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HIERARCHICAL AGGREGATION OF DIFFUSION PROCESSES WITH MULTIPLE EQUILIBRIUM POINTS

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SUMMARY

When a dynamical system with multiple equilibrium points is perturbed by continous wide-band noise, it is known that transitions between different equilibrium points occur with probability one. An important problem associated with the analysis of these systems is the statistical characterization of the jump process which represents the transitions between different domains of attraction. A physical example of a dynamical system with multiple equilibrium point are common is an interconnected power system, where the swing equations [1] represent a system with many possible equilibrium angles, defined by a power balance between electrical supply and demand. When the demand fluctuations and unmodeled effects are represented as additive random noise, the resulting system undergoes transitions between the equilibrium points.

In this paper, we study the long-term behavior of a class of models with multiple equilibrium points and additive white noise distrubances. These models are characterized by the presence of a small parameter in the description of the process. The objective of the paper is to obtain a simplified aggregate model of the original process.

Specifically, we study the long-term behavior of the trajectories of the diffusion process

$$dx_t = f(x_t)\, dt + \varepsilon\, dw_t \tag{1}$$

where the function f is a gradient vector field, as

$$f(x_t) = \nabla g\,(x_t)$$

We construct a simple finite state process $A(x_t)$ by aggregating the state space of x about the equilibrium points. We proceed to show that this aggregated process converges in distribution to a Markov process z(T). In order to do this, we identify the appropriate time scales of evolution as a function of the potential g(x) and the exit times of the process in equation (1). In addition, we use expressions for the relevant exit distributions to define the transition rates of the approximate Markov process z. The accuracy of this approximation is established in the main proposition, which states that, in the slower time scales, $A(x_t)$ converges to z(T).

Let $h_i(\varepsilon)$ denote the expected mean time for the process to transition from equilibrium point x^i to another equilibrium point. Define $h(\varepsilon)$ as

$$h(\varepsilon) = \min_{i=1,..n} h_i(\varepsilon)$$

where n is the number of stable equilibrium points.

Let $P_{ij}(\varepsilon)$ be the probability that the process x_t leaves the domain of attraction of equilibrium point i for the domain of attraction of equilibrium point j. Define the finite state Markov process z(T) on the state space $\{x^1,\ldots x^n\}$ by its infinitesimal transition rates

$$r_{ij}(\varepsilon) = \frac{P_{ij}(\varepsilon)\, h(\varepsilon)}{h_i(\varepsilon)} \tag{2}$$

The main result can be stated as follows:

Let $A(x_t) = x^i$ if x_t is in the domain of attraction of the equilibrium point x^i.

Define the initial distribution of z(o) as:

$$\Pr\{z(o) = x^i\} = P\,\{A(x_o) = x^i\}.$$

<u>Theorem</u>: Consider any scale function $g(\varepsilon) \leq h(\varepsilon)$.

The finite dimensional distributions of the process $A(x_{t/g(\varepsilon)})$ converge as $\varepsilon \to o$ to the distributions of the process $z\left(T\,\frac{h(\varepsilon)}{g(\varepsilon)}\right)$ for all t in (δ,∞) for $\delta > o$. Furthermore, the finite dimensional distributions of the process $x_{t/g(\varepsilon)}$ will converge to those of $z\left(\frac{T\,h(\varepsilon)}{g(\varepsilon)}\right)$ also.

The theorem implies that the approximation z(T) captures <u>all</u> of the slow time behavior of the evolution of the x_t process. Under additional restrictions, this evolution can be decomposed further into a hierarchy of time scales, along the lines of [2]. Based on the above approximations, analytical expressions for the ergodic distributions can be derived. It is conjectured that the above approximations be used to construct useful approximate solutions to nonlinear filtering problems with small diffusion intensities.

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