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ALTERNATIVE METHODS OF OIL SUPPLY FORECASTING

by

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1. THE ROLE OF "PRICE-TAKER" SUPPLIERS IN THE OIL MARKET

Analysis of likely developments in the world oil market is ultimately dependent on some method of forecasting oil supply from key regions. Unfortunately, data problems tend to dominate work in this area, and much of the analysis task reduces to making the best use of the limited information that is available. Here we report on two alternative approaches to this forecasting problem, both avowedly data-oriented.

Petroleum exporters need to be grouped into two rough categories. First, there are what we will call "price-taker" suppliers. This is a group of petroleum exporters who appear to act as price-takers in the sense that each takes the world price (which is being set by others) as given. Each makes supply decisions according to his own parochial interest, without concern for their impact on the world price. This

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group includes non-OPEC sources of the North Sea, the USSR, China, and Mexico. It also may include members of OPEC who have low per-capita incomes such as Algeria, Indonesia, and Nigeria. Second, there is the "cartel core"--a small group of nations who are the price-makers. This core includes Saudi Arabia, Kuwait, and others on the Arabian Peninsula; it also may include Iraq, Iran, Libya, and Venezuela. These countries face a residual demand for world oil, which is the total demand less that supplied by the price taker exporters.

These groupings are not hard and fast; indeed a major focus of our inquiry is the circumstance in which a given exporter would change from one to another camp. The world oil scene is a dynamic interplay among these importers and suppliers wherein the oil price is set by the members of the cartel core, who assume the task of controlling oil production so it does not outstrip the world demand forthcoming at that price.

In this paper our focus is on the price-takers. And, since the desired form of a supply function depends on its intended use, we begin with a brief look at the broader market studies for which these supply analysis methods are designed. The structure of the overall study is shown in Figure 1; the figure also is a simple flow diagram of the simulation model framework we are using to tie the various pieces of work together.[1] The heart of the project is the supply and demand studies shown in the middle of the figure. These studies seek to improve our understanding of the fundamental market forces, and to provide estimates of supply functions for price-taker suppliers and demand functions for importers. These functions are then incorporated into a simulation model of overall market performance.

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[1] An overview of the research method, and the results of the early simulation studies, are shown in the work of Eckbo [8].

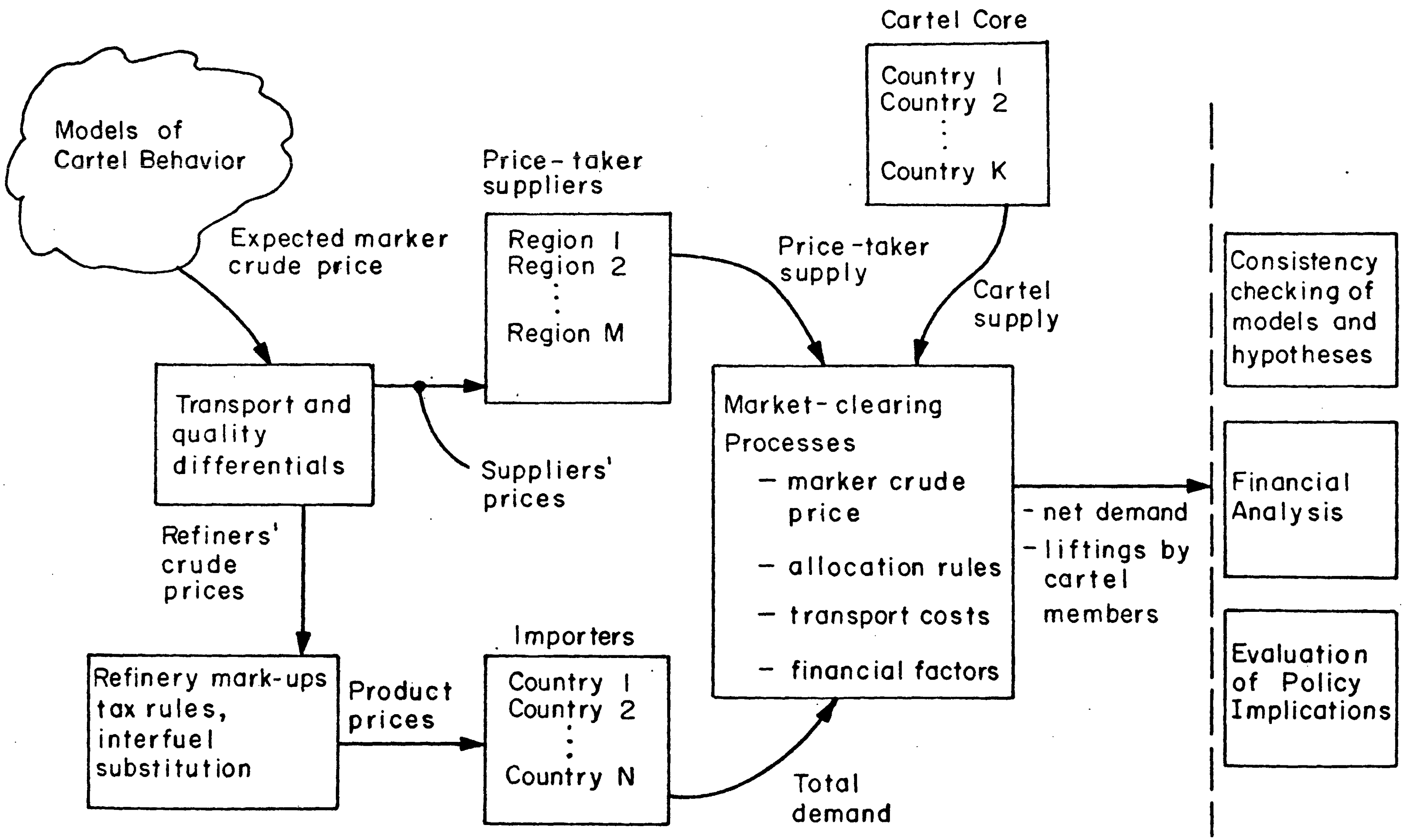

Figure 1 Overall Analysis Framework

The simulation framework is designed to accept an anticipated oil price trajectory, and to compute the resulting demands, supplies, and other market characteristics over the study period. Hypotheses about likely cartel price behavior are developed using a separate set of behavioral models,[1] as shown in the cloud at the upper left of Figure 1.

Thus we approach the problem with two types of models--analytical representations of cartel behavior, and a detailed simulation of market supply and demand. The reason for the division is analytical convenience. The determinants of import demand and price-taker supply are varied and complex; they involve cost and price, along with the effects of tax and regulatory policies. To analyze the likely response of the market to one or another price pattern, one needs a method that can accept unwieldy functional relationships. This requirement leads to a simulation framework for the overall analysis of market demand and supply outside the cartel. On the other hand, study of the cartel itself, and its pricing decisions, often involves some form of static or dynamic optimization calculation. For this part of the analysis, drastically simplified supply-demand relationships are needed so that many formulations of cartel behavior may be simply and cheaply tested. The two analyses feed one another, as shown in Figure 1.

In keeping with our emphasis on the underlying forces in the market, the simulation framework is based on what we call a "bathtub" approximation to the world oil market. That is, the market is treated as a single pool,

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[1] Examples of this type of model include those by Pindyck [18], Hnyilicza, and Pindyck [10], and Cremer and Weitzman [7]. Price scenarios based on judgment or the analysis of others also can be tested using the simulation framework.

where exporters put oil in and importers draw it out. The details of the transportation network and the refinery and distribution sector are almost neglected. Our aim is to match demand for products with the supply of crude, treating the intervening margin as a buffer, exogenously determined. Data and simple models of these factors are part of the simulation framework, as shown in the two boxes at the left of Figure 1. We plan to add more complex representations of these subsectors only as necessary.

The result of the overall simulation calculation is a forecast of net demand for oil produced by the cartel core--supplemented by work on core country supply, which is part of the overall supply-studies effort. Together, these components form the basis for study of current market characteristics, and forecasting of possible future developments.

The estimation of price-taker supply is a critical aspect of this analysis, and it is to this topic that we now turn.

2. PROBLEMS OF SUPPLY ANALYSIS

Several types of analysis have been used to explain and forecast petroleum supply. One approach that has gained acceptance in recent years, particularly in the United States, is the use of econometrics.[1] This technique has been applied in circumstances where hundreds of large fields, each containing a number of reservoirs, have given the productive systems the stability of large numbers, and where the depletion effect (tending to raise costs as less of a reserve remains) has for a long time been offset by new discoveries and improvements in technology. Recently this balance seems to have been lost. Also, in the data used in the econometric studies there seems to have been some ambiguity about the meaning of reported "reserves" and changes therein, so that a given year's reported "discoveries" bore little relation to what had actually been found. Moreover, there was no explicit attention to costs, which might cause a given price to be profitable in one place but not in another.

In studying supply from many areas of the world, the conditions for econometric analysis are even less favorable than in the U.S. In many countries the oil fields are both fewer and younger, and even the short histories are poorly documented. Another limitation is the fact that the price series are so fragmentary and untrustworthy. The so called "posted prices" of the past were rendered meaningless around 1960, when they became artifacts used for the calculation of taxes. Moreover, data on arms-length

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[1]For an example, see the work of MacAvoy and Pindyck [11].

sales of crude oil are insufficient and are ridden with too many errors to serve as a basis for econometric investigation. Were those problems not serious enough, current prices are far outside the historical range, and the inputs needed for development are not necessarily available in easy supply, either at constant or predictably changing prices. Finally, econometric calculations assumed, correctly, the existence of a competitive industry in the United States, hence a competitive supply curve. But our task is the modeling of a cartel, where price changes may have perverse effects on output.

Another approach to supply forecasting, which also involves an orderly summation of the past, is that typified by the work of the National Petroleum Council [15], and subsequently applied by the Federal Energy Administration [20]. Under this approach, the experience of past exploratory drilling is summarized and a trend in the finding of reserves per foot drilled is established. Based on estimates of the costs of exploration and development, calculations are made of the relative attractiveness of exploratory activity, conditioned on some assumption about the price of oil. Given an estimate of exploratory drilling, the forecast of barrels added per foot drilled, a reserve-to-production ratio, and hoped-for stability in reserve expansion in old "fully-developed" fields, it is possible to forecast supply into the future.

Unfortunately, many of the shortcomings of the econometric approach apply as well to the NPC-type format. In many areas of the world the exploratory histories are poorly documented, and several of the relationships which are required for this approach may be estimated only very approximately. This is because many important supply areas of the world are relatively new, and the experience that makes the NPC method believable simply does not exist. Moreover, the more productive potential areas in the world often are located in offshore or otherwise inaccessible areas, and the cost of development-production of particular resources weighs very heavily in the supply relation, as opposed to the phenomenon of exploration and finding which is emphasized in the NPC method.

Finally, there are the methods of resource estimation used by oil companies in evaluating prospective areas, and in constructing global estimates of regional or world resources. These methods, which draw on detailed geologic and geophysical data as well as on past drilling experience, seem to be rarely used for supply estimation of the type being carried out here. Where they are so applied, it usually is not possible to gain access to the details of underlying data and assumptions. They do, nonetheless, contain important components of concept, information and analytic method, and use is made of these approaches below.

2.1 Key Factors in the Analysis

To a very great extent, therefore,the mechanisms we have chosen to use are determined by what we perceive as the severity of data limitations in the main variables. The first and fundamental problem is in the reserve data. Viewed as an economic process, oil supply is the depletion of a stock, which is constantly being renewed by adding new reservoirs and expanding the limits of the old ones. "Reserves" have for years been reported by The American Petroleum Institute (API) for the United States on a consistent and meaningful basis [1]. Essentially they represent "money in the bank": in effect, the organized consensus of industry personnel as to the amount to be produced from existing installations. Elsewhere, one must rely on government estimates whose basis is rarely revealed, and on the trade press, which also is essentially an informal consensus of company opinion. The main difficulty in using non-API estimates results from various conceptions and definitions of what is likely to be added, how soon, to "proved reserves." In all cases, "reserves" are not a direct measurement but an inference drawn from data on geological structure plus observations on production volumes, pressures and temperatures. There will be legitimate differences of opinion in the interpretation of such data, hence in the estimation even of "proved" reserves. These variations are magnified as one moves from reserves proved to those considered as "probable" in the existing cluster of reservoirs.

