ESTIMATING A POLICY MODEL OF U.S. COAL SUPPLY

by

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Introduction

It is becoming evident that traditional methods of econometric analysis are inadequate in estimating the long-run supply of minerals. Econometrics is limited to extracting estimates from observations of the past. Economic theory predicts that the least cost deposits will be exploited first.\(^1\) The estimation of parameters from past data then must lead to biased projections of the future. Mining conditions in the future will be worse than in the past and it is the impact of the future conditions on cost that we need to know.

In this paper we present a methodology for dealing with this problem. We estimate a "cumulative cost" function that yields the long-run marginal cost of mining as a function of cumulative output produced over time.\(^2\) This is relatively easy to do for coal mining in the United States where exploration for new deposits is not a major factor in supply. The first step of the procedure relates cost to the rate of output and to the geological conditions of the mine. The second step combines geological information on remaining deposits with the cost function to yield the cumulative cost function. A central theme of both these steps is the interaction of economics with geology. The best deposits are selected first and in each step of the estimation allowance must be made for this fact.

The problem of depletion is part of the general problem of structural change. The change through depletion in the nature of the exploitable mineral stock is only one factor that calls for an analysis sensitive to the changing circumstances of the industry. The factors impinging most heavily on the coal industry today are the strict environmental regulations legislated
by Congress. Strip-mining and air pollution standards limit the coal that can be supplied and burned. National air pollution regulations were first introduced in 1967 and are still changing. National strip mining legislation has just become law. Econometric estimation of supply functions based on past data would not be adequate to estimate the effects of these changing regulations. The modeling technique described below deals with this changing regulatory environment. Specific results are presented for eastern and midwestern deep-mining and for western and midwestern strip-mining. These results, in particular, have a bearing on current policy issues.

The analysis begins with a discussion of coal-mining technology. This provides the technical knowledge necessary for the cost estimation. After estimating cost functions for both strip-mining and deep-mining, the functions are combined with available data on remaining coal deposits to yield cumulative cost functions. The paper concludes with a discussion of the likely effects of environmental regulation on the evolution of coal costs.

Technology

There are two primary techniques for producing coal. Underground methods are used for coal seams lying deep (usually more than 150 feet) underground. Openings to the seam are constructed and mining takes place beneath the surface. When the coal seams lie close to the surface, the earth and rock above the seam, or the overburden, is removed and the coal is excavated at the surface.

Deep-Mining Technology

There are three types of deep mine: drift, shaft and slope. When the coal outcrops on a hill, a drift opening is constructed that provides access
at the level of the seam. When the seam does not outcrop on a hillside, shafts or slopes are constructed (see Figure 1). Once the seam is reached, the mining of coal can begin.

The dominant technique for mining coal underground in the United States and the one upon which the following estimation is based is continuous mining. In continuous mining, a large machine rips the coal from the seam and loads it onto shuttle cars in one continuous operation. The shuttle cars transfer the coal to a central transport network for removal to the surface. A mining machine, two shuttle cars, and a complement of miners comprise a mining section. A mine consists of a number of sections, each working independently, but sharing a common haulage system, ventilation system and a set of openings to the seam that provide access for miners, for supplies, and a means for removing the coal.

Within a mining section, the ratio of capital to labor is relatively fixed. Non-continuous techniques are also used, but, at current factor prices, the large new mine, the mine that determines the cost of coal at the margin, will use the continuous mining method. This will be true for a wide variation in factor prices.

Surface Mining Technology

In surface mining, the overburden is removed with large earth-moving equipment. Then, using smaller shovels, coal is removed and loaded onto trucks. After coal removal, the overburden is returned to the pit and the ground is recontoured and reclaimed. Equipment used varies from region to region in the United States. In the west and midwest, where relatively
Principal Types of Coal Mines

Figure 1

Source: S.M. Cassidy, Ed., Elements of Practical Coal Mining
large parcels of reserves lie under flat or gently rolling terrain, large
draglines and shovels with capacities of up to 300 cubic yards are used.
In the hills of Appalachia smaller equipment is the norm. The hilly terrain
forces operations on narrow hillsides with frequent moving of machines. This
can be accomplished only with smaller, more mobile machinery.

Surface mining is more capital intensive than deep mining. The size of
draglines and shovels has increased dramatically over time, so that one
operator with one machine can move a large amount of material in one machine
cycle. In the last five years, because of their great flexibility, large
draglines have come to dominate the large-scale surface mining operation. The estimation below is based on dragline operation, although this is not
important and consideration of shovels would lead to only minor changes.

Estimation Procedure

Although the specifics differ, there is a common structure to the pro-
cedure used to estimate cost functions for both deep and strip mining.
Mining costs are determined by the interaction of geology and of economic
behavior. The rate of output is an economic choice and it is affected by
the geology of the mine. This interaction is the subject of the following
sections.

Physical Characteristics and Costs

The U.S. Bureau of Mines collects data on coal reserves classified broadly
by seam thickness for deep reserves, and by the overburden ratio, that is feet
of overburden to feet of coal seam, for strip reserves [13,14]. The latter ratio
is simply a measure of the material that must be removed to expose a foot of coal seam. Clearly, these factors correlate with cost. However, one is impressed in reading a coal mining manual by the multiplicity of factors affecting costs:

"Natural conditions involve roof, floor, grades, water, methane and the height of the seam .... In addition to these normal conditions, there are, in some mines, rolls in the roof or floor, and clay veins of generally short horizontal distance that intersect the coal seam. All these must be taken into account.

"It is possible for an experienced engineer to examine previous conditions of the sections and the immediate area of the section and assess proper penalties. As an example, if the roof is poor, production is reduced by as much as 15% of the available face working time. If the floor is soft, fine clay and water is present, the production handicap could be as much as 15%. If a great deal of methane is being liberated, so that it is necessary to stop the equipment until the gas has been bled off, this delay could run as high as 10%. Fortunately, only a few mines in the United States have such severe conditions. The same remarks apply to all the other natural conditions."[1]

The following partial list of factors to be considered when planning a surface mine is from an authoritative mining manual.[8]

1. Total coal reserves and general arrangement of deposit ...
2. Annual capacity
3. Nature of terrain
4. Maximum depth
5. Nature of overburden ...
6. Type of overburden ...

Unfortunately, data are not generally available on all these cost-determining variables. Ideally, the following cost function would be estimated:

\[ c = \phi(G_1, G_2, G_3 \ldots G_n, Q) \]

where \( G_i \) = the \( N \) different appropriate geological characteristics and \( Q \) =
output. For deep mining the only data available are on seam thicknesses. Assuming seam thickness is independent of other factors represented by \( \epsilon \), the following relationship is estimated:

\[
c = \psi(Th, Q) + \epsilon
\]

where \( Th = \) seam thickness

and \( \epsilon = \) disturbance term, reflecting other natural mining conditions.

Similarly, assuming overburden ratios are independent of other mining conditions, a strip-mining cost function is estimated as:

\[
c = \phi(R, Q) + \eta
\]

where \( R = \) the ratio of feet of overburden to feet of coal seam, and \( \eta = \) a disturbance term. The disturbance terms reflect the non-observable mining conditions. The more important and variable these factors are, the greater will be their variance. The variance, estimated together with the cost functions, will be directly incorporated in the long-run cumulative cost curves.

A Summary of the Procedure

No data are available detailing cost by mine. One is forced to rely on an indirect procedure. The above citations suggest a method for estimating cost. The production process within the mine is modeled as the sum of the operations of individual processing units. In deep mining, these are the sections. In strip mining the relevant unit is more complicated and is described below. The first step of the procedure is to estimate a relation-
ship between productivity of the relevant producing units that comprise a mine and output and geological factors:

\[
\frac{Q}{u} = f(Q, G_i) + \epsilon
\]  

(1)

where \( Q \) = output of the mine 
\( u \) = the number of production units within the mine 
\( G \) = observable geological characteristics 
\( \epsilon \) = unobservable geological characteristics

Capital expenditures and expenditures for labor and supplies will be seen to be functions of the number of producing units:

\[
E_i = f(u) \quad i = 1, \ldots, N
\]  

(2)

where \( E_i \) refers to the class of expenditures. Finally, using (1) and (2), \( E \) can be expressed as a function of \( Q, G \) and \( \epsilon \). This then yields cost as a function of output, observable, and unobservable geological characteristics. While the procedure, in the abstract, is the same for deep and strip mining, the actual estimation differs significantly.

