

ENGINEERING AND ECONOMETRIC APPROACHES TO INDUSTRIAL
ENERGY CONSERVATION AND CAPITAL FORMATION:
A RECONCILIATION

by

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TABLE OF CONTENTS

| | <u>Page</u> |
|--|-------------|
| I. Introduction | 1 |
| II. Definitions and Review of Underlying Theory | 5 |
| III. Two Input Energy-Capital Models | 16 |
| IV. An Interpretation of Energy-Capital Complementarity | 22 |
| V. Towards a Reconciliation of Seemingly Disparate Econometric Findings | 40 |
| VI. A Reconciliation Without the Separability Assumption | 46 |
| VII. Concluding Remarks | 49 |
| Appendix I | 56 |

"Finally, on the list of deterrents to capital spending, there is the significantly increased cost of building and operating new plants and equipment because of the higher price of energy of all types. The average economist may have forgotten his micro-economics, but the average businessman has not; he pays close attention to the relative cost of factors of production. And over the past three years it has become more expensive to increase capacity by adding machinery and equipment than it has by adding workers."¹

I. Introduction

The optimal mix of inputs in production processes has traditionally attracted the professional interests of industrial engineers and economists. The principal focus of attention for engineers is the design, construction and implementation of known production processes that produce outputs at minimum cost. Thus engineering science seeks to solve an optimization problem involving detailed knowledge of technological possibilities and constraints as well as the expected costs of factor inputs.² Economists are also concerned with cost minimization objectives, but assume that the engineering optimization problem has been solved. The economic theory of cost and production provides an explicit framework for characterizing the effects upon factor inputs of alternative input prices. In this fundamental sense, then, the economic theory of cost and production characterizes the results of engineering reoptimization in terms of the effects of changes in factor prices and technology on the derived demands for factor inputs.

Both engineers and economists are concerned with optimal responses to large and sustained price changes of some factor inputs. For example, in response to the recent OPEC induced energy price increases, a significant number

¹Irwin Kellner, Vice President and Economist, Manufacturers Hanover Trust, Quarterly Business Conditions Analysis, March 1977, p. 3.

²It is important to distinguish engineering design from basic scientific research, which involves physical laws, such as the Second Law of Thermodynamics. If all non-energy inputs were free, then physical laws would determine the optimal process designs for energy use. When the non-energy inputs are not free, physical laws provide the ultimate technological limits within which an optimal process design can be developed.

of engineering and econometric studies have developed estimates of energy conservation possibilities. The engineering approach has largely been to investigate the redesign or retrofiting potential of durable capital to facilitate inter-fuel substitution or improved energy efficiency (reduced energy consumption per unit of output). Economists have extended the analysis to consider the effects of increased energy prices not only upon capital and energy, but also on output and other factor inputs such as labor and non-energy intermediate materials. From the perspective of both engineering and economic studies, if one looks at energy and capital and ignores all other inputs, then generally energy and capital have a substitutable relationship. For example, a considerable number of recent technologically-based engineering studies have shown that investment in more energy efficient equipment could significantly reduce demand for energy, although in many cases the required capital outlay may be larger than that required for less energy efficient equipment.³ This technology-based engineering insight into the capital-energy relationship is supported by economic theory which states that whenever one limits attention to two inputs, they must be substitutable.

What engineers and economists think we know about the relationship between energy and capital can easily become confused when we consider simultaneously all other inputs in the production process. For example, are energy and capital substitutable as discussed above, or do energy and capital move together in a complementary fashion, both being substitutable with labor? To the best of our knowledge, this issue of energy conservation and capital formation in the con-

³It is not possible to provide an exhaustive set of literature references. In addition to references cited in the much publicized studies of the Ford Foundation Energy Policy Project [1974] and the U.S. Federal Energy Administration Project Independence Report [1974]; see U.S. Office of Emergency Preparedness [1972], U.S. National Academy of Sciences [1977], National Petroleum Council [1971, pp. 24-31], Lee Schipper and Joël Darmstadter [1977], Eric Hirst et al. [1976], [1977], A.H. Rosenfeld [1977], E.P. Gyftopoulos et al. [1974], [1977], and R.H. Socolow [1977].

text of other inputs such as labor has only been addressed by economists.⁴ Unfortunately, the available econometric evidence is apparently inconclusive. A principal finding of our earlier econometric study (E.R. Berndt and D.O. Wood [1975]) involving four inputs (capital-K, labor-L, energy-E, and non-energy intermediate materials-M) was that in U.S. manufacturing, 1947-71, E and K were complementary, while E and L were substitutable. Similar econometric findings have been reported in the four-input KLEM studies by E.R. Berndt and D.W. Jorgenson [1973] based on slightly different industrial data for the same time period, by M.G.S. Denny and C. Pinto [1976] based on 1949-70 time series for Canadian manufacturing, and by M. Fuss [1977] using pooled cross-section time-series data of Canadian manufacturing by region, 1961-71. In contrast, J.M. Griffen and P.R. Gregory [1976] and Robert S. Pindyck [1977] have examined three input KLE international pooled cross-section and time series data for industry which yield E-K substitutability. Thus the econometric evidence is apparently inconclusive.

The above remarks suggest that it would be desirable to develop an analytical framework which embodies the engineering notion of E-K substitutability and the economic analysis that allows for other inputs, and which also reconciles the seemingly disparate econometric evidence. That is the purpose of this paper. In Section II we present a summary of the underlying economic theory of cost and production, functional separability, derived factor demand functions, and various measures of the elasticity of substitution between inputs. Readers familiar with this literature may wish to move directly on to Section III, where we focus attention on a weakly separable two input energy-capital model, and introduce the notion of "utilized capital" -- a composite of energy and capital. In Section IV we imbed this utilized capital subfunction into a four input KLEM model and

⁴Engineering design studies for investments in, for example, completely new plants will continue to focus upon the optimal combination of all inputs. Unfortunately, to our knowledge the results of such studies have not been presented in terms permitting evaluation of inter-factor substitution elasticities and possibilities for energy conservation.

demonstrate that although the relationship between energy and capital is one of apparent substitutability, in fact it is one of energy-capital complementarity (in the sense of Hicks-Allen). In Section V we derive implications of our analytical framework for a reconciliation of the seemingly disparate econometric findings noted above. In Section VI we present a model which does not rely on the utilized capital separability specification, yet still is able to reconcile the engineering notion of energy-capital substitutability with the Hicks-Allen concept of energy-capital complementarity. Finally, in Section VII we provide concluding remarks and offer suggestions for further research.

II. Definitions and Review of the Underlying Theory

Reconciling engineering and econometric approaches to evaluating the potential for energy conservation requires first that we compare and reconcile the underlying theory and analytical concepts, and second that we compare and contrast estimates which are based upon common measurement and theoretical concepts. As will become apparent, seeming differences in the results of these two approaches, as well as some variations within the econometric studies, are due to differences in what is being measured. Both approaches begin with the concept of a production function, a technical concept characterizing the possibilities for combining input factors to produce a given level of output¹. Likewise, both approaches have the same objective, to produce a given level of output at minimum total cost, subject to the technical constraints of the production function and such institutional restrictions that the producer must satisfy. While the underlying concept and the objective are the same, the methods of analysis are very different.

We begin with a discussion of the production function and the closely related concepts of the cost function, functional separability, derived factor demand functions, and various measures of the elasticity of substitution between factor inputs. In the course of this discussion we will show how similar are the engineering and econometric approaches, and that well defined measures of factor substitution possibilities exist which can be compared unambiguously.

We begin by considering a positive, twice differentiable, strictly quasi-concave production function with a finite number of inputs,

$$(1) \quad Y = F(x) = F(x_1, x_2, \dots, x_n), \quad x_i > 0,$$

relating the maximum possible output, Y , obtainable from any given set of inputs.

The set of n inputs is denoted $N = [1, \dots, n]$, and $\partial F / \partial x_i = F_i$, $\partial^2 F / \partial x_i \partial x_j = F_{ij}$.

¹A more stylized caricature of engineering and economic notions of production functions, as well as a related bibliography, is found in Thomas G. Cowing (1974).

In the case of engineering design studies, all the inputs are considered since the objective is to develop and ultimately implement a detailed plan for the production process. Subsequent efforts to improve upon this plan will focus upon any input or subset of inputs which appear to provide opportunities for substantial cost reduction. In contrast, engineering energy conservation studies have tended to concentrate on the efficiency of energy use in the production process. Hence the focus is on a small subset of the inputs to the production process. For example, as will be discussed in greater detail later, a number of engineering studies have shown that energy savings of a particular fuel input, say x_i , are possible if certain new types of equipment inputs, say x_j , are employed. Such detailed engineering process analysis studies typically either ignore all other inputs x_k , $k \neq i, j$, in the production function (1) or else implicitly assume that these other input quantities all remain constant. Econometric studies, on the other hand, often aggregate the myriad of inputs into a much smaller number of composite inputs. Both the engineering process analysis and the econometric approaches frequently rely on the notion of functional separability, which we now define.

Let us partition the set of n inputs, $N = [1, \dots, n]$ into r mutually exclusive and exhaustive subsets $[N_1, N_2, \dots, N_r]$, a partition we shall call R .¹ The production function $F(x)$ is said to be weakly separable with respect to the partition R if the marginal rate of substitution between any two inputs x_i and x_j from any subset N_s , $s=1, \dots, r$, is independent of the quantities of inputs outside of N_s ,

i.e.

$$(2) \quad \frac{\partial (F_i/F_j)}{\partial x_k} = 0, \text{ for all } i, j \in N_s, \text{ and } k \notin N_s.$$

¹The separability discussion is presented more fully in E.R. Berndt and L.R. Christensen [1973].

The production function $F(x)$ is said to be strongly separable with respect to the partition R if the marginal rate of substitution between any two inputs from subsets N_s and N_t does not depend on the quantities of inputs outside of N_s and N_t , i.e.

$$(3) \quad \frac{\partial(F_i/F_j)}{\partial x_k} = 0, \text{ for all } i \in N_s, j \in N_t, k \notin N_s \cup N_t.$$

These separability conditions can alternatively be written as

$$(4) \quad F_j F_{ik} - F_i F_{jk} = 0$$

where subscripts follow the pattern noted in (2) and (3).

Weak separability, as employed in engineering and econometric studies of the relationships among inputs factors, has several important implications. First, weak separability with respect to the partition R is necessary and sufficient for the production function $F(x)$ to be of the form $F(X_1, X_2, \dots, X_r)$ where X_s is a positive, strictly quasi-concave, homothetic function of only the elements in N_s , i.e.

$$(5) \quad X_s = f_s(x_i), \quad i \in N_s, \quad s=1, \dots, r$$

When f_s is linear homogeneous in x_i , X_s is called a consistent aggregate index of the inputs in N_s . Thus a consistent aggregate index of a subset of inputs exists if and only if the subset of inputs is weakly separable from all other inputs. Engineering process analysis studies which focus only on a small subset of the inputs and ignore all other inputs are appropriate if and only if the subset of inputs are weakly separable from all others. Similarly, econometric studies which utilize input aggregates such as labor (or energy) are valid if and only if the components of labor (energy) are weakly separable from all other non-labor (non-energy) inputs.¹

¹We rule out the unlikely possibility that prices of all other inputs and output are perfectly correlated.

A second closely related implication of weak separability is that it permits sequential optimization.¹ More specifically, if $F(x)$ is weakly separable, then in production decisions relative factor intensities can be optimized within each separable subset, and finally overall optimal intensities can be attained by holding fixed the within-subset intensities and optimizing the between-set intensities; the corresponding factor intensities will be the same as if the entire production optimization decision had been made at once. For example, if electricity and refrigerators are weakly separable from all other inputs, then both engineers and economists do not have to worry about other inputs such as labor or natural gas, but can simply choose refrigerators of the optimal electricity-efficient design, knowing that their optimal energy/refrigerator ratio is independent of other input optimizations.

