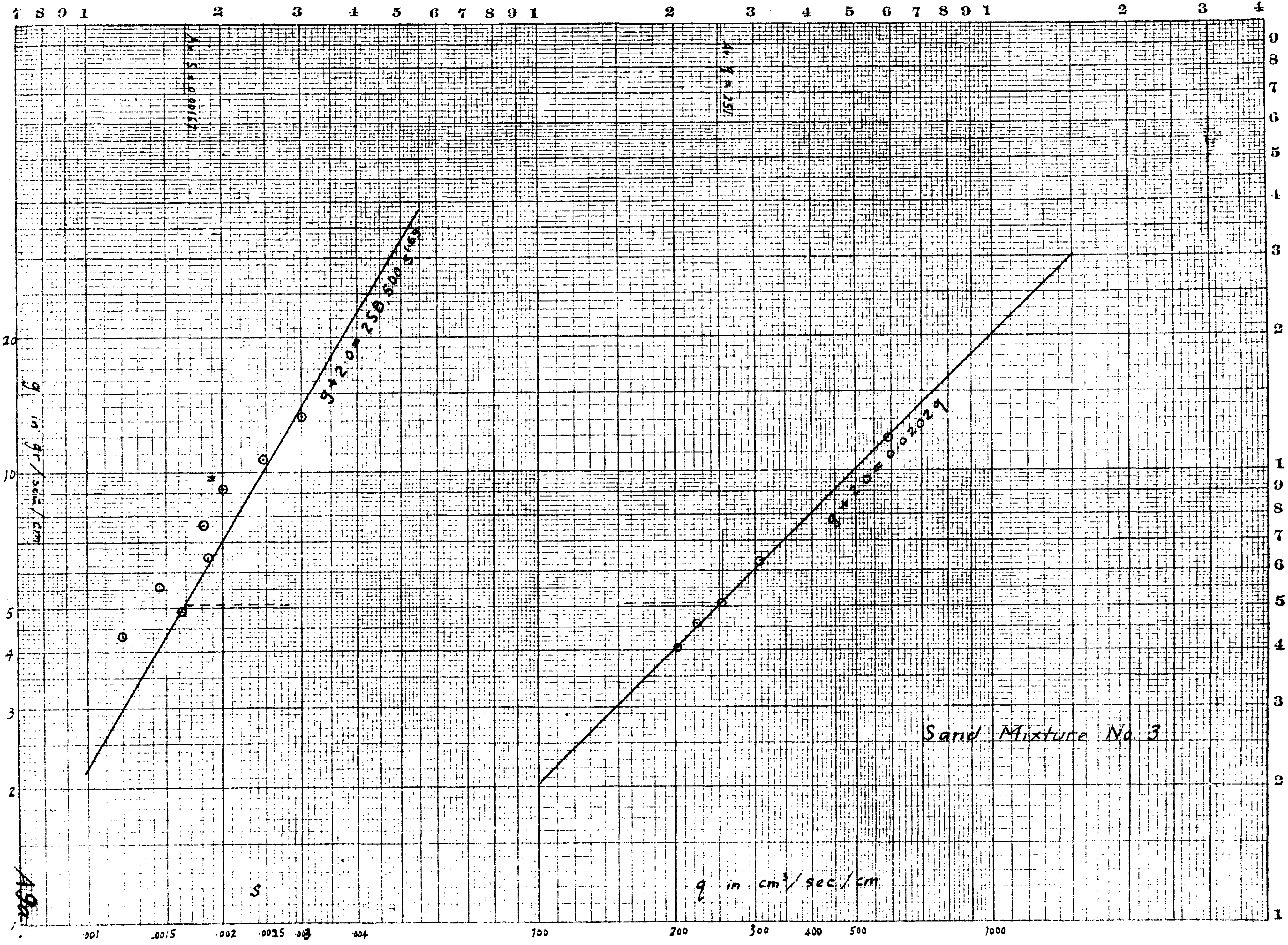


Sand Mixture No. 2
$g = 7.12\,(S^{0.885}q - 0.435)$
g in gr/sec/cm
$S^{0.885}q$ (q in cm³/sec/cm)
⌧ Poor running condition
0
1
2
3
4
5
6
7
8
9
10
11
0.5
1.0
1.5
2.0
19
18
20
17
16
21
15
11
14
23

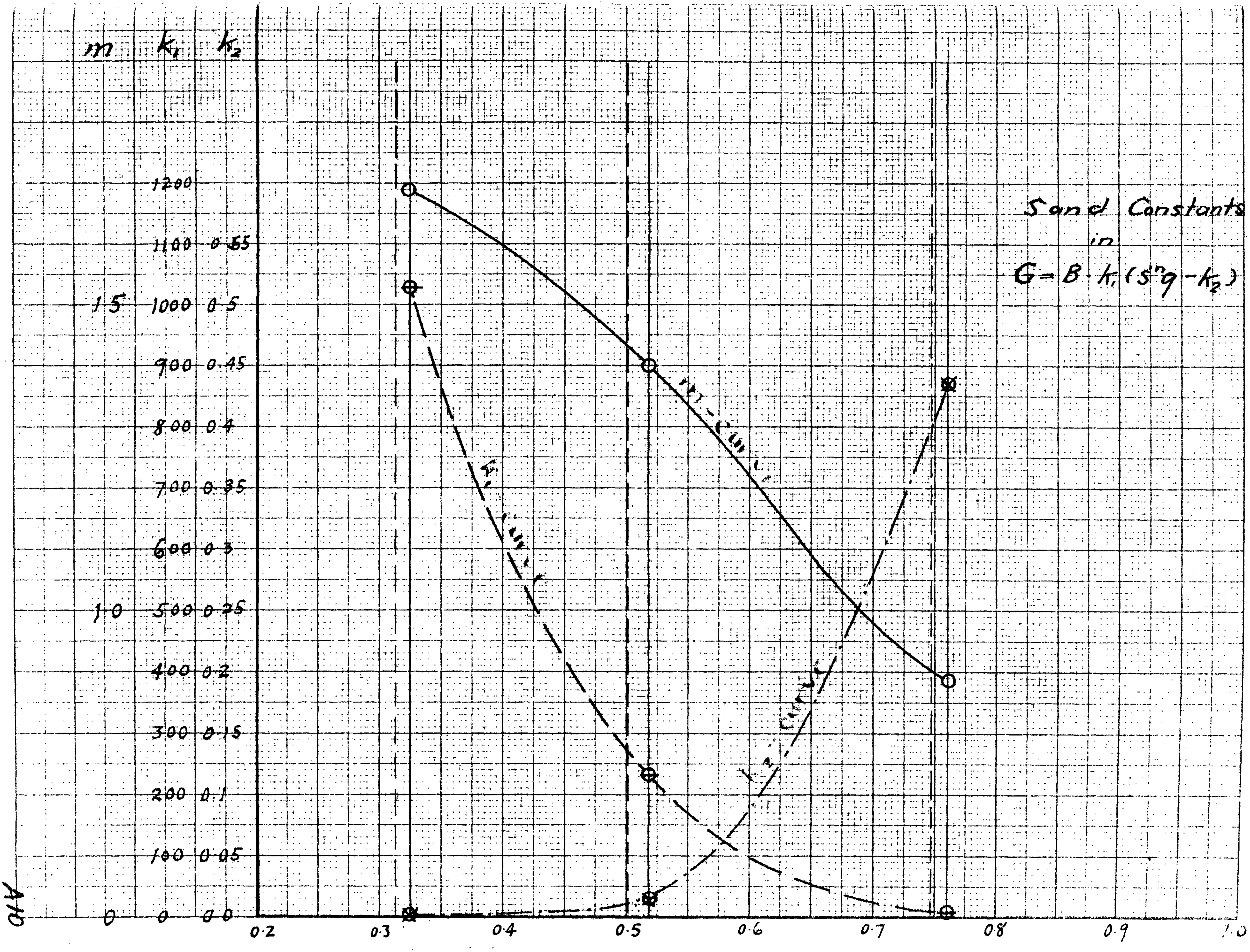

Sand Mixture No 2
g in gr/sec/cm
s
q in cm³/sec/cm
g × 3.1 = s^.885 2225
g × 3.1 = 0.0252 q
Av. s = .00167
Av. q = 312
.001
.0015
.002
.0025
.003
.004
100
200
300
400
500
1000
1.0
2
3
4
5
10
20