**Deep Mining Cost Functions**

**The Productivity Equation**

The estimation begins with an analysis of drift mines where the depth is effectively zero. Once the costs of a drift mine are determined, the added costs of deeper mining are considered.

The relevant unit in an underground mine is the producing section. The productivity of a unit in isolation is written as:

\[
q = A T h^\gamma \epsilon
\]  

(3)
where $q =$ output per section shift

$Th =$ seam thickness

$\gamma, A =$ constants

$\epsilon =$ disturbance reflecting the impact of unobserved mining conditions.

The multiplicative form for the equation expresses the interaction between natural mining conditions and thickness described in the above citation. One rarely observes a unit in isolation. The available data force one to measure $q$ as total mine output divided by the number of unit shifts. A mine is usually comprised of many units. Since these units share common equipment for haulage, ventilation, etc., their productivity is not independent of one another. Larger mines will have greater logistics problems. The larger the mine, the greater the travel time to the working sections. For these reasons, scale will affect the observed productivity per unit. To capture this effect (3) is rewritten as:

$$\frac{Q}{S} = q = A Th^\gamma S^\alpha \epsilon \quad (4)$$

where $S =$ the number of producing units, or sections

$Q =$ total mine output.

Production per section should decline as congestion takes its toll. However the problems faced by larger mines can be mitigated by capital expenditure. As congestion effects increase costs, new openings to the seam can be constructed, lessening the difficulties. In short, the observed scale effects will reach a limit as new shafts are opened to the seam. Equation (4) is rewritten to capture the effect of increasing the number of openings:
\[ \frac{Q}{S} = A \theta S^\beta Op^\alpha \epsilon \]  

where \( Op \) is the number of openings (drift, shaft, slope) to the seam.

**Econometric Difficulties**

(a) Truncation.

The fact that mining proceeds from the least costly deposits to ever more costly deposits presents a complication for estimation. The economic system generates the sample. Future mines cannot be observed. The observations consist of mines already opened. Since mines will open only if they can earn a profit, the sample consists of those mines with values of seam thickness and \( \epsilon \) such that they earn a profit at current prices. For mines opening in thin seams the value of \( \epsilon \) must be relatively favorable. This condition can be written as

\[ A \theta S^\beta Op^\alpha \epsilon \geq T \]

where \( T \) is the minimum productivity rate necessary for profitable operation. Therefore, while in nature \( \theta \) and \( \epsilon \) are assumed uncorrelated, in the sample this cannot be true, and therefore ordinary least squares is biased. This is another manifestation of the inadequacy of past data in forecasting mineral supply.

The extent of the OLS bias depends upon how close the bulk of the observations are to the truncation point. In the U.S. east of the Mississippi River, the observations represent mines opened over many years. Most mines will be non-marginal, that is, far from the truncation point. Consequently, the truncation of the sample is not likely to lead to a large bias.
However, in order to eliminate this potential source of bias in OLS, a maximum-likelihood estimator is used that takes explicit account of the truncation problem.  

The Choice of T  

The level of productivity sufficient for profitable operation is not known. An estimate of that level requires knowledge of the parameters to be estimated. The assumption used here is that the lowest productivity rate observed in the sample is the truncation point. This is an extreme assumption, since the actual rate could be lower. This assumption allows an estimate of the maximum bias of least squares.

Heteroskedasticity  

The second problem arises because of the manner in which the data are collected. The data come from mine inspection reports and production is the output of the entire mine. Recall that the productivity figure is derived by dividing total mine production by the number of sections in the mine. A problem in a large mine that shuts down one or two sections will have a relatively small effect on observed production per section. A small mine, on the other hand, could see production per section cut substantially. This means that the variance in production per section will be inversely proportional to the number of sections. Assume that the variance can be written as:

\[ \nu(\log \epsilon) = \frac{\sigma^2}{S} \]

where \( \sigma^2 \) is a constant and \( S \) the number of sections. The adjustment for heteroskedasticity thus simply weights all variables by the square root of \( S \).
Results

The maximum likelihood results are as follows:

\[
\sqrt{s} \log \frac{Q}{s} = 0.7568\sqrt{s} + 1.1071(\log \text{Th})\sqrt{s} - 0.2185(\log s)\sqrt{s}
\]

(S.E.) (.4842) (.1205) (.0594)

(t-statistic) (1.5630) (9.1906) (-3.6762)

\[+ 0.0283(\log \text{Op})\sqrt{s}\]

(S.E.) (.0655)

(S.E.R.) = .9799

\[\chi^2 = 232.4285\]

N = 244

These results indicate the importance of seam thickness and confirm our a priori expectations. A 1% decrease in seam thickness leads to a 1.10 percent decline in productivity per section. The negative value of the coefficient of \(\log s\) implies decreasing returns to the number of sections. The positive value of the coefficient of Op means, as expected, that additional openings to the seam stem the productivity decline. Also, the standard error of the regression, the estimate of \(\sigma\), indicates a substantial dispersion about the conditional mean of productivity per section. The variance of productivity per section is:

\[
v(\varepsilon) = e^{(2\mu + \frac{\sigma^2}{S})} (e^{(\sigma^2/S)} - 1)
\]

where \(\mu\) is the mean of the distribution of \(\log \varepsilon\) and is zero by assumption. For a single unit:

\[
v(\varepsilon) = e^{\sigma^2} (e^{\sigma^2} - 1) = 4.21
\]
This is a substantial dispersion and its implications are discussed below.

The productivity equation can be rewritten as:

\[ q = 2.1314 \text{Th}^{1.1071} \text{S}^{-0.2185} \text{Op}^{0.0283} \epsilon \]  

(8)

The number of units necessary to maintain a given rate of annual production in a given seam thickness, assuming three shifts per day and a work year of 245 days is:

\[ S = \frac{\bar{Q}}{Q \times 3 \times 245} \]  

(9)

where \(\bar{Q}\) = annual output.

Solving (9) for \(S\) by substituting equation (8) in equation (9) yields:

\[ S = \left( \frac{\bar{Q}}{1566.579 \text{Th}^{1.1071} \text{Op}^{0.0283} \epsilon} \right)^{1.2796} \]  

(10)

Underground Mine Expenditures

Once the number of sections is determined, the expenditures on labor, capital, and supplies can be determined. In addition to the capital equipment used in each producing section, a great deal of equipment is shared by all sections. This shared equipment includes ventilation equipment, transport for miners and machines, etc. The number of units determines the extent of the mine underground and should therefore determine the extent of this haulage, and ventilation equipment and support material such as rescue equipment.

This hypothesis is tested using engineering cost estimates prepared by the Bureau of Mines[11]. These are estimates of the equipment, labor and supplies necessary for hypothetical mines. They represent estimates for mines assumed to produce a given level of output in a seam with a thickness either 48 or 72 inches thick under some unspecified set of mining conditions.
Because the mining conditions are unspecified, it is difficult to assess how the number of necessary units was arrived at. These engineering estimates serve here only to establish the relationship between the number of units, however arrived at, and necessary expenditure.

The following equation was estimated for each class of expenditure:

\[ E = \alpha + \beta S + \epsilon \]

where \( E \) = expenditure

\( S \) = number of sections

\( \epsilon \) = disturbance term

The results are presented in Table 1. In spite of the small number of observations, the results are good. The high \( R^2 \) indicates an implicit or explicit relationship used by the engineers in constructing these model mines.

