Mortgagor Behavior and Prepayment Models

by

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Abstract

We measure the efficiency of mortgagors over time and the implications that this increased efficiency may have on prepayments. The problem was approached using two methodologies: a) econometric techniques to measure the refinancing incentive elasticity of prepayments and b) simulations based on a model of the refinancing incentive using system dynamics modeling.

The results suggest that there is a structural change in the prepayment function used in the study over a period of 6 years. However, the results of the simulations model play down the effects that increased mortgagor efficiency may have on prepayments and reaffirms the effect of a sustained downward drift of the mortgage rate as the most important prepayment rates explanatory factor.

Thesis Supervisor: Gordon M. Kaufman
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1. Introduction.

We study mortgagors' behavior over time as they respond to changes in the mortgage rate and trace the implications of these changes on prepayments and on prominent prepayment models used for valuation and forecasts of mortgage backed securities.

Part I of the thesis reviews relevant literature and describes some of the most important prepayment models, characterizing each model in terms of its principal objectives, the variables used and the model's ease of implementation and usefulness from an end-user perspective.

Part II examines, from an econometric perspective, structural changes of the relationship between refinancing incentives and prepayments over a year period using a prepayment model constructed from the basic functional relationships embraced by models reviewed in Part I.

In Part III we develop a simple causal model for the refinancing incentive effect based on techniques from System Dynamics Modeling. A Monte Carlo approach is used to study the underlying causal factors driving the refinancing incentive effect that in turn affects prepayments. Given the discrete nature of the model, we focus on pool age as a proxy for aggregate prepayments over time and we draw our conclusions from the behavior of this variable.

We conclude that there is a structural change in mortgagor's behavior during the period studied and that refinancing incentive elasticity has an upward trend over time. This has important implications for calibration of prepayment models and for forecasts of prepayments. We assert that the dominant factor in observed decreases in pool age is drift in the mortgage rate. Other factors are volatility of rates, refinancing costs and mortgagor efficiency measured as a time delay in the response of mortgagors to changes in the mortgage rate. These factors are included in our study.
2. Review of Prepayment Models

The secondary mortgage market in the US is largely responsible for the liquidity and strength of the real estate market. The importance of this market is evidenced on the size of the secondary mortgage market, which is close to 3 trillion dollars. This is an amazing magnitude considering that this market was born in the late 70’s and early 80’s.

As with any other security, investors want to correctly value and price the securities that they trade, namely mortgage backed securities (MBS). For the most part, the tools used to price MBS are similar to those used to value other fixed income securities: term structure models, cash flow information, yield curve information and discount techniques among others.

A distinctive feature of the MBS is that the mortgagor, and not the lender, holds an option to prepay the outstanding balance of the mortgage at any moment, without incurring in any penalty. This leads to “prepayment risk” for the MBS investor and poses an important problem for the valuation of MBS. Moreover, prepayments themselves cannot be explained with precision. The MBS market suffers from the same problem as option markets prior to Black-Scholes: there is no closed formula for value of a MBS. Some of the difficulties in valuing MBS are:

- **Prepayments reduce expected cash flows**: Prepayments decrease outstanding principal so that the interest and cash flow the investor will receive in the future decreases.

- **MBS cash flows are path dependent**: The path of interest rates influences prepayment. Prepayments depend not only on the current level of interest rates but also on all previous states of the interest rate.

- **Housing turnover influences prepayments**: There is a “normal” level of prepayments due to regional economic activity and people who move for “non-economic” reasons such as divorce, death, retirement, etc.
Prepayment models designed to account for these special features of MBS were developed in order to price MBS. A prepayment model allows the long-term investor to evaluate alternative investments. Other players in the market, such as speculators and dealers, could use a prepayment model to forecast prepayments and make short-term bets on a particular security: If a speculator can forecast correctly the prepayment rate in a given month, he can take long/short positions in that security so as to assure a gain when the prepayment information is available to the public. We believe that the best way to evaluate the prepayment models presented below is to focus on the rationale of the end user of the model. The problem can be posed in two ways: One set of investors may be interested in long-term prepayment forecasts to arrive at an expected value of the MBS. On the other hand, a speculative trader may be interested in short-term prepayment forecasts to decide whether to hold or to sell a position in an MBS security.

We review several prepayment models: Markov Model (Zipkin), Salomon Smith Barney (Hayre), Goldman Sachs (Richard-Roll), Wharton (Zenios-Kang), Andrew Davidson & Co, Schwartz and Tourus (Proportional Hazard), Lacour-Little (Non-Parametric regression), Jegadeesh and Ju (Non parametric GAM model). One drawback is that most of these papers do not give details of the functions that describe the relationship between variables since the models were developed for financial institutions and are therefore proprietary. The Wharton Prepayment Model, the Markov based prepayment model and the logit-regression model are exceptions.

Some of these models reviewed emphasize valuation and don't incorporate detailed causes of prepayments. Others incorporate many variables that may potentially affect prepayments. This latter class of models is better suited for trading/dealer purposes. As with any model of reality there are tradeoffs that must be made: computational simplicity vs. model precision, long-term fundamental relationships vs. periodical recalibration or fit to historical data, focus on major trends vs. focus on the error estimate. We review only a few of the models most relevant for our purposes.
2.1 The Salomon Smith Barney Model

In terms of number of variables involved and the number of relationships among variables, this model is the most comprehensive of the models we review. It is divided into four sub-models: a Refinancing model, a Housing turnover model, a Defaults model and a Curtailments and payoffs model. It is a hybrid model composed of an econometric fit to data and a probabilistic model.

This model is clearly oriented towards a trader/dealer perspective: it incorporates a wide array of variables that can affect prepayments and tries to capture their effects on prepayments with a “prepayment duration” calculation¹. The large number of variables included in the model may lead to pseudo-estimation; i.e. attribution of an increase/decrease of prepayment rates to a change variable, when in fact there is no such effect. For example, parameter fitting or the lack of granularity of the model could prevent a correct measurement of “prepayment duration”.

Variables.
The main variables used in the model can be seen in the graph 1.1. The diagram gives a quick overview of the model and the relationship among the variables.

Advantages and disadvantages.

Refinancing Model.
Most practitioners agree that the refinancing incentive effect is the largest and most volatile effect on prepayments. The Salomon Smith Barney refinancing model is rigorous in the sense that it is based on a probability distribution of the “propensity to prepay” across the population of mortgagors. Changes in this probability density function measure the burnout effect. The refinancing rate for the pool is given by the expected value of the refinancing rate across the population of mortgagors given: a) the refinancing incentive, b) the propensity to prepay of each mortgagor.

¹ Prepayment duration is a measure of the change in the prepayment rate with respect to the change in a particular variable included in the model.
Figure 2.1 Salomon Smith Barney Model

Another factor that increases the refinancing effect is the **media effect**, which in effect changes the distribution of the propensity to prepay of the mortgagors. The media effect addresses the marketing campaigns of the lenders when an important reduction in the mortgage rate occurs.

SSB gives no details about how they derived the probability density functions (p.d.f.) of propensity to prepay. The measure of the refinancing incentive is defined as the present value of savings, expressed in % terms, if the mortgagor takes a new loan at the prevailing market rates.

**Housing Turnover Model.** The **housing turnover model** is not as well defined as the refinancing model. There is a lot of interaction between the variables.

The **main driver of the turnover model** is the ratio of existing home sales to housing stock. The housing stock is taken from census projections that may be a **source of error** in the model. At the same time, the Home sales / stock ratio is driven itself by the
affordability and the desirability factors, where the **affordability factor** is a ratio of median income/median mortgage payment and the **desirability factor** is measured as the changes of the weighted average of nominal home prices, thus effectively incorporating inflation as a variable in the housing turnover.

