CYCLICAL VARIATION IN THE PRODUCTIVITY OF LABOR

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I. Introduction

Numerous macroeconomic studies, like those of Okun [17], Perry [19], and Nordhaus [14], agree that, for a given cyclical change in GNP, labor demand changes less than proportionately. Thus, output per man-hour changes in the same direction as output, in other words the average productivity of labor shows a procyclical pattern of movement. This was formulated by Okun as "Okun's law": For each percentage point of reduction in unemployment, GNP will rise by three per cent. Even after correction for variations in the workweek this elasticity is well above unity. Perry, after adjustment for demographic shifts in the labor force, gets it around 1.4; Nordhaus' estimate of the inverse is around .9. Outside the U.S., similar results have been reported e.g. by Brechling [5] and Ball and St. Cyr [1] for British manufacturing; and by Brechling and O'Brien [6] in an international comparison of several European countries, Canada, and the U.S.

These results are of course puzzling in the light of the neoclassical theory of the firm, since they seem to violate the law of diminishing returns. A defense of the validity of this law is given by Draghi [8], who claims that previous results can be explained as a specification bias. Rather than improving the specification, Draghi reestimates Nordhaus' specification with instrumental variables and thus obtains a short-run output elasticity of labor demand that exceeds unity.

This paper attempts to take a fresh look at this problem with more rigor in theory and measurement than has been done in the past. Firstly, Draghi's criticism is accommodated by a more complete specification. A short-run aggregate cost function with four inputs is estimated with a flexible functional form. The four inputs are capital, which is fixed in the short run, and
labor, energy, and agricultural goods. As a consequence of the multi-input specification, output is defined as gross output, not value added.

Secondly, a parameter is identified and estimated which measures the cyclical variation in the productivity of labor. The identifying condition comes from long-run cost minimization and a weak assumption of long-run competition. In this sense, the law of diminishing returns is a maintained hypothesis of this paper. The cyclical fluctuation in productivity thus defined is found to be large.

Thirdly, an attempt is made to explain the phenomenon, but without much success empirically. It is demonstrated that it is not an artifact caused by aggregation, although a small part can be explained by cyclical shifts in employment between high- and low wage industries. I am also unable to explain it by labor hoarding. A slight part can be explained by overtime and shiftwork compensation, which cause cyclical fluctuations in the marginal wage rate. But we are mostly left with a puzzle whose explanation will have to await further work.

Section II of the paper presents the empirical and theoretical framework of the paper, with special emphasis on the definition and identification of the cyclical productivity parameter. Section III discusses various hypotheses for explanation of cyclical productivity variation and presents some independent empirical evidence on aggregation effects; whereas the estimates of the model of this paper are presented and discussed in section IV.

II. Formal and Empirical Framework

1. Empirical Framework

For the purpose of this paper, the private U.S. economy is divided into three sectors: agriculture, energy production, and a third remaining sector
which will be referred to as the goods sector. The focus of interest is on the short-run technology of the goods sector; the outputs of the other two will be treated as inputs to the goods sector along with labor and capital. Thus, in contrast to all existing short-run macro studies that I am familiar with, this is a study of the production of gross output of the goods sector, not just its value added.

The energy sector is defined as the production of energy in its crudest form, so that refining, conversion into electricity, and distribution are all included in the goods sector. This is done because it is considered desirable that the price of energy in the model contain as little as possible of labor and capital cost. The agricultural sector is defined as farming alone, thus not including fisheries and forestry. The goods sector is then defined residually as the remaining part of the business nonfarm sector plus the household sector.

Imports of energy and agricultural goods have been added to the deliveries from the respective domestic sectors. For completeness, these export figures contain refined energy products and processed agricultural goods. Exports of raw energy and unprocessed agricultural products have been subtracted off.

Details of data and definitions are given in Appendix A.

2. The Short-Run Cost Function

The technology of the goods sector is estimated in the form of a short-run (or restricted) cost function. As has been shown by McFadden [13], this function will, under certain regularity conditions, give a complete representation of the technology. Thus, if $x$ is a vector of variable inputs, $K$ is the fixed factor, and $Q$ is output; then the technology can be represented either
by the production function

\[(2.1.) \quad Q = F(x,K),\]

or equivalently by the restricted cost function

\[(2.2.) \quad C(w,Q,K),\]

where \(w\) is the vector of variable factor prices.

When \(F\) exhibits constant returns to scale, the following properties of \(C\) can be shown to hold: It is (a) increasing in \(w\) and \(Q\) and decreasing in \(K\), (b) homogeneous of degree 1 in \(w\) and in \((Q,K)\), (c) concave in \(w\) and convex in \((Q,K)\).

Particularly useful is Shephard's lemma, which in logarithmic form gives the "share equations":

\[(2.3.) \quad w_i x_i/C = \partial \log C/\partial \log w_i\]

One of the regularity conditions underlying McFadden's theory is that the technology is "putty-putty". An alternative modeling of the short-run technology is presented by Johansen [9], as the ex post production function in a putty-clay technology. In general, this will not be stable over time and consequently not estimable by time-series. It will be stable only if the net addition to utilized capacity will be distributed in the same way - over the space of variable factor intensities - as the already existing capacity. One might hope that this condition would be more likely to be satisfied - in an approximate sense - in the aggregate than for a single product.

If such stability exists, Johansen suggests (op. cit., p. 208) that the stock of existing (i.e. not only utilized) capital can be used as a single shift parameter for changes over time. I.e. we can write
This is not a long-run (or ex ante) production function, but it is homogeneous of degree one in \( x, K \). Formally, then, we are back to (2.1.) and can deduce the short-run cost function in much the same way as above. Hence, although the two cases are different, there does not seem to be any reason to distinguish between them here.

There is a problem of how to treat trend technical progress in the putty-clay framework. Without a specific treatment of different vintages of capital - which would complicate the problem enormously - embodied technical progress cannot be incorporated. However, using Johansen's terminology, capacity-increasing technical change could be taken care of by a time trend multiplying capital; and input-saving progress can similarly be represented by a time trend multiplying each variable factor price.\(^1\) Since the same procedure fits into the neoclassical framework also, we have in either case

\[
(2.5.) \quad C(w,Q,K,t) = C(e^{-\lambda_1 t}w_1, \ldots, e^{-\lambda_n t}w_n, Q, e^{\lambda K t}).
\]

3. **Modeling of the Cyclical Variation in the Productivity of Labor**

Define an efficiency unit of labor as a natural unit\(^2\) (i.e. man-hour) times a cyclical factor. A natural choice of cyclical variable is the output-capital ratio, which is the cyclical variable in the cost function when homogeneity in output and capital is imposed. Hence, labor demand may be expressed in efficiency units as

\(^1\)Strictly speaking, it is price-diminishing rather than input-saving progress that is modeled here (c.f. Ohta [15]). However, since the difference between the two concepts is unimportant here, I see no reason to emphasize it.

\(^2\)After correction for trend productivity, which has been done in (2.5.).
(2.6.) \[ L^* = L(Q/K)^h, \; h > 0, \]  
where the parameter \( h \) is a measure of the cyclical variation in the productivity of labor.

For a dual formulation, define the price of labor per efficiency unit as \( w^* \) so that

\[ w^* L^* = wL, \]

where \( w \) is the wage rate per man-hour. Substitution from (2.6.) then gives

(2.7.) \[ \log w^* = \log w - h \log (Q/K). \]

However, the parameter \( h \) will not be identified in a system consisting only of the cost function and the cost share equations. This may be seen most easily in a Cobb-Douglas formulation, which gives the cost function

(2.8) \[ \log (C/K) = a_L \log w + a_A \log p_A + a_E \log p_E + (a_Q - h a_L) \log (Q/K), \]

where \( a_i = \alpha_i/(1 - \alpha_K), i = L, A, E, \) are the short-run cost shares; \( a_Q = \alpha_K/(1 - \alpha_K); \) and \( \alpha_i \) is the long-run cost share of factor \( i. \) In (2.8.) the elasticity of the output-capital ratio is biased downward because of the cyclical productivity factor, but the bias cannot be determined without further information.