The equipment expenditures are reported in equations (11) through (14). Capital equipment has been subdivided into equipment with different operating lifetimes. Thus, equation (11) represents capital goods with 5-year lifetimes, equation (12) represents capital with 10-year lifetimes, etc. This breakdown is necessary since the correct treatment of depreciation must adjust for the useful life of the equipment. Associated with the direct expenditures are expenditures for engineering, overhead, and various small construction tasks. These are assumed as a fixed 28 percent of initial direct capital expenditures by the Bureau of Mines [11]. Operating and labor costs are shown as equations (15) and (16) of Table 1. Labor costs reflect union wage agreements. In addition, following the Bureau of Mines,
<table>
<thead>
<tr>
<th>Equation</th>
<th>Item</th>
<th>Constant</th>
<th>Slope</th>
<th>$R^2$</th>
<th>$F(1/5)$</th>
<th>S.E.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(11)</td>
<td>$I_5$</td>
<td>88380.5</td>
<td>25528.0</td>
<td>.9312</td>
<td>67.6843</td>
<td>46202.7</td>
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<td>5-Yr Capital</td>
<td></td>
<td>(39528.5)</td>
<td>(3102.92)</td>
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<td></td>
<td></td>
<td>(2.23587)</td>
<td>(8.22708)</td>
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<td></td>
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<tr>
<td>(12)</td>
<td>$I_{10}$</td>
<td>2,193,350</td>
<td>546842</td>
<td>.9934</td>
<td>751.994</td>
<td>296930</td>
</tr>
<tr>
<td>10-Yr Capital</td>
<td></td>
<td>(254037)</td>
<td>(19941)</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(8.6340)</td>
<td>(27.4224)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(13)</td>
<td>$I_{20}$</td>
<td>597,135</td>
<td>53,870.7</td>
<td>.9861</td>
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<td>20-Yr Capital</td>
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<td>(36493.1)</td>
<td>(2864.65)</td>
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<td>(16.3629)</td>
<td>(18.8053)</td>
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<td>(14)</td>
<td>$I_T$</td>
<td>2,878,870</td>
<td>626,241</td>
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<td>(210178)</td>
<td>(16498.6)</td>
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<tr>
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<td></td>
<td>(13.6973)</td>
<td>(37.9571)</td>
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<tr>
<td>(15)</td>
<td>LAB</td>
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<td>509,088</td>
<td>.9995</td>
<td>11026.5</td>
<td>72,189.2</td>
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<td>(61761.2)</td>
<td>(4848.14)</td>
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<td>(6.59974)</td>
<td>(105.007)</td>
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<td>(16)</td>
<td>SUP</td>
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<td>352,102</td>
<td>.9978</td>
<td>2250.23</td>
<td>110523</td>
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<td>(7422.6)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>(-2.12265)</td>
<td>(47.4365)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are the Standard Error and t-Statistic respectively.
35 percent is added to account for overhead and fringe benefits.

Allowance is made for the Union welfare charge (80¢ per ton in 1977), as well as indirect operating costs. The latter is estimated by the Bureau of Mines as 15 percent of total operating costs. The costs of loading the coal into trains must be added. The cost of a facility capable of loading up to 5 million tons per year is about $1,000,000. Annual labor costs for this facility add $57,000. Finally, working capital, that is funds necessary to begin operations, is estimated by the Bureau of Mines as 25% of annual direct operating costs plus 33% of annual indirect operating costs plus 3% of initial direct investment.

In summary, the underground drift mine has been modeled as a conglomerate of producing sections. The productivity of these units was related to seam thickness, the number of sections, and the number of openings. In turn, expenditures were related to the number of sections.

Slope and Shaft Costs

For deep mines there are the added costs of sinking shafts and slopes, as well as the additional operating costs involved in mining deep underground. A recent estimate for a mine 796 feet below the surface indicates that a set of one intake and one outtake shaft cost $6,217,500 in 1974[16]. The cost of additional depth is small since fixed costs are large and the average cost per cubic foot of shaft declines rapidly as cubic feet increase. There is an additional ventilation cost and additional haulage costs, however the total is small[12].
Costs in the Long Run

Summing equations, adding indirect costs, and the cost per shaft, or slope, and loading equipment yields total expenditures on equipment as a function of sections and openings. This is converted to an annual capital cost, based on the assumption of a 10% rate of return after tax, all equity financing, the sum-of-years depreciation and allowance for a 7% investment tax credit. The capital costs are added to operating costs, and all costs are adjusted for inflation to yield the total cost function in 1977 dollars:

\[
\text{TOTAL Annual Underground Cost} = 1,743,222 + 2,122,480(S) + 1,085,771(\text{OP})
\]

Substituting (10) into the above equation yields:

\[
\text{TOTAL Annual Underground Cost} = 1,743,222 + 2,122,480\left(\frac{\tilde{Q}}{1567 \text{ Th} 1.1071 \text{ OP} 0.0283} \right) + 1,085,771(\text{OP})
\]

(17)

In the long run, the marginal cost of producing coal is the minimum average cost of a new mine. The task is thus to establish the minimum efficient scale of a mine, and evaluate the average cost at that point. Dividing by \(Q\), differentiating with respect to \(Q\) and setting the resulting equation equal to zero, yields an expression for the minimum efficient scale:

\[
Q^* = \left(\frac{1,743,222 + 1,085,771(\text{OP})}{593,445}\right)^{0.7815} 1567 \text{ Th}^{1.1071 \text{ OP} 0.0283} \epsilon
\]

(18)
The minimum efficient scale depends upon OP, the number of openings, and upon Th, the thickness of the seam. The latter is determined by geology; the former is an economic choice. To determine the number of openings, we differentiate (17) with respect to openings, substitute (18) into the resulting equation, and solve for openings. The result, however, yields less than one opening as a result. Clearly, there must be a constraint. A minimum number of openings is necessary to produce. Two openings is taken as the minimum number, yielding equations for minimum efficient size and for minimum average cost:

\[ Q^* = 6981 \, Th^{1.1071} \]

\[ AC^* = \frac{2567}{Th^{1.1071} \epsilon} \]  

(19)
The resulting minimum average cost level of output is a function of seam thickness and $\varepsilon$. This is depicted in figure (2) as locus AC*. The values of $Q^*$ shown in Table 2 indicate minimum efficient sizes for deep mines that fall short of the largest mines in actual operation. This is because the largest mines will represent some duplication of efficient units. Also, the influence of $\varepsilon$ means that for any given seam thickness, there will be a distribution of minimum efficient sizes. The locus AC* reflects the interaction of economics and geology. As seam thickness declines, the cost increase is mitigated, to some extent, by reducing the rate of output.

**Strip-mining Costs**

The procedure followed in estimating the strip-mining cost functions is analogous to that used for estimating deep mining cost functions. The analysis begins with the productivity of a mining unit, except that a unit is defined in a rather different way.

The dragline is the most commonly used equipment for overburden removal in large-scale strip mining. In the last five years it has dominated new equipment purchases. The cost functions developed below are therefore based on that technology. The size of draglines used in the industry has been increasing steadily over time. Since draglines have become larger and larger, the relevant unit is not the dragline itself, but rather its size. Productivity is defined as the output, or overburden removed, per unit of size.
### TABLE 2

Minimum Efficient Size and Minimum Average Cost
as a Function of Th for Deep Shaft Mines

<table>
<thead>
<tr>
<th>Th</th>
<th>Q*</th>
<th>AC*</th>
</tr>
</thead>
<tbody>
<tr>
<td>28 inches</td>
<td>279,279</td>
<td>64.16</td>
</tr>
<tr>
<td>36 inches</td>
<td>369,870</td>
<td>48.58</td>
</tr>
<tr>
<td>42 inches</td>
<td>437,512</td>
<td>40.96</td>
</tr>
<tr>
<td>60 inches</td>
<td>649,355</td>
<td>27.60</td>
</tr>
<tr>
<td>72 inches</td>
<td>794,591</td>
<td>22.55</td>
</tr>
<tr>
<td>90 inches</td>
<td>1,017,261</td>
<td>17.62</td>
</tr>
</tbody>
</table>

Source: Equations (18) and (19) where OP = 2

ε assumed equal to 1
FIGURE 2

Locus of Minimum Average Cost Points - $AC^*$
What is needed is a measure of the capacity of the machine. Engineers put emphasis on what is called the maximum usefulness factor or MUF: 

\[ MUF = d \times r \]  

(20)

where \( d \) = the capacity of the dragline bucket

\( r \) = the dumping reach.

These variables are shown in relation to a dragline in Figure (3). This is a measure, in effect, of the working capacity of the machine: How much material can be moved and how far can it be moved in one operation of the machine? How large a machine is necessary to remove a given quantity of overburden in a year? This will depend on how efficient a machine is.

Efficiency can be expected to vary with size, since large size will affect how far a machine must move, and how long it takes for a machine cycle. These are engineering relationships that are measured in the following regression:

\[ MUF = A(OV)^\alpha \epsilon \]  

(21)

where \( OV \) = total overburden removed, measured in cubic yards

\( \epsilon \) = disturbance term reflecting natural conditions

\( A, \alpha \) = constants

Again, \( \epsilon \) reflects the importance of characteristics that are not observed.

It is reasonable to expect that this relation will also be multiplicative.

Natural factors will interact with the quantity of overburden, making each cubic yard more difficult to move.

In many mines several draglines are used. The working capacity of two
Figure 3

"Working Diagram"
draglines each with one-half the capacity of a large dragline is equal, according to the above definition, to the capacity of one large dragline. However, in actual operation, using more than one dragline could be a more complicated process involving slower operations. As the overburden is removed from one part of the seam, the dragline moves to a new part of the seam. Draglines with larger reach need move less often than smaller draglines since their long reach allows them to uncover overburden over a wider radius. In order to capture this effect, equation (21) is rewritten as:

$$\sum MUF = A(OV)^{\alpha} N^\beta$$  \hspace{1cm} (22)

where $\sum MUF$ = the total capacity of all machines\(^{18}\) and $N$ = the number of machines in use.