A8a


Sand Mixture No 3
g in gr/sec/cm
$S^{1.69}q$ (q in cm^3/sec/cm)
g = 1030 ($S^{1.69}q$ - 0.00194)
[g = 886 ($S^{1.69}q$ - .0002)]
Poor running condition
Without dune movement
0
1
2
3
4
5
6
7
8
9
10
11
12
0.005
0.01
0.015
0.02
1
2
3
4
5
6
7
8
10
11
12
13

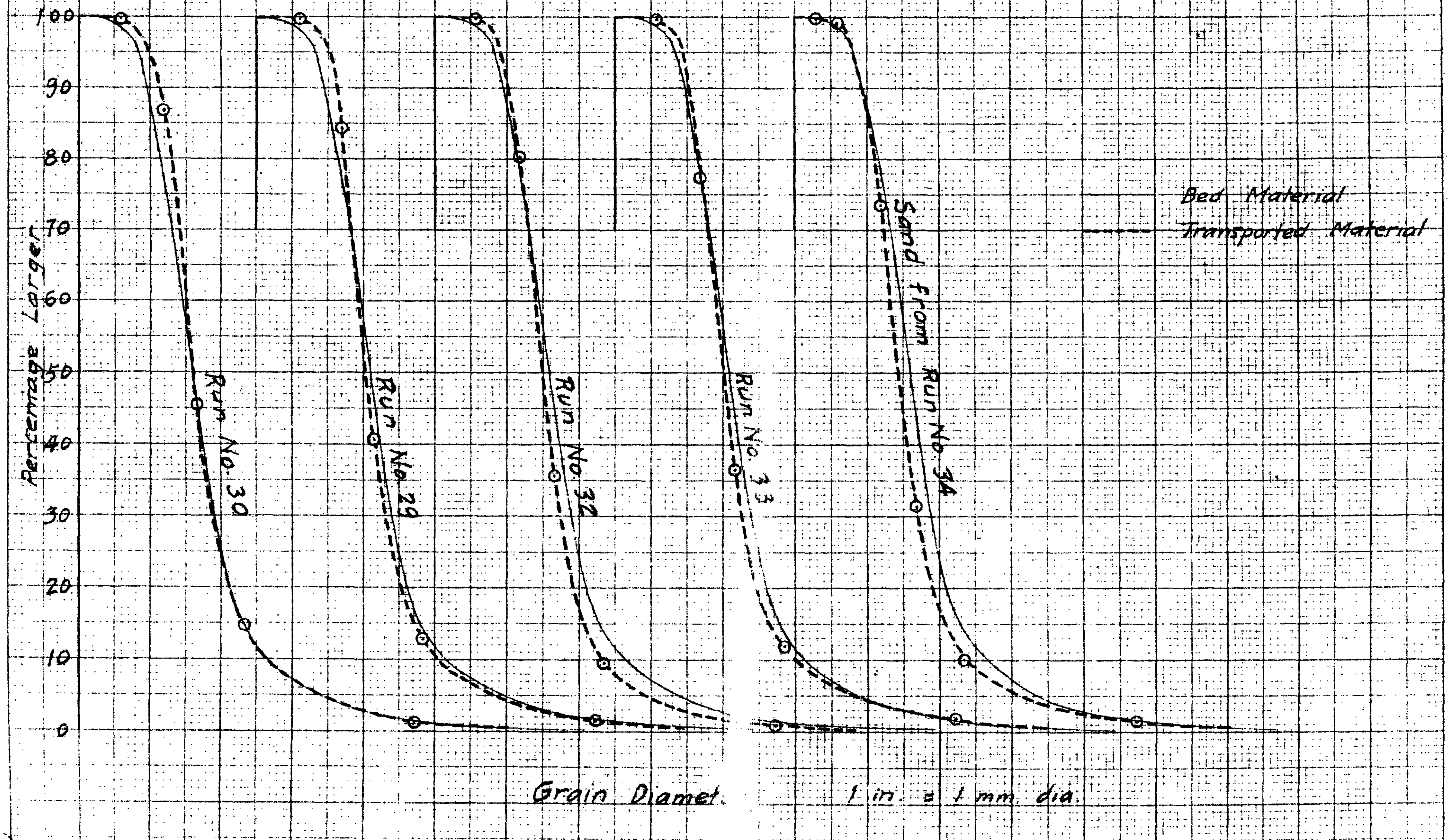

Sand Mixture No 3
g + 2.0 = 258,500 S^1.63
g + 2.0 = 0.0202 q
Av. S = 0.00167
Av. q = 257
g in gr/sec/cm
S
q in cm³/sec/cm
.001
.0015
.002
.0025
.003
.004
100
200
300
400
500
1000
2
3
4
5
10
20