The most important implication of strong separability is that with respect to the partition R it is a necessary and sufficient condition for the production function to be of the form $F(x) = F(X_1 + X_2 + \dots + X_r)$, where X_s , $s = 1, \dots, r$, is a function of the elements of N_s only. Notice that strong separability implies weak separability but in general the reverse is not true:

Both economists and engineers analyze production processes with the objective of minimizing the cost of production. In particular, it is very useful to specify the optimization problem as that of minimizing the costs of producing a given level of output, subject of course to the exogenous input prices p_1, p_2, \dots, p_n and the technological constraints embodied in $F(x)$. W. Erwin Diewert [1974] has shown that when $F(x)$ is strictly quasi-concave and twice differentiable, the cost minimization assumption implies the existence of a dual cost function

$$(6) \quad C = G(Y, p_1, p_2, \dots, p_n)$$

relating the minimum possible total cost of producing output Y to the positive

¹For further discussion, see Charles Blackorby, Daniel Primont, and R. Robert Russell [1975].

input prices p_1, p_2, \dots, p_n and the output level Y . Notice in particular that the cost function (6) reflects the technological parameters in (1). Also, when the production function is weakly separable with respect to the partition R and when each of the f_s functions is linearly homogenous, the corresponding dual cost function has the same partition in input prices; i.e., the dual cost function is

$$(7) \quad C = G(Y, P_1, P_2, \dots, P_r)$$

where the input price aggregator functions P_s are positive, strictly quasi-concave linearly homogenous functions of only the elements in N_s , i.e.,

$$(8) \quad P_s = g_s(p_i), \quad i \in N_s, \quad s=1, \dots, r.$$

The usefulness of the cost function in demand theory is that the cost minimizing derived demand equations are directly obtainable from the derivatives of G . As before, $\partial G / \partial p_i = G_i$, $\partial^2 G / \partial p_i \partial p_j = G_{ij}$. By Shephard's Lemma,¹ the optimal derived demands are

$$(9) \quad x_i = G_i, \quad i=1, \dots, n.$$

A question of considerable interest to economists is the sensitivity of the optimal derived demand for x_i to a change in the price of the same input ((the "own price response") or to a change in the price of another input j , $i \neq j$ ((the "cross price response")). Such sensitivity measures will vary of course depending on what other variables are held fixed. The most common elasticity measure, due to J.R. Hicks [1933] and R.G.D. Allen [1938], is the demand price

¹For further discussion, see W.E. Diewert [1974].

elasticity defined as

$$(10) \quad \epsilon_{ij} = \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i} = \frac{\partial \ln x_i}{\partial \ln p_j}$$

where output Y is held constant, only p_j changes, but all factors are allowed to adjust to their optimal levels. Hicks-Allen defined inputs x_i and x_j as substitutes, independent, or complements according as ϵ_{ij} was positive, zero, or negative, respectively. Furthermore, when there are only two inputs in the production function (say x_1 and x_2), the strict quasi-concavity condition on the underlying production function F will ensure that $\epsilon_{12} > 0$, i.e. the two inputs must be substitutable.

In general, $\epsilon_{ij} \neq \epsilon_{ji}$. The Allen partial elasticities of substitution σ_{ij} are essentially normalized price elasticities,

$$(11) \quad \sigma_{ij} = \epsilon_{ij} / M_j$$

where M_j is the cost share of the j^{th} input in total costs, i.e.

$$(12) \quad M_j = p_j x_j / \sum_{i=1}^n p_i x_i.$$

The effect of this normalization is to make the Allen partial elasticities of substitution symmetric, i.e. $\sigma_{ij} = \sigma_{ji}$, even though $\epsilon_{ij} \neq \epsilon_{ji}$. H. Uzawa [1962] has shown that the σ_{ij} can be derived conveniently from the cost function (6) as

$$(13) \quad \sigma_{ij} = \frac{G_i G_j}{G_i G_j}, \quad i, j = 1, \dots, n.$$

Two less familiar but nonetheless interesting alternative substitution elasticity measures are the direct and shadow elasticities of substitution.

Daniel McFadden [1963] and Ryuzo Sato and Tetsunori Koizumi [1976], among others, have shown that the direct elasticity of substitution d_{ij} , defined as

$$(14) \quad d_{ij} = \frac{\frac{1}{F_i x_i} + \frac{1}{F_j x_j}}{-\frac{F_{ii}}{F_i^2} + \frac{2 F_{ij}}{F_i F_j} - \frac{F_{jj}}{F_j^2}}, \quad i \neq j,$$

can be rewritten, making use of the cost minimization assumption, as

$$(15) \quad d_{ij} = \frac{\partial \ln (x_i/x_j)}{\partial \ln (p_j/p_i)}.$$

Hence the direct elasticity measures the percentage change in x_i/x_j given an exogenous percentage change in p_j/p_i . We would expect that in general d_{ij} would differ from σ_{ij} or ϵ_{ij} , since with the d_{ij} only the two inputs x_i and x_j move to their optimal levels, while with the σ_{ij} or ϵ_{ij} all factor quantities -- not just x_i and x_j -- are allowed to adjust to their cost-minimizing levels, all with output fixed. Two implications of this are worth noting. First, the strict quasi-concavity "curvature" restrictions on $F(x)$ require that all $d_{ij} > 0$, $i \neq j$, even though certain ϵ_{ij} or σ_{ij} may be negative. Hence we can simultaneously have negative Allen partial elasticities of substitution indicating that inputs i and j are complementary and positive direct elasticities of substitution reflecting two-space substitutability. Second, when there are only two inputs, x_1 and x_2 , $\sigma_{12} = d_{12}$.

The final elasticity of substitution measure we consider is the shadow elasticity of substitution S_{ij} , defined as

$$(16) \quad S_{ij} = \frac{-\frac{G_{ii}}{G_i^2} + \frac{2 G_{ij}}{G_i G_j} - \frac{G_{jj}}{G_j^2}}{\frac{1}{p_i G_i} + \frac{1}{p_j G_j}}, \quad i \neq j,$$

where imputed or shadow prices of the remaining factors and imputed total cost are fixed. Equation (16) can be rewritten as

$$(17) \quad S_{ij} = \frac{\partial \ln (p_i/p_j)}{\partial \ln (x_j/x_i)}$$

The S_{ij} therefore measure the percentage change in the ratio of the input prices p_i/p_j in response to an exogenous percentage change in the quantity ratio x_j/x_i , where prices of the remaining factors x_k ($k \neq i, j$) and total shadow cost are held fixed. Again, the curvature conditions on $F(x)$ imply that $S_{ij} > 0$, even though ϵ_{ij} and σ_{ij} may be negative. Also when there are only two factors of production x_1 and x_2 , $d_{12} = \sigma_{12} = S_{12}$. With more than two production factors, however, in general d_{ij} , S_{ij} , and σ_{ij} will differ.¹ Thus considerable care must be taken in discussing "substitution elasticities," for unless the context makes it clear what variables are being held constant, needless confusion can easily result.

In practice both process engineers and econometricians interested in measuring energy conservation potential may choose to analyze only a subset of the inputs or may be unable to obtain sufficiently detailed or reliable data on all n inputs in (1). For this and other reasons researchers may choose to focus attention on estimating substitution elasticities among only a subset of the n inputs. As an

¹ Relationships among these three alternative elasticity of substitution concepts in the multiple input case have been analyzed by R. Sato and T. Koizumi [1973a].

example, suppose a researcher only has data on the inputs belonging to the N_s subset in the partition R , and that he wishes to obtain estimates of the price elasticities ϵ_{ij} among the q inputs within the N_s subset. Although the marginal rate of substitution F_i/F_j , $(i,j \in N_s)$ is independent of all $x_k (k \notin N_s)$ when inputs in N_s are weakly separable from all other inputs, we now demonstrate that unless additional information is available the researcher cannot obtain estimates of the Hicks-Allen price elasticities ϵ_{ij} $(i,j \in N_s)$ based only on data for inputs belonging to the weakly separable N_s subset.

Let us define the gross price elasticity ϵ_{ij}^* $(i,j \in N_s)$ as the derivative $\partial \ln x_i / \partial \ln p_j$ where X_s , the aggregate input or "output" of the weakly separable f_s subfunction (5) is held constant, all $x_k (k \in N_s)$ are allowed to adjust to their optimal levels, but all $x_m (m \notin N_s)$ are held fixed. Let us also define the net price elasticity ϵ_{ij} as $\epsilon_{ij} \equiv \frac{d \ln x_i}{d \ln p_j}$, $i,j \in N_s$, where now the output Y is held constant at $Y = \bar{Y}$ and all other inputs -- not just those in N_s -- are allowed to move to their new cost minimizing positions. Notice that the gross price elasticity ϵ_{ij}^* treats X_s as fixed (say, $X_s = \bar{X}_s$), whereas the net price elasticity allows X_s to respond to the change in p_j . More specifically,

$$(18) \quad \frac{d \ln x_i}{d \ln p_j} \Big|_{Y = \bar{Y}} = \frac{\partial \ln x_i}{\partial \ln p_j} \Big|_{X_s = \bar{X}_s}$$

$$+ \frac{\partial \ln x_i}{\partial \ln X_s} \cdot \frac{\partial \ln X_s}{\partial \ln P_s} \cdot \frac{d \ln P_s}{d \ln p_j} \Big|_{Y = \bar{Y}}, \quad i, j \in N_s.$$

The left-hand side of (18) is the net price elasticity and the first term on the right-hand side is the gross price elasticity; we shall call the last expression on the right side of (18) the scale elasticity. The three terms comprising the scale elasticity may be interpreted as follows. First, because f_s is linearly

homogeneous, $\partial \ln x_i / \partial \ln X_s = 1$. Next, the derivative $\partial \ln X_s / \partial \ln P_s$ is the price elasticity for the aggregate X_s given a change in the price of the aggregate input P_s , where $Y = \bar{Y}$. Let us denote this own price elasticity as ϵ_{ss} . Finally, since $P_s = \sum_i p_i x_i / X_s$, $i \in N_s$, and using Shephard's Lemma $\frac{\partial P_s}{\partial p_i} = \frac{x_i}{X_s}$, we have

$$(19) \quad \frac{\partial \ln P_s}{\partial \ln p_j} = \frac{p_j}{P_s} \frac{\partial P_s}{\partial p_j} = \frac{p_j x_j}{P_s X_s}$$

Equation (19) indicates that $\partial \ln P_s / \partial \ln p_j$ is simply the cost share of the j^{th} input in the total cost of producing the input aggregate X_s ; hereafter we denote this share as

$$(20) \quad N_{sj} = \frac{p_j x_j}{\sum_i p_i x_i}, \quad i, j \in N_s.$$

Combining these expressions and substituting into (18), we have

$$(21) \quad \epsilon_{ij} = \epsilon_{ij}^* + \epsilon_{ss} N_{sj}.$$

Thus the net effect on the derived demand for x_i given a change in the price of x_j ($i, j \in N_s$) is the sum of a gross effect which holds the input aggregate X_s fixed plus the cost share of the j^{th} input in the total cost of input aggregate X_s times the price elasticity of demand for X_s .

The implication of this for the researcher dealing only with data on inputs in the weakly separable subset N_s is that in general he cannot estimate ϵ_{ij} , given data for the variables comprising the subset N_s ; all that can be done is to estimate the gross price elasticity ϵ_{ij}^* and of course the cost share N_{sj} .

