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Some Min-Max Optimization Problems in Infinite-Dimensional Control Systems[1]

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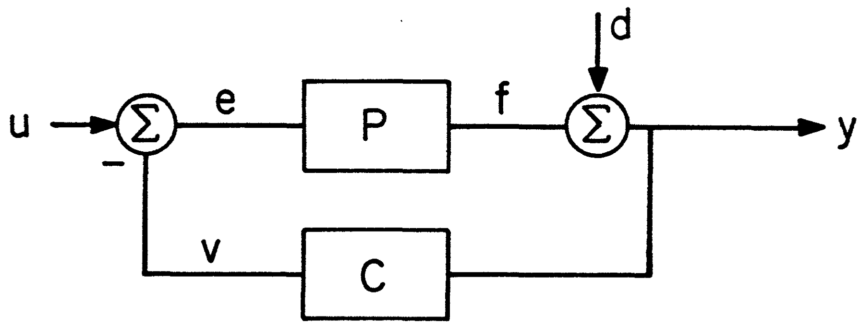
1. Problem Formulation

Consider the following feedback control system:



Here P(s) is a causal transfer function, the plant, and it is required to find a compensator, with causal transfer function C(s) which achieves internal stability and minimizes the weighted sensitivity function in the infinity norm:

$$\|W(s)[1+P(s)C(s)]^{-1}\|_{\infty}, \tag{1.1}$$

where W(s) is a proper, stable, outer rational function, normalized to be 1 at infinity. The plant P(s) is not assumed to be a rational function. In this paper we concentrate on the case

$$P(s) = e^{-S\Delta}P_0(s) \tag{1.2}$$

where $P_0(s)$ is a strictly proper rational function which is outer.

The choice of W as proper (and not strictly proper) makes the problem difficult, since in general there is a high degree of non-uniqueness in the solution.

2. Notation, Mathematical Formulation and Preliminaries

We follow the basic notation of Hoffman [1962] and Garnett [1981].

Let $D = \{z \in C \mid |z|<1\}$ and $\pi^+ = \{z \in C \mid \text{Re } z>0\}$. By $H^p(D)$ or $H^p(\pi^+)$, $1 \le p \le \infty$, we denote the Banach space of holomorphic functions in D or π^+ with the usual norms. Given a $g \in H^p(D)$, let

$$h(w) = (1+w)^{-2/p} g((w-1)(w+1)^{-1}). \tag{2.1}$$

Then $h \in H^p(\pi^+)$. The mapping $g \to h$ is an isometric isomorphism of $H^p(D)$ to

$H^p(\pi^+)$. We shall usually work with $H^p(\pi^+)$ and the argument will be omitted.

We assume the plant admits a co-prime factorization:

$$\begin{cases} P(s) = \varphi(s)\phi^{-1}(s), \quad \varphi,\phi \varepsilon H^\infty \\ \exists\ a,b,\varepsilon H^\infty, \text{ such that, } a\varphi+b\phi = 1. \end{cases} \tag{2.2}$$

By the Corona Theorem (cf. Garnett [1981], p. 324) this will be true if, there exists a $\delta>0$, such that

$$|\varphi(w)| + |\phi(w)| \geq \delta \quad \forall\ w\varepsilon\pi^+. \tag{2.3}$$

We also assume that P(s) is the transfer function corresponding to a causal, time-invariant, linear bounded operator

$$T:L_e^2(R) \rightarrow L_e^2(R), \tag{2.4}$$

where L_e^2 denotes the extended L^2-space (cf. for example Desoer and Vidyasagar [1975]. A characterization of such transfer functions can be given in terms of the Paley-Wiener-Schwartz theorem (c.f. Segal [1955], Theorem 3).

The Youla parametrization of compensators achieving stability is then valid for this more general class. We say that the causal feedback compensator C(s) renders the feedback system stable of

$$u,d\varepsilon L^2 \rightarrow e,v,f,y\varepsilon L^2. \tag{2.5}$$

Such a compensator will be termed admissible and (2.5) is true if and only if

$$(1+P(s)C(s))^{-1},\ P(s)(1+P(s)C(s))^{-1},\ C(s)(1+P(s)C(s))^{-1}\ \varepsilon\ H^\infty. \tag{2.6}$$

Note however internal asymptotic stability of the canonical state-space realizations corresponding to each of these transfer functions is not necessarily guaranteed in this general setting. Some form of exact controllability and observability conditions will have to be verified for this to be true. We then have

