Understanding Estimation and its Relation to Engineering Education

by

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ABSTRACT

A wide variety of engineering activities benefit from the use of rough estimates of the type commonly referred to as back-of-the-envelope calculations. These include evaluating the feasibility of an idea, planning experiments, sizing components, and setting up and checking detailed analyses. The overall goals of this thesis were to understand how people make rough estimates for physical quantities and to understand how that activity relates to undergraduate engineering education.

The specific objectives of this thesis were to describe the nature and extent of mechanical engineering students' estimation capabilities, to develop a framework describing estimation activity and to characterize the relationship between rough estimation activities and learning activities. The intent of these objectives was to develop conceptual knowledge useful for assessing and teaching rough estimation skills as well as for guiding estimation activity in practice.

Students were found to have considerable difficulty making estimates for common engineering quantities, such as force and energy. Students were also found to have difficulty applying basic engineering concepts in rough estimation situations even at the senior level. In order to identify concepts that give students difficulty, a new assessment method based on students' ability to associate correct units with common engineering quantities was developed.

The mediated action framework that was developed consists of three components: effective actions people take when they make estimates, mediating characteristics and the resulting limitations imposed on these actions, and compensation methods people use to circumvent these limitations. The primary focus of this thesis was on identifying the effective actions. A set of effective actions was identified that was sufficient to describe a large number of people's solutions to a variety of estimation problems.

The relationship between rough estimation and engineering curricula was examined by comparing rough estimation activities in practice and learning activities in curricula. Rough estimation activities were found to be incongruent with typical undergraduate engineering curricula. The differences between these activities suggest ways in which curricula might be changed to improve students' estimation skills.

Thesis Supervisor: Woodie Flowers
Title: Pappalardo Professor of Mechanical Engineering

Thank you Woodie, Jeanne, Warren, Karl, Julie, Matt and especially Connie.

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Chapter 1 Introduction

1.1 A few scenarios

Suppose that you are making your way through the grocery store buying vegetables for a dinner you are preparing that evening for several guests. The next item on your shopping list is salad greens. You find that salad greens are sold by the bag and are somewhat pricey. The label on the bag indicates that there are four ounces (113 grams) of greens in each bag. You have never bought one of these bags before. How many bags should you buy to feed your guests?

You have just analyzed the deflection of a mounting bracket for a senior engineer in your company. You obtained the solution by using a new finite-element-analysis software package recently purchased by the company. The solution comes out to be 0.01 mm, an order of magnitude less then the allowable deflection. This result is favorable, but a whole order of magnitude? Is the result right?

While driving to work you have a clever idea for a small, hand-held product. Excited about the idea, you plan to tell your co-workers about it later that morning. For the product to be practical, however, it has to run for a long time on a small battery. You wonder if one 9-volt "transistor" battery would provide enough energy, or would the product require a bigger battery, making it too big and impractical?

Imagine you are working for a bicycle-accessories company as a project manager. One of your new projects is the design of a fairing for commuter bicycles. An engineer working on the project reports that a bicycle and rider traveling at 9 m/s (20 mph) experiences a drag force of 95 N without a fairing. The engineer indicates that this number should be used as a basis for evaluating all fairing designs. Does the engineer's number seem right?

These scenarios characterize situations that frequently arise in our personal and professional lives. In these situations, we need to make a rough estimate for a physical quantity, usually for the purpose of making a decision or improving our understanding. A rough estimate is acceptable because it provides useful information, whereas a more detailed analysis is unnecessary, impractical or impossible because the situation does not provide enough time, information or other resources to perform one.

In the simple case of buying salad greens, it is only necessary to estimate how many bags of greens your guests will eat. In fact, it would be difficult, if not impossible, to determine how many ounces they would eat. If you had time, you could call each of your dinner guests and ask them how much salad they will eat at dinner. However, your guests could be more or less hungry at dinnertime, possibly as a result of the other food you choose to serve them at dinner! In the case of the stress analysis, a quick estimate of the order of magnitude of the answer would help you decide if you should delay reporting a result and redo the analysis. If the estimate and the finite element analysis results were different, understanding gained from making the estimate could help you understand what is wrong.

1.2 Goals and motivation

The overall goals of this thesis are to understand how people make rough estimates for physical quantities and to understand how that activity relates to undergraduate engineering education. Although rough estimation is an important topic of study in its own right, setting these goals was motivated by pragmatic issues. Rough estimates are a significant aspect of engineering practice, as well as of participation in a technology-based society. Yet, observations of engineering students indicate that they have considerable difficulty making rough estimates for basic engineering quantities.

In engineering, a wide variety of activities benefit from the use of rough estimation, including evaluating the feasibility or sensibility of an idea; planning projects or experiments; sizing and selecting materials and components; and setting up, finding parameters for and checking detailed analyses. Evaluating the feasibility of innovative concepts in product design is an example of the first activity. Using rough estimates in conjunction with the Navier-Stokes equation in fluid dynamics is a well-known, prototypical example of the last activity.

Students have the most opportunity to demonstrate their estimation skills within design courses. And, it is in these courses that students were observed to have considerable difficulty performing rough estimates while working on their design projects. Students seldom used estimates to determine the feasibility of an idea or to justify material or component selections, even when give explicit instructions to do so. Many students in the senior design course were found to need considerable help with their estimates to be successful.

Even if we accept that an undergraduate engineering degree is a general education and not a professional one, the degree can be considered no less than preparation for participation in a profession and in our technological society. At the very least, this preparation should enable students to think critically using basic laws and principles of engineering and science. Rough estimates require the usage of these basic laws and principles to make simple calculations, which often constitute the first evidence for or against an idea or policy. If engineering graduates have difficulty making rough estimates, then the efficacy of the education they received is questionable.

1.3 Summary

In order to address the issue of students have difficulty making estimates, this thesis attempted to answer the following three questions. Answers to these questions provide useful information for future efforts to teach estimation and assess students' skills, as well as for practicing estimation.

> What is the nature and extent of students' capabilities?
>
> What are people doing when they make rough estimates?
>
> What is the relationship between rough estimation and engineering curricula?

It is important to address the first question in order to establish whether students are really having difficulties, and if so, how widespread these difficulties may be. To do this, the performance of mechanical engineering students on rough estimation problems was documented, as well as that of engineering practitioners for comparison. Subjects' written solutions, which are essentially abbreviated protocols (Ericsson and Simon 1993), were the primary form of data collected. Students, including seniors, were found to have considerable difficulty making simple rough estimates. The performance data is summarized in Chapter 1.

A more detailed look at their answers revealed that a surprising number of seniors lacked facility with the most basic engineering concepts. For example, many seniors confused energy with power by, for instance, using a relationship defining power to calculate energy. In order to identify concepts that give students difficulty, a new assessment method based on students' ability to associate correct units with common engineering quantities was developed. This finding and the units survey are also presented in Chapter 1.

Determining where students are having difficulty requires an understanding of what people are doing when they make estimates, i.e. addressing the second question listed above. This was accomplished by developing a framework to describe rough estimates and rough estimation activity. The framework consists of three components: effective actions people take when they make estimates, mediating characteristics and the resulting limitations imposed on these actions, and compensation methods people use to circumvent these limitations. In this thesis, the primary emphasis was placed on identifying the effective actions. This set of actions has been used to effectively describe a large number of people's solutions to a variety of estimation problems. The mediated action framework is presented in Chapter 1.

The third question listed above was addressed by comparing rough estimation activities to the learning activities of undergraduate engineering curricula. The comparison reveals several differences that collectively make engineering curricula incongruent with simple rough estimation, if not rough estimation in general. These differences are likely reasons why students are having difficulty making estimates. Fortunately, they also suggest areas in curricula to place emphasis on so as to improve students' estimation skills. The comparison is presented in Chapter 1.

The questions addressed by this thesis are broad and admit several possible areas wherein research efforts might be focused. Thus, it is necessary to describe the particular focus of this thesis to establish the context in which the results were obtained. This focus is better understood by first establishing the meaning of the term rough estimation as it is used in this thesis, which is presented in the next section. The focus of this thesis is then described in the subsequent section.

1.4 Rough estimation

The term estimation is synonymous with analysis in the sense that all quantities are determined to some level of specificity. An analysis that obtains higher specificity is generally less economical to perform, i.e. it requires more resources in the form of time, information, formalization or computation. This trade-off between specificity and economy describes a continuum of analyses, as shown in Figure 1. See for example (Starfield, Smith et al. 1994), (O'Connor and Spotila 1992) and (Palm 1986) for similar discussions for modeling.

The term rough estimates describes the range of analyses that provide relatively low specificity with high economy, while the term detailed analyses describes the opposite range. These terms will have different meanings for different groups of practitioners because of different analysis capabilities and standards for specificity.

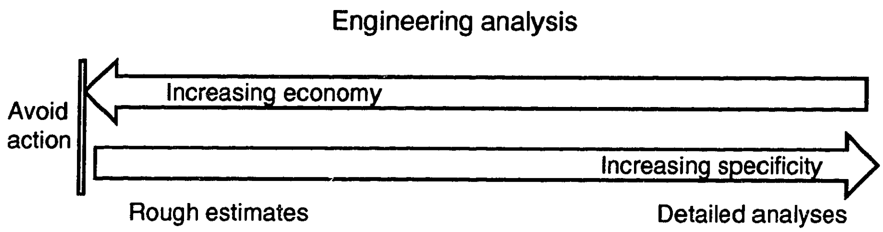

Figure 1. The general trade-off of analyses between specificity of solutions and the economy of performing the analyses.

In this thesis, specificity is defined as the combination of resolution and certainty. Specificity can also be characterized by accuracy where accuracy is a measure of how close a given solution is to the actual solution. Resolution and certainty, however, are useful in a wider range of situations. The actual solutions to most quantity problems are never known or are not knowable and accuracy is not definable. On the other hand, resolution and certainty apply to a solution regardless of whether or not an actual solution is knowable and still capture the notion of accuracy. A solution that is known to high resolution with certainty must be an accurate solution.

As suggested in Section 1.1, the choice of what analysis to perform depends upon the situation in which a problem is addressed. To see why, consider the following problem.

`Determine the thrust of a Boeing 747 jet engine.`

The thrust could be determined with either a rough estimate or a detailed analysis. The approach used and the specificity obtained are determined by the purpose for finding the value, the nature of the situation, and the capabilities of the solver(s). Over lunch in a cafe, I would determine an answer with a simple rough estimate probably with factor-of-two resolution, while a flight engineer at one of Boeing's rival companies would make a more detailed estimate. My answer would be sufficient to help me understand how jets fly but would probably not provide new information to the flight engineer.

In practice, rough estimates are commonly referred to by several different names, including order-of-magnitude estimate, ballpark estimate, simple calculation, back-of-the-envelope calculation and rough guess. The words and phrases used in these names are often suggestive of the relative level of resources involved, the resolution desired or action to be taken. See Table 1.

Table 1. Terms common to names describing rough estimation loosely grouped by meaning.

Resources (adjective)	Resolution (adj. and adv.)	Action (verb) or Result (noun)
Quick	Approximate	Guess
Short	Crude	Estimate (-ation)
Simple	Rough	Guesstimate (-ation)
Back of the envelope	Factor of two	Calculate (-ation)
Cocktail napkin	Order of magnitude	Approximate (-ation)
Paper and pencil	Ballpark	Determine (-ation)
Hand (as in by hand)	Wild	Figure

1.5 Focus

This thesis focused on the information and actions involved in making simple rough estimates. These estimates correspond to the left end of the rough estimates range (and the far-left end of the analysis continuum) as shown in Figure 2. Simple rough estimates may be performed with or without limited supporting resources such as paper and pencil and access to relevant physical objects. The subjects studied for this thesis were always allowed to use the former and sometimes provided access to the latter.

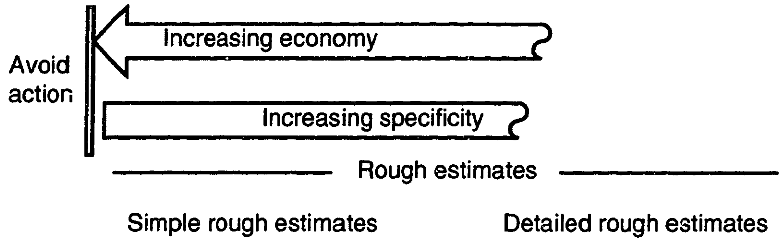

Figure 2. Simple rough estimates correspond to the left end of the rough estimates range.

This thesis focused on physical quantities that are common to and characteristic of mechanical engineering, such as force and energy. However, basic physical quantities, such as length and area, which are common outside of engineering were also covered. Quantities that are specific to a sub-discipline of mechanical engineering, such as viscosity, were not studied directly, although the results obtained are expected to be applicable to these quantities as well. See the horizontal axis in Figure 3. See (Sonin 1997) for a formal discussion of physical quantities.

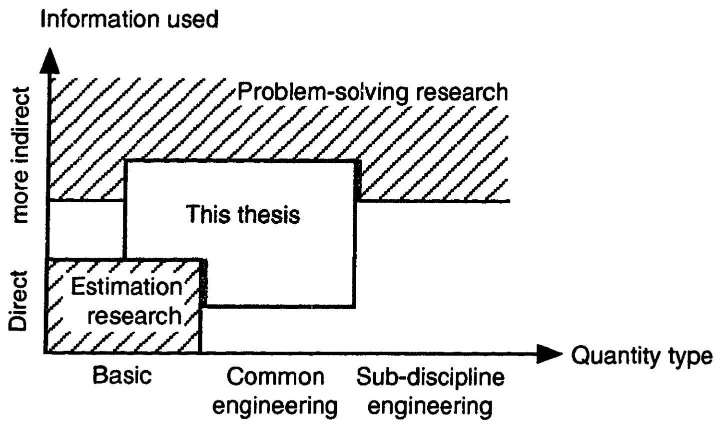

Figure 3. Estimation and problem-solving research organized by the type of quantities studied and the amount of information used to obtain solutions.

This thesis focused on seniors in mechanical engineering as subjects. However, data was also collected for mechanical engineering students at other levels as well as for engineering practitioners. See Figure 4.

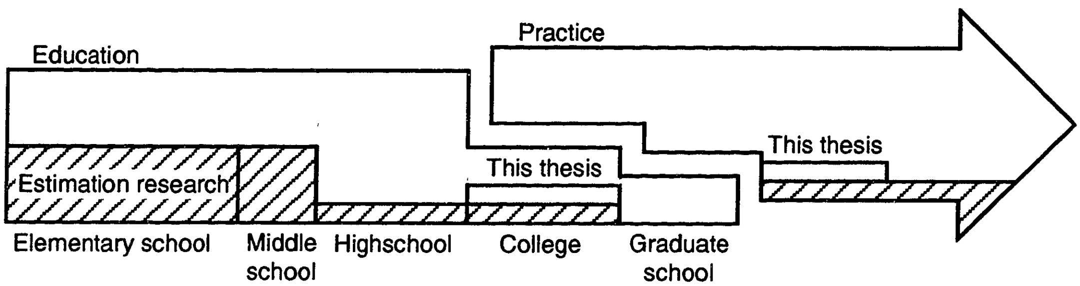

Figure 4. The subjects covered by estimation research.

The related research indicated by the crosshatched areas in Figure 3 and Figure 4 are discussed in the next section.

1.6 Related research

A number of studies have been conducted that asked people to give simple estimates for quantities. See (Joram, Subrahmanyam et al. 1998) and (Brown and Siegler 1993) for reviews as well as (Plous 1993). The majority of these have come from the cognitive psychology and education fields. These studies are characterized in this section by 1) the types of quantities that were investigated, 2) the level of education or experience of the subjects studied and 3) the amount of information and corresponding actions subjects were prompted to use to make estimates.

Almost all of the studies investigated basic quantities and values that are common to everyday life and that can be directly experienced. These quantities included primarily numerosity, length, and time as well as frequency, probability and percentage. Area, volume, weight, and temperature also received some attention. The values to be estimated for the physical quantities were almost always on the human scale, including the values for numerosity. Brown's studies on the estimates for country land areas is an exception (Brown and Siegler 1993).

The majority of studies on numerosity and length focused on elementary and middle school students, however, several studies have focused on high school students, college students and adults. The studies on frequency and probability have focused more on college students and adults. See the crosshatched areas in Figure 4. (Smith, D'Angelo et al. 1998) documented performance of mechanical engineering college students' estimates for several mechanical engineering quantities. This study was prompted in part by an earlier publication from this thesis research documenting mechanical engineering students' difficulties (Linder and Flowers 1996). They reported that their results confirmed the findings of this research.

Responses that subjects can give for an estimation problem span a continuum that ranges in the amount of information and corresponding actions they use to make an estimate. This continuum is represented by the vertical axis in Figure 3. At one extreme, a subject simply provides an answer value directly. Almost all estimation studies are clustered at this end of the continuum. At the other extreme, a subject carries out extensive problem-solving activity to determine an answer value indirectly.

At the direct end of the continuum, a number of studies have focused on asking subjects to provide values directly or nearly directly. Both (Joram, Subrahmanyam et al. 1998) and (Brown and Siegler 1993) summarize studies of this type. Many studies of this type simply determine if people know values for certain quantities. Several studies of this type have identified cognitive effects or heuristics (different than problem-solving heuristics), including availability, anchoring and representativeness (Plous 1993), that mediate and often bias our knowledge of values and other information.

Brown and Seigel focused on estimates slightly further along the continuum (Brown and Siegler 1993). They developed a framework for describing estimation activity that incorporates direct determination of values, cognitive heuristics and a small amount of additional information that they refer to as domain knowledge. They provided information to subjects in the form of reference values and considered their influence relative to the influence of heuristics on estimates. Reference values are of the same quantity type as the value to be estimated. They do not mention how the additional information was used to make an estimate.

Yet further along the continuum, several studies focused on how reference values are used to make estimates, particularly of length or distance. (Joram, Subrahmanyam et al. 1998) provide a substantial review of these studies. In these studies, the subjects usually generated the reference values. The actions taken to estimate basic physical quantities using these values often correspond to measurement actions, hence the term measurement estimation is used to refer to this type of estimation. See Section 3.5.2 for an example. (Joram, Subrahmanyam et al. 1998) summarize the measurement estimation strategies identified by these studies. They also present a framework involving the mental representation of numbers that incorporates direct knowledge of values with the use of reference values. Cognitive heuristics were not mentioned in this framework.

