Essays on Internet Markets and Information Goods

by

Yu (Jeffrey) Hu

B.S. Finance, Tsinghua University, 1997
M.S. Economics, University of Wisconsin-Madison, 1999

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Signature of Author: __________________________

Alfred P. Sloan School of Management
October 6, 2004

Certified by: __________________________

Erik Brynjolfsson
George and Sandi Schussel Professor of Management
Thesis Supervisor

Accepted by: __________________________

Birger Wernerfelt
Professor of Management Science
Chair, Doctoral Program
ABSTRACT

This dissertation consists of three essays on Internet markets and information goods.

The first essay, "Consumer Surplus in the Digital Economy: Estimating the Value of Increased Product Variety at Online Booksellers", presents a framework and empirical estimates that quantify the economic impact of increased product variety made available through electronic markets. While efficiency gains from increased competition significantly enhance consumer surplus, for instance by leading to lower average selling prices, our present research shows that increased product variety made available through electronic markets can be a significantly larger source of consumer surplus gains. One reason for increased product variety on the Internet is the ability of online retailers to catalog, recommend and provide a large number of products for sale. For example, the number of book titles available at Amazon.com is over 23 times larger than the number of books on the shelves of a typical Barnes & Noble superstore and 57 times greater than the number of books stocked in a typical large independent bookstore. Our analysis indicates that the increased product variety of online bookstores enhanced consumer welfare by $731 million to $1.03 billion in the year 2000, which is between seven to ten times as large as the consumer welfare gain from increased competition and lower prices in this market. There may also be large welfare gains in other SKU-intensive consumer goods such as music, movies, consumer electronics, and computer software and hardware.

The second essay, "Performance-based Pricing Models in Online Advertising", applies the economic theory of incentive contracts to the study of performance-based pricing models in online advertising and provides explanations as to when and how incorporating them into advertising deals can be profitable. We argue that using these pricing models appropriately can give both the publisher and the advertiser proper incentives to make non-contractible efforts that may improve the effectiveness of advertising campaigns. It also allows the publisher and the advertiser to share the risk caused by uncertainty in the product market. We show that key factors that influence the use of performance-based pricing models are the importance of the publisher’s incremental efforts, precision of click-through measurement, uncertainty in the product market, and risk aversion parameters. We also clarify issues that are being debated in the industry, such as how the importance of the advertiser’s incremental efforts and existence of non-immediate purchases affect the use of performance-based pricing models.
The third essay, "Renting versus Selling Durable Information Goods", studies whether a monopoly producer of a durable information good should sell or rent its good to consumers. We study whether the producer obtains a higher profit under a selling strategy or a renting strategy. Our analysis shows that the conventional wisdom that a durable good monopolist would always prefer renting to selling is no longer valid in the context of durable information goods, because of the existence of "individual depreciation". We find that a renting strategy leads to a higher producer surplus than a selling strategy does, when this individual depreciation parameter is high, i.e., the utility a durable information good provides to consumers decreases relatively slowly from the first consumption to the second consumption and so on. But when the individual depreciation parameter is low, a renting strategy may lead to a lower producer surplus than a selling strategy does. Whether a monopoly producer of a durable information good should adopt a renting strategy depends on the individual depreciation parameter of the good.

Thesis Committee:

Erik Brynjolfsson, George and Sandi Schussel Professor of Management, Chair
Jerry A. Hausman, John and Jennie S. MacDonald Professor of Economics
Duncan Simester, Professor of Management Science
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Estimating the Value of Increased Product Variety at Online Booksellers

Doctoral Thesis Chapter 1

Yu (Jeffrey) Hu  
MIT Sloan School of Management

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Consumer Surplus in the Digital Economy: Estimating the Value of Increased Product Variety at Online Booksellers

ABSTRACT

We present a framework and empirical estimates that quantify the economic impact of increased product variety made available through electronic markets. While efficiency gains from increased competition significantly enhance consumer surplus, for instance by leading to lower average selling prices, our present research shows that increased product variety made available through electronic markets can be a significantly larger source of consumer surplus gains.

One reason for increased product variety on the Internet is the ability of online retailers to catalog, recommend and provide a large number of products for sale. For example, the number of book titles available at Amazon.com is over 23 times larger than the number of books on the shelves of a typical Barnes & Noble superstore and 57 times greater than the number of books stocked in a typical large independent bookstore.

Our analysis indicates that the increased product variety of online bookstores enhanced consumer welfare by $731 million to $1.03 billion in the year 2000, which is between seven to ten times as large as the consumer welfare gain from increased competition and lower prices in this market. There may also be large welfare gains in other SKU-intensive consumer goods such as music, movies, consumer electronics, and computer software and hardware.

(Consumer surplus; Product Variety; Electronic Commerce; Welfare; Internet)
1. Introduction

"Clearly, new goods are at the heart of economic progress. But that realization is only the beginning of an understanding of the economics of new goods. The value created by new goods must somehow be converted into an exact quantitative measure..."

Timothy F. Bresnahan and Robert J. Gordon (1997, p. 1)

"The Internet is providing access for people who just can't find the book they are looking for in a store."


Information technology facilitates the delivery of many new products and services over electronic networks. As these electronic networks develop and mature, it will be important to quantify their value for customers, merchants, shareholders, and society. While much of the attention in academic research and in the press has been on the relative operational efficiency of the online channel versus traditional channels, we believe that important benefits lie in new products and services made available through these channels. While it has been relatively easy to quantify the operational costs of each channel, the value of new products and services made available through electronic networks has remained unquantified. By default, this value has been ignored, effectively treating convenience and selection as if its value were zero.

Our research focuses on increased product variety, which is one category of new products and services made available through electronic networks. Internet retailers have nearly unlimited "virtual inventory" through centralized warehouses and drop-shipping agreements with distributors (e.g., Bianco 1997 and Mendelson and Meza 2002). Because of this, they can offer convenient access to a larger selection of products than brick-and-mortar retailers can. Table 1 shows the difference between the number of items available at Amazon.com and a typical large
brick-and-mortar retailer for several consumer product categories.\textsuperscript{1} For example Amazon.com and Barnesandnoble.com provide easy access to each of the 2.3 million books in print (and millions more used and out of print titles) while conventional brick-and-mortar stores carry between 40,000 and 100,000 unique titles on their shelves. Thus, online consumers are easily able to locate, evaluate, order, and receive millions of books that are not available on the shelves of local bookstores. Large differences in product variety are also seen in music, movies, and consumer electronics products. Even Wal-Mart Supercenters, which range in size from 109,000 to 230,000 square feet, only carry one-sixth of the number of SKUs that are carried by Walmart.com (Owen 2002).

<table>
<thead>
<tr>
<th>Product Category</th>
<th>Amazon.com</th>
<th>Typical Large Brick-and-Mortar Store</th>
</tr>
</thead>
<tbody>
<tr>
<td>Books</td>
<td>2,300,000</td>
<td>40,000 – 100,000</td>
</tr>
<tr>
<td>CDs</td>
<td>250,000</td>
<td>5,000 – 15,000</td>
</tr>
<tr>
<td>DVDs</td>
<td>18,000</td>
<td>500 – 1,500</td>
</tr>
<tr>
<td>Digital Cameras</td>
<td>213</td>
<td>36</td>
</tr>
<tr>
<td>Portable MP3 players</td>
<td>128</td>
<td>16</td>
</tr>
<tr>
<td>Flatbed Scanners</td>
<td>171</td>
<td>13</td>
</tr>
</tbody>
</table>

While some of these products may be available from specialty stores or special ordered through brick-and-mortar stores, the search and transaction costs to locate specialty stores or place special orders are prohibitive for most consumers.\textsuperscript{2} In addition, the enhanced search features and

\textsuperscript{1} Inventory values for Amazon.com were obtained from discussions with industry executives, industry estimates and Bowker’s Books in Print database (books), wholesale suppliers to Amazon.com (CDs), and direct counting of normally stocked items. Inventory values for brick-and-mortar stores were obtained from interviews with managers and direct observation of inventory for Barnes and Noble Superstores (Books, CDs, DVDs), Best Buy (CDs, DVDs, Digital Cameras, Portable MP3 Players, Flatbed Scanners) and CompUSA (Digital Cameras, Portable MP3 Players, Flatbed Scanners).  

\textsuperscript{2} To illustrate this difference, on November 26, 2001 one of the authors ordered the same book through Barnesandnoble.com and through a special order at a local Barnes & Noble Superstore. The Barnesandnoble.com order process took 3 minutes to place, arrived in 3 days, and cost $31.99. The Barnes and Noble order took nearly 1 hour to place, took 8 days to arrive, and cost $37.45. The store was located 4.6 miles away from the author's house (note that Brynjolfsson and Smith (2000) found that the average person in the United States lives 5.4 miles away.
personalized recommendation tools offered by Internet retailers allow consumers to locate products that would have remained undiscovered in brick-and-mortar stores. For instance, Amazon uses at least seven separate “recommender systems” to help advise customers on their purchases. Via these systems, new products, including obscure books, are brought to the attention of shoppers visiting their site or emailed as suggestions to past shoppers.

Recommender systems have the potential to automate “word of mouth”, speeding the discovery and diffusion of new goods (Resnick and Varian 1997).

In effect, the emergence of online retailers places a specialty store and a personalized shopping assistant at every shopper’s desktop. This improves the welfare of these consumers by allowing them to locate and buy specialty products they otherwise would not have purchased due to high transaction costs or low product awareness. This effect will be especially beneficial to those consumers who live in remote areas that do not have specialty retailers.

As one might expect, the lower transactions costs offered by the Internet have led to increased orders for many titles not previously stocked in brick-and-mortar stores. Frank Urbanowski, Director of MIT Press, attributed the 12% increase in sales of backlist titles directly to increased accessibility to these titles through the Internet (Professional Publishing Report 1999). Similarly, Nora Rawlinson, the editor of Publishers Weekly, observes:

“Publishers are finding that books on their backlists are suddenly selling well. Bookstores are great for browsing but they are difficult places to find a specific title…The Internet is providing access for people who just can’t find the book they are looking for in a store.” (Lyster 1999)

from the closest bookstore). The time to place the order included 21 minutes of driving time (round-trip) to place the order; 8 minutes to park, search for the book in the store, search for a sales person, and place the order; 20 minutes of driving time to pick up the order; and 9 minutes to park and pay for the special order.
The differences in variety reflect underlying differences in the technology and economics of conventional and Internet retailers. As noted by Saul Hansell in the New York Times:

"The average book may sit on the shelf of a store for six months or a year before it is bought. The cost of this inventory in a chain of hundreds of stores is huge. Amazon can keep just one or two copies in its warehouse — and still make the title available to the whole country — and restock as quickly as customers buy books." (Hansell 2002)

Further, anecdotal evidence suggests that consumers place a high value on the convenience offered by Internet retailers when locating and purchasing obscure products. For example, Yahoo/ACNielsen's 2001 Internet Confidence Index lists “wide selection of products” as one of the top three drivers of consumer ecommerce based on a survey of Internet purchasers. However, no systematic estimates exist to empirically quantify the dollar value consumers place on the increased product variety available through Internet markets.\(^3\)

This paper represents a first effort to apply a methodology for estimating this value to one prominent category of products offered by Internet retailers — obscure book titles. Our methodology uses a small set of generally available statistics that track how consumers “vote with their dollars,” and thus may find application in a variety of product categories. The resulting estimates of consumer surplus will have important economic and public policy implications, especially as investors and managers try to understand and evaluate the value proposition of Internet-based commerce.

The remainder of the paper proceeds as follows. Section 2 presents the economic literature pertaining to consumer welfare gains from new goods and increased product variety. Section 3 develops a methodology to measure consumer welfare from increased product variety offered in

\(^3\) Israilevich (2001) uses a theoretical model of product differentiation to calibrate how lower fixed costs of selling books on the Internet may have led to increased variety, and estimates the welfare implications of it.
Internet markets. Section 4 applies this methodology to obscure book sales over the Internet and section 5 concludes with some broader implications. Appendix A summarizes both the model development (Section 3) and the data necessary to calibrate the model in a general market environment (Section 4).

2. Literature Review

The development of an empirical methodology to estimate the welfare change resulting from price changes can be traced to Hicks' (1942) compensating variation measure. Historically compensating variation has been difficult to measure because it involves integration of the unobservable Hicksian compensated demand curve. However, Hausman (1981) develops a closed-form solution for measuring compensating variation under standard linear or log-linear demand functions. More recently, Hausman (1997a) shows that the welfare effect of the introduction of a new product is equivalent to the welfare effect of a price drop from the product’s “virtual price”, the price that sets its demand to zero, to its current price. Applying this technique, he estimates that the FCC’s decision to delay the introduction of two telecommunication services has reduced U.S. consumer welfare by billions of dollars a year. Subsequently researchers have examined the welfare effects of other new products in traditional markets, using similar or more refined models. Examples include Hausman (1997b), Nevo (2001), Goolsbee and Petrin (2001), Petrin (2001), Hausman and Leonard (2001). In addition, Bresnahan (1986) and Brynjolfsson (1995) have looked at welfare gains from information technology investments.

Researchers in the field of macroeconomics have also started to pay attention to new products or new varieties of products. Bils and Klenow (2001) find that consumer spending has shifted away
from products that have shown little variety gain. The Stigler commission (NBER 1961) and the Boskin Commission (Boskin Commission Report 1996) conclude that the greatest flaw in the Consumer Price Index is its failure to account adequately for new goods and quality improvements in existing goods.

It is also worth noting that there is a large body of marketing literature examining the relationship between perceived variety and actual assortment. Most researchers agree that consumers generally prefer more variety when given a choice (e.g., Baumol and Ide 1956 and Kahn and Lehmann 1991). More recently, researchers have shown that consumers’ perception of variety is influenced not only by the number of distinct products, but also by the repetition frequency, organization of the display, and attribute differences (e.g., Dreze, Hoch and Purk 1994; Broniarczyk, Hoyer and McAlister 1998; Hoch, Bradlow and Wansink 1999; Van Herpen and Pieters 2002). In this paper, we focus on the impact that increased availability of products in the online channel has on consumers’ actual purchase behavior. Thus, questions of shelf space and consumer perceptions are muted relative to the actual assortment of products and observed consumer behavior.

3. Methodology

This paper applies and extends existing welfare estimation techniques to measure the consumer welfare gain from the increased product variety made available through electronic markets. To do this, we define the total effect of the introduction of new products in online markets on consumer welfare as the difference in the consumer’s expenditure function before and after the introduction, measured at the level of post-introduction utility:

\[ CV = e(p_e0, p_n0, u_1) - e(p_{e1}, p_{n1}, u_1), \]  

(1)
where $p_{e0}$ and $p_{el}$ are the vectors of pre- and post-introduction prices of existing products respectively, $p_{n0}$ is the virtual price of the new product (the price that sets demand to zero), $p_{n1}$ is the post-introduction price of the new product, and $u_i$ is the post-introduction utility level. In effect, equation (1) measures how much a pre-Internet consumer would need to be compensated in order to be just as well off as they would be after the emergence of online markets.

We then follow Hausman and Leonard’s (2001) derivation to break the total effect into the variety effect resulting from the availability of the new product and the price effect resulting from changes of prices of existing products:

$$CV = [e(p_{el}, p_{n0}, u_i) - e(p_{el}, p_{n1}, u_i)] + [e(p_{e0}, p_{n0}, u_i) - e(p_{el}, p_{n0}, u_i)].$$

(2)

When the vector of prices of existing products does not change before and after the introduction of the new product, i.e., $p_{e0} = p_{el} = p_e$, one only needs to measure the variety effect and we can redefine the expenditure function such that $e(p_{e},.,.) = e'(.,.)$:

$$CV = e(p_{e}, p_{n0}, u_i) - e(p_{e}, p_{n1}, u_i) = e'(p_{n0}, u_i) - e'(p_{n1}, u_i).$$

(3)

The assumption that $p_{e0} = p_{el} = p_e$ appears to be valid in our empirical context because the overwhelming majority of book prices charged by brick-and-mortar stores have not changed as a result of the emergence of online markets. Nearly all brick-and-mortar stores sold most titles at the manufacturer’s suggested list price before the emergence of online markets and continue to do so today. Moreover, most studies have shown that, if anything, Internet retailers tend to increase competition and place downward pricing pressure on brick-and-mortar retailers (e.g., Brynjolfsson and Smith 2000; Scott Morton, Zettelmeyer, and Silva-Risso 2001; Brown and Goolsbee 2002; Baye, Morgan and Scholten 2002). Thus, if brick-and-mortar prices were to
change at all, we would expect them to decline. Our calculations under the zero price change assumption would therefore underestimate true gains in consumer surplus.

To apply equation (3) in practice, we specify a standard log-linear demand function for the new product made available by the Internet,

$$x(p, y) = Ap^\alpha y^\delta,$$  \hspace{1cm} (4)

where \(p\) is the price of the new product, \(y\) is the income, \(\alpha\) is the price elasticity, and \(\delta\) is the income elasticity. This specification is the most widely used specification in the literature of demand estimation and it fits a wide variety of data well (e.g., Brynjolfsson 1995, Hausman 1997a, 1997b, and Hausman and Leonard 2001).\(^4\)

Following Hausman (1981), we can use Roy’s identity to write equation (4) as

$$x(p, y) = -\frac{\partial v(p, y)}{\partial p} / \frac{\partial v(p, y)}{\partial y},$$  \hspace{1cm} (5)

where \(v(p, y)\) is the indirect utility function.