In order to estimate the net price elasticity ϵ_{ij} , additional information on ϵ_{ss} is necessary. Since the curvature conditions on $F(x)$ require that $\epsilon_{ss} < 0$ and since the cost share $N_{sj} > 0$, it follows that $N_{sj} \epsilon_{ss} < 0$ and therefore that $\epsilon_{ij} < \epsilon_{ij}^*$. Notice that if inputs x_i and x_j are gross complements then they also are net complements. However, gross substitutability ($\epsilon_{ij}^* > 0$) does not necessarily imply net substitutability ($\epsilon_{ij} > 0$), for if the absolute value of the the scale elasticity $N_{sj} \epsilon_{ss}$ is greater than the positive gross substitution elasticity, inputs i and j will be gross substitutes but net complements.

The implications of this discussion for reconciling engineering and econometric studies of energy conservation should now be clear. Both engineering and econometric approaches employ the concepts of a production function, weak separability of inputs, and assumptions as to the optimizing behavior of producers. Engineering studies tend to focus upon the changes in capital design, and the manner in which capital and energy are jointly utilized. Their emphasis is on the subset of inputs including capital and energy inputs, with the assumption of weak separability between these inputs and the other factors of production. Thus, engineering studies tend to provide information to evaluate gross elasticities of substitution between the inputs comprising a subset of factor inputs. Econometric studies, on the other hand, have tended to focus on the relations between all inputs, employing the assumption of weak separability to reduce the number of subsets to a manageable number. When all inputs are included, the econometric studies can provide information on the gross elasticities of substitution between factors within a subset, as well as scale elasticities between subsets and consequently the net elasticities between any pair of inputs. Thus it is apparent that both approaches to evaluating the potential for energy conservation use the same analytical principles so that summary measures, such as the various elasticity concepts, may be related and compared.

III Two Input Energy-Capital Models

The recent engineering literature contains numerous blueprint examples of how equipment and appliances could be redesigned or retrofitted to consume less energy, but at the cost of a larger initial capital outlay. For example, E.P. Gyftopoulos et al. [1974] have compared actual fuel use in industry with the theoretically most efficient use based on the Second Law of Thermodynamics; in E.P. Gyftopoulos and T.F. Widner [1977], percentage changes in fuel efficiency are compared with percentage changes in initial capital cost outlay. Other studies (see, for example, U.S. Environmental Protection Agency [1973]), have attempted to rank alternative capital-energy use combinations using "life cycle" costing. On the basis of such two-input studies, some economists have been led to conclude that energy (E) and capital (K) are substitutes. For example, J.M. Griffen and P.R. Gregory [1976] state that ". . .in the long run, one might expect K and E to be substitutes since new equipment could be redesigned to achieve higher thermal efficiencies but at greater capital costs."¹

Engineering examples of the reduced energy consumption-higher initial cost tradeoff are numerous.² To our surprise, very little econometric work on this issue has been published. Makoto Ohta (1975) reports that the initial capital costs of boilers and turbogenerators purchased by U.S. steam generating electric utilities have varied positively with energy efficiency (essentially, the ratio of electrical output to fuel input per time period). Using U.S. data for twenty-two 1976 models of automatic frost-free refrigerators with freezers,³ we have fitted the following least squares semi-logarithmic regression of the log of the list price in dollars (LNPRICE) against refrigerator volume in cubic feet (REFVOL), freezer volume in cubic feet (FZVOL), and average monthly electricity consumption in kilowatt hours (KWH):

¹J.M. Griffen and P.R. Gregory (1976), p. 856.

²In addition to the Gyftopoulos references cited above, see Eric Hirst et al. (1977), Eric Hirst and Janet Carney (1977), Joel Darmstadter et al. (1977), and R.H. Socolow (1977).

³The data is drawn from A.H. Rosenfeld (1977), and is reproduced in Appendix A of this paper.

$$\text{LNPRICE} = 5.577 + .043 \text{ REFVOL} + .092 \text{ FZVOL} - .003 \text{ KWH}$$

$$(10.446) \quad (1.222) \quad (1.372) \quad (-2.863)$$

$$R^2 = .406 \quad \text{t-values are in parentheses}$$

Although this equation is admittedly naive as an hedonic price specification,¹ the significant negative coefficient on the KWH variable illustrates the tradeoff between lower electricity consumption and higher initial capital cost. Based on these coefficient estimates, the list price of a 16.5 cubic feet refrigerator (say, 12 cubic feet of REFVOL and 4.5 cubic feet of FZVOL) is predicted to increase from about \$425 to about \$460 as average monthly electricity required decreases from 150 to 125 KWH.²

A final example is that of Jerry A. Hausman (1978). Using U.S. trade journal data for 65 models of 1976 room air conditioners, Hausman obtains the regression equation

$$\text{LNPRICE} = \text{constant} + .0598 \text{ Btu} + .09765 \text{ EER}$$

$$(11.3355) \quad (4.96314)$$

where Btu is cooling capacity in thousands of British thermal units and EER is the energy efficiency ratio (computed as cooling capacity in Btu per hour over rated power consumption in KWH). The statistically significant coefficient on the EER variable indicates that, holding cooling capacity fixed, initial capital cost outlay for air conditioners and energy efficiency (consumption) are positively (negatively) correlated.

These examples do not necessarily imply, however, a tradeoff between the flow of capital services and the quantity of energy. Initial capital cost must first be decomposed into a quantity service flow and a rental price per unit of time. In order to construct a capital quantity flow, additional informa-

¹See M. Ohta and Z. Griliches (1976) for a discussion of issues involved in the construction of hedonic price indices. Our choice of a simple semilogarithmic equation specification is based on their procedure. For an hedonic study of refrigerators not accounting directly (but implicitly allowing) for energy consumption, see J.E. Triplett and R.J. McDonald (1975).

²The reduction in KWH requirements, given FZVOL and REFVOL, is possible through increased insulation, improved compression efficiency, use of an anti-sweat heater switch, etc. See the various Hirst references and A.H. Rosenfeld (1977) for further discussion.

tion would be needed on, for example, durability and physical deterioration over time; similarly, capital rental price measurement would require information on economic depreciation, the opportunity cost of capital, and in the industrial sector, effective rates of corporate taxation. Nonetheless, the proposition that there is a tradeoff between capital quantity flow and energy consumption per unit of time, other inputs not considered, seems reasonable to us. In the next section of this paper we will present some estimates of this engineering energy-capital substitutability. Our first task, however, is to develop a more precise definition of the engineering energy-capital substitutability notion.

It is obvious that a great deal of industrial equipment relies for power on some type of energy. Therefore let us hypothesize a two-input weakly separable function which combines the inputs of aggregate capital and aggregate energy and produces an output called "utilized capital."¹ In the context of refrigerators (or air conditioners) utilized capital services could be the number of hours in which a specified amount of space is cooled to a certain temperature; such utilized capital services would be the output of a production process with two inputs -- a refrigerator (or air conditioner) and KWH of electricity.

More formally, assume that gross output Y is produced by a cost-minimizing competitive firm according to the positive, twice differentiable, strictly quasi-concave production function

$$(22) \quad Y = F(K, L, E, M),$$

where K is an input aggregate of capital services, L of labor services, E of energy, and M of non-energy intermediate materials. Notice that (22) is a highly aggregated form of the production function (1). Within the production function (22), we assume there exists a weakly separable linearly homogeneous utilized capital subfunction with only two inputs, aggregate capital and aggregate energy,

$$(23) \quad K^* = f(K, E)$$

¹For other two-input E-K discussions, see Paul W. Zarembka (1974) and Thomas G. Cowing (1974). Some economists have used electricity or other energy consumption indexes as a "proxy" measure of actual capital services; e.g., see Carlisle E. Moody, Jr. (1974) and the references cited therein.

which together produce the utilized capital output K^* . With output quantity and input prices exogenous, the regularity conditions on (22) and (23) imply the existence of a gross output dual cost function relating total cost $C = P_K K + P_L L + P_E E + P_M M$ to gross output Y and to the input prices $P_K, P_L, P_E,$ and $P_M,$

$$(24) \quad C = G(Y, P_K, P_L, P_E, P_M);$$

the existence of the separable subfunction (23) in the production function (22) implies the existence of a dual separable utilized capital cost subfunction in the same input partition,

$$(25) \quad C_{K^*} = g(K^*, P_K, P_E)$$

where $C_{K^*} = P_K K + P_E E$. It might be noted here that the assumption of a linearly homogeneous weakly separable utilized capital subfunction implies that the optimal E/K ratios within the utilized capital subfunction depend solely on $P_K, P_E,$ and not on the other input prices P_L, P_M or the level of gross output Y . Hence, under the above separable utilized capital specification, engineers interested in energy conservation issues are able to choose K/E ratios so as to minimize the unit cost C_{K^*}/K^* of producing utilized capital services, without having to consider prices of other inputs such as P_L or P_M . In turn, the firm will then determine the cost-minimizing total amount of utilized capital K^* it demands -- a decision which will depend of course on the price of utilized capital services (P_{K^*}), and on $P_L, P_M,$ and Y .

Based on the utilized capital cost subfunction (25), we can define gross price elasticities as follows:

$$(26) \quad \epsilon_{KK}^* = \frac{\partial \ln K}{\partial \ln P_K}, \quad \epsilon_{EE}^* = \frac{\partial \ln E}{\partial \ln P_E}$$

$$(27) \quad \epsilon_{KE}^* = \frac{\partial \ln K}{\partial \ln P_E}, \quad \epsilon_{EK}^* = \frac{\partial \ln E}{\partial \ln P_K}$$

where utilized capital output K^* is fixed. These gross price elasticities must be interpreted carefully. For example, ϵ_{KE}^* measures the percentage change in the quantity of capital demanded in response to a percentage change in the price of energy, assuming P_K and utilized capital output K^* is fixed. Hence these gross price elasticities do not allow for the scale effect mentioned in the previous section, wherein the energy price change affects the unit cost of producing utilized capital services, which in turn brings about a change in the amount of K^* demanded and thus results in a change in the derived demand for K .

Since the utilized capital subfunction has only two inputs, the regularity conditions on (25) require E and K to be substitutes; hence ϵ_{KE}^* and ϵ_{EK}^* must be positive. Equivalently, in this two-input model E and K must be substitutable along a strictly convex utilized capital isoquant. We construe much of the recent literature dealing with possibilities for energy conservation as focussing on the real world possibilities for movement along (or a shift in) the utilized capital isoquant. As noted earlier, either through equipment retrofitting or through new equipment design, the engineering-economic energy conservation literature illustrates the fact that a given amount of utilized capital services can be produced with less E but more K .

We are aware of no econometric research which provides estimates of the gross E-K price elasticities in (26) and (27).¹ One point, however, is very

¹A related measure has been estimated by Cowing (1974), Table III, P. 149.

clear: engineering or econometric research on (26) and (27) could not report E-K complementarity, for curvature restrictions on (25) rule this possibility out. In order to be able to discover E-K complementarity, it is necessary to specify a production function with more than two inputs. We now turn to a discussion of complementarity in the four input KLEM model.

IV. An Interpretation of Energy-Capital Complementarity

Although economics students typically learn about complementary inputs or commodities in their principles courses, the underlying intuitive basis for complementarity remains surprisingly elusive. Paul A. Samuelson [1974] illustrates the possible confusion with a classic coffee, tea, and lemon example:

"... we 'know' that coffee and tea are 'substitutes' because we can drink one or the other; in the same way, we know that tea and lemon are 'complements', because tea with lemon makes up our desired brew. And probably we feel that tea and salt are somewhere between being substitutes and being complements: relatively speaking, tea and salt are in the nature of 'independents.'

