AGGREGATE ENERGY, EFFICIENCY AND PRODUCTIVITY MEASUREMENT

by

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INTRODUCTION AND BACKGROUND

"We leave it to the Political Arithmetician to compute, how much Money will be sav'd to a Country, by its spending two thirds less of Fuel; how much Labour sav'd in Cutting and Carriage of it; how much more Land may be clear'd for Cultivation; how great the Profit by the additional Quantity of Work done, in those Trades particularly that do not exercise the Body so much, but that the Work folks are obliged to run frequently to the Fire to warm themselves: and to Physicians to say, how much healthier thick built Towns and Cities will be, now half suffocated with sulphury Smoke, when so much less of that Smoke shall be made, and the Air breath'd by the Inhabitants be consequently so much purer."


When Benjamin Franklin offered his newly invented fireplace for sale to the public in 1744, he promoted it largely on the basis of its energy conservation potential: Franklin's fireplace saved wood and augmented the benefit of fire. Moreover, as the above quotation from the last page of his promotional material indicates, Franklin believed that the more efficient use of fire fuel had enormously beneficial social and economic implications. Franklin's statement indicates that although public concern over the efficient and productive use of energy may have been heightened since the 1973 Organization of Petroleum Exporting Countries (OPEC) oil price increases, interest in the implications of increases in fuel efficiency dates back for centuries.
The more narrow issue of simply measuring energy or fuel efficiency also dates back more than 100 years. Around the year 1824, an engineer named Lazar Carnot observed that for certain well-defined physical tasks (such as the raising of a mass of material from one level to a higher one in the presence of a gravity field), there existed a maximum possible efficiency of available energy in that a minimum amount was required to perform the given task. Furthermore, this minimum amount of energy could be derived analytically, and more than the minimum amount would be required whenever any part of the physical mechanism or process was less than perfect. Carnot's observations have led to the development of generalized laws of thermodynamics which provide yardsticks for measuring the energy efficiency of a wide variety of physical tasks.¹ Today the laws of thermodynamics form the basis of measures for the technological potential of fuel efficiency improvements.

In the economics literature, until recently relatively little attention has been focused on specific aspects of energy efficiency measurement. The principal historical economic studies (1, 2, 3, 4, 5) measured energy efficiency simply as energy-output ratios, where energy was measured in terms of aggregate British thermal units (Btus). In terms of other

¹For a brief history of thermodynamic measures of energy efficiency, see (6); also see (7).
productivity studies, for several centuries in the economics discipline attention has been focused primarily on the productivity of labor rather than on that of energy. Presumably the reason why economists have directed attention primarily to labor productivity is because of their belief that economic activity is directed to the benefit of mankind, and labor productivity (output per capita, or more recently, output per man-hour) measures the fruitfulness of human toil and labor under varying circumstances. Economists have also stressed, however, that labor is not the only scarce resource, and that it is preferable to measure productivity in a way that compares output with the combined wise use of all scarce resources -- not just labor. Furthermore, economists have observed that measures of labor productivity depend critically on the amount of capital plant and equipment available to workers, and also that to some extent capital and labor are substitutable inputs. For these reasons the concept of total factor productivity has come into being.

The basic idea behind the total factor productivity measure is that it is more useful to measure how productively society utilizes all its scarce resources, rather than just its labor inputs. Following the pathbreaking work of Tinbergen (8), aggregate national total factor productivity indexes have typically been computed as changes over time in the ratio of
output to total capital and labor inputs. Since the early total factor productivity studies were conducted at the aggregate national level, output was measured as value added; intermediate inputs were ignored and the only inputs considered were capital (which occasionally included land) and labor. In particular, energy inputs were not considered.

The major exception to the exclusion of the intermediate goods from total factor productivity studies was the agricultural productivity research. Although attention in these studies often focused on yield per acre, again it was soon realized that land was not the only scarce input, and that yield per acre depended on the scarce inputs of labor, capital (such as tractors), seeds, fertilizers, and, in some cases, fuels. Hence in these more disaggregated productivity studies, the measure of output became gross output (total farm production) rather than value added (total farm production minus such intermediate inputs as fertilizers, seeds, and fuel). Total factor productivity was then measured as changes over time in the ratio of gross output to total capital, labor, and intermediate inputs.

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2Intermediate inputs include energy, raw materials inputs, and other materials produced by one domestic sector but purchased by another domestic sector.

3The first American study of aggregate total factor productivity appears to be that of Stigler (9); for brief histories of economic research on productivity, see (10) and (11).

4For a brief history of agricultural productivity research, see (12).
The above discussion might seem to indicate that unlike physics, historically economics has little to say concerning the measurement and implications of energy efficiency or energy productivity. Such a conclusion, however, would be grossly inaccurate. The approach taken in this paper is that there are persuasive reasons why most economists have tended to go beyond a myopic measure of labor productivity to a more panoramic analysis of total factor productivity, and that even in the current excitement over energy efficiency, attention is best focused on productivity indices that compare output with the combined use of all scarce resources -- not just energy, or labor, or land.\(^5\) While measures of energy efficiency are of considerable interest, they are best analyzed in the larger context of energy and nonenergy inputs. Economic theory together with recent econometric evidence provides a framework in which the different factor-specific productivity measures (e.g. labor productivity, energy productivity, yield per acre, etc) can be related to one another and to total factor productivity. One of the benefits of such a framework is that it provides an analysis of how, for example, recent energy price increases are likely to affect trade-offs between energy productivity and labor productivity.

\(^5\)Unfortunately, the U.S. Department of Labor still uses labor productivity as its "official" measure; the U.S. Department of Agriculture, however, publishes total factor productivity indices.
The purpose of this paper is to provide a framework within which alternative energy productivity measures can be analyzed and interpreted. In the following section I consider thermodynamic and economic foundations, and discuss economic implications of Second Law efficiency measures. In the third section I consider the assumptions implicit in aggregate Btu measurement, and propose an alternative indexing procedure more consistent with economic theory. In the fourth section I provide a brief historical survey of several major aggregate average energy productivity studies and then go on to a discussion of the economic theory underlying average input and total factor productivity. The elasticity of average productivity is shown to be the negative of the economist's traditional price elasticity of demand. In the fifth section I survey recent econometric evidence bearing on the quantitative magnitudes of factors affecting productivity movements. In the final section I conclude and summarize.
The state of the matter is as follows: -- Where coal is dear, but there are other reasons for requiring motive power, elaborate engines may be profitably used, and may partly reduce the cost of the power."

"But if coal be dear in one place and cheap in another, motive power will necessarily be cheaper where coal is cheap, because there the option of using either simple or perfect engines is enjoyed. It is needless to say that any improvement of the engine which does not make it more costly will readily be adopted, especially by an enterprising and ingenious people like the Americans."

--W. Stanley Jevons

Energy is a complex concept. The popular usage of the word energy typically refers to something that makes automobiles and airplanes move, provides heat and illumination, and enables factories to transform raw materials into finished consumer goods. In short, the popular notion of energy relates to the capacity of certain materials to perform useful tasks. Such a notion of energy is inadequate for the precise measurement of energy quantities.

The thermodynamic definition of energy is very precise and differs from popular usage of the word. Essentially, the thermodynamic concept of energy is derived from an independent law of nature. The First Law of Thermodynamics is a

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6See (13), Preface to the second edition (1866), pp. xxxv-xxxvi.

7"Independent" here means that this law cannot be derived from other laws of nature or principles of physical science. For a further clarification and enunciation of the basic laws of thermodynamics and a brief history of their development, see (6).
statement on the existence of a property called energy which is based on the concept of work. More specifically, a property of a system is defined, called energy, such that its change of value between two states, say 1 and 2, is equal to the work involved in a process that has 1 as the beginning state and 2 as the end state. An implication of the First Law is that the total amount of energy in the universe is constant. Energy cannot be created or destroyed. When energy is transformed from one form into another -- say from the potential energy of a weighty object elevated above the earth's surface to the kinetic energy of the falling object, no energy is lost or gained; the friction of the falling object produces an energy equivalent amount of heat, which might however be dissipated into the atmosphere. Hence, an implication of the First Law definition of energy is that energy conservation is assured. Moreover, energy consumption and efficiency cannot be computed on the basis of changes over time in the amount of energy, for the amount of energy is always constant.

Energy efficiency measurement attempts to provide a numerical basis for determining how effectively energy is used. The often-used First Law efficiency measure, $\varepsilon_1$, is defined as

\begin{equation}
\varepsilon_1 = \frac{\text{energy transfer achieved by a device or system}}{\text{energy input to the device or system}}.
\end{equation}
An example is provided by the conventional home oil-burning 
furnace, where First Law efficiency is calculated as the ratio of 
the amount of heat delivered to specified rooms in a house to the 
amount of heat initially provided when the fuel oil is burned in 
the furnace. In this case the First Law efficiency simply 
measures the proportion of energy input that reaches its final 
destination. Because of heat loss up the smokestack and through 
the outside walls of the house as heat is transferred, $\varepsilon_1$ 
measures for home furnaces are less than 100%. According to an 
American Institute of Physics publication (14, p. 28), for 
typical residential oil-burning furnaces supplying warm air to a 
home at 43°C (110°F) when the outside temperature is 0°C 
(32°F), $\varepsilon_1$ is about 60%. Thus, based on the First Law 
efficiency measure it would appear that the maximum possible 
energy efficiency gain in home oil-burning furnaces is limited to 
less than a doubling of current efficiency levels.\(^8\)

Although the First Law efficiency measure has been used 
extensively by heating engineers and, more recently, by energy 
conservation analysts, the $\varepsilon_1$ measure is not without fault. 
Essentially the First Law efficiency measure is based on a 
definition of energy that implies constancy over time, even when 
certain properties or attributes of that constant energy amount 
vary. What is missing from the First Law efficiency measure is a 
recognition of changes in the quality of energy -- more 
specifically, its ability to do useful work. Consider, for

\(^8\)A recent detailed discussion of home furnace efficiency based on 
First Law principles is found in (15).
example, the air around us; it contains an enormous amount of energy and yet it has very limited ability to heat our homes or power our factories. In short, the capacity or availability of the energy-rich air around us to perform useful tasks is very low. Another example is the common battery: the ability of a charged cooled battery to perform useful work is greater than that of a discharged battery having the same energy by virtue of being hot. A final example is the following: a given amount of Btus at high temperature, say, in the steam main of a power plant, has greater capacity to do work than an equal amount at lower temperature -- say, in the circulating water of the power plant. Even though the quantity of energy is the same in either case, the transformation to a lower temperature environment involves a reduction in the quality of the Btus -- a loss in their ability to do work. Somehow a measure of "useful energy" consumed must account for the changes in the ability of energy to perform useful tasks.

The basis for this alternative measurement is provided by the Second Law of Thermodynamics, which deals with, among other things, a property of energy called availability (also available useful work or available energy) which in turn is uniquely related to another important property called entropy (also called unavailable energy).\(^9\)

\(^9\)For a rigorous definition of availability to do work, entropy, and the related concepts of enthalpy and Gibbs free energy, see (7).
A prominent feature of available energy is that although energy quantity is constant when energy is transferred from one material to another, as in the heat transfer from hot steam to cooler water, an amount of available energy may be (and usually is) irretrievably lost. This reduction in availability is measured by the increase in entropy. In a reversible process, the available useful work is conserved; however, an implication of the Second Law of Thermodynamics is that almost all processes are irreversible and thus involve reductions in available useful work and increases in entropy. In most cases the available useful work remaining in the material operated on is a small fraction of that available at the beginning of the task.

On the basis of the Second Law of Thermodynamics, two important relationships can be quantified. First, the maximum amount of useful work obtainable through oxidation from a given quantity of fuel can be calculated. This maximum would of course be realized only if the oxidation process were perfect. Second, for any physical task that is to be performed within an environment that is essentially in a stable equilibrium state, a certain minimum amount of available useful work is required and can be calculated. Together these two results determine the Second Law efficiency of a specified task, hereafter denoted \( \epsilon_2 \), and defined as

\[
\epsilon_2 = \frac{\text{minimum work required to perform a given task}}{\text{maximum possible work that could be extracted from the fuel being consumed in performing the given task}}.
\]
In the context again of home oil furnaces, while $\varepsilon_1 = .60$, $\varepsilon_2 = .082$. Hence, according to Second Law efficiency, considerably more potential for fuel conservation exists than is implied by the $\varepsilon_1$ measure. One advantage of the $\varepsilon_2$ measure can be illustrated by noting that the Second Law efficiency of an engine-driven heat pump with waste heat recovery for home heating purposes is estimated to be about .202, which is almost two and one-half times the Second Law efficiency of the conventional home oil-burning furnace. Such a multiple gain in efficiency would appear to be impossible using the First Law efficiency criterion, for the $\varepsilon_1$ measure of the home oil-burning furnace is already .60.