A considerably greater leap, and the one calling for more specialized knowledge, is the estimation of "undiscovered reserves." Less than a decade ago, such estimations were an exercise in method; or in the language of a distinguished geologist, Lewis Weeks, they were merely an indication of where an exploration department ought to go look. These estimates had, in short, only a relative meaning, and it was a plain error to compare them with, or add them to, proved or probable reserves in known reservoirs. But today one can estimate the ultimate reserves for a "trend" or "play" (i.e., a population of reservoirs, generated by a geological sequence) provided that enough is already known about the area to furnish a reliable sample. Such a method is presented in Section 3 below. The combination of mathematical statistics and geologists' knowledge is not easily created, however, and we have been able thus far to apply the method in only one area, the North Sea. A number of estimates have been made, much more approximately, for larger areas. These are discussed in Section 4.

Rather than rejecting or ignoring them as not good enough, we regard them as a considerable advance on simple extrapolation on the basis of cubic yards of sedimentary rocks, etc.

An equally important data limitation is from reservoir engineering: how much can be produced out of a given set of reservoirs in any given time. In the past few years these limits have been perceived as much more tightly binding. When the price of oil in the United States was around $3 per barrel there was little dissent from the view that if the price were $6, vast new reserves could be created by applying more capital and extracting much more than the average 30 percent of the oil in place. These hopes are not dead, but it is now seen that too little was known of the processes by which additional oil could be recovered from a given reservoir, in the field rather than the laboratory. In the United States, drilling has responded to price, but reservoirs have not.

There also has been some unpleasant learning about the amount which can be produced per day or per year without damaging the reservoir and lessening the ultimate recovery to the point where present value is also less. Iran is one example. We happen to have obtained the capital budgets of the Iranian Consortium for over a decade [6]. Reading them in succession makes it plain that for years there was no felt need to know what would happen if production were raised by several times. It was reasonable to foresee, at prices much less than now rule at the Persian Gulf, capacity of 10 million barrels daily (mbd). The maximum will probably be a third to fourth less. Instead of a continuum, with higher prices bringing out higher output rates, the marginal cost appears to become nearly vertical in the neighborhood of 6 to 7 mbd. Given the strategic position of Iran among oil producing nations, this change has had important consequences. We can only be sure it is not the only one nor the last.

Finally, there is the influence of government policies. In competitive industries, supply and demand will be equated by price; in non-competitive industries, by marginal revenue. World oil is a good deal more complex. At current prices, the margin of price over costs is very great even in the highest-cost areas. Where the industry is operated by private companies, payments to the government greatly exceed payments to factors, including capital charges. Hence the most important economic variable, sometimes by factors of 10 or even 100, is the government's perception of how great a rent exists, and how high a price they can charge without reducing their total take. But the government take may also be in the form of participation or joint control. There is much room for misunderstanding and deadlock, so that a given country's actual rate of development may be much below where it would be under a government which was better informed and free to maximize, without political or ideological pressures. Matters are simpler in those countries where the oil industry is owned entirely by the government. As a first approximation, given knowledge of development costs and of known and probable reserves, one can calculate the rate of output which would maximize the present value of the current reserves--as well as estimate the finding rate which maximizes the present value of reserves in reservoirs to be discovered in the known areas. But one may need to modify the approximation to accomodate cartel solidarity, or other objectives.

In some countries, there is a backward bending supply curve, where higher prices lead to less supply. A government may simply overreach itself, to take such different cases as Canada and Malaysia. Higher prices may promise to generate so much revenue as to disrupt the desired rate of social adaptation; as a result, the higher the price, the less the target rate. A government with certain plans or obligations can meet them, given higher prices, with less output; hence is willing to reduce output or at least to accept reductions. Finally, price increases always generate expectations of still further increases. This raises the present perceived value of any reserves, and lowers the optimal rate of development.

Thus the three basic determinants--reserves, development costs, and government policies--must be put into a framework where they can be acted upon by current and expected prices. The framework must be modular to an extreme degree, since there is hardly a piece of the data base which we may not need to replace at any time, as more becomes known, or as data become outmoded.

2.2 Two Approaches to Price-Taker Supply

Here we present two of the analysis methods that we are exploring. One of these, the "disaggregated pool analysis," is the most detailed of the models developed and requires the most data, geological interpretation, and computational capacity. The other, which we call "aggregated country analysis" is among the most simple of the methods formulated. Various extensions, modifications, and combinations of the two approaches remain to be explored.

Though one approach is far more ambitious than the other, essentially they are variations of the same model. That is, the supply function is based on a simulation of the process of exploration, development, and production within a petroleum region. Both take account of geological data, though one has a more complex hypothesis about geological deposition and the nature of the exploratory process. Both take account of economic factors such as costs and future petroleum prices, and both include an approximation of oil-developer decisions, though the aggregated method necessarily treats these in a highly summarized manner. Both allow for the effects of tax regimes and other aspects of producer country oil policy, though once again the details of tax structure must be sacrificed in the aggregated model.

Figure 2 shows how the two analyses fit into the sequence of activities which compose the oil production industry, and the kinds of statistics generated. Looking first at the left-hand column, an immense store of experience, combined with formal science and technology, gives rise to judgmental estimates of what may be contained, and is worth producing, in various parts of the world. The unknown areas are judged by analogy with the known. The kinds of pools which may be generated and the relative size distributions, constitute the estimates of "ultimate production," as compiled by several oil companies.

These estimates have a direct effect on the direction of geological and geophysical prospecting. The knowledge gained thereby feeds back into the judgmental estimates. Geological-geophysical results also determine exploratory drilling, in new and in old areas; and the good or bad results again feed into judgmental estimates, both of "ultimate production," and of what may be thought, in old areas, of likely new discoveries there, i.e., "probable reserves."

Exploratory drilling generates dry holes, which are not necessarily bad news, since the information about successive layers may be of great value;

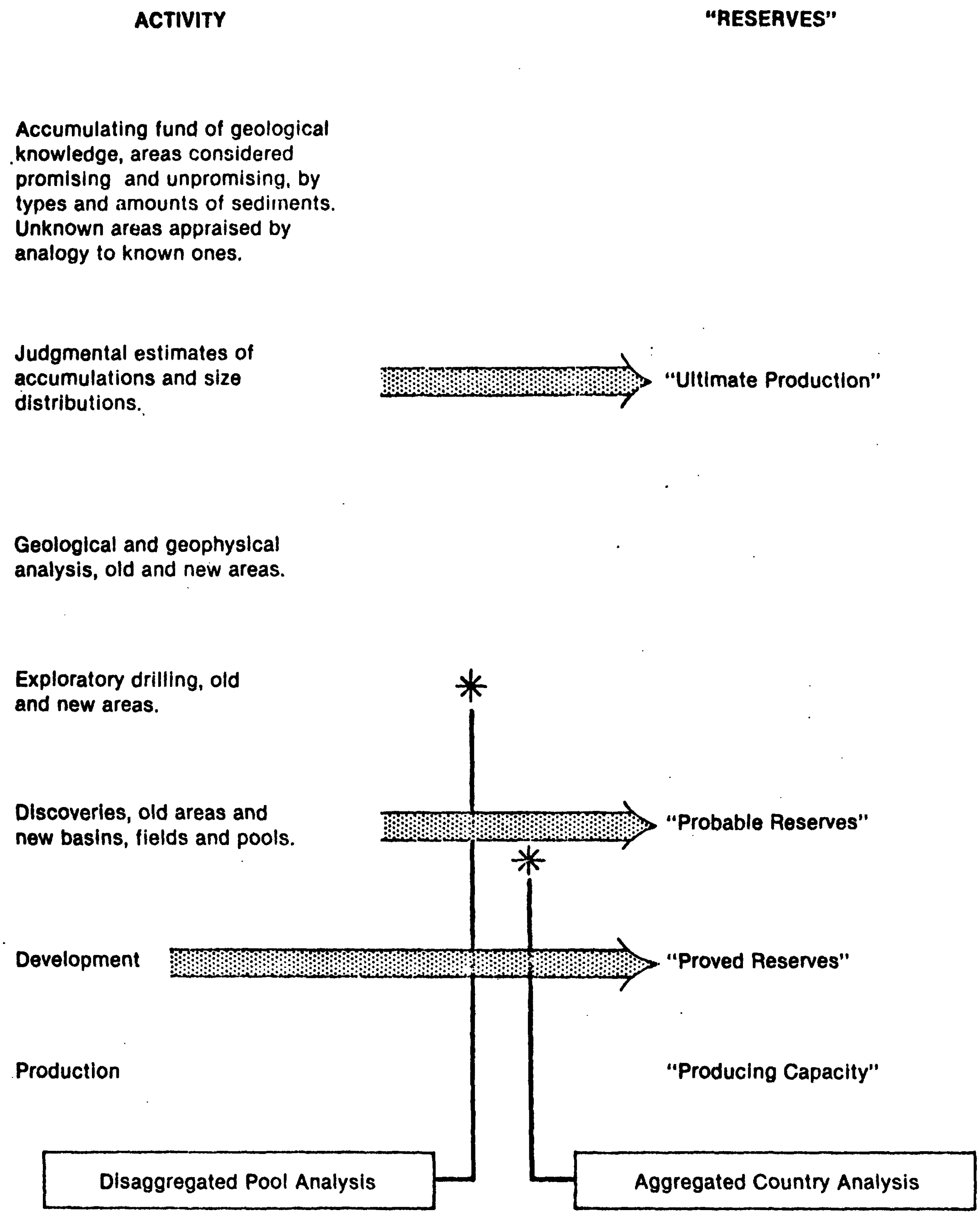

Figure 2. Sequence of Oil Activities and Kinds of Statistics Generated

it also leads to new fields in old areas, again affecting estimates of "probable reserves" and, perhaps more weakly of "ultimate production."

At this point the Disaggregated Pool Analysis can begin. Given enough wells drilled to furnish a reliable sample, and given geological knowledge to certify the existence of a population from which the sample is drawn, a forecast is made of the underlying distribution, the number of reservoirs to be found, and their size distribution. Given knowledge of costs, prices, and tax policies, one can make a forecast of the rate of exploratory drilling, and the rate of development of the area.

Stated in terms of the steps of our analysis procedure, the disaggregated pool approach is shown in the left half of Figure 3. First, an estimate is made of the number of exploratory wells in the region. Then the exploratory process itself must be approximated, and an estimate made of the number of reservoirs found and their characteristics--e.g., recoverable reserves, well productivity, depth to pay, and (in the case of offshore areas) water depth and distance to shore. The key variable is recoverable reserves, and here the method draws on research on statistical analysis of the exploratory process carried out by Kaufman and Barouch [2,3]. The economics are then calculated, reservoir-by-reservoir. Development costs are estimated from the reservoir characteristics and applicable tax rules, and these are combined with an evaluation of revenues based on expected oil prices. Built into the analysis is an evaluation of the likely production profile for reservoirs with different characteristics. The overall supply estimate for an area is the sum of the production profiles of its individual reservoirs; and the overall supply from a country (or other region) is the aggregate of the results from the various areas distinguished for analysis.