At the indirect end of the continuum, values are determined with the use of a considerable amount of indirect information and corresponding actions. In these situations, additional resources are necessary to determine what information to consider, to obtain that information, and to make use of it. These situations have not been studied by estimation research but have been widely studied by problem-solving research. A considerable number of studies have been performed, and many reviews are available. The reviews by (Woods and al. 1997) and (Wankat and Oreovicz 1993)provide an engineering education perspective. The review by (Schoenfeld 1992) provides a mathematics education perspective. And, (Smith 1993) discusses several main areas of problem-solving research relative to problem definition. The broad views on problem solving delineated by (Schön 1983) as well as (Newell and Simon 1972) have been found to be useful. See (Dorst 1997) for a study that compares the two views.

1.7 Additional efforts

The usefulness of rough estimation skills is recognized in most communities of practice that routinely deal with physical quantities. In fact, it is difficult to find anyone in the traditional engineering disciplines that does not agree with the usefulness of estimation skills. Despite this widespread view, only a few people have taken steps to address student acquisition of these skills at the college level, and much of this activity does not appear to be recorded in published literature. For example, there have reportedly been seminars at MIT in the past on order-of-magnitude estimation, and there is one scheduled in an engineering department for the upcoming calendar year. In personal communications, several faculty members at MIT and other universities indicated that they have used estimation problems in their classes.

In a few cases, regularly appearing columns on estimation have been run in educational journals associated with a particular profession: two in physics (Hobart 1963; Purcell 1983-85; Weisskopf 1984-86) and one in geology (Triplehorn 1994, 1995). These columns provided a few problems in each issue with solutions typically being given in the subsequent issue. The purpose of these columns was to heighten awareness of estimation, provide a forum for practicing the skill, and provide a source of worked problems for use in the classroom. Triplehorn's examples correspond to the left end of the rough estimates range, while Weisskopf and Purcell's examples correspond more to the right end, illustrating that not everyone has the same level of estimates in mind when they discuss rough estimation.

In physics, Enrico Fermi was well known for the pleasure he took in posing and solving rough estimation problems and was known to engage his graduate students in solving them. The physicist Philip Morrison coined the term Fermi problem (Morrison 1998), and it probably first appeared in the American Journal of Physics in his letter to the editor emphasizing the importance of rough estimation problems to learning physics (Morrison 1963). Problems originally posed by Fermi have come to be known as authentic Fermi problems. For example, how many piano tuners are there in the city of Chicago is one such problem. (Morrison 1963).

In engineering design, Woodson's introductory text includes a section on estimation (Woodson 1966). He discusses the importance of estimation in engineering and gives several worked examples. He also gives provides some guidelines for expectations when making estimates.

In environmental studies, Harte's book, "Consider a spherical cow," (Harte 1988) has been well received and is known outside his field. It is ostensibly a text on rough estimation. However, Harte primarily describes different models useful for making estimates in his field and gives a number of examples. Thus, the book is more aptly described as a text on modeling well suited for use in estimation. The book, like the journal columns, portrays the form of good estimates in a particular profession and heightens the awareness of estimation. In zoology, the similarly titled journal article "Consider a spherical Lizard: Animals, models and approximations" provides similar advice to a research audience (O'Connor and Spotila 1992).

Similar to Harte's book is the introductory modeling book, "How to model it," (Starfield, Smith et al. 1994). It provides a well-presented introduction to modeling for a general audience in a way that is congruent with rough estimation. Its title is based on Polya's well-known problem-solving book "How to solve it" (Polya 1973). Articles have also been written giving advice to a general audience on how to make estimates. For example, see (Meledin 1991). References to rough estimation have also appeared in popular science and mathematics publications covering numeracy and innumeracy.

Chapter 2 Student performance

2.1 Introduction

The performance of MIT mechanical engineering students was documented to ascertain the extent and nature of the difficulties they have when making estimates. This was accomplished by asking a large number of students to solve estimation problems and then collecting their responses. Responses were obtained from students at other universities to determine if difficulty making rough estimates is a wide spread problem. Responses were also obtained from engineering practitioners to set expectations for student performance. These responses are summarized in this chapter. Students were found to have considerable difficulty making simple rough estimates. These difficulties are due in part to a lack of understanding of basic engineering concepts. An assessment method developed to probe students' understanding of basic concepts is also presented.

2.2 Background

2.2.1 Method

The basic method used for collecting data was the same for each group of subjects studied. Subjects were given problems in situations where a solution could only be obtained via a simple rough estimate. These problems focused on physical quantities common to mechanical engineering. Each of the problems used was informally piloted with a few students and faculty members to ensure that the problems were understandable and solvable in the time available.

Mechanical engineering students were asked to solve estimation problems at the beginning of class lectures and then their solutions were collected. Each student was given a problem statement and allowed to use paper and pencil to solve the problem. In most cases, the time available to solve the problem was limited to five minutes. The physical objects identified by the problems were not made available unless they were needed to solve a problem. Students did not know in advance that they would be solving an estimation problem. Students were asked to provide their name to motivate a thoughtful response. The use of calculators was not allowed primarily because not everyone had one available in each situation.

When possible, these problems were given in lectures of design courses. The context in which a problem is solved can influence how the problem is solved. If the problems were given in lectures of engineering science courses, such as courses on fluid mechanics or statics, students' thinking might be biased towards the subject material of those courses. Design courses provide a more balanced environment because knowledge from all of the engineering sciences is used in design activities.

Although rough estimates are not typically made in a classroom, many engineers solve a wide range of problems under similar conditions. Situations where simple rough estimates are needed can arise in brainstorming sessions, planning meetings and even grocery stores with little time or resources available. The estimates are often made with only the aid of paper and pencil and sometimes just mentally. Similarly, the physical context in which an estimate is made is often not the same as the context of the physical object identified by the problems because estimates are often carried out to make predictions about unknown or future situations.

2.2.2 Problems

The problems were designed based on the following considerations.

The problems should cover a range of physical quantities in order to capture a range of estimation activity. This was accomplished by asking students to estimate basic quantities, such as length and area, as well as quantities commonly used throughout mechanical engineering, such as force and energy. Similarly, the problems should cover a range of values for a given quantity. This was accomplished simply by asking students to estimate values of different magnitudes. The problems should not, however, ask students to estimate values that normally vary by a significant amount. Otherwise, it is not possible to determine where the variance in that answer values comes from. This was accomplished by choosing quantities whose values were known to have much lower variance than that generated by students.

The problems should indicate that a simple rough estimate is to be made. This was accomplished through a combination of methods. The problem statements were worded to suggest that a low-resolution answer was acceptable and to suggest that an estimate should be made. This was done be using phrases such as "order-of-magnitude" and words such as "estimate". And in most cases, only enough time to make a simple rough estimate was provided.

Finally, only information sufficient for making a simple rough estimate was made available in the problem statement and problem context, which is consistent with many estimation situations in practice. For example, consider the jet engine problem mentioned in Chapter 1.

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Determine the thrust of a Boeing 747 jet engine.
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Notice that the problem is not explicit about the thing in question or the quantity being estimated. Does "thrust" refer to the output at takeoff, to the output at cruising speed or to the maximum output the engine can produce? Which "747 jet engine" should be considered? In a rough estimation situation, this amount of information and level of uncertainty is appropriate and common. The missing information is either not knowable in the situation or an answer for any of the possibilities would be acceptable. In a situation requiring a detailed analysis, the missing information would either be a tacit part of the situation or resources would be utilized to obtain it.

The problems should not require unique knowledge or knowledge that could not readily be used in the time available. Otherwise, the problems would be testing this knowledge and not rough estimation skills. Several things were done to minimize this effect. The problems were chosen to be about objects or processes known to be familiar to students. The problems were designed to only ask for values of physical quantities frequently studied in engineering. The problems were chosen so that they could be solved using basic engineering or science principles with little information and few computations. And, only problems that could be solved in at least two ways were used.

2.2.3 Students

The population of students chosen for study was the set defined by a class year. A class year is the largest clearly distinguishable group of students in an educational program and is the basic unit of operation for a program. Lectures of required courses provided an opportunity to reach almost all of the students for a given class year at once. The only students missed were those that did not come to class the day the questions were asked. By obtaining data for the whole population there was no need to generalize results from a smaller sample. Also, having students solve a problem all at once eliminated any variability between responses that might result from asking different students the same question at different times and locations.

Particular interest was directed towards the senior class for two reasons. First, these students have completed a majority of their course work. Thus, not only will they have taken the core engineering, science, and math courses, they will have had an opportunity to develop experience with the material of these courses in subsequent courses. Second, the end of a program provides a decisive point for assessing students' knowledge and skills. Before this point, it could be argued that students have not had an opportunity to develop their knowledge and skills. After this point, the program no longer has an opportunity to improve students' knowledge and skills.

2.3 Pilot study

Sophomore students were asked to solve several rough estimation problems. Based on their responses, improvements were made to the method used for studying seniors. Responses for two of these problems are presented in this section. The problems were given at the beginning of a lecture of a required sophomore design course. Each problem referred to a physical object from a kit of components and supplies the students were given and required to use in their term design project.

2.3.1 The aluminum bar problem

The students were shown a rectangular aluminum bar, 12 x 2 x 1/4 inches in size, and were verbally told to "give a rough estimate for each of the dimensions". The 159, 158 and 117 responses obtain for the length, width and thickness respectively are shown as relative frequency histograms in Figure 5.

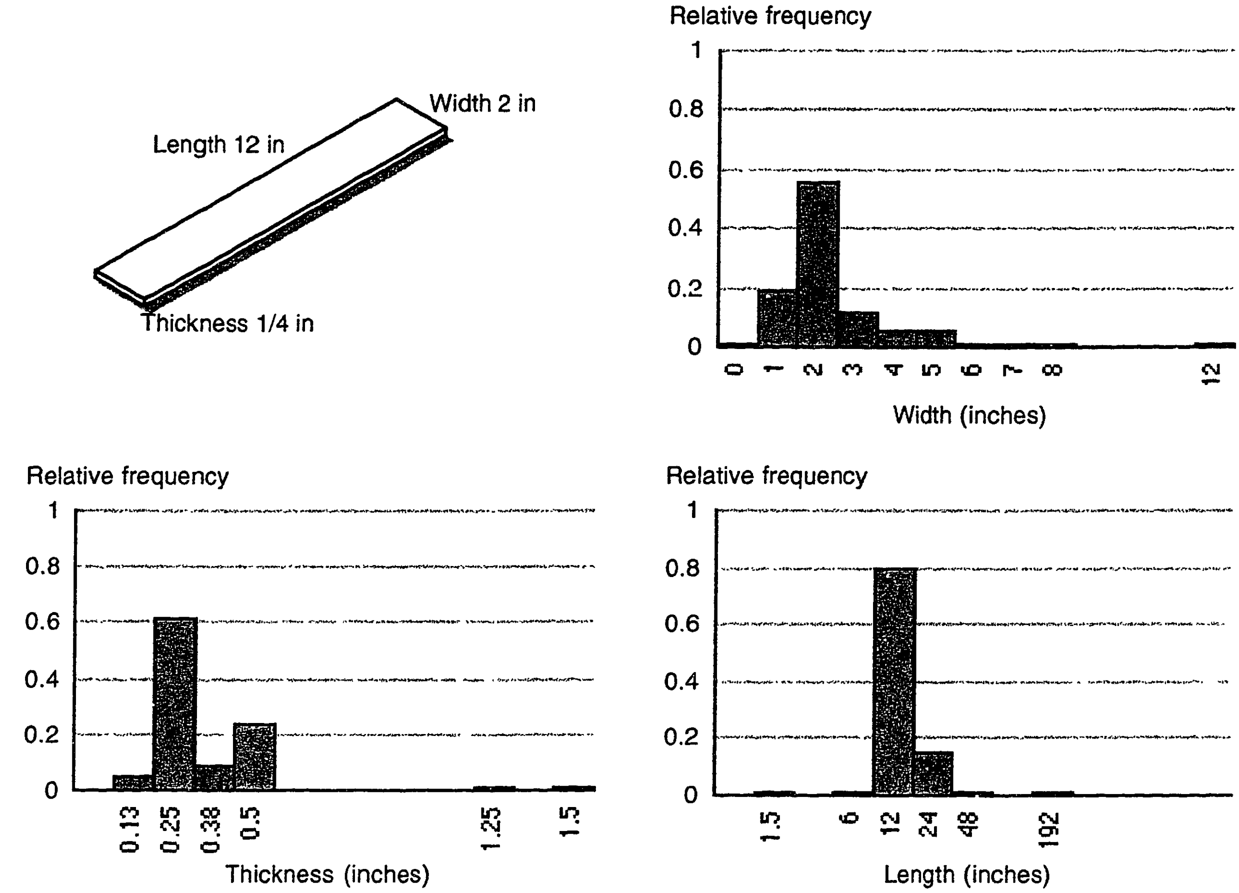


Figure 5. Estimates for the length (top right), width (bottom left), and thickness (bottom right) of an aluminum bar (top left).

2.3.2 The motor problem

The students were shown a DC permanent magnet motor, approximately 2 inches in diameter and 3 inches in length, and were verbally told to "give an order-of-magnitude estimate of the motor's maximum continuous power output". They were asked to work quickly but were allowed time to finish. All of the students finished answering the question within 3 minutes or less. A total of 161 responses were obtained. 137 students provided values with units of power, which are shown as a relative frequency histogram in Figure 6. 19 students gave a response with incorrect units for power. Five students did not attempt the question. The responses varied by six orders of magnitude. The motor had an actual peak continuous power output of approximately 10 watts.

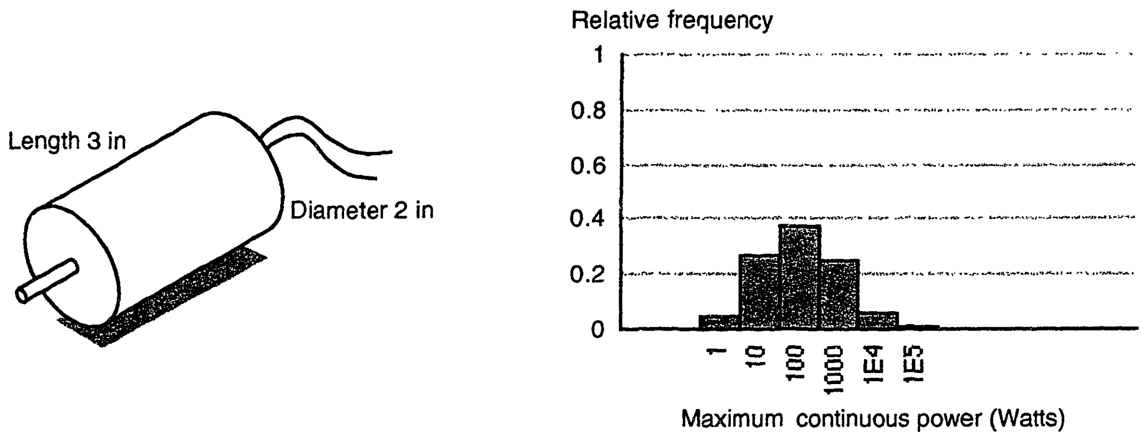


Figure 6. Estimates for the maximum continuous power output (right) of a DC permanent magnet motor (left). The actual value was approximately 10 Watts.

While the students did well on the aluminum bar problem, they were unsuccessful on the motor problem. This relative performance was expected. People know much more information about lengths than they do power. And, simple visual comparisons to references available in the room could have been used to determine length but not to determine power. Furthermore, students are unfamiliar with motor sizes and how motors function.

Mechanical power is both common to and characteristic of the mechanical engineering field, unlike basic quantities like length. One can make good estimates for length without engineering or science knowledge, but estimates for mechanical power are difficult to make without this knowledge. Given the sophomore students' difficulty with power, emphasis was placed on quantities of this type in subsequent problems given to seniors.

Providing equal access to a physical object for a large number of people proved to be difficult. The aluminum bar and the motor were placed at the front of the classroom. Although students had seen the objects up close in the preceding days, they could not see the objects equally well when making their estimates. This difference could lead to variation in the responses. Thus, subsequent problems did not require the presence of an object in order to solve them.

The students were not asked to explain their estimation process, nor did they give any written justification for their answers to either problem. This makes it difficult to know how they made their estimates. Apparently, they did not use methods that required the use of paper and pencil to carry out. They may have provided values directly or based their answer values on comparisons to reference values. It is possible, especially for the motor problem, that students guessed and therefore had no justification to write down. Some students were found to associate low specificity estimates, as in "order-of-magnitude estimates", with guesses. In subsequent problems, students were explicitly asked to show or explain their estimation processes. Also, the problem statements were made less explicit about the specificity desired.

2.4 Summary of responses

Mechanical engineering students and engineering practitioners were asked to solve a number of different estimation problems during the course of this research. The responses for two of these problems are summarized in the following sections. Of the problems studied, the largest amount of data was collected for these two.

2.4.1 Subjects

The students studied were seniors in mechanical engineering at MIT and five top engineering universities. The responses from students at the other universities were obtained by sending written problems along with instructions to 15 colleagues in mechanical engineering departments around the country. Five of these colleagues administered the problems and returned the results. They did not all obtain responses for an entire class year. In one case, responses from a combination of juniors and seniors were obtained. All five universities were ranked within the top 12 engineering universities in the country by U.S. News and World Report in the year the problems were administered.

The practitioners studied were all attendees of a plenary talk at an American Society of Engineering Education (ASEE) conference. They were asked to solve problems at the beginning of the talk. Practitioners involved in academics were chosen because they have knowledge and backgrounds similar to senior students. The setting was chosen because of its similarity to the lecture setting used for the students.

The practitioners were asked to answer the following two questions after they had solved the estimation problems.

```
What is your area of expertise? (Example: Mechanical Engineering)
_____________________________ Engineering
_____________________________ Science
_____________________________ Other
```

```
How many years have you been an active practitioner and/or educator
in your area of expertise?
___________ Years
```

The general areas of expertise reported by the 117 subjects are given in Table 2. The breakdown of the engineering areas of expertise is given in Table 3. Engineers who reported a combination of two or three of the engineering areas are listed in the combined category. Figure 7 is a histogram of the years of experience reported by 96 of the 99 Engineering subjects.

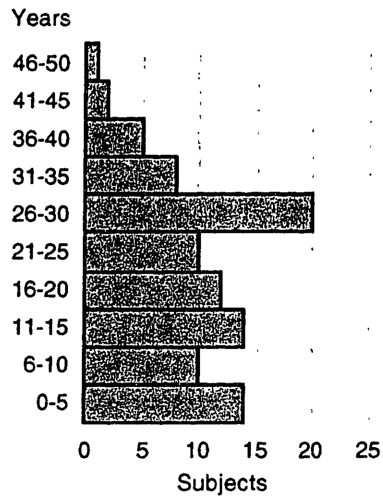
Table 2. General areas of expertise reported.