Solving this partial differential equation gives

$$v(p, y) = -A \frac{p^{1+\alpha}}{1+\alpha} + y^{1-\delta}$$  \hspace{1cm} (6)

and the expenditure function

$$e(p, u) = \left[ (1-\delta) \left( u + A \frac{p^{1+\alpha}}{1+\alpha} \right) \right]^{1/(1-\delta)}.$$

\(^4\) The final result of welfare estimates will depend on the adopted specification of demand function. However, earlier research (e.g. Brynjolfsson 1995 and Hausman and Newey 1995) finds that using a nonparametric specification with complete freedom to fit the data may not significantly improve the accuracy of welfare estimates over estimates using a standard log-linear specification.
Using equations (3) and (7), it can be shown (Hausman 1981) that the welfare impact of the introduction of a new product is given by

\[ CV = \left[ \frac{1 - \delta}{1 + \alpha} y^{-\delta} \left( p_{n0} x_0 - p_{n1} x_1 \right) + y^{(1-\delta)} \right]^{1/(1-\delta)} - y, \]  

(8)

where \( CV \) is the compensating variation, \( \delta \) is the income elasticity estimate, \( \alpha \) is the price elasticity, \( y \) is income, \((p_{n0}, x_0)\) are the post-introduction price and quantity of the new product, and \((p_{n0}, x_0)\) are the pre-introduction virtual price and quantity of the new product.

Prior research has shown that income elasticity effects can be ignored for typical consumer products where purchases are a small fraction of the consumer’s annual income (e.g. Hausman 1997a, Brynjolfsson 1995). Applying this assumption, i.e. \( \delta = 0 \), equation (8) simplifies to

\[ CV = - \frac{p_{n1} x_1}{1 + \alpha}, \]  

(9)

since the pre-introduction quantity is zero and \( p_{n0} x_0 = 0 \). If income elasticity were positive, as is likely for books, including income elasticity in our calculations would increase our consumer surplus estimates by a small amount (Varian 1992).

4. Data and Results

We use this methodology to measure the consumer surplus gain in Internet markets from access to books not readily available through brick-and-mortar retailers. As noted above, for many consumers, these obscure books can properly be categorized as “new” products because, while they are readily available in Internet markets, the transactions costs necessary to acquire these goods in physical markets are prohibitively high. The availability of these books to Internet consumers reflects, in part, the increased inventory carrying capacity of Internet retailers.
Furthermore, recommendation lists, customer and industry reviews, images of the book jacket and selected book pages, and convenient search facilities allow Internet consumers to discover and evaluate obscure books that likely would have remained undiscovered in conventional retail environments where these books would be unavailable for browsing.

This product category also provides a useful starting point for surplus measurement because it represents a relatively mature Internet market, and because prior research has already measured the reduction in prices from increased competition on the Internet (e.g., Brynjolfsson and Smith 2000), providing a point of reference for our surplus measurements.

In the following sections, we discuss how we estimate the parameters necessary to calculate the consumer surplus resulting from increased accessibility to obscure books on the Internet: the price elasticity of demand and the price and quantity of sales of obscure books on the Internet.

4.1. Elasticity of Demand

The most straightforward approach to calculate elasticity of aggregate demand would be through direct empirical estimation. In the conclusion section we discuss how elasticity of demand might be obtained by partnering with a book publisher or with a retailer with dominant market share to conduct a direct pricing experiment. Unfortunately, we were unable to obtain cooperation from either publishers or retailers to conduct such an experiment. In the absence of this data, we estimate the elasticity of aggregate demand by taking advantage of the characteristics of the book industry structure and available industry statistics on gross margins.

To do this, we first note that the book industry is vertically structured as shown in Figure 1, where $c$ is the marginal cost of a book and $p_{wi}$, $q_{wi}$, $P_{ri}$, and $q_{ri}$ are wholesale price and quantity
and retail price and quantity for retailer $i$ ($i=1,2,\ldots N$) respectively. Publishers set both list prices and wholesale prices of the books they publish. They sell books to retailers, either directly or through distributors, at prices that are a set percentage of the books' list prices, typically between 43-51% off list prices. Thus, a change in the list price of a book would also result in a proportional change in the wholesale price of the book. Further, wholesale prices charged on an individual book are almost the same across retailers, regardless of the channel that the retailer operates in or the size of the retailer (e.g., Clay, Krishnan, and Wolff 2001). Thus we have $p_{wi} = p_{w}$ for $i=1,2,\ldots N$.

Figure 1: Vertical Industry Structure in Book Sales

Retailer $i$ ($i=1,2,\ldots N$) receives books from either publishers or distributors, and then sells these books to consumers at some discount off list price. Books in different categories are sold at

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5 This structure is accurate for the vast majority of consumer purchases, which are made through bookstores. However, in a few cases, customers choose to special-order books directly from the publisher. For our purposes, as long as these books are available from the publisher, at the same prices both before and after Internet book retailers introduce the increased selection of books, publisher special-orders will not affect our underlying model. This follows because, as noted above, the unchanged prices of existing products are irrelevant to consumer surplus calculations from the introduction of new products.

different pre-set discounts off their list prices. For a particular book, the discount off its list price will not change as a result of a change in its list price.\(^7\) Most obscure books are sold at their full list prices.\(^8\) Therefore, for a given obscure title, there exists a stable relationship between the book's wholesale price and retailer \(i\)'s retail price, \(p_{wi} = k_{ri}\), where \(k_i\) is a constant between 0 and 1. In addition, we assume that \(q_{wi} = q_{ri}\). This holds for two reasons. First, most obscure books are ordered by retailers from publishers and wholesalers after a customer initiates a purchase. Second, according to several publishers we interviewed, as well as the American Wholesale Booksellers Association, the vast majority of books are sold on consignment. Typically, retailers can return unsold or returned books to publishers or distributors without penalty (except for return shipping). Given this, if we define \(q_r = \sum_{i=1}^{N} q_{ri}\) as the total quantity sold by retailers to consumers and \(q_w = \sum_{i=1}^{N} q_{wi}\) as the total net quantity sold by the book’s publisher to retailers, we easily get \(q_r = q_w\).

\(^7\) Moreover, this discount off list price is usually set by retailers in multiples of 10%. For example, in a representative sample of 23,744 titles sold at Amazon.com in late 1999, 88.5% of them follow such a pricing pattern — 29.5% have 0% discount, 1.4% have 10% discount, 34.3% have 20% discount, 18.4% have 30% discount, 1.6% have 40% discount, 3% have 50% discount, and 0.1% have 60, 70, 80, or 90% discounts. (See Smith 2001 for more information on this sample of titles.)

\(^8\) We selected 100 books at random from a sample of all customer searches at Dealtime.com on July 2, 2001. Among the 37 books with Amazon.com sales ranks greater than 100,000, 86% are sold at their respective list prices at Amazon.com (versus 41% for the remaining titles). Lee and Png (2002) also collect data showing that bookstores typically offer zero discounts on non-bestseller titles.
If we define retailer $i$’s market share on this book as $s_i = \frac{q_{ri}}{q_r}$, then the weighted retail market price can be written as $\bar{p} = \sum_{i=1}^{N} s_i p_{ri} = \sum_{i=1}^{N} \frac{S_i}{k_i} p_w$. One can show that the elasticity of aggregate demand in the retailing market equals the elasticity of demand faced by the publisher:

$$\frac{-pdq_r}{q_r dp} = \frac{-pdq_w}{q_w dp} = \frac{\left( \sum_{i=1}^{N} \frac{S_i}{k_i} p_w \right) dq_w}{q_w \left( \sum_{i=1}^{N} \frac{S_i}{k_i} dp_w \right)} = \frac{\left( \sum_{i=1}^{N} s_i p_w \right) dq_w}{q_w \left( \sum_{i=1}^{N} s_i dp_w \right)} = \frac{p_w dq_w}{q_w dp_w}. \tag{10}$$

Since the publisher of a particular title has total control over establishing the title’s list and wholesale price, it is reasonable to apply the well-known Lerner index formula to estimate the price elasticity of demand faced by the publisher:

$$\frac{p_i - C_i}{p_i} = -\frac{1}{\alpha_{ii}}. \tag{11}$$

Publishers sell books to both online retailers and brick-and-mortar retailers, either directly or through distributors, at wholesale prices that are a set percentage of books’ list price, typically between 43-51% off list prices. Publishers incur the same production costs whether books are sold to an online retailer or to a brick-and-mortar retailer. Therefore, publishers sell obscure books to online retailers and brick-and-mortar retailers at the same gross margins. Discussions

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9 The elasticity of aggregate demand can be thought of as the percentage change in total market sales if all retailers changed their price by one percentage point. In general, this will be less than the elasticity of demand faced by a particular retailer acting independently.

10 This form of the Lerner index is applicable to single product monopolists, multiproduct monopolists who maximize profits on a per product basis, or in instances where cross elasticity is zero. In the more general multiproduct monopolist case, the Lerner index for product $i$ is given by $\frac{p_i - C_i}{p_i} = -\frac{1}{\alpha_{ii}} \sum_{j \neq i} \frac{(p_j - C_j)q_j}{p_i \alpha_{ij}}$, where $i$ and $j$ are indexes for products. However, in the Internet book market all available evidence suggests that prices are set on an individual book basis and thus we use estimates based on equation (11) for our elasticity calculations. If the publisher is not a monopolist for the book title being sold, then this formula will overestimate the true price elasticity of demand (in absolute value) and underestimate true consumer surplus.
with various publishers indicate that the gross margin of a typical obscure title is between 56-64%. Thus, using (11), the elasticity of demand faced by the publisher is between -1.56 and -1.79, and by (10) this is also the aggregate demand in the retailing market.

This estimate can also be compared with what other researchers have obtained, albeit using retailer data. For example, Brynjolfsson, Dick, and Smith (2002) use shopbot data to calculate an own-price elasticity of -1.47 for retailers listing their prices at a popular shopbot, which is somewhat lower in absolute value than our estimates. Similarly, Chevalier and Goolsbee (2003) estimate a demand system for two online book retailers: Amazon and BarnesandNoble.com. The imputed demand elasticity using their calculations is also lower than our elasticity estimate. As noted in (9), a smaller elasticity will translate to a larger consumer surplus estimate.

4.2. Sales of Obscure Titles on the Internet

Internet retailers are extremely hesitant about releasing specific sales data, and we were unable to obtain sales data from a major Internet retailer that would allow us to estimate the sales of obscure titles on the Internet. However, we were able to obtain data from one publisher that allow us to estimate the proportion of sales of obscure titles in total sales at Amazon.com. This proportion should generalize to the overall Internet book market, given that Amazon.com has approximately a 70% share of the Internet book market (Ehrens and Markus 2000).

This publisher provided data matching the publisher’s weekly sales for 321 titles to the sales rank observed at Amazon.com’s web site during the same week. According to Amazon.com’s frequently asked questions page:

---

11 For example, data from the American Association of Publishers suggest that the gross margin of a typical book is between 56-58% depending on whether shipping is included. A typical MIT Press book has a gross margin of approximately 63% (source: conversation with Vicki Jennings, MIT Press). A large publisher of technical books revealed that each of their books had gross margins of between 58-64% over the past several years. A large publisher of trade books revealed that each of their books had gross margins of approximately 60%.
"The [rank] calculation is based on Amazon.com sales and is updated regularly. The top 10,000 best sellers are updated each hour to reflect sales in the preceding 24 hours. The next 100,000 are updated daily."  

These data, gathered for three weeks in the summer of 2001, provide a fairly robust basis for correlating sales and sales rank at Amazon.com. The observed weekly sales range from 1 to 481 units sold and the observed weekly sales rank ranges from the 238 to 961,367. Summary statistics for this data are shown in Table 2. 

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly Sales</td>
<td>861</td>
<td>19.17</td>
<td>30.63</td>
<td>1</td>
<td>481</td>
</tr>
<tr>
<td>Weekly Sales Rank</td>
<td>861</td>
<td>31,532.85</td>
<td>58,350.92</td>
<td>238</td>
<td>961,367</td>
</tr>
</tbody>
</table>

We fit our data on sales and sales rank to a log-linear (Pareto) distribution:

\[
\log(\text{Quantity}) = \beta_1 + \beta_2 \cdot \log(\text{Rank}) + \epsilon, \tag{12}
\]

where \( \epsilon \) is orthogonal to \( \log(\text{Rank}) \) and is spherical, following the standard OLS assumptions.

This approach was suggested by Madeline Schnapp of O'Reilly Books who reported excellent success estimating competitors' unit books sales by comparing their books' sales ranks to O'Reilly's. Chevalier and Goolsbee (2003) also fit sales and sales rank data to a (slightly different) log-linear distribution with good success. Earlier applications include Pareto (1896), who found that income can be approximated such a log-linear distribution, and Zipf (1949) who suggested that city size also follows a log-linear distribution with a slope of -1.

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12 Available at http://www.amazon.com/exec/obidos/tg/browse/-/525376/104-2977251-9307125. Further experimentation demonstrated that the sales rank does not include used book sales or sales through Amazon's marketplace sellers.

13 The panel of titles changed somewhat during the sample period and as a result not all titles were tracked in all weeks.
Regressing log(Quantity) onto a constant and log(Rank), we obtain an estimate of 10.526 for $\beta_1$ and -0.871 for $\beta_2$ as shown in Table 3 below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>10.526</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
</tr>
<tr>
<td>Log(Rank)</td>
<td>-0.871</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.8008</td>
</tr>
</tbody>
</table>

The coefficients in this regression are highly significant and the $R^2$ value suggests that our model is precisely estimated. Furthermore, the estimates lead to plausible sales-rank results. Given our estimates, a book with a rank of 10 is estimated to get 5,000 sales per week and a book with a rank of 100,000 gets, on average, 1.6 sales per week. Likewise, integrating under the curve for titles with rank from 1 to 2.3 million suggests that Amazon.com was selling books at a rate of 99.4 million per year in the summer of 2001. This estimate compares well with industry statistics.\textsuperscript{15}

These estimates also compare favorably with Pareto slope parameter estimates obtained by Chevalier and Goolsbee (2003) using a clever and easily executed experiment. To conduct this experiment, they first obtained information from a publisher on a book with relatively constant weekly sales, then purchased 6 copies of the book in a 10-minute period, and tracked the Amazon rank before and shortly after the purchases. Using the sales and sales rank before and

\textsuperscript{14} A graphical analysis suggests that the size of the residuals increases in rank, and a Breusch-Pagan test confirms the presence of heteroskedasticity in the residuals. Thus we use White heteroskedasticity consistent estimator (see Greene 2000, p.463) to estimate both coefficients. We also performed a test for structural change by interacting log(Rank) with a dummy variable that took on the value of one for ranks larger than 40,000. The coefficient on this variable was positive (but statistically insignificant) suggesting that our results would, if anything, be strengthened if we based our $\beta_2$ on only high rank books.

\textsuperscript{15} The 2001 Book Industry Trends lists 2000 total unit sales of books at 2.5 billion and their study also shows that the Internet makes up 6\% of total book purchases. Amazon.com has a 70\% share of the Internet book market (Ehrens and Markus 2000).
after the experiment, they estimated \( \beta_2 \) as -0.855 (note that the \( \theta \) reported by Chevalier and Goolsbee corresponds to \(-1/\beta_2\)). They also estimated \( \beta_2 \) from similar sales-rank data reported by Weingarten (2001) and Poynter (2000) as -0.952 and -0.834 respectively. We performed a similar purchase experiment in September 2002 and calculated \( \beta_2 \) as -0.916.\(^{16}\) It is significant that while these parameter estimates rely on only 2 points, they are remarkably similar to the results calculated in Table 3 above, which are based on over 800 points.

We can use the Pareto slope parameter estimate from our data to calculate the proportion of unit sales at Amazon that fall above a particular rank as

\[
r(x, N) = \frac{\int_{x}^{N} \beta_1 t^{\beta_2} \, dt}{\int_{1}^{N} \beta_1 t^{\beta_2} \, dt} = \frac{x^{(\beta_2 + 1)} - N^{(\beta_2 + 1)}}{N^{(\beta_2 + 1)} - 1},
\]

where \( x \) is the rank, and \( N \) is the total number of books available.

What rank cutoff is appropriate for our purposes? As noted above, we wish to estimate the gain in consumer surplus from access to books on the Internet that are not normally stocked by brick-and-mortar stores. Thus, our rank figure should approximate the average number of books a consumer could readily locate in local physical stores.