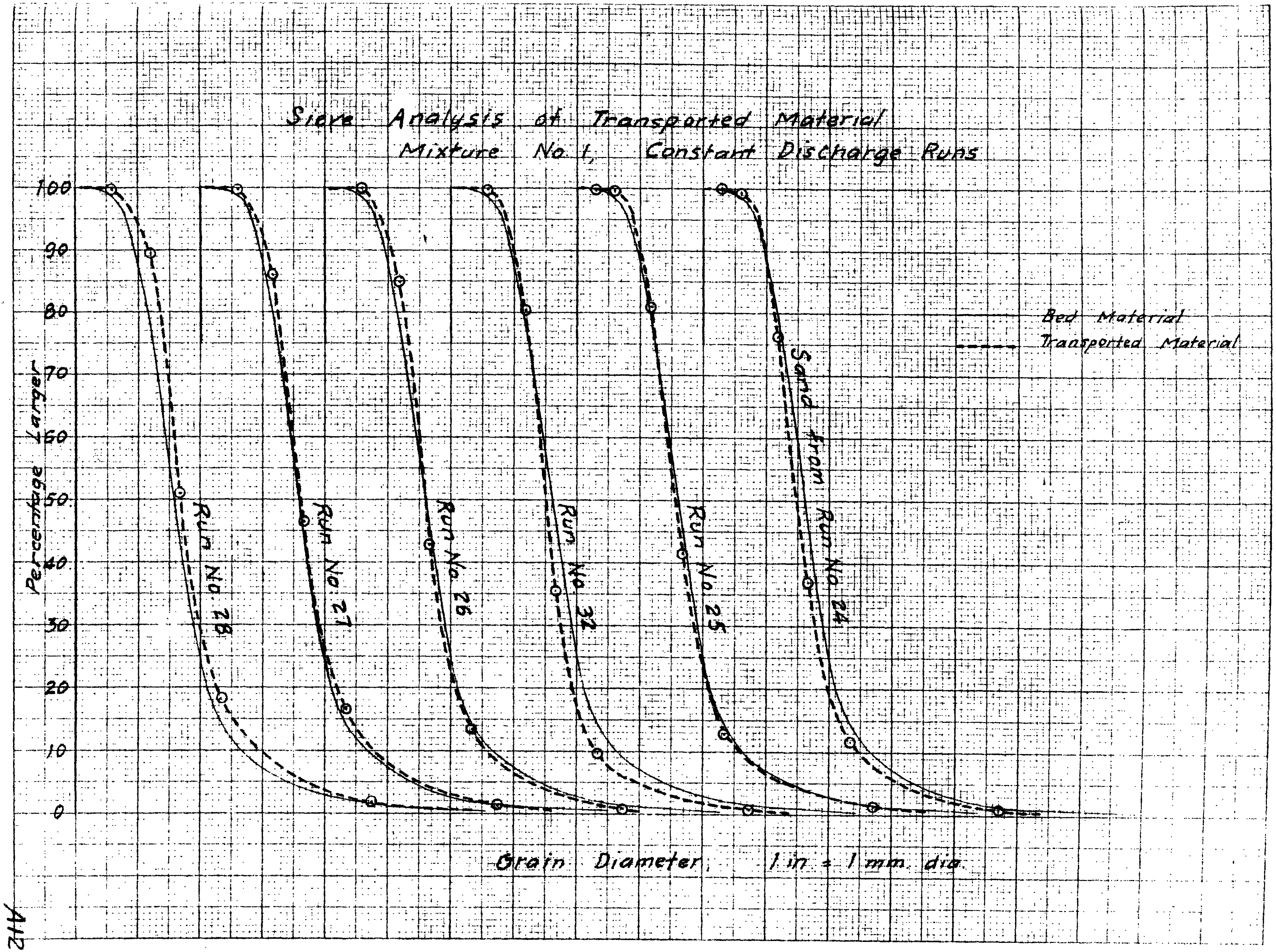
A9a

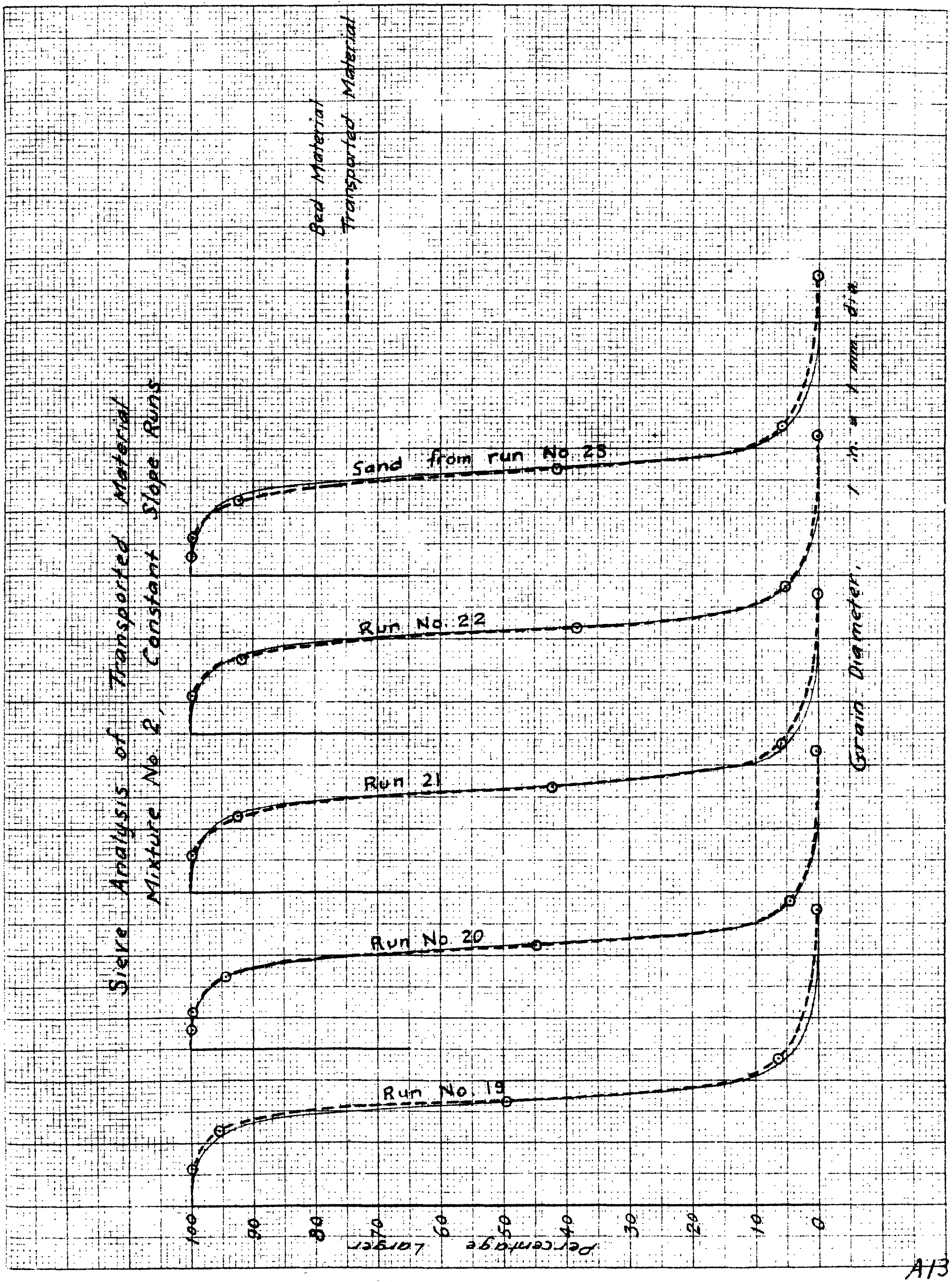


Sand Constants in $G = B \cdot k_1 (S^n q - k_2)$

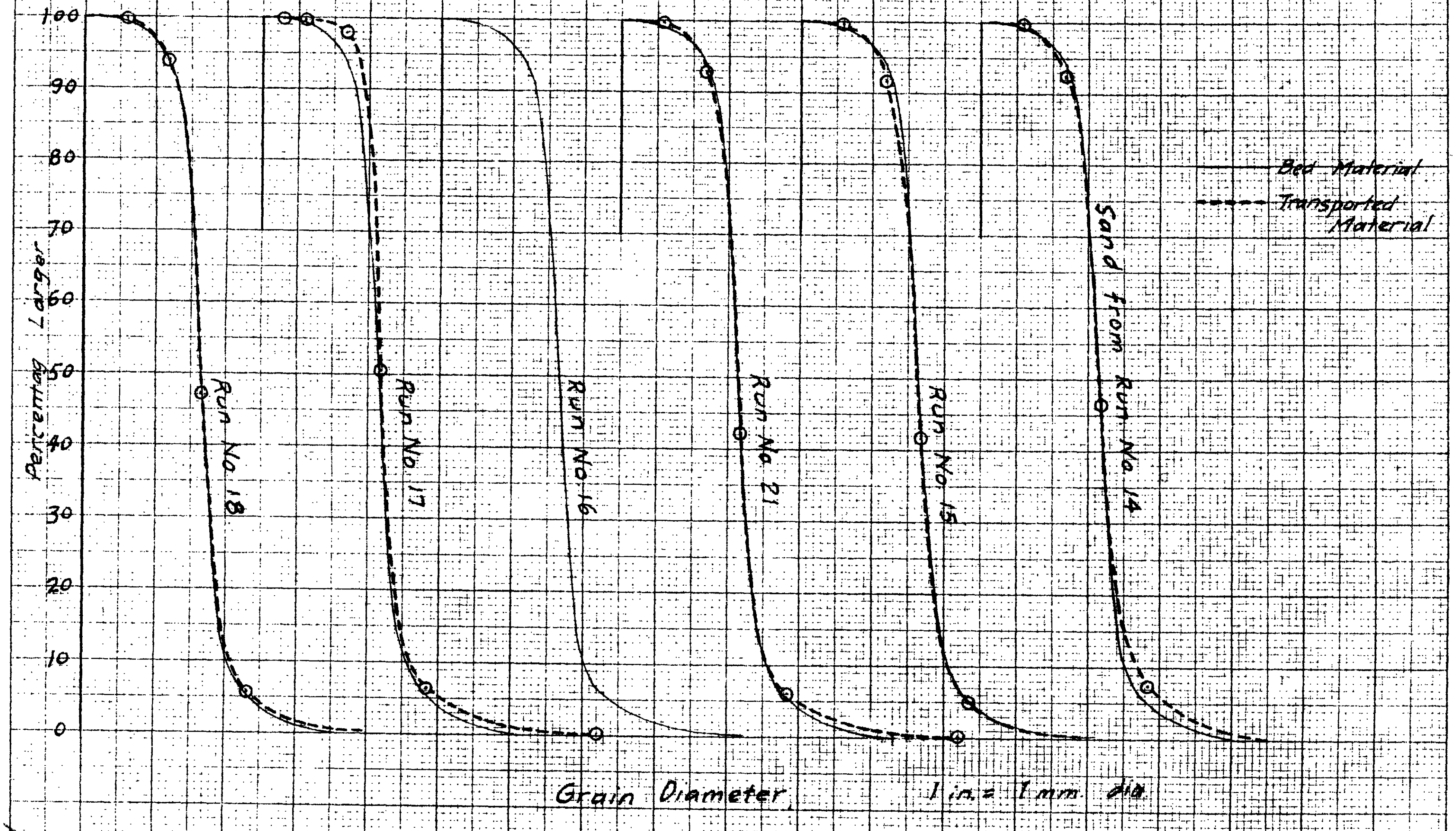
Sieve Analysis of Transported Material
Mixture No. 1, Constant Slope Runs