Affordability and desirability define the major trends in the housing turnover model: **seasoning, lock-in and relative mobility.** Seasoning refers to the speed in which a pool reaches a steady state of prepayments, ceteris paribus. Lock-in refers to the low probability that a prepayment will occur given that the market mortgage rates are higher than the contract mortgage rate. Finally, relative mobility refers to the turnover rates that are embedded in the choices of loan types (fixed, variable, jumbo) that mortgagors make.

**Default Model.** The defaults are not a major component of prepayment speeds. The model uses the Standard default assumptions to forecast prepayments due to this cause.

**Curtailments model.** According to the model, curtailments and full payoffs are very low at the earlier periods of the pool. At the end they tend to increase their weight in monthly prepayments. On one hand, curtailments are a function of the ability to refinance of the mortgagor. If he/she cannot refinance because the costs of refinancing are too high relative to the savings, then the best strategy would be to decrease the principal outstanding. On the other hand curtailments are a function of age, as the mortgagors tend to decrease their risk by decreasing their overall debt.
2.2 The Wharton Model.

Objective and motivation: Zenios and Kang explain the model in detailed form. They describe the variables used, functions to fit the data, and how the data was sorted in order to use the functions and retrieve the parameters for each of the variables. This model is geared to the valuation of MBS, this means that if the model performs 100%, monthly errors will average out over time leaving a value estimate close to true value. We can characterize the method used as Sort-approximate – fit: Sort data, approximate a function and fit the data.

Variables.
This model is very simple in the number of parameters to estimate and the number of variables used. The factors are:

- Refinancing effect: A function of the ratio of Coupon rate/ Mortgage rate (C/R)
- Seasonality effect: A function of the month of year and C/R
- Aging effects: A function of the pool age.
- Seasoning effect: A function of the ratio C/R, the age of the mortgage (t) and the history of C/R up to (t).
- Burnout effect: A function of the ratio C/R, the age of the mortgage (t) and the history of C/R up to (t).

Advantages, Disadvantages.
The modelers first state the general relationship between the factors:

\[
CPR [t, m, (C/R)_0] = \sigma[m, C/R] \cdot \rho[(C/R)_t, C/R] \cdot \sigma[(C/R)_t, t, C/R] \cdot \beta[(C/R)_t, C/R] \cdot \varepsilon
\]

- \( t \) = life of mortgage
- \( m \) = month of year
- \( C \) = mortgage contract rate
- \( R \) = prevailing mortgage rate
- \( s \) = seasonality variation
- \( \rho \) = refinancing incentive
- \( \sigma \) = seasoning of the mortgage
- \( \beta \) = burnout effect for old mortgages.
For the **seasonality factor** they sort the data holding C/R constant and the month of the year constant. They measure the seasonality factor for month \(m\) as the ratio of prepayments for that particular month to the overall prepayments for that C/R ratio, with both terms weighted by the outstanding balance.

For the **refinancing factor** effect first they limit the prepayment rates (CPR) observations to a specific age range. Then they adjust the CPR by \(s(m, C/R)\). After that they sort all observations by C/R and develop intervals of C/R, and fit a line for that interval using **least squares** and using **basis functions** developed for the refinancing factor. In effect they are “fitting” the sorted data using the basis functions that are step like functions. To achieve the best fit, they divide the data into high C/R ratio(Premium) and low C/R ratio (Discounts) and choose the specific age range for these 2 groups. Again they sort and fit. Finally they repeat this process if the CPR’s do not fit the aging constraint:

\[
\sigma((C/R)_t, t, C/R) \cdot \beta((C/R)_t, t, C/R) < 1
\]

and choose the age range again until it meets the constraints. This iterative process of choosing age range-fitting-verifying constraints is the main distinctive procedure of Zenios and Pang approach.

For the **seasoning factor** they remove the previously calculated factors from the CPR. They group the data into five categories, deep discount, discount, par, premium and high premium. They make subgroups by grouping pools that have the same C/R at the same point in time. They are trying to group the data so they can assume a vector of \((C/R)_t\) that is assumed constant. Once this is done, least squares and the basis functions for this effect are used to estimate the fitted parameters or factors.

The **burnout factors** are obtained similarly but using the end of the life of the pool and a decaying exponential basis functions to fit the data.
The drawback of this approach is that the factors are highly dependent on the choice of the age range in the case of the refinancing factor; and the choice of groups and C/R intervals in the case of the seasoning and burnout factors. Since there is evidence that burnout can occur at early stages of the pool age and the authors specifically limit the seasoning to the earlier periods of the pool and the burnout to the latter, parameter values can be misleading.

Golub and Pohlman (1994) describe this iterative procedure to be similar to the Cochrane-Orcutt procedure and say that “if both data and function are well behaved, this stepwise procedure should result in the same solution one would obtain by estimating all coefficients simultaneously”. If such is the case, why not perform the GLS estimation? The Cochrane-Orcutt procedure does not include a choice of the age-range, which seems to be an important part of the Wharton Model. As a consequence, the parameters estimated using Cochrane-Orcutt will very likely yield different results.

In addition to these drawbacks, the factors must be recalibrated frequently because the parameters and functions are approximated and fit to the data. If there are significant changes in the seasoning process or the burnout process as a result of circumstances that
were not evident at the time the data was produced, then the basis functions might not fit the new data.

Finally, another drawback deals with the aging effect. When the grouping of the data was made, the five data groups are identified arbitrarily and there is no reason to believe that these groups represent the right level of granularity to capture the final effects on prepayments.

2.3 The Markov Chains Based Model

Objective or motivation: The focus of this model is to value Mortgage Backed securities. This model is based on a Markov process where the transition probabilities arise from parameters of a term structure model with an interest rate process (Cox-Ingersoll-Ross). Zipkin proposes a methodology based on a mathematical argument to reduce a set of equations that are difficult to evaluate computationally. Based on this methodology he is able to simulate the cash flows of the MBS and value them.

Variables.
- A discrete time, discrete space birth-death Markov Process derived from the Cox-Ingersoll-Ross model.
- The interest rates and the prepayment rates are both functions of this Markov process. The prepayments are a function of a particular interest rate.
- The transition probabilities of the Markov process are derived from the C-I-R model parameters $\kappa, \theta, \sigma$
- The states of the interest rates are from 0.5% to 19.5% increments of 1

Description, Advantages, Disadvantages.
Zipkin starts by expressing the general valuation of a future cash flow in terms of a recursion equation and building on this to express the valuation of an MBS in the same
fashion.

From the basic concepts of bond valuation and arbitrage, Zipkin derives recursive formulas for bond values. Given a constant interest rate, the value of a cash flow \( cf \) maturing in \( \tau \) period is, by definition:

\[
v(\tau) = z^\tau \cdot cf
\]

where \( z \) is a discount factor given by \( z = (1/1+\text{interest rate}) \). Similarly the value of the cash flow 1 period from now and the resulting recursion is:

\[
v(\tau + 1) = z \cdot v(\tau)
\]

A second stage is to derive the same recursive formulas when interest rates follow a stochastic Markov process. The interest rates result follow a Markov process whose possible states describe economic conditions. As in the previous case, it is assumed that there are no defaults on the obligations. The same conditions of certainty for the payment of the cash flows apply. The value of the cash flow \( (cf) \) one period from now and the resulting recursion are:

\[
v(\tau) = z^\tau \cdot cf
\]

\[
v(\tau + 1) = \mathbb{E}[v(\tau+1) \mid v(\tau), P, x_i] \cdot A
\]

where \( P \) is the transition probability matrix for the states \( x \), and \( A=(z_1 p_{ij}) \) is the discount factor matrix computed for each of the states \( i \).

Given the previously stated conditions and no prepayments, Zipkin derives the recursive formulas for mortgage backed securities:

\[
y(t+1) = \left[(1+c) - (c/(1-\beta^\tau))\right] y(t)
\]

where \( \beta \) is the discount factor on mortgage rate, \( \tau = T - t \); \( T \) is the term of the mortgage, \( c = \text{mortgage rate} \) and \( y(t) \) is the remaining principal at the start of period \( t \).