The First and Second Law efficiency criteria differ then simply because they measure distinct phenomena and approach the measurement problem from diverse vantages. The $\varepsilon_1$ measure is concerned with energy, while $\varepsilon_2$ focuses on a property of energy called available useful work. The First Law approach myopically compares energy input to the actual energy transfer achieved by a given appliance, device or system, whereas the Second Law approach compares the minimum work required to perform a given

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10 The various empirical comparisons of $\varepsilon_1$ and $\varepsilon_2$ in the American Institute of Physics (14, Chapter 2) for specified tasks all result in $\varepsilon_2 < \varepsilon_1$. Although this inequality relationship is typical, in general it is not ensured on the basis of physical laws. Incidentally, the recent revival of interest in $\varepsilon_2$ measures is attributed in (14) to the MIT conference paper read by Keenan, Gyftopoulos & Hatsopoulos (6).
task to the more panoramic maximum possible transfer of useful work given the same fuel input. The current marked preference within the scientific community for the \( E_2 \) measure over \( E_1 \) is based primarily on the reasoning that (a) available useful work rather than energy is the more relevant concept since, unlike energy, the amount of available useful work decreases with the performance of tasks, and that (b) the \( E_2 \) measure provides a basis for reckoning how efficiently the remaining stock of available useful work is being consumed.

Since the approaches and concepts underlying \( E_1 \) and \( E_2 \) are diverse, the two efficiency measures can yield very different implications. For example, suppose we wish to consider the physical efficiency of \( n \) distinct tasks -- heating of a home, refrigeration, water heating, production of process steam, etc. For each of these tasks, let us compute typical First and Second Law efficiency measures, denoting them as \( e_{11}, e_{12}, \ldots, e_{1n} \) and \( e_{21}, e_{22}, \ldots, e_{2n} \), respectively. Suppose now that in searching for tasks associated with the greatest possibilities for "energy conservation," analysts ranked the \( n \) tasks in descending order of First Law efficiency, i.e.,

\[
e_{11} > e_{12} >= e_{12} \ldots > e_{1n}.
\]

The analyst may be tempted to conclude that certain tasks are carried out rather efficiently already, that others are typically
performed much less efficiently, and therefore that additional research or tax incentives should be focused on the currently less efficient tasks since they offer the greatest possibilities for fuel conservation. Although there is a number of errors in such reasoning, at this point it is worth noting that if the ranking of tasks were done using the Second Law efficiency measure, the $\epsilon_1$ and $\epsilon_2$ rankings might differ; presumably the $\epsilon_2$ ranking would be preferred in that it relates to the available energy concept.

This example raises the issue of what are the implications of the Second Law efficiency criterion for fuel conservation programs. Wide dissemination of the Second Law efficiency figures for various energy-using processes common in our industrialized society identifies areas for potential substantial fuel conservation, and thereby provides a very valuable educational function. It may also help us to reorganize and redirect technological skills toward fuel conservation and to provide target values for the fuel efficiency of particular tasks. However, it is also important to realize that the Second (or First) Law efficiency measures are basically engineering or physical measurements rather than economic indices, and therefore that the information content of $\epsilon_1$ and $\epsilon_2$ is not sufficient to answer questions such as the following: Which energy-using industrial equipment should a plant manager replace most quickly, that with an $\epsilon_2$ of 0.3 or that with an $\epsilon_2$ of 0.1? Which
residential energy-conserving devices are the best investment for a homeowner? Should government or private fuel conservation research funds be allocated on a priority basis with those energy uses with lowest $\epsilon_2$ receiving the highest priority? What is the optimal level of $\epsilon_2$ (alternatively, what is the optimal increase in entropy)?

To demonstrate why the Second (or First) Law efficiency criterion is not sufficient to answer these questions, we now provide a simple geometric and economic interpretation. Recall that on the basis of the Second Law one can determine the minimum amount of fuel required in an ideal or perfect equilibrium environment to perform a specific well-defined task. Let us denote the task to be performed by $T^*$. For example, in the residential sphere, $T^*$ could be the amount of energy transferred at a specified temperature to a home of given space dimensions in the winter months; in the industrial arena, $T^*$ could be the amount of process steam delivered within a specified environment. The task $T^*$ is performed using the inputs of hydrocarbon fuel (denoted $F$) and other inputs, principally capital structures and equipment (e.g. furnaces or boilers, pipes, insulation, heat pumps, etc). We denote the other largely durable capital inputs by $K$. Using functional notation, we have

\[ T^* = g(F,K). \]
In Figure 1 below, the specific task $T^* = T_0^*$ can be performed with numerous combinations of $F$ and $K$; the various fuel-capital combinations technologically capable of performing the task $T_0^*$ are found by plotting the coordinates of the isoquant curve along which $T^* = T_0^*$. The isoquant has been drawn to be strictly convex to the origin, which reflects the fact that to a limited extent fuel and physical capital are substitutable inputs.\(^{11}\)

The minimum possible amount of fuel required to perform the task $T^* = T_0^*$ is denoted by $F_{0_{\text{min}}}$; notice that the line emanating vertically from $F_{0_{\text{min}}}$ constitutes an asymptote approached by the isoquant. Hence the asymptote represents the minimum theoretically possible fuel requirement; the actual fuel used in performing the task $T_0^*$ will generally be considerably larger than $F_{0_{\text{min}}}$, since as $F$ falls, $K$ increases and thus $F_{0_{\text{min}}}$ could be attained only with virtually an infinite expenditure on capital plant and equipment.\(^{12}\) Naturally, if a larger amount of the same task were considered, say where $T_1^* > T_0^*$, then another isoquant curve could be drawn. Figure 1 shows such an isoquant along which $T^* = T_1^*$; it delineates the various combinations of $F$ and $K$ technologically capable of performing $T_1^*$. Since $T_1^* > T_0^*$, the $T_1^*$ isoquant appears above and to the right of the $T_0^*$ isoquant. Moreover, 

\(^{11}\)Technological examples of fuel-capital substituability are discussed in, for example, (16) and (17). \(^{12}\)This discussion ignores the amount of time that may be necessary to attain $F_{0_{\text{min}}}$. 
FIGURE 1

\[ T^* = T^*_1, \quad T^*_1 > T^*_0 \]

FIGURE 2
the minimum possible fuel requirement associated with the larger
task $T^*_1$, denoted $F^1_{\text{min}}$, is to the right of $F^0_{\text{min}}$.

Figure 1 displays the various combinations of $F$ and $K$
technologically capable of performing specified physical tasks.
It does not, however, provide us with any information on what
combination of $F$ and $K$ will be chosen. Economists typically
employ the behavioral assumption of cost minimization. In the
present context, and ignoring other inputs for the moment, let us
define total discounted capital-fuel costs $C$ as
$$C = P_k K + P_f F$$
where $P_k$ and $P_f$ are the exogenous prices of capital services
and fuels, respectively. An isocost line is now defined as a
line in $F$-$K$ space which delineates the various combinations of $F$
and $K$ purchases that sum to the same total cost. For example,
let the constant total cost be $C_0$. Then based on the relation
$$C_0 = P_f F + P_k K$$
oncide one can easily derive the equation of the
corresponding isocost line as
$$K = \frac{C_0}{P_k} - \frac{P_f}{P_k} F.$$  

Such an equation is plotted in Figure 2 as isocost line I.
Notice that the intercept term of this isocost line is at
$C_0/P_k$, while the negative slope is equal to $-P_f/P_k$. The
isocost line corresponding to the same total cost but increased
fuel prices is the steeper isocost line II. Finally, an isocost
line corresponding to the original input prices but increased
total cost $C_1$ ($C_1 > C_0$) is line III.
The economic problem facing the manager is, given the input prices $P_f$ and $P_k$ and the assigned task $T^*_0$, to find the cost-minimizing combination of $F$ and $K$ technologically capable of performing the task $T^*_0$. More formally, the economic optimization problem is to minimize $C = P_kK + P_fF$ subject to $T^*_0 = g(F,K)$.

This optimization problem and its solution are represented geometrically in Figure 3. Since input prices $P_f$ and $P_k$ are exogenous, all isocost lines are parallel to line AB. The cost-minimizing manager chooses that combination of $K$ and $F$ corresponding to the point where the isoquant curve (along which $T^* = T^*_0$) is tangent to the lowest isocost line. In Figure 3, the minimum cost combination is at point D where AB is tangent to the isoquant curve, and thus where $T^*_0$ is performed optimally using $K_0$ units of capital and $F_0$ units of fuel input. The resulting derived demand for fuel at $F_0$ is much larger than the minimum possible amount based on the Second Law of Thermodynamics. Indeed, the ratio of line segments $OF_{min}$ to $OF_0$ in Figure 3 represents the actual efficiency of the fuel usage, and in this case it is considerably smaller than unity.

It is also worth noting that since the slope of the isocost line is $-P_f/P_k$, increases in the price of fuel and/or decreases in $P_k$ (such as federal investment incentives for energy conservation) will make the negatively sloped isocost line steeper, and thus will decrease the economically optimal amount...
FIGURE 3
of fuel demanded. For example in Figure 3 let the combination of fuel price increases and investment incentives change the isocost line from the original AB to a new isocost line A'B', which may also represent a different level of total cost C. The new cost-minimizing input combination to produce T* = T*0 is at E where the optimal derived demands for K and F are at K1 and F1. Hence, as a result of the fuel prices increases and/or investment incentives, the same task T* is performed based on a demand for fuel that drops from F0 to F1 while demand for capital services increases from K0 to K1; some fuel conservation is attained, but at the cost of increased demand for capital services. Notice, however, that since F1 is still greater than Fmin, some fuel "waste" remains economically optimal. The economic and thermodynamic optimal demands for F will coincide at Fmin only as the isocost line becomes virtually vertical, i.e., only as \(-P_F/P_K\) approaches negative infinity.13

A number of implications of Figure 3 are worth noting. First, from an economic vantage, the optimal amount of energy conservation is that combination of capital and fuels where the present value of any additional capital expenditure is just equal

13After writing a first draft of this paper in late 1977, the author learned from Paul A. Samuelson and V. Kerry Smith that a similar asymptotic argument has been made independently by Berry, Heal & Salamon (18). The latter paper also considers cases of more than one fuel type, but does not deal with implications of the economic-thermodynamic optima.
to the present value of fuel savings. If present values of additional capital expenditures were less than present values of fuel savings, then (ignoring other inputs) the firm could increase its profits (it could minimize costs further) by investing in energy conservation equipment with larger fuel savings; if present values of additional capital expenditures were larger than fuel savings, the energy conservation investments would not be compatible with cost minimization. In this context it is useful to note that engineers such as Gyftopoulos & Widmer (19), among others, have shown that because of recent fuel price increases, a large number of investments in energy conservation plant and equipment are now economical. Future fuel price increases will likely reinforce this trend.

Second, investment incentives will of course reduce $P_k$ and thus bring about a substitution of capital for fuel along the isoquant $T^* = T^*_0$. However, it is not realistic to assume that the size of the task $T^*$ remains unchanged when investment incentives are offered. Given $P_f$, investment incentives which reduce $P_k$ also reduce the total cost $C$ of performing the task $T^*_0$; this will induce cost-minimizing firms to increase their derived demand for $T^*$ from, say $T^*_0$ to $T^*_1$ where $T^*_1 > T^*_0$, since the cost per unit of $T^*$ relative to labor and other input prices has fallen. In such a case the outward shift in the isoquant and consequent increase in the derived demands for both $F$ and $K$ may more than offset the reductions in fuel demand along
the initial $T^* = T^*_0$ isoquant. Thus it is not necessarily the case that introduction of investment incentives will reduce total demand for fuels.\textsuperscript{14} In order to assess the effects of investment incentives on overall fuel conservation, it is necessary to include consideration of interactions among the capital-fuel and other inputs, such as labor, raw materials, nonfuel intermediate materials, etc as well as changes in the composition of overall output. A further analytical and empirical discussion of possible perverse effects of investment incentives on fuel conservation is presented in Berndt & Wood (20).