"Beyond these simple classifications the plain man may hesitate to go. Thus, sometimes I like tea and lemon; sometimes I like tea and cream. What would you say is the relation between lemon and cream for me? Probably substitutes. I also sometimes take cream with my coffee. Before you agree that cream is therefore a complement to both tea and coffee, I should mention that I take much less cream in my cup of coffee than I do in my cup of tea. Therefore a reduction in the price of coffee may reduce my demand for cream, which is an odd thing to happen between so-called complements; at least this is in contrast to the case of the tea-and-lemon complements where we should expect a reduction in the price of either to increase the demand for both (as I am induced to consume more cups of lemoned tea).

"Things are not so plain sailing after all. We . . . are not so sure what it is that we know."¹

It is fitting, therefore, that we attempt to develop a more precise intuition for E-K complementarity consistent with the Hicks-Allen demand framework, yet also simultaneously consistent with the engineering-economic notion of gross E-K substitutability.

¹P. A. Samuelson [1974], p. 1255.

Recall that we have specified a weakly separable linearly homogeneous utilized capital production sub-function $K^* = f(K, E)$ imbedded within a "master" production function $Y = F(K, L, E, M) = F(K^*, L, M)$ and the corresponding dual unit cost subfunction for utilized capital $C_{K^*}/K^* = g(P_K, P_E)$ nested within a master unit cost function $C/Y = G(Y, P_K, P_L, P_E, P_M)$. In (26) and (27) of the previous section, we also defined the gross price elasticities between E and K.

The net price elasticities for the general case have been derived and defined in (20) and (21) of Section II. In the present context, the net price elasticities for E and K are

$$(28) \quad \epsilon_{KE} = \epsilon_{KE}^* + \epsilon_{uu} N_E, \quad \epsilon_{EK} = \epsilon_{EK}^* + \epsilon_{uu} N_K$$

$$(29) \quad \epsilon_{KK} = \epsilon_{KK}^* + \epsilon_{uu} N_K, \quad \epsilon_{EE} = \epsilon_{EE}^* + \epsilon_{uu} N_E$$

where ϵ_{uu} is the price elasticity of demand for utilized capital services and N_K and N_E are the cost shares of K and E in the total cost of producing output K^* . Equation (28) indicates, for example, that the net price elasticity ϵ_{KE} along a gross output isoquant (where $Y = \bar{Y}$) is equal to the positive gross substitution elasticity ϵ_{KE}^* along a utilized capital isoquant (where $K^* = \bar{K}^*$) plus another term which reflects the cost share of energy in the K^* subfunction (N_E) times ϵ_{uu} , the price elasticity of demand for K^* ,

$$(30) \quad \epsilon_{uu} = \frac{\partial \ln K^*}{\partial \ln P_{K^*}}, \quad \text{gross output held constant.}$$

Notice in particular that even though the gross substitution term ϵ_{KE}^* is positive, the sign of the net elasticity ϵ_{KE} is indeterminate a priori. If the negative scale effect $\epsilon_{uu} N_E$ is larger in absolute value than the positive gross price elasticity ϵ_{KE}^* , then energy and capital will be gross substitutes but net complements.

At this point it might be useful to demonstrate this gross substitute-net complement phenomenon geometrically. For simplicity, we first specify another weakly separable linearly homogeneous production subfunction with two inputs,

$$(31) \quad L^* = h(L, M)$$

where L^* is the output of the labor-materials production subfunction. Hence the "master" production function can be written as

$$(32) \quad Y = F(K, L, E, M) = F(K^*, L^*),$$

and the dual master cost function as

$$(33) \quad C = G(Y, P_K, P_L, P_E, P_M) = G(Y, P_K^*, P_L^*)$$

where

$$(34) \quad C_{L^*}/L^* = h^*(P_L, P_M), \quad C_{L^*} = P_L L + P_M M,$$

and

$$(35) \quad P_L^* = C_{L^*}/L^* , \quad P_{K^*} = C_{K^*}/K^* .$$

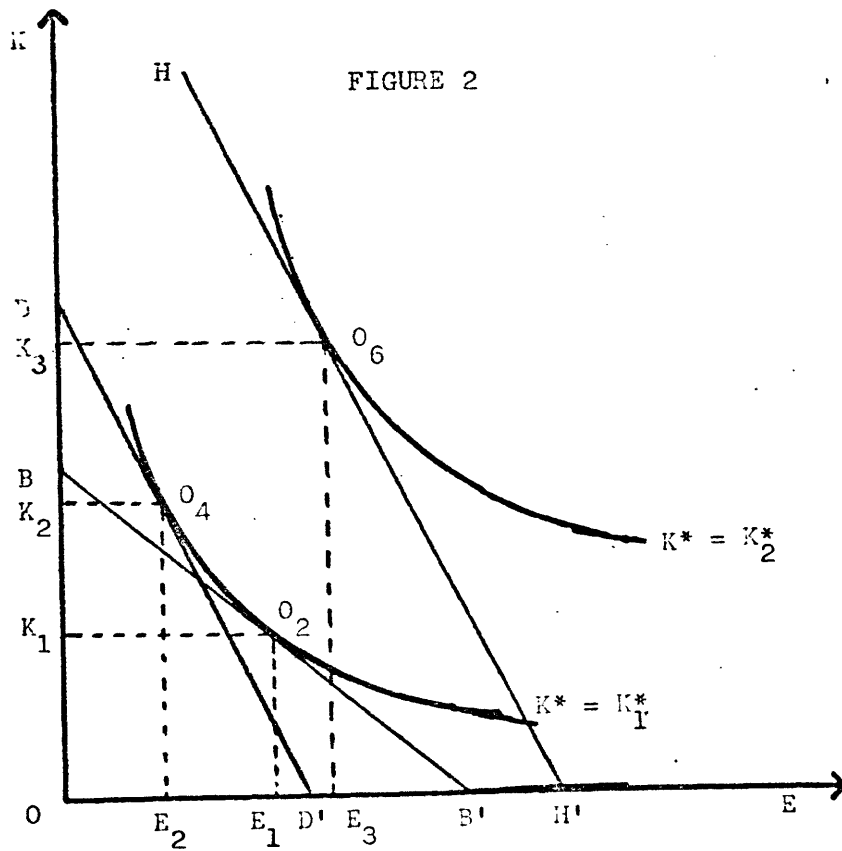
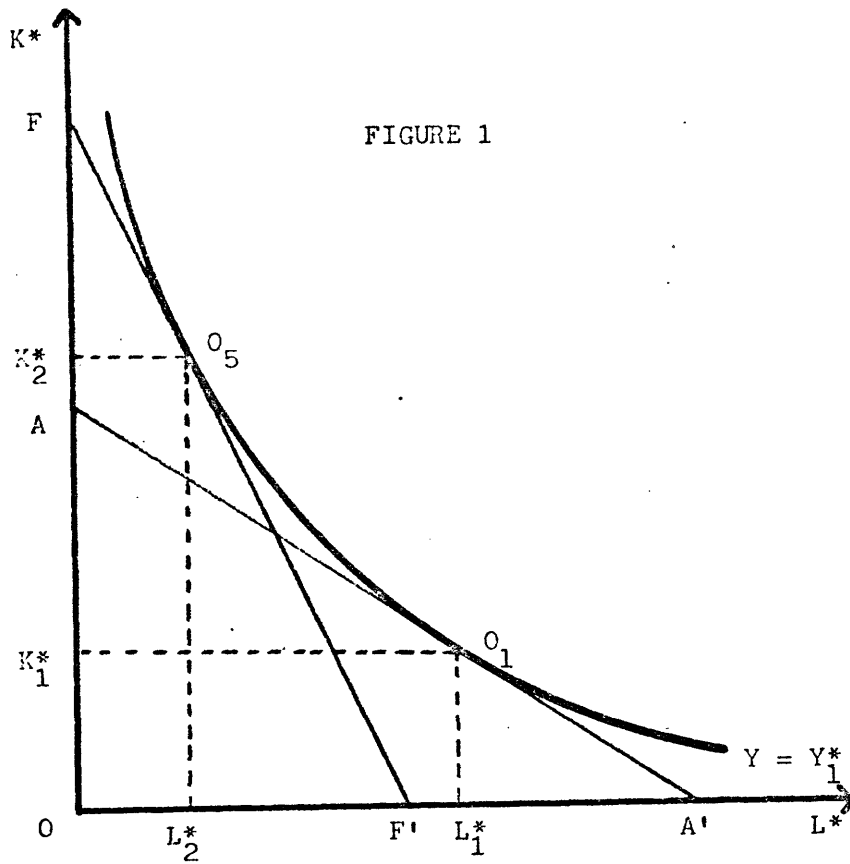
We now illustrate (28) and (29) with the following example. Suppose that a cost-minimizing competitive firm was in equilibrium producing gross output $Y = Y_1^*$. Given the original input prices P_K^* and P_{L^*} , the slope of the isocost line AA' in Figure 1 is $-P_{L^*}/P_{K^*}$, and the firm minimizes costs of producing Y_1^* at O_1 using K_1^* units of utilized capital and L_1^* units of the labor-materials composite. Given the original prices P_K and P_E as reflected in the isocost line BB' in Figure 2, the firm produces the K_1^* output at O_2 using K_1 units of capital and E_1 units of energy; similarly, the L_1^* output, given P_L and P_M as reflected in the isocost line CC' in Figure 3, is produced at O_3 using L_1 units of labor and M_1 units of materials.

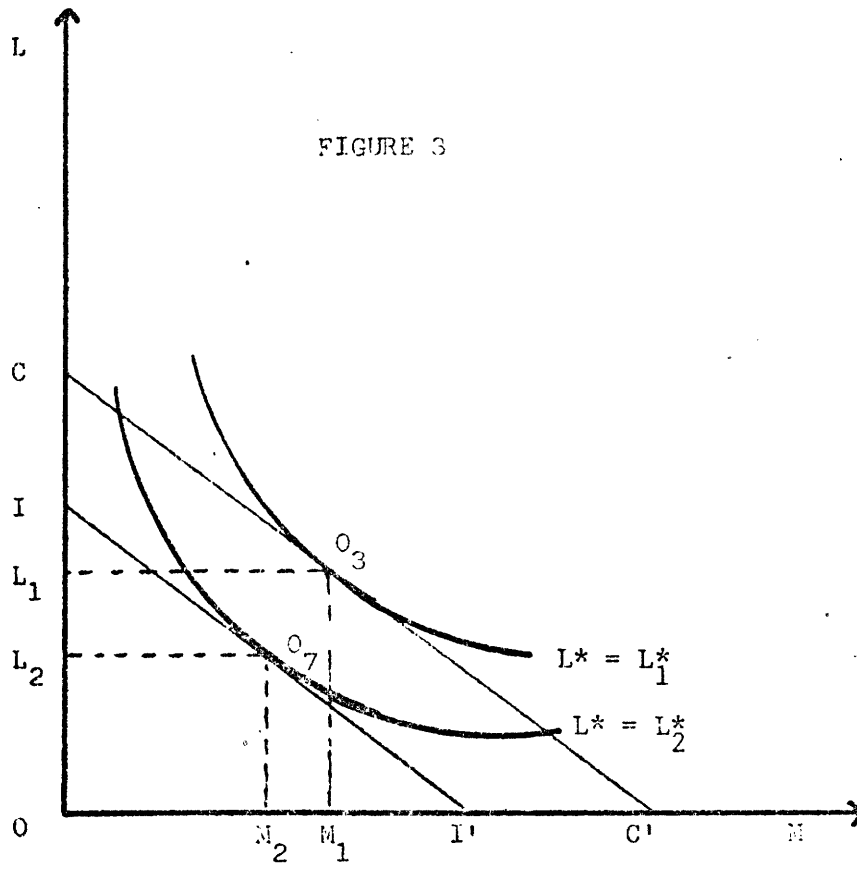
Now let us assume that the federal government introduces investment incentives that lower P_K . The total effect on the elasticity of demand for capital, ϵ_{KK} , and on the demand for energy, ϵ_{EK} , consists of two components -- as shown in (28) and (29). First, holding fixed the output of the utilized capital subfunction $K^* = K_1^*$, the steeper isocost line DD' in Figure 2 (due to the lower P_K) indicates that demand for capital would increase from K_1 to K_2 , and that demand for energy would fall from E_1 to E_2 ; in (28) and (29), these gross substitution effects are represented by ϵ_{KK}^* and ϵ_{EK}^* , respectively. Second, since the investment incentives decrease P_K , this reduces the cost C_{K^*} of producing utilized capital services, and by (35) lowers P_{K^*} . This changes the isocost line in Figure 1 from AA' to a steeper isocost line FF' , and results in a new cost-minimizing equilibrium at O_5 where derived demand for utilized capital increases