In order to make the model workable, he relaxes the no prepayments condition and reformulates the previous recursion to include prepayments in the dynamics of \( y(t) \), the remaining principal balance, resulting in the following equations:
\[ y(t+1) = [1-\pi(t)] \left[ (1+c) - (c/(1-\beta^t)) \right] y(t) \]

defining \( \pi = \) prepayment rate as function of interest rates. Finally, the recursion is solved resulting in:

\[ y(t) = (1-\beta^t) z(t) \]

where \( z(t) = \left[ \Pi \left[ 1-\pi(s) \right] y(0) \right] / \left[ 1 - \beta^t \right] \); for \( s < t \). We will not discuss the additional complications that arise from including cash flows in the equation. Zipkin’s contribution is in the mathematical techniques used to solve the resulting relationships after the inclusion of the cash flows. In this relationship, the prepayments rates are a function of the specific path of interest rates over time. Therefore we cannot infer anything about underlying causes of prepayments.

The main problem with this model is that prepayment rates are a function of an underlying Markov process which has all the relevant information that determine prepayments rates. The model is a tool for valuating the MBS only after we have knowledge of the relationship between prepayments and interest rates. Simulating the model will give us a prepayment rate trend, which on average might be accurate but it is not very useful for forecasts of prepayments using current data.

2.4 The Goldman Sachs Model (Richard and Roll)

Objective or Motivation: Undoubtedly, the Richard and Roll Model (1989) influenced much of the work on prepayments oriented towards MBS valuation. Their model uses non-linear regression to estimate prepayments as a function of seasoning, the refinancing incentive, burnout and the seasonality factor. This model has an explicit objective: to forecast prepayment rates, given a set of economic conditions.

For Richard and Roll, the value of mortgagors’ payoff when interest rates change is measured by the ratio of the present value of an annuity per dollar of monthly payment calculated at the mortgage coupon rates to the present value of an annuity per dollar of monthly payment calculated at the prevailing rates in the market mortgage. An
approximation is taken with the ratio of the coupon rate to the market rate. Consequently, the value of the option to prepay can be calculated if the costs for refinancing were known. The disparity of costs across mortgagors is what gives rise to the need for prepayment models otherwise the calculations are straightforward.

*Variables used.*

Figure 2.3 shows the main structure of the model. The structure of Wharton Model is similar to the Richard and Roll model since the former was developed as a response to this and other models available in the literature at that time. The CPR is determined by age of the loan, mortgagor costs, coupon and market rates, month of the year and the interaction between these variables.

The advantages of this model are its simplicity and ease of implementation. Richard and Roll generate fitted functions to estimate factors for each of the effects that influence the CPR and relate them in a multiplicative manner to the prepayments. The C/R ratio acts as the "base" and the other effects, seasonality, burnout, seasoning, are calculated as multipliers.

The seasonality factor is derived as a function of age and the refinancing incentive. The same is true for the burnout factor. The refinancing incentive is a function of the ratio of C/R and finally, seasonality factors are calculated for each month. None of the functions to derive these factors are specified in the Richard and Roll paper.

*Advantages and disadvantages.*

The simplicity of implementation of this model is one of the major advantages of this model. If the sample is large enough, the model will give reasonable estimates of coefficients for prepayment equation variables.
The results of this model will depend on how well the future data "behaves" with respect to the data on which model parameters are calibrated. If mortgagors behave differently from the past and increase their response time to the changes in interest rates, the model will lose accuracy. The main problem with this feature of the model is that these changes in mortgagor behavior occur over a long period of time and therefore it is difficult to capture the changes that are relevant at any certain time for the securities being held.
3. Evidence for Mortgagor Efficiency?

We know that not every mortgagor that has a positive refinancing incentive will refinance. This does not imply that those who do not refinance are inefficient since some subset of these mortgagors may have costs greater than the possible savings generated by the incentive. Moreover, in some cases mortgagors with no refinancing incentives may prepay the mortgages because the costs of refinancing become negative (Richard and Roll, 1989).

Most models are based on the premise that mortgagor behavior and efficiency is constant. We assume that calibration of the models is frequent and required every few years. The implications for increased efficiency of mortgagors are important. First, faster refinancing should be expected in the future for all prepayment models and therefore corresponding changes in the price of mortgage backed securities. These securities may be overpriced if the prepayment models are calibrated using data that do not reflect structural changes in the behavior of mortgagors. Second, mortgage bankers would expect a faster response when interest rates are adjusted downward. Third, if taken to the limit, this increased efficiency can cause mortgages to behave more like callable corporate bonds. In place of prepayment models valuation can then be done using binomial trees and some adjustments due to non-efficient exercise of these options caused by mortgagors predilections to move to can move for a variety of reasons.

How can we measure if mortgagors are becoming more efficient when there is a change in the refinancing incentive? We suggest that it can be achieved by measuring the effects that the refinancing incentive has on the prepayment rates, analyzing the data during consecutive periods of time. In this case, the refinancing incentive measure we use is the ratio of weighted average coupon to the prevailing market mortgage rate (Richard and Roll, 1989). Also, the prepayment rate is the fraction of the pool that terminated in a given month or single monthly mortality (SMM) or its annual equivalent, the conditional prepayment rate (CPR).
We look at how elasticities of the refinancing incentive change over a period of six years using 31,504 records of cohorts build from Fannie Mae mortgage pools. We use a series of regression and statistical techniques designed to identify these elasticities and decompose their effects through time.

3.1 The Sample

The sample is comprised of 31504 records from MBS cohorts. Each record contains information about the status of the cohort at any given month from January 1997 until September 2002. The sample is NOT balanced, meaning that the status of each cohort is not tracked through time. Instead we may have any number of records from 1 to 70 for any given cohort. If we add the total original balance of each record in the database and compare this with the sum of total original balances for a selection of 3,000 records, we will find that these 3,000 records account for 84% of the sum of total original balances and also 91% of the sum of the current balances of the records in the database. The rest of the records (approximately 28,000) complete the balances. Because a large number of the cohorts with larger original balances are also the ones with larger current balances, the same case arises when we look at current balances. These same 3,000 records contain 91% of current balances. Figure 3.1 characterizes the data and makes evident the rationale for our selection procedure.

![Records Sorted by Original Balance](image_url)
It is not a coincidence that these records are also the most relevant for the study since a mortgagor with a large loan balance will be more aware of potential savings that might arise from the changes in the mortgage rates. The data presents the same disparity in the variables that are of most interest to us: the refinancing incentive, CPR, age, time and current balances.

The conditional distributions of the data give a sense of the behavior of the average and variance of prepayment rates across different scenarios of explanatory variables such as age, month, etc. The following figures characterize these distributions.

Figure 3.2 Average CPR given a refinancing incentive ratio and month

Conditional Distribution of Average CPR

![Conditional Distribution of Average CPR](image)

Figure 3.2 presents the level of interest rates and also the conditional distribution of the prepayment rates (CPR) during the period of study. The highest CPR averages are concentrated around two clusters: The first from Sep/98-April/99, and the second from Sep/01-Apr/02. Note that the decreasing periods of interest rates are accompanied by the higher C/R averages, as expected.
Figure 3.3 is an 3D surface graph of the standard deviation of CPR, given a refinancing ratio (C/R) and time. In this case, the standard deviation is clustered around the periods of high prepayment activity. There seems to be a conditional heteroskedasticity characteristic in the distribution where the current variance is explained by volatilities in previous periods.