A third implication of the above framework is that the shape of the isoquant $T^* = T^*_0$ may be expected to vary with the particular task being considered. Hence it is possible that although the Second Law efficiency of task 1 is less than that of task 2 indicating greater fuel conservation possibility for task 1, the additional fuel savings from a dollar invested in redesign of task 2 equipment could be larger than fuel savings attained

\textsuperscript{14}A related example in the residential sphere is the following. Suppose the federal government offered all households a 10% tax credit if they purchased a new "energy-efficient" refrigerator-freezer this year. The household finding itself with an "energy-inefficient" 16.5 cubic foot refrigerator might be induced by this tax policy to purchase a new 16.5 cubic foot refrigerator using less electricity (a movement along a given isoquant), but might also be tempted to purchase instead a 20.0 cubic foot refrigerator with additional accessories that uses more electricity than the original "energy-inefficient" 16.5 cubic foot refrigerator (a shift in the isoquant).
from investing an additional dollar in redesigning task 1 equipment. This implies that the ranking of alternative tasks on the basis of the Second Law efficiency criterion does not necessarily coincide with the ranking of most promising investments (in terms of fuel savings) for the private or public sector.

Let me now summarize and conclude the discussion of this section. The First Law of Thermodynamics defines energy in such a way that the total energy in the universe is constant. Since by definition conservation is assured, this energy concept is not appropriate for the purpose of measuring and assessing policies for "energy conservation." The Second Law of Thermodynamics considers a property of energy called available energy or available useful work which is of critical importance since even though total energy is constant, the ability of this constant amount of energy to do useful work irreversibly declines as society performs its tasks through time. Thus the Second Law distinction between energy and available useful work is critical in focusing attention and analysis in what it is that we must conserve -- namely, available energy, not energy.

The Second Law efficiency measure provides a useful yardstick for measuring the efficiency with which we currently utilize available energy. A number of recent empirical studies indicate that on average Second Law efficiency is very low -- currently near 8%. This result is not surprising to economists,
since nonfuel inputs are also scarce and costly. Actual fuel used by firms and households is greater than the minimum possible amount implied by the laws of thermodynamics, simply because firms and households seek to minimize the sum of fuel costs and non-fuel costs. It is reasonable to expect that in the future Second Law efficiency levels will improve as long as fuel prices increase more rapidly than non-fuel prices. However, the economic cost-minimizing derived demand for fuels and the thermodynamic optimum will coincide only as the price of fuel inputs relative to nonfuel inputs (particularly durable capital goods) approaches infinity. Because Second Law efficiency is basically a physical concept devoid of economic or social content, its role in the formation of fuel conservation policy is circumscribed. It focuses attention on an extremely important property of energy, provides a clear and rigorous basis for measuring the efficiency with which available useful work is consumed, draws attention to enormous technological possibilities for additional fuel conservation, and if properly used serves as a target efficiency level for industry and households. However, Second Law efficiency levels and their thermodynamic foundations cannot provide the basis for determining the optimal cost-minimizing consumption mix of available energy and other inputs; this latter resource allocation issue is clearly within the domain of joint economic-engineering analysis.
The thermodynamic distinction between energy and available work has unfortunately been largely ignored by economic analysts. For example, numerous economic studies measure energy demand in heat units such as the Btu, and then assess possibilities for energy conservation. Care must be taken in interpreting such studies, for Btu energy conservation is assured by definition. This raises the issue of whether it is possible to devise an economic measure of energy demand or supply that takes into account not only the quantity of energy, but also its other "quality" properties such as availability to do useful work, weight, cleanliness, safety, and amenability to storage. Thus in the next section I consider issues in the construction of an economic measure of aggregate energy demand or supply.
AGGREGATE ENERGY ACCOUNTING AND INDEXING

Since the various fossil fuels and electricity are to some extent substitutable in supply and end-use demands, it has long been recognized that there is a need to forecast not only the demand for specific fuels, but also the demand for aggregate energy. For example, Barnett (2) states:

"The economy's functional requirements for energies could be satisfied with a variety of energy commodity combinations. These commodities could themselves be produced from a variety of domestic natural resources and imports. Therefore, as a practical matter, projections have to be made for all the energy supplies (and demands) simultaneously, as each projection depends on the others."15

In turn, the need to develop forecasts of total energy demand has focused attention on properties of alternative procedures to account for and index aggregate energy flows. Virtually all of the initial research in this area has utilized quantity flows of energy measured in diverse physical units (e.g., barrels of oil, tons of coal, kWh of electricity) and in a common thermal unit, the Btu. Available energy is typically not measured.

Although the practice of measuring total energy supply in Btus has a long history, the complete national accounting of both energy demand and supply is a relatively new phenomenon, with intellectual roots in the input-output framework introduced by

15See (2), p. 7.
Leontief. The development of a complete national supply and demand energy framework is primarily due to the early research of Barnett (2) who introduced the notion of an energy balance table. In energy balance tables, a complete accounting is made of energy Btu flows from original supply sources through conversion processes to end-use demands with all double counting avoided. Since all conversion losses are incorporated into the accounting framework, the energy balance table provides an exhaustive accounting framework for itemizing the sources and uses of energy.

Using energy balance tables, Barnett analyzed a great deal of United States historical data; the same energy balance accounting framework provided the basis for his energy supply and demand forecasts. Although Barnett's approach has been refined and extended by others, it still serves as the basic framework around which most energy demand and supply projects are presented.

Traditionally the energy balance methodology has been used for projection purposes in the following "bottoms-up" fashion:

"Independent estimates of demand by each of the major end-use sectors for each of the detailed energy types are developed by relating demand to aggregate economic activity

16See (2), p. 31 and (21).
17See, for example, Morrison & Readling (22) and Dupree & West (23). A recent useful summary discussion of energy balance accounting and conversion formulae is found in Guyol (24).
and trends in energy consumption. Independent estimates of supply of major energy types are developed and compared with the demand estimates. Differences are resolved, usually in a judgmental way, by assuming that one energy type is available to fill any gap that may exist between supply and demand. This energy type is normally assumed to be imported petroleum, including crude oil and refined petroleum products.18

Based on the projected volumes of the various energy types, researchers have typically converted to a common unit such as the Btu and then have aggregated over energy types to obtain the total Btu demand and supply forecasts. It should be noted that this total Btu figure which in some sense represents the aggregate level of energy activity, is the outcome of the specific energy type forecasts and in particular is not projected or forecasted initially.

One potential problem with this bottoms-up forecasting procedure is that by myopically focusing attention on the individual energy types, it is frequently difficult to incorporate the fact that energy types are to some extent substitutable in end-use demands. Thus it is possible that when viewed in isolation each of the individual energy type forecasts might be reasonable, but when the implied total Btu figure is computed the aggregate is not as reasonable. In practice, forecasters using the bottoms-up procedure typically examine the

resulting total Btu figure to ensure compatibility with prior expectations and aggregate economic-demographic trends; if the aggregate Btu projection is "too large" or "too small," it is changed and suitable adjustments to specific energy type forecasts are made so that accounting identities are preserved.

Of interest to us at this point is the fact that these total Btu figures in bottoms-up forecasts, although not projected initially, typically provide a check on the overall reasonableness of the individual forecasts. Hence the accounting framework provides a total figure which in some sense indexes the overall level of energy supply or demand. What is not clear, however, is the economic foundations for using aggregate Btus as an index of aggregate energy demand or supply. Although the double-entry accounting framework embodied in the energy balance tables is internally consistent in delineating Btu flows, it does not necessarily follow that the total of the Btu's provides an economic index of aggregate energy demand or supply.

An alternative forecasting procedure uses the "top-down" approach. In the top-down procedure, aggregate energy demand is projected initially based on, among other things, assumptions about future demographic-economic activity and price trends. Once total energy demand is forecasted for each end-use sector, demands for specific energy types are calculated on a "market share" basis. The advantage of this procedure is that it ensures
that total energy demand is consistent with the underlying national demographic and economic trends. The obvious problem, however, is how one initially computes the aggregate energy demand forecast. Most projections using the top-down approach index aggregate energy demand using the total Btu index, and then calculate the corresponding energy balance tables.\(^{19}\) Again this raises the issue of the theoretical foundations for using aggregate Btu as an index of aggregate energy demand or supply.

The above remarks suggest then that regardless of whether one uses bottoms-up, top-down, or some (not discussed) simultaneous forecasting procedure, indexes of aggregate energy demand and supply play a central role. Traditionally, aggregate energy has been indexed using a total Btu framework measuring energy rather than available energy. Note that both energy and available energy can be measured using British thermal units. However, the quality of the various Btus within the total energy aggregate will of course vary. Such variation in quality should affect any economic index of aggregate energy demand or supply. Thus in the remaining pages of this section I consider theoretical and analytical foundations of alternative economic indexes of aggregate energy demand and supply.

\(^{19}\) See, for example, Canadian forecasts in (26).
At the outset, it is useful to note that the price per Btu of the various primary and secondary energy products is not equal among energy types. Although "parity pricing" is the announced (perverse) goal of some governments, in North America for example the price of natural gas per Btu, the price of electricity per (thermally equivalent) Btu, and the price of fuel oil or coal per Btu all differ from one another. Reasons for these price differentials include of course the institutional idiosyncrasies of regulation, market concentration and government policy, but also and more fundamentally the variation among energy types in such attributes as weight, cleanliness, safety, amenability to storage, relative costs of conversion and cooperating end-use technology, and capacity to do useful work. Even if there were ideal competitive markets everywhere and no government regulation, we would expect energy prices per Btu to differ among energy types simply because

20The Btu equivalent of electricity remains a somewhat contentious issue which illustrates inherent problems of indexing aggregate energy. Based on the theoretical equivalence of heat and motive power, it is of course possible to obtain the primary energy equivalent of electricity as .86 Mcal per kWh or 3412 Btu per kWh. Some analysts, however, have computed the heat equivalent of electricity on an "embodied energy" basis by calculating the actual heat content of the fossil fuels burned in the generation of electricity via steam. This latter procedure typically produces a much larger energy equivalent of electricity -- as much as five times the 3412 Btu per kWh (First Law efficiency of thermal generation was about .19 in 1939, .2 in 1947, .3 in 1965 and is about .375 today). The economic index number approach outlined later in this section produces a basis for resolving this issue of how to weight electricity in total energy calculations.
energy forms vary in their attribute combinations.\textsuperscript{21}

The different prices of energy forms per Btu illustrate the fact that end-users of energy are concerned not only with the Btu heat content of the various energy types, but also with other attributes. Thus an aggregate index of energy based only on Btu heat content fails to capture the effect of the other attributes of energy -- weight, cleanliness, safety, volatility, amenability to storage, quality, etc.

Because of this variation in attributes among energy types, the various fuels and electricity are less than perfectly substitutable -- either in production or consumption. For example, from the point of view of the end-user, a Btu of coal is not perfectly substitutable with a Btu of electricity; since the electricity is cleaner, lighter, and of higher quality, most end-users are willing to pay a premium price per Btu for electricity. However, coal and electricity are substitutable to a limited extent, since if the premium price for electricity were too large, a substantial number of industrial users might switch to coal. Alternatively, if only heat content mattered and if all energy types then were perfectly substitutable, the market would

\textsuperscript{21}Furthermore, price differentials per Btu among energy types would continue to exist even if all industrial and residential users had multiple-burning capacity (i.e., if they could use any of coal, oil, natural gas, etc in their end-uses) -- simply because other characteristics such as weight, volatility, cleanliness, and quality would remain important.
tend to price all energy types at the same price per Btu. Since available energy can also be measured in Btus, if only available energy mattered the market would tend to price all available energy sources at the same price per Btu, and would ignore characteristics such as cleanliness, weight, etc.

One implication of the limited and less than perfect substitutability among energy types is that the end-use choice of a particular energy type is an economic phenomenon affected by variations in relative fuel prices, technology, income, and preferences for certain attributes. It is eminently reasonable to insist, therefore, that aggregate energy supply or demand measures should reflect the partial but imperfect substitutability among energy types and that the weights used in constructing energy aggregates should reflect the relative value or worth of the various energy types to end-users.