A possible identifying assumption for \( h \) is marginal cost pricing, which gives the additional equation

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3 Prooper scaling eliminates the need for a multiplicative constant in (2.6.). Also, \( Q/K \) need not be detrended because time is a separate argument in the cost function.

4 This formulation seems to suggest that \( w^* \) is endogenous to the firm so that the cost function is ill defined. However, \( w^* \) should be thought of as exogenous to the firm and (2.7.) merely as a formula for computation of \( w^* \) from observable data (cf. also footnote 9).
(2.9.) \[ \frac{pQ}{C} = \log C / \log Q \bigg|_{w^*}, \]

which will be referred to as the profit equation. In the Cobb-Douglas case, this takes the form

(2.10.) \[ \frac{pQ}{C} = aQ. \]

It is obvious that \( h \) is identified by the difference between the slope of \( \log(Q/K) \) in (2.8.) and the constant in (2.10.). A translog formulation, which is used for the estimations in this paper (cf. section II.4. below), only adds second order terms that do not alter this property.

The assumption of marginal cost pricing is considered too strong and too controversial to be adopted here. However, it turns out that an argument based on long-run cost minimization and a weak assumption of long-run competition gives almost the same result. The argument goes as follows. Firms, facing stochastic demand, plan their capacity so as to minimize rationally expected unit cost in an \textit{ex ante} sense, i.e. the expectation is made conditional upon the information available at the time the capital stock is planned.\(^6\)

Letting the symbol \( E \) denote expectations in this \textit{ex ante} sense, this means that every firm seeks to minimize

\[ E[(C + p_K K)/Q] \]

with respect to \( K \).\(^7\) The first order condition for an interior maximum is

\(^5\)On the form of this partial derivative, cf. footnote 4 above.

\(^6\)There is a subtlety stemming from Jensen's inequality that makes a difference between minimization of expected unit cost and expected total cost under uncertainty. The assumption of expected unit cost minimization is based on the argument that it is the only behavior that can survive long-run competition, since a firm that minimizes expected unit cost on average can afford to undersell others that do not and thus drive them out of business in the long run.

\(^7\)With a putty-clay technology this minimization process also includes a choice of the optimal form of the function \( C \). However, minimization with respect to \( K \) will still be part of the problem, so that the implications below will hold in either case.
Using the homogeneity of $C$ in $Q$ and $K$, rearranging and multiplying by $K$ (which is non-stochastic), gives

\[(2.11.) \quad E[\frac{\partial C}{\partial Q} - (C + pK)/Q] = 0.\]

It is further natural to argue that long-run competition makes price equal unit cost in the same expectational sense. Note that this assumption implies nothing as to price rigidity or flexibility in the short run. This gives

\[(2.12.) \quad E(\frac{\partial C}{\partial Q} - p) = 0.\]

Following common exonometric practice, this can be converted into the more convenient logarithmic form

\[(2.13.) \quad \frac{pQ}{C} = \alpha \log C/\alpha \log Q + u\]

and $u$ be treated as having zero mean, although this does not follow strictly from (2.12.). But this is of course the profit equation (2.9.) with an error term added. It is important, however, that the equation will have to be estimated with lagged instruments with a lag at least as long as the investment lag, since the expectation is taken conditionally on information available at the time the capacity was planned.

The explanation of how cyclical productivity is identified may now be expressed as follows. The profit equation gives an estimate of the output elasticity of variable cost that is consistent with long-run cost minimization, i.e. a value around 1.5. However, when this elasticity is estimated in the cost function itself without any correction for cyclical productivity, it is
expected to get a value less than one. This seemingly increasing return to variable factors is due to the cyclical variation in the productivity of labor, and the difference between the two estimates can thus be interpreted as a measure of this productivity variation.

4. **Algebraic Specification**

The algebraic specification chosen here is a translog approximation of the short-run cost function. The translog specification was first introduced by Christensen, Jorgenson, and Lau [7], and has later been used successfully in many studies. The translog function is a quadratic form in natural logarithms. When the cost function thus is log-quadratic it follows that the cost share and profit equations are log-linear. Homogeneity and symmetry across equations are imposed throughout. Concavity in variable factor prices is imposed locally by a method developed by Lau [11], details are given in Appendix B. The concavity restriction and the formulation of cyclical productivity in (2.7.) introduces some non-linearity in the system. The point of approximation is chosen to be the observations of the second quarter of 1972, which are therefore normalized to be zero. Measuring the log of variable cost as the deviation from its 1972 II value eliminates the constant from the cost function. In the presentation of the results, the first order parameters of the translog cost function are referred to as a's with single subscripts and the second order terms as b's with double subscripts.

III. **Hypotheses and Previous Findings**

This section will discuss five possible sources of explanation for the cyclical variation in the productivity of labor. The first two are labor hoarding and cyclical variation in the marginal wage rate as a result of overtime and shiftwork. The conclusion for the discussion of labor hoarding
is somewhat ambiguous, but it is clear that labor hoarding cannot account for cyclical productivity variation in any simple way. Overtime and shiftwork compensation do, on the other hand, offer logically plausible explanations because they call for a cyclical correction of the marginal wage rate of the same type as in (2.7.).

The last three points all go under the heading of aggregation effects, caused by cyclical shifts in employment or output between industries. These are shifts between industries with high and low wage levels, with high and low productivity variation, or with high and low average variable cost, respectively. Each of the three aggregation effects has a logical potential of explaining parts of the observed productivity fluctuation in macro. However, computations based on previous results suggest that the last two effects are empirically unimportant, and if anything add to the problem rather than explaining it. Shifts in employment between high- and low-wage industries, on the other hand, cause a procyclical fluctuation in the aggregate wage rate of the same form as (2.7.) and is thus potentially capable of explaining part of the observed cyclical variation in labor productivity.

1. **Labor Hoarding**

One may think of cyclical variations in labor productivity as the result of movements along a decreasing part of the average variable cost (AVC) curve. Oi's [16] theory of the quasi-fixity of labor, by introducing a U-shape in the AVC curve, implies that such a decreasing segment exists. Because a part of the cost of labor is fixed cost, employment is likely to be decreased less than proportionately when output falls below normal, which gives a procyclical movement in output per man-hour.

However, the power of this explanation is limited because it considers only the downward sloping part of the U-shaped curve. From (2.11.) it follows
that, in a large enough sample, the sample mean of short-run marginal cost will exceed that of average variable cost, so that the AVC curve will be increasing in this average sense. This does not rule out the possibility that labor hoarding is responsible for cyclical productivity in some other, more indirect way. But it is clearly indicated that the relationship, if it exists, is not simple, and further analysis is left for future research.

2. Overtime, Shiftwork, and Cyclical Variation in the Marginal Wage Rate

When a firm can regulate its labor input by using overtime and shiftwork, it will face a wage schedule rather than a fixed wage rate. Letting \( w \) denote the average wage rate and \( w^0 \) the rate for straight-hour daytime work, this can be written as

\[
\bar{w} = w^0 f(L).
\]

The marginal wage rate is then defined as

\[
w' = (1 + e)w^0 f(L),
\]

where \( e > 0 \) is the elasticity of the function \( f \). Obviously, \( w' \) fluctuates cyclically with employment. Assuming for simplicity that labor is the only variable factor, short-run cost is defined as

\[
C = \bar{w}L = w^0 f(L)L,
\]

and

\[(3.1.) \quad \frac{a \log C}{a \log Q} |_{w^0} = (1 + e) \frac{a \log L}{a \log Q}.
\]

Thus, the output elasticity of cost is higher than what is given by the technologically determined return to the variable factor, because the price of that variable factor...
factor increases when its quantity is increased. In fact, it is possible to have increasing returns to labor and increasing cost at the same time. This may explain part of the paradox of much empirical literature because it is mostly the output elasticity of labor, not cost, that has been estimated.