Similar to figure 3.2, figure 3.4 presents iso-contours of the average CPR. This time we account for the current balances in the conditional distribution. Note that we took the log of current balances because of the differences in scale between the cohorts that was discussed earlier. It is interesting to point out the pattern of CPR and current balances: for a lower balance we need a larger refinancing incentive(C/R) to result in a large CPR. It means that large pools will be more affected than small pools, given the same level of CPR.
Figure 3.5. The behavior of the variance follows the pattern of the previous case, in which large variances correspond to large CPR occurrences. The same shape and similar relative heights are observed.
Following the formats of Figures 3.2 and 3.4, figure 3.6 shows a surface graph with the average CPR across refinancing ratios (C/R). We include the weighted average loan age as a factor. The refinancing activity (CPR) seems to decrease with age and the highest activity is concentrate during the first 60 months of pool life. As in figure 3.4, the refinancing activity shifts to higher C/R ratios when the pool life increases above 80 months.

The last graph shows the standard deviation for this conditional distribution. Again the variance follows the refinancing activity (CPR), but in this case the variance seems to be more stable across the loan age factor than the observed variances related to the other two factors.
3.2 Treatment of Seasonality

For all of these graphs, the CPR is adjusted by the seasonality factor before we compute the averages and standard deviation for these graphs. The procedure used to calculate the seasonality factor is the following: The factors in the sample were calculated using a seasonal adjustment with a geometric mean and the current balances as the weights for the observations. For a given range of C/R and a given month we add up the current balances and take the ratio of the total current balances for a given month to the geometric mean for all the months. The sum of the factors is 12.18, which we could scale so the factors add up to 12. They present a higher peak in April than in September. This may be because for the calculations of the factors we limited the sample to cohorts greater than 30 months or seasoned cohorts. The seasonality factors are as follows.
3.3 The specification.

From Richard And Roll(1989) we assume that the model takes the form of:

\[ Y = X_1 \cdot X_2 \cdot X_3 \cdot X_4 \]  

(1)

where \( Y \) = Conditional Prepayment Rate  
\( X_1 \) = Refinancing Incentive  
\( X_2 \) = Seasonality Multiplier  
\( X_3 \) = Seasoning Multiplier  
\( X_4 \) = Burnout Multiplier

We transform this specification into the linear equation.

\[ \ln Y = \ln X_1 + \ln X_2 + \ln X_3 + \ln X_4 \]  

(2)

For the purpose of the study we are most interested in the behavior of the coefficients of the refinancing incentive over time, which in this case reflects the response of mortgagors to different values of the refinancing incentive. After this first stage, we can look at the results and explore what other effects that might be present, namely the seasoning and the refinancing incentive. Since we are also working with very current pools, we hope that the burnout effects are not dominant in these pools and we will be able to see the changes in the refinancing incentive over time. Finally, we mention that we will also look for omitted variables problems. This first specification yielded the following results using Eviews:

\[ \begin{array}{lllll}
\text{Variable} & \text{Coefficient} & \text{Std. Error} & \text{t-Statistic} & \text{Prob} \\
C & 2.29 & 0.01 & 176.06 & 0.0000 \\
\text{LOG(WAC2MTGRADE)} & 2.78 & 0.06 & 47.80 & 0.0000 \\
\text{LOG(SEASONALITY_FAC)} & 0.38 & 0.05 & 7.88 & 0.0000 \\
\end{array} \]

(Table 3.1)

\[ \begin{array}{lll}
R\text{-squared} & 0.41 & \text{Mean dependent var} & 2.70 \\
\text{Adjusted R-squared} & 0.41 & \text{S.D. dependent var} & 0.87 \\
\text{S.E. of regression} & 0.52 & \text{Akaike Info criterion} & 1.52 \\
\text{Sum squared resid} & 802.44 & \text{Schwarz criterion} & 1.53 \\
\text{Log likelihood} & -2278.76 & \text{F-statistic} & 1034.72 \\
\text{Durbin-Watson stat} & 0.71 & \text{Prob(F-statistic)} & 0.00 \\
\end{array} \]
The t-stats are very significant and the signs of the elasticities are as expected: a 1% change in the refinancing incentive will result in a 2.78% increase of the CPR. However, the Durbin Watson statistic indicates the presence of serial correlation. An approach to this problem would be to try to correct using the Cochrane-Orcutt Procedure, but these data are not in time series, so we cannot regress the residuals from the regression using $\varepsilon_t = \rho \varepsilon_{t-1} + \nu_t$ as our regression for the error term. We will touch on this problem later.

3.4 An alternative approach.

A semi log equation (Green, pg 239) was used as well, based on the reading of Radakrishnan(1992), where the logitsmm or the odds ratio is:

$$\logit(smm) = \frac{s_{mm}}{1 - s_{mm}} = X_1 + X_2 + X_3 + X_4$$

(3)

and $s_{mm}$ is the single monthly mortality or the proportion of the pool that prepaid in a given month. The $s_{mm}$ will always be less than one since mortgagors cannot prepay more than the amount owed so this is a valid odds ratio.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-6.75</td>
<td>0.08</td>
<td>-84.75</td>
<td>0.0000</td>
</tr>
<tr>
<td>WAG2MTGRATE</td>
<td>1.70</td>
<td>0.05</td>
<td>35.46</td>
<td>0.0000</td>
</tr>
<tr>
<td>SEASONALITY_FACTOR</td>
<td>0.93</td>
<td>0.05</td>
<td>10.02</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The log-log form is a better fit so we will keep using this specification.

3.5 Other effects

If we go back to our log-log formulation, we can suggest that it would be implausible that the refinancing incentive effects do not have a limit. Beyond a threshold, the effects of the refinancing incentive on prepayments should decrease because people may have
already refinanced, at which stage other effects become the dominant force behind prepayments. This relationship can be captured if we include a squared term in the formulation as follows:

\[ \ln Y = \ln X_1 + \ln X_2 + \ln^2 X_1 \]  

(4)

This formulation yields the following results...

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.30</td>
<td>0.01</td>
<td>173.34</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(WAC2MTGRATE)</td>
<td>4.75</td>
<td>0.12</td>
<td>39.97</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(SEASONALITY_FAC)</td>
<td>0.36</td>
<td>0.04</td>
<td>7.99</td>
<td>0.0000</td>
</tr>
<tr>
<td>TOR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOG(WAC2MTGRATE)^2</td>
<td>-6.49</td>
<td>0.29</td>
<td>-22.50</td>
<td>0.0000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.48</td>
<td>Mean dependent var</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.48</td>
<td>S.D. dependent var</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.48</td>
<td>Akaikoe info criterion</td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>703.97</td>
<td>Schwarz criterion</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-2082.37</td>
<td>F-statistic</td>
<td>925.73</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>0.78</td>
<td>Prob(F-statistic)</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Again all the coefficients are strongly significant and the fit improved considerably. The regression yields a reasonable pattern of CPR when we look at the predicted CPR across increasing refinancing incentives. However we still have a low Durbin Watson statistic which signals the presence of serial correlation.

In a case-exercise, Berndt (1991, pg 93) explores the information that the Durbin Watson statistic can give when used on a cross-sectional data. The exercise suggests that the ordering of the data can lead to a change in the pattern of residuals which is reflected in the Durbin Watson statistic. In our case, the pattern of residuals from regression (3.3) resembled the pattern of the refinancing incentives ratios, that were ordered by year in the following fashion (1996...RefInc1, RefInc2, RefInc3,..., RefIncn), (1997...RefInc1, RefInc2, RefInc3,..., RefIncn), and so on...
We reordered the sample using refinancing incentive as the sort order and ran the regression again resulting in the output (3.4). The Durbin Watson statistic improved considerably, although we still cannot reject the presence of serial correlation. Note that none of the coefficients from our regression changed. A graph of the residuals explains the results clearly. The residuals from output (3.3) show the pattern that caused the low Durbin Watson statistic. The residuals from output (3.4) show the change in the residual pattern that corrected the problem.