The economic index number approach to this issue is, in the context of demand, to weight the various energy types using their relative (marginal) price per unit in consumption; in the context of energy supply, the weights are the relative (marginal) costs per unit in production, including costs of conversion. Under ideal market conditions, the relative prices per unit in consumption and relative costs per unit in production are equal, and in this sense these relative prices (costs) reflect the relative worth per unit of the various energy types. It is of
course true that the actual economic marketplace is less than ideal, and that the use of actual market prices in weighting various energy types must be viewed as an approximation to calculations based on ideal weights. In some cases it is possible to adjust the actual market prices partially to take account of market imperfections such as utility pricing which departs from marginal cost. In general, then, while the use of prices to weight the various energy types has a solid theoretical foundation, in practice the construction of energy aggregates based on actual market prices must be viewed as an approximation to the ideal. There is some evidence that the use of actual relative prices imparts a reasonably good approximation to the ideal prices, and in particular that aggregate indices based on actual market prices are demonstrably preferable to those based on simple Btu measures.22 This issue of the approximation quality of actual market to perfect market prices remains, however, a topic worthy of additional research.

Although the reader may be attracted to the economic approach for aggregate energy measurement because of its use of prices as weights, he may also have noted that I have not discussed precisely how one incorporates prices into index number formulae. It turns out that the choice of an index number formula implies certain assumptions on the degree of substitutability among energy types.

22 See Turvey & Nobay (27).
To illustrate this, let us first denote the quantities of \( n \) distinct types of energy inputs at time \( t \) as \( E_{1t}, E_{2t}, \ldots, E_{nt} \) and the corresponding input prices as \( P_{1t}, P_{2t}, \ldots, P_{nt} \), where each of the prices is deflated by an aggregate consumer price index, i.e., the prices are real rather than nominal. Total expenditure on the \( i \)th energy type at time \( t \) is of course price times quantity, i.e., \( P_{it}E_{it} \), and total expenditure on the \( n \) energy types is

\[
\sum_{i=1}^{n} P_{it}E_{it}.
\]

A simple aggregation procedure would involve computing aggregate energy at time \( t \), denoted \( E^*_t \), as a weighted sum of the individual energy types, where the weights are denoted as \( \lambda_{it} \):

\[
(3.1) \quad E^*_t = \lambda_{1t}E_{1t} + \lambda_{2t}E_{2t} + \ldots + \lambda_{nt}E_{nt}.
\]

For example, Turvey & Nobay (27) have advocated and utilized a procedure in which one energy type, say the first, is numeraire and where the weights are computed as

\[
(3.2) \quad \lambda_{it} = \frac{P_{it}}{P_{1t}}.
\]

This yields an aggregate energy quantity index \( E^*_t \) in terms of the numeraire as
(3.3) \( E_t^* = E_{1t} + \left( \frac{P_{2t}}{P_{1t}} \right) E_{2t} + \ldots + \left( \frac{P_{nt}}{P_{1t}} \right) E_{nt} \).

The corresponding aggregate energy price index in terms of the numeraire is computed as total energy expenditure divided by aggregate energy quantity, i.e.

(3.4) \( P_{Et}^* = \frac{\sum_{i=1}^{n} p_{it} E_{1t}}{E_t^*} \).

According to equation (3.3), one unit of \( E_{2t} \) is equivalent to (i.e. is weighted equally as or is perfectly substitutable with) \( \frac{P_{2t}}{P_{1t}} \) units of \( E_{1t} \); similarly, one unit of \( E_{3t} \) is equivalent to \( \frac{P_{3t}}{P_{1t}} \) units of \( E_{1t} \), etc. A particularly interesting special case of (3.3) occurs if the price per Btu were the same for all energy types and if each of the energy types \( E_{it} \) were measured in Btu equivalents. In such a special case the \( \lambda_{it} \) weights would all equal unity and (3.3) would collapse to the simple Btu summation formula used in energy balance tables:

(3.5) \( E_t^* = E_{1t} + E_{2t} + \ldots + E_{nt} \).

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Footnote 23: Turvey & Nobay (27) also used a still simpler index in which \( \lambda_{1} = p_{1t}/p_{1} \) where \( p_{1} \) is the price of energy type 1 in the base year. Incidentally, an aggregation procedure similar to (3.3) has been used by, among others, Denison (28) and Bowles (29) to obtain an aggregate labor input index from diverse labor inputs. In the case of Denison and Bowles, the relative price weights are relative earnings or wage rates.
This demonstrates that the Btu summation formula and the economic index of aggregate energy demand or supply would be identical only if prices and unit costs per Btu were equal and if all energy types were or available energy types perfectly substitutable. Hence even though the Btu summation procedure is attractive in that it utilizes clearly defined physical units which are not plagued by price changes over time, the Btu index encounters formidable economic and aggregation problems; in particular, it assumes perfect substitutability among energy types and strict parity pricing per Btu.24

Notice that these Btu summation problems remain regardless of whether one measures energy or available energy in Btus. The basic measurement issue is not one of measurement unit -- Btus -- but rather what it is that is being measured. From an economic point of view, one wants a composite index of all energy

24It might be useful to note here that some researchers have measured and forecasted final demand for energy using "useful Btus" or "output Btus" which represent the "actual" energy demanded after adjusting the "input Btus" by end-use First Law efficiency ratios. This output Btu procedure implicitly assumes parity pricing among output Btus and perfect substitutability, and must be interpreted carefully since the relationship between input Btus and output Btus depends on the choice of conversion and end-use technology which in turn is affected by, among other things, the relative prices of energy and durable capital (see previous section). Choice of an input-output efficiency ratio therefore implies a choice along the capital-energy isoquant. Thus forecasts based on output Btus implicitly assume a forecast of investment in energy-using equipment of given energy efficiency.
attributes -- quality, quantity, weight, cleanliness, amenability to storage, safety, volatility and relative costs of conversion and end-use technology. Thus, to the extent that relative energy prices reflect variations in quality and their other attributes, the price-weighted aggregation procedure (3.3) is preferable to the simple Btu summation formula (3.5).

Although equation (3.3) is preferable to the simple summation formula (3.5), equation (3.3) suffers from one restrictive implication. In particular, according to (3.3), one unit of energy type 2 is completely equivalent or perfectly substitutable with \( \frac{P_2}{P_1} \) units of \( E_1 \); more generally, one unit of energy type \( E_i \) is specified to be perfectly equivalent or perfectly substitutable with \( \frac{P_i}{P_1} \) units of \( E_1 \). While some substituability among energy types is possible, I have already noted that it is unrealistic to assume that diverse energy types are perfectly substitutable -- even by any factor of proportionality. This suggests that it would be desirable to employ an aggregate indexing procedure that weighted the constituent fuels according to their value and usefulness as reflected in relative fuel prices, that allowed for substitutability among energy types but that did not a priori constrain the substitutability to be perfect.

One such aggregate indexing formula is known as the Cobb-Douglas index,
which can be written in linear logarithmic form as

\[(3.7) \quad \ln E^* = \sum_{i=1}^{n} w_i \ln E_{it},\]

where the \(w_i\) are constant weights which typically are positive and sum to unity. Notice that in (3.7), the partial derivative \(\frac{\partial \ln E^*}{\partial \ln E_{it}}\) equals a constant \(w_i\). Hence if the \(i\)th energy type were increased by, say \(x\) percent, then other things being equal, total energy \(E^*_t\) would be increased by \(w_i x\) percent. The advantage of the Cobb-Douglas index (3.7) is that it allows for limited substitutability among the diverse energy types.\(^{25}\) The two principal disadvantages, however, are the following: (a) the Cobb-Douglas index assumes that possibilities for substitution among energy types are constant,\(^{26}\) and (b) it implies that expenditure shares

\(^{25}\)In terms of the price elasticities defined in the following section, the Cobb-Douglas index (3.7) implies that all cross-price elasticities \(\varepsilon_{ij}(ij)\) equal the constant expenditure share of the \(j\)th input in total energy expenditures, and that all own-price elasticities \(\varepsilon_{ii}\) equal the constant cost share of the \(i\)th energy input minus one.

\(^{26}\)See the previous footnote.
of the various energy types are also constant. The first feature -- constant possibilities for interfuel substitutability -- is unattractive simply because it is arbitrary and unnecessarily restrictive. The second feature -- constant expenditure shares -- is also unnecessarily restrictive and in addition is inconsistent with the recent historical evidence. In particular, it is inconsistent with post-World War II increases in the expenditure shares of electricity and natural gas, and decline in the market share of coal. This suggests that a yet more general aggregate indexing formula is desirable which allows for limited substitutability but does not constrain substitution possibilities and expenditure shares to be constant.

Such general index number formulae have in fact been developed. The classic work on the economic theory of index numbers is Fisher (30). Among other things, Fisher developed several stringent criteria for evaluating alternative index

27At first glance it might be conjectured that constancy of substitution possibilities is inconsistent with the ultimate limits implied by the Second Law of Thermodynamics, in that after some point no further substitution against fuel inputs is possible. This limiting possibility rules out further substitution of available energy for non-energy inputs, but does not rule out continued interfuel substitution in which one type of available energy is substituted for another.
numbers and showed that although many commonly used index number formulae failed on at least one of his criteria, a whole class of "ideal" index number formulae satisfied his tests. Moreover, according to Fisher, choice among the various ideal indices was not a serious problem, since on the basis of numerous empirical examples it became clear to Fisher that all members of this class of ideal index number formulae "...give results so nearly alike that it matters little or nothing, for practical purposes, which form is used. Any one of these forms is as accurate as many instruments that are universally employed in other sciences."28 Fisher's contributions have recently been extended by Diewert (31), (32) who, among other things, has derived the substitutability implications among constituent elements (in our case, among energy types) of alternative aggregate index number generalizations. Although Diewert focuses attention on a number of highly general index number formulae, he places particular emphasis on the following frequently used discrete approximation to the continuous Divisia index:29

\[
\ln E_t^* - \ln E_{t-1}^* = \sum_{i=1}^{n} \bar{w}_{it}(\ln E_{i,t} - \ln E_{i,t-1})
\]

---

29 This approximation is usually attributed to Törnqvist (33), although Theil (34) has traced its development to earlier writers.
where the $w_{it}$ are the nonconstant expenditure shares of the $i$th energy component,

$$(3.9) \quad w_{it} = \frac{p_{it}E_{it}}{\sum_{i=1}^{n} p_{it}E_{it}}$$

and

$$(3.10) \quad w_{it} = \frac{1}{2} \left( w_{it} + w_{i,t-1} \right).$$

Notice that prices enter the discrete Divisia index via expenditure shares. The heuristic interpretation of the discrete Divisia index (3.8) is as follows: the percentage (logarithmic) change in the aggregate energy quantity index is a weighted average of the percentage (logarithmic) quantity changes of the component energy types, where the weights are the time-varying "chained" mean expenditure or cost shares. It is noted that if expenditure shares were constant, then the discrete Divisia index (3.8) would collapse to the more restrictive Cobb-Douglas index (3.7). The generality of this discrete Divisia index has been delineated in greater detail by Diewert (31). In particular, Diewert has shown that the discrete Divisia index (3.8) permits variable substitution possibilities among the components, yet does not impose any a priori restrictions on the substitutability
parameters. Although other index number formulae with comparable generality exist, the discrete Divisia index (3.8) is frequently used by economists and productivity analysts because of its convenient computational features. Moreover, Diewert (1976) finds that for practical purposes, differences among the alternative comparably general index number formulae are typically very small.