The correct measure of the output elasticity of cost in (3.1.) can be compared to

\[
(3.2.) \quad \frac{\partial \log C}{\partial \log Q} \bigg|_{w} = \frac{\partial \log L}{\partial \log Q},
\]

which is what will be estimated when the average wage rate \( \bar{w} \) is used. This measure is obviously downward biased and may violate (2.11.). The bias can be eliminated however, by correcting the wage rate for overtime and shiftwork compensation.\(^9\)

This can be interpreted as saying that part of the adjustment of labor cost in (2.7.) is in fact due to cyclical fluctuations in the wage rate per man-hour rather than to productivity fluctuations proper. It follows from this that the estimated value of \( h \) is expected to decrease when this is corrected for in the data.

3. Aggregation Effects

Economists have long suspected (cf. Kuh [10], Nordhaus [14], and Okun [18]) that the solution to the mysterious cyclical productivity shifts lies hidden somewhere in the aggregation from micro to macro. Thus, Okun points out that cyclical fluctuations in employment tend to be stronger in industries with high wages and high output per man-hour, such as durable manufacturing. Intuitively, at least, this seems to offer an explanation of the phenomenon.

\(^9\)It should be noted that this correction makes the cost function conceptually different from the case of fixed factor prices. It is easy to show, however, that all the formal properties carry over. In particular, the labor share equation is given as

\[
\frac{\partial \log C}{\partial \log w} = \frac{w^0 f(L)}{C} = \frac{wL}{C}.
\]
Nordhaus, on the other hand, reports evidence indicating that the cyclical fluctuation in employment tends to take place in industries with low cyclical productivity variation, so that the fluctuation in productivity is in fact lower in macro than the average across industries. It will be shown below that three different aggregation effects can be isolated and analyzed separately. Of these, only the cyclical shift in employment between high- and low-wage industries seems able to offer an unambiguous contribution to the explanation of cyclical productivity variation. The two other effects, which represent cyclical shifts between industries with high and low productivity variation and with high and low average variable cost, respectively, are shown to offer no further insight and, if anything, to add to the problem to be explained.

a. Cyclical Shifts in Employment Between High- and Low-Wage Industries

Cyclical fluctuations in employment tend to be stronger in high wage industries, c.f. Okun (op. cit.). Thus, even if industry wages stay constant over the cycle there will be a procyclical movement in the observed aggregate wage rate per man-hour. Empirically, then, part of the cost increase will be ascribed to the apparent wage increase and the estimated output elasticity of cost will be underestimated.

Formally, note that the aggregate wage rate is defined as

$$\bar{w} = \sum w_i L_i / L = \Sigma L_i w_i,$$

where the $w_i$ are industry wage rates and $L_i$ and $L$ industry and total manhours, respectively. Observe that we can write

$$\left. \frac{\partial \log C}{\partial \log Q} \right|_{w_i} = \left. \frac{\partial \log C}{\partial \log Q} \right|_{\bar{w}} + \left( \frac{\partial \log C}{\partial \log w} \right) \left( \frac{\partial \log \bar{w}}{\partial \log Q} \right)_{w_i}.$$

Furthermore,
\[
\frac{\partial \log w}{\partial \log Q} \bigg|_{w_i} = \frac{1}{w} \frac{\partial (\Sigma w_i L_i/L)}{\partial \log Q} \bigg|_{w_i}
\]
\[
= \Sigma \left( L_i/L \right) \left( w_i/w \right) \frac{\partial \log (L_i/L)}{\partial \log Q}
\]
(3.3.)
\[
= \Sigma \frac{L_i}{w_i} \left( \frac{\partial \log L_i}{\partial \log Q} - \frac{\partial \log L}{\partial \log Q} \right).
\]

But this can be interpreted as the covariance across industries between the sensitivity of industry employment to fluctuations in aggregate output on the one hand and relative industry wage levels on the other. It follows immediately from the empirical observations above that this covariance term is positive. Thus, \( \frac{\partial \log C}{\partial \log Q} \) will be biased downward when estimated directly from the cost function. It is seen from (2.8.) and (2.10.) that this gives an upward bias in the productivity parameter \( h \). The bias can be removed by adjusting the wage rate for interindustry shifts in employment, i.e. by using fixed weights \( L_i \) in the construction of the aggregate wage rate so that the covariance term (3.3.) disappears.

In the same way as for overtime and shiftwork compensation, this correction can be viewed as part of the correction of the wage rate in (2.7.). Again, this is a correction in the wage rate per man-hour and has nothing to do with productivity proper, but failure to make the correction will bias \( h \) upwards. Hence, it is expected that the estimate of \( h \) will be reduced when the shifts are corrected for.

b. Cyclical Shifts Between Industries with High and Low Productivity Fluctuation

Aggregate cost and output can be decomposed by industry as
Further, define industry cost shares

\[ b_i = \frac{C_i}{C} . \]

It is then possible to write

\[ \frac{\partial \log C}{\partial \log Q} \bigg|_{w_i} = \left( \sum_i b_i \frac{\partial \log C_i}{\partial \log Q_i} \right) \left( \sum_i b_i \frac{\partial \log Q_i}{\partial \log Q} \right) + \text{COV} \left( \frac{\partial \log C_i}{\partial \log Q_i}, \frac{\partial \log Q_i}{\partial \log Q} \right) . \]

Since, as can be seen from (2.8.), \( \frac{\partial \log C_i}{\partial \log Q_i} \), estimated without the correction in (2.7.), can be interpreted as an inverse measure of cyclical productivity variation within an industry, the whole expression can be read as the non-central covariance between this and industry sensitivity to cyclical changes in overall demand. Thus, it can be written as

(3.4.)

10 Implicit in this notation is an assumption that the industry cost functions, which will also include deliveries to and from other industries within the goods sector, are separable so that

\[ \ddot{C}_i = C_i(w_i, p_A, p_E, Q_i, K_i) + \sum_{j \neq i} p_j X_{ji} \]

where \( X_{ji} \) are cross deliveries, and

\[ Q_i = \ddot{Q}_i - \sum_{j \neq i} X_{ij}, \]

where \( \ddot{Q}_i \) is gross industry output.
The covariance term in (3.4.) is practically the same composition term as the one Nordhaus discusses and claims to be positive. Since an estimate of this covariance term that is conceptually consistent with the rest of the model of this paper would require very detailed industry data, it has not been computed here. Instead, it is included in the residual measure $h$.

It is possible, however, to use more crude data to get an impression of its order of magnitude. Treating labor as the only variable factor, total compensation will be industry variable cost and GNP originating will be industry output. This gives easily numbers for $b_i$. $\alpha \log Q_i/\alpha \log Q$ has been estimated on annual data by running OLS of $\log$ of industry GNP on $\log$ of total GNP in the industries considered, a time trend, and a constant; corrected for serial correlation. Industry estimates of $\alpha \log C_i/\alpha \log Q_i$ (without the correction in (2.7.)) are provided by Nordhaus (op. cit.) and Draghi [8]. The relevant numbers are listed in table 1.