Proposition 2.1

C is an admissible controller $\longleftrightarrow \exists$ $Z \varepsilon H^{\infty}$, $Z \neq b\varphi^{-1}$ such that

$$C = \frac{a+\phi Z}{b-\varphi Z}. \tag{2.7}$$

□

As is well known, the original sensitivity minimization problem does not admit a solution and it is necessary to consider a relaxed problem. In the standard way, we consider the relaxed problem

$$\operatorname*{Min}_{h \varepsilon H^{\infty}} ||W+\varphi h||_{\infty}, \tag{2.8}$$

where $W \varepsilon H^{\infty}$ and is outer and $\varphi \varepsilon H^{\infty}$ inner. (2.8) is equivalent to the problem of minimizing the distance

$$d(\bar{\varphi}W,\ H^{\infty}), \tag{2.9}$$

where $\bar{\varphi}$ denotes the conjugate of φ.

If W is proper and not strictly proper, $\bar{\varphi}W \varepsilon L^{\infty}$ and in general there is no unique solution. If W is strictly proper, $\bar{\varphi}W \varepsilon H^{\infty}+C$ ($H^{\infty}+C$ = set of functions f+g, $f \varepsilon H^{\infty}$, $g \varepsilon C$ (the space of continuous functions)), then there is a unique solution (c.f. Garnett [1981], Theorem 1.7). If φ has an essential singularity on the imaginary axis at the point μ then $\bar{\varphi}W \varepsilon H^{\infty}+C \longleftrightarrow$ $W(\mu) = 0$. In this case there is a unique solution to (2.8).

3. Generalized Interpolation and the Theory of Hankel Operators

The results of this section are contained in Sarason [1967, 1985] and Adamjan, Arov and Krein [1968].

Let H be a Hilbert space and K a closed subspace. Let $S \varepsilon \mathscr{L}(H)$, the space of bounded linear operators from H to H. Let

$$T = P_K S|_K. \tag{3.1}$$

T is the compression of S on K and S is the dilation of T on H. Now let

$H=H^2$ and $K=(\varphi H^2)^{\perp}$ where φ is an inner function. Let $W\varepsilon H^{\infty}$ and let M_W be the corresponding multiplication operator on H^2. Clearly $M_W \ \varepsilon\ \mathscr{L}(H^2)$ and $||M_W||_{\mathscr{L}(H^2)} = ||W||_{\infty}$.

Let $T = P_K M_W|_K$. W is termed the interpolating symbol of T. Consider the semigroup of translation $s \rightarrow e^{-s\Delta}$ and the compressions $S_\Delta = P_K M_{e^{-s\Delta}}|_K$. Then,

<u>Proposition 3.1</u>. $T\varepsilon\mathscr{L}(K)$, $TS_\Delta = S_\Delta T\ \ \forall\Delta>0 \rightarrow \exists\ Z\varepsilon H^{\infty}$ s.t. $T = P_K M_Z|_K$ and $||T||_{\mathscr{L}(K)} = ||Z||_{\infty}$. □

Z is the minimal interpolating symbol of T. Returning to the relaxed sensitivity minimization problem (2.8), if we take $K = (\varphi H^2)$, $T = P_K M_W|_K$, then

$$\min_{h\varepsilon H^{\infty}} ||W+\varphi h||_{\infty} = ||T||_{\mathscr{L}(K)}. \tag{3.2}$$

□

Let $f\varepsilon K$ and $f\neq 0$. f is called a maximal vector of T of

$$||Tf||=||T||.||f|| \tag{3.3}$$

<u>Proposition 3.2</u>. Let f be a maximal vector T. Then

$$\phi = \frac{Tf}{f}$$

is a minimal interpolating symbol of T, which is a constant times an inner function.

It is clear that T has a maximal vector iff T^*T has a maximal eigenvalue, and $\rho(T^*T) = ||T^*T|| = ||T||^2$ and hence $||T|| = (\rho(T^*T))^{1/2}$. Finally if T is compact then T has a maximal vector and T is compact $\longleftrightarrow \bar{\varphi}W\varepsilon H^{\infty}+C$.