At one end of the spectrum one would want to consider consumers who do not have easy access to bookstores with a broad selection of titles. In Appendix C of Brynjolfsson and Smith (2000)\(^{16}\)

\[^{16}\text{We selected a book whose Amazon.com rank on September 13, 2002 was 606,439. We checked the rank of this book each day between September 14 and September 30 and noted 3 changes: on Monday September 16 the book jumped from 606,439 to 596,625; on Monday September 23 the book dropped from 596,625 to 599,352; and on Monday September 30 the book dropped from 599,352 to 601,457. We infer from this that Amazon updates its sales rankings on low selling books each Monday and that a sale occurred sometime during the week ending September 15 and no sales occurred during the remaining weeks. On September 30 one of the authors ordered 5 copies of this book using 5 different Amazon user accounts. The next morning the book had a sales rank of 4,647.}\]
the authors calculated that the average consumer in the United States lives 5.4 miles away from the closest general selection bookstore. Using the same dataset, we find that 14% of U.S. consumers live more than 10 miles away from the nearest general selection bookstore and 8% live more than 20 miles from their nearest bookstore. For such customers the relevant rank might be near 0. That is, without the Internet such customers are not able to easily purchase even general selection books.

More typically, consumers will have at least one and possibly multiple bookstores close-by. However, these brick-and-mortar bookstores vary significantly in size. Small bookshops and mall-based stores stock approximately 20,000 unique titles, large independent booksellers stock approximately 40,000 unique titles, Barnes and Noble and Borders superstores stock approximately 100,000 unique titles, and the Barnes and Noble superstore in New York City, reported to be the “World’s Largest Bookstore,” carries 250,000 unique titles on its shelves.\(^{17}\)

\[
\begin{array}{|c|c|c|}
\hline
\text{Sales Rank} & \text{Proportion in Total Sales} & \text{Standard Error}\(^{18}\) \\
\hline
>40,000 & 47.9% & 2.7% \\
>100,000 & 39.2% & 2.5% \\
>250,000 & 29.3% & 2.0% \\
\hline
\end{array}
\]

In Table 4 we estimate the proportion of total sales at Amazon.com that lies above a particular rank (i.e., titles that are not available at a typical brick-and-mortar bookstore) for each of the reference points discussed above. These calculations are based on equation (13) along with the

\(^{17}\) Stock figures for Barnes and Noble were obtained from correspondence with Mary Ellen Keating, Senior Vice President of Corporate Communications and Public Affairs, Barnes and Noble, December 3, 2001. Stock figures for independent stores were obtained from multiple industry sources and discussions, including Ritchie (1999).

\(^{18}\) Since the proportion of Amazon unit sales that fall in titles with ranks above \(x\) is a function of \(\beta\) and we obtain the standard error of \(\beta\) from the regression, we calculate the standard error of our estimate using the Delta Method (see Greene 2000, p.118).
estimate from Table 3 for $\beta_2$ and 2,300,000 (the number of books in print) for $N$. This table shows that 47.9% of Amazon’s unit sales fall in titles with ranks above 40,000 and 39.2% of sales fall in titles with ranks above 100,000, as Figure 2 illustrates. It is unlikely that every consumer will live within reasonable driving distance to the largest Barnes and Noble superstore in New York City and have access to the 250,000 titles stocked there, but using that number as the cutoff point only reduces the proportion down to 29.3%.

**Figure 2: Share of Amazon Sales Above Rank 100,000**

In subsequent calculations, we use a rank of 100,000 as our point-of-reference for consumer surplus estimates. This cutoff can be interpreted either in terms of the average stock levels at a Barnes and Noble or Borders superstore or as taking into account the possibility that consumers shop at multiple smaller independent stores. For example, if there were only a 50% overlap in
stocked titles at large independent bookstores, a consumer would have to shop at a minimum of five such stores to have access to 100,000 titles.

This large cutoff point seems fairly conservative on two dimensions. First, it is unlikely that most consumers, particularly rural consumers as mentioned above, have access to this number of unique titles through local bookstores. Second, even if all consumers had access to these larger stores, the 100,000 cutoff will underestimate true consumer surplus if, as seems likely, superstores do not stock exactly the same 100,000 most popular books that Amazon.com stocks.

4.3. Consumer Welfare

According to 2001 Book Industry Trends, book revenue in year 2000 was $24.59 billion (Book Industry Study Group 2001). Given that the Internet makes up 6% of total book sales (Book Industry Study Group 2001), we estimate that the total Internet book sales in 2000 were $1.475 billion. If we assume that obscure titles account for about the same proportion of total sales at other Internet book retailers as at Amazon, the sales of titles that are not available at a typical brick-and-mortar bookstore are $578 million based on the estimates in Section 4.2.

Since these estimates are based on aggregate figures, it is further necessary to ensure that the average prices of obscure books sold on the Internet are not lower than the average prices of more popular books sold on the Internet. If this were not true, we would overestimate the true consumer surplus by using aggregate figures. To analyze the relative prices of obscure and more popular books we selected 100 books at random from a sample of all customer searches at Dealtime.com on July 2, 2001. We then categorized the books into obscure and regular titles based on whether their Amazon.com sales rank was greater than (obscure) or less than (regular)
Table 5: Price Comparison for Obscure Titles and Regular Titles on the Internet

<table>
<thead>
<tr>
<th>Amazon Sales Rank</th>
<th>&lt;100,000</th>
<th>&gt;100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average List Price</td>
<td>$34.53</td>
<td>$42.18</td>
</tr>
<tr>
<td>Average Amazon Price</td>
<td>$29.26</td>
<td>$41.60</td>
</tr>
<tr>
<td>Average Price at Dealtime</td>
<td>$29.52</td>
<td>$39.06</td>
</tr>
<tr>
<td>Average Minimum Price at</td>
<td>$20.03</td>
<td>$29.52</td>
</tr>
<tr>
<td>Dealtime</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>63</td>
<td>37</td>
</tr>
</tbody>
</table>

With these estimates of elasticity and revenue from obscure book sales, we use equation (9) to calculate that the introduction of obscure books in online markets has increased consumer welfare by between $731 million and $1.03 billion in the year 2000 alone, with standard errors of $46.7 million and $65.8 million respectively.\(^2\)

It is worth noting that our log-linear demand curve does not restrict consumers' valuation to be below a certain dollar amount. One concern, therefore, is that our consumer surplus estimates could be driven by a small number of consumers with very high valuations. It might be reasonable to exclude some of these consumers with very high valuations on the assumption that they might have been motivated to find a way to gain access to the book without using the Internet, even if that entailed significant personal effort. On the other hand, they might never have learned of the book in the first place without the recommendation engines, search tools and

\(^{19}\) Analogous results are obtained using a cutoff of 40,000, the number of books stocked at a typical large independent bookseller.

\(^{20}\) Using a cutoff of 250,000 would reduce our consumer surplus estimates to between $547 and $772 million in the year 2000, with standard errors of $37.3 million and $52.7 million respectively. Using a cutoff of 40,000 would increase our consumer surplus estimates to between $894 and $1.26 billion in the year 2000, with standard errors of $50.4 million and $71.1 million respectively.
other aids provided by successful online booksellers. Discussions with a publisher suggest that this latter effect is more important than any substitution away from conventional channels by high value consumers.

Nonetheless, as a check on the robustness of our results, we can also conduct an analysis in which we restrict our consumer estimates by excluding high value consumers. Excluding all consumers with valuations above five times a book's current price would reduce our current consumer surplus estimates by 28.0%-40.6% while excluding consumer valuations above ten times a book's current price would reduce our current consumer surplus estimates by 16.2%-27.5%.

We also calculate the consumer surplus gain from increased competition and operational efficiency in Internet markets as a point of reference to the consumer surplus gains estimated above. Brynjolfsson and Smith (2000) calculated that prices on the Internet including shipping and handling charges were 6% lower than prices in brick-and-mortar retailers due to increased competition and increased operational efficiency. A fractional price change of \( \phi \) will lead to a \( \phi \alpha \) change in quantity, according to the definition of price elasticity of demand. Thus we have

\[
CV = \frac{p_1x_1 - p_0x_0}{1 + \alpha} = \frac{-p_1x_1 - (1 + \phi)p_1(1 + \phi \alpha)x_1}{1 + \alpha},
\]

(14)

where \( CV \) is the change in consumer surplus, \( \alpha \) is the price elasticity, \((p_1, x_1)\) are the price and quantity after the price change, and \((p_0, x_0)\) are the price and quantity before the price change.

Using the same estimates as were used above (i.e., \( p_1x_1 = $1.475 \) billion and \( \alpha \) between -1.56 and -1.79), equation (14) shows that the consumer welfare gain from a 6% drop in price for all titles on the Internet is between $100.5 million and $103.3 million. Thus, the consumer welfare gain
from the introduction of obscure books in online markets is between 7.3 (with a standard error of 0.5) and 10.0 (with a standard error of 0.6) times as large as the consumer welfare gain from increased competition and lower prices on the Internet.

4.4. Discussion

While the magnitude of the consumer welfare gain from increased variety is large both in absolute terms and relative to the savings from lower prices, our approach is imperfect and is likely to underestimate the total welfare benefits for a number of reasons.

First, it is important to note that the book market is just one of many markets affected. Online sales of other consumer product categories, like music CDs, movies, and electronic products, are likely to also show significant gains in consumer surplus. Furthermore, gains in all product categories will increase as more customers gain access to the Internet channel and as new technologies such as print-on-demand, digital content delivery, mobility services, and broadband access further reduce consumer search and transactions costs. Finally, it is possible that the ability to sell obscure books through Internet channels that would not have been stocked in physical stores will allow some books to be published that otherwise would not have been viable.21

Second, there is some evidence that the Internet may have reduced the effective cost of special orders even in offline stores, including the consumer time and effort required to identify the relevant books. Some obscure titles were available in brick-and-mortar stores through customer initiated special orders, even before the rise of the Internet as a channel for books. However,

21 While making more and more titles available online will result in higher sales, it is important to note that our calculations demonstrate that there are diminishing returns to adding titles. For example, according to our Pareto curve estimates, titles ranked from 100,000 to 200,000 account for 7.3% of sales at Amazon.com while titles ranked between 200,000 to 300,000 account for only 4.6% of sales.
according to several bookstore owners we spoke to, special orders for items not normally stocked account for less than 1% of sales through the physical channel. This low level of special orders should not be surprising given that the special order process in a conventional store is inconvenient and time-consuming, as discussed above.

However, it is interesting to note that the availability of obscure titles on the Internet has apparently led to somewhat increased sales through special orders at brick-and-mortar stores. Several brick-and-mortar retailers we spoke to said that the Internet has allowed brick-and-mortar customers to locate and evaluate books they would not have been able to find otherwise. Mary Ellen Keating, Barnes and Noble Senior Vice President of Corporate Communication and Public Affairs put it as follows with regard to sales in Barnes and Noble’s brick-and-mortar stores:

“Sales from special orders are up, and customers are ordering a broader range of titles in a number of different categories. What some customers tend to do is their own research on the Web and then special order the book from our stores.”

If the cost of special orders is unaffected by the Internet, then our consumer surplus calculations can ignore changes in the quantity of special orders, while our estimates will be too low if the effective cost of special orders were reduced as suggested by the preceding quotation.

Lacking precise data on the costs or quantities of special ordering sales of obscure titles at brick-and-mortar stores, this potential consumer welfare gain is left out in our calculation. However, given that the Internet has apparently led to a net increase in special order sales through the physical channel, our calculations will underestimate the true consumer surplus from the availability of obscure titles on the Internet.

22 Source: E-mail communication with Mary Ellen Keating, December 3, 2001.
The Internet may also lower the cost of placing special orders in other ways. For example, on October 26, 2000 Barnes and Noble announced a plan to install Internet service counters in all its superstores. These service counters would allow in-store customers to place orders from Barnes and Noble’s Internet site for home delivery. While not included in our calculations, the availability of this service will increase consumer surplus by providing Internet access in new and convenient locations and thus lowering the cost of placing special orders to in-store consumers and consumers who otherwise would not have access to the Internet.

Last but not least, our calculations only focus on consumer welfare gains. There may also be significant gains in producer welfare from the additional sales. Indeed, retailers like Amazon, book publishers, printers, and authors, all stand to benefit and earn a slice of the growing pie created by lower search and transactions costs. In contrast, consumer welfare gains from lower prices come largely at the expense of producers. This suggests that increased product variety creates a total welfare gain, including both consumer and producer welfare, which exceeds the total welfare gain from lower prices by even more than the ratio we estimated for consumer welfare gains alone. It would be interesting to adapt the methods of this paper to also explore the implications for producer welfare.

5. Conclusions

While lower prices due to increased market efficiency in Internet book markets provide significant benefits to consumers, we find that the increased online availability of previously hard-to-find products represents a positive impact on consumer welfare that is seven to ten times larger. Limited shelf-space in conventional retail outlets constrains the types of products that can be discovered, evaluated, and easily purchased by consumers. Limits on the number of titles
Internet retailers can present and sell to consumers are substantially lower. As a result, Internet customers have easy access to millions of products that they could not easily locate or purchase through brick-and-mortar retailers.

To date, the economic effect of increased product variety on the Internet has been ignored, effectively setting to zero the value consumers place on increased selection at Internet retailers. Recent econometric advances have allowed for the measurement of the economic impact of such new products. Our research applies and extends these methodologies to quantify an important welfare impact of online markets. Preliminary calculations for one product category sold in U.S. markets show that the welfare gains are between $731 million and $1.03 billion for the year 2000 alone. These welfare gains dwarf the consumer welfare gain from increased competition and lower prices uncovered in previous research (Brynjolfsson and Smith 2000).

There are a variety of ways our results can be extended by future research. First, while our results use the well-known Lerner index to obtain price elasticity estimates, it may be possible for future research to directly estimate price elasticity using an experiment in cooperation with a publisher of obscure books or possibly a retailer with a dominant market position. Such an experiment would change wholesale prices on a randomly selected set of titles and track the resulting levels of demand. These price changes would be exogenous if both the titles and price change levels (positive and negative) were selected at random for the purposes of this experiment.

Second, it would be interesting to analyze whether consumer surplus gain from access to increased product variety online are primarily from the reduced transactions cost to order obscure products in Internet markets or from the lower search costs to discover books using
Internet search and collaborative filtering tools. The example mentioned above — that Barnes and Noble attributes a net increase in special orders in brick-and-mortar stores to consumers discovering new books online — may shed some light on this question. If consumers gained more value from lower transactions costs, we would expect to see a shift away from in-store special orders to Internet purchases. The fact that the opposite seems to have occurred suggests that the value of discovery may significantly outweigh the value of lower transactions costs. This question deserves further analysis.

It also should be possible to extend this methodology to measure welfare contributions of other product categories sold on the Internet or other new products made available through Internet markets. For example, one could easily extend our results to the online sale of movies, music CDs, or consumer electronics products. It also might be possible to estimate consumer surplus gains from the distribution of digital products such as downloadable e-books, music, movies, and software. Moreover, consumers should also benefit from easy access to formerly localized markets such as RealAudio broadcasts of local radio stations or eBay auctions for products that would otherwise have been sold in yard sales. The results of this paper suggest that ultimately the most important benefits of Internet retailing are not fully reflected in lower prices, but rather are due to the new goods and services made readily available to consumers.
References


Appendix A: Summary of Derivation and Necessary Data

1. Derivation of the formula to calculate consumer surplus
(Follows Hausman 1981 and Hausman and Leonard 2000)

\[
CV = e(p_{e0}, p_{n0}, u_1) - e(p_{el}, p_{nl}, u_1)
\]

Assume \( p_{e0} = p_{el} = p_e \)

\[
CV = e(p_e, p_{n0}, u_1) - e(p_e, p_{nl}, u_1)
\]

\[e(p, u) = (1 - \delta)(u + \frac{Ap^{1+\alpha}}{1 + \alpha})^{1/(1-\delta)} - y\]

Assume \( \delta = 0 \)

\[
CV = - \frac{p_{nl}x_1}{1 + \alpha}
\]

Notes:
- If post-introduction prices of existing products are lower than pre-introduction prices (i.e., \( p_{e0} > p_{el} \)) results under equality assumptions will underestimate true consumer surplus.
- If \( \delta > 0 \) (i.e., if the good is a luxury good as opposed to a necessity good) results under \( \delta = 0 \) will underestimate true consumer surplus.
2. Derivation of the price elasticity of aggregate demand

Assume \( p_{wi} = p_w \)

\[ p_w = k_i p_{ri} \]

Assume \( p_{wi} = k_i p_{ri} \)

\[ s_i = \frac{q_{ri}}{q_r} \]

\[ p = \sum_{i=1}^{N} s_i p_{ri} = \sum_{i=1}^{N} \frac{s_i}{k_i} p_w \]

Assume \( q_{wi} = q_{ri} \)

\[ q_r = \sum_{i=1}^{N} q_{ri} \]

\[ q_r = q_w \]

\[ q_w = \sum_{i=1}^{N} q_{wi} \]

\[ \frac{p}{q} \frac{d q}{d p} = \frac{p}{q} \frac{dq}{dp} = \left( \frac{\sum_{i=1}^{N} s_i p_w}{\sum_{i=1}^{N} k_i} \right) \frac{dq_w}{dq} = \left( \frac{\sum_{i=1}^{N} s_i}{\sum_{i=1}^{N} k_i} \right) \frac{p_w dq_w}{q_w dp} = \frac{p_w dq_w}{q_w dp} \]

3. Derivation of the formula to calculate total sales of obscure products on the Internet

\[ \log(Quantity) = \beta_1 + \beta_2 \cdot \log(Rank) + \epsilon \]