The amount of overburden can be rewritten as a function of the product of the annual output, and the overburden ratio, $R$.

$$OV = ZRQ$$

where $Z$ = constant $= .89$ (see Footnote 19)

$R$ = overburden ratio = feet of overburden per foot of coal

$Q$ = annual output rate.

Equation (22) is now written as:

$$\sum MUF = AZ^{\alpha}(RQ)^{\alpha} N^\beta$$,  \hspace{1cm} (23)

de the form actually estimated.\(^{20}\)
Results

The resulting estimates are:

\[ \log MUF = -.446684 + .612306 \log RQ + .506967 \log N \]  
(Std. Error) (2.31895) (.145663) (.229134) 
(t-statistic) (-.192623) (4.20357) (2.21254)

\[ R^2 = .8595 \]

\[ F(2/7) = 21.4089 \]

\[ \text{S.E.R.} = .287762 \]

The results confirm what one expects based on the history of increasing dragline size. The value of the coefficient of \( \log(RQ) \) indicates there are increasing returns. It takes only a 0.6% increase in size to handle a 1% increase in overburden removal. The value of the coefficient of \( \log N \) indicates that one large machine is better than two small ones having the same total capacity. This creates the incentive for larger and larger sizes.

In this case dispersion is smaller than for deep mining, indicating that other natural conditions explain a smaller proportion of mining productivity. In this case the variance of \( \varepsilon \) is .094. Because of the small number of observations, these results have been checked against more extensive data available from an earlier period. In 1959, a survey of operating experience on draglines was conducted. The resulting equation for that period is:

\[ \log MUF = -5.01166 + .858326 \log(\text{cu-yds}) + .711493 \log N \]  
(Std. Error) (2.29250) (.150932) (.266627) 
(t-statistic) (-2.18611) (5.68683) (2.66852)

\[ R^2 = .6161 \]  
\[ \text{S.E.R.} = .288071 \]

\[ F(2/21) = 16.8488 \]
While the parameters have changed, the character of the results is the same. The surprising result is that the estimated variance is also the same. The larger sample causes more confidence in the results, while the parameter values used are those for the 1975 data. Rewriting (24),

\[
\text{MUF} = 0.6397(RQ)^{612306 N^{-506967}} e
\]

(24B)

Expenditures

Following the procedure used for deep mining, expenditures are related to the number of producing units, in this case the size of the dragline. The size of dragline is a function of the amount of overburden to be removed. However, there will also be expenditures solely dependent on the amount of coal produced. Therefore, each class of expenditure is related to both the size of dragline and to the annual production of coal. An equation of the following form is estimated:

\[
E = \alpha + \beta(\text{MUF}) + \gamma Q
\]

(25)

where \( Q \) is annual output in tons. The results are presented in Table 3. Equation (25) is the dragline cost equation. This relates the cost to size. The data come from the major dragline manufacturers. Again, for calculating depreciation, capital goods are divided according to lifetime. The other capital cost data are from hypothetical mine analyses of the U.S. Bureau of Mines [12]. Equation (25) suggests that MUF's do determine the cost of the dragline. Equation (26) relating all other capital expenditures to MUF and \( Q \) also provides a good fit. When the expenditures are subdivided according to equipment lifetimes, the fit is less good. In two cases (27 and 29) there is a negative coefficient. The fact that the total investment equation is
a good approximation, but the individual investment equations are not, indicates a good deal of substitutability among types of equipment. Some allocation of the total investment by life of capital good is necessary for depreciation accounting and equations (27) through (30) play this role here. About half of total capital expenditures are represented by the dragline so that incorrectly depreciating a fraction of the remainder of capital equipment will not lead to serious error. Comparing the strip expenditure equations to the deep expenditure equations indicates more flexibility in choice of equipment in planning a strip mine.

Costs in the Long Run

A total cost equation is constructed as for deep mining, by converting the capital expenditures to an amortized annual cost, adding labor and operating costs, and adding loading costs. Indirect costs are included on the same basis as they were for deep mining. The resulting cost equation is:

\[ TC = 3,170,223 + 467,262 \text{ MUF} + .96Q \] (33)
TABLE 3

Strip Mine Expenditures

<table>
<thead>
<tr>
<th>Equation</th>
<th>Item</th>
<th>Constant</th>
<th>Coeff. of MUF</th>
<th>Coeff. of Q</th>
<th>( R^2 )</th>
<th>( F )</th>
<th>S.E.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(25)</td>
<td>Dragline Cost (s.e.)</td>
<td>1,586,500</td>
<td>593.48</td>
<td>.9175</td>
<td>177.910</td>
<td>1,885,590</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(t-statistic)</td>
<td>(702532)</td>
<td>(44.4945)</td>
<td>(13.3383)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(26)</td>
<td>( I_T ) (All capital other than dragline)</td>
<td>3,107,220</td>
<td>17.2718</td>
<td>.213019</td>
<td>.9335</td>
<td>35.0953</td>
<td>222357</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(142884)</td>
<td>(5.72053)</td>
<td>(.0577415)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(21.7465)</td>
<td>(3.01926)</td>
<td>(3.68919)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(27)</td>
<td>( I_5 )</td>
<td>268,374</td>
<td>-12.0284</td>
<td>.247652</td>
<td>.8680</td>
<td>16.4385</td>
<td>171719</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(110344)</td>
<td>(4.41771)</td>
<td>(.0445918)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.43215)</td>
<td>(-2.72274)</td>
<td>(5.55374)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(28)</td>
<td>( I_{10} )</td>
<td>596,687</td>
<td>1.72287</td>
<td>.0112230</td>
<td>.4681</td>
<td>2.20031</td>
<td>65819.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(110344)</td>
<td>(1.69333)</td>
<td>(.0170920)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(14.1078)</td>
<td>(1.01745)</td>
<td>(6.56623)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(29)</td>
<td>( I_{201} )</td>
<td>1,762,480</td>
<td>22.0264</td>
<td>-.114750</td>
<td>.8461</td>
<td>13.7415</td>
<td>167271</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(107486)</td>
<td>(4.30334)</td>
<td>(.0434368)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(16.3973)</td>
<td>(5.11844)</td>
<td>(-2.64176)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(30)</td>
<td>( I_{20}^* )</td>
<td>479,674</td>
<td>5.55102</td>
<td>.0688944</td>
<td>.7736</td>
<td>8.54082</td>
<td>145369</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(93412.2)</td>
<td>(3.73987)</td>
<td>(.0377493)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5.13502)</td>
<td>(1.48428)</td>
<td>(1.82505)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(31)</td>
<td>LAB</td>
<td>448,156</td>
<td>-11.7455</td>
<td>.0427509</td>
<td>.8727</td>
<td>17.1455</td>
<td>107454</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(68894.9)</td>
<td>(4.40273)</td>
<td>(.0315682)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6.50492)</td>
<td>(2.66778)</td>
<td>(1.35424)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(32)</td>
<td>SUP</td>
<td>-61,161.9</td>
<td>62.0065</td>
<td>.228441</td>
<td>.9753</td>
<td>98.8775</td>
<td>237134</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(152040)</td>
<td>(9.71610)</td>
<td>(.0696658)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-.4002276)</td>
<td>(6.38183)</td>
<td>(3.27910)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are the Standard Error and t-Statistic respectively.

* \( I_{20}^* \) represents capital ineligible for the investment tax credit.
Substituting (24B) into (33), dividing by Q, and setting N = 1 yields:

\[ AC = \frac{3,170,223}{Q} + \frac{299}{Q^{.3877}} R^{.6123} \epsilon + .96 \]  

(34)

Average costs are declining over the range in our sample. This suggests an unstable industry. However, while these economies of scale have theoretically not been exhausted, in reality there are always barriers in the short-run to building larger and larger equipment. Transportation of such large components presents a difficulty. Welding huge components might not be feasible. Therefore at any point in time average costs will turn up and the constraint is reached. Over time these constraints are overcome and the equipment increases in size.

One constraint on the size of machine therefore is technological. However, there are other constraints operating on the observed size of machines. Large draglines are costly to transport over large distances. They are costly to erect and then to knock down. It is prohibitively expensive to erect the largest feasible dragline unless it will be amortized over a sufficiently large amount of cumulative production over time. In other words, only if the parcel of reserves is large enough, will the technological constraint be binding. Otherwise, something smaller will be erected.