<table>
<thead>
<tr>
<th>Dependent Variable: LOG(CPR)</th>
<th>(Table 3.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method: Least Squares</td>
<td></td>
</tr>
<tr>
<td>Date: 04/12/03 Time: 18:59</td>
<td></td>
</tr>
<tr>
<td>Sample: 1 3000</td>
<td></td>
</tr>
<tr>
<td>Included observations: 3000</td>
<td></td>
</tr>
<tr>
<td>White Heteroskedasticity-Consistent Standard Errors &amp; Covariance</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.30</td>
<td>0.01</td>
<td>173.34</td>
<td>0.00</td>
</tr>
<tr>
<td>LOG(WAC2MTGRATE)</td>
<td>4.75</td>
<td>0.12</td>
<td>39.97</td>
<td>0.00</td>
</tr>
<tr>
<td>LOG(SEASONALITY_FAC TOR)</td>
<td>0.36</td>
<td>0.04</td>
<td>7.99</td>
<td>0.00</td>
</tr>
<tr>
<td>LOG(WAC2MTGRATE)**2</td>
<td>-6.49</td>
<td>0.29</td>
<td>-22.50</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| R-squared                         | 0.48        | Mean dependent var | 2.70 |
| Adjusted R-squared                | 0.48        | S.D. dependent var | 0.67 |
| S.E. of regression                | 0.48        | Akaike info criterion | 1.39 |
| Sum squared resid                 | 703.97      | Schwarz criterion | 1.40 |
| Log likelihood                    | -2082.37    | F-statistic | 925.73 |
| Durbin-Watson stat                | 1.76        | Prob(F-statistic) | 0.00 |

Now we can use this specification and run it over the time span of our sample recording the yearly coefficients and comparing their confidence intervals to confirm if the elasticities for the refinancing incentive have change over time.
3.5 Behavior of coefficients through time.

If mortgagors are becoming more efficient, we can expect that the coefficient of Log(Wac2Migr) will increase over time and that this increase will be significant. APPENDIX I contains the results of six regressions for each year from 1997 until 2002 and a summary table.

<table>
<thead>
<tr>
<th>Reg_ID</th>
<th>C</th>
<th>log C/R</th>
<th>log Seasonality</th>
<th>log C/R Squared</th>
<th>Std Errs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>1.852</td>
<td>3.678</td>
<td>-0.033</td>
<td>-2.363</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.088</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.964</td>
</tr>
<tr>
<td>1998</td>
<td>1.614</td>
<td>13.104</td>
<td>0.213</td>
<td>-23.392</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.519</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.124</td>
</tr>
<tr>
<td>1999</td>
<td>2.413</td>
<td>5.982</td>
<td>0.496</td>
<td>-9.786</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.234</td>
</tr>
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<td></td>
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<td></td>
<td>0.098</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.692</td>
</tr>
<tr>
<td>2000</td>
<td>2.169</td>
<td>1.893</td>
<td>0.245</td>
<td>-1.345</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.069</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.704</td>
</tr>
<tr>
<td>2001</td>
<td>2.291</td>
<td>6.800</td>
<td>0.053</td>
<td>-12.824</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.380</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.104</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.894</td>
</tr>
<tr>
<td>2002</td>
<td>2.221</td>
<td>9.944</td>
<td>-0.001</td>
<td>-18.274</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.416</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.213</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.836</td>
</tr>
</tbody>
</table>

To interpret the results we can compute the values for the predicted CPR given a fixed refinancing incentive and seasonality factor. Moreover, it makes sense to fix them at their average value for the entire sample, 1.17 and 1.03 respectively. The results are graphed in figure 3.8. The peak-valley pattern is due to the mortgage rate trends for those particular periods.

![Predicted CPR for @ 1.2 C/R](image)

Figure 3.10

Given these computations, there is an increasing trend for the CPR over time. However we cannot conclude that the coefficients increase in value over time without exploring the alternative hypothesis that the coefficients are constant over time. Hence, we need to look at the coefficients relative to a decreasing or increasing trend using other techniques.
We can test for structural changes in the specified relationship between two different periods with a Chow breakpoint test\(^2\). To perform the Chow-test, we partition the sample into three periods, 1997-98, 1999, 2000-02 and apply the relevant regression based on our model. The test result is distributed as an F-statistic with (4, 2992) degrees of freedom and has critical value is 2.37 at the 95% level. We can reject the hypothesis that the vector of coefficients is the same across time.

<table>
<thead>
<tr>
<th>Chow Breakpoint Test: 880 1380</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-statistic</td>
</tr>
<tr>
<td>Log likelihood ratio</td>
</tr>
</tbody>
</table>

A similar test was performed using the model from regression (3.1) were we only included two variables and the hypothesis was rejected as well at the 95% level.

Given these results for the structural change in the regressions, we can turn to the issues of omitted variables and ask ourselves if their effect will change these results.

3.6 Including the Seasoning Effect

From the residual graph for regression (3.4) in figures 3.8, we see here that there is a clump of residuals in the left part of the graph that precisely contains the refinancing incentives with the lowest refinancing ratios—we remind the reader of the sorting procedure performed for regression (3.4). This may be due to other effects that are not included in the model such as the seasoning effect or the burnout. If we can capture the seasoning effect, for example, we can reaffirm the structural change in the previous section.

\(^2\) The idea of the breakpoint Chow test is to fit the equation separately for each subsample and to see whether there are significant differences in the estimated equations. A significant difference indicates a structural change in the relationship. (EViews Reference Guide, p. 380)
To capture the seasoning effect we need a function that describes a directly proportional relationship between age and seasoning effect. Nonetheless, these effects must be decreasing on the margin as they approach some limit defined by pool age. We also need a shift in the slope of the function as the refinancing incentive increases or decreases.

A linear function of the refinancing incentive and the age is not a natural choice to capture this effect. But there are functions that do describe these movements. For example, Richard and Roll(1989) mention do not describe explicitly in their paper a similar function used for their model.

We make use of the logistic growth function which we think describes this behavior. As an example we graph two logistic functions with different rates of growth. Adapted to our case, this function will tell us how seasoned is the pool as a function of its age and the refinancing incentive of the pool. We included this new calculated variable in the regression using the following relationship to estimate the seasoning effect.

\[
Seasoning = \frac{1}{1 + (360/2) \cdot e^{CPR \cdot AGE}}
\]  
(6)
The results of the regression are as follows:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.34</td>
<td>0.012</td>
<td>202.30</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(WAC2MTGRATE)</td>
<td>4.62</td>
<td>0.11</td>
<td>43.98</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(SEASONALITY_FACCTOR)</td>
<td>0.41</td>
<td>0.05</td>
<td>8.38</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(WAC2MTGRATE)^2</td>
<td>-6.49</td>
<td>0.30</td>
<td>-21.90</td>
<td>0.0000</td>
</tr>
<tr>
<td>(1/(1+180*EXP(WAC2MTGRATE^WALA)))</td>
<td>-4932.53</td>
<td>238.68</td>
<td>-20.67</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared                                      | 0.55        | Mean dependent var | 2.70  |
Adjusted R-squared                             | 0.55        | S.D. dependent var  | 0.67  |
S.E. of regression                             | 0.45        | Akaike info criterion | 1.26 |
Sum squared resid                               | 616.12      | Schwarz criterion   | 1.27  |
Log likelihood                                  | -1882.42    | Durbin-Watson stat  | 1.81  |

All the coefficients are highly significant. We gain considerable explanatory power by including this variable on the regression. Graphically comparing the residuals from (3.4)

![Figure 3.12](image1.png)  ![Figure 3.13](image2.png)

with the residuals from this regression (3.7) we can see a reduction in the residuals, particularly in the beginning periods of the regression in which the seasoning effect is greater.
We can also look at the yearly regressions results. These results—see APPENDIXII— are improved from the ones based on regression (3.4). The table shows the summary results.

