Least-Cost Ground Holding Strategies with Departure and Arrival Delay Uncertainties

by

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Abstract

Ground Holding Programs (GDP) were introduced in the early eighties to lower the air traffic controllers' workload when airports are congested and aircraft have to wait in the air before they can land. The underlying idea is that it is cheaper (i.e. lower operating costs for the airline) and safer to wait on the ground at the origin airport than wait in the air at the destination airport when both departure and arrival airports have uncertain capacities.

In the future free flow management environment flights subject to a GDP will be assigned, only a controlled time of arrival (CTA), i.e., target time by which each flight should arrive at the terminal airspace of its airport of destination. It will then be up to the airlines to determine the times at which their flights should leave the gates at their airports of origin, in order to meet the CTAs at the airport of destination. In other words, the FAA will no longer issue estimated departure clearance times or controlled times of departure for flights subject to GDPs.

The objective of this research is to assist the airlines in developing a methodology for determining for themselves the optimum gate departure time for a flight aiming at meeting a given CTA. The proposed Departure and Arrival stochastic model (D/A model) takes into account the two main sources of uncertainty namely departure and arrival delay uncertainties. This model builds on and improves considerably some earlier research that dealt only with the uncertainty about delay on arrival. A closed form solution for the optimum ground time is found without making any assumption on the distribution of delays for both the linear and nonlinear cost of delay cases. Finally, numerical computations comparing the performance of the different strategies at hand are presented and the proposed strategy outperforms all others for all cases.

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TABLE OF CONTENTS

Section **Page**

1. INTRODUCTION.....9

1.1 OBJECTIVES.....9

1.2 OVERVIEW OF THE METHODOLOGY.....11

1.3 OVERVIEW OF THE THESIS.....11

2. LITERATURE REVIEW.....13

2.1 STOCHASTIC GROUND DELAY PROBLEM RESEARCH EFFORT.....13

2.2 ANDREWS' STOCHASTIC ARRIVAL DELAY MODEL.....14

3. LINEAR DEPARTURE AND ARRIVAL STOCHASTIC MODEL.....16

3.1 INTRODUCTION.....16

3.2 MODEL FORMULATION.....17

3.3 OPTIMALITY CONDITIONS FOR THE GENERAL CASE.....24

3.4 CASE OF GAUSSIAN DELAY DISTRIBUTIONS.....27

3.4.1 Introduction.....27

3.4.2 Model Formulation and Optimality Conditions.....28

3.4.3 Sensitivity Analysis and Model Comparison.....30

3.5 CASE OF UNIFORM DELAY DISTRIBUTIONS.....38

3.5.1 Introduction.....38

3.5.2 Model Formulation and Optimality Conditions.....39

3.5.3 Sensitivity Analysis and Model Comparison.....45

4. NONLINEAR DEPARTURE AND ARRIVAL STOCHASTIC MODEL.....53

4.1 INTRODUCTION.....53

4.2 MODEL FORMULATION.....54

4.3 OPTIMAL CONDITIONS FOR THE GENERAL CASE.....57

TABLE OF CONTENTS (Continued)

Section **Page**

4.4 CASE OF GAUSSIAN DELAY DISTRIBUTIONS.....59

4.4.1 Introduction.....59

4.4.2 Sensitivity Analysis and Model Comparison.....59

5. CONCLUSIONS AND RECOMMENDATIONS.....65

5.1 CONCLUSIONS AND DISCUSSION.....65

5.2 RECOMMENDATIONS FOR FUTURE RESEARCH.....67

5.2.1 Explore Alternative Cost Minimization Strategies.....67

5.2.2 Incorporate Gaming Component in Model Formulation.....67

5.2.3 Study the Dynamic Case.....68

References.....69

LIST OF TABLES

Table	Page
Model Implementation: First Scenario	31
Model Implementation: Second Scenario	32

LIST OF FIGURES

Figure	Page
3.1 Model of Scheduling Process for the Passive Strategy	21
3.2 Model of Scheduling Process for the D/A Strategy	22
3.3 Linear Gaussian Case: Cost Savings vs. Marginal Cost Ratio	34
3.4 Linear Gaussian Case: Cost Savings vs. Departure Uncertainty	35
3.5 Linear Gaussian Case: Cost Savings vs. Arrival Uncertainty	36
3.6 Linear Gaussian Case: Expected Costs vs. Arrival Uncertainty	37
3.7 Joint Cumulative Distribution Function	39
3.8 Area Over Which the Integral is Calculated	41
3.9 & 3.10 Linear Uniform Case: Cost Index vs. Marginal Cost Ratio	46 & 47
3.11 Linear Uniform Case: Ground Holding vs. Marginal Cost Ratio	48
3.12 & 3.13 Linear Uniform Case: Average Cost vs. Arrival Uncertainty	51 & 52
4.1 Nonlinear delay Cost Function	54
4.2 Nonlinear Gaussian Case: Cost Index vs. Incremental Slope α	60
4.3 Nonlinear Gaussian Case: Cost Index vs. Airborne Delay Threshold B	62
4.4 & 4.5 Nonlinear Gaussian Case: Cost Index vs. Marginal Cost Ratio	63 & 64

CHAPTER 1

INTRODUCTION

1.1 Objectives

One of the stated (medium-term) objectives of Traffic Flow Management is to give more freedom to the airlines in determining the amount of airborne delay and ground delay to be assigned to a flight delayed due to flow management actions. In the future flow management environment, flights subject to GDP will be assigned, via CDM, only a controlled time of arrival (CTA), i.e., a target time by which each flight should arrive at the terminal airspace of its airport of destination. It will then be up to the airlines to determine the times at which their flights should leave the gates at their airports of origin, in order to meet the CTAs at the airport of destination (the GDP airport). In other words, the FAA will no longer issue estimated departure clearance times (EDCTs) or controlled times of departure (CTDs) for flights subject to GDPs.

The objective of this thesis is to assist the airlines in developing a methodology for determining for themselves the optimum gate departure time for a flight aiming at meeting a given CTA. Note that this time must be determined in the face of two principal sources of uncertainty:

(a) Uncertainty about how much delay the flight will experience in departing from the airport of origin, due to the fact that many airports are congested and it is very difficult to estimate in advance exactly how much taxi-out time will be needed to reach the departure runway and how much queuing there will be for take-off at the time of departure.

(b) Uncertainty about how much delay the flight will experience after arriving at the terminal airspace of the airport of destination, due to the fact that it is really

impossible to predict accurately several hours in advance, how much landing delay a flight will suffer at a congested airport.

There are other sources of uncertainty, such as en route travel times, but they are typically dominated, in terms of magnitude, by the uncertainty about the delay on departure and the delay for landing.

In summary, for each flight the airlines will face the following decision problem: how much airborne delay and how much ground delay to plan for, in the face of uncertainty about delay on departure and about delay on arrival.

This thesis presents a model for determining the optimal ground holding strategy when departure and arrival airports are subject to stochastic delays. This model is intended to help understand the strategies that airlines may use in order to strike a balance between the amount of delay absorbed on the ground (ground delay) and the amount absorbed in the air (airborne delay).

The model presented may also serve as a first step in developing relevant decision support tools for airlines to make tactical decisions on ground holding. The following four points summarize the main contributions of this thesis:

- Model the uncertainty in both the departure and arrival delays in order to determine the optimal ground holding time that would minimize the total expected cost of delays.
- Test the sensitivity of the model output to changes in delay characteristics (i.e. means, standard deviations), as well as changes in the marginal costs of ground and airborne delays.
- Compare the performance of alternative ground holding strategies.
- Quantify the benefits of having better (i.e. more accurate) delay prediction systems.

1.2 Overview of the Methodology

The model takes advantage of the First Scheduled First Served (FSFS) scheduling policy that would most probably be adopted under the "Free Flight" environment. This scheduling policy makes it possible to perform an analysis for each flight separately. Unlike the First Come First Served (FCFS) policy, FSFS preserves the original takeoff and landing sequence of aircraft even when there are delays at both the departure and arrival airports. The proposed model determines the ground holding time that minimizes the expected total cost of delays. We note that, in practice, there will undoubtedly be deviations from FSFS, but the strategies derived through our model may still be near optimal as long as FSFS is adhered to in broad terms.

Departure and arrival delays are modeled as static[1] random variables. It is assumed that the probability density functions (pdfs) of these random variables are given inputs. Marginal costs of ground and airborne delays (i.e. cost of one minute of delay) are also assumed known to airlines and are therefore exogenous parameters to the model.

A first model will be presented where the total cost is assumed to be linear as a function of delay, meaning that the marginal disutility of being delayed on the ground or in the air is assumed to be constant regardless of the amount of delay. A second model where the restriction on linearity is relaxed for airborne delays will also be presented. In this case the nonlinearity will be modeled using a piecewise approximation.

1.3 Overview of the Thesis

In Chapter 2, a brief review of current research on stochastic ground holding problems is provided. A brief description of Andrews' model (A model), which provided the base for the proposed model (D/A model), is also presented. In Chapter 3, the linear

[1] The dynamic case where delay random variables are a function of time is a more accurate representation of reality but was not studied as part of this thesis and is mentioned amongst the recommendations for future research.

case for the D/A model will be presented including a general result on optimality along with implementations for the cases where the probability distributions for the arrival and departure delays are either Gaussian or uniform. In Chapter 4, the nonlinear case will be presented including a general result on optimality along with an implementation for the specific case of Gaussian probability distributions. Finally, Chapter 5 provides conclusions and recommendations for future research.

CHAPTER 2

LITTERATURE REVIEW

2.1 Stochastic Ground Delay Problem Research Effort

Most of the research that has been undertaken on ground delay programs when departure and arrival airports are subject to uncertain capacities has dealt with scheduling ensembles of aircraft and adopted a mathematical programming approach. Uncertainty in airport capacities has been captured through the use of a limited number of capacity scenarios that were assigned probabilities (i.e. likelihood of occurring). The expected cost of delay for the entire ensemble of aircraft is then minimized subject to a set of constraints. Most of this research has focused on capacitated arrival airports without taking into account departure airport congestion.

Most of this research was initiated by the seminal paper on the static stochastic holding problem "Solving Optimally the Static ground-holding Policy Problem in Air Traffic Control", by Richetta and Odoni [4]. This paper deals with stochasticity by assuming that a probabilistic distribution of "scenarios", or possible realizations of capacity, is known it then minimizes the expected cost of delays. A subsequent paper "Dynamic Ground-Holding Policies for a Network of Airports" by Vranas, Bertsimas and Odoni [10] has extended the previous model to a network of airports. Finally, a paper "The Air Traffic Flow Management Problem with Enroute Capacities" by Bertsimas and Stock [2] has included enroute capacities in modeling the ground holding problem.

In his paper "Impact of weather event uncertainty upon an optimum ground-holding strategy" Andrews [1] has introduced a purely probabilistic modeling approach to the ground holding problem. This model assumes a FSFS scheduling policy, which allows it to model delays for individual aircraft without taking into account the remaining aircraft and thus has reduced considerably the number of variables required to model delays for a

single aircraft. The A model assumes that departure delay is deterministic and equal to zero and the proposed D/A model is an extension to it taking into account departure uncertainty. The following section will give a brief description of the A model as background to the presentation of the D/A model in Chapter 3.

2.2 Andrews' Stochastic Arrival Delay Model

The Andrews model (A model) assumes a FSFS scheduling policy which means that a decision to hold an individual aircraft on the ground converts airborne delay to ground delay without loss of landing sequence position. The delay cost for this model is computed as the sum of ground delay and airborne delay:

$$C = C_g \times d_g + C_a \times D_a \qquad (2.1)$$

where C is the total cost, C_g and C_a are respectively the marginal costs of ground and airborne delays, and d_g and D_a are respectively ground delay (deterministic decision variable) and airborne delay random variable (R.V.). This model assumes that the decision-maker is given the probability distribution for the "slot delay" D[1] (R.V.) and is asked to impose the ground delay time that will minimize the total expected cost of delays.

The total cost that is incurred for a given ground delay can then be written as follows:

$$C = \begin{cases} C_g \times d_g & \text{if} \quad D \leq d_g \qquad (2.2\text{ a}) \\ C_g \times d_g + C_a \times (D - d_g) & \text{if} \quad d_g \leq D \qquad (2.2\text{ b}) \end{cases}$$

[1] Corresponds to the difference between an aircraft's initially scheduled arrival time and the time when the aircraft will actually be able to land.

If D is assumed to have a Gaussian distribution, we obtain the following optimality condition for the ground holding time:

$$P\left(D \geq d_g^*\right) = \frac{C_g}{C_a} \quad with \quad D \sim N(\mu, \sigma)$$

Therefore, the optimal ground holding time is given by:

$$d_g^* = \max\left\{0, \mu - \sigma \times F_Y^{-1}\left(\frac{C_g}{C_a}\right)\right\}$$

where $Y \sim N(0,1)$ and F_Y is the cumulative distribution function (cdf) of Y.

CHAPTER 3

LINEAR DEPARTURE AND ARRIVAL STOCHASTIC MODEL

3.1 Introduction

We have summarized in the previous chapter the ground holding problem developed by Andrews, where the departure delay is assumed to be deterministic and equal to zero and the delay cost function linear. This model could apply to flights departing from an airport that has good weather conditions during the time period over which the decision on ground holding is made. This may make reasonable the deterministic assumption on departure delays. However, in real life situations and given the accuracy of existing weather forecast systems very few US airports would always fall under this category. It is therefore important to consider the case in which departure delays are considered as uncertain.

This chapter presents the Linear Departure and Arrival Stochastic Model (D/A model), which is an extension of the A model. The D/A model takes into account both the uncertainty on both the departure and arrival delays. The D/A model, like the A model, assumes that the delay cost function is linear. It also assumes that delay random variables are static[1] (i.e., the probability distributions of delay random variables are assumed to be constant during the time period over which the decision on ground holding is made). Furthermore, this model assumes that aircraft takeoffs and landings follow a FSFS sequencing policy for both takeoff and landing. As in the case of the A model, this assumption allows delays for an aircraft to be modeled independently of other aircraft and therefore reduces considerably the number of variables and parameters required to model delays.

[1] This assumption could be relaxed by reducing the length of the time period over which the decision on ground holding is made and thus allow the delay random variables to vary (i.e. different pdfs with different means and standard errors could be considered every time a decision on ground holding is made).

We will start by giving the mathematical formulation for the D/A model without making any assumption on the specific pdf of the delay random variables. A general result on optimality conditions for the ground holding decision variable will be proven and the model will be implemented with both Gaussian and uniform delay random variables. Finally, three strategies (i.e., passive, A, and D/A) will be compared as part of the numerical analysis to gain some insights on their relative performance.

3.2 Model Formulation

We will call the departure and arrival airports D and A, respectively, and use these letters to index the different variables and parameters associated with these two airports. In formulating the D/A model, we define the departure delay D_d as the total departure delay (i.e. difference between the flight's original departure time and its actual departure time), whereas the arrival delay D_a is the delay in the originally scheduled landing time (i.e. the difference between the flight's original landing time and the earliest time when its landing slot can be served).