An important factor affecting the size of dragline actually chosen is the contour of the terrain. In very hilly terrain, only smaller machines will be stable. Also, in extremely hilly terrain equipment must be more mobile as mining proceeds from hill to hill. For all these reasons, the technological constraint is not likely to be binding in the majority of mines in a given area. If the technological constraint is substituted in
equation (33), the resulting cost equation is a lower limit. Since non-
technological constraints are likely to be binding, these cost functions
will be biased. Although data on machine sizes in new mines are unavailable,
the available data can be used to estimate a non-technological constraint. 24
Equation (24B) gives the relations between R, Q, ε and \( \overline{\text{MUF}} \). New mines
opening in a given area will push actual machine size to the constraint, \( \overline{\text{MUF}} \). If one could observe \( \overline{\varepsilon} \), along with R and Q, one would know \( \overline{\text{MUF}} \).
All that is actually observed is R and Q. Since \( \varepsilon \) is lognormal, the best
guess about \( \overline{\text{MUF}} \) will come from substituting the geometric mean of the
observed RQ in equation (24B), along with a value of one for \( \varepsilon \). The average
cost equation results from inserting the \( \overline{\text{MUF}} \) thus estimated into (33) and di-
viding by Q. Solving (24B) for Q after inserting \( \overline{\text{MUF}} \) yields an equation for
minimum efficient scale.

\[ \text{WEST (Powder River Basin):} \]
\[ \begin{align*}
\overline{\text{AC}} & = .52 R^1.63317 + .96 \\
\overline{\text{Q}} & = \frac{25,966,542}{R^1.63317}
\end{align*} \]  

\[ \text{MIDWEST:} \]
\[ \begin{align*}
\overline{\text{AC}} & = .67 R^1.63317 + .96 \\
\overline{\text{Q}} & = \frac{16,277,052}{R^1.63317}
\end{align*} \]  

Table 4 summarizes the above equations. The huge mine sizes predicted
for very low overburden ratios are consistent with new strip mines opening
in the western part of the United States. The greater open areas allow, in
general, larger machines and lead to lower costs. The higher overburden
ratios of the midwest yield the smaller mines.
TABLE 4

Minimum Efficient Size and Minimum Average Cost
as a Function of Overburden Ratios for Surface Mines

Powder River Basin

<table>
<thead>
<tr>
<th>R</th>
<th>Q*</th>
<th>AC*</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5,193,234</td>
<td>3.57</td>
</tr>
<tr>
<td>10</td>
<td>2,596,618</td>
<td>6.17</td>
</tr>
<tr>
<td>15</td>
<td>1,731,079</td>
<td>8.78</td>
</tr>
<tr>
<td>20</td>
<td>1,298,309</td>
<td>11.38</td>
</tr>
</tbody>
</table>

Midwest

<table>
<thead>
<tr>
<th>R</th>
<th>Q*</th>
<th>AC*</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3,255,410</td>
<td>4.31</td>
</tr>
<tr>
<td>10</td>
<td>1,627,705</td>
<td>7.66</td>
</tr>
<tr>
<td>15</td>
<td>1,085,137</td>
<td>11.01</td>
</tr>
<tr>
<td>20</td>
<td>813,853</td>
<td>14.36</td>
</tr>
</tbody>
</table>

Source: Equation (35)

$C$ assumed equal to 1

Costs shown exclude UMW welfare ton charge, cleaning costs, and reclamation costs under new law.
Reserves and Cumulative Costs

Equations (20) and (35) can be rewritten in log form as:

\[ \log \frac{AC_*}{D} = \log \kappa_D - \gamma_D \log Th - \log \epsilon \]

\[ \log(AC_* - \gamma_S) = \log \kappa_S + \log R + \frac{1}{\alpha} \log \eta \]

where \( D \) refers to deep- and \( S \) to strip-mined coal.

The distribution of underground coal according to the cost of mining depends upon \( \gamma_D, \kappa_D \), the distribution of seam thickness, \( Th \), and upon the distribution of natural factors, \( \epsilon \). The parameters \( \gamma_D \) and \( \kappa_D \) were estimated in equation (6). The variable \( \epsilon \) is, by assumption, lognormally distributed. Its variance was estimated in equation (6) and the mean of \( \log \epsilon \) is zero. All that remains to be estimated is the distribution of \( Th \).

Similarly, for strip mining, the parameters \( \kappa_S, \gamma_S \) and \( \alpha \) were estimated in equation (24). The distribution of \( \eta \) is lognormal, with the mean of \( \log \eta \) equal to zero, and variance as estimated in equation (24). All that remains to be estimated is the distribution of \( R \), or the distribution of strip coal according to the overburden ratio.

The Distribution of \( Th \)

The United States Geological Survey presents coal reserve data in broad seam thickness intervals. They record coal lying in seams between 28 inches and 42 inches thick and seams greater than 42 inches in thickness[13]. Considering the impact of seam thickness on cost, this is inadequate for a complete description of the cumulative cost function. An approximation is used that establishes a method for more accurate estimation as more data become available.

There are data available on the distribution of tons of coal in the ground according to the thickness of the seam in which it lies for Pike County, Kentucky 26. This is a large and important coal producing country in East
Kentucky.27 The object is to approximate this actual distribution by a well-known statistical distribution. The lognormal distribution has been used toward similar purpose in studies of other minerals and proves useful here [3].

Aitchison and Brown in their book on the lognormal distribution offer a test for lognormality [2]. Plot the distribution on log-probability paper. If lognormality is a good approximation, the points will lie approximately along a straight line. This is done in Figure 4. Thus, for example, 50% of coal reserves lie in seams thicker than 42 inches. The fit is poorest in the tails, a common problem in work using the lognormal. However, the approximation appears adequate.

The variance is measured from the data shown in Figure 4. It is assumed that the lognormal distribution applies state by state and the variance remains constant from state to state. The mean in each state is estimated separately using Bureau of Mines data on seams lying in seams thicker and thinner than 42 inches[13]. The log of seam thickness distribution can be converted to a standard normal distribution:

\[
\frac{\log T_{th} - \log \bar{T}}{\sigma_{\log T}} = u_{\log T}
\]

where

\[
\begin{align*}
\log T_i &= \log \text{of thickness } i \\
\log \bar{T} &= \text{mean of the logs of seam thickness} \\
\sigma_{\log T} &= \text{variance of the log of seam thickness} \\
u_{\log T} &= \text{the point on the standard normal distribution corresponding to } \log T_{th}
\end{align*}
\]
Figure 4

The Distribution of Coal Reserves by Seam Thickness in Pike County, Kentucky
or:

\[
\log \text{Th} = \log \text{Th}_1 - \sigma_{\log \text{Th}} \cdot u_{\log \text{Th}_1}
\]

Using the percentage of coal in seams greater than 42 inches to establish \(u_{\log \text{Th}_1}\), one can solve for the mean of the log of seam thickness in each state. Thus, if 50 percent of the coal lies in seams greater than 42", \(u_{\log \text{Th}_1}\) is the fifty-percent point on the standard normal, or zero. Substituting the value of \(u_{\log \text{Th}_1}\) allows one to solve for \(\log \text{Th}\).

It is assumed that sulfur is distributed independently of seam thickness, so that this mean applies to all sulfur contents.\(^{28}\)

Since tons of coal by the log of seam thickness is distributed normally, the distribution of tons of coal in the ground according to the log of the cost of mining is the sum of two normal distributions and therefore itself normal.

The mean is equal to \(\log \kappa - \gamma_D \log \text{Th}\), and its variance is equal to \(\gamma_D^2 \sigma_{\log \text{Th}}^2 + \sigma_{\log \varepsilon}^2\).

**Strip Reserves**

Again there is a paucity of data. There is information for the Powder River basin area of Montana, and for the state of Illinois on the distribution of coal according to the overburden ratio [7,9]. This data is plotted on log-probability paper and is shown in Figures (5) and (6). Again the fit is good over most of the range, but diverges in the tail. Since the tails represent the highest cost coal and in the western region and
midwestern region, this coal will not be used for hundreds of years, the lognormal approximation appears adequate. The Montana distribution is assumed to hold over the western states. The Illinois distribution is assumed to hold over the midwestern states.