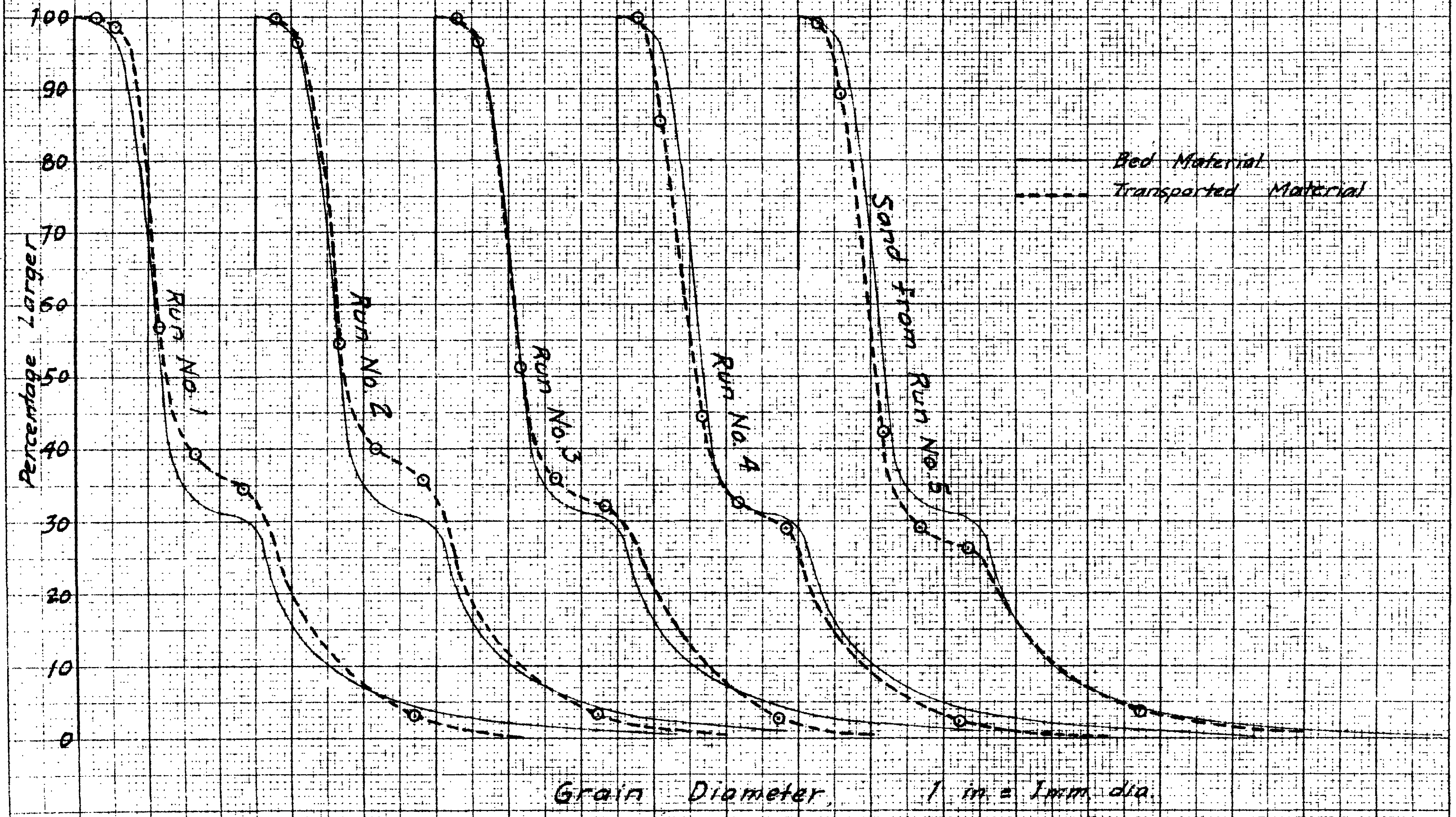

Sieve Analysis of Transported Material
Mixture No 1, Constant Discharge Runs

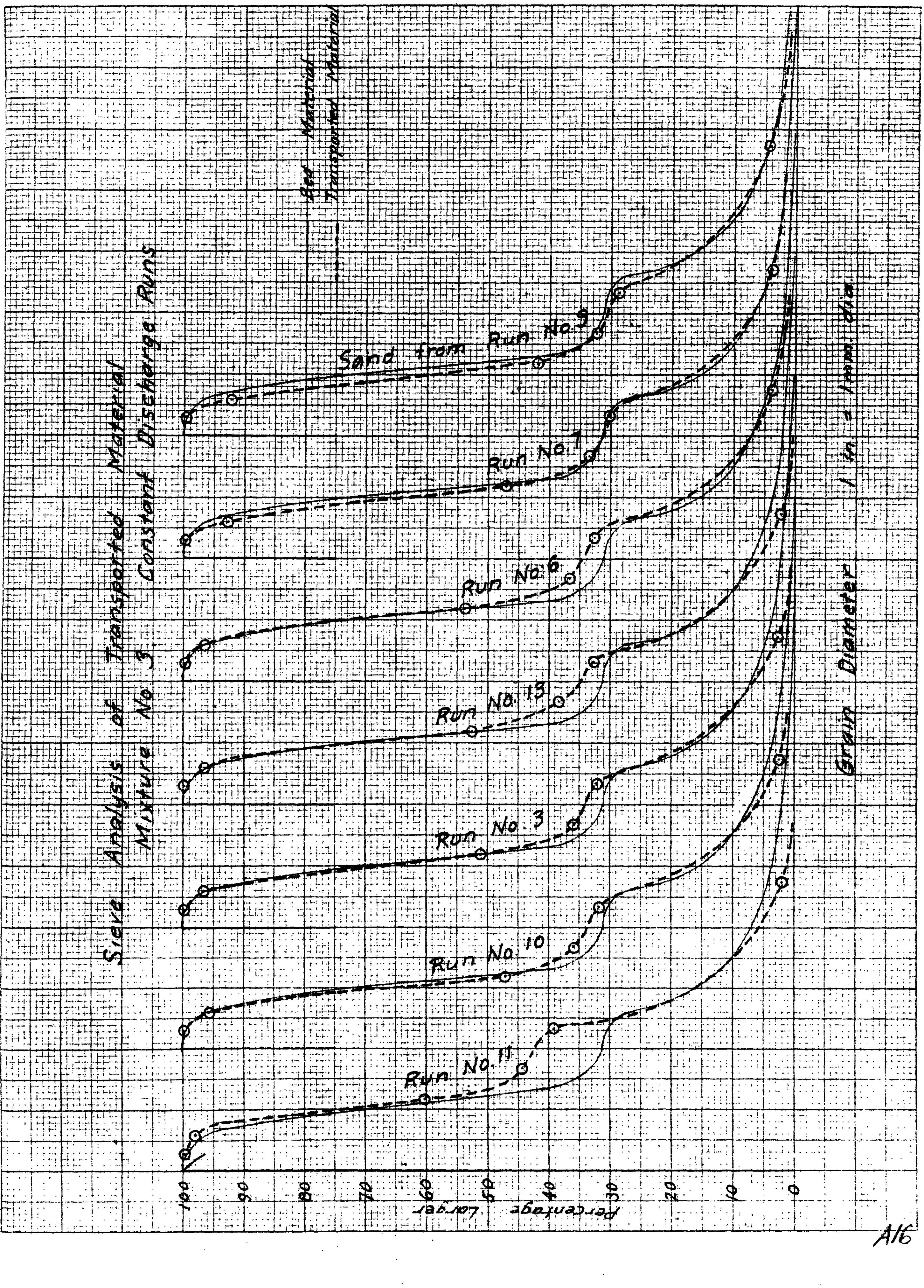

Sieve Analysis of Transported Material
Mixture No. 2, Constant Slope Runs
Bed Material
Transported Material
Sand from run No 23
Run No. 22
Run 21
Run No 20
Run No. 19
100
90
80
70
60
50
40
30
20
10
0
Percentage Larger
Grain Diameter, 1 in. = 1 mm. dia.