Figure 3.1 provides a graphical illustration of the scheduling process for the case where no ground holding is used (i.e. passive strategy). In this figure, X is a R.V. representing the unavoidable delay in the departure time associated with congestion at airport D. For this strategy, X is equal to the departure delay D_d since no additional ground holding d_g is imposed on the aircraft. Then the airborne delay faced by the aircraft is given by the following equations:

$$\text{Airborne Delay} = \begin{cases} 0 & \text{if} \quad D_a \leq X \qquad (3.1\text{ a}) \\ D_a - X & \text{if} \quad X \leq D_a \qquad (3.1\text{ b}) \end{cases}$$

Equation (3.1a) means that when the departure delay is larger than the delay in the landing slot the flight does not need to wait in the air since it reaches airport A after its

landing slot can be served and the FSFS policy allows it to land right away. Equation (3.1b) corresponds to the case where the flight reaches airport A before its landing slot can be served and therefore it has to wait in the air until it can land.

Figure 3.2 provides a graphical illustration of the scheduling process for the case of the D/A strategy where additional ground holding d_g is imposed in order to absorb part of the expected airborne delay on the ground with the objective of minimizing the total expected cost of delays. On the departure side, the decision-maker assigns a rescheduled departure time to the flight corresponding to its original departure time plus some positive ground holding d_g. However, this does not mean that the flight will be able to depart at its rescheduled departure time and some additional unavoidable delay X is taken. On the arrival side, things work the same way as for the passive strategy.

The previous discussion allows us to calculate the total ground and airborne delays as follows:

Departure delay:

$$D_d = d_g + X \qquad (3.2)$$

$$\text{Airborne delay} = \begin{cases} 0 & \text{if} \quad D_a \leq D_d \qquad (3.3a) \\ D_a - D_d & \text{if} \quad D_d \leq D_a \qquad (3.3b) \end{cases}$$

It should be noted that for the static case, X is the same for both the D/A and passive strategies which is of course not the case for a dynamic situation where X would vary with time. Furthermore, when X is set equal to zero with probability one, the D/A model corresponds to the A Model.

(3.3a) corresponds to the case where either too much ground holding d_g was taken or too much ground delay X in the rescheduled departure time was experienced, and the aircraft arrived after its rescheduled landing time and was therefore able to land right

away. (3.3b) corresponds to the case where the total ground delay was less than the delay in the rescheduled landing time and therefore the flight had to wait in the air at the arrival airport A.

Figure 3.1: Model of Scheduling Process for the Passive Strategy

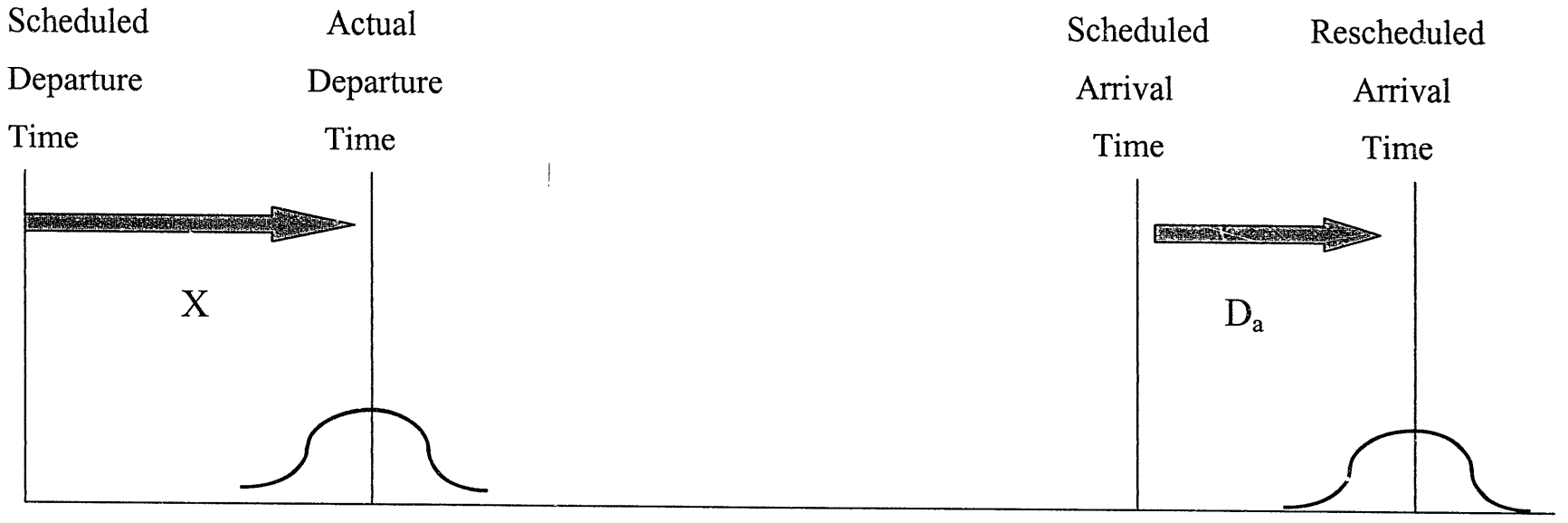
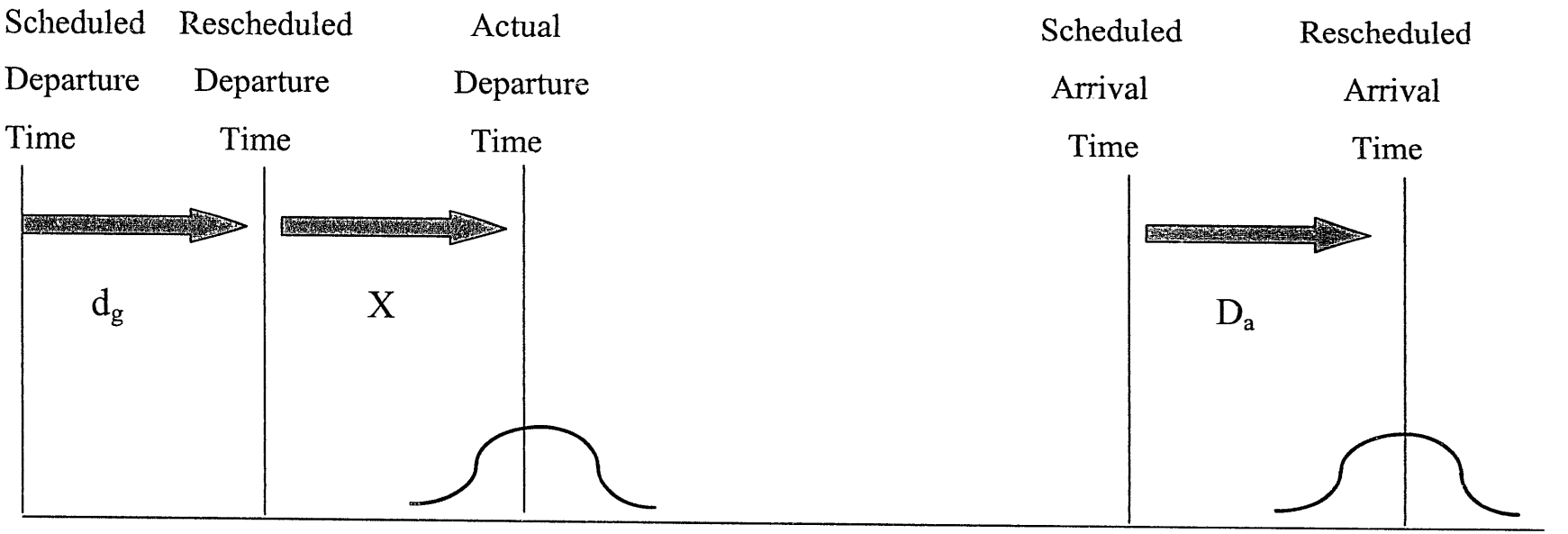


Figure 3.2: Model of Scheduling Process with the D/A Strategy



The total cost of delays is then given by the following equation:

$$\text{Total Delay Cost } C = \begin{cases} C_g \times D_d & \text{if } D_a \leq D_d \quad (3.4\text{ a}) \\ C_g \times D_d + C_a \times (D_a - D_d) & \text{if } D_d \leq D_a \quad (3.4\text{ b}) \end{cases}$$

where C_g and C_a are respectively the marginal costs (i.e., cost of one minute of delay) for ground and airborne delays. This total delay cost can be broken down into ground and airborne costs as follows:

$$\text{Ground Cost} = C_g \times D_d \qquad (3.5)$$

$$\text{Airborne Cost} = \begin{cases} 0 & \text{if } D_a \leq D_d \quad (3.6\text{ a}) \\ C_a \times (D_a - D_d) & \text{if } D_d \leq D_a \quad (3.6\text{ b}) \end{cases}$$

Under a Collaborative Decision Making (CDM) environment airlines will have to decide how much ground holding they want to impose on flights. Different airlines may follow different strategies in terms of what to minimize. A reasonable strategy could be to minimize the expected cost of delays. The A and D/A models focus on the minimization of this objective function. Another strategy could be to ignore costs and simply minimize the total expected delay time. In this latter case, it can be easily shown that the optimum corresponds to the no ground holding strategy. In fact, under a static delay assumption, if the pilot's objective is to take the least amount of total expected delay (ground and airborne), the sooner he takes-off the better.

Throughout this thesis it will be assumed that the objective is to minimize the expected cost of delay. Using (3.4), (3.5) and (3.6) this expected cost is given by the following formula:

$$E[C] = E[\text{Ground Cost}] + E[\text{Airborne Cost}] \qquad (3.7)$$

The objective of the D/A model is to find the value of d_g that minimizes (3.7) subject to the non-negativity constraint on d_g.

We can observe that the objective function has two parts, a linear one that corresponds to the expected ground delay cost and a nonlinear part corresponding to the expected airborne delay cost. The nonlinearity in the latter makes the optimization quite intricate. However the next section will derive some interesting properties regarding this function which will allow us to formulate a general (i.e., no assumption on the delay distributions) optimality result.

3.3 Optimality Conditions for the General Case

In this section we shall derive necessary and sufficient optimality conditions for the minimization problem described above. In order to do that we need to first derive some results on random variables.

Lemma 1: *Let V be a random variable that has a pdf and a cdf F and let us define the functions $W_h(x)$ and $U(h)$ as follows:*

$$W_h(x) = \begin{cases} 0 & \text{if } x \le h \\ x-h & \text{if } x \ge h \end{cases}$$

$U(h) = E[W_h(V)]$ *where E[ ] denotes the expected value of a R.V.*

Then U satisfies $$\left.\frac{\partial U(h)}{\partial h}\right|_{h=a} = F(a) - 1$$

Proof: We can write the function U as follows:

$$U(h) = \int_{-\infty}^{+\infty} W_h(v) f(v) dv = \int_h^{+\infty} (v-h) \times f(v) dv$$

So

$$U(h) = h \times (F(h) - 1) + \int_h^{+\infty} u \times f(u) du$$

Therefore

$$\left. \frac{\partial U(h)}{\partial h} \right|_{h=a} = F(a) - 1$$

[End of Proof]

Theorem 3.1: Existence and Uniqueness of Optimal Ground Holding Time

If we consider the departure and arrival delay random variables for the D/A model then the optimal ground holding time exists, is unique and is given by the following equation:

$$d_g^{Optimal} = Max\left(0, F_Z^{-1}(1-r)\right) \quad (3.8)$$

where $Z = D_a - X = D_a - D_d + d_g$ *and* $r = \frac{C_g}{C_a}$.

Proof: Let us first start by proving that the expected total delay cost function is globally convex.

If we use equations (3.5), (3.6), (3.7) and the definition of the function W_h given in Lemma 1, we have that :

$$E[C] = C_g \times E[D_d] + C_a \times E[W_{d_g}(Z)] \qquad (3.9)$$

So by using (3.9) and the result of Lemma 1 we have:

$$\frac{\partial E[C]}{\partial d_g} = C_g + C_a \times \left(F_Z(d_g) - 1\right) \qquad (3.10)$$

Therefore

$$\frac{\partial^2 E[C]}{\partial d_g^2} = C_a \times f_Z(d_g) \geq 0$$

This proves that the function is globally convex. So a necessary and sufficient condition for a global minimum is to have the first derivative equal to zero.
Furthermore, since the marginal cost of ground delay is by definition smaller than the marginal cost of airborne delay, the following equation always has a solution.

$$\frac{\partial E[C]}{\partial d_g} = C_g + C_a \times \left(F_Z(d_g) - 1\right) = 0$$

Therefore the optimal ground holding time exists, is unique and is given by the following equation:

$$d_g^{Optimal} = Max\left(0, F_Z^{-1}(1-r)\right)$$

[End of Proof]

This theoretical result is applicable to any choice of delay random variables and will be useful when we shall implement the D/A model for particular choices of delay random variables (e.g., uniform random variables). Another interesting observation concerns the fact that the result does not depend explicitly on the departure and arrival random variables but depends on their difference Z.

When the delay random variable X for the rescheduled departure is set equal to zero with probability one we obtain the same optimality result as the A model. Furthermore, we can also note that when the ratio of ground and airborne marginal costs takes on large values (i.e. close to 1), the D/A strategy coincides with the no ground holding one. This last observation is consistent with intuition. In fact, we would expect no ground holding when the marginal cost of ground delay is equal to the marginal cost of airborne holding. One drawback of this model is that it does not penalize explicitly the flight for never reaching its destination[1] when the marginal cost of ground delay is equal to zero.

3.4 Case of Gaussian Delay Distributions

3.4.1 Introduction

We have presented in the previous section the optimality conditions for the general case where no assumptions were made on the delay distributions. This section will apply these results to the case where the departure and arrival random variables are assumed to have Gaussian distributions.

We will start by deriving the mathematical formulation for the Gaussian case and then conduct numerical computations to compare the performance of the different models (i.e. no ground holding, A model and D/A model). We will also test the sensitivity of these models to changes in the problem parameters such as the mean and standard deviation of the departure and arrival delays as well as changes in the ground and airborne marginal costs. Finally, we will quantify the benefits of having 'better' weather forecasts allowing delays to be forecast more accurately (i.e., with smaller standard errors).

[1] This scenario happens when the R.V. Z has a pdf defined over the entire set of real numbers (e.g., Gaussian distribution) and the ratio of marginal cost goes to zero. Under this circumstances (3.8) gives an infinite value for the optimal ground holding time, which means that the aircraft never reaches its destination.

It should be noted that Gaussian distributions make calculations much easier and allow us to implement the model even when departure and arrival random variables are correlated[1]. It should also be noted that the choice of Gaussian distributions might be a reasonable approximation in some cases and not in others. For example, when arrival and departure delays are bounded, the Gaussian distribution assumption is not reasonable since all delays (even very large ones) have a nonzero probability of occurring. For bounded delays a uniform or other bounded distribution might be a better approximation. The next section will provide a treatment of the uniform case.