An operator $\mathscr{H}: H^2 \rightarrow (H^2)^{\perp}$ is a Hankel operator $\longleftrightarrow \exists$ a symbol $\phi\varepsilon L^{\infty}$ such that $\mathscr{H}=P\_M_\phi|_{H2}$, where P_ is the projection on $(H^2)^{\perp}$. If $\mathscr{H}$ is a Hankel operator with symbol ϕ, then ϕ' is a symbol of $\mathscr{H} \longleftrightarrow \phi-\phi'\ \varepsilon\ H^{\infty}$. The operator $T = P_K M_W|_K$ defines a Hankel operator $\mathscr{H}= P\_M_{\bar{\varphi}}T$, with symbol $\bar{\varphi}W$. Clearly ϕ is a symbol of $\mathscr{H} \longleftrightarrow \varphi\phi$ is a symbol of T.

Let $\mathscr{H}$ be a Hankel operator with $||\mathscr{H}|| = s$. Then on $H^2(D)$, we have the following criterion for the uniqueness of the minimal symbol of $\mathscr{H}$.

Theorem 3.3. H has a unique minimal symbol $\longleftrightarrow$

$$\lim_{\rho \downarrow s} \langle (\rho^2 I - \mathcal{H}^* \mathcal{H})^{-1} .1, 1 \rangle = \infty,$$

where 1 is the function identically 1 on $H^2(D)$. □

The function 1 transforms to $\frac{1}{s+1}$ on $H^2(\pi^+)$. In case there is no unique symbol, one can obtain a parametrization of all minimal symbols of by the following generalization of the Schur algorithm for interpolation. Let

$$\mathcal{H}_\varepsilon = (1-\varepsilon)\mathcal{H},\ 0<\varepsilon<1. \quad \text{Let} \tag{3.4}$$

$$q_\varepsilon = \| (I - \mathcal{H}_\varepsilon^* \mathcal{H}_\varepsilon)^{-1/2} 1 \|_2^{-1} (I - \mathcal{H}_\varepsilon^* \mathcal{H}_\varepsilon)^{-1} 1.$$

$$r_\varepsilon = \overline{\mathcal{H}_\varepsilon q_\varepsilon}$$

The sequences $\{q_\varepsilon\}$ and $\{r_\varepsilon\}$ are uniformly bounded on compacts of D and hence there exists a sequence $\varepsilon_n \to 0$ such that q_ε r_ε converge uniformly on the compacts of D and hence to q, r in the space of holomorphic functions on D. Let

$$U = \begin{pmatrix} \overline{q} & \overline{r} \\ r & q \end{pmatrix}. \tag{3.5}$$

Theorem 3.4. ϕ is a minimal symbol for $\mathcal{H} \longleftrightarrow \exists\ \varphi \varepsilon \mathcal{B}(H^\infty)$ (the unit ball of $H^\infty(D)$) such that

$$\phi = U\varphi = \frac{\overline{q}\varphi + \overline{r}}{r\varphi + q}$$

ϕ is unimochular $\longleftrightarrow \varphi$ is inner. □

On $H^2(\pi^+)$, we have to use q' and r' in the definition of U, where

$q = (s+1)q'$

$r = (s+1)r'$.

4. Sensitivity Minimization for Delay Systems.

This section follows Flamm [1985], Flamm and Mitter [1986], and Fagnani [1986]. See also Foias, Tannenbaum and Zames [1986].

In the first instance, we take the plant to be $P(s) = e^{-s\Delta}$, and the weighting function $W(s) = n(s)d^{-1}(s)$, with $n(s) = \prod_{i=1}^{n}(s+a_i)$ and $d(s) = \prod_{i=1}^{n}(s+b_i)$, a_i, $b_i > 0$. In this case $K = (e^{-s\Delta}H^2)^{\perp}$ and $T = P_K M_W|_K$. It is easy to show that

$$\mathscr{L}^{-1}(K) = L^2(0,\Delta) \subseteq L^2(0,\infty), \text{ where}$$

$\mathscr{L}$ is the Laplace Transform considered as a unitary operator between $L^2(0,\infty)$ and H^2 and $\mathscr{L}^{-1}$ denotes the inverse Laplace Transform. Defining

$$V = \mathscr{L}^{-1}T\mathscr{L}\Big|_{L^2(0,\Delta)}, \text{ we see that } V \in \mathscr{L}(L^2(0,\Delta))$$

and

$$V = I+S \text{ where } S \text{ is the Volterra operator} \tag{4.1}$$

$$(Sf)(t) = \int_0^t G(t-s)f(s)ds \quad \forall t \in [0,\Delta] \text{ and} \tag{4.2}$$