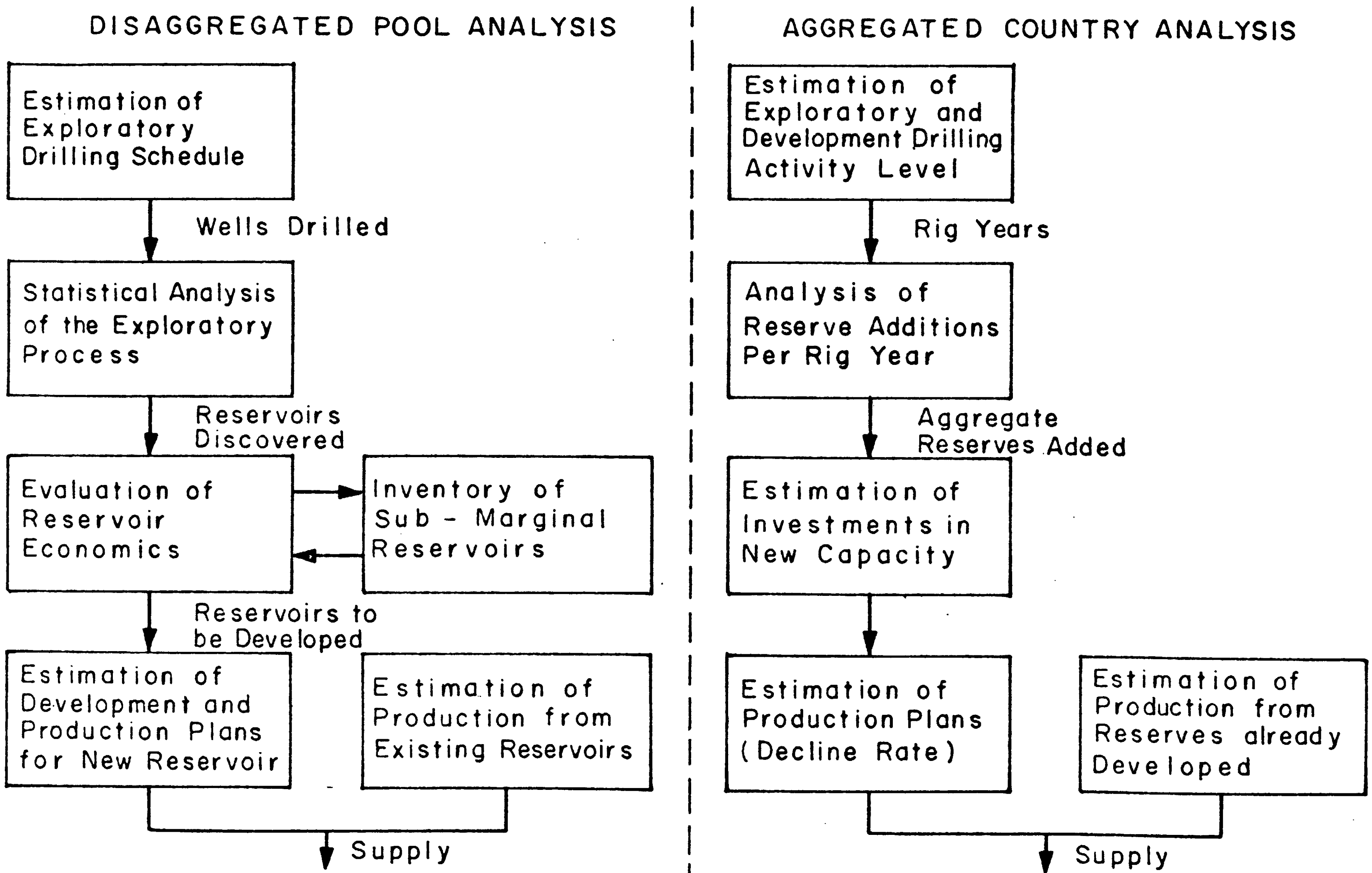

Figure 3 Outline of Alternative Supply Analysis Methods

The disaggregated pool analysis is a formal, precise, reproducible version of what happens in industry every time a discovery is made. Estimates of probable reserves are changed, as well as of "ultimate production." As development wells are drilled, providing new producing capacity and more detailed knowledge of reservoirs, the forecast of their production is crystallized into the industry estimates of "proved reserves."

The aggregated analysis is an attempt to tap into the calculations which the industry has already made, which are in substance the same as the disaggregated pool analysis. By considering prices, costs, and taxes, the companies have already made estimates of what they expect to produce. Recent experience indicates how fast a given amount of drilling effort will produce the associated productive capacity, i.e. the ability to deplete a given reservoir in a given length of time. And the "probable reserves" and "ultimate production," both affected by recent experience, are imprecise measures of the larger "pool" out of which are being impounded the newly "proved" reserves. The smaller the proportion impounded up to the present, the more is left to be taken into "proved reserves" before rising marginal costs are felt. This is strictly analagous to the discovery decline rate which is made explicit in the statistical analysis at the pool level.

Once again, the approach can be expressed in terms of the explicit steps to be worked out in Section 4. As indicated in the right-hand panel of Figure 3, exploratory and development activity are measured in rig years for the area in question, and an estimate is made of proved reserves added per rig year. Instead of making a detailed study of reservoir economics, we factor in the industry's estimates, since <u>all</u>

proved reserves are by definition slated to be developed. The focus is on the amount of capacity that will be installed, considering costs and expected oil prices. The cost data are not in the form of cost functions, as required by the disaggregated model, but simple estimates for each country of the capital coefficients per daily barrel of capacity and of operating cost per barrel of production. As with the disaggregated analysis, separate estimates are made of supply from reserves already in production, as shown in the lower part of the diagram. And, of course, the method is flexible enough to take account of information about other aspects of planned activity.

Both methods are based on the same idea of how reservoirs are distributed in the earth and on the same sequence of industry operations. Whether we use one or the other depends altogether on the available data on the one side, and the money and manpower available to use them, on the other.

3. DISAGGREGATED POOL ANALYSIS

As noted above, the key to the disaggregated method of forecasting is the statistical analysis of the exploratory process, and economic evaluation of the pools that are found.[1] Methods have been developed whereby a forecast can be made not only of the total recoverable reserves to be discovered by a given level of exploratory effort, but of the distribution of pool size itself and of the sequence of discoveries by size. Pool size is a key to the economics of supply, particularly in offshore areas (or under certain tax regimes) where small pools (and under some conditions relatively large ones) may be uneconomic to develop. Pool size also is a significant influence on the speed of development and the time profile of production from reserves that are economic to produce.

3.1 Analysis of the Exploratory Process

As currently implemented, the analysis begins with a postulated level of exploratory effort within an oil "play." A play is defined as the pre-drilling and drilling exploration of a geological configuration, generated by a series of geological events, and conceived or proved to contain hydrocarbons. The wells drilled form a statistical population. An important element of the research is to discover the difference in

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[1]What follows is an abbreviated version of the disaggregated method. Its development and initial applications are described in a paper by the M.I.T. World Oil Project [19], and the methodology is further developed in a paper by Eckbo, Jacoby, and Smith [9].

results that may accompany alternative ways of drawing these geographical boundaries. For example, the North Sea results shown here treat the entire region as a single play, but work is under way to perform the same analysis with the area divided into four areas that correspond more closely to the geologist's definition of a "play."

Exploratory effort is measured in terms of the number of exploratory wells, W_t, to be sunk in period t. To this drilling effort we apply a "dry hole risk," δ_t, which is the probability that an exploratory well will fail to find a field. For purposes of the example developed here, δ_t, is held constant over the forecast period, and reflects a judgmental estimate based on historical experience. The expected number of discoveries in a period can then be expressed as $W_t(1 - \delta_t)$. The purpose of the analysis of the exploratory process is to determine the expected characteristics of the next set of discoveries in a petroleum play over some period to be analyzed. This is the first step in calculating the contribution that new pools may make to future supply, as shown in Figure 3. This estimate can be combined with data on pools already discovered to produce an overall supply forecast for the play.

The analysis of the exploratory process is based on two hypotheses about the natural process of resource deposition. Following the work of Barouch and Kaufman [2,3], one assumes that the number of pools in a play, N, and their size distribution (characterized by the mean and variance, μ and σ^2) are generated according to a probability law whose functional form is dictated by the way in which nature deposited the oil in the first place. Many distributions may be analyzed using the methods applied here, but the customary assumption is that pool size in terms of recoverable reserves, r, is a random variable, and that its density function normal ; i.e., log r is normally distributed with mean μ and variance σ^2.

To this hypothesis about nature, then, is added another hypothesis about the process by which oil operators search for and find these reservoirs. Once again following the work of Kaufman and others [2,3] the exploratory process is characterized as one of random sampling, without replacement, in proportion to pool size, r. With these hypotheses it is possible to predict the characteristics of discoveries number n+1, n+2,..., N--and also total recoverable reserves in the play, $\sum_{j=1}^{N} r_j$ -- if we know there have been n discoveries of size $r_1,\ldots,r_n$ and the order in which they were discovered.

The procedure is as follows. We can specify joint distribution function of the n discoveries to date conditional on the parameters μ,σ^2, and N:

$$D(r_1, \ldots, r_n \mid \mu, \sigma^2, N) \tag{1}$$

Then, using the actual sequence, $r_1, \ldots, r_n$ as one sample from this distribution we can estimate (using maximum likelihood techniques) the parameters μ, σ^2 and N. Table 1 presents the sequence of discoveries in the North Sea, which we use as the basis of the sample calculations presented below.[1]

Next, it is possible to specify the density function of discovery number n+1 conditional upon the exploratory history already experienced, $r_1, \ldots, r_n$, and given estimates of the parameters $\hat{\mu}$, $\hat{\sigma}^2$, and $\hat{N}$. This

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[1] The estimates are treated as measures of "proved" reserves, although the strict conditions for such a definition often have not been met (i.e., limiting the estimate to reserves actually enclosed by development wells). The assumption is that a small number of exploratory and development wells, when coupled with geophysical data on structure size, gives a reasonably accurate estimate of what will ultimately be proved in a given specific reservoir.

Table 1. Northern North Sea Discoveries, Recoverable Reserves

Oil Equivalent (Millions of Barrels)

Order	Name or Location	Date	Size
1	Cod	2/68	156
2	Montrose	4/69	180
3	Ekofisk	9/69	1713
4	Josephine	6/70	100
5	Tor	8/70	243
6	Eldfisk	8/70	910
7	Forties	8/70	1800
8	W. Ekofisk	8/70	490
9	Auk	9/70	50
10	Frigg	4/71	1325
11	Brent	5/71	2500
12	Argyll	6/71	70
13	Bream	12/71	75
14	Lomond	2/72	500
15	S.E. Tor	4/72	34
16	Beryl	5/72	525
17	Cormorant	6/72	165
18	Edda	6/72	98
19	Heimdal	7/72	300
20	Albuskjell	7/72	357
21	Thistle	7/72	375
22	Piper	11/72	638
23	Maureen	11/72	500
24	Dunlin	4/73	435
25	3/15-2	4/73	150
26	Hutton	7/73	250
27	Alwyn	7/73	350
28	E. Frigg	8/73	623
29	Heather	8/73	150
30	Brisling	8/73	75

Order	Name or Location	Date	Size
31	Ninian	9/73	1000
32	Statfjord	12/73	4960
33	Odin	12/73	178
34	Bruce	3/74	450
35	Magnus	4/74	800
36	N.E. Frigg	4/74	71
37	Balder	4/74	100
38	Andrew	4/74	300
39	Claymore	4/74	375
40	E. Magnus	6/74	250
41	9/13-4	6/74	220
42	15/6-1	9/74	150
43	Brae	9/74	800
44	Sleipner	9/74	50
45	Hod	11/74	75
46	211/27-3	11/74	450
47	Gudrun	11/74	450
48	2/10-1	11/74	100
49	3/4-4	12/74	100
50	14/20-1	1/75	75
51	Crawford	1/75	150
52	9/13-7	1/75	350
53	3/8-3	1/75	100
54	Tern	2/75	175
55	21/2-1	2/75	175
56	3/2-1A	3/75	200
57	Valhalla	4/75	50
58	3/4-6&3/9-1		200
59	15/13-2		200
60	211/26-4		175

Source: Beall [4], and estimates by the M.I.T. World Oil Project as of June 1976.

we may write, defining $\underline{r} = (r_1, \ldots, r_n)$, as

$$D(r_{n+1} \mid \underline{r}; \hat{\mu}, \hat{\sigma}^2, \hat{N}). \tag{2}$$

An estimate of the expectation of r_{n+1}, the $(n+1)^{st}$ discovery, given that $\underline{r}$ has been observed, is

$$E(r_{n+1} \mid \underline{r}) = \int_0^\infty xD(x \mid \underline{r}; \hat{\mu}, \hat{\sigma}^2, \hat{N})dx. \tag{3}$$

The nature of this calculation can be seen in Figure 4, based on data from the North Sea. When this analysis was done there had been 60 discoveries in the North Sea, so n = 60. The figure shows the rough shape of the density function for the 61st discovery along with the conditional expectation $E(r_{61} \mid \underline{r})$ of the size of the 61st discovery, which in this case was 258 million barrels. The calculation for n+2, n+3, etc. is a straightforward extension of this procedure.