<table>
<thead>
<tr>
<th>Reg_ID</th>
<th>In Wac1</th>
<th>Seasonality</th>
<th>In Wac2</th>
<th>In seasonality</th>
<th>In Wac2</th>
<th>Seasonality</th>
<th>In seasonality</th>
<th>In Wac1</th>
<th>Seasonality</th>
<th>In Wac2</th>
<th>Seasonality</th>
<th>In seasonality</th>
<th>DW</th>
<th>R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>1.87</td>
<td>3.69</td>
<td>-0.03</td>
<td>-2.70</td>
<td>-8446.88</td>
<td>0.02</td>
<td>0.25</td>
<td>0.08</td>
<td>0.90</td>
<td>1036.96</td>
<td>1.90</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>1.74</td>
<td>12.01</td>
<td>0.25</td>
<td>-21.44</td>
<td>-5709.54</td>
<td>0.05</td>
<td>0.49</td>
<td>0.09</td>
<td>1.05</td>
<td>599.13</td>
<td>1.04</td>
<td>0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>2.49</td>
<td>5.48</td>
<td>0.59</td>
<td>-9.06</td>
<td>-5906.74</td>
<td>0.02</td>
<td>0.19</td>
<td>0.08</td>
<td>0.57</td>
<td>375.44</td>
<td>1.66</td>
<td>0.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>2.20</td>
<td>1.93</td>
<td>0.28</td>
<td>-2.31</td>
<td>-5235.31</td>
<td>0.01</td>
<td>0.09</td>
<td>0.06</td>
<td>0.52</td>
<td>397.11</td>
<td>2.09</td>
<td>0.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>2.37</td>
<td>6.22</td>
<td>0.10</td>
<td>-11.86</td>
<td>-4241.17</td>
<td>0.04</td>
<td>0.36</td>
<td>0.10</td>
<td>0.85</td>
<td>484.98</td>
<td>2.43</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>2.33</td>
<td>9.16</td>
<td>0.14</td>
<td>-17.13</td>
<td>-3768.38</td>
<td>0.04</td>
<td>0.37</td>
<td>0.19</td>
<td>0.73</td>
<td>304.82</td>
<td>1.27</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now we turned to perform the Chow Break Point test with this new regression. In this case, we have a new critical F-value of 2.21 which compared with the chow test value of 38.3 results in a rejection of the hypothesis of equal vector of coefficients for the same three periods as before.

The inclusion of the seasoning effect did not change our conclusions about the structural changes over time in the relationship expressed by the formulation (3.7) and (3.4). The upward trend leads us to believe that the elasticities are increasing over time and that the direction of this structural change would indicate an increase in mortgagor's efficiency.
4. Simulating mortgagor’s efficiency and its effects on prepayments.

The efficiency theory of markets assumes perfect information of the market participants. With the mortgage markets that prevailed before and even during the 1980’s this efficiency was not possible. The costs of the refinancing process for small mortgagors where too high to make it worth their time, search costs, evaluation of the available options, etc. The evolution of the secondary mortgage market and deregulation made the efficiency goal attainable to the mortgagors by changing the structure of the lending environment and the competitive rules.

The system is now able to pass on to small mortgagors the effects of better mortgage rates at a faster rate. Mortgage bankers offer these new rates because they quickly sell and/or securitize their portfolio of mortgage loans. The bulk of their source of funding is quoted at the new rates allowing them to offer new refinancing to the public. In essence, the source of value is in the service of offering the best rates possible to the mortgagors consequently dividing the added value between them. To appropriate the potential value, the mortgagor must go through the refinancing process and lock in the new rate.

Before the 1990’s, mortgagors were not comfortable with the refinancing process. However, this situation is quickly changing as the cost of completing the process has decreased for bankers and mortgagors alike. What changes more slowly is the perception by mortgagors that the process is cumbersome and time consuming, that the interest rates will not go down further, etc... These perceptions and expectation changes cause delays in the responses to positive changes in the refinancing incentives like lower mortgage rates. If the mortgagors reduce this response time to changes in the interest rate we can infer that they are becoming more efficient. Moreover, the lower the response time the more refinancing we will see on aggregate.

The behavior of refinancings is similar to the behavior of a diffusion processes (Sterman, 1995) by word of mouth and advertising. The number of refinancings can increase dramatically, after a critical mass of people has gone thru the process. For this diffusion
to work, the conditions leading to the beginning of the process should be stable over time. Two periods can exemplify this argument. From April/98 to April/99, as well as from Nov/01 until Apr/02 interest rates maintained a decreasing trend. During these periods, refinancing exploded across the range of refinancing ratios. The same experience is the cause for the current (Early 2003) refinancing activity.

4.1 Modeling the Refinancing Incentive.
A very simplified model can serve to study the behavior of some of the most important variables that play a roll in the refinancing activity. The model was prepared using Vensim Simulation Software and it has three basic components.

4.1.1 The mortgage schedule
The mortgage schedule is a model of the traditional payment schedule of mortgages. All things constant after the moment of signing, this part of the model exhibits the typical movement of the scheduled payments of interest and principal that any mortgage will produce. The principal payments as a proportion of monthly payments are increasing at
an increasing rate and the opposite is true for interest. Standard formulas were used for the equations used in this part of the model.

4.1.2 The mortgage rate movements

This part of the process assumes a geometric random walk for the interest rate movements during a period of 360 months. The parameters were calibrated using data from Fannie Mae of published mortgage market rates during the last five years. The interest rates movements that were replicated during the simulation were credible and within the rage of rates. We used monthly rates instead of annual values and adjusted the growth rate and standard deviation accordingly. A sample run from one of the
simulations shows the confidence intervals for the mortgage rate. The 95% confidence interval shows that most of the rates will fall between the blue areas.

The initial value for the "last rate" parameter was set to equal the initial coupon rate for the mortgage pool so that its initial state is par. Note the negative drift that is characteristic of the last few years or rate environments.

4.1.3 The refinancing decision.

The refinancing decision links the two previous models and is responsible for the dynamics of the process, since mortgagors adapt to new mortgage rate environments by refinancing their mortgage consequently decreasing the pool balance and pool life.

The refinancing decision is based on some simplifying rules about mortgagor behavior. We assume that mortgagors compare the present value of the annuity at the market rate with the present value of the annuity at the coupon rate and identify potential savings that could be captured by refinancing. Afterwards, a decision will be made to refinance the mortgage if the savings from taking this decision at least compensate the costs of doing so.

Some complications arise from the fact that the decision to refinance is not immediate. Mortgagors do not "absorb" the news about changes in the mortgage rate immediately. There is a delay to realize these potential savings. To capture this delay we include a parameter that measures this time delay in the decision to refinance. As usual, reality goes even further than the models, because we are assuming that mortgage rates will not affect this decision further which is an oversimplification that will only work if the delay time is shorter than the time it takes for mortgage rates to increase.
Another complication arises from the heterogeneity of refinancing costs across mortgagors. For the simulation we assume that the refinancing costs follow a normal distribution with mean $5,000 and standard deviation of $3,000. A flatter distribution is assumed because both monetary and non-monetary costs are included.

Finally, we assume that mortgagors will only consider refinancing after a threshold value of 10% above the savings is reached as an arbitrary margin of error imposed on the model. The complete model used in the simulation is shown in figure 4.5.
The model has 3 types of mortgagors: The low, mid and high coupon mortgagors who can have different coupon rates, balances and refinancing costs. In the base case, we choose the same original loan balance of $200,000 for all of them.

4.2 Simulation

For simulation purposes we are interested in the pool age and the market rate behavior. The control variables are the variables that we change during our simulation to verify their effect on the pool age and mortgage rate. Our interest in the mortgage rate is only to maintain realistic scenarios to match the current coupon rates that are available today. On the other hand, pool age reflects the age of the pool at the time the poll reaches zero value. We focus on pool age because this final value has embedded the prepayment rates behavior over a large period of time and because in a simple model of three mortgagors the CPR rates may not be as meaningful. Below is a summary of simulation results.