3.4.2 Model Formulation and Optimality Conditions

We have shown in section 3.3 that for any given departure and arrival delay random variables, the optimal ground holding time is given by the following equation:

$$d_g^{Optimal} = Max\left(0, F_Z^{-1}(1-r)\right) \qquad (3.11)$$

For the case of Gaussian delays (i.e. $D_d \sim N(d_g + \mu_d, \sigma_d)$ and $D_a \sim N(\mu_a, \sigma_a)$), the random variable Z has a Gaussian distribution (i.e. $Z \sim N(\mu, \sigma)$) with mean and standard error defined as follows:

$$\mu = \mu_a - \mu_d \quad and \quad \sigma = \sqrt{\sigma_d^2 + \sigma_a^2 - 2 \times \sigma_{da}}$$

Where:

$$\sigma_{da} = Cov\,(D_d, D_a)$$

If we call $Y \sim N(0,1)$, we have the following result:

$$Z = \mu + \sigma \times Y$$

[1] When the two random variables X and D_a defined in the previous section are assumed to be Gaussian the random variable $Z = D_a - X$ that governs the ground holding optimality condition is also Gaussian even when the two random variables are correlated.

Then:

$$F_Y(-x) = 1 - F_Y(x) \quad and \quad F_Y(1-x) = -F_Y(x)$$

$$F_Z(x) = F_Y\left(\frac{x-\mu}{\sigma}\right) \quad and \quad f_Z(x) = \frac{1}{\sigma} \times f_Y\left(\frac{x-\mu}{\sigma}\right)$$

We can then rewrite the closed form solution for the optimal ground holding time as follows:

$$d_g^{Optimal} = Max\left(0, \mu - \sigma \times F_Y^{-1}(r)\right) \qquad (3.12)$$

The value r_0 of r where the optimal ground holding time is equal to zero is given as follows:

$$r_0 = F_Y\left(\frac{\mu}{\sigma}\right) \qquad (3.13)$$

With this definition of r_0 we can re-write (3.9) as follows:

$$\begin{cases} d_g^{Optimal} = \mu - \sigma \times F_Y^{-1}(r) & for \quad r \leq r_0 \qquad (3.14\text{ a}) \\ d_g^{Optimal} = 0 & for \quad r \geq r_0 \qquad (3.14\text{ b}) \end{cases}$$

(3.13) and (3.14) suggest the following observations:

- When the expected arrival delay is larger than the expected delay in the rescheduled departure time (i.e. $\mu \geq 0$) and σ is equal to 0, the optimal ground holding time is equal to μ which means that the total expected delay is taken on the ground.

- When the expected arrival delay is smaller than the expected delay in the rescheduled departure time (i.e. $\mu \leq 0$) and σ is equal to 0, the optimal ground holding time is equal to zero. This last result means that no ground holding is taken since the average unavoidable ground delay μ_d is higher than the average delay in the arrival landing slot μ_a.

- When r is larger than 0.5 the optimal ground holding time is a linear and downward sloping function of uncertainty. However, when r is smaller than 0.5 the optimal ground holding time is linear and upward sloping as a function of uncertainty. This result is quite intuitive since it says that when ground holding is "cheap" a good response to more uncertainty is to ground hold more, whereas if ground holding is expensive a response to a similar situation is to ground hold less. When r is equal to 0.5 the optimal ground holding time does not depend on the amount of uncertainty that the system is facing. It is quite easy to verify this last result from the above equation. However, it is less easy to understand intuitively why this 0.5 value for r plays such an important role.

3.4.3 Sensitivity Analysis and Model Comparison:

This section will present the results of the numerical analysis undertaken for the Gaussian linear case. The computations are mainly intended to confirm certain observations that were made on the different mathematical equations derived in the previous section. They are also intended to test the sensitivity of model outputs, in terms of both cost savings and optimal ground holding time to variations in problem parameters, namely average delays, standard errors, and to the ratio of marginal costs. Finally, a comparison between the A and D/A stochastic models will be presented along with the sensitivity analyses.

The spreadsheet Excel was used to implement the numerical computations and a macro was developed to this effect. In order to compare the departure and arrival

stochastic model with Andrews' model, it was assumed that the D/A model is the "true" one in the sense that its average cost function was used as the basis for comparison.

A/ Model Implementation for Simple Scenarios:

Let us consider a flight from airport D to airport A with the two airports having delay random variables assumed independent and Gaussian distributed with the following characteristics:

Airport D: $\mu_d = 15$ min and $\sigma_d = 5$ min

Airport A: $\mu_a = 30$ min and $\sigma_a = 10$ min

We further assume that the marginal costs for ground and airborne delays are respectively equal to \$590 and \$2200 per hour ,which are the same costs Andrew used in his paper. This assumption corresponds to a ratio r equal to about 27%. The three strategies at hand (i.e. passive, A and D/A) yield the following results:

Table 3.1: First Scenario	Passive Strategy	A Strategy	D/A Strategy
Ground Holding d_g (min)	0	36	22
Expected Airborne Holding (min)	15	0	2
Expected Total Delay (min)	30	51	39
Expected Cost (\$)	715	508	430

As expected the D/A strategy outperforms the other two by substantial margins. In fact, about a 40% cost saving is achieved when compared to the passive strategy and about 15% when compared to the A strategy. Furthermore, when compared to the passive strategy, the A strategy increases the total expected delay substantially more than the D/A one does. Overall for this example, the D/A model outperforms the A strategy on both cost savings as well as the expected amount of delays.

If we now consider the same airports as before with the same delay characteristics but with ground and airborne marginal costs equal respectively to $1760 and $2200 per hour which corresponds to a cost ratio r equal to about 80% we obtain the following results:

Table 3.2: Second Scenario	Passive Strategy	A Strategy	D/A Strategy
Ground Holding d_g (min)	0	22	6
Expected Airborne Holding (min)	15	2	11
Expected Total Delay (min)	30	39	31
Expected Cost ($)	1,007	1,144	998

We can note for this choice of marginal costs that the Passive and D/A strategies have similar performances. Furthermore, the Passive strategy outperforms the A one by about 12%.

B/ Sensitivity to System Parameters:

B.1/ Cost Saving Ratio Sensitivity to r:

We define the cost saving ratio for both A and D/A strategies as being the expected cost saving achieved by these two strategies when compared to the no-ground-holding strategy (i.e. passive). These are given by the following formulae:

$$A\ Cost\ Saving = 1 - \frac{E(C/d_g^{A})}{E(C/d_g^{Passive})} \qquad (3.15)$$

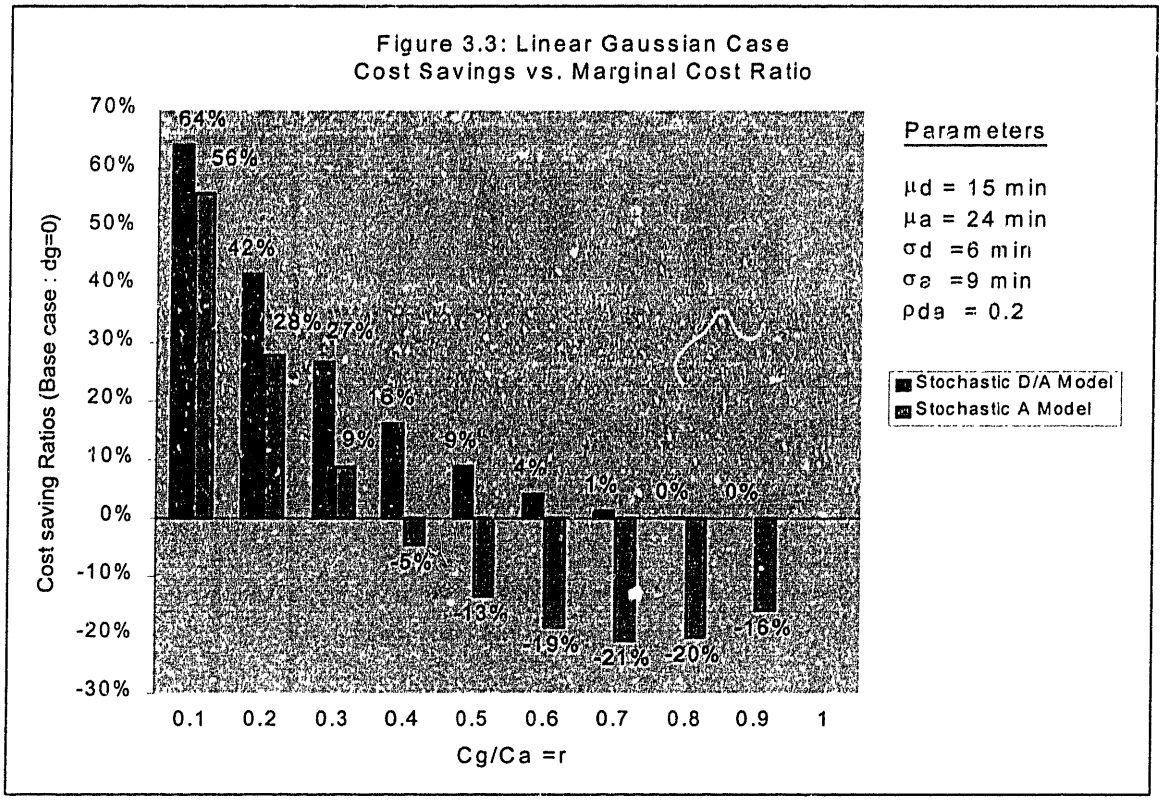
$$D/A\ Cost\ Saving = 1 - \frac{E(C/d_g^{D/A})}{E(C/d_g^{Passive})} \qquad (3.16)$$

Figure 3.3 summarizes the results for the numerical computation of cost saving ratios for both A and D/A strategies for different values of r. The following points are worth noting:

- The D/A strategy outperforms both the A and passive strategy for all values of r. Furthermore, substantial savings (i.e. about 60%) are achieved for small values of r. This last result is consistent with the intuition that when the relative cost of ground holding is small more cost savings are achieved.

- For values of r larger than 40%, the A strategy is less attractive than the passive one. This last result could be explained by the fact that ignoring departure delay uncertainties corresponds to ground holding the aircraft more than required and thus yields a sub-optimal strategy when compared to the passive strategy.

- The threshold point r_0 introduced in section 3.4.2, beyond which no ground holding is warranted is equal to approximately 70% for this case.

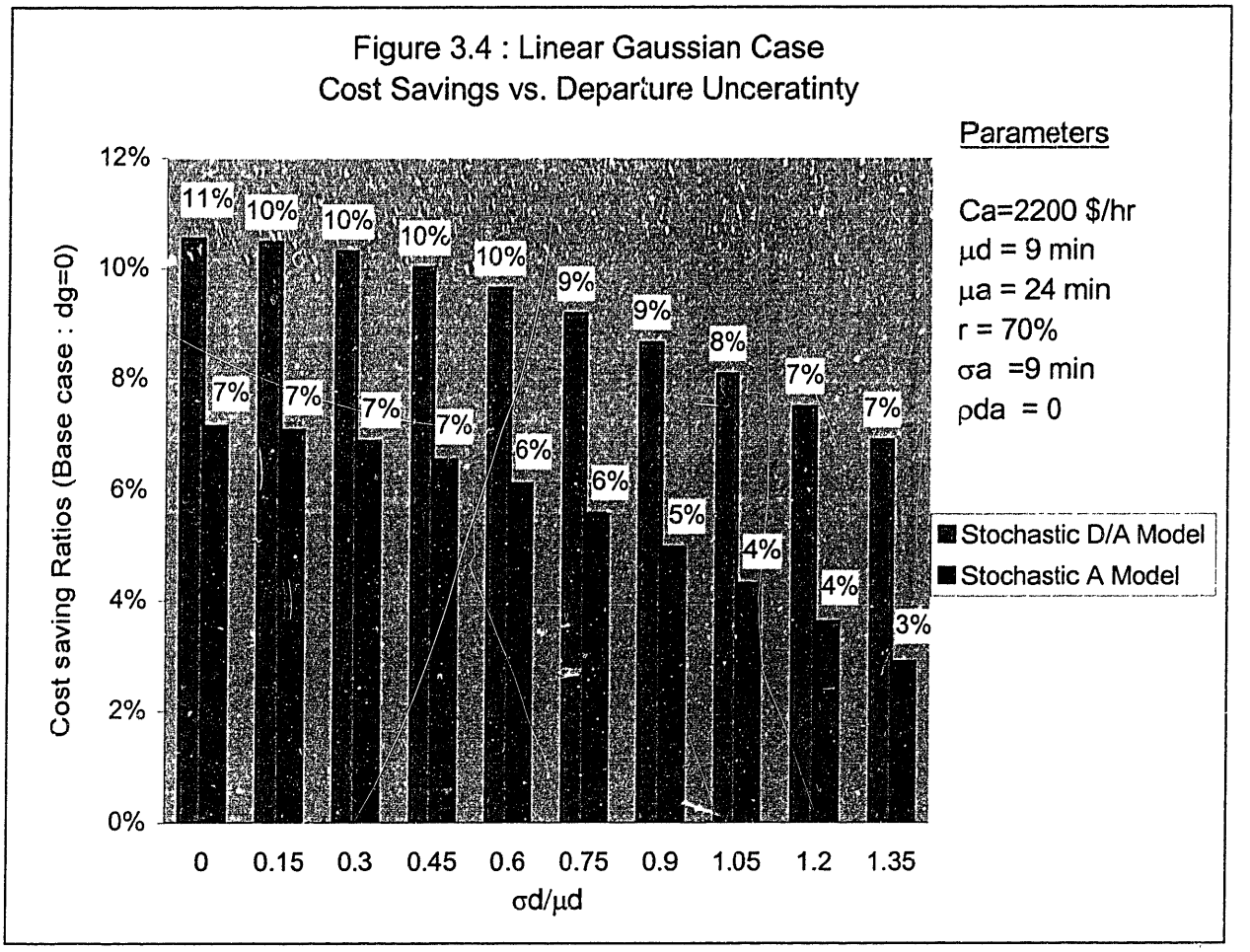


B.2/ Cost Saving Ratio Sensitivity to Departure Uncertainty $\sigma d/\mu d$:

Figure 3.4 summarizes the results for the numerical computation of the cost saving ratios for both A and D/A strategies for different values of the departure uncertainty. Departure uncertainty is described by the ratio σ_d/μ_d. Clearly the higher the ratio, the greater the uncertainty. The parameters chosen to conduct this analysis are given in the figure. The following points are worth noting:

- As expected the D/A strategy outperforms the other two for all values of the departure delay uncertainty. For the deterministic case (i.e. $\sigma_d = 0$), the A strategy has a lower cost saving ratio than the D/A one. This is because the A strategy does not account for the fact that the departure delay is equal to a nonzero value μ_d with probability one.
- We can also note that, when the departure delay uncertainty increases, the cost savings for both A and D/A strategies decrease. This last point confirms the rule of thumb

that "better delay prediction yields more cost savings". This will be discussed in more detail in the following section.