In the above discussion I have presented an economic index number approach to aggregating over diverse energy inputs, and have shown that in this context the traditional Btu summation approach is a highly restrictive special case. Implicitly this discussion has assumed the existence of a meaningful concept such as aggregate energy; indeed, the issues I have discussed are basically how to go about indexing an aggregate assuming that such an aggregate number does in fact have meaning. Obviously, one should be aware of the possibility that an aggregate energy index may be very difficult to interpret and may not be well defined. The economic theory of aggregation provides a rigorous framework within which the notion of a consistent aggregate index is well defined and therefore clearly interpreted. Although beyond the scope of this survey paper, it is worth mentioning that in general consistent aggregate indexes of diverse inputs

30More precisely, Diewert has shown that (3.8) is an exact index number representation of the widely used linear homogeneous translog production function.
exist if and only if certain proportionality or functional separability restrictions are satisfied. A discussion of these restrictions and their relationship to aggregate indexes is found in Leontief (35, 36, 37). Intuitively, and in the present context of energy inputs, a consistent aggregate index of diverse energy inputs exists if and only if substitution possibilities between energy and each non-energy input are the same for every energy input.\textsuperscript{31}

Finally, it is worth remarking again that the focus of this section has been on how one might aggregate diverse energy inputs for purposes of measuring and modeling aggregate energy demand or supply. Although the simple Btu summation procedure has been shown to be inappropriate for this purpose, the usefulness of the Btu summation procedure and energy balance tables has not been vitiated. Obviously, the usefulness of any measurement depends on the purposes for which it is designed. Energy balance tables are extremely informative in itemizing the sources and uses of Btus. However, since energy types vary in availability and other characteristics, and since choice among energy types is an economic phenomenon affected by technology, tastes and prices, the economic concept of aggregate energy is best measured by an

\textsuperscript{31}For a more rigorous and detailed discussion, see E.R. Berndt and L.R. Christensen (38).
indexing procedure consistent with basic economic theory. The index number approach discussed above is attractive in that it employs prices as weights, and is flexible in that it places no a priori restrictions on the extent of substitutability among energy types. It can also be used in conjunction with energy balance tables, so that the valuable distinctive features of both procedures may be preserved.  

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32 An example of a model that utilizes both economic index numbers and energy balance tables is the econometric model of the U.S. economy constructed by Jorgensen et al (39). In this model aggregate energy is computed using the discrete Divisia indexes, and all econometric modeling, optimizations and forecasts involve the Divisia index. Within the model, the aggregate energy index is broken down into shares of coal, crude petroleum, refined petroleum products, natural gas and electricity on the basis of technological considerations and relative energy prices. At the end of all optimizations and calculations (and partly for reasons of comparability with other studies), energy balance tables and an aggregate Btu index are obtained based on the fuel share data and Btu conversion factors. For a further discussion and application of this model, see Hudson & Jorgenson (40).
AGGREGATE ENERGY PRODUCTIVITY MEASUREMENT

"Wood, our common Fewel, which within these 100 years might be had at every Man's Door, must now be fetch'd near 100 Miles to some Towns, and makes a very considerable Article in the Expense of Families.

As therefore so much of the Comfort and Conveniency of our Lives, for so great a Part of the Year, depends on the Article of Fire;--since Fewel is become so expensive, and (as the Country is more clear'd and settled) will of course grow scarcer and dearer; any new Proposal for Saving the Wood, and for lessening the Charge and augmenting the Benefit of Fire, by some particular method of Making and Managing it, may at least be thought worth Consideration."


Having discussed alternative procedures for indexing aggregate energy, I now turn attention to the problem of measuring how intensively economic units (firms, industries, national economies) utilize aggregate energy. I begin with some definitions and a brief historical survey, and then turn to a more theoretical analysis.

Productivity analysis derives from the observation that the production of output is possible only when inputs of capital, labor, energy, and other materials are combined in a technologically feasible manner. Hence in its general form, productivity analysis deals with all inputs -- not just labor, or capital, or energy. Energy productivity analysis focuses attention more narrowly on energy inputs, and typically involves examining variations over time and space in the ratio of output
(hereafter denoted $Y$) to aggregate energy consumption ($E$). Average energy productivity is thus defined as output per unit of energy input, i.e.

\[(4.1) \quad aE = \frac{Y}{E} .\]

The reciprocal of (4.1) is of course simply average energy intensity, $E/Y$. Viewed in the context of production, there is no compelling reason to restrict one's attention to $Y/E$ instead of say, average capital productivity $Y/K$ or average labor productivity $Y/L$. Indeed, classical economists paid little attention to energy. David Ricardo and Thomas Malthus, for example, are well known for their concerns regarding the perceived decline in the average productivity of land, while Karl Marx devoted great efforts toward analyzing the value and average productivity of labor.

A complete intellectual history of energy productivity analysis is beyond the scope of this present survey. Readers interested in pursuing such an historical investigation might begin with references cited in Jevons (13). Jevons' melancholy "Limits to Growth" book, first published in 1865, contained a number of statistical tables but was largely an effort to convince the suspicious British public that disastrous economic, social, intellectual and moral decay would occur within Britain since her finite coal supplies were being exhausted at an alarming rate.
To the best of my knowledge, the first extensive empirical study of average energy productivity in the United States is that of Tryon (1). Tryon introduced energy productivity analysis by stating simply that

"Anything as important in industrial life as power deserves more attention than it has yet received from economists. A theory of production that will really explain how wealth is produced must analyze the contribution of this element of energy."\(^{33}\)

In order to measure average energy productivity, it was necessary for Tryon first to construct indexes of aggregate energy and aggregate output. Tryon's aggregate energy index (which he called power) for the industrial sector in the U.S., 1870-1926, was based on Btu measures of fuel consumption and the Btu equivalent of other energy sources including water power, wind power and animal power,\(^{34}\) but for data availability reasons, excluding firewood. The construction of an output data series also presented difficulties, since official government statistics on output by industry were not available in 1927. Tryon used and updated the output measures of Stewart (41) covering the physical volume of agricultural production, manufacturing output, and

\(^{33}\)See (1), p. 271.

\(^{34}\)Tryon's ingenuity and thoroughness in developing data is partly reflected in his animal power series, which is based on an agricultural study of horsepower-hours of animal power on farms and is then converted into Btu equivalents "...assuming a very low thermal efficiency." (p. 276).
railroad transportation as well as a combined index of all production. It is interesting to note that the industry output measures used by Tryon represented gross output rather than value added (defined as gross output minus intermediate goods), partly because use of the value-added concept was not yet widespread in 1927.

In his empirical analysis of energy and output, Tryon found that from 1870 to about 1910 aggregate energy (measured in Btus, but not accounting for energy quality variations) grew at a more rapid rate than combined production, implying that over this time period average energy productivity fell (average energy intensity increased). A portion of this measured fall in average energy productivity was attributed by Tryon to the exclusion of firewood from his aggregate energy series, since firewood was more important in the earlier years. Tryon also reported that average energy productivity stabilized after 1910, and that after 1916 the ratio began to increase. This increase in average energy productivity intrigued Tryon, and caused him to speculate that

"The high prices of fuel which began in 1916 and the actual shortages of the war itself stimulated interest in fuel economy and greatly accelerated the tendency to get more work out of the same quantity of coal, which had been present, though in a less degree, from the beginning." 35

35See (1), p. 278.
Tryon's speculative comments are significant, for they indicate his awareness of the fact that fuel prices could affect average energy productivity measures.

Tryon's early work in energy productivity analysis has been followed by other empirical studies too numerous to document completely. Almost all of these more recent studies measure aggregate energy in thermal units or in some physical unit using Btu conversion rates, and do not account for changes in useful energy or quality changes due to interfuel substitution. In the next few pages I will briefly survey the empirical findings of several major studies. As will be seen, the studies vary in terms of level of data aggregation, and recognition of the effects on consumption of energy price changes.

Earlier it was noted that the classic work of Barnett (2) represented a substantial achievement because of its complete accounting of Btu flows within the U.S. economy. Since Barnett dealt primarily with the aggregate U.S. economy, his measure of national output was gross national product (GNP); this measure avoided, of course, problems with double counting the intermediate flows of goods and services. Barnett found that

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36 Barnett's GNP data for 1939 and 1947 were based on official U.S. Department of Commerce publications, but since official GNP data for 1929 were not available, Barnett relied heavily on measures constructed by Simon Kuznets at the National Bureau of Economic Research and several other unpublished studies.
aggregate energy-GNP ratios (the reciprocal of average energy productivity) fell successively from 1929 to 1939 and 1947. Hence, even though his output measure differed from Tryon and even though Barnett's data encompassed larger aggregates, Barnett obtained evidence extending the post-1916 United States trends first reported by Tryon. Because his efforts were devoted primarily to the factual task of accounting for supply and energy use, Barnett did not elaborate significantly on the underlying economic theory and the role of fuel and nonfuel prices affecting energy-GNP ratios. Barnett projected, however, that the aggregate energy-GNP ratio in the U.S. would fall from an index of 100 in 1947 to 78 in 1965.

The first detailed and complete Canadian study of aggregate energy supply and demand is that of Davis (3). Davis' work draws heavily on the accounting framework developed by Barnett, and is of additional interest because of its regional detail and numerous U.S.-Canadian comparisons. Davis reported that over the 1929-1953 time period the energy-GNP ratio in Canada fell slightly, but at a smaller rate than in the U.S.⁷ Although Davis presented no theoretical framework for movements in energy-GNP ratios, he expected the ratio to fall as economic development proceeded. He also acknowledged that energy usage

was affected by price (Davis emphasized the siting decision of industry), but did not estimate the magnitude of price effects on aggregate energy-GNP ratios.

Although the earlier aggregate American research by Tryon and Barnett was suggestive and important, it raised a number of issues whose resolution required the availability and analysis of more detailed and disaggregated data. Thus in 1960 Schurr & Netschert (4) published a massive statistical study of energy supply and demand in the American economy. Again, energy was measured in thermal units such as Btus, and available energy was not distinguished from energy. Interestingly, Schurr & Netschert substantiated a number of Tryon's earlier findings or conjectures. For example, they found that the increase in the use of energy per unit of output during the 1880-1910 period in the U.S. was reduced but remained to a smaller extent after allowance was made for use of firewood as fuel, thereby confirming Tryon's conjecture. Tryon's empirical findings beyond 1916 were also corroborated:

"The record between 1880 and 1910 is one of persistent increases in the input of energy per unit of GNP; between 1920 and 1955, the record appears to be one of persistent decline. The decade 1910-20, which separates the two long periods, appears to be transitional, with almost no change in the relationship between the input of energy and the output of the economy."38

Schurr & Netschert devoted considerable attention to reasons for the post-1920 decline in the aggregate energy-GNP ratio.
Factors cited by them include output compositional changes in the total economy, overall increases in national productivity, and changes within the energy economy such as increases in the thermal efficiency of energy use and the shifting composition of the energy mix. Unlike Tryon and Davis, however, Schurr & Netschert completely overlooked the effects of relative fuel and nonfuel prices on energy-GNP ratios. For example, nowhere in their lengthy section entitled "Some Factors Involved in Long Run Changes in the Relationship Between Energy and GNP" did Schurr & Netschert even mention these prices. Finally, in terms of output measures, when particular industries were being analyzed, Schurr & Netschert used both value-added and gross output measures; but for national aggregates GNP was employed.

Concern over energy consumption and economic growth was not confined to North America. In the 1960s the OECD gathered and began publishing energy supply, demand and trade data for member countries. A massive collection of international energy supply, demand and trade data covering most areas of the world was published by Joel Darmstadter in 1971. Although the primary purpose of the research was to set forth a quantitative record of

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39 Ibid., pp. 164-190.

40 Also see the interesting data discussion by Adams & Miovic (42).
long-term historical energy developments throughout the world, Darmstadter also made preliminary aggregate comparisons of energy-GNP ratios among countries. Darmstadter measured aggregate energy in kilograms coal equivalent, following the United Nations measurement practice. Among the industrialized countries, Darmstadter found that Canada and the U.S. had higher energy-GNP ratios than Sweden, Denmark, Switzerland, West Germany, France, Belgium, Netherlands and Japan. Interestingly, Darmstadter noted that energy-GNP ratios seemed to fluctuate considerably over time and space.\textsuperscript{41} For example, he pointed out that in the most recent four-year period for which he had data (1965-1969), the long historical trend of a falling aggregate energy-GNP ratio for the U.S. was apparently reversed.

The subject of international comparisons of energy-GNP ratios became much more topical after the 1973 OPEC oil embargo, partly because Americans and Canadians observed that other industrialized countries had comparably high standards of living, but lower energy-GNP ratios. Partly in response to the oft-heard query, "if the Swedes (or Germans or French or still others) can do it, why can't we consume less energy?", Darmstadter, Dunkerley, & Alterman (43) published results of an analysis of the quantitative magnitudes and reasons underlying differences

\textsuperscript{41}A number of these results were quite sensitive to the choice of the Btu equivalent of electricity for hydro power. The two choices are discussed briefly in footnote 20 above.
among industrialized countries in energy consumption patterns. Among the principal findings of this highly informative and valuable study are the following: (a) the smallest amount of intercountry variability in energy-output ratios occurs in the industrial sector, while the greatest intercountry differences occur in the transport sector; (b) the prices of fuels and power, traditionally much higher in Europe and Japan than in the U.S., are of considerable significance in explaining energy-output variations among countries, especially within the transportation and residential sectors; (c) national aggregate energy-GNP ratios also depend critically on the composition of a country's output, which in turn appears to be decisively influenced by relative user costs.