Area of expertise	Subjects
Engineering	99
Science	6
Other	9
Blank	3

Table 3. Engineering areas of expertise reported.

Engineering area	Subjects
Electrical	27
Mechanical	26
Chemical	9
Civil	7
Industrial	3
Aerospace	3
Computer	3
Manufacturing	2
Systems	2
Combined	9
Other	4
Blank	4

Figure 7. Years of expertise reported by the engineers.



2.4.2 The bicycle problem

Subjects were asked to solve the following written estimation problem. They were given five minutes and allowed to use paper and pencil.

```
Estimate the drag force on a bicycle and rider traveling at 20 mph (9
m/s).
Please show your estimation process.
```

The responses are summarized in Table 4 and Figure 8 through Figure 11. These figures are open bin, relative frequency histograms of the answer values given with valid units of force. Each bin represents an order of magnitude. For example, 66 of the 96 MIT mechanical engineering seniors provided a value with units of force, which are shown in Figure 8. Published measurements for the drag force on a 180 lb. adult male rider range from 20 to 40 N.

The students' answer values indicate the right order of magnitude; however, their answers cover six orders of magnitude not including outliers. The mechanical engineering practitioners, and even the electrical engineering practitioners, did substantially better. All of the mechanical engineers' answer values were on the right order of magnitude.

Table 4. Summary of responses for the bicycle problem.

	MIT mech. eng. seniors	All engineering practitioners	Mech. eng. practitioners	Elec. eng. practitioners
Responses	96	99	26	27
Answers	66	56	14	16
Incorrect units	5	4	2	0
Median (N)	36†	37†	40	37†
Stdev (N)	12,000†	690†	24	61†

† extreme outlier not included.

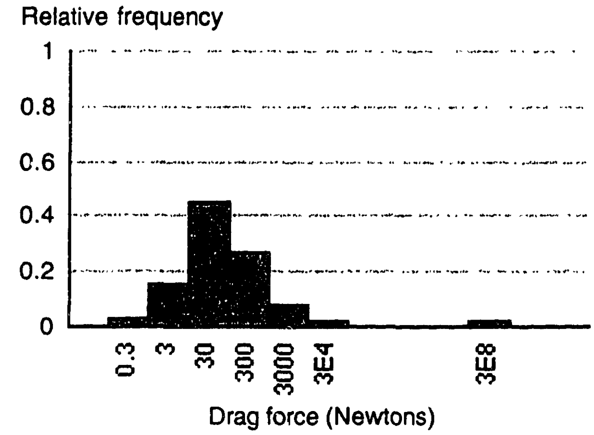


Figure 8. MIT mechanical engineering seniors.

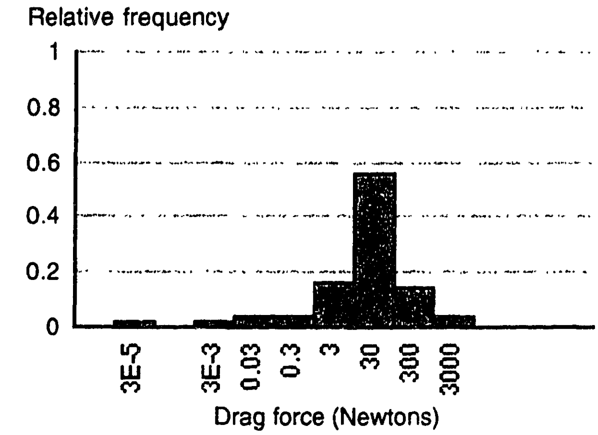


Figure 9. All engineering practitioners.

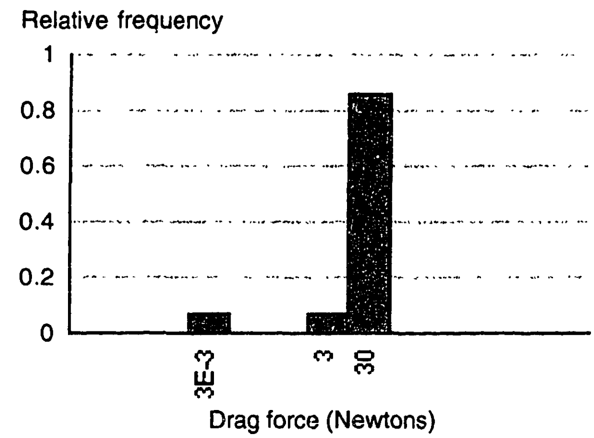


Figure 10. Mechanical engineering practitioners

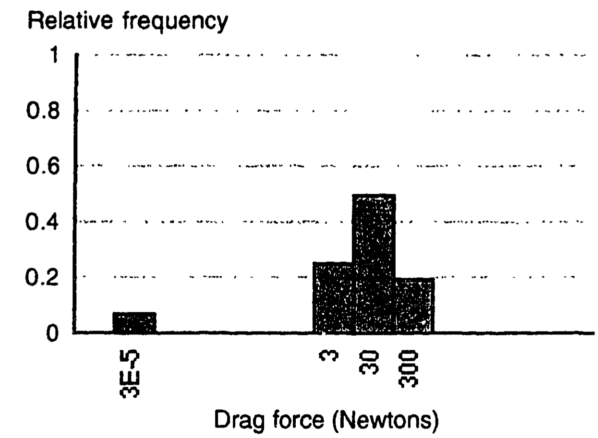


Figure 11. Electrical engineering practitioners.

2.4.3 The battery problem

Subjects were asked to solve the following written estimation problem. They were given five minutes and allowed to use paper and pencil.

```
Estimate the energy stored in a new 9-volt "transistor" battery.
Please show you estimation process.
```

The responses are summarized in Table 5 and Figure 12 through Figure 16. These figures are open bin, relative frequency histograms of the answer values given with valid units of energy. Each bin represents an order of magnitude. For example, 123 of the 135 MIT mechanical engineering seniors provided a value with units of energy, which are shown in Figure 12. At the time of this study, a Panasonic carbon zinc battery and 9V alkaline battery were calculated to have approximately 7,000 J and 17,000 J of available energy respectively based on values published by Panasonic. A Duracell alkaline battery was calculated to have 7,000 J of available energy based on values published by Duracell.

The MIT seniors' answer values do not indicate the right order of magnitude and cover a range of nine orders of magnitude not including outliers. The seniors at other universities responded similarly. Their answer values range by ten orders of magnitude. The mechanical engineering practitioners did noticeably better, and the electrical engineering practitioners did substantially better. Their answer values indicate the right order of magnitude and do not vary by more than four orders of magnitude.

A higher percentage of the MIT students reported answers than did the students at the other universities and the practitioners. It is possible that the students at MIT felt the attention of their professors more strongly than did the other students. The MIT students may have been concerned that their performance might affect their grade. Our colleagues at the other universities may not have influenced their students in a similar way. The practitioners would not have felt the pressure that grades create. And, inadvertently, they were not asked to provide their names as the students were. Anonymity may have allowed them to reduce their effort.

Table 5. Summary of responses for the battery problem.

	MIT mech. eng. seniors	Other univ. m.e. seniors	All engineering practitioners	Mech. eng. practitioners	Elec. eng. practitioners
Responses	135	161	99	26	27
Answers	123	78	56	16	21
Incorrect units	12†	29	18	1	3
Median (J)	160	5,900	7,200	2,800	13,000
Stdev (J)	3.3E8	2.5E7	4.5E6	8.1E6	2.4E6

† The MIT students were provided the correct units and still made 12 mistakes.

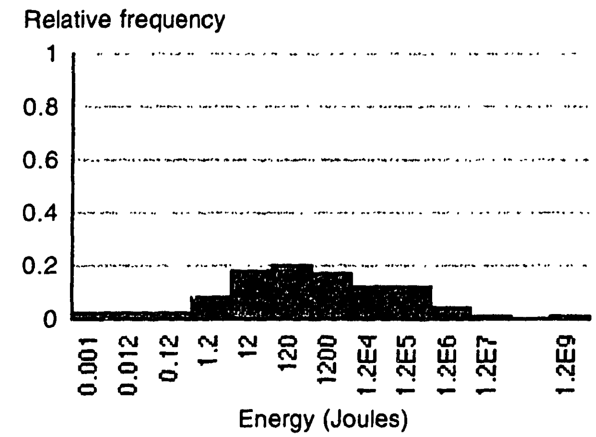


Figure 12. MIT mechanical engineering seniors.

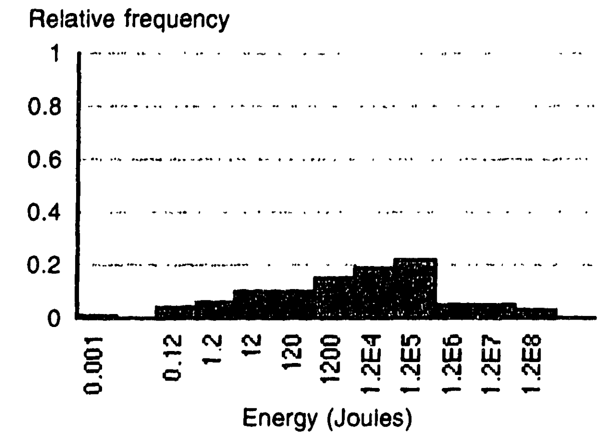


Figure 13. Other university seniors.

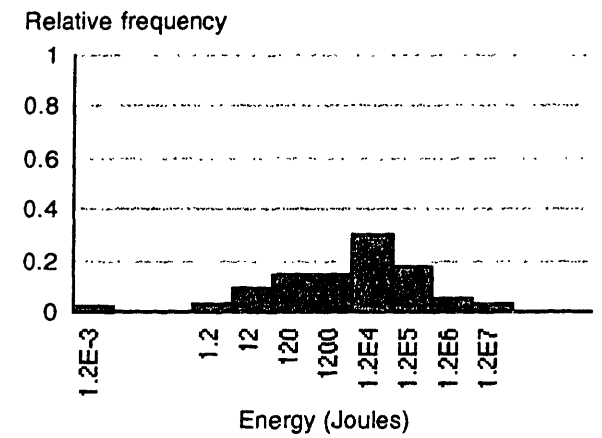


Figure 14. All engineering practitioners.

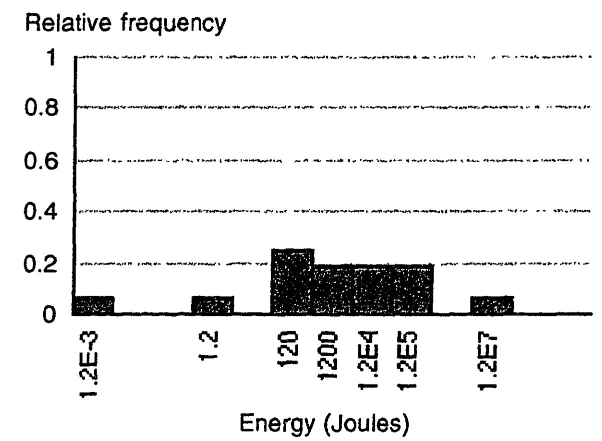


Figure 15. Mechanical engineering practitioners

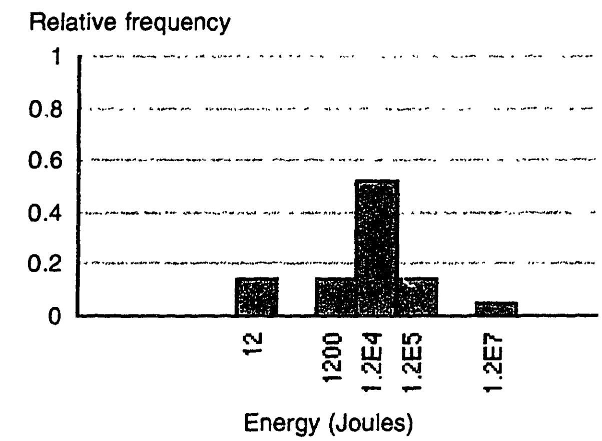


Figure 16. Electrical engineering practitioners.

The responses for the battery problem for each university are summarized in Table 6 and Figure 17 through Figure 22. The original MIT responses were randomly sampled to produce a comparable number of responses. The responses for the different schools are very similar.

Table 6. Summary of responses for the battery problem for all universities.

	University					
	A	E	D	C	B	MIT (reduced)
Responses	42	31	35	29	24	36
Answers	24	24	9	13	8	30
Incorrect units	6	2	8	9	4	3
Median (J)	8,100	3,400	20	1.1E5	3,800	47
Stdev (J)	2.0E7	4.0E7	3.3E4	1.6E5	1.2E5	3.7E5

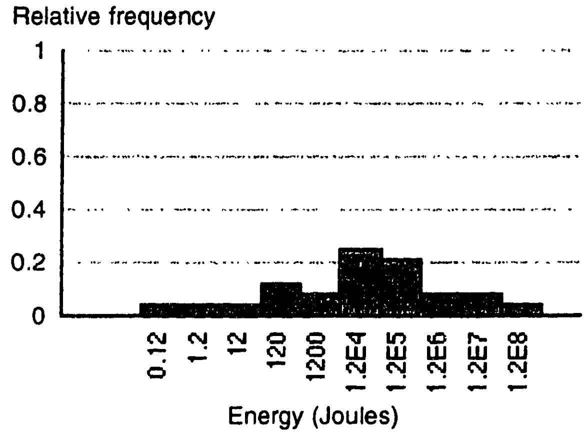


Figure 17. University A.

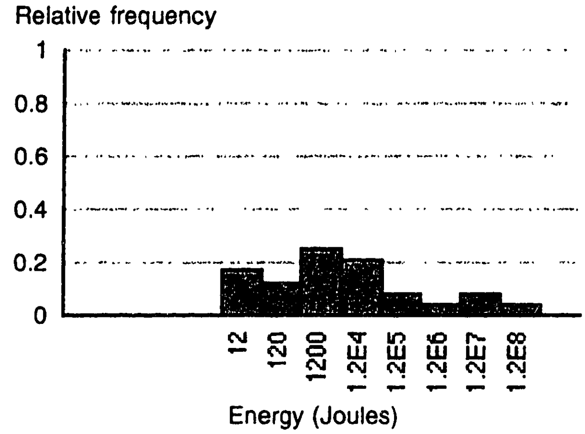


Figure 18. University E.

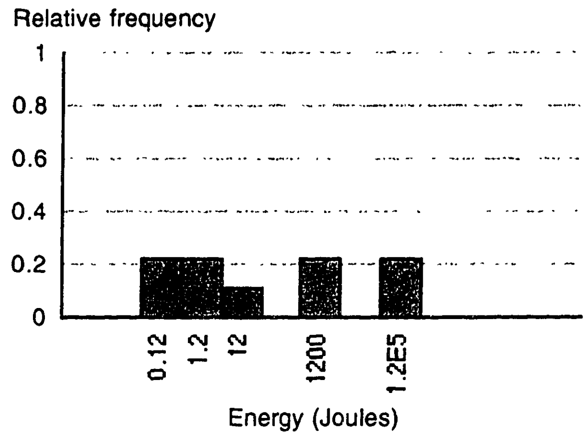


Figure 19. University D.

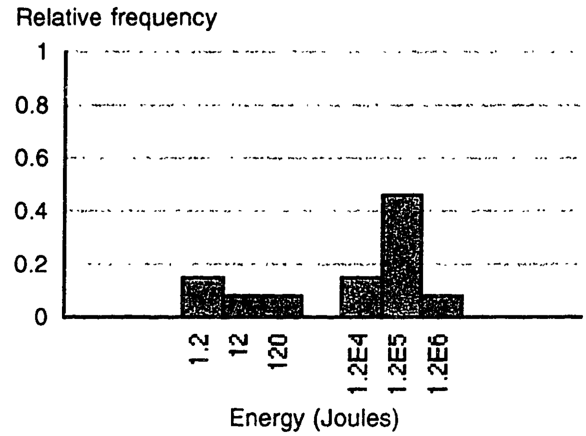


Figure 20. University C.

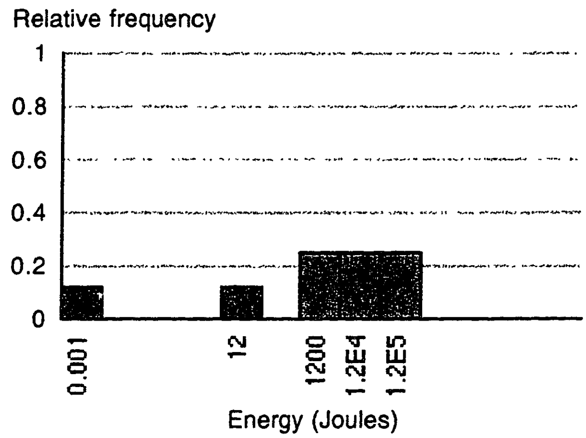


Figure 21. University B.

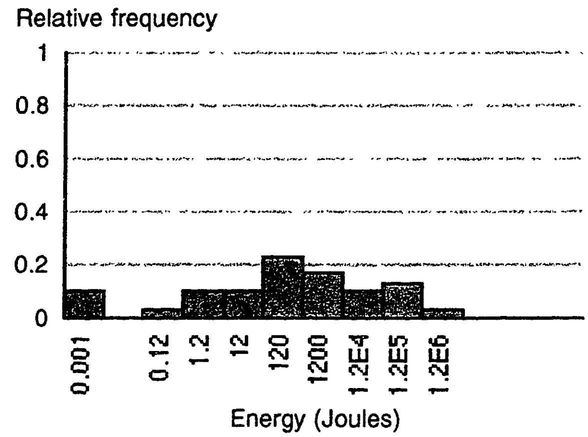


Figure 22. MIT (produced from the original data)

2.5 Detailed descriptions

The previous section summarized subjects' responses in terms of the answer values they provided. More detailed descriptions than these are necessary to understand what the students did to solve the problems. This section presents a method used during this research for describing estimates in more detail.