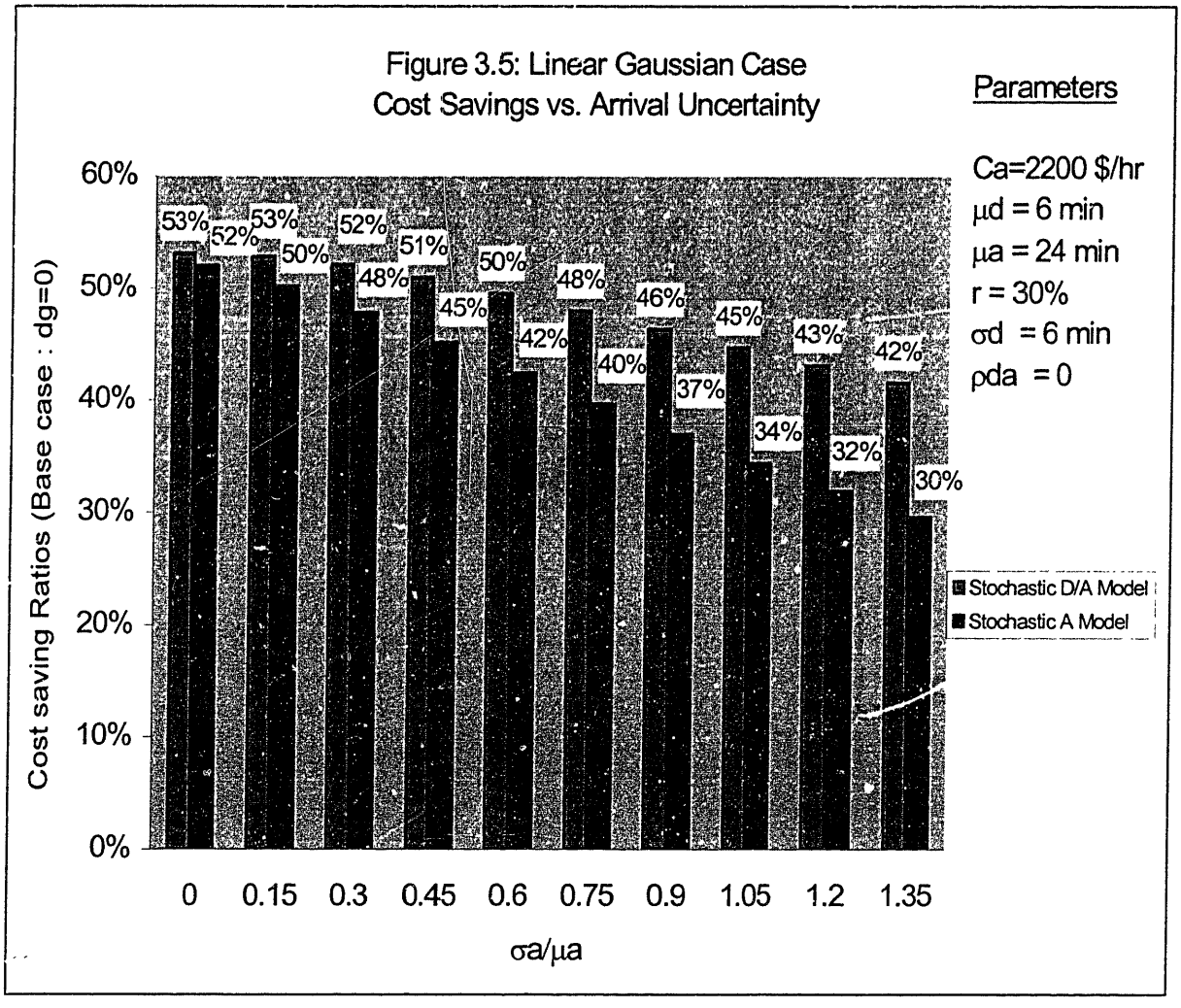


B.3/ Cost Saving Ratio Sensitivity to Arrival Uncertainty σa/μa:

Figure 3.5 summarizes the results for the numerical computation of the cost saving ratios for both A and D/A strategies for different values of the arrival uncertainty as described by the ratio σ_a/μ_a. The parameters chosen to conduct this analysis are given in the figure. The following points are worth noting:

- As in all previous sensitivity analyses, the D/A strategy outperforms the other two for all levels of arrival delay uncertainty.

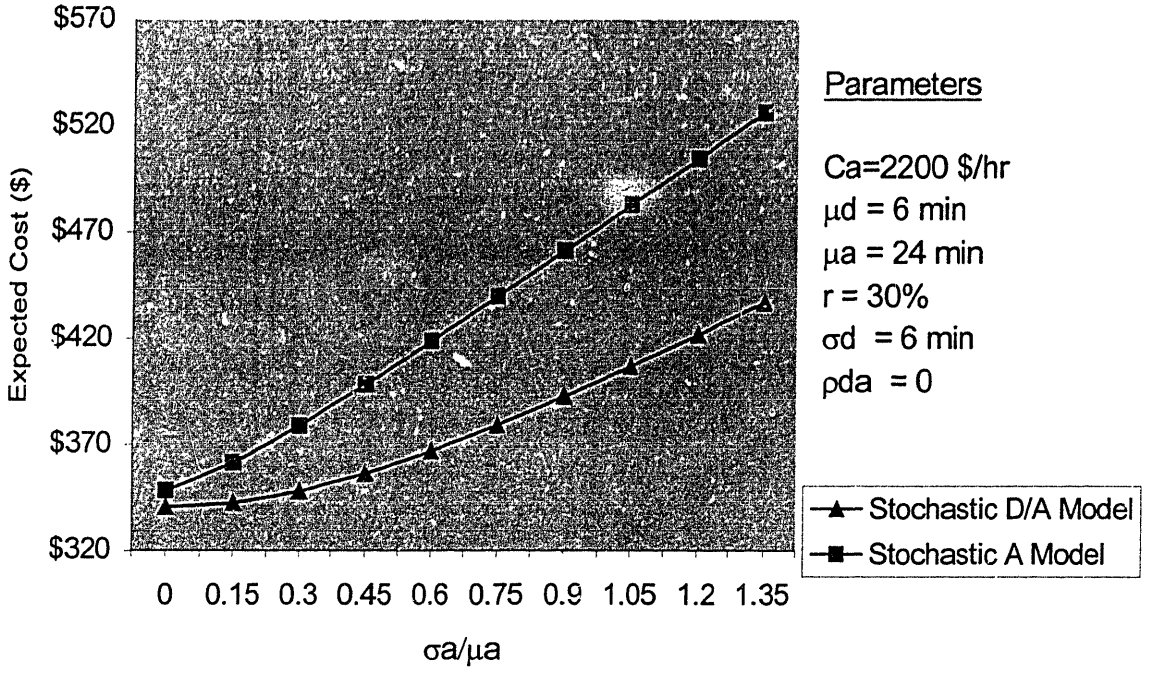
- As in the case of the previous sensitivity analysis, the cost saving ratios for both A and D/A strategies decrease when the arrival delay uncertainty increases.



- Figure 3.6 illustrates the expected cost for both A and D/A strategies as a function of arrival delay uncertainty. It can be noted that these two functions are upward slopping which means that more uncertainty results in more average cost of delays. This supports the rule of thumb "better delay prediction yields more cost savings" which

was mentioned earlier. Furthermore, we can also note that for values of uncertainties larger than 0.75 the two expected cost functions are almost linear with slopes equal to \$58/min and \$14/min respectively for the A and D/A strategies. This last observation means that if we improve the arrival delay prediction by one minute (i.e. reduce σ_a by one minute) we shall obtain a cost saving of \$58 per flight for the D/A strategy.

Figure 3.6: Linear Gaussian Case
Expected Costs vs. Arrival Uncertainty



3.5 Case of Uniform Delay Distributions

3.5.1 Introduction:

This section provides a treatment of the case in which the departure and arrival delay random variables are assumed to be independent and uniformly distributed (Fig 3.6). The independence can be justified by the fact that in many instances the weather patterns at departure and arrival airports are independent[1].

The assumption that departure and arrival delay random variables are uniformly distributed could be justified by the fact that in some instances both arrival and departure delays may be known to fall within a certain bounded time window with all delay values approximately equally probable. In this last case, a uniform distribution captures both the time window and the equal probability features.

We will first derive the optimality conditions and provide a closed form solution for the optimal ground holding time, as well as the associated optimal average cost of delays. A sensitivity analysis will then be presented in which different problem parameters will be changed and their impact on the optimal solution will be tested.

It should be noted from Figure 3.7 that the joint probability density function is constant inside the square and equal to $1/(12*\sigma_d*\sigma_a)$ where σ_d and σ_a are respectively the departure and arrival random variables' standard deviations. Outside this area, the joint pdf is equal to zero.

[1] As it was pointed out earlier this is a reasonable assumption in the case where the departure and arrival airports are far away from each other. In addition, weather events that impact departure and arrival capacities such as low ceilings or fog are generally local ones (i.e. limited to specific airport terminal areas).

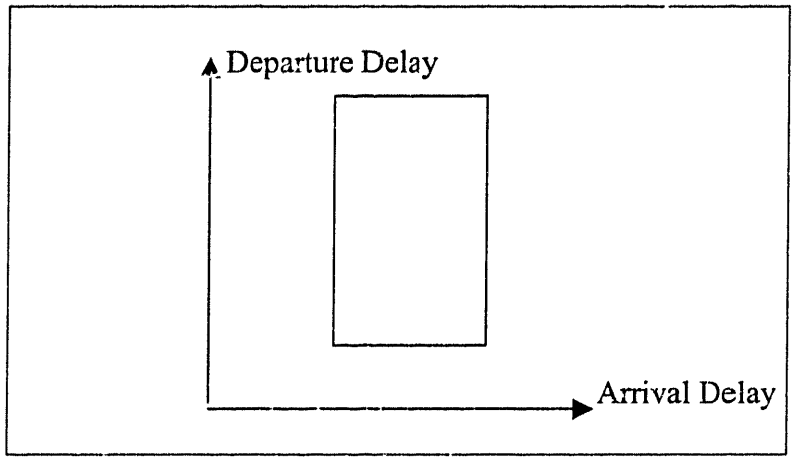


Fig 3.7: Joint Cumulative Distribution Function

A more general treatment of this case in which the two delays are correlated was also tried. However, the algebraic derivations of optimality conditions turned out to be too complex to allow an explicit solution of this case. This complexity stems from the fact that the joint pdf for the two delay random variables is not uniformly distributed when they are correlated.

3.5.2 Model Formulation and Optimality Conditions

If we use the result (3.11), we can easily prove[1] with the notation defined below that will be used for the rest of this section on the uniform case that:

$$d_g^{Optimal} = Max(\,0\ , F_z^{-1}(1-r)+\mu) \qquad (3.17)$$

where:

$$r = Cg / Ca$$
$$D_d \sim U(d_g + \mu_d, \sigma_d)$$
$$D_a \sim U(\mu_a, \sigma_a)$$
$$\mu = \mu_a - \mu_d$$
$$X = D_d - d_g - \mu_d \Rightarrow E[X] = 0$$
$$Y = D_a - \mu_a$$
$$Z = Y - X$$

F_z denotes the cdf of Z.

[1] (3.11) was formulated in terms of Z' = D_a-D_d+d_g and therefore with the notation defined in this section we have that Z = Z'-μ. If we substitute in condition (3.11) Z' with Z we obtain result (3.17).

In order to use (3.17), we need to derive a closed form solution for the inverse cumulative distribution function of Z. Let us start by deriving the formula for the cdf of Z. The cdf is given by the following equation

$$F_z(u) = \int_{-\infty}^{u} f_z(t)dt \quad (3.18)$$

A direct computation of this equation requires us to derive the pdf for Z, which is difficult to do. An alternative and easier way to compute this integral, consists of using the joint probability function (i.e. for the R.V. (X,Y)) to calculate this integral. The following equation provides the desired result:

$$F_z(u) = \iint_{\{(x,y)/\,y-x\le u\}} pdf(x,y)\ dx\,dy \qquad (3.19)$$

(3.16) can be rewritten as follows:

$$F_z(u) = \iint_{S\cap\{(x,y)/\,y-x\le u\}} \frac{1}{12\times\sigma_d\times\sigma_a}\,dx\,dy \qquad (3.20)$$

where S is the area defined by the square shown in Figure 3.7.

The area over which we calculate the integral is shown in the following figure (Figure 3.8). It consists of the area lying under the line (Y-X=U) and inside the square.

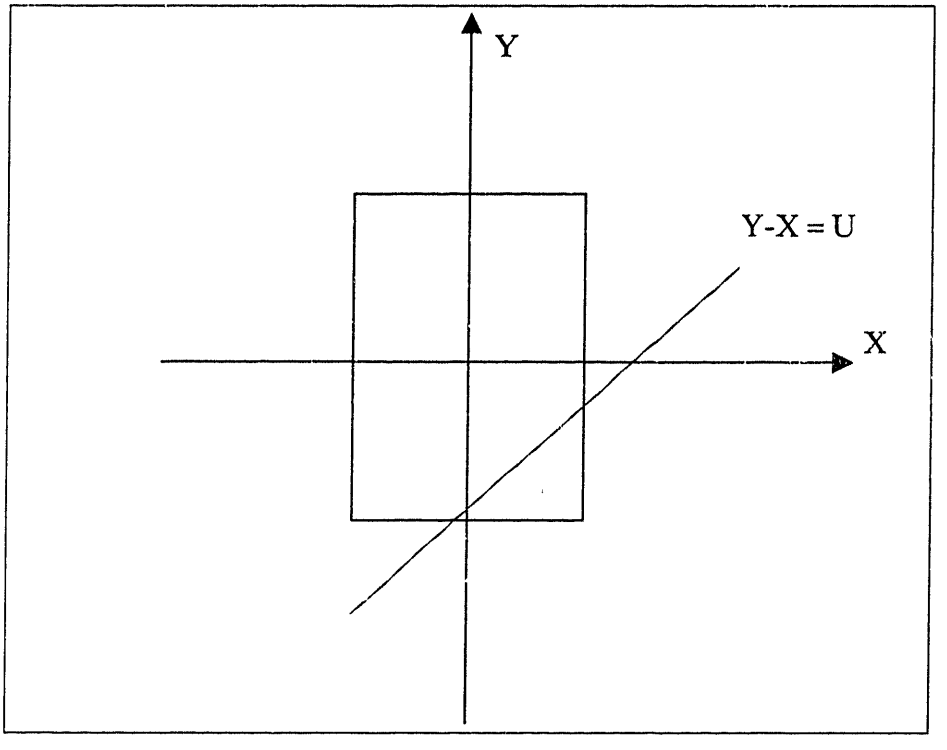


Figure 3.8: Area over which the integral is calculated.

The square is by definition centered at zero since the two R.V. X and Y have mean zero for the notation adopted for this section. Furthermore, the square crosses the Y-axes at values $\pm\sqrt{3}\ \sigma_a$ and the X-axes at values $\pm\sqrt{3}\ \sigma_d$. We can note from Figure 3.8 that the integral is equal to the product of the area lying under the line and inside the square and the joint pdf of (X,Y) which is equal to $\frac{1}{12\times\sigma_d\times\sigma_a}$.

In order to compute this integral, we will assume that the standard error for the arrival delay is larger than the one for the departure delay (i.e. $\sigma_d \leq \sigma_a$). This last assumption can be justified by the fact that the time horizon is longer for the arrival forecast than the departure one and therefore the accuracy of the departure delay forecast is expected to be better. The derivation for the case where $\sigma_a \leq \sigma_d$ could also be done following a similar reasoning.

