Integrated Driving Behavior Modeling

by

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Abstract

A framework for integrated driving behavior modeling, based on the concepts of short-term goal and short-term plan is proposed. Drivers are assumed to conceive and perform short-term plans in order to accomplish short-term goals. This behavioral framework captures drivers' planning capabilities and allows decisions to be based on anticipated future conditions.

An integrated driving behavior model, which utilizes these concepts, is developed. This model captures both lane changing and acceleration behaviors. The driver's short-term goal is defined by the target lane. Drivers who wish to change lanes but cannot change lanes immediately, select a short-term plan to perform the desired lane change. Short-term plans are defined by the various gaps in traffic in the target lane. Drivers adapt their acceleration behavior to facilitate the lane change using the target gap. Hence, inter-dependencies between lane changing and acceleration behaviors are captured.

The lane changing portion of the model integrates mandatory and discretionary lane changing considerations in a single model. Hence, allowing trade-offs between these considerations to be captured. Moreover, the integrated lane changing model overcomes the difficulty in defining conditions that trigger a mandatory lane changing situation. Model components that describe the choice of target gaps and acceleration behaviors to facilitate lane changing are introduced.

The parameters of all components of the driving behavior model are estimated jointly using detailed vehicle trajectory data collected in a freeway in Arlington, VA. The result is a driving behavior model applicable to the behavior of all freeway traffic. Validation results of the proposed model using a microscopic traffic simulator are also presented.

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Chapter 1

Introduction

1.1 Motivation

Traffic congestion is a major problem in urban areas. It has a significant adverse economic impact through deterioration of mobility, safety and air quality. A recent study (FHWA 2001) estimated that 32% of the daily travel in major US urban areas in 1997 occurred under congested traffic conditions. The annual cost of lost time and excess fuel consumption during congestion was estimated at $72 billion, over $900 per driver. These numbers represent a 300% increase from 1982.

Schrank and Lomax (2001) estimated that 1,800 new freeway lane-miles and 2,500 new urban street lane-miles would have been required in the US in order to keep congestion from increasing from 1998 to 1999. The budgets required for such infrastructure investments far exceed available resources. Moreover, in many urban areas, land scarcity and environmental constraints would limit construction of new roads or expansion of existing ones even if funds were available.

As a result, the importance of better management of the road network to efficiently utilize existing capacity is increasing. In recent years, a large array of traffic management schemes have been proposed and implemented. Methods and algorithms proposed for traffic management need to be calibrated and tested. In most cases, only limited, if any, field tests are feasible because of prohibitively high costs and lack of public acceptance. Furthermore, the usefulness of such field studies is deterred by the inability to fully control the conditions under which they are performed. Hence, tools to perform such evaluations in a laboratory environment are needed.

Intelligent Transportation Systems (ITS) applications, such as dynamic traffic control and route guidance, have emerged as efficient tools for traffic management. These applications involve information dissemination from a traffic management center to drivers and deployment of management and control strategies. The impact of information and control strategies on traffic flow can be realistically modeled only through the response of individual drivers to the information. For example, evaluation of different incident response strategies that utilize lane-use signs requires modeling of drivers' response to the signs and a plausible model of their lane changing behavior. Microscopic traffic simulation models, which analyze traffic phenomena through explicit and detailed representation of the behavior of individual drivers, have been widely used to that end by both researchers and practitioners. Hence, microscopic traffic simulation is an important tool for traffic analysis and particularly valuable in the context of ITS technologies and dynamic traffic management systems. The detailed level of behavior modeling in microscopic simulation models is particularly critical when disaggregate relations between vehicles are more important than the aggregate traffic flow characteristics. For example, An example is the study of safety impacts, for which headway distribution, frequency of emergency braking and the number and locations of lane changes may provide better indication of the impact on safety of different geometric design plans than aggregate traffic characteristics, such as average speed, flow and density.

Driving behavior models describe drivers' decisions with respect to their vehicle movement under different traffic conditions. These models include speed/acceleration models, which describe the movement of the vehicle in the longitudinal direction, and lane changing models, which describe drivers' lane selection and gap acceptance behaviors.

Driving behavior models are an important component of microscopic traffic simulation tools. They are also important to several other fields of transportation science and engineering such as safety studies and capacity analysis, in which aggregate traffic flow characteristics may be deduced from the behaviors of individual drivers. For example, the capacities of different road facilities may be determined by studying the relations between two vehicles, a leader and a follower, traveling through the facility.

State-of-the-art driving behavior models do not capture drivers' planning capabilities and proactive behaviors. Implementation of these models in micro-simulation tools may lead to unrealistic traffic flow characteristics: underestimation of bottleneck capacities and over-estimation of congestion (e.g. DYMO 1999, Abdulhai et al 1999). Hence, there is a need to develop more realistic driving behavior models that will capture the complexity of human decision-making processes.

1.2 Problem description

The driving task is a hierarchical process with three levels of performance (Lunenfeld and Alexander 1990, Koppa 1997):

1. Navigation or Planning (Strategic): Route choice and trip schedule decisions drivers make pre-trip and en-route. These decisions are affected by the driver's knowledge of and familiarity with the transportation network and traffic conditions as well as real-time information available to the driver.
2. Guidance (Tactical): Determination of the two dimensional movement of the vehicle in traffic. These decisions are affected by the vehicle's driving environment and by strategic considerations. This behavior is driven by goals that include safety, adhering to the path plan and a desire to maintain an acceptable driving experience in terms of speed and comfort.
3. Control (Operational): Continuous activities the driver performs to control and direct the vehicle (e.g. steering, throttle, braking). These activities are skill-based and mostly done automatically with little conscious effort.

Interactions between the different driving tasks are shown in Figure 1.1. The driver makes strategic decisions: chooses a path and determines a schedule for the trip (e.g. in terms of desired arrival time). Tactical decisions are affected by the vehicle's driving neighborhood and by the strategic considerations: the driver has to be in the correct lanes in order to follow the path plan; the trip schedule affects desired speeds. If the trip schedule is not kept or in the presence of traffic information the driver may decide to re-evaluate the path plan and switch paths. The choices of speed and lane are translated to mechanical actions to control the vehicle. In turn, the outcome of these actions affects the positioning of the vehicle within its neighborhood.

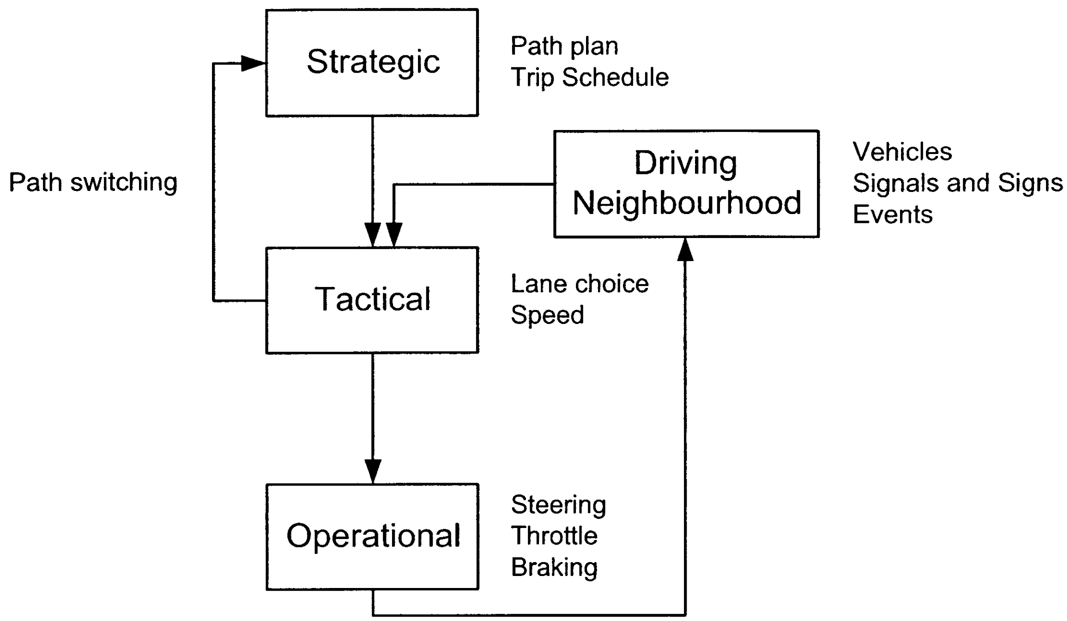


Figure 1.1 - Interactions between the driving tasks

Travel behavior researchers study drivers' strategic choices. The operational behavior is studied in human factors research. Driving behavior models capture tactical decisions. The most notable driving behavior models are acceleration and lane changing models. Other important driving behaviors include negotiation of intersections and merging areas and response to signals and signs.

Early acceleration models were car following models. These models describe the behavior of a vehicle while it is following the vehicle in front of it (the leader). The subject vehicle is assumed to react to the leader's actions. More recently, general acceleration models that also capture the behavior of drivers in other situations have been developed. These models define behaviors for different driving regimes such as car following and free-flow. For example, drivers that are not close to their leaders may apply a free-flow acceleration focused on attaining their desired speed. Lane changing models have mostly been developed specifically for micro-simulation models. The lane changing process is normally modeled in two steps: (i) the decision to consider a lane change and (ii) the execution of the lane change. Lane changes are usually classified as either mandatory (MLC) or discretionary (DLC). MLC are performed when the driver must leave the current lane. DLC are performed to improve driving conditions. Gap acceptance models are used to model the execution of lane changes. Little rigorous work has been done to calibrate and validate driving behavior models and in particular lane changing models.

Existing driving behavior models have several important limitations. Most models assume that drivers make instantaneous decisions based on current or past traffic conditions. These decisions are purely reactive responses to the situation. In reality, human drivers may conceive an action plan and perform it over length of time based on anticipated future conditions. This is particularly important in lane changing behavior, in which drivers try to anticipate the behavior of other vehicles and to adjust their own to facilitate completion of a desired lane change. Moreover, in order to model a more sophisticated driving behavior it is necessary to account for inter-dependencies between behaviors. However, in existing models, different behaviors are modeled separately and so, inter-dependencies are not captured. Most significantly the effect of lane changing behavior on the acceleration is ignored. Similarly, the classification of lane changes as either MLC or DLC does not allow trade-offs between mandatory and discretionary considerations to be modeled. The result is a rigid behavior structure that does not permit, for example, overtaking when mandatory considerations are active. Applying these models to a simulation model may result in unrealistic traffic flow characteristics.

1.3 Thesis contributions

The objective of this research is to improve modeling of driving behavior and in particular capture inter-dependencies between different behaviors. This thesis contributes to the state-of-the-art in driving behavior modeling in the following aspects:

- A framework for integrated driving behavior is proposed. This framework is based on the concepts of short-term goals and short-term plans. Rather than make instantaneous decisions based strictly on current conditions, driver are assumed to conceive and execute short-term plans over a length of time in order to achieve short-term goals. This behavioral framework captures drivers' planning capabilities and allows decisions to be based on anticipated future conditions.

- Based on these concepts, an integrated driving behavior model structure is developed. This model captures both lane changing and acceleration behaviors. Drivers' short-term goals are defined by their target lane. Driver that target a lane change but cannot change lanes immediately, choose a short-term plan, and adapt their acceleration behavior to facilitate the lane change. This model structure accounts for inter-dependencies between the different decisions. The generic structure of the model allows it to be used to specify models for different driving environments, such as freeways (as developed and estimated in this thesis) or urban streets.
- The lane changing portion of the model integrates mandatory and discretionary lane changing considerations in a single model. This approach differs from existing lane changing models, which model MLC and DLC separately. The integrated model structure allows trade-offs between mandatory and discretionary considerations, which were previously ignored, to be captured. Moreover, the integrated lane changing model does not require specification of the conditions that trigger MLC situations, which is necessary when separate MLC and DLC models are used. These conditions were not specified and estimated rigorously in the literature, and therefore estimation results and the subsequent applicability of existing lane changing models is restricted to special cases in which MLC situations could be reasonably assumed (e.g. vehicle merging from a freeway on-ramp).
- A new driving behavior component within the integrated driving behavior model captures drivers' choice of target gaps for lane changing (short-term plans). In this model, drivers choose gaps in traffic in the target lane that they will use to change lanes.
- New acceleration models are introduced to capture drivers' acceleration behavior to facilitate lane changing using the target gap. Estimation results show these acceleration behaviors are significantly different from the behaviors of drivers who are not trying to change lanes.
- The parameters of all components of the driving behavior model, including behaviors first introduced in this thesis, are estimated jointly using detailed vehicle trajectory data from all vehicles in a freeway section. The result is a driving behavior model applicable to the behavior of all freeway traffic, rather than only special cases or

specific groups of vehicles (e.g. only vehicles merging to the freeway or vehicle which do not make any lane changes).

- Estimation results of the lane changing model show that variables related to the driver's path plan are significant in lane selection. Thus, demonstrating the effect of travel behavior on driving behavior.
- Estimation results provide a second case in support of contributions made by Ahmed (1999) with respect to acceleration behavior and in particular indicating that enhancements to the GM car following model are significant. In particular, results support Ahmed's non-linear specification of the car following stimulus term over the linear GM specification. Estimation results also assert the important effect of traffic conditions ahead of the vehicle, captured by the density variable, on car following. Similarly, the conclusion that the speed of the subject vehicle does not affect braking decelerations is strengthened.

1.4 Thesis outline

In Chapter 2, a literature review of existing driving behavior models, including acceleration, lane changing and gap acceptance behaviors is presented. Chapter 3 introduces the concepts that form the basis to the proposed modeling framework. The structure of an integrated driving behavior model based on these concepts is described. Detailed specifications of the various components within the integrated driving behavior model are presented in Chapter 4. The dataset used to estimate the parameters of the proposed model and the methodology used to extract the required information from the raw data are presented in Chapter 5. Estimation results of the integrated driving behavior model are presented in Chapter 6. Validation of the model using a microscopic traffic simulator is presented in Chapter 7. Finally, conclusions and directions for future research are presented in Chapter 8.

Chapter 2

Literature Review

Most of the driving behavior modeling literature focuses on a few key aspects. Acceleration behavior is by far the most extensively studied driving task. Lane changing behavior has also received considerable attention, particularly as part of the development of microscopic traffic simulation models in recent years. This section summarizes some of the relevant literature in these two areas.

2.1 Acceleration models

The acceleration a driver applies depends on several parameters. These include roadway and vehicle characteristics, response to signals and signs, relations with the leader and other vehicles and the desired speed. Acceleration models can be broadly classified in two groups:

- Car following models, which describe the behavior of drivers reacting to the behavior of their leaders.
- General acceleration models, which also describe behaviors in non car following situations.

2.1.1 Car following models

The concept of car following was first proposed by Reuschel (1950) and Pipes (1953). A car following situation is illustrated in Figure 2.1. The subject vehicle follows the leader (vehicle in front) and reacts to its actions. Pipes assumed that the follower wishes to maintain safe time headway equal to 1.02 sec. from the leader. This value was derived from the California Vehicle Code recommendation that drivers maintain a distance of 15 ft for every 10 mph of speed. Using Laplace transformations, he developed theoretical expressions for the accelerations applied by the follower given the leaders' behavior described by a mathematical function.

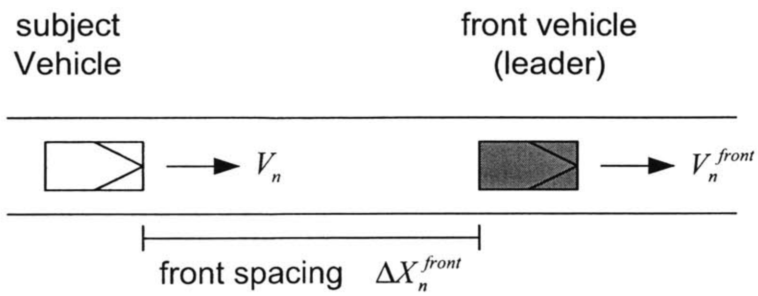


Figure 2.1 - A Car following situation

GM model

Researchers at the GM Research Laboratories introduced the sensitivity-stimulus framework that is the basis for most car following models to date. According to this framework a driver reacts to stimuli from the environment. The response (acceleration) the driver applies is lagged to account for reaction time and is given by:

$$response_n(t) = sensitivity_n(t) \times stimulus_n(t - \tau_n) \tag{2.1}$$

Where, t is the time of observation and τ_n is the reaction time for driver n. The reaction time includes perception time (time from the presentation of the stimulus until the foot starts to move) and foot movement time.

The GM models assume that the stimulus is the leader relative speed (the speed of the leader less the speed of the subject vehicle). Several models, which differ in the specification of the sensitivity term, were developed. The simplest of these models is the linear car following model (Chandler et al 1958, Herman et al 1959), which assumes a constant sensitivity term:

$$a_n(t) = \alpha \Delta V_n^{front}(t - \tau_n) \tag{2.2}$$

Where, $a_n(t)$ is the acceleration applied by driver n at time t. $\Delta V_n^{front}(t-\tau_n)$ is the leader relative speed measured at time $t-\tau_n$. α is a parameter.

This model was estimated using correlation analysis with microscopic data of 8 drivers traveling in real traffic. The data included discrete measurements of acceleration, speed, space headway and the relative leader speed. For each driver in the dataset, the correlation between observed and predicted accelerations was computed for various combinations of values of α and τ. The combination that yielded the highest correlation was selected for that driver. The reported parameter estimates, which are the averages of the individual driver's values, were 0.37 sec.$^{-1}$ and 1.55 sec. for α and τ, respectively.

The main advantage of the linear GM model is its simplicity. However, the assumption that the response to the relative leader speed is independent of the spacing between the vehicles is unrealistic. Moreover, steady state equations derived from this model yield a linear flow-density relationship, in which capacity is obtained at zero density. To overcome this problem Gazis et al (1959) proposed a nonlinear model, in which the response is inversely proportional to the spacing:

$$a_n(t)=\alpha\frac{\Delta V_n^{front}(t-\tau_n)}{\Delta X_n^{front}(t-\tau_n)} \tag{2.3}$$

Where, $\Delta X_n^{front}(t-\tau_n)$ is the spacing between the subject vehicle and its leader measured at time $t-\tau_n$.

The model was also calibrated using correlation analysis using datasets from 3 locations: the GM test-track and the Lincoln and Holland tunnels in New York. Parameter estimates are presented in Table 2.1.

Steady state equations derived by integrating this model correspond to the Greenberg model (1959). Edie (1961) noted that Greenberg's model yields infinite speeds when the density tends to zero. He suggested a modified model, in which the response is proportional to the speed of the subject vehicle.

Table 2.1 - Estimation results for the model proposed by Gazis et al (1959)

Site	Number of drivers	α (mph)	τ (sec.)
GM test track	8	27.4	1.5
Holland tunnel	10	18.3	1.4
Lincoln tunnel	16	20.3	1.2

The most general form of the GM model, the non-linear GM model, is due to Gazis et al (1961). This model allows non-linearity in the sensitivity of the response to the spacing and the subject speed:

$$a_n(t) = \alpha \frac{V_n(t)^\beta}{\Delta X_n^{front}(t-\tau_n)^\gamma} \Delta V_n^{front}(t-\tau_n) \tag{2.4}$$

Where, $V_n(t)$ is the speed of the subject vehicle measured at time t. α, β and γ are parameters.

May and Keller (1967) estimated the non-linear GM model using the corresponding macroscopic speed-density equations derived by Gazis et al (1961). Two sets of data were used: freeway data and tunnel data. Their results are summarized in Table 2.2. Reaction time could not be identified with the macroscopic data.

Table 2.2 - Estimation results for the non-linear GM model (May and Keller 1967)

Parameter	Freeway model	Tunnel model
α	0.000133	0.0127
β	0.8	0.6
γ	2.8	2.1

Ozaki (1993) used a sequential procedure to estimate the non-linear GM model. First, He used regression analysis to estimate reaction times for 4 different actions, two for deceleration and two for acceleration conditions:

- Start of deceleration: time lag from the time the subject becomes faster than the leader to the beginning of the deceleration maneuver.

- Maximum deceleration: time lag from the time the relative speed reached its minimum value (negative) to the time the subject applies maximum deceleration.
- Start of acceleration: time lag from the time the subject becomes slower than the leader to the beginning of the acceleration maneuver.
- Maximum acceleration: time lag from the time the relative speed reached its maximum value (positive) to the time the subject applies maximum acceleration.

Ozaki found that reaction times vary depending on the situation and in particular between acceleration and deceleration decisions. One of the explanatory variables contributing to this result is the activation of brake lights by a decelerating leader. These reaction times were used to estimate car following parameters with correlation analysis. Ozaki estimated separate sets of parameters for acceleration and deceleration decisions. Estimation results are summarized in Table 2.3. Spacing speed and acceleration measurements are in meters, km/hr and m/sec^2, respectively.

Table 2.3 - Estimation results for the non-linear GM model (Ozaki 1993)

Parameter	Acceleration	Deceleration
α	1.1	1.1
β	-0.2	0.9
γ	0.2	1.0

Extensions to the GM model

Over the years, several extensions to the GM model were proposed. These extensions aimed at overcoming some of the limitations of the model.

Acceleration and deceleration asymmetry

Herman and Rothery (1965) noted that vehicles' acceleration and deceleration capabilities are different. Therefore, they used different sets of parameters for acceleration and deceleration decisions. Different sets of parameters may also account for differences in drivers' alertness to an increase in the relative leader speed as opposed to a decrease in it.

Memory functions

Lee (1966) developed a variation of the GM model that addresses the way drivers process the relative speed information. The driver reacts to the relative leader speed over a period of time rather than at an instant. The mathematical model is:

$$a_n(t) = \int_0^t M(t-s)\Delta V_n^{front}(s)\,ds \tag{2.5}$$

Where, $M(\;)$ is a memory (or weighting) function, which represents the way the driver acts on the information that has been received over time.

Lee proposed several functional forms of the memory function and analyzed the stability of the resulting response to periodic changes in the leader speed. Darroch and Rothery (1972) empirically estimated the shape of the memory function using spectral analysis. They found that a dirac-delta function, which corresponds to the linear GM model, is a reasonable approximation.

Multiple car following

Herman and Rothery (1965) and Bexelius (1968) hypothesized that drivers follow vehicles in front of their leader as well as the immediate leader. Assuming different sensitivities to the relative speed with respect to each one of these leaders, they proposed the following linear model:

$$a_n(t) = \sum_{i=1}^{n-1} \alpha_i \Delta V_n^{front\,i}(t-\tau_n) \tag{2.6}$$

Where, $\Delta V_n^{front\,i}(t-\tau_n)$ is the relative speed with respect to the i^{th} nearest leader measured at time $t-\tau_n$. α_i are parameters.

Herman and Rothery (1965) report inconclusive results regarding the effect of the second-nearest leader on the subject behavior.

Other models

While GM-type models have dominated the literature, other models have also been proposed and studied.

Spacing models

Newell (1961) proposed an alternative modeling framework, assuming that the speed of the subject vehicle is a non-linear function of the spacing between the subject and the leader:

$$V_n(t) = G_n\left[\Delta X_n^{front}(t-\tau_n)\right] \tag{2.7}$$

Where, G_n is a function that specifies the car following behavior. Newell analysed the theoretical properties of the functional form:

$$G_n\left[\Delta X_n^{front}(t-\tau_n)\right] = V_{max}\left[1 - exp\left(\frac{\lambda}{V_{max}}\left(\Delta X_n^{front}(t-\tau_n)+d\right)\right)\right] \tag{2.8}$$

Where, V_{max}, λ and d are parameters. V_{max} and d can be interpreted as the maximum speed and the minimum space headway, respectively.

Newell assumed different functional forms for acceleration and deceleration decisions. No attempt to estimate this model was made.

Komentani and Sasaki (1958) developed a model based on the assumption that the subject speed is determined so as to keep a minimum safe spacing, and is therefore a function of the leader space headway and the leader speed:

$$V_n(t) = f\left[\Delta X_n^{front}(t-\tau_n),\ V_n^{front}(t-\tau_n)\right] \tag{2.9}$$

They proposed linear and quadratic (in the subject speed) formulations of the model and studied the stability of the predicted motion of the subject in response to disturbances in the speed of the leader.

Desired measures models

Several models were developed assuming that the driver tries to attain some desired measure. Helly (1961) suggested that the driver seeks to minimize both the leader relative speed and the difference between the actual space headway and a desired one:

$$a_n(t) = \alpha_1 \Delta V_n^{front}(t-\tau_n) + \alpha_2 \left[\Delta X_n^{front}(t-\tau_n) - D(V_n)\right] \tag{2.10}$$

Where, $D(V_n)$ is the desired space headway, which depends on the subject speed.

This model addresses a deficiency of the GM model that if the two vehicles travel at the same speed, any value of the spacing between them is acceptable. Bekey et al (1977) developed a similar model from optimal control considerations and assuming that the dynamics of the two vehicles are identical. Gabard et al (1982) implemented Helly's model in SITRA-B, a microscopic traffic simulation model. In their model the desired space headway is given by:

$$D(V_n) = L_{front} + \Delta T_n^{front} V_n(t-\tau_n) \tag{2.11}$$

Where, L_{front} is the length of the leader vehicle and ΔT_n^{front} is the desired time headway of the subject, which is assumed constant.

A non-linear extension of Helly's model was proposed by Koshi et al (1992):

$$a_n(t) = \alpha_1 \frac{\Delta V_n^{front}(t-\tau_n)}{\Delta X_n^{front}(t-\tau_n)^l} + \alpha_2 \frac{\left[\Delta X_n^{front}(t-\tau_n) - D(V_n)\right]}{\Delta X_n^{front}(t-\tau_n)^m} \tag{2.12}$$

Where, l and m are parameters.

Aycin and Benekohal (1998) developed a car following model for use in time-based simulation tools. They hypothesized that drivers try to attain preferred time headways with respect to their leader and to equalize the leader speed. To ensure a continuous acceleration profile, they compute the rate of change in the acceleration for the next simulation time step based on the current spacing, speeds and accelerations of the subject vehicle and the leader using equations of laws of motion. The model was calibrated as follows: the preferred time headway was computed as the average of observations in which the absolute relative speed was less then 5 ft/sec. Reaction time was assumed equal to 80% of the preferred time headway. A driver was assumed to be car following if the leader space headway was less then 250 ft. All these values were rather arbitrarily selected based on values found in the literature.

Bando et al (1995) assumed that the acceleration drivers apply is proportional to the deviation of their actual speed from a desired speed, which depends on the leader spacing. Reaction times are ignored. The model is given by:

$$a_n(t) = \alpha\left[V_{des}\left(\Delta X_n^{front}(t)\right) - V_n(t)\right] \tag{2.13}$$

Where, $V_{des}\left(\Delta X_n^{front}(t)\right)$ is the desired speed for a given space headway. The following function was proposed:

$$V_{des} = tanh\left(\Delta X_n^{front} - 2\right) + tanh(2) \tag{2.14}$$

No behavioral justification for this functional form is presented.

The main difficulty with these models is estimating the desired measure. None of the above mentioned models were rigorously estimated.

The Ohio model

Hanken and Rockwell (1967) and Rockwell et al (1968) developed a piecewise linear empirical car following model. The linear subspaces are defined by speed and acceleration ranges. The acceleration the driver applies is a function of the deviation of the space headway, leader speed and subject speed from their respective means in the subspace:

$$\begin{aligned} a_n(t) = a_{nm} + b_0\left[\Delta X_n^{front}(t-\tau_n) - \Delta X_{nm}^{front}\right] + \\ + b_1\left[V_{front}(t-\tau_n) - V_{front\,m}\right] + b_2\left[V_n(t-\tau_n) - V_{nm}\right] \end{aligned} \tag{2.15}$$

Where, the index *m* denotes the mean value of the respective variable over the subspace.

An analysis of the variance showed that the nonlinear effects captured by the piecewise linear function were insignificant.

Psycho-physical models

Weidmann (1974) and Leutzbach (1988) identified two unrealistic behavioral implications of the GM models: The model assumes that drivers follow their leader even when the spacing between them is large, and it assumes perfect perception and reaction even to small changes in the stimulus. They introduced the term perceptual threshold to define the minimum value of the stimulus the driver will react to. The perceptual threshold value increases with the space headways. This captures both the increased alertness of drivers at small headways and the lack of car following behavior at large headways. Perceptual thresholds were found to be different for acceleration and deceleration decisions.

Figure 2.2 illustrates how car following proceeds under these assumptions. A vehicle traveling faster than its leader will get closer to it until the deceleration perceptual threshold is crossed (a). The driver will than decelerate in an attempt to match the leader speed. However, the driver is unable to do this accurately and the headway will increase until the acceleration threshold is reached (b). The driver will again accelerate and so on. This model is able to explain the oscillating phenomenon observed in car following experiments. However, no rigorous framework for calibration of this model was proposed.

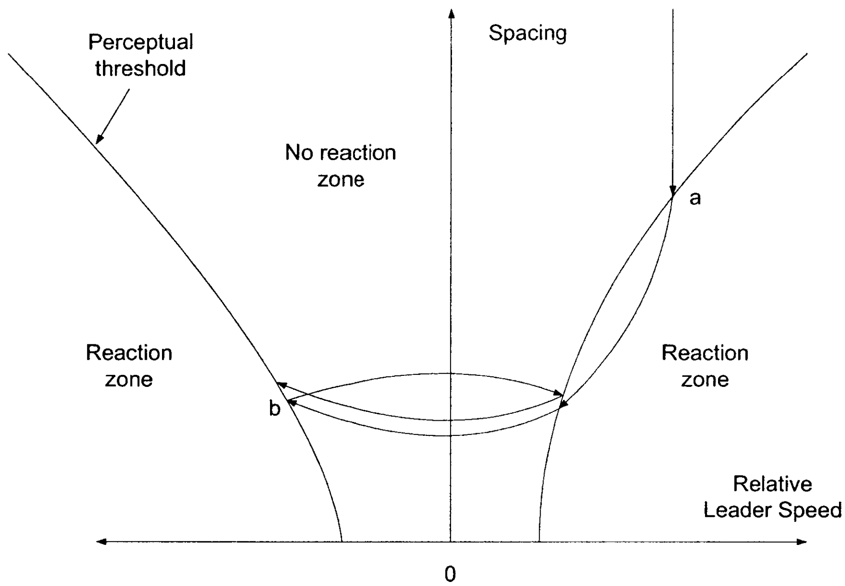


Figure 2.2 - Behavior of the Psycho-physical model

Fancher and Bareket (1998) developed a similar model for the study of automatic cruise control (ACC) systems. Drivers respond to changes in the leader relative speed only when the perceptual thresholds are exceeded. In their model, drivers try to attain desired space headways rather than match the leader speed. In addition, two more response zones are defined: a free-flow zone and a comfort zone. Drivers in the free-flow zone try to attain their desired speed. This behavior applies when the space headway is large and the subject vehicle is traveling faster than its leader. The comfort zone is used when the driver is within ±12% of his desired headway and is unable to perceive the relative speed (i.e. it is within the perceptual thresholds). In this case the driver will maintain his current speed. Perceptual thresholds for this model were derived from known results in human factors studies. No effort to calibrate or validate the model was made. Similar models were also used in the study of front-to-rear-end collisions (Kourjanski and Misener 1998, Misener et al 2000).

2.1.2 General acceleration models

Interest in acceleration models renewed with the development of microscopic simulation tools starting in the 1980s. New models intended for use in micro-simulation tools needed to address non car following situations. Gipps (1981) developed the first general acceleration model that applies to both car following and free-flow conditions. The maximum applicable acceleration is determined based on two constraints: the desired speed may not be exceeded and a minimum safe headway must be kept. The safe headway is the minimum that allows the driver to avoid a collision with his leader, if the leader applies emergency braking. Calculations are based on equations of laws of motion. Vehicle characteristics are captured through upper bounds on acceleration and deceleration values.

Benekohal and Treiterer (1988) developed a similar model for the CARSIM simulation tool. The acceleration is calculated separately for 5 different situations. The driver applies the most constraining acceleration. The 5 situations are:

- The vehicle is moving but has not reached its desired speed.
- The vehicle has reached its desired speed.
- The vehicle accelerates from a standing position.
- The vehicle is in a car following state with the space headway constraint satisfied.
- The vehicle is car following with an active collision avoidance constraint.

Maximum and comfortable deceleration values are used. Reaction times are randomly distributed in the population. Calculations are based on equations of laws of motion.

Yang and Koutsopoulos (1996) developed a general acceleration model and implemented it in MITSIM, a microscopic traffic simulator. The driver is assigned to one of three regimes based on time headway: emergency, car following and free-flow.

In the emergency regime, the driver applies the necessary deceleration to avoid collision with its leader. The car following and free-flow regimes currently implement a model developed by Ahmed (1999). The car following component of this model is a generalization of the GM model that allows non-linearity in the stimulus term and different reaction times for the sensitivity and the stimulus:

$$a_n(t) = \alpha \frac{V_n(t-\xi\tau_n)^{\beta}}{\Delta X_n^{front}(t-\xi\tau_n)^{\gamma}} k_n(t)^{\delta} \Delta V_n^{front}(t-\tau_n)^{\rho} + \varepsilon_n(t) \tag{2.16}$$

Where, $k_n(t)$ is the density of traffic ahead of the subject. $\xi \in [0,1]$ is a sensitivity lag parameter. $\varepsilon_n(t)$ is a normally distributed error term.

In the free-flow regime the driver tries to attain its desired speed:

$$a_n(t) = \lambda\left[V_n^*(t) - V_n(t-\tau_n)\right] + \upsilon_n(t) \tag{2.17}$$

Where, λ is a constant sensitivity term. $\upsilon_n(t)$ is a normally distributed error term. $V_n^*(t)$ is the desired speed given by:

$$V_n^*(t) = \alpha + \beta V_n^{front}(t-\tau_n) + \gamma\delta_n^{heavy} + \rho\delta_n^{LOS\ A} \tag{2.18}$$

Where, $V_n^{front}(t-\tau_n)$ is the leader speed at time $t-\tau_n$. δ_n^{heavy} is an indicator variable with value 1 if the subject vehicle is heavy (its length exceeds 30 ft) and 0 otherwise. $\delta_n^{LOS\ A}$ is an indicator variable with value 1 if the level of service in the road is A (density less than 19 veh/km/lane) and 0 otherwise. α, β, γ and ρ are parameters.

Both the time headway threshold and the reaction time are modeled as random variables. The parameters of all components of the model were jointly estimated using the maximum likelihood estimation method with data of individual vehicle trajectories collected from a Boston freeway. Estimation results are summarized in Table 2.4.

Zhang et al (1998) implemented a multi-regime acceleration model in MRS, a microscopic traffic simulator. They define several different driving regimes based on space headways. The regimes are emergency, normal car following, uncomfortable car following and free-flow. The emergency regime is invoked when the space headway is smaller than a pre-specified threshold. Emergency braking is driven by collision avoidance considerations and bounded by the capabilities of the vehicle. The normal car following model uses the non-linear GM model (Gazis et al 1959). Uncomfortable car following is applied when the acceleration calculated by the normal car following model is positive and the headway is unsafe based on Pipes' definition (1953). In this case, the driver applies a normal deceleration. Normal accelerations and decelerations are also applied in the free-flowing regime in an attempt to attain the desired speed. Ludmann et al (1997) used similar driving regimes in the microscopic traffic simulator PELOPS.

Table 2.4 - Estimation results for the acceleration model proposed by Ahmed (1999)

Car following model		
Parameter	Acceleration	Deceleration
α	0.0225	-0.0418
β	0.722	-
γ	0.242	0.151
δ	0.682	0.804
ρ	0.600	0.682
ε_n	$\sim N(0,0.825^2)$	$\sim N(0,0.802^2)$
Free-flow model		
Parameter	Value	
λ	0.309	
α	3.28	
β	0.618	
γ	-0.670	
ρ	7.60	
υ_n	$\sim N(0,1.13^2)$	
Headway threshold distribution		
Truncated normal distribution	$\sim(3.17,0.870^2)$	
Truncation bounds (sec.)	$[0.5,6.0]$	
Reaction time distribution		
Truncated log normal distribution	$\sim(0.272,0.212^2)$	
Truncation bounds (sec.)	$[0,3.0]$	

Reaction time

Reaction time is the time lag between the detection of a stimulus and the application of the response. Further distinction is made in human factors research between reaction time and response time (Koppa 1997). The latter includes the duration of the response itself, while the former does not. Several studies have been conducted to estimate drivers' reaction times in different situations. Their results are summarized in Table 2.5.

Table 2.5 - Reaction time estimation results

Study	Stimulus	Mean	Median	Standard deviation
Fambro et al (1998), Review	Unexpected	1.28		0.20
	Expected	0.73		0.16
Fambro et al (1998)	Unexpected object	1.10	1.11	0.21
Lerner et al (1995)	Expected object	0.54	0.53	0.1
	Unexpected object	1.31	1.18	0.61
Ahmed (1999)	Speed difference	1.34	1.31	0.31

Reaction times to expected and unexpected stimuli are different. Fambro et al (1998) summarize earlier results for both these categories. Mean reaction times for expected and unexpected stimuli are 1.28 and 0.73 seconds, respectively. Similarly, Johansson and Rumar (1971) found a ratio of 1.35 between mean reaction times to expected and unexpected stimuli.

Ahmed (1999) estimated the reaction time associated with acceleration behavior jointly with the other parameters of the acceleration model. He assumed a truncated lognormal distribution. Estimated parameters and summary statistics are presented in Table 2.4 and Table 2.5 respectively.

Headway threshold

The headway threshold defines the driving regime, car following or free-flow, the driver is in. Herman and Potts (1961) estimated a deterministic space threshold of 61 meters (200 feet). This threshold does not capture the effect of the subject speed on the headway threshold. Ahmed (1999) used time headway rather than space headway threshold to overcome this limitation. He also accounted for heterogeneity in the driver population by estimating the distribution of headway thresholds. Ahmed hypothesized a truncated normal time headway threshold distribution. This functional form allows the distribution to be skewed either to the right or to the left. The distribution parameters were estimated jointly with other components of the acceleration model (Table 2.4). The estimated median headway threshold was 3.17 seconds.

2.1.3 Summary

In the past, interest in acceleration models originated from an attempt to predict aggregate traffic flow characteristics from the behaviors of individual drivers. For example, the capacities of different road facilities may be determined by integration of the relations between a leader and a follower traveling through the facility and therefore early acceleration models focused on car following behavior. Several specifications have been proposed for car following models, however, they share the assumption that the speed/acceleration of the subject vehicle depends only on the relations with its leader (i.e. spacing and relative speeds). Ahmed (1999) relaxed this assumption by introducing traffic density downstream of the subject as an explanatory variable to capture the effect of macroscopic traffic conditions on car following.

In recent years, acceleration models are also being developed as a basis for microscopic traffic simulation tools. This shift in the focus of application lead to development of free-flow acceleration models, which capture the behavior of drivers who are not closely following their leaders as well as to further splitting car following behavior to sub-regimes (e.g. acceleration and deceleration or reactive and non-reactive car following regimes). The Introduction of multiple driving regimes to acceleration models requires definition of boundaries to determine which regime the driver is in. For example, headway thresholds are used to determine whether a vehicle is in the car following or free-flow regimes. However, in most models these thresholds are modeled deterministically. Similarly, reaction time is explicitly represented in acceleration models, but is often assumed deterministic and assigned arbitrary values. Moreover, there has been very little rigorous treatment of estimation of these models. Most models presented in the literature either completely ignore the issue of estimation or assume values for some parameters and use ad-hoc procedures to determine values for others. Ahmed (1999) proposed a general acceleration model, which addresses these deficiencies. The model captures both car following and free-flow behaviors. Heterogeneity in the headway threshold and reaction time are captured by introducing distributions of these variables in the driver population. Ahmed also developed a maximum likelihood estimation framework, which allows joint estimation of all parameters of the acceleration model including parameters of the car following acceleration, car following deceleration and free-flow acceleration models as well as the parameters of the headway threshold and reaction time distributions.

Still, all existing models capture only the effect of the leader (car following) and the desired speed (free-flow) on acceleration behavior. Such models represent myopic behavior, which fails to incorporate other considerations and goals, such as the impact of lane changing behaviors. New models need to be developed, with an increasing number of driving regimes to represent drivers' behavior under different conditions in order to capture more diverse behaviors and improve realism. For example, new acceleration models may be specified for drivers who are accelerating or decelerating to facilitate lane changing.

2.2 Lane changing models

Lane changing behavior has not been studied as extensively as acceleration behavior. but interest in this field has grown with the development of micro-simulation tools.

2.2.1 General models

Gipps (1986) presented the first lane changing decision model intended for microscopic traffic simulation tools. The model covers various urban driving situations, in which traffic signals, transit lanes, obstructions and presence of heavy vehicles affect drivers' lane selection. The model considers three major factors: necessity, desirability and safety of lane changes. Drivers' behavior is governed by two basic considerations: attaining the desired speed and being in the correct lane to perform turning maneuvers. The relative importance of these considerations varies with the distance to the intended turn. Gipps defines three zones: when the turn is far away it has no effect on the behavior and the driver concentrates on attaining the desired speed. In the middle zone, lane changes will only be considered to the turning lanes or lanes that are adjacent to them. In the last zone, close to the turn, the driver focuses on keeping the correct lane and ignores speed considerations. Zones are defined deterministically, ignoring variability between drivers and inconsistencies in the behavior of a driver over time. When more than one lane is acceptable the conflict is resolved deterministically by a priority system considering, in order of importance, locations of obstructions, presence of heavy vehicles and potential speed gain. No framework for rigor estimation of the model parameters was proposed.

Rorbech (1976) developed a model of lane changing behavior in two-lane motorways. A vehicle may be in one of four states based on two characteristics: the lane it is in (right or left) and traffic conditions (free-flow or constrained). A stochastic Markov process is used to model transitions between the states. An important observation he made is that lane changing behaviors from the right lane and from the left lane are different.

CORSIM (Halati et al 1997, FHWA 1998) is a microscopic traffic simulation model developed by FHWA. In CORSIM, lane changes are classified as either mandatory (MLC) or discretionary (DLC). An MLC is performed when the driver must leave the current lane (e.g. in order to exit to an off-ramp, avoid a lane blockage). A DLC is performed when the driver perceives that driving conditions in the target lane are better, but a lane change is not required. A risk factor is computed for each potential lane change. This factor is defined in terms of the deceleration a driver would have to apply if its leader brakes to a stop. The risk is calculated for the subject with respect to its intended leader and for the intended follower with respect to the subject. The risk is compared to an acceptable risk factor, which depends on the type of lane change and its urgency. Variability in gap acceptance behavior is ignored.

Yang and Koutsopoulos (1996) implemented in MITSIM a rule based lane changing model. Lane changes are classified as mandatory (MLC) or discretionary (DLC). Drivers perform MLC in order to connect to the next link on their path, bypass a downstream lane blockage, avoid entering a restricted-use lane and comply with lane use signs and variable message signs. Conflicting goals are resolved probabilistically based on utility theory models. DLC are considered when the speed of the leader is below a desired speed. The driver then checks the opportunity to increase speed by moving to another lane.

Hidas and Behbahanizadeh (1998) implemented a similar model in the micro-simulator SITRAS. Downstream turning movements and lane blockages may trigger MLC or DLC, depending on the distance to the point where the lane change must be completed. MLC are also taken in order to obey lane use regulations. DLC are performed in an attempt to obtain speed advantage or queue advantage, which are defined as the adjacent lane allowing faster traveling speed and having a shorter queue, respectively. In addition they introduced a cooperative lane changing model. A vehicle in an MLC situation under heavily congested traffic conditions may change lanes through cooperation with the intended follower. The willingness of the follower to allow the subject vehicle to change lanes is a function of his aggressiveness. A cooperative follower will start following the subject vehicle and the subject will start following the intended leader in the target lane. As a result a gap will open in the target lane and the subject vehicle will be able to change lanes.

Ahmed et al (1996) and Ahmed (1999) developed a general lane changing model framework that captures both MLC and DLC situations. The structure of the model is shown in Figure 2.3. The lane changing process is modeled with three-steps: a decision to consider a lane change, choice of a target lane and acceptance of gaps in the target lane. A discrete choice framework is used to model these decisions. Logit models are used to capture the various choices. The model allows different gap acceptance parameters for DLC and MLC situations. The utility of responding to an MLC situation is affected by the time delay since the MLC situation arose and a bias against using the first gap available to the driver. If an MLC situation does not apply or the driver chooses not to respond to it a decision whether to consider a DLC is made. A two-step decision process is assumed: First, drivers examine their satisfaction with driving conditions in the current lane, which is affected by the difference between the subject speed and its desired speed. The model also captures differences in the behavior of heavy vehicles and the effect of the presence of a tailgating vehicle. If the driver is not satisfied with driving conditions in the current lane, he compares conditions in neighboring lanes to those in the current lane in order to decide whether to change lane and to choose the target lane. The utilities of neighboring lanes are affected by the speeds of the lead and lag vehicles in these lanes and the current and desired speed of the subject vehicle. A gap acceptance model is also included within the lane changing framework.