Detailed descriptions were created by coding subjects' responses for a set of descriptors that characterize progress towards a solution. The set of descriptors developed is given in Figure 23. The descriptors are independent of any particular method of solving a problem. Once a set of responses has been coded with these descriptors it can be plotted as a river diagram to reveal how the responses are grouped. For example, all of the student responses for the battery problem were coded to produce the river diagram shown in Figure 24. Only the reduced MIT data was used so that the diagram would represent all universities equally. Of the 197 students that attempted the problem, 160 attempted to determine a value indirectly. Of those, 94 used a valid procedure, but only 18 obtained an answer value on the right order of magnitude without error. One student provided an answer value directly that was on the right order of magnitude.

Type of approach taken

- No approach - no approach discernable, minimal information given
- Explain - provided an explanation, no attempt to provide a solution
- Direct - provided an answer value directly
- Indirect - determined a value indirectly from additional information

Validity of a procedure

- No procedure - no procedure discernable
- Invalid - used invalid procedure
- Valid - used valid procedure

Completeness of a procedure

- Incomplete - completed a procedure
- Complete - did not compete a procedure

Presence of detailed errors

- No error - made no errors using information such as relationships and units
- Errors - made errors using information

Figure 23. Descriptors and their values used to code responses to estimation problems.

These more detailed descriptions reveal that the summaries given in the last section are misleading by suggesting that the students did better than they actually did. Based on the histograms, 38 of the 96 students that attempted the bicycle problem obtained an answer on the right order of magnitude (based on 30 N to 40 N actual value range). The more detailed description, however, revealed that only 29 obtained an answer on the right order of magnitude without making a mistake such as using an invalid procedure, relationship or conversion factor. For the battery problem, only 19 out of 197 students that attempted the problem obtained an answer on the right order of magnitude without making a mistake (based on 7,000 J to 17,000 J actual value range).

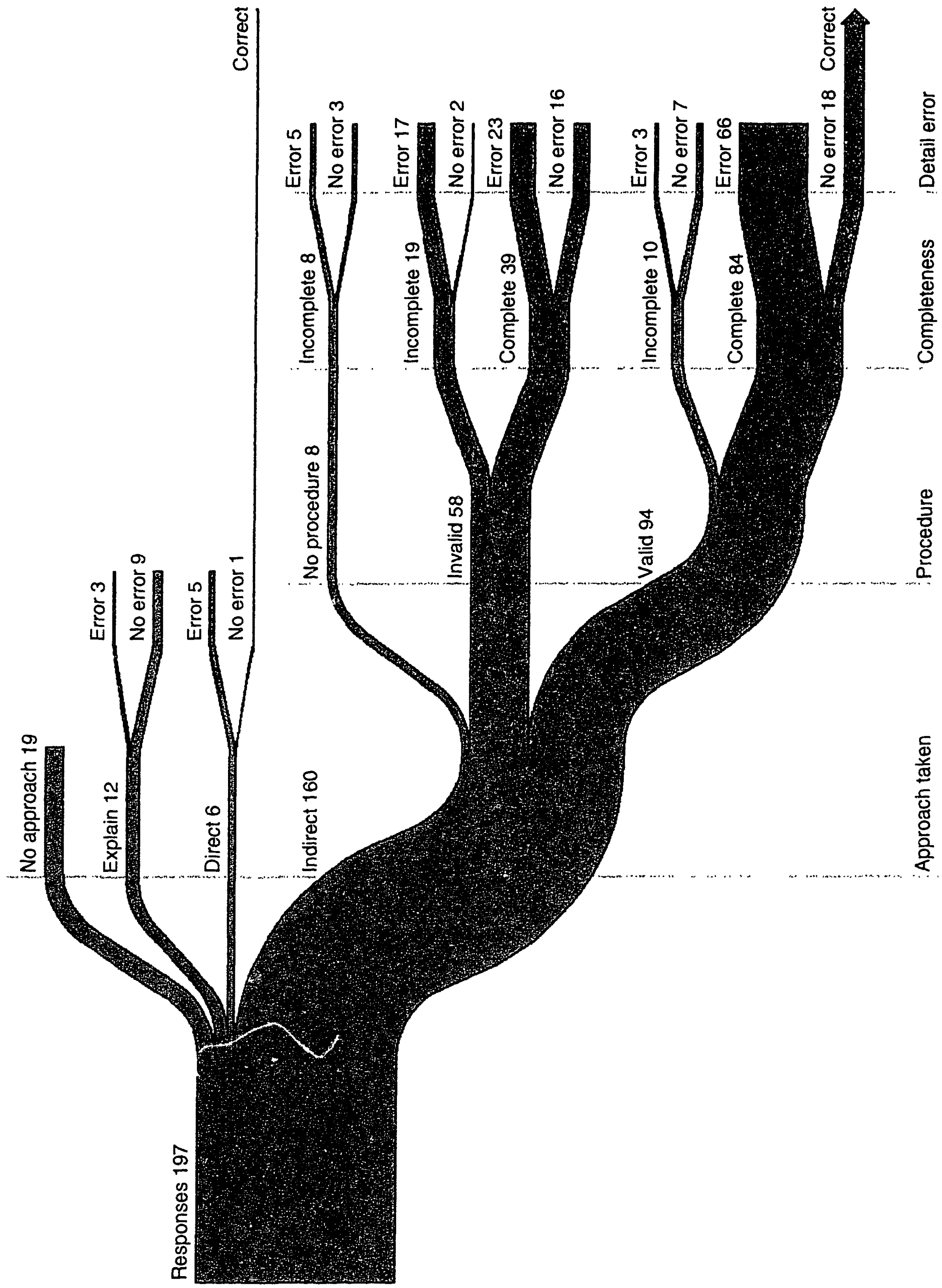


Figure 24. Detailed description of the students' responses to the battery problem. The 19 "correct" responses had answer values on the right order of magnitude (based on 7,000 J to 17,000 J actual value range).

2.6 Knowledge of fundamentals

The detailed descriptions of the responses make clear that students have difficulty making simple rough estimates in part because they lack facility with fundamental engineering concepts, even at the senior level. Seniors appear to have significantly less understanding of basic concepts than was expected. Several examples are given in this section to illustrate the lack of understanding they demonstrated for values, quantities and relationships.

Students did not associate the values they obtained for quantities with their physical significance. The values reported by the MIT seniors for the energy stored in a new 9-volt "transistor" battery ranged from the amount needed to turn this page (approx. 10^{-3} J) to the amount stored in a barrel of crude oil (approx. 10^{9} J) (Strauss 1995). They were equally likely to give any answer value between 10 J to 10^{5} J. Recall Figure 12. This situation is possible if students do not know any reference values for the quantity they are estimating. This is probably the case for energy. However, students do know numerous reference values for weight and force. Yet, 12% of the answer values reported by the MIT seniors for the drag force for the bicycle problem were higher than the weight of a small adult (100 lbs.). Recall Figure 8.

When the students that solved the bicycle problem were informally asked if their answers made sense, the ones that answered in Newtons often said that they did not know. For example, one student simply said, "I don't really know Newtons". All but one of the students that gave values higher than an adult's weight answered in Newtons. In other words, students do not have reference values for forces in Newtons and do not know how to think about numbers in those units. It is still possible that students knew their answers were unreasonable but did not mention it. Only a few students out of hundreds mentioned that their answers might not be right.

Students also had difficulty working with units. 12% of the MIT sophomores in the pilot study reported incorrect units for power including ft/lb, N, ft-lb, J, N/s, W/m and V. 9% of the MIT seniors reported incorrect units for energy, despite having been given the correct units on the answer blank. This could happen if they generated different units in their calculation and did not realize that they were wrong. Five seniors each wrote one of the following surprising statements.

```
# of volts in a Newton? ~100 N/Volt (Guess)

9Volts -> convert to torque.  N-m  Can't remember the conversion
for the life of me

Conversion factor from volts to Joules = ?

I don't know conversion from 450mWh to Nm

1V = ? J
```

Of the 135 MIT seniors' responses obtained for the battery problem, 30 responses demonstrated a lack of understanding of quantities or relationships. The following three responses were selected from these. The first example illustrates how some students appeared to use symbolic operations without regard to meaning to obtain answers. The second and third examples illustrate how students confused different physical quantities.

Example 1

```
?what uses a 9-volt battery?
small battery powered car
mass of car ~ 0.5lbs
acceleration ~ 10 ft/sec^2
distance until battery dies ~ (30 ft)(200)
~ 6000 (if it goes in circles of r ~ 5 ft)
E=Fx=max=(0.5)(10)(6000)=30,000 lb/ft
```

This student found a valid way to think about the battery problem but did not calculate the energy in a battery. Depending on how the solution is interpreted, he has either 1) calculated the potential energy of the toy car at 6,000 feet above the ground but used the wrong value for gravitational acceleration, or 2) calculated the energy to move the toy car at constant acceleration for 6,000 feet neglecting air drag, which would result in the car reaching a speed of 240 mph and completing one loop around the circle in less than 1/10 of a second.

Example 2

```
Energy=V^2/R (Voltage^2/Resistance)
I assume that Resistance R=5 ohms
=> Energy stored = 9^2/5 = 81/5 ~ 16 joules = newton-meters
```

This student recalled a relationship for power but used it to calculate energy, omitting the use of time to calculate energy from power. Many students used energy and power interchangeably.

Example 3

```
What is a "transistor" battery?
If it is a little alkaline, I think at 1 foot, 1lb would stall out a
motor.
Figure 50% efficiency of the motor, so somewhere near maybe 2 ft-lbs.
```

This student neglected that torque must be applied through an angle to obtain energy. The student most likely worked until she obtained the units of ft-lbs on the answer blank and then stopped.

2.7 Gauging students' knowledge

It is proposed that immediate "shallow" knowledge of units may be a useful indicator of "deep" knowledge of quantities and engineering concepts. By surveying students' knowledge of units it should be possible to gauge which quantities they are having trouble with, and by extension which engineering principles they might be having trouble with. For example, students should have trouble giving units for quantities such as weight and entropy that are widely known to give them difficulty. They often confuse weight with mass. They are also known to have far more difficulty with entropy than weight.

Immediate knowledge of units may also be a useful indicator of the ability to solve estimation problems. Knowledge of units is necessary for making simple rough estimates. If students can not associate the correct units with a quantity they will naturally have more difficulty making an estimate involving that quantity. Since estimates involve several quantities, students that have more difficulty providing units should have more difficulty making estimates.

2.7.1 Units survey

A survey was developed that assesses students' immediate knowledge of units by asking them to give the units for several engineering quantities in a short amount of time. 30 quantities were randomly selected from a list of 80 quantities and placed on a page in random order (length was inadvertently omitted making the number actually 79). An example page is given in Figure 25. Each student was allowed 6 minutes to give the units for all 30 quantities. They were told they could give each answer in any unit system they preferred. They were not required to give their answers as combinations of base units.

The list of 80 quantities was generated by reviewing texts for several undergraduate mechanical engineering subjects and selecting what were believed to be the more commonly used quantities. The names of the quantities were kept consistent with the names used in the texts. Some quantities were used twice with different names to check if students had difficulty recognizing the quantities by different names. Furthermore, some of the quantity names such as "shear" were left ambiguous to see if how much difficulty the ambiguity added. The amount of time selected gave students had enough time to consider all 30 quantities and give answers but not enough time to work out the answers by, say, figuring out base units. This was done so that the survey would test immediate knowledge about quantities more so than procedural knowledge associated with units.

FULL NAME ____________________ SECTION____________________

Give the units for each quantity using the unit system you prefer in each case. Show your work. If you don't know the answer put a "?" next to that quantity.

modulus of elasticity	efficiency
entropy	angular momentum
safety factor	centripetal acceleration
fatigue strength	phase
lift	thermal emittance
kinematic viscosity	electrical current
force	heat flow rate
mass density	linear momentum
surface tension	bulk modulus of elasticity
heat	moment
mass	specific weight
fracture toughness	frequency
gravity	electrical capacitance
power	coefficient of friction
area moment of inertia	strength

Figure 25. A sample page from the units survey. Each student received a set of 30 quantities in random order that were randomly selected from a list of 80 quantities common to undergraduate mechanical engineering texts.

2.7.2 Results

The survey was administered to 115 students in a required senior design course in the mechanical engineering department at MIT. Any units that could reasonably be accepted for a quantity were accepted, and in cases of ambiguity, the units in question were accepted. For shear, for example, units of force, stress and angle in addition to unitless were accepted. The complete results are shown in Figure 26. As expected, students had difficulty giving the units for weight and entropy. Only 70% of the students gave a force unit for weight. Only 7.5% gave the correct units for entropy. Overall, the results are surprising and unsettling. For example, only 81% gave the correct units for work, a quantity that is ubiquitous in mechanical engineering. Only 56% gave the correct units for Reynolds number. Given this limited availability of knowledge for the most basic mechanical engineering concepts, it is not surprising that students have difficulty making estimates for all but the simplest quantities.

The results are consistent with the students' performance on the estimation problems. 13% of the seniors gave invalid units for power compared to 12% for sophomore students for the motor problem. 15% gave incorrect units for energy compared to 9% that did so for the battery problem. However, the seniors were actually given the correct units on the answer blank for the battery problem. 97% gave correct units for force consistent with the better performance on the bicycle problem than the battery problem.

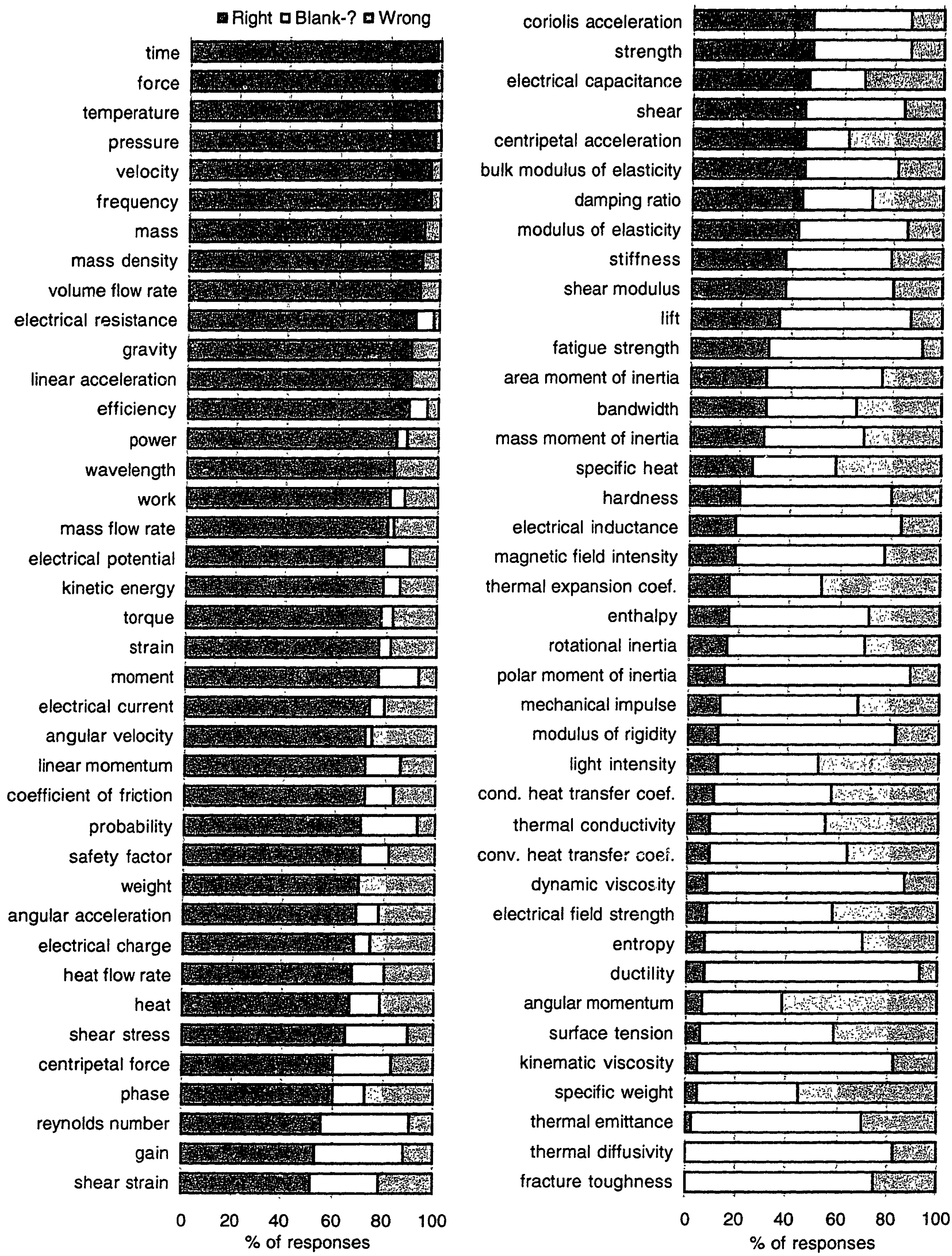
Several groups of quantities can be identified indicating potential areas of confusion for students. For example, the correct responses for the basic quantities associated with angle, including torque, angular velocity and angular acceleration, ranged from 77% down to 69%. Six of the quantities associated with heat transfer, not counting heat and heat flow rate, were given valid units at rates less than 25%.

Students had difficulty when faced with ambiguity, something that is common in estimation problems. While 65% and 52% gave valid units for shear stress and shear strain respectively, only 44% gave valid units for shear. Only 37% gave valid units for shear modulus, however, this appears to be because students are unclear about modulus in general. For example, only 42% gave valid units for modulus of elasticity.

Students responded at the same rates for the same quantity identified by different names, indicating that the choice of names probably did not strongly influence results. For example, 11% and 9.1% of students gave valid units for conduction heat transfer coefficient and thermal conductivity. 78% and 77% of students gave valid units for torque and moment respectively.

It is likely that the result are influenced by the availability effect (Tversky and Kahneman 1973). Students probably have more exposure to some quantities over the course of their total education than other quantities making the units for these quantities easier to remember. Thus, the survey potentially identifies which quantities and units are more commonly encountered in addition to which ones are easier to work with. It is plausible, however, that the two effects are correlated. Units for quantities that are easier to recall might get more attention in a curriculum allowing students more opportunities to learn them. No attempt was made to account for each effect separately. A separate measure of availability for each quantity would be necessary to do so.

Figure 26. (Next page) Results for the units survey. The left bar, middle and right bar indicate the percentage of students that provided the right units, provided no units and provided the wrong units respectively. The quantities are sorted in descending order based on the percentage of correct units.