The above survey, though admittedly brief and less than complete provides I think the essential flavor of average energy productivity analysis. The common feature of these studies is their concern with explaining the observed variations among countries and over time in average energy productivity levels. None adjust their energy consumption data for changes in quality. Thus it is not clear whether aggregate energy grew faster or slower than aggregate available energy consumption. Although some of the studies mention in passing the role of price, none provides theoretical underpinnings that show analytically how average energy productivity is affected by fuel
and nonfuel prices, as well as by overall total factor productivity. Thus in the remaining pages of this section I focus attention on the economic theory underlying average and total factor productivity movements. This focus on economic theory will hopefully make more clear what it is that is being measured and thereby will enable us to interpret variations in average and total factor productivity.

As noted in the first part of this section, productivity analysis derives from the observation that the production of output is possible only when inputs of capital, labor, energy and other inputs are combined in a technologically feasible manner. Moreover, economic analysis builds on the assumption that firms choose among the various technologically feasible input combinations using the criterion of cost minimization. More formally, let us define a production function which relates the technologically maximum possible amount of output obtainable for a firm given various combinations of n distinct inputs, $X_1$, $X_2$, $X_3$ ..., $X_n$; write this production in implicit form as

$$Y = f(X_1, X_2, X_3 \ldots, X_n, t).$$

One can think of this production function as containing a very large number of alternative blueprint designs by which the firm could produce its gross output $Y$ in a technologically feasible manner. The variable $t$ enters the production function, since the
set of feasible blueprint input combinations is affected by time-dependent technological progress. For pedagogical purposes, let us specify that there are four input aggregates: \( \text{capital services (K), labor (L), energy (E) and non-energy intermediate materials (M).} \) Hence we rewrite (4.2) as

\[(4.3) \quad Y = f(K, L, E, M, t).\]

The firm's optimization problem, given output \( Y \) and exogenous input prices \( P_K, P_L, P_E \) and \( P_M \), is to choose its input combination of \( K, L, E, \) and \( M \) so as to minimize total production costs \( C = P_KK + P_LL + P_EE + P_MM \) subject to the technological constraint that output is feasible, i.e. that \( Y = f(K, L, E, M) \). A basic result of the economic theory of duality \( ^4 \) is that when (4.3) is positive, strictly monotone, twice differentiable and strictly quasi-concave in input quantities, then corresponding to the primal production function (4.3) there exists a dual cost function of the form

\[(4.4) \quad C = G(Y, P_K, P_L, P_E, P_M, t)\]

which relates the minimum possible cost of producing the output \( Y \) to the input prices, output quantity, and the state of

\( ^4 \)Each of these input aggregates is of course indexed in a manner consistent with basic economic theory; see previous section.

\( ^3 \)For further discussion, see Shephard (44, 45).
technology. Obviously the technological constraints expressed in
the primal production function (4.3) are embodied in the
parameters of the dual cost function (4.4).

A principal lemma due to Shephard (44) is that the firm's
cost-minimizing derived demand for the ith input \( X_i \) is simply
the partial derivative of the cost function (4.4) with respect to
the price of the ith input, i.e.

\[
(4.5) \quad X_i^* = \frac{\partial G(Y, P_K, P_L, P_E, P_M, t)}{\partial p_i}, \quad i = K, L, E, M.
\]

The simple derivation of optimal input demands makes the cost function
very attractive for empirical research. Note that the optimal
(cost-minimizing) derived demand for the ith input depends on technology,
the level of output, the prices of all inputs and time. This raises the
issue of how optimal input demands change in response to exogenous input
price variations. The sensitivity of the derived demand for \( X_i \) to a
change in the price of input \( j \) is called the price elasticity of demand
\( \epsilon_{ij} \) and is defined as the partial derivative

\[
(4.6) \quad \epsilon_{ij} = \frac{\partial \ln X_i}{\partial \ln p_j},
\]

where output quantity and all input prices \( P_k \) (\( k \neq j \)) are
fixed. This price elasticity measures the percentage change in
the cost-minimizing derived demand for \( X_i \) in response to a
change in the price of input j when gross output Y and all other input prices are held fixed, but after all input quantities are allowed to adjust to their new cost-minimizing levels. Notice that in general, \( e_{ij} = e_{ji} \). When \( e_{ij} \) is positive, inputs i and j are called substitutes; when \( e_{ij} < 0 \), they are called complements, and when \( e_{ij} = 0 \), \( X_i \) and \( X_j \) are called independent. The curvature restrictions on the production function impose the condition that all "own-price" elasticities \( e_{ii} \) must be negative.

Since output is exogenous in (4.4), it is possible to divide through by \( Y \) and thereby define a unit cost function \( c = C/Y \); this then yields optimal cost-minimizing input-output coefficients \( X_i/Y \) in (4.5). An interesting feature to note about these cost-minimizing input-output coefficients \( X_i/Y \) is that they are simply the reciprocal of the average input productivity measures defined as

\[
(4.7) \quad a_i = Y/X_i .
\]

Thus maximizing the average productivity of the ith input is equivalent to minimizing its input-output coefficient \( X_i/Y \). Note that average input productivity for the ith input will

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44 Allen (46, pp. 503-509), has defined a transformation of the price elasticity (4.6) that is symmetric; his Allen partial elasticity of substitution \( \sigma_{ij} \) is equal to \( e_{ij}/w_j \), where \( w_j = P_jX_j/C \), and thus has the property that \( \sigma_{ij} = \sigma_{ji} \).
depend on technology, output, factor prices and technological progress. This implies that maximizing energy productivity \( Y/E \) is economically inefficient and wasteful, since minimizing \( E/Y \) to the greatest extent technologically possible would in general be inconsistent with the goal of minimizing total resource costs \( C = P_K K + P_L L + P_E E + P_M M \). Hence to the extent that prices reflect social costs, maximizing energy productivity is an inappropriate goal and is inefficient, for if it were followed, goods and services would then be produced at greater than necessary social costs. Of course, energy price increases by themselves will reduce \( E/Y \) and thus will increase average energy productivity; but even with higher energy prices, \( E/Y \) will be greater than the minimum possible technologically.

This raises the interesting issue of whether it is possible, given an estimate of the own-price elasticity of demand for energy \( c_{EE} \), to predict quantitatively the increase in average energy productivity as a result of energy price increases. It turns out that such a calculation is exceedingly simple. To see this, let us use our definition of average productivity for the \( i \)th input as \( a_i = Y/X_i \) and then define the elasticity of the average productivity of the \( i \)th input with respect to a change in the price of the \( j \)th input (hereafter, average productivity elasticity) as the logarithmic partial derivative \( \eta_{ij} \).
where output quantity $Y$ and input prices $P_k (k \neq j)$ are fixed. Since output $Y$ is constant,

$$\ln (Y/X) = \ln (X/Y) = \ln X$$

and we have that

$$n_{ij} = - \varepsilon_{ij},$$

i.e. the average productivity elasticity is simply the negative of the familiar price elasticity. For example, if the own price elasticity of demand for energy $\varepsilon_{EE}$ is $-0.5$, then $n_{EE} = 0.5$; a small (say 1%) increase in the price of energy, if all other input prices and output quantity fixed, will produce a 1/2% increase in average energy productivity. Also, if in addition energy and labor are substitutable inputs so that $\varepsilon_{LE} > 0$, then an increase in the price of energy will improve energy productivity (reduce energy intensity) since $n_{EE} = -\varepsilon_{EE} > 0$, but will reduce labor productivity (increase labor intensity) since $n_{LE} = -\varepsilon_{LE} < 0$.

The above comments have dealt with the average productivity of individual inputs. Obviously, we must also be concerned with
how productively society uses its inputs in combination or in total. Thus we now turn our attention to the concept of total factor productivity. Before proceeding further, however, we must define yet a few more concepts. Recall that earlier we defined a primal production function and the corresponding dual cost function. From the point of view of the primal production function, returns to scale are said to be increasing when an equiproportional simultaneous change in all inputs (but not t) results in a greater than proportional change in output; returns to scale are decreasing when the equiproportional change in all inputs results in a less than proportional change in output. Finally, when returns to scale are constant, an equiproportional change in all inputs results in the same proportional change in output. Using the dual cost function (4.4), let us define the elasticity of cost with respect to output \( \epsilon_{CY} \) as

\[
(4.10) \quad \epsilon_{CY} = \frac{\partial \ln C}{\partial \ln Y}.
\]

Then the degree of returns to scale is simply the reciprocal of (4.10), i.e. returns to scale are

\[
(4.11) \quad \epsilon^{-1}_{CY} = 1/\epsilon_{CY}.
\]

Hence if, for example, returns to scale are greater than one
(increasing), the elasticity of costs with respect to output is less than one. In such a case, a doubling of output would less than double total costs, given input prices. When returns to scale are constant, total cost and output increase at the same rate, i.e. \( \epsilon_{CY} = \epsilon_{-1} = 1 \). Finally, when returns to scale are less than one (decreasing), total cost increases more rapidly than output.

We now turn to definitions of productivity for inputs in total. Since output is produced by a number of inputs -- not just one input -- let us define the primal notion of total factor productivity \( \epsilon_{ft} \) as the partial derivative

\[
\epsilon_{ft} = \frac{\partial \ln f(K, L, E, M, t)}{\partial t},
\]

input quantities held constant. Hence primal factor productivity is the percentage increase in output due to "technical progress." As a practical matter, Diewert (31) has shown that when constant returns to scale exist, (4.12) can be approximated empirically by

\[
\epsilon_{ft} = \frac{\dot{Y}}{Y} - \frac{\dot{X}}{X}
\]

where \( \frac{\dot{Y}}{Y} \) is the percentage change in aggregate output and \( \frac{\dot{X}}{X} \) is the percentage change in aggregate input.\(^{45}\)

\(^{45}\)In practice both \( \frac{\dot{Y}}{Y} \) and \( \frac{\dot{X}}{X} \) are often computed using the discrete approximation to the Divisia quantity index similar to that discussed in the previous section.
The dual notion of primal total factor productivity is called the dual rate of total cost diminution $\varepsilon_{Ct}$ and is defined as the partial derivative

\begin{equation}
(4.14) \quad \varepsilon_{Ct} = \frac{\partial \ln G(Y, P_K, P_L, P_E, P_M, t)}{\partial t},
\end{equation}

where input prices and output quantity are held fixed. Thus the dual rate of cost diminution is the percentage reduction in total costs (given output quantity and input prices) brought about by "technical progress." As a practical matter, $\varepsilon_{Ct}$ can be approximated empirically as

\begin{equation}
(4.15) \quad \varepsilon_{Ct} = \frac{\dot{P}/P - \dot{c}/c}{c/c}
\end{equation}

where unit costs $c = C/Y$, $\dot{c}/c$ is the percentage change in unit costs and $\dot{P}/P$ is the percentage change in the aggregate input price index.\(^{46}\)

Naturally the question arises as to the relationship between total factor productivity (4.12) viewed from the primal production function and total cost diminution (4.14) viewed from the dual cost function. Ohta (47) has shown that in general

\begin{equation}
(4.16) \quad \varepsilon_{ft} = \varepsilon_{CY} \varepsilon_{Ct},
\end{equation}

\(^{46}\)In practice, $\dot{P}/P$ is often computed using the discrete approximation to the Divisia price index which is similar to that for energy discussed in the previous section except that input quantities are replaced by input prices.
i.e. total factor productivity viewed from the primal side is
equal to the returns to scale times the rate of total cost
diminution. Notice that if constant returns to scale are
imposed, then $\varepsilon_{CY} = \varepsilon_{CY}^{-1} = 1$ and $\varepsilon_{ft} = \varepsilon_{Ct}$, i.e.
primal and dual measures of total factor productivity are
equivalent.