One further step is necessary before proceeding to economic and financial analysis of the reservoirs themselves. As stated earlier, smaller pools may be uneconomic to develop, and the pace of extraction may differ among larger pools depending on size. Therefore we need some indication of the expected number of barrels to be found in pools of various sizes. The data for such a calculation is contained in the conditional distribution of Equation 3, and using this function we can calculate the partial expectations of the numbers of barrels in reservoirs of various sizes. To do this we define a set of k class sizes S_k, where the size limits are defined as shown in the following table.

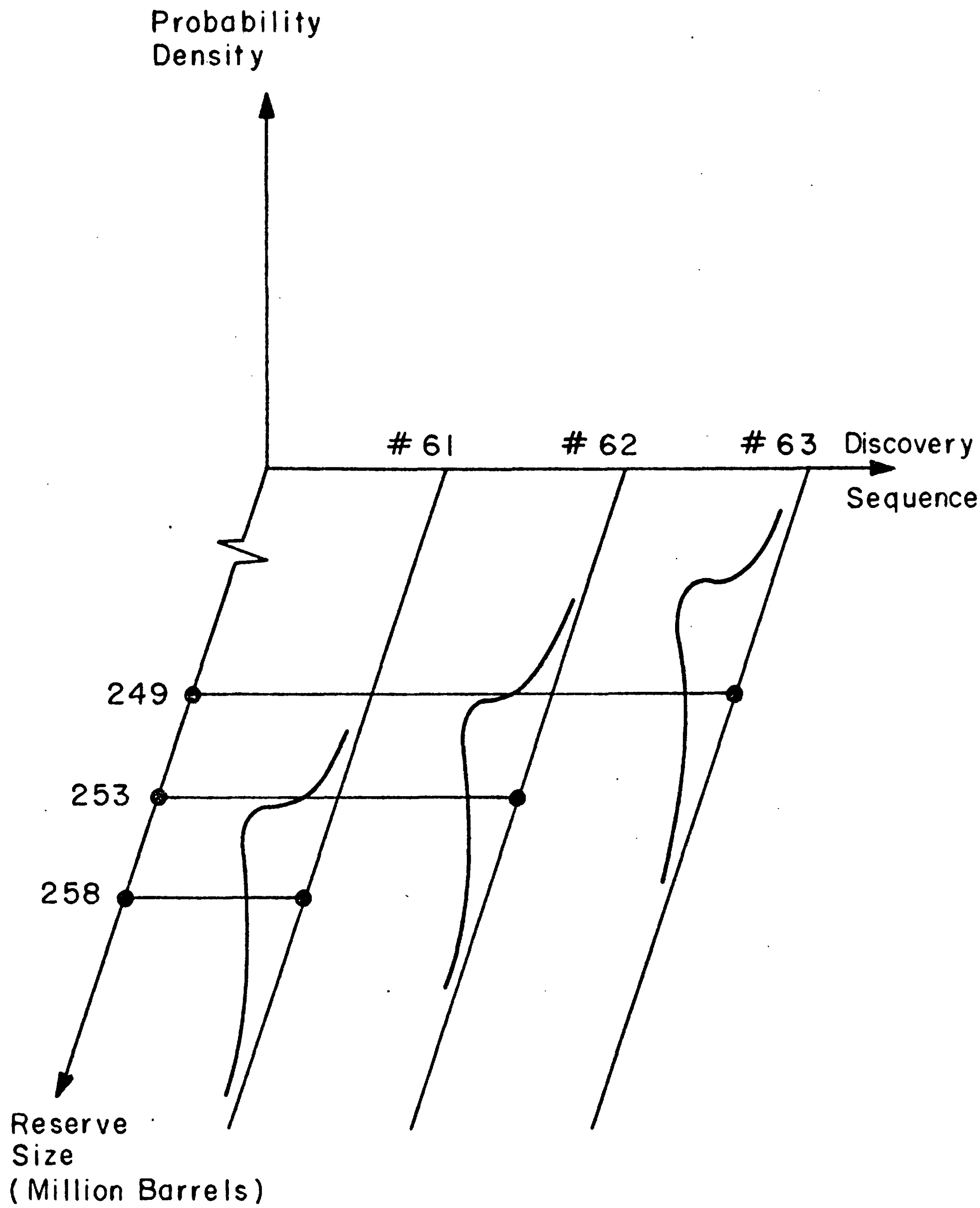

Figure 4 The Sequence of Predictive Discovery Distributions

Class (k)	Lower Limit	Upper Limit
1	a_1	a_2
2	a_2	a_3
3	a_3	a_4
.		
.		
.		

Then for the $(n+1)^{st}$ discovery the expected number of barrels to be found in pools in size class k, or the partial expectation of size class k, is

$$P_{n+1,k} = \int_{a_k}^{a_{k+1}} xD(x \mid \underline{r};\ \hat{\mu},\ \hat{\sigma}^2,\ \hat{N})dx \qquad (4)$$

The results of this calculation are illustrated in Table 2, once again using data from our North Sea example. Four size categories are used and the table shows the partial expectations of the number of barrels to be discovered in each category in the next five successful exploratory wells.

Though the table shows only the first few discoveries from a longer sequence that must be generated for supply forecasting, several characteristics of the process are evident in the data shown. First, most of the oil is expected to be found in larger reservoirs, and the difference in <u>economic</u> reserves which would result were the smaller size reservoirs infeasible to develop is not great--though it is significant. Second, the table plainly shows a process which we refer to as "discovery decline." That is, as the province is drilled up, the expected finding from each additional success-

Table 2. Predictive Discovery Distribution

(Millions of Barrels Oil Equivalent)

Size Category, k	Limits	Partial Expectation, P_{ik}, for discovery number				
		61	62	63	64	65
1	125 to 250	18	18	18	17	17
2	250 to 375	26	25	25	25	24
3	375 to 500	31	31	30	30	30
4	over 500	176	173	169	166	163
Expected Value, $E(r_i)$		258	253	249	244	240

ful exploratory well tends to decrease. This is the behavior one would expect in practice, and it falls out of the analysis because the fundamental geological facts of life are built into the method through the two key hypotheses introduced earlier.[1]

Referring again to Figure 3, the results in Table 2 constitute the data on "reservoirs discovered." The next step, then, is to estimate what will be done with them once their location and size are known.

3.2 Evaluation of Reservoir Economics

There are a number of attributes of an oil reservoir that influence its economics. The most important is recoverable reserves,[2] and this we predict by the methods above. Reservoir depth and (in offshore areas) water depth and distance to shore also are imporant determinants of cost--though these usually do not vary significantly over a region of the size on which this analysis is based. Likewise, average well productivity is an important factor, though it also is reasonably assumed to be constant over the unit of analysis when a relatively disaggregated approach is taken. These attributes--together with cost factors, tax rules, and the expected oil price--determine the economic viability of a reservoir.

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[1]A similar phenomenon is built into the aggregated analysis as well, where the amount added to reserves is a decling function of the number of rig years in an area.

[2]Note that the discovery process has been defined in terms of recoverable reserves, and the analysis of Equations 1 through 4 carried out apart from concern for the oil price. In principle, the amount of oil that is "recoverable" from the oil in place is a function of price and cost factors: at higher oil prices it is worthwhile to spend more to recover a larger percentage of the oil. In practice, given the state of the art at any moment, and in the relevant range of prices, elasticity of recovery factor to price appears to be very low.

We are concerned here with the behavior of price-taker suppliers, and in particular with the forecasting of their response to the price strategy of the cartel core. Since smaller reservoirs are more expensive to develop, per barrel produced, one may expect that higher cartel prices will bring more resources to the point of economic viability and call forth increased price-taker supply. To analyze this phenomenon, we construct a cash-flow analysis of the reservoir from the operator's viewpoint.

A profile of capital expenditures typical of oil reservoir development is shown by the solid line E_t in the top half of Figure 5. We have prepared an analysis of reservoir economics to generate estimates of E_t for reservoirs of different size [9, 19]. Relationships were estimated for the various components of capital cost (e.g., development drilling, platform structures, platform equipment, pipelines, terminals) and of operating cost. In this analysis of the North Sea, extensive use was made of data prepared by Wood, Mackenzie, and Co. In addition, detailed consideration was given to the tax systems of the U.K. and Norway.

Estimates were also prepared of the typical patterns of annual production, Q_t, from reservoirs of different size, as shown by the solid curve in the bottom half of Figure 5. These data are then combined into a cash flow analysis of reservoir investment and production, both to determine economic viability and to establish the likely production profile, reservoir by reservoir. (The dashed lines in Figure 5 represent a simplified or "collapsed" version of development expenditures and the associated production profile. The collapsed model is used in constructing the aggregated model of oil supply, discussed in Section 4.)