Right
Blank-?
Wrong
time
force
temperature
pressure
velocity
frequency
mass
mass density
volume flow rate
electrical resistance
gravity
linear acceleration
efficiency
power
wavelength
work
mass flow rate
electrical potential
kinetic energy
torque
strain
moment
electrical current
angular velocity
linear momentum
coefficient of friction
probability
safety factor
weight
angular acceleration
electrical charge
heat flow rate
heat
shear stress
centripetal force
phase
reynolds number
gain
shear strain
0
20
40
60
80
100
% of responses
coriolis acceleration
strength
electrical capacitance
shear
centripetal acceleration
bulk modulus of elasticity
damping ratio
modulus of elasticity
stiffness
shear modulus
lift
fatigue strength
area moment of inertia
bandwidth
mass moment of inertia
specific heat
hardness
electrical inductance
magnetic field intensity
thermal expansion coef.
enthalpy
rotational inertia
polar moment of inertia
mechanical impulse
modulus of rigidity
light intensity
cond. heat transfer coef.
thermal conductivity
conv. heat transfer coef.
dynamic viscosity
electrical field strength
entropy
ductility
angular momentum
surface tension
kinematic viscosity
specific weight
thermal emittance
thermal diffusivity
fracture toughness
0
20
40
60
80
100
% of responses

2.8 Discussion

2.8.1 Expectations

Students were expected to have difficulty making simple rough estimates, but not with the severity discovered. After all, direct attention is not given to conceptual or procedural knowledge of estimation in their undergraduate engineering courses. Nevertheless, they were expected to draw on their extensive problem-solving experience to obtain reasonable estimates. They have solved more than an estimated 3,000 engineering problems by the end of their degree program (10 problems/class week x 10 week/term x 4 classes/term x 2 terms/year x 4 years, see also (Schoenfeld 1992; Woods and al. 1997)).

The battery problem could be considered too difficult for mechanical engineering students. Even the mechanical engineering practitioners had some difficulty solving the problem. The problem appears to be more readily solved using electrical engineering knowledge that the students would not be expected to have. This is supported by the fact that the electrical engineering practitioners did well on the problem. The bicycle problem, however, asks students to estimate the value of a force for an everyday mechanical system. The value asked for is on the human scale and force is a quantity that can be perceived by the senses. Force is perhaps the most fundamental quantity in mechanical engineering. The fact that a majority of mechanical engineering seniors at a top university can not solve this problem raises doubts about the quality of the curriculum in which they were enrolled.

2.8.2 Limitations of the method

The method used to study estimation activity should be representative of estimation activities carried out in practice. Students found to have difficulty making estimates would also have difficulty making estimates in engineering practice. And, the estimation behavior solicited by the method would be similar to that exhibited in practice. To this end, the estimation situations were designed to be similar to those believed to be encountered in practice. However, some of the difficulties students had making rough estimates may have resulted from the method used. This section discusses some of the limitations that could have contributed to this outcome.

In practice, people solving estimation problems might be thinking about material related to a problem before encountering one. This could happen if a problem came up in the context of an ongoing project. In this case, related information would be more readily available for use in the problem. In the studies, subjects were not made aware of problem topics before they were asked to make estimates. The topics of the problems could have been incongruent with what was on their minds at the time. They would have had to bring to mind any information they needed in the time available.

Knowledge useful for solving a problem may not be prompted in the minds of students by the classroom context. Conversely, the context of the classroom could have prompted knowledge that would make a problem more difficult. For example, students that used SI units to solve the bicycle problem gave values that were less accurate than the values given by students that used English units. SI units are characteristic of the academic knowledge associated with the classroom. English units (in the U.S.) are characteristic of experiential knowledge not usually associated with the classroom. The academic knowledge may have been harder to apply to this problem then experiential knowledge.

In practice, people would have a contextually meaningful purpose for solving an estimation problem. One effect of purpose is to guide the determination of the specificity required for an estimate to be useful. In the method used, subjects were either told the resolution required or had access to other information, such as the time available, to guide this decision. However, the subjects had little incentive to obtain a value with high certainty, something that would have been influenced by a meaningful purpose. For the students, the lecture context provided little opportunity for a meaningful purpose other than the sake of learning or requirement. For the practitioners, the only purpose for solving the problems was to support the study. There are probably other effects of purpose that were unaccounted for by the method.

In practice, the limited time available to make an estimate is a natural aspect of an estimation situation. In the method used, however, there was an explicit time limit. Awareness of this limit could have created anxiety over finishing on time that could have affected performance, even if the time available was sufficient for solving the problems. The problems were tried with several students and faculty to check that the time available was reasonable. The majority of these people solved the problems in the time available or could not solve the problems regardless of how much time they were given. However, it was difficult to determine if a majority of the students and practitioners studied had enough time or not.

The issues mentioned above are absent when students solve estimation problems in the context of their design projects. Estimation situations arising during these projects have a meaningful context and purpose and a natural time limit. Students were already known to have trouble making estimates in these situations. However, the method used did not reduce the effects of these issues over the design project situations. Therefore, it is possible that the results obtained overstate the difficulties that the students would have when making estimates in practice.

Chapter 3 A framework for describing estimates

3.1 Introduction

A framework for describing what people do when they make rough estimates would be useful for at least two reasons. It would provide a basis for identifying where people have difficulty when making estimates. And, it would provide conceptual knowledge useful for teaching estimation skills and guiding estimation activity. To these ends, a new framework for describing simple rough estimates is presented in this chapter.

3.2 Overview

3.2.1 Basic ideas

The framework consists of the following three ideas or components. First, a relatively small set of effective actions and corresponding information describe most estimation activity. Combinations of these actions can account for the variety of solutions people generate. Second, the use of these actions is mediated by the characteristics of a person, such as knowledge, mental abilities and beliefs, as well as by the characteristics of a situation, such as context and resources, in which an estimate is made. The particular solutions given by individuals can be accounted for by considering how these characteristics influence the actions they take. Third, the limitations imposed by these mediating characteristics can be overcome by various compensation methods, such as guessing and brainstorming. These compensation methods and the effective actions account for much of what people do when they make estimates. The primary focus of this thesis was on identifying the set of effective actions. These actions are listed in Table 8 and discussed in Section 3.3 along with examples. Mediating characteristics and their influences as well as compensation methods are discussed briefly in Section 3.4.

3.2.2 Method

The framework was developed by examining peoples' estimation processes and their solutions to estimation problems and identifying common actions and information used. An initial set of actions was identified based on studying a small set of problems. Then, these actions were tested by using them to describe how people solved a new set of problems. The actions were then modified until they accounted for the solutions to these new problems. This process was repeated until a set of actions was found that was able to describe peoples solutions to a large number of problems without further adjustment.

The primary data used in this process were the written solutions people provided, some of which are summarized in Chapter 2. However, interviews and direct observations of people making estimates were frequently used to clarify issues and check the ideas generated from studying the written solutions.

A range of people and problems were studied to develop the actions. This was done to insure that a general set of actions was identified. The primary focus was on people with a mechanical engineering background, including undergraduate students, graduate students, educational practitioners and industry practitioners. However, engineers with other backgrounds as well as people outside of engineering were also studied. The problems studied were drawn from several sources, including problems common to engineering practice, problems studied by other researchers and problems developed for this research. The problems chosen covered a range of values, quantities and physical things. Solutions to different problems as well as different solutions to a given problem were considered. For example, at least twelve distinctly different solutions for the bicycle problem were found. These solutions are briefly summarized in Table 7.

Table 7. Different solutions for the bicycle problem given by the students and practitioners studied.

1.	Provide a value for the answer
2.	Feels like a certain amount of force when riding at the given speed
3.	Human energy output balanced by the drag energy loss
4.	Drag force balances gravitational force on the bicycle on a slope
5.	Drag force decelerates bicycle while coasting to a slower speed
6.	Drag force lowers the momentum while coasting to a slower speed
7.	Drag energy loss removes kinetic energy until the bicycle coasts to a stop
8.	Foot force on the pedal balances drag force through the bicycle transmission
9.	Fluid drag force on the person and bicycle given by the drag equation
10.	Fluid drag force is the same as the force felt while standing in a wind of same speed
11.	Fluid drag force is proportional to the weight of a human falling at terminal velocity
12.	Effort at the given speed is equivalent to the effort on an uphill slope at a slow speed

3.3 Effective actions

A person's solution process includes everything they do between the time they start a problem and the time they finish it. These processes can be richly diverse and complex. No attempt was made to describe literally what people do in these processes. Instead, the actions that they effectively take to get from a problem statement to a solution were identified. This made it possible to describe what people do at a level of abstraction high enough to make the diversity and complexity manageable while still providing useful insight into their activities.

The actions were identified in part by considering the types of information people introduced that were necessary for their solutions to be complete. As a simple example, a person's solution may include the use of a particular relationship. Regardless of how much time or how many steps in their process they spent coming up with it, their effective action was to identify a relationship.

Because the actions describe what was effectively done and not actually done, a set of actions describing a solution do not correspond uniquely to a solution process. Thus, two people may go through different processes that result in the same set of effective actions for a given problem. One person may do little to achieve an effective action, while another may do a considerable amount to achieve the same effective action. The effective actions and associated information provide a way to summarize a solution process that preserves important detail and the overall structure of what was done and what was required to solve an estimation problem.

Table 8. The effective actions identified for describing estimates.

1.	Identify a problem system
2.	Identify a quantity with a system
3.	Provide a value for a quantity
4.	Count a set of things
5.	Compare two systems for a quantity
6.	Identify a relationship between quantities
7.	Change a system scope (The "has a" action)
8.	Identify a similar system (The "is a" action)

The effective actions that were identified are listed in Table 8. The actions at the beginning of the list are more direct and lead to convergence of a solution process. The actions at the end of the list are more indirect and lead to divergence of a solution process. The use of an indirect action requires additional actions to be carried out for a problem to be solved. Thus, the simplest complete solution to a problem consists of the first three actions. (Recall the discussion of direct and indirect actions and information in Section 1.6.)

Estimation problems involve determining values for quantities associated with things. The terms object and system are used in this chapter generally to refer to these things. This is because several of the actions involve thinking of things as systems of objects or as objects imbedded in larger systems. When the distinction is unnecessary, the word system is used to refer to one or more objects for the sake of brevity.

3.3.1 Identify a problem system

In order for a solution to a problem to be meaningful, a system must be identified that is consistent with the problem formulation. Some problem formulations make clear what system to consider, while other formulations leave considerable room for interpretation.

In the simplest case, the objects referred to by a problem are physically present with their important characteristics readily apparent. For example, a problem may entail estimating the area of a field in which one is standing or the weight of an object held in one's hand. More likely, however, the objects are not physically present, or physical presence is not helpful.

In some cases, a problem is focused on a particular object, as is the case for the battery problem. A representation of the object may simply come to mind as a result of reading the problem. If the object were unfamiliar, however, more effort would be required to identify it. For example, one student wrote, "What is a “transistor” battery? If it is a little alkaline..." at the beginning of their solution, indicating that they took time to identify the battery. The result of this action is shown in Figure 27.

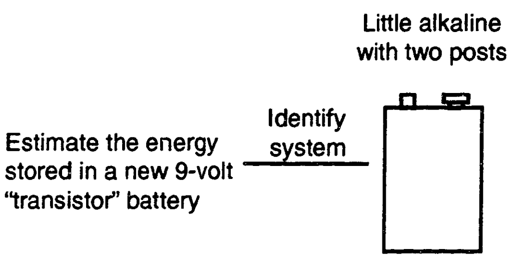


Figure 27. The result of identifying a system for the battery problem.

Many problem formulations, however, are not explicit. A reason for this was discussed in Section 2.2.2 for the jet engine problem. In these situations, several possible systems can usually be identified that are consistent with a formulation, and at least one must be identified. Identifying such a system requires more effort; however, it offers the opportunity to choose a system that is familiar or easy to think about.

The bicycle problem is an example of this type of problem formulation. The problem can be interpreted as involving a person pedaling along a flat surface, a person coasting down a shallow slope, or a person coasting to a stop on a flat surface or a person pedaling up an incline. Subjects in the studies identified all of these options. The first two options are represented in Figure 28.

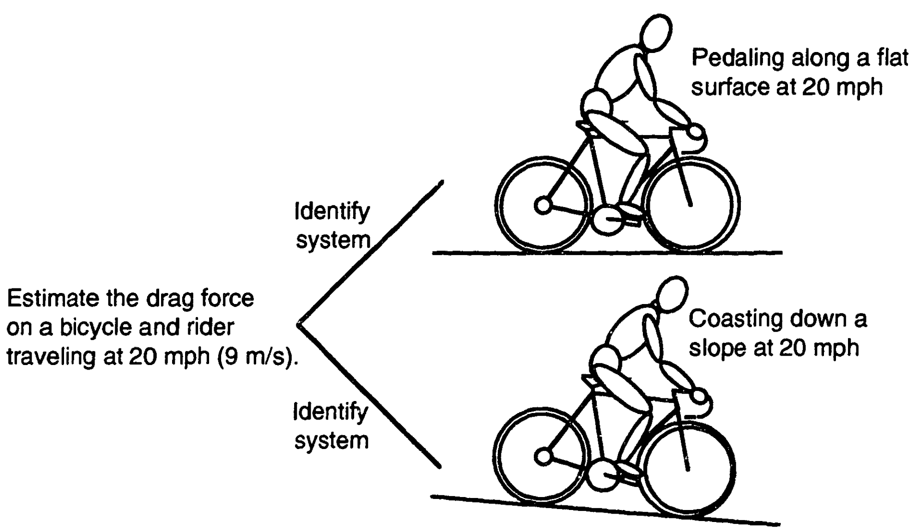


Figure 28. Two of the many systems that can be identified for the bicycle problem.

A problem can also be solved without considering a particular object or system. Instead, the problem is solved for a class of things. For example, one could estimate the thrust of a large jet engine without considering a particular situation involving a jet plane, let alone a particular engine. In this case, the solver would use information such as values and relationships that applies to the entire class to make an estimate.

Because most objects are not present or physical presence is not helpful, people usually work completely with mental representations of the objects. These representations may consist only of mental images. For the bicycle problem, several subjects indicated that they pictured a person on a bicycle from a distance from the side and/or the front. Correspondingly, subjects often made sketches of these views. More likely, however, mental representations also involve schema for how physical objects function or behave. In any case, these representations are often very limited, involving only a small fraction of the information associated with an object or system. As people carry out actions they add to these limited representations.

3.3.2 Identify a quantity with a system

Quantities that are introduced in the process of solving a problem must in general be identified with aspects of the system under consideration. These include the quantities introduced by a problem statement as well as those introduced by another action, such as when a relationship is identified.

For example, the "drag force" in the bicycle problem can be identified as the aerodynamic drag, the rolling friction, or a combination of the two. See Figure 29. Many subjects chose to consider the aerodynamic drag only. They may have known that the rolling friction is small in comparison, or they may not have thought of rolling friction at all. Many subjects chose to use the empirical drag relationship commonly presented in fluid dynamics texts to determine the aerodynamic drag. This relationship is given by $F_d = C_d(1/2\rho V^2)A$. By choosing this relationship, they then had to identify the other quantities, namely C_d, ρ, and A, in the system they were considering.

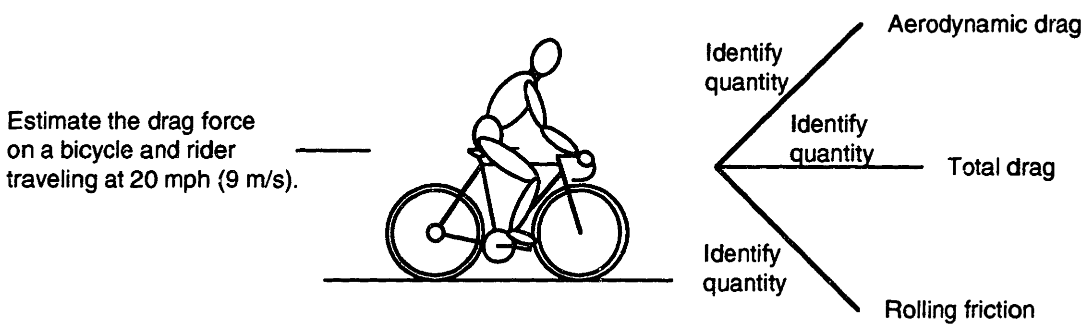


Figure 29. Three possible ways to identify the quantity drag force reference by the bicycle problem.

For the battery problem, most subjects identified the "energy stored" as the electrical energy available to a circuit powered by a battery. This interpretation ignores the energy dissipated directly by the battery. Some subjects identified the energy stored as chemical energy. A few subjects identified the energy as the mass energy given by $E = MC^2$.

3.3.3 Provide a value for a quantity

The most direct way to establish the value for a quantity is simply to provide one directly. In fact, values must be provided for most of the quantities involved in making an estimate. There are usually several possible values that a person can provide for a quantity. For example, a person may choose a particular value or a representative one, such as a minimum, maximum or average value, or a range defined by these values. Of course, whenever a value is provided, units must also be provided although standard units need not be used. People often use familiar objects as non-standard units to qualify values. For example, people often speak of room sizes in terms of a number of their paces.

People tend to have more knowledge of values on the human scale and values for quantities that can be directly experienced by the senses. The sophomores discussed in Section 0 clearly knew more about the dimensions of small objects than they did about the power output of motors. People may know values for quantities for particular physical objects or they may know values for classes of those things. For the battery problem, a few electrical engineers indicated that devices of the kind that run on small batteries draw about 10 milliamperes. Similarly, quantities for some objects have "standard" values, such as standard bolt sizes, for which people may have knowledge.

Many people know values for quantities with high specificity but cannot verbalize why they know them. They may have learned the values through experience and then forgotten how they learned them. This form of knowing can be confused with guessing, a compensation method, because it also results in an unsupported value being provided.

In some cases, people know a numerical value for a quantity. In other cases, people know the value for a quantity in a non-numerical form, such as a mental image or other form resulting from sensory experience. A few subjects indicated that they imagined themselves on a bicycle and what the wind felt like. In these cases, a numerical value can be obtained by a combination of counting and comparing actions discussed in the following sections.

3.3.4 Count a set of things

The numerosity of a set of thing is often determined by counting. When objects are physically present people can look over them and count them directly. When they are not present, people may have mental images of them and the objects can be counted using these images. In effect, non-numerical knowledge of a number of objects can be converted to a numerical value by counting. For example, this method is often used to determine the number of seats in a room one has seen before.

The counting action is often used in the act of making comparisons, which is discussed in the next section. It is also often used in conjunction with the scope change action described in Section 3.3.7. For example, to estimate the number of seats in a room, one can start by counting the number of rows in the room and then the number of seats in a row. Including rows in the representation of a room of seats is a scope change action.