![Figure 4.6: Interest rate effects on pool age](image)

**Base Case.** For our base case we verify the confidence intervals for the age of the pool. The base case includes simulated values for the seed used to generate the random numbers for the growth in mortgage rates. As a result of this first case, the expected pool age is 115 months or 9.6 years a reasonable approximation considering the simplicity of the model. The graph containing the confidence intervals was presented in figure 4.6.
section whereas the resulting 95% confidence intervals for the pool age are presented in blue.

- **Drift in rates.** In the second case we reduced the drift in half from -0.0022 down to -0.0011. The result was an increase in the lower bound of the pool age consistent with the fact that the probability of a larger pool age is increased with the higher interest rates. With the prevailing mortgage rates, this is a plausible scenario looking forward if one believes that the mortgage rates will not keep declining at the present rate and either stabilize or reverse its slope.

![Figure 4.7](image)

- **Volatility.** Referring to the previous scenario, the next change was in the volatility parameter. We doubled the volatility from 0.0234 to 0.0468 and computed the resulting CI of pool age. Doubling the volatility seems rather extreme, but it is another possible scenario. The effect on the CI was a considerable reduction in the 95% lower bound although the average pool completed 360 months of age. The past scenarios shows that while the drift is the most important effect in terms of prepayments, and confirms the evidence reflected during April/98 to April/99, as well as from Nov/01 until Apr/02.
• **Time to perceive savings.** As modeled, the potential savings from a decrease in rates must first be "absorbed" by mortgagors. The time to perceive savings is measured in months and will delay the prepayments and extend pool life. We increased this parameter from 1 to 6 months and recorded the effects. Referred to our base case scenario, the effect is that both the average pool life is increased and a narrower 95% confidence intervals. This effect is only secondary to the effect that drift had on pool age.

• **Refinancing Costs.** The last effect studied was the effect that refinancing costs had on pool age. In our base case the refinancing costs were static at $5,000 dollars. For what follows, the new base case is defined by a distribution of refinancing costs with mean $5,000 and standard deviation of $3,000. We then performed the simulation allowing
for 3 cases: a shifted and more concentrated distribution, a larger spread and a lower average refinancing cost.

- In the first case we find that the confidence intervals become narrower relative to the base case, with a slight increase in average loan age. This is consistent with our intuition considering we lowered the refinancing costs to half the previous values.

- The next case causes the CI to widen on the long end, basically reflecting the lower probability for negative refinancing costs of mortgagors.

- Finally, in the last case we increased the standard deviation and made the distribution even flatter. The result is not very different from the preceding case, with only a slight narrowing of the confidence intervals.
We are leaving out the possibility of all possible simultaneous combinations of these changes, with a plausible one being the simultaneous decrease in the time to perceive savings and lower refinancing costs. A more realistic calibration is needed to compute realistic scenarios, but more data on mortgagors' costs and behavior is needed for this.
5. Conclusions

Prepayment models reflect the motivations and objectives of the modeler. Even though most of the models studied here can be used for valuation of mortgage-backed securities, some will serve a specific user better than others. For example, a trader taking short-term positions might find that an estimate of prepayment duration with regard to an underlying cause of prepayments is more useful than an estimate of the prepayment vector for the MBS.

A good balance between theoretical models and data specific models must be reached in order to make the model both useful and flexible enough to capture long-term trends in prepayment rates. Very data-specific models require frequent recalibration, whereas more general probabilistic models don’t provide enough detail for some end users.

Using as a starting point the Richard and Roll model for prepayment rates, we studied how changes in the refinancing incentive coefficient changed over a period of six years. For this model, the refinancing incentive coefficient represents the refinancing incentive elasticity of prepayments. Both the trend of the coefficient and a Chow Breakpoint test confirms that this elasticity is increasing over time. As expected, the inclusion of more effects in the formulation, apart from the refinancing incentive, did not change this conclusion. If this elasticity is increasing it means that mortgagors are indeed more efficient.

We studied the implications of this increased efficiency by modeling the refinancing incentive using tools from System Dynamics and Monte-Carlo simulation. Given the refinancing model developed in chapter 4, we can infer the crucial importance of changes in drift rate of the prepayment rates. This effect is much more important than any increase or change in mortgagor efficiency or behavior.

A sustained decrease in the mortgage rate, even on a small scale, can cause a considerable increase in the prepayment rate which had pool age as a proxy for prepayment rate. In other runs of the simulation runs, increasing the volatility of rates had
an important effect on the prepayment rates but again, a secondary one relative to a sustained decrease in the drift rates. We explored the proxy for the increased efficiency of mortgagors, modeled as a decrease in the time it takes mortgagors to perceive the savings that arise from lower interest rates. Although important, this proxy's importance would be greatly decreased if the downward trend of rates is not sustained for a reasonable period of time. In the next and final set of runs, we changed the distribution of the refinancing costs, we found that none of these would impact prepayments as much as the changes in the drift rate.
6. References

6. Golub, Bennet; Pohlman, Lawrence. Mortgage Prepayments and an Analysis of the Wharton Prepayment Model. 6/1/94. Interfaces volume 24, issue 3, pgs 80-90
19. Tuckman, Bruce. Fixed Income Securities. 1/1/02. Wiley and Sons. 512 pgs
**APPENDIX I**

**Dependent Variable: LOG(CPR) Records for 1997**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
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<tr>
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<td>1.852164</td>
<td>0.024637</td>
<td>75.17777</td>
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<tr>
<td>LOG(WAC2MTGRATE)</td>
<td>3.678189</td>
<td>0.268443</td>
<td>13.70194</td>
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<tr>
<td>LOG(SEASONALITY_FACT OR)</td>
<td>-0.033028</td>
<td>0.085529</td>
<td>-0.386162</td>
<td>0.6996</td>
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<tr>
<td>LOG(WAC2MTGRATE)²</td>
<td>-2.383140</td>
<td>0.867695</td>
<td>-2.746517</td>
<td>0.0063</td>
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</tbody>
</table>

**R-squared** 0.606402  Mean dependent var 2.218587
**Adjusted R-squared** 0.603104  S.D. dependent var 0.509866
**S.E. of regression** 0.320562  Akaike info criterion 0.572204
**Sum squared resid** 41.82337  Schwarz criterion 0.611314
**Log likelihood** -113.5879  F-statistic 209.0167
**Durbin-Watson stat** 1.658877  Prob(F-statistic) 0.000000

**Dependent Variable: LOG(CPR) Records for 1998**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
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<td>LOG(SEASONALITY_FACT OR)</td>
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<td>-23.39237</td>
<td>1.520243</td>
<td>-15.38726</td>
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**R-squared** 0.635461  Mean dependent var 2.919132
**Adjusted R-squared** 0.633110  S.D. dependent var 0.646665
**S.E. of regression** 0.391589  Akaike info criterion 0.971793
**Sum squared resid** 71.34032  Schwarz criterion 1.007193
**Log likelihood** -223.8856  F-statistic 270.1951
**Durbin-Watson stat** 1.195521  Prob(F-statistic) 0.000000

**Dependent Variable: LOG(CPR) Records for 1999**

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**R-squared** 0.676572  Mean dependent var 2.801564
**Adjusted R-squared** 0.674612  S.D. dependent var 0.674458
**S.E. of regression** 0.384729  Akaike info criterion 0.935431
**Sum squared resid** 73.26830  Schwarz criterion 0.969200
**Log likelihood** -229.3901  F-statistic 345.1602
**Durbin-Watson stat** 1.478066  Prob(F-statistic) 0.000000
### Dependent Variable: LOG(CPR)  
**Records for 2000**

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<td>LOG(WAC2NTGRATE)</td>
<td>1.892666</td>
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<td>LOG(SEASONALITY_FACT</td>
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R-squared: 0.404254  
Adjusted R-squared: 0.400963  
S.E. of regression: 0.285954  
Akaike info criterion: 0.343312  
Sum squared resid: 44.40094  
Schwarz criterion: 0.372789  
Log likelihood: -89.30882  
F-statistic: 122.8208  
Durbin-Watson stat: 2.086939  
Prob(F-statistic): 0.000000