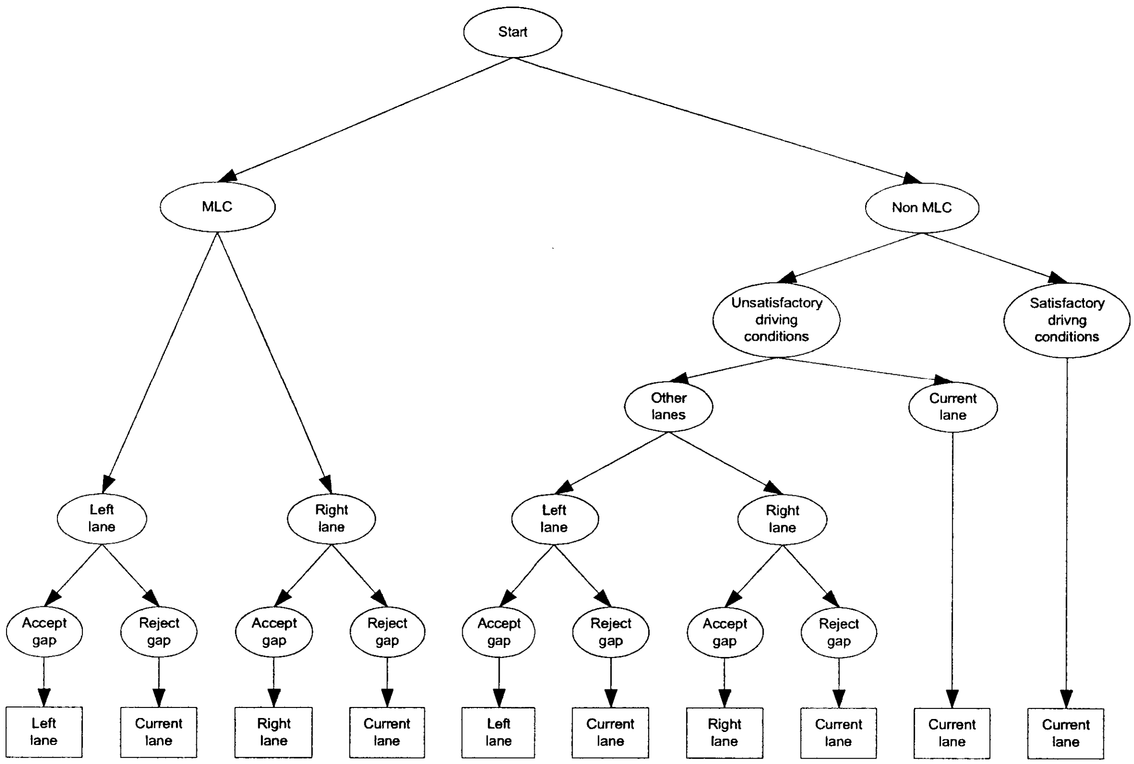


Figure 2.3 - Structure of the lane changing model proposed by Ahmed (1999)

It is difficult to explain the choice to react to an MLC situation (the upper level decision in Figure 2.3). To overcome that problem, Ahmed (1999) separately estimated parameters of the discretionary and mandatory components of the model. Gap acceptance parameters were estimated jointly with the other components for each case. He used maximum likelihood estimation techniques with second-by-second vehicle trajectory data. The DLC component was estimated with data collected from a 200 meter long freeway section in Boston. The MLC component was estimated for a special case of vehicles merging to a freeway from a ramp. Estimation results for the DLC and MLC models are summarized in Table 2.6 and Table 2.7, respectively.

Table 2.6 - Estimation results for the DLC model proposed by Ahmed (1999)

Variable	Parameter value
Utility of unsatisfactory driving conditions	
Constant	0.225
(Subject speed - desired speed), m/sec.	-0.0658
Heavy vehicle dummy	-3.15
Tailgate dummy	0.423
Utility of left lane	
Constant	-2.08
(Lead speed - desired speed), m/sec.	0.0337
(Front speed - desired speed), m/sec.	-0.152
(Lag speed - subject speed), m/sec.	-0.0971
Desired speed model	
Average speed, m/sec.	0.768
Lead critical gap	
Constant	0.508
Min (0, lead speed - subject speed), m/sec.	-0.420
$\sigma_{\varepsilon}^{lead,DLC}$	0.488
Lag critical gap	
Constant	0.508
Min (0, lag speed - subject speed), m/sec.	0.153
Max (0, lag speed - subject speed), m/sec.	0.188
$\sigma_{\varepsilon}^{lag,DLC}$	0.526

Table 2.7 - Estimation results for the MLC model proposed by Ahmed (1999)

Variable	Parameter value
Utility of mandatory lane change	
Constant	-0.654
First gap dummy	-0.874
Delay (sec.)	0.577
Lead critical gap	
Constant	0.384
$\sigma_{\varepsilon}^{lead,MLC}$	0.859
Lag critical gap	
Constant	0.587
Min (0, lag speed - subject speed), m/sec.	0.0483
Max (0, lag speed - subject speed), m/sec.	0.356
$\sigma_{\varepsilon}^{lag,MLC}$	1.073

Ahmed (1999) also developed and estimated a forced merging model. This model captures drivers' lane changing behavior in heavily congested traffic, where acceptable gaps are not available. In this situation, drivers are assumed to change lanes either through courtesy yielding of the lag vehicle in the target lane or by forcing the lag vehicle to slow down. Important factors affecting this behavior include lead relative speed, the remaining distance to the point the lane change must be completed and existence of a total clear gap in excess of the subject vehicle length.

Zhang et al (1998) developed a multi-regime traffic simulation model (MRS). Their definitions of MLC and DLC and the gap acceptance logic are similar to Ahmed et al (1996). MLC critical gaps are randomly distributed across the population. The mean critical gap is a function of the remaining distance to the point where the lane change must be completed. Drivers in MLC situations may adapt their acceleration in order to be able to accept available gaps. The following cases are considered:

- No change in acceleration: The adjacent gap is acceptable as is.

- The subject needs to accelerate: Either the total length of the adjacent gap is sufficient but the lag gap is too small or the total length of the adjacent gap is unacceptable but the gap between the lead vehicle and its leader is acceptable.
- The subject needs to decelerate: Either the total length of the adjacent gap is sufficient but the lead gap is too small or the total length of the adjacent gap is unacceptable but the gap between the lag vehicle and its follower is acceptable.

The model also considers courtesy yielding. The authors performed a validation study but did not suggest a framework for calibration of the model.

Wei et al (2000) developed a model of drivers' lane selection process when turning into two-lane urban arterials and their subsequent lane changing behavior. Depending on the driver's path plan the arterial lanes are classified according to three criteria:

- Target (non-target) lane: a lane (not) connecting to the intended turn at the next intersection.
- Preemptive (non-preemptive) lane: a lane (not) connecting to the intended turn at a downstream intersection.
- Closest (farther) lane: the lane closest to (farther away from) the curb on the side the driver is turning onto the arterial from.

A set of deterministic lane selection rules were developed using observations from Kansas City, Missouri:

- Drivers that intend to turn at the next intersection choose the target lane.
- Drivers that intend to turn farther downstream choose the preemptive lane if it is the closest. If the preemptive lane is the farthest, the lane choice is based on the aggressiveness of the driver.
- Drivers that are already traveling in the arterial remain in their lanes.

The lane changing behavior of drivers in the arterial is influenced by the lane classification (e.g. target, preemptive) and is governed by another set of rules. Analysis of the field observations showed that passing is an important behavior that needs to be modeled. Vehicles in the target lane may perform a passing maneuver (double lane change to a non-target lane and back) in order to gain speed. The model requires that both the adjacent gap in the non-target lane and the gap in the target lane between the subject's leader and its leader be acceptable for passing to take place. Most of the decision rules do not account for variability in the driver population.

2.2.2 Gap acceptance models

Gap acceptance is an important element in most lane changing and unsignalized intersection behavior models. The driver assesses the positions and speeds of the lead and lag vehicles (see Figure 2.4) and decides whether the gap between them can be used to perform the lane change.

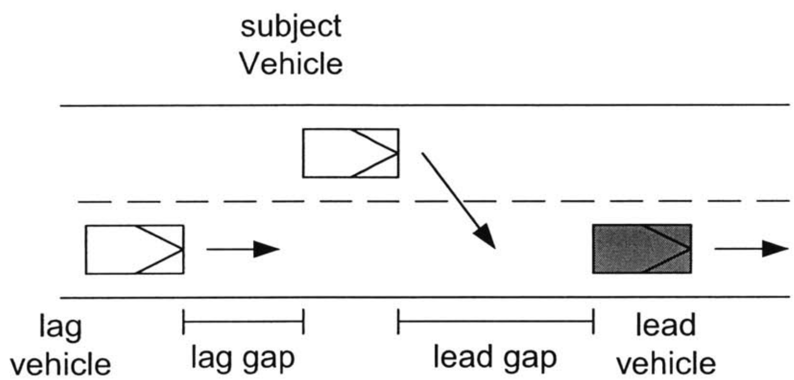


Figure 2.4 - Gap acceptance elements

Gap acceptance models are formulated as a binary choice problem. The driver will either accept or reject the available gap, based on comparison of the gap with an unobserved critical gap:

$$Y_n(t) = \begin{cases} 1 & \text{if } G_n(t) \geq G_n^{cr}(t) \\ 0 & \text{if } G_n(t) < G_n^{cr}(t) \end{cases} \tag{2.19}$$

Where, $Y_n(t)$ is the choice indicator variable with value 1 if the gap is accepted and 0 otherwise. $G_n(t)$ is the available gap and $G_n^{cr}(t)$ is the critical gap.

Critical gaps are modeled as random variables in order to capture the probabilistic nature of gap acceptance decisions. Herman and Weiss (1961) assumed an exponential distribution, Drew et al (1967) assumed a lognormal distribution and Miller (1972) assumed a normal distribution. Daganzo (1981) proposed a framework to capture critical gap variation in the population as well as in the behavior of a driver over time. He used a multinomial probit model formulation appropriate for panel data to estimate parameters of the multivariate normal distribution of critical gaps. The critical gap for driver n at time t is given by:

$$G_n^{cr}(t) = G_n + \varepsilon_n^{cr}(t) \tag{2.20}$$

Where, G_n is a driver specific random component of the critical gap, which captures the within driver variability over time. $\varepsilon_n^{cr}(t)$ is the random term associated with variability across drivers. G_n and $\varepsilon_n^{cr}(t)$ are assumed to be mutually independent normally distributed random variables.

Mahmassani and Sheffi (1981) introduced impatience functions to gap acceptance models. The mean of the distribution of critical gaps is a function of explanatory variables. This framework allows incorporating the impact of different factors on drivers' gap acceptance behavior. The model was estimated for a stop-controlled intersection under the assumption that critical gaps are normally distributed. The number of rejected gaps (or waiting time at the stop line) was found to have a significant impact on gap acceptance behavior. This variable captures the impatience and frustration of drivers standing at the stop line. Madanat et al (1993) used total queuing time to capture impatience. Velan and Van Aerde (1996) implemented a decaying critical gap function in INTEGRATION, a mesoscopic traffic simulator. Critical gaps decay linearly with waiting time. Two parameters determine the shape of the function: the initial critical gap and the max waiting time. Default values are based on recommendations in the Highway Capacity Manual (HCM 1994). Variability across drivers is not considered.

Cassidy et al (1995) used a logit model to capture gap acceptance behavior at stop controlled T-intersections. They differentiated lags (the first gap) from subsequent gaps and gaps in the near lane from gaps in the far lane. These variables significantly improved the fit of the model. Several other parameters that may affect the critical gap are discussed in the literature (e.g. Brilon 1988 1991, Adebisi and Sama 1989, Saad et al

1990, Hamed et al 1997). These include the type of maneuver, speeds of vehicles in the major road, geometric characteristics and sight distances, type of control in the intersection, presence of pedestrian, police activities and daylight conditions. However, most of the discussion is qualitative and addresses macroscopic characteristics rather than microscopic drivers' behavior.

In the context of lane changing, Kita (1993) estimated a logit gap acceptance model for the case of vehicles merging to a freeway from a ramp. He found that important factors are the length of the available gap, the relative speed of the subject with respect to mainline vehicles and the remaining distance to the end of the acceleration lane.

Ahmed (1999), within the framework of the lane changing model described earlier (see Figure 2.3), assumed that drivers consider the lead gap and the lag gap separately. The model requires that both gaps be acceptable. The critical gap functional form guarantees that it is always non-negative:

$$G_n^{cr,g}(t) = exp\left(X_n^g(t)\beta^g + \alpha^g \upsilon_n + \varepsilon_n^g(t)\right) \qquad g=lead,\ lag \qquad (2.21)$$

Where, $X_n^g(t)$ and β^g are vectors of explanatory variable and the corresponding parameters. υ_n is an individual specific random term assumed to be distributed standard normal. α^g is the parameter of υ_n. $\varepsilon_n^g(t)$ is a normally distributed generic random term.

Gap acceptance parameters were estimated jointly with other components of the model. As expected, lead and lag critical gaps under MLC situations were lower than under DLC situations. Estimation results for DLC and MLC situations are shown in Table 2.6 and Table 2.7, respectively.

2.2.3 Summary

Interest in lane changing behavior has grown with the development of microscopic traffic simulation tools. Lane changing behavior is usually modeled in two steps: the decision to consider a lane change and the decision to execute the lane change. In all existing models, lane changes are classified as either mandatory (MLC) or discretionary (DLC). MLC are performed when the driver must leave the current lane. DLC are performed to improve driving conditions. Gap acceptance models are used to model the execution of lane changes.

The distinction between MLC and DLC is artificial and prohibits capturing trade-offs between mandatory and discretionary considerations. Moreover, it requires specification of the conditions that trigger MLC to determine which behavior is active. However, none of the existing models addresses this issue and the conditions that trigger MLC have not been estimated. Micro-simulation tools, which implement these models, use simple rule-based models to determine whether MLC conditions apply. The parameters of these models are usually based on the modelers' judgment.

As with acceleration models, the parameters of most lane changing models proposed in the literature were not estimated rigorously. Ahmed (1999) proposed a framework to jointly estimate parameters of the lane selection and gap acceptance components of lane changing models. However, he estimated the MLC and DLC models separately. Moreover, the MLC model he estimated applies only to the special case of vehicles merging to the freeway from an on-ramp. Thus, a lane changing model that accounts for the behaviors of all vehicles in a road section has not been estimated.

Inter-dependencies between lane changing and acceleration behaviors have not been accounted for in existing models. More generally, all existing models address isolated behaviors. There has been no effort to develop an integrated driving behavior modeling framework that allows combining different behaviors and captures inter-dependencies between these behaviors.

2.3 Limitations of existing driving behavior models

The literature review reveals several limitations of existing driving behavior models. Specifically, the following assumptions are made in existing models:

- Independent behaviors - Interactions between the various decisions a driver makes are ignored. However, such interactions exist and may be important. For example, the acceleration behavior may be affected by lane changing considerations.
- Instantaneous behavior - Most models assume that drivers make instantaneous decisions. At each point in time the driver assesses the situation and selects the

immediate action. In reality, human drivers may conceive and perform action plans over a length of time.

- Reactive behavior - Existing models assume that driving decisions are based only on present or past conditions. In reality, many decisions are based on anticipated future conditions.
- Myopic behavior - Driving behavior in existing models is affected, almost exclusively, by considerations related to the local driving neighborhood (e.g. the relations between the subject vehicle and surrounding vehicles). In some situations, broader considerations may be important. For example, consider a two-lane urban arterial with signalized intersections and permissive left-turns. Even if driving conditions are locally better in the left lane, drivers may prefer the right lane to avoid being delayed behind left-turning traffic. This consideration may affect drivers' behavior long before they reach the intersection and without knowing whether or not left-turning vehicles are actually present. Similarly, freeway drivers may choose to avoid the right-most lane knowing that merging traffic may slow traffic down in this lane. Again, this behavior may take place before the merge is visible and regardless of whether or not merging traffic is present.

Application of these models in micro-simulation tools may result in unrealistic traffic flow characteristics. The negative effects of these assumptions on the realism increase with the complexity of the driving behavior being modeled. Car following is, to a large extent, a passive behavior performed with little conscious attention and so the above may be acceptable modeling assumptions. Lane changing is a more involved behavior that requires the driver to evaluate the situation, anticipate the behavior of other drivers, decide on a course of action and perform these actions over time. However, existing models assume that drivers are purely reactive when responding to the situation they face: drivers who wish to change lanes evaluate the existing gaps and decide whether to accept or reject them, but make no effort to adapt their position (by changing speed and acceleration) in order to be able to accept an available gap.

The situation described in Figure 2.5 illustrates the limitations of such behavior. Consider a situation in which driver A tries to change to the right lane, and suppose that the total lengths of the gaps between vehicles B and C and between vehicles C and D are both acceptable. In current models, driver A only considers the adjacent gap (C-D). This gap is rejected because the lead gap (gap between A and C) is unacceptable, and therefore vehicle A does not change lanes. The acceleration vehicle A applies is determined by the acceleration model, which ignores the lane changing goal. The process is repeated until vehicle A is able to execute the lane change, which, depending on the speeds of vehicles A and C, may take some time. In reality, to accomplish the lane changing goal, driver A may adapt his vehicle's acceleration over a length of time. For example, vehicle A may decelerate and accept gap C-D or accelerate and accept gap B-C.

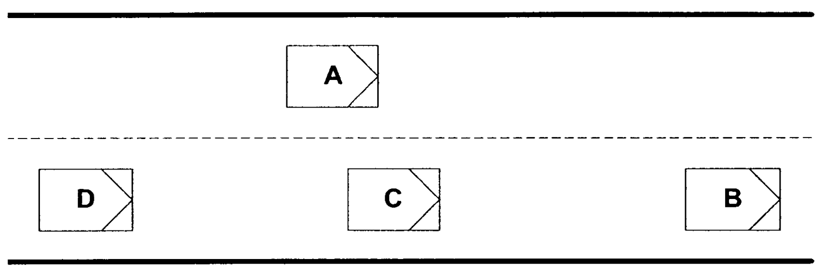


Figure 2.5 - A lane changing situation illustrating the limitations of existing models

Chapter 3

Modeling Framework

In this chapter, a conceptual framework for developing driving behavior models, which allows limitations of existing models to be addressed, is presented. This framework is based on the concepts of a short-term goal and short-term plan. The structure of a driving behavior modeling framework, based on these concepts is developed. The presentation is organized as follows: first, the concepts of short-term goal and short-term plan are introduced. These concepts are then utilized to structure the driving behavior modeling framework, which overcomes the limitations of existing models. Finally, mechanisms that are available within the model structure to capture inter-dependencies between the various components of the integrated framework are discussed.

3.1 The short-term goal and short-term plan

The literature review presented in Chapter 2 demonstrates the need for an integrated framework for modeling driving behavior, which will capture inter-dependencies between behaviors, such as lane changing and acceleration. The concepts of short-term goal and short-term plan will be used as a mechanism that will enable to capture such inter-dependencies and account for drivers' planning capabilities.

Sukthankar (1997) defines a short-term plan as a sequence of actions a driver performs in order to complete a desired tactical maneuver. This desired maneuver is the short-term goal.

Consider the situation discussed in Section 2.3 and shown again in Figure 3.1. Suppose that driver A tries to change to the right lane, and that the total lengths of the gaps between vehicles B and C and between vehicles C and D are both acceptable, but the adjacent gap (C-D) is rejected because the lead gap (gap between A and C) is unacceptable, and therefore vehicle A cannot change lanes. The short-term goal of driver A is to be in the right lane (for example, in order to take an off-ramp). The short-term plan may be, for example, to use gap B-C to change lanes and accomplish the short-term goal. The sequence of actions required to execute the plan may involve accelerating in order to pass vehicle C and then accepting the gap between vehicles B and C. A model that captures this behavior must overcome the limitations of existing models discussed above: short-term plans require that different behaviors be inter-dependent. In this example, the acceleration of vehicle A is determined to facilitate lane changing. The behavior cannot, by definition, be instantaneous since the plan will be executed over a length of time. Moreover, in order to choose a plan and decide its actions the driver must predict future conditions by anticipating the behaviors of other vehicles.

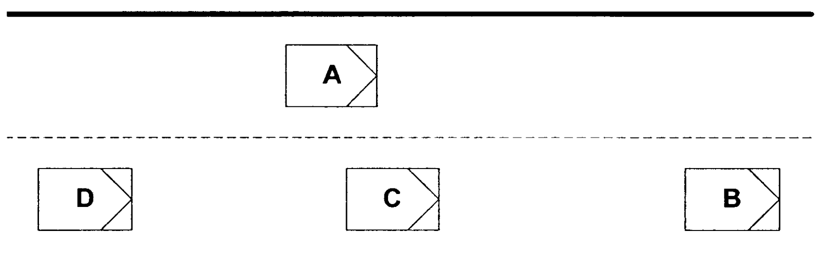


Figure 3.1 - A lane changing situation illustrating the limitations of existing models

3.2 Model structure

The proposed integrated driving behavior modeling framework based on the concepts of short-term goal and short-term plan is shown in Figure 3.2. Driving behavior consists of three main elements: the short-term goal, the short-term plan and the driver's actions. The short-term goal is defined by the driver's target lane. The driver constructs a short-term plan, which is defined by the target gap in the target lane that the driver wishes to use in order to accomplish his goal (i.e. move to the target lane). The actions are the 2-dimensional movements of the vehicle (accelerations and lane changes) that the driver performs in order to execute the short-term plan. In the case that the driver does not intend to change lanes, he may be viewed as executing a passive plan to follow the leader or attain the desired speed.

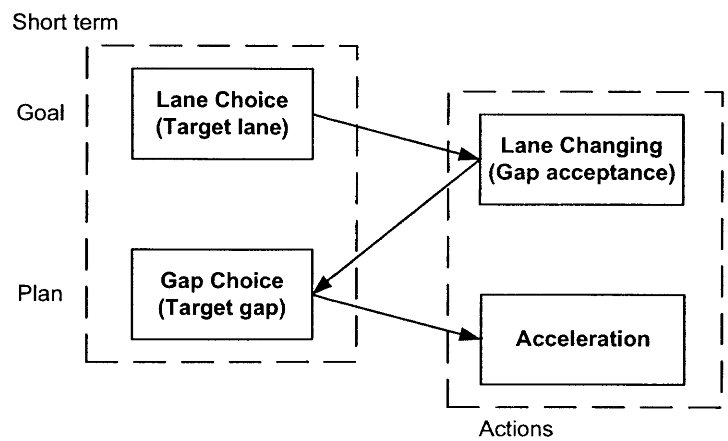


Figure 3.2 - Conceptual framework for the driving behavior process

The detailed model structure is shown in Figure 3.3. The model hypothesizes four levels of decision-making: target lane, gap acceptance, target gap and acceleration. This decision process is latent. The short-term goal (target lane) and short-term plan (target gap) are both unobservable. Only the driver's actions (lane changes and accelerations) are observed. Latent choices are shown as ovals in Figure 3.3. Observed choices are shown as rectangles.

At the highest level the driver chooses a target lane. The target lane is the lane the driver perceives as best to be in. This is the choice of short-term driving goal. The CURRENT branch corresponds to a situation in which the driver decides not to pursue a lane change. In this situation car following or free-flow behaviors are active based on the relations with the vehicle in front. In the case that either the right lane or the left lane are chosen (the RIGHT and LEFT branches, respectively), the driver evaluates the adjacent gap in the target lane and decides whether this gap can be used to execute the lane change or not. If the gap is accepted (CHANGE RIGHT or CHANGE LEFT), the lane change is immediately executed and the short-term goal is accomplished. The acceleration the driver applies in this case is determined by car following or free-flow behavior with respect to the leader in the new lane.

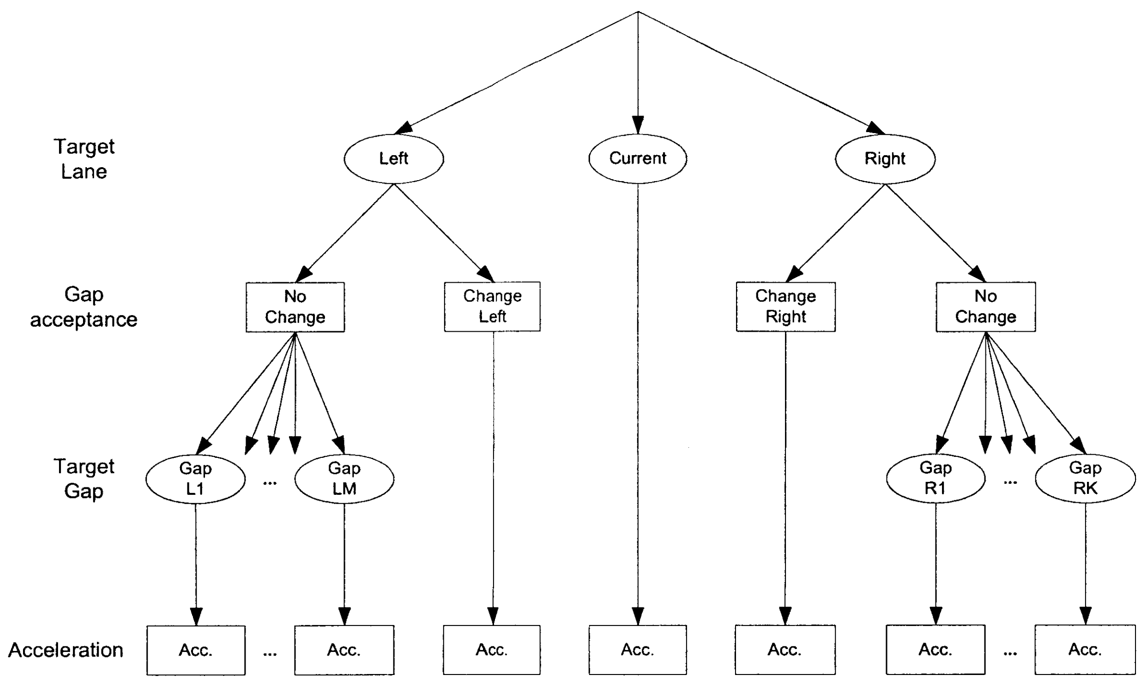


Figure 3.3 - The structure of the driving behavior model

If the available gap is rejected (NO CHANGE), the driver evaluates available gaps in the target lane and chooses the one that would be used to perform the desired lane change (GAP R1 to GAP RK or GAP L1 to GAP LM). This choice defines the short-term plan that the driver would follow. The target gap choice set may include alternatives that do not correspond to specific gaps in traffic. For example, the driver may decide to look for gaps in traffic upstream of his current location (i.e. gaps between vehicles that are currently behind him) without committing to a specific gap. The acceleration the driver applies is determined such that to facilitate the execution of the short-term plan (i.e. the driver tries to position the vehicle in a way that will increase the probability that the target gap will be acceptable). The acceleration may also be constrained by car following considerations, since the lane change is not immediate.

It is worth noting that lane changes and accelerations are represented differently in the model. Lane changes are modeled as discrete events occurring at specific points in time. Therefore, a lane changing observation (or absence of one) is assumed to take place *during* the time interval being modeled. The assumption, similar to Markov models, is that only one lane change may be executed during each time interval. This assumption is reasonable since in reality lane changes are not performed instantaneously and given that the length of time intervals is short relative to the execution time of lane changes. Acceleration is modeled as a continuous function in time. Observations are therefore *instantaneous* measurements sampled from the acceleration profile at specific points in time (corresponding to the beginning of each time interval).

3.2.1 Inter-dependencies across driving decisions

In this section, mechanisms within the model structure that allow capturing inter-dependencies between the various decisions made are discussed. More specifically, three such mechanisms are described:

- Causality
- Unobserved driver/vehicle characteristics
- State dependency

Causality

Decisions made at lower levels of the driving behavior decision process are conditional on those made at higher levels (e.g. the acceleration behavior is conditional on the short-term plan). The effects of lower level choices on higher-level decisions may be accounted for by introducing variables that capture the expected maximum utility (*EMU*) of the alternatives at the lower level in the specification of higher-level choices. *EMU* values increase with the utilities of the lower level alternatives, and therefore, the inclusion of these variables in the utility functions of the corresponding higher-level alternatives captures the utility that the driver may extract from the lower level choices that are becoming feasible to him when choosing a higher-level alternative. For example, the likelihood that the driver would be able to perform a lane change affects the target lane decision. The gap acceptance *EMU* variable represents this likelihood, thus capturing the effect of gap acceptance decisions on the target lane choice.

Unobserved driver/vehicle characteristics

Unobserved driver and vehicle characteristics, such as the aggressiveness and level of driving skill and intelligence of the driver and maximum comfortable speeds and acceleration capabilities of the vehicle introduce correlations between the various observations obtained from a given driver/vehicle (over time and choice dimensions). A Driver/vehicle specific latent variable may be introduced in the model to capture these correlations. The model assumes that conditional on the value of this latent variable, the error terms of different observations are independent. This specification is given by:

$$U_n^d(t) = X_n^d(t)\beta^d + \alpha^d \upsilon_n + \varepsilon_n^d(t) \tag{3.1}$$

Where, $U_n^d(t)$ is the utility of decision d to individual n at time t. $X_n^d(t)$ is a vector of explanatory variable. β^d is a vector of parameters. υ_n is an individual-specific latent variable assumed to follow some distribution in the population. α^d is the parameter of υ_n. $\varepsilon_n^d(t)$ is a generic random term with i.i.d. distribution across decisions, time and individuals.

The resulting error structure is given by:

$$\begin{aligned} & cov\left(\varepsilon_n^d(t), \varepsilon_{n'}^{d'}(t')\right) = \begin{cases} \sigma_d^2 & if\ t = t', n = n'\ and\ d = d' \\ 0 & otherwise \end{cases} \\ & cov\left(\varepsilon_n^d(t), \upsilon_{n'}\right) = 0 \qquad \forall\, t, d, n, n' \end{aligned} \tag{3.2}$$

$$cov\left(U_n^d(t), U_{n'}^{d'}(t')\right) = \begin{cases} \left(\alpha^d\right)^2 + \sigma_d^2 & if\ n = n',\ d = d'\ and\ t = t' \\ \left(\alpha^d\right)^2 & if\ n = n',\ d = d'\ and\ t \neq t' \\ \alpha^d \alpha^{d'} & if\ n = n',\ d \neq d'\ and\ \forall t \\ 0 & otherwise \end{cases} \tag{3.3}$$

Where, σ_d^2 is the variance of $\varepsilon_n^d(t)$.

A discussion of this formulation including identification and normalization issues related to it is presented in Appendix A.

In the acceleration component of the model, reaction times (τ_n) and time headway thresholds (h_n^*), which are assumed to be randomly distributed in the population, also capture correlations between the various acceleration decisions.

State dependency

The execution of short-term plans to achieve short-term goals involves making decisions and performing actions over time. However, traffic is a dynamic and uncertain environment and therefore drivers may need to re-evaluate and possibly modify their short-term goals and short-term plans as conditions change. Moreover, short-term goals and short-term plans are generally unobservable and therefore need to be modeled probabilistically as latent variables. The combination of latent short-term goals and short-term plans and a dynamic driving environment introduces considerable modeling complexity. For example, several combinations of short-term goals and short-term plans may lead to observing a vehicle in the same lane over a period of time:

- The driver was satisfied in that lane and was not trying to change lane.
- The driver targeted changing either to the right lane or to the left lane and adopted any of a number of feasible short-term plans to execute the lane change, but did not consider the available gaps in the target lane acceptable at any time.
- Any combination of the above short-term goals and short-term plans over the period of observation.

One possible modeling approach would be to define latent states as combinations of a short-term goal and a short-term plan and capture the dynamics of the behavior by modeling state dependencies in the driving process. The joint probability of a latent state (s_t) and observed action (o_t, i.e. acceleration and lane changing) of a given vehicle at time *t*, conditional on the sequence of previous states is given by:

$$p(o_t, s_t \mid S_{t-1}, X) = p(o_t \mid s_t, S_{t-1}, X)\, p(s_t \mid S_{t-1}, X) \tag{3.4}$$

Where, S_t is the sequence of states up to time *t*, $S_t = \{s_i : i = 0,1,...,t\}$. X are explanatory variables.

The probability of the entire sequence of states (S_T) and observations (O_T) is given by:

$$p(O_T, S_T \mid s_0, X) = \prod_{t=1}^{T} p(o_t, s_t \mid S_{t-1}, X) \tag{3.5}$$

Where, *T* is the number of time periods the vehicle is observed.

Finally, the joint marginal probability of observations is calculated by summation over all possible state sequences:

$$p(O_T \mid s_0, X) = \sum_{\substack{state \\ sequences}} p(O_T, S_T \mid s_0, X) \tag{3.6}$$

There are two practical difficulties with this formulation:

- It assumes that initial conditions (s_0) are either observed or represent a steady state (i.e. the static probability mass function of s_0 is known or can be estimated). However, in most cases, the first time a vehicle is observed does not correspond to any natural starting point that would support this assumption. Instead, it is usually determined by the location and capabilities of the data collection system.
- The number of possible sequences in the summation of Equation (3.6) is $|s|^T$, where $|s|$ is the number of possible states. The total number of probabilities to calculate is $2T|s|^T$. Except for degenerate cases with a very small set of possible states (combinations of a short-term goal and a short-term plan) or a very short observation period, modeling all possible combinations of states is prohibitively expensive.

An alternative approach is based on the concept of a partial short-term plan. Here, the assumption is that the driver executes one step of the short-term plan, re-evaluates the situation and decides the next action to take. To illustrate this approach, consider the situation described in Figure 3.4: suppose that vehicle B is a slow-moving vehicle and that the goal of vehicle A is to overtake it. The short-term plan may consist of the following steps:

- Change to the left lane.
- Accelerate and pass vehicle B.
- Change back to the right lane.

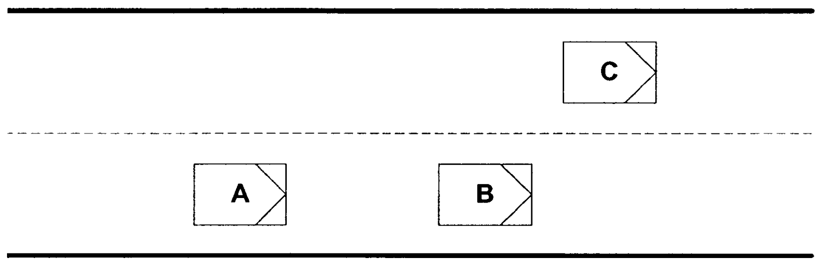


Figure 3.4 - A lane changing situation illustrating partial short-term planning

Assuming partial short-term planning, vehicle A will perform the first step: change to the left lane and then re-evaluate the situation and decide what to do next. For example, depending on the behavior of vehicle C, vehicle A may continue with the previous plan, or abandon the goal of overtaking vehicle B and follow vehicle C in the left lane.

This approach captures the effect of evolving conditions on driving behavior while being significantly more efficient computationally since it does not require explicit enumeration of all the possible sequences of combinations of short-term goals and short-term plans. This is done at the expense of assuming that all state dependencies are captured by the explanatory variables. This assumption may not be restrictive since explanatory variables that are derived from the positions and speeds of the subject vehicle and surrounding vehicles are important in all driving behavior models. The values of these variables depend on the past actions of the vehicle (e.g. the current speed and position of a vehicle are a function of previously applied accelerations) and so, capture the effects of previous actions and states.

Under the partial short-term planning assumption, the joint probability of a latent state and the observed actions, at time t, is given by:

$$p(o_t, s_t \mid X) = p(o_t \mid s_t, X)\, p(s_t \mid X) \tag{3.7}$$

The marginal probability of an observation is given by:

$$p(o_t \mid X) = \sum_{s_t} p(o_t, s_t \mid X) \tag{3.8}$$

The joint probability of the entire sequence of observations is given by:

$$p(O \mid X) = \prod_{t=1}^{T} p(o_t \mid X) \tag{3.9}$$

The computational burden associated with the partial short-term approach is significantly lower since this calculation only requires $2T \mid s \mid$ probability calculations.

The model specification presented in this thesis assumes partial short-term planning, mainly due to the computational consideration. However, as shown above, the model structure presented in this chapter also allows for state dependence to be represented.

3.3 Conclusions

In this chapter, an integrated driving behavior modeling framework was presented. Drivers are assumed to perform short-term plans to accomplish short-term goals. The short-term goal is defined by a target lane, which is the lane the driver perceives as best to be in. A target gap, which the driver intends to use to change lane, defines the short-term plan. The acceleration the driver applies is adapted to facilitate the short-term plan. This modeling framework supports specification and estimation of models that capture inter-dependencies between lane changing and acceleration behaviors and represent drivers' planning capabilities.

Several mechanisms are available within the model structure to capture inter-dependencies between the various decisions made. Decisions made at lower levels of the driving behavior decision process are conditional on those made at higher levels (e.g. the acceleration behavior is conditional on the short-term plan). The effects of lower level choices on higher-level decisions may be captured by introducing the expected maximum utility (*EMU*) of the alternatives at the lower level in the specification of higher-level choices. In addition, individual-specific latent variables may be introduced in the various choice models to capture correlations between the decisions made by a given driver that are the result of unobserved driver and vehicle characteristics. Finally, a model specification that accounts for state dependency in short-term goals and short-term plans is presented. However this model is computationally demanding. An alternative specification, which is based on the concept of partial short-term plan, is proposed. This approach captures the effect of evolving conditions on driving behavior, but assumes that all state dependencies are captured by the explanatory variables.

Chapter 4

Model Components

In this chapter, mathematical formulations of the various components of the integrated driving behavior model are presented.

Some of these model components, such as the model describing the short-term plan choice (i.e. the target gap model) and the acceleration behaviors to facilitate lane changing, may only be captured within an integrated driving behavior model that captures both lane changing and acceleration behaviors and the inter-dependencies between them. Hence, these behaviors have not been addressed in the literature, and so new models are developed here for the first time. In addition, a new model is also developed for the short-term goal model (i.e. the choice of target lane). This model integrates mandatory lane changing (MLC) and discretionary lane changing (DLC) into a single model, thus capturing trade-offs between mandatory and discretionary considerations. The merit of this model extends beyond its role in the integrated framework since it may also be applied independently. For other components, such as gap acceptance behavior and car following behavior, existing models that have been proposed for isolated behaviors may be adequate. In these cases, state-of-the-art, tested and validated models are used.

4.1 Factors affecting driving behavior

We first discuss and categorize variables that may be used to explain drivers' decisions within the integrated driving behavior model. The expected impact of these variable categories will be further discussed within the specification of each model component.

An important limitation of existing driving behavior models, discussed in Section 2.3, is that in most cases the behavior is explained only by variables related to the vehicle's driving neighborhood, such as the relations between the subject and surrounding vehicles (e.g. expressed by relative speeds and spacing). In reality, a wider range of considerations explains the driving process. These considerations may be broadly classified into four categories:

- Neighborhood variables - The vehicle's surroundings significantly affect driving behavior. Most importantly, the presence of other vehicles on the road and their behaviors directly influence drivers' decisions. The choices of target lanes and target gaps, gap acceptance decisions and acceleration behaviors all depend on the relative positions and speeds of the subject vehicle with respect to neighboring vehicles: the vehicle in front and the vehicle following the subject, lead and lag vehicles in neighboring lanes and other vehicles that define gaps in traffic. Additional variables related to neighboring vehicles, such as the presence of heavy vehicles and tailgating behavior, may also affect the subject's behavior. Other elements in the vehicle's surrounding that may affect the behavior include geometry elements, signals and signs and police presence.
- Path plan variables - These variables capture the effect of the path plan and trip schedule on drivers' decisions. It is assumed that the driver has already defined the destination and schedule (e.g. desired arrival time) for the trip and chose an appropriate path. These decisions have an important effect on driving behavior: drivers choose lanes and perform lane changes to be able to follow their paths, desired speeds and accelerations choices are directly affected by the desired arrival time. Variables in this group may include the distance to a freeway off-ramp the driver needs to take or to locations of turns at urban intersections, the number of lane changes required to be in a correct lane to follow the path, indicators to whether the driver needs to take the next exit (or turn at the next intersection) and so on.
- Network knowledge and experience - Variables in this class capture considerations and preferences that are based on the driver's knowledge and experience with the transportation system. For example, freeway lane choices may be affected by the driver's preference to avoid using the lane closest to an on-ramp to avoid interacting

with merging traffic. The knowledge that determines such behaviors is built over time. Commuters repeatedly travel the same parts of the network and thus learn the specific attributes of their paths. With experience, drivers also develop a more general knowledge base that they can use while traveling in networks they are not familiar with. Knowledge considerations may influence the behavior before the situation actually arises. For example, the presence of an on-ramp merging lane may affect lane choices long before the vehicle actually arrives at the merging point and regardless of the presence of traffic on the ramp. Other examples of situations in which such behaviors may occur include urban arterials with permissive left turning movements, bus stops and bus traffic and toll plazas.

- Driving style and capabilities - Individual characteristics of the driver, such as aggressiveness, motoric capabilities (e.g. reaction time) and vision (e.g. sight distances) and of the vehicle, such as braking and acceleration capabilities affect driving behavior. In many cases these characteristics are not directly measurable. However, their effects may still be captured through introduction of latent variables that capture correlations between the various decisions made by a driver/vehicle.

4.2 The target lane model

At the highest level of the driving process, the driver chooses a short-term goal. The short-term goal is defined in terms of a target lane (*TL*). The target lane choice set includes up to three alternatives: The driver may decide to stay in the current lane (*CL*) or to target changing to either the right lane (*RL*) or the left lane (*LL*). The utilities of the current, right and left lanes, respectively, to driver n at time t are given by:

$$U_n^{CL}(t) = X_n^{CL}(t)\beta^{CL} + \alpha^{CL}\upsilon_n + \varepsilon_n^{CL}(t) \tag{4.1}$$

$$U_n^{RL}(t) = X_n^{RL}(t)\beta^{RL} + \alpha^{RL}\upsilon_n + \varepsilon_n^{RL}(t) \tag{4.2}$$

$$U_n^{LL}(t) = X_n^{LL}(t)\beta^{LL} + \alpha^{LL}\upsilon_n + \varepsilon_n^{LL}(t) \tag{4.3}$$

Where, $X_n^{CL}(t)$, $X_n^{RL}(t)$ and $X_n^{LL}(t)$ are vectors of explanatory variables affecting the utilities of the current, right and left lanes, respectively. β^{CL}, β^{RL} and β^{LL} are the corresponding vectors of parameters. $\varepsilon_n^{CL}(t)$, $\varepsilon_n^{RL}(t)$ and $\varepsilon_n^{LL}(t)$ are the random terms associated with the lane utilities. υ_n is an individual specific error term that captures correlations between the observations of a single driver over time. α^{CL}, α^{RL} and α^{LL} are the parameters of υ_n. In model estimation, not all the α values can be identified. Instead, one of these parameters must be normalized to zero. A discussion of this formulation is presented in Appendix A.

This model integrates mandatory and discretionary considerations into a single utility function for each target lane. This approach is different from current models in which lane changing behavior (lane choice and gap acceptance) is treated separately for MLC and DLC situations. The integrated utility overcomes two important weaknesses of the separate models:

- Current models do not capture trade-offs between mandatory and discretionary considerations.
- These models assume that the existence (or non-existence) of MLC situations is deterministic and known. However, except for very special cases, the emergence of a MLC situations is unobservable. For example, Ahmed (1999) studied the lane changing behavior of traffic merging to a freeway from an on-ramp. He assumed that vehicles enter the MLC state as soon as they enter the acceleration lane.

To illustrate the advantage of the integrated model, consider the situation shown in Figure 4.1. Suppose that vehicle A is planning to exit the freeway and that vehicle B is a slow moving heavy vehicle. In current models once vehicle A enters an MLC state it will change to the right lane and stay there until the off-ramp. The presence of vehicle B does not affect this behavior. The proposed model captures the trade-off between the utility of being in the correct lane (mandatory consideration) and the speed advantage of the left lane (discretionary consideration). Hence, the driver may choose to stay in the left lane until it passes vehicle B.

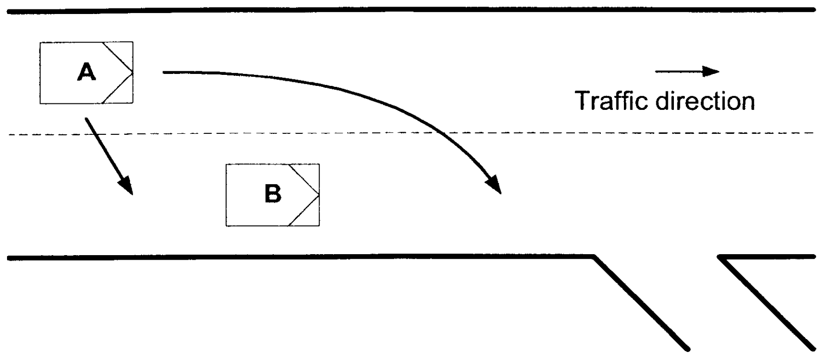


Figure 4.1 - A lane changing situation illustrating the integrated target lane utility

Lane utility functions may depend on explanatory variables from all four categories discussed above. Variables should reflect the conditions in the immediate neighborhood in each lane (e.g. relative leader speed in each lane and presence of heavy vehicles) path plan considerations (e.g. the distance to a point where the driver must be in certain lane(s) and the number of lane changes needed in order to be in these lanes) and knowledge of the system (e.g. avoiding the left lane before a permissive left turn or avoiding an on-ramp merging lane). In most cases, information about the driver's style and characteristics is not available. Nevertheless, these characteristics are captured by the individual specific error term υ_n.

One of the considerations in choosing a short-term goal is the relative ease or difficulty of accomplishing this goal. Thus, the expected maximum utility (*EMU*) of the available gaps in the right lane and in the left lane are included in the corresponding target lane utilities to capture the effect of gap acceptance decisions (i.e. the likelihood of being able to execute the desired lane change) on the target lane choice. The values of *EMU* variables increase with the probability of accepting the available gap in the target lanes.