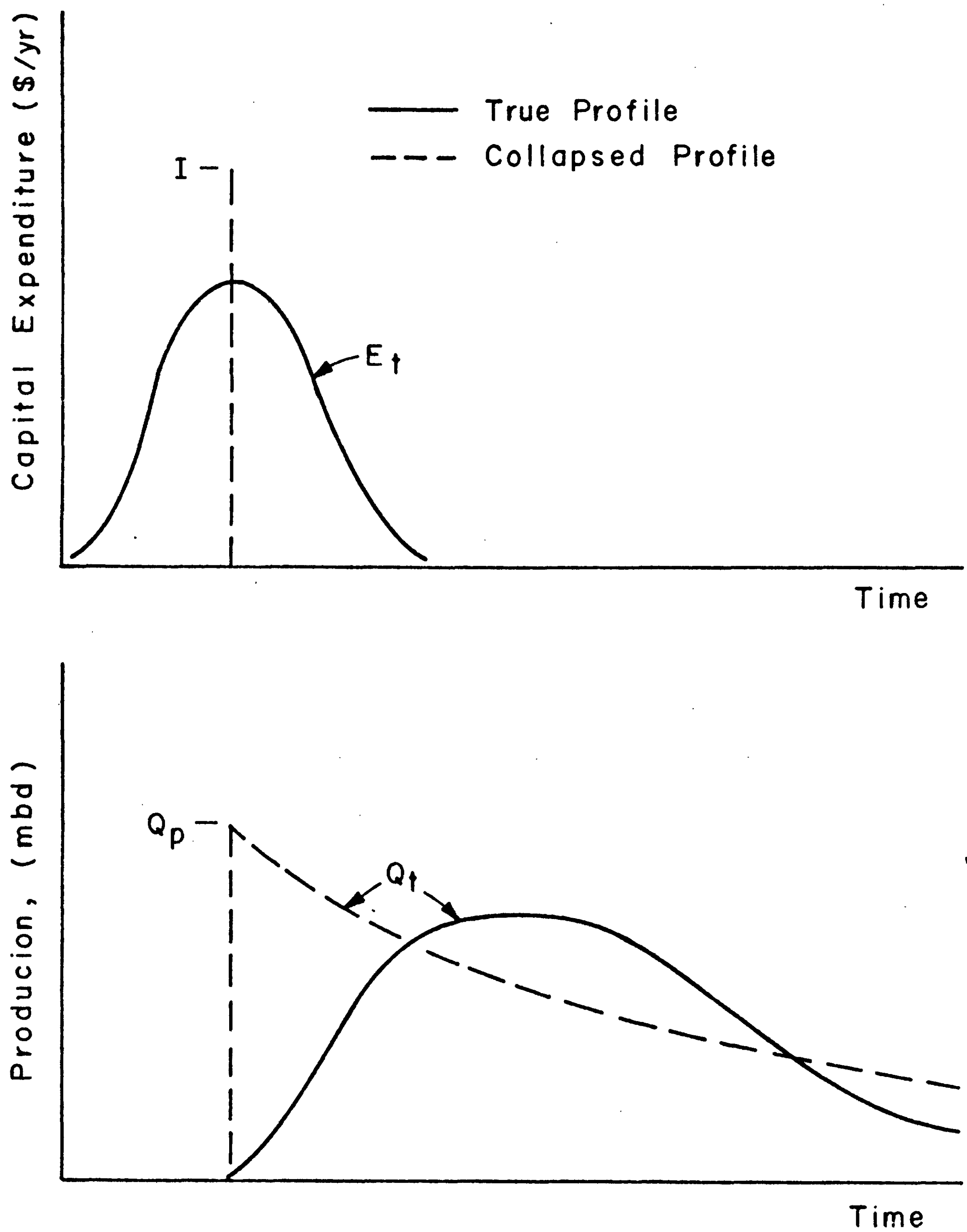

Figure 5 Profiles of Capital Expenditure and Production

The result of the reservoir analysis is shown in Figure 6, once again using data from the North Sea. The figure shows a plot of the minimum sized pool which it is feasible to develop given an expected level of oil price. At $6 per barrel (1976 prices), no reservoir below 200 million barrels will be developed given current costs and tax rules in the North Sea. At a $12 price, the marginal reservoir decreases to 90 million barrels. Similarly, it is possible to hold price constant and calculate the effect of changing tax rules.

3.3 Simulation of Supply

The analysis of the exploratory process, presented in Table 2, and the evaluation of reservoir economics, summarized by Figure 6, then constitute the building blocks for simulation analysis of price-taker supply. Refer again to the left-hand side of Figure 3. Based on an estimate of exploration drilling activity (which for this example we estimate from announced plans) a discovery sequence generated. It represents the expected value of the reserves to be found in pools of various sizes. These expected quantitites are then subjected to analysis of economic viability. Submarginal reservoirs are held aside until such time as rising oil prices or postulated revisions in tax rules change their attractiveness for development. Those reserves that are in pools above the minimum pool size are then converted (based on analysis of reservoir economics) into a time profile of production. To the estimates of supply from new discoveries, then, are added data on expected production from reservoirs already in production or under development. These, of course, constitute the more accurate portion of any forecast. In the North Sea we depend heavily on Wood, Mackenzie data for these estimates.

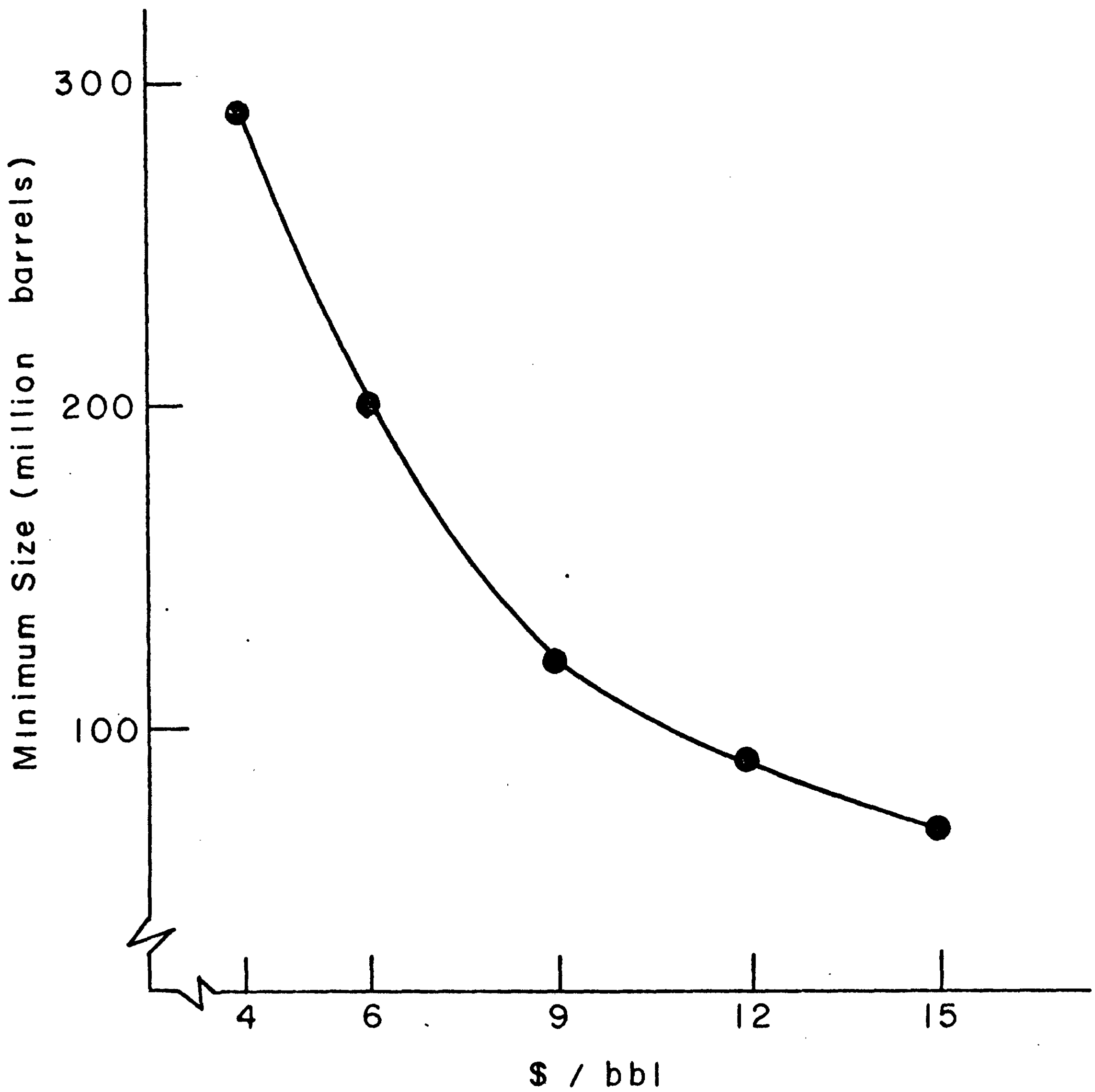

Figure 6 Minimum Economic Reservoir Size as a Function of Oil Price (1976 prices)

The final result is a supply forecast built up pool by pool for existing fields and (on an expected value basis) discovery by discovery for new reservoirs. If there is more than one play in the supply region, then the regional production is built up play by play. The resulting supply estimtes for the North Sea are shown in Table 3, and compared with estimates of other analysts.

Table 3. North Sea Supply Estimates (Million Barrels Per Day, oil equivalent)

	1980		1985	
	$9	$12	$9	$12
Our Sample Analysis[a]				
Existing Reservoirs	2.82	2.82	2.50	2.50
Recent Discoveries	1.68	1.82	1.57	1.66
1977-78 Discoveries	0.47	0.48	1.68	1.72
Total	4.97	5.13	5.75	5.88
OECD[b] [17]	3.9	4.06	5.16	5.3
BP[c] [5]	3.46		6.8	
Odell and Rosing[c] [16]	4		12	

Notes: (a) 1976 Prices

(b) 1974. Based on 1972 prices of $6 and $9 in the Persian Gulf, which are used here as proxies for 1976 prices of $9 and $12 in the North Sea

(c) Price assumption not specified

4. AGGREGATED COUNTRY ANALYSIS

It is only in special circumstances where we have the data needed for the disaggregated analysis, so we are developing a greatly simplified, but consistent, aggregated method sketched in Figure 3. It is modular, in that the estimates for any given country, region, or field can be replaced by more precise knowledge when available from other sources.

As noted earlier, the method is based on forecasts of rig activity and analysis of proved-reserves added per rig year. Reserve additions then become an input to calculations of capacity expansion and likely oil production. Therefore we turn first to the data sources for worldwide reserves, and the way they may be used to forecast new additions.

4.1 Analysis of Reserve Additions

As used by the American Petroleum Institute (API) the concept of proved reserves has a definite economic meaning: a highly accurate forecast of what will be produced from wells and facilities already installed. Since variable costs are normally only a small part of the total, it would take an unusually severe price drop to abort much production. It is this concept of reserves that forms the basis of the model developed below.

However, outside the U.S. published "proved reserve" estimates generally include a substantial element of what the API calls "indicated additional reserves from known reservoirs." At end-1975 in the U.S. (excluding Alaska) this category included 5.0 billion barrels compared with 22.7 billion proved reserves, or an additional 22% [1]. Usually the published "proved reserves" go farther. Interesting data were gathered in a survey of oil companies conducted by the National Petroleum Council. The companies made estimates of the discrepancies between API "proved reserves" and the reserves as published by the Oil and Gas Journal (OGJ). The results are shown in Table 4. Since the North Sea was insignificant in 1970, the close agreement on Europe is now outdated. Essentially, OGJ reserves for certain large Persian Gulf and African countries include a large amount of oil not yet developed into proved reserves, and the companies were not unanimous on its size.

We gain a different and more useful perspective on reserve estimation by noting that reserves credited to individual developed fields fall substantially below the OGJ national totals. Table 5 shows a comparison of OGJ data and estimates of the largest fields as published by the International Petroleum Encyclopedia (IPE), along with an estimate based on the IPE figures. The estimate of proved reserves (Column 4) was prepared by dividing IPE reserves of identified largest fields by the portion of total country production accounted for by those fields. Finds accounting for less than 3 percent of current production were excluded. (In Lybia the proportion was based on cumulative not current production.) These authors' estimates as a percentage of the OGJ figure is shown in Column 5; the results are consistent with those in Table 4.

Prior to 1975, the percentages in column (5) would have been lower, since OGJ in that year revised its estimates downward, and for the first time labelled them "proved reserves" and stated that "probable and possible reserves [are] not included." Given this definition, we should consider that the excess of OGJ reserves over those estimated on an IPE basis includes largely the undeveloped portions of known fields, including some known but undeveloped reservoirs. For our purposes, the IPE-based

Table 4. Oil Company Estimates of Proved Reserves (API Concept) As A Percent Of Published Reserves (Oil and Gas Journal), 1970

Area	Proved Reserves	Proved Plus Probable Reserves
Latin America	.97 to .99	
Europe	.97 to .98	Not
Africa	.50 to .73	Available
Middle East	.67 to .80	
Total	.66 to .81	.88 to .97
Rough Point Estimate	about .75	about .95

Source: National Petroleum Council Committee On U.S. Energy Outlook: an Interim Report. An Interim Appraisal by the Oil Supply Task Group, 1972, pp. 21-24.

Table 5. Reserve Estimates End-1975 For Selected Countries

(billions of barrels)

Country	(1) OGJ Published Total Reserves	(2) IPE Identified Largest Fields No. of Fields	(3) Reserves	(4) Total Estimated Proved Reserves	(5) Column (4) as Percent of Column (1)
Abu Dhabi	29.5	4	4.8	6.4	21.5
Iraq	34.3	6	24.7	27.4	79.9
Kuwait	68.0	all (8)	54.6	52.2	76.8
Saudi Arabia	148.6	12	112.6	118.1	79.5
Iran	64.5	17	49.6	52.5	81.3
Nigeria	20.2	12	3.4	10.6	52.3
Libya	26.1	13	21.3	23.0	88.1
Venezuela	17.7	21	14.7	16.4	92.7

figures are preferred for they are closer to the API definition, though considerable judgment may be involved in making estimates for particular countries.