from K_1^* and K_2^* , while demand for L^* falls from L_1^* to L_2^* . This results in an outward shift of the K^* isoquant, as shown in Figure 2, increasing derived demand both for capital and for energy; at the new equilibrium O_6 , this scale effect increases derived demand for capital from K_2 to K_3 and demand for energy from E_2 to E_3 . For capital, the gross substitution effect (K_1 to K_2) and scale effect (K_2 to K_3) reinforce each other, but for energy the two effects work in opposite directions; the gross substitution effect decreases energy demand from E_1 to E_2 , whereas the scale effect increases demand for energy from E_2 to E_3 . Note that E_3 is larger than E_1 . In the particular example of Figure 2, the scale effect $N_K \epsilon_{uu}$ dominates the gross substitution effect c_{EK}^* , and thus, although in this example E and K are gross substitutes ($\epsilon_{EK}^* > 0$), they are net complements ($\epsilon_{EK} < 0$). Notice that net complementarity implies that the investment incentives contribute to increased (not reduced) energy demand. It might also be noted that the effect of the investment incentives is to lower L^* from L_1^* to L_2^* ; as seen in Figure 3, at the new equilibrium O_7 the scale effect results in a reduction of derived demand both for L and M from L_1 to L_2 and from M_1 to M_2 .¹

Our simple model suggests then that whether net K-E substitutability or net K-E complementarity exists depends on whether the gross substitution

¹In terms of the refrigerator example mentioned earlier, the investment incentives would induce a shift toward a more capital and less energy intensive refrigerator of a given type (the gross substitution effect), but because these investment incentives would reduce the unit cost (price) of utilized refrigerator services, they would also encourage purchases of a larger, or more sophisticated self-defrosting refrigerator that uses more capital and more energy, but less labor time (the scale effect).





effect or the scale effect is dominant.¹ This is, of course, an empirical issue. In order to implement our simple model empirically, we must specify functional forms for the master function (32) and for the K* and L* subfunctions (23) and (31). For convenience, we will specify forms for the dual cost functions (33), (34), and (25). In specifying the separable cost subfunctions (25) and (34), we wish to employ a rather flexible or unrestrictive function. For both the K* and L* cost subfunction, we employ the translog form

$$(36) \quad \ln(C_{K^*}/K^*) = \ln\delta_{K^*} + \alpha_K \ln P_K + \alpha_E \ln P_E + \frac{1}{2} \gamma_{KK} (\ln P_K)^2 \\ + \gamma_{KE} \ln P_K \ln P_E + \frac{1}{2} \gamma_{EE} (\ln P_E)^2$$

and

$$(37) \quad \ln(C_{L^*}/L^*) = \ln\delta_{L^*} + \alpha_L \ln P_L + \alpha_M \ln P_M + \frac{1}{2} \gamma_{LL} (\ln P_L)^2 \\ + \gamma_{LM} \ln P_L \ln P_M + \frac{1}{2} \gamma_{MM} (\ln P_M)^2$$

where

$$(38) \quad \alpha_K + \alpha_E = \alpha_L + \alpha_M = 1 \\ \gamma_{KK} + \gamma_{KE} = \gamma_{KE} + \gamma_{EE} = 0 \\ \gamma_{LL} + \gamma_{LM} = \gamma_{LM} + \gamma_{MM} = 0.$$

The translog form is attractive in that it imposes no a priori restrictions on the Allen partial elasticities of substitution, and can be interpreted as

¹This result is analogous to the familiar substitution and income effects of consumer demand theory.

a second order Taylor's series approximation in logarithms to an arbitrary cost function.

To complete our empirical model specification, we assume the master production function is the familiar strongly separable linearly homogeneous Cobb-Douglas function with two inputs, K^* and L^* . We also assume that any technical change is constant exponential Hicks-neutral. The dual master unit cost function is then written as

$$(39) \quad \ln(C/Y) = \ln\beta_0 + \alpha_t t + \beta_{K^*} \ln P_{K^*} + \beta_{L^*} \ln P_{L^*}$$

where P_{K^*} and P_{L^*} are defined as equalling the unit cost of K^* and L^* (see (34) and (35)), t represents time, and where

$$(40) \quad \beta_{K^*} + \beta_{L^*} = 1.$$

Substituting (36) and (37) into (39) and using (35), we can write the master cost function in terms of the separable subfunction prices and parameters:

$$(41) \quad \ln(C/Y) = \ln\alpha_0 + \alpha_t t + \sum_i \beta_i \ln P_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln P_i \ln P_j, \quad i, j = K, L, E, M$$

where $\beta_{ij} = \beta_{ji}$, $\sum_i \beta_{ij} = \sum_j \beta_{ij} = 0$, $\sum_i \beta_i = 1$, and

$$(42) \quad \ln\alpha_0 = \ln\beta_0 + \beta_{K^*} \ln\delta_{K^*} + \beta_{L^*} \ln\delta_{L^*}$$

$$\begin{aligned} \beta_{KL} = \beta_{KM} = \beta_{LE} = \beta_{EM} = 0, & \quad \beta_M = \beta_{L^*} \alpha_M \\ \beta_{KK} = \beta_{K^*} \gamma_{KK} & \quad \beta_{KE} = \beta_{K^*} \gamma_{KE} & \quad \beta_{EE} = \beta_{K^*} \gamma_{EE} \\ \beta_{LL} = \beta_{L^*} \gamma_{LL} & \quad \beta_{LM} = \beta_{L^*} \gamma_{LM} & \quad \beta_{MM} = \beta_{L^*} \gamma_{MM} \\ \beta_K = \beta_{K^*} \alpha_K & \quad \beta_L = \beta_{L^*} \alpha_L & \quad \beta_E = \beta_{K^*} \alpha_E \end{aligned}$$

In summary, our simple empirical model includes a strongly separable Cobb-Douglas master cost function of K^* and L^* , where the K^* and the L^* subfunctions are translog with inputs K and E , L and M , respectively. This simple model appears to have substitutable relationships everywhere -- between K and E in the two input utilized capital subfunction, between the L and M in the two input L^* subfunction, and between K^* and L^* in the two input master Cobb-Douglas gross output cost function. As we shall now show, however, this simple model is completely consistent with energy-capital complementarity.

Based on the master cost function specification (41)-(42), we utilize (13) and compute Allen partial elasticities of substitution σ_{ij} and price elasticities along a gross output isoquant as

$$(43) \quad \sigma_{ij} = \frac{\beta_{ij} + M_i M_j}{M_i M_j}, \quad i, j = K, L, E, M, i \neq j$$

$$\sigma_{ii} = \frac{\beta_{ii} + M_i^2 - M_i}{M_i^2}, \quad i = K, L, E, M$$

and

$$(44) \quad \epsilon_{ij} = M_j \sigma_{ij} \quad i, j = K, L, E, M$$

where the M_i are the cost shares of the i^{th} input in the total cost C of producing gross output, obtained by logarithmically differentiating (41) and using Shephard's Lemma:

$$(45) \quad M_i \equiv \frac{\partial \ln C}{\partial \ln P_i} = \frac{P_i X_i}{C} = \beta_i + \sum_j \beta_{ij} \ln P_j, \quad i, j = K, L, E, M$$

The price elasticities in (44) are of course net price elasticities. It is of interest to rewrite the net price elasticities (44) in terms of gross price elasticities ϵ_{ij}^* and scale elasticities. Using (42) and the fact that

$$(46) \quad M_K = \beta_{K^*} N_K, \quad M_E = \beta_{K^*} N_E, \quad M_L = \beta_{L^*} N_L, \quad M_M = \beta_{L^*} N_M$$

where from (36) and (37)

$$(47) \quad N_K = \frac{\partial \ln (C_{K^*}/K^*)}{\partial \ln P_K} = \frac{P_K K}{C_{K^*}} = \alpha_K + \gamma_{KK} \ln P_K + \gamma_{KE} \ln P_E$$

$$N_E = \frac{\partial \ln (C_{K^*}/K^*)}{\partial \ln P_E} = \frac{P_E E}{C_{K^*}} = \alpha_E + \gamma_{KE} \ln P_K + \gamma_{EE} \ln P_E$$

$$N_L = \frac{\partial \ln (C_{L^*}/L^*)}{\partial \ln P_L} = \frac{P_L L}{C_{L^*}} = \alpha_L + \gamma_{LL} \ln P_L + \gamma_{LM} \ln P_M$$

$$N_M = \frac{\partial \ln (C_{L^*}/L^*)}{\partial \ln P_M} = \frac{P_M M}{C_{L^*}} = \alpha_M + \gamma_{LM} \ln P_L + \gamma_{MM} \ln P_M$$

we can rewrite the net price elasticity ϵ_{ij} in terms of the gross price elasticity ϵ_{ij}^* and the scale elasticity. In the context of the E-K net price elasticities, we have

$$(48) \quad \epsilon_{KE} = \epsilon_{KE}^* - N_E \beta_{L^*}, \quad \epsilon_{EK} = \epsilon_{EK}^* - N_K \beta_{L^*}$$

$$(49) \quad \epsilon_{KK} = \epsilon_{KK}^* - N_K \beta_{L^*}, \quad \epsilon_{EE} = \epsilon_{EE}^* - N_E \beta_{L^*}$$

Equation (48) indicates that, for example, the net price elasticity ϵ_{KE} along a gross output isoquant is equal to the gross price elasticity ϵ_{KE}^* along a utilized capital isoquant plus the scale elasticity term ($-N_E \beta_{L^*}$) which reflects the cost share of energy in the K^* subfunction times the price elasticity of demand for K^* which in this particular model is $\beta_{K^*} - 1 = -\beta_{L^*}$. Since $-N_E \beta_{L^*}$ is negative, $\epsilon_{KE} < \epsilon_{KE}^*$. Even though the gross substitution term ϵ_{KE}^* is positive, the sign of the net elasticity ϵ_{KE} depends on whether the absolute value of the scale elasticity is larger or smaller than the positive gross price elasticity ϵ_{KE}^* .