### Dependent Variable: LOG(CPR)  
**Records for 2001**

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<td>LOG(SEASONALITY_FACT</td>
<td>0.052746</td>
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<td>-12.84231</td>
<td>0.958588</td>
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R-squared: 0.378362  
Adjusted R-squared: 0.375227  
S.E. of regression: 0.449687  
Akaike info criterion: 1.246127  
Sum squared resid: 120.3202  
Schwarz criterion: 1.275478  
Log likelihood: -369.2152  
F-statistic: 120.7161  
Durbin-Watson stat: 2.418305  
Prob(F-statistic): 0.000000

### Dependent Variable: LOG(CPR)  
**Records for 2002**

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<th>Coefficient</th>
<th>Std. Error</th>
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<th>Prob.</th>
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<td>9.944374</td>
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<td>-18.27441</td>
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R-squared: 0.551055  
Adjusted R-squared: 0.546195  
S.E. of regression: 0.383260  
Akaike info criterion: 0.928170  
Sum squared resid: 69.18425  
Schwarz criterion: 0.963237  
Log likelihood: -216.4422  
F-statistic: 192.7086  
Durbin-Watson stat: 1.285871  
Prob(F-statistic): 0.000000

---

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### APPENDIX II

**Dependent Variable:** LOG(CPR)

**Method:** Least Squares

**Date:** 04/21/03  Time: 12:07

**Sample (adjusted):** 78,2793 IF ASOFDATE360>=0 AND ASOFDATE360<360

**Included observations:** 411 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>CoefficientStd. Error</th>
<th>Statistic</th>
<th>Prob.</th>
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<tr>
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<td>-8.145786 0.0000</td>
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**R-squared: 0.661693 Mean dependent var 2.219587**

**Adjusted R-squared: 0.658360 S.D. dependent var 0.509086**

**S.E. of regression: 0.297561 Akaike info criterion:425635**

**Sum squared resid: 35.948247 Schwarz criterion: 0.474533**

**Log likelihood: -82.48029 F-statistic: 198.5231**

**Durbin-Watson stat: 1.897586 Prob(F-statistic): 0.000000**

---

**Dependent Variable:** LOG(CPR)

**Method:** Least Squares

**Date:** 04/21/03  Time: 12:07

**Sample (adjusted):** 439,2986 IF ASOFDATE360>=360 AND ASOFDATE360<360

**Included observations:** 460 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>CoefficientStd. Error</th>
<th>Statistic</th>
<th>Prob.</th>
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</tr>
<tr>
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<td>-9.52987 0.0000</td>
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</table>

**R-squared: 0.695132 Mean dependent var 2.919132**

**Adjusted R-squared: 0.692504 S.D. dependent var 0.646655**

**S.E. of regression: 0.358585 Akaike info criterion:797304**

**Sum squared resid: 59.66276 Schwarz criterion: 0.041554**

**Log likelihood: -181.9579 F-statistic: 264.4926**

**Durbin-Watson stat: 1.042241 Prob(F-statistic): 0.000000**

---

**Dependent Variable:** LOG(CPR)

**Method:** Least Squares

**Date:** 04/21/03  Time: 12:07

**Sample (adjusted):** 30,2963 IF ASOFDATE360>=720 AND ASOFDATE360<720+360

**Included observations:** 499 after adjusting endpoints

<table>
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<th>Variable</th>
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<tr>
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<td>-15.73277 0.0000</td>
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**R-squared: 0.784533 Mean dependent var 2.801564**

**Adjusted R-squared: 0.782788 S.D. dependent var 0.574588**

**S.E. of regression: 0.314338 Akaike info criterion:533272**

**Sum squared resid: 48.81127 Schwarz criterion: 0.575483**

**Log likelihood: -128.0514 F-statistic: 449.6728**

**Durbin-Watson stat: 1.656196 Prob(F-statistic): 0.000000**
### Dependent Variable: LOG(CPR)

**Method:** Least Squares  
**Date:** 04/21/03  
**Time:** 12:07

Sample (adjusted): 1 2553 IF ASOFDATE360>=1080 AND ASOFDATE360<1080+360  
Included observations: 547 after adjusting endpoints

| Variable                      | Coefficient | Std. Error | t-Statistic | Prob.  
<table>
<thead>
<tr>
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<th></th>
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<td>0.0000</td>
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R-squared: 0.548910  
Mean dependent var: 2.171044

Adjusted R-squared: 0.545581  
S.D. dependent var: 0.369461

S.E. of regression: 0.249056  
Akaikes info criterion: 10.66822

Sum squared resid: 33.61970  
Schwartz criterion: 0.169168

Log likelihood: -13.27580  
F-statistic: 164.8833

Durbin-Watson stat: 2.086638  
Prob(F-statistic): 0.000000

---

### Dependent Variable: LOG(CPR)

**Method:** Least Squares  
**Date:** 04/21/03  
**Time:** 12:07

Sample (adjusted): 118 2998 IF ASOFDATE360>=1440 AND ASOFDATE360<1440+360  
Included observations: 599 after adjusting endpoints

| Variable                      | Coefficient | Std. Error | t-Statistic | Prob.  
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.368140</td>
<td>0.035211</td>
<td>67.25486</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(WAC2MTRGATE)</td>
<td>6.220663</td>
<td>0.364168</td>
<td>17.08185</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(SEASONALITY_FACT00.096252)</td>
<td>0.098482</td>
<td>0.977357</td>
<td>0.3288</td>
<td></td>
</tr>
<tr>
<td>LOG(WAC2MTRGATE)^2</td>
<td>-11.86192</td>
<td>0.849662</td>
<td>-13.96404</td>
<td>0.0000</td>
</tr>
<tr>
<td>(1/(1+(180*EXP(WAC2M-1241.1655A))</td>
<td>484.9792</td>
<td>-8.745044</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

R-squared: 0.449267  
Mean dependent var: 2.867017

Adjusted R-squared: 0.445558  
S.D. dependent var: 0.568918

S.E. of regression: 0.423521  
Akaikes info criterion: 1.128358

Sum squared resid: 106.5962  
Schwartz criterion: 1.165046

Log likelihood: -332.9432  
F-statistic: 121.1406

Durbin-Watson stat: 2.434165  
Prob(F-statistic): 0.000000

---

### Dependent Variable: LOG(CPR)

**Method:** Least Squares  
**Date:** 04/21/03  
**Time:** 12:07

Sample (adjusted): 191 3000 IF ASOFDATE360>=1800 AND ASOFDATE360<1800+360  
Included observations: 475 after adjusting endpoints

| Variable                      | Coefficient | Std. Error | t-Statistic | Prob.  
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.333850</td>
<td>0.043374</td>
<td>53.80762</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(WAC2MTRGATE)</td>
<td>9.155555</td>
<td>0.367657</td>
<td>24.90245</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(SEASONALITY_FACT00.135609)</td>
<td>0.185523</td>
<td>0.730954</td>
<td>0.4652</td>
<td></td>
</tr>
<tr>
<td>LOG(WAC2MTRGATE)^2</td>
<td>-17.12782</td>
<td>0.733115</td>
<td>-23.36309</td>
<td>0.0000</td>
</tr>
<tr>
<td>(1/(1+(180*EXP(WAC2M-3768.3751A))</td>
<td>304.8242</td>
<td>-12.36245</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

R-squared: 0.661217  
Mean dependent var: 3.188962

Adjusted R-squared: 0.658334  
S.D. dependent var: 0.570188

S.E. of regression: 0.332873  
Akaikes info criterion: 0.650847

Sum squared resid: 52.20780  
Schwartz criterion: 0.694671

Log likelihood: -149.5761  
F-statistic: 229.3293

Durbin-Watson stat: 1.268270  
Prob(F-statistic): 0.000000