Given an estimate of current proved reserves, we proceed to develop a method for forecasting this quantity. We consider gross additions to proved reserves in any year as an output, and rig-years as a proxy or indicator of investment input which generates these reserves. We let R_t be proved reserves in an area in year t, Q_t be the area's production, and RY_t be rigs operating there. Then reserves-added per rig time unit, RA, is calculated as

$$RA = [R_{75} - R_{72} + \sum_{t=73}^{75} Q_t] / \sum_{t=73}^{75} RY_t. \qquad (5)$$

The numerator is gross additions to proved reserves over the 3-year period, 1972-1975, where additions include new-field discoveries, new pool discoveries, and revisions and extensions of known fields, often from development drilling. The denominator is the number of rig years during the same period. Rig time is superior to feet drilled because it is a better predictor of investment, although it is necessary to calculate RA separately for onshore and offshore areas. Rig time includes all time-related elements, including not only depth but also time used in moving rigs[1]; interruptions for lack of an essential part or service or any other reason; unusually difficult drilling conditions, etc.

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[1]In the Soviet Union, which operates nearly as many rigs as the United States, 25-40 percent of the time is used in moving them. See World Oil, August 15, 1976, p. 126.

Equation 5 applied to a subsequent year's drilling rate yields a forecast of reserve additions:

$$\Delta R_t = RY_t \cdot RA. \tag{6}$$

The method is a simple extrapolation of recent experience, with only inertia (which is considerable) to justify its use.

We need to take care to avoid some obvious inaccuracies. Thus, for example, we disregard the 1973-74 Venezuelan "marking up" of proved reserves to the extent of about 4.5 billion barrels.[1] The validity of the changes seems questionable. Elsewhere, no price effect is perceptible. More particularly, in the United States the recovery factor was not significantly different in 1975 from 1972.[2] It would appear that the effect of higher prices on supply makes itself felt only by increasing investment, i.e., drilling and reserves-added.

Of course, the yield from new investment cannot go on forever, undiminished by the effects of depletion. Thus we define a coefficient b_t to reflect this phenomenon. Let Cum R_t encompass all past production plus current proved reserves in year t. Ultimate recoverable reserves, Ult R_t, are then Cum R_t <u>plus</u> all future additions to reserves (and, ultimately, to production). If 1975 is the most recent year when reserves data are available, then b_t may be defined as

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[1] Petroleo Y Otros Datos Estadisticos, 1974 (Republic of Venezuela, Ministry of Mines and Hydrocarbons), Table-page 45. Although revisions were 5.5 billion, nearly one billion must have been contributed outside the "markup."

[2] See [1], Table III, respective years.

$$b_t = [1 - \frac{\text{Cum } R_t}{\text{Ult } R_t}]/[1 - \frac{\text{Cum } R_{75}}{\text{Ult } R_{75}}]. \quad (7)$$

And the expression for reserve additions is modified to take this factor into account:

$$\Delta R_t = RY_t \cdot RA \cdot b_t \quad (6')$$

As new reserves are created by drilling, they are essentially a transfer out of the pool of ultimate production, Ult R_t, for the country or area. Thus, if cumulative production plus the amount already impounded into reserves at any moment were the same as the ultimate production, then the numerator of Equation 7 would be unity, b_t would be zero, and no amount of drilling could add anything to reserves. The closer the numerator is to zero, the smaller is the fraction, and therefore the less is the return to drilling effort, relative to the 1973-75 showing.[1] If, and as, the ultimate reserve estimates are changed up or down, or if we are confronted with varying estimates of ultimate reservers, we substitute them into Equation 7 and see what difference it makes.

This aggregated method, with national entities as building blocks, treats as one reservoir what may be a collection of hundreds. Small countries may be left that way. But larger ones must be, as soon as possible, divided into rational subgroups. (In every case, we divide between onshore and offshore.) We can illustrate the method using the

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[1]For a fixed Ult R_t (i.e., no major change in the ultimate prospect) the "discovery decline" is linear. As Cum R_t goes from Cum R_{75} to Ult R_t, b_t goes (linearly) from one to zero.

important new Reforma area of Mexico.[1] In this region, the new reserves--proved, probable and ultimate--have all been added since 1972, and therefore we treat the area separately from the rest of Mexico. To apply to the newly developing areas the coefficients derived from areas a half-century old or more would have been right only by chance.

The Reforma results are shown in Table 6. The top part of the table shows the key parameters needed for Equations 5-7; the bottom portion presents the results. Column 5 shows cumulative reserves added totalling 19 billion barrels (bb) by 1985. Also shown in the table is a forecast of annual production (to be discussed in the following section) which is based on this forecast of reserves-added. The same analysis performed for the rest of Mexico, or for any other nation or sub-national region, will look very much like Table 6.

In this example, an assumed constant rate of development drilling, interacting with government estimates of proved reserves and ultimate recovery, yields a peak production rate of 2.39 million barrels per day (mbd) reached in 1984, declining thereafter. Yet high-ranking officials forecast a production rate of 7 mbd in the year 2000. This implies considerably larger reserves-added, through more drilling activity, or more effective drilling (reduced rig time per well) than assumed in the model. Note we have assumed a constant number of development rigs. More development drilling would mean proportionately more production, lagged about 3 years. Thus we have identified one or two key variables in Mexico's plan which will change, and to which we must adjust our data set accordingly.

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[1]The Mexican authorities use a definition of proved reserves which is very close to the strict API concept: "reserves which are expected to be produced by existing wells through primary and secondary recovery;" from Prospectus, Mexico External Bonds Due 1983 (September 1976), p. 17.

Table 6. Sample Calculation of Reserves Added For Reforma Fields, Mexico

a.	Reserved added, 1973-75	5.7 bb[1]
b.	Development rig years, 1973-75	99[2]
c.	Reserves added per rig year [RA][3]	0.58 bb
d.	Ultimate recoverable reserves [Ult R_{75}]	60.0 bb[4]
e.	Cumulative production plus proved reserves, Reforma only, end-1975 [Cum R_{75}]	5.8 bb[5]

Yr.	Develop. Rigs	Reserves-Added Decline Coefficient, b_t	Proved Reserves Added, RA_t (bb)	Cumulative Proved Reserves Added (bb)	Ttl. Annual Production Capacity[6] (mbd) Depletion Rate 8%	5%	Actual Production[7] (mbd)
1974	33	1.000	1.91	1.91	0.121	0.077	0.113
1975	33	0.965	1.85	3.76	0.349	0.224	0.288
1976	33	0.931	1.78	5.54	0.713	0.461	0.432
1977	33	0.898	1.72	7.26	1.030	0.677	N/A
1978	33	0.866	1.66	8.92	1.320	0.875	
1979	33	0.835	1.60	10.52	1.570	1.050	
1980	33	0.806	1.54	12.06	1.790	1.220	
1981	33	0.778	1.49	13.55	1.970	1.360	
1982	33	0.750	1.44	14.99	2.104	1.500	
1983	33	0.724	1.38	16.37	2.280	1.620	
1984	33	0.698	1.34	17.71	2.390	1.720	
1985	33	0.673	1.29	19.00	2.290	1.820	

Notes to Table 6

1. International Petroleum Encyclopedia (IPE), 1976, p. 222.

2. Petroleos Mexicanos, Report of Director General (March 18, 1974 and 1975), Chiapas-Tabasco only. According to Hughes Tool Co. report, there was no substantial increase of rigs in the South Zone from 1974 to 1975. Hence, we assume the same number of development rigs throughout.

3. Line (a) divided by line (b); see Equation 5.

4. Minister of National Patrimony, Francisco Javier Alejo, quoted in El Tiempo, May 17, 1976, and Petroleum Economist, June 1976, as expecting 7 million mbd by 2000 A.D. A linear growth, and assumption of 10 percent depletion rate in 2000 A.D. indicates ultimate added reserves of just over 60 bb.

5. Original Reforma reserves. Differs from line (a) by 100 million barrels already produced.

6. Assumes: (i) a lag structure whereby new proved reserves are fully developed at the following rates: same year (t) 30%, year (t+1) 30%, year (t+2) 40%. This estimation is based on observed lags to full capacity in publicly-reported proved reserves. And (ii) either (a) 8% depletion annually of pre-existing and of new capacity, which is a weighted average of all large producing Mexico fields excluding Reforma (as calculated from IPE), or (b) 5% depletion annually, which is the announced objective. See IPE, op. cit., p. 191.

7. Reforma fields average production for the first six months of each year (calculated from OGJ). IPE reports total year averages of 0.0 mbd in 1974, 0.274 mbd in 1975, and 0.553 mbd in 1976. The estimates for 1974 and 1975 are probably inaccurate. The Prospectus for Mexican External Bonds, September 1976, is presumably authoritative, and reports 0.400 mbd average for the month of December 1975. The 1974 PEMEX: Memoria De Labores, p. 13, reports 0.171 mbd as the total 1974 average with a rate of 0.275 mbd reached in December 1974.

4.2 Analysis of New Capacity and Production Plans

We need now to go from proved reserves-added to attainable new capacity, as indicated in Figure 3. In some areas we have the companies' own plans. For actual forecasting, they dominate any estimates we could make. Moreover, they are a valuable check on our own calculations, which we make by two possible methods. We may rely again on the inertia of the system, and base production forecasts on the historical relation of output to proved reserves. We show this method first. Or, it may be possible to analyze the economic forces underlying observed production behavior, and methods of doing that also are discussed below.

But whichever method we use, application of a single depletion rate to a reserve estimate--to obtain a production profile--is a strong simplification, as Figure 5 shows. In the lower part of the figure, the solid line shows a typical production profile for a field; the area under the curve is the total proved reserves R. The dashed line is our simplified version of oil exploitation where production jumps immediately to an initial (and presumed peak) capacity Q_p and declines at a constant percentage rate thereafter. Once again, the area under the curve is R, so that

$$R = Q_p \int_o^T e^{-at} dt. \qquad (8)$$

As $T \rightarrow \infty$, $R \rightarrow Q_p/a$; or in the limit, $a = Q_p/R$. The question for analysis then is: what is the value of the depletion rate, a, (and therefore of Q_p) that is appropriate for new additions to reserves? Once a is determined it is a simple step to forecasts of supply. The remaining issue is the lag between the time proved reserves are "booked" and the point when the new capacity Q_p is on line. In a mature producing area the lag from the former to the latter is short, but not in a new province. We use, tentatively, 2 years onshore and 4 years offshore, but in every case including only those fields where development drilling has been started.[1]

4.2.1 Historical Production-Reserve Ratio

As a first approximation, capacity plans can be calculated on the assumption that new additions to proved reserves will be depleted at the same rate as existing fields. Under the assumption of a uniform policy regarding depletion, the depletion rate for an area can be calculated as

$$a = \sum_{t=1973}^{1975} Q_t / R_{73-75} \tag{9}$$

where 1973-75 is taken as a reasonable base period for estimation, and R_{73-75} is the average level of proved reserves over that period. Then for any new addition to proved reserves in year t, the new installed capacity is

$$Q_{pt} = a\Delta R_t. \tag{10}$$

Assuming no excess capacity is installed, production from R_t begins at the level $Q_t = Q_{pt}$ and declines at a percent per year.

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[1] Of course, we need to worry about the degree of error this simplified model may involve. The North Sea affords a check. The end-1973 IPE reserves (fields under development only) were less than 2 billion barrels. Preliminary indications then (which have, incidentally, been well borne out) were of a 9 percent depletion rate. This would predict 495 tbd in 1977, somewhat over a year too late, since 1976 and 1977 are now estimated at 585 and 1230 tbd respectively. The end-1975 IPE developed reserves were 9.4 billion barrels, and the same method, again assuming 9 percent, would predict 2.3 mbd in 1979, which seems about on target so far, since late-1976 estimates are for 2.5 mbd in 1979.