Empirical research is therefore necessary in order to determine the relative magnitudes of the gross substitution and scale elasticities. An interesting feature of (42) is that it constitutes a set of parametric restrictions on the more general four input KLEM translog unit cost function with Hicks-neutral constant exponential technical change:

$$(50) \ln(C/Y) = \ln \alpha_0 + \alpha_t t + \sum_i \beta_i \ln P_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln P_i \ln P_j, \quad i, j = K, L, E, M$$

where $\beta_{ij} = \beta_{ji}$ and $\sum_i \beta_i = 1$, $\sum_i \sum_j \beta_{ij} = \sum_j \beta_{ij} = 0$. E.R. Berndt and David O. Wood have called the set of restrictions (42) on (50) linear separability restrictions for [(K,E),(L,M)] separability. They report these restrictions could not be rejected with their data -- annual U.S. manufacturing, 1947-71.¹ Furthermore, Berndt-Wood tested for many different types of separability among the K,L,E and M inputs; all forms except that represented in (42) were rejected. These results therefore provide some empirical support for our simple model specification.^{2,3}

In Table 1 we present maximum likelihood estimates of the parameters in the share equations (45) with the utilized capital model restrictions (42) imposed

¹In Berndt-Wood [1975], we presented estimated elasticities based on iterative three stage least squares estimation. In the present paper, we assume input prices and gross output quantity are exogenous, and estimate the parameters of the share equations (45) using maximum likelihood procedures; the I3SLS and maximum likelihood estimates are virtually identical. The likelihood ratio test statistic of the four independent restrictions in (42) using maximum likelihood estimation is 10.326, while the .01 chi-square critical value is 13.277; under I3SLS estimation, the Wald test statistic is 9.038 and the .01 chi-square critical value remains 13.277.

²We have also tested for the validity of a related utilized capital separability specification using the three input K,L,E data of Griffen-Gregory, which they kindly provided us. Based on their data, the chi-square test statistic for the two restrictions is 3.2505, while the .01 chi-square critical value is 9.210. Hence, using the Griffen-Gregory KLE data, we cannot reject the null hypothesis of nonlinear [(K,E),L] separability.

³We note that although our principal concern focuses on the K* subfunction specification, with a translog gross output KLEM unit cost function the specification of such a separable K* subfunction necessarily implies a symmetric specification for a separable L* subfunction. For further discussion of possible nested specifications within a translog framework, see Blackorby, Primont, and Russell [1977], and Denny-Fuss [1977].

TABLE 1

Maximum Likelihood Parameter Estimates with the Separable Utilized Capital Specification
 [(K,E),(L,M)] Linear Separability Restrictions Imposed,
 U.S. Manufacturing, 1947-1971

| <u>Parameter</u> | <u>Parameter Estimate</u> | <u>Ratio of Parameter Estimate to Asymptotic Standard Error</u> |
|--|---------------------------|---|
| β_{K^*} | .0983 | 89.78 |
| β_{L^*} | .9017 | 823.46 |
| α_K | .5702 | 231.46 |
| α_E | .4298 | 174.48 |
| α_L | .2800 | 118.16 |
| α_M | .7200 | 303.84 |
| $\gamma_{KK} = -\gamma_{KE} = \gamma_{EE}$ | .1851 | 15.16 |
| $\gamma_{LL} = -\gamma_{LM} = \gamma_{MM}$ | .0868 | 12.87 |
| <u>Implied Estimates</u> | | |
| β_K | .0561 | 75.79 |
| β_L | .2525 | 124.42 |
| β_E | .0423 | 93.13 |
| β_M | .6492 | 352.84 |
| $\beta_{KK} = -\beta_{KE} = \beta_{EE}$ | .0182 | 15.38 |
| $\beta_{LL} = -\beta_{LM} = \beta_{MM}$ | .0782 | 12.88 |
| <u>Fit Statistic</u> | | |
| $\tilde{R}^2 = .9844$ | | |

based on the Berndt-Wood data. All coefficients appear to be significantly different from zero. We measure goodness of fit using the generalized R^2 measure discussed by E.R. Berndt [1977]. Here R^2 is computed as

$$(51) \quad \tilde{R}^2 = 1 - \frac{|\hat{r}_1|}{|\hat{r}_0|}$$

where $|\hat{r}_0|$ is the determinant of the residual covariance matrix when all "slope" coefficients are zero, and $|\hat{r}_1|$ is the determinant of the residual covariance matrix for the model being estimated. This \tilde{R}^2 figure collapses to the traditional R^2 measure when only one equation is estimated. In the present context, our \tilde{R}^2 figure of .9844 indicates that the model "explains" a very high proportion of the generalized variance in our share equations.

Based on these parameter estimates, we compute maximum likelihood estimates of selected net, scale, and gross elasticities for the last year of our sample (1971).¹ These are presented in the top panel of Table 2. There it is seen that although E and K are gross substitutes in U.S. manufacturing, they also are net complements. The gross substitution effect (.133) is dominated by the scale effect (-.462), resulting in a value for the net elasticity ϵ_{EK} of -.329.

The above results were based on U.S. manufacturing time series data. To investigate the robustness of our net complementarity findings within a utilized capital framework, we now estimate a slightly generalized model using pooled cross-section time series data for Canadian manufacturing, by region, 1961-1971. Recently Melvyn A. Fuss [1977] published estimates of substitution elasticities

¹Since all our elasticity estimates are very stable over the 1947-71 time period the year 1971 can be interpreted as representative. It is also worth noting that all our fitted shares were positive and that the strict quasi-concavity curvature conditions were satisfied for all years in our sample.

Table 2

NET, SCALE, AND GROSS SUBSTITUTION ELASTICITIES IN UTILIZED CAPITAL MODEL[(K,E),(L,M)] SEPARABILITY RESTRICTIONS IMPOSEDU.S. AND CANADIAN MANUFACTURING, 1971

(Estimated Asymptotic Standard Errors in Parentheses)

| <u>Net Elasticity</u> | <u>Gross Substitution Elasticity</u> | <u>Scale Elasticity</u> | <u>Value of Net Elasticity</u> |
|---|--------------------------------------|-------------------------|--------------------------------|
| U.S. Manufacturing, 1971 | | | |
| ϵ_{KK} | -.126 (.024) | -.462 (.003) | -.588 (.026) |
| ϵ_{EE} | -.133 (.026) | -.440 (.003) | -.573 (.024) |
| ϵ_{KE} | .126 (.024) | -.440 (.003) | -.314 (.027) |
| ϵ_{EK} | .133 (.026) | -.462 (.003) | -.329 (.026) |
| Canadian Manufacturing- Ontario, 1971 | | | |
| ϵ_{KK} | -.039 (.009) | -.705 (.238) | -.744 (.238) |
| ϵ_{EE} | -.505 (.115) | -.054 (.018) | -.559 (.117) |
| ϵ_{KE} | .039 (.009) | -.054 (.018) | -.015 (.020) |
| ϵ_{EK} | .505 (.115) | -.705 (.238) | -.200 (.264) |
| Canadian Manufacturing- British Columbia, 1971 | | | |
| ϵ_{KK} | -.121 (.011) | -.664 (.206) | -.785 (.206) |
| ϵ_{EE} | -.650 (.052) | -.123 (.038) | -.773 (.066) |
| ϵ_{KE} | .121 (.011) | -.123 (.038) | -.002 (.040) |
| ϵ_{EK} | .650 (.052) | -.664 (.206) | -.014 (.213) |

for Canadian manufacturing based on a nonhomothetic KLEM translog cost function.¹ Because Fuss specified a nonhomothetic translog cost function and estimated using an error components estimation procedure (the "covariance method"), the conditions (42) for separability are not directly applicable or testable.²

To preserve the distinguishing features of Fuss' paper -- a nonhomothetic translog specification and the covariance estimation method -- we proceed with separate estimation of the gross, net, and scale elasticities as follows. First the energy-capital (K^*) and labor-materials (L^*) subfunctions are again specified as (36) and (37). Using Canadian manufacturing data, 1961-71, by region,³ we estimate the N_K and N_L equations in (47) using the covariance method. We then insert the resulting parameter estimates into (36) and (37) and form fitted data series for P_{K^*} and P_{L^*} ; these values are unique up to a multiplicative scaling, since the intercept terms in (36) and (37) cannot be identified.⁴ Secondly, for our master function we follow Fuss and specify a nonhomothetic translog cost function

¹Fuss' principal findings for Canadian manufacturing were similar to the Berndt-Wood results for U.S. manufacturing. In particular, Fuss' price elasticity estimates (calculated at the mean values for Ontario) are $\epsilon_{EE} = \epsilon_{EE} = -.486$, $\epsilon_{KK} = .762$, $\epsilon_{EK} = .050$, $\epsilon_{KE} = .004$, $\epsilon_{EL} = .554$, and $\epsilon_{LE} = .043$. Hence like Berndt-Wood [1975], Fuss finds E-K complementarity and E-L substitutability.

²We note that in the context of a translog gross output function, it is not possible to test for nonhomothetic separability; see Blackorby, Primont, and Russell [1977] and Denny-Fuss [1977].

³The Canadian manufacturing data was kindly provided us by Melvyn Fuss. Using this data, we successfully replicated the KLEM results reported by Fuss ([1977], Table 4, p. 109), except for a typographical error on his reported estimate of γ_{MM} . The correct estimate of γ_{MM} is .0618, and the correct standard error estimate for γ_{MM} is .0140.

⁴This approach is equivalent to that used by Fuss in forming an aggregate P_E series from data on constituent fuel prices.

$$\begin{aligned}
(52) \quad \ln C &= \ln \alpha_0 + \delta_Y \ln Y + \frac{1}{2} \delta_{YY} (\ln Y)^2 + \beta_{K^*} \ln P_{K^*} + \beta_{L^*} \ln P_{L^*} \\
&+ \frac{1}{2} \beta_{K^*K^*} (\ln P_{K^*})^2 + \beta_{K^*L^*} \ln P_{K^*} \ln P_{L^*} + \frac{1}{2} \beta_{L^*L^*} (\ln P_{L^*})^2 \\
&+ \beta_{K^*Y} \ln P_{K^*} \ln Y + \beta_{L^*Y} \ln P_{L^*} \ln Y
\end{aligned}$$

where

$$\begin{aligned}
(53) \quad \beta_{K^*} + \beta_{L^*} &= 1 \\
\beta_{K^*Y} + \beta_{L^*Y} &= 0 \\
\beta_{K^*K^*} + \beta_{K^*L^*} &= \beta_{K^*L^*} + \beta_{L^*L^*} = 0.
\end{aligned}$$

In summary, the nonhomothetic separable specification is simply a nonhomothetic master translog cost function with linearly homogeneous translog K^* and L^* sub-functions whose components are K and E , L and M , respectively.

Finally, logarithmically differentiating (52) and using Shephard's Lemma, we obtain the estimable share equations for the master function:

$$\begin{aligned}
(54) \quad M_{K^*} &\equiv \frac{P_{K^*} K + P_E E}{C} = \beta_{K^*} + \beta_{K^*K^*} \ln P_{K^*} + \beta_{K^*L^*} \ln P_{L^*} + \beta_{K^*Y} \ln Y \\
M_{L^*} &\equiv \frac{P_{L^*} L + P_M M}{C} = \beta_{L^*} + \beta_{K^*L^*} \ln P_{K^*} + \beta_{L^*L^*} \ln P_{L^*} + \beta_{L^*Y} \ln Y.
\end{aligned}$$

We estimate (54) subject to the restrictions (53) using the covariance method, and then compute the associated gross, scale, and net substitution elasticities. Estimates of these elasticities in 1971 for two provinces -- Ontario and British Columbia -- are presented in the bottom two panels of Table 2.

As shown in Table 2, for Canadian manufacturing E and K are gross substitutes but net complements. The net substitution effect for Ontario in 1971 (-.200)

is negative while for British Columbia the gross substitution and scale effects almost offset each other, resulting in only a very slight net complementarity value of $-.014$.¹ Both net elasticity estimates are, however, insignificantly different from zero.

In summary, the utilized capital models presented in this section provide a useful analytical and empirical foundation for reconciling energy-capital complementarity with the economic-engineering notion of gross energy-capital substitutability.

¹It should be noted that with this specification and Fuss' data, the estimated parameters and fitted cost shares satisfy the strict quasi-concavity conditions in all regions for all years, 1961-71.