Depending on the value of U the areas over which we compute the integral have different shapes, which yields a function with different expressions depending on where it is evaluated. The following set of equations summarizes the result:

$$\begin{cases} F_z(u) = 0 & \text{for} \quad u \le -\sqrt{3}\times(\sigma_d+\sigma_a) \quad (3.21a) \\ F_z(u) = \dfrac{1}{24\times\sigma_a\times\sigma_a}\left[\sqrt{3}\times(\sigma_d+\sigma_a)+u\right]^2 & \text{for} \quad -\sqrt{3}\times(\sigma_d+\sigma_a) \le u \le \sqrt{3}\times(\sigma_d-\sigma_a) \quad (3.21b) \\ F_z(u) = \dfrac{\sqrt{3}}{6\times\sigma_a}\cdot\left[\sqrt{3}\times(\sigma_a-\sigma_d)+u\right]+\dfrac{\sigma_d}{2\times\sigma_a} & \text{for} \quad \sqrt{3}\times(\sigma_d-\sigma_a) \le u \le \sqrt{3}\times(\sigma_a-\sigma_d) \quad (3.21c) \\ F_z(u) = 1-\dfrac{1}{24\times\sigma_a\times\sigma_a}\left[\sqrt{3}\times(\sigma_d+\sigma_a)-u\right]^2 & \text{for} \quad \sqrt{3}\times(\sigma_a-\sigma_d) \le u \le \sqrt{3}\times(\sigma_d+\sigma_a) \quad (3.21d) \\ F_z(u) = 1 & \text{for} \quad u \ge \sqrt{3}\times(\sigma_d+\sigma_a) \quad (3.21e) \end{cases}$$

We can see from the previous set of equations that the cdf for the random variable Z has a quite complex shape and it is not easy to tell a priori how the optimal ground holding time will vary when the different problem parameters vary. It is also interesting to note that this function is linear over certain ranges and quadratic over others.

It can be easily verified that the previous function is invertible for the three ranges where it is not constant and the inverse is given by the following set of equalities:

$$\begin{cases} F_z^{-1}(u) = \sqrt{24\times\sigma_d\times\sigma_a\times u} - \sqrt{3}\times(\sigma_d+\sigma_a) & \text{for} \quad 0 \le u \le \dfrac{\sigma_d}{2\times\sigma_a} \quad (3.22a) \\ F_z^{-1}(u) = 2\sqrt{3}\times\sigma_a\times u - \sqrt{3}\times\sigma_a & \text{for} \quad \dfrac{\sigma_d}{2\times\sigma_a} \le u \le 1-\dfrac{\sigma_d}{2\times\sigma_a} \quad (3.22b) \\ F_z^{-1}(u) = \sqrt{3}\times(\sigma_d+\sigma_a) - \sqrt{24\times\sigma_d\times\sigma_a\times(1-u)} & \text{for} \quad 1-\dfrac{\sigma_d}{2\times\sigma_a} \le u \le 1 \quad (3.22c) \end{cases}$$

It can be noted from the previous set of equations that when the departure and arrival random variables have the same mean ($\mu_d = \mu_a$), the optimal ground holding is bounded by zero and $\sqrt{3} \times (\sigma_d + \sigma_a)$. This last observation could intuitively be understood by the fact that our boundedness assumption on the departure and arrival random variables implies that certain delays, typically large ones, have probability zero of occurring, which explains why the optimal ground holding is bounded. It is interesting to note that this upper bound is an upward sloping function of both the departure and arrival delay standard deviations. Intuitively, this last observation makes sense since it is expected that more uncertainty could potentially force the optimal ground holding time to take-on larger values. This observation also holds when the two delay random variables do not have the same mean, but it is more difficult to give an explicit expression for the upper bound on the optimal ground holding.

Another interesting particular case is when the two random variables are identically distributed (i.e. same means and standard deviations). In this case the optimal ground holding is given by the following two equations:

$$\begin{cases} d_g^{Optimal} = 2\sqrt{3} \times \sigma \times \left(1 - \sqrt{2r}\right) & for \quad r \leq \frac{1}{2} \qquad (3.23a) \\ d_g^{Optimal} = 0 & for \quad r \geq \frac{1}{2} \qquad (3.23b) \end{cases}$$

Intuitively, it makes sense that beyond a certain value of the ratio of marginal costs it is not advantageous to ground hold the aircraft and thus the optimal decision variable is equal to zero. Furthermore, when some ground holding is warranted ($r \leq 1/2$), it is upward sloping as a function of uncertainty. This is the result one would expect since more uncertainty is expected to lead to more ground holding within a certain range of r. Over the same range, the optimal ground holding decreases as the square root of r. This last observation is consistent with the intuition that when the marginal cost of ground

holding increases, it is less advantageous to ground-hold the aircraft. However, it is more difficult to explain the square root relationship.

Using result (3.22) on the inverse cdf and given a set of values for the problem parameters we can compute the optimal ground holding time. Furthermore, we can also compute the optimal average cost of delays and compare it with the strategy in which no ground holding is used.

The average cost function for any amount of ground holding is given by the following equation:

$$E[C] = C_g \times (\mu_d + d_g) + C_a \times \int_{d_g - \mu}^{+\infty} (z + \mu - d_g) \times f_Z(z)\ dz \qquad (3.24)$$

However, the pdf for Z is equal to zero when z is larger than a certain value, so (3.24) can be rewritten as follows:

$$E[C] = C_g \times (\mu_d + d_g) + C_a \times \int_{d_g - \mu}^{\sqrt{3} \times (\sigma_d + \sigma_a)} (z + \mu - d_g) \times f_Z(z)\ dz \qquad (3.25)$$

The pdf for Z is given by the following set of equations, which were obtained by differentiating the cdf of Z:

$$\begin{cases} f_z(u) = 0 & \text{for} \quad u \le -\sqrt{3} \times (\sigma_d + \sigma_a) \quad (3.26a) \\ f_z(u) = \dfrac{1}{12 \times \sigma_d \times \sigma_a} \left[\sqrt{3} \times (\sigma_d + \sigma_a) + u \right] & \text{for} \quad -\sqrt{3} \times (\sigma_d + \sigma_a) \le u \le \sqrt{3} \times (\sigma_d - \sigma_a) \quad (3.26b) \\ f_z(u) = \dfrac{\sqrt{3}}{6 \times \sigma_a} & \text{for} \quad \sqrt{3} \times (\sigma_d - \sigma_a) \le u \le \sqrt{3} \times (\sigma_a - \sigma_d) \quad (3.26c) \\ f_z(u) = \dfrac{1}{12 \times \sigma_d \times \sigma_a} \left[\sqrt{3} \times (\sigma_d + \sigma_a) - u \right] & \text{for} \quad \sqrt{3} \times (\sigma_a - \sigma_d) \le u \le \sqrt{3} \times (\sigma_d + \sigma_a) \quad (3.26d) \\ f_z(u) = 0 & \text{for} \quad u \ge \sqrt{3} \times (\sigma_d + \sigma_a) \quad (3.26e) \end{cases}$$

We have now derived all the required mathematical equations to undertake the sensitivity analysis, which will be presented in the next section.

3.5.3 Sensitivity Analysis and Model Comparison

This section will present the results of the numerical analysis undertaken for the uniform case. The numerical results are mainly intended to confirm certain observations that were made in the previous section. They are also intended to test the sensitivity of model outputs both in term of cost savings and optimal ground holding time to variations in problem parameters, namely expected delays, standard deviations of delays, and ratio of marginal costs. Finally, a comparison between the D/A model and the A model extended to the uniform case (i.e., departure delay assumed to be deterministic) will be presented along with the sensitivity analyses.

The software Matlab was used to obtain the numerical results and a code was written to this effect. The trapeze method[1] was used to compute the integrals in the average cost of delays. In order to compare the D/A model with the uniform extension of the A model, it was assumed that the former is the "true" model in the sense that its average cost function was used as the basis for comparison. Finally, these computations were undertaken assuming an arrival delay standard deviation larger than the standard deviation of the departure delay.

A. Sensitivity to the Cost ratio *r*

A.1/ Cost Index sensitivity to the Ratio r:

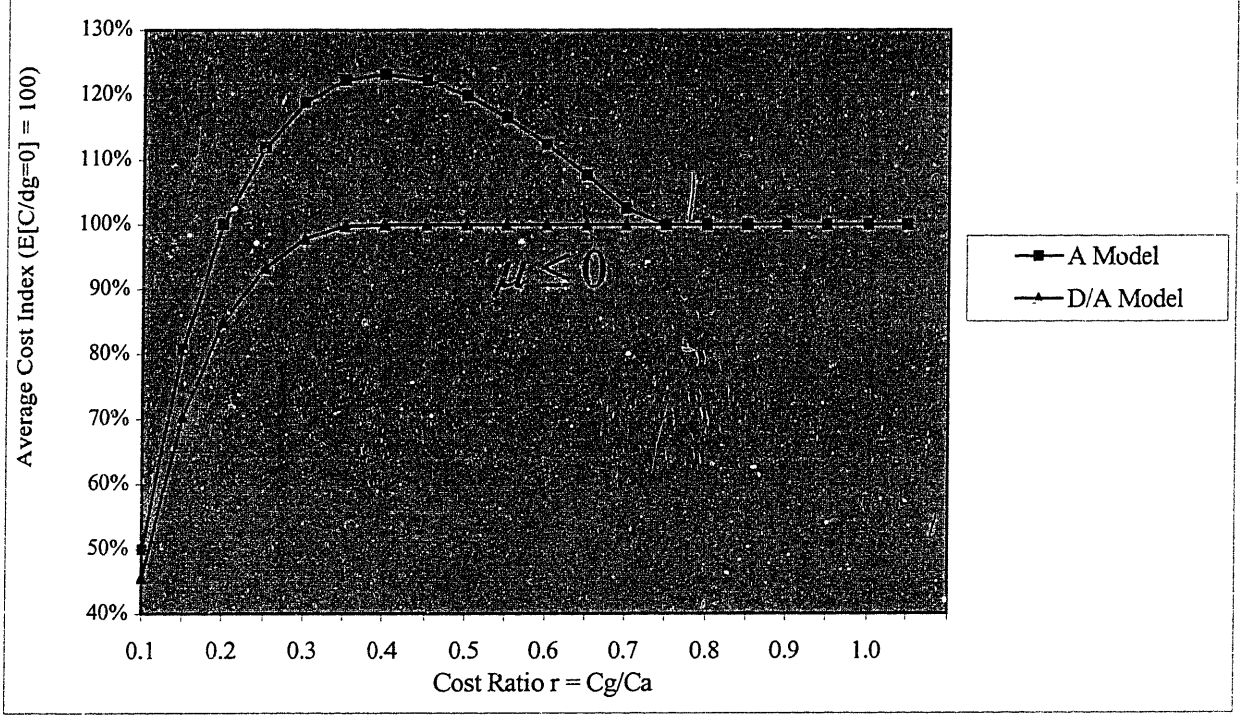
Figure 3.9 shows the average delay cost index (i.e. average cost for the model considered divided by the average cost for the strategy where no ground holding is used) for the two models under comparison. In this figure the departure and arrival random

[1] This method consists of approximating the integral of a function over a bounded interval by the sum of the areas of trapezes whose bases constitute the whole interval and whose heights are function values taken at the bases of the trapezes.

variables were uniformly distributed and have respectively (30,20) and (15,25) as mean and standard deviations expressed in minutes. The range of values for the two delays is actually very large in reality but it was selected to make the point. The following points are worth noting:

- The cost saving ratio (i.e. 1-(cost index)) decreases as the ratio r increases. This result is consistent with the intuition that when the cost ratio r increases, it is less advantageous to ground hold the aircraft. Furthermore, it can be noted that beyond a certain value for the ratio r no ground holding is warranted. For our choice of parameters, this value of r is approximately equal to 30%.

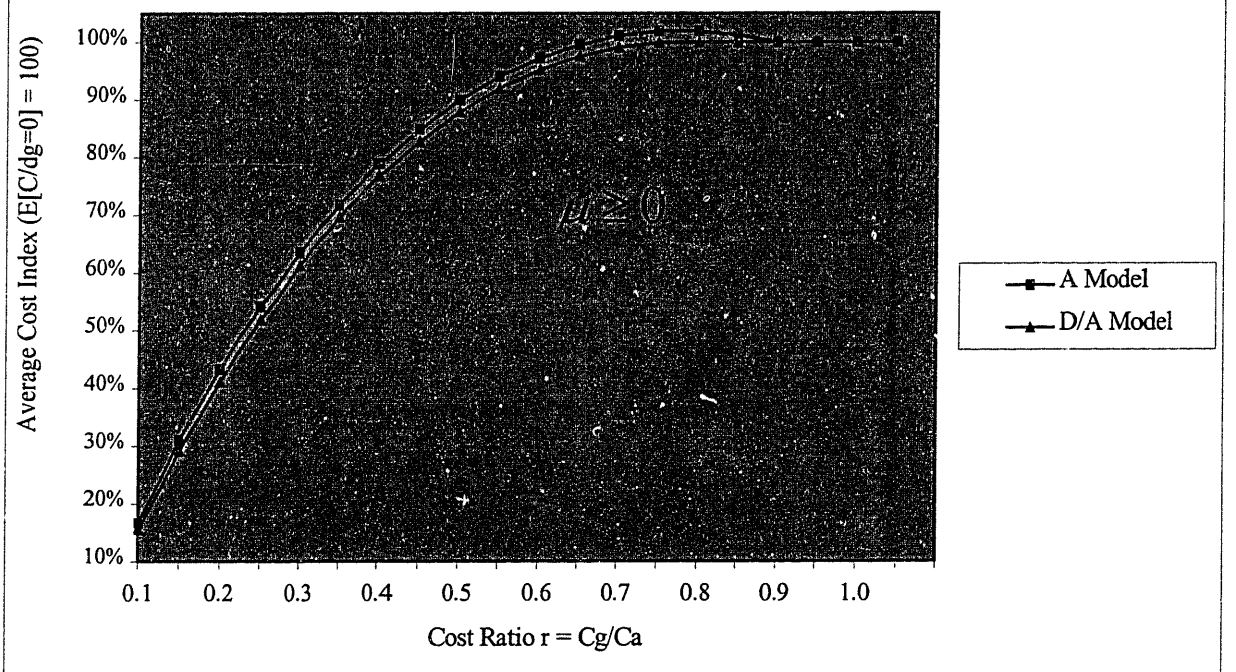
Figure 3.9: Linear Unifrom Case
Cost Index vs. Marginal Cost Ratio



- It can be noted that the A model performs worse in all instances than the D/A model. However, for small values of r (typically less than 15%) or for large values of r (typically more than 70%), the two models perform almost equally. For the remaining values (i.e.

between 15 and 70 percent), the D/A model performs much better than the A model. Furthermore, for values of r ranging between 20 and 70 percent, the A model performs worse than the *no ground holding* strategy with a high average cost index of 122 % for $r \approx 0.4$. This last observation might not be true in all cases and could be due to the particular choice of parameters in this simulation. But the example points out, once again, the importance of considering uncertainty in departure delay when determining ground delay strategies.