3.3.5 Compare two objects for a quantity

Two objects can be compared if they have a quantity of the same type in common. Comparing two objects determines the ratio of the values for that quantity, without necessarily knowing the absolute values. The comparison is accomplished by visualizing two objects juxtaposed. This visualization is easier if either of the objects is physically present.

When two objects are not comparable in dimension, it is necessary to picture the smaller object repeated until the larger object is matched in dimension and to count how many times the smaller object was repeated. The visualization becomes more difficult if the smaller object must be repeated more than a few times. For the battery problem, one subject found the ratio of the volumes of the "transistor" battery and a "AA" battery by visually picturing two "AA" batteries filling a "transistor" battery.

Subjects used this action for spatial quantities such as length, area, volume and angle. Subjects often made estimates for area, volume and angle (and even time) by taking recourse to length estimates, suggesting that comparisons for length are the easiest to make. For quantities that are difficult to visualize, subjects were found to use absolute numerical values to calculate ratios instead. Although the subjects studied only used this action for spatial quantities, it may be possible to make similar comparisons for other types of quantities. A person would have to have some form of sensory experience that they could recall and "visualize" for these quantities.

In general, objects with the same orientation are easier to compare because they require fewer mental operations before a comparison can be made. For example, comparing the length of a bed to the height of a room would first require a mental rotation. Objects are also easier to compare when they both belong to the same conceptual scope. People tend to have more knowledge of the relative dimensions of such objects, and these dimensions are probably easier to recall and visualize together. For example, it is easier to compare the height of a living room to the height of a person rather than to the height of a car.

3.3.6 Identify a relationship between quantities

A value for a quantity can be determined indirectly by identifying a relationship between it and other quantities. This action is necessary if a value for a quantity cannot be determined by one of the more direct actions (provide, count and compare) discussed in the previous sections. Introducing a relationship provides opportunities to use additional actions to solve a problem.

People can identify and use many kinds of relationships to solve a problem. For engineering systems these include, among others, definitions, geometric relations, physical laws and constitutive relations. Subjects often used more than one relationship to solve a problem and seldom used more than a few. Relationships may be stated as equalities, inequalities or proportionalities. Subjects frequently used proportionality relationships in addition to the more commonly used equality relationships. The different forms require different information to be obtained by subsequent actions.

For the bicycle problem, many subjects used the equality form of the drag equation introduced in most undergraduate fluid dynamics texts, given by $F_d = C_d(1/2)\rho V^2 A$. This usually required using another relationship to establish the area, A. Other subjects used a proportionality form of this relationship such as $F_d \propto V^2$. A few subjects converted relationships they had stated as proportionalities to equalities by explicitly identifying a constant of proportionality. For example, one subject converted $F_d \propto AV^2$ to $F_d \propto kAV^2$. For the battery problem, many subjects used the equality relationship $P = ET$. However, some subjects used the proportionality relationship $E \propto V$.

In order to complete the use of a relationship, some form of computation must be carried out to evaluate the relationship. In simple rough estimation situations, subjects often use computational estimation techniques to make these evaluations. A computational estimate is made when the result of a computation is approximated by a simpler computation. For example, to compute the area of U.S. letter paper, one must multiply 8.5 inches by 11 inches. A typical estimate for this computation would be to multiply 9 inches by 10 inches yielding 90 square inches. In this case, two computational estimation strategies have been used together. First, the original numbers were adjusted to numbers that are easier to work with. Second, the numbers were changed in opposite directions so that one change compensated for the effect of the other on the answer. See (LeFevre, Greenham et al. 1993) for a review of research on computational estimation.

3.3.7 Change a system scope (the "has a" action)

When people think of an object or system, they are thinking of a partial representation of those things. Initially, a representation usually does not include enough aspects for a person to carry out an estimate. To facilitate making an estimate, the scope of the representation can be changed to include more or different information. The change is useful if it allows additional actions to be made.

People often make two complementary types of changes. One is to consider additional aspects of the object or system. The other is to consider the object or system in mind as an aspect of a larger system. For the battery problem, one can think of the chemical constituents of the battery, or one can think of the battery as powering a device such as a small radio, as shown in Figure 30. These changes can be characterized as "has a" changes. A battery has a chemical subsystem. A radio has a battery. Many subjects had enough knowledge of devices that used batteries to make an estimate, but none had enough knowledge about the chemistry of a battery to do so, although several tried this approach.

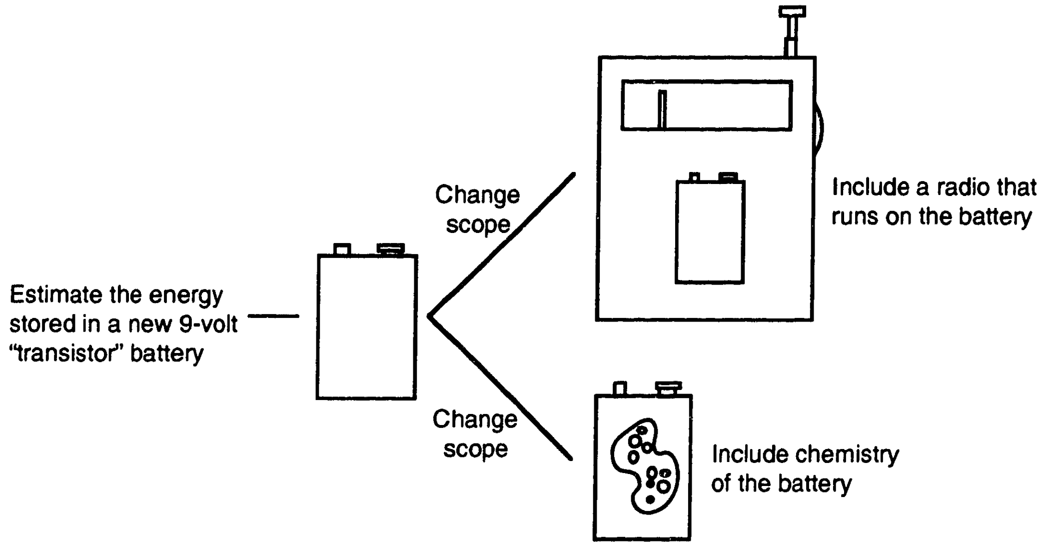


Figure 30. Two scope changes identified by subjects for the battery problem.

The aspects that are identified may or may not be physical features of the object involved. To estimate the height of a building, one could expand the scope to include the floors or to include another building. See Figure 31. These changes both involve identifying physical features of the building. However, one could have identified the midpoint of the height of the building, which is not a physical aspect of the building. In general, one can divide spatial characteristics of objects, such as length, area, volume and angle, into regular sections and then work with these sections. For example, one subject imagined half of the "transistor" battery and compared it to the volume of a "AA" battery.

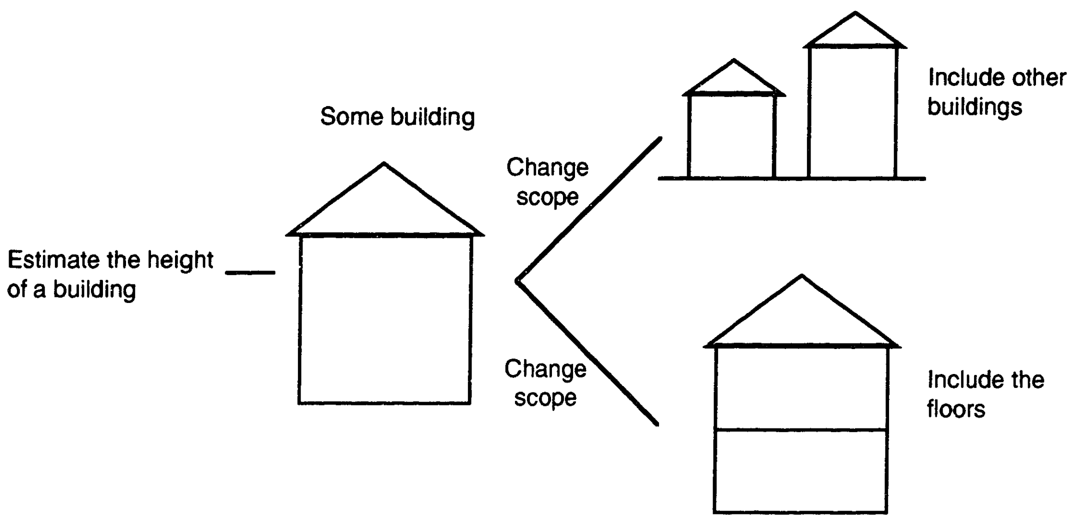


Figure 31. Two changes of scope for estimating the height of a building.

Objects often belong to several different conceptual systems. Thus, several possibilities for a scope change exist as a consequence of these different perspectives. For the building height problem, thinking of a building as part of a spatial system leads to considering other buildings or perhaps the city block in which the building exists. However, a building can also be thought of as part of a city with zoning laws, a very different kind of system. Some cities have laws limiting the height of buildings. If one happens to know this limit, it could be used to establish a value for the height of a building in question. Similarly, for the battery problem, a battery can be thought of as a product in a larger economic system. The energy density might be related to the energy density of other batteries based on arguments of product competitiveness in a commodity market. The estimate might then involve knowledge of market rates for units of energy.

3.3.8 Identify a similar system (the "is a" action)

People often find it useful to identify another system that is similar to the one under consideration. The new system often provides opportunities to perform actions that a person cannot make with the original system. Results of actions taken on the new system apply to the original system if they are sufficiently similar. To be sufficiently similar, the two objects must have quantities of the same type in common, and these quantities must also be in the same relation to each other when more than one is quantity considered. When two or more quantities are considered, this condition is referred to in engineering and science as geometric or physical similarity depending on the types of quantities involved. See (Szirtes and Rosza 1998) for discussion and numerous examples. The new systems can be characterized as "is a" systems because they are the same as the original systems in some way.

The simplest type of similar system is one that has only one quantity in common with the original system. These systems are used as references for making comparisons. For example, when estimating the height of a room one can identify a person or a doorway as an object and compare it to the room.

For similar systems involving more than one quantity, the results of actions taken on the similar system must be related back to the original system. This amounts to establishing proportionality factors for the relationships common to both systems or to finding the ratios of quantities common to both systems. Which factors and ratios must be determined depends on which values are known for each system. In any case, they are either calculated using known relationships or by making comparisons. In a sense, these similar systems are more sophisticated reference systems requiring more complicated comparisons.

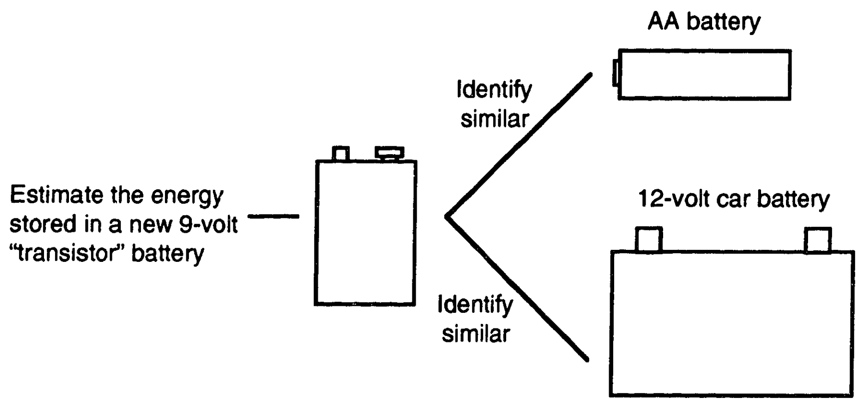


Figure 32. Two similar systems identified by subjects for the battery problem.

Subjects identified several similar systems involving more than one quantity. For the battery problem, subjects identified an "AA" battery and a 12-volt "car" battery as similar objects to a "transistor" battery. See Figure 32. An "AA" battery is a chemical battery as the "transistor" battery is a chemical battery. For the bicycle problem, some subjects thought of a person standing in the wind, while others thought of a person falling at terminal velocity. See Figure 33. A person falling is acted on by air as a person on a bicycle is acted on by air. One subject thought of the similar system of their hand held out a window while driving a car.

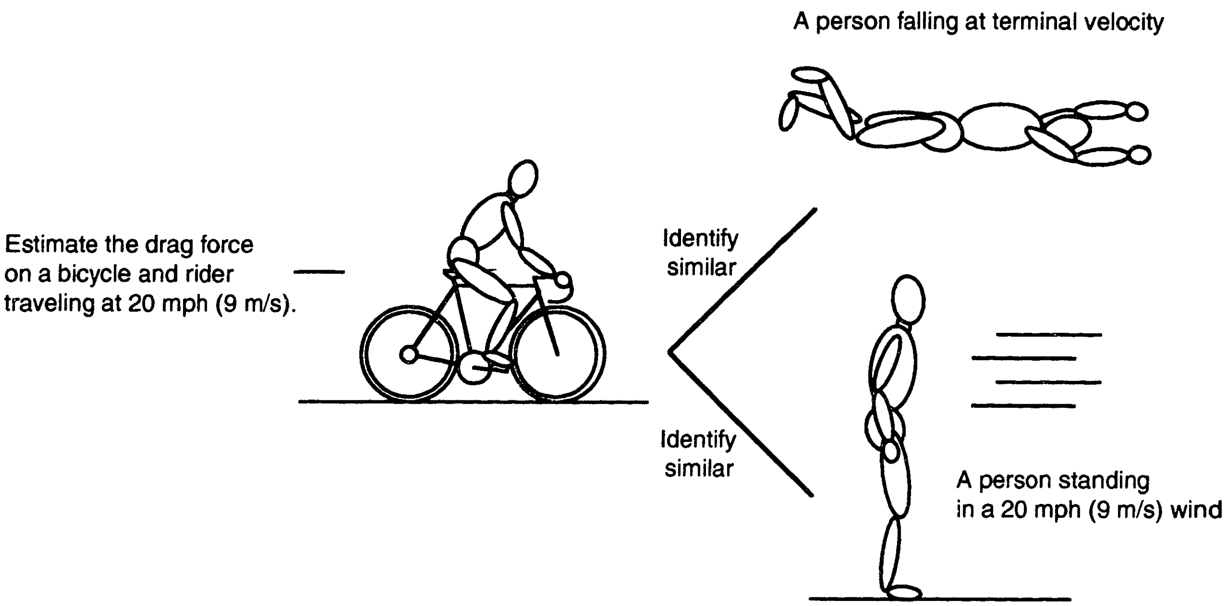


Figure 33. Two similar systems identified by subjects for the bicycle problem.

The differences between the original system and the similar system provide opportunities to use additional actions involving other knowledge. However, the larger the difference, the harder it is to use this technique. A hand-out-a-window based solution requires more effort than a person-standing-in-the-wind based solution because the system involved is not as similar to the original system. One subject chose to consider the bicycle and rider as a sphere, as shown in Figure 34. Although this similar system is simple, it is sufficiently different from the original system that it gave the subject some difficulty. (A flat plate would probably have been a better choice.)

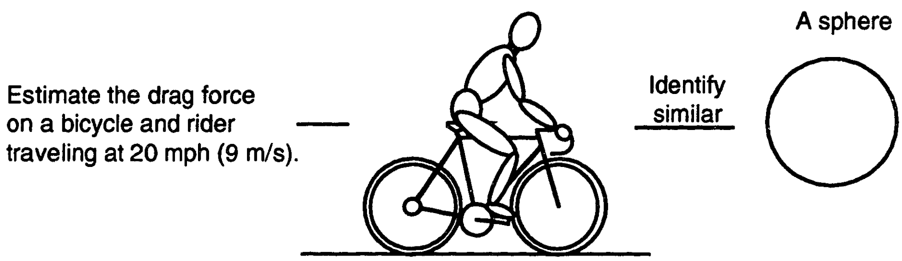


Figure 34. A similar system identified by a subject for the bicycle problem.

3.3.9 Examples

This section contains three complete examples to illustrate how the effective actions can be used to describe estimation activity.

Example 1

The written response given by a subject to the bicycle problem is transcribed below. The description that follows is depicted graphically in Figure 35.

```
Human can produce ≈ 200 watts
200 = 9 x F
F = 200 / 9 ≈ 20 newtons
```

The subject decided that a human can produce approximately 200 watts (provide value) of mechanical power (identify quantity), which is established by the first and second lines of the response. Furthermore, he decided that all of this power is needed to propel a bicycle at a speed of 9 m/s (identify quantity) and that it all goes to overcoming the drag force (identify quantity). These ideas are established by the form of the relationship, given on the second line of the response, which relates the human power output to the drag force (identify relationship). From this information, he calculated the drag force. It can be inferred from this solution that he thought of a person pedaling along a flat surface (identify system).

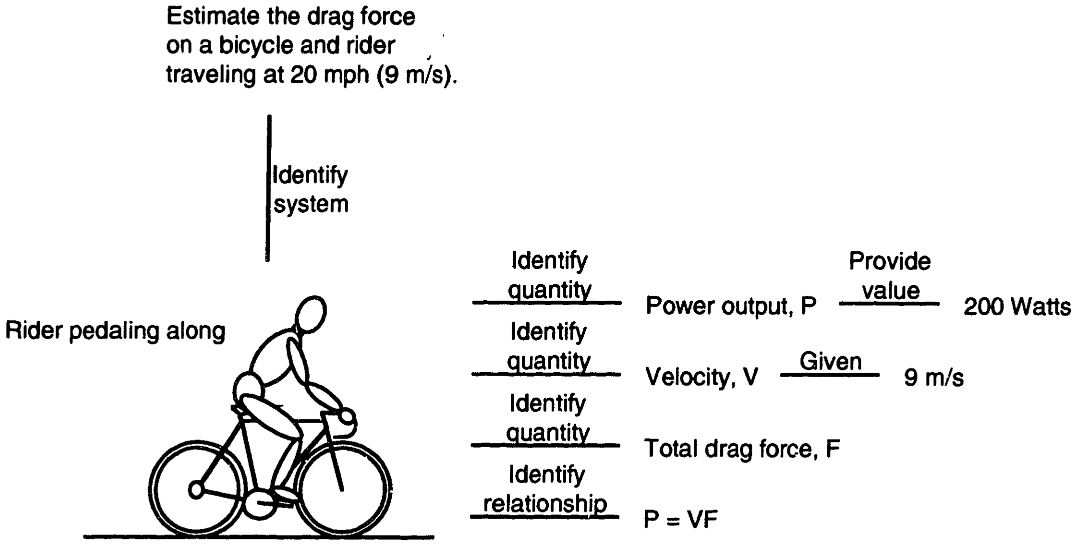


Figure 35. A depiction of a subject's written response to the bicycle problem.