V. Towards a Reconciliation of Seemingly Disparate Econometric Findings

In the preceding paragraphs we have emphasized that results of engineering process analysis are consistent with our E-K complementarity econometric findings. In effect, we have shown that the Griffen-Gregory intuitive argument for E-K substitutability can be misleading: the engineering notion of E-K substitutability does not necessarily imply net E-K substitutability in the sense of Hicks-Allen. Griffen-Gregory (hereafter, GG) have, however, published econometric findings which appear to report Hicks-Allen E-K substitutability. Thus it remains to reconcile our econometric findings with those of GG.

GG have estimated a three input (K, L, and E -- but not M) translog cost function based on data for the manufacturing sector of nine industrialized OECD countries in four benchmark years -- 1955, 1960, 1965, and 1969.¹ Parameters are estimated using a maximum likelihood procedure. For reasons unspecified, however, GG do not use maximum likelihood methods in estimating elasticities.

Recall that price elasticities in the translog model are based on the relations

$$\begin{aligned} \epsilon_{ij} &= \frac{\beta_{ij} + M_i M_j}{M_j}, & i \neq j, \\ \epsilon_{ii} &= \frac{\beta_{ii} + M_i^2 - M_i}{M_i}, \end{aligned} \quad (55)$$

where the cost shares M_i are

$$M_i = \alpha_i + \sum_j \beta_{ij} \ln P_j, \quad i, j = K, L, E. \quad (56)$$

Maximum likelihood (ML) estimates of the price elasticities are of course obtained by inserting the ML parameter estimates into (56) and then using these "fitted" or

¹The nine countries are Belgium, Denmark, France, West Germany, Italy, Netherlands, Norway, United Kingdom, and the United States.

"predicted" shares along with the ML parameter estimates in (55) to compute estimates of ϵ_{ii} and ϵ_{ij} . Instead of using predicted shares, GG insert actual data shares into (55). The difference between their procedure and the ML approach would be negligible if the estimated model fitted their data closely, but unfortunately this is not the case with the GG model -- especially for the United States. The \tilde{R}^2 for the GG preferred Model I is .41, and the difference between fitted and actual cost shares for the United States is considerable. For example, in 1965 -- the year for which GG report elasticity estimates -- the GG predicted capital cost shares in the U.S. is .2205, while the actual share is a 35% lower .1436; the corresponding predicted (actual) cost shares for labor and energy are .6622 (.7311) and .1174 (.1253). Table 3 indicates that the difference between the GG and ML elasticity estimates is quite small for all countries except the U.S. Since GG compared their results with our U.S. manufacturing findings, it is useful to examine their U.S. estimates more closely. The GG (ML) estimates for ϵ_{KK} in the U.S. are -.18 (-.34), for ϵ_{KL} , .05 (.22) and for ϵ_{EK} , .15 (.23). These differences in the ML and GG estimates reflect the rather poor fit of the GG model to their U.S. data, and ought to make one cautious in comparing their U.S. results with those of Berndt-Wood.¹ With respect to the ϵ_{EK} elasticity, it might also be noted that the crucial β_{KE} parameter reported in GG has a very large standard error estimate. This leads to two standard error confidence intervals for ϵ_{EK} which include E-K complementarity; for the U.S. the confidence intervals are $-.05 \leq \epsilon_{EK} \leq .51$ (ML) and $-.13 \leq \epsilon_{EK} \leq .43$ (GG). The above comments suggest then that differences between the econometric results of Griffen-Gregory and Berndt-Wood may well be statistically insignificant.

An even stronger analytical argument can be made that all σ_{KE} estimates based on three input KLE models are upwards biased. To see this, recall that

¹This computational nuance and not the speculative "rather intriguing explanation" by GG (see GG [1976], p. 853) may explain their "unexpected" low σ_{KL} estimate for the U.S.

TABLE 3

Selected Price Elasticity Estimates for OECD Countries: A Comparison of
Griffen-Gregory (GG) and Maximum Likelihood (ML) Estimates*

| Country | ϵ_{KK} | | ϵ_{KL} | | ϵ_{EK} | |
|--|-----------------|------|-----------------|-----|-----------------|-----|
| | GG | ML | GG | ML | GG | ML |
| Belgium | -.38 | -.37 | .20 | .23 | .32 | .38 |
| Denmark | -.37 | -.38 | .29 | .25 | .39 | .32 |
| France | -.37 | -.37 | .26 | .24 | .27 | .35 |
| West Germany | -.36 | -.38 | .26 | .24 | .40 | .34 |
| Italy | -.38 | -.36 | .23 | .23 | .33 | .39 |
| Netherlands | -.38 | -.37 | .22 | .24 | .32 | .35 |
| Norway | -.38 | -.38 | .21 | .25 | .33 | .30 |
| United Kingdom | -.37 | -.38 | .25 | .24 | .27 | .33 |
| United States | -.18 | -.34 | .05 | .22 | .15 | .23 |
| Standard Error Estimate for the U.S. | .28 | | .08 | | .14 | |

* - The GG estimates are taken from Griffen-Gregory [1976], Table 3, p. 852.

the linear homogeneous gross output production function is $Y = f(K, L, E, M)$. In general, the three input KLE specification is valid if and only if the inputs K, L, and E are weakly separable from M, i.e. if and only if we can write $Y = f(K, L, E, M) = h[h^*(K, L, E)M] = h^{**}(V, M)$, where V is the output of the $h^*(K, L, E)$ production subfunction. A number of authors -- among them Griffen-Gregory, Robert S. Pindyck [1977] and Jan R. Magnus [1977] -- have been unable to obtain sufficiently reliable data on M, and for this reason have estimated substitution elasticities among K, L, and E assuming that these three inputs are weakly separable from M. Even if this untested restrictive KLE specification were valid, however, the resulting elasticity estimates are not in general directly comparable to those based on four input KLEM models.

The economic intuition on this issue is similar to that utilized in Section IV. Suppose that the price of energy increases, other input prices remaining fixed. If within the KLE subfunction K and E are gross substitutes, then the increased energy prices will induce substitution toward capital, holding fixed the output of the KLE subfunction. But the increased energy prices will raise the cost of producing this output V, and this will induce a substitution away from $V = h^*(K, L, E)$ and toward M, holding the $Y = f(K, L, E, M)$ output fixed. This latter scale effect reduces the derived demand for all three inputs in the $V = h^*(K, L, E)$ subfunction. The net effect of the energy price increase on the demand for capital is therefore indeterminate; the sign of ϵ_{KE} depends on whether the positive gross substitution effect or the negative scale effect is dominant.

Hence by analogy with the gross substitution and scale effect argument developed in Section IV, we can relate the three input K-E gross substitution elasticities ϵ_{ij}^* to the four input K-E net substitution elasticities ϵ_{ij} as

$$\epsilon_{EK} = \epsilon_{EK}^* + N_K \epsilon_{VV}$$

$$\epsilon_{KE} = \epsilon_{KE}^* + N_E \epsilon_{VV}$$

(57)

where now N_K and N_E are the cost shares of K and E, respectively, in the total cost of producing the $V = h^*(K,L,E)$ output and ϵ_{VV} is the price elasticity of demand for the output V along a four input $Y = f(K,L,E,M) = h^{**}(V,M)$ isoquant. Since N_K and N_E are positive cost shares and ϵ_{VV} is non-positive, $\epsilon_{EK}^* \geq \epsilon_{EK}$ and $\epsilon_{KE}^* \geq \epsilon_{KE}$, implying that unless the output price elasticity $\epsilon_{VV} = 0$, the K-E substitution elasticity estimates based on a three input KLE specification are upwards biased. Furthermore, ceteris paribus, this upward bias will be larger the more capital and energy intensive is the industry.

It would of course be interesting to obtain some idea of the potential quantitative magnitude of this upward bias. The three input KLE studies by Griffen-Gregory and Pindyck report estimates for the U.S. of about 1.1 and .8, respectively, while Magnus' KLE study for the Netherlands finds a σ_{KE}^* estimate of about -4.4. Hence the three-input study reporting the greatest amount of E-K substitutability is Griffen-Gregory. Let us insert into (57) reasonable values of N_K , N_E , ϵ_{VV} and the GG "high" estimates of ϵ_{EK}^* and ϵ_{KE}^* . The 1965 values of N_K and N_E in GG's U.S. data are about .14 and .13 respectively, while their reported ϵ_{EK}^* and ϵ_{KE}^* estimates are .15 and .13. Reasonable estimates of ϵ_{VV} are more difficult to obtain. We can proceed by letting ϵ_{VV} take on three alternative values: -0.5, -1.0, and -1.5. Inserting these alternative estimates of ϵ_{VV} into (57), and using the GG values for N_K , N_E , ϵ_{EK}^* and ϵ_{KE}^* , we obtain three alternative net elasticity estimates: .08, .01, and -.06 for ϵ_{EK} , and .065, .0, and -.065 for ϵ_{KE} . Notice that even with the high ϵ_{EK}^* and ϵ_{KE}^* estimates of GG, we obtain values for the net elasticities ϵ_{KE} and ϵ_{EK} in the U.S. that include negative (complementary) estimates. Thus the positive GG estimates for ϵ_{EK}^* and ϵ_{KE}^* are not necessarily inconsistent with the E-K complementarity estimates obtained by Berndt-Wood, Berndt-Jorgenson, and Fuss. Moreover, since GG's positive estimates of ϵ_{KE}^* are the largest of those reported in the various three-input

KLE studies, the K-E elasticity estimates of the various KLE and KLEM studies can be reconciled.¹ GG note that comparison of their elasticity estimates with those of Berndt-Wood might be questioned since, unlike Berndt-Wood, they omit M and justify this omission by assuming weak separability of the form [(K,L,E),M]. GG correctly note that "...this omission may bias our findings if our weak separability assumption...is invalid" (GG, [1976], p. 852). They fail to recognize that even if this weak separability assumption were valid, all their estimates reflect gross substitution elasticities and therefore all are upward biased.

We conclude that the seemingly inconsistent Berndt-Wood energy-capital complementarity and Griffen-Gregory energy-capital substitutability econometric results may simply be due to the fact that different elasticities are being compared; when the distinction between net and gross elasticities is acknowledged and the same output is held constant, the various net elasticity estimates are reasonably consistent with one another. Any remaining discrepancies are likely to be statistically insignificant, especially since standard errors for the GG energy-capital elasticity estimates are large.

¹At this point the only other four input KLEM study of which we are aware is that by Paul Swain and Gerhard Friede [1976] for manufacturing in West Germany. Swain and Friede also find E-K complementarity.

VI. A Reconciliation Without the Separability Assumption

In Section IV we showed that if one assumes [(K,E),(L,M)] separability, the resulting "utilized capital" specification enables us analytically and empirically to reconcile the engineering notion of E-K substitutability with the Hicks-Allen concept of E-K complementarity. Then in Section V we demonstrated that the Griffen-Gregory [(K,L,E),M] separability assumption and their use of only KLE data implies that the GG elasticity estimates are not directly comparable with those of Berndt-Wood, since different outputs are being held constant; when the elasticities are properly compared, the seemingly disparate empirical findings can be reconciled.

Separability has played a prominent role in both of these discussions. This raises the issue of whether our reconciliation of engineering E-K substitutability with Hicks-Allen E-K complementarity is dependent on the separability assumption.

We shall now show that the separability assumption, although useful for purposes of exposition and pedagogy, is not necessary for the reconciliation of engineering E-K substitutability with Hicks-Allen E-K complementarity. Thus our analytical findings are considerably generalized and strengthened.