Sieve Analysis of Transported Material
Mixture No. 2, Constant Discharge Runs



Sieve Analysis of Transported Material
Mixture No. 3, Constant Slope Runs



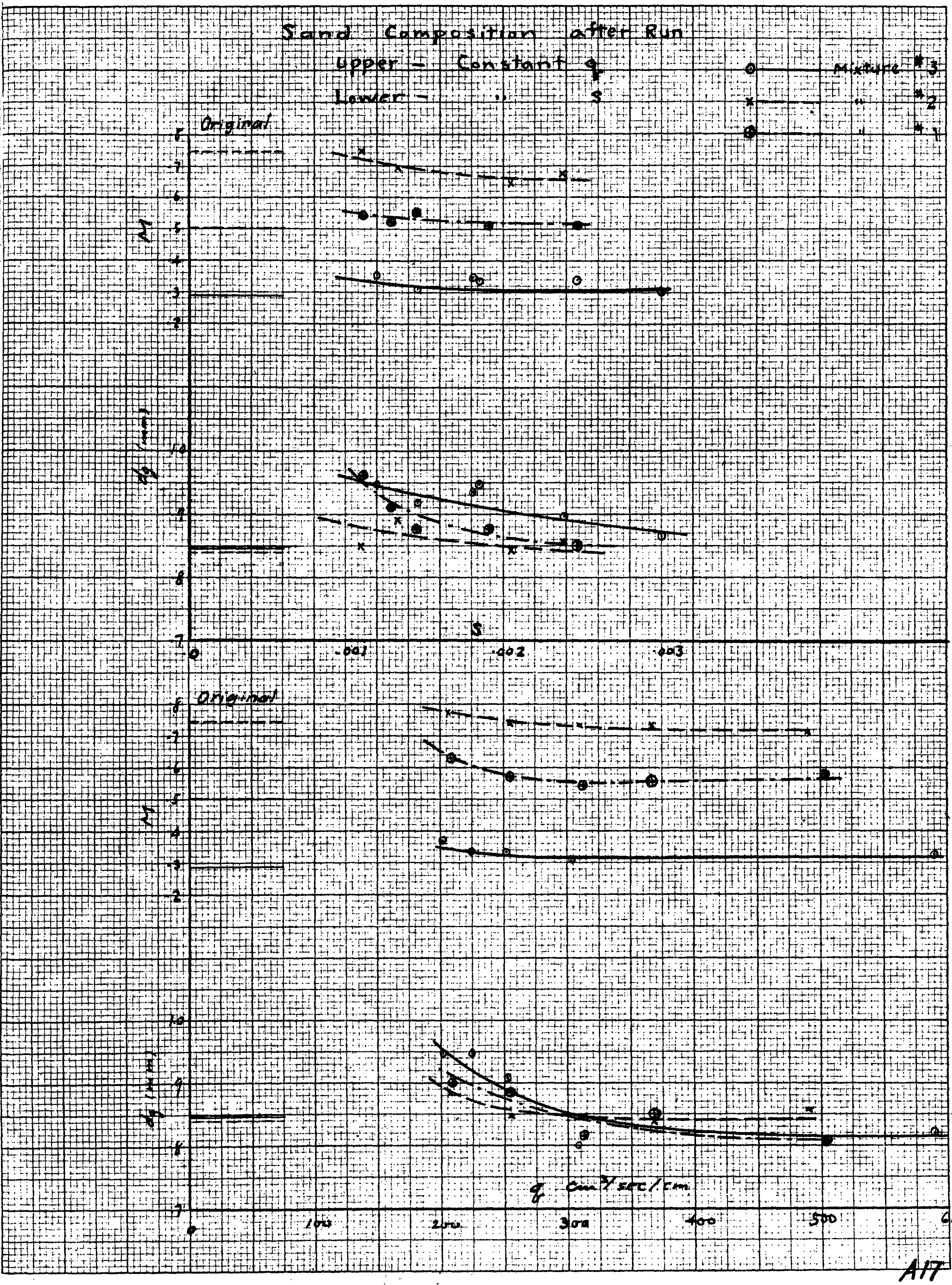


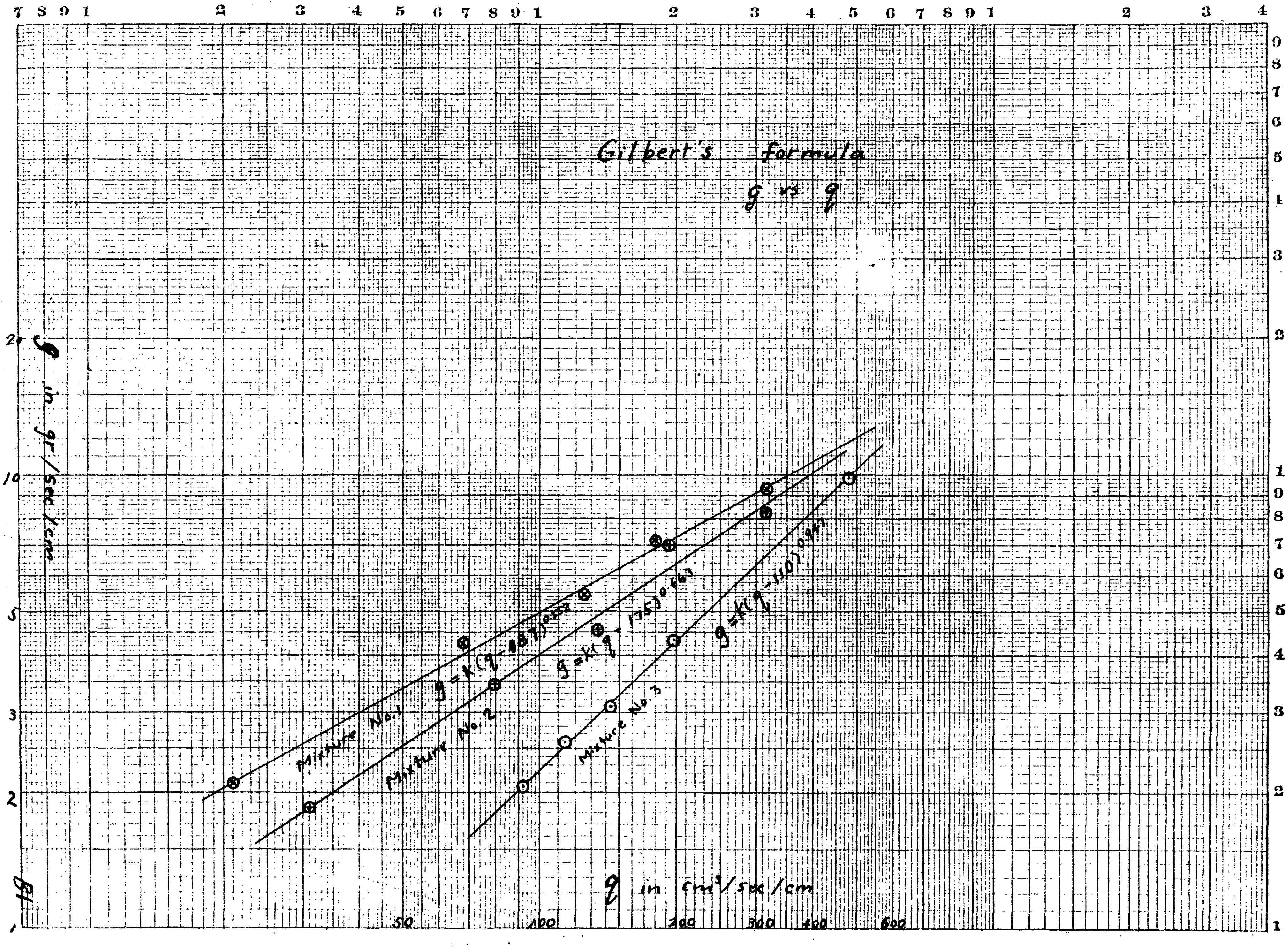
Sieve Analysis of Transported Material
Mixture No 3 Constant Discharge Runs