Together, the above remarks imply the following. In
general, the firm's average energy productivity will vary in
response to price changes, output variations, "neutral"
technological progress that increases the productivity with which
all inputs are utilized, and "biased" technological progress that
increases the average productivity of some inputs more than
others. More specifically, if the production function were
characterized by constant returns to scale, the cost-minimizing
choice of $X_i/Y$ would not depend on $Y$ and thus average input
productivity would remain unchanged when output varied, i.e.
$\frac{\partial \ln a_i}{\partial \ln Y} = 0$. If increasing returns to scale were present,
then as gross output increases $X_i/Y$ would fall for every input,
thereby raising the average productivity of each input. In such
a case, $\frac{\partial \ln a_i}{\partial \ln Y} > 0$. Finally if returns to scale were
decreasing, then as output increased $X_i/Y$ would increase for
each input, thereby lowering average input productivity,
i.e. $\frac{\partial \ln a_i}{\partial \ln Y} < 0$.47

47Implicitly, this discussion assumes homotheticity of the
production function. For a more general discussion with
nonhomothetic technology, see Berndt & Khaled (48).
In the context of technological change, if technological progress were neutral then the effects on average input productivity would be the same as those of increasing returns to scale in that both would improve average energy productivity, i.e. $\ln a_t / a_t > 0$. If technological change were non-neutral (biased), then the effects of technological progress on average input productivity would depend on the nature of the bias and the parameters of the production technology. However, unless technological change were of a rather extreme input-i using form, we would expect even non-neutral technological progress to improve the average input productivity of all inputs, although of course the rate of improvement would vary among inputs.48

The above analytical framework is microeconomic in the sense that it refers to the individual firm. Typically this theory is extended with some loss to the analysis of relatively homogeneous industries or sectors. However, it quite clearly is less applicable to a national economy whose components are very diverse sectors and industries. Ideally, it would be preferable to analyze each industry separately and also to model carefully the interindustry flows of intermediate goods and services. If this were done, aggregate energy consumption (net of interindustry flows within the aggregate) would rely not only on industry-specific input prices and technology but also on the

48 For further discussion, see Be-ndt & Khaled (48).
composition of output among industries. In turn, output composition would be affected by output prices in each industry and consumers' preferences and income. Thus, to analyze variations in aggregate energy-GNP ratios, it would be preferable to have available very disaggregated data as well as information on input prices, technology, output prices, interindustry interactions and consumers' preferences and income. Unfortunately, such detailed data are usually not available, and researchers are typically forced to work with data and models on a more aggregated level.

Before leaving this largely analytical discussion of energy productivity analysis, it is worth commenting briefly on the merits and drawbacks of energy productivity calculations based on "net energy analysis." Net energy has been defined as the amount of energy that remains for consumer use after the energy costs of finding, producing, upgrading and delivering the energy have been paid. Some net energy analysts have advocated that economies ought to maximize the net energy of their GNP; indeed, the U.S. Congress has mandated that "the potential for production of net energy by the proposed technology at the stage of commercial application shall be analyzed and considered in evaluating

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49See, for example, Gilliland (50) and the references cited therein.
proposals."\textsuperscript{50} Since net energy analysts measure energy in Btus,\textsuperscript{51} they claim their measure of "net output" is not distorted by prices and thus is more helpful to policymakers because it is less ambiguous. From the point of view of the physical sciences, the net energy analysis measurement of energy is unfortunate, since it is available energy rather than energy that declines over time. From an economic vantage, as shown in the previous section, Btu aggregation does not avoid aggregation and evaluation problems but arbitrarily imposes the assumption of strict parity pricing among energy types per Btu and perfect substitutability. Thus at first sight it might seem that net energy analysis would be credible if it maximized net available energy rather than net energy. However, the problems with net energy analysis are much greater than this. It can easily be shown that the unrestricted competitive market would maximize net energy (or, say, net available energy) only if relative prices of all goods and services were determined solely by the ratio of their energy content; a simple proof of this is given by Huettner (49). Hence, net energy analysis implicitly views all non-energy commodities as transformed energy. In such an energy theory of

\textsuperscript{50} Non-Nuclear Energy Research and Development Act of 1974 (PL 93-577, 93rd Congress, 1974, Section 5).

\textsuperscript{51} See, for example, Hannon (51).
value, energy is the only scarce input. The analogy to Marx's labor theory of value should be obvious. Instead of pricing all goods on the basis of embodied labor as was advocated by Marx, net energy analysts suggest pricing all goods on the basis of their energy content. Both myopic approaches are fundamentally flawed; the resource constraints faced by society at any point in time are not those of a single input, but of a host of inputs -- capital, land, labor, raw materials, and energy, to name but a few. Energy is but one of many scarce inputs; maximizing energy productivity is inconsistent with the principle of minimizing total resource costs. Although net energy analysis can provide useful information relative to the ultimate energy consumption consequences of alternative production patterns, attention is best focused on productivity indices that compare output with the combined wise use of all scarce resources -- not just energy, or labor, or land.52

52 For a related useful discussion on the logic underlying energy conservation, see Schipper & Darmstadter (52).
AVERAGE ENERGY EFFICIENCY AND TOTAL FACTOR PRODUCTIVITY: ECONOMETRIC EVIDENCE

In the previous section I showed that the effect of price changes on average energy productivity is simply the negative of the familiar price elasticity. The major accounting studies of energy productivity (2, 3, 4, 5, 43) all considered factors contributing to average energy productivity, but did not attempt to quantify the effects of fuel and nonfuel prices. A number of econometric studies have recently been published that provide estimates of price elasticities and thus of average energy productivity elasticities. I now consider briefly some of the recent econometric evidence, and also attempt to relate econometric energy price elasticity estimates to engineering-economic energy efficiency calculations. This econometric survey is not intended to be exhaustive, but will, I hope, be faithful to much of the flavor of recent research.53

I begin with estimates of the own-price elasticity of demand for aggregate energy $\varepsilon_{EE}$,

$$
\varepsilon_{EE} = \frac{\partial \ln E}{\partial \ln P_E}
$$

where output $Y$ and all other input prices are held constant, but all input quantities are allowed to adjust to their new

53 More exhaustive surveys can be found in, for example, Berndt & Wood (53), Taylor (54), Pindyck (55-56), and Berndt (57).
cost-minimizing levels. Although econometric estimates still vary somewhat, evidence based on total manufacturing time series data for the United States\textsuperscript{54} and on total manufacturing pooled cross-section time-series data for Canada\textsuperscript{55} suggests energy own-price elasticities of about -0.5. This implies that, other things being equal, a 1% increase in the price of energy will improve average energy productivity by 1/2 of 1%. Pindyck (55) finds that when international (OECD) pooled cross-sectional time-series data for aggregate industry are used, the estimated own-price elasticity of energy demand is a somewhat larger (in absolute value) -0.8 to -1.0; similar findings have been reported by Griffin & Gregory (59).\textsuperscript{56}

At first glance, these estimates might be considered too large; in particular, on the basis of numerous recent engineering studies it might appear reasonable to assume that such implied energy savings are possible and economic for only some kinds of equipment, but not on the average for all equipment. These engineering studies typically focus on a two-input production function, where the two inputs are capital and fuel and output is something like the task $T^*$ defined earlier; let us call the

\textsuperscript{54}See Berndt & Wood (58).

\textsuperscript{55}See Fuss (60).

\textsuperscript{56}The Berndt & Wood (58), Fuss (60), and Pindyck (55-56) studies use Divisia indices of energy, while Griffin & Gregory (59) employ Btu aggregation.
output of these fuel-capital production functions "utilized capital." The engineering and econometric estimates can be reconciled once it is noted that engineering energy-conservation studies typically hold fixed the utilized capital output of the capital-energy input bundle and look only at compositional changes between energy and capital, whereas the econometric estimates examine not only this energy-capital compositional change but also incorporate price-induced changes in the amount of the total utilized capital bundle demanded. This latter effect measures utilized capital substitutability with labor and other intermediate material inputs. More formally, define the production

\[
K^* = k(K, E)
\]

which is nested within the "master" production function

\[
Y = f(K^*, L, M).
\]

The engineering price elasticity estimates (hereafter called gross price elasticity estimates and denoted as \( \varepsilon_{EE}^* \)) can be viewed as holding \( K^* \) fixed at, say, \( R^* \),

\[
\varepsilon_{EE}^* = \frac{3 \ln E}{3 \ln P_E} \left|_{K^*=R^*} \right.
\]

while the econometric price elasticity estimates (hereafter called net price elasticity estimates, and denoted as \( \varepsilon_{EE} \)) hold gross output \( Y \) fixed at, say, \( Y^* \).
Berndt & Wood (20) have shown that the engineering and econometric elasticities are related as follows:

\[
\varepsilon_{EE} = \varepsilon^{*}_{EE} + \left( \frac{p_{E}E}{p_{E}E + p_{K}K} \right) \frac{\partial \ln K^{*}}{\partial \ln P_{K}^{*}} \bigg|_{Y=Y}.
\]

In words, the net elasticity \( \varepsilon_{EE} \) is the sum of the gross elasticity \( \varepsilon^{*}_{EE} \) plus a scale elasticity which represents the share of energy in total capital-energy costs times the own-price elasticity of demand for utilized capital (the energy-capital composite). Alternatively, econometric own-price elasticity estimates can be interpreted as the sum of two price responses: (a) the engineering compositional substitution between E and K holding fixed the utilized capital output \( K^{*} \), and (b) the effect on energy demand of a price-induced change in the size of demand for the utilized capital (energy-capital) input bundle, since labor and other intermediate materials are substituted for the increasingly higher priced utilized capital. If one believes that a reasonable estimate of engineering gross price elasticity estimates for \( \varepsilon^{*}_{EE} \) is about -0.2, then the econometric \( \varepsilon_{EE} \) net price elasticity estimate of -0.5 implies a scale elasticity estimate of -0.3. The econometric evidence cited by Berndt & Wood (20) suggests that scale effects in U.S. manufacturing are
larger than engineering compositional effects. In any case, once it is realized that econometric energy own-price elasticity estimates are the sum of these two effects, $\varepsilon_{EE}$ estimates of -0.3 to -0.8 seem reasonable. The implication is that, due to higher energy prices, we can expect substantial improvements in energy productivity. Because of the scale effect energy productivity $Y/E$ will likely increase more than the measured technical energy efficiency of capital equipment, which holds $K^*$ fixed.

Since energy is but one of many inputs in the production process, it is also important to examine cross-price elasticities. To the best of my knowledge, all of the published econometric studies on aggregate energy demand have reported substantial energy-labor substitutability. This result agrees well with basic intuition. In the field of transport, for example, energy consumption and time are clearly substitutable. Truck drivers have become militant over the United States' 55-miles-per-hour speed limit, since this energy conservation policy implies a greater amount of time required to travel given distances. Similarly, the supersonic Concorde utilizes more fuel per passenger mile than other similarly sized planes, but saves on time. In the residential sector, self-defrosting refrigerators or self-cleaning ovens save on human toil and labor, but utilize more energy. In the industrial sector there
are numerous examples of motive power (a composite of energy and capital) being substitutable with human toil and labor.

Econometric estimates of energy-labor substitutability indicate that $\varepsilon_{EL}$ and $\varepsilon_{LE}$ are positive and significant; moreover, as long as the cost share of labor is larger than the cost share of energy $\varepsilon_{EL} > \varepsilon_{LE}$. The Berndt & Wood (58) estimates, for example, are $\varepsilon_{EL}$ of about .15 and $\varepsilon_{LE}$ of about .03. Two implications of this energy-labor substitutability are worth noting. First, energy price increases by themselves will lead to substitution of labor for energy. This implies that energy price increases will likely bring about improvements in energy productivity, but average labor productivity will grow at a smaller rate than it would in the absence of energy price increases. Secondly, the amount of energy conservation actually realized in coming years will depend considerably on the extent at which wage rates rise. If wage rates rise more rapidly than energy prices in the future (as occurred in the post-World War II period in the United States), then energy will continue to be substituted for increasingly expensive labor. The recently enacted Social Security tax increases in the United States are likely to result in a substantial increase in the price of labor to employers. Thus the commendable goal of making the Social Security system financially solvent will increase the difficulty of attaining stated energy conservation targets. While new
equipment is likely to be more energy-efficient, the higher wage rates induce cost-minimizing firms to increase their capital-energy intensities and substitute against labor.