Based on these figures, $\text{cov} \left( \frac{\alpha \log C_i}{\alpha \log Q_i}, \frac{\alpha \log Q_i}{\alpha \log Q} \right)$ comes out as .06 based on Nordhaus' estimates and .04 based on Draghi's. It is reasonable to say that the latter is upward biased because the output elasticity of cost for finance, insurance and real estate, which has the lowest cyclical sensitivity, is estimated by OLS only. When this is taken into account, the number based on Draghi's results is only trivially different from zero. The covariance term based on Nordhaus' industry estimates is substantially lower than Nordhaus' own "composition term" of .156. The difference can probably be explained by the slight difference in definition, but it does indicate that Nordhaus' result is not very robust. Thus, it seems safe to conclude that cyclical shifts in production between industries with high and low cyclical variation in productivity hardly offers any explanation of the phenomenon in the aggregate. But neither does it add significantly to the problem.
Table 1.
Data needed for crude estimation of
the covariance terms in (3.4.) and (3.5.)

<table>
<thead>
<tr>
<th>Industry</th>
<th>$b_i$ (1972)</th>
<th>$q_i$ (1972)</th>
<th>$\alpha \log Q_i$</th>
<th>$\alpha \log C_i$ (Nordhaus)</th>
<th>$\alpha \log C_i$ (Draghi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>.0784</td>
<td>.0594</td>
<td>1.008</td>
<td>1.480</td>
<td>1.406</td>
</tr>
<tr>
<td>Non-Durable Manufacturing</td>
<td>.1394</td>
<td>.1225</td>
<td>1.150</td>
<td>0.923</td>
<td>1.221</td>
</tr>
<tr>
<td>Durable Manufacturing</td>
<td>.2311</td>
<td>.1803</td>
<td>2.479</td>
<td>0.830</td>
<td>1.191</td>
</tr>
<tr>
<td>Transportation</td>
<td>.0582</td>
<td>.0485</td>
<td>1.479</td>
<td>0.575</td>
<td>1.71</td>
</tr>
<tr>
<td>Communication</td>
<td>.0266</td>
<td>.0308</td>
<td>0.284</td>
<td>1.242</td>
<td>1.418</td>
</tr>
<tr>
<td>Electric, Gas and Sanitary Services</td>
<td>.0170</td>
<td>.0294</td>
<td>0.373</td>
<td>0.413</td>
<td>0.474$^a$</td>
</tr>
<tr>
<td>Trade</td>
<td>.2102</td>
<td>.2111</td>
<td>0.704</td>
<td>0.433</td>
<td>1.041</td>
</tr>
<tr>
<td>Finance, Insurance and Real Estate</td>
<td>.0695</td>
<td>.1769</td>
<td>0.133</td>
<td>0.717</td>
<td>0.717$^b$</td>
</tr>
<tr>
<td>Services</td>
<td>.1692</td>
<td>.1411</td>
<td>0.403</td>
<td>0.548</td>
<td>1.343</td>
</tr>
</tbody>
</table>

$^a$Not corrected for serial correlation and with slightly different instruments.

$^b$OLS estimate, replication of Nordhaus (for explanation, see Draghi (op. cit. p. 20)).
c. **Cyclical Shifts Between Industries with High and Low Average Variable Cost**

Next consider the term

\[ \sum b_i \frac{\partial \log Q_i}{\partial \log Q}, \]

which appears in (3.4.) and is the average elasticity of industry output with respect to total output with industry cost shares as weights. This will in general differ from unity because the weights are not output shares. Define output shares as \( q_i = Q_i/Q \). We can then write the term as

\[ \sum (C_i/C) \frac{\partial \log Q_i}{\partial \log Q} = \sum (Q_i/Q) \left( \frac{C_i/C}{Q_i/Q} \right) \frac{\partial \log Q_i}{\partial \log Q} \]

\[ = \sum_q \left( \frac{C_i/C}{Q_i/Q} \right) \frac{\partial \log Q_i}{\partial \log Q} \]

(3.5.)

\[ = 1 + \text{cov} \left( \frac{C_i/Q}{Q_i/Q}, \frac{\partial \log Q_i}{\partial \log Q} \right), \]

where the covariance is computed with output shares as weights. The last equality is obtained because

\[ \sum_q \left( \frac{C_i/Q}{Q_i/Q} \right) = \sum (Q_i/Q)(C_i/Q_i)(Q/C) = \Sigma b_i = 1, \]

and

\[ \sum_q \frac{\partial \log Q_i}{\partial \log Q} = \sum (Q_i/Q)(Q_i/Q) (\partial Q_i/\partial Q) = 1. \]

This covariance term can be interpreted as follows. If it is true that it is negative, it means that an upswing in the economy will imply a shift to low variable cost industries and vice versa. Hence, the observed aggregate marginal cost in an upswing will reflect the net effect of two forces, namely the sum of cost increases in each industry minus the savings made possible by a shift to low variable cost industries. Thus, this would be a source of downward bias in the observed aggregate output elasticity compared to the
industry average, and a corresponding upward bias in \( h \).

This is similar to the point made by Okun about shifts toward industries with high output per man-hour, because average variable cost equals the average wage rate times the inverse of output per man-hour when labor is the only variable factor. However, the multiplication by the wage rate makes an important difference because, if labor markets are perfect, it amounts to adjusting labor for differences in marginal productivity. One should therefore be cautious about interpreting average variable cost as a measure of productivity. It is more accurate to interpret it as a measure of intensity in variable factors in value terms, and that has really very little to do with productivity.

For this reason, the difference from Okun's argument is more than one of interpretation. When this covariance term is computed on the basis of the figures in table 1, it actually turns out to be positive, but as low in numerical value as .14. The reason why it is positive, contrary to Okun's suggestion, is exactly that it is calculated in terms of labor cost rather than just man-hours. This is seen most easily for durable manufacturing, which is the cyclically most sensitive industry. Although its output per man-hour is high, its cost share \( b_i \) exceeds its output share \( q_i \) because of its high wage rate.

On the background of these observations it seems hard to escape the conclusion that the only aggregation effect that can explain any of the observed cyclical fluctuation in labor productivity is the cyclical shift in employment between high and low wage industries. Thus, when the wage rate is corrected for this (and for overtime payment), the resulting estimate of the parameter \( h \) can truly be interpreted as a measure of cyclical productivity, even when the estimations are made with aggregate data.
IV. **Empirical Estimation**

1. **Estimation Method**

The estimation method used is non-linear three stage least squares, as described by Berndt, Hall, Hall, and Hausman [3]. As argued in section II.3, it was necessary to use lagged instruments in order to get consistent estimates of the profit equation. Lags of various length were tried out initially. The problem was to pick a lag that is sufficiently long so that marginal cost pricing has not in fact been assumed and at the same time sufficiently short so as to minimize the loss of efficiency in the estimation procedure. It turned out that twelve and eight quarter lags gave practically identical point estimates, whereas shorter lags gave slightly different results. Based on the assumption that the short run lasts no more than three years I decided to use eight quarter lagged instruments. In addition to time, time squared and a constant, the instrument list consisted of government expenditure, money supply \( (M_1) \), capital stock, and relative factor prices, all in log form.

As shown by Barten [2], the present system is singular so that one of the share equations must be dropped for estimation purposes. However, when the non-linear three stage least squares procedure is iterated until convergence on the variance-covariance matrix as well as on the structural parameters, the results are invariant with respect to which equation is dropped. I then arbitrarily dropped the energy share equation.

Serial correlation turned out to be serious enough to require correction. It was assumed that cross-equation serial correlation is zero. As shown by Berndt and Savin [4] this implies that the serial correlation parameter \( \rho \) must be the same for all the share equations. Because of the magnitude of the problem no attempt was made to estimate the \( \rho \)'s simultaneously with the structural parameters. This is also unnecessary for obtaining a consistent estimate
of their variance-covariance matrix. The following procedure was adopted to preserve the invariance property. The system was first estimated by iterative non-linear three stage least squares without correction for serial correlation. Consistent estimates of the \( \rho \)'s were then derived from the Durbin-Watson statistics of the respective equations. Because the system was iterated until convergence, these are all invariant with respect to which equation was dropped. Furthermore, regression of the implied residuals of the energy share equation on their lagged values gave an equally invariant, and consistent, estimate of the serial correlation parameter of this equation. The common value of \( \rho \) for the share equations was then taken to be the arithmetic mean of the estimate from each equation.