Figure 3.10: Linear Unifrom Case
Cost Index vs. Marginal Cost Ratio



- Figure 3.10 shows the result of another numerical computation where the departure and arrival random variables have respectively (10,5) and (30,25) as mean and standard error expressed in minutes. This figure confirms the observation made earlier regarding the importance of the particular choice of parameters. In this case, the expected arrival delay is larger than the expected departure delay. As a consequence the A model almost always outperforms the no ground holding strategy and, for the few instances where it

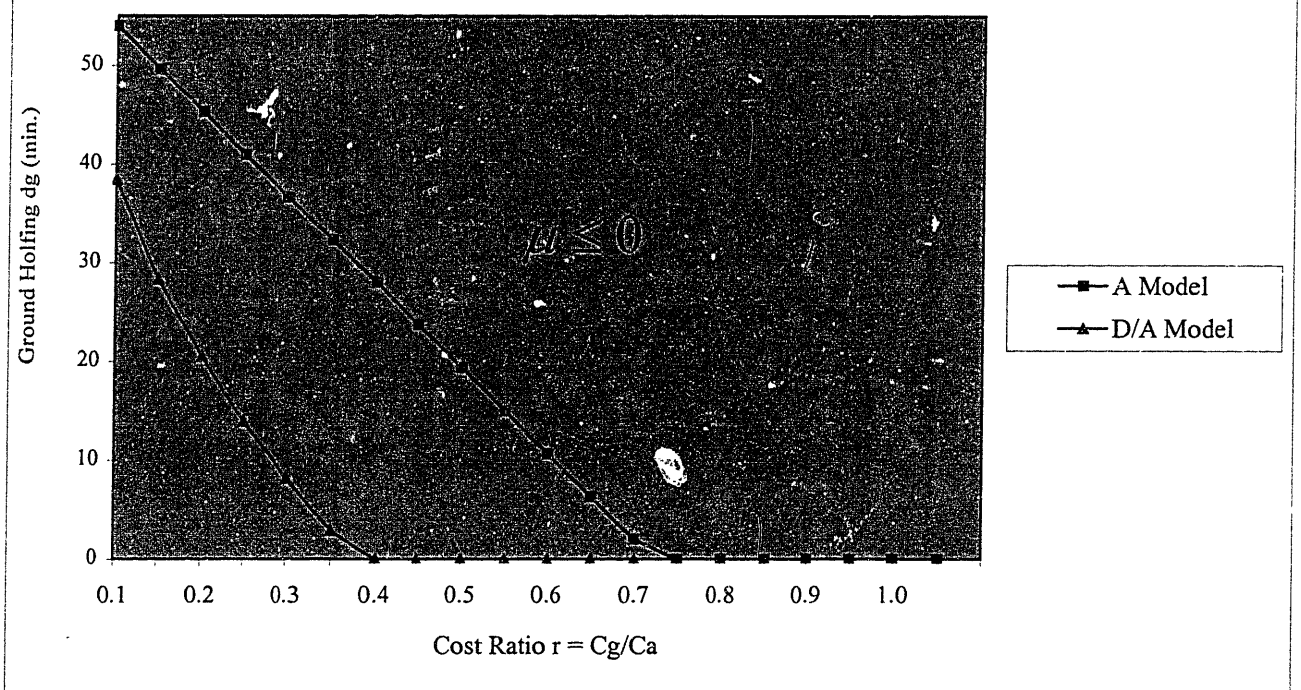
does not, the difference is very small. Furthermore, we can also note that in this case the D/A and the A models have very similar levels of performance for all values of r.

- We can also note from the previous two figures that the cost index is concave as a function of r, which means that the cost saving function is convex. This last observation proves that the marginal cost saving (differential of the cost saving ratio as a function of r) increases when r decreases, meaning that the smaller r gets the more savings we obtain per unit of variation. This last result could be extended to any choice of parameters.

A.2/ Ground Holding Sensitivity to Cost Ratio r:

Figure 3.11 illustrates the variation of the optimal ground holding time as a function of the ratio r; it also shows the variation of the optimal ground holding time for the A model as a function of the same parameter. The following points are worth noting:

Figurte 3.11: Linear Uniform Case
Ground Holding vs. Marginal Cost Ratio



- We can see that the optimal ground holding time is a downward sloping function of the ratio r. This observation is somewhat equivalent to the one made earlier regarding the decreasing cost saving ratio. In fact, when r increases. ground holding becomes more expensive and thus less attractive in monetary terms. It has been pointed out in the discussion of the particular case where departure and arrival random variables are assumed to be identically distributed, that the optimal ground holding time becomes equal to zero above a certain value for r. In the current choice of parameters, departure and arrival R.V. are not identically distributed but the result still holds with a threshold value for r equal to about 40%. This can be seen as an illustration of the general result derived from the optimality theorem for any choice of distributions.

- We can also note that the optimal ground holding time exhibits a less than linear shape as a function of r. This result is also very similar to the one obtained for the particular case of identically distributed departure and arrival random variables and where the optimal d_g was proportional to the square root of r. However, it can be noted that the optimal ground holding time for Andrews' model exhibits a linear shape as a function of r. This last observation is explained by the fact that Andrews' optimum is obtained by applying the optimality condition with a departure random variable set equal to d_g with probability one, which removes the nonlinearity (see expression (3.19) where $D_d = d_g$ with probability one).

- Unlike the Gaussian case, where the model provided an infinite ground holding time for r equal to zero, the uniform case will in all instances provide a bounded ground holding time regardless of the value of r. The upper bound U_o is given by the following equation:

$$U_o = Max\left(0, \sqrt{3} \times (\sigma_d + \sigma_a) + \mu\right) \qquad (3.27)$$

One interesting consequence of (3.27) is that when the expected departure average delay is larger than the expected arrival delay plus $\sqrt{3} \times (\sigma_d + \sigma_a)$ and $\sigma_d \leq \sigma_a$, no

ground holding is warranted. This could form the basis for a practical and simple-to-implement rule that can be followed in order to minimize the expected cost of delays.

B. Sensitivity to the arrival standard error σ_a

An ongoing debate within the aviation community concerns the value of having better delay forecasts. The following two numerical examples contribute to answering to this question by quantifying the dollar savings achieved by better delay forecasts in the case where the D/A model is used as the "baseline" and where departure and arrival delays are assumed to be independent and uniformly distributed.

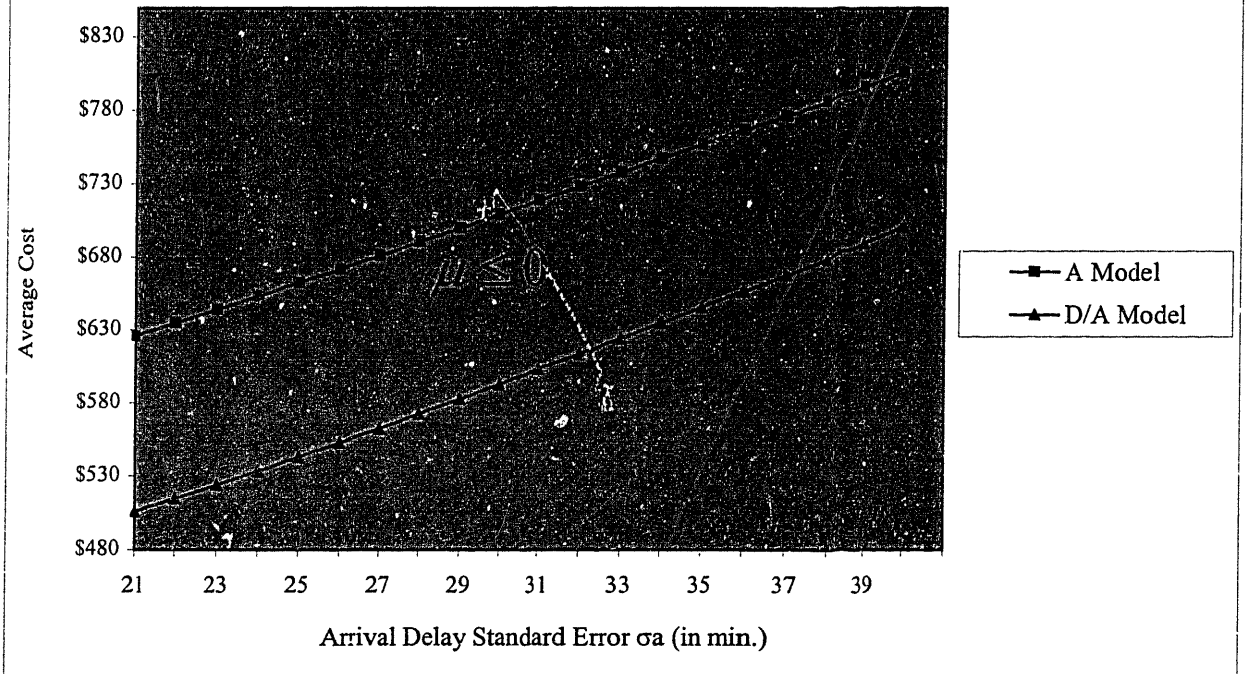
Figure 3.12 shows the average cost of delays (ground and airborne) for the two models being compared (D/A and Andrews) as a function of the arrival delay uncertainty. In this figure the departure and arrival random variables were uniformly distributed the former having (30,20) as mean and standard deviation which corresponds to a delay ranging from about –5 to 64 minutes and the latter having 15 as mean. The following points are worth noting:

- As expected the average cost of delays is an upward sloping function of arrival uncertainty. This observation confirms the rule of thumb that less uncertainty lowers the cost of delays.
- It can also be noted that the average cost of delays for both the optimal ground holding strategy and Andrews' model vary linearly as functions of the arrival delay standard error. As a consequence, the marginal cost of delays is constant which means that the elasticity of cost to uncertainty is equal to one. Practically, this implies that if we improve the performance (in terms of standard error) of the arrival delay forecast system by 10% we would reduce the cost of delays by an equal percentage.
- It is also worth noting that the average cost of delays for the optimal ground holding strategy and for Andrews' model have the same slope. This last observation means that the extra cost incurred by following Andrews' strategy as compared to the D/A

strategy is constant regardless of the amount of uncertainty that we have in the delay-forecast system.

- For a cost ratio of about 30%, the slope of the average delay cost function is equal to $ 9.5 per minute meaning that for this choice of parameters one minute of uncertainty costs us $9.5.

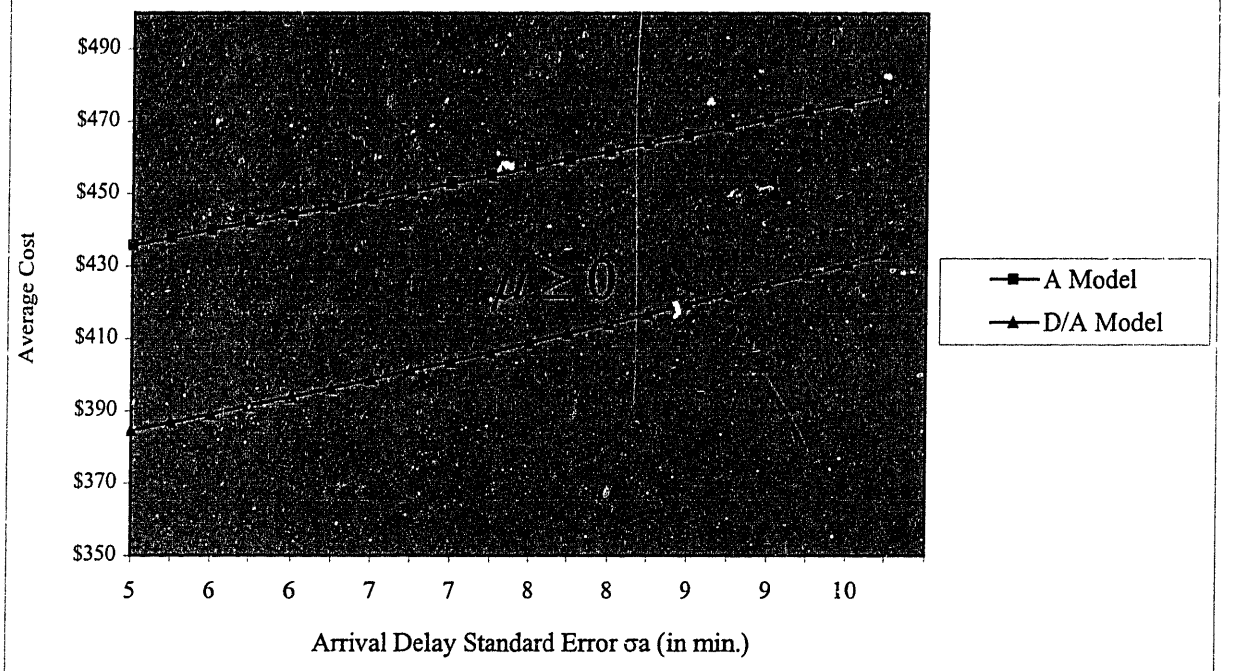
Figure 3.12: Linear Uniform Case
Average Cost vs. Arrival Uncertainty



- Figure 3.13 shows the average cost of delays (ground and airborne) for the two models under comparison (i.e., A and D/A models) as a function of the arrival delay uncertainty for a different choice of parameters. In this figure the departure and arrival random variables were uniformly distributed, the former having (10,5) as mean and standard errors and the latter having 30 as mean. We can observe that the

comments made on the previous simulation still apply to this case as well, regardless of the sign of μ.

Figure 3.13: Linear Uniform Case
Average Cost vs. Arrival Uncertainty



CHAPTER 4

NONLINEAR DEPARTURE AND ARRIVAL STOCHASTIC MODEL

4.1 Introduction:

We have studied in Chapter 3 the optimal ground holding problem with both departure and arrival uncertainties under the assumption of a linear delay cost function. That chapter presented results on the expected cost savings, as compared to an ATM system where no ground holding is used, as well as on the increase in the expected total delay under the proposed model. It also presented a mathematical expression for the optimal ground holding time where no assumption is made on the probability distribution of the departure and arrival delay random variables.

The cost linearity assumption made earlier applies to both the costs of ground and airborne delays. In real word situations both costs are usually nonlinear. In fact, the marginal disutility (i.e. marginal cost) of delays is expected to increase when delays increase. We will explore in this Chapter the implications of having a nonlinear cost function for the airborne delay part but we will still assume the ground delay cost to be linear. In modeling the former as nonlinear we want to discourage (i.e. penalize) excessive airborne delay. A more general treatment of the case where both delays are nonlinear was not done as part of this research, but could be undertaken following a similar approach as the one presented in this Chapter.

We will consider in this Chapter a nonlinear and convex cost function for the airborne delay, approximated by a piecewise linear function (Figure 4.1) with an associated sequence of thresholds (B_i). The main advantage in using a piecewise linear cost function is the ability to capture both the convexity of the function and the threshold feature, while at the same time making it possible to derive explicitly the optimality conditions and to solve them to find the optimal ground holding time.