The variances are estimated from the data in Figures (5) and (6). The means are calculated separately for each state. The Bureau of Mines has estimated how much strip coal lies at overburden ratios of less than what they call the economic limit and how much lies at greater ratios. This economic limit varies from state to state. These data represent one point on the normal distribution function. These data are utilized to estimate the mean the same way that the mean of the seam thickness distribution was obtained. Since $\eta$ and $R$ are distributed lognormally, $(C - \gamma_S)$ is also lognormally distributed with mean equal to $\log \kappa_S + \log R$ and variance equal to $\sigma_{\log R}^2 + (1/\alpha)^2 \sigma_{\log \eta}^2$.

**Truncation Again**

There is one remaining complication. The source of the difficulty is again the fact that the least cost deposits are mined first. That implies that the coal remaining in the ground must be at least as costly to exploit as today's long-run marginal cost. If the distribution of coal according to cost is $c$, and if today's long-run marginal cost is $\bar{c}$, then the distribution of coal according to the log of the cost of mining can be written as:

\[
\frac{\phi(\log c)}{1 - \int_{-\infty}^{\log \bar{c}} \phi(\log c)}
\]

(38)

where $\phi(\log c)$ is normal. This is simply the truncated normal distribution.
Figure 5
Distribution of Coal According to Overburden Ratio
(Powder River Basin)
Figure 6
Distribution of Strip-Mineable Coal According to Overburden Ratio (Illinois)
The previous analysis indicates that in underground mining there is a trade-off between seam thickness and $\varepsilon$. For strip mining, there is a trade-off between $R$ and $\eta$. At any constant level of cost, thinner seams and higher overburden ratios are compensated for by more favorable values of $\varepsilon$ and $\eta$. If $\varepsilon$ and $\eta$ were observable, it would be a simple matter to calculate $\bar{c}$. The last mine that must be opened to satisfy demand would be the incremental mine.

Since $\varepsilon$ and $\eta$ are not observed, $\bar{c}$ must be estimated. New mines opening in a given area and producing a given quality coal should all be approximately equal-cost mines. The distribution of seam thicknesses of the new deep mines and the distribution of overburden ratios of the new strip mines will be disperse, reflecting the influence of the other natural conditions. This is, in fact, the case. Data were collected on the seam thickness and overburden ratios of new deep mines in the east opened between 1973 and 1975 inclusive. The sample was divided by region and by sulfur content. In the western states, a data base was available indicating seam thicknesses and overburden ratios for mines now in the planning stage.

The data are summarized in Table (5). The wide dispersion confirms the earlier statistical estimates of the importance of $\varepsilon$ and $\eta$.

The remaining problem is to estimate $\bar{c}$, given these observations. This involves selecting a value of $Th$ to characterize the underground mine data and a value of $R$ to characterize the surface mine. The underground cost estimate will then be given by $\kappa_u/Th^{1.1071}$ where $\kappa_u$ is the constant from equation (19) and $\bar{Th}$, the chosen value of $Th$. Similarly, the strip mine cost will be given by $\kappa_s\bar{R} + \gamma_s$ where $\kappa_s$ and $\gamma_s$ are constants estimated in equation (35) and $\bar{R}$ is the chosen $R$. Again, given the lognormality of the
TABLE 5

Deep Mines

<table>
<thead>
<tr>
<th>Area/Sulfur content/type of mine</th>
<th>Number of Observations</th>
<th>Range</th>
<th>Geometric Mean Th or R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Appalachia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High sulfur (&gt;1.5%) (Shaft)</td>
<td>7</td>
<td>42-85</td>
<td>56.8 29.96</td>
</tr>
<tr>
<td>High sulfur &quot; (Drift)</td>
<td>10</td>
<td>36-78</td>
<td>53.3 27.54</td>
</tr>
<tr>
<td>Southern Appalachia (Drift)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low sulfur (&lt;.8%)</td>
<td>29</td>
<td>35-72</td>
<td>44.1 33.25</td>
</tr>
<tr>
<td>Medium sulfur (.9 to 1.5%)</td>
<td>16</td>
<td>31-131</td>
<td>49.6 29.20</td>
</tr>
<tr>
<td>High sulfur (&gt;1.5%)</td>
<td>17</td>
<td>39-108</td>
<td>53.2 27.00</td>
</tr>
<tr>
<td>Midwest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High sulfur (Shaft)</td>
<td>5</td>
<td>49-87</td>
<td>74.1 21.84</td>
</tr>
</tbody>
</table>

Strip Mines

<table>
<thead>
<tr>
<th>Area</th>
<th>Number of Observations</th>
<th>Range</th>
<th>Geometric Mean Th or R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midwest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High sulfur</td>
<td>4</td>
<td>7.5-22</td>
<td>17.0 12.46</td>
</tr>
<tr>
<td>Montana-Wyoming</td>
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<td></td>
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</tr>
<tr>
<td>Low sulfur</td>
<td>23</td>
<td>.07-11.5</td>
<td>5.2 3.76</td>
</tr>
</tbody>
</table>

Source: Equations (19) and (35) in text.

N.B.: These costs exclude UMW welfare fee, coal cleaning costs and state taxes. In strip mining they exclude reclamation costs. Drift mine costs are calculated excluding shaft costs.
distribution, the cost estimate that maximizes the likelihood of observing the distributions of Th and of R is obtained using the geometric means of the observed Th and of R respectively. These values are reproduced in Table (5).

Table 5 indicates that the incremental cost of coal varies inversely with sulfur content, and that high sulfur midwestern coal is less expensive than high sulfur eastern coal. These results are confirmed by market relationships. The lower strip mining costs underestimate the cost of strip-mined coal. However, an important item excluded from Table 5 is reclamation cost (see Footnote 22). The addition of reclamation cost would narrow considerably the difference between the estimate of c for midwestern strip-mined coal and for the deep coal. The cost of western strip coal in Table 5 is also low. Actual price will include state taxes, which are high in Montana. Nevertheless, it appears there is a downward bias in the strip cost estimates. In the discussion that follows, we deal only with percentage increases in cost for strip mining and for deep mining separately. Consequently, the underestimate of the absolute cost level does not affect the results.
Long-Run Costs

We are now able to calculate the distribution of coal in the ground according to the cost of exploitation. The distribution is given by equation (38). The amount of coal available at up to 10% more than current cost in a given state is the total state tonnage multiplied by the probability that the cost of production is between $c$ and $1.10c$:

$$T_i \int_{\log c}^{\log 1.10c} \left[ \phi(\log c)/(1 - \int_{\infty}^{\log c} \phi(\log c) \, dc) \right] \, dc$$

where $T_i$ is total state tonnage.

The data on the tons of coal in the ground come from the U.S. Bureau of Mines[13,14]. In addition to thickness and depth intervals, the Bureau divides coal reserves into certainty categories. The certainty categories are called measured, indicated and inferred. The first two are estimates based on observations at most 1-1/2 miles apart, due to mining or drilling. Beyond that point the reserves are inferred based on extrapolation. The Bureau of Mines calls all coal in the measured and indicated categories, lying in seams thicker than 28 inches and no more than 1000 feet below the surface, the demonstrated reserve base. Only coal meeting these physical limits is considered technically mineable using present technology. The Bureau further divides the total into coal exploitable by underground methods, and coal exploitable by surface methods. The vast amount of coal in
the demonstrated reserve base has been used to justify assumptions of perfect elasticity for long-run coal supply. In the East, the area of interest here, past mining and drilling has been extensive, meaning that most of the technically mineable coal will be in the measured and indicated categories. In the west, the numbers are probably an underestimate because previous activity has not been extensive. However, given the great elasticity of supply in western coal seen below, this is not a serious bias. Consequently, we follow previous practice [15] and use the demonstrated reserve base.

Equation (39) defines implicitly the cumulative cost function. If a cumulative output total is specified, equation (39) can be solved for the upper limit of integration. The upper limit will be the marginal cost of mining resulting from having mined that volume of coal.

The results of this analysis are summarized in Tables (6) and (7). Table (6) presents estimates of how fast costs would rise in eastern underground mining if current rates of underground output were maintained for 5 years, 10 years, 20 years, 30 years and 50 years. Thus, at current rates of output it is estimated it would take 20 years for the costs of high sulfur coal to rise 10% in northern Appalachia. In southern Appalachia high sulfur coal costs are estimated to rise by 11.4% in 20 years. A particularly interesting result is the greater elasticity of midwestern deep output compared to midwestern strip production. This implies that we can expect to see an increase in the relative share of deep output in that region, a reversal from previous history. However, planned new mine openings, in fact, confirm this result.