Estimate of \( \beta_2 \)

\[ r(x, N) = \frac{\int_{1}^{N} \beta_1 t^{\beta_3} dt}{\int_{1}^{N} t^{\beta_3} dt} = \frac{N^{(\beta_3+1)} - x^{(\beta_3+1)}}{N^{(\beta_3+1)} - 1} \]

\[ p_{ntx} = (\bar{P} Q) \ast r(x, N) \]
### 4. Data Requirements and Potential Sources:

<table>
<thead>
<tr>
<th>Data Requirement</th>
<th>Potential Data Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of Aggregate Demand ($\alpha$)</td>
<td>- Lerner index and information on publisher/manufacturer margins.</td>
</tr>
<tr>
<td></td>
<td>- Experimental estimation through partnership with publisher/manufacturer (see pp. 10, 25-26).</td>
</tr>
<tr>
<td></td>
<td>- Experimental estimation through partnership with retailer with a dominant market position (see pp. 10, 25-26).</td>
</tr>
<tr>
<td></td>
<td>- Industry estimates of elasticities.</td>
</tr>
<tr>
<td>Total sales of products on the Internet ($PQ$)</td>
<td>- Market research firms.</td>
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<tr>
<td></td>
<td>- Department of commerce reports.</td>
</tr>
<tr>
<td></td>
<td>- Industry consortia.</td>
</tr>
<tr>
<td>Proportion of sales of obscure products at Internet retailers ($r(x,N)$)</td>
<td>- Estimation using log-linear relationship between sales rank and sales and data obtained from publisher/manufacturer (pp. 14-16).</td>
</tr>
<tr>
<td></td>
<td>- Experimental estimation using power-law relationship between sales rank and sales and observation of changes in rank after purchase of goods (Chevalier and Goolsbee 2003, pp. 16-17).</td>
</tr>
<tr>
<td></td>
<td>- Direct observation from representative publisher/manufacturer.</td>
</tr>
<tr>
<td></td>
<td>- Direct observation from retailer (or retailers) with dominant market share.</td>
</tr>
<tr>
<td>Total sales of obscure products on the Internet ($p_{n;x}$)</td>
<td>- Total sales of product category on the Internet * proportion of sales of obscure products at Internet retailers.</td>
</tr>
<tr>
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<td>- Direct observation from representative publisher/manufacturer.</td>
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<td>- Direct observation from retailer (or retailers) with dominant market share.</td>
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Potential data sources in bold indicate those used in this paper.
Performance-based Pricing Models in Online Advertising

Doctoral Thesis Chapter 2

Yu (Jeffrey) Hu

MIT Sloan School of Management

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Performance-based Pricing Models in Online Advertising

ABSTRACT

The Internet is an accountable and measurable medium with bidirectional information flows. This makes possible performance-based pricing models that tie online advertising payments directly to campaign measurement data such as click-throughs and purchases. These pricing models have become increasingly popular in the online advertising industry.

This paper applies the economic theory of incentive contracts to the study of these pricing models and provides explanations as to when and how incorporating them into advertising deals can be profitable. We argue that using these pricing models appropriately can give both the publisher and the advertiser proper incentives to make non-contractible efforts that may improve the effectiveness of advertising campaigns. It also allows the publisher and the advertiser to share the risk caused by uncertainty in the product market. We show that key factors that influence the use of performance-based pricing models are the importance of the publisher’s incremental efforts, precision of click-through measurement, uncertainty in the product market, and risk aversion parameters. We also clarify issues that are being debated in the industry, such as how the importance of the advertiser’s incremental efforts and existence of non-immediate purchases affect the use of performance-based pricing models.

(Online advertising; Pricing model; Incentive; Performance)
1. Introduction

"Half the money I spend on advertising is wasted. The problem is that I don't know which half it is."

Attributed to Lord Lever, founder of Lever Brothers

“(Web marketing) is accountable and measurable—the perfect direct-response medium. If 50 percent of advertising isn’t working, with the Web you know which 50 percent.”


The Internet has emerged as an important medium for advertising. According to Interactive Advertising Bureau’s recent report (Interactive Advertising Bureau 2003), U.S. advertisers spent $6.0 billion on online advertising in 2002, and online advertising accounted for 2.5 percent of total U.S. advertising spending, surpassing advertising in traditional media such as outdoor advertising and business papers advertising. Jupiter Research forecasts that online advertising revenue will grow to $14.8 billion in 2008, when it will account for 6 percent of total U.S. advertising spending. (Jupiter Research 2003)

While there is no doubt about the future of the Internet as an important advertising medium, there has been much confusion on which pricing model should be used. In the early days of online advertising, online advertisers and publishers have simply borrowed the widely used CPM (cost-per-thousand-impressions) pricing model which is the standard in traditional media advertising. In this model, every time an advertisement is displayed, the publisher can collect money from the advertiser. It does not matter if consumers notice it, let alone interact with it.

Recently, both advertisers and publishers have started to realize that the Internet is a much more accountable and measurable medium than traditional media. In traditional media advertising, information flows only in one direction: from the advertiser to the publisher, and from the
publisher to consumers. Thus the advertiser does not have direct contact with consumers, and this makes the measurement of an advertisement’s effectiveness very difficult. For some traditional media like TV, companies like Nielsen provide measurement data through consumer surveys. For other traditional media, even such data are unavailable. In contrast, the Internet is a bidirectional medium that allows the advertiser to have direct contact with consumers. The advertiser can track how consumers respond to its advertisement through various interactivity metrics, such as click-throughs, unique visitors, average viewing time, and purchases (Hoffman and Novak 2000b, Interactive Advertising Bureau 2002). This property of the Internet being a bidirectional medium has enabled performance-based pricing models that let the advertiser pay more for advertisements that perform well and pay less for advertisements that do not perform.

Currently there are two performance-based pricing models that are widely used. The first model is called a CPC (cost-per-click-through) model. Under this model, the publisher receives no guaranteed payment for each impression delivered. Instead, the publisher receives a payment for each click-through that has occurred. The second model is known as a revenue-sharing or CPA (cost-per-action) model. Under this model, the publisher receives no guaranteed payment for each impression delivered. Instead, the publisher receives a payment from the advertiser for each purchase that has occurred and can be traced to advertisements delivered by the publisher.

As these performance-based pricing models emerge, the online advertising industry is engaged in a debate over which pricing model should be used. Should the industry stick to the traditional CPM model, or should the industry use performance-based pricing models, such as CPC and CPA models? (see Digitrends 2001 and Meskauskas 2001) On one side of the debate, many publishers prefer the CPM model because of its low financial risk for them. In addition, they argue that they are only good at attracting and retaining an audience by informing and
entertaining them, and that they cannot control many factors that affect the performance of an advertisement, such as the design of an advertisement, attractiveness of the offer and the sales process. On the other side of the debate, many advertisers prefer performance-based pricing models because these models deliver measurable ROI and pose little risk to them. Advertisers argue that it does not make sense to pay for advertisements that generate no value, when the Internet makes it possible to measure performance. (see Braud 2001, Hallerman 2002, Heyman 2001, McCrea 2000a, 2000b, and Sisney 2000)

As the debate goes on, we have observed that performance-based pricing models become more and more widely used. Leading portals like Google and Yahoo now all offer performance-based advertising in which advertisers only pay for clicks and conversions. Google says more than 100,000 advertisers have signed up for its performance-based advertising, and Overture, a subsidiary of Yahoo, has about 80,000 advertisers using performance-based advertising. Pay-for-performance search advertisements are expected to generate $1.5 billion to $2 billion in 2003, up from $100 million in 2000. (Liedtke 2003) A Forrester report (Nail et al. 2001) says that deals that include performance-based elements, already accounting for 50 percent of online advertising spending in 2001, will account for 53 percent of online advertising spending in 2006.

Why are performance-based pricing models becoming so popular? What factors affect the use of performance-based pricing models? This paper applies a formal model to these problems and provides potential explanations as to when and how incorporating performance-based pricing models into advertising deals can be profitable. We argue that online publishers can make non-contractible efforts that may improve the effectiveness of advertising campaigns. However, these efforts are costly to publishers, thus publishers will not make these efforts unless they are given proper incentives to do so. Therefore, online advertisers have to offer publishers incentives
through performance-based pricing models that tie publishers' advertising revenue to the measurable effectiveness of advertising campaigns. The greater the importance of publishers' non-contractible efforts to the effectiveness of advertising campaigns, the more incentives publishers should be given, which translates to a higher reliance on performance-based pricing models. On the other hand, if advertisers can make some of these efforts contractible and include them in their contracts with publishers, they should rely less on performance-based pricing models.

This paper explains some commonly-observed phenomena in the industry and provides guidance to managers on how to use performance-based pricing models, by matching our model to current industry practices. It shows that key factors that influence the use of these pricing models are the importance of the publisher's incremental efforts, precision of click-through measurement, uncertainty in the product market, and risk aversion parameters. It also clarifies issues that are being debated in the industry, such as how the importance of the advertiser's incremental efforts and existence of non-immediate purchases affect the use of these pricing models.

The remainder of the paper proceeds as follows. Section 2 reviews the related management and economics literature and discusses the differences in modeling between this paper and the existing literature. Section 3 develops a model of the publisher, advertiser, and advertising campaign measurement. Section 4 solves for the optimal contract. Section 5 uses this result to explain why and how performance-based pricing models should be used and examine what factors may affect their use. Section 6 extends the formal model to discuss how the advertiser's efforts and non-immediate purchases affect the optimal contract. Section 7 concludes with some broader implications.

2. Literature Review
Only recently have researchers started to study online advertising. Baye and Morgan (2000) explain why publishers tend to derive the bulk of their revenue from advertising rather than subscription fees. Dewan, Freimer and Zhang (2002) study the optimal amount of advertising an online publisher should place on its pages. Dreze and Zufryden (1998) show that the design of web pages can affect the effectiveness of advertising. However, there is very little literature that directly studies pricing models in online advertising. Hoffman and Novak (2000a) describe various pricing models in online advertising and provide a lot of industry insights. Hoffman and Novak (2000b) present a case study of CDNOW’s successful pay-for-performance affiliate program. Neither paper formally models this problem. This paper represents a first effort to apply a formal model to this problem.

We conceptualize performance-based pricing models in online advertising as contracts that provide publishers and advertisers with incentives to make non-contractible efforts, and allow publishers and advertisers to share risks. We draw upon the economics literature that studies incentive contracts when moral hazard exists in a principal-agent model, in particular, the models proposed by Holmstrom (1979) and Holmstrom and Milgrom (1987, 1991). These types of models have been applied to the study of incentive contracts in the context of agricultural sharecropping (see Allen and Lueck 1992 for a review), retail franchising (e.g., Lafontaine and Slade 1996), executive compensation (see Murphy 1999 for a review), sales-force compensation (e.g., Banker, Lee and Potter 1996), and customer satisfaction incentives (e.g., Hauser, Simester and Wernerfelt 1994).

We apply and extend a model that is used by Holmstrom and Milgrom (1987) to the study of performance-based pricing models in online advertising. Most of the papers that have studied incentive contracts in various contexts consider contracting problems between a firm and its
employees. Thus they use a restricted version of the model that assumes that the principal is risk neutral (the agent is still risk averse), partly for simplicity and partly because of the belief that individuals are usually more risk averse than firms. Nonetheless the assumption that the principal is risk neutral will both distort the optimal contract by having the principal shoulder most of the risks, and prevent the study of how the principal’s risk aversion affects the optimal contract. We consider the contracting problem between an advertiser and a publisher, both being firms. Thus we use a more general version of the model suggested by Holmstrom and Milgrom (1987) that allows both parties to be risk averse. Using this more general model allows us not only to obtain an un-distorted optimal contract, but also to study how the principal’s risk aversion affects the optimal contract. More importantly, it reveals a previously obscured fact that signals can differ in terms of whether their use adds uncertainty to both parties’ payoffs, or their use shifts uncertainty from one party to another. We find that these two different types of signals are used differently in the optimal contract. These results have not been obtained in the existing literature. In Section 6, we further consider a problem in which both parties are risk averse and both parities can make non-contractible efforts.

3. A Model of Publisher, Advertiser, and Campaign Measurement

In our model, we focus on two entities that are involved in an online advertising contract: an online advertiser and an online content publisher. The advertiser sells a product (or service) to consumers through the online channel. In order to boost its sales, the advertiser launches an online advertising campaign by designing an advertisement and contracting with a publisher so that

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1 Holmstrom and Milgrom (1987) prove that a linear contract is optimal in a more generalized setting where both the principal and the agent are risk averse, but they introduce the assumption that the principal is risk neutral for simplicity when they derive comparative statistics for an example. This may have influenced many papers that follow them to assume that the principal is risk neutral.

2 I thank Rajiv Banker for suggesting using this more general version of the model.

3 This double-sided moral hazard problem is first raised by Reid (1977) and has been studied by others.
that the publisher would deliver its advertisement to consumers who may be interested in the product it sells. An impression is defined as an instance of the advertisement being served to a consumer’s browser.

Every time the advertisement is served to a consumer’s browser, the consumer may choose to ignore the advertisement, or to click on the advertisement and be taken to the advertiser’s store, which we refer to as a click-through. If the consumer is taken to the advertiser’s online store, the consumer may make a purchase, or leave without making a purchase. Click-through rate (\( \theta_c \)) is defined as the ratio of click-throughs to impressions, and purchase rate (\( \theta_p \)) is defined as the ratio of purchases to impressions.

The Publisher’s Efforts \( e_p \)

The publisher can make efforts (decisions and actions) to improve the effectiveness of the advertising campaign. Some of these efforts are contractible. For example, the publisher can experiment with the size, background color, animation style, and placement of the advertisement in the content page and find a combination that attracts the most attention of consumers.\(^4\) These efforts—an advertisement’s size, placement, and rotation schedule—are usually stipulated in the contract between the advertiser and the publisher. The existence of these efforts adds more clauses to the contract, but will not affect the choice of various pricing models. Thus we will not focus on these efforts.

However, a lot of the publisher’s efforts are non-contractible. For example, whether the publisher closely associates the advertisement with its surrounding content and whether the publisher chooses appropriate wording in its pitch to consumers both affect the advertisement’s

\(^4\)Dreze and Zufryden (1998) show that the effectiveness of promotional content on a website is influenced by the website’s background color and pattern, image size, style of display, and use of Java Applets and frames.
effectiveness. More importantly, the publisher can serve the advertisement to consumers who are the most likely to be interested in it by using a targeting technology based on its knowledge of consumers’ demographics, geographical location, expressed interests and other information. (Needham 1998, Maislin 2001) These efforts are either non-observable or too expensive and too difficult for the advertiser to observe and monitor. Aside from the difficulty and cost of direct monitoring, it may be in the advertiser’s best interest to give the publisher some freedom to make decisions and actions, because of the publisher’s better knowledge of consumers who visit its website. We focus on these non-contractible efforts and call them \( e_p \).

It is worth noting that the publisher’s efforts \( e_p \) that we focus on are incremental efforts above and beyond what the publisher will make without incentives. We assume that purchase rate \( \theta_p \) is a linear function of the publisher’s efforts \( e_p \), and we model the impact of standard efforts on purchase rate as a positive intercept. The influence of other factors on purchase rate is modeled as uncertainty \( \varepsilon_p \), which is distributed normally with a mean of zero and a variance of \( \sigma_{pp} \).

Formally, we have:

\[
\theta_p = \alpha_p + \beta_p e_p + \varepsilon_p, \quad \alpha_p > 0, \quad \beta_p > 0. \tag{1}
\]

The Advertiser’s Measures

The advertiser can observe the result of the publisher’s efforts, i.e., purchase rate. Another possible measure the advertiser can use is click-through rate. It is reasonable to assume that click-through rate is positively influenced by the publisher’s incremental efforts. Formally, we assume that click-through rate \( \theta_c \) equals a linear function of the advertiser’s efforts \( e_p \), plus random noise \( \varepsilon_c \) that is distributed normally with a mean of zero and a variance of \( \sigma_{cc} \). Thus
\[ \theta_c = \alpha_c + \beta_c \epsilon_p + \epsilon_c, \quad \alpha_c > 0, \quad \beta_c > 0. \] (2)

We assume that \( \epsilon_p \), uncertainty in the product market, is uncorrelated with \( \epsilon_c \), random noise in the measurement of click-through rate.

**The Publisher's Motivation**

Incremental efforts are costly to the publisher, and become more costly as total effort level increases. We model the publisher's cost of incremental efforts by a quadratic cost function that is widely used in the literature of incentive contracts. Formally, the cost of efforts \( e_p \) is

\[ C(e_p) = \frac{e_p^2}{2}. \] (3)

We assume that the publisher acts in its own best interest when it decides on the level of incremental efforts that it will make. When the publisher is paid purely on a per-impression basis, it will focus on creating better content and attracting a bigger audience. The publisher has no incentive to make any incremental efforts that may improve the effectiveness of the advertising campaign. This highlights the fact that the interest of the advertiser and that of the publisher are misaligned under a pure CPM model.

When an advertising contract includes performance-based elements like a per-click-through payment or a per-purchase payment, the publisher starts to weigh in the cost and benefit of making incremental efforts, and decide the level of incremental efforts it will make. However, performance-based elements expose both parties to uncertainties. For example, when the payment from the advertiser to the publisher is tied to click-throughs, both parties are exposed to the random noise in the measurement of click-through rate—when click-through rate is unexpectedly low, the publisher loses advertising revenue; when click-through rate is
unexpectedly high, a contract with per-click-through payment can send costs through the roof and break the advertiser's budget. In parallel, when the advertising payment is tied to purchases, both parties are exposed to the uncertainty in the product market—when sales are unexpectedly strong, a contract with per-purchase payment can force the advertiser to pay too much; when sales are unexpectedly weak, the publisher loses advertising revenue.