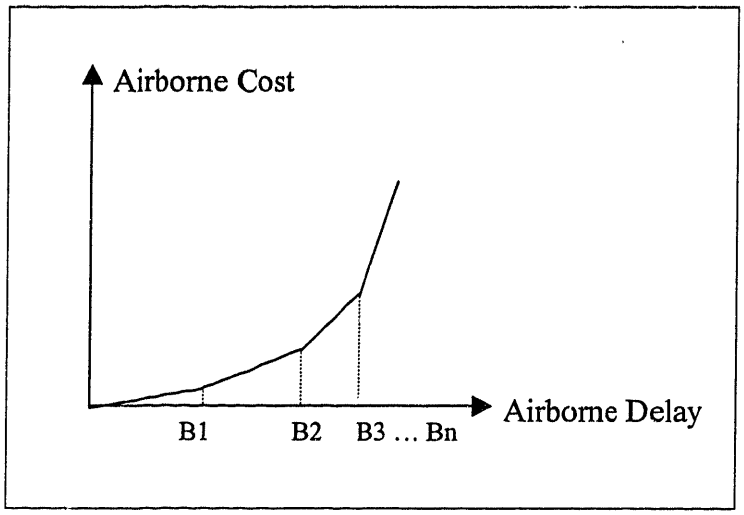


Figure 4.1: Nonlinear Delay Cost Function

4.2 Model Formulation:

We will use throughout the analysis the notation defined in Chapter 3. Furthermore, we will first derive optimality conditions for the piecewise linear case with only one threshold denoted as B. Under this last condition the total cost function can be written as follows:

$$C = \begin{cases} C_g \times D_d & \text{for} \quad D_d \geq D_a \quad (4.1a) \\ C_g \times D_d + C_a \times (D_a - D_d) & \text{for} \quad 0 \leq D_a - D_d \leq B \quad (4.1b) \\ C_g \times D_d + C_a \times B + (1+\alpha) \times C_a \times (D_a - D_d - B) & \text{for} \quad B \leq D_a - D_d \quad (4.1c) \end{cases}$$

where α indicates the increase in the slope for the airborne cost once a delay of B is reached. In order to satisfy the convexity assumption, we will require α to be positive. Furthermore, if we assume both D_d and D_a to be normally distributed, we obtain equation (4.2) for the total expected cost of delays.

$$E[C] = C_g \times (\mu_d + d_g) + C_a \times \int_{dg}^{+\infty} (z - dg) \times f_Z(z)\, dz + \alpha \times C_a \times \int_{dg+B}^{+\infty} (z - dg - B) \times f_Z(z) dz \quad (4.2)$$

So, by using (4.2) and noting that the average linear cost function satisfies

$$\bar{C}_L(d_g) = E[C] \qquad for \quad B = +\infty \qquad (4.3)$$

we obtain by decomposing (4.1) into a sum of linear cost functions and using (4.3) that:

$$E[C] = \bar{C}_L(d_g) + \alpha \times \bar{C}_L(d_g + B) - \alpha \times C_g \times (\mu_d + d_g + B) \quad (4.4)$$

This last expression for the expected cost of delays allows us to derive easily the optimality conditions for the nonlinear case using the optimality conditions already obtained for the linear one. By differentiating (4.4) and using Lemma 1 from section 3.3, we obtain the optimality condition P2.

$(P1)$ $d_g \geq 0$ $\quad (4.5a)$

$(P2)$ $F_Z(d_g) + \alpha \times F_Z(d_g + B) = 1 + \alpha - r$ $\quad (4.5b)$

$(P3)$ $\dfrac{\partial^2 E[C / d_g]}{\partial d_g^2} \geq 0$ $\quad (4.5c)$

The other two conditions (i.e. P1 and P3) are associated with the non-negativity of the ground holding decision variable and the cost convexity condition for the cost function.

It should be noted that condition P3 is always true given that the average cost function is globally convex. We can also note from the complexity of condition P2 that a closed form solution to this problem cannot be derived in the general case (i.e., for any distribution) and thus this problem has to be solved numerically.

We define a new function G_z as follows:

$$G_z(X) = F_Z(X) + \alpha \times F_Z(X + B) - (1 + \alpha) + r$$

We can easily verify that this function is strictly upward sloping and continuous and therefore invertible. Furthermore, it satisfies the following two conditions:

$$G_Z(-\infty) = -(1+\alpha) + r \leq 0 \quad and \quad G_Z(+\infty) = r \geq 0$$

We can then conclude that condition P2 has a unique solution. This last result allows us to derive the optimal value of the ground holding time as follows:

$$d_g^{Optimal} = Max\ (0\ , G_Z^{-1}(0)) \qquad (4.6)$$

Result (4.6) suggests the following observations:

- If we set α equal to zero or B equal to $+\infty$ in (4.5b) we find the expression derived for the linear case. In fact, these two particular examples correspond to the linear case.
- Unlike the linear case, if we set r = 1 (same marginal cost for both the ground and airborne delays) the optimal strategy is not bound to have no ground holding. We could have instances where, even though $C_g = C_a$, some ground holding is warranted.
- As in the case of the linear cost function, a higher value for the ratio r implies lower optimal ground holding, all other things being equal. This is obviously the result one would expect, since increasing the marginal cost of ground delay makes it less attractive. This result could easily be proven using the fact that the function G previously defined is upward sloping.
- If we consider the optimal value of the ground holding time as a function of the value of the threshold B, all other things (i.e. α, μ and σ) being equal, we can easily prove that this value is an upward sloping function. In fact, the higher the value of B, the less impact the end tail of the airborne cost curve (i.e. the part that has a slope of $(1+\alpha)\times C_a$) has on the optimal value of the ground holding. When B is given a large value, the optimal ground holding time is very close to the one obtained with the linear cost function. This last result could be intuitively understood by the fact that, when B is large, the impact of

the extra marginal cost penalty $(1+\alpha)\times C_a$ occurs for very large delays corresponding to the end tail of the delay distribution which is usually associated with small probabilities and thus has a small impact on the total expected cost.

4.3 Optimality Conditions for the General Case:

So far we have considered only a piecewise linear function with only one threshold. However, in reality, we might have two or more such thresholds and we need to extend the results we found in the previous section to such cases.

If we consider a piecewise linear cost function for the airborne delay with several thresholds we have the following proposition.

Theorem 4.1:

We denote (B_i) for i varying from 1 to N the increasing sequence of thresholds for the airborne delay cost function with the corresponding positive incremental slope (α_i) and Z the random variable defined in Chapter3. We have the following optimality conditions:

$$(P1) \quad d_g \geq 0 \quad (4.7a)$$

$$(P2) \quad F_Z(d_g) + \sum_{i=1}^{N} \alpha_i \times F_Z(d_g + B_i) = 1 + \sum_{i=1}^{N} \alpha_i - r \quad (4.7b)$$

$$(P3) \quad \frac{\partial^2 E[C]}{\partial d_g^2} \geq 0 \quad (4.7c)$$

Proof:

We will prove this result by induction on the number of thresholds N for the optimality condition (P2). The other two conditions are straightforward to prove. In fact, condition (P1) is imposed on d_g only to avoid having negative solutions that do not make sense, while condition (P3) is similarly related to the fact that the cost function is convex. The condition (P2) has been proven in the previous section for N=1. Let's assume that the

result is true for rank N and try to prove it for N+1. For N+1 we have an airborne delay cost function C^{N+1} that has N+1 thresholds with the corresponding N+1 incremental slopes. This function can be decomposed as the sum of a piecewise linear function C^N with N thresholds B_1 to B_N with the associated incremental slopes α_1 to α_N and a piecewise cost function with one threshold B_{N+1} and an incremental slope α_{N+1} as follows:

$$C^{N+1} = C^N + \alpha_{N+1} \times C_a \times (D_a - D_d - B_{N+1}) \times 1_{\{D_a - D_d - B_{N+1} \geq 0\}} \qquad (4.8)$$

Note that the second term in the summation (4.8) is proportional to the airborne cost of a linear problem where the decision variable is equal to $d_g + B_{N+1}$. So, if we take the expected value of the total cost of delays (ground and airborne) and use (4.3) and the observation we just made, we obtain the following result:

$$E[C^{N+1}] = E[C^N] + \alpha_{N+1} \times \bar{C}_L (d_g + B_{N+1}) - \alpha_{N+1} \times C_g \times (\mu_d + d_g + B_{N+1}) \qquad (4.9)$$

So by differentiating (4.9) and using (3.10) and the assumption we made with regard to the rank N we obtain that:

$$F_Z(d_g) + \sum_{i=1}^{N+1} \alpha_i \times F_Z(d_g + B_i) = 1 + \sum_{i=1}^{N+1} \alpha_i - r$$

[End of Proof]

It should be noted that Theorem 4.1 does not make any assumption on the distributions for departure and arrival delays and thus it is valid for any choice of distributions.

4.4 Case of Gaussian Delay Distributions

4.4.1 Introduction

The optimal ground holding time must be derived from a fairly complex equation, which does not have a closed form solution that can be used to compute the value of the optimum. It was therefore necessary to solve this problem numerically for a choice of parameters (i.e. r, α, μ, σ and B). The software Matlab was used to solve this problem numerically and a code was written to this effect.

This section summarizes the results of numerical examples intended to test both the sensitivity of the nonlinear model to different parameters, as well as to compare the performance of the different models that have been considered so far, namely no ground holding, the A model, and the linear and nonlinear D/A models. In all the following examples, the average delay cost index (i.e., average cost of the model considered divided by the average cost if no ground holding is used) was derived for the different models as a function of the given parameters in each case.

4.4.2 Sensitivity Analysis and Model Comparison

A. Sensitivity to the Parameter α:

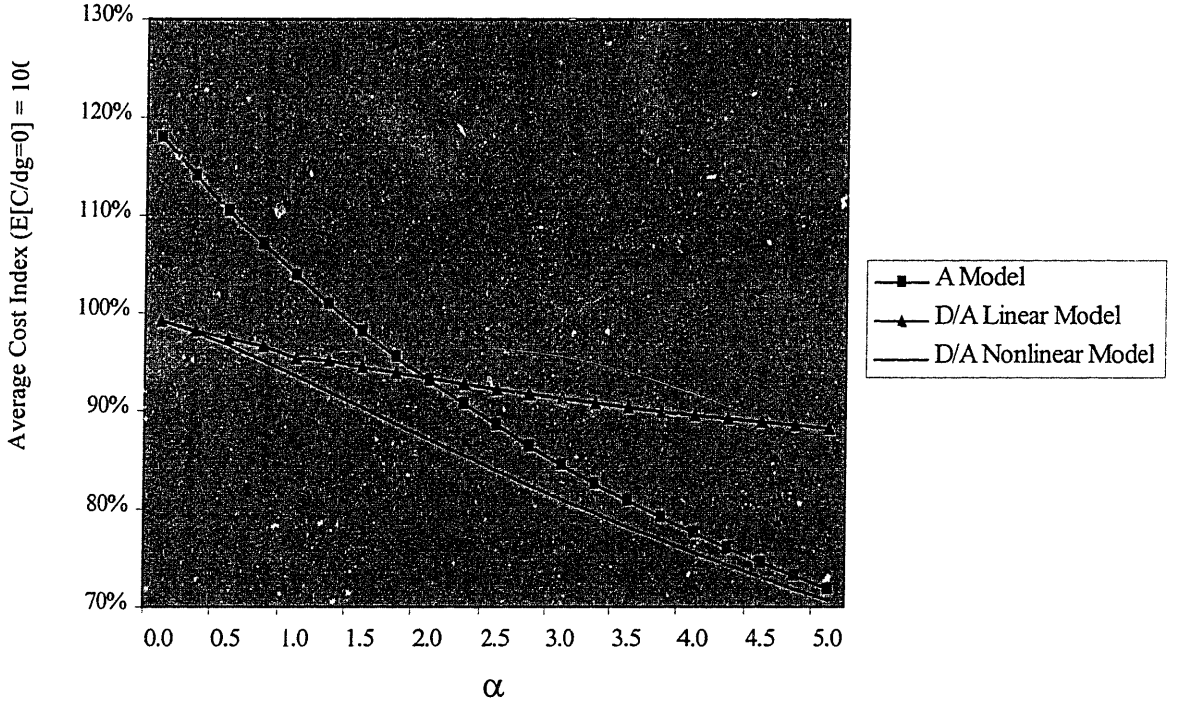
Figure 4.2 shows the average delay cost index for the three models considered so far. In this figure the departure and arrival R.V. were assumed Gaussian and have respectively (30,20) and (15,25) as mean and standard error expressed in minutes. No correlation was assumed, the cost ratio used was about 30% and a single threshold was set at 20 minutes. The following points are worth noting:

- Overall the nonlinear D/A model performs better than all the others. We can see that for small values of α, this model gives similar results to the linear D/A model. This should be expected since, as it has been pointed out earlier, when α is equal to zero the nonlinear cost function coincides with the linear one. On the other hand, for large values

of α, the nonlinear model gives similar results to the A model. It is not obvious why this is happening and it might be simply due to the choice of parameters. Thus, this may not be a general result.

- As one would expect, we obtain the intuitive result that higher values of α yield more average cost savings than the model where no ground holding is used. The intuition behind this result stems from the fact that a higher α penalizes more the airborne delay (beyond the threshold B) and thus encourages more ground holding.
- For small values of α, the linear D/A model outperforms Andrews' and vice versa for large values of the same parameter, with a crossing point for a value of α close to two. It is also worth noting that the A model performs worse than the no ground holding strategy for values of α less than 1.5.