Different choice models are obtained depending on the assumption made about the distribution of the random terms $\varepsilon_n^{CL}(t)$, $\varepsilon_n^{RL}(t)$ and $\varepsilon_n^{LL}(t)$. Assuming that these random terms are independently and identically Gumbel distributed, choice probabilities for the various lanes, conditional on the individual specific error term (υ_n) are given by a logit model:

$$P_n(lane\ i_t \mid \upsilon_n) = \frac{\exp\left(V_n^{lane\ i}(t) \mid \upsilon_n\right)}{\sum_{j \in I} \exp\left(V_n^{lane\ j}(t) \mid \upsilon_n\right)} \qquad lane\ i \in I = \{CL, RL, LL\} \qquad (4.4)$$

Where, $V_n^{lane\ i}(t) \mid \upsilon_n$ are the conditional systematic utilities of the alternatives, given by:

$$V_n^{lane\ i}(t) \mid \upsilon_n = \left[X_n^{lane\ i}(t)\right]\beta^{lane\ i} + \alpha^{lane\ i}\upsilon_n \qquad lane\ i = CL, RL, LL \qquad (4.5)$$

4.3 The gap acceptance model

In the target lane model the driver may choose to stay in the current lane or to target changing either to the right lane or to the left lane. Conditional on the target lane choice, the gap acceptance model captures the decision whether or not to change lanes immediately using the adjacent gap. The model assumes that if the adjacent gap in the target lane is acceptable the driver performs the lane change and does not consider any other gaps. This assumption is in agreement with satisficing behavior theory (Simon 1955), which states that human behavior is not an optimal behavior, but a satisficing one: if an available option (i.e. using the adjacent gap to change to the target lane) is satisfactory the driver does not try to find a better one.

The adjacent gap in the target lane is defined by the lead and lag vehicles in that lane as shown in Figure 4.2. The lead gap is the clear spacing between the rear of the lead vehicle and the front of the subject vehicle. Similarly, the lag gap is the clear spacing between the rear of the subject vehicle and the front of the lag vehicle. Note that one or both of these gaps may be negative if the vehicles overlap.

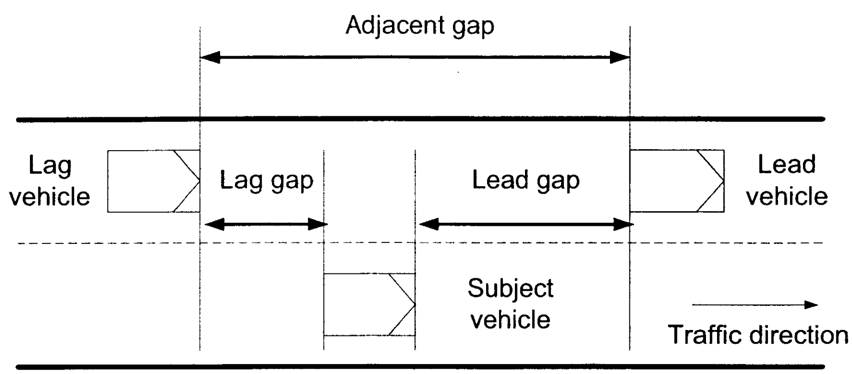


Figure 4.2 - The adjacent gap, subject, lead and lag vehicles and the lead and lag gaps

The available lead and lag gaps are compared to the driver's corresponding critical gaps, which are the minimum acceptable gaps. The available gap is accepted only if it is greater than the critical gap. Critical gaps vary for different individuals and with the situation. They are modeled as random variables whose means are functions of explanatory variables. The individual specific error term captures correlations between the critical gaps of the same individual over time. In order to ensure that critical gaps are always positive, they are assumed to follow a lognormal distribution:

$$\ln\left(G_n^{lead\ TL,cr}(t)\right) = X_n^{lead\ TL}(t)\beta^{lead} + \alpha^{lead}\upsilon_n + \varepsilon_n^{lead}(t) \tag{4.6}$$

$$\ln\left(G_n^{lag\ TL,cr}(t)\right) = X_n^{lag\ TL}(t)\beta^{lag} + \alpha^{lag}\upsilon_n + \varepsilon_n^{lag}(t) \tag{4.7}$$

Where, $X_n^{lead\ TL}(t)$ and $X_n^{lag\ TL}(t)$ are vectors of explanatory variables affecting the lead and lag critical gaps, respectively. β^{lead} and β^{lag} are the corresponding vectors of parameters. $\varepsilon_n^{lead}(t)$ and $\varepsilon_n^{lag}(t)$ are normally distributed random terms associated with the critical gaps: $\varepsilon_n^{lead}(t) \sim N\left(0,\sigma_{lead}^2\right)$ and $\varepsilon_n^{lag}(t) \sim N\left(0,\sigma_{lag}^2\right)$. α^{lead} and α^{lag} are the parameters of the individual specific random term υ_n for the lead and lag critical gaps, respectively.

The gap acceptance model assumes that both the lead gap and the lag gap must be acceptable in order for the vehicle to change lanes. The probability of accepting the gap and performing a lane change, conditional on the individual specific term and the target lane is given by:

$$\begin{aligned}&P_n\left(\text{change to target lane}|TL_t,\upsilon_n\right)=P_n\left(l_t^{TL}=1\,|\,TL_t,\upsilon_n\right)=\\&P_n\left(\text{accept lead gap }|TL_t,\upsilon_n\right)P_n\left(\text{accept lag gap }|TL_t,\upsilon_n\right)=\\&P_n\left(G_n^{lead\,TL}(t)>G_n^{lead\,TL,cr}(t)\;|TL_t,\upsilon_n\right)\cdot P_n\left(G_n^{lag\,TL}(t)>G_n^{lag\,TL,cr}(t)\;|TL_t,\upsilon_n\right)\end{aligned}\tag{4.8}$$

Where, $TL=RL,LL$ is the target lane (which requires a lane change). l_t^{TL} is the lane changing indicator for the target lane:

$$l_t^{TL}=\begin{cases}1 & a\ change\ to\ lane\ TL\ is\ performed\ at\ time\ t\\0 & otherwise\end{cases}$$

$G_n^{lead\,TL}(t)$ and $G_n^{lag\,TL}(t)$ are the available lead and lag gap in the target lane, respectively. $G_n^{lead\,TL,cr}(t)$ and $G_n^{lag\,TL,cr}(t)$ are the corresponding critical gaps.

Under the assumption that critical gaps follow a lognormal distribution, the conditional probability that the lead gap is acceptable is given by:

$$\begin{aligned}&P_n\left(G_n^{lead\,TL}(t)>G_n^{lead\,TL,cr}(t)\;|TL_t,\upsilon_n\right)=\\&P_n\left(\ln\left(G_n^{lead\,TL}(t)\right)>\ln\left(G_n^{lead\,TL,cr}(t)\right)\;|TL_t,\upsilon_n\right)=\\&\Phi\left[\frac{\ln\left(G_n^{lead\,TL}(t)\right)-\left(X_n^{lead\,TL}(t)\beta^{lead}+\alpha^{lead}\upsilon_n\right)}{\sigma_{lead}}\right]\end{aligned}\tag{4.9}$$

Where, $\Phi[\cdot]$ denotes the cumulative standard normal distribution.

Similarly the conditional probability that the lag gap is acceptable is given by:

$$\begin{aligned}
&P_n\left(G_n^{lag\,TL}(t) > G_n^{lag\,TL,cr}(t)\ |TL_t, \upsilon_n\right) = \\
&P_n\left(\ln\left(G_n^{lag\,TL}(t)\right) > \ln\left(G_n^{lag\,TL,cr}(t)\right)\ |TL_t, \upsilon_n\right) = \\
&\Phi\left[\frac{\ln\left(G_n^{lag\,TL}(t)\right) - \left(X_n^{lag\,TL}(t)\beta^{lag} + \alpha^{lag}\upsilon_n\right)}{\sigma_{lag}}\right]
\end{aligned} \tag{4.10}$$

The gap acceptance decision is primarily affected by neighborhood variables, but it may also be affected by path plan variables, capturing the necessity and urgency of the lane change. Explanatory variables may include the subject vehicle's speed, relative speeds with respect to the lead and lag vehicles in the target lane, traffic density and indicators for the urgency of the lane change (e.g. the distance to the point the lane change must be completed).

Gap acceptance decisions may be affected by the expected maximum utility (*EMU*) of target gap alternatives, similar to the effect of gap acceptance decisions on the target lane choice. The target gap *EMU* variable represents the usefulness of short-term plans that may be used if the available gap is rejected, and so captures the likely ease of changing lanes later if the available gap is rejected.

4.4 The target gap model

If the adjacent gap is rejected, the driver cannot change lanes immediately. It is at this point that the driver creates the short-term plan by choosing a target gap in the target lane traffic. This is the gap the driver plans to use in order to execute the lane change in the (short-term) future.

The alternatives in the target gap choice set include available gaps in the vicinity of the subject vehicle (e.g. the adjacent gap, forward gap and backward gap shown in Figure 4.3). Note that the adjacent gap, although not acceptable at the time of the decision, may still be chosen in anticipation that it will be acceptable in the future. Although the definition of short-term plans in terms of target gap selection is simple and intuitive, it is not a requirement of the model structure. For example, alternatives in which the short-term plan is to look for gaps between vehicles that are currently either downstream or upstream of the subject vehicle, without committing to a specific gap may also be incorporated.

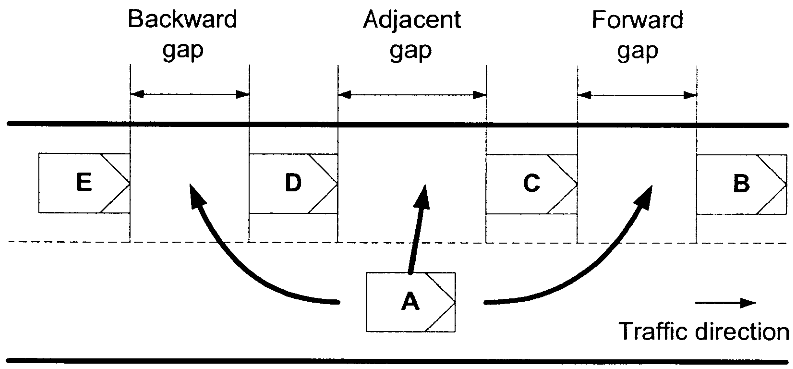


Figure 4.3 - The target gap choice set: adjacent, forward and backward gaps

The utilities of the different target gaps to driver n at time t are given by:

$$U_n^{gap\ i}(t) = X_n^{gap\ i}(t)\beta^{gap\ i} + \alpha^{gap\ i}\upsilon_n + \varepsilon_n^{gap\ i}(t) \qquad gap\ i \in I_n(t) \tag{4.11}$$

Where, $X_n^{gap\,i}(t)$ is a vector of explanatory variables affecting the utility of gap i. $\beta^{gap\,i}$ is the corresponding vector of parameters. $\varepsilon_n^{gap\,i}(t)$ are the random terms associated with the gap utilities. $\alpha^{gap\,i}$ are the parameters of the individual specific error term υ_n. $I_n(t)$ is the choice set of target gaps for driver n at time t.

The utilities of the different gaps are expected to be affected by driving neighborhood explanatory variables, such as the size of the gap, the gap trend, i.e., whether it is expanding or narrowing (captured by the relative speed between the intended lead and lag vehicles) and the subject's relative speed with respect to these vehicles. It is expected that the effect of path plan variables would not be as important as it is on higher-level decisions (i.e. the target lane and gap acceptance decisions). Nevertheless, variables such as the distance to the point where the lane change must be completed may affect the target gap choice, for example, biasing drivers to prefer backward gaps to forward gaps when the lane change becomes urgent.

Assuming a logit error structure and conditional on the individual specific term, the target lane and rejection of the adjacent gap, the choice probabilities for the various alternatives are given by:

$$P_n\left(gap\ i_t \mid TL_t, l_t^{TL}=0, \upsilon_n\right)=\frac{exp\left(V_n^{gap\,i}(t)\right)}{\sum_{j\in I_n(t)} exp\left(V_n^{gap\,j}(t)\right)} \tag{4.12}$$

Where, $V_n^{gap\,i}(t)$ is the systematic utility of gap *i*.

4.5 Acceleration models

The acceleration behavior is expected to be different depending on the driver's short-term goal and short-term plan. Therefore, different acceleration models are used for the various combinations of target lane, gap acceptance decision and target gap. More specifically, three different types of accelerations are considered:

- Stay-in-the-lane acceleration (conditional on the current lane being the target lane, no lane change is desired). This is the acceleration applied if the driver wishes to stay in the current lane.
- Acceleration during a lane change. This acceleration is applied when the target gap is different than the current lane, the driver accepts the available adjacent gap ($l_t^{TL}=1$) and executes the lane change.
- Target gap acceleration (conditional on rejecting the adjacent gap and a choice of a target gap). This acceleration is applied if the driver wishes to change lanes but rejects the adjacent gap (i.e. does not change lanes immediately). In this case different models are used depending on the target gap choice.

The overall acceleration model is expressed by:

$$a_n(t)=\begin{cases} a_n^{s}(t) & if\ TL=CL \\ a_n^{lc}(t) & if\ TL=RL,\ LL\ and\ l_t^{TL}=1 \\ a_n^{tg}(t) & otherwise \end{cases} \tag{4.13}$$

Where, $a_n(t)$ is the acceleration vehicle *n* applies at time *t*. $a_n^s(t)$ is the stay-in-the-lane acceleration. $a_n^{lc}(t)$ is the lane changing acceleration. $a_n^{tg}(t)$ is the target gap acceleration.

In order to capture the effect of the subject's leader (the vehicle in front of it) on the acceleration behavior, two driving regimes, constrained and unconstrained, are defined within each one of these acceleration behaviors. In the constrained driving regime the subject vehicle is close to its leader and therefore the driver reacts to the behavior of the leader. This model corresponds to car following behavior, which has been studied extensively in the literature. In the unconstrained regime the subject is not close to its leader and therefore can determine the acceleration based on considerations related to the short-term goal and short-term plan. The regime the driver is in is determined by the time headway between the subject and the leader. If the time headway is less than a threshold, the driver is in the constrained regime, otherwise the driver is in the unconstrained regime. Mathematically, the subject's acceleration is expressed by:

$$a_n^k(t) = \begin{cases} a_n^{cf,k}(t) & \text{if } h_n(t-\tau_n) \leq h_n^* \\ a_n^{uc,k}(t) & \text{otherwise} \end{cases} \tag{4.14}$$

Where, $a_n^{cf,k}(t)$ and $a_n^{uc,k}(t)$ are the constrained (car following) and unconstrained accelerations under situation *k* (stay-in-the-lane, lane changing, target gap), respectively. τ_n is the reaction time of driver *n*. The reaction time includes perception time (time from the presentation of the stimulus until the foot starts to move), foot movement time and vehicle response time. $h_n(t-\tau_n)$ and h_n^* are the time headway at time $t-\tau_n$ and the headway threshold for driver *n*. The time headway is defined as $h_n(t) = \frac{\Delta X_n(t)}{V_n(t)}$. $\Delta X_n(t)$ is the clear spacing between the subject vehicle and its leader. The definitions of the subject vehicle and leader vehicle, their speeds and the clear spacing and space headway between them are shown in Figure 4.4.

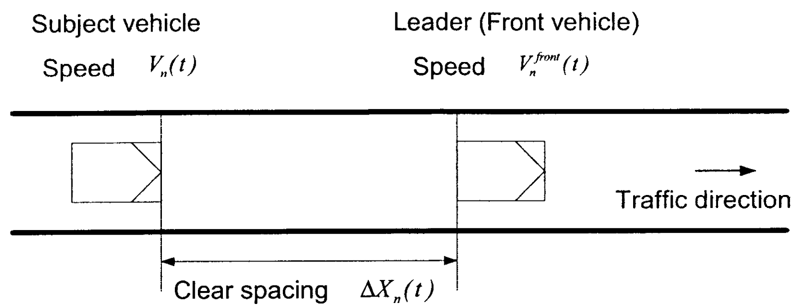


Figure 4.4 - The subject and leader vehicles, their speeds, spacing and space headway

Different models describe the acceleration behavior under the various situations. However, in order to create a consistent set of acceleration behaviors, the stimulus-sensitivity framework proposed within the GM model (see Section 2.1.1) for the car following regime is adapted for all these acceleration models. Thus, the acceleration driver n applies in each regime r is assumed to be a response to stimuli from the environment:

$$response_n^r(t) = sensitivity_n^r(t) \times stimulus_n^r(t - \tau_n) \tag{4.15}$$

The driver reacts to different stimuli in the various situations, depending on constraints imposed by the driving neighborhood and on the driver's short-term goal and short-term plan.

4.5.1 Stay-in-the-lane acceleration model

This model captures the acceleration behavior of drivers who are not trying to change lanes. This corresponds to the type of acceleration models that have been formulated in the literature. The structure of the model is based on the one proposed, estimated and validated by Ahmed (1999). The constrained and unconstrained driving regimes assume car following and free-flow behaviors, respectively. In the car following regime the driver reacts to the leader relative speed. In the free-flow regime the driver tries to attain a desired speed. The regime the driver is in is defined by the leader time headway as discussed above. Thus, the stay-in-the-lane acceleration is expressed by:

$$a_n^s(t)=\begin{cases} a_n^{cf}(t) & \text{if } h_n(t-\tau_n)\le h_n^* \\ a_n^{ff}(t) & \text{otherwise} \end{cases} \tag{4.16}$$

Where, $a_n^{cf}(t)$ and $a_n^{ff}(t)$ are the car following and the free-flow accelerations, respectively.

The car following model

The GM stimulus-sensitivity framework (Gazis et al 1961) was originally developed for this situation. In this model, the stimulus is given by the subject relative speed with respect to the leader (defined here as the speed of the leader less the speed of the subject vehicle). The sensitivity is a function of explanatory variables.

The expected value of the response to the stimuli is positive (acceleration) for positive leader relative speeds, i.e., when the leader is faster than the subject vehicle and negative (deceleration) for negative leader relative speeds. However, the response to positive and negative stimuli may be different because of the different nature of these situations: the main consideration in the reaction to a negative leader relative speed is safety, whereas the acceleration applied in a positive leader relative speed situation may be affected by speed advantage considerations and by inertia (i.e. humans tendency to conform with the actions of others). To capture these differences the model allows the coefficients of explanatory variables to be different for positive and negative stimuli. The car following acceleration is therefore given by:

$$a_n^{cf}(t)=\begin{cases} a_n^{cf,acc}(t) & \text{if } \Delta V_n(t-\tau_n)\ge 0 \\ a_n^{cf,dec}(t) & \text{otherwise} \end{cases} \tag{4.17}$$

Where, $a_n^{cf,acc}(t)$ and $a_n^{cf,dec}(t)$ are the car following acceleration and car following deceleration models, respectively. $\Delta V_n(t-\tau_n)$ is the leader relative speed at time $t-\tau_n$, which is defined as $\Delta V_n(t-\tau_n)=V_n^{front}(t-\tau_n)-V_n(t-\tau_n)$.

The car following acceleration and car following deceleration models are given, respectively, by:

$$a_n^{cf,acc}(t) = s^{acc}\left[X_n^{cf,acc}(t)\right] f^{acc}\left[\Delta V_n(t-\tau_n)\right] + \varepsilon_n^{cf,acc}(t) \tag{4.18}$$

$$a_n^{cf,dec}(t) = s^{dec}\left[X_n^{cf,dec}(t)\right] f^{dec}\left[\Delta V_n(t-\tau_n)\right] + \varepsilon_n^{cf,dec}(t) \tag{4.19}$$

Where, $s^{acc}\left[X_n^{cf,acc}(t)\right]$ and $s^{dec}\left[X_n^{cf,dec}(t)\right]$ are the sensitivity functions for car following acceleration and car following deceleration, respectively. $X_n^{cf,acc}(t)$ and $X_n^{cf,dec}(t)$ are vectors of explanatory variables describing the two sensitivity functions. $f^{acc}\left[\Delta V_n(t-\tau_n)\right]$ and $f^{dec}\left[\Delta V_n(t-\tau_n)\right]$ are the respective stimulus functions. $\varepsilon_n^{cf,acc}(t)$ and $\varepsilon_n^{cf,dec}(t)$ are the random terms associated with the car following acceleration and car following deceleration of driver *n* at time *t*, respectively.

The random terms $\varepsilon_n^{cf,acc}(t)$ and $\varepsilon_n^{cf,dec}(t)$ capture unobserved effects on car following acceleration and car following deceleration, respectively. It is assumed that these terms follow a normal distribution and that they are independent of each other (acceleration and deceleration), for different drivers, and for the same driver over time. The model assumes that the correlations between acceleration decisions from the same driver over time are captured by the reaction time and the time headway threshold distributions. The resulting error structure is given by:

$$\begin{aligned} &\varepsilon_n^{cf,g}(t) \sim N\left(0, \sigma_{cf,g}^2\right) \\ &cov\left(\varepsilon_n^{cf,g}(t), \varepsilon_{n'}^{cf,g'}(t')\right) = \begin{cases} \sigma_{cf,g}^2 & if\ g = g', t = t', n = n' \\ 0 & otherwise \end{cases} \end{aligned} \tag{4.20}$$

Where, $\sigma_{cf,g}^2$ are the variances of the car following error terms for $g = acc,\ dec$.

Under these assumptions the probability density functions of the car following acceleration and car following deceleration are given by:

$$f\left(a_n^{cf,g}(t)\mid\tau_n\right)=\frac{1}{\sigma_{cf,g}}\phi\left(\frac{a_n^{cf,g}(t)-s^g\left[X_n^{cf,g}(t)\right]f^g\left[\Delta V_n(t-\tau_n)\right]}{\sigma_{cf,g}}\right) \tag{4.21}$$

The distribution of the combined car following model is given by:

$$f\left(a_n^{cf}(t)\mid\tau_n\right)=f\left(a_n^{cf,acc}(t)\mid\tau_n\right)^{\delta\left[\Delta V_n(t-\tau_n)\right]}f\left(a_n^{cf,dec}(t)\mid\tau_n\right)^{\left(1-\delta\left[\Delta V_n(t-\tau_n)\right]\right)} \tag{4.22}$$

Where, $\delta\left[\Delta V_n(t-\tau_n)\right]$ is the relative speed indicator:

$$\delta\left[\Delta V_n(t-\tau_n)\right]=\begin{cases}1 & \text{if } \Delta V_n(t-\tau_n)\geq 0\\ 0 & \text{otherwise}\end{cases}$$

The free-flow model

The subject vehicle is in the free-flow regime if the leader time headway is larger than a threshold, and so the subject is not affected by the leaders' behavior. Instead, the driver is trying to attain a desired speed. The desired speed itself is unobservable. Two classes of variables may affect the desired speed: driving neighborhood variables, such as the speed limit, weather conditions, road geometry and condition of the pavement surface and characteristics of the driver, such as aggressiveness and level of driving skill, and of the vehicle, such as maximum comfortable speeds and acceleration capabilities. Variables related to the trip schedule (e.g. the preferred arrival time at the trip destination) may also be important. The desired speed is expressed by:

$$V_n^{DS}(t-\tau_n)=X_n^{DS}(t-\tau_n)\beta^{DS}+\alpha^{DS}\upsilon_n \tag{4.23}$$

Where, $V_n^{DS}(t-\tau_n)$ is the desired speed of driver n at time $t-\tau_n$. $X_n^{DS}(t-\tau_n)$ is a vector of explanatory variables describing the desired speed. β^{DS} is the corresponding set of parameters. α^{DS} is the parameter of the individual specific random term υ_n, which captures correlations between the various decisions made by the same driver.

The free-flow stimulus is a function of the difference between the desired speed and the actual speed. The acceleration the driver applies is given by:

$$a_n^{ff}(t) = s^{ff}\left[X_n^{ff}(t)\right] f^{ff}\left[V_n^{DS}(t-\tau_n) - V_n(t-\tau_n)\right] + \varepsilon_n^{ff}(t) \tag{4.24}$$

Where, $a_n^{ff}(t)$ is the free-flow acceleration driver n applies at time t. $X_n^{ff}(t)$ is a vector of explanatory variables describing the free-flow sensitivity function $s^{ff}\left[X_n^{ff}(t)\right]$. $f^{ff}\left[V_n^{DS}(t-\tau_n) - V_n(t-\tau_n)\right]$ is the corresponding stimulus function. $\varepsilon_n^{ff}(t)$ is the random term associated with the free-flow acceleration.

The random term $\varepsilon_n^{ff}(t)$ captures the effect of omitted variables on the free-flow acceleration. It is assumed to follow a normal distribution. Error terms are independent for different drivers, and for the same driver over time. As in the car following model, it is assumed that correlations between acceleration decisions from the same driver over time are captured by the reaction time and the time headway threshold distributions, which are individual specific. The free-flow error term is also independent of the car following error terms $\varepsilon_n^{cf,acc}(t)$ and $\varepsilon_n^{cf,dec}(t)$.

The resulting error structure is given by:

$$\begin{aligned} &\varepsilon_n^{ff}(t) \sim N\left(0, \sigma_{ff}^2\right) \\ &cov\left(\varepsilon_n^{ff}(t), \varepsilon_{n'}^{ff}(t')\right) = \begin{cases} \sigma_{ff}^2 & if\ t = t', n = n' \\ 0 & otherwise \end{cases} \end{aligned} \tag{4.25}$$

and

$$cov\left(\varepsilon_n^{ff}(t),\varepsilon_{n'}^{cf,g}(t')\right)=0 \qquad \forall t,t',n,n',g \tag{4.26}$$

Where, σ_{ff}^2 is the variance of the free-flow error term.

The distribution of the free-flow acceleration is, therefore, given by:

$$f\left(a_n^{ff}(t)\,|\,\tau_n,\upsilon_n\right)=\frac{1}{\sigma_{ff}}\phi\left(\frac{a_n^{ff}(t)-s^{ff}\left[X_n^{ff}(t)\right]f^{ff}\left[V_n^{DS}(t-\tau_n)-V_n(t-\tau_n)\right]}{\sigma_{ff}}\right) \tag{4.27}$$

Combined with the car following model [Equation (4.22)], the probability density function of the overall stay-in-the-lane acceleration model is given by:

$$f\left(a_n^{s}(t)\,|\,h_n^*,\tau_n,\upsilon_n\right)=f\left(a_n^{cf}(t)\,|\,\tau_n\right)^{\delta[h_n(t-\tau_n)]}f\left(a_n^{ff}(t)\,|\,\tau_n\right)^{(1-\delta[h_n(t-\tau_n)])} \tag{4.28}$$

Where, $\delta\left[h_n(t-\tau_n)\right]$ is the time headway indicator:

$$\delta\left[h_n(t-\tau_n)\right]=\begin{cases}1 & \text{if } h_n(t-\tau_n)\le h_n^* \\ 0 & \text{otherwise}\end{cases}$$

4.5.2 Lane changing acceleration model

This model captures the acceleration behavior of drivers during the time they are performing the lane changing maneuver. The lane change is observed only after committing to this maneuver. The model therefore assumes that the driver determines the acceleration by evaluating the relations with the target lane leader (the leader in the lane the subject is changing to). The subject vehicle, the target lane leader and the variables defining their relations are shown in Figure 4.5.

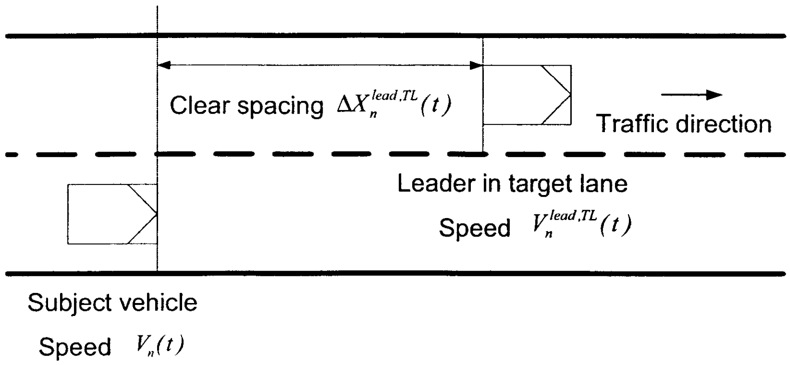


Figure 4.5 - The subject vehicle, target lane leader and their relations

The driving regimes and the models describing the behavior are assumed to have the same definitions and functional forms as in the stay-in-the-lane acceleration model. The difference is that the variables are defined with respect to the leader in the new lane instead of the front vehicle. The drivers' lane changing acceleration is therefore expressed by:

$$a_n^{lc}(t) = \begin{cases} a_n^{lc,cf}(t) & \text{if } h_n^{lead,TL}(t-\tau_n) \le h_n^* \\ a_n^{ff}(t) & \text{otherwise} \end{cases} \tag{4.29}$$

Where, $a_n^{lc}(t)$ is the acceleration vehicle n applies if it changes lanes at time t. $a_n^{lc,cf}(t)$ is the car following acceleration during lane changing. $h_n^{lead,TL}(t-\tau_n)$ is the time headway to the leader in the target lane at time $t-\tau_n$. The target lane leader time headway is defined as $h_n^{lead,TL}(t) = \frac{\Delta X_n^{lead,TL}(t)}{V_n(t)}$. $\Delta X_n^{lead,TL}(t)$ is the clear spacing between the subject vehicle and the leader in the target lane (see Figure 4.5).

The car following lane changing acceleration is expressed by:

$$a_n^{lc,cf}(t)=\begin{cases} a_n^{lc,cf,acc}(t) & if\ \Delta V_n^{lead,TL}(t-\tau_n)\geq 0 \\ a_n^{lc,cf,dec}(t) & otherwise \end{cases} \tag{4.30}$$

Where, $a_n^{lc,cf,acc}(t)$ and $a_n^{lc,cf,dec}(t)$ are the car following lane changing acceleration and deceleration models, respectively. $\Delta V_n^{lead,TL}(t-\tau_n)$ is the target lane leader relative speed at time $t-\tau_n$, which is defined as $\Delta V_n^{lead,TL}(t-\tau_n)=V_n^{lead,TL}(t-\tau_n)-V_n(t-\tau_n)$.

The car following lane changing acceleration and deceleration models are given by:

$$a_n^{lc,cf,g}(t)=s^{lc,cf,g}\left[X_n^{lc,cf,g,TL}(t)\right]f^{lc,cf,g}\left[\Delta V_n^{lead,TL}(t-\tau_n)\right]+\varepsilon_n^{lc,cf,g}(t) \tag{4.31}$$

Where, $g=acc,dec$. $X_n^{lc,cf,g,TL}(t)$ are vectors of explanatory variables, $s^{lc,cf,g}[\cdot]$ and $f^{lc,cf,g}[\cdot]$ are the lane changing car following sensitivity and stimulus functions, respectively. $\varepsilon_n^{lc,cf,g}(t)$ are the corresponding random terms.

The free-flow acceleration behavior does not depend on the identity of the leader (whether it is in the current lane or a different lane). The expressions used in the lane changing acceleration model are therefore identical to those used in the stay-in-the-lane acceleration model, as described in previous section.

The error structure and the probability density functions for the target lane acceleration are similar to those of the stay-in-the-lane model. The lane changing car following acceleration and deceleration distributions are given by:

$$f\left(a_n^{lc,cf,g}(t)\mid\tau_n\right)=\frac{1}{\sigma_{lc,cf,g}}\phi\left(\frac{a_n^{lc,cf,g}(t)-s^{lc,cf,g}\left[X_n^{lc,cf,g,TL}(t)\right]f^{lc,cf,g}\left[\Delta V_n^{lead,TL}(t-\tau_n)\right]}{\sigma_{lc,cf,g}}\right) \tag{4.32}$$

The distribution of the combined lane changing car following model is given by:

$$f\left(a_n^{lc,cf}(t)\mid\tau_n\right)=f\left(a_n^{lc,cf,acc}(t)\mid\tau_n\right)^{\delta\left[\Delta V_n^{lead,TL}(t-\tau_n)\right]}f\left(a_n^{lc,cf,dec}(t)\mid\tau_n\right)^{\left(1-\delta\left[\Delta V_n^{lead,TL}(t-\tau_n)\right]\right)} \quad (4.33)$$

Where, $\delta\left[\Delta V_n^{lead,TL}(t-\tau_n)\right]$ is the target lane leader relative speed indicator:

$$\delta\left[\Delta V_n^{lead,TL}(t-\tau_n)\right]=\begin{cases}1 & if\ \Delta V_n^{lead,TL}(t-\tau_n)\geq 0\\ 0 & otherwise\end{cases}$$

The distribution of the free-flowing acceleration is given in Equation (4.27). Combined with the car following model [Equation (4.33)], the probability density function of the lane changing acceleration model is given by:

$$f\left(a_n^{lc}(t)\mid h_n^*,\tau_n,\upsilon_n\right)=f\left(a_n^{lc,cf}(t)\mid\tau_n\right)^{\delta\left[h_n^{lead,TL}(t-\tau_n)\right]}f\left(a_n^{ff}(t)\mid\tau_n\right)^{\left(1-\delta\left[h_n^{lead,TL}(t-\tau_n)\right]\right)} \quad (4.34)$$

Where, $\delta\left[h_n^{lead,TL}(t-\tau_n)\right]$ is the target lane time headway indicator:

$$\delta\left[h_n^{lead,TL}(t-\tau_n)\right]=\begin{cases}1 & if\ h_n^{lead,TL}(t-\tau_n)\leq h_n^*\\ 0 & otherwise\end{cases}$$

4.5.3 Target gap acceleration model

This model captures the behavior of drivers who want to change lanes (i.e. their target lane is either the right lane or the left lane) but reject the available adjacent gap and therefore cannot immediately perform the lane change. In this case, the driver constructs and executes a short-term plan. The short-term plan is defined by a target gap in the target lane traffic. This gap will be used to negotiate the lane change. The acceleration the driver applies is determined to facilitate executing the short-term plan. Therefore, it depends on the target lane and target gap choices. However, while the detailed specification of the various models within this group differs, they all incorporate the common assumption that, if unconstrained, the driver targets a desired position with respect to the target gap, which would allow the lane change to be performed. This desired position is defined as the point, relative to the target gap, that the driver perceives as optimal in terms of facilitating the lane change. The stimulus the driver reacts to is the difference between the desired position and the vehicle's current position.

The situation shown in Figure 4.6 illustrates this behavior with an example of a driver who targets the forward gap. The subject vehicle (vehicle A) is committed to changing to the left lane using the forward gap, the gap between vehicles B and C. The acceleration this vehicle applies depends on its relations with the vehicle in front of it in the current lane (vehicle D) and the vehicles defining the target gap in the target lane (vehicles B and C).

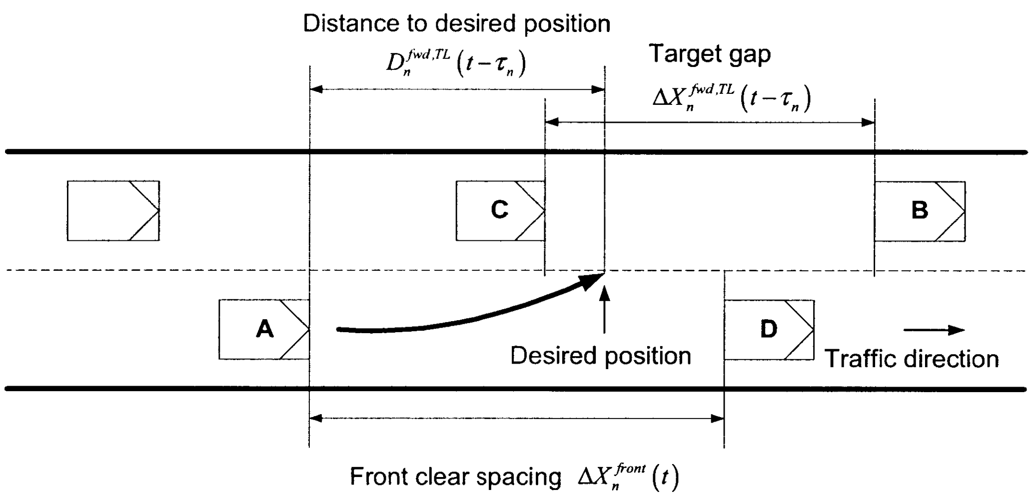


Figure 4.6 - Target gap acceleration situation and variables

The model assumes that vehicle A would try to accelerate to be able to complete the short-term plan. However, the behavior of driver A may be constrained by his current leader, vehicle D. Similar to the acceleration models presented earlier, constrained and unconstrained driving regimes are considered to capture the effect of the front vehicle presence. The car following model, presented in Section 4.5.1, captures the acceleration behavior in the case that the front vehicle is a constraining factor. In the case that the front vehicle is not constraining, the driver determines the acceleration to enable execution of the short-term plan.

The structure of the target gap acceleration model is expressed by:

$$a_n^{tg,TL}(t) = \begin{cases} a_n^{cf}(t) & \text{if } h_n(t-\tau_n) \le h_n^* \\ a_n^{uc,tg,TL}(t) & \text{otherwise} \end{cases} \tag{4.35}$$

Where, $a_n^{tg,TL}(t)$ is the acceleration vehicle *n* applies if it committed, at time *t*, to changing to the target lane *TL*, using gap *tg*. Examples of possible target gaps are the forward, adjacent and backward gaps shown in Figure 4.3. $a_n^{cf}(t)$ is the car following acceleration. $h_n(t-\tau_n)$ and h_n^* are the time headway at time $t-\tau_n$ and time headway threshold, respectively. $a_n^{uc,tg,TL}(t)$ is the unconstrained target gap acceleration.

The driver's objective in the unconstrained situation is to put the vehicle in a position to use the target gap to perform the lane change. The model assumes that the desired position with respect to the target gap is expressed as a fraction of the total length of the target gap. The stimulus is the difference between the desired position and the vehicle's current position (shown in Figure 4.6 for the forward gap). The acceleration applied in this case is affected by the relations between the subject vehicle and the leader in the target lane. To capture these effects the following functional form is used:

$$a_n^{uc,tg,TL}(t) = s^{tg}\left[X_n^{tg,TL}(t)\right] f^{tg}\left[D_n^{tg,TL}(t-\tau_n)\right] + \varepsilon_n^{tg}(t) \tag{4.36}$$

Where, $TL = RL,\ LL$. $s^{tg}\left[X_n^{tg,TL}(t)\right]$ and $f^{tg}\left[D_n^{tg,TL}(t-\tau_n)\right]$ are the target gap acceleration sensitivity and stimulus functions, respectively. $\varepsilon_n^{tg}(t)$ is the random term associated with the unconstrained target gap acceleration.

The random terms $\varepsilon_n^{tg}(t)$ are assumed to follow a normal distribution. Error terms are independent across target gaps, for different drivers and for the same driver over time. The resulting error structure is given by:

$$\varepsilon_n^{tg}(t) \sim N\left(0, \sigma_{tg}^2\right)$$

$$cov\left(\varepsilon_n^{tg}(t), \varepsilon_{n'}^{tg'}(t')\right) = \begin{cases} \sigma_{tg}^2 & if\ tg = tg', t = t', n = n' \\ 0 & otherwise \end{cases} \tag{4.37}$$

Where, σ_{tg}^2 is the variance of the target gap acceleration error term.

The corresponding distribution of the unconstrained target gap acceleration is given by:

$$f\left(a_n^{uc,tg,TL}(t) \mid \tau_n\right) = \frac{1}{\sigma_{tg}} \phi\left(\frac{a_n^{uc,tg,TL}(t) - s^{tg}[\cdot] f^{tg}[\cdot]}{\sigma_{tg}}\right) \tag{4.38}$$

Combined with the car following model [Equation (4.22)], the probability density function of the overall forward gap acceleration model is given by:

$$f\left(a_n^{tg,TL}(t) \mid h_n^*, \tau_n\right) = f\left(a_n^{cf}(t) \mid \tau_n\right)^{\delta[h_n(t-\tau_n)]} f\left(a_n^{uc,tg,TL}(t) \mid \tau_n\right)^{(1-\delta[h_n(t-\tau_n)])} \tag{4.39}$$

4.5.4 The time headway threshold distribution

The time headway threshold determines the driving regime. If the leader time headway is less than the threshold the driver is in the constrained regime, otherwise the appropriate unconstrained regime applies. The time headway threshold varies as a function of driver characteristics (e.g. aggressiveness and alertness), vehicle characteristics (e.g. braking capability) and environmental variables (e.g. weather conditions, visibility and road surface conditions). The environmental effects may only be captured if data from different sites and collection times are used. The driver and vehicle effects are captured by assuming that the time headway threshold is distributed in the population.

Previously, Ahmed (1999) assumed that the time headway threshold follows a truncated normal distribution with truncation on both sides. The truncation is needed since the time headway threshold must be positive and is finite. Ahmed (1999) noted that this form allows the distribution to be skewed in either direction, thus not restricting the frequency of drivers with relatively small thresholds to be larger than that of drivers with a large threshold (skew to the left, as in a lognormal distribution) or vice versa. The probability density function of the time headway threshold is given by:

$$f\left(h_n^*\right)=\begin{cases}\dfrac{\frac{1}{\sigma_h}\phi\left(\frac{h_n^*-\mu_h}{\sigma_h}\right)}{\Phi\left(\frac{h_{max}^*-\mu_h}{\sigma_h}\right)-\Phi\left(\frac{h_{min}^*-\mu_h}{\sigma_h}\right)} & \text{if } h_{min}^* \le h_n^* \le h_{max}^* \\ 0 & \text{otherwise}\end{cases} \tag{4.40}$$

Where, h_n^* is the time headway threshold. h_{min}^* and h_{max}^* are the minimum and maximum values of the threshold, respectively. μ_h and σ_h are the mean and standard deviation of the un-truncated distribution, respectively. $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function and the cumulative density function of a standard normal random variable, respectively.

Using the distribution in Equation (4.40), the probability that driver n is in the constrained driving regime at time t is given by:

$$P_n(\text{constrained at time } t)=P\left(h_n(t)\le h_n^*\right)=$$

$$=\begin{cases}1 & \text{if } h_n(t)\le h_{min}^* \\ 1-\dfrac{\Phi\left(\frac{h_n(t)-\mu_h}{\sigma_h}\right)-\Phi\left(\frac{h_{min}^*-\mu_h}{\sigma_h}\right)}{\Phi\left(\frac{h_{max}^*-\mu_h}{\sigma_h}\right)-\Phi\left(\frac{h_{min}^*-\mu_h}{\sigma_h}\right)} & \text{if } h_{min}^*\le h_n(t)\le h_{max}^* \\ 0 & \text{otherwise}\end{cases} \tag{4.41}$$

4.5.5 The reaction time distribution

The reaction time is the time delay from the appearance of the stimulus to the application of the response. It includes the perception time, foot movement time and vehicle response time. The perception time is the time the driver takes to recognize the stimulus and decide how to react to it. The foot movement time is the time required for the physical movement to be taken. The vehicle response time is the time the vehicular systems take to respond to the driver's inputs (brake, throttle and steering). The reaction time is affected by characteristics of the driver (e.g. age, alertness, physical condition) and the vehicle as well as by environmental conditions (e.g. weather conditions, visibility, road geometry). Reaction time is modeled as a random variable in order to capture its variability in the population. The lognormal probability function is widely accepted and used to describe the distribution of reaction times (Koppa 1997). A truncated lognormal distribution is used to account for the finiteness of the value of the reaction time. The probability density function of the reaction time is therefore given by:

$$f(\tau_n) = \begin{cases} \dfrac{\frac{1}{\sigma_\tau \tau_n}\phi\left(\frac{ln(\tau_n)-\mu_\tau}{\sigma_\tau}\right)}{\Phi\left(\frac{ln(\tau_{max})-\mu_\tau}{\sigma_\tau}\right)} & if\ 0 < \tau \leq \tau_{max} \\ 0 & otherwise \end{cases} \tag{4.42}$$

Where, τ_n is the reaction time. μ_τ and σ_τ are the mean and standard deviation of the distribution of $ln(\tau)$, respectively. τ_{max} is the maximum value of the reaction time.

In this specification it is assumed that the reaction time distribution is common to the various acceleration behaviors. This assumption may be worth further exploration. For example, reaction times associated with accelerations that are the result of the short-term plan may be shorter than those in cases that do not involve planning. Similarly, reaction times may vary depending on the level of drivers' alertness and therefore reaction times in car-following and lane changing situations may be shorter than free-flow reaction times. One possible way to relax this assumption is to introduce explanatory variables to the mean of the reaction time distribution, which could reflect the effects of planning, alertness and other factors on reaction times.

4.5.6 Summary

Different acceleration behaviors are assumed for various situations. The situations considered in the model are defined by the driver's short-term goal (target lane) and short-term plan (target gap). The situations are stay-in-the-lane, lane changing and targeting a lane change using a target gap. For each one of these situations the model considers two driving regimes: constrained and unconstrained behavior. The driving regime is determined by the relations between the subject vehicle and its leader. Reaction time is explicitly accounted for in all acceleration models.

The situation considered, driving regimes and the acceleration models that apply for each one of them are summarized in Table 4.1.

Table 4.1 - Summary of the acceleration model components

Regime / Situation	Constrained (car following) acceleration	Constrained (car following) deceleration	Unconstrained	Probability density function
Stay-in-the-lane	$a_n^{cf,acc}$ Eq. (4.18)	$a_n^{cf,dec}$ Eq. (4.19)	a_n^{ff} Eq. (4.24)	$f(a_n^s)$ Eq. (4.28)
Lane changing	$a_n^{lc,cf,acc}$ Eq. (4.31)	$a_n^{lc,cf,dec}$ Eq. (4.31)	a_n^{ff} Eq. (4.24)	$f(a_n^{lc})$ Eq. (4.34)
Target gap	$a_n^{cf,acc}$ Eq. (4.18)	$a_n^{cf,dec}$ Eq. (4.19)	$a_n^{uc,tg,TL}$ Eq. (4.36)	$f(a_n^{tg})$ Eq. (4.39)

4.6 Conclusions

In this chapter, mathematical formulations of the different components of the integrated driving behavior model were presented. The integrated modeling of lane changing and acceleration behaviors model allows behaviors that were not modeled previously, such as the choice of short-term plan (i.e. the target gap model) and acceleration behaviors to facilitate lane changing to be captured. In addition, a new model was developed for the target lane choice. This model integrates MLC and DLC considerations into a single model, thus capturing trade-offs between mandatory and discretionary considerations. The merit of this model extends beyond its role in the integrated framework since it may also be applied independently. For other components, such as gap acceptance behavior and car following behavior, existing models that have been proposed for isolated behaviors may be adequate. In these cases, state-of-the-art, tested and validated models were used.