The next step was to use these estimates to write the system in quasi-difference form and iterate on the transformed system until convergence. After quasi-differencing of the variables and lagging of the instruments, the sample for estimation was 49II - 75IV. The instruments were also quasi-differenced using the mean value of the \( \hat{\rho} \)'s.

The system was estimated with three different wage concepts, which are described in more detail in Appendix A:

\( \bar{w} \): Unadjusted compensation per man-hour for all persons engaged in the goods sector.

\( w' \): Compensation per man-hour in the goods sector adjusted for inter-industry shifts in employment between eight major sectors.

\( w^0 \): Compensation per man-hour in the goods sector, adjusted for overtime in manufacturing and for interindustry shifts in employment.

The question of efficiency is blurred by the fact that the system is non-linear and maximum likelihood estimation cannot be applied easily given the stochastic character of the profit equation.
In preliminary runs, the coefficient of time squared in the cost function and the time trend in the agricultural share equation were both statistically indistinguishable from zero and constrained to have this value in the final runs.

2. **Estimates of Cyclical Productivity Variation**

The estimated parameters of the cost function with their standard errors are presented in table 2. These estimates are obtained using the wage rate \( w^0 \), which is corrected for interindustry shifts in employment and for overtime in manufacturing. The two other wage concepts gave practically identical results with the exception of the cyclical productivity parameter \( h \). This was estimated as 1.0418 (s.e. .0692) with the unadjusted wage rate \( \bar{w} \), and .9724 (s.e. .0844) with the wage rate adjusted for interindustry shifts in employment \( w' \).

The first thing to observe is that this parameter is significantly positive, which demonstrates clearly that the cyclical productivity variation is a fact of life. As mentioned above, Draghi [8] argues that it is an artifact stemming from incomplete model specification. In the present model, however, great pain has been taken to specify the model completely and to use consistent estimators. The remaining problem of those pointed out by Draghi is the one of measuring capital, since both the benchmark and the depreciation rate are unknown. I strongly suspect, however, that the possible bias introduced by this measurement problem is substantially smaller than the misspecification bias when this and other variables are left out, even with instrumental variable estimation, because the instruments may well be correlated with the left-out variables.

The next point to note is the magnitude of \( h \), which is in the neighborhood of unity for all three wage concepts. The fact that the cyclical productivity
Table 2
Estimates of the parameters of the system based on the wage rate $w^0$ (corrected for interindustry shifts in employment and for overtime in manufacturing).

Estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_L$</td>
<td>.8785</td>
<td>.0013</td>
</tr>
<tr>
<td>$b_{LQ}$</td>
<td>.0790</td>
<td>.0371</td>
</tr>
<tr>
<td>$h$</td>
<td>.9503</td>
<td>.0839</td>
</tr>
<tr>
<td>$a_A$</td>
<td>.0774</td>
<td>.0013</td>
</tr>
<tr>
<td>$b_{AQ}$</td>
<td>-.0885</td>
<td>.0397</td>
</tr>
<tr>
<td>$a_t$</td>
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<td>.00006</td>
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<tr>
<td>$b_{tQ}$</td>
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<td>$a_Q$</td>
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<td>.0060</td>
</tr>
<tr>
<td>$b_{QQ}$</td>
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<td>.0952</td>
</tr>
<tr>
<td>$b_{tL}$</td>
<td>-.00029</td>
<td>.00002</td>
</tr>
</tbody>
</table>

Parameter values implied by restrictions:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_E$</td>
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<td>.0007</td>
</tr>
<tr>
<td>$b_{LL}$</td>
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<td>$b_{AA}$</td>
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<td>.0011</td>
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<td>$b_{EE}$</td>
<td>.0421</td>
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<td>.0147</td>
</tr>
<tr>
<td>$b_{tE}$</td>
<td>.00029</td>
<td>.00002</td>
</tr>
</tbody>
</table>

$b_{tA} = 0.0$  
$b_{tt} = 0.0$

<table>
<thead>
<tr>
<th>Labor share</th>
<th>Agr. share</th>
<th>Profit eq.</th>
<th>Cost fn.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$ a)</td>
<td>.9314</td>
<td>.7142</td>
<td>.9961</td>
</tr>
<tr>
<td>s.e. a)</td>
<td>.0073</td>
<td>.0143</td>
<td>.0105</td>
</tr>
<tr>
<td>$\rho$</td>
<td>.6782</td>
<td>.8237</td>
<td>.7800</td>
</tr>
<tr>
<td>DW b)</td>
<td>1.8420</td>
<td>1.7361</td>
<td>1.8832</td>
</tr>
</tbody>
</table>

a) Computed from the untransformed, i.e. serially correlated residuals.

b) Computed from transformed residuals.
variation is so large has mainly been ignored in the literature. The puzzle that has been discussed is that employment (or short-run cost) is less than unit elastic with respect to output. My estimated cost function also exhibits this property, since the estimated output elasticity of cost with the measured wage rate ($\bar{w}$, $w'$, or $w^0$) held constant is well below unity for the whole sample. For $w^0$, which gives the highest value, it varies between .62 and .70, which is rather on the low side of what has been found in the literature. However, a value greater than or equal to one, although indicating decreasing return to short-run factors, would not imply absence of cyclical variation in the productivity of labor. This is seen most easily at the point of approximation, for which

$$\frac{\partial \log C}{\partial \log Q} \bigg|_w = a_Q - h a_L, \; w = \bar{w}, \; w', \; w^0.$$ 

Since very robust estimates of $a_Q$ and $a_L$ are provided by the profit and labor share equations as 1.49 and .88, respectively, a value of unity of this elasticity would imply a value of $h$ of .56, which is well above zero.

This high value of $h$ stems of course from the identifying restriction (2.12.), which says that price will equal marginal cost in an expectational sense *ex ante*. This implies that the output elasticity of cost with the observed wage rate held constant would have to be around 1.5 rather than 1.0 in order to give zero cyclical fluctuation in the productivity of labor. It is true that an elasticity greater than or equal to unity would have satisfied the restriction from theory that the short-run cost function be decreasing in capital, since then

$$\frac{\partial \log C}{\partial \log K} \bigg|_w = 1 - \frac{\partial \log C}{\partial \log Q} \bigg|_w \leq 0.$$ 

However, to obtain a full understanding of the problem, a link is required between the short and the long run. This is provided by (2.12.), with the inevitable
result of a high value of h.

The third point to note about this parameter is how it changes when the aggregate wage rate is corrected for interindustry shifts in employment and for overtime in manufacturing. Since the three estimates really are obtained in three different models which are not submodels of each other or of any "master" model, the changes in the estimate of h cannot be tested statistically in a rigorous sense. Heuristically speaking, the changes are within two standard errors. They do, however, go in the direction predicted by the theory.

The reduction obtained by correction for overtime (about 2%) is very small. Correction for interindustry shifts in employment gives a change (about 6%) that is somewhat more significant in an economic sense, but not large.