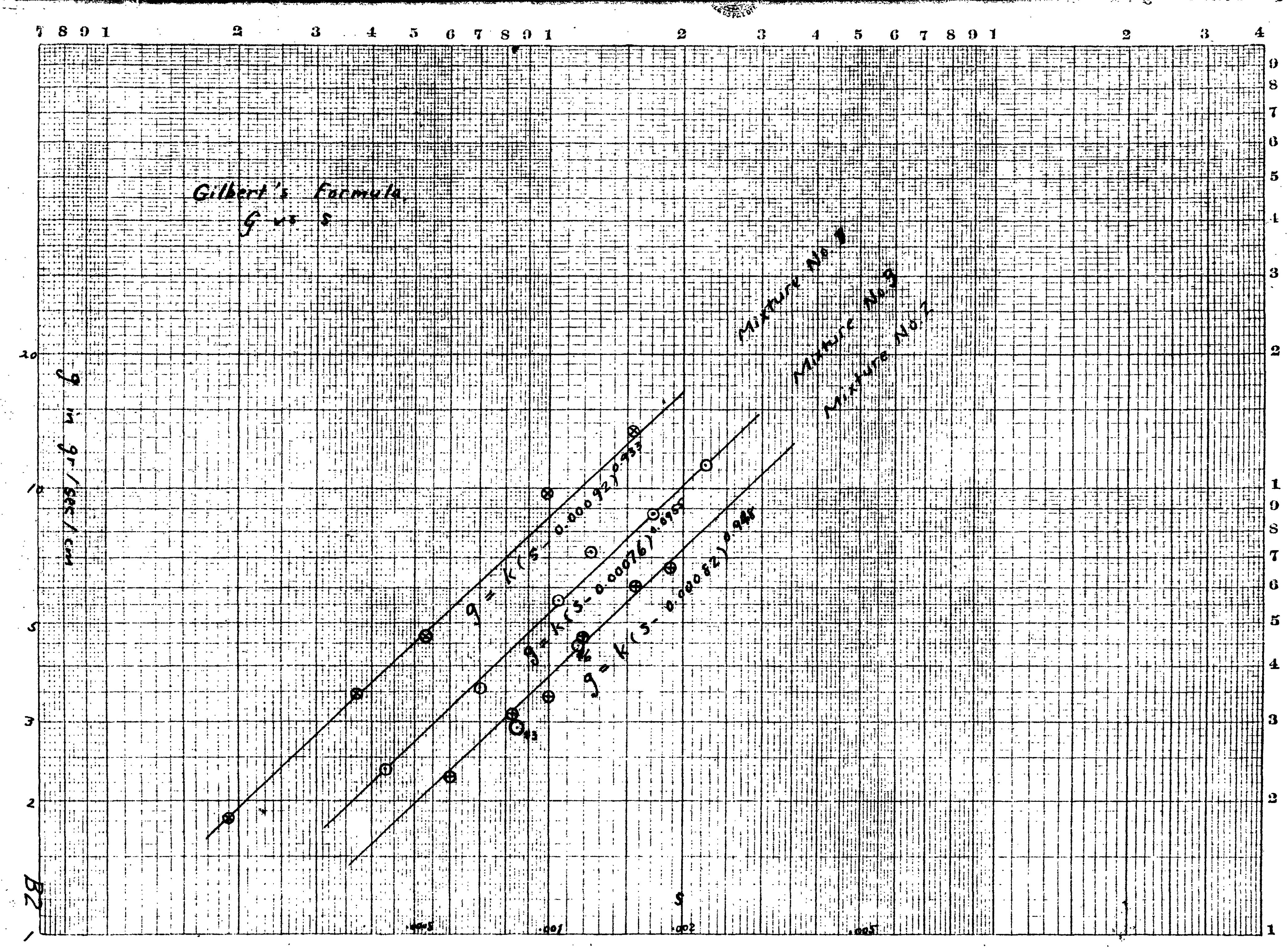
Sand Composition after Run

Upper – Constant q

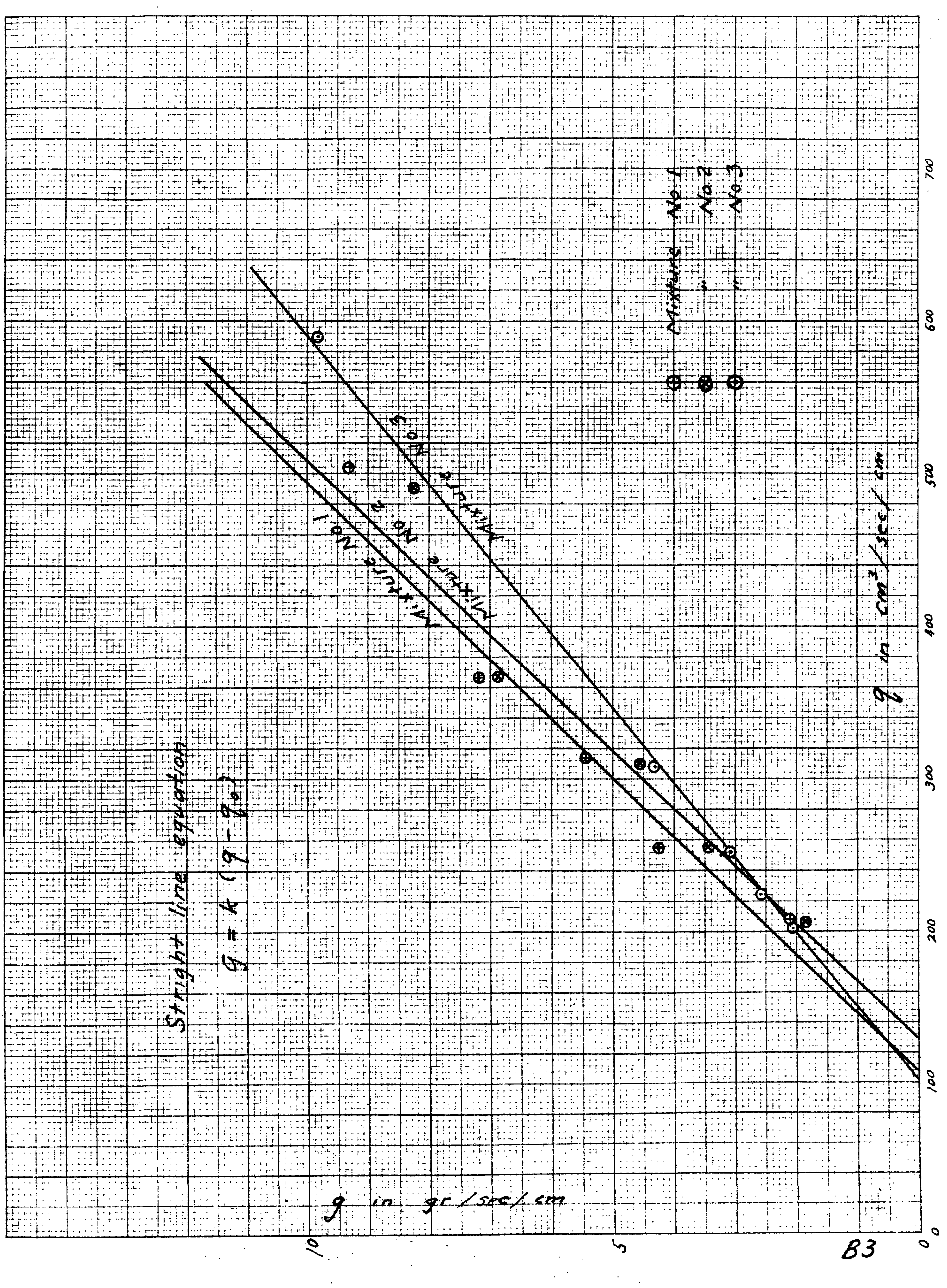
Lower – " S

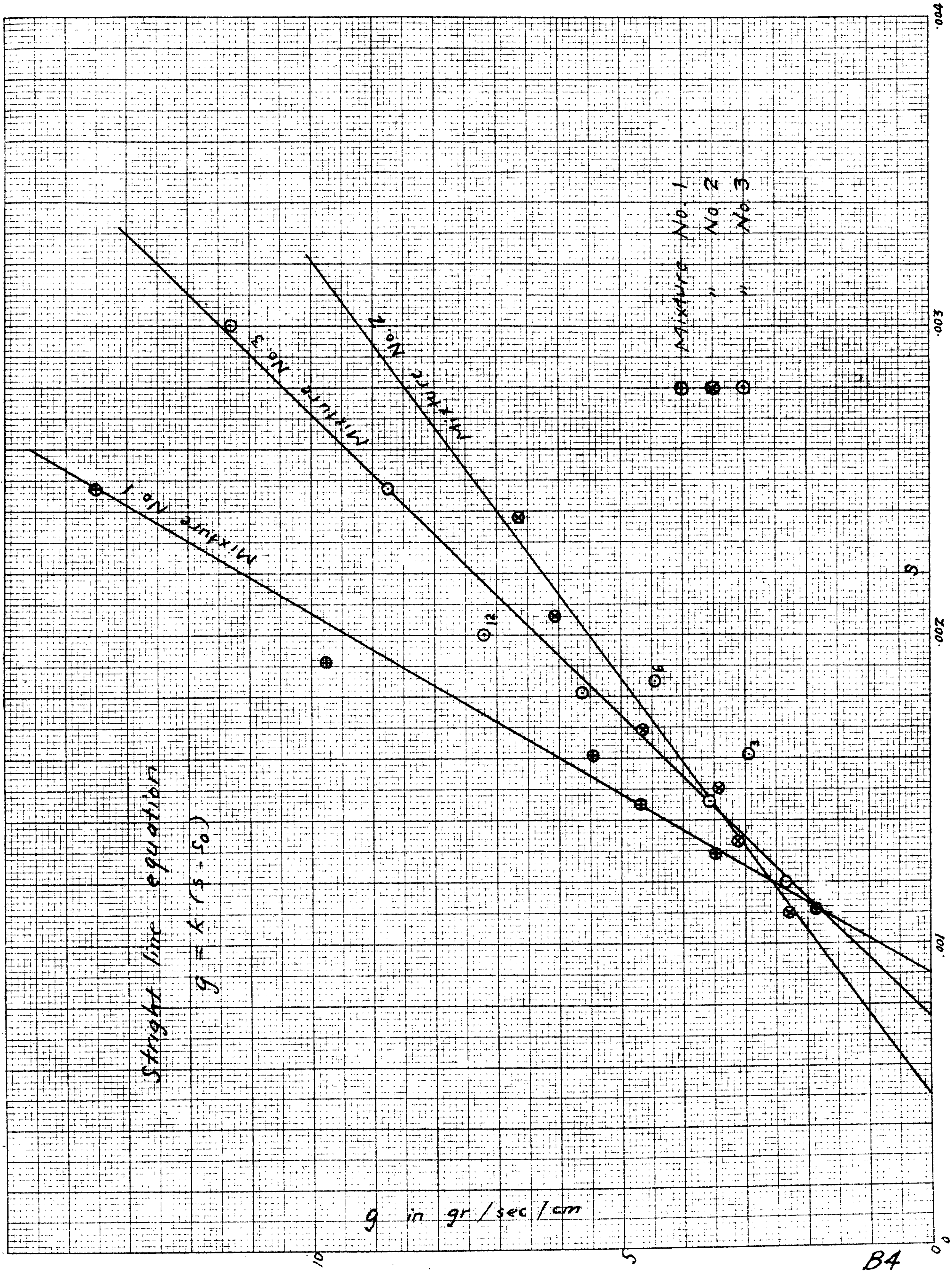


Gilbert's formula
g vs q
g in gr/sec/cm
q in cm³/sec/cm
Mixture No. 1
Mixture No. 2
Mixture No. 3
g = K(q − 87)^0.552
g = K(q − 175)^0.663
g = K(q − 110)^0.947
50
100
200
300
400
600
1
2
3
5