Chapter 5

Data for Model Estimation

In this chapter, the data requirements for estimation of the integrated driving behavior model, techniques used to obtain it and a methodology applied to improve the data quality are discussed. Also, the characteristics of the collection site and the dataset used for model estimation in this thesis are summarized.

5.1 Data requirements

Estimation of the integrated driving behavior model, described in the previous chapters, as well as other models with similar level of detail, requires detailed data. Recall that the following groups of factors were identified in Section 4.1 as important explanatory variables affecting driving behavior:

- Neighborhood variables, which describe the subject vehicle and its relations with surrounding vehicles. Variables in this group may include the subject speed, relative speeds and spacing with respect to the vehicle in front of and behind it and lead and lag vehicles in neighboring lanes, the presence of heavy vehicles and so on. Variables that capture traffic conditions in the extended environment of the vehicle, such as measures of densities and average speeds and their distributions by lane are also included in this group.
- Path plan variables, which capture the effect of the path plan and trip schedule on drivers' decisions. Variables in this group include distances to the point where the driver must be in specific lanes to follow his path, the number of lane changes required to be in the correct lanes and indicators of whether the driver needs to take the next off-ramp.

- Network knowledge and experience variables, which capture considerations and preferences that are based on the driver's knowledge and experience with the transportation system. Examples may include avoiding the right-most freeway lane to minimize interactions with weaving traffic or preferring the right lane in an urban arterial to avoid delays caused by left-turning traffic.
- Driving style and capability variables, which capture the individual characteristics of the driver, such as aggressiveness and motoric capabilities (e.g. reaction time), and of the vehicle, such as speed and acceleration capabilities.

Trajectory data, which consists of observations of the positions of vehicles at discrete points in time, provides useful information about some of these variables. Trajectory data points are equally spaced in time with short time intervals between them, typically 1 second or less. Speeds, accelerations and lane changes are extracted from the time series of positions. Additional explanatory variables required by the model, such as relations between the subject and other vehicles (e.g. relative speeds, time and space headways, lengths of gaps in traffic) may also be inferred from the raw dataset.

A wide range of collection and sensing technologies, such as aerial photography, laser video, ultrasound and microwave sensors, GPS and cellular location technologies have been utilized to collect trajectory data. The configurations of collection systems can be broadly classified in two classes:

- Fixed systems - A road segment is equipped with sensing systems, most often video based, which record the positions of all vehicles on the section. The raw data is reduced to discrete observations.
- Moving systems - Rather than observing a road segment, instrumented vehicles are used. These vehicles are equipped with sensing systems, which record the position of the subject vehicle and its relations with its surrounding.

At present, neither of these system configurations is capable of collecting the complete set of data required for estimation of the integrated driving behavior model. A fixed collection system can provide the required information about the position of the subject vehicle and its relations with surrounding vehicles. However, most available datasets only cover short road segments, up to 300-400 meters long. Thus, extended view variables, such as downstream densities and speeds cannot be accurately calculated. Also, geometry, weather, surface conditions and other similar factors are uniform within a short section and so, their effects on driving behavior cannot be captured. More significantly, only limited information about the effect of the path plan may be obtained since the path is not observed. In addition, no information about the driver and only limited information about the vehicle (e.g. its length or classification) is available, and so, network knowledge and driving style variables may not be inferred.

Data generated from instrumented vehicles may alleviate some of the deficiencies of a fixed system. Observations of complete trips allow the effects of the path plan and trip schedule to be captured. The system will record the behavior of drivers traveling through multiple road facilities and therefore geometry effects may be captured. Driver and vehicle characteristics may be directly observed. However, available datasets collected by instrumented vehicles only provide limited and partial information about the driving neighborhood. In many cases only the vehicle in front of the subject is observed. Other vehicles as well as extended view traffic characteristics are not available. Therefore, while instrumented vehicles are a promising source of rich datasets for estimation of driving behavior models, these systems are still unable to produce some of the fundamental variables required within the integrated driving behavior model (e.g. lane changing behavior cannot be modeled unless vehicles in the adjacent lanes are observed).

In conclusion, datasets collected through a fixed system, with all the limitations discussed above, need to be used. To alleviate some of these limitations, these datasets should be collected in sections that are as long as possible and contain road facilities that allow capturing some of the path plan effects.

5.2 Data processing

Raw measurements of the positions of vehicles at discrete points in time need to be processed and smoothed to reduce measurement errors and to estimate speeds and accelerations. The methodology to estimate these variables consists of two steps that are repeated for each vehicle:

1. Estimation of a smooth time-continuous trajectory function from the discrete position observations.
2. Calculation of speeds and accelerations by differentiating the trajectory function once and twice, respectively.

The trajectory function is estimated using the local regression procedure. This procedure was developed by Cleveland (1979) and Cleveland and Devlin (1988) and used in the context of vehicle trajectories by Ahmed (1999).

The local regression procedure smoothes the time-series position measurements by fitting a polynomial in time curve to each measurement point. The regression curve is estimated using measurements in the neighborhood of the point being fitted.

Consider a time-series of position measurements of a given vehicle (the vehicle index is omitted for simplicity): X_t, $t = 1,...,T$. The local regression estimator of the measurements is a polynomial function in time:

$$\hat{X}_t = \boldsymbol{T}_t(\boldsymbol{t})\boldsymbol{\beta}_t + \varepsilon_t = \sum_{i=0}^{I} \beta_{t,i}(t)^i + \varepsilon_t \tag{5.1}$$

Where, $\hat{X}_t$ is the estimated position at time t. $\boldsymbol{T}_t$ is a matrix of independent variables for the observations used to estimate $\hat{X}_t$. $\boldsymbol{T}_t(\boldsymbol{t})$ is a row in that matrix corresponding to the observation at time t. It includes the polynomial in time independent variables: $\boldsymbol{T}_t(\boldsymbol{t}) = \begin{bmatrix} 1 & t & t^2 & t^3 & \dots & t^I \end{bmatrix}$. I is the order of the polynomial to be estimated. $\boldsymbol{\beta}_t = \begin{bmatrix} \beta_{t,0} & \beta_{t,1} & \beta_{t,2} & \dots & \beta_{t,I} \end{bmatrix}$ is a vector of the $I+1$ parameters of the polynomial function. ε_t are normally distributed error terms.

A regression curve of the form (5.1) is estimated for each measurement point t by weighted least squares (WLS) using a subset of the N position measurements that are closest (in time) to t. These observations are weighted by a function, $w(s,t)$, which depends on the distance (time difference) of point s from t.

A tri-cube weight function recommended by Cleveland et al (1988) is used. The weight assigned to the measurement at time s in fitting a curve for time t is given by:

$$w(s,t) = \left(1 - u(s,t)^3\right)^3 \tag{5.2}$$

Where, $u(s,t) = \frac{|s-t|}{d}$. d is the distance from t to the furthest point within the window of N points to be considered in fitting the curve.

Note that $w(s,t)$ decreases as the distance between s and t increases and that $w(t,t) = 1$. An odd number of points is chosen to ensure that the window is symmetric about t. In this case d is given by:

$$d = \frac{N+1}{2} + \delta \tag{5.3}$$

Where, $\delta > 0$ is a small constant, which ensures that the weight of the furthest point in the window will be positive.

The problem of finding the local regression trajectory by estimating fitted positions is formulated as a set of minimization problems (one for each trajectory point t). The mathematical formulation of each one of these problems is given by:

$$\min_{\boldsymbol{\beta}_t \in R^{I+1}} \; [\boldsymbol{X}_t - \boldsymbol{T}_t\boldsymbol{\beta}_t]' \boldsymbol{W} [\boldsymbol{X}_t - \boldsymbol{T}_t\boldsymbol{\beta}_t] \tag{5.4}$$

Where, $\boldsymbol{X}_t$ is the vector of position observations used to estimate a trajectory function at time t. $\boldsymbol{W}$ is an $[(I+1)\times(I+1)]$ diagonal matrix, with elements corresponding to the weights $w(s,t)$ of the observations used for the local estimation.

In this application, the local regression procedure serves not only to smooth the data and reduce measurement errors but also to estimate a continuous trajectory function, which may be used to derive the subject's speeds and accelerations at arbitrary points in time. In order to ensure that the estimated speeds and accelerations are acceptable, suitable constraints may be added to the WLS formulation. These constraints are:

- Non-negativity speed constraints.
- Upper bounds on the speed.
- Upper (acceleration) and lower (deceleration) bounds on the vehicle's acceleration to reflect vehicle capabilities.

Instantaneous speeds and accelerations are estimated as the first and second derivatives of the smoothed position, respectively:

$$\hat{V}_t = \frac{d\hat{X}_t}{dt} \tag{5.5}$$

$$\hat{a}_t = \frac{d^2\hat{X}_t}{dt^2} \tag{5.6}$$

Where, $\hat{V}_t$ and $\hat{a}_t$ are the estimated speed and acceleration at time t, respectively. $\hat{X}_t$ is the estimated trajectory function.

5.3 The estimation dataset

5.3.1 The collection site

The dataset used in this study was collected in 1983 by FHWA in a section of I-395 Southbound in Arlington VA, shown in Figure 5.1. The four-lane highway section is 997 meter long. It includes an on-ramp and two off-ramps. An hour of data at a rate of 1 frame per second was collected through aerial photography of the section. A detailed technical description of the systems and technologies used for data collection and reduction is found in FHWA (1985). The reduced dataset contains observations of the position, lane and dimensions of every vehicle within the section every 1 second.

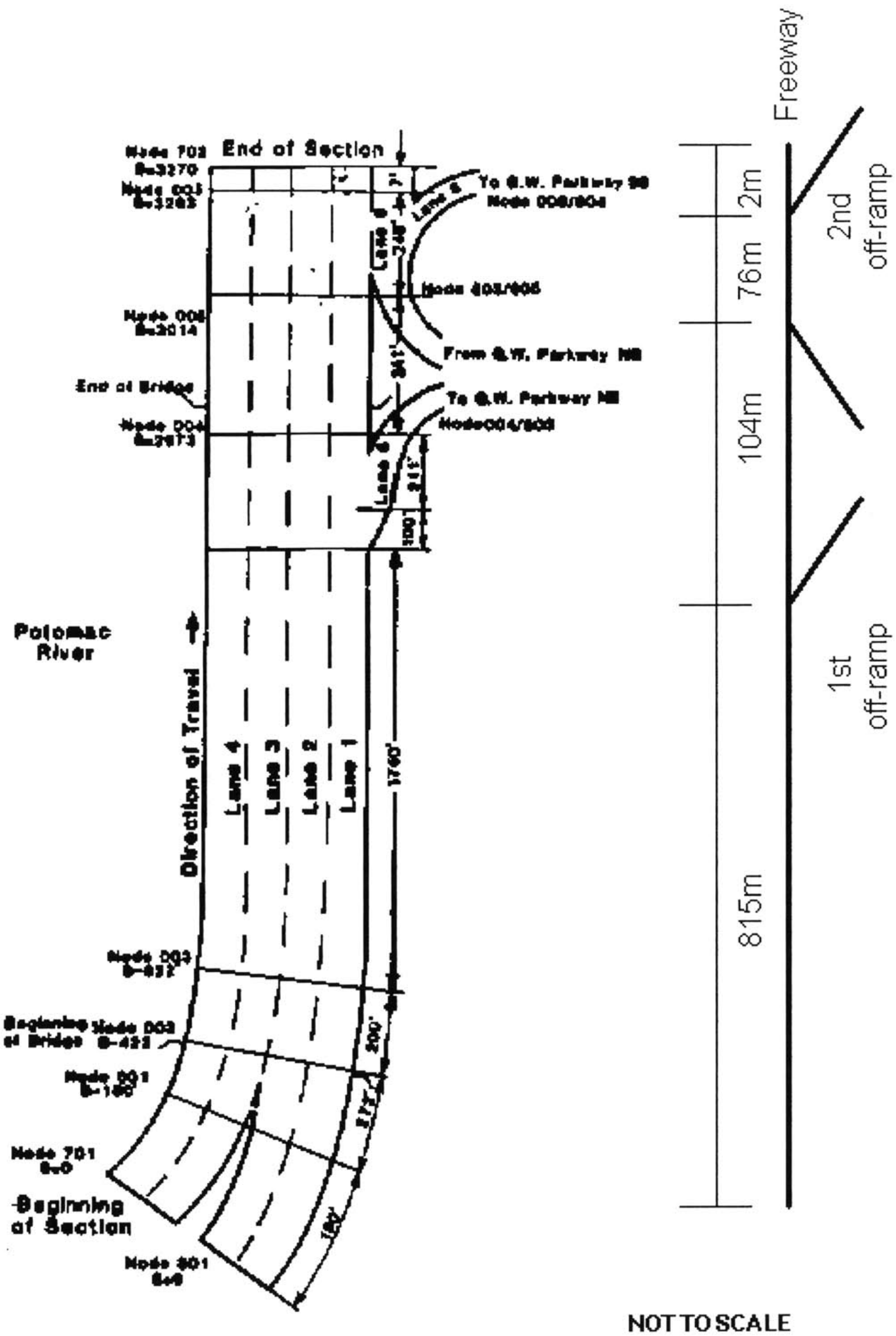


Figure 5.1 - Data collection site (Source: FHWA 1985)

As discussed above, this dataset is particularly useful for estimation of the integrated driving behavior model because of the geometric characteristics of the site: it is 997 meters long with two off-ramps and an on-ramp. It is by far the longest site for which trajectory data is available, and so, is best suited to capture elements of the integrated driving behavior: inter-dependencies between driving behaviors, short-term planning, anticipation and non-myopic considerations. The ramps within the site provides path plan information for the various vehicles. Moreover, the fact that three distinct path plans (i.e. staying in the freeway or taking the first or second off-ramp) are represented within the site creates the variability that is needed to capture these effects.

5.3.2 Characteristics of the estimation dataset

The vehicle trajectory data of the various vehicles in the section and the speeds and accelerations derived from these trajectories are used to generate the required variables. The resulting estimation dataset includes 442 vehicles for a total of 15632 observations at a 1 second time resolution. On average a vehicle was observed for 35.4 seconds (observations). All the vehicles are first observed at the upstream end of the freeway section. At the downstream end, the majority of traffic (76%) stays in the freeway. The 8% and 16% of vehicles, which exit the section using the first and second off-ramps (shown in Figure 5.1) respectively, are useful to capture the effect of the path plan on driving behavior. The breakdown of the destinations of vehicles is shown in Table 5.1.

Table 5.1 - Breakdown of vehicles by destination

Destination	# of vehicles
Freeway	337 (76%)
1^{st} ramp	35 (8%)
2^{nd} ramp	70 (16%)

Speeds in the section range from 0.4 to 25.0 m/sec. with a mean of 15.6 m/sec.. Densities range from 14.2 to 55.0 veh/km/lane with a mean of 31.4 veh/km/lane. 2% of the vehicles are categorized as heavy vehicles (length over 9.14 meters or 30 feet). Acceleration observations vary from -3.97 to 3.99 m/sec^2. Drivers are accelerating in 52% of the observations. The level of service in the section is D-E (HCM 2000). The vehicles the subject interacts with and the variables related to these vehicles are shown in Figure 5.2. Relative speeds with respect to various vehicles are defined as the speed of these vehicles less the speed of the subject. Table 5.2 summarizes statistics of the variables related to the subject vehicle and the vehicle in front. The distributions of speed, acceleration, density and time headway are shown in Figure 5.3.

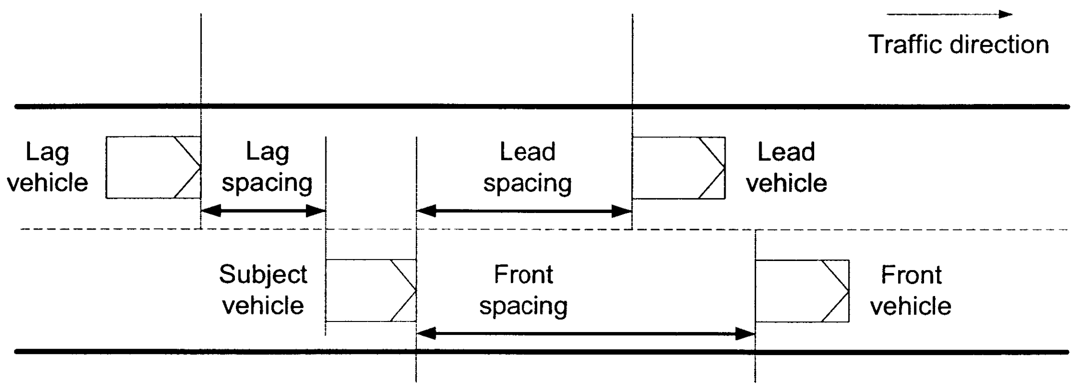


Figure 5.2 - The subject, front, lead and lag vehicles and related variables

Table 5.2 - Statistics of variables related to the subject vehicle and the vehicle in front

Variable	Mean	Std	Median	Minimum	Maximum
Speed (m/sec)	15.6	3.1	15.8	0.4	25.0
Acceleration (m/sec^2)	0.05	1.21	0.05	-3.97	3.99
Positive	0.96	0.76	0.78	0	3.99
Negative	-0.93	0.75	-0.74	-3.97	0
Density (veh/km/lane)	31.4	6.5	30.8	14.2	55.0
Relations with the front vehicle					
Speed (m/sec)	15.8	3.2	16.0	0.2	25.0
Relative speed (m/sec)	0.2	1.7	0.2	-8.6	9.7
Spacing (m)	26.6	21.2	20.4	1.4	250.5
Time headway (sec)	2.0	1.4	1.7	0.3	27.3

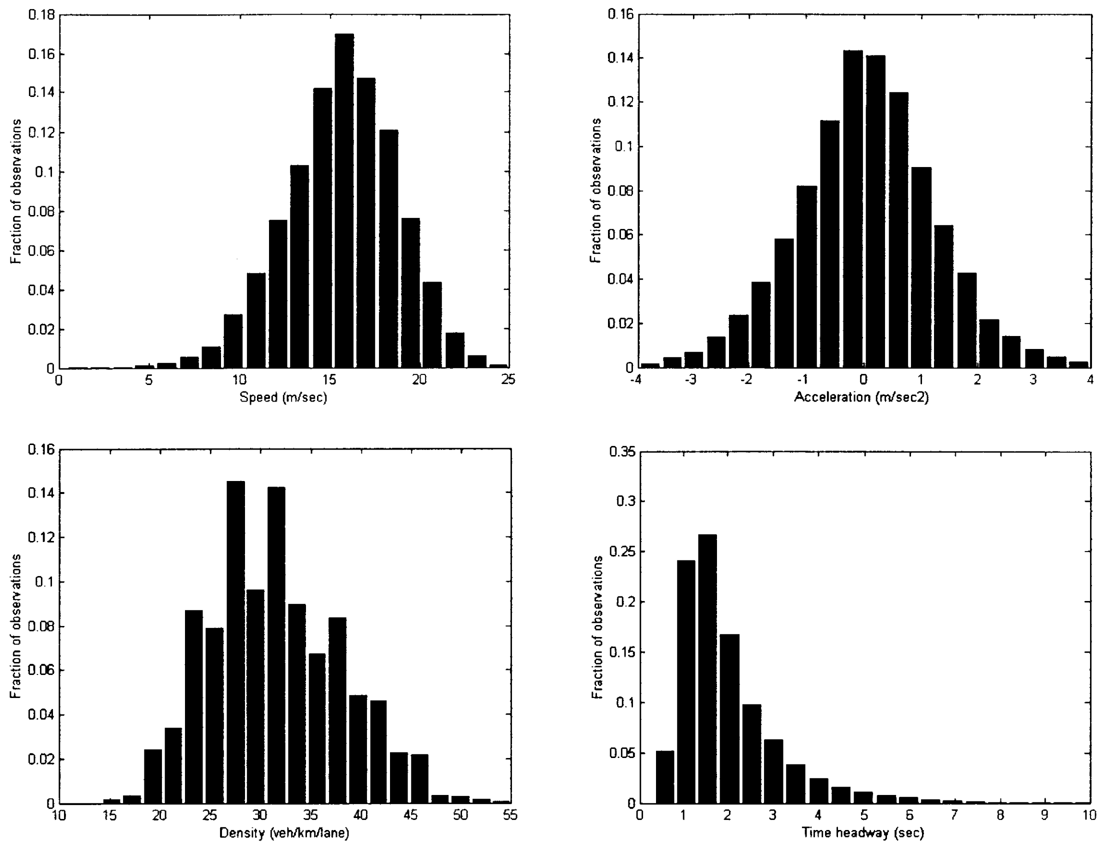


Figure 5.3 - Distributions of speed, acceleration, density and time headway in the data

Lane selection and gap acceptance behaviors are captured by observing lane changes drivers perform. An important factor in these behaviors is drivers' desire to follow their path. In this dataset drivers have three possible destinations, each with a corresponding path following behavior:

- Exiting the section through the first off-ramp.
- Exiting the section through the second off-ramp.
- Staying in the freeway at the downstream end of the section.

Table 5.3 describes the distribution of observed lane changes by direction (right, left) and by destination. It is worth noting that many of the vehicles that exit the section through the off-ramps are observed in the right-most lane at the upstream end of the section. This indicates that they may have started considering the path plan constraint earlier. As a result the coefficients of explanatory variables related to the path plan may be biased towards aggressive behaviors since the more timid drivers are discounted in the dataset.

Table 5.3 - Distribution of lane changes by direction and destination

Destination	Right	Left
Total	123	74
Freeway	71	71
1st ramp	12	0
2nd ramp	40	3

Table 5.4 - Statistics describing the lead and lag vehicles

Variable	Mean	Std	Median	Minimum	Maximum
Relations with Lead vehicle					
Relative Speed (m/sec)	0.2 (0.0)	2.6 (2.9)	0.5 (0.1)	-17.3 (-17.5)	8.1 (15.5)
Lead spacing (m)	22.2 (19.6)	21.9 (39.9)	14.1 (13.0)	0.04 (-18.1)	117.9 (268.9)
Relations with Lag vehicle					
Relative Speed (m/sec)	-0.4 (0.0)	2.2 (2.7)	-0.3 (0.0)	-6.7 (-15.0)	5.2 (14.1)
Lag spacing (m)	23.1 (18.6)	20.6 (23.0)	16.6 (12.0)	1.7 (-18.1)	110.1 (232.6)
Statistics are for the accepted gaps only, in parentheses for the entire dataset					

The relations between the subject and the lead and lag vehicles in the lanes to its right and to its left affect the gap acceptance and gap choice behaviors. Table 5.4 summarizes statistics of the accepted lead and lag gaps (i.e. the gaps vehicle changed lanes into). Accepted lead gaps vary from 0.04 to 117.9 meters, with a mean of 22.2 meters. Accepted lag gaps vary from 1.7 to 110.1 meters, with a mean of 23.1 meters. No significant differences were found between the right and left lanes. Relative speeds are defined as the speed of the lead vehicle or the lag vehicle less the speed of the subject. Statistics for the entire dataset are also shown. With these statistics, negative spacing values indicate that the subject and the lead vehicle partly overlap (this is possible because they are in different lanes). As expected, the mean accepted gaps are larger than the mean gaps in the traffic stream. Similarly, lead relative speeds in accepted gaps are larger than in the mean of the dataset and lag relative speeds are smaller in the entire dataset (i.e. on average, in accepted gaps the subject vehicle is slower relative to the lead vehicle and faster relative to the lag vehicle compared to the entire dataset). The distributions of relative speeds and spacing, with respect to the front, lead and lag are shown in Figure 5.4 and Figure 5.5, respectively.

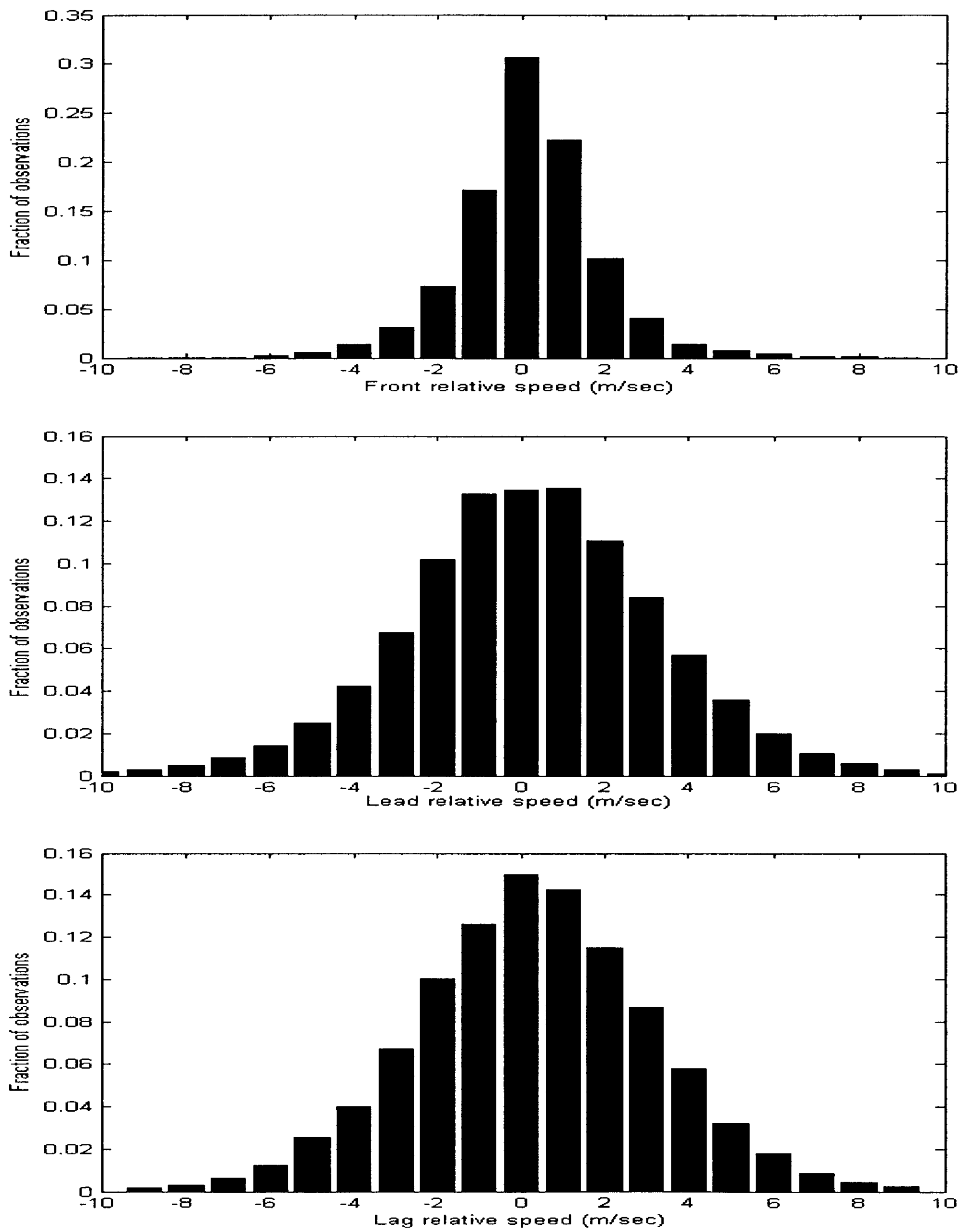


Figure 5.4 - Distributions of relative speed with respect to the front, lead and lag vehicles

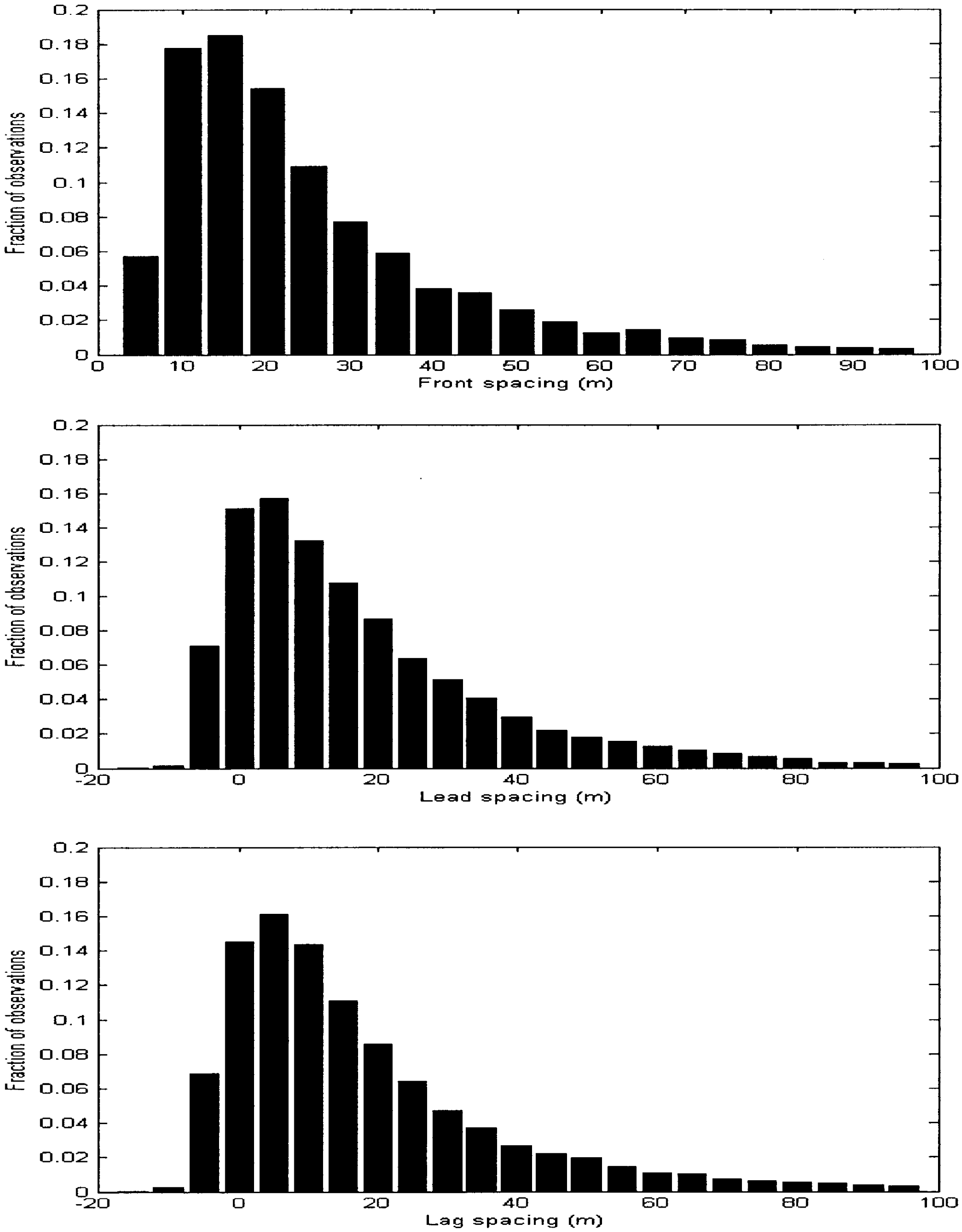


Figure 5.5 - Distributions of spacing with respect to the front, lead and lag vehicles

5.4 Conclusions

In this chapter, the data requirements for estimation of the integrated driving behavior model were discussed. Trajectory data, which consists of observations of the positions of vehicles at discrete points in time, is a useful basis to infer variables that may explain driving behavior. A methodology to improve the quality of trajectory data and to estimate instantaneous speeds and accelerations was presented. It is based on estimating a trajectory function using the local regression procedure. This procedure was developed by Cleveland (1979) and Cleveland and Devlin (1988) and first used in the context of vehicle trajectories by Ahmed (1999).

The characteristics of the collection site and the dataset used for model estimation in this thesis were summarized. This dataset is particularly useful for estimation of the integrated driving behavior model because of the geometric characteristics of the site: the site is 997 meters long with two off-ramps and an on-ramp and therefore includes weaving sections that may exhibit behaviors represented in the integrated model, such as inter-dependencies between lane changing and acceleration, short-term planning and anticipation and capture the effect of the path plan on driving behavior. The data represents a wide range of traffic conditions. Speeds range from 0.4 to 25.0 m/sec. Densities range from 14.2 to 55.0 veh/km/lane. The level of service in the section is D-E.

Chapter 6

Estimation Results

In this chapter, estimation results of the integrated driving behavior model using the Arlington, VA dataset are described. Statistical assessment and behavioral interpretation of the results are also presented. All components of the model were estimated jointly using a maximum likelihood estimation procedure. The likelihood function derived for this dataset is presented in the next section followed by the discussion of estimation results of the various components.

6.1 Likelihood function

In this section, the joint likelihood function for the accelerations and lane changes observed in the trajectory data is presented. As discussed in the previous chapter, the following important limitations of the dataset need to be addressed:

- No information about the driver/vehicle characteristics is available except for the length of the vehicle.
- Only limited information about drivers' path plans is available. In particular, the path plans of drivers who remain in the freeway at the downstream end of the section are unknown.

Driver/vehicle specific latent variables are introduced in the model to overcome the lack of driver/vehicle characteristics data. These variables capture correlations between the decisions made by the same driver over time. The individual specific error term υ_n is included in the specification of the target lane, gap acceptance and target gap utility functions and in the desired speed function. The parameters associated with this variable in the various model components are estimated jointly, and so, capture correlations between these decisions, which may be attributed to unobserved driver/vehicle characteristics. Similarly, the acceleration component of the model assumes that reaction times (τ_n) and time headway thresholds (h_n^*) are randomly distributed in the population. These distributions capture unobserved correlations between the various acceleration decisions over time due to unobserved driver and vehicle characteristics.

Variables related to the path plan, such as the distance to an off-ramp the driver needs to use, are important in the integrated driving behavior model. However, this information is not available for vehicles that exit the freeway downstream of the observed section. In order to capture the effect of these variables, a distribution of the distance from the downstream end of the road section being studied to the exit points is used. The parameters of this distribution are estimated jointly with the other parameters of the model. In this study, a discrete distribution of the distances is used based on the locations of off-ramps downstream of the section. The alternatives considered are the first, second and subsequent off-ramps. The probability mass function of distances to the off-ramps, beyond the downstream end of the segment is given by:

$$p(d_n)=\begin{cases}\pi_1 & \textit{first downstream exit}\left(d^1\right)\\ \pi_2 & \textit{second downstream exit}\left(d^2\right)\\ 1-\pi_1-\pi_2 & \textit{otherwise}\left(d^3\right)\end{cases} \tag{6.1}$$

Where, π_1 and π_2 are the proportions of drivers using the first and second downstream off-ramp, respectively. These are parameters to be estimated. d^1, d^2 and d^3 are the distances beyond the downstream end of the section to the first, second and subsequent exits, respectively.

The discrete distribution is preferred to a continuous distribution because it exploits additional information available from the geometric layout of the road. The first and second exit distances (d^1 and d^2) are measured directly from geometric information. For the subsequent exits an infinite distance is used ($d^3=\infty$), which corresponds to an assumption that the driver ignores this consideration in the lane choice.

With these mechanisms in place to handle missing data and unobserved driver/vehicle characteristics, the joint likelihood function of lane changing and acceleration observations can be formulated. The joint probability density of a combination of target lane (*TL*), lane action (*l*), target gap (*TG*) and acceleration (*a*) observed for driver *n* at time *t*, conditional on the individual specific variables (d_n, υ_n, τ_n and h_n^*) is given by:

$$\begin{aligned} f_n\left(TL_t, l_t, TG_t, a_t | d_n, \upsilon_n, \tau_n, h_n^*\right) = P_n\left(TL_t \mid d_n, \upsilon_n\right) P_n\left(l_t \mid TL_t, \upsilon_n\right) \cdot \\ \cdot P_n\left(TG_t \mid TL_t, l_t, \upsilon_n\right) f_n\left(a_t \mid TL_t, l_t, TG_t, \upsilon_n, \tau_n, h_n^*\right) \end{aligned} \tag{6.2}$$

Where, $P_n(TL_t \mid \cdot)$, $P_n(l_t \mid \cdot)$ and $P_n(TG_t \mid \cdot)$ are given by Equations (4.4), (4.8) and (4.12), respectively. The equations defining $f_n(a_t \mid \cdot)$ in the various situations are summarized in Table 4.1.

Only the lane changing and acceleration decisions are observed. The marginal probability of these two variables is given by summing the target lane and target gap out of the joint probability:

$$f_n\left(l_t, a_t | d_n, \upsilon_n, \tau_n, h_n^*\right) = \sum_{TL_t} \sum_{TG_t} f_n\left(TL_t, l_t, TG_t, a_t | d_n, \upsilon_n, \tau_n, h_n^*\right) \tag{6.3}$$

The behavior of driver *n* is observed over a sequence of *T* consecutive time intervals. The joint probability of the sequence of observations is the product of the probabilities given by Equation (6.3):

$$f_n\left(\mathbf{l}, \mathbf{a} | d_n, \upsilon_n, \tau_n, h_n^*\right) = \prod_{t=1}^{T} f_n\left(l_t, a_t | d_n, \upsilon_n, \tau_n, h_n^*\right) \tag{6.4}$$

Where, **l** and **a** are the sequences of lane changing decisions and accelerations, respectively.

The unconditional individual likelihood function is acquired by integrating (or summing, for the discrete variable d_n) the conditional probability over the distributions of the individual specific variables:

$$L_n = \int_{\upsilon}\int_{\tau}\int_{h^*}\sum_{d} f_n\left(\mathbf{l},\mathbf{a} \mid d_n,\upsilon_n,\tau_n,h_n^*\right) p(d) f\left(h^*\right) f(\tau) f(\upsilon)\, dh^* d\tau d\upsilon \qquad (6.5)$$

Where, $p(d)$, $f\left(h^*\right)$ and $f(\tau)$ are given by Equations (6.1), (4.40) and (4.42), respectively. $f(\upsilon)$ is the standard normal probability density function.

Assuming that observations of different drivers are independent, the log-likelihood function for all *N* individuals observed is given by:

$$L = \sum_{n=1}^{N} \ln\left(L_n\right) \qquad (6.6)$$

The maximum likelihood estimates of the model parameters are found by maximizing this function. In this study, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization algorithm implemented in the statistical estimation software GAUSS (Aptech Systems 1994) was used. BFGS is a quasi-Newton method, which maintains and updates an approximation of the Hessian matrix based on first-order derivative information (see, for example, Bertsekas 1999). GAUSS implements a variant of BFGS due to Gill and Murray (1972), which updates the Cholesky decomposition of the Hessian (Aptech Systems 1995).

The integrals in the likelihood function were calculated numerically using the Gauss-Legendre quadrature method (Aptech Systems 1994). Numerical integration is expected to performs better than Monte-Carlo integration in the application at hand because of the presence of the reaction time dimension: Monte-Carlo integration would require explanatory variable values, lagged by the reaction time, to be calculated for each draw. In contrast, with numerical integration only the explanatory variables values for the (much fewer) points used for the integration need to be calculated.

The likelihood function is not globally concave. For example, if the signs of all the coefficients of the individual-specific error term υ_n are reversed, the solution is unchanged due to its symmetric distribution function. To avoid obtaining a local solution, different starting points were used in the optimization procedure.

6.2 Estimation results

All components of the model were estimated jointly using a maximum likelihood estimation procedure as described in the previous section. However, in order to simplify the presentation, estimation results for the various components of the model are presented and discussed separately. The presentation order follows the hierarchy of the hypothesized decision-making process: the target lane model is presented first, followed by the gap acceptance model, the target gap model and the various acceleration models.

6.2.1 The target lane model

This model describes drivers' choice of lane they would want to travel in. A driver chooses one of up to three alternatives: to stay in the current lane (*CL*) or to target changing either to the right lane (*RL*) or to the left lane (*LL*). The estimated systematic utilities of the current, right and left lane, respectively, for driver *n* at time *t*, are given by:

$$V_n^{CL}(t) = X_n^{CL}(t)\beta^{CL} + \alpha^{CL}\upsilon_n \tag{6.7}$$

$$V_n^{RL}(t) = X_n^{RL}(t)\beta^{RL} + \alpha^{RL}\upsilon_n \tag{6.8}$$

$$V_n^{LL}(t) = X_n^{LL}(t)\beta^{LL} \tag{6.9}$$

Where, $X_n^{CL}(t)$, $X_n^{RL}(t)$ and $X_n^{LL}(t)$ are vectors of explanatory variables affecting the utilities of the current, right and left lanes, respectively. β^{CL}, β^{RL} and β^{LL} are the corresponding vectors of parameters. υ_n is an individual specific error term, which is assumed to be standard normally distributed in the population. α^{CL} and α^{RL} are the parameters of υ_n. Conditional on the value of υ_n, the target lane choice probabilities are defined by a logit model.

Estimation results of the target lane model are presented in Table 6.1.

Path plan variables are the most important of the four groups of variables discussed in Section 4.1. The effect of the path plan is captured by a group of variables, which combine a function of the distance to the point where the driver needs to be in a specific lane (i.e. in order to take an off-ramp) and the number of lane changes required to be in the correct lane. The estimation dataset is from a four-lane freeway, in which the off-ramps are in the right-most lane. Hence, three variables are defined:

$$path\_plan\_impact\_j_n^{lane\,i}(t) = \left[d_n^{exit}(t)\right]^{\theta^{MLC}} \delta_n^{j\,i}(t) \qquad \begin{matrix} i = CL, RL, LL \\ j = 1,2,3 \end{matrix} \tag{6.10}$$

Where, $d_n^{exit}(t)$ is the distance from the position of vehicle n at time t to the point where it needs to be in a specific lane (i.e. its exit point from the freeway) in kilometers. θ^{MLC} is a parameter to be estimated. $\delta_n^{j,i}(t)$ are indicators of the number of lane changes required to follow the path:

$$\delta_n^{j,i}(t) = \begin{cases} 1 & j \text{ lane changes are required from lane } i \\ 0 & otherwise \end{cases} \tag{6.11}$$

As expected, the utility of a lane decreases with the number of lane changes the driver needs to make from it. This effect is magnified as the distance to the off-ramp decreases (θ^{MLC} = -0.358). The use of a power function to capture the effect of the distance from the off-ramp ensures that at the limits, the path plan impact approaches 0 when $d_n^{exit}(t) \to +\infty$ and approaches $-\infty$ when $d_n^{exit}(t) \to +0$. Figure 6.1 shows the impact of path plan lane changes on the utility of a lane as a function of the distance from the off-ramp.

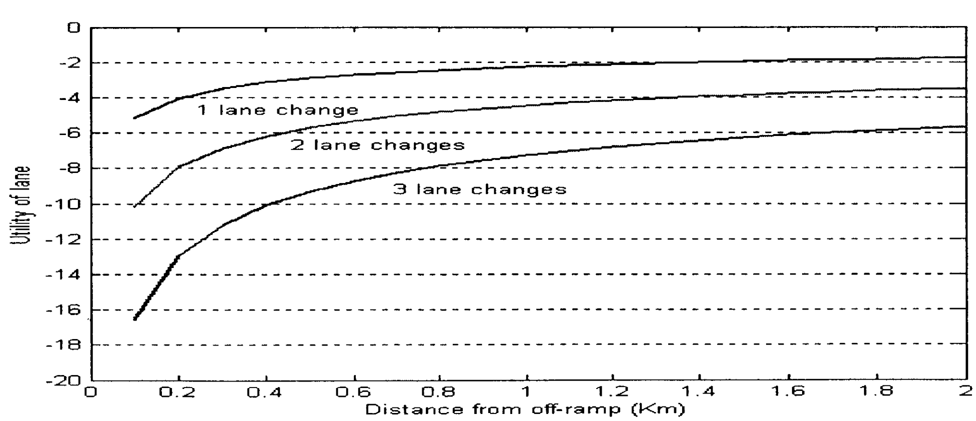


Figure 6.1 - Impact of path plan lane changes on the utility of a lane

Table 6.1 - Estimation results for the target lane model

Variable	Parameter value	t-statistic
CL constant	2.128	2.68
RL constant	-0.369	-1.28
Right-most lane dummy	-1.039	-3.85
Front vehicle speed, m/sec.	0.0745	1.78
Front vehicle spacing, m.	0.0225	3.68
Lane density, veh/km/lane	-0.0018	-1.45
Heavy neighbor dummy	-0.218	-0.93
Tailgate dummy	-3.793	-1.83
Gap acceptance expected maximum utility	0.0052	0.41
Path plan impact, 1 lane change required	-2.269	-5.57
Path plan impact, 2 lane changes required	-4.466	-7.18
Path plan impact, 3 lane changes required	-7.265	-8.34
Next exit dummy, 1 lane change required	-1.264	-2.92
Next exit dummy, each additional lane change	-0.252	-1.36
π_1	0.0063	0.57
π_2	0.0406	1.16
θ^{MLC}	-0.358	-2.74
α^{CL}	0.539	5.07
α^{RL}	1.035	5.15

Drivers' perception and awareness of path plan considerations is likely to be a function of the geometric elements of the road. In particular, drivers are more likely to respond to constraints that involve the next road element they will encounter. In the road section used for estimation, such behavior would present itself for drivers who exit the freeway using the next off-ramp, as opposed to drivers who will use subsequent exits. As with the impact of the distance, explanatory variables are generated by interaction of a next-exit dummy variable with the number of lane changes required:

$$next\_exit\_impact\_1_n^{lane\,i}(t) = \delta_n^{next\,exit}(t)\delta_n^{1,i}(t) \quad i = CL, RL, LL \tag{6.12}$$

$$next\_exit\_impact\_2+_n^{lane\,i}(t) = \delta_n^{next\,exit}(t)\delta_n^{add,i}(t) \quad i = CL, RL, LL \tag{6.13}$$

Where, $\delta_n^{1,i}(t)$ is defined in equation (6.11). The indicator variables $\delta_n^{next\,exit}(t)$ and $\delta_n^{add,i}(t)$ are given by:

$$\delta_n^{next\,exit}(t) = \begin{cases} 1 & \textit{the next off-ramp is used} \\ 0 & \textit{otherwise} \end{cases} \tag{6.14}$$

$$\delta_n^{add,i}(t) = \begin{cases} 2 & \delta_n^{3,i}(t) = 1 \\ 1 & \delta_n^{2,i}(t) = 1 \\ 0 & \textit{otherwise} \end{cases} \quad i = CL, RL, LL \tag{6.15}$$

The specification of the variable *next_exit_dummy_2+* forces a restriction that each additional lane change is penalized with the same disutility. This specification was chosen because there are few observations of vehicles that required three lane changes and therefore a separate coefficient could not be estimated.