An alternative perspective is obtained by comparing the three estimates of the elasticity

$$\frac{\partial \log C}{\partial \log Q} \bigg|_w = w, w', w^0,$$

which can be interpreted as inverse measures of the cyclical variation in the productivity of labor. These are not constant within the translog formulation but can easily be computed for each observation. The same systematic pattern is found: the elasticity increases for every adjustment that is made in the wage rate. The observed ranges are (.55, .63), (.60, .69), (.62, .70) for $w, w'$ and $w$, respectively, with sample means of .59, .65, and .67. Some more insight into the aggregation effects can be obtained by comparison of the former two of these elasticities. Since they can be identified as

$$\frac{\partial \log C}{\partial \log Q} \bigg|_w \quad \text{and} \quad \frac{\partial \log C}{\partial \log Q} \bigg|_{w_i},$$
respectively, the difference between them, .06, (on average) can be interpreted as the aggregation effect from shifts between high and low wage industries. Assume further that the aggregation effect from shifts between industries with high and low cyclical productivity variation is zero. Then, using (3.4.), (3.5.), and the numbers of section III.3.c., we have

$$0.65 = \frac{\partial \log C}{\partial \log Q}|_{W}^{|W} = \left( \sum b_i \frac{\partial \log C_i}{\partial \log Q_i} \right) \left[ 1 + \text{cov}(Q/Q, \frac{\partial \log Q_i}{\partial \log Q}) \right]$$

$$= \left( \sum b_i \frac{\partial \log C_i}{\partial \log Q_i} \right) (1.14) .$$

This gives

$$\sum b_i \frac{\partial \log C_i}{\partial \log Q_i} = 0.57 < \frac{\partial \log C}{\partial \log Q}|_{W}^{|W} = 0.59$$

In other words, the aggregation effect from shifts between high and low wage industries is more than offset by the other aggregation effect from shifts between industries with high and low average variable cost. Thus, the net result is that aggregation contributes nothing to the explanation of cyclical variation in the productivity of labor.

The discussion of labor hoarding in section III.1. suggests that the AVC curve is U-shaped. This implies that

$$\frac{\partial \log C/\partial \log Q}{\partial \log Q}|_{W}^{|W} = \frac{(\partial C/\partial Q)(Q/C)}{SRMC/AVC}$$

can take on values below as well as above unity. This elasticity is the non-stochastic part of the profit equation, i.e. its fitted value. The positive value of $b_{QQ}$, which is the slope of log(Q/K) in this equation, makes it possible for the elasticity to be less than one when output is low enough, i.e. the

12 This conclusion depends on the assumption that the crude estimations of section III.3. are consistent with the rest of the model.
curve is indeed U-shaped. Interestingly enough, however, no points are observed on the decreasing part of the curve.

It is possible that improved data could lower the estimated value of \( h \). More detailed industry data could give a better correction for interindustry shifts in employment than the eight sector breakdown used here. Adjustment for overtime has been made only for durable and nondurable manufacturing. These are probably the sectors with most overtime, but it does also occur in other sectors. The assumption made in the data that the overtime premium always is 50\% may also be inaccurate. Finally, it has not been possible to correct for shiftwork compensation.

However, the possibilities of reducing the estimate of \( h \) by improving the data seem limited. Inspection of the data for \( w^0 \) reveals extremely low cyclical fluctuations, which does not seem to be consistent with an assertion that the correction in (2.7.) can be understood mainly as cyclical variation in the wage rate. One is thus left with the conclusion that most of the estimate of .95 of the \( h \) parameter is a measure of productivity fluctuations proper, whose explanation will have to await future work.

3. The Output Elasticity of Labor Demand

Previous studies on this issue have almost invariably found the demand for labor to be less than unit elastic with respect to short-run fluctuations in output. Although theoretically possible within a non-homothetic multi-input model, this is highly implausible, and it might be expected that the correction for cyclical productivity made in this paper will change this.

This is indeed what happens. The demand for labor in efficiency units is derived from the labor cost share equation:

\[
\hat{L}^* = \hat{M}_L C/w^* ,
\]
where $\hat{M}_L$ is the fitted value of the share equation. Hence, the elasticity can be found as

$$\frac{a \log L^*}{\log Q^*} = b_{LQ}/\hat{M}_L + a \log C/a \log Q^*_w$$

$$= b_{LQ}/\hat{M}_L + \hat{M}_Q,$$

where $\hat{M}_Q$ is the fitted value of the profit equation. This elasticity is clustered around its sample mean of 1.57, which is exactly in the region that one would expect.

On the other hand, it is expected that the computed output elasticity of labor measured in natural units with the observed wage rate held constant would lie in the neighborhood of previous estimates. This is also true. The elasticity is computed as

$$\frac{a \log L/\log Q}{w_0} = (b_{LQ} - hb_{LL})/\hat{M}_L$$

$$+ a \log C/a \log Q|_{w_0},$$

which gives values around .65. This is reasonably close to the elasticity of .54 with a standard error of .05 that is obtained by regressing $\Delta \log L$ on $\Delta \log Q$ and a constant with ordinary least squares on the same data. They are both slightly on the low side of previous finding, e.g. Perry's .7 or Nordhaus' .9. The difference may be due to the difference in the definition of output.

V. Conclusions

Cyclical variation in the productivity of labor has been shown empirically to be substantial. This makes it an important factor for determination of short-run cost. It has been demonstrated that the productivity fluctuations are larger than what seems to have been indicated by previous authors.

It may be helpful to review the way this estimate has been obtained. The
identifying restriction is the result that firms' capacity is planned so as to equalize price and short-run marginal cost in an expectational sense ex ante. This implies that the mean observed price should, in a sample containing several "short runs" equal the sample mean of marginal cost. Unrestricted estimates of marginal cost are, however, consistently and substantially lower than price. It is the maintained hypothesis of this paper that this difference can be ascribed to cyclical variation in the productivity of labor. Thus, heuristically speaking, it has been identified as the productivity variation that can account for the difference between the sample means of price and unrestricted estimates of short-run variable cost.

The high magnitude of the estimate of cyclical productivity variation follows directly from this method. The concern in the literature has been with the question of whether short-run cost is more or less than unit elastic in output. Whereas an elasticity greater than or equal to unity would make the short-run cost function well-behaved in a narrow sense, there would still be a contradiction between short-run and long-run behavior if price is consistently lower than short-run marginal cost. This contradiction is resolved in this paper by the estimate of cyclical productivity.

It has been hard, however, to find a plausible explanation of this phenomenon. It is not simply an aggregation phenomenon. A little can be explained by cyclical shifts in employment between high- and low wage industries. But the effect is not large and seems actually to be outweighed by the bias in opposite direction from another aggregation effect, namely shifts between industries with high and low average variable cost. Even less appears to be explained by variations in the marginal cost of labor caused by overtime payment, and labor hoarding is not easily seen to add any insight at all. We are left with a genuine and important puzzle that can only be solved by further research.
References:


Appendix A: Data and Definitions

Definitions of the Three Sectors

The Energy Sector was defined as Coal Mining and Oil and Gas Extraction. This sector includes all production of "raw energy", except hydroelectric and nuclear power. These were included in energy demand and considered produced by the energy sector, but with zero value added.

The Agricultural Sector was defined as Farming. It was considered desirable to aggregate this with other raw materials into a materials sector. Important such materials are ferrous and non-ferrous metals and non-metal, non-fuel minerals. For these, annual value data are available for domestic production and imports. However, since a substantial portion of this is processed and semi-precessed imports, these value figures contain a disproportionately large part of price changes other than changes in raw materials proper. In particular, since these commodities are very energy intensive, deflation by the prices of crudes would result in a spurious increase in quantity during the 1973-75 energy price increase. This, in addition to the lack of quarterly data, led to the conclusion that it would be better not to include these inputs. It should also be noted that the cost of these inputs never exceeded 2% of variable cost, so that the error caused by this omission can hardly be large.

The Goods Sector was defined as the remaining part of the Nonfarm Business Sector plus the Household Sector. Its inputs are capital, labor, energy, and agricultural goods. The latter two are defined as the output of the corresponding sectors, including imports and excluding exports and deliveries to final demand.