Due to all the uncertainties mentioned above, we now must model the publisher's risk aversion. We assume that the publisher has exponential utility with CARA (constant absolute risk aversion) parameter of \( r_p (r_p > 0) \), which means, \( u(y_p) = 1 - \exp(-r_p y_p) \) when the publisher's payoff is \( y_p \).\(^5\) The certainty equivalence of the publisher's payoff is

\[
CE(y_p) = E(y_p) - r_p Var(y_p)/2.
\]

**The Advertiser's Utility Function**

We assume that the advertiser also has exponential utility with CARA (constant absolute risk aversion) parameter of \( r_a (r_a > 0) \), which means, \( u(y_a) = 1 - \exp(-r_a y_a) \). The certainty equivalence of the advertiser's payoff is \( CE(y_a) = E(y_a) - r_a Var(y_a)/2 \).

**Timeline**

First, the advertiser offers a contract to the publisher. Second, the publisher can decide to accept the contract, or to decline the contract in which case it obtains a utility of \( u_0 \) and the game is over.\(^6\) Third, if the publisher accepts the contract, the publisher decides the level of its

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\(^5\) This type of utility function has proved to be a good approximation in a variety of cases.

\(^6\) The publisher has this outside option because it could potentially advertise for other advertisers. Such a competition between the advertiser and other potential advertisers prevents the advertiser from making a “low-ball” offer to the publisher.
incremental efforts $e_p$. Finally, the advertiser and the publisher observe click-through rate and purchase rate, and payoffs to the advertiser and the publisher are realized.

4. The Optimal Contract

In principle, an advertising contract could be any function of click-through rate and purchase rate. Fortunately, our mathematical formulation implies that the optimal contract is a linear function of these two variables. This result follows from our assumptions that random noise in the measurement of click-through rate and uncertainty in the product market both conform to Normal distributions and that the advertiser and the publisher both have exponential utility with constant absolute risk aversion. Holmstrom and Milgrom (1987) provide a detailed proof, and Banker and Datar (1989) identify necessary and sufficient conditions for other formulations to have linear optimal contracts. Although the exact linearity of the optimal contract depends on technical assumptions, linear contract is a good approximation to a variety of incentive contracts. More importantly, linear contract, because of its simplicity, is the most widely used form of contract in online advertising (Hoffman and Novak 2000a).

We first specify a linear contract that includes a CPM element and two performance-based elements, and then we study the propensity of each element being used in the optimal contract. Formally, such a contract specifies a guaranteed payment for each impression, $t_m$, a per-click-through payment, $t_c$, and a per-purchase payment, $t_p$. Under this contract of $(t_m, t_c, t_p)$, the publisher can expect to receive from the advertiser, for each impression delivered, a payment of $t_m + t_c \theta_c + t_p \theta_p$, if click-through rate is $\theta_c$ and purchase rate is $\theta_p$.

The Optimal Behavior of the Publisher
If the publisher accepts a contract of \((t_m, t_c, t_p)\) and makes efforts \(e_p\), the publisher's payoff is:

\[
y_p = t_m + t_c \theta_c + t_p \theta_p - e_p^2 / 2 = t_m + t_c (\alpha_c + \beta_c e_p + \varepsilon_c) + t_p (\alpha_p + \beta_p e_p + \varepsilon_p) - e_p^2 / 2.
\] (4)

The certainty equivalence of the publisher's payoff is:

\[
CE(y_p) = E(y_p) - r_p Var(y_p) / 2 = t_m + t_c (\alpha_c + \beta_c e_p) + t_p (\alpha_p + \beta_p e_p) - e_p^2 / 2 - r_p (t_c^2 \sigma_{cc} + t_p^2 \sigma_{pp}) / 2.
\] (5)

The publisher who is assumed to be rational will act in its own best interest and choose a level of incremental efforts that maximizes the certainty equivalence of its payoff. The first-order-condition of (5) gives the publisher's incentive compatibility (IC) constraint, which is:

\[
e_p^* = \arg\max CE(y_p) = t_c \beta_c + t_p \beta_p.
\] (6)

Equation (6) shows that a larger per-click-through payment or per-purchase payment will induce the publisher to make a higher level of incremental efforts.

In order for the publisher to accept the contract, the contract must give the publisher a utility of at least \(u_0\). In other words, the publisher's individual rationality (IR) constraint is:

\[
CE(y_p) \geq u_0.
\] (7)

The Optimal Contract

Let \(m\) be the profit that the advertiser makes on each purchase. The advertiser's payoff is:

\[
y_a = (m - t_p) \theta_p - t_m - t_c \theta_c = (m - t_p) (\alpha_p + \beta_p e_p + \varepsilon_p) - t_m - t_c (\alpha_c + \beta_c e_p + \varepsilon_c).
\] (8)

The certainty equivalence of the advertiser's payoff is:
CE(y_a) = E(y_a) - r_a Var(y_a) / 2
= (m - t_p)(\alpha_p + \beta_p e_p) - t_m - t_c(\alpha_c + \beta_c e_p) - r_a[t_c^2 \sigma_{cc} + (m - t_p)^2 \sigma_{pp}]/2.

(9)

We define the optimal contract as the contract that maximizes the advertiser's utility (equation (9)) under the condition that both the publisher's incentive compatibility (IC) constraint (equation (6)) and its individual rationality (IR) constraint (equation (7)) are satisfied.\(^7\)

An increase of the per-impression payment, \(t_m\), lowers the advertiser's utility while increasing the publisher's utility by the same amount. The optimal contract that maximizes the advertiser's utility will set the per-impression payment, \(t_m\), such that the publisher obtains a utility of \(u_0\).

This implies
\[
t_m^* = u_0 + e_p^2/2 + r_p(t_c^2 \sigma_{cc} + t_p^2 \sigma_{pp})/2 - t_c^* (\alpha_c + \beta_c e_p) - t_p^* (\alpha_p + \beta_p e_p).
\]

(10)

Substituting (6) and (10) into (9) gives us
\[
CE(y_a) = m[\alpha_p + \beta_p(t_c \beta_c + t_p \beta_p)] - (t_c \beta_c + t_p \beta_p)^2/2 - (r_p + r_a)t_c^2 \sigma_{cc}/2
- r_a(m - t_p)^2 \sigma_{pp}/2 - r_p t_p^2 \sigma_{pp}/2 - u_0.
\]

(11)

Finding the optimal contract is now a matter of solving the first order conditions of (11), with respect to \(t_c\) and \(t_p\), together.

**Proposition 1.** The optimal contract, which maximizes the advertiser's utility under the condition that both the publisher's incentive compatibility (IC) constraint and its individual rationality (IR) constraint are satisfied, is given by

\(^7\) We can also think of another formulation of the optimal contract that maximizes the sum of the advertiser's utility and the publisher's utility. By Kuhn-Tucker Theorem, the second formulation is just a special case of our current formulation in which the multiplier on the publisher's individual rationality (IR) constraint is set at 1. Therefore, the optimal contract as we have defined also maximizes the sum of the advertiser's utility and the publisher's utility.
For the proof of this and all other propositions, please see the appendix.

5. Why and How Should Performance-based Elements Be Used?

We first discuss why performance-based elements should be used in the contract between the advertiser and the publisher. We then study how these performance-based elements should be used by examining how the optimal contract is affected by various factors.

Should Performance-based Elements Be Used?

**Proposition 2.** The advertiser obtains a higher utility by including performance-based elements, in addition to a CPM element, in its contract with the publisher, as opposed to using only a CPM element.

To understand this proposition, let us first consider the case of a pure CPM contract. Under this contract, the publisher has no incentives to make incremental efforts. As a result, the advertising campaign is unlikely to be very effective, and the advertiser will not achieve high sales and a high utility. However, under a contract that includes performance-based elements, these performance-based elements give the publisher incentives to make incremental efforts that may improve the effectiveness of the advertising campaign. Proposition 2 says simply that it is possible to select appropriate parameters for performance-based elements such that the
advertiser’s utility increases. It is worth noting that including performance-based elements in the contract between the advertiser and the publisher increases the sum of the advertiser’s utility and the publisher’s utility. In our model we assume that the publisher always obtains a reservation utility of \( u_0 \) and the advertiser reaps all the benefit from using performance-based elements. But it is reasonable to think of scenarios in which the publisher shares the benefit from such a change by negotiating a higher per-impression payment. Proposition 2 explains why performance-based pricing models have become more and more widely used in online advertising. (Liedtke 2003, Nail et al. 2001)

**How Should Performance-based Elements Be Used?**

**Proposition 3.** *If the importance of the publisher’s incremental efforts to the effectiveness of the advertising campaign is smaller, the optimal contract places a smaller weight on click-throughs and purchases.*

Performance-based elements are used in online advertising because they can provide the publisher incentives to make incremental efforts that may improve the effectiveness of the advertising campaign. The greater the importance of the publisher’s incremental efforts to the effectiveness of the advertising campaign, the more incentives the publisher should be given to make these efforts, which translates to a higher reliance on performance-based elements. On the opposite side, if some portion of the publisher’s incremental efforts can be made contractible and be included in the advertising contract, the importance of the publisher’s incremental efforts will decrease. This alleviates the need to give the publisher incentives and leads to a lower reliance on performance-based elements. The online advertising industry has continued to standardize online advertising formats and terminology. For example, Interactive Advertising Bureau has recently developed standards and guidelines on advertisement size, placement, measurement, and
auditing. These new standards and guidelines aim to define standard efforts that the publisher must make and limit incremental efforts at the publisher's discretion. According to Proposition 3, their development will lower the industry's reliance on performance-based pricing models.

It is shown in Proposition 1 that the optimal contract places positive weights on both click-through rate and purchase rate—two signals of the publisher's incremental efforts. This result follows from Holmstrom (1979)'s informativeness condition. In our model, neither click-through rate nor purchase rate is a sufficient statistic for the pair of them. Therefore, both should be used in the optimal contract. In addition, the weight placed on a signal depends on its precision. An increase in the precision of a signal leads to an increase in the weight placed on that signal. This rule also applies to our model.

**PROPOSITION 4. If click-through rate is measured with greater precision, a) the optimal contract places a greater weight on click-throughs and a smaller weight on purchases, and b) the advertiser obtains a higher utility.**

Proposition 4 shows that click-through rate can be relied more heavily upon if it is a more precise measure, because this exposes the advertiser and the publisher to lower risks. In addition, this proposition helps explain the efforts over the past few years by industry practitioners to obtain a more precise measure. When the cost-per-click pricing model was first introduced, there was no uniform standard on the definition of click-through rate. Some publishers, in order to inflate their click-through rate, used various tricks like displaying input boxes that are really pictures to deceive people into clicking. This action made click-through rate an unreliable measure of the publisher's incremental efforts. Since then, the online advertising industry has started to standardize advertising measurement and to use other interactivity metrics to refine the measurement of click-through rate (Interactive Advertising Bureau 2002). These efforts aimed at
obtaining a more precise measure will likely make both the advertiser and the publisher better off.\(^8\)

**PROPOSITION 5.** *As uncertainty in the product market decreases, a) the optimal contract places a greater weight on purchases and a lower weight on click-throughs, and b) the advertiser obtains a higher utility.*

Proposition 5 shows that purchase rate can be relied more heavily on if uncertainty in the product market decreases. The intuition is that, as uncertainty in the product market decreases, purchase rate becomes a more accurate signal of the publisher’s incremental efforts, and relying on a more accurate signal exposes the advertiser and the publisher to smaller uncertainty and lower risk.

This proposition also sheds light on what products are good candidates for deals that tie advertising payments to purchases. According to this proposition, products that are suitable for these cost-per-action (CPA) deals are products that are mature, have steady and predictable sales, and have a low level of market uncertainty. In contrast, these deals are not suitable for products that are new and unproven, unless there is a mechanism that can limit these products’ uncertainty. The online advertising industry shares our view on this problem, and has taken actions to limit products’ uncertainty. For example, Affiliate Fuel, a CPA advertising network, requires all new advertisers to run a test campaign before they can enter a larger scale contract. (Affiliate Fuel 2003) Similar ideas of providing historical data on an advertiser’s past advertisements and performing an upfront test before entering a larger CPA deal are expressed in Digitrends (2001) and Hallerman (2002).

**How Do Risk Aversion Parameters Affect the Use of Performance-based Elements?**

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8 In our model we assume that the publisher always obtains a reservation utility of \(u_0\) and the advertiser reaps all the benefit from a more precise measure. But it is reasonable to think of scenarios in which the publisher shares the benefits from such a change by negotiating a higher per-impression payment.
We first examine how the assumption of the advertiser (principal) being risk neutral would affect the optimal contract. Substituting \( r_a = 0 \) into (12) and (13) gives

\[
t_p^* = m \left( \frac{r_p \sigma_{pp}}{\beta_p^2} + \frac{\beta_c^2 \sigma_{pp}}{\beta_p^2 \sigma_{cc}} \right),
\]

(14)

\[
t_c^* = m \left( \frac{\beta_c \sigma_{cc}}{\beta_p \sigma_{pp}} + \frac{r_p \sigma_{cc}}{\beta_p \beta_c} \right).
\]

(15)

The comparison of (14) with (12) shows that the optimal per-purchase payment when the advertiser is risk averse has one additional term, \( r_a/(r_a + r_p) \), than when the advertiser is risk neutral. This term exhibits the role of the cost-per-action pricing model as a mechanism for the advertiser and the publisher to share risks, in addition to its role as a mechanism to give the publisher incentives to make incremental efforts. This risk-sharing term is reduced to zero when the advertiser is risk neutral, because in that case it is desirable for the advertiser to shoulder most of the risks. However, this risk-sharing term does not surface in the optimal per-click-through payment, either in (13) or in (15), because the cost-per-click pricing model does not act as a risk-sharing mechanism. Although tying advertising payments to purchases shifts uncertainty from the advertiser to the publisher, tying advertising payment to click-throughs adds uncertainty to both parties’ payoffs. These two signals—purchases and click-throughs—have different properties, as a result, they should be used differently in the optimal contract. The difference between these two signals would have been obscured if the advertiser were assumed to be risk neutral.

Next we study how risk aversion parameters affect the use of performance-based pricing models.
PROPOSITION 6. a) The optimal per-purchase payment increases as the advertiser becomes more risk averse, and it decreases as the publisher becomes more risk averse. b) The optimal per-click-through payment decreases as the advertiser becomes more risk averse. However, the relationship between the optimal per-click-through payment and the publisher's risk aversion parameter is inverted-U-shaped.

A higher per-purchase payment from the advertiser to the publisher means the publisher shoulders a larger proportion of the risk caused by uncertainty in the product market. This becomes more desirable as the advertiser becomes more risk averse, and less desirable as the publisher becomes more risk averse. This is shown in Proposition 6.

However, a higher per-click-through payment would expose both the publisher and the advertiser to a higher level of risks associated with the measurement of click-throughs. This becomes less desirable as either the advertiser or the publisher becomes more risk averse. But we also have to consider whether the publisher is given enough incentives to make incremental efforts. In the case of the advertiser becoming more risk averse, the publisher will be given a higher per-purchase payment that translates to a higher level of incentives. Thus, the incentive consideration is unnecessary in this case, and the optimal per-click-through payment decreases. In the case of the publisher becoming more risk averse, the publisher will be given a lower per-purchase payment that translates to a lower level of incentives. Thus, the incentive consideration becomes effective, and the balancing of the incentive consideration and the undesirability of using per-click-through payment leads to an inverted-U-shaped relationship between the publisher's risk aversion and the optimal per-click-through payment.\(^9\)

\(^9\) If we assumed that the advertiser (principal) is risk neutral, we would not have obtained this result. Instead, we would have obtained the traditional result that the optimal per-click-through payment decreases as the publisher...
6. Extensions

In this section, we make extensions to the formal model to clarify issues that are being debated in the online advertising industry regarding the use of performance-based pricing models.

The Advertiser's Efforts

In our basic model, we focus on non-contractible efforts that can be made by the publisher to improve the effectiveness of the advertiser's advertising campaign. However, one can argue that a campaign's effectiveness depends on the advertiser's efforts as well. We assume that the advertiser can make efforts to convert more click-throughs to purchases by designing a convenient-to-browse storefront, making attractive offers and having good customer services.¹⁰

For example, a Forrester report (Nail et al. 1999) claims that advertisers can "entice consumers with tempting product displays, irresistible offers, and a satisfactory customer experience to close the sale". Maislin (2001) holds a similar view and he says that "no matter how well the advertisement is written and targeted, if the product or even the transaction experience itself is terrible, no conversion will take place." These efforts made by the advertiser are either non-observable or too expensive and too difficult for the publisher to observe. Thus they are non-contractible, and the advertiser will not make these efforts unless it is in its best interest to do so.

We focus on these efforts and modify our basic model to study how the existence of these efforts affects the optimal contract.