10
20
7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4
9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
B1

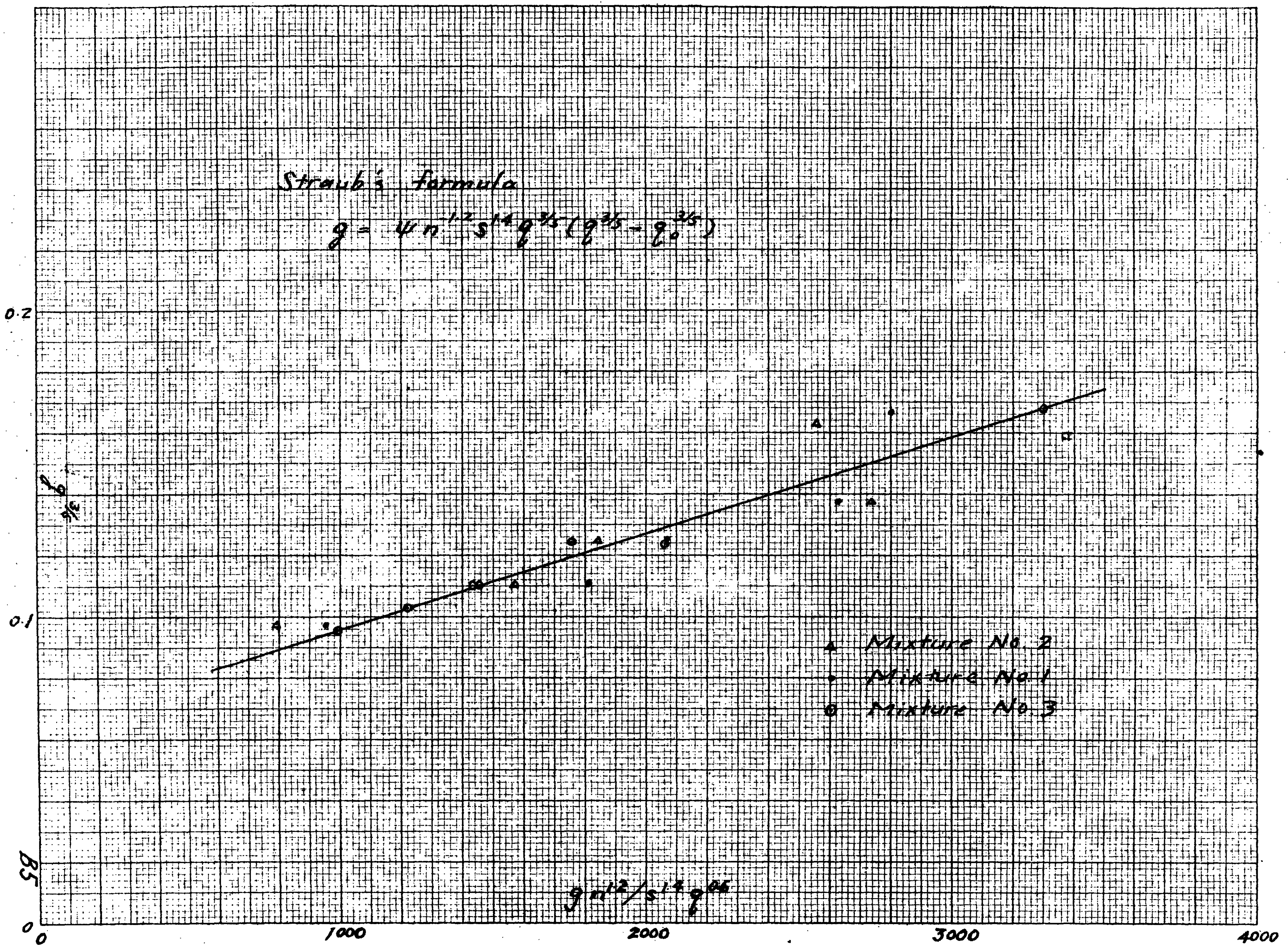

Gilbert's Formula
G vs S
Mixture No. 1
Mixture No. 3
Mixture No. 2
$g = K(S - 0.00092)^{0.933}$
$g = K(S - 0.00076)^{0.955}$
$g = K(S - 0.00082)^{0.948}$
46
43
g in gr/sec/cm
S
.0005
.001
.002
.005
1
2
3
5
10
20