As expected, the estimated coefficients for these variables are negative and the utility of a lane decreases with the number of lane changes required. It is worth noting that the magnitude of the marginal disutility associated with a need for one lane change to take the next off-ramp is larger (in absolute value) than that of any additional lane changes. This implies that drivers perceive being in the wrong lane as a more significant factor compared to the number of lane changes that are required.

A second group of variables are those capturing driving conditions in the immediate and extended neighborhood of the vehicle. These include the speed of the vehicle in front of the subject and the spacing between them, densities in the various lanes, presence of heavy neighbor vehicles and the expected maximum utilities of the available gaps in the lanes to the right and to the left of the subject vehicle. The speed $\left(V_n^{front}(t)\right)$ and spacing $\left(S_n^{front}(t)\right)$ of the front vehicle (only appearing in the utility of the current lane) capture the likely satisfaction of the driver with conditions in the current lane. The utility of the current lane increases with the speed of the front vehicle and with the spacing between the two vehicles. This implies that the subject is less likely to perceive the front vehicle as a constraint when the front vehicle speed is higher and the spacing is larger.

The expected maximum utilities (*EMU*) of the available gaps in the right lane and in the left lane capture the impact of gap acceptance decisions on the target lane choice. Mathematical expressions for the maximum expected utilities are developed in Appendix B. The values of these variables increase with the probability that the subject vehicle will be able to accept the gap in the right lane or in the left lane, if one of these lanes is chosen as the target lane. Therefore *EMU* values increase with the spacing to the lead and lag vehicle and with the relative lead speed and decreases with the relative lag speed.

Another variable that captures driving conditions in the subject's neighborhood is the presence of heavy vehicles. This dummy variable is defined separately for each candidate lane by:

$$\delta_n^{heavy\ neighbor,\ i}(t) = \begin{cases} 1 & \textit{lead and/or lag in lane i is heavy} \\ 0 & \textit{otherwise} \end{cases} \quad i = CL, RL, LL \qquad (6.16)$$

A vehicle is defined as heavy if its length exceeds 9.14 meters (30 feet). The utility of a lane decreases if one of the neighboring vehicles in that lane is heavy. This captures drivers' preference to avoid interacting with heavy vehicles. Interestingly, a variable defined by the subject vehicle itself being heavy was insignificant in this model.

The tailgating dummy variable captures drivers' tendency to move out of their current lane if they are being tailgated. Tailgating is not directly observable in the data. Instead, tailgating behavior is assumed if a vehicle is close to the vehicle in front of it (the subject vehicle) when traffic conditions permit a longer headway (i.e. free-flow conditions apply). Mathematically, the tailgate dummy variable is defined by:

$$\delta_n^{tailgate}(t) = \begin{cases} 1 & \textit{gap behind} \leq 10m \textit{ and level of service is A, B or C} \\ 0 & \textit{otherwise} \end{cases} \qquad (6.17)$$

Levels of service definitions are based on densities (HCM 2000). The estimated coefficient of the tailgate dummy is negative and its magnitude is large relative to the coefficients of other variables. It implies a strong preference to avoid these situations. This result is comparable with those of Ahmed (1999), who also found tailgating to be an important explanatory variable.

The variable *density_in_lane* $\left(d_n^{lane}(t)\right)$ captures conditions in an extended neighborhood. The utility of a lane decreases with congestion in that lane (indicated by higher densities).

The right-most lane dummy variable captures the preference of freeway drivers to avoid the right-most lane because of the merging and weaving activity that takes place in that lane. The right-most dummy variable is defined by:

$$\delta_n^{right\text{-}most,\, i}(t) = \begin{cases} 1 & \text{lane } i \text{ is the right-most lane} \\ 0 & \text{otherwise} \end{cases} \qquad i = CL, RL \tag{6.18}$$

The heterogeneity coefficients, α^{CL} and α^{RL}, capture the effects of the individual specific error term υ_n on the target lane choice, thus accounting for correlations between observations of the same individual due to unobserved characteristics of the driver/vehicle. Both estimated parameters are positive and so, υ_n can be interpreted as positively correlated to the driver's timidity. A timid driver (i.e. $\upsilon_n > 0$) is more likely to choose the right lane and the current lane over the left lane compared to a more aggressive driver.

In summary, the target lane utilities are given by:

$$\begin{aligned} V_n^{CL}(t) = {} & 2.128 - 1.039\delta_n^{right\ most,\ CL}(t) + 0.0745V_n^{front}(t) + 0.0225S_n^{front}(t) - \\ & - 0.0018d_n^{CL}(t) - 0.218\delta_n^{heavy\ neighbor,\ CL}(t) - 3.793\delta_n^{tailgate} - \\ & - \left[d_n^{exit}(t)\right]^{-0.358}\left(-2.266\delta_n^{1,\,CL}(t) - 4.466\delta_n^{2,\,CL}(t) - 7.265\delta_n^{3,\,CL}(t)\right) - \\ & - 1.264\delta_n^{next\ exit,\ CL}(t) - 0.252\delta_n^{add,CL}(t) + 0.539\upsilon_n \end{aligned} \tag{6.19}$$

$$
\begin{aligned}
V_n^{RL}(t) = & -0.369 - 1.039\delta_n^{right\ most,\ RL}(t) - 0.0018 d_n^{RL}(t) - 0.218\delta_n^{heavy\ neighbor,\ RL}(t) - \\
& -\left[d_n^{exit}(t)\right]^{-0.358}\left(-2.266\delta_n^{1,\ RL}(t) - 4.466\delta_n^{2,\ RL}(t) - 7.265\delta_n^{3,\ RL}(t)\right) - \\
& -1.264\delta_n^{next\ exit,\ RL}(t) - 0.252\delta_n^{add,RL}(t) + 0.0052 EMU_n^{RL}(t) + 1.035\upsilon_n
\end{aligned} \tag{6.20}
$$

$$
\begin{aligned}
V_n^{LL}(t) = & -0.0018 d_n^{LL}(t) - 0.218\delta_n^{heavy\ neighbor,\ LL}(t) - \\
& -\left[d_n^{exit}(t)\right]^{-0.358}\left(-2.266\delta_n^{1,\ LL}(t) - 4.466\delta_n^{2,\ LL}(t) - 7.265\delta_n^{3,\ LL}(t)\right) - \\
& -1.264\delta_n^{next\ exit,\ LL}(t) - 0.252\delta_n^{add,\ LL}(t) + 0.0052 EMU_n^{LL}(t)
\end{aligned} \tag{6.21}
$$

Where, $EMU_n^{RL}(t)$ and $EMU_n^{LL}(t)$ are right lane and left lane gap acceptance expected maximum utilities, respectively.

6.2.2 The gap acceptance model

The gap acceptance behavior is conditioned on the driver targeting either the right lane or the left lane. In these cases, the driver is assumed to evaluate the available adjacent gap in the target lane and decide whether to change lanes immediately or not. In order for the gap to be acceptable both the lead and lag gaps, shown in Figure 6.2, must be acceptable.

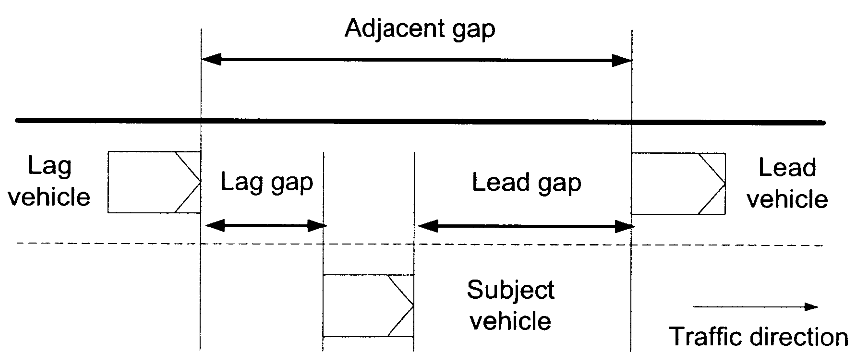


Figure 6.2 - The adjacent gap, subject, lead and lag vehicles and the lead and lag gaps

The lead (or lag) gap is acceptable only if the available gap is larger than an unobservable critical lead (or lag) gap, which is the minimum acceptable gap. In order to ensure that critical gaps are always positive, they are assumed to follow a lognormal distribution:

$$\ln\left(G_n^{lead\,TL,cr}(t)\right) = X_n^{lead\,TL}(t)\beta^{lead} + \alpha^{lead}\upsilon_n + \varepsilon_n^{lead}(t) \qquad (6.22)$$

$$\ln\left(G_n^{lag\,TL,cr}(t)\right) = X_n^{lag\,TL}(t)\beta^{lag} + \alpha^{lag}\upsilon_n + \varepsilon_n^{lag}(t) \qquad (6.23)$$

Where, $X_n^{lead\,TL}(t)$ and $X_n^{lag\,TL}(t)$ are vectors of explanatory variables affecting the lead and lag critical gaps, respectively. β^{lead} and β^{lag} are the corresponding vectors of parameters. $\varepsilon_n^{lead}(t)$ and $\varepsilon_n^{lag}(t)$ are the random terms associated with the critical gaps for driver n at time t. These error terms are normally distributed: $\varepsilon_n^{lead}(t) \sim N\left(0, \sigma_{lead}^2\right)$ and $\varepsilon_n^{lag}(t) \sim N\left(0, \sigma_{lag}^2\right)$. α^{lead} and α^{lag} are the parameters of the individual specific random term υ_n for the lead and lag critical gaps, respectively.

Estimation results for the lead and lag critical gaps are presented in Table 6.2.

Table 6.2 - Estimation results for the gap acceptance model

Variable	Parameter value	t-statistic
Lead Critical Gap		
Constant	1.127	2.78
$Max\left(\Delta V_n^{lead}(t), 0\right)$, m/sec.	-2.178	-0.63
$Min\left(\Delta V_n^{lead}(t), 0\right)$, m/sec.	-0.153	-1.86
Target gap expected maximum utility	0.0045	1.29
α^{lead}	0.789	2.46
σ^{lead}	1.217	2.55
Lag Critical Gap		
Constant	0.968	4.18
$Max\left(\Delta V_n^{lag}(t), 0\right)$, m/sec.	0.491	5.95
Target gap expected maximum utility	0.0152	1.65
α^{lag}	0.107	0.47
σ^{lag}	0.622	4.53

Both the lead critical gap and the lag critical gap are a function of the subject relative speed with respect to the corresponding vehicles. Relative speed with respect to a vehicle is defined as the speed of that vehicle less the speed of the subject.

The lead critical gap decreases with the relative lead speed, i.e., it is larger when the subject is faster relative to the lead vehicle. The effect of the relative speed is strongest when the lead vehicle is faster than the subject. In this case, the lead critical gap quickly reduces to almost zero, as the relative speed is increasingly positive. This result suggests that drivers perceive very little risk from the lead vehicle when it is getting away from them.

Inversely, the lag critical gap increases with the relative lag speed: The faster the lag vehicle is relative to the subject, the larger the lag critical gap is. In contrast to the lead critical gap, the lag gap does not diminish when the subject is faster. An explanation may be that drivers have a less reliable perception of the lag gap compared to the lead gap (due to the indirect observation of lag gaps through mirrors). Therefore, drivers may keep a minimum critical gap as a safety buffer.

The expected maximum utility (*EMU*) of target gap utilities captures the effects of available gaps in the vehicle neighborhood on critical gaps. The target gap *EMU* increases with the utilities of the alternative target gaps, which capture the usefulness and ease of changing lanes into these gaps. Both the lead and lag critical gaps increase with the target gap *EMU*. This suggests risk aversive behavior: when traffic conditions are such that useful gaps are available (i.e. larger values of target gap *EMU*) drivers tend to accept lower risks in lane changing, therefore requiring larger critical gaps compared to the case where available gaps are not as useful (lower *EMU* values). The effect of this variable is stronger in the lag critical gap relative to the lead critical gap. This may again be explained by the higher uncertainty and extra caution associated with the lag gap.

Median lead and lag critical gaps, as a function of the relative speeds and *EMU* are presented in Figure 6.3 and Figure 6.4, respectively.

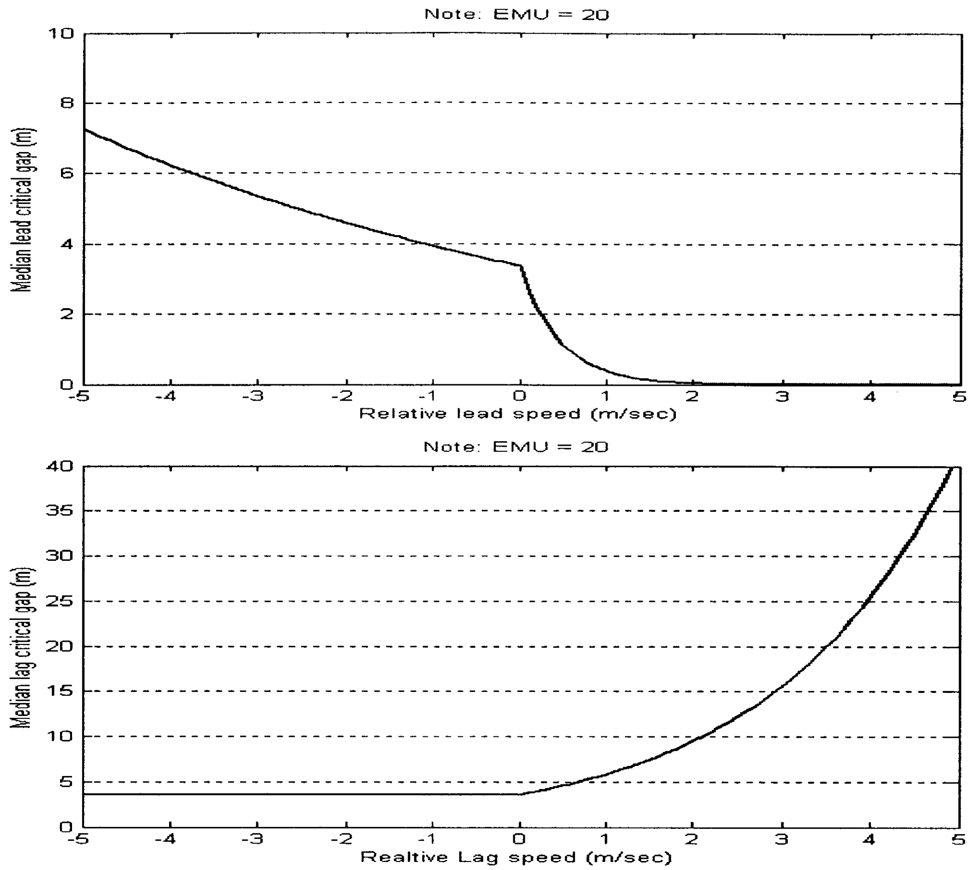


Figure 6.3 - Median lead and lag critical gaps as a function of relative speed

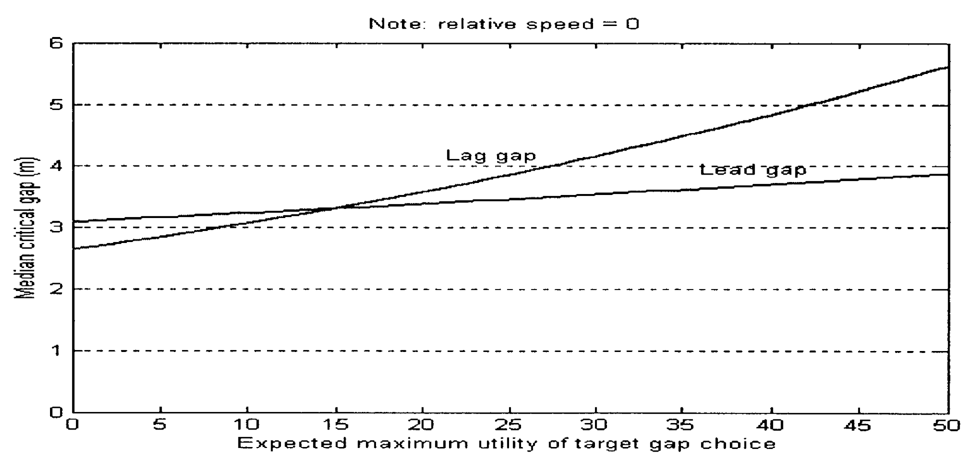


Figure 6.4 - Median lead and lag critical gaps as a function of the target gap expected maximum utility

Estimated coefficients of the unobserved driver characteristics variable υ_n are positive for both the lead and the lag critical gaps. Hence, consistent with the case of the target lane model, the variable can be interpreted as positively correlated with the characteristics of a timid driver who requires larger gaps for lane changing compared to more aggressive drivers.

In summary, the estimated lead and lag critical gaps are given by:

$$G_n^{lead\ TL,cr}(t) = \exp\begin{pmatrix} 1.127 - 2.178 Max\left(0, \Delta V_n^{lead,TL}(t)\right) - \\ -0.153 Min\left(0, \Delta V_n^{lead,TL}(t)\right) + 0.0045 EMU_n^{TG,TL}(t) + \\ +0.789 \upsilon_n + \varepsilon_n^{lead}(t) \end{pmatrix} \quad (6.24)$$

$$G_n^{lag\ TL,cr}(t) = \exp\begin{pmatrix} 0.968 + 0.491 Max\left(0, \Delta V_n^{lag,TL}(t)\right) + \\ +0.0152 EMU_n^{TG,TL}(t) + +0.107 \upsilon_n + \varepsilon_n^{lag}(t) \end{pmatrix} \quad (6.25)$$

Where, $EMU_n^{TG,TL}(t)$ is the target gap expected maximum utility of the target lane $(TL = RL, LL)$. $\varepsilon_n^{lead}(t)$ and $\varepsilon_n^{lag}(t)$ are random terms, $\varepsilon_n^{lead}(t) \sim N\left(0, 1.217^2\right)$ and $\varepsilon_n^{lag}(t) \sim N\left(0, 0.622^2\right)$.

6.2.3 The target gap model

Conditional on targeting a lane change and rejecting the currently available adjacent gap, the driver selects a short-term plan to perform the desired lane change. The short-term plan is defined by a choice of a target gap. The model assumes that the three gaps shown in Figure 6.5 are considered: the adjacent gap, the forward gap and the backward gap.

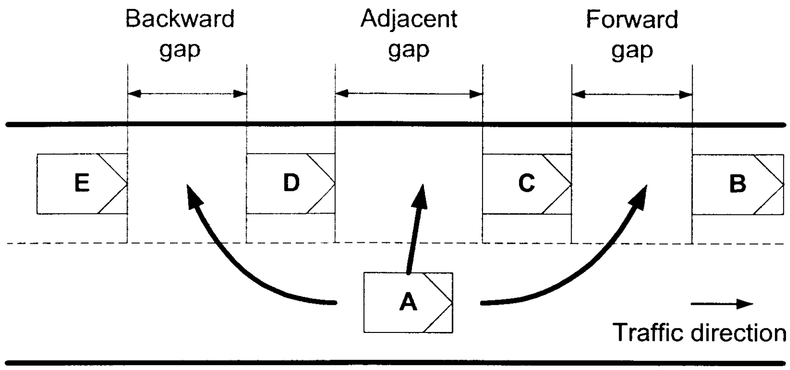


Figure 6.5 - The target gap choice set: adjacent, forward and backward gaps

The utilities of these gaps are given by:

$$U_n^{fwd}(t) = X_n^{fwd}(t)\beta^{fwd} + \alpha^{fwd}\upsilon_n + \varepsilon_n^{fwd}(t) \tag{6.26}$$

$$U_n^{adj}(t) = X_n^{adj}(t)\beta^{adj} + \alpha^{adj}\upsilon_n + \varepsilon_n^{adj}(t) \tag{6.27}$$

$$U_n^{bck}(t) = X_n^{bck}(t)\beta^{bck} + \varepsilon_n^{bck}(t) \tag{6.28}$$

Where, $X_n^{fwd}(t)$, $X_n^{adj}(t)$ and $X_n^{bck}(t)$ are vectors of explanatory variables affecting the utilities of the forward, adjacent and backward gaps, respectively. β^{fwd}, β^{adj} and β^{bck} are the corresponding vectors of parameters. $\varepsilon_n^{fwd}(t)$, $\varepsilon_n^{adj}(t)$ and $\varepsilon_n^{bck}(t)$ are i.i.d. Gumbel distributed random terms associated with the respective utilities. α^{fwd} and α^{adj} are parameters of the individual specific error term υ_n, which follows a standard normal distribution.

Estimation results for this model are presented in Table 6.3.

Table 6.3 - Estimation results for the target gap model

Variable	Parameter value	t-statistic
Forward gap constant	-0.837	-0.50
Backward gap constant	0.913	4.40
Distance to gap, m.	-2.393	-7.98
Effective gap length, m.	0.816	2.20
Front vehicle dummy	-1.662	-1.53
Relative gap speed, m/sec.	-1.218	-4.00
α^{bck}	0.239	0.81
α^{adj}	0.675	0.95

The effective gap length and the relative gap speed variables describe the size of the gap and its rate of change. The effective gap is defined by the minimum of the length of the gap in question (e.g. the forward gap B-C, shown in Figure 6.6) and the spacing between the front vehicle and the vehicle at the rear of the gap (C-D). It is mathematically defined by:

$$EG_n^{gap\ i}(t) = Min\left(\Delta X_n^{gap\ i,TL}(t), \Delta X_{n,front}^{gap\ i,TL}(t)\right) \quad i = fwd, adj, bck \tag{6.29}$$

Where, $EG_n^{gap\ i}(t)$ is the effective length of gap i. $\Delta X_n^{gap\ i,TL}(t)$ and $\Delta X_{n,front}^{gap\ i,TL}(t)$ are the length of the gap and the spacing between the vehicle at the rear of the gap and the front vehicle, respectively.

The utility of a gap increases with the effective length of the gap since the subject vehicle is more likely to be able to merge into a larger gap relative to a smaller gap.

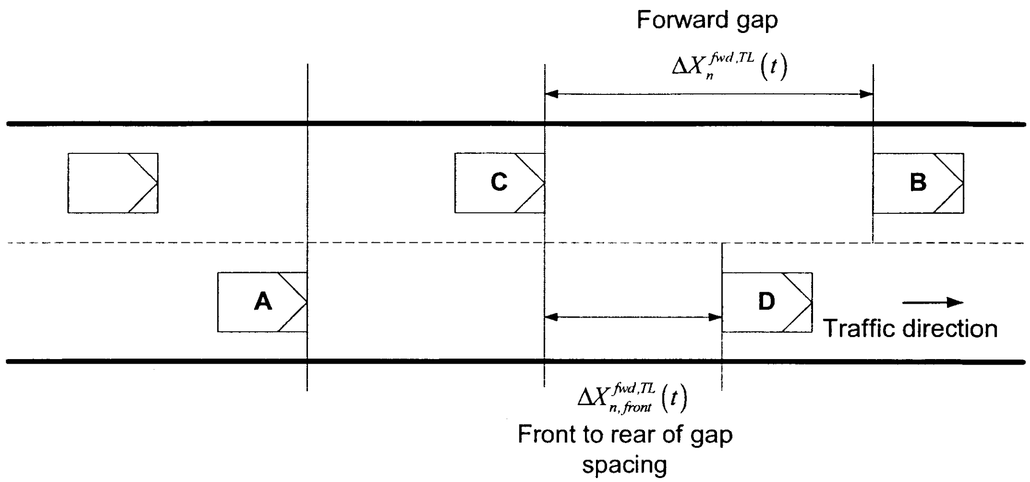


Figure 6.6 - The effective gap and relative gap speed

The relative gap speed is defined by the speed of the vehicle at the rear of the gap (C) less the speed of the vehicle that determines the front of the gap, (either B or D, B as drawn). It is an indicator to the anticipated usefulness of the gap. A positive relative gap speed value implies that the gap is getting smaller, whereas a negative value implies that it is getting larger. The relative gap speed is given by:

$$\Delta V_n^{gap\ i,TL}(t) = \begin{cases} V_n^{rear\ i,TL}(t) - V_n^{front\ i,TL}(t) & \Delta X_n^{gap\ i,TL}(t) \le \Delta X_{n,front}^{gap\ i,TL}(t) \\ V_n^{rear\ i,TL}(t) - V_n^{front}(t) & otherwise \end{cases} \qquad (6.30)$$
$$i = fwd, adj, bck$$

Where, $\Delta V_n^{gap\ i,TL}(t)$ is the relative gap speed. $V_n^{front\ i,TL}(t)$ and $V_n^{rear\ i,TL}(t)$ are the speeds of the vehicle at the front and the rear, respectively, of gap i in lane TL. $V_n^{front}(t)$ is the speed of the vehicle in front of the subject.

The estimated coefficient of this variable is negative, suggesting that drivers do try to anticipate the likely future situation when choosing their target gap.

The front vehicle dummy variables capture the effect of the presence of the front vehicle on the target gap choice. These variables are defined by:

$$\delta_{n,front}^{gap\ i,\ TL}(t) = \begin{cases} 1 & \Delta X_{n,front}^{gap\ i,\ TL}(t) \le \Delta X_n^{gap\ i,\ TL}(t) \\ 0 & otherwise \end{cases} \qquad (6.31)$$
$$i = fwd, adj; \quad TL = RL, LL$$

Where, $\delta_{n,front}^{gap\,i,\,TL}(t)$ is an indicator that the front vehicle affects the utility of gap i. The value of this variable is equal to 1 when the front vehicle is the constraining vehicle in defining the effective gap length (in Figure 6.6, this would be the case for the forward gap but not for the adjacent gap). The estimated coefficient for this variable is negative. It may indicate that drivers prefer to avoid the more complex situation that arises from having to consider their relationship with an additional vehicle.

Choice probabilities of the various gaps as a function of the gap lengths and relative gap speeds are shown in Figure 6.7. In the figure the default lengths of all gaps are 5 meters and the default relative gap speeds are 0 m/sec. The distances to the forward and backward gap are assumed to be equal.

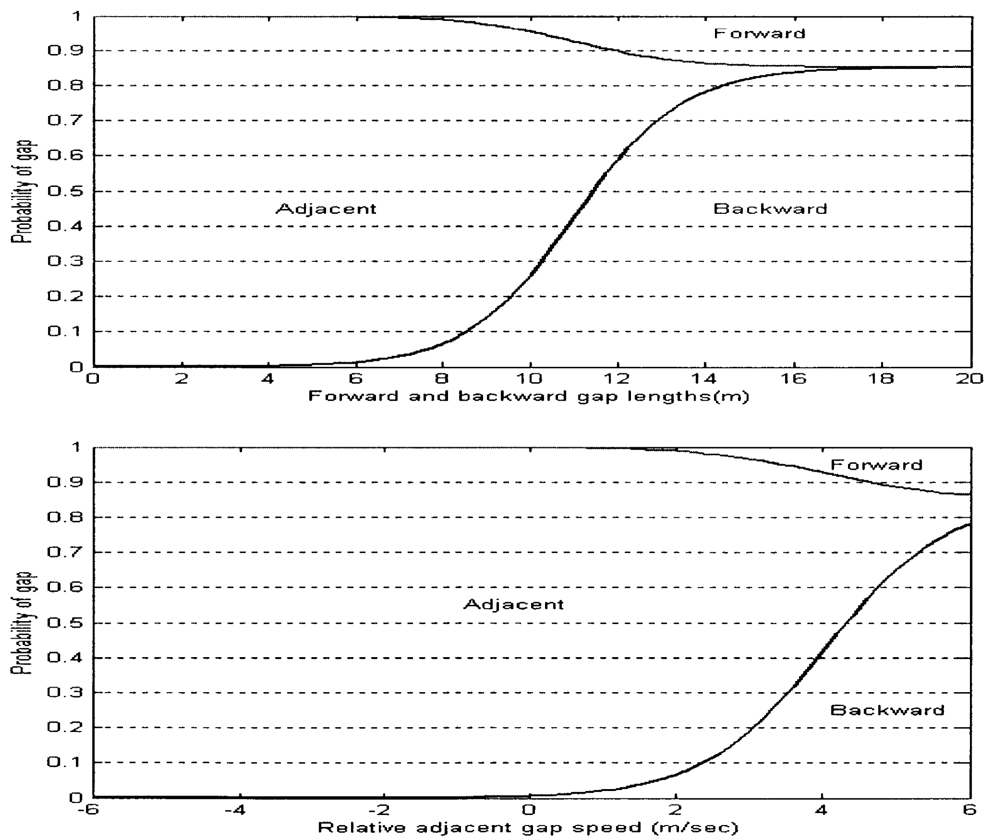


Figure 6.7 - Gap choice probabilities as a function of gap lengths, relative gap speeds

The variable *distance_to_gap* is defined by the space headway between the subject vehicle and the target gap as shown in Figure 6.8. These distances are both non-negative. The distance to the adjacent gap is by definition equal to 0. The estimated coefficient of this variable is negative. It is also important relative to other variables in the target gap utility function. A possible explanation is that as the distance to the gap increases, the short-term plan using that gap is likely to be executed over a longer period of time and therefore be more complex and involve more uncertainty with respect to the behavior of other vehicles. This also implies a strong preference for the adjacent gap over the alternative gaps. In addition, the forward gap alternative specific constant is negative and the one for the backward gap is positive. Coupled together, these results imply that, everything else being equal, drivers tend to prefer the adjacent gap and backward gaps to the forward gap. Such behavior reflects risk averse behavior.

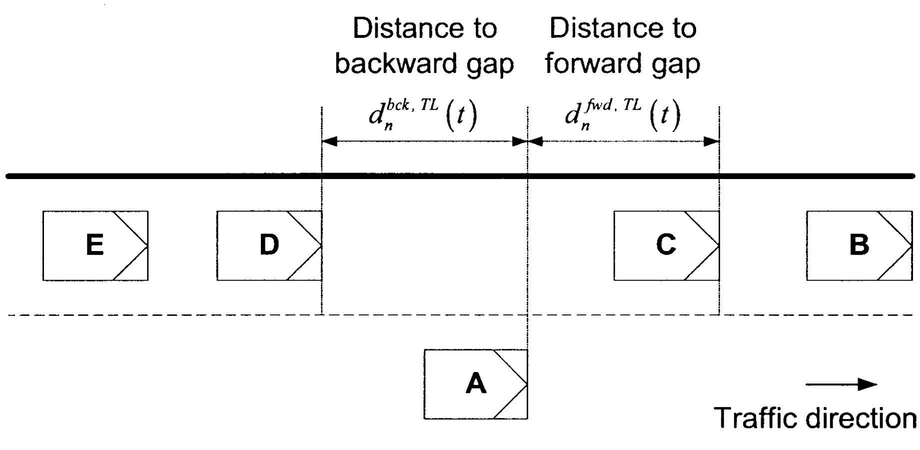


Figure 6.8 - The distance to the forward and backward gaps

Gap choice probabilities, as a function of the distances to gaps, are shown in Figure 6.9. The length of the adjacent gap is 5 meters and the forward and backward gaps are 5 and 10 meters on the top and bottom figures, respectively.

The estimated coefficients of the unobserved driver characteristics variable, υ_n, for the forward and adjacent gaps are both positive. This result is consistent with the positive correlation of υ_n with timid drivers who are more likely to choose the adjacent and backward gap over the forward gap relative to more aggressive drivers.

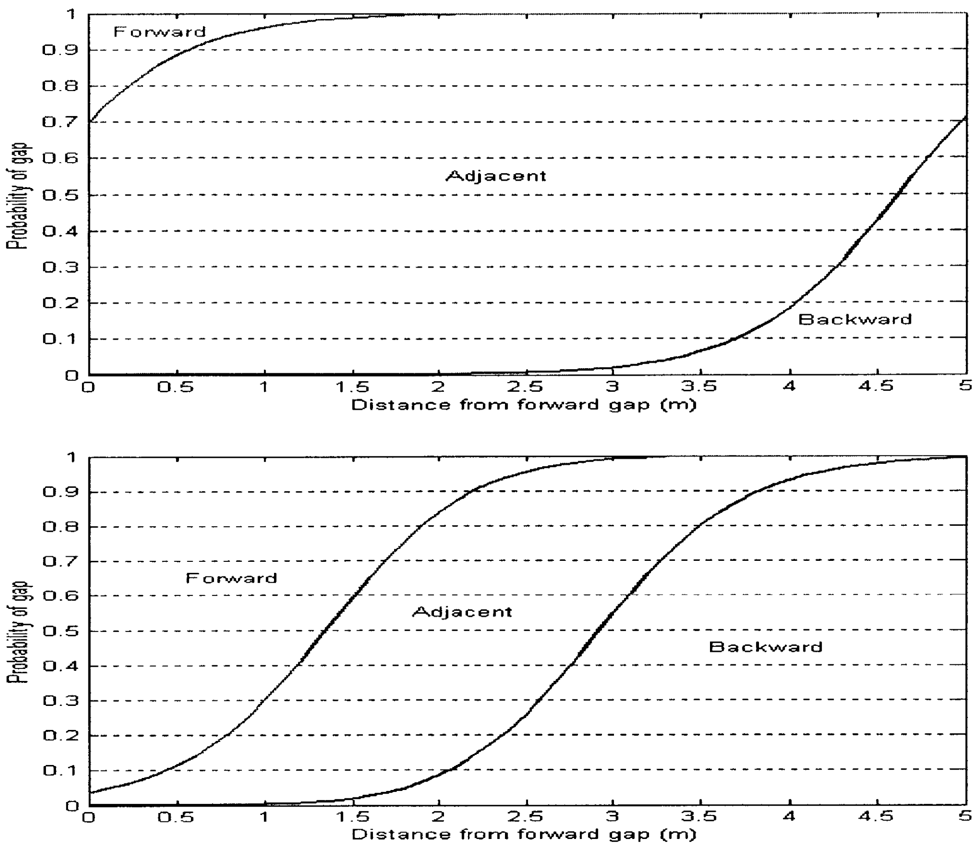


Figure 6.9 - Gap choice probabilities as a function of the distance to gaps

In summary, the target gap utilities are given by:

$$\begin{aligned} V_n^{fwd,TL}(t) = & -0.837 - 2.393 distance\_to\_gap_n^{fwd,TL}(t) + 0.816 EG_n^{fwd,TL}(t) - \\ & -1.662\delta_n^{fwd,TL}(t) - 1.218\Delta V_n^{fwd,TL}(t) \end{aligned} \tag{6.32}$$

$$V_n^{adj,TL}(t) = 0.816 EG_n^{adj,TL}(t) - 1.662\delta_n^{adj,TL}(t) - 1.218\Delta V_n^{adj,TL}(t) + 0.675 \upsilon_n \tag{6.33}$$

$$\begin{aligned} V_n^{bck,TL}(t) = & \, 0.913 - 2.393 distance\_to\_gap_n^{bck,TL}(t) + 0.816 EG_n^{bck,TL}(t) - \\ & -1.218\Delta V_n^{bck,TL}(t) + 0.239 \upsilon_n \end{aligned} \tag{6.34}$$

6.2.4 Acceleration models

Different acceleration behaviors are applied depending on the driver's short-term goal and plan: stay-in-the-lane acceleration, lane changing acceleration and target gap accelerations. Within each one of the acceleration behaviors the driver is assumed to be either in a constrained regime or in an unconstrained regime. A constrained regime applies when the driver is close to the vehicle in front and therefore affected by its behavior. The stimulus-sensitivity framework proposed within the GM model (see Section 2.1.1) is adapted for all these acceleration models. The driver reacts to different stimuli in the various situations depending on constraints imposed by the driving neighborhood and on the driver's short-term goal and short-term plan. The reaction time and time headway threshold distributions are common to all components of the acceleration model.

Stay-in-the-lane acceleration model

The car following model

The functional forms of the stimulus and sensitivity functions of the car following model are adopted from Ahmed (1999) who extended the non-linear GM model (Gazis et al 1961). The stimulus term is a non-linear function of the relative leader speed given by:

$$f^{g}\left[\Delta V_{n}(t-\tau_{n})\right]=\left|\Delta V_{n}(t-\tau_{n})\right|^{\lambda^{g}} \tag{6.35}$$

Where, $g = acc,\ dec$. λ^{g} is the corresponding parameter.

A positive correlation between the relative leader speed and the acceleration the driver applies is expected a-priori. The parameters λ^{g} are, therefore, expected to be positive for both acceleration and deceleration. The effect of the relative leader speed on the acceleration for different values of λ^{g} is illustrated in Figure 6.10. The absolute values of the acceleration and deceleration a driver may apply are bounded by the performance capabilities of the vehicle. Therefore, it is also expected that λ^{g} be less than 1 so that the absolute value of the acceleration does not tend to infinity. The linear stimulus function ($\lambda^g = 1$) corresponds to the specification of the GM model (Gazis et al 1961).

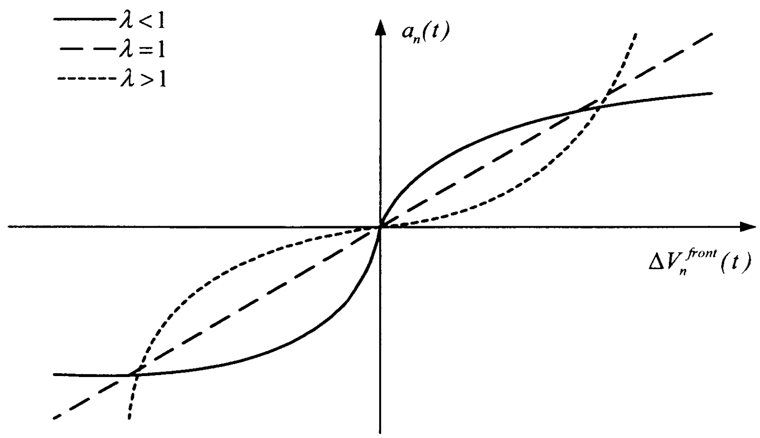


Figure 6.10 - Effect of the relative leader speed on the acceleration for different λ values

The sensitivity term is a non-linear function of explanatory variables that may include the speed of the subject vehicle, the spacing between the subject vehicle and its leader and traffic density. Similar to the case of the stimulus function, the effect of these explanatory variables on the car following acceleration and car following deceleration may be different. Following Ahmed (1999) the functional form adopted for the sensitivity term is:

$$s^g\left[X_n^g(t)\right] = \alpha^g V_n(t)^{\beta^g} \Delta X_n(t)^{\gamma^g} k_n(t)^{\rho^g} \tag{6.36}$$

Where, $g = acc,\ dec$. $V_n(t)$ and $\Delta X_n(t)$ are the subject speed and the spacing between the subject and its leader. $k_n(t)$ is the traffic density ahead of the subject vehicle within his view. α^g, β^g, γ^g and ρ^g are parameters.

The a-priori expectations with respect to the parameters of the model are the following: for the car following acceleration model the constant α^{acc} is expected to be positive. The car following acceleration is related to speed advantage. The perceived gain associated with increased speeds is higher at lower speeds. Therefore, the speed of the subject vehicle is expected to be negatively correlated with the acceleration, i.e., the driver is likely to apply a lower acceleration at higher speeds relative to lower speeds. The parameter β^{acc} is therefore expected to be negative. This effect may be offset by vehicle acceleration characteristics: vehicles are able to accelerate faster in high gear which also implies high speeds relative to low gear and low speeds. The effect of the spacing between the subject vehicle and its leader is not obvious. On one hand, a larger spacing allows the driver more room to maneuver and therefore may lead to higher accelerations. On the other hand, as the spacing increases the attention given to the leader and the perception of his behavior as a stimulus decreases, which may result in lower accelerations. The traffic density captures the effects of the broader traffic conditions on drivers' acceleration. Higher traffic densities imply more congested conditions that may have different effects on the behavior: the probability that the driver would be able to sustain significant gains in speed over-time is lower at higher traffic densities. This suggests that the acceleration a driver applies may be higher for low traffic densities relative to high traffic densities. However, higher traffic densities may also cause drivers to be more attentive to their environment and result in reduced variability in speeds. Thus, causing the sensitivity term to increase. Overall the acceleration applied is expected to increase with traffic density and the parameter ρ^{acc} is expected to be positive.

The car following deceleration constant α^{dec} is expected to be negative. This behavior is mainly governed by safety considerations. Therefore, the deceleration at higher speeds is expected to be larger relative to lower speeds. It is also expected to be higher for small spacing relative to large spacing. The parameters β^{dec} and γ^{dec} are therefore expected to be positive and negative, respectively. The effect of the traffic density variable on car following deceleration is similar to its effect on car following acceleration and the parameter ρ^{dec} is also expected to be positive.

The estimated car following acceleration model is given by:

$$a_n^{cf,acc}(t) = 0.0355 V_n(t)^{0.291} \Delta X_n(t)^{-0.166} k_n(t)^{0.550} \Delta V_n(t-\tau_n)^{0.520} + \varepsilon_n^{cf,acc}(t) \qquad (6.37)$$

Where, $\varepsilon_n^{cf,acc}(t)$ is a normally distributed error term, $\varepsilon_n^{cf,acc}(t) \sim N\left(0, 1.134^2\right)$.

The estimated car following deceleration model is given by:

$$a_n^{cf,dec}(t) = -0.860\Delta X_n(t)^{-0.565} k_n(t)^{0.143} \Delta V_n(t-\tau_n)^{0.834} + \varepsilon_n^{cf,dec}(t) \qquad (6.38)$$

Where, $\varepsilon_n^{cf,dec}(t) \sim N(0, 1.169^2)$.

Estimated car following parameters, with the exception of the acceleration constant and space headway coefficient, have significant t-statistics at the 5% level of significance.

The effects of different variables on the mean car following acceleration and deceleration are shown in Figure 6.11 and Figure 6.12, respectively. In these figures the following default values are assumed: the subject speed is 15 m/sec., space headway is 25 meters, density is 30 veh/km/lane and the relative leader speed is 3 (or -3) m/sec.

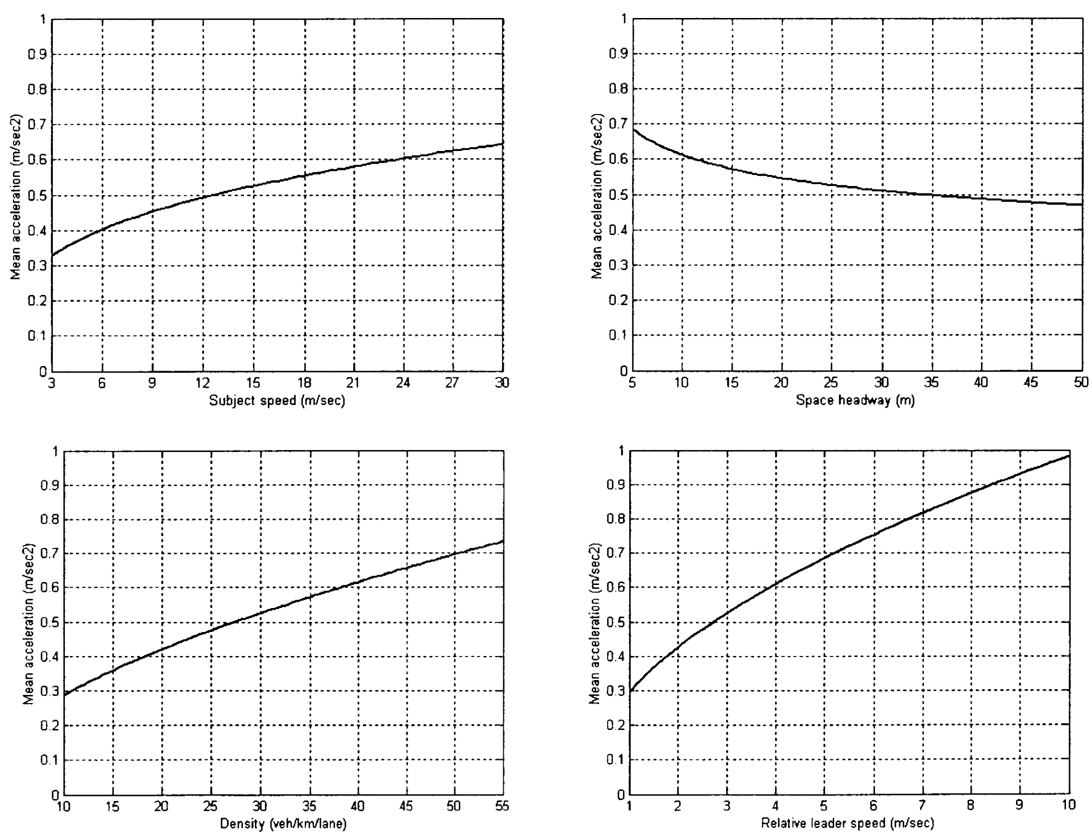


Figure 6.11 - Effects of different variables on the mean car following acceleration

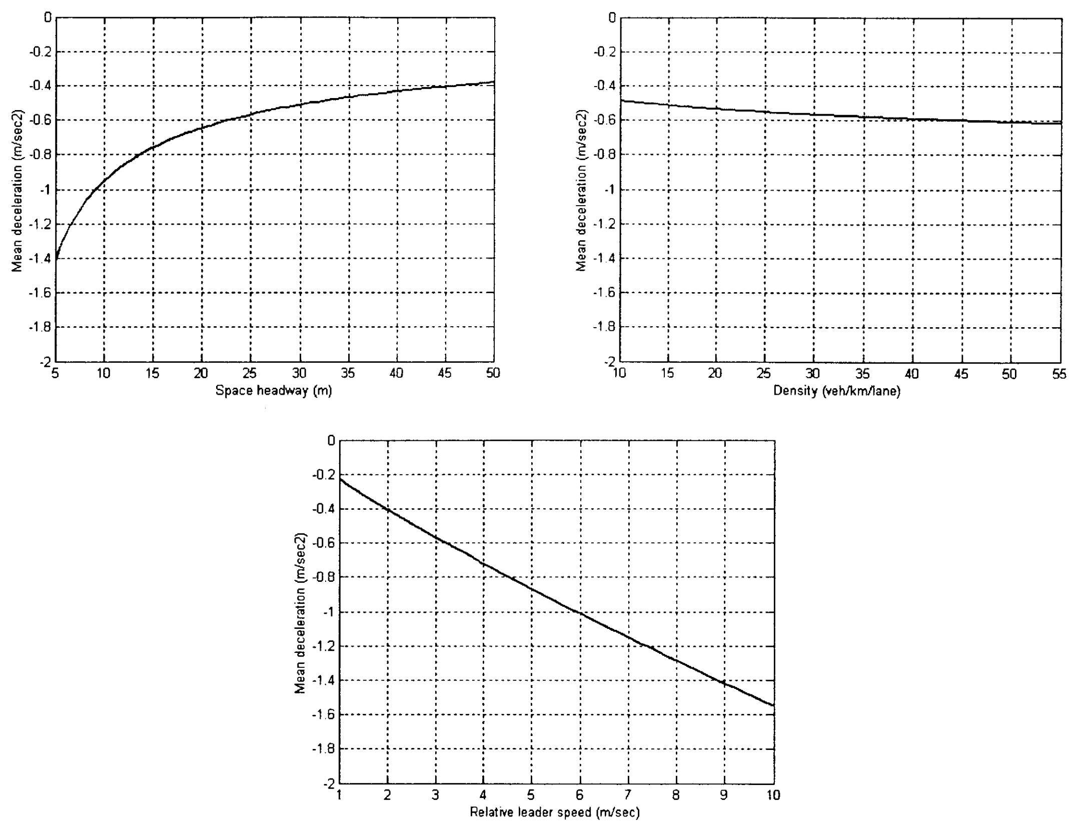


Figure 6.12 - Effects of different variables on the mean car following deceleration

Car following acceleration increases with the subject speed, the density and relative leader speed and decreases with the headway spacing. Car following deceleration increases (in absolute value) with the density and relative leader speed and decreases with the space headway.