¹⁰ One may also think that the advertiser can make efforts to improve click-through rate. The advertiser controls the design of the advertisement—"creative", and the advertiser can make efforts to improve the attractiveness of the design so that the advertisement will draw consumers' attention and result in a higher click-through rate. These efforts are observable by the publisher when the advertiser submits its advertisement to the publisher. Thus, the publisher can ensure the advertiser will make a certain level of these efforts, by setting a standard the advertiser's design must meet in the advertising contract. The existence of these efforts will not affect the choice of various pricing models. Thus we will not focus on these efforts.
We make two modifications. First, we assume that purchase rate $\theta_p$ is now a linear function of the publisher’s efforts $e_p$ and the advertiser’s efforts $e_a$. Formally, we have

$$\theta_p = \alpha_p + \beta_p e_p + \gamma_p e_a + e_p, \quad \alpha_p > 0, \quad \beta_p > 0, \quad \gamma_p > 0.$$  \hfill (16)

Second, the advertiser’s efforts are costly to the advertiser, and become more costly as total effort level increases. We model the advertiser’s cost of efforts by a quadratic cost function that is widely used in the literature of incentive contracts. Formally, the cost of efforts $e_a$ is

$$C(e_a) = e_a^2 / 2.$$  \hfill (17)

We solve for the optimal contract, and compare it with the optimal contract when the advertiser cannot make efforts that may convert more click-throughs to purchases.

**Proposition 7.** The optimal contract places a lower weight on purchases and a greater weight on click-throughs when the advertiser can make efforts to convert more click-throughs to purchases, as opposed to when the advertiser cannot make these efforts.

When the advertiser can make efforts to convert more click-throughs to purchases, the contracting problem becomes a double-sided moral hazard problem. In such a problem, the optimal contract should give both parties incentives to make their efforts, and the size of a party’s incentive should be proportional to that party’s role in deciding the outcome of joint efforts. To reflect the increased role of the advertiser, the optimal contract should have a higher percentage of the advertiser’s payoff depending on purchases. This results in a smaller per-purchase payment from the advertiser to the publisher. However, a smaller per-purchase payment would reduce the publisher’s incentives. In order to maintain an appropriate level of incentives for the publisher, the publisher should be given a higher per-click-through payment.
Non-immediate Purchases

In the basic model, we focus on immediate purchases made by consumers, and we assume away non-immediate purchases. In other words, we assume that a consumer who sees the advertiser's offer will either make an immediate purchase, or make no purchase and forget the offer. This assumption of no non-immediate purchases becomes a concern in two scenarios. First, non-immediate purchases can be significant for products that have high value or products that are difficult to be evaluated, such as cars and electronics. It is likely that a consumer who sees an offer of these products will not make an immediate decision on whether to make a purchase. But such a consumer may remember this offer, later come back directly to the advertiser's store, and make a purchase. A study cited by Briggs (2003) shows that an advertiser gets 80 percent of its conversions from these returning consumers. Second, some products, for instance, medicines, office supplies, and insurance policies, feature high repeat purchase rates. Consumers usually go directly to the advertiser's store when making repeat purchases. These repeat purchases can also be thought of as non-immediate purchases and can be a significant part of the total purchases.

In both these scenarios, online advertising increases both immediate purchases and non-immediate purchases—purchases that are not immediate and cannot be traced to an advertisement at a certain publisher. We modify our basic model to consider how the existence of non-immediate purchases may affect the optimal contract. We introduce a variable \( \lambda (0 < \lambda < 1) \) which stands for the percentage of immediate purchases in the total purchases and we assume that the advertiser knows this percentage. Thus, \( \lambda \theta_p \) is the immediate purchase rate, and \( (1 - \lambda)\theta_p \) is the non-immediate purchase rate. We use \( \lambda \theta_p \) to substitute \( \theta_p \) in the

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11 I thank Erik Brynjolfsson and Jerry Hausman for suggesting studying non-immediate purchases.
optimization problem, and we also add \((1 - \lambda)\theta_p\) to the advertiser’s payoff. We solve for the optimal contract when non-immediate purchases exist, and compare it with the optimal contract when non-immediate purchases do not exist.

**Proposition 8.** The optimal per-purchase payment is higher when non-immediate purchases exist, as opposed to when non-immediate purchases do not exist. However, the optimal per-click-through payment is independent of non-immediate purchases.

The intuition for Proposition 8 is as follows. The advertiser benefits from non-immediate purchases just as it does from immediate purchases. However, since the publisher does not get credit for these purchases, the old optimal contract, if it were used, would not have given the publisher enough incentives to make incremental efforts. In order to solve this problem, the new optimal contract sets the per-purchase payment as if the advertiser’s profit from each purchase were \(m/\lambda\), which means giving the publisher full credit for non-immediate purchases, while the per-click-through payment is kept intact.

It is worth noting that the online advertising industry is evolving in a direction of giving the publisher credit for non-immediate purchases. When CPA pricing model was introduced, the publisher got per-purchase commission only for purchases that happen immediately after a consumer’s click-through. Recently, many advertisers have started to give the publisher longer commission duration—the amount of time the publisher can receive commission for a purchase after a consumer has first clicked-through. (See Commission Junction 2003, Franco and Miller 2003) Although the industry is leaning toward addressing the problem of non-immediate
purchases through the use of cookies, this trend of giving the publisher credit for non-immediate purchases is certainly consistent with what Proposition 8 suggests.\textsuperscript{12}

Summarizing the analysis in Section 5 and 6, Table 1 shows how various factors can influence the use of performance-based pricing models in online advertising.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Per-purchase Payment</th>
<th>Per-click-through Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importance of publisher's incremental efforts</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Precision of click-through measurement</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Uncertainty in the product market</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Publisher's risk aversion</td>
<td>-</td>
<td>inverted-U-shaped</td>
</tr>
<tr>
<td>Advertiser's risk aversion</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Importance of advertiser's incremental efforts</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Existence of non-immediate purchases</td>
<td>+</td>
<td>unchanged</td>
</tr>
</tbody>
</table>

7. Conclusions

Because the Internet is a medium with bidirectional information flows, performance-based pricing models that tie online advertising payments directly to campaign measurement data such as click-throughs and purchases are possible. These pricing models have become increasingly widely used in the online advertising industry. This paper attempts to provide a formal structure to analyze issues related with this new phenomenon.

\textsuperscript{12} When a consumer clicks-through an advertisement, this consumer is assigned a cookies number and is classified as belonging to the publisher who has delivered the advertisement. Thus, this publisher will receive commission for all the purchases made by this consumer within the commission duration.
More specifically, we focus on non-contractible efforts that the publisher and the advertiser can make to improve the effectiveness of advertising campaigns, and we define measurement data such as click-throughs and purchases as noisy signals of these non-contractible efforts. Using well-established premises and a simple model, we provide an explanation of why performance-based pricing models are desirable. An online advertising contract that includes appropriate performance-based elements gives the publisher and the advertiser proper incentives to make their efforts, and helps align the interests of the publisher and the advertiser. We use this model to explore factors that may influence the use of performance-based pricing models and to clarify issues that are being debated in the industry.

There are a variety of ways our results can be extended by future research. First, we only analyze the incentive contract problem between one advertiser and one publisher. Researchers may explore cases that have oligopoly players. In addition, it may be interesting, but perhaps technically challenging, to make extensions to different utility functions, nonlinear mappings from efforts to click-throughs, and nonlinear mappings from efforts to purchases. Some of these extensions can be analyzed in the context of a linear contract between the advertiser and the publisher, but others may require nonlinear contracts. We suspect that most of the results, as summarized in Table 1, would be qualitatively unchanged.

This paper has a number of propositions that predict how the use of performance-based pricing models is influenced by various factors such as the importance of the publisher's incremental efforts, precision of click-through measurement, uncertainty in the product market, and risk aversion parameters. These factors differ across different advertisers and different publishers. It would be interesting to have these propositions tested using empirical data.

References


Appendix

**Proof of Proposition 1.** The Hessian matrix of (11) is

\[
\begin{bmatrix}
-\beta_c^2 + (r_a + r_p)c \sigma_{cc} & -\beta_p \beta_c \\
-\beta_p \beta_c & -[\beta_p^2 + (r_a + r_p)c \sigma_{pp}]
\end{bmatrix},
\]

and it is negative definite. This guarantees the existence of a unique global maximum. First order conditions of (11), with respect to \( t_c \) and \( t_p \), are

\[
(m - t_p) \beta_p \beta_c - [\beta_c^2 + (r_a + r_p) \sigma_{cc}] t_c = 0, \tag{A1}
\]

\[
mr_a \sigma_{pp} + (m - t_p) \beta_p^2 - t_p (r_a + r_p) \sigma_{pp} - t_c \beta_p \beta_c = 0. \tag{A2}
\]

Solving (A1) and (A2) together gives us (12) and (13).

**Proof of Proposition 2.** We need to prove that \((t_p = 0, t_c = 0)\) is not an optimal solution. Since Proposition 1 has proved that \((t_p = t_p^*, t_c = t_c^*)\) is a unique global maximum, we only need to prove that \(t_p^*\) and \(t_c^*\) are nonzero. This is implied by \(r_p > 0\) and \(r_a > 0\).
PROOF OF PROPOSITION 3. Let $\beta_c = \beta \delta_c$ and $\beta_p = \beta \delta_p$. Plugging $\beta_c = \beta \delta_c$ and $\beta_p = \beta \delta_p$ into (12) and (13) and differentiating the new $t_p^*$ and $t_e^*$ with respect to $\beta$ gives us

\[
\frac{dt_p^*}{d\beta} = m \frac{r_w}{r_a + r_w} \left[ \frac{1 + \frac{(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}}{1 + \frac{(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}} \right]^{\frac{(-1)(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}} > 0 ,
\]

\[
\frac{dt_e^*}{d\beta} = m \frac{r_w}{r_a + r_w} \left[ \frac{\delta_c + \frac{\delta_p \sigma_{cc}}{\beta^2 \delta_p^2} + (r_a + r_w)\sigma_{cc}}{\delta_p + \frac{\delta_c \sigma_{cc}}{\beta^2 \delta_p^2} + (r_a + r_w)\sigma_{cc}} \right]^{\frac{(-2)(r_a + r_w)\sigma_{cc}}{\beta^3 \delta_p^2}} > 0.
\]

PROOF OF PROPOSITION 4. a) Differentiating $t_p^*$ and $t_e^*$ with respect to $\sigma_{cc}$ gives us

\[
\frac{dt_e^*}{d\sigma_{cc}} = m \frac{r_w}{r_a + r_w} \left[ \frac{\delta_c + \frac{\delta_p \sigma_{cc}}{\beta^2 \delta_p^2} + (r_a + r_w)\sigma_{cc}}{\delta_p + \frac{\delta_c \sigma_{cc}}{\beta^2 \delta_p^2} + (r_a + r_w)\sigma_{cc}} \right]^{\frac{(-1)(r_a + r_w)\sigma_{cc}}{\beta^3 \delta_p^2}} < 0 ,
\]

\[
\frac{dt_p^*}{d\sigma_{cc}} = m \frac{r_w}{r_a + r_w} \left[ \frac{1 + \frac{(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}}{1 + \frac{(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}} \right]^{\frac{(-1)(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}} > 0.
\]

b) By Envelope Theorem, we can directly differentiate $CE(y_a)$ with respect to $\sigma_{cc}$. Doing so give us $dCE(y_a) / d\sigma_{cc} = -(r_p + r_a)t_e^2 / 2 < 0$.

PROOF OF PROPOSITION 5. a) Differentiating $t_p^*$ and $t_e^*$ with respect to $\sigma_{pp}$ gives us

\[
\frac{dt_e^*}{d\sigma_{pp}} = m \frac{r_w}{r_a + r_w} \left[ \frac{\delta_c + \frac{\delta_p \sigma_{cc}}{\beta^2 \delta_p^2} + (r_a + r_w)\sigma_{cc}}{\delta_p + \frac{\delta_c \sigma_{cc}}{\beta^2 \delta_p^2} + (r_a + r_w)\sigma_{cc}} \right]^{\frac{(-1)(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}} > 0 ,
\]

\[
\frac{dt_p^*}{d\sigma_{pp}} = m \frac{r_w}{r_a + r_w} \left[ \frac{1 + \frac{(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}}{1 + \frac{(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}} \right]^{\frac{(-1)(r_a + r_w)\sigma_{pp}}{\beta^2 \delta_p^2}} < 0.
\]
b) By Envelope Theorem, we can directly differentiate $CE(y_a)$ with respect to $\sigma_{pp}$. Doing so give us $\frac{dCE(y_a)}{d\sigma_{pp}} = -r_a(m - t_p)^2 / 2 - r_p t_p^2 / 2 < 0$.

**PROOF OF PROPOSITION 6.**

a) Differentiating $t_p^*$ with respect to $r_a$ and $r_p$ gives us

$$
\frac{dt_p^*}{dr_a} = \frac{m}{(r_a + r_p)^2} \left\{ \frac{1}{1 + \frac{(r_a + r_p)\sigma_{pp}}{\beta_p^2} + \frac{\beta_c^2 \sigma_{pp}}{\beta_p^2 \sigma_{cc}}} \right\} < 0.
$$

b) Differentiating $t_c^*$ with respect to $r_a$ and $r_p$ gives us

$$
\frac{dt_c^*}{dr_a} = \frac{m}{(r_a + r_p)^2} \left\{ \frac{-1}{\left( \frac{\beta_c}{\beta_p \sigma_{pp}} + \frac{(r_a + r_p)\sigma_{cc}}{\beta_p \sigma_{cc}} \right)^2} \right\} < 0,
$$

The relationship between $t_c^*$ and $r_p$ is inverted-U-shaped.

**PROOF OF PROPOSITION 7.** The advertiser's payoff is $y_a = (m - t_p)\theta_p - t_m - t_c \theta_c - e_a^2 / 2$, and its certainty equivalence is
The first-order-condition of (A3) gives the advertiser’s incentive compatibility (IC) constraint, which is

\[ e_a^* = \arg \max CE(y_a) = (m-t_p)\gamma_p. \]  

(A4)

The publisher’s payoff is \( y_p = t_m + t_c \theta_c + t_p \theta_p - e_p^2 / 2 \), and its certainty equivalence is

\[ CE(y_p) = E(y_p) - \frac{r_a}{2} Var(y_p) \]

\[ = t_m + t_c (\alpha_c + \beta_c e_p) + t_p (\alpha_p + \beta_p e_p + \gamma_p e_a) - e_p^2 / 2 - r_p (t_c^2 \sigma_{cc} + t_p^2 \sigma_{pp}) / 2. \]  

(A5)

The first-order-condition of (A5) gives the publisher’s incentive compatibility (IC) constraint, which is

\[ e_p^* = \arg \max CE(y_p) = t_c \beta_c + t_p \beta_p. \]  

(A6)

The optimal contract will set \( t_m \) such that the publisher obtains a utility of \( u_0 \). This implies

\[ t_m^* = u_0 + e_p^2 / 2 + r_p (t_c^2 \sigma_{cc} + t_p^2 \sigma_{pp}) / 2 - t_c^* (\alpha_c + \beta_c e_p) - t_p^* (\alpha_p + \beta_p e_p + \gamma_p e_a). \]  

(A7)

Substituting (A4), (A6) and (A7) into (A5) gives us

\[ CE(y_a) = m[\alpha_p + \beta_p (t_c \beta_c + t_p \beta_p) + \gamma_p^2 (m-t_p)] - (t_c \beta_c + t_p \beta_p)^2 / 2 - (m-t_p)^2 \gamma_p^2 / 2 \]

\[ - (r_p + r_a) t_c^2 \sigma_{cc} / 2 - r_a (m-t_p)^2 \sigma_{pp} / 2 - r_p t_p^2 \sigma_{pp} / 2 - u_0. \]  

(A8)

Solving the first order conditions of (A8), with respect to \( t_c \) and \( t_p \), together give us

\[ t_p^{**} = m \frac{[\beta_c^2 + (r_a + r_p) \sigma_{cc}] r_a \sigma_{pp} + \beta_p^2 (r_a + r_p) \sigma_{cc}}{[\beta_c^2 + (r_a + r_p) \sigma_{cc}] [(r_a + r_p) \sigma_{pp} + \gamma_p^2] + \beta_p^2 (r_a + r_p) \sigma_{cc}}, \]  

(A9)

\[ t_c^{**} = m \frac{(r_p \sigma_{pp} + \gamma_p^2) \beta_p \beta_c}{[\beta_c^2 + (r_a + r_p) \sigma_{cc}] [(r_a + r_p) \sigma_{pp} + \gamma_p^2] + \beta_p^2 (r_a + r_p) \sigma_{cc}}. \]  

(A10)
Notice that when $\gamma_p = 0$, $t_p^{**}$ and $t_c^{**}$ are exactly $t_p^*$ and $t_c^*$ in Proposition 1.