Energy Data

Raw energy is defined as crude petroleum, natural gas, natural gas liquids, coal, and hydroelectric and nuclear power. As mentioned in the main text, refined imports was added to this for computation of energy input to the goods
sector. Quantity data for industrial consumption of most raw energy are available in abundance from the US Bureau of Mines, published in the Minerals Yearbook. Some supplementary data and updates were found in various mimeographed publications from the Bureau, in the Survey of Current Business, and in press releases from the Federal Power Commission. Some remaining updates were obtained by telephone.

For crude petroleum, imported refined petroleum products and natural gas liquids, data on industrial consumption (including imports) was available in pure quantity figures (i.e. barrels) monthly for the whole period 1947-75. This was converted to quarterly data and adjusted seasonally. For bituminous and lignite coal, similar consumption data was available annually for 1947-50 and monthly from then on. One half of "retail deliveries" was subtracted off under the assumption that it went to final demand. For 1951-75, the monthly figures were converted to quarterly and adjusted seasonally. For 1947-50 I assumed quarterly fluctuations to be proportional to production of the same goods, seasonally adjusted. For Pennsylvania anthracite, which was of some importance in the early years, there are monthly series for production, exports, and imports, which I converted to a quarterly basis. The imports series showed small numbers all the time and was discontinued in September 1963. I then assumed imports to be zero from 1963 III and computed industrial demand as production plus imports minus exports, seasonally adjusted. For natural gas, demand figures at the wellhead are available, but only annually. For 1967-75, for which reasonably reliable data are available, I let it fluctuate quarterly like production plus imports minus exports, seasonally adjusted. For 1947-66, I used linear interpolation. This was probably not too far off the mark since commercial utilization of natural gas rose during that period, and annual figures were hardly affected by cyclical fluctuations in the economy.
Demand for hydroelectric power was measured as production of the same, which was available as monthly figures of kilowatt-hours in the Business Statistics supplements of the Survey of Current Business. Again, these were converted to quarterly and corrected for seasonal variation. Nuclear power production was negligible before 1970 and assumed equal to zero. For 1970-75, annual figures were available in kilowatt-hours from press releases of the Federal Power Commission. Quarterly figures for 1974-75 were obtained from Monthly Energy Review and for 1970-73 by linear interpolation of the FPC data.

This gives quite good quantity data. To obtain good price data was much more of a problem. The main difficulties were: (i) Transportation costs, especially for coal, are substantial and give large regional variations. (ii) For both coal and gas, long-term contracts dominate the market. In addition, price controls have kept gas prices at an artificially low level. It is therefore hard, both to determine what the relevant price concept is and, if one can define it, to find proper data. (iii) Even for markets like crude oil, where there are good series for price quotations, these may differ from prices actually paid. (iv) A special problem arises for hydroelectric and nuclear power, since user price contains generation and distribution cost, which was intended to be kept outside the energy sector.

In face of these difficulties, the following compromise was made to obtain a price index for energy. First, I used the price data of the Bureau of Mines for 1972 to construct two aggregate series in 1972 dollars, one "oil aggregate" and one "coal aggregate". The "oil aggregate" consisted of crude oil, imported refined petroleum products, natural gas and natural gas liquids. No good price quotation was available for refined petroleum imports. However, since the volume of oil is changed but little by refining, and since we are interested in the "raw energy" content of these imports, I just added the figures in barrels to those of crude oil consumption. A value series for the "oil aggregate" was then
obtained by assuming that the price movements of all these products followed the crude petroleum WPI. The price index was adjusted upwards for 1973-75 to include imports, since the WPI quotation covers domestic oil only for this period.

The "coal aggregate" consisted of bituminous and lignite coal, Pennsylvania anthracite and hydroelectric and nuclear power. The two latter were valued at the average fuel cost per kwh for fossil fuel generated electricity in 1972. The rationale for this is that this can be said to represent the replacement cost of these two energy sources. It is included in the "coal aggregate" because coal is the quantitatively most important fossil fuel for electricity generation.

A value series was then obtained by multiplying the "coal aggregate" by the coal WPI. As a next step, an energy value aggregate was obtained by adding the two value series. An aggregate energy price index was constructed as a divisia index based on the two aggregates and the oil and coal WPI's; and an aggregate quantity series for energy demand was constructed residually by dividing value by price.

Data for Agricultural Goods

For inputs from the agricultural sector I started with nominal figures. For domestic agricultural inputs I used the monthly series of Farm Income (Receipts form marketing and CCC loans, excluding government payments) published by the US Department of Agriculture, converted to a quarterly basis. To this I added the value of agricultural imports, subtracted agricultural exports, adjusted the outcome for seasonal variation, and subtracted the already seasonally adjusted series of changes in farm inventories in the National Income and Product Accounts (NIPA).

This procedure deserves the following comment. Using the Farm Income series is equivalent to assuming that all farm sales, except exports, are made
to the goods sector and none to final demand, which represents a deviation from the convention of published input-output tables. I see this as an advantage because consumers actually buy their food from stores, not from farmers. The subtraction of exports means that exports of farm products are assumed to be made directly from the farm sector. The subtraction of inventory changes is made as a correction of the fact that the Farm Income series contains receipts from CCC loans with stored crops as collateral.

The data for imports and exports were taken from the publications of the US Bureau of the Census on foreign trade. Because the classifications of these publications do not exactly match those of the present model and also have been changed over time, the extraction of these data had to rely on judgement to some extent. The objective was, for exports, to sort out unprocessed agricultural goods. For imports, processed food etc. was included for the same purpose of completeness as for refined petroleum imports.

As the input price of agricultural goods I used the WPI for Farm Products. I think this is a reasonably good series and suitable because it also covers imports.

Labor Data

The basic source for labor data is the US Bureau of Labor Statistics (BLS). The two most important series are "Hours of all persons" in the private nonfarm sector \( (\text{MH}_{\text{PNF}}) \) and "Hourly compensation" for all persons in the same sector \( (\text{CPM}_{\text{PNF}}) \). These series, quarterly 1947-75, were obtained on request directly from BLS.

These figures had to be modified slightly for employment and wages in the energy sector. Labor data for the energy sector was obtained in the following way. From the NIPA tables of hours worked by employees and compensation of employees I constructed an annual series of compensation per man-hour in the
energy sector (CPM_E) as the ratio of the two. This implies an assumption that hourly compensation is the same for entrepreneurs as for employees in this sector, which probably is fairly innocuous. Man-hours for all persons engaged was available for mining as a whole only, and I assumed that, for each year, the ratio of man-hours for all persons to man-hours of employees was the same in the energy sector as in mining as a whole. This gave an annual series of man-hours of all persons engaged in the energy sector (MH_E). Quarterly figures were constructed by linear interpolation. For CPM_E, quarterly fluctuation was assumed proportional to a separate series of CPM for employees in mining, obtained directly from BLS.

For the goods sector (referred to with subscript Q), man-hours could then be constructed as

\[ MH_Q = MH_{PNF} - MH_E \]

Compensation per man-hour in the same sector is defined as

\[ CPM_Q = \left( \frac{MH_{PNF}}{MH_Q} \right)^* CPM_{PNF} - \left( \frac{MH_E}{MH_Q} \right)^* CPM_E, \]

and total labor cost in nominal terms is

\[ CPM_Q^* MH_Q. \]

Two more wage concepts were constructed, on which is adjusted for inter-industry shifts in employment, and one that is adjusted both for this and for overtime in manufacturing. This could not be done directly with the CPM series, since industry data and data excluding overtime are not available. It could, however, be done for the less comprehensive series of Average Hourly Earnings of Production or Nonsupervisory Workers on Private Nonagricultural Payrolls (AHE). This is available by industry for the total private economy in the BLS publication "Employment and Earnings 1909-72", and in the monthly issues of Employment and Earnings. The industry division used was Mining,
Contract Construction, Durable Manufacturing, Nondurable Manufacturing; Finance, Insurance, and Real Estate; Wholesale and Retail Trade; Transportation and Public Utilities; and Services. For the durable and nondurable goods manufacturing industries there are also series for AHE excluding overtime, based on the assumption that overtime pays 50% more than regular hours. These industry data permitted computation of average hourly earnings adjusted for interindustry shifts in employment (AHEAS), and adjusted for overtime in manufacturing and for interindustry shifts in employment (AHEAOS). Indirectly, then, it is possible to do the same adjustments on compensation per man-hour by defining

\[ CPMAS_Q = CPM_Q \times \frac{AHEAS}{AHE_{TP}} \]

\[ CPMAOS_Q = CPM_Q \times \frac{AHEAOS}{AHE_{TP}} \]

where the subscript TP stands for "total private".