The stimulus term in the car following regime is a function of the relative leader speed. As expected, the parameter associated with this term is positive for both acceleration and deceleration, which implies a positive correlation between the relative leader speed and the acceleration the subject applies.

The sensitivity terms are positive and negative for car following acceleration and car following deceleration, respectively. However, the magnitude of sensitivity to a negative relative leader speed is much larger than the sensitivity to a positive one. This is expected since a negative relative speed stimulus may have safety implications whereas a positive relative leader speed stimulus only suggests a possible speed advantage to the driver.

The estimated coefficients of the space headways are negative for both acceleration and deceleration car following. For deceleration car following this is expected since the underlying safety concern increases when the spacing is reduced. In the case of acceleration car following, it may be related to a reduced perception of the leader as a stimulus the driver needs to react to. Similar to the sensitivity constants, the magnitude of the coefficient for deceleration is larger than that for acceleration.

The estimated coefficient of the subject's speed in the acceleration model is positive, which is contrary to what was expected. This coefficient is strongly significant (t-statistic 5.63). This suggests that drivers apply higher accelerations at high speeds and high densities relative to lower speeds and densities. A possible explanation may be related to the acceleration capabilities of vehicles, which are higher at high speeds (and gear) relative to low speeds. Mean accelerations applied at high densities are higher relative to lower densities for both car following acceleration and car following deceleration. This may be explained by drivers' being more attentive to their environment when traveling in dense traffic.

While the specification of the car following model follows Ahmed (1999), the parameter estimates obtained in this study differ significantly from the ones he reported, especially for the car following deceleration model (see Table 2.4 in Section 2.1.2). Mean accelerations predicted by the two models are shown in Figure 6.13. These differences may in part be explained by the introduction of additional types of acceleration models in this study, which were not accounted for in his model. For example, a vehicle planning to change lanes using the backward gap may apply a different acceleration than the one predicted by a stay-in-the-lane model. Failing to model this behavior may lead to biased estimation of car following behaviors. The differences in model estimates between the two studies also suggest that further work is required to identify additional factors affecting the behavior such as the type of road facility (e.g. freeways, urban streets, tunnels, bridges) and geometric characteristics (e.g. curvature, slope, visibility).

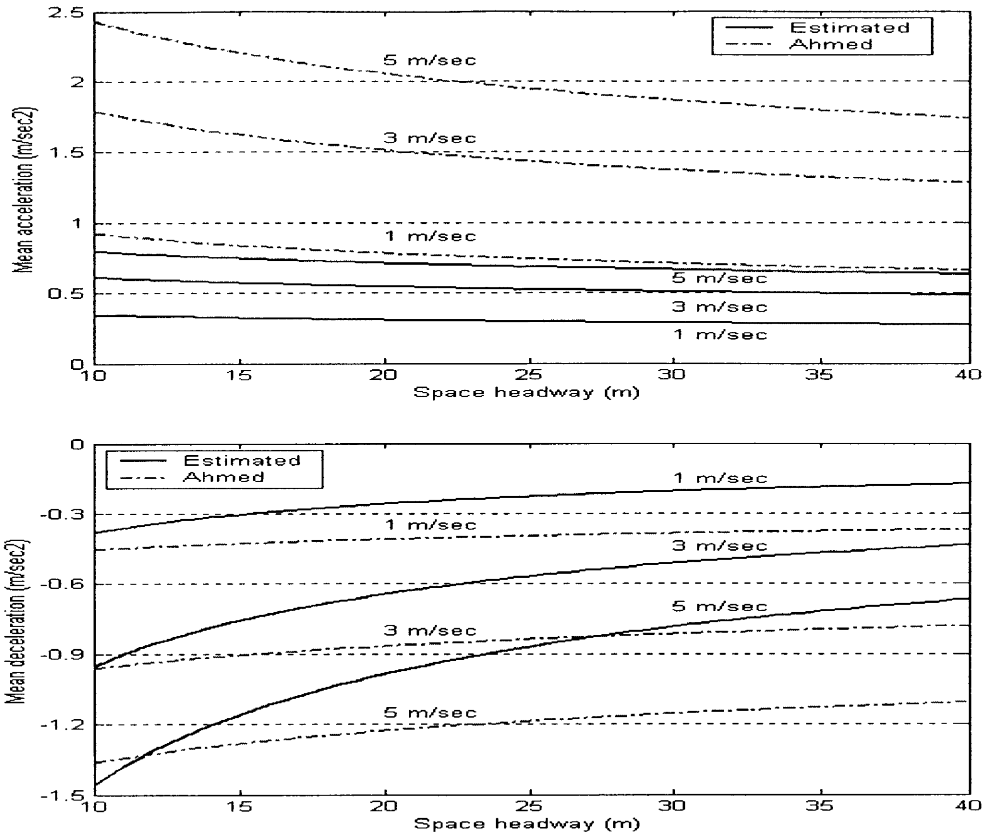


Figure 6.13 - Comparison of accelerations predicted by the car following models estimated in this study and by Ahmed (1999)

Ahmed's specification includes the following enhancements to the non-linear GM model (Gazis et al 1961):

1. A non-linear specification of the stimulus term.
2. Introduction of the density variable into the model.

Despite the differences in parameter estimates, our results support both these enhancements and strengthen the conclusion that this specification is superior to the GM model: The coefficients of the stimulus terms (relative leader speed) are significantly smaller than one (which corresponds to a linear specification) for both acceleration and deceleration. t-statistic values for a test against a unit value of the coefficients are -7.36 and -2.53, respectively, thus rejecting the linear model at the 5% significance level in both cases. Similarly the density variables are significant in both acceleration and deceleration models. Moreover, the variable headway spacing was rejected in both studies as having counter-intuitive and insignificant effect on the car following deceleration model.

The free-flow model

The stimulus in this case is the difference between the desired speed and the actual speed. A constant sensitivity term is assumed, and therefore the acceleration the driver applies is given by:

$$a_n^{ff}(t) = \lambda^{ff}\left[V_n^{DS}(t-\tau_n) - V_n(t-\tau_n)\right] + \varepsilon_n^{ff}(t) \tag{6.39}$$

Where, $a_n^{ff}(t)$ is the free-flow acceleration driver n applies at time t. λ^{ff} is a constant sensitivity term. $V_n^{DS}(t-\tau_n)$ is the subject's desired speed. $\varepsilon_n^{ff}(t)$ is the random term associated with the free-flow acceleration.

The driver is expected to accelerate if the actual speed is lower than the desired speed and to decelerate if it is higher. The parameter λ^{ff} is therefore expected to be positive.

The estimated free-flow acceleration model is given by:

$$a_n^{ff}(t) = 0.0881\left[V_n^{DS}(t-\tau_n) - V_n(t-\tau_n)\right] + \varepsilon_n^{ff}(t) \tag{6.40}$$

Where, $\varepsilon_n^{ff}(t) \sim N\left(0, 1.184^2\right)$. The subject's desired speed is given by:

$$V_n^{DS}(t-\tau_n) = 17.636 - 1.458\delta_n^{heavy} - 0.105\upsilon_n \tag{6.41}$$

Where, δ_n^{heavy} is a heavy vehicle dummy variable defined by:

$$\delta_n^{heavy} = \begin{cases} 1 & \textit{subject is heavy} \\ 0 & \textit{otherwise} \end{cases} \tag{6.42}$$

The desired speed is a function of attributes of the road facility, such as the curvature, grade, surface conditions and posted speed and of characteristics of the driver and the vehicle. The estimation data represents conditions on a single facility over a short period of time. Therefore, the effects of changing conditions on the desired speed cannot be identified. The heavy vehicle dummy variable captures the impact of the limited speed capabilities of these vehicles. The desired speed of heavy vehicles is lower by 1.458 m/sec (5.2 km/h) relative to other vehicles. The effect of the unobserved driver characteristics variable υ_n on the desired speed is negative. This is consistent with the positive correlation between this variable and the driver's timidity.

The mean accelerations predicted by the free-flow model as well as by the model estimated by Ahmed (1999) are shown in Figure 6.14. Ahmed uses the leader speed as an explanatory variable in the desired speed model. In the figure, it is assumed that the leader speed is equal to the subject's speed. The speed at which the mean acceleration is equal to 0 (i.e. the subject speed and the desired speed are equal) is 17.636 m/sec (63.5 km/h) in our model, but only 8.586 m/sec (30.9 km/h) in Ahmed's model.

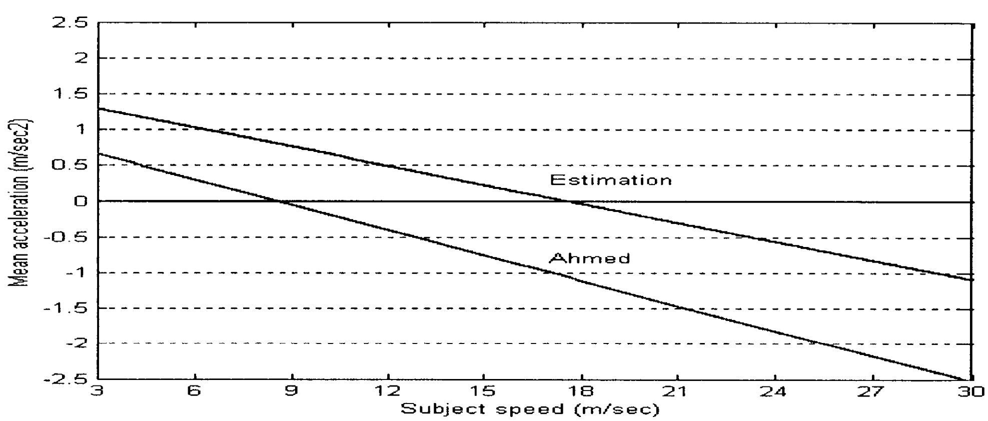


Figure 6.14 - Accelerations predicted by the free-flow model and by Ahmed (1999)

Estimation results for the stay-in-the-lane acceleration model are summarized in Table 6.4.

Table 6.4 - Estimation results for the stay-in-the-lane acceleration model

Variable	Parameter value	t-statistic
Car following acceleration		
Constant	0.0355	1.21
Speed, m/sec.	0.291	5.64
Space headway, m.	-0.166	-1.68
Density, veh/km/lane	0.550	2.50
Relative speed, m/sec.	0.520	7.97
$\ln\left(\sigma_{cf,\ acc}\right)$	0.126	12.05
Car following deceleration		
Constant	-0.860	-3.92
Space headway, m.	-0.565	-9.51
Density, veh/km/lane	0.143	2.04
Relative speed, m/sec.	0.834	12.68
$\ln\left(\sigma_{cf,\ dec}\right)$	0.156	14.87
Free-flow acceleration		
Sensitivity constant	0.0881	11.20
$\ln\left(\sigma_{ff}\right)$	0.169	10.36
Desired speed		
Constant	17.636	61.26
heavy vehicle dummy	-1.458	-1.12
α^{DS}	-0.105	-0.40

Lane changing acceleration model

The lane changing acceleration model captures the behavior of drivers during the time a lane change is performed. The model assumes that the behavior is similar to stay-in-the-lane acceleration behavior, but with respect to the leader in the lane the subject is changing to. The constrained (car following) model is given by:

$$a_n^{lc,cf,g}(t) = \alpha^g V_n(t)^{\beta^g} \Delta X_n^{lead,TL}(t)^{\gamma^g} k_n(t)^{\rho^g} \left|\Delta V_n^{lead,TL}(t-\tau_n)\right|^{\lambda^g} + \varepsilon_n^{cf,g}(t) \qquad (6.43)$$

Where, $g \in \{acc, dec\}$. α^g, β^g, γ^g, ρ^g and λ^g are parameters. $\varepsilon_n^{cf,g}(t)$ are the random terms associated with the car following behavior of driver *n* at time *t*.

The free-flow acceleration behavior does not depend on the identity of the leader (whether it is in the current lane or a different lane). Therefore the lane changing free-flow model is identical to the stay-in-the-lane free-flow acceleration model given by Equations (6.40) and (6.41).

The estimation data ignores the duration of lane changing maneuvers. Instead, a lane change is reported only at the end of the time period in which it was completed. In contrast, the acceleration data is instantaneous. Hence, the available information is not sufficient to study the lane changing acceleration behavior in detail. Therefore, the model assumes that parameters of the lane changing acceleration model are the same as those of the stay-in-the-lane model. This restriction only applies to the car following regime since the free-flow regime is lane-independent and does not depend on the relations with any vehicles.

The estimated lane changing car following acceleration model is given by:

$$a_n^{lc,cf,acc}(t) = 0.0355 V_n(t)^{0.291} \Delta X_n^{lead,TL}(t)^{-0.166} k_n(t)^{0.550} \Delta V_n^{lead,TL}(t-\tau_n)^{0.520} + \varepsilon_n^{cf,acc}(t) \qquad (6.44)$$

Where, $\Delta X_n^{lead,TL}(t)$ and $\Delta V_n^{lead,TL}(t-\tau_n)$ are the headway spacing and relative speed with respect to the leader in the target lane. $\varepsilon_n^{cf,acc}(t)$ is the car following acceleration error term, $\varepsilon_n^{cf,acc}(t) \sim N(0, 1.134^2)$.

The estimated lane changing car following deceleration model is given by:

$$a_n^{lc,cf,dec}(t) = -0.860 \Delta X_n^{lead,TL}(t)^{-0.565} k_n(t)^{0.143} \Delta V_n^{lead,TL}(t-\tau_n)^{0.834} + \varepsilon_n^{cf,dec}(t) \qquad (6.45)$$

Where, $\varepsilon_n^{cf,dec}(t) \sim N(0, 1.169^2)$.

Target gap acceleration model

The target gap acceleration model captures the behavior of drivers who target a lane change and follow a short-term plan to accomplish it. Drivers are assumed to modify their acceleration behavior to facilitate completing their short-term plan. Acceleration models for the three alternative target gaps, the forward, backward and adjacent gaps, are estimated.

The forward gap acceleration model

This model describes the behavior of drivers whose short-term plan is to use the forward gap to change to the target lane. The situation in which this behavior applies is shown in Figure 6.15. The subject vehicle (vehicle A) is committed to changing to the left lane using the forward gap, the gap between vehicles B and C. The acceleration this vehicle applies depends on its relations with the vehicle in front of it in the current lane (vehicle D) and the vehicles defining the forward gap in the target lane (vehicles B and C).

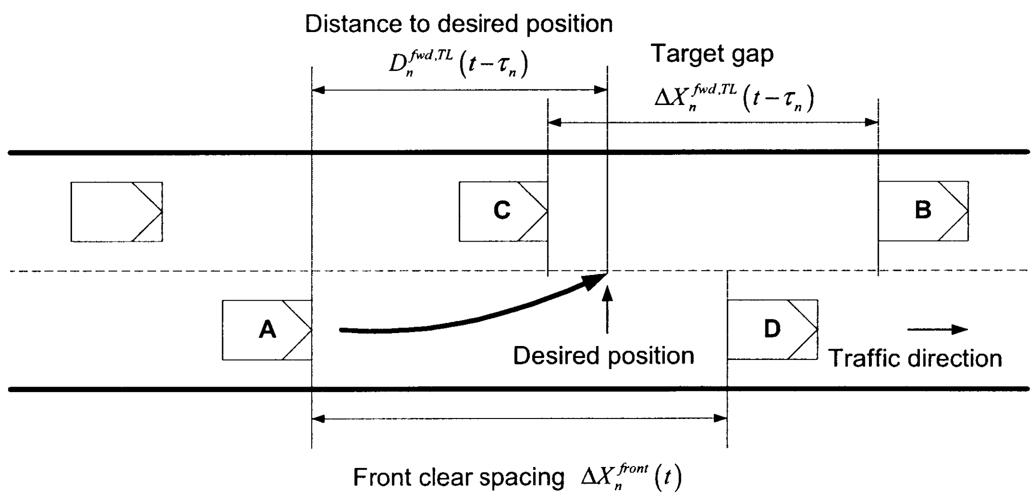


Figure 6.15 - Forward gap acceleration situation and variables

The driver may be in a constrained regime, in which it is affected by the vehicle in front, or in an unconstrained regime. In the constrained regime the driver is assumed to focus on the stimulus created by its leader and apply car following behavior. In the unconstrained regime, rather than applying a free-flow acceleration, the driver chooses an acceleration that enables accomplishing the short-term plan. The stimulus the driver reacts to in the unconstrained regime is the difference between the desired position with respect to the forward gap and the vehicle's current position (see Figure 6.15). A non-linear specification is used for the stimulus term:

$$f^{fwd}\left[D_n^{fwd,TL}\left(t-\tau_n\right)\right]=\left(D_n^{fwd,TL}\left(t-\tau_n\right)\right)^{\gamma^{fwd}} \tag{6.46}$$

Where, γ^{fwd} is a constant parameter. $D_n^{fwd,TL}\left(t-\tau_n\right)$ is the desired position with respect to the forward gap. It is determined as a fraction of the total length of the gap (measured from the front of the gap lag vehicle) and is given by:

$$D_n^{fwd,TL}\left(t-\tau_n\right)=\Delta X_n^{lead,TL}\left(t-\tau_n\right)+l_n^{lead,TL}+\beta^{DP}\Delta X_n^{fwd,TL}\left(t-\tau_n\right) \tag{6.47}$$

Where, $\Delta X_n^{lead,TL}\left(t-\tau_n\right)$ and $l_n^{lead,TL}$ are the target lane leader space headway and length, respectively. $\Delta X_n^{fwd,TL}\left(t-\tau_n\right)$ is the length of the forward gap. The parameter β^{DP} captures the desired relative position of the vehicle with respect to the forward gap. It is expressed as a fraction of the gap and therefore is expected to be positive and constrained $0<\beta^{DP}<1$.

The forward gap sensitivity term is a non-linear function of the subject speed and the target lane leader relative speed. The functional form used for the sensitivity is:

$$s^{fwd}\left[X_n^{fwd}\left(t\right)\right]=\alpha^{fwd}V_n\left(t\right)^{\beta^{fwd}}\ exp\left[\lambda_+^{fwd}\Delta V_n^{lead,TL}\left(t\right)_+\right]exp\left[\lambda_-^{fwd}\Delta V_n^{lead,TL}\left(t\right)_-\right] \tag{6.48}$$

Where, α^{fwd}, β^{fwd}, λ_+^{fwd} and λ_-^{fwd} are parameters. $\Delta V_n^{lead,TL}\left(t\right)_+$ and $\Delta V_n^{lead,TL}\left(t\right)_-$ are the positive and negative relative target lane leader speeds, respectively. These are defined as $\Delta V_n^{lead,TL}\left(t\right)_+=max\left(0,\Delta V_n^{lead,TL}\left(t\right)\right)$ and $\Delta V_n^{lead,TL}\left(t\right)_-=min\left(0,\Delta V_n^{lead,TL}\left(t\right)\right)$.

This formulation allows the sensitivity of the acceleration to the relative speed of the leader in lane for the situation in which the leader is faster than the subject to be different than the sensitivity in the situation in which it is slower than the subject. The exponential form guarantees continuity of the acceleration when the relative target leader speed approaches zero.

The forward gap acceleration constant α^{fwd} is expected to be positive. The driver is likely to accelerate in order to overtake the target lane leader and be adjacent to the forward gap. The effect of the subject speed is expected to be negative. The driver is likely to apply a lower acceleration at higher speeds relative to lower speeds, considering that it is generally easier change lanes at lower speeds and accounting for the effect of the desired speed. The parameter β^{fwd} is therefore expected to be negative. The effect of the space headway between the subject vehicle and the target lane leader is expected to be positive. The driver is committed to using the forward gap and therefore has to cover the space headway. This implies that in order to complete the maneuver quickly the acceleration applied is likely to be higher for long space headways relative to shorter ones. The relative target lane leader speed is expected to be positively correlated with the acceleration. The driver is likely to accelerate more aggressively when the relative target lane leader speed is increasingly positive (i.e. the target lane leader is faster) in order to make the short-term plan feasible. Similarly, the acceleration required by the driver diminishes as the relative target lane leader speed is increasingly negative (i.e. the subject vehicle is faster). The parameters λ_{+}^{fwd} and λ_{-}^{fwd} are, therefore, both expected to be positive.

The estimated forward gap acceleration model is given by:

$$a_n^{uc,fwd,TL}(t) = 0.385 D_n^{fwd,TL}\left(t-\tau_n\right)^{0.323} \exp\left(0.0678 \Delta V_n^{lead,TL}(t)_{+}\right) \cdot \\ \cdot \exp\left(0.217 \Delta V_n^{lead,TL}(t)_{-}\right) + \varepsilon_n^{fwd}(t) \tag{6.49}$$

Where, $\varepsilon_n^{fwd}(t)$ is the forward gap acceleration error term, $\varepsilon_n^{fwd}(t) \sim N\left(0, 0.583^2\right)$. $D_n^{fwd,TL}\left(t-\tau_n\right)$ is the distance to the forward gap desired position, given by:

$$D_n^{fwd,TL}\left(t-\tau_n\right) = \Delta X_n^{lead,TL}\left(t-\tau_n\right) + l_n^{lead,TL} + 0.604 \Delta X_n^{fwd,TL}\left(t-\tau_n\right) \tag{6.50}$$

The mean forward gap acceleration is positive and increases with the distance to the desired position and with the target lane leader relative speed. The sensitivity of the forward acceleration to these variables is shown in Figure 6.16, which makes the default assumptions that the speeds of the subject and lead vehicles are equal and that the distance to the desired position is 10 meters.

Positive correlation between the distance to the desired position and the forward acceleration implies that drivers try to keep the duration of time to complete their short-term plan short. Thus, applying a larger acceleration when the distance they need to cover is longer.

The forward acceleration increases with the target lane leader relative speed. This was expected since a driver who targets the forward gap must overtake the target lane leader to be able to merge into the forward gap. Therefore, the driver needs to accelerate more aggressively in the case that the target lane leader is faster (positive relative speed) compared to the case it is slower.

In Figure 6.17, the acceleration predicted by the forward gap acceleration model for different relative leader speeds is compared to the acceleration predicted by the free-flow model, which would have been used if the short-term plan was not modeled. The figure shows that the desire to change lanes and the short-term plan may cause drivers to apply larger accelerations than they would otherwise.

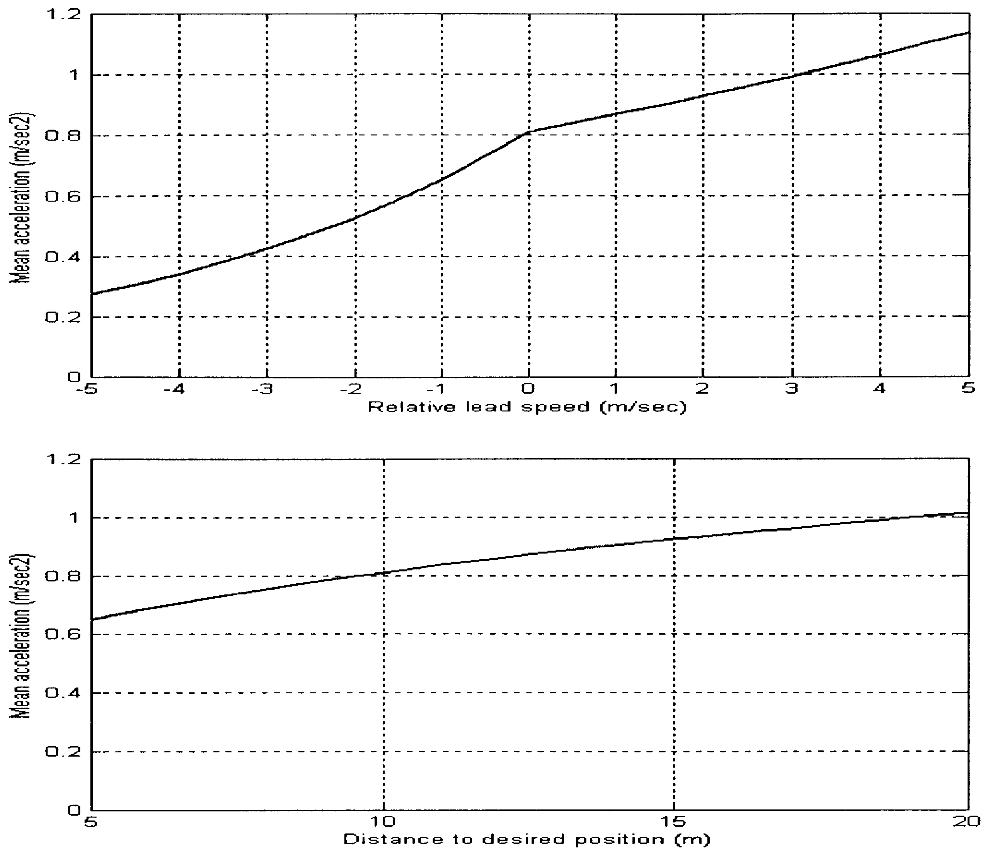


Figure 6.16 - Effects of relative lead speed and distance to desired position on the forward gap acceleration

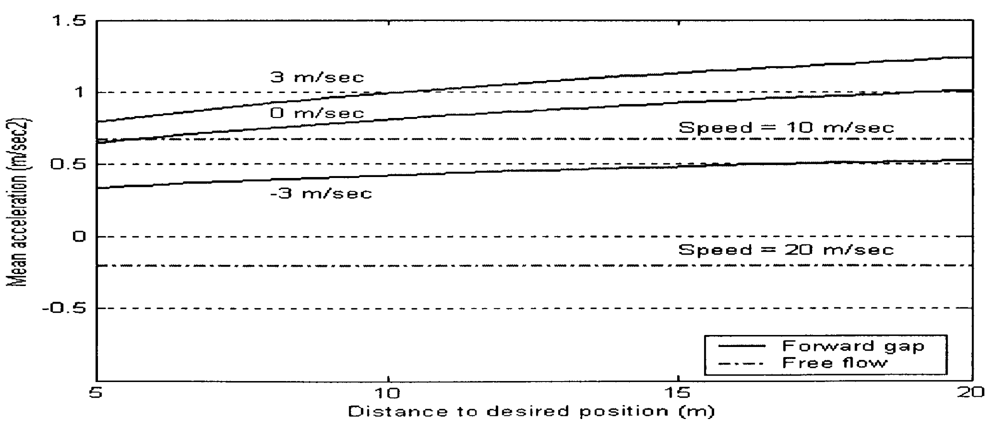


Figure 6.17 - Comparison of accelerations predicted by the forward gap acceleration model and the free-flow acceleration model

The backward gap acceleration model

This model is similar to the forward gap acceleration model. The short-term plan in this case is to use the backward gap to change to the target lane. This situation is shown in Figure 6.18. The subject vehicle (vehicle A) is committed to changing lanes to the left lane using the backward gap, the gap between vehicles B and C. The acceleration this vehicle applies depends on its relations with the current lead vehicle (vehicle D) and the vehicles defining the backward gap in the target lane (vehicles B and C).

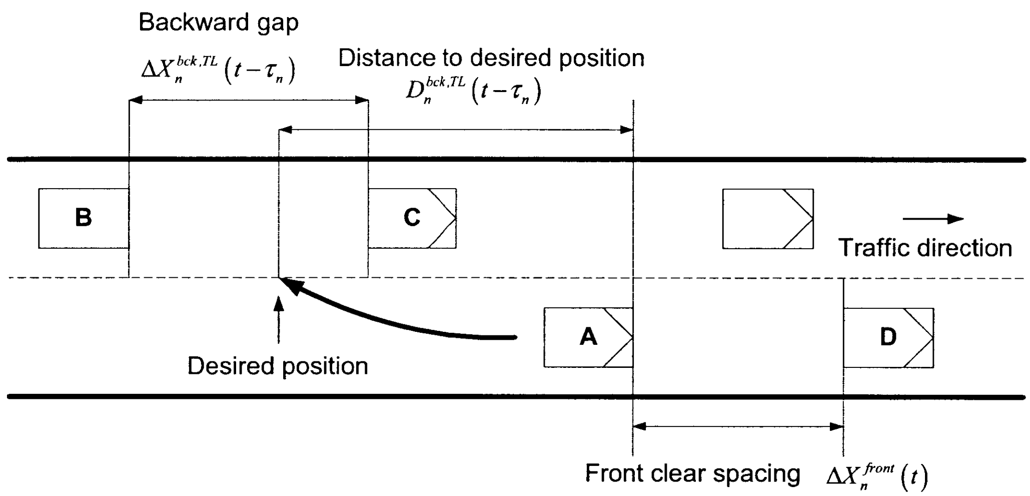


Figure 6.18 - Backward gap acceleration situation and variables

Similar to the forward gap acceleration model, a driver in the constrained regime is assumed to apply car following behavior. Unlike the forward gap model the car following acceleration regime is considered unconstrained since a driver targeting the backward gap is expected to decelerate. In the unconstrained regime the driver chooses an acceleration that facilitates accomplishing the short-term plan. The driving regimes are determined by the leader time headway and the time headway threshold. The backward gap acceleration model is expressed by:

$$a_n^{bck,TL}(t)=\begin{cases} a_n^{cf,dec}(t) & \text{if } h_n(t-\tau_n)\le h_n^* \text{ and } \Delta V_n(t-\tau_n)<0 \\ a_n^{uc,bck,TL}(t) & \text{otherwise} \end{cases} \tag{6.51}$$

Where, $a_n^{bck,TL}(t)$ is the acceleration vehicle n applies if it committed, at time t, to changing to the target lane TL, using the backward gap. $a_n^{cf,dec}(t)$ is the car following deceleration model. $a_n^{uc,bck,TL}(t)$ is the unconstrained backward gap acceleration.

The stimulus the driver reacts to in the unconstrained regime is the difference between the current position and the desired position relative to the backward gap (see Figure 6.18). The sensitivity and stimulus functions, respectively, are expressed by:

$$s^{bck}\left[X_n^{bck,TL}(t)\right] = \alpha^{bck} V_n(t)^{\beta^{bck}} \exp\left(\lambda_+^{bck} \Delta V_n^{lag,TL}(t)_+\right) \exp\left(\lambda_-^{bck} \Delta V_n^{lag,TL}(t)_-\right) \qquad (6.52)$$

$$f^{bck}\left[D_n^{bck,TL}(t-\tau_n)\right] = \left(D_n^{bck,TL}(t-\tau_n)\right)^{\gamma^{bck}} \qquad (6.53)$$

Where, α^{bck}, β^{bck}, λ_+^{bck}, λ_-^{bck} and γ^{bck} are parameters. $\Delta V_n^{lag,TL}(t)_+$ and $\Delta V_n^{lag,TL}(t)_-$ are the positive and negative relative target lane lag speeds, respectively. These are defined as $\Delta V_n^{lag,TL}(t)_+ = max\left(0, \Delta V_n^{lag,TL}(t)\right)$ and $\Delta V_n^{lag,TL}(t)_- = min\left(0, \Delta V_n^{lag,TL}(t)\right)$.

The desired position relative to the backward gap is defined as a fraction of the total length of the gap (measured from the gap lag vehicle). The distance to the desired position is given by:

$$D_n^{bck,TL}(t-\tau_n) = \Delta X_n^{lag,TL}(t-\tau_n) + l_n + l_n^{lag,TL} + \left(1-\beta^{DP}\right)\Delta X_n^{bck,TL}(t-\tau_n) \qquad (6.54)$$

Where, $\Delta X_n^{lag,TL}(t-\tau_n)$ is the target lane leader space headway. l_n and $l_n^{lag,TL}$ are the length of the subject vehicles and the target lane lag, respectively. $\Delta X_n^{bck,TL}(t-\tau_n)$ is the length of the backward gap. β^{DP} is a parameter capturing the desired position relative to the gap.

This formulation, similar to the forward gap acceleration, allows the sensitivity of the acceleration to the relative target lane lag speed to be different depending on whether the lag is faster or slower than the subject.

The backward gap acceleration constant α^{bck} is expected to be negative. The driver is likely to decelerate in order to let the target lane lag vehicle pass it, so that the subject is adjacent to the backward gap. The effect of the speed of the subject vehicle is expected to be positive. The driver is likely to apply a higher deceleration at higher speeds relative to lower speeds, in order to allow the lag vehicle to pass it quickly. Moreover, everything else being equal it is easier for the subject vehicle to perform a lane change at lower speeds relative to higher speeds. The parameter β^{bck} is therefore expected to be positive. The effect of the space headway between the subject vehicle and the target lane lag is not obvious. On one hand, the driver is committed to using the backward gap and therefore has to cover the space headway. This implies that in order to complete the maneuver quickly the absolute value of the deceleration applied is likely to be higher for long space headways relative to shorter ones. On the other hand, when the space headway is relatively high the driver may apply a lower deceleration, knowing that the maneuver will take some time and in order to maintain some speed.

A negative correlation is expected between the relative target lane lag speed and the acceleration. The driver is likely to accelerate more aggressively when the relative target lane leader speed is increasingly negative (i.e. the target lane lag is slower) in order to make the short-term plan feasible. Similarly, the acceleration required by the driver diminishes as the relative target lane leader speed is increasingly positive (i.e. the subject vehicle is slower). The parameters λ_{+}^{bck} and λ_{-}^{bck} are, therefore, both expected to be negative.

The estimated backward gap acceleration model is given by:

$$\begin{aligned} a_n^{uc,bck,TL}(t) = -0.596 D_n^{bck,TL}\left(t-\tau_n\right)^{-0.219} \exp\left(-0.0832 \Delta V_n^{lag,TL}(t)_{+}\right) \cdot \\ \cdot \exp\left(-0.170 \Delta V_n^{lag,TL}(t)_{-}\right) + \varepsilon_n^{bck}(t) \end{aligned} \qquad (6.55)$$

Where, $\varepsilon_n^{bck}(t)$ is the backward gap acceleration error term, $\varepsilon_n^{bck}(t) \sim N\left(0, 1.478^2\right)$. $D_n^{bck,TL}\left(t-\tau_n\right)$ is the distance to the backward gap desired position, given by:

$$D_n^{bck,TL}\left(t-\tau_n\right)=\Delta X_n^{lag,TL}\left(t-\tau_n\right)+l_n+l_n^{lag,TL}+0.396\Delta X_n^{bck,TL}\left(t-\tau_n\right) \tag{6.56}$$

The mean backward gap acceleration is negative and decreases (in absolute value) with the distance to the desired position and with the relative target lane lag speed. The sensitivity of the backward acceleration to these variables is shown in Figure 6.19, which has the default assumptions that the speeds of the subject and lag vehicles are equal and that the distance to the desired position is 10 meters.

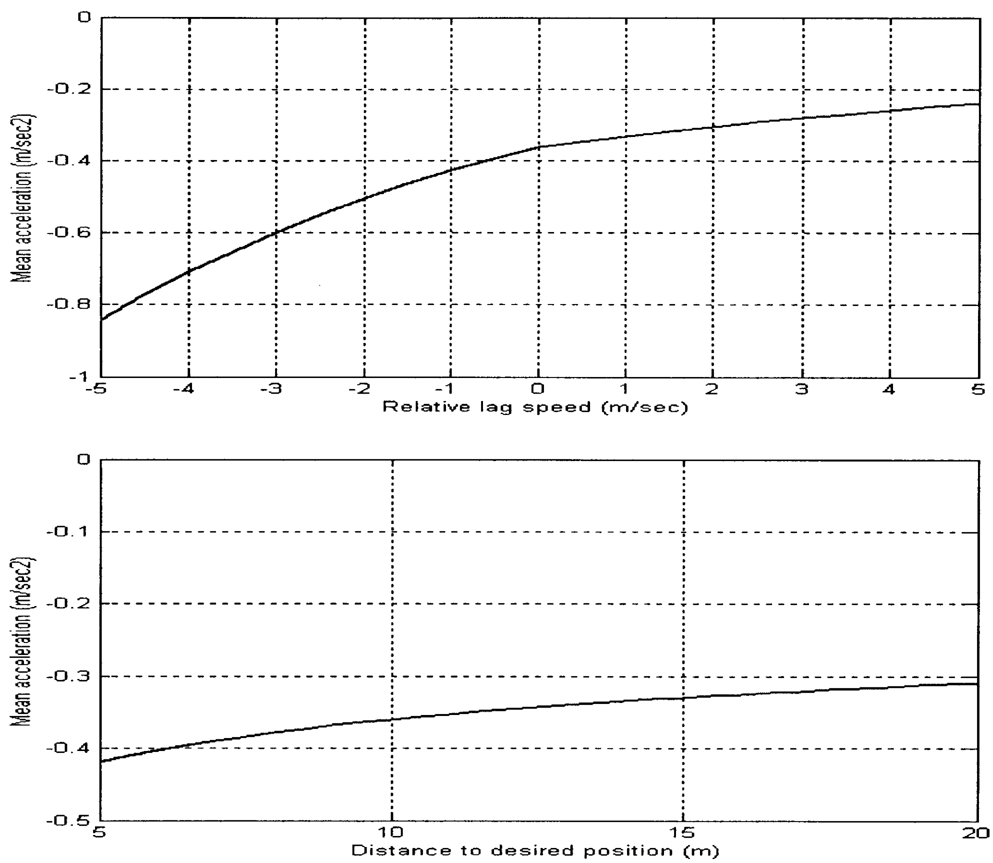


Figure 6.19 - Effects of relative lead speed and distance to desired position on the backward gap acceleration

Contrary to a-priori expectations, the backward gap acceleration is negatively correlated with the distance to the desired position. This may be because drivers prefer to maintain their speed relative to the lead and lag vehicles to facilitate gap acceptance.

The backward acceleration also decreases with the relative target lane leader speed. A driver who targets the backward gap must let the lag vehicle overtake it. Therefore, the driver needs to decelerate more aggressively in the case that the target lane lag is slower (negative relative speed) compared to the case it is faster.

Similar to the case of the forward gap acceleration, accelerations predicted by the backward gap acceleration model for different relative leader speeds are significantly different than those predicted by the free-flow model. Figure 6.20 shows the accelerations predicted by the two models.

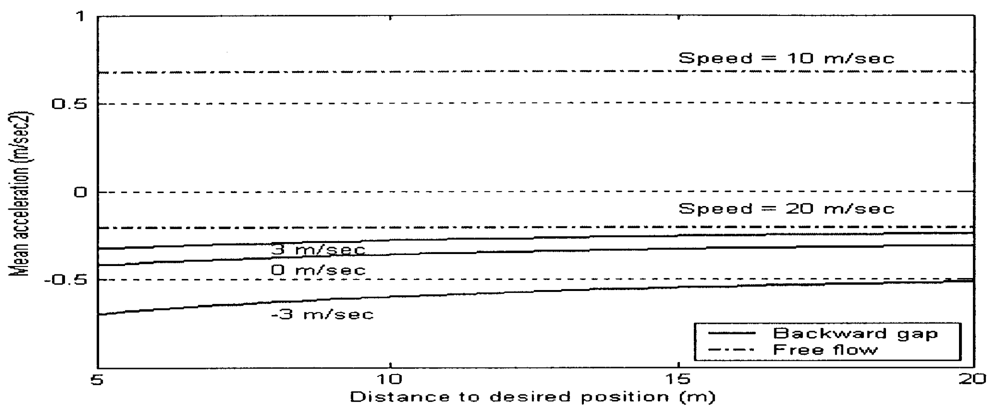


Figure 6.20 - Comparison of accelerations predicted by the backward gap acceleration model and the free-flow acceleration model

The adjacent gap acceleration model

The adjacent gap acceleration model describes the behavior of drivers who target the currently adjacent gap in order to change lanes. A driver in the constrained regime would apply car following behavior, both in the acceleration and deceleration regimes. The unconstrained acceleration is aimed at maneuvering the vehicle to an optimal position in terms of being able to accept the available gap. The situation in which this model applies is shown in Figure 6.21.

The subject vehicle (vehicle A) is committed to using the adjacent gap (the gap between vehicles B and C) to change to the left lane. The acceleration the subject applies depends on its relations with the lead vehicle (vehicle D) and its position relative to the adjacent gap.

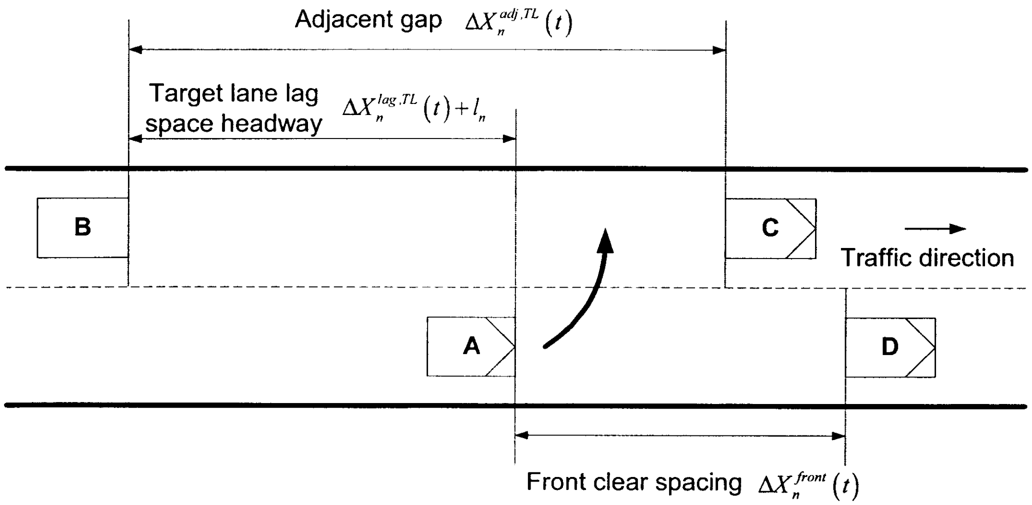


Figure 6.21 - Adjacent gap acceleration situation and variables

The acceleration applied in the unconstrained regime is affected by the relative position with respect to the adjacent gap. The model assumes that the driver tries to attain a desired position, expressed as a fraction of the total adjacent gap. The stimulus is the difference between the desired position and the vehicle's current position, given by:

$$f^{adj}\left[X_n^{adj,TL}(t-\tau_n)\right]=\beta^{DP}\Delta X_n^{adj,TL}(t-\tau_n)-\left(\Delta X_n^{lag,TL}(t-\tau_n)+l_n\right) \tag{6.57}$$

Where, β^{DP} is the desired relative position parameter. $\Delta X_n^{adj,TL}(t-\tau_n)$ is the clear adjacent gap spacing. $\Delta X_n^{lag,TL}(t-\tau_n)$ is the target lane lag space headway. l_n is the length of the subject vehicle. The adjacent gap variables are shown in Figure 6.21.

A constant sensitivity term α^{adj} is used in this model, i.e., the adjacent gap acceleration is proportional to the difference between the desired and the actual position. The driver is likely to accelerate if the stimulus term is positive (i.e. the relative desired position is ahead of the vehicle's current position) and to decelerate if it is negative (i.e. the relative desired position is behind the vehicle's current position). Therefore, the adjacent gap acceleration sensitivity constant α^{adj} is expected to be positive.

The estimated adjacent gap acceleration model is given by:

$$a_n^{uc,adj,TL}(t) = 0.131\left[0.604\Delta X_n^{adj,TL}(t-\tau_n) - \left(\Delta X_n^{lag,TL}(t-\tau_n) + l_n\right)\right] + \varepsilon_n^{adj}(t) \qquad (6.58)$$

Where, $\varepsilon_n^{adj}(t)$ is the adjacent gap acceleration error term, $\varepsilon_n^{adj}(t) \sim N\left(0, 1.158^2\right)$.