B2

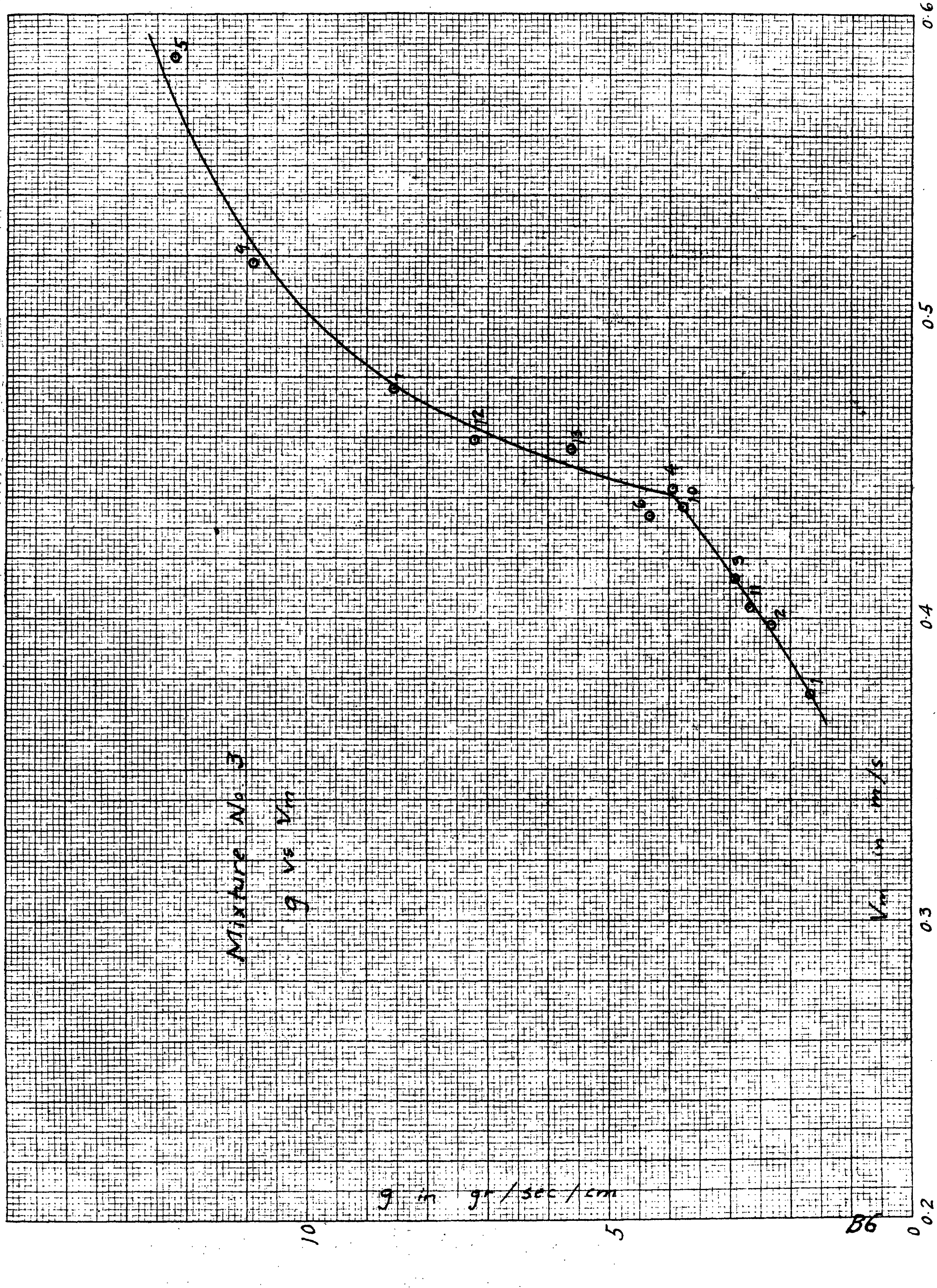

Stright line equation
$G = k\,(q - q_0)$
G in gr/sec/cm
q in cm³/sec/cm
0
5
10
0
100
200
300
400
500
600
700
Mixture No 1
Mixture No 2
Mixture No 3
⊕ Mixture No. 1
⊗ " No. 2
⊙ " No. 3

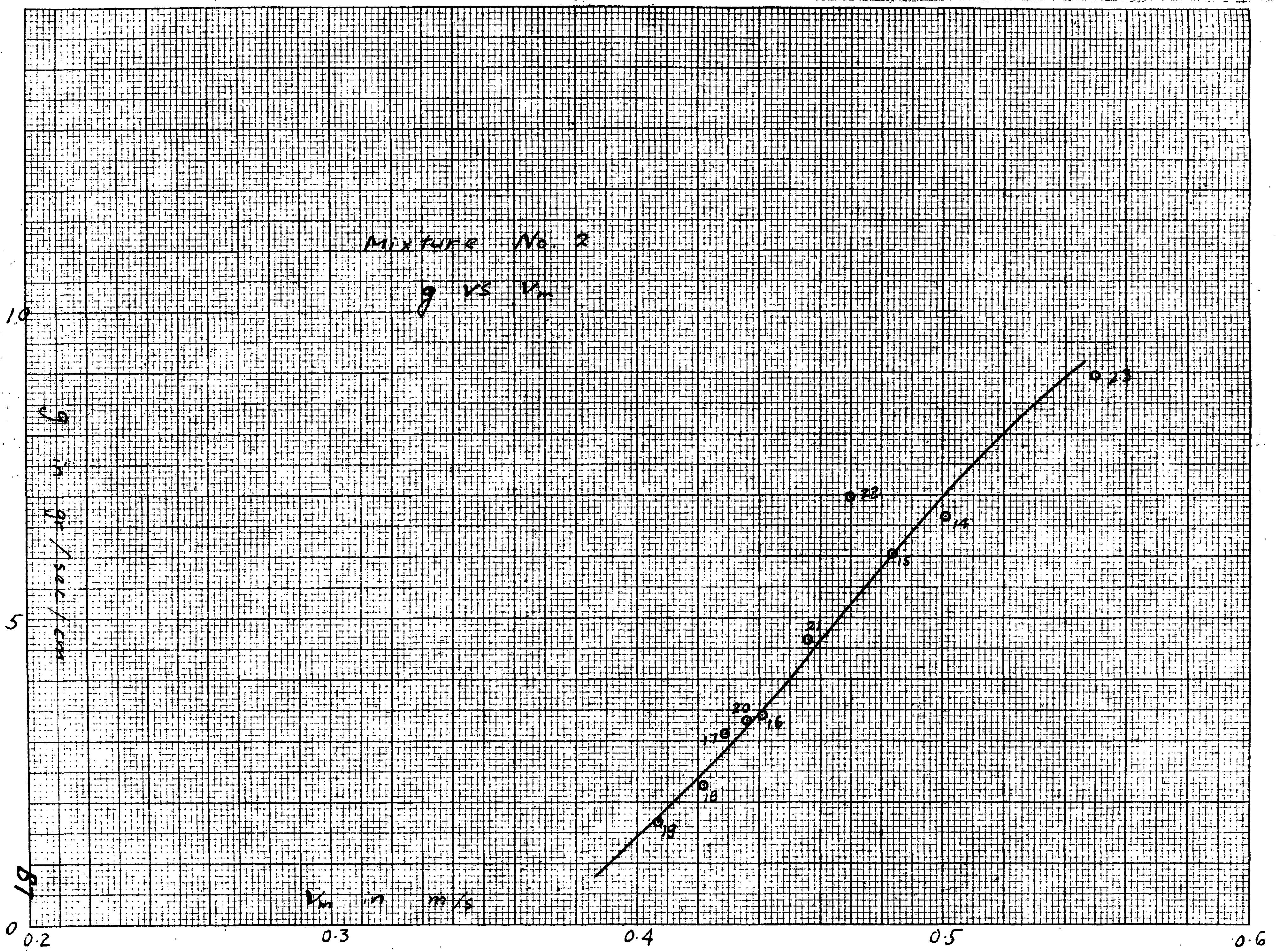

Stright line equation
$g = k(s - s_0)$
g in gr / sec / cm
s
0
5
10
0
.001
.002
.003
.004
Mixture No. 1
Mixture No. 2
Mixture No. 3
⊕ Mixture No. 1
⊗ " No. 2
⊙ " No. 3
12
6
3

B4


Straub's formula
$g = 4 n^{-1.2} s^{1.4} q^{3/5} (q^{3/5} - q_o^{3/5})$
0.2
0.1
$g^{3/5}$
0
0
1000
2000
3000
4000
$g n^{1.2} / s^{1.4} q^{0.6}$
▲ Mixture No. 2
● Mixture No. 1
⊙ Mixture No. 3

B5


Mixture No 3
g vs Vm
g in gr/sec/cm
Vm in m/s
0
0.2
0.3
0.4
0.5
0.6
5
10
1
2
3
4
5
6
7
9
10
11
12
13
B6


Mixture No. 2
g vs V_m
g in gr/sec/cm
V_m in m/s
0
5
10
0.2
0.3
0.4
0.5
0.6
14
15
16
17
18
19
20
21
22
23
BT