Capital Data

The capital stock for the goods sector was based on investment in the same sector in 1972 bill. dollars. The data were taken from the NIPA tables. Gross investment in the goods sector was defined as Private Purchases of Producers' Durable Equipment; minus Tractors, Agricultural Machinery (except tractors), and Mining and Oilfield Machinery; plus Private Purchases of Structures; minus Nonresidential structures for Mining Explorations, Shafts and Wells, and minus Residential and Nonresidential Farm Structures. This series was available annually and was assumed to fluctuate quarterly like total private fixed investment. I then used the perpetual inventory method with an annual depreciation rate of .1 to construct a quarterly series of real capital after I had obtained a benchmark for 1974 I in the following way.

A preliminary benchmark was based on a 1929 benchmark from the Survey of Current business and historic investment. But this was too low, because the public sector did most of the investment during WWII and gave much of the
equipment away to the private sector after the war. I then used the following reasoning. Since we have
\[ \frac{dK}{dt} = I - \delta K, \]
we can solve for $K$ to get, in any period,
\[ K = I \frac{d \log K}{dt} + \delta. \]

The growth rate $d \log K/dt$ was first computed as the sample average resulting from the capital series with the preliminary benchmark. After having checked that the investment-output ratio was close to average in 1947 I, I recomputed $K_{47I}$ from the above formula. This gave a new average growth rate, which again gave a new value of $K_{47I}$. After a few iterations this procedure converged and gave a benchmark and an average growth rate that were mutually consistent.

Output data

The only variable left to define is output of the goods sector, denoted as $Q$. This is defined as gross output, not value added in the Q-sector. There are no data directly available for this gross output, so it had to be constructed from NIPA data and the others above. In terms of the NIPA I define the Q-sector as the business nonfarm sector (BNF) plus the household sector (H) minus the energy sector (E). GNP originating in the Q-sector can then be defined as
\[ GNP_Q = GNP_{BNF} + GNP_H - GNP_E. \]

The accounting identity for the Q-sector gives
\[ GNP_Q = Q - A - E, \]
where $A$ and $E$ are deliveries from agriculture and energy, respectively, including imports. Inverting this relation and substituting from the preceding formula
gives gross output as

\[ Q = GNP_{BNF} + GNP_{H} - GNP_{E} + A + E. \]

This definition is useful in the sense that data are published for all the RHS variables and can thus be used for construction of a data series for \( Q \).

As a practical matter, then, we measure \( pQ \) in value terms as

\[ pQ = GNP_{BNF}^N + GNP_{H}^N - GNP_{E}^N + p_A + p_E, \]

where the superscript \( N \) denotes nominal figures and \( p_A, p_E, A \) and \( E \) are as defined above. \( GNP_{BNF}^N \) and \( GNP_{H}^N \) are taken directly out the existing NIPA tables. \( GNP_{E}^N \) was not directly available. An annual series for total mining (\( M \)) is published. This was compared with value added in coal mining and crude petroleum and natural gas as published in the I-O tables for 1947, 58, 63, 67, 68, 69 and 70. Since no significant time trend was detected in the ratio of the two, \( GNP_{E}^N \) was assumed to be the average ratio of the two, times \( GNP_{M}^N \). This annual series was converted to a quarterly one by linear interpolation and proper extrapolation at the endpoints. Since \( GNP_{E}^N \) corresponds to a very small fraction of \( pQ \) I do not think this procedure introduced much error.

Gross output in real terms was computed in 1972 bill. dollars as

\[ Q = GNP_{BNF}^R + GNP_{H}^R - GNP_{E}^R + A + E. \]

The superscript \( R \) stands for "real", i.e. measured in 1972 bill. dollars, and the GNP figures are taken from the NIPA tables in the same way as above.
Appendix B: Testing and Imposing Concavity

Concavity in factor prices is an inherent property of any cost function but is not satisfied automatically by the translog specification. When it fails to be satisfied, it can be tested and imposed locally at the point of approximation by a technique developed by Lau [11], which makes the Hessian matrix in prices negative semidefinite at this point.

The elements of this matrix are, at the point of approximation,

\[ C_{i i} = \left( \frac{C}{w_i^2} \right) \left[ b_{i i} + \left( \frac{\partial \log C}{\partial \log w_i} \right) \right] \]
\[ = b_{i i} + a_i (a_i - 1), w_i = w, P_A, P_E \]

\[ C_{i j} = \left( \frac{C}{w_i w_j} \right) \left[ b_{i j} + \left( \frac{\partial \log C}{\partial \log w_i} \right) \right] \]
\[ = b_{i j} + a_i a_j, i \neq j . \]

Thus, at the point of approximation, the elements of the Hessian are functions of the constant parameters only.

The basic idea is that the Hessian matrix H can be reparametrized in terms of its Choleski factorization,

\[ H = T D T', \]

where T is lower triangular and has 1's along the diagonal, and D is a diagonal matrix. The diagonal elements of D are called the Cholesky values, and Lau proves that the number of positive, zero and negative Cholesky values will be the same as the number of positive, zero and negative eigenvalues.
Concavity can be tested for locally by estimating the system in its original form computing the Choleski values and their asymptotic covariance matrix from the solution of (B.3.) and, if the point estimates are positive, testing the hypothesis that they are zero. From homogeneity, one of the Choleski values will be identically zero. By arbitrary ordering, this was chosen to be \(d_E\). The two others are found as

\[
\begin{align*}
  d_L &= b_{LL} + a_L(a_L - 1) \\
  d_A &= b_{AA} + a_A(a_A - 1) - (b_{LA} + a_La_A)^2/(b_{LL} + a_L(a_L - 1))
\end{align*}
\]

Table B.1. shows the estimated values of \(d_L\) and \(d_A\) for each of the three systems (based on the three different wage concepts, respectively), their asymptotic standard errors, and the asymptotic \(\chi^2\) test statistic for the null hypothesis that they are both zero. As the 10% critical value of \(\chi^2\) is 4.61, it is clearly seen that there is no reason to reject this hypothesis. Concavity with zero Choleski values was then imposed locally, which amounts to the non-linear constraints

\[
\begin{align*}
  b_{LL} &= -a_L(a_L - 1) \\
  b_{LA} &= -a_La_A \\
  b_{AA} &= -a_A(a_A - 1)
\end{align*}
\]
Table B.1.
Results of tests for concavity

<table>
<thead>
<tr>
<th>Wage concept</th>
<th>$d_L$</th>
<th>$d_A$</th>
<th>$\chi^2_2$</th>
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</thead>
<tbody>
<tr>
<td>$\bar{w}$</td>
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<td>.0177</td>
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<tr>
<td>$w'$</td>
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<td>.0162</td>
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<td>(.0109)</td>
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<td>.0132</td>
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<tr>
<td></td>
<td>(.0290)</td>
<td>(.0124)</td>
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</tbody>
</table>