Differentiating $t_p^{**}$ and $t_c^{**}$ with respect to $\gamma_p$ gives us

$$\frac{dt_p^{**}}{d\gamma_p} = m \frac{-2\gamma_p \{(\beta_c^2 + (r_a + r_p)\sigma_{cc})r_a\sigma_{pp} + \beta_p^2 (r_a + r_p)\sigma_{cc}\}\{(\beta_c^2 + (r_a + r_p)\sigma_{cc})[\gamma_p^2 + \beta_p^2 (r_a + r_p)\sigma_{cc}]\} < 0,}$$

$$\frac{dt_c^{**}}{d\gamma_p} = m \frac{2\gamma_p \beta_p \beta_c \{(\beta_c^2 + (r_a + r_p)\sigma_{cc})r_a\sigma_{pp} + \beta_p^2 (r_a + r_p)\sigma_{cc}\}\{(\beta_c^2 + (r_a + r_p)\sigma_{cc})[\gamma_p^2 + \beta_p^2 (r_a + r_p)\sigma_{cc}]\} > 0.}$$

Therefore, we have proved that $t_p^{**} < t_p^*$ and $t_c^{**} > t_c^*$.

PROOF OF PROPOSITION 8. The advertiser's payoff is

$$y_a = m(1 - \lambda)\theta_p + (m - t_p)\lambda \theta_p - t_m - t_c \theta_c = (m - \lambda t_p)\theta_p - t_m - t_c \theta_c.$$ 

The publisher's payoff is $y_p = t_m + t_c \theta_c + t_p \lambda \theta_p - e_p^2 / 2$.

The new optimization problem is exactly the optimization problem in Proposition 1 with $t_p$ replaced by $\lambda t_p$. Thus, if we call the solutions of the new optimization problem $t_p^{***}$ and $t_c^{***}$, in order to distinguish them from the optimal solutions $t_p^*$ and $t_c^*$ in Proposition 1, we have $\lambda t_p^{***} = t_p^*$ and $t_c^{***} = t_c^*$. Because $0 < \lambda < 1$, we have $t_p^{***} = t_p^*/\lambda > t_p^*$ and $t_c^{***} = t_c^*$. 

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Renting versus Selling Durable Information Goods

Doctoral Thesis Chapter 3

Yu (Jeffrey) Hu

MIT Sloan School of Management

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Renting versus Selling Durable Information Goods

ABSTRACT
A monopoly producer of a durable information good can either sell or rent its good to consumers. We study whether the producer obtains a higher profit under a selling strategy or a renting strategy. Our analysis shows that the conventional wisdom that a durable good monopolist would always prefer renting to selling is no longer valid in the context of durable information goods, because of the existence of “individual depreciation”. We find that a renting strategy leads to a higher producer surplus than a selling strategy does, when this individual depreciation parameter is high, i.e., the utility a durable information good provides to consumers decreases relatively slowly from the first consumption to the second consumption and so on. But when the individual depreciation parameter is low, a renting strategy may lead to a lower producer surplus than a selling strategy does. Whether a monopoly producer of a durable information good should adopt a renting strategy depends on the individual depreciation parameter of the good.

(Renting; Selling; Durable good; Information good)
1. Introduction

Today the renting channel is an important channel for firms to distribute their information goods to consumers. A significant proportion of the movie industry’s revenue comes from movies rentals. Software vendors have increasingly moved toward an Application Service Provider (ASP) revenue model. In this revenue model, users, instead of taking the ownership of software, rent software from vendors and pay monthly fees for their usage of software. However, research that studies the distribution of information goods through the renting channel is scarce.¹ Almost all the prior research on the pricing of information goods has focused on the distribution of information goods through the retailing channel. For example, Bakos and Brynjolfsson (1999) have studied the pricing of bundled information goods and the implication of bundling on firm profit and social welfare; Varian (1997) has developed a model for the sharing of information goods among individual consumers; Bakos and Brynjolfsson (1997) have provided a framework for the aggregation and disaggregation of information goods across products, time, and individuals; and similar ideas have been explored in Chuang and Sirbu (1997). This paper aims to study the strategy of a producer that rents its information good to consumers.

There is an extensive literature on the selling and renting strategies of durable goods. Recent examples include Desai and Purohit (1998) and Huang et. al. (2001). The literature can be traced back to the conjecture made by Coase (1972) and formalized by other researchers such as Bulow (1982) and Stokey (1981). Coase conjectures that a firm that sells a durable good will lose its monopoly power and be forced to lower its price from the beginning, if buyers of its good have rational expectations and realize that the firm has an incentive to lower its price over time. It is

¹ An exception is Choudhary et. al. (1998) that studies the benefit of renting software due to software’s network externality.
very difficult for the firm to make a credible commitment that it will not sell the good at a lower price later on, unless it destroys its manufacturing capability. However, this problem does not exist if the firm rents the good to consumers. The conventional wisdom is that a monopolist producer would achieve a higher profit through renting its durable good than selling it durable good. We extend this literature by examining whether the unique properties of information goods could lead to a different conclusion.

We focus on information goods that are durable and can provide utilities to consumers over multiple time periods. Normal durable goods such as automobiles, apartments, and refrigerators provide reduced utilities to consumers over time. The reduction in utilities happens mainly because of quality degradation, and it applies to all consumers. For example, a car provides a lower utility in the second year than in the first year, and this reduction in utility applies to all consumers. Durable information goods also provide reduced utilities to consumers over time. However, the reduction in utilities happens depending on whether a consumer has consumed the good before or not. A movie DVD provides a lower utility when it is consumed for the second time than when it is consumed for the first time. But this lower utility only applies to consumers who have consumed it before. To consumers who have not consumed it before, this movie DVD would provide the same utility as a new movie DVD would, if it is not damaged and still contains all the original information. We focus on this unique property of “individual depreciation” possessed by durable information goods.

Our analysis shows that the conventional wisdom that a durable good monopolist would always prefer renting to selling is no longer valid in the context of durable information goods, because of the existence of “individual depreciation”. We find that a renting strategy leads to a higher producer’s profit than a selling strategy does, when this individual depreciation parameter is
high, i.e., the utility a durable information good provides to consumers decreases relatively slowly from the first consumption to the second consumption and so on. But when the individual depreciation parameter is low, a renting strategy may lead to a lower producer surplus than a selling strategy does. Whether a monopoly producer of a durable information good should adopt a renting strategy depends on the individual depreciation parameter of the good.

The remainder of the paper proceeds as follows. In Section 2, we develop a two-period model in which a monopoly producer distributes a durable information good to consumers who have heterogeneous valuations for the good. We solve for the optimal prices under a selling strategy, with and without the possibility of making a credible commitment of not selling the good at a lower price in Period 2, as well as the optimal rental fees under a renting strategy. Section 3 compares the results under various strategies and gives explanations for why one strategy may dominate another. Section 4 concludes with some broader implications.

2. Model

In this section, we lay out basic assumptions of the model. We use a two-period model in which a monopoly producer distributes a durable good to consumers who have heterogeneous valuations for the good. As in most literature that study durable goods, we assume that the producer has a zero marginal cost of production. The durable good can provide utilities to consumers in both periods. We assume that consumers are heterogeneous in terms of their valuations for the good. Their valuations for the good, denoted by \( v \), are distributed uniformly between zero and one.

We assume that the durable good may have individual depreciation, and we assume away the quality degradation from Period 1 to Period 2. Take the consumer who has a valuation \( v \) for the good for example. The good can provide a utility of \( v \) to this consumer in Period 1. If the
consumer has consumed the good in Period 1, the good can provide a utility of $\theta v$ to this consumer in Period 2. $\theta$ represents the good's individual depreciation and it is bounded between zero and one ($0 \leq \theta \leq 1$). Notice that if $\theta$ equals one, the good is just a normal durable good. If $\theta$ is positive but less than one, the good is a durable information good. If $\theta$ is zero, the good is a non-durable good. If this consumer has not consumed the good in Period 1, the good can still provide a utility of $v$ to this consumer in Period 2.

Baseline Case: Selling with Commitment

We first consider a baseline case in which the monopoly producer sells the durable good to consumers and the producer is assumed to be able to make a credible commitment of not selling the good at a lower price in Period 2. Researchers have argued that this kind of commitment is not time consistent—i.e., ex post, the producer would like to sell the good at a lower price. That is, the commitment is not credible, unless the producer destroys its manufacturing capacity. We only use this case as a baseline case, so that we compare the results in this case to the results in other more realistic cases.

Consumers have two choices: either buy the good in Period 1 at price $p$ and consume it in both Period 1 and Period 2, or not buy the good at all. Suppose that a consumer with a valuation for the good that is equal to $v_1$ is indifferent between buying the good in Period 1 and not buying the good at all. Such a consumer will obtain a utility of $v_1$ in Period 1 and a utility of $\theta v_1$ in Period 2 if she buys the good in Period 1. Thus we have the following equation: $v_1(1 + \theta) - p = 0$. All consumers who have valuations that are greater than $v_1$ will prefer to buy the good in Period 1.
We can derive the demand for the good which is \( D(p) = 1 - \frac{p}{1 + \theta} \). Therefore, the producer's profit is \( \pi(p) = D(p)p = \left(1 - \frac{p}{1 + \theta}\right)p \).

Solving the producer's profit maximization problem gives us the optimal price which is \( p^* = \frac{(1 + \theta)}{2} \), along with the producer's maximum profit which is \( \pi^* = \frac{(1 + \theta)}{4} \).

**Selling without Commitment**

A more realistic case is that the monopoly producer cannot make a credible commitment of not selling the good at a lower price in Period 2. In this case, consumers have three choices: either buy the good in Period 1 at price \( p_1 \) and consume it in both Period 1 and Period 2, or buy the good in Period 2 at price \( p_2 \) and consume it in Period 2, or not buy the good at all. Suppose that a consumer with valuation for the good that is equal to \( v_1 \) is indifferent between buying the good in Period 1 and buying the good in Period 2, and that a consumer with valuation for the good that is equal to \( v_2 \) (\( v_2 \leq v_1 \)) is indifferent between buying the good in Period 2 and not buying the good at all. It is straightforward to show that all consumers who have valuations that are greater than \( v_1 \) will buy the good in Period 1, consumers who have valuations that are between \( v_2 \) and \( v_1 \) will buy the good in Period 2, and consumers who have valuations that are lower than \( v_2 \) will not buy the good at all. We have the following two equations: \( v_1 (1 + \theta) - p_1 = v_1 - p_2 \), and \( v_2 - p_2 = 0 \).

We solve the producer’s optimization problem by backward induction. First, we solve the producer’s optimization problem in Period 2, given that consumers who have valuations that are greater than \( v_1 \) have bought the good in Period 1. We will get the optimal price in Period 2 for
each possible value of $v_1$, that is, $r_2^*(v_1)$. Given this optimal selling strategy in Period 2, we then solve for the optimal price in Period 1. We have the following proposition that characterizes the optimal strategy for renting the information good.

**PROPOSITION 1.** *The optimal selling strategy for a monopoly producer of information goods is the following: the optimal price in Period 1 is $p_1^* = \frac{\theta + 1/2}{2\theta + 1/2}$, the optimal rental fee in Period 2 is also $p_2^* = \frac{\theta + 1/2}{4\theta + 1}$, and the producer's profit is $\pi^* = \frac{(\theta + 1/2)^2}{4(\theta + 1/4)}$ under this optimal strategy.*

Proofs of the proposition and other propositions can be found in the Appendix.

**Renting**

We now consider the case in which the monopoly producer distributes the durable good to consumers by renting it. Consumers do not take the ownership of the good, but they can use the good for one period by paying a rental fee in each period. We let the rental fee in Period 1 be $r_1$ and the rental fee in Period 2 be $r_2$. Consumers have four choices: either rent the good in both Period 1 and Period 2, or rent the good in Period 1 only, or rent the good in Period 2 only, or not rent the good at all.

We solve this case through backward induction. Suppose that consumers with valuation that are greater than $v_1$ have rented the good in Period 1 and consumers with valuations that are less than $v_1$ have not rented the good in Period 1. In Period 2, we will have two different groups of consumers: consumers who have rented the good in Period 1 and consumers who have not rented the good in Period 1. Consumers who have rented the good in Period 1 have valuations that are distributed between $\theta v_1$ and $\theta$, while consumers who have not rented the good in Period 1 have
valuations that are distributed between 0 and $v_1$. The total demand for the good in Period 2 is the sum of the demands from these two different groups of consumers. The demand in Period 2 has different forms, depending on whether $v_1$ is greater than $\theta$. Figure 1 shows these two different cases.

When $v_1 < \theta$, the demand in Period 2 is

$$D_2(r_2, v_1) = \begin{cases} 
1 - \frac{r_2}{\theta} & \text{if } v_1 < r_2 \leq \theta \\
1 - \frac{r_2}{\theta} + v_1 - r_2 & \text{if } \theta v_1 < r_2 \leq v_1 \\
1 - r_2 & \text{if } 0 \leq r_2 \leq \theta v_1
\end{cases}$$

When $v_1 \geq \theta$, the demand in Period 2 is

$$D_2(r_2, v_1) = \begin{cases} 
v_1 - r_2 & \text{if } \theta < r_2 \leq v_1 \\
1 - \frac{r_2}{\theta} + v_1 - r_2 & \text{if } \theta v_1 < r_2 \leq \theta \\
1 - r_2 & \text{if } 0 \leq r_2 \leq \theta v_1
\end{cases}$$

Figure 1: Consumers’ Valuations for the Good in Period 1 and Period 2 in Two Cases
We first solve for the optimal rental fee in Period 2 for each possible value of $v_1$, that is, $r_2^*(v_1)$. Given this optimal renting strategy in Period 2, we then solve for the optimal rental fee in Period 1, that is, $r_1^*$. We have the following proposition that characterizes the optimal strategy for renting the information good.

**PROPOSITION 2.** The optimal renting strategy for a monopoly producer of information goods is the following: 1) when $\theta \geq 1/3$, the optimal rental fee in Period 1 is $r_1^* = \frac{\theta}{1+\theta}$, the optimal rental fee in Period 2 is also $r_2^* = \frac{\theta}{1+\theta}$, and the producer’s profit is $\pi^* = \frac{\theta}{1+\theta}$ under this optimal strategy; 2) when $\theta < 1/3$, the optimal rental fee in Period 1 is $r_1^* = 1$, the optimal rental fee in Period 2 is $r_2^* = \frac{1}{2}$, and the producer’s profit is $\pi^* = \frac{1}{4}$ under this optimal strategy.

3. **Comparing the Optimal Renting and Selling Strategies**

In this section, we compare the results under a renting strategy with the results under a selling strategy for. We show that a renting strategy leads to a higher producer surplus than a selling strategy does, when individual depreciation parameter is high, that is, the utility a durable information good provides to consumers decreases relatively slowly from the first consumption to the second consumption and so on. But when individual depreciation parameter is low, a renting strategy may lead to a lower producer surplus than a selling strategy does. We provide explanations for why this happens. We also compare the results in these two cases with the results in the baseline case, in which the monopoly producer sells the durable good to consumers,
and the producer is assumed to be able to make a credible commitment of not selling the good at a lower price in Period 2.

![Graph showing profit under different strategies](image)

**Figure 1: A Monopoly Producer's Profit in Three Cases**

Figure 1 shows the maximum profit a monopoly producer can obtain under different individual depreciation parameter $\theta$. The solid line is the maximum profit when the producer sells the durable good to consumers and the producer is assumed to be able to make a credible commitment of not selling the good at a lower price in Period 2. The dotted line is the maximum profit when the producer sells the durable good to consumers and the producer cannot make a credible commitment of not selling the good at a lower price in Period 2. The dashed line is the maximum profit when the producer rents the durable good to consumers.

*When the Good is Normal Durable Good ($\theta = 1$)*

When $\theta$ equals one, the good is just a normal durable good. In this case, the maximum profit under a selling strategy is lower than the maximum profit under a renting strategy. A monopoly
producer would prefer renting to selling. This result has been obtained by existing literature on durable goods, e.g., Bulow (1982).

The intuition is the following. First, if the monopoly producer sells the good to consumers and cannot make a credible commitment of not selling the good at a lower price in Period 2, then the producer has an incentive to sell the good at a lower price in Period 2. The good sold in Period 2 is a substitute for the good sold in Period 1, because the good sold in Period 2 can provide utility to consumers just like the good sold in Period 1 can, although the former only provides utility in Period 2 while the latter provides utility in both periods. Knowing that the producer would like to lower its price once Period 1 has passed, consumers have an option to wait until the price drops in Period 2. Thus the producer loses some of its monopoly power and has to lower its price in Period 1. The producer makes additional profit from sales in Period 2, but the additional profit in Period 2 is not enough to offset the profit decrease in Period 1. Overall, the producer achieves a lower profit if it cannot make a credible commitment of not selling the good at a lower price in Period 2 than if it can make such a credible commitment.

However, a renting strategy would remove this inefficiency. The good rented in Period 2 is not a substitute for the good rented in Period 1 at all. This is because a consumer who has valuation \( v \) for the good in Period 1 still has the same valuation for the good in Period 2, no matter whether the consumer has rented the good in Period 1 or not. A renting strategy would allow the producer to achieve the profit that could have been achieved had the producer been able to make a credible commitment of not selling the good at a lower price in Period 2 in the case of selling.