Example 2

This example is based on the verbal response given by a subject to the bicycle problem. The following description is depicted graphically in Figure 36. The subject considered a rider pedaling at constant speed on a flat surface (identify system). She chose to focus on the transmission of the bicycle (change scope), which included the rear wheel, a rear gear, a front gear and a pedal crank arm. Finding this system too difficult to work with, she considered it to be made up of two smaller transmissions (change scope). She then determined the transmission ratios for these two systems. First, she compared the wheel radius to the rear gear radius (identify quantities, compare quantities) to obtain their ratio. Then, she compared the crank length to the front gear radius (identify quantities, compare quantities).

Next, she returned to the original transmission system by established a relationship between the force of the road on the wheel and the foot force that included the two ratios that she had already established (identify quantities, identify relationship). Finally, she finished the problem by deciding that the force of the road on the wheel was the same as the drag force (identify quantity, identify relationship) and by providing a value for the foot force (provide value). Note that two more identify-quantity actions might be added to this description for the two gear ratios, depending on the interpretation of the subject's response.

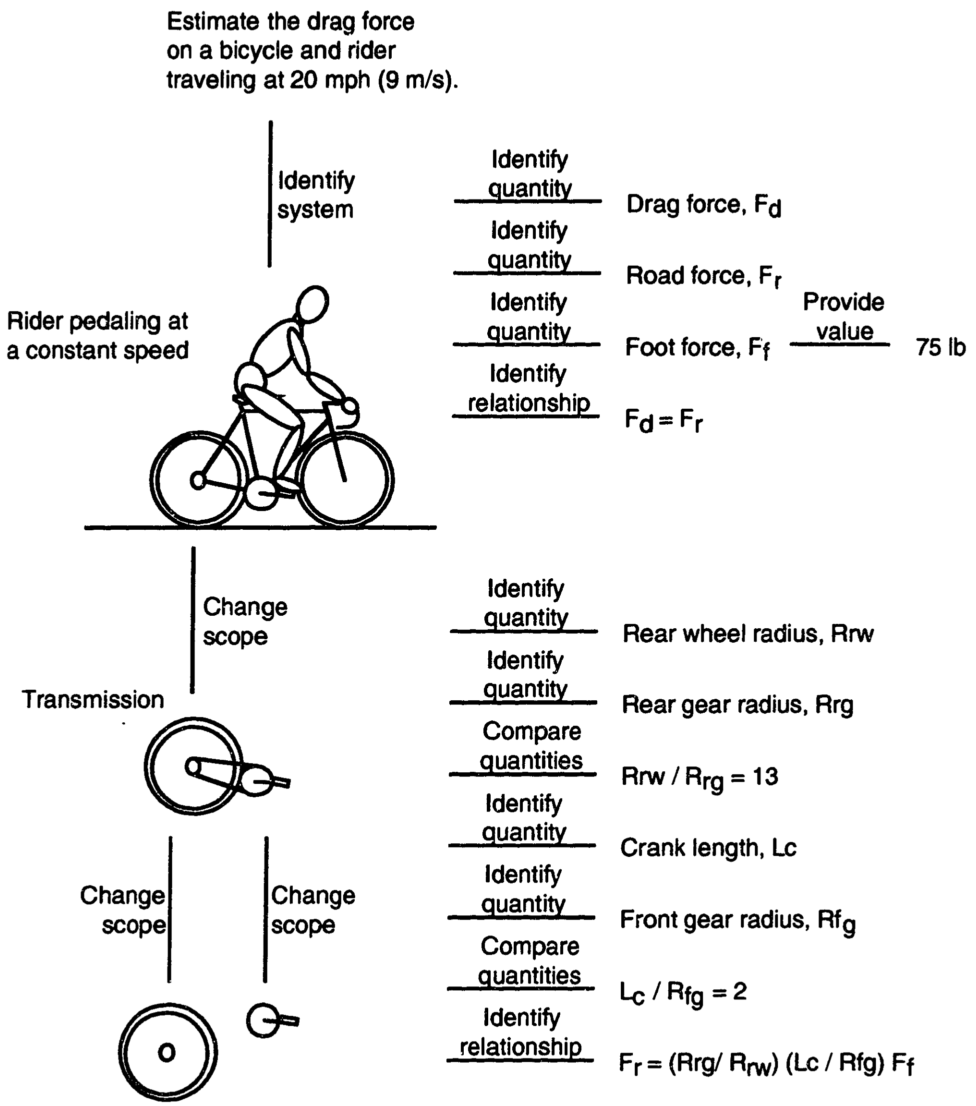


Figure 36. A depiction of a subject's verbal response to the bicycle problem.

Example 3

This example is based on observations of a subject's solution process for the battery problem. This example was chosen because it illustrates all of the effective actions. The following description is shown graphically in Figure 37. The subject identified a 9-volt "transistor" battery as one of those little alkaline batteries with two posts (identify system). Like most people, she did not know the value for the stored energy (provide value). She also could not think of anything that uses a "transistor" battery (change scope). However, she was familiar with "AA" batteries (identify similar) because she recalled using them in a small flashlight (change scope).

She realized that the energy in the "AA" batteries (identify quantities) was used up over time (identify quantity) by the flashlight bulb and that she knew that the flashlight could be used for about 2 hours on new batteries (provide value). These thoughts triggered the knowledge that energy per unit time is power which led to a relationship for the energy provided by the batteries (identify relationship). She was not sure of the power rating of the light bulb (identify quantity) but guessed that it was about 5 watts (provide value). She then calculated the energy provided by the "AA" batteries.

To calculate the energy provided by one "AA" battery, she had to account for the number of batteries (identify quantity) used by the flashlight. The number of batteries was found by picturing the batteries in the flashlight and counting them (count things). She then divided the energy provided by the batteries by this number (identify relationship).

Once she had the energy for the "AA" battery, she realized she needed to relate that to the energy for the "transistor" battery. After some thinking, she decided that the batteries were the same (earlier identify similar finalized) and just had different volumes (identify quantities). Thus, she reasoned that since the "transistor" battery was about twice (compare quantities) the volume of the "AA" battery the energy was about twice also (identify relationship).

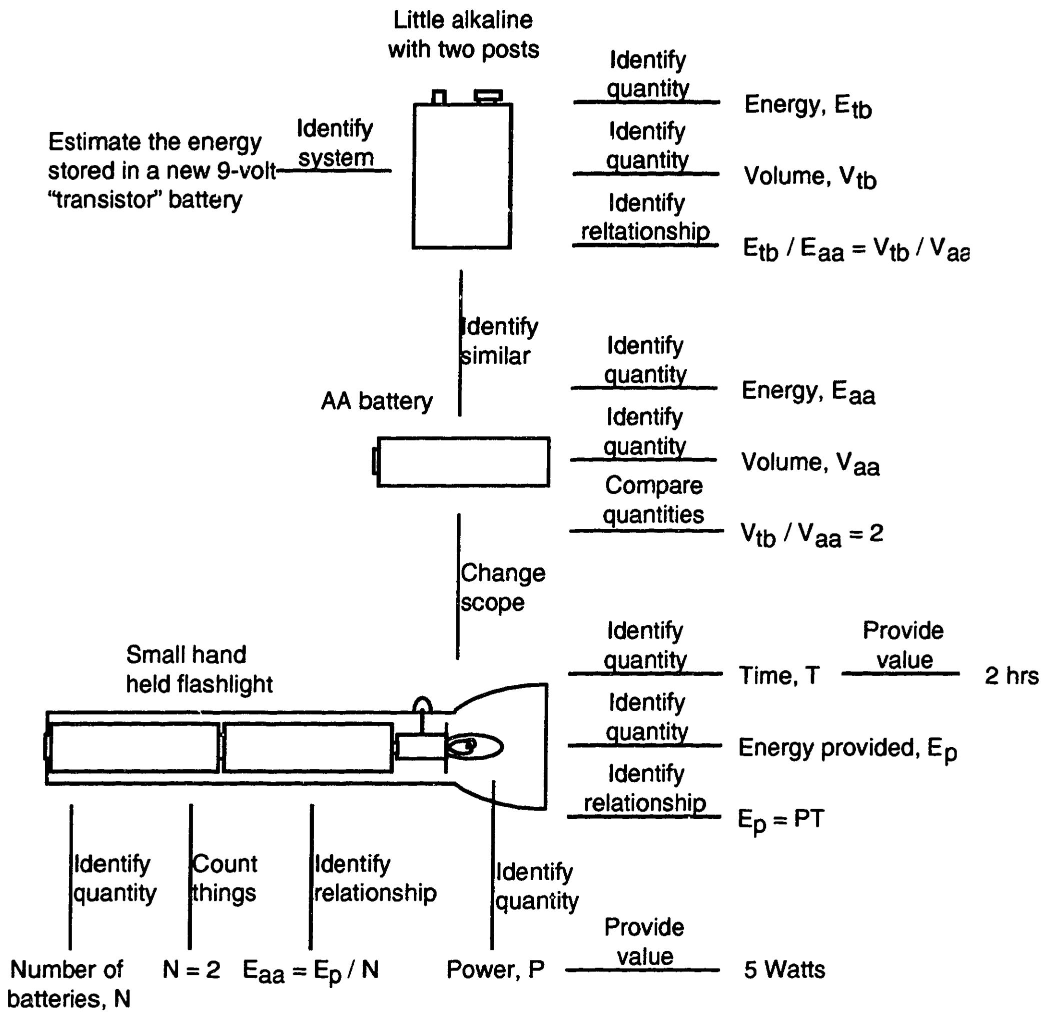


Figure 37. A depiction of a subject's solution to the battery problem.

3.4 Mediation and compensation

The characteristics of people and estimation situations mediate their solution processes and consequently the effective actions they take. The characteristics of people that affect their solution processes include aspects of their knowledge, mental abilities and beliefs. Characteristics of situations include the resources available and the nature of the surroundings. The combination of these characteristics allows a person to solve a problem in some ways and not others. Consequently, people can only generate some of the possible solutions to a problem, and they can not all generate the same solutions. For the bicycle problem, most subjects could generate at most one or two solutions, but overall they generated the twelve solutions given in

Table 7. A few examples are given in Section 3.4.1 to illustrate how characteristics of people mediate solution processes.

People have developed a variety of methods to compensate for the limitations these characteristics impose on their solution processes. A few examples are given in Section 3.4.2 to illustrate these methods. In fact, a significant portion of estimation activities consist of compensation activities. Typically, a person will encounter a difficulty while solving a problem that prompts them to use one of these methods to circumvent it. See Figure 38. Unless a difficulty arises, people are often not aware of the limitations imposed by mediating characteristics.

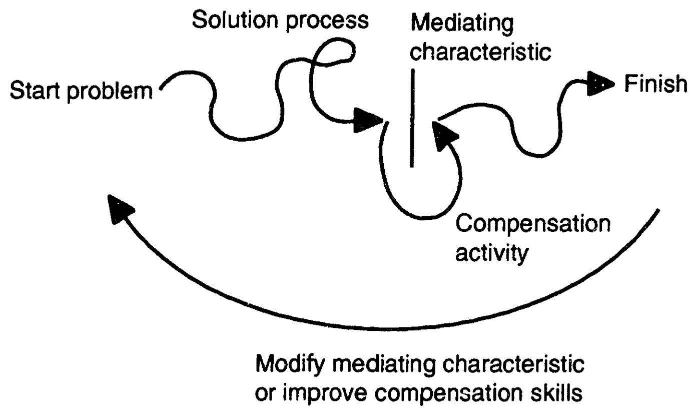


Figure 38. The influenced of a mediating characteristic on a solution process is overcome by compensation activity.

Estimation skills can be improved by reducing the influence of mediating characteristics or by improving compensation skills. See Figure 38. For example, a person could choose to learn more reference values so that they can more easily provide values. They could resolve to carry a calculator so they can more easily make computations in estimation situations. Or, they might decide to practice a compensation method to improve their compensation skills. Thus, it is important to understand which mediating characteristics have the most influence on peoples' solution processes and how they affect these processes. Likewise, it is important to understand which compensation methods people can use and which are the most effective.

3.4.1 Examples of mediating characteristics

A brief discussion based on examples of how the characteristics of people mediate their solution processes is given in this section.

Particular characteristics affect the performance of certain actions more than others do. The comparison action, for example, is easier to carry out by someone with better visualization skills. Anchoring effects have more influence on people who provide a value based on less knowledge (see Section 1.6).

Some characteristics make certain actions possible and others impossible. Different people have different knowledge and experience. A person that has never used a 9-volt "transistor" battery in any device lacks knowledge that would allow a change of scope and consequently, a whole set of solutions is not possible for that person. Some subjects studied believed they did not have enough knowledge to solve a problem, indicated by statements such as "not enough info for an EE" for the bicycle problem, even though the bicycle problem can be readily solved without special knowledge. This belief prevented them from solving the problem.

Some characteristics select for certain actions. People think in different ways, depending on their background, which leads to preferences or biases that favor certain actions. 23 of the 26 mechanical engineering practitioners studied (88%) attempted to solve the bicycle problem using the drag equation relationship while only 5 of the 27 electrical engineering practitioners (19%) did so. The electrical engineers may not have known about the drag equation. Even though the mechanical engineers most likely had the knowledge to carry out many of the other simpler solutions, they preferred the drag equation approach. Indeed, when several subjects who had just solved the bicycle problem were given more time, and asked to think of other solutions, they could not think of any. However, when shown the other solutions, they had no difficulty understanding them, and actually agreed that they had the knowledge needed to complete these other solutions.

Because of these different effects, choices of which actions to take become important. For example, identifying a relationship is an expansive action that leads to additional actions that must be completed, which require more information and time to complete. The effect of this choice becomes important when time or knowledge related to the relationship is limited. Of the

23 mechanical engineering practitioners that tried to use the drag equation for the bicycle problem only eleven completed the problem using this approach. Seven did not finish the problem and five abandoned the approach and provided a value for the answer directly. Only six of the eleven that finished the problem with the drag equation obtained an answer value on the right order of magnitude. All five that provided an answer value directly obtained an answer on the right order of magnitude.

3.4.2 Examples of compensation methods

Several examples of compensation methods are given in this section. Each example identifies a particular difficulty subjects encountered and the method they used to get around it.

Most subjects found it difficult to solve the battery and bicycle problems without using some form of external representation, such as writing or sketching. These subjects may have been compensating for limited working memory or limited visualization skills. External representations appear to be widely used to compensate for the influences of different characteristics.

Some subjects needed information that they could not produce, such as a value or a relationship, and compensated by guessing the information. They may not have had knowledge of the information or the ability to recall it in the situation they were in. In some cases, guessing functions as a placeholder action that allows one to continue and finish a solution process. In this case, the accuracy of a solution is limited by the accuracy of the guess unless the answer value is checked. Some subjects revised their guessed value when they deemed their answer value to be unreasonable. In other cases, guessing may serve as a means of facilitating recall. Guessing may access knowledge that is otherwise unavailable to a person.

Some subjects could not think of a way to proceed and used brainstorming techniques to compensate. For example, they ask themselves questions such as "what quantities do I know" and "how could I use weight" and then tried to answer them. These questions helped them think of information they could use to make progress, possibly by facilitating recall or a process of generating new knowledge.

Many subjects had difficulty providing values in situations calling for single values. To compensate, they provided a range of values that they were confident in. This is a simple example of using approximations to compensate for difficulties. Approximations allow one to reduce the specificity of a solution for ease of obtaining the solution. Approximations are widely used to overcome the limitations of a variety of characteristics.

Many subjects did not realize that their initial actions would require subsequent actions that they would not be able to complete. For example, several subjects almost completed the bicycle problem using the drag equation only to realize that they did not know the density of air and could not finish. However, some subjects were observed compensating for this effect by explicitly considering the consequence of an action before using it.

Some subjects could not recall a relationship completely or accurately. Some of these subjects compensated by using other knowledge such as that of units to help them correct the relationship. This happened even for simple relationships like $P = I^2R$ for the battery problem. This also happened for the drag equation for the bicycle problem because it has several terms that can be hard to remember. Some people did not have difficulty recalling the drag equation because they thought of it in a way that is similar to the structure of the well-known relationship $F = PA$ that made it easy to recall. They thought of the drag force as equal to dynamic pressure ($1/2 \rho V^2$) multiplied by area (A) with a coefficient (C_d).

3.5 Discussion

3.5.1 Actions and solution processes

The effective actions were presented separately in order to describe them. However, the actions do not necessarily correspond to separate, distinct portions of a solution process, nor do they necessarily correspond to explicit portions.

Some actions may effectively be carried out together during a process. For the jet engine problem discussed in Section 2.2.2, both the problem system and the problem quantity are ambiguous. One could start by considering different "thrusts" such as the thrust at takeoff or the thrust at cruising speed. This consideration introduces two potential problem systems, a plane taking off or a plane cruising. Choosing one of the quantities also chooses the system and vice versa. A decision results in the two actions happening together.

Some actions may happen implicitly in a process. For the battery problem, for example, a person may identify the problem system as a single battery and then change the scope of the system to consider the battery inside a radio without first identifying the "energy stored" in the battery. With the radio in mind, they may then identify a relationship between the energy consumed by the radio and other quantities and proceed to establish this value. Many people often stop here, and in so doing, implicitly identify the "energy stored" in the battery as the energy consumed by the radio.

In some cases, a person may know so much about a problem that there is little explicit process to describe. Nevertheless, the solution can still be described in terms of the effective actions. For example, when subjects were asked to estimate the area of U.S. letter paper most immediately wrote down the product of 8.5 x 11. Many people in the U.S. know "U.S. letter paper" as "8.5 x 11 paper". In this case, just knowing the problem system corresponds with knowing everything one needs to solve the problem.

It is sometimes possible to describe a part of a process in different ways. For example, some people may only be able to identify a "transistor" battery by thinking of it in the context of a radio that runs on it. Only the battery corresponds to the system referred to by the problem statement. This could be described as two actions taken simultaneously, identifying a problem system and effecting a scope change. Or, it could simply be described as identifying a problem system that has aspects beyond the problem statement.

3.5.2 Related research

The mediated action framework is consistent with the related estimation and problem-solving research discussed in Section 1.6. The estimation studies that have focused on asking subjects to provide values directly or nearly directly provide insight into how the provide value, count and compare actions are carried out and which characteristics may mediate these actions.