$$G(t) = \mathscr{L}^{-1}(W(s)^{-1}). \tag{4.3}$$

The kernel $G(t)$ has the explicit representation

$$G(t) = \sum_{i=1}^{n} \alpha_i e^{-b_j t}, \quad \text{where} \tag{4.4}$$

$$\alpha_j = [n(-b_j)-d(-b_j)] \prod_{i\neq j}(b_i - b_j)^{-1}. \tag{4.5}$$

Since T and V are unitarily equivalent, in order to calculate the maximal eigenvalue of T^*T (if it exists) it is useful to obtain a state-space realization of V^*V.

Proposition 4.1. The operator V^*V is realized as the map $f \rightarrow z$, where

$$\begin{cases} \dot{x}_1 = -Dx_1 + \alpha' f; \quad x_1(0) = 0 \\ y = c'x_1 + f \end{cases} \tag{4.6}$$

$$\begin{cases} \dot{x}_2 = Dx_2 - \alpha' y; \quad x_2(\Delta) = 0 \\ z = c'x_2 + y , \end{cases}$$

where $D = \text{diag.}\ (b_1,\ldots,b_n)$ $c = (1,1,\ldots,1)'$ and $\alpha = (\alpha_1,\ldots,\alpha_n)'$. □

Now,

$$V^*V = I + (S+S^*+S^*S)$$

$$= \text{Identity + Compact operator, and hence}$$

$$\sigma(V^*V) = 1+\sigma(S+S^*+S^*S).$$

Therefore, the spectrum of V^*V consists of a succession of eigenvalues with a possible point of accumulation at 1 and the point 1 whose spectral type is not known a priori.

From this we may arrive at the following criteria for the existence of maximal eigenvalues:

(i) If $U = S+S^*+S^*S$ is a non-negative operator, then V^*V has a maximal eigenvalue.

(ii) If $\exists\ M\geq 0$ such that $|W(i\omega)|>1$, $\forall\omega\geq M$, then V^*V has a maximal eigenvalue greater than 1.

Now,

$$|W(i\omega)| > 1 \leftrightarrow \prod_{i=1}^{n} (\omega^2+a_i^2) > \prod_{i=1}^{n} (\omega^2+b_i^2), \tag{4.7}$$

and we may conclude that

$$\sum_{i=1}^{n} a_i^2 > \sum_{i=1}^{n} b_i^2 \rightarrow V^*V \text{ has a maximal eigenvalue.} \tag{4.8}$$

Also,

$$\sum_{i=1}^{n} a_i \geq \sum_{i=1}^{n} b_i \rightarrow V^*V \text{ has a maximal eigenvalue.} \tag{4.9}$$

Let us denote by λ^2 the maximal eigenvalue and g the corresponding eigenvector. Then from the theory of Sarason it follows that the optimal sensitivity is given by

$$\chi = \frac{\mathscr{L}(Vg)}{\mathscr{L}(g)} . \tag{4.10}$$

Indeed, we have the following expression for the optimal sensitivity.

<u>Proposition 4.2</u>. There exist polynomials v,c of degree (n-1) such that $v\tilde{v} = \lambda^2 c\tilde{c}$ and

$$\chi(s) = \lambda^2 \frac{n(s)c(s) - e^{-s\Delta}\tilde{d}(s)v(s)}{\lambda^2 d(s)c(s) - e^{-s\Delta}\tilde{n}(s)v(s)} , \tag{4.11}$$

where for a polynomial P(s), $\tilde{P}(s) = (-1)^{\text{degree } P}\, P(-s)$.

<u>Proof</u>: From (4.6)

$$\hat{f}(s) = \mathscr{L}(f(s)) = \frac{c'(sI-D)^{-1}x_2(0)}{\lambda^2-(I-c'(sI-D)^{-1}b)(I+c'(sI+D)^{-1}b)}$$

$$= \frac{c(s)d(s)}{\lambda^2 d(s)\tilde{d}(s)-n(s)\tilde{n}(s)}$$

where c(s) is a polynomial of degree (n-1), and hence

$$f(t) = \sum_{i=1}^{n} (\alpha_i e^{\beta_i t} + \tilde{\alpha}_i e^{-\beta_i t}), \tag{4.12}$$

where β_i are the solutions of

$$\lambda^2 d\tilde{d}-n\tilde{n} = 0 \text{ and } \alpha_i, \tilde{\alpha}_i \tag{4.13}$$

are appropriate coefficients.