Although econometric evidence on energy-labor substitutability is reasonably consistent, at first glance there appears to be a lack of agreement on energy-capital relationships. The engineering-technological evidence as discussed above suggests that energy conservation is possible but only at the cost of a larger initial capital outlay. Some interpret this as providing justification for concluding that energy and capital are substitutable. On the other hand, as industrialized societies have become more capital-intensive, they have also become more energy-intensive. Thus it could be argued that energy and capital are complements. The two seemingly inconsistent positions can be reconciled once one again distinguishes the compositional change within the energy-capital bundle from the "scale" effect of utilized capital.

Define the engineering gross price elasticities as

\[ (5.6) \quad \epsilon_{KE}^* = \frac{\partial \ln K}{\partial \ln P_E} \bigg|_{K^K\times K^K} , \quad \epsilon_{EK}^* = \frac{\partial \ln E}{\partial \ln P_K} \bigg|_{K^K\times K^K} \]

and the econometric net price elasticities as

\[ (5.7) \quad \epsilon_{KE} = \frac{\partial \ln K}{\partial \ln P_E} \bigg|_{Y=Y} , \quad \epsilon_{EK} = \frac{\partial \ln E}{\partial \ln P_K} \bigg|_{Y=Y} . \]
Berndt & Wood (20) have related the gross and net energy-capital price elasticities as follows:

\[
\varepsilon_{KE} = \varepsilon_{KE}^* + \left( \frac{P_{KE}}{P_{KE} + P_{K}} \right) \frac{\partial \ln K^*}{\partial \ln P^*_K} \bigg|_{Y=Y}
\]

\[
\varepsilon_{EK} = \varepsilon_{EK}^* + \left( \frac{P_{EK}}{P_{EK} + P_{K}} \right) \frac{\partial \ln K^*}{\partial \ln P^*_K} \bigg|_{Y=Y}
\]

The engineering gross price elasticity estimates of \( \varepsilon_{EK}^* \) and \( \varepsilon_{KE}^* \) are positive, indicating gross energy-capital substitutability. However, since the scale effects are always negative (cost shares of \( E \) and \( K \) are always positive and the own-price elasticity of demand for \( K^* \) must be negative), whether energy and capital are net substitutes (\( \varepsilon_{EK}^*, \varepsilon_{KE}^* > 0 \)) or net complements (\( \varepsilon_{EK}^*, \varepsilon_{KE}^* < 0 \)) depends on whether the positive gross elasticity is larger than the negative scale elasticity. If the compositional or gross substitution energy conservation effect is larger than the scale effect, then energy and capital are net substitutes; however, if the energy savings due to the compositional or gross substitution effect are smaller than the increased energy demanded because of the scale effect, then energy and capital are net complements.

The econometric evidence on this net energy-capital relationship indicates that results tend to vary among the various sectors of the U.S. economy. In manufacturing, for
example, both Berndt & Wood (20, 58) and Berndt & Jorgenson (61) find that energy and capital are net complements; similar findings for Canadian and West German manufacturing have been reported by Fuss (60) and by Swaim & Friede (62), respectively. This implies that in the manufacturing sector, scale effects tend to dominate gross substitution or compositional effects. However, Berndt & Jorgenson (61) also report that in other sectors of the economy, notably in the service sector, energy and capital tend to be net substitutes. The effect of energy price increases on capital formation in the aggregate, multisector U.S. economy depends therefore on relative sizes of various sectors, their technology, consumer preferences and income, and the nature of interindustry flows. The simulations performed by Hudson & Jorgenson (40) for the aggregate multisector U.S. economy indicate that while energy and labor in the aggregate are substitutable, energy and capital are complementary.\(^5\)

If the energy-capital complementarity finding turns out to be true over other bodies of data as well, it will have important implications. First, with energy-capital complementarity, energy

\(^5\)The seemingly contradictory econometric findings of Griffin & Gregory (59) have also been reconciled with those of Berndt & Wood (58), Fuss (60), and Berndt & Jorgenson (61). In particular, Berndt & Wood (20) have shown that the elasticity estimates of Griffin & Gregory (59) hold a different output constant, and that when the various studies are compared holding the same output fixed, all these econometric findings are consistent with energy-capital complementarity.
price increases, other things equal, will reduce the derived demand for capital and for energy, and thus will increase the average productivity of capital and of energy. Second, capital-energy complementarity implies that because of energy price increases there occurs a reduction in the rate of investment in new plant and equipment, although energy-labor substitutability implies an increase in the demand for labor. The very recent economic behavior of the U.S. economy is consistent with this set of relationships; because of the recent energy price increases, capital-energy complementarity and energy-labor substitutability, the post-1975 recovery of the U.S. economy has been characterized by more employment and less investment than previous recoveries -- especially those in the 1960s. In particular, since 1975 energy productivity has improved, but labor productivity has grown at smaller rates than previous recoveries. The lower investment occurring currently because of energy price increases implies that the capital stock passed on to future decades and generations will be smaller than would be passed on in the absence of energy price increases. In turn, the smaller future capital stock implies that future output will be smaller. Via investment and capital accumulation, therefore, higher energy price increases have a dampening effect on future economic growth.\footnote{For further discussion and quantification, see Hudson & Jorgenson (63) and Hogan (64).} Finally, although investment
incentives such as investment tax credits may improve the average technical energy efficiency of the capital stock, because of the scale effect these investment incentives might also bring about an increase in the derived demand for energy and for capital, and thus produce a decrease in the average productivity of energy. Investment incentives for energy conservation are likely to be most effective if they are confined to specific investments for which the gross substitution or compositional effect is large relative to the scale effect, e.g. tax credits for residential insulation. With net energy-capital complementarity, the effect of general investment tax credits would be an increase in the capital and energy intensiveness of production processes, i.e. a reduction in the average productivity of both capital and energy, since the scale effect would be larger than the compositional effect by which equipment becomes more energy-efficient.

To obtain a better grasp of the above discussion, it might be useful to refer briefly to some recent empirical data. In Table 1 I present price and quantity indexes for K, L, E, and M in total U.S. manufacturing, 1947-1971. Although more disaggregated data by individual industries would be preferable, such reliable data are not yet available. In Table 1 it is seen that over the 1947-71 time period, the prices of energy and

59This table is taken directly from Berndt & Wood (58), Table 1, p. 263.
<table>
<thead>
<tr>
<th>YEAR</th>
<th>$P_K$</th>
<th>$P_L$</th>
<th>$P_E$</th>
<th>$P_M$</th>
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**Average Annual Growth Rate, 1947–1971**

<table>
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<th>Year</th>
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<tr>
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capital rose less rapidly than the price of labor. Incidentally, the energy price and quantity series in Table 1 is based on a Divisia index which allows for changes in energy quality and other attributes; it is not a simple Btu index. Significantly, the quantities of E and K grew more rapidly than the demand for (increasingly more costly) labor. This table clearly suggests that the relatively rapid growth in E and K as compared to L is due partly to price trends, in particular, the small increases in $P_E$ and $P_K$ relative to $P_L$ over this time period. Increases in energy demand were partly the result of low price increases for energy and its complementary input capital, as well as large price increases in the substitutable labor input.

This simple economic explanation of variations in growth rates for inputs is complicated slightly when one examines trends in input-output coefficients over the same time period. These figures are displayed in Table 2. From Table 2 it is clear that over the 1947-71 time period in U.S. manufacturing, the energy-input coefficient has been relatively constant, implying of course that average energy productivity has also been constant. This stability of average energy productivity differs somewhat from the conclusions of other studies (cited earlier) which typically reveal secular declining trends in energy-output

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60 Table 2 is taken from Berndt & Khaled (48), p. 14.
Table 2: Gross output quantity and input/output coefficients in U.S. manufacturing, 1947-71

<table>
<thead>
<tr>
<th>Year</th>
<th>Y</th>
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<th>L/Y</th>
<th>E/Y</th>
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ratios; the difference in results is due partly to the fact that energy here is quality-adjusted using price weights whereas in most other studies energy is not adjusted for quality-attribute variations.

Another interesting feature of Table 2 is that if one looks at the input-output coefficients for the smaller time period 1959-71, one notices that over this time period energy intensity fell (energy average productivity increased), as did the K, L, and M intensities. Although average labor productivity (the input whose price grew most rapidly) increased at the greatest rate, the 1959-71 trends for all inputs suggest that movements in average input productivity were affected by determinants in addition to relative prices. The Berndt & Khaled (48) analysis suggests tentatively that there is some evidence supporting significant increasing returns of scale, but also notes that separate quantification of technological change and scale economies is difficult given current data constraints. If one imposes the restriction of constant returns to scale, however, then the econometric estimates imply that total factor productivity in U.S. manufacturing 1947-71 has been increasing at the rate of about 0.7% per year, and technical change has been biased in the sense that labor has been saved more than energy (in a relative sense, technical change has been labor-saving and
The determination of the extent to which technological change, relative price variations and returns to scale separately and jointly are responsible for post-1959 improvements in average energy productivity is a promising area for further research and may provide clues regarding future trends in average energy productivity.

\[61\text{In the context of thermal (steam) electricity generation, Christensen & Greene (65) have reported that while substantial increasing returns to scale existed in the 1950s, by the late 1960s most of these scale economies had been exploited. Unfortunately, the Christensen & Greene (65) data base was not sufficiently rich to permit quantification of the effects of technological change.}\]
CONCLUDING REMARKS

This essay has surveyed a number of important issues -- economic implications of thermodynamic measures of energy efficiency, problems involved in developing an aggregate index of energy consumption, and determinants of average input and total factor productivity. Because of space limitations, the discussion has necessarily been brief. A few underlying themes are, however, worth repeating.

First, energy is a complex concept. In particular, available energy is not the same as energy. The measurement of aggregate energy must somehow take account of variations in energy quality and other characteristics. The reason that simple physical measures such as total Btu are unsatisfactory for indexing aggregate energy is that energy inputs are not homogeneous, they are less than perfectly substitutable, and choice among fuels is affected by prices, preferences, and technology. The economic theory of indexing involves price-weighting and thus provides a basis for alternative aggregate energy measurement.

Second, energy is but one of many inputs entering the production process. Maximizing the average productivity of energy (or maximizing net energy) is inappropriate, since it is generally inconsistent with the more appropriate goal of minimizing the total resource costs of all inputs. It is
important that we use all our scarce inputs wisely, not just energy.

Third, thermodynamic measures of energy efficiency (especially Second Law efficiency measures) are extremely valuable in pointing out substantial possibilities for fuel conservation and in focusing attention on the more fuel-efficient technological redesign of equipment and industrial processes. However, as noted by physical scientists such as Keenan, Gyftopoulos & Hatsopoulos (6), Second Law efficiency measures do not provide a basis for determining the economically optimal amount of fuel conservation. Since Second Law efficiency is basically a physical concept, it is not adapted to determining which investments are consistent with total cost minization, nor does it provide a criterion on which to assign priorities for further research and development. The resolution of such questions requires examination of both economic and technological feasibility, not just Second Law efficiency measures.

Further, the effect of price variations on average input productivity can be summarized succinctly by the negative of the familiar price elasticity of demand. In this context, the response of aggregate energy demand to an increase in energy price can be viewed as the sum of two separate responses -- a compositional substitution of capital for energy within the capital-energy bundle (the gross substitution effect typically
examined by engineers and physicists) and a scale effect that reduces the demand for the higher-priced energy-capital composite, and thus reduces the derived demand for both capital and energy. This decomposition implies that in response to energy price increases, average energy productivity will increase more rapidly that the average technical energy efficiency measures for capital equipment.

Finally, although econometric studies report that energy and labor are substitutable inputs, the evidence also is that energy and capital are complementary inputs. Hence recent energy price increases have improved the average productivity of energy, but have also reduced the growth rate of average labor productivity (since they increased the labor intensity of production) and have reduced the growth rate of private investment in fixed plant and equipment. The econometric evidence, though based on pre-1973 data, is nonetheless consistent with the post-1975 high-employment, low-investment recovery of the U.S. economy and suggests that higher energy prices will eventually lead to slightly dampened economic growth.
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55. Pindyck, R.S. 1977. Interfuel substitution and the industrial demand for energy: an international comparison. MIT Energy Laboratory Working Paper no. MIT-EL 77-026WP.