However, the production-reserve ratio for any country is an aggregate of many fields. Hence a simple division of national production by national reserves may be seriously misleading. A good example is Table 7, showing Abu Dhabi. Total national production was 1.7% of proved-plus-probable reserves. But the bulk of those "reserves" are still undeveloped; the working inventory is being drawn down much faster. If we confine ourselves to proved reserves, the simple quotient Q/R = .08 gives equal weight to every barrel of developed reserves. But our true objective is to give equal weight to every barrel produced. If we want to estimate how much Abu Dhabi is capable of producing next year, the Bu Hasa field (156 million barrels) is approximately twice as important as Zakum (82 million).

Accordingly, we weight the production-reserve ratio for each field by its production, and divide the total for all fields by the total weight. This calculation is shown in Table 7. Use of the weighted average mean lets us escape from some of the ill effects of poorly estimated reserves. Since it is a better predictor of future production, it yields more accurate cost data, which depend on our estimated production profile (Figure 5). We have, of course, a minor sampling problem since we are using the average of the listed fields for the whole country. However, the finite population multiplier is a powerful ally; since we have accounted for 77 percent of the national total of production (394/512), the error cannot be great.

It is likely that outside the United States and Canada, reserves are overstated and therefore decline rates understated. This is not because of errors of optimism, but because undeveloped reservoirs in known fields tend to be counted in.

Table 7. Weighted Mean Depletion Rate, Abu Dhabi, 1975

(millions of barrels)

Field, Discovery Date	Production Q	Reserves R	Q/R	Q^2/R
Asab, 1965	90	500	.180	16.2
Bu Hasa, 1962	156	1,289	.121	18.9
Mubarras, 1971	7	150	.047	0.3
Umm Shaif, 1958	59	1,706	.035	2.1
Zakum, 1964	82	1, 314	.062	5.1
Sub Total	394	4,959	.080	42.6

Unweighted Mean, total Abu Dhabi: $\Sigma Q'/\Sigma R' = 512/29,500 = 1.7\%$

Unweighted Mean, Large Fields: $\Sigma Q/\Sigma R = 394/4,959 = 8.0\%$

Weighted Mean: $\Sigma[Q(Q/R)]/\Sigma Q = 42.6/394 = 10.8\%$

Sources: Unweighted Mean, OGJ; others, IPE.

Table 8 shows the results of this simple approximation, using the weighted average depletion rate as the decline rate. We have chosen four widely differing areas to show the meaning of the concepts, and to test two capacity forecasts. The first case is Mexico, which draws on the data used earlier. It is assumed that Antarctica contains 20 billion barrels available but will not be explored or developed. Hence it has, and will have, zero reserves. Abu Dhabi drilling effort, continued at current levels, leaves them with a 1985 production about where they are now. Iraq attains 4.5 million barrels daily, double current production, but less than the government's announced objective for the year 1980 or 1981.[1]

4.2.2 Optimal Decline Rate

The value of a observed in historical data, and calculated by Equation 9, is the result of a particular set of past conditions of cost, price, and tax. It may or may not be an appropriate guide to future behavior under different conditions, and therefore we should like to be able to calculate this parameter. We can do this by assuming profit-maximizing behavior on the part of oil operators, and solving for the optimal depletion rate, a*, based on estimates of cost per barrel and future price.

First, it is assumed that the profile of capital expenditures E_t shown in Figure 5 can be collapsed to a single-period outlay I. Data on operating costs are rather sketchy; fortunately the bulk of costs

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[1]The equating of capacity with production assumes that these cartel nations are not forced to hold excess capacity in order to help support the price.

Table 8. Reserves and Production of Crude Oil for Selected Countries, 1975-85

(billions of barrels)

Country	(1) Cumulative Production	(2) Current Reserves	(3)	(4)	(5) Ultimate*	(6) Additional Proved Reserves 1976-85	(7) Production Forecast	(8)
		Existing Fields		Probable				
		Proved	Indicated Additional				1976-85 Total	1985
Mexico (Reforma Only)	0.1	5.7	N.A.	N.A.	60.0	19.0	6.4	0.84
Antarctica	0.0	0.0	0.0	0.0	20.0	0.0	0.0	0.0
Abu Dhabi	2.7	5.8	24.2	incl. (3)	49.0	11.0	5.5	0.55
Iraq	10.9	27.8	7.2	incl. (3)	70.0	15.0	12.0	1.64

*Columns 1 + 2 + 3 + 4 plus new fields

Sources: Col. 1, IPE

Col. 2, IPE, large fields reserves, multiplied by ratio of their production to total national production, as in Table 5.

Col. 3, OGJ, estimate of "proved reserves" less column 2.

Col. 5, Private correspondence, and John Moody and R.W. Esser, "An Estimate of the World's Recoverable Crude Oil Resources," paper delivered to World Petroleum Congress, Tokyo, 1975; for Mexico, see Table 6.

Col. 6 - 8 calculated by methods described in text.

are usually capital outlays, and they can be approximated by an adjustment to I, as shown below. Also, to begin, we assume that the capital coefficient, I/Q_p is a constant; later we consider the effects of a coefficient which is higher at higher depletion rates.

By our simplified model of depletion, annual production is $Q_t = Q_p e^{-at}$, and given a constant expected future oil price, P, and discount rate, r, the net present value of a block of reserves becomes

$$\begin{aligned} NPV &= PQ_p \int_o^T e^{-(a+r)t}\, dt - I \\ &= \frac{P\, Q_p}{a + r} - I \end{aligned} \tag{11}$$

By setting $\partial NPV/\partial Q_p = 0$, we can derive the first-order conditions for maximizing the value of the reserves, and the result can be stated in terms of an optimal rate of depletion, a*. That is,

$$a^* = \sqrt{\frac{Q_p}{I} \cdot Pr} - r \tag{12}$$

Thus the optimal depletion rate is a function of the capital coefficient (or, more properly, its reciprocal), the oil price and the discount rate.

Before proceeding to estimate values of I/Q_p and a*, it is worthwhile to question the accuracy of the simplified or "collapsed" model of oil exploitation in Figure 5. We have reduced a complex process to only three parameters--total proved reserves, total investment and peak output--and we need to make sure that this model yields a reasonable approximation to reality in view of the wide variations in production profiles among fields, and the dependence of the results on the discount rate assumed.

We can get a rough check on this issue by comparing the average per-barrel cost of oil as calculated by our "collapsed" model against actual data on expenditure and production profiles for the North Sea. Referring to Figure 5, we can define an oil price, C, which would meet the condition

$$NPV = \int_o^T E_t \, e^{-rt} \, dt - \int_o^T CQ_t \, e^{-rt} \, dt = 0 \qquad (13)$$

This C is the supply price of oil from the reserves illustrated in Figure 5. Or, more usefully for our purposes, C may be referred to as the average cost per barrel of oil from a new project in the area shown. We are able to calculate the "true" value of average per-barrel cost from detailed data for 17 North Sea fields. Similarly, by setting NPV = 0 in Equation 11, inserting equivalent values of the "collapsed" parameters, and solving for P, we can calculate a comparable set of figures for the simplified model. When we estimate $C = \alpha_o + \alpha_1 P$ using the values of C and P thus calculated, the results are as follows:

Discount Rate, r	α_o	α_1	R^2	SE	CV
10	-.028 (-0.19)	0.972 (16.5)	.95	.177	.078
12	-.033 (-0.18)	0.989 (14.4)	.93	.224	.089
15	-.046 (-0.18)	1.013 (12.0)	.91	.310	.107
20	-.055 (-0.13)	1.067 (9.5)	.86	.487	.133

(Numbers in parentheses are t-statistics, SE = standard error of regression, CV = coefficient of variation. Source of data: Wood, MacKenzie and Company, Section 2 of the North Sea Service," February 1976).

For example, with a 12 percent discount rate, the "collapsed" unit cost is a little higher on the average than the true cost: to get from "collapsed" to true, one multiplies collapsed by 0.989 and subtracts 3.3 cents. Thus estimated, the calculated cost will be within 9 percent of the true cost in about two-thirds of all cases. Whatever the discount rate, however, the collapsed cost is a reasonable approximation to the true cost--though this good fit depends critically on the strict definition of reserves as "planned cumulative output from planned facilities," as discussed above.

Analysis Assuming a Constant Capital Coefficient. Unfortunately, outside the North Sea, we do not have anything remotely resembling these detailed investment data. However, capital coefficients for differing producing areas can be estimated very approximately from historical data. Equation 14 shows the method used most frequently to calculate I. For any country, i, we take the proportion of its rig time RY_i to total regional rig time RY. We then multiply this by the Chase Manhattan Bank estimates of total regional production capital expenditures in the year CMB, and divide by the capacity increase. That is,

$$I_i = [\frac{RY_1}{RY} \cdot CMB] \tag{14}$$

This method overstates oil development costs, since it includes the cost of all wells, including exploratory and gas wells. In terms of Table 5, the cost of adding to total reserves (column 1) is included in the cost of adding to proved reserves (column 4).

In addition to this bias, there is also year-to-year inaccuracy caused by the lumpy nature of much development investment, especially in loading facilities. The investment needed per unit of additional daily output will be exaggerated in a year with much work in process, and conversely understated in the year facilities are completed, and begin operating with little new money being spent. More important: the method overstates costs in any area with an above-average percent of onshore drilling and understates them in an area with an above-average percent offshore, as compared with the whole region. Where this is a problem, corrections can be made using American data.

Additions to capacity Q_p may be calculated in one of two ways (or both as a mutual check). The first is to multiply the number of newly completed wells by the average flow rate of all existing wells.[1] To the maximum extent possible one should divide the completions by separate fields. This method tends to understate capacity increments because new wells are always drilled with better knowledge of the reservoirs than old wells, and to overstate increments because one would expect lower well productivities as well interference began to be significant and as lower-quality strata and reservoirs were developed.

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[1] Our estimate of the weighted average daily flow rate per well for each country is calculated as follows:

$$\text{estimated flow rate} = \sum_{i=1}^{M} \left(\frac{Q_i}{Q_M}\right)\left(\frac{Q_i}{W_i}\right),$$

where M is the number of major fields (a major field is defined as one which accounts for at least 3% of the country's total production), Q_i is the production of field i, Q_M is the total production of all major fields, and W_i is the number of producing wells in field i. Thus

$\frac{Q_i}{Q_M}$ is our weighting factor and $\sum_{i=1}^{M} \frac{Q_i}{Q_M} = 1.$

The alternative is to take the difference in production between year t and (t-t') and add to it the estimated loss of productive capacity over the interim. This involves the calculation of an appropriate decline rate, to be discussed below. Wherever there is irregular fluctuation in output, and any appreciable amount of excess capacity, however, the method cannot be used at all, since change in output is no indication of change in capacity. In this paper, therefore, we used the method of multiplying new completions by average flow rate. However, since independent estimates of capacity are available for recent years, for some countries, we will be able to apply the second method to them.

Table 9 shows capital coefficients for selected countries for the year 1973. (Data for 1974, the only later year, are extremely "noisy.") Since we have used only single-year data, the estimates are subject to such large error factors that they should not be used for any purpose except to check on the calculation of a*, which we now propose to do.

As an example, take a North Sea field where investment per peak daily barrel is calculated to be $7,300. We will take r as 10 percent.[1] The oil price is assumed to be $12.50 per barrel, and initial operating cost (at peak production) is $.95 per barrel,[2] i.e., $347 per year over the life of the field. We can equate this to a capital sum.

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[1]During 1975-76, dollar denominated Eurobonds had a 9 percent yield, which we consider as the prime business-risk rate on long-term finance. We add 1 percent, which is the usual premium above the LIBOR (London Inter-Bank Rate) on secured financing for the North Sea development projects.