These estimates put the popular interpretation of "reserves" into perspective. Popular impressions as well as authoritative estimates have
## TABLE 6

Estimated Percent Increases in Cost Over Time in Underground Coal Mining if Current Rates of Output Are Maintained

<table>
<thead>
<tr>
<th>Region</th>
<th>5 Years</th>
<th>10 Years</th>
<th>20 Years</th>
<th>30 Years</th>
<th>50 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Appalachia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>medium sulfur</td>
<td>4.5</td>
<td>8.6</td>
<td>17.0</td>
<td>25.0</td>
<td>41.0</td>
</tr>
<tr>
<td>high sulfur</td>
<td>2.3</td>
<td>5.2</td>
<td>10.0</td>
<td>15.0</td>
<td>24.9</td>
</tr>
<tr>
<td>Southern Appalachia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low sulfur</td>
<td>7.9</td>
<td>15.0</td>
<td>31.8</td>
<td>51.2</td>
<td>98.6</td>
</tr>
<tr>
<td>medium sulfur</td>
<td>6.9</td>
<td>13.2</td>
<td>25.7</td>
<td>40.9</td>
<td>68.0</td>
</tr>
<tr>
<td>high sulfur</td>
<td>2.9</td>
<td>5.9</td>
<td>11.4</td>
<td>16.8</td>
<td>29.6</td>
</tr>
<tr>
<td>Midwest</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>high sulfur</td>
<td>0.6</td>
<td>1.5</td>
<td>2.7</td>
<td>4.1</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Source: Text
## TABLE 7

Estimated Percent Increases in Cost Over Time
Midwestern and Western Surface-Mined Coal
if Current Rates of Output Are Maintained

<table>
<thead>
<tr>
<th></th>
<th>5 Years</th>
<th>10 Years</th>
<th>20 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midwest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High sulfur</td>
<td>10.1</td>
<td>15.6</td>
<td>25.7</td>
</tr>
<tr>
<td>Montana-Wyoming*</td>
<td>1.3</td>
<td>2.5</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Source: Text

*These figures represent cost increases at five times current output levels.
used the large amount of coal in the reserve base to justify the assumption of no increase in costs over time due to depletion. In fact, the estimated cost increase for high sulfur coal are not dramatic, but it is far from negligible. The increase in low-sulfur costs is significant.

Tables (6) and (7) sum up the central policy dilemma with respect to the large-scale substitution of coal for liquid and gas fuels. Environmental regulations limit emissions of sulfur dioxide. This had led to a large demand for coal low in sulfur content. As Table (6) indicates, low-sulfur eastern coal is in relatively inelastic supply. This had led to the great interest in the low-sulfur western coal. Table (7) shows how greatly western output can be expanded at a cost close to current levels. In the absence of sulfur regulations, eastern high-sulfur coal could expand. In particular, Table 6 indicates midwestern deep coal would likely expand and capture new markets. Sulfur regulations then increase the attractiveness of western coal. Yet, the movement to western coal involves another environmental problem. Western coal, in large part, must be strip-mined. In the arid climate of these western states, reclamation of the land is a difficult task. It is uncertain what happens once the fragile ecology of the region is disturbed. This uncertainty has led some to question the advisability of developing western coal. Others, fearful of the effect of western competition on eastern output, also wish to retard development.

The results of this paper indicate that there is a policy choice that must be made. The output of low-sulfur coal can be increased, but at the expense of rapid development of western coal. This is a particularly difficult trade-off to be made since the costs of each action are borne in different regions. Air pollution in the cities can be reduced, but at the
expense of strip-mining in rural areas. Only the advent of a low-cost desulfurizing device can slice through this gordian knot.

These environmental trade-offs are made more expensive by the desire to increase the use of coal as a substitute for oil and gas. An oft repeated goal is to double the United States' production of coal. This would mean that costs would rise at twice the rate shown in Tables (6) and (7). Doubling output and retarding the development of western coal would put severe pressure on eastern low-sulfur coal. The country might not be willing to bear the price.

Other Factors Influencing Costs

This paper has dealt with only one element leading to increased costs: depletion. Depletion interacts strongly with environmental regulation so that the regulations we choose will, in part, determine the evolution of coal costs. Other factors will also importantly determine coal prices.

The Health and Safety Act of 1969, which specified safety regulations, has been cited as the cause of the decline in productivity experienced since 1970. The estimation of this paper relies on 1975 data, so that some of this impact has been incorporated. However, the decline has continued since 1975. As a first approximation this can be treated as a negative rate of technological change, causing the entire distribution of coal according to exploitation cost to shift to higher cost levels. How long this will continue, no one knows. In the long history of mineral supply technological change has worked to stem the cost increases imposed by nature. The technological regress of recent years can be overcome with new techniques.
Finally, the prices of input factors will affect costs over time. The coal industry itself is competitive, but important inputs are not competitively supplied. As oil prices increase, the United Mine Workers union will be in a position to capture some of the rents created.

Summary

This paper has developed a method for dealing with structural change in a mineral industry. Depletion and environmental regulations are continually changing the supply responsiveness of the coal industry in the United States. Traditional econometric analysis is inadequate in the presence of this structural change. To deal with depletion, this paper has based its estimates of long-run supply upon the geology of remaining deposits. To deal with environmental regulations, cumulative cost curves were estimated separately by type of mining and by type of coal. The evolution of coal costs will depend crucially upon policy choices. There are, however, other factors this paper has not discussed, that will greatly affect the cost of coal. Finally, technological change over the truly long run will have the greatest impact on the evolution of coal prices.
Footnotes

I am indebted to R.I. Gordon for comments and to Jerry Hausman, who provided his program for truncated estimation. Michael Baumann and Christopher Alt provided research assistance par excellence. Financial support of the U.S. Energy Research and Development Administration and the Council on Wage and Price Stability is gratefully acknowledged.

1. The theory of depletable resources predicts that lower cost resources will be exploited first. This has been discussed frequently in the literature. See, for example [4], for a discussion of behavior in a competitive industry. Similarly, it would never pay a monopolist to move a unit of lower cost production to a future period in exchange for a high cost in an earlier period since the present value of profit stream would be reduced. Unless all is produced in a single period, any optimal set of prices and outputs will exploit the lower cost deposits first.

2. The cumulative cost curve first appears in Hotelling's theoretical analyses of exhaustible resources [5]. Hotelling considered the problem of the behavior of price over time when cumulative output led to cost increases. It can be shown that the following equation describes the behavior of price over time in a competitive industry:

\[ rp = \frac{d(x \cdot c)}{dt} \cdot \frac{dx}{dt} + \frac{dp}{dt} \]

where 
- \( r \) = rate of interest
- \( p \) = net price (gross price minus cost)
- \( x \) = cumulative output
- \( c \) = marginal cost
- \( t \) = time

In this expression, \( \frac{dc}{dx} \) is the cumulative cost function.

3. A unit is defined here as a mining machine, two shuttle cars, and a complement of miners.


The continuous mining machine has been in use since the 1950's and the dominant technique in new mines since the early 1960's. In other words although relative factor prices have varied in this period, continuous mining has still been preferred.
4. Draglines have increased steadily in size. In the 1940's the largest bucket was 25 cu. yards. By 1965 it had reached 100 cu. yards and today it is over 220 cu. yards. There was a concomittant use in dumping radius [8].

5. Draglines have come to dominate large-scale shipping. One major manufacturer informed us that there have been no orders for large stripping shovels in the last five years. A complete analysis could treat other types of overburden removal equipment, but the results would be only slightly altered.

6. The data represent deep mines producing more than 100,000 tons per year. Smaller mines generally respond to spot market signals, entering and exiting as spot prices change. The smaller mines do not compete in the long-term contract market and represent a different production function. The data on seam thickness, number of openings and number of sections and shafts worked per day, days worked per year, and output come from mine inspection reports of the Mining Enforcement Safety Agency.

In order to be sure ignoring depth in the productivity equations was not a difficulty, an earlier data sample was used to test for the impact of depth. Data on a sample of mines for 1954 collected by the Bureau of Mines allowed a test. The results indicated that the null hypothesis that depth had no effect on productivity could not be rejected at a 50-percent level of confidence. There was no significant change in the coefficients or in the standard error of the regression when depth was added to the independent variables.