The eigenvalues of V^*V are obtained by restricting the functions given by (4.12) to the interval $[0,\Delta]$ and are of the form

$$F(s) = \mathscr{L}(f|_{[0,\Delta]}) = \frac{cd-e^{-s\Delta}b}{\lambda^2 d\tilde{d}-n\tilde{n}}, \tag{4.14}$$

where b is an appropriate polynomial. One can then compute

$$Vf = \sum_{i=1}^{n} (\alpha_i W(\beta_i)e^{\beta_i t} + \tilde{\alpha}_i \tilde{W}(\beta_i)e^{-\beta_i t}). \tag{4.15}$$

By computing, $V^*Vf = Vf + S^*Vf$, and imposing the condition $V^*Vf = \lambda^2 f$, we obtain

$$\sum_{i=1}^{n} \frac{\alpha_i W(\beta_i)e^{\beta_i}}{-b_j+\beta_i} + \frac{\tilde{\alpha}_i \tilde{W}(\beta_i)e^{-\beta_i}}{-b_j-\beta_i} = 0 \quad \forall j. \tag{4.16}$$

By computing, $\mathscr{L}((Vf))$, one obtains the expression for the optimal sensitivity

$$\chi(s) = \frac{n(s)c(s)-e^{-s\Delta}v(s)\tilde{d}(s)}{d(s)c(s)-e^{-s\Delta}b(s)} . \qquad (4.17)$$

It remains to find the relation between the polynomials c,v and b. It can be shown

$$b(s) = \frac{1}{\lambda^2}\tilde{n}(s)v(s), \text{ and} \qquad (4.18)$$

$$v(s)\tilde{v}(s) = \lambda^2 c(s)\tilde{c}(s), \qquad (4.19)$$

which concludes the proof. □

4.1 The One-Pole One-Zero Case

In this case $W(s) = \frac{s+1}{s+\beta}$. We have the following theorem.

Theorem 4.3. If $\beta<1$, then there exists a maximal vector and the optimal sensitivity is given by

$$\chi(s) = \lambda\,\frac{n-\lambda e^{-s\Delta}\tilde{d}}{\lambda d-e^{-s\Delta}\tilde{n}} . \qquad (4.20)$$

If $\beta>1$, no unique solution exists. All minimal symbols are given by

$$\chi_\phi(s) = \frac{n(s)-e^{-s\Delta}\tilde{d}(s)\phi(s)}{d(s)-e^{-s\Delta}\tilde{n}(s)\phi(s)}, \quad \phi \in \mathscr{B}(H^\infty). \qquad (4.21)$$

χ_ϕ is inner $\longleftrightarrow \phi$ inner.

Proof. The first part of the theorem follows from Proposition 4.2. The second part of the theorem follows from Theorem 4.3 and 4.4. □

Remark 4.4. If we take $\phi=0$ in (4.14) then we recover $W(s) = \frac{n(s)}{d(s)}$ as a minimal symbol and this corresponds to applying open-loop control. By taking $\phi=1$, we obtain the minimal symbol

$$\chi(s) = \frac{n(s)-e^{-s\Delta}\tilde{d}(s)}{d(s)-e^{-s\Delta}\tilde{n}(s)},$$

which is inner.

4.2 The Case where $W(s) = \frac{(s+1)^2}{(s+\alpha)(s+\beta)}$, $\alpha, \beta > 0$.

This case is far more complicated. If $\alpha+\beta \geq 2$, we obtain a unique solution. This follows by applying the criterion (4.9). Now

$$||W||_\infty = 1 \longleftrightarrow \alpha\beta \geq 1.$$

In this case, one can show that W is a minimal symbol. Now assuming $\alpha \neq 1$, $\beta \neq 1$, we obtain the following theorem which is the analog of Theorem 4.3

Theorem 4.4. $\alpha\beta \geq 1 \longrightarrow$ no unique solution exists. All minimal symbols are given by

$$\chi_\phi = \frac{n-e^{-s\Delta}\tilde{d}\varphi_\phi}{d-e^{-s\Delta}\tilde{n}\varphi_\phi}, \text{ where} \tag{4.22}$$

$$\chi_\phi = \frac{b(s+1) + \tilde{V}\phi}{V+b(s-1)\phi}, \ \phi \in \mathscr{B}(H^\infty), \text{ and} \tag{4.23}$$

V is a polynomial of degree 1 and b is a constant.