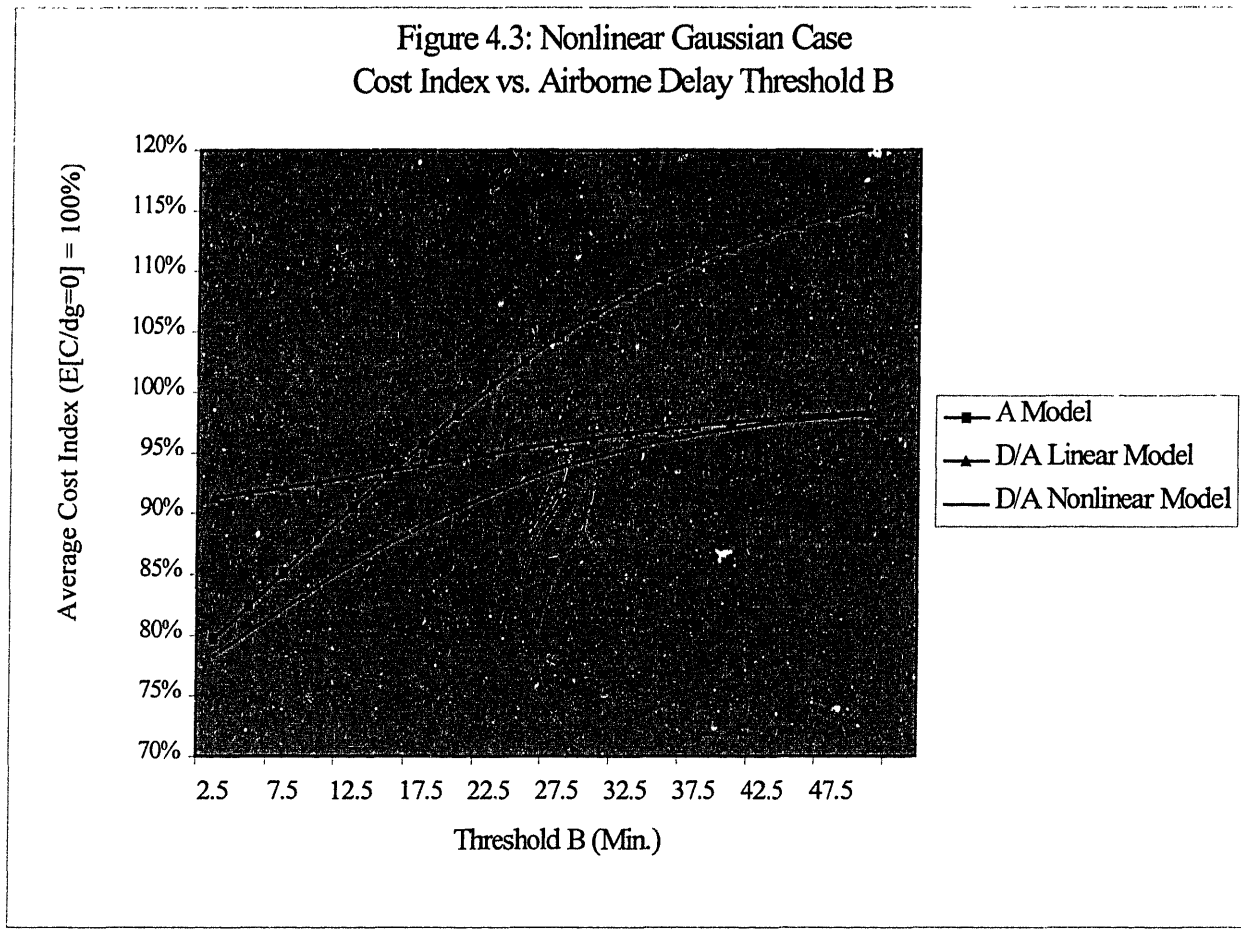
Figure 4.2: Nonlinear Gaussian Case
Cost Index vs. Incremental Slope α



B. Sensitivity to the Threshold Parameter B:

Figure 4.3 shows the average delay cost index for the three models considered so far. In this figure the departure and arrival R.V. were assumed Gaussian and have respectively (30,20) and (15,25) as mean and standard error expressed in minutes. No correlation was assumed, the cost ratio used was about 30% and α=1.5. The following points are worth noting:

- Overall the nonlinear D/A model outperforms the other models. However, we can see that for large values of the threshold B this model gives results similar to the linear D/A model. The reason for this is the same as the one mentioned in the previous section and stems from the fact that the linear and nonlinear models coincide when B goes to infinity. For small values of B the nonlinear model gives results similar to the A model. It is not easy to explain why this is happening and it might again only be due to the choice of data.
- As one would expect, we obtain the intuitive result that small values of B yield more average cost savings than a model where no ground holding is used. The intuition behind this result stems from the fact that a small value of B penalizes more small airborne delays since the threshold B is more likely to be reached.
- For large values of B, the linear D/A model outperforms the A model and vice versa for small values of the same parameter with a crossing point for a value of B close to 17 minutes. It is also worth noting that the A model performs worse than the no-ground-holding strategy for values of B greater than 25 minutes.
- It is also worth noting that the cost saving ratio function (1-Cost Index) for the nonlinear D/A model is a convex function. This last result means that the marginal cost saving ratio increases when B decreases. This is a result one should expect since low values of B are more likely to be reached and, as it has been pointed out earlier, to provide more of an advantage to the optimal ground holding strategy.



C. Sensitivity to the Cost Ratio r:

Figure 4.4 shows the average delay cost index for the nonlinear D/A, linear D/A and A models. In this figure the departure and arrival R.V. were assumed Gaussian and have respectively (30,20) and (15,25) as mean and standard error expressed in minutes. No correlation was assumed, a value of 1.5 was chosen for the parameter α and a value of 20 minutes for the threshold B. The following points are worth noting:

- Overall the nonlinear D/A model outperforms other models. However, we can see that the three models considered perform similarly for small values of the ratio r. This could be explained by the fact that when r is small any ground holding strategy will yield substantial savings as compared to no ground holding. It is important to point out that the

models we have developed so far do not introduce any cost penalty in case the airplane does not reach its destination which obviously is a major limitation for small values of r.

- For large values of r (close to one) the nonlinear D/A model and the linear model perform similarly and do not warrant any ground holding (Cost Index = 100%). As it has been pointed out earlier this is not always the case (see optimality condition P2) but depends on the values chosen for the parameters.
- Figure 4.4 also shows that the D/A linear model performs better than the A model. This result is not always true and does depend on the value chosen for the parameter α. For example, Figure 4.5 is analogous to the case of Figure 4.4, except that α=0.5. We can see in this case that there are instances where the A model performs better than the D/A linear one.
- It is also interesting to note that the cost saving ratio function (1-Index) is downward sloping, as one would expect.

Figure 4.4: Nonlinear Gaussian Case
Cost Index vs. Marginal Cost Ratio

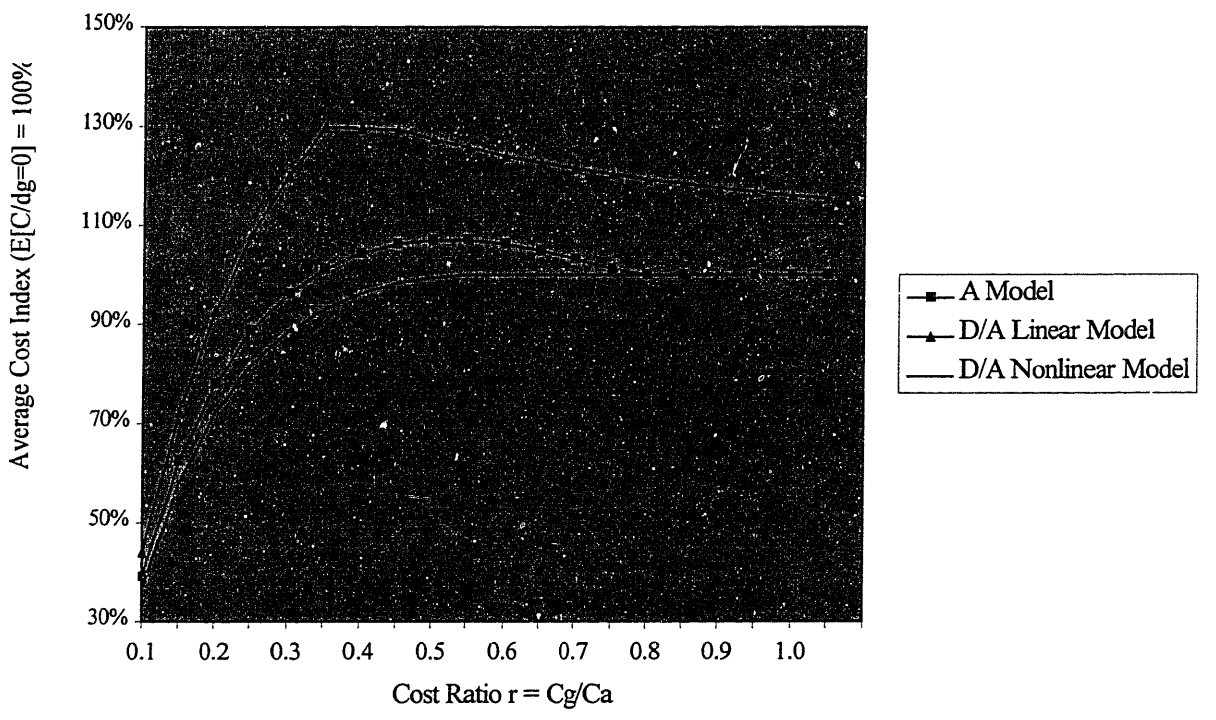
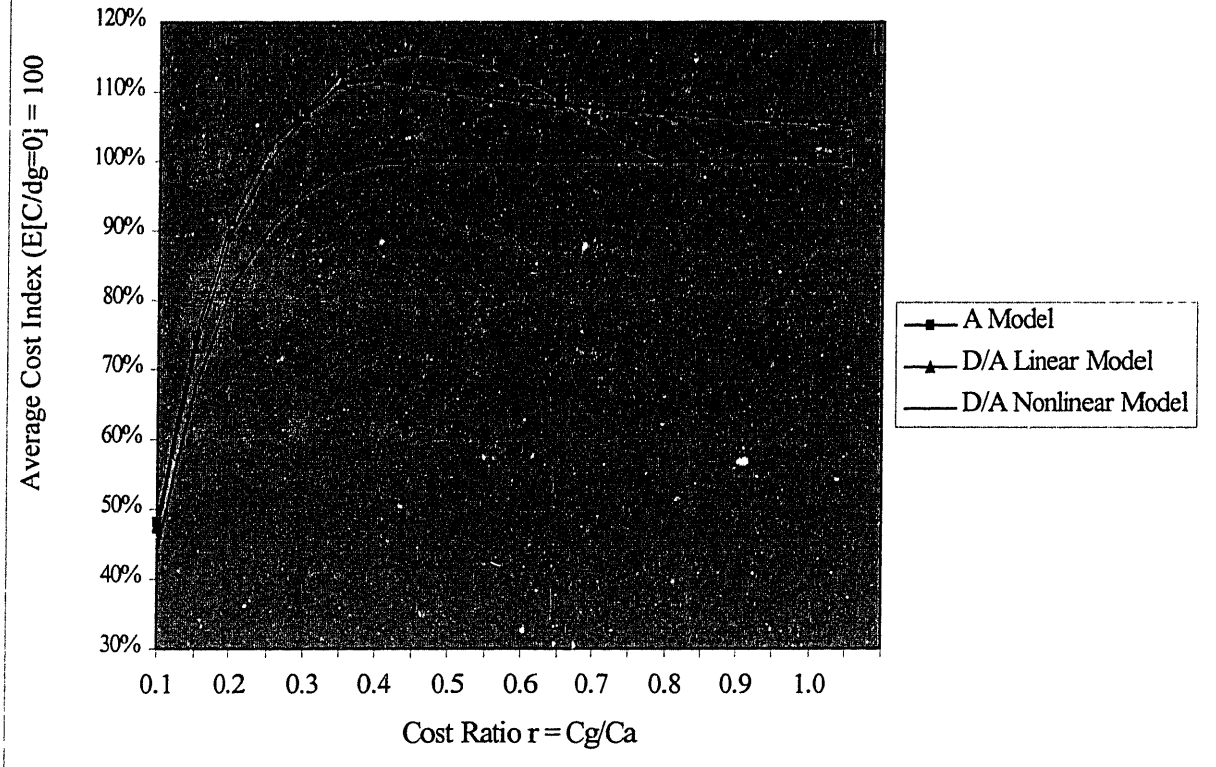


Figure 4.5: Nonlinear Gaussian Case
Cost Index vs. Marginal Cost Ratio



CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions and Discussion

The objective of this thesis was to assist the airlines in developing a methodology for determining for themselves the optimum gate departure time for a flight aiming at meeting a given CTA. This time was determined in the face of two principal sources of uncertainty:

(a) Uncertainty about how much delay the flight will experience in departing from the airport of origin, due to the fact that many airports are congested and it is very difficult to estimate in advance exactly how much taxi-out time will be needed to reach the departure runway and how much queuing there will be for take-off at the time of departure.

(b) Uncertainty about how much delay the flight will experience after arriving at the terminal airspace of the airport of destination, due to the fact that it is really impossible to predict accurately several hours in advance, how much landing delay a flight will suffer at a congested airport.

This thesis has shown that uncertainty in departure and arrival delays can be modeled in an analytical way and that the adoption of the FSFS policy leads to a tractable model. The model developed allows the decision-maker to determine ground holding times that minimize the expected cost of delays and to assess performance in terms of cost savings.

The sensitivity analyses carried out in this thesis showed that the D/A strategy outperformed both the A and passive ones for all cases considered. The D/A strategy yielded significant cost savings (i.e. more than 10%) for cases where uncertainty was relatively low or cases where the ratio of marginal costs was small. Furthermore, the D/A

model derived a couple of simple-to-implement rules for situations where the optimal strategy coincides with the passive one.

For the linear case, no ground holding is warranted when either the expected departure delay is much larger than the arrival one or when the ratio of marginal costs is larger than a certain value. For the nonlinear case, the no-ground holding optimal rule for large values of the ratio of marginal costs applies. Finally, when delays are uniformly distributed it was proved that the optimal ground holding time is bounded by a function of delay standard deviations.

More generally optimal ground holding times and their associated expected costs increase with uncertainty which means that more accurate delay forecasts will certainly yield cost savings. These cost savings have been quantified in the numerical computations carried out in this thesis. Given the large number of flights operated between pairs of congested US airports, much would be gained by investing in R&D to develop more accurate delay prediction systems.

Finally, in today's hub and spoke network a significant number of travelers connect at hub airports in order to reach their final destinations and large delays can make travelers miss their connections. Given this consideration, the linearity assumption in the cost of delays may, in some cases be inappropriate. The treatment of the nonlinear case proved that significant increase in costs could occur if the nonlinearity is not accounted for. The development of ground holding decision support tools should therefore build on the nonlinear case in order to capture the hub and spoke network structure and provide optimal strategies with lower expected cost of delays.

5.2 Recommendations for Future Research

There are many ways to improve the model developed in this thesis to account for practical concerns that may face the decision maker when he decides on how much ground holding he should impose on flights. This section presents some of the improvement opportunities that can be followed-up in future research.

5.2.1 Explore Alternative Cost Minimization Strategies

The strategy described in this thesis consisted in minimizing the expected cost of delays. However, another strategy could be to minimize the variance in the cost of delays or more generally devise a set of strategies that would span the whole range between this two strategies. This can be done by modifying the objective function and formulating it as a weighted sum of both the delay cost's mean and standard error. Every choice of the weight parameters would define a separate strategy depending on how much aversion the decision-maker has to fluctuations around the mean. The following equation captures this idea by defining a generalized cost $\underline{C}$ as follows:

$$\underline{C} = \alpha_1 \times E[C] + \alpha_2 \times Std\,[C] \quad \text{with} \quad \alpha_1 + \alpha_2 = 1$$

Moreover, the airline might be interested in preserving the connectivity of their network by making sure that connecting flights at hub-airports are not delayed too much so passengers can catch their connections. One way to capture this idea consists in determining the optimal ground-holding time that maximizes the probability that total delay is less than a certain value (typically the maximum amount of delay a flight can take without causing downstream effects in the network).

5.2.2 Incorporate Gaming Component in Model Formulation

An increasing concern regarding the adoption of the "Free Flight" concept and giving more decision-making powers to airlines in making takeoff decisions are potential negative safety impacts. The model developed does not account explicitly for that and ways to incorporate the safety element should be thought of which would make it more

reliable. A first step in achieving that could be to identify potentially "undesirable" incentives that airlines may have if the proposed D/A strategy is adopted. One example of a situation that should be avoided is to have too many airborne airplanes at the arrival airport, which might happen if everybody decides to forgo ground-holding at the departure airport.

5.2.3 Study the Dynamic Case

The D/A model developed assumed delays to be static which is often not true in the real world. The extension of this model to the dynamic case, where departure and arrival delays are functions of time would be a better representation of reality and would introduce the feedback loop feature that is missing in its current form. However, it is expected that an analytical treatment of the dynamic case could be very difficult and it is suggested that a simulation approach be adopted.

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