Perhaps a better inflation allowance would start from the fact that the sterling LIBOR was generally 4 percentage points above the dollar LIBOR in 1975-76 (9.5 and 5.5 percent respectively) before the sterling slide in late 1976. An interesting confirmation of our 9 percent discount rate is in the LASMO-SCOT "package" rate, in late 1976, which can be shown to approximate 14 percent in sterling (see the LASMO-SCOT Prospectus, February 1976).

[2]Wood-MacKenzie report, October 1976, Section Two, Table 1, average for all proved fields.

Table 9. Capital Coefficients: Dollars Per Barrel of Initial Capacity, Selected Countries, 1973

Country	Capital Coefficient, I/Q_p
Algeria	$ 758
Libya	144
Egypt	483
Iran	186
Iraq	147
Nigeria	210
Venezuela	1,021
Brazil	5,622
Australia	5,185
Indonesia	335

Sources: Expenditures from Chase Manhattan Bank; Rig-time from Hughes Tool Company and AAPG Bulletin; other data from OGJ, "Worldwide Oil," annual issue, 1972-1974, and World Oil, "International Outlook," annual issue, 1974.

Assuming the typical North Sea 20-year life, and a 10 percent discount rate, a stream of outlays or receipts of \$347 dollars per year is worth \$347/.118 = \$2,950. Investment plus operating cost now equates to a capital sum of \$7,268 + \$2,950 = \$10,218. Then

$$a^* = \sqrt{\frac{365(\$12.50)(.10)}{\$10,218}} - .10 = .111$$

In fact, outside of Auk and Argyll, North Sea Q_p/R ratios (which are an approximation of a*) are about 9 percent. We will see shortly why they should be expected to be a bit lower than optimal.

As another example, we may take the hypothetical field "discovered" by Conoco on the Georges Bank (Oil and Gas Journal, July 19, 1976, p. 60). It contains 200 million barrels, to be drained in 15-20 years (say, 17.5) and requires an investment of \$397 million. We assume operating costs will be slightly lower than Montrose and Heather (slightly smaller fields) in the North Sea, i.e., \$25 million annually, over a 17.5 year period, hence with a present value of \$203 million, for a total investment of \$600 million. Initial output is 60 thousand barrels per day, hence the total capital coefficient is \$10,000 per daily barrel. Conoco assumes P = \$14 per barrel, and hence a* = 12.6 percent. Conoco's estimate is 10.95 percent.

Another comparison is possible, using a paper prepared at the U.S. Geological Survey [12]. The authors hypothesize a reservoir with a given quantity of producible gas, reservoir pressure, price-cost parameters, and four possible rates of water encroachment. The operator's decision on number of wells determines the initial Q_p, the ratio Q_p/R, the production profile over time, and the NPV of the deposit. Since price, operating costs, taxes, and the capital coefficient can be approximated from their data, and the interest rate is given, we can calculate a* = .135. In fact, the ratio which would maximize NPV is in the range between .157 and .173. (Although ultimate recovery is sensitive to the rate of withdrawal, there is no well interference, and initial withdrawal is explicitly stated as strictly proportional to the number of wells drilled.)

Variable Capital Coefficients. So far, it would appear that a* gives values not too far out of line with reality. But the three validation examples have been very high-cost areas. Typical capital coefficients are only a small fraction of such values. Yet observed values of the depletion rate are not many times those calculated; in fact they are typically lower.

Many of the low observed rates are apparently not real, because of overstated reserves, as shown above. But we might consider the United States, where reserves are estimates strictly on a very large sample of reservoirs. Since 1972 when the United States essentially went on maximum efficient rate (MER) capacity production, the ratio of production to proved reserves has been very stable in the neighborhood of 11.8 percent. It is expected to remain there, and production out of proved reserves to decline at that rate, [14, p. 214].

The incentive to speed up recovery has greatly increased in 1972-75, but the rate has stayed constant. Here we might reckon that a worldwide average limit to the depletion rate out of proved reserves is approximately 12 percent, and in few fields will it exceed 20 percent. One obvious reason is that too high a rate would induce gas or water breakthrough to the well bore, leaving much oil behind, reducing reserves.

Moreover, we have so far assumed that the investment per daily barrel remains constant regardless of how intensively the field is developed. In fact, in many cases the value of I/Q_p may not be a constant over wide-ranging values of a. It is in fact constant over some range, wide or narrow, for the whole hydrodynamic system is involved: the oil in place (including dissolved gas), any associated gas, and water. The volume and pressure of the hydrocarbon may be a relatively minor fraction of the whole. But usually, past some point, the higher the output, the lower the output per well. Hence, there is a margin where the output gained by drilling a well declines, perhaps sharply. Therefore, we may need to make I, the investment an increasing function of Q_p/R. To do this, James L. Smith as devised the following method.

We specify the reservoir investment function as

$$I = Ka^{\varepsilon}R \tag{15}$$

with K the proportionality constant and ε (epislon) the elasticity of investment with respect to the depletion rate. Since $aR = Q_p$, a varying ε in $a^{\varepsilon}R$ will vary the investment as a function of chosen output (i.e., the depletion rate). The constant K merely translates planned initial output into dollar requirements. Thus $\varepsilon = 1$ implies constant (linear) costs; $\varepsilon > 1$ implies increasing costs, etc.

The equivalent of Equation 11 is

$$NPV = \frac{P\,Q_p}{a + r} - Ka^{\varepsilon}R \tag{11a}$$

and the optimal decline rate can be expressed in the following relation

$$a^* = \sqrt{\frac{Pr}{\varepsilon K(a^*)^{\varepsilon-1}}} - r \tag{12a}$$

Note that Equation 12 is just a special case of 12a, where $\varepsilon = 1$ and therefore $K = I/Q_p$. The a* in Equation 12 implied that well productivity had not yet entered the phase of diminishing returns. But equation 12a now lets us obtain a marginal cost which allows for the increasing costs imposed by well interference, and to derive an optimal initial producing rate accordingly.

With an estimate of ε, we have all the parameters needed to solve for a*, with the exception of the constant K. By Equation 15 we can state K as a function of I/Q_p and a, and we have the historical value of the capital coefficient, $\hat{I}/\hat{Q}_p$, and the depletion rate $\hat{a}$, which are proxies for the true values. Thus

$$K = [\hat{I}/\hat{Q}_p]\hat{a}^{1-\varepsilon} \qquad (15a)$$

With K as estimated by 15a, we can solve Equation 12a (we use an iterative algorithm that permits a numerical approximation) for the optimal <u>future</u> depletion rate a*.

Table 10 shows some of the results, assuming the historical depletion rate was 0.1, and for P = \$10.50 and r = .12. The reason why we need this procedure is not far to seek. The range of observed capital coefficients is enormous. The top line, which puts the interference effect to zero (i.e., $\varepsilon = 1$), would indicate that with particularly low capital requirements (\$300 per initial daily barrel) the oil should be brought out in less than a year. If the operator of the reservoir acts as a competitive price-taker, then he may be restrained by the loss of ultimate recovery, or by well interference expressed in the ε coefficient. If the operator is not inhibited, and yet is producing at "too low" a depletion rate,

Table 10. Optional Depletion Rate (a*), for $\hat{a}$ = 0.1, P = $10.50, r = .12

Elasticity	Historical Capital Coefficient ($\hat{I}/\hat{Q}_p$)			
(ε)	$300	$800	$1500	$8000
1.0	1.12	0.64	0.43	0.12
1.25	0.74	0.44	0.31	0.10
1.5	0.54	0.34	0.24	0.08
2.0	0.35	0.23	0.18	0.08
3.0	0.21	0.16	0.13	0.07

then there is slack in the system which we would expect him to use up, by expanding output. If none of these inhibitors is sufficient, then he may be restrained by fear of spoiling the market.

The decline in per-well productivity is a prominent feature of many field production histories. But these are all after the fact. The basic data that would permit a calculation of an ex-ante a* are rarely, if ever, published. Filling this gap is an obviously high-priority task.

A final qualification: an expectation of higher prices would ceteris paribus justify delaying output. This is a problem in optimal control theory, and we intend to work out the functional relation.

4.2.3 The Influence of Taxes

The taxation may lower the optimal output rate. Looking again at the North Sea example and assuming output to have been pushed to where $\varepsilon = 1.0$, NPV is maximized by depleting at 11 percent per annum. From Equation 11 and the data on p. 54, we can calculate the DCF rate of return r'. That is, if NPV = 0 in Equation 14, then

$$\frac{P\,Q_p}{a + r'} = I \tag{16}$$

and in this case $4,562/(.11 + r') = $10,218 and thus r' = 0.34.

The objective of the government is to capture as much as possible of the difference between a DCF of 34 percent, and the operator's minimum acceptable DCF of, say, 10 percent, which is how we define the bare cost of production. But royalty or excise payments may make the operator change his plans, to make everybody worse off. For example, if they simply took a 50 percent royalty, thus cutting P to $6.25, the operator would maximize NPV at a* = .05.

Thus the investment and the peak output would be only 5/11 of optimal. It is not worth the operator's while to spend more money to get the oil out faster, though he will get it out eventually.[1] The total NPV per barrel is less:

$$NPV/Q = ((5/11)(365 \times \$12.50)/.15) - \$10,218 = \$3,608$$

This is only 45 percent of the optimal NPV.

The operator's DCF is now: $\$10,218 + \dfrac{(\$6.25(365)5/11)}{.05 + r'} = 0$

$$r' = .10 - .05 = .05$$

which is of course unacceptably low, and the government must prepare a lower royalty. Where costs are very low, the dampening effect of a royalty or excise does not matter nearly as much. Given a not-infrequent case of poor information and mutual mistrust, one can easily write such a scenario of deadlock as is being played out in several countries today. An alternative would be to set NPV as the upper limit to taxation, and aim to get some maximum practical fraction of it. The chief barrier is uncertainty. Where there is agreement on the cost and revenue data in our example, the government could bargain for a lump sum payment of somewhat less than \$11,500, payable in installments at the convenience of both parties, but independent of the volume of production, so that the operator would have no incentive to change his plan. Another possibility

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[1] Note that one barrel per day, declining at 10 percent, cumulates to 9.09 barrels of original reserves. If we produce only (5/11) of a barrel, declining at 5 percent, this too cumulates to 9.09 barrels.

would be to calculate total NPV as expected, then to provide a sliding scale of government take, in order to give the operator an incentive to reduce costs or increase output.

We may eventually devise some plan, but for the present we need only note that since governments have not followed it, they must have reduced the rate of production and NPV below what is attainable. Again, we follow a simplified approach: translate the tax into a royalty-equivalent, of the type of Equation 16, and calculate with the resulting depletion rate. In practice this becomes very complex, because the usual arrangement is for recovery of costs at an early stage. This reduces, often drastically, the present-value equivalent excise, and, therefore, the impact on NPV.

4.3 Conclusion

Oil costs are everywhere only a small fraction of prevailing prices. Hence even substantial price changes would have little effect on supply. Moreover, the owners of the resource are governments, with more than the usual number of degrees of freedom to choose investment and pricing policies. Hence a model driven by some assumed price-cost-profit equilibrium will probably not capture the essentials of the supply side of the market. Furthermore the basic determinants of cost and supply--the investment needed to find, delineate, and exploit reservoirs, and the time for this operation--are so imperfectly known that the need to respect data limitations has dominated our model. We have perforce adopted a rough and often ad hoc scheme for predicting the amount of reserves to be developed, calculating real production costs, and predicting government policy on capturing the profits on incremental investment, and deciding on the rate of growth of capacity.

For most countries, with relatively small reserves, a rather mechanical procedure and assumed zero supply elasticity results in no substantial error. For those relatively few countries that matter, we hope to supplant our first working assumptions with more realistic and complex hypotheses, including even backward bending supply responses.

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