7. Loge_i is normally distributed by assumption. Therefore the probability of loge_i is given by the following:

\[
f(\log \varepsilon_i) = 0, \text{ if } \log \varepsilon_i < \log T - \log A - \gamma \log Th - \beta \log S - \alpha \log OP
\]

\[
\frac{f(\log \varepsilon)}{T \log A - \gamma \log Th - \beta \log S - \alpha \log OP}
\]

\[1 - \int \Phi(\log \varepsilon) \, \text{d}e\]

if \(\log e_i \geq \log T - \log A - \gamma \log Th - \beta \log S - \alpha \log OP\)

Jerry Hausman kindly provided the computer program for performing this estimation and Michael Bauman modified it to deal with the present problem. For a discussion of this problem see Hausman, J. and Wise, D., "Social Experimentation, Truncated Distributions and Efficient Estimation," Econometrica, forthcoming.

8. The results also confirm expectations about the bias of OLS in this case. The maximum bias was, in fact, very small. The results are available from the author.

9. The depreciation rule used in these calculations is the sum of years digits. This is the most favorable method of depreciation for a company. See Hall, R.E. and Jorgensen, D.W., "Tax Policy and Investment Behavior," American Economic Review," June 1967, pp. 391-414.

10. TRW, Inc., Coal Program Support Report (prepared for the Federal Energy Administration), June 28, 1973, Figure 3-5A.

12. To haul a ton of coal up 1000 feet requires 1.3276 Kwh. If the efficiency of the engine is 80%, the effective power needed is 1.66 KWh. At 1¢ per KWh this is a trivial amount. Ventilation costs increase according to the following formula:

\[ HP = \frac{KOV^3(\lambda)}{33,000} \]

where \( HP \) = increase in horsepower
\( O \) = area of shaft (24)
\( V \) = velocity of air (774 cu. ft. per minute)
\( \lambda \) = incremental length of airway (2,900 ft)
\( K \) = coefficient of friction \( (2 \times 10^{-8}) \)

This yields an additional power cost of $5,540 per year, again a trivial amount.

13. Cost of capital is calculated in the following way:

\[ I = \int_0^T (c -\text{taxes}) e^{-rt} dt \]

where \( I \) = initial investment
\( c \) = annual return necessary to realize an after-tax return of \( r \) percent
\( T \) = life of capital good

Let \( u \) = corporate income tax
\( v \) = depletion allowance in percent of gross profit, total depletion = \( v(c - \text{depreciation}). \)

Then:
\[ \text{taxes} = u(c - \text{depreciation} - \text{depletion}) \]

\[ I = \int_0^T [c - u(c - \text{depreciation} - \text{depletion})] e^{-rt} dt \]

\[ = \int_0^T ce^{-rt} dt - \int_0^T uce^{-rt} dt + \int_0^T u(\text{depreciation})e^{-rt} dt \]

\[ + \int u(\text{depletion})e^{-rt} dt \]

\[ = cF - ucF + u\int_0^T (\text{depreciation})e^{-rt} dt + u\int_0^Tv(c - \text{depreciation})e^{-rt} dt \]

where \( F = \int_0^T e^{-rt} dt \)

\[ I = cF - ucF + uvcF + (u - uv)(\text{present value of depreciation}) \quad (1) \]

The last term on the right hand side is

\[ (u - uv) \left\{ \frac{2}{rt} \left[ 1 - \frac{1}{rt} (1 - e^{-rt}) \right] \right\} \]

Substituting the above expression into equation (1) yields:

\[ \frac{c}{I} = \text{cost of capital} = \frac{[1 - (u - uv)(1 - F/t)2/rt]}{F(1 - u + uv)} \]

Costs were adjusted to 1977 prices using the BLS construction machinery wholesale price index. Wages were adjusted by the income in the BLS bituminous coal mining average wage.
14. The choice of two openings reflects the minimum number observed in the sample, and what is necessary for intake and outtake.

15. See, for example, the description of the Wabash Mine, Coal Age, September 1974, p. 102, for an indication of duplication of units.


18. Some mines were using shovels and draglines. The comparable MUF number for a shovel were added to those of the draglines. The adjustment for shovels simply considers the equivalent dumping reach. This is discussed in [17], p. 420.

19. The value of Z is derived in the following way:

\[
\text{TOTAL Cubic Yards} = \frac{\text{Feet of overburden} \times \text{acres mined} \times 43,560}{27} \quad (1)
\]

where \(43,560 = \text{square feet in an acre}\)
\(27 = \text{cu. ft in a cu. yd.}\)

But:

\[
\text{Acres mined} = \frac{Q}{\text{Th} \times 1800} \quad (2)
\]

Where \(Q = \text{annual output}\)
\(\text{Th} = \text{thickness of seam in feet}\)
\(1800 = \text{tons per acre-foot}\)

Substituting (2) into (1) yields:

\[
\text{Total cu. yds} = \frac{\text{ft of overburden} \times (Q/\text{Th} \times 1800) \times 43}{27} = .89 \text{ QR}
\]

20. The data on production and overburden ratios are from State of Illinois, Department of Mines and Minerals, Annual Report 1975. The information on equipment size is from direct correspondence with the companies involved. Experimentation indicated that truncation made little difference in the estimates. The OLS estimates are reported here.

21. The earlier data are from [17], p. 408.

22. The sample excludes the contour mines, since they represent a different technology. Also, recent estimates for three mines are available, but they include equipment and labor for reclamation cost. The idea here is to exclude reclamation cost so that the impact of new reclamation legislation can be added on separately. The data used here represent practice as of 1969, when reclamation requirements were not nearly as stringent as at present.
23. For an example of this same phenomenon in the petrochemical industry, see Adelman, M.A. and Zimmerman, M., "Prices and Profits in the Petrochemical Industry," Journal of Industrial Economics, June 1974, pp. 245-254.

24. The best approach would be to establish a relation between contour, parcel size, etc. and the size of dragline chosen. Then, reserves would be classified along these additional dimensions. Unfortunately, the requisite data do not exist.

25. If the mean is not zero, the constant term will incorporate the bias.

26. The data include coal that is recoverable with present underground techniques, that is, coal lying in seams at least 28 inches thick. Thus, the distribution should be interpreted as the distribution of coal that is mineable with today's techniques. Exploiting seams thinner than 28 inches would involve using other techniques and would lead to a large discontinuous jump in the cost function. Excluding the thin coal as we do here means that the upper tail of the cost distribution is somewhat underestimated. More coal is available at very high costs. However, this does not change the qualitative results. Furthermore, exclusion of this very thin coal is consistent with the Bureau of Mines' Reserve Base measure discussed below as well as with the FEA's 1974-1976 work on coal supply. The data are from: Paul Weir Company, Economic Study of Coal Reserves in Pike County, Kentucky and Belleville District, Illinois ..., Job No. 1555, January 1972. The data from Belleville District consider reserves in only one seam and thus are too restrictive to establish a distribution.

According to the United States Geological Survey Bulletin 1120, 1963, past mining had depleted about 12 percent of the original reserves lying in seams greater than 28 inches thick in Pike County. This means the present distribution is a slightly biased version of the original distribution. Past mining depleted, on average, thicker seams which probably accounts for the relatively poorer fit in the upper tail of the estimated distribution as noted below. Alternatively, the distribution can be viewed as approximating today's distribution.

27. The total underground reserves in seams 28" or more in the measured and indicated categories is given by the Bureau of Mines as 2.2 billion tons. This is larger than the reserves of the coal-producing states of Alabama, Virginia, Tennessee, and Maryland. It represents 73% of the reserves of Eastern Kentucky. United States Bureau of Mines, Information Circular, 8655, The Reserve Base of Bituminous Coal and Anthracite for Underground Mining in the Eastern United States.

28. We adjust below for the differences by sulfur content in total tons of coal and for the different levels of past depletion by sulfur content.

The data classify reserves into the following sulfur intervals, where the limits are percent sulfur by weight:
29. The reason for the behavior in the tail for the Montana data is that data were not recorded for higher overburden ratios. An arbitrary cut-off on overburden ratios was used. The cut-off overburden ratio decreased as depth of seam increased, so that the upper tail of the overburden distribution was truncated. So, the approximation is better than the data show. See [7]


32. Recent work of the Federal Energy Administration [17] does attempt to deal with depletion. The essence of their procedure is the assumption that all mines produce at a constant rate of output for twenty years. Depletion occurs as low-cost mines are exhausted at the end of twenty years. While certainly an improvement over earlier efforts, this assumption leads to anomalous results. For example, since the bulk of the expansion in low-sulfur production has taken place in the last 5 years, no low-sulfur mines are exhausted until 1990. The effect of cumulative output is zero for all low-sulfur coal in all regions until 1990. In that year, if output rates were to remain constant, there would be a large discontinuous jump in cost. This problem is contradicted by the recent rapid increases in low-sulfur coal prices, and differs substantially from the results of this paper.
References