Strategies identified by measurement estimation research can also be described by the effective actions identified in this research. As a simple example of measurement estimation, suppose that you wish to estimate the ceiling height of your living room. This can be accomplished by considering your height as a reference value and visualizing how many of you placed end-to-end would reach from floor to ceiling. Your height forms a non-standard unit, which is used to mentally measure the ceiling height. When described in terms of the effective actions, these steps are broken down into more detail. This description is presented graphically in Figure 39.

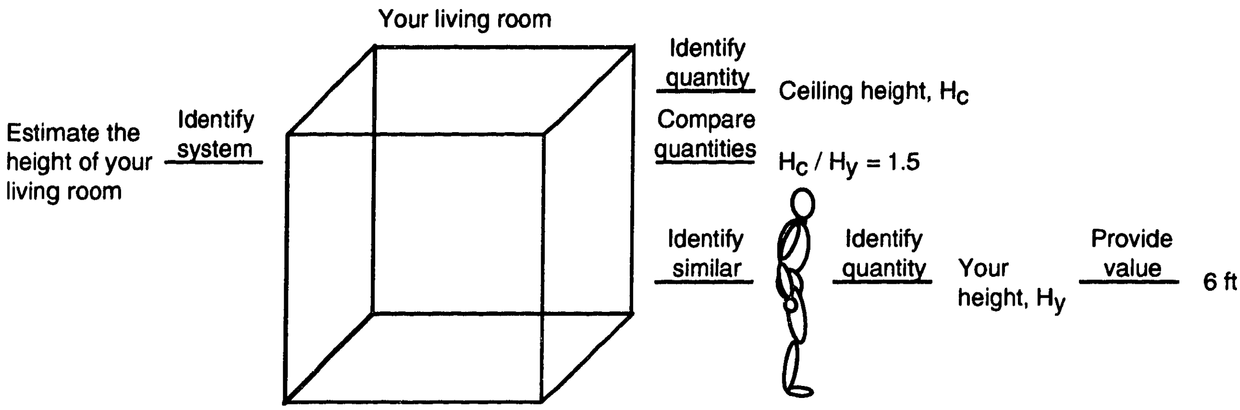


Figure 39. The effective actions used to estimate the height of your room.

Human problem-solving research is generally concerned with understanding peoples' solution process somewhat independent of the subject area in which the problems are solved. For example, Simon developed a problem-space-goal-action framework while Schön developed a frame-move-evaluate framework for describing peoples' solution processes, and both applied these frameworks to a number of subject areas. The framework described in this chapter captures the specifics of what people do in the subject area of rough estimation and does not explain people's overall problem-solving processes. Thus, the estimation framework can be combined with different problem-solving frameworks to describe estimation activity more completely.

3.5.3 Guiding estimation activity

The mediated action framework is useful for prescribing estimation activity in addition to describing it. The framework provides conceptual knowledge that can be used by a problem solver to guide their solution process. The actions identified provide an explicit set of options available at each point during a process. Furthermore, each action suggests the kind of information that one needs to establish. Knowledge of mediating characteristics provides an opportunity for a person to be selective about what they choose to do, making it easier to avoid more difficult or time-consuming actions for example. Knowledge of compensation methods helps a person overcome difficulties when they can not be avoided.

Suppose, for example, that you were one of the many people that could not make progress on the battery problem. Selecting from the list of actions, you could choose to brainstorm to generate ideas for how to change the scope of the system you are working with. What device "has a" 9-

volt battery that you know of? Subjects that solved the problem thought of clocks, radios, toys, flashlights, smoke alarms and garage door openers. Or, you might generate ideas for similar systems. What "is a" battery that you do know about? Subjects that solved the problem thought of "AA", "C" and 12-volt "car" batteries. You could also combine the two actions and try to think of any device that uses a battery.

3.5.4 Completeness

A set of actions describing estimation is useful to the extent that it captures a significant portion of estimation activity in a meaningful and universal way. The small number of effective actions identified in this thesis were sufficient to describe the large number of solutions generated by subjects for the problems studied in a consistent manner. They capture estimation activity at a level of abstraction that is both insightful and practical.

However, the set of actions identified is not necessarily a complete or unique set of actions. They were identified by focusing on particular situations involving particular subjects and problems. Other actions may very well have to be identified or the current ones modified in order to account for other situations. These actions have not yet been used by many people to describe or guide estimation activity. They may have to be revised to meet the needs of other students, educators, practitioners and researchers.

Chapter 4 Rough estimation and engineering curricula

4.1 Introduction

The relationship between rough estimation and undergraduate engineering curricula is discussed in this chapter to provide insight into why students have difficulty with rough estimation activities. This relationship is examined by comparing and contrasting rough estimation activities and the learning activities used in engineering curricula. Learning activities consist of the homework assignments, exams, projects, lectures, etc. that curricula engage students in. Based on this comparison, rough estimation activities are found to be incongruent with typical undergraduate engineering curricula.

4.2 Comparison and contrast

Rough estimation activities and the learning activities used in undergraduate engineering curricula have much in common. A large amount of the basic engineering knowledge covered by these learning activities is needed to make rough estimates for engineering quantities. This includes knowledge of quantities, units systems, physical laws, appropriate assumptions, mathematical concepts, geometry principles, material properties and physical devices. Both activities ask students to use this knowledge to solve similar problems. "Determine the value of a quantity for a physical thing" is a description that fits the problems encountered in either type of activity.

Students that have experienced a mechanical engineering curriculum find the content of the estimation problems used in this research to be familiar. They readily understand the solutions to these problems when they see them. Yet, they have considerable difficulty solving these problems on their own. This suggests that important differences exist between rough estimation activities and the learning activities used in undergraduate engineering curricula. Several of these differences are identified in this section by contrasting characteristics of the two activities. In so doing, both activities are necessarily highly caricatured. The main characteristics considered are summarized in Table 9.

Table 9. Relative characteristics of rough estimation and learning activities.

Characteristic	Rough estimation activities	Learning activities
Supporting resources	low	high
Relevant Information	selected	provided
Uncertainty	high	low
Required knowledge	balanced	focused
Situations	uncontrolled	controlled

The learning activities considered are primarily those of core mechanical engineering science subjects, such as statics, solid mechanics, dynamics, kinematics, fluid mechanics, heat transfer and thermodynamics. A brief comparison of learning activities associated with design and rough estimation activities will be presented in the discussion section.

4.2.1 Supporting resources

Rough estimates are usually carried out with limited supporting resources. In fact, a lack of resources is often the very reason a rough estimate is chosen over a more detailed analysis. The needed resources may not be available in the context in which an analysis must be made, or the resources may not exist at all. Thus, when making rough estimates, people must be able to work with a limited amount of resources. In the case of simple rough estimates, these resources are usually just paper and pencil, which require people to provide much of the information needed to make an estimate from memory.

Most learning activities, however, are well supported in comparison. The supporting resources available to students include textbooks, lecture notes, example problems, computer programs, teaching assistants, classmates and a relatively large amount of time. Thus, a student normally draws on a variety of resources to carry out learning activities.

On the surface, exams and quizzes appear to be an exception. They are both activities where access to supporting resources during the activities is limited. However, students are almost always told in advance when these activities will take place and what subject material they will cover, giving them opportunities to prepare. Students can focus on relevant material in a way that allows them to effectively "bring" supporting material with them to such activities.

4.2.2 Relevant information

A person making a rough estimate usually does not know a priori what information is relevant to a particular estimation problem. Even if resources are available, they may not be specific to the estimate being made. A person must decide how to think about or frame a problem and decide what information is relevant. The fact that rough estimation problems are naturally ill-defined and open-ended makes this task more difficult. There is usually more than one way to make an estimate and more than one acceptable answer. A person making an estimate must overcome a fair amount of uncertainty to obtain a solution.

Most learning activities, however, indicate to students which information should be used. For example, most activities are carried out in the context of a particular chapter of a particular course outlined by a syllabus. A student primarily searches a relatively small set of relevant support material to solve problems. Furthermore, the problems tend to be well-defined and closed-ended. There is usually only one way to solve a problem and only one right answer. Students solve the same problems with the same information and should get the same answer.

In contrast to estimation activities, learning activities are designed to limit the uncertainty faced by students. The primary goal of learning activities is to help students learn new information and learn how to use it to solve problems. For example, they generally learn one relationship by applying it to many different problems. A person making an estimate, however, potentially considers many different relationships for one problem. Students are not learning how to choose what known information to use on a given problem. Consequently, students have difficulty choosing relevant information and framing problems.

The following example illustrates the contrast between estimation activities and learning activities. The subjects that solved the bicycle problem were not told what information to use or how to frame the problem. They found twelve different solutions. Suppose, on the other hand, that this problem were an end-of-chapter problem in a textbook. Given as a homework assignment, it is likely that all of the students in a class would give the same solution.

Unfortunately, the contexts established for students also lead to corresponding ways of thinking that mediate their problem-solving actions. For example, the majority of students saw the battery problem as an "electrical" problem. This is probably because they primarily encountered batteries in courses covering electrical engineering information. This view makes the problem more difficult because students then do not consider relevant information from their extensive mechanical engineering background. In fact, the problem can be solved using only information from this background and everyday experience.

4.2.3 Required knowledge

A person making rough estimates must have a balanced knowledge of values, quantities, relationships and things. These are the basic informational elements of the effective actions presented in Section 3.3. Solutions to estimation problems involve all of these elements.

Learning activities, however, are focused primarily on relationships. This is because relationships are the primary way that engineering principles are codified. While relationships are emphasized, values, quantities and things are de-emphasized. Information about these other elements is still necessary for solving engineering problems, and teachers provide it to students through supporting materials. Consequently, students develop limited knowledge of these other elements.

Students are more likely to have knowledge of values, quantities and things from personal experience as opposed to knowledge of relationships. Unfortunately, because such information is provided to them, there is little room for their personal experience in learning activities. Thus, students do not realize that they must draw information from their personal experience to make estimates or have difficulty doing so. When teaching estimation, one has to remind students frequently that they know more than they think they know and to point out the relevancy of their everyday experience.

This imbalance of knowledge is exacerbated by the general focus of engineering education on more detailed analyses. By focusing on detailed analyses, teachers implicitly see knowledge of values and quantities as elementary, which makes providing information of this type seem harmless. Knowledge of things is not easily codified and activities are often designed to require limited information of this type. For example, descriptions of things are often highly abstracted with iconic pictures. Educational time and resources that would be spent on teaching values, quantities and things are made available to teach more detailed relationships. Most solutions to simple rough estimation problems, however, make use of relationships involving only a few quantities. Recall that many practitioners abandoned the more detailed drag equation for simpler methods to solve the bicycle problem as discussed in Section 3.4.1.

4.2.4 Situations

Rough estimates are usually carried out at a particular time or place when and where an analysis is needed. Thus, the situations in which a person must make rough estimates can not necessarily be controlled by that person. They might turn out to be public situations. For example, an estimate may be needed in a planning meeting or a brainstorming session where others can observe a person's performance. Or, they might turn out to be situations where an estimate must be made impromptu leaving no opportunity for preparation. In fact, these are the conditions under which simple rough estimates are particularly valuable.

Learning activities, however, are either defined to allow students to carry them out at their own pace, or the time and nature of the activity is established in advance. For example, homework assignments allow students to solve problems at their own pace, almost always in private. Although exams must be carried out at a particular time and place, they are almost always announced in advance. Students are told what will be covered by the exam and when it will be given, and therefore can prepare accordingly. Consequently, students do not need to have the ability to solve problems in varied situations. Some teachers do give impromptu quizzes during their classes that are similar to rough estimation situations. However, they are dissimilar because students are aware of the relevant material they need to know. Regardless, such quizzes do not currently appear to represent a significant fraction of educational activities.

4.3 Dependent knowledge

Each of the characteristics covered in the previous sections indicates potential areas where students are have difficulty making rough estimates. In this section, these characteristics are considered together and additional reasons why students are having difficulty are proposed.

Students solve over three thousand problems in the course of an engineering program and most do reasonably well on these problems. However, they usually solve these problems privately and then hand in solutions. Their solutions are outcomes of problem-solving processes that are unseen and unguided by teachers. They also solve these problems with access to a considerable amount of relevant supporting resources. In this situation, their solutions could be more a result of an orchestration of resources than an understanding of engineering concepts.

It is proposed that students are developing partial knowledge of basic engineering concepts combined with substantial mental "indices" of the supporting material that they use to solve problems. Instead of developing independent knowledge, students develop knowledge that is dependent upon and shaped by the presence of supporting resources. In learning activities, students make up for knowledge they do not have with the resources that are always available, even when they take exams. In estimation situations, however, the supporting material referred to by their mental index is missing, and the knowledge they have is difficult to use without this material. Consequently, they find estimation problems difficult.

It is further proposed that there is no requirement for the partial knowledge students develop to be accurate, clear or well structured. They seldom solve problems with this knowledge alone. They use supporting resources to compensate for things that they are confused or uncertain about. Unfortunately, learning activities that also provide guidance and allow privacy further support and conceal their misunderstandings. Confusion of concepts can persist and mental structures can remain cluttered. These misunderstandings are exposed in estimation situations, as was documented in Section 2.6. Recall that many seniors confused energy with power by, for instance, using a relationship defining power to calculate energy.

The solutions in the three examples given in Section 2.6. look like good solutions in form but are wrong in the details. This suggests that the students are learning the mechanics of putting together problem solutions but not the basic principles the problems are meant to help them learn. This finding is similar to that found by (Miller 1995) for students' solutions to simple design problems. For example, Miller asked students to design a small truss to hold a weight over the edge of a table and a pulley transmission to pull a pin out of a latch. He found that their solutions looked right iconically but were wrong in the details. For example, students developed inoperative transmissions. And, when asked if these transmissions were good design, students did not notice any problems.

The idea that students are developing the ability to orchestrate resources is also supported by the observation that some students use symbolic manipulation to solve problems, a well-known problem in engineering education. While the use of symbolic manipulation by students is often considered a fault of students, it is more likely a consequence of the learning activities that are commonly used. Students can carry out symbolic manipulation and obtain valid solutions in the context of relevant supporting resources, even though they may not understand their solutions. This is because the resources available for a problem provide a limited, consistent and sufficient set of information (values, quantities, relationships, example problems, etc.) suited to that problem. When students solve estimation problems, they do not have access to this kind of information. Consequently, attempts to use symbolic manipulation lead to spectacularly confused estimates, an example of which was given in Section 2.6.

4.4 Discussion

Although rough estimation activities and engineering science learning activities have much in common, they are different in several important ways. The two activities place people in different situations and make different demands on them. As a result, students do not learn important skills from the engineering science learning activities necessary for making rough estimates. Furthermore, students are developing engineering knowledge in ways that do not support estimation activity. Thus, to the extent that a curriculum's learning activities fit the caricatured description in this chapter, the students enrolled in that curriculum will likely have difficulty making rough estimates.

4.4.1 Design activities

This chapter focused primarily on the differences between rough estimation and engineering science learning activities. Learning activities used to teach design were not included. Engineering science activities, unlike design activities, cover the majority of engineering concepts students need to solve estimation problems. However, the relationship between rough estimation and design learning activities is also important to understand, especially since design activities represent a significant and increasing number of the activities within engineering curricula.

Where rough estimation and engineering science activities differ, design activities are similar. For example, design activities involve more emphasis on values, quantities and things and less on relationships. Students are placed in resource-constrained situations, naturally providing opportunities for students to make estimates. Particular disciplines of engineering science are not necessarily favored, so students are required to make decisions about relevant material. Design activities can be much more open-ended and ill-defined than engineering science activities, especially if they are partially or completely defined by students.

Design activities imbue a curriculum with qualities important to estimation activity. Thus, it is reasonable to expect these activities to benefit students' estimation skills. However, as long as engineering science activities cover the concepts and principles used in estimation activities, students will learn engineering concepts in ways incongruent with estimation activities and design activities as well. Unless design and engineering science activities are well integrated, design activities will probably not strongly improve students' estimation skills.

Unfortunately, design activities are often not well integrated with engineering science activities in engineering curricula. Possible reasons for this have been previously presented (Linder and Flowers 1999). One reason is that design activities support a growing list of design learning objectives that includes fewer and fewer objectives involving the use of engineering analysis. This situation may change if design activities continue to define more and more of engineering programs. Otherwise, it may be necessary to redefine engineering programs to remove the separation between engineering science and design activities all together.

4.4.2 Options

At a minimum, the following three options exist for improving students' estimation skills. Note that these options have not been experimentally explored.

First, teach conceptual knowledge of estimation and estimation problem-solving skills. The framework presented in Chapter 2 can provide a basis for this activity. Teaching this knowledge will probably not have a significant impact on students' abilities without combining it with one or both of the other two options. Estimation problem solving is a form of human problem solving, and its understanding should be facilitated by this more general knowledge. Thus, teaching general problem-solving knowledge should also be considered (Woods and al. 1997).

Second, increase the number of rough estimation activities addressed by students, particularly simple rough estimation problems. Considerable benefit appears to be gained just with these shorter problems. All of the different aspects of estimation can be addressed by these short problems. The shorter length allows feedback to be given to students in a timely fashion. Many students may find these problems difficult so the shorter length makes the problems more manageable. Also, it is difficult to find problems or activities that incorporate all aspects of estimation at once. Using shorter activities would allow more activities to be performed and would allow more aspects of estimation as well as engineering concepts and topics to be covered.

Third, include learning activities that have characteristics like those of rough estimation activities. Engineering analysis activities should be the primary focus, although a range of activities should be considered, including those that focus on sketching, building, explaining and diagnosing. This means designing activities that have students use their knowledge instead of supporting resources, including knowledge from personal experience. Activities should have students select relevant information from larger sets of information. The activities should have students use a balance of different types of information. Students should be exposed to a considerable amount of uncertainty, including open-ended, ill-defined engineering science analysis problems. Activities should involve impromptu and public situations. Activities with these characteristics will also have to be short because they will be more difficult for students.

Implementing these options requires significant changes to a curriculum. For example, consider making changes that would have students use a balance of engineering information. This would probably require teachers to spend more time on values, quantities and things and significantly less time on relationships. And, this change would need to be made to most of the engineering science courses in the curriculum. Or, consider changing courses so that students are not led to relevant information to solve problems. To be successful, it may be necessary to have students solve problems in courses completely separate from the courses in which they learn engineering principles. Just a few changes like these will lead to significant changes in the overall design of a curriculum. However, they should also lead to considerable improvements in students' abilities to make estimates.

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