The adjacent gap acceleration is positively correlated with the mis-positioning of the vehicle, i.e., the spatial difference between the location of the vehicle and the desired position. The sensitivity of the forward acceleration to distance to the desired position is shown in Figure 6.22, which also shows the free-flow acceleration. Again, the gap acceleration exhibits more aggressive behavior relative to the free-flow acceleration.

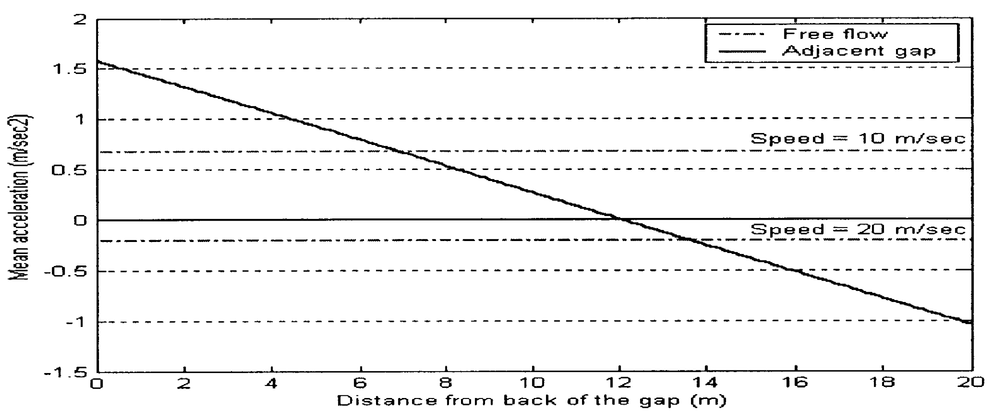


Figure 6.22 - Comparison of accelerations predicted by the adjacent gap acceleration model and the free-flow acceleration model

Estimation results for the target gap acceleration model are summarized in Table 6.5.

Table 6.5 - Estimation results for the target gap acceleration model

Variable	Parameter value	t-statistic
Forward gap acceleration		
Constant	0.385	1.39
Distance to desired position, m.	0.323	2.03
$\exp\left(\Delta V_n^{lead,TL}(t)_+\right)$, m/sec.	0.0678	1.13
$\exp\left(\Delta V_n^{lead,TL}(t)_-\right)$, m/sec.	0.217	-2.52
$\ln\left(\sigma_{fwd}\right)$	-0.540	-0.72
Backward gap acceleration		
Constant	-0.596	-1.56
Distance to desired position, m.	-0.219	-3.34
$\exp\left(\Delta V_n^{lag,TL}(t)_+\right)$, m/sec.	-0.0832	-1.15
$\exp\left(\Delta V_n^{lag,TL}(t)_-\right)$, m/sec.	-0.170	1.44
$\ln\left(\sigma_{bck}\right)$	0.391	1.86
Adjacent gap acceleration		
Constant	0.131	2.29
$\ln\left(\sigma_{adj}\right)$	-1.202	-2.50
Desired relative position		
Constant	0.604	5.59

Distribution parameters

All components of the acceleration model are conditional on two driver characteristics: the reaction time and the headway threshold. Both are modeled as random variables reflecting their distributions in the population and thus capturing heterogeneity. Parameter estimation results for the reaction time and headway threshold distributions are presented in Table 6.6.

Table 6.6 - Estimation results for the reaction time and headway threshold distributions

Variable	Parameter value	t-statistic
Reaction time distribution		
Constant	-0.160	-3.08
$\ln(\sigma_\tau)$	-0.294	-1.20
Headway threshold distribution		
Constant	2.579	45.85
$\ln(\sigma_h)$	-0.799	-7.87

Reaction time distribution

The reaction time captures the time lag between the emergence of the stimulus and the application of the response. The estimated reaction time distribution is given by:

$$f(\tau)=\begin{cases}\dfrac{1}{0.743\tau\sqrt{2\pi}}exp\left[-\dfrac{1}{2}\left(\dfrac{ln(\tau)+0.160}{0.746}\right)^2\right] & if\ 0<\tau\leq 6\\ 0 & otherwise\end{cases} \tag{6.59}$$

The probability density function and the cumulative distribution function of the reaction time are shown in Figure 6.23. The median, mean and standard deviation of the reaction time are 0.85, 1.10 and 1.00 seconds, respectively. These numbers are well within the range reported in the literature (see Table 2.5 in Section 2.1.2), in particular estimates of reaction times to expected stimuli. This can be expected since speed difference stimuli do not, in most cases, change suddenly. Reaction times are lower than the estimates reported by Ahmed (1999). One explanation may be that while in Ahmed's work reaction time was applied to stay-in-the-lane accelerations, the same distribution is also used for lane changing and target gap acceleration models in this study. These accelerations are related to the lane changing process. Drivers' reaction times in these situations may be shorter since they are more attentive to the situation while trying to change lanes. However, there may be many other possible explanations for this including differences in traffic conditions, geometry of the road, time and geographic location of the data collection. For example, the more complex geometry in this section may cause drivers to be more alert to their surroundings and therefore require lower reaction times.

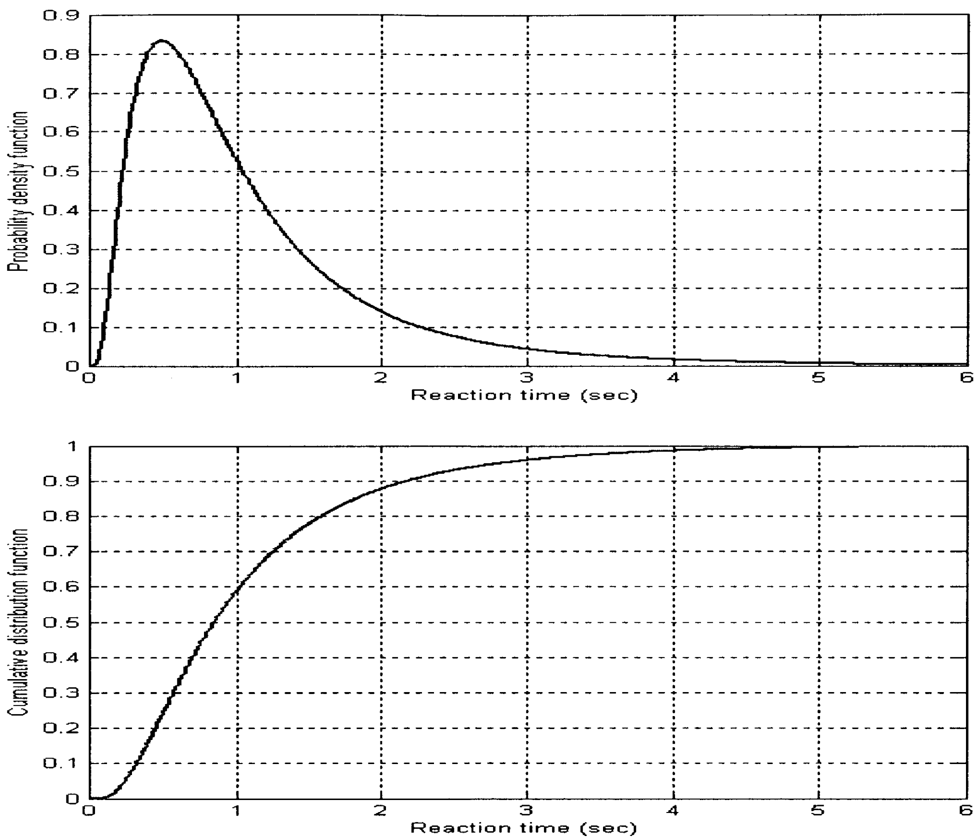


Figure 6.23 - The probability density function and the cumulative distribution function of the reaction time

Time headway threshold distribution

The time headway threshold defines whether a vehicle is in the constrained (car following) or unconstrained regime. The estimated headway threshold distribution is given by:

$$f\left(h^{*}\right)=\begin{cases}\dfrac{1}{0.450\sqrt{2\pi}}exp\left[-\dfrac{1}{2}\left(\dfrac{h^{*}-2.579}{0.450}\right)^{2}\right] & if\ 0<h^{*}\leq 6\\ 0 & otherwise\end{cases} \qquad (6.60)$$

Given a headway of $h_n(t)$, the probability that the driver is in the constrained (car following) regime is given by:

$$P_n(\text{car - following at time } t) = \begin{cases} 1 - \Phi\left(\frac{h_n(t) - 2.579}{0.450}\right) & \text{if } h_n(t) \leq 6 \\ 0 & \text{otherwise} \end{cases} \tag{6.61}$$

The probability density function and the cumulative distribution function of the time headway threshold are shown in Figure 6.24.

These headway threshold estimates are smaller than the ones reported by Ahmed (1999). This may be for several different reasons, similar to the ones discussed for the case of the reaction time distribution.

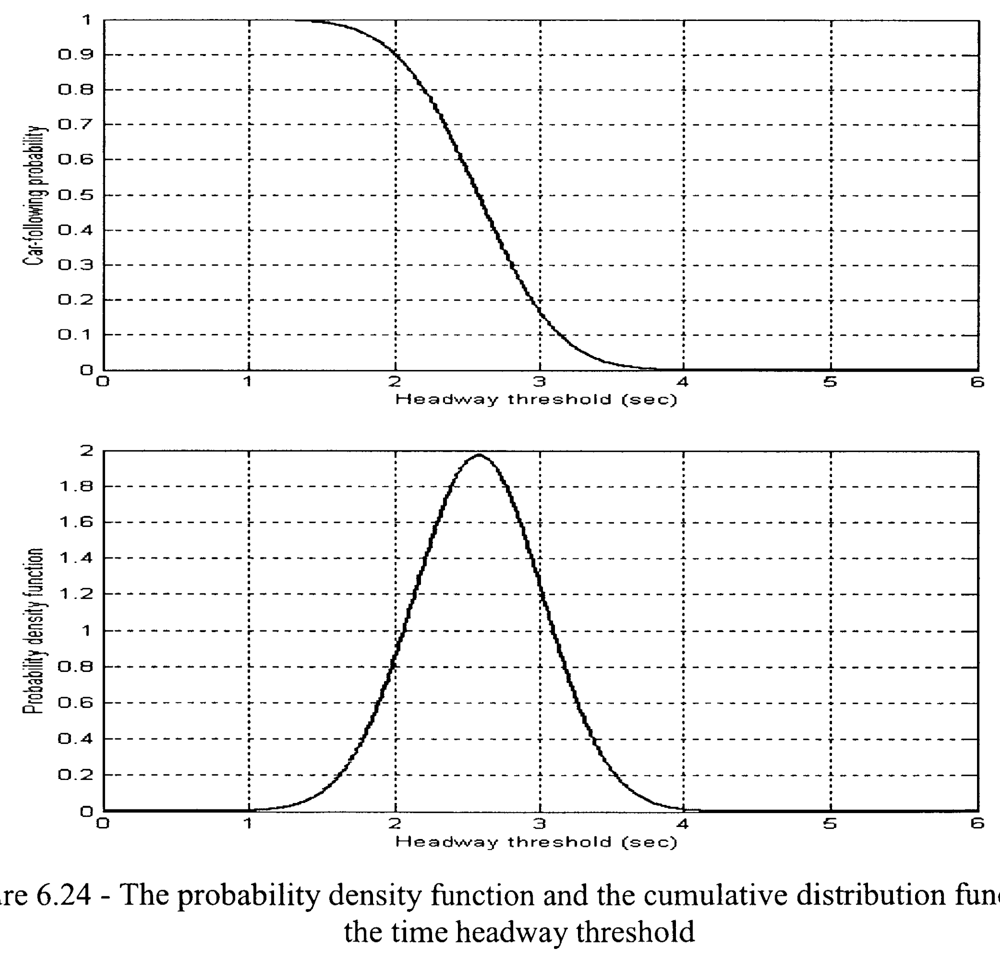


Figure 6.24 - The probability density function and the cumulative distribution function of the time headway threshold

6.3 Model selection

In this section, model selection tests are performed based on the likelihood function values at convergence of the integrated driving behavior model and a combination of the independent acceleration and lane changing models. Estimation results for the independent model are presented in Appendix C.

The independent models differ from the integrated model in the following:

- The independent models do not capture drivers' planning capabilities and therefore the target gap choice and acceleration behaviors to facilitate lane changing are excluded from the independent models.
- Acceleration and lane changing behaviors are modeled independently. Thus, the effect of lane changing on acceleration behaviors is not modeled. Correlations between the various decisions drivers make are captured within each one of the independent models by the unobserved driver/vehicle characteristics, reaction times and headway thresholds, but correlations between lane changing and acceleration decisions are not captured.
- The independent lane changing model considers MLC and DLC separately and therefore does not capture trade-offs between mandatory and discretionary considerations.
- The conditions that trigger an MLC were not estimated previously. The model used in this research assumes that the probability of being in an MLC state depends only on the distance from the relevant off-ramp. This model corresponds to the level of detail used in many microscopic traffic simulation tools.

Neither of the two models can be viewed as nested within the other due to the different structures of the lane changing models. Therefore, likelihood ratio tests for model selection are not applicable. The models can only be compared on the basis of maximum likelihood values and numbers of parameters, which are presented in Table 6.7.

Table 6.7 - Likelihood values of the estimated models

Model	Likelihood value	Parameters
Integrated driving behavior model	-25469.7	71
Independent lane changing model	-932.2	28
Independent acceleration model	-24591.9	19
Independent models combined	-25524.1	47

Akaike (1973, 1974) developed the Akaike information criterion (AIC) as a tool for selecting between competing model specifications. The AIC penalizes the maximum likelihood value of each model to account for model complexity:

$$AIC = -2L\left(\beta^*\right) + 2K \tag{6.62}$$

Where, $L(\beta^*)$ is the maximum log-likelihood value. K is the number of estimated parameters.

In model selection, AIC is computed for all candidate models and the model with the smallest AIC is selected. In this case, the AIC values for the integrated model and the combined independent models are 51081.4 and 51142.2, respectively. Thus, the 60.8 points difference recommends the integrated model over the independent ones.

6.4 Conclusions

In this chapter, the joint likelihood function for the accelerations and lane changes observed in the trajectory data was derived. Estimation results of the integrated driving behavior model were presented.

Estimation results for the target lane model indicate significant trade-offs between discretionary considerations, captured by the lane density, front vehicle speed and spacing and presence of heavy vehicles and tailgaters, and mandatory considerations captured by the impact of the path plan and next exit dummy variables. Thus, the results justify the integration of MLC and DLC considerations in a single model. The significant effect of variables related to the driver's path plan on lane selection demonstrates the impact of travel behavior on driving behavior. The expected maximum utility variable captures the effect of gap acceptance decisions on the target lane choice.

Gap acceptance decisions are affected by the subject relative speeds with respect to the lead and lag vehicle in the target lane and by the expected maximum utility of target gap choices, which captures the effect of available gaps in the subject vehicle neighborhood on critical gaps.

Important factors that affect the target gap choice are the position of the subject vehicle relative to the candidate gaps, the effective length of the candidate gaps and the relative speeds of the vehicles that define the gap.

Estimation results for the stay-in-the-lane acceleration model support enhancements Ahmed (1999) introduced to the GM car following model. Results support Ahmed's non-linear specification of the car following stimulus term over the linear GM specification and assert the important effect of traffic conditions ahead of the vehicle, captured by the density variable, on car following. Similarly, the conclusion that the speed of the subject vehicle does not affect car following decelerations is strengthened. However, the parameter estimates obtained in this study differ significantly from the ones Ahmed reported, which suggest that further work is required to identify additional factors affecting the behavior such as the type of road facility (e.g. freeways, urban streets, tunnels, bridges) and geometric characteristics (e.g. curvature, slope, visibility).

Drivers' acceleration behaviors to facilitate lane changing using the target gap depend on their relations with the vehicle in front and the vehicles defining the target gaps. Important variables that capture these behaviors are the distance from the subject's current position to the desired position relative to the target gap and relative speeds with respect to the vehicles defining target gaps. Estimation results show these acceleration behaviors are significantly different from the behaviors of drivers who are not trying to change lanes.

A model selection test was performed based on the likelihood function values at convergence of the integrated driving behavior model and a combination of independent acceleration and lane changing models. Test results recommend the integrated model and reject the independent models.

Chapter 7

Model Validation

In this chapter, validation results of the integrated driving behavior model are reported and compared against those obtained for a combination of independent lane changing and acceleration models. The validation tests are performed within the framework of a microscopic traffic simulation tool, MITSIMLab. The integrated model and the independent models were implemented in MITSIMLab and the two MITSIMLab versions were applied to the same data used for estimation of the model and to a freeway corridor in Southampton, UK. Outputs from these simulations are compared with respect to their ability to replicate observed traffic patterns.

This chapter is organized as follows: an overview of the MITSIMLab simulation tool is presented first. Next, methodologies for the calibration of the simulator and for the validation study are presented. Finally, results from the two case studies are presented and discussed.

7.1 MITSIMLab

MITSIMLab is a microscopic traffic simulation laboratory developed to evaluate Advanced Traffic Management Systems (ATMS) and Advanced Traveler Information Systems (ATIS) at the operational level. MITSIMLab can represent a wide range of traffic management systems and model the response of drivers to real-time traffic information and control. This enables MITSIMLab to simulate the dynamic interactions between traffic management systems and drivers. MITSIMLab consists of three main modules:

1. Microscopic Traffic Simulator (MITSIM)

2. Traffic Management Simulator (TMS)
3. Graphical User Interface (GUI)

MITSIM represents traffic and network elements. It represents the movements of individual vehicles in detail. The road network is represented by nodes, links, segments (links are divided into segments with uniform geometric characteristics) and lanes. Traffic controls and surveillance devices are represented at the microscopic level. Travel demand is input in the form of time-dependent origin to destination (OD) flows, from which, individual vehicles wishing to enter the network are generated. A probabilistic model is used to capture drivers' route choice decisions. Behavior parameters (e.g. desired speed, aggressiveness) and vehicle characteristics are assigned to each vehicle/driver. MITSIM moves vehicles according to acceleration and lane changing models. The acceleration model captures drivers' response to conditions ahead as a function of relative speed, headway and other traffic measures. The lane changing model distinguishes between mandatory and discretionary lane changes. Merging, drivers' responses to traffic signals, speed limits, incidents, and tollbooths are also captured. The driving behavior models implemented in MITSIMLab are those estimated by Ahmed (1999). These acceleration and lane changing models are described in detail in Sections 2.1.2 and 2.2.1, respectively.

TMS mimics the traffic control system in the network under consideration. A wide range of traffic control and route guidance systems can be simulated. These include intersection controls, ramp control, freeway mainline control, lane control signs, variable speed limit signs, portal signals, variable message signs and in-vehicle route guidance. TMS can represent different designs of such systems with logic at varying levels of sophistication (pre-timed, actuated or adaptive). An extensive graphical user interface is used for both debugging purposes and demonstration of traffic impacts through vehicle animation. A detailed description of MITSIMLab appears in Yang and Koutsopoulos (1996) and Yang et al (2000).

7.2 Calibration methodology

In general, calibration and validation of microscopic traffic simulation models should be based on the framework illustrated in Figure 7.1, which consists of two steps: first, the individual models the simulation consists of (e.g. driving behavior and route choice models) are specified and estimated using disaggregate data, independent of the overall simulation model. Disaggregate data includes detailed driver behavior information such as vehicle trajectories. In the second step, aggregate data (e.g. time headways, speeds, flows) is used to fine-tune parameters and calibrate general parameters in the simulator.

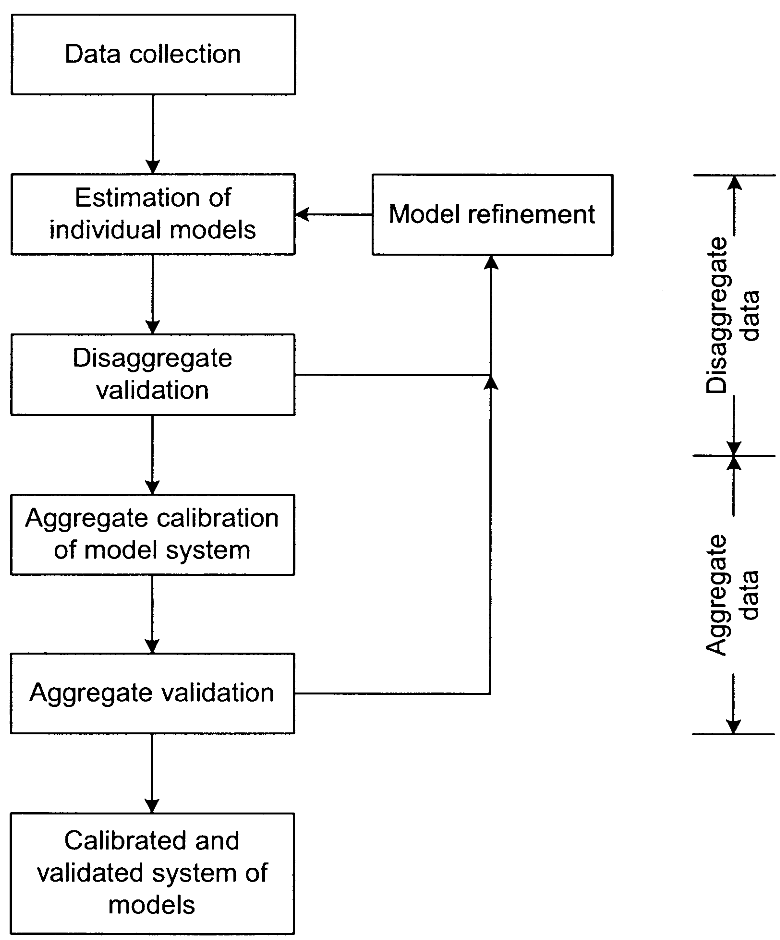


Figure 7.1 - Overall calibration and validation framework

Results from the first step of this process with respect to the integrated driving behavior model are reported in Chapter 6. In the second step, discussed in this chapter, aggregate calibration is performed using data available from loop detectors. Aggregate calibration is based on a formulation of an optimization problem, which seeks to minimize a measure of the deviation between observed and corresponding simulated measurements. The reason for this approach is that, in general, it is not feasible to isolate the contribution of individual models to the overall error. For example, OD estimation methods require an assignment matrix as input. The assignment matrix maps OD flows to counts at sensor locations. Usually the assignment matrix is not readily available and needs to be generated from the simulation model. Therefore, the assignment matrix is a function of the route choice and driving behavior models used. Similarly, an important explanatory variable in route choice models is route travel times, which are flow-dependent. Simulated flows are a function of the OD flows, driving behavior and the route choice model itself. Hence, the following optimization problem, which simultaneously calibrates the parameters of interest (OD flows, route choice and driving behavior parameters) may be formulated:

$$\begin{aligned} &\min_{\beta,\theta,OD} \; f\left(M^{obs}, M^{sim}\right) \\ &s.t. \quad M^{sim} = g\left(\beta,\theta,OD\right) \\ &\qquad\;\; OD = \arg\min_{X} \left\| AX - Y^{obs} \right\| \end{aligned} \tag{7.1}$$

Where, β, θ and OD are vectors of parameters to be calibrated: driving behavior, route choice and OD flows, respectively. M^{obs} and M^{sim} are vectors of observed and simulated traffic measurements, respectively. $g(\cdot)$ represents the simulation model. Y^{obs} are observed traffic counts at sensor locations. A is the assignment matrix.

Problem (7.1) is very difficult to solve exactly. The OD constraint, for example, is a fixed-point problem, which is a hard problem on its own merits (Cascetta and Postorino 2001). Hence the iterative heuristic approach outlined in Figure 7.2 is used. This approach accounts for interactions between driving behavior, OD flows and route choice behavior by iteratively calibrating driving behavior parameters and travel behavior elements. At each step the corresponding set of parameters is calibrated, while other parameters remain fixed at their previous values. Calibration of the route choice model requires a set of reasonable paths for each OD and expected link travel times used as explanatory variables in the model. OD estimation requires generation of an assignment matrix. Hence, the travel behavior calibration step is also iterative: based on the existing OD flows, parameters of the route choice model are calibrated. The calibrated route choice model is used to generate an assignment matrix, and perform OD estimation. The new OD flows are used to re-calibrate route choice parameters and so on. In summary the calibration process proceeds as follows:

1. Initialize parameters, β_0, θ_0 and OD_0.
2. Estimate the OD matrix and calibrate route choice parameters assuming fixed driving behavior parameters.
3. Calibrate driving behavior parameters assuming the OD matrix and route choice parameters estimated in Step 2.
4. Update habitual travel times using the OD matrix, route choice and driving behavior parameters estimated in Steps 2 and 3.
5. Check for convergence: if converged, terminate.
   else, return to step 2.

Route choice behavior is not represented in the application discussed here and so the steps shown faded in Figure 7.2 are skipped.

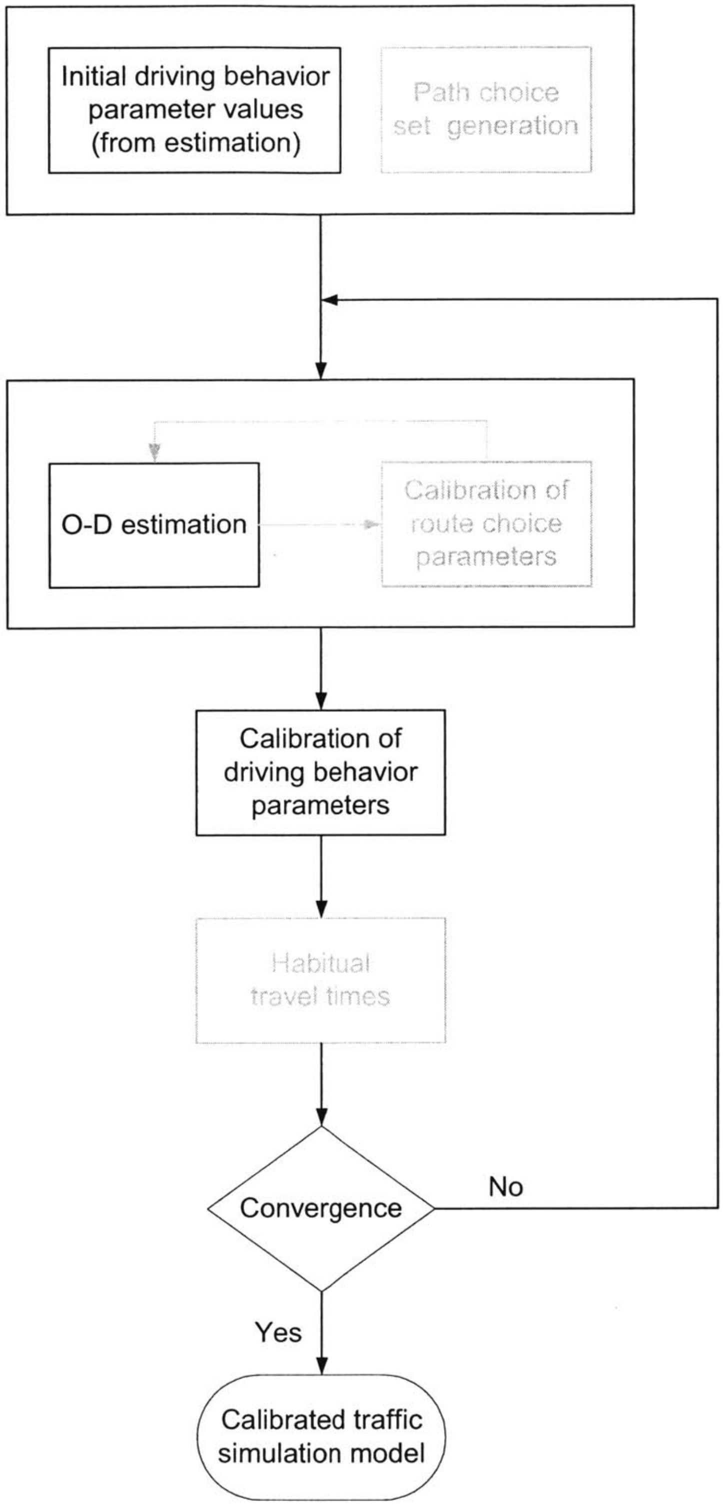


Figure 7.2 - Methodology for aggregate calibration of micro-simulation models

7.2.1 OD estimation

The OD estimation problem is often formulated as a generalized least squares (GLS) problem. The GLS formulation minimizes the deviations between estimated and observed sensor counts while also minimizing the deviation between the estimated OD flows and seed OD flows (see Cascetta et al (1993) for more detail). The corresponding optimization problem is:

$$\min_{X \geq 0} \quad (AX - Y^H)^T W^{-1} (AX - Y^H) + (X - X^H)^T V^{-1} (X - X^H) \qquad (7.2)$$

Where, X and X^H are vectors of estimated and historical (seed) OD flows, respectively. Y^H are the historical (observed) sensor counts. W and V are the variance-covariance matrices of the sensor counts and OD flows, respectively.

However, in the problem at hand, the assignment matrix is not known, hence, the iterative process shown in Figure 7.3 is used. First, the simulation is run, using the calibrated parameters and a set of seed OD flows to generate an assignment matrix. This assignment matrix is in turn used for OD estimation. Due to congestion effects, the assignment matrix generated from the seed OD may be inconsistent with the estimated OD. Therefore, the OD estimation process must be iterative.

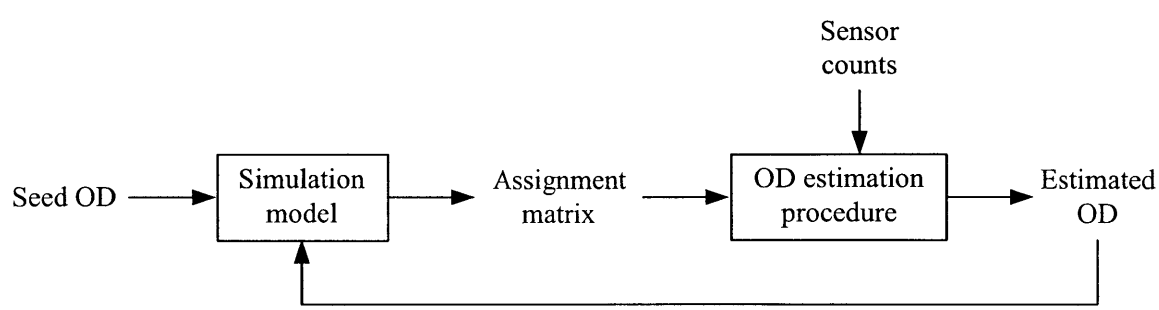


Figure 7.3 - OD estimation process

7.2.2 Calibration of driving behavior parameters

In this step the estimated values of various parameters may be modified to support two purposes:

1. Ensuring that the interactions between the individual models within the simulation tool are captured correctly.
2. Refining previously calibrated parameter values for the specific site being studied.

Driving behavior parameters are calibrated by minimizing a function of the deviations of simulated measurements from observed ones:

$$\min_{\beta} \sum_{i=1}^{I} \left(\frac{m_i^{sim} - m_i^{obs}}{m_i^{obs}} \right)^2 \tag{7.3}$$

Where m_i^{obs} and m_i^{sim} are the i^{th} observed and simulated measurements, respectively. The I measurements may represent different locations (sensors), time periods and/or vehicles. β are parameters to be calibrated.

Ideally, the calibration objective function should be independent of sensor counts used in OD estimation.

While the initial estimation of driving models included a wide range of parameters, during this step only a limited set of parameters may be calibrated. Hence, given OD flows and route choice parameters, a subset of driving behavior parameters are calibrated using the formulation given in Equation (7.3).

7.3 Validation Methodology

The purpose of validation is to determine the extent to which the simulation model replicates the real system. This is done by comparing measures of performance (MOPs), which are statistics of outputs of interest from the two systems.

7.3.1 Goodness of fit measures

Different goodness-of-fit measures may be used to quantify the similarity between observed and simulated MOPs. The presentation here is adapted from Pindyck and

Rubinfeld (1997). The root mean square error (RMSE) and root mean square percent error (RMSPE) quantify the overall error of the simulator. These measures penalize large errors at a higher rate than small errors. The two measures are given by:

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(Y_n^s - Y_n^o\right)^2} \tag{7.4}$$

$$RMSPE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[\frac{Y_n^s - Y_n^o}{Y_n^o}\right]^2} \tag{7.5}$$

Where, Y_n^o and Y_n^s are the observed and simulated measurements at space-time point *n*, respectively. The simulation observations are averages of the replications made.

The mean error (ME) and mean percent error (MPE) statistics indicate the existence of systematic under- or over-prediction in the simulated measurements. These measures are calculated by:

$$ME = \frac{1}{N}\sum_{n=1}^{N}\left(Y_n^s - Y_n^o\right) \tag{7.6}$$

$$MPE = \frac{1}{N}\sum_{n=1}^{N}\left[\frac{Y_n^s - Y_n^o}{Y_n^o}\right] \tag{7.7}$$

The mean error statistics are most useful when applied individually to measurements at a single point in space rather than to all measurements jointly. This way they provide insight into the spatial distribution of errors on the network and may point to deficiencies in the model.

Percent error measures are often preferred to their absolute error counterparts because they provide information on the magnitude of the errors relative to the average measurement. Another measure that provides information on the relative error is Theil's inequality coefficient, given by:

$$U=\frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(Y_n^s-Y_n^o\right)^2}}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(Y_n^s\right)^2}+\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(Y_n^o\right)^2}} \tag{7.8}$$

U is bounded, $0 \leq U \leq 1$. $U=0$ implies perfect fit between observed and simulated measurements. $U=1$ implies the worst possible fit. Theil's inequality coefficient may be decomposed into three proportions of inequality: the bias (U^M), the variance (U^S) and the covariance (U^C) proportions given, respectively by:

$$U^M=\frac{\left(\bar{Y}^s-\bar{Y}^o\right)^2}{\frac{1}{N}\sum_{n=1}^{N}\left(Y_n^s-Y_n^o\right)^2} \tag{7.9}$$

$$U^S=\frac{\left(s^s-s^o\right)^2}{\frac{1}{N}\sum_{n=1}^{N}\left(Y_n^s-Y_n^o\right)^2} \tag{7.10}$$

$$U^C=\frac{2\left(1-\rho\right)s^s s^o}{\frac{1}{N}\sum_{n=1}^{N}\left(Y_n^s-Y_n^o\right)^2} \tag{7.11}$$

Where, ρ is the correlation between the two sets of measurements.

By definition, the three proportions sum to 1 ($U^M+U^S+U^C=1$). The bias proportion reflects the systematic error. The variance proportion indicates how well the simulation model is able to replicate the variability in the observed data. These two proportions should be kept as close to zero as possible. The covariance proportion measures the remaining error and therefore should be close to 1. Note that since the various measurements are taken from non-stationary processes, the proportions can only be viewed as rough indicators to the sources of error.

7.3.2 Replications

The validation uses stochastic simulation models. Therefore, results of several simulation runs need to be used. Assuming that the outputs from different simulation runs are normally distributed, the minimum number of replications required to achieve an allowable error is:

$$R_i = \left(\frac{\sigma_i^s t_{\alpha/2}}{\mu_i^s \varepsilon} \right)^2 \tag{7.12}$$

Where μ_i^s and σ_i^s are the mean and standard deviation of the distribution of simulated MOP i, respectively. ε is the allowable error specified as a fraction of the mean. $t_{\alpha/2}$ is the critical value of the t-distribution at significance level α.

Both μ_i^s and σ^s are unknown. To get initial estimates, the simulation is run a number of times (e.g. 10 replications). The required number of replications is calculated for all MOPs of interest. The most critical (highest) value of R_i is used.

7.4 Case studies

The simulation model was applied to two case studies: the same road section that was used to estimate the parameters of the driving behavior model and a freeway corridor in Southampton, UK. The integrated driving behavior model is compared with a combined model of independent lane changing and acceleration models.

The independent models differ from the integrated model in the following:

- The independent models do not capture drivers' planning capabilities and therefore the target gap choice and acceleration behaviors to facilitate lane changing are excluded from the independent models.
- Acceleration and lane changing behaviors are modeled independently. Thus, the effect of lane changing on acceleration behaviors is not modeled. Correlations between the various decisions drivers make are captured within each one of the independent models by the unobserved driver/vehicle characteristics, reaction times and headway thresholds, but correlations between lane changing and acceleration decisions are not captured.

- The independent lane changing model considers MLC and DLC separately and therefore does not capture trade-offs between mandatory and discretionary considerations.
- The conditions that trigger an MLC were not estimated previously. The model used in this research assumes that the probability of being in an MLC state depends only on the distance from the relevant off-ramp. This model corresponds to the level of detail used in many microscopic traffic simulation tools.

The parameters of the independent models were estimated using the same data used to estimate the integrated model. Estimation results for the independent models are presented in Appendix C.

7.4.1 Arlington, VA case study

In this case study the simulation model is applied to the same road section used to estimate the parameters of the driving behavior model. The purpose of this exercise is two-fold: to verify the implementation of the model in the micro-simulator and to compare the structure of the integrated driving behavior model with the independent lane changing and acceleration models.

Detailed travel demand information was extracted directly from the trajectory data such that individual vehicles enter the network at the exact times they appeared in the real system and in the correct lanes. Since both models were estimated using detailed trajectory data for the case study in question, no further calibration was performed other than adjustment of scale parameters of the acceleration models. This adjustment is performed to account for the simplified treatment of reaction time in the simulation model. The objective function used in the calibration step was to minimize the deviation of simulated travel times from observed ones:

$$\min_{\beta} \sum_{i=1}^{I} \left(\frac{tt_i^{sim} - tt_i^{obs}}{tt_i^{obs}} \right)^2 \tag{7.13}$$

Where tt_i^{obs} and tt_i^{sim} are the observed and simulated travel times of vehicle i, respectively. I is the number of vehicles observed. β are the scale parameters to be calibrated.

The two models are compared based on travel times in the section and on lane distributions at key locations.

Travel times

Travel times are a result of both the disaggregate interactions between vehicles and aggregate traffic characteristics. Thus, these are important indicators of the performance of driving behavior models. However, it should be noted that travel times were used in model calibration. The goodness-of-fit statistics are shown in Table 7.1. Although both models exhibit good fit with observed travel times, the fit of the integrated model is higher. Travel times are compared at the level of individual vehicles without any aggregation: the simulated travel time of each vehicle is compared with the corresponding observed value. Thus, some of the error may be attributed to unobserved characteristics of drivers in the sample. This may explain the smaller mean error measures relative to their root mean square error counterparts. The MITSIMLab version with independent behaviors shows more congestion relative to the integrated driving behavior model and to the observed data. This explains the larger bias in travel times: 4.8 sec. and 9.5% against 0.9 sec. and 3.2%. A similar bias is also indicated by Theil's bias proportion (0.165 against 0.044). The only measure that seems to indicate an advantage of the independent models over the integrated one is Theil's variance proportion. However, given the large difference in the bias proportions between the two versions, this result is of very little importance. Moreover, the covariance proportion ($U^C = 1 - U^M - U^S$), which captures the pure noise in the model is larger for the integrated model compared to the independent models (0.870 and 0.831, respectively).

Table 7.1 - Statistics for the travel time comparison in the Arlington, VA network

Statistic	Integrated model	Independent models
RMSPE (%)	15.3	20.2
RMSE (sec.)	9.4	11.7
MPE (%)	3.2	9.5
ME (sec.)	0.9	4.8
U (Theil's inequality coefficient)	0.075	0.091
U^M (bias proportion)	0.044	0.165
U^S (variance proportion)	0.086	0.004

Lane distribution

The distribution of vehicles across lanes was calculated for the four locations shown in Figure 7.4. Each location contains four lanes. These locations cover all the elements of the road section and allow capturing lane changing behavior. The fraction of vehicles in each lane was measured from the data and from the simulation. The calculation of goodness-of-fit statistics based on all lanes at all locations is presented in Table 7.2. Since the total fraction of vehicles in all four lanes is equal to 1 at each location, the mean error (ME) and Theil's bias proportion (U^M) statistics are by definition equal to zero and therefore omitted from the table.

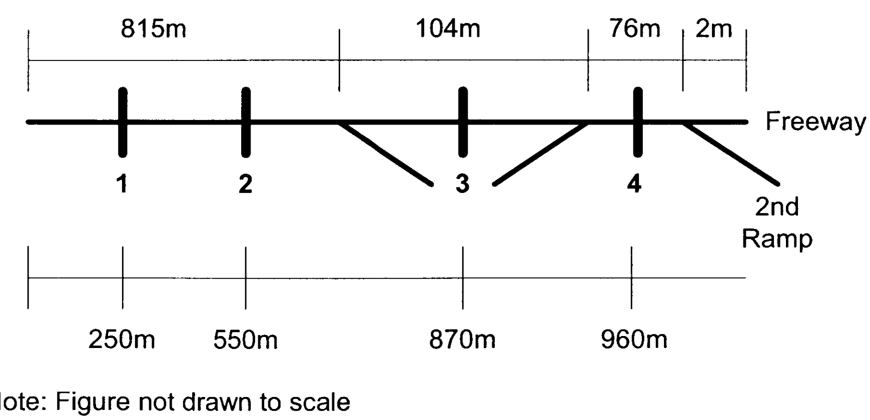


Figure 7.4 - Lane distribution measurement locations in the Arlington, VA network

Table 7.2 - Statistics for the lane distribution comparison in the Arlington, VA network

Statistic	Integrated model	Independent models
RMSPE (%)	10.8	11.2
RMSE (fraction)	0.030	0.042
MPE (%)	9.0	9.3
U (Theil's inequality coefficient)	0.059	0.091
U^S (variance proportion)	0.556	0.775

As in the case of travel times, the integrated model outperforms the independent models in all the goodness of fit measures calculated. The lane distributions at all locations are shown in Figure 7.5. Both models, but especially the independent models, overestimate the usage of the right-most lane and underestimate the usage of the two left-most lanes. This may suggest that the tendency of vehicles that are not using any of the off-ramps to move to the left is stronger than captured in the models. This behavior is most evident at location 4 (see Figure 7.4). Some of the error at this location may be explained by the lack of information about downstream effects on the behavior of vehicles that enter the network from the on-ramp: in the simulation, these vehicles ignore any considerations downstream of the network boundary (e.g. downstream speeds and densities) and therefore have no incentive to change lanes.

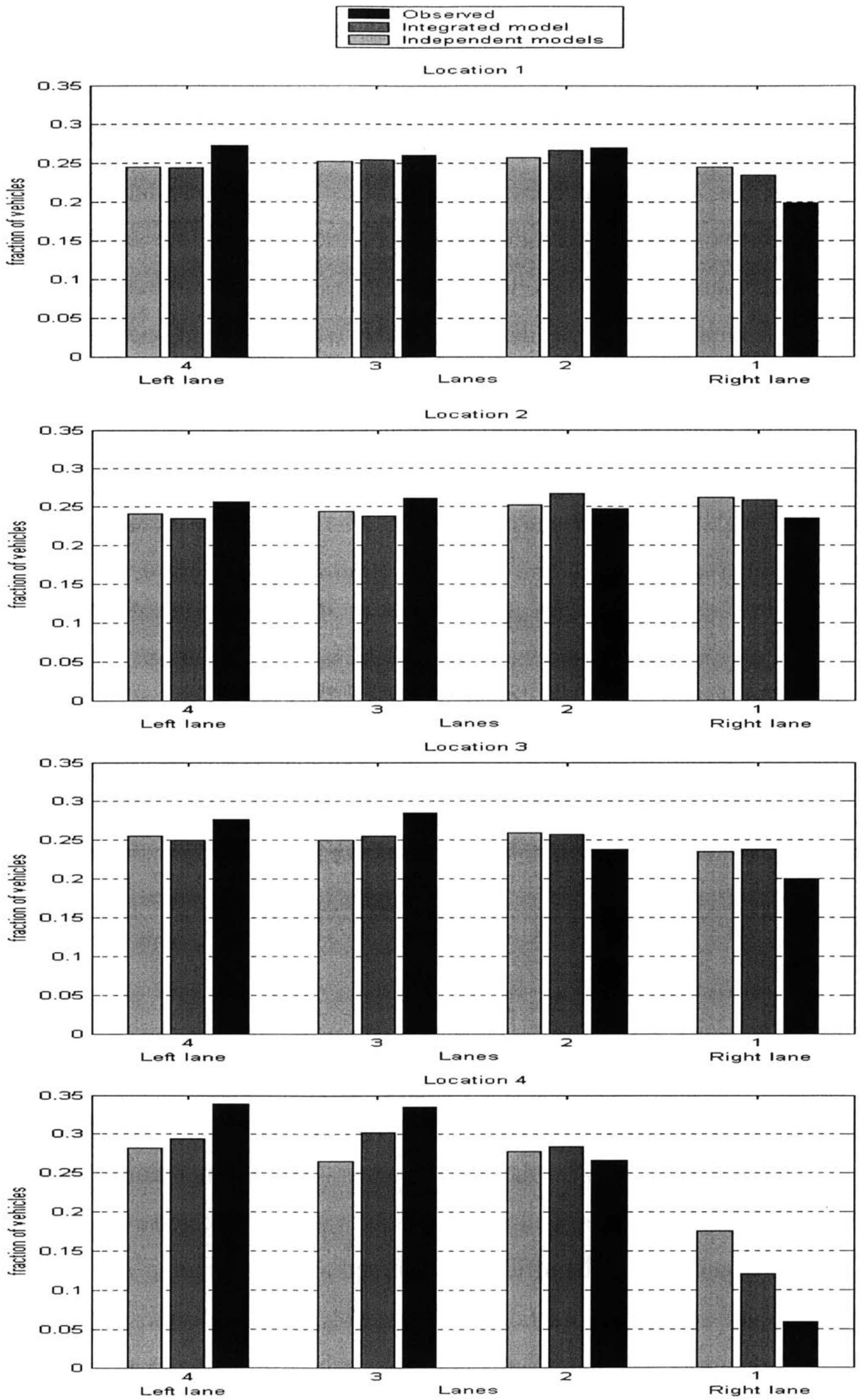
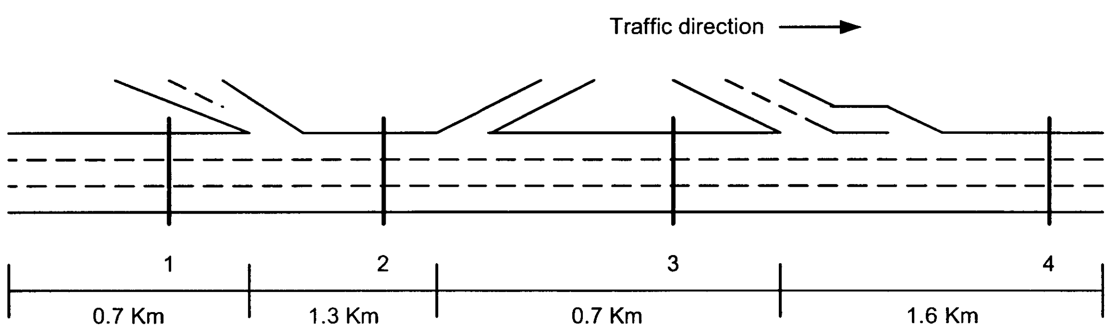


Figure 7.5 - Observed and simulated lane distributions in the Arlington, VA network

7.4.2 Southampton, UK case study

This case study provides independent validation of the integrated driving behavior model. The MITSIMLab versions using the integrated model and the independent models were both applied to the freeway corridor shown schematically in Figure 7.6. The section is a 4.3 kilometer long, three-lane freeway that includes two on-ramps and an off-ramp. Both on-ramps are two-lane, but with different geometric layout. In the upstream ramp, the two ramp lanes merge into a single lane, which then merges into the mainline. In the other ramp the left ramp lane merges into the freeway while the right one remains as an additional lane, physically separated from the mainline, for another 500 meters and only then merges into the freeway. This network exhibits several behaviors: vehicles merging into the freeway from the two ramp configurations described above, weaving behaviors, mainline vehicles making lane changes to use the off-ramp and to avoid the merging traffic in the left-most lane. The two on-ramps are controlled by ramp metering with the control logic implemented in the simulation model.