Proof. The proof of this theorem follows from a detailed application of Theorems 4.3 and 4.4. □

Remark 4.5. It should be noted that in the above φ does not necessarily lie in H^∞. □

Remark 4.6. One may conclude that in the "neighbourhood" of $\alpha+\beta=2$, a unique solution exists. However in the region $\{(\alpha,\beta)\,|\,\alpha+\beta>2,\ \alpha\beta<1\}$, it is not known whether the solution is unique or non-unique. It appears to be very difficult to calculate the norm of the operator T in this region.

5. Concluding Remarks.

A far more general theory can be obtained when we combine the ideas presented in this paper (and earlier in Flamm [1986], Flamm and Mitter [1985], [1986]) with the Scattering Theory ideas implicit in Adamjan, Arov and Krein [1968] and the theory of realizations of infinite-dimensional systems, as for example discussed in Fuhrmann [1981]. The basis for these ideas are the following:

We define:

$$K = H^2 \ominus \varphi H^2 \doteq (\varphi H^2)^{\perp} \quad \text{(as before)},$$

$$K^{\#} = (H^2)^{\perp} \ominus \bar{\varphi}(H^2)^{\perp} \doteq (\bar{\varphi}(H^2)^{\perp})^{\perp}.$$

We have the decomposition:

$$L^2 = \varphi H^2 \oplus K \oplus K^{\#} \oplus \bar{\varphi}(H^2)^{\perp}.$$

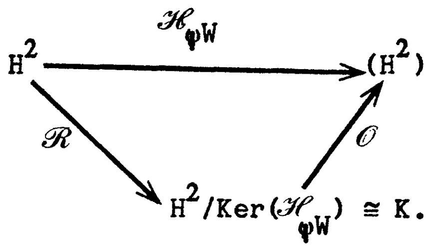
Define the Hankel operator with symbol $\bar{\varphi}W$

$$\mathscr{H}_{\bar{\varphi}W} : H^2 \to (H^2)^{\perp}.$$

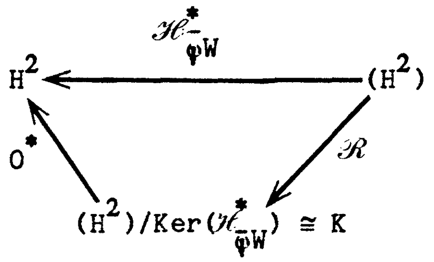
Then since $W \in H^{\infty}$ is proper (but not strictly proper) and outer, $W^{-1} \in H^{\infty}$. Hence

$$\mathrm{Ker}(\mathscr{H}_{\bar{\varphi}W}) = \varphi H^2$$

Therefore we have the canonical factorization



where $\mathcal{R}$ and $\mathcal{O}$ are the reachability and observability operators. Moreover $_{\bar{\varphi}W}$ with domain restricted to K has image $K^{\#}$. We also have $\mathcal{H}^*_{\bar{\varphi}W}: (H^2 \to H^2$, with symbol $\varphi\bar{W}$, with domain restricted to $K^{\#}$ has range K and the diagram



To conform to systems theory, one should regard $\mathcal{H}^*_{\bar{\varphi}W} = P_+(\bar{W}\varphi f)$ as a causal operator and $\mathcal{H}_{\bar{\varphi}W}$ as an anti-causal operator. The canonical state space realizations corresponding to these two Hankel operators will be exactly controllable and observable (at least when Range $(\mathcal{H}_{\bar{\varphi}W})$ is closed, which can be ensured by a Corona condition on the pair (φ, W)). Finally, $P_K \mathcal{H}^*_{\bar{\varphi}W}\mathcal{H}_{\bar{\varphi}W}P_K = T^*T$, and therefore the eigenvalue problem for T^*T is the same as the eigenvalue problem for $\mathcal{H}^*_{\bar{\varphi}W}\mathcal{H}_{\bar{\varphi}W}$ considered as an operator from K to K. One should therefore work with the realizations of $\mathcal{H}_{\bar{\varphi}W}$ and $\mathcal{H}^*_{\bar{\varphi}W}$ instead of T and T^* as done in this paper. It is also clear that a large part of the state space constructions of Glover [1984] admit a generalization to this setting.

The details of these ideas will be presented elsewhere.

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