Figure 1 confirms what we have just discussed as it shows that the solid line (selling with commitment) intersects the dotted line (renting) at \( \theta = 1 \) and both are higher than the dashed line (selling) at \( \theta = 1 \).
When the Good is Durable Information Good ($0 < \theta < 1$)

When $\theta$ is less than one, the good is no longer a normal durable good. Instead, it is a durable information good. In this case, the maximum profit under a selling strategy is not always lower than the maximum profit under a renting strategy. When individual depreciation parameter $\theta$ is high, a monopoly producer would prefer renting to selling. But when individual depreciation parameter $\theta$ is low, a monopoly producer would prefer selling to renting. Figure 1 shows that the dotted line (renting) is higher than the dashed line (selling) when $\theta$ is high, and that the dotted line (renting) is lower than the dashed line (selling) when $\theta$ is low. Both lines are lower than the solid line (selling with commitment).

The intuition is the following. When $\theta$ is less than one, the good rented in Period 2 becomes a substitute for the good rented in Period 1. This is because consumption of the good in Period 1 lowers the valuation of the good in Period 2. Take the consumer who has valuation $v$ for the good in Period 1 for example. This consumer will have valuation $v$ for the good in Period 2 if the consumer has not rented the good in Period 1, and valuation $\theta v$ for the good in Period 2 if the consumer has not rented the good in Period 1. When $\theta$ is high, this substitution effect is small. Thus, a renting strategy would allow the producer to achieve a profit level that is close to what could have been achieved had the producer been able to make a credible commitment of not selling the good at a lower price in Period 2 in the case of selling. But when $\theta$ is low, this substitution effect becomes significant, and renders a renting strategy very inefficient. A renting strategy would lead to a profit level that is much lower than what could have been achieved had the producer been able to make a credible commitment of not selling the good at a lower price in Period 2 in the case of selling. In fact, the producer's profit under a renting strategy is even lower
than the producer’s profit under a selling strategy that does not allow the producer to make a credible commitment of not selling the good at a lower price in Period 2.

*When the Good is Non-durable Good (θ = 0)*

When θ is zero, the good is a non-durable good. All three strategies are equivalent in this case. The producer would achieve the same profit no matter what strategy is adopted. This is reflected in Figure 1 as all three lines intersect at θ = 0.

4. **Discussions**

In this section we discuss the implications of the results derived in Section 3 and how our results can be extended by future research.

*Disney’s Selling Strategy*

Walt Disney Co. has long used a strategy of releasing its classic animated films and then making them unavailable for up to ten years, in hopes to build consumers’ demand for the films’ future re-releases. (Orwall 1999) In order to publicize this policy, Disney commercials frequently deliver lines such as “This is your last chance to own this enchanting DVD, then it goes into the vault for ten years!” Why does Disney have this policy? Does this policy cause Disney to lose revenue or gain revenue? The framework developed in this model will help answer these questions.

We argue that Disney’s classic animated films are durable information goods that have a high individual depreciation parameter (θ is high). This is because the utilities these films can provide to consumers decrease very slowly over time, as they are played again and again by their consumers that are mainly children. For this type of goods, the monopoly producer obtains a much higher profit if the producer can make a credible commitment of not selling the good at a
lower price later on than if the producer cannot make such a commitment. Researchers have argued that this kind of commitment is not credible, because they are time-inconsistent. But if Disney can leverage its reputation and its long-standing policy of retiring films for up to ten years to make this commitment credible to consumers, Disney will certainly obtain a higher profit than it can otherwise.

What Products Should Be Sold and What Products Should Be Rented?

The decision of whether a durable good should be sold or rented has traditionally been made by the good's producer based on its evaluation of the good's cost of production, cost of delivery, and cost transaction in the retailing channel and in the renting channel. Information technology has greatly reduced these costs, and in turn their effect on the producer's profit. This paper suggests that in such a new environment, firms that produce durable goods, especially durable information goods, should pay attention to their goods' individual depreciation parameter, when they make a decision of whether to sell or rent their goods to consumers. For goods that can consistently provide utilities to an individual consumer, such as classic movies, music, and software that has a long life span, a renting strategy can lead to a higher profit to the producer than a selling strategy does. But for goods that provide greatly reduced utilities to an individual consumer after the first consumption, such as non-classic movies, fiction books, and software that has a short life span, a renting strategy can lead the producer to a lower profit to the producer than a selling strategy does.

There are a variety of ways our results can be extended by future research. First, we only focus on how the individual depreciation parameter of durable goods affects a producer's strategy in our analysis. Researchers may explore cases in which other properties of durable goods, such as quality degradation and the experience good property, interact with individual depreciation. In
addition, it may be interesting to study how the existence of secondary market may change some of the current results.

References


Appendix

PROOF OF PROPOSITION 1. We solve the producer’s optimization problem by backward induction.

First, we solve the producer’s optimization problem in Period 2, given that consumers who have valuations that are greater than $v_1$ have bought the good in Period 1. The producer’s profit function in Period 2 is $\pi_2(v_1, p_2) = (v_1 - p_2)p_2$. Solving the producer’s optimization problem in Period 2 gives us the optimal price in Period 2, given that consumers who have valuations that are greater than $v_1$ have bought the good in Period 1, which is $p_2^* (v_1) = \frac{v_1}{2}$. It also gives us the producer’s maximum profit in Period 2, given that consumers who have valuations that are greater than $v_1$ have bought the good in Period 1, which is $\pi_2^* (v_1) = \frac{v_1^2}{4}$. Substituting the producer’s optimal price in Period 2 into the equation that defines the consumer who has an evaluation that is equal to $v_1$ gives us $p_1 = (\theta + 1/2)v_1$. 

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Now we solve the producer’s optimization problem in Period 1. The producer’s total profit in both periods is \[ \pi(v_1) = \pi_1(v_1) + \pi_2^*(v_1) = (1 - v_1) p_1 + \frac{v_1^2}{4} = (1 - v_1)(\theta + 1/2)v_1 + \frac{v_1 v_1}{2}. \] Solving the producer’s optimization problem in Period 1 gives us that the optimal strategy is to set \[ v_1^* = \frac{\theta + 1/2}{2\theta + 1/2}, \] which is also equivalent to setting \[ p_1^* = \frac{(\theta + 1/2)^2}{2\theta + 1/2}. \] The producer’s maximum profit in this case is \[ \pi^* = \frac{(\theta + 1/2)^2}{4(\theta + 1/4)}. \] We can also calculate the producer’s optimal price in Period 2, which is \[ p_2^* = \frac{\theta + 1/2}{4\theta + 1}. \]

**Proof of Proposition 2.**

Case 1: when \( v_1 < \theta \), the demand in Period 2 is

\[
D_2(r_2, v_1) = \begin{cases} 
1 - r_2 / \theta & \text{if } v_1 < r_2 \leq \theta \\
1 - r_2 / \theta + v_1 - r_2 & \text{if } \theta v_1 < r_2 \leq v_1 \\
1 - r_2 & \text{if } 0 \leq r_2 \leq \theta v_1
\end{cases}
\]

We solve for the optimal rental fee in Period 2 for each possible value of \( v_1 \), that is, \( r_2^*(v_1) \).

Case 1.1 when \( \theta > \sqrt{3}/2 \)

\[ r_2^*(v_1) = \begin{cases} 
\theta / 2 & \text{if } v_1 \leq (1 - \sqrt{1 - \theta}) / (2\theta) \\
\theta v_1 & \text{if } (1 - \sqrt{1 - \theta}) / (2\theta) < v_1 \leq 1 / (2\theta) \\
1 / 2 & \text{if } 1 / (2\theta) < v_1 < \theta
\end{cases}
\]

Case 1.2 when \( \sqrt{2}/2 < \theta \leq \sqrt{3}/2 \)
Case 1.3 when $1/2 < \theta \leq \sqrt{2}/2$

\[
    r_2^*(v_1) = \begin{cases} 
    \frac{\theta}{2} & \text{if } v_1 \leq \sqrt{1+\theta} - 1 \\
    (1 + v_1)/(2 + 2/\theta) & \frac{\sqrt{1+\theta} - 1}{\theta} < v_1 \leq 1/(2\theta + 1) \\
    \theta v_1 & \text{if } 1/(2\theta + 1) < v_1 \leq \theta \\
    1/2 & \text{if } 1/(2\theta) < v_1 < \theta 
    \end{cases}
\]

Case 1.4 when $\theta \leq 1/2$

\[
    r_2^*(v_1) = \begin{cases} 
    \frac{\theta}{2} & \text{if } v_1 \leq \sqrt{1+\theta} - 1 \\
    (1 + v_1)/(2 + 2/\theta) & \frac{\sqrt{1+\theta} - 1}{\theta} < v_1 < \theta \\
    \theta v_1 & \text{if } 1/(2\theta + 1) < v_1 < \theta \\
    1/2 & \text{if } 1/(2\theta) < v_1 < \theta 
    \end{cases}
\]

Case 2: when $v_1 \geq \theta$, the demand in Period 2 is

\[
    D_2(r_2, v_1) = \begin{cases} 
    v_1 - r_2 & \text{if } \theta - r_2 \leq v_1 \\
    1 - r_2 / \theta + v_1 - r_2 & \text{if } \theta v_1 < r_2 \leq \theta \\
    1 - r_2 & \text{if } 0 \leq r_2 \leq \theta v_1 
    \end{cases}
\]

We solve for the optimal rental fee in Period 2 for each possible value of $v_1$, that is, $r_2^*(v_1)$.

Case 2.1 when $\theta > \sqrt{2}/2$

\[
    r_2^*(v_1) = 1/2 \quad \text{if } \theta \leq v_1 \leq 1
\]

Case 2.2 when $1/2 < \theta \leq \sqrt{2}/2$

\[
    r_2^*(v_1) = \begin{cases} 
    \theta v_1 & \text{if } \theta \leq v_1 \leq 1/(2\theta) \\
    1/2 & \text{if } 1/(2\theta) < v_1 \leq 1
    \end{cases}
\]

Case 2.3 when $(\sqrt{2} - 1)/2 < \theta \leq 1/2$
\[ r_2^*(v_1) = \begin{cases} \frac{(1 + v_1)}{(2 + 2/\theta)} & \text{if } \theta \leq v_1 \leq 1/(2\theta + 1) \\ 0/2 & \text{if } 1/(2\theta + 1) < v_1 \leq \theta/(\theta^2 + 1/4) \\ v_1 / 2 & \text{if } \theta/(\theta^2 + 1/4) < v_1 \leq 1 \end{cases} \]

Case 2.4 when \( \theta \leq (\sqrt{2} - 1)/2 \)

\[ r_2^*(v_1) = \begin{cases} \frac{(1 + v_1)}{(2 + 2/\theta)} & \text{if } \theta \leq v_1 \leq 1/(\sqrt{1 + 1/\theta} - 1) \\ v_1 / 2 & \text{if } 1/(\sqrt{1 + 1/\theta} - 1) < v_1 \leq 1 \end{cases} \]

We can now combine these eight cases to five cases: \( \theta > \sqrt{3}/2, 1/2 < \theta \leq \sqrt{3}/2, 1/4 < \theta \leq 1/2, (\sqrt{2} - 1)/2 < \theta \leq 1/4, \theta \leq (\sqrt{2} - 1)/2 \). In each case, we can solve for the optimal rental fee in Period 1 \( r_1^* \), given the optimal renting strategy in Period 2.

Case A1 \( \theta > \sqrt{3}/2 \)

1) if \( v_1 \leq (1 - \sqrt{1 - \theta})/(2\theta) \), we have

\[ r_1^* = (1 - \sqrt{1 - \theta})/(2\theta), r_2^* = \theta/2, \pi^* = (1 - \sqrt{1 - \theta})/(2\theta)*[1 - (1 - \sqrt{1 - \theta})/(2\theta)] + \theta / 4 \]

2) if \( (1 - \sqrt{1 - \theta})/(2\theta) < v_1 \leq 1/(2\theta) \), we have \( r_1^* = \theta/(1 + \theta), r_2^* = \theta/(1 + \theta), \pi^* = \theta/(1 + \theta) \)

3) if \( 1/(2\theta) < v_1 \leq 1 \), we have \( r_1^* = 1/2, r_2^* = 1/2, \pi^* = (3\theta - 1)/(4\theta) \)

The maximum profit is achieved in 2).

Case A2 \( 1/2 < \theta \leq \sqrt{3}/2 \)

1) if \( v_1 \leq \sqrt{1 + \theta} - 1 \), we have

\[ r_1^* = \sqrt{1 + \theta} - 1, r_2^* = \theta/2, \pi^* = (\sqrt{1 + \theta} - 1)*(2 - \sqrt{1 + \theta}) + \theta / 4 \]
2) if $\sqrt{1+\theta} - 1 < v_1 \leq 1/(2\theta + 1)$, we have

$$r_1^* = \theta/(1 + 2\theta), r_2^* = \theta/(1 + 2\theta), \pi^* = \theta(3\theta + 1)/(1 + 2\theta)^2$$

3) if $1/(2\theta + 1) < v_1 \leq 1/(2\theta)$, we have $r_1^* = \theta/(1 + \theta), r_2^* = \theta/(1 + \theta), \pi^* = \theta/(1 + \theta)$

4) if $1/(2\theta) < v_1 \leq 1$, we have $r_1^* = 1/2, r_2^* = 1/2, \pi^* = (3\theta - 1)/(4\theta)$

The maximum profit is achieved in 3).

Case A3 $1/4 < \theta \leq 1/2$

1) if $v_1 \leq \sqrt{1+\theta} - 1$, we have $r_1^* = \sqrt{1+\theta} - 1, r_2^* = \theta/2, \pi^* = (\sqrt{1+\theta} - 1)(2 - \sqrt{1+\theta}) + \theta/4$

2) if $\sqrt{1+\theta} - 1 < v_1 \leq 1/(2\theta + 1)$, we have

$$r_1^* = \theta/(1 + 2\theta), r_2^* = \theta/(1 + 2\theta), \pi^* = \theta(3\theta + 1)/(1 + 2\theta)^2$$

3) if $1/(2\theta + 1) < v_1 \leq \theta/((\theta^2 + 1/4)$, we have $r_1^* = \theta/(1 + \theta), r_2^* = \theta/(1 + \theta), \pi^* = \theta/(1 + \theta)$

4) if $\theta/((\theta^2 + 1/4) < v_1 \leq 1$, we have $r_1^* = 1/2, r_2^* = 1/2, \pi^* = 1/4$

The maximum profit is achieved in 3) if $1/3 < \theta \leq 1/2$ and in 4) if $1/4 < \theta \leq 1/3$.

Case A4 $(\sqrt{2} - 1)/2 < \theta \leq 1/4$

1) if $v_1 \leq \sqrt{1+\theta} - 1$, we have $r_1^* = \sqrt{1+\theta} - 1, r_2^* = \theta/2, \pi^* = (\sqrt{1+\theta} - 1)(2 - \sqrt{1+\theta}) + \theta/4$

2) if $\sqrt{1+\theta} - 1 < v_1 \leq 1/(2\theta + 1)$, we have

$$r_1^* = \theta/(1 + 2\theta), r_2^* = \theta/(1 + 2\theta), \pi^* = \theta(3\theta + 1)/(1 + 2\theta)^2$$

3) if $1/(2\theta + 1) < v_1 \leq \theta/((\theta^2 + 1/4)$, we have

$$r_1^* = \theta^2/((\theta^2 + 1/4), r_2^* = \theta^2/((\theta^2 + 1/4), \pi^* = \theta^2(\theta^2 - \theta + 1/2)/(\theta^2 + 1/4)^2$$
4) if $\theta/(\theta^2 + 1/4) < v_1 \leq 1$, we have $r_1^* = 1/2, r_2^* = 1/2, \pi^* = 1/4$

The maximum profit is achieved in 4).

Case A5 $\theta \leq (\sqrt{2} - 1)/2$

1) if $v_1 \leq \sqrt{1 + \theta} - 1$, we have $r_1^* = \sqrt{1 + \theta} - 1, r_2^* = \theta/2, \pi^* = (\sqrt{1 + \theta} - 1)(2 - \sqrt{1 + \theta}) + \theta/4$

2) if $\sqrt{1 + \theta} - 1 < v_1 \leq 1/(\sqrt{1 + \theta} - 1)$, we have

$$r_1^* = [1 + 1/(\sqrt{1 + \theta} - 1)]/(2 + 2/\theta), r_2^* = [1 + 1/(\sqrt{1 + \theta} - 1)]/(2 + 2/\theta),$$
$$\pi^* = [1 + 1/(\sqrt{1 + \theta} - 1)][3 - 1/(\sqrt{1 + \theta} - 1)]/(4 + 4/\theta)$$

3) if $\theta/(\theta^2 + 1/4) < v_1 \leq 1$, we have $r_1^* = 1/2, r_2^* = 1/2, \pi^* = 1/4$

The maximum profit is achieved in 3).

Summarizing the optimal solution in Case A1 through A5, we have: 1) when $\theta \geq 1/3$, the optimal rental fee in Period 1 is $r_1^* = \frac{\theta}{1 + \theta}$, the optimal rental fee in Period 2 is also $r_2^* = \frac{\theta}{1 + \theta}$, and the producer's profit is $\pi^* = \frac{\theta}{1 + \theta}$ under this optimal strategy; 2) when $\theta < 1/3$, the optimal rental fee in Period 1 is $r_1^* = 1$, the optimal rental fee in Period 2 is $r_2^* = \frac{1}{2}$, and the producer's profit is $\pi^* = \frac{1}{4}$ under this optimal strategy.