It is well known that separability places restrictions on the Hicks-Allen partial elasticities of substitution σ_{ij} and price elasticities ϵ_{ij} .¹ In the present context, [(K,E),(L,M)] separability implies $\sigma_{KL} = \sigma_{EL} = \sigma_{KM} = \sigma_{EM}$.² Moreover, holding the $K^* = f(K,E)$ output constant and using only the K-E data, this separability specification implies that the Allen gross substitution elasticity σ_{KE}^* is independent of L and M.³ Hence, under [(K,E),(L,M)] separability, σ_{KE}^* can be computed without any consideration of inputs L and M.

¹For further discussion of this point, see E.R. Berndt and L.R. Christensen [1973].

²Our use of the translog function in the empirical application also constrained these σ_{ij} to equal unity.

³The gross elasticity σ_{KE}^* would of course be computed applying (13) to the utilized cost subfunction (25).

In Section II we defined the direct elasticity of substitution d_{ij} (15) and the shadow elasticity of substitution S_{ij} (17). There we also noted that in general with more than two inputs, σ_{ij} , d_{ij} and S_{ij} differ from one another. In the present $Y = F(K,L,E,M)$ context, d_{KE} and S_{KE} are

$$(58) \quad d_{KE} = \frac{\partial \ln (K/E)}{\partial \ln (P_E/P_K)}, \quad Y, L, \text{ and } M \text{ fixed,}$$

$$(59) \quad S_{KE} = \frac{\partial \ln (P_K/P_E)}{\partial \ln (E/K)}, \quad \tilde{C}, \tilde{P}_L, \text{ and } \tilde{P}_M \text{ fixed,}$$

where \tilde{C} , \tilde{P}_L , and \tilde{P}_M are the shadow total cost and shadow prices of L and M, respectively. Note that the direct elasticity d_{KE} is computed conditional on the values of Y, L, and M, in contrast to the gross substitution elasticity σ_{KE}^* which is computed completely independent of L and M. Whether the recent economic-engineering literature on energy-capital tradeoffs totally ignores independent inputs such as L or M or merely assumes they are fixed is of course a matter of interpretation. The important role of separability is that if $[(K,E),(L,M)]$ separability holds, then $\sigma_{KE}^* = d_{KE} = S_{KE}$. Hence, under this type of separability, the "conditional" and "independent" elasticity measures coincide.

When the production function $Y = f(K,L,E, M)$ is not separable, the various elasticity measures will generally differ from one another, i.e., conditional and independent elasticity measures will usually not coincide. In particular the d_{KE} elasticity which measures E-K substitutability conditional on L and M will of course be positive, even though K and E may be complementary inputs in the sense of Hicks-Allen. If one interprets engineering-economic studies as measuring E-K substitutability conditional on L and M, then the resulting positive d_{KE} estimates can be completely consistent with negative, complementary values for the

"unconditional" σ_{KE} estimates. Hence in this sense engineering E-K substitutability and Hicks-Allen complementarity can be mutually reconciled without the assumption of separability.

We now illustrate these points empirically. When [(K,E),(L,M)] separability is imposed on the Berndt-Wood data for total U.S. manufacturing, 1947-71, the estimates of σ_{KE}^* , d_{KE} , and S_{KE} coincide; their common value in 1965 is .243. However, without imposing these [(K,E),(L,M)] separability restrictions, we have estimated the conditional elasticities d_{KE} and S_{KE} and the unconditional Hicks-Allen elasticity of substitution σ_{KE} . The results, based on the Berndt-Wood data and the Fuss pooled cross-section time series data for Canadian manufacturing are reported in Table 4 below.¹ There it is seen that the positive conditional d_{KE} and S_{KE} estimates are completely consistent with the negative σ_{KE} estimate. We conclude that a reconciliation of engineering E-K substitutability with Hicks-Allen E-K complementarity does not depend on the $Y = F[(K,E),(L,M)]$ separability assumption.

Table 4

ESTIMATES OF ALLEN, DIRECT AND SHADOW PARTIAL ELASTICITIES OF SUBSTITUTION
BETWEEN CAPITAL AND ENERGY IN U.S. AND CANADIAN MANUFACTURING
NO UTILIZED CAPITAL SEPARABILITY RESTRICTIONS IMPOSED

| <u>Data Set</u> | <u>Direct Elasticity</u> <u>d_{KE}</u> | <u>Shadow Elasticity</u> <u>S_{KE}</u> | <u>Allen Elasticity</u> <u>σ_{KE}</u> |
|---|--|--|--|
| Berndt-Wood, U.S. manufac- turing, 1965 | .308 | .320 | -3.193 |
| Fuss, Canadian manufacturing, Ontario, 1965 | .464 | .501 | -0.207 |

¹In computing these elasticities, we have employed the formulae of R. Sato and T. Koizumi [1973a], who express the d_{ij} and S_{ij} in terms of σ_{ij} and cost shares.

VII. Concluding Remarks

The purpose of this paper has been to provide a rather general analytical and empirical foundation for E-K complementarity consistent with the process engineering-technological view of E-K substitutability. Along the way we have developed the notion of utilized capital and have reconciled some of the seemingly disparate econometric findings. Above all this research has emphasized to us that care must be taken in interpreting and properly comparing alternative elasticity estimates.

A particularly interesting implication of our analytical framework is that if one is willing to assume E-K separability, then it would be possible to use either engineering or econometric estimates of gross E-K substitution elasticities. The engineering estimates might be preferable, especially in the context of longer term forecasts, or for regions or countries which lack reliable economic data but utilize known technologies.

At this point in time there appears to be a substantial and growing body of econometric evidence supporting the notion of Hicks-Allen E-K complementarity.¹ In our view, however, the empirical issue of E-K complementarity is still far from settled. A number of data and basic economic model specification problems are worthy of particular attention in future research.

First, all the econometric evidence available to date is based on data that does not include the post-1973 dramatic energy price increases. It would be useful to examine the robustness of the E-K complementarity findings with the more recent data. However, it is worth noting that E-K complementarity would seem to be consistent with the view (articulated in the opening quotation of this paper) that higher energy prices are partly responsible for the very recent slow-down in rates of fixed capital formation.

¹A recent study by William Hogan [1977] argues for the plausibility of a different type of E-K complementarity; using a three input KLE model of the aggregate U.S. economy, Hogan argues that if L and P_K are fixed, then the recent increases in P_E will result in reduced rates of capital formation.

Second, there remains the concern, expressed vigorously by GG, that E-K complementarity estimates based on annual time series data actually reflect short-run variations in capacity utilization, and that the "true" long run relationship is one of E-K substitutability. For this reason GG and Pindyck [1977] prefer pooled international cross-section time series elasticity estimates to estimates based solely on time series data. We have shown, however, that the GG and Pindyck pooled cross-section time series elasticity estimates must be interpreted carefully, and that in particular they are not inconsistent with E-K complementarity. Moreover, Fuss' pooled cross-section time series data yields E-K complementarity. Hence the short-run, long-run E-K substitutability-complementarity issue does not seem to be simply one of pooled versus time series data.¹ In our view, this issue cannot be resolved at this time, for even if extremely reliable data were available, we still would need an economic model of the disequilibrium or adjustment process. At the present time, the econometric literature on dynamic adjustment processes relies largely on ad hoc, constant coefficient adjustment specifications, rather than on explicit dynamic optimization.²

Third, a number of data problems arise. In their international pooled cross-section time series studies, both GG and Pindyck were unable to take into account variations in effective corporate and property tax rates among OECD countries and over time. Also, both studies computed the value of capital services as

¹In background work for his [1976b] paper, Edwin Kuh ([1976a], Table 6c) finds that when the six "recession" or "excess capacity" years of 1949, 1954, 1958, 1961, 1970, and 1971 are dropped from the Berndt-Wood 1947-71 data set, E-K complementarity still results, albeit in a smaller absolute magnitude. Also, E.R. Berndt and M.S. Khaled [1977] use the Berndt-Wood data and find that E-K complementarity is robust even when the assumptions of constant returns to scale and Hicks neutral technical change are relaxed, and when flexible functional forms other than the translog cost function are employed.

²Research on this topic is presently underway; see E.R. Berndt, M.A. Fuss, and L. Waverman [1977].

value added minus the wage bill. This procedure has been criticized by, among others, E.R. Berndt [1976], for the resulting residual captures not only the return to capital equipment and structures, but also the returns to land, inventories, economic rent, working capital, and any errors in the measurement of value added or wage bill. Berndt [1976] finds that elasticity estimates are very sensitive to such data errors and to the choice of rate of return. Interestingly enough, in a recent unpublished KLE study, Barry Field and Charles Grebenstein [1977] use total U.S. manufacturing cross-section data for states in 1971 and obtain E-K substitutability when the return to capital is computed as value added minus wage bill, but find E-K complementarity when the capital rental price measure refers only to plant and equipment. Clearly, research on this topic merits additional attention.

Finally, we wish to emphasize that we are not rigidly beholden to the view that in all industries over any time period, E and K are complements. We expect variations in σ_{KE} estimates across industries and over time. The post-1973 tripling in energy prices may in fact induce a change in the relative magnitudes of the E-K gross substitution and scale elasticities. Such changes are of course compatible with our analytical framework. While these data and specification issues remain problems for future research, we believe the present paper contributes substantially to clarifying and showing that engineering and econometric approaches are mutually consistent, and that seemingly disparate econometric estimates can be reconciled.

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APPENDIX I

1976 REFRIGERATOR MODEL DATA - AUTOMATIC DEFROST, REFRIGERATOR, AND FREEZER

| <u>Brand</u> | <u>List Price</u> | <u>Cubic Feet of Refrigerator Volume</u> | <u>Cubic Feet of Freezer Volume</u> | <u>Electricity Use (KWH/month)</u> |
|---------------------------|-------------------|--|-------------------------------------|------------------------------------|
| Coldspot 7655110 | \$365 | 10.92 | 4.25 | 161 |
| Coldspot 7657110 | 360 | 12.30 | 4.77 | 169 |
| Coldspot 7657010 | 360 | 12.40 | 4.60 | 136 |
| Coldspot 7657411 | 455 | 12.31 | 4.75 | 175 |
| Coldspot 7657210 | 385 | 12.31 | 4.75 | 182 |
| Frigidaire FPS-170TA | 470 | 12.26 | 4.75 | 144 |
| General Electric TBF16VR | 400 | 11.28 | 4.30 | 139 |
| General Electric TBF18ER | 450 | 12.92 | 4.65 | 155 |
| Gibson RT17F3 | 470 | 12.40 | 4.60 | 136 |
| Kelvinator TSK170KN | 488 | 12.40 | 4.60 | 136 |
| Kelvinator TSK 170KN | 520 | 12.40 | 4.60 | 136 |
| Philco Cold Guard RD-1667 | 550 | 11.99 | 3.62 | 103 |
| Philco Cold Guard RD-1767 | 510 | 12.40 | 4.65 | 101 |
| Signature UFO-1525-00 | 355 | 10.44 | 4.74 | 146 |
| Signature UFO-1715-20 | 385 | 12.28 | 4.74 | 153 |
| Signature UFO-1625-00 | 450 | 10.46 | 6.05 | 196 |
| Westinghouse RT170R | 470 | 12.45 | 4.65 | 127 |
| Whirlpool EAT17NK | 400 | 12.31 | 4.75 | 175 |
| Whirlpool EAT15PK | 415 | 10.86 | 4.19 | 160 |
| Whirlpool EAT171HK | 440 | 12.31 | 4.75 | 110 |
| Whirlpool EAT17PM | 418 | 12.46 | 4.75 | 175 |
| Philco Cold Guard RD-1768 | 550 | 12.37 | 4.65 | 104 |

Source: A. H. Rosenfeld [1977], Table 2.