Note: Figure not drawn to scale

Figure 7.6 - The Southampton, UK case study network

Traffic data for that network was available for the four sensor locations indicated by the numbers 1-4 in Figure 7.6 and for the two on-ramps. The sensors recorded minute by minute traffic counts and speeds on multiple days. The validation exercise focused on the 7:00-9:00 AM peak period. Light traffic was observed at the beginning and end of the simulation period with congested traffic within the AM peak. To further capture initial conditions the simulation was started at 6:45. Thus when the collection of simulated traffic conditions started at 7:00, the network has already "warmed-up".

OD estimation and calibration were performed, for each one of the models, using the methodology presented in Section 7.2 and with data from September 28, 2001. The formulation given in Equation (7.2) was used to estimate OD flows at 15 min. intervals from sensor counts. Calibration was performed to match mainline traffic speeds using observations from the same sensors. The objective function in this stage is given by:

$$\min_{\beta} \sum_{n=1}^{N} \sum_{t=1}^{T} \left(\frac{V_{nt}^{sim} - V_{nt}^{obs}}{V_{nt}^{obs}} \right)^2 \tag{7.14}$$

Where, V_{nt}^{obs} and V_{nt}^{sim} are the observed and simulated speeds, respectively, measured at sensor n during time period t. N and T are the number of sensors and time periods, respectively. β are parameters to be calibrated, which in this case study were the acceleration scale parameters and the constants of the target lane and gap acceptance models.

Prior to the application of the calibration methodology, the models were adapted to left-hand driving by reversing the definitions of variables and utilities associated with the right and left directions. Thus, in the target lane utilities the alternative specific constants of the right and left lanes were switched and the right-most lane variables were replaced by similar left-lane variables.

Data from a different day, November 21, 2001 was used for validation. An OD matrix was estimated for this day from the traffic counts, but no further calibration was performed. The validation is based on comparison of traffic counts and average speeds at the 4 sensor locations. The results are presented next.

Traffic counts

Simulated traffic counts obtained from both the integrated model and the independent models versions of MITSIMLab and the corresponding observed values are shown in Figure 7.7. Traffic count goodness-of-fit statistics for the two models are presented in Table 7.3. Both models match observed flows very well, with a slightly better fit with the integrated model compared to the independent models. The good fit to observed traffic counts is not surprising since they were used to estimate the OD matrix. Thus, the results indicate that there is little error from the OD estimation and that both models are able to process the level of travel demand in the network. In general, traffic flows are underestimated in the earlier time periods and overestimated later. This suggests that the build-up of congestion in the simulation is faster than it should be. The resulting flow deficit is recovered during the later periods, in which travel demand is reduced.

Table 7.3 - Statistics for the traffic flow comparison in the Southampton, UK network

Statistic	Integrated model	Independent models
RMSPE (%)	2.8	3.4
RMSE (veh/15min.)	34.6	42.2
MPE (%)	-0.7	-0.7
ME (veh/15min.)	-8.6	-9.0
U (Theil's inequality coefficient)	0.014	0.017
U^M (bias proportion)	0.062	0.045
U^S (variance proportion)	0.001	0.001

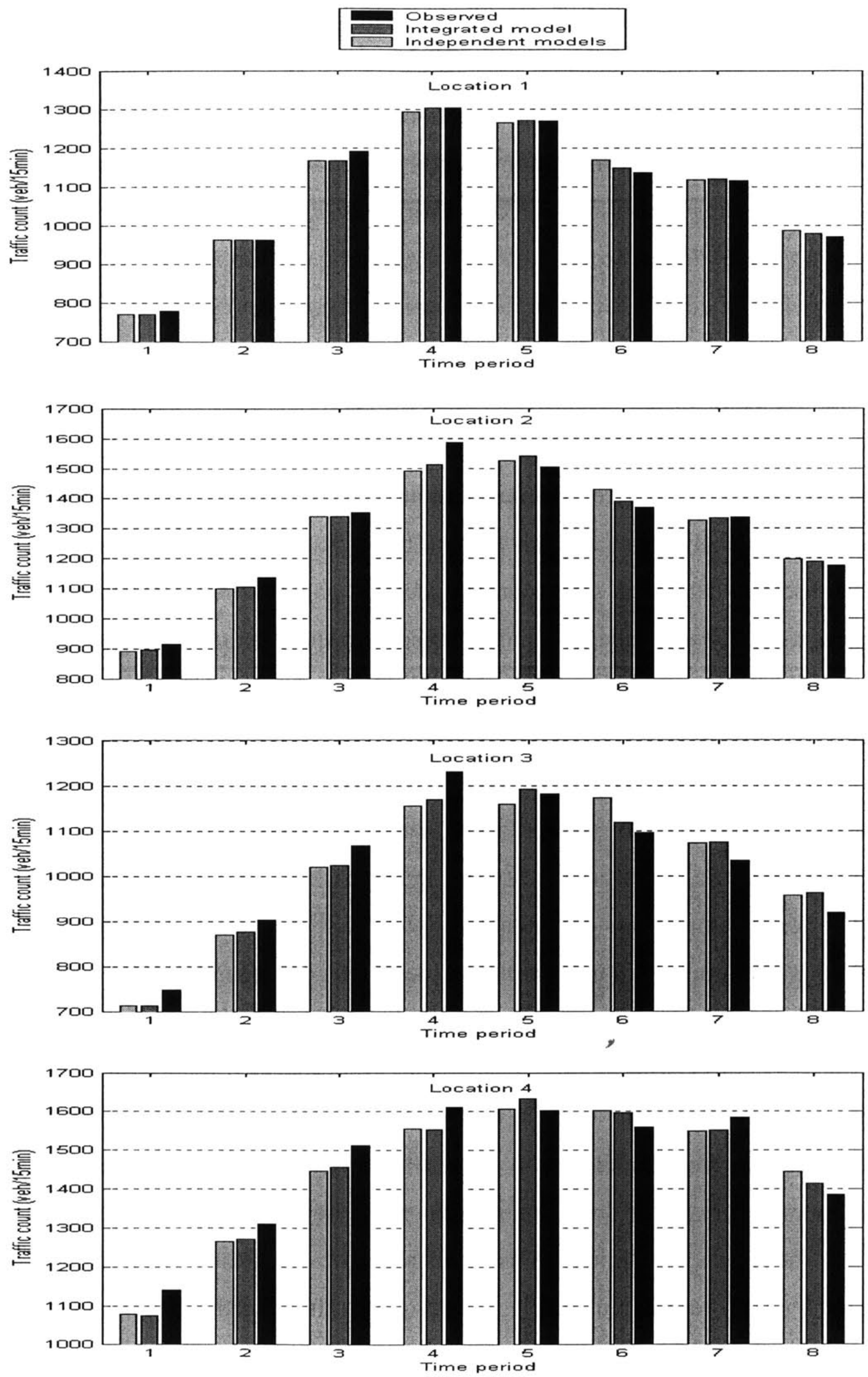


Figure 7.7 - Observed and simulated traffic counts in the Southampton, UK network

Traffic speeds

Observed and simulated traffic speeds are shown in Figure 7.8 with the corresponding goodness-of-fit statistics presented in Table 7.4. These measurements were not used for the OD estimation and calibration and therefore facilitate an independent validation.

The integrated model performed consistently better than the independent models. Goodness of fit measures expressing the total error (RMSE, RMSPE and Theil's number) of the integrated model are all around 15% lower than the corresponding statistics for fit of the independent models. Perhaps more significantly, the measures related to bias in the models (ME, MPE and the bias proportion) are around 50% lower for the integrated model compared for the independent models.

At all sensor locations, congestion build-up (in time periods 1-4) occurs faster with the independent models relative to the integrated model. As a result, simulated speeds are lower with this model. In general, both models underestimate observed speeds at this stage. In the observed data, high speeds are maintained longer but the reduction in speed is steeper once capacity is reached. A similar effect is observed in the dissipation stage: simulated traffic takes longer to recover speed. This effect is again more pronounced with the independent models relative to the integrated model. Hence, in both MITSIMLab versions and in particular with the independent models, changes in traffic speeds are more gradual compared to the observed data. These observations are also consistent with the comparison of traffic flows at these sensor locations. A possible explanation is that under semi-congested conditions drivers are able to adjust their behavior to avoid speed loss. The integrated model captures some of these effects through behaviors such as short-term planning and acceleration to facilitate lane changing. However, these behaviors are not captured with the independent behavior models.

It should be noted that in the furthest downstream location (location 4), both models produce a flat speed profile and fail to capture the dynamics of the observed speed profile. This may be a result of downstream phenomena beyond the limit of the network. However, this sensor is 1.9 kilometers downstream of the nearest upstream sensor (location 3) and therefore it is reasonable to assume that the other measurements are not affected by the downstream boundary conditions.

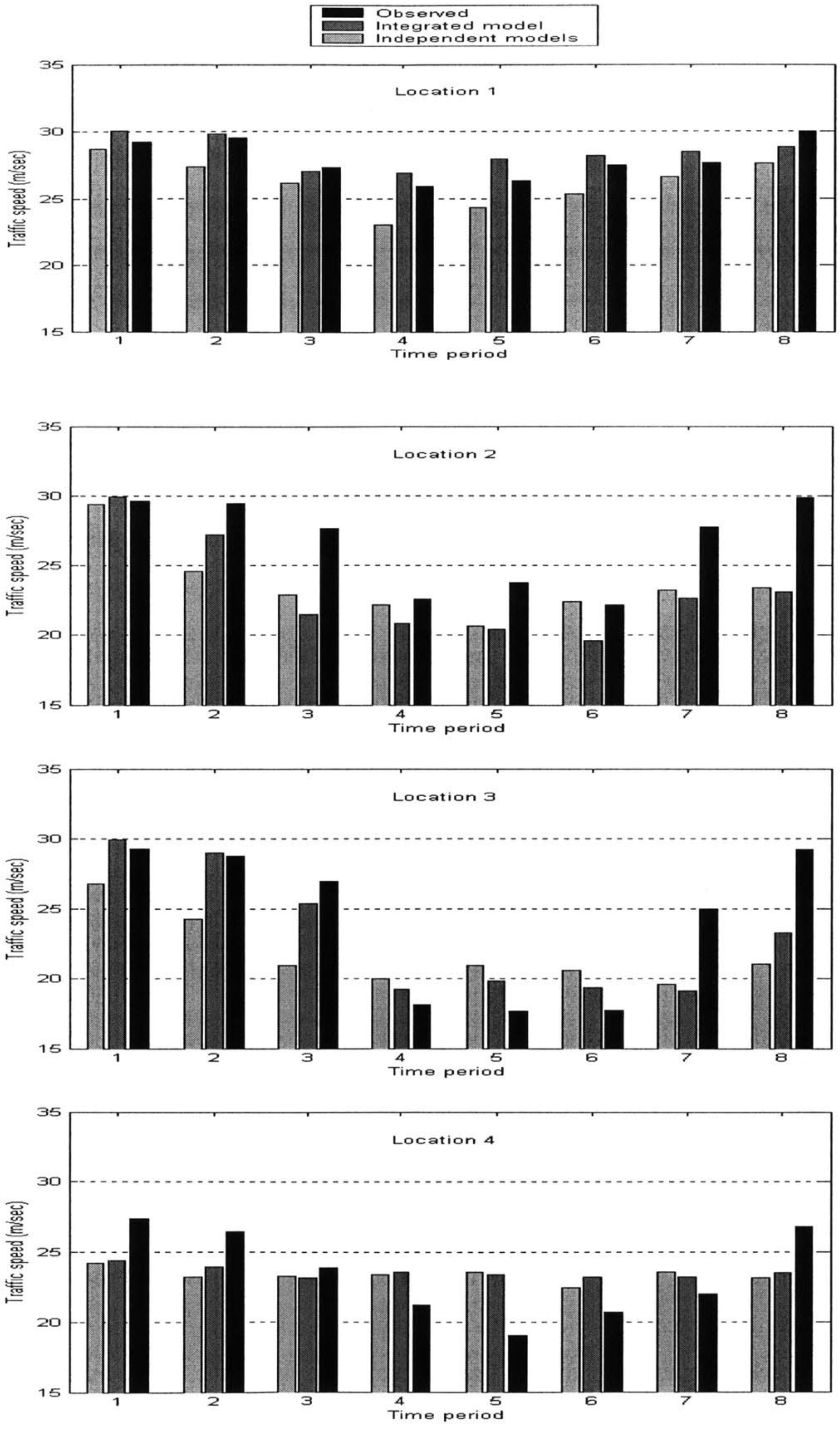


Figure 7.8 - Observed and simulated traffic speeds in the Southampton, UK network

Table 7.4 - Statistics for the traffic speed comparison in the Southampton, UK case study

Statistic	Integrated model	Independent models
RMSPE (%)	11.7	13.6
RMSE (m/sec.)	3.0	3.5
MPE (%)	-2.9	-5.6
ME (m/sec.)	-1.0	-1.8
U (Theil's inequality coefficient)	0.059	0.071
U^M (bias proportion)	0.106	0.264
U^S (variance proportion)	0.014	0.156

7.5 Conclusions

The integrated driving behavior model was validated and compared against a combination of independent lane changing and acceleration models using the microscopic traffic simulator, MITSIMLab. The two models were implemented within the micro-simulator. A methodology for joint estimation of OD demands and aggregate calibration of the model parameters to match observed traffic characteristics was discussed.

The outputs of the two MITSIMLab versions were compared against two sets of real-world data: the estimation dataset collected in Arlington, VA and data collected from a freeway corridor in Southampton, UK.

The integrated model performed consistently better than the independent models. Congestion build-up was faster with the independent models relative to the integrated model. Both models overestimate the observed congestion. Similarly, dissipation of congestion was slowest with the independent models and fastest in the observed data. A possible explanation is that under semi-congested conditions drivers are able to adjust their behavior to avoid speed loss. The integrated model captures some of these effects through behaviors such as short-term planning and acceleration to facilitate lane changing. However, these behaviors are not captured with the independent behavior models.

Chapter 8

Conclusions

This chapter summarizes the research reported in this thesis and highlights the major contributions. Directions for future research are suggested.

8.1 Research Summary

An integrated driving behavior modeling framework was presented. Drivers are assumed to perform short-term plans to accomplish short-term goals. The short-term goal is defined by a target lane, which is the lane the driver perceives as best to be in. A target gap, which the driver intends to use to change lanes, defines the short-term plan. The acceleration the driver applies is adapted to facilitate the short-term plan. This modeling framework supports specification and estimation of models that capture inter-dependencies between lane changing and acceleration behaviors and represent drivers' planning capabilities.

The lane-changing component of this model integrates MLC and DLC considerations into a single model, thus capturing trade-offs between mandatory and discretionary considerations. This integrated approach is justified by the estimation results for the target lane model, which indicate significant trade-offs between discretionary considerations, captured by the lane density, front vehicle speed and spacing and presence of heavy vehicles and tailgaters, and mandatory considerations captured by the impact of the path plan and next exit dummy variables. The significant effect of variables related to the driver's path plan on lane selection demonstrates the impact of travel behavior on driving behavior. The expected maximum utility variable captures the effect of gap acceptance decisions on the target lane choice.

Drivers that target lane changing evaluate the available adjacent gap in the target lane to decide whether they can immediately change lanes or not. The gap acceptance model requires that both the lead gap and the lag gap are acceptable. Their decision is based on comparison of the available gaps to corresponding critical gaps, which are functions of explanatory variables. Critical gaps depend on the subject relative speeds with respect to the lead vehicle and the lag vehicle in the target lane and by the expected maximum utility of target gap choices, which captures the effect of available gaps in the target lane traffic.

If the adjacent gap is rejected the driver chooses a short-term plan to accomplish the desired lane change by selecting a target gap from the available gaps in the target lane traffic. The choice of target gap is affected by the position of the subject relative to the candidate gaps, the effective length of the candidate gaps and the relative speeds of the vehicles that define the gap.

Different acceleration behaviors apply depending on the driver's short-term goal and plan: stay-in-the-lane acceleration, lane changing acceleration and target gap accelerations. Within each one of the acceleration behaviors the driver is assumed to be either in a constrained regime or in an unconstrained regime. A constrained regime applies when the driver is close to the vehicle in front and therefore affected by its behavior. The stimulus-sensitivity framework is adapted for all these acceleration models. The driver reacts to different stimuli in various situations depending on constraints imposed by the driving neighborhood and on the driver's short-term goal and short-term plan. Reaction time and time headway thresholds are explicitly modeled in the acceleration model.

The car following and free-flow models use the specification proposed by Ahmed (1999). The car following model extends the GM nonlinear model. Different sets of parameters are used when the leader is faster than the subject and when it is slower. The free flow model assumes that drivers respond to the difference between their current speed and a desired speed.

Drivers' acceleration behaviors to facilitate lane changing using the target gap depend on their relations with the vehicle in front and the vehicles defining the target gaps. Important variables that capture these behaviors are the distance from the subject's current position to his desired position relative to the target gap and relative speeds with respect to the vehicles that define the target gap. Estimation results show these acceleration behaviors are significantly different from the behaviors of drivers who are not trying to change lanes.

The integrated driving behavior model was validated and compared against a combination of independent lane changing and acceleration models using the microscopic traffic simulator, MITSIMLab. The outputs of the two MITSIMLab versions were compared against two sets of real-world data: the estimation dataset collected in Arlington, VA and data collected from a freeway corridor in Southampton, UK. The integrated model performed consistently better than the independent models. Congestion build-up was faster with the independent models relative to the integrated model. This results along with a model selection test suggest that the independent models should be rejected.

8.2 Contributions

The objective of this research is to improve the modeling of driving behavior and in particular the relations between different behaviors. More reliable simulation of traffic flow requires integrated driving behavior models that capture dependencies and trade-offs between behaviors. This thesis contributes to state-of-the-art in driving behavior modeling in the following respects:

- A framework for integrated driving behavior is proposed. This framework is based on the concepts of short-term goals and short-term plans. Rather than make instantaneous decisions based strictly on current conditions, driver are assumed to conceive and execute short-term plans over a length of time in order to accomplish short-term goals. This behavioral framework captures drivers' planning capabilities and allows decisions to be based on anticipated future conditions.
- Based on these concepts, an integrated driving behavior model structure is developed. This model captures both lane changing and acceleration behaviors. Drivers' short-term goals are defined by their target lane. Drivers that target a lane change but cannot change lanes immediately, choose a short-term plan, and adapt their acceleration behavior to facilitate the lane change. This model structure accounts for

inter-dependencies between the various decisions. The generic structure of the model allows it to be used to specify models for different driving environments, such as freeways (as developed and estimated in this thesis) or urban streets.

- The lane changing portion of the model integrates mandatory and discretionary lane changing considerations in a single model. This approach differs from existing lane changing models, which model mandatory lane changes (MLC) and discretionary lane changes (DLC) separately. The integrated model structure allows trade-offs between mandatory and discretionary considerations, which were previously ignored, to be captured. Moreover, the integrated lane changing model does not require specification of the conditions that trigger MLC situations, which is necessary when separate MLC and DLC models are used. These conditions were not specified and estimated rigorously in the literature, and therefore estimation results and the subsequent applicability of existing lane changing models is restricted to special cases in which MLC situations could be reasonably assumed (e.g. vehicle merging to a freeway from an on-ramp).
- A new driving behavior component within the integrated driving behavior model captures drivers' choice of target gaps for lane changing (short-term plans). In this model, drivers choose gaps in traffic in the target lane that they will use to change lanes.
- New acceleration models are introduced to capture drivers' acceleration behavior to facilitate lane changing using the target gap. Estimation results show that these acceleration behaviors are significantly different from the behaviors of drivers who are not trying to change lanes.
- The parameters of all components of the driving behavior model, including behaviors first introduced in this thesis, are estimated jointly using detailed vehicle trajectory data from all vehicles that were observed in a freeway section. The result is a driving behavior model applicable to the behavior of all freeway traffic, rather than only special cases or specific groups of vehicles (e.g. only vehicles merging to the freeway or vehicle which do not make any lane changes).

- Estimation results of the lane changing model show that variables related to the driver's path plan are significant in lane selection. Thus, demonstrating the effect of travel behavior on driving behavior.
- Estimation results provide a second case in support of contributions made by Ahmed (1999) with respect to acceleration behavior and in particular indicating that enhancements to the GM car following model are significant. The results support Ahmed's non-linear specification of the car following stimulus term over the linear GM specification. Estimation results also assert the important impact of traffic conditions ahead of the vehicle, captured by the density variable, on car following. Similarly, the conclusion that the speed of the subject does not affect car following decelerations is strengthened.

8.3 Directions for future research

The emergence of microscopic traffic simulation tools in the last few years has brought about increasing interest in driving behavior modeling. However, much more remains to be learned about drivers' behavior. Some of the directions in which further research is needed are presented below.

- The main contribution of this thesis is the presentation of a conceptual modeling framework that supports integrated driving behavior modeling and accounts for drivers' planning and anticipation capabilities. This approach is used to develop the structure of a driving behavior model. The same ideas may be applied to model other driving situations, which involve similar behaviors. For example, intersection gap acceptance models assume that drivers are standing at the stop line and decide whether to accept or reject gaps in traffic. However, these models do not capture the effect of gap acceptance on the acceleration behavior of drivers who are approaching the intersection. These drivers may adapt their acceleration in order to be able to accept available gaps.
- Virtually all published estimation results of acceleration and lane changing models are for freeway traffic. Similar models need to be developed for urban streets, in which other factors and considerations such as bus traffic and pedestrians may affect the behavior. While again in this thesis the proposed model structure is used to

estimate a model of freeway driving behavior, the modeling framework and the model structure are general enough to be applied to other environments, including modeling of driving behavior in urban streets. The effects of features of urban streets, such as signals and signs and interactions with pedestrians and other travel modes may be incorporated within the modeling framework as explanatory variables within the selection of short-term goals. Short-term plans and acceleration behaviors that support these plans may also be introduced in the model to capture drivers' response to the different stimuli that arise in urban environments.

- The model proposed in this research does not account for state dependence between observations of a given driver over time. The difficulty in modeling state dependence, described in Section 3.2.1, is a computational one. Further work is needed to find ways to overcome the computational problem.
- The lane changing model assumes that traffic conditions are such that acceptable gaps exist. This may not be the case in heavily congested traffic. Models of forced merging and yielding have been proposed in the literature. These should be integrated into the lane changing model.
- One of the results of this research is the important impact of drivers' path plan on driving behavior. The effects of strategic trip planning choices on the tactical level driving behavior need to be studied further. For example, the trip schedule (e.g. desired arrival time at the destination) is another element in the trip plan. Drivers' adherence to the trip schedule may affect driving behavior.
- Various assumptions made in the specification of the model need to be further studied. For example, the specification presented in this thesis assumes that the reaction time and time headway threshold distributions are identical in the various acceleration behaviors. However, accelerations applied to facilitate lane changing may exhibit shorter reaction times relative to stay-in-the-lane accelerations because these are planned behaviors. Similarly, the target gap acceleration models presented here assume that drivers apply car following behaviors in the constrained regime. However, drivers may consider their lane changing goal even when they are constrained by their leader. The impact of these assumptions and the possibility of relaxing them should be further investigated.

- Target gap acceleration models are formulated in this thesis for the first time. The underlying assumptions and specifications of the different components of these models need to be further studied, with different datasets, under different traffic conditions.
- Currently, the most significant constraint in driving behavior modeling is the data. The availability of detailed vehicle trajectory data is very limited. Due to limitations of the collection equipment, available sets were collected on relatively short freeway segments. Further study of driving behaviors, including some of the research directions proposed above, requires observations from longer sections with more versatile geometric characteristics. In addition to the traditional video and film methods, several technologies seem promising to that end. Instrumented vehicles and GPS systems have the potential to collect data on the behavior of drivers during long time periods. The behavior may be observed on different types of road facilities with different geometric characteristics. Therefore, path plans, trip schedules, behaviors in urban streets and other elements may be observed. Driving simulators may be used to collect data in situations that are otherwise difficult to observe, such as emergency situations, and to control some of the latency in the behavior (e.g. by asking drivers to perform a specific maneuver, thus eliminating the uncertainty in modeling the drivers short-term goal).

Appendix A

Unobserved heterogeneity models for panel data

The driving behavior model presented in this thesis is estimated using panel data. In panel data, an individual is observed making one or more decisions. Unobserved driver and vehicle characteristics introduce correlations between observations obtained from a given driver. Therefore, random terms for different observations may not be i.i.d. distributed. The unobserved heterogeneity model (Heckman 1981) accounts for such correlations. It assumes that these correlations are captured by an unobserved individual specific term υ_n. Conditional on this latent variable, the error terms of different observations are independent. This specification is given by:

$$U_n^d(t) = X_n^d(t)\beta^d + \alpha^d \upsilon_n + \varepsilon_n^d(t) \tag{A.1}$$

Where, $U_n^d(t)$ is the utility of decision *d* to individual *n* at time *t*. $X_n^d(t)$ is a vector of explanatory variable. β^d is a vector of parameters. υ_n is an individual-specific latent variable assumed to be distributed standard normal in the population. α^d is the parameter of υ_n. $\varepsilon_n^d(t)$ is a generic random term with i.i.d. distribution across decisions, time and individuals.

The resulting error structure is given by:

$$cov\left(\varepsilon_n^d(t),\varepsilon_{n'}^{d'}(t')\right)=\begin{cases}\sigma_d^2 & if\ t=t', n=n'\ and\ d=d' \\ 0 & otherwise\end{cases}$$
$$cov\left(\varepsilon_n^d(t),\upsilon_{n'}\right)=0 \qquad \forall\, t,d,n,n' \tag{A.2}$$

$$cov\left(U_n^d(t),U_{n'}^{d'}(t')\right)=\begin{cases}\left(\alpha^d\right)^2+\sigma_d^2 & if\ n=n',\ d=d'\ and\ t=t' \\ \left(\alpha^d\right)^2 & if\ n=n',\ d=d'\ and\ t\neq t' \\ \alpha^d\alpha^{d'} & if\ n=n',\ d\neq d'\ and\ \forall t \\ 0 & otherwise\end{cases} \tag{A.3}$$

Where, σ_d^2 is the variance of $\varepsilon_n^d(t)$. Different assumptions on the distribution of $\varepsilon_n^d(t)$ lead to different choice models (e.g. logit, probit), conditional on υ_n.

Walker (2001) discussed identification and normalization issues related to the specification of various unobserved heterogeneity models. The analysis presented here is based on her findings. Consider an individual making a choice between three alternatives (i=1,2,3) over two time periods (t=1,2). Omitting the index for the individual, the utilities U_{it} are given by:

$$\begin{aligned}U_{11}&=V_{11}+\alpha_1\upsilon+\varepsilon_{11}\\ U_{21}&=V_{21}+\alpha_2\upsilon+\varepsilon_{21}\\ U_{31}&=V_{31}+\alpha_3\upsilon+\varepsilon_{31}\\ U_{12}&=V_{12}+\alpha_1\upsilon+\varepsilon_{12}\\ U_{22}&=V_{21}+\alpha_2\upsilon+\varepsilon_{22}\\ U_{32}&=V_{32}+\alpha_3\upsilon+\varepsilon_{32}\end{aligned} \tag{A.4}$$

Where, V_{it} are the systematic utilities. α_i are alternative specific parameters. υ is the individual specific error term distributed $\upsilon \sim N(0,1)$. ε_{it} are i.i.d. error terms, independent of υ, with standard deviation g.

The variance-covariance matrix of the alternative utilities is given by:

$$\Omega_n = \begin{bmatrix} \alpha_1^2 + g^2 & & & & & \\ \alpha_1\alpha_2 & \alpha_2^2 + g^2 & & & & \\ \alpha_1\alpha_3 & \alpha_2\alpha_3 & \alpha_3^2 + g^2 & & & \\ \alpha_1^2 & \alpha_1\alpha_2 & \alpha_1\alpha_3 & \alpha_1^2 + g^2 & & \\ \alpha_1\alpha_2 & \alpha_2^2 & \alpha_2\alpha_3 & \alpha_1\alpha_2 & \alpha_2^2 + g^2 & \\ \alpha_1\alpha_3 & \alpha_2\alpha_3 & \alpha_3^2 & \alpha_1\alpha_3 & \alpha_2\alpha_3 & \alpha_3^2 + g^2 \end{bmatrix} \tag{A.5}$$

Rewritten in difference form these utilities are given by:

$$\begin{aligned} U_{11} - U_{31} &= (V_{11} - V_{31}) + (\alpha_1 - \alpha_3)\upsilon + \varepsilon_{11} - \varepsilon_{31} \\ U_{21} - U_{31} &= (V_{21} - V_{31}) + (\alpha_2 - \alpha_3)\upsilon + \varepsilon_{21} - \varepsilon_{31} \\ U_{12} - U_{32} &= (V_{12} - V_{32}) + (\alpha_1 - \alpha_3)\upsilon + \varepsilon_{12} - \varepsilon_{32} \\ U_{22} - U_{32} &= (V_{22} - V_{32}) + (\alpha_2 - \alpha_3)\upsilon + \varepsilon_{22} - \varepsilon_{32} \end{aligned} \tag{A.6}$$

The variance-covariance matrix of utility differences is given by:

$$\Omega_{n,\Delta} = \begin{bmatrix} (\alpha_1 - \alpha_3)^2 + 2g^2 & & & \\ (\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3) + g^2 & (\alpha_2 - \alpha_3)^2 + 2g^2 & & \\ (\alpha_1 - \alpha_3)^2 & (\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3) & (\alpha_1 - \alpha_3)^2 + 2g^2 & \\ (\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3) & (\alpha_2 - \alpha_3)^2 & (\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3) + g^2 & (\alpha_2 - \alpha_3)^2 + 2g^2 \end{bmatrix} \tag{A.7}$$

There are six different elements in this matrix:

$$\begin{bmatrix} (\alpha_1-\alpha_3)^2+2g^2 \\ (\alpha_2-\alpha_3)^2+2g^2 \\ (\alpha_1-\alpha_3)^2 \\ (\alpha_2-\alpha_3)^2 \\ (\alpha_1-\alpha_3)(\alpha_2-\alpha_3)+g^2 \\ (\alpha_1-\alpha_3)(\alpha_2-\alpha_3) \end{bmatrix} \tag{A.8}$$

All these elements are functions of the differences $(\alpha_1-\alpha_3)$ and $(\alpha_2-\alpha_3)$. This result also holds for the Jacobian of these six elements with respect to α_1, α_2, α_3 and g:

$$\begin{bmatrix} 2(\alpha_1-\alpha_3) & 0 & -2(\alpha_1-\alpha_3) & 2 \\ 0 & 2(\alpha_2-\alpha_3) & -2(\alpha_2-\alpha_3) & 2 \\ 2(\alpha_1-\alpha_3) & 0 & -2(\alpha_1-\alpha_3) & 0 \\ 0 & 2(\alpha_2-\alpha_3) & -2(\alpha_2-\alpha_3) & 0 \\ (\alpha_2-\alpha_3) & (\alpha_1-\alpha_3) & -(\alpha_1-\alpha_3)-(\alpha_2-\alpha_3) & 1 \\ (\alpha_1-\alpha_3) & (\alpha_1-\alpha_3) & -(\alpha_1-\alpha_3)-(\alpha_2-\alpha_3) & 0 \end{bmatrix} \tag{A.9}$$

Therefore, only these differences may be identified. In general, given a choice model between J alternatives, only the *J-1* differences between the α^d parameters are identified in model estimation. A normalization must be imposed on one of the variance terms. If the conditional model is multinomial logit, the natural normalization is to set one of the terms to zero. However, this normalization may not arbitrary. The following conditions must hold for a normalization to be valid (Walker 2001):

$$\Omega^N_{n,\Delta} = \Omega_{n,\Delta} \tag{A.10}$$

Σ^N_n positive semi-definite. (A.11)

Where, $\Omega_{n,\Delta}^{N}$ and $\Omega_{n,\Delta}$ are the variance-covariance matrices of utility differences of the normalized model and the non-normalized model, respectively. Σ_n^N is the individual specific component of the variance-covariance matrix of the normalized model.

Suppose now that the proposed normalization is $\alpha_3 = 0$.

Clearly, $\alpha_1^* = (\alpha_1 - \alpha_3)$ and $\alpha_2^* = (\alpha_2 - \alpha_3)$ would guarantee that condition (A.10) holds.

Σ_n^N is given by:

$$\Sigma_n^N = \begin{bmatrix} \alpha_1^{*2} & & & & & \\ \alpha_1^*\alpha_2^* & \alpha_2^{*2} & & & & \\ 0 & 0 & 0 & & & \\ \alpha_1^{*2} & \alpha_1^*\alpha_2^* & 0 & \alpha_1^{*2} & & \\ \alpha_1^*\alpha_2^* & \alpha_2^{*2} & 0 & \alpha_1^*\alpha_2^* & \alpha_2^{*2} & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.12}$$

The eigenvalues of this matrix are $\left(2\alpha_1^{*2} + 2\alpha_2^{*2}, 0, 0, 0, 0, 0\right)$, which are all non-negative for every values of α_1^* and α_2^*. Therefore, Σ_n^N is always positive semi-definite and condition (A.11) holds. Therefore, in this case the choice of alternative for normalization is arbitrary.

Appendix B

Gap acceptance expected maximum utilities

In this appendix, mathematical expressions for the expected maximum utility (*EMU*) of the available gaps in the right lane and in the left lane are developed. These variables, which appear in the utility functions of the respective target lane alternatives, capture the impact of gap acceptance decisions on the target lane choice.

For simplicity, the notation for the observation (individual and time) is omitted in the following derivation. The model assumes that both the lead and lag gaps must be acceptable for the driver to change lanes. The probability of accepting the available gap in the target lane is therefore given by:

$$p(change) = p(accept\ lead) \times p(accept\ lag) \tag{B.1}$$

The probabilities of accepting the lead gap and the lag gap are given by:

$$p(accept\ g) = p(U^g_{acc} > U^g_{rej}) = p(U^g > 0) \qquad g = lead, lag \tag{B.2}$$

Where, U^g_{acc} and U^g_{rej} are the utilities of accepting and rejecting gap g, respectively. U^g is the difference between the values of these utilities. In this model it is given by:

$$U^g = ln(G^g) - X^g \beta^g + \varepsilon^g = V^g + \varepsilon^g \tag{B.3}$$

Where, G^g is the length of the available g (lead or lag) gap. X^g and β^g are vectors of explanatory variables and the corresponding parameters. V^g is the systematic utility (difference) of accepting the gap. ε^g are random terms, $\varepsilon^g \sim N(0,\sigma^g)$.

In model estimation only the utility differences can be identified. A natural assumption to set the location of the utilities is that the utility of rejecting the gap is zero:

$$\begin{aligned} U^g_{rej} &= 0 + \varepsilon^g_{rej} \\ U^g_{acc} &= V^g + \varepsilon^g_{acc} \end{aligned} \tag{B.4}$$

Where, ε^g_{acc} and ε^g_{rej} are the random terms associated with accepting and rejecting gap g, respectively. $\varepsilon^g_{acc} \sim N(0,\sigma^g_{acc})$ and $\varepsilon^g_{rej} \sim N(0,\sigma^g_{rej})$ such that:

$$\sigma^{g2} = \sigma^{g\ 2}_{acc} + \sigma^{g\ 2}_{rej} - 2\rho^g_{acc,rej}\sigma^g_{acc}\sigma^g_{rej} \tag{B.5}$$

Where, $\rho^g_{acc,rej}$ is the correlation between the two error terms.

Under these assumptions, Clark's equation (Clark 1961), which calculates the expected value of the greatest of a set of normally distributed random variables, can be used to calculate the expected maximum utility of the gap accepting decision with respect to the g (lead or lag) gap:

$$E(max(U^g_{acc},U^g_{rej})) = V^g\Phi\left(\tfrac{V^g}{\sigma^g}\right) + \sigma^g\phi\left(\tfrac{V^g}{\sigma^g}\right) \tag{B.6}$$

Where, $\phi(.)$ and $\Phi(.)$ denote the standard normal probability density function and cumulative distribution function, respectively.

Finally, since the decisions with respect to the lead gap and the lag gap are assumed to be independent of each other, the overall expected maximum utility of gap acceptance is given by:

$$E(max(U_{gap\ acceptance})) = E(max(U^{lead}_{acc},U^{lead}_{rej})) + E(max(U^{lag}_{acc},U^{lag}_{rej})) \tag{B.7}$$

Appendix C

Estimation results of the independent lane changing and acceleration models

In this appendix, estimation results of the independent lane changing and acceleration models using the Arlington, VA data are presented. These models are based on the model structures proposed by Ahmed (1999).

The lane changing and acceleration models are independent of each other and were estimated separately.

C.1 Acceleration model

Depending on the time headway to the leader, the driver is assumed to be either in a car following regime or in a free-flow regime. Car following applies when the driver is close to the vehicle in front and therefore affected by its behavior. In the free-flow regime the driver tries to attain his desired speed. Reaction time is explicitly modeled.

The specifications of the car following and free-flow models are described in detail in Section 2.1.2. The specifications of the reaction time and time headway threshold distributions are similar to those described in Sections 4.5.5 and 4.5.4, respectively. This specification is identical to the one used in this thesis for the stay-in-the-lane acceleration model. Estimation results for the acceleration model are presented in Table C.1.

Table C.1 - Estimation results for the independent acceleration model

Variable	Parameter value	t-statistic
Car following acceleration		
Constant	0.0270	0.45
Speed, m/sec.	0.364	0.83
Space headway, m.	-0.167	-2.72
Density, veh/km/lane	0.571	2.00
Relative speed, m/sec.	0.525	8.18
$\ln\left(\sigma_{cf,\,acc}\right)$	0.131	12.92
Car following deceleration		
Constant	-0.830	-1.65
Space headway, m.	-0.561	-9.49
Density, veh/km/lane	0.152	0.92
Relative speed, m/sec.	0.825	12.78
$\ln\left(\sigma_{cf,\,dec}\right)$	0.155	15.14
Free-flow acceleration		
Sensitivity constant	0.0788	10.64
$\ln\left(\sigma_{ff}\right)$	0.183	11.86
Desired speed		
Constant	17.546	55.81
Heavy vehicle dummy	-1.345	-1.07
Reaction time distribution		
Constant	-0.124	-1.90
$\ln\left(\sigma_{\tau}\right)$	-0.121	-1.05
Headway threshold distribution		
Constant	2.574	45.78
$\ln\left(\sigma_{h}\right)$	-0.807	-8.41

C.2 The lane changing model

The structure of the lane changing model is shown in Figure C.1. The lane changing process is modeled using three steps: a decision to consider a lane change, choice of a target lane and acceptance of gaps in the target lane. The decision whether or not to respond to an MLC is modeled using a binary logit model. The utility of responding to an MLC situation is a function of the distance to the point where the lane change must be completed. If an MLC situation does not apply or the driver chooses not to respond to it a decision whether to consider a DLC is made. This decision process includes two steps, both modeled with logit models. First, drivers examine their satisfaction with driving conditions in the current lane. Drivers that are not satisfied with driving conditions in the current lane compare the driving conditions of neighboring lanes to the current one and decide whether to change lane or not and the target lane. A gap acceptance model is also included within the lane changing framework.

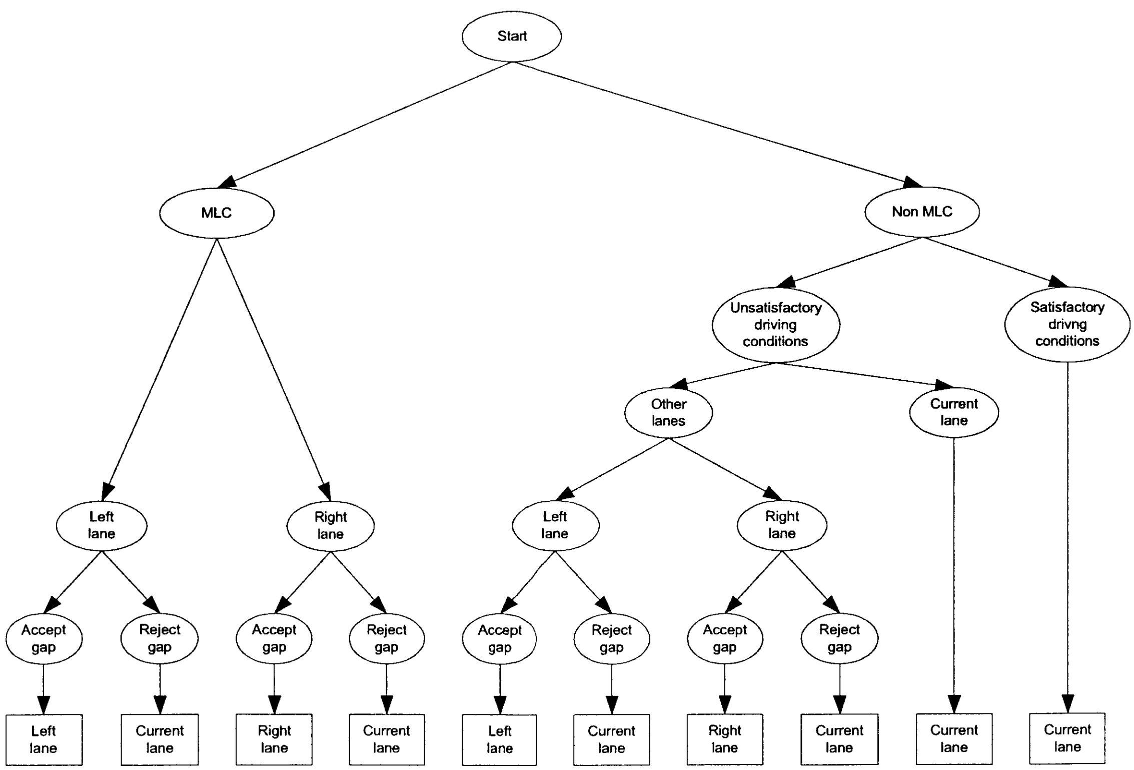


Figure C.1 - Structure of the independent lane changing model

All components of the model were estimated jointly. Estimation results of the target lane choice and gap acceptance components of this model are summarized in Table C.2 and Table C.3, respectively. The definitions of the various variables and functional forms used are identical to those described in Sections 6.2.1 and 6.2.2.

Table C.2 - Estimation results for the independent lane changing model: target lane

Variable	Parameter value	t-statistic
Response to MLC model		
Constant	-3.399	-0.94
Path plan impact	1.133	0.34
θ^{MLC}	-0.584	-0.53
α^{MLC}	0.651	1.44
Utility of unsatisfactory driving conditions		
Constant	-1.409	0.45
Heavy vehicle dummy	-0.485	-1.69
Tailgate dummy	3.570	1.60
$\alpha^{unsatisfactory}$	-2.072	-2.21
Lane utilities		
Current lane constant	-0.377	-0.24
Right lane constant	0.688	0.81
Front speed (current lane only), m/sec.	0.1920	3.44
Lag speed (right and left lane only), m/sec.	-0.171	12.78
Heavy neighbor dummy	-0.127	0.81
Lane density, veh/km/lane	-0.0217	-2.20
Lead vehicle spacing (all lanes), m.	0.0022	2.65
π_1	0.0062	0.52
π_2	0.0361	1.03
α^{CL}	0.089	0.17
α^{RL}	3.832	3.02

Table C.3 - Estimation results for the independent lane changing model: gap acceptance

Variable	Parameter value	t-statistic
Lead critical gap		
Constant	1.729	4.30
Max (0, lead speed - subject speed), m/sec.	-4.873	-1.77
Min (0, lead speed - subject speed), m/sec.	-0.194	-2.40
α^{lead}	0.902	1.68
σ^{lead}	1.027	2.49
Lag critical gap		
Constant	1.352	6.69
Max (0, lag speed - subject speed), m/sec.	0.462	4.61
α^{lag}	0.029	0.12
σ^{lag}	0.691	4.55

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