POSITIONS AND KINEMATICS OF QUASARS AND RELATED
RADIO OBJECTS INFERRED FROM VLBI OBSERVATIONS

by

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ABSTRACT

Very-long-baseline interferometry observations at 3.8 cm, with left circular polarization, have been used to probe the fine structure and gross motion of quasars and related radio objects.

Correlated-flux and closure-phase measurements made between 1971 October and 1974 October of the quasars 3C 345 and NRAO 512 and the blue stellar object 1633+38 indicate that 3C 345 is composed of two components with a separation of about 1.0 (in late 1971) and 1.3 (in late 1974) milliarcseconds and with a position angle of about 105°, with the smaller, weaker, component to the east (direction of increasing right ascension) and south of the larger, stronger one; the latter two sources contain an unresolved compact component with a Gaussian half-width at half-power of ≤0.2 and 0.3 milliarcseconds, respectively. Based on the two-component interpretation of the 3C 345 structure, the observations suggest that the component separation is increasing at a rate of about 1.2 to 1.6 x 10^{-17} radians/sec, corresponding to a projected velocity of expansion of 2 to 3c based on the proper motion distance.
The separation of the quasars 3C 345 and NRAO 512 was determined to an accuracy of about 0.5 to 1.0 milliarcseconds in each of four experiments between 1971 October and 1974 May using differenced fringe-phase observations. The separation of the two sources showed no trend with time to an accuracy of about 0.6 milliarcseconds per year. Taking this uncertainty as an upper limit on the rate of change of the separation of the sources, we can estimate a minimum distance to the sources by two methods: if one source is assumed to be extragalactic, the parallax distance of the second source is greater than about 2 kpc; if both sources have transverse velocities of the order of 100 km/sec, and the difference of the velocity vectors is also of the order of 100 km/sec, the proper motion distance of both sources is at least about 35 kpc.

Thesis Supervisor: Irwin I. Shapiro

Title: Professor of Geophysics and Physics
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>2</td>
</tr>
<tr>
<td><strong>CHAPTER I.</strong> INTRODUCTION</td>
<td>12</td>
</tr>
<tr>
<td><strong>CHAPTER II.</strong> OBSERVABLES</td>
<td>15</td>
</tr>
<tr>
<td>II.1 Source Structure</td>
<td>16</td>
</tr>
<tr>
<td>II.2 Source Positions</td>
<td>19</td>
</tr>
<tr>
<td><strong>CHAPTER III.</strong> EXPERIMENTS</td>
<td>24</td>
</tr>
<tr>
<td>III.1 General Description</td>
<td>24</td>
</tr>
<tr>
<td>III.2 Radiometry Data</td>
<td>29</td>
</tr>
<tr>
<td>III.3 Scheduling</td>
<td>31</td>
</tr>
<tr>
<td><strong>CHAPTER IV.</strong> SOURCE STRUCTURE - METHODS FOR ANALYSIS</td>
<td>33</td>
</tr>
<tr>
<td>IV.1 Amplitude Data Preparation</td>
<td>33</td>
</tr>
<tr>
<td>IV.2 Definition of Closure Phase</td>
<td>38</td>
</tr>
<tr>
<td>IV.3 Methods of Source Modelling</td>
<td>39</td>
</tr>
<tr>
<td>IV.3.a The Standard Model</td>
<td>39</td>
</tr>
<tr>
<td>IV.3.b Modelling Using Fourier Series and Transform Inversion</td>
<td>43</td>
</tr>
<tr>
<td><strong>CHAPTER V.</strong> SOURCE STRUCTURE RESULTS</td>
<td>46</td>
</tr>
<tr>
<td>V.1 NRAO 512</td>
<td>46</td>
</tr>
<tr>
<td>V.2 1633+38</td>
<td>60</td>
</tr>
<tr>
<td>V.3 3C 345</td>
<td>67</td>
</tr>
<tr>
<td>V.3.a Discussion of the Individual Experiments</td>
<td>97</td>
</tr>
<tr>
<td>V.3.b Results of Fourier Modelling</td>
<td>109</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (continued)

CHAPTER V (continued)

V.3.c Secular Trends in the Structure of 3C 345 115
V.3.d Other Simple Models 120

CHAPTER VI. RELATIVE SOURCE POSITIONS FROM DIFFERENCED PHASE MEASUREMENTS

VI.1 Phase Connection 126
VI.2 Removal of Source Structure 135
VI.3 Source Separation Results 138
VI.4 Discussion 153

CHAPTER VII. FURTHER DISCUSSION OF 3C 345

VII.1 Summary of Available Information 156
VII.1.a Optical Data 156
VII.1.b X-Ray Data 162
VII.1.c Radio Data 163
VII.2 Topics for Further research 177

Appendix A: CALCULATION OF THE COMPLEX FRINGE VISIBILITY FROM THE BRIGHTNESS DISTRIBUTION 179

LIST OF SYMBOLS 182

BIBLIOGRAPHY 189
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV.1</td>
<td>Geometry for the Two-Component Model</td>
<td>41</td>
</tr>
<tr>
<td>V.1</td>
<td>The u-v Tracks of 3C 345</td>
<td>49</td>
</tr>
<tr>
<td>V.2</td>
<td>1972 Correlated Flux of NRAO 512</td>
<td>52</td>
</tr>
<tr>
<td>V.3a-f</td>
<td>1974 Correlated Flux of NRAO 512</td>
<td>52</td>
</tr>
<tr>
<td>V.4</td>
<td>1974 Closure Phase of NRAO 512</td>
<td>52</td>
</tr>
<tr>
<td>V.5a-f</td>
<td>Correlated Flux of 1633+38</td>
<td>61</td>
</tr>
<tr>
<td>V.6a-c</td>
<td>Closure Phase of 1633+38</td>
<td>61</td>
</tr>
<tr>
<td>V.7a&amp;b</td>
<td>HG Correlated Flux of 3C 345 from 1971 October through 1973 March</td>
<td>68</td>
</tr>
<tr>
<td>V.8a&amp;b</td>
<td>HN Correlated Flux of 3C 345</td>
<td>72</td>
</tr>
<tr>
<td>V.9a&amp;b</td>
<td>HG Correlated Flux of 3C 345 from 1973 May through 1974 October</td>
<td>72</td>
</tr>
<tr>
<td>V.10a&amp;b</td>
<td>NG Correlated Flux of 3C 345</td>
<td>72</td>
</tr>
<tr>
<td>V.11a-d</td>
<td>HS Correlated Flux of 3C 345</td>
<td>72</td>
</tr>
<tr>
<td>V.12a&amp;b</td>
<td>NS Correlated Flux of 3C 345</td>
<td>72</td>
</tr>
<tr>
<td>V.13</td>
<td>GS Correlated Flux of 3C 345</td>
<td>72</td>
</tr>
<tr>
<td>V.14a-d</td>
<td>HNS Closure Phase of 3C 345</td>
<td>72</td>
</tr>
<tr>
<td>V.15a&amp;b</td>
<td>HNG Closure Phase of 3C 345</td>
<td>72</td>
</tr>
<tr>
<td>V.16</td>
<td>HGS Closure Phase of 3C 345</td>
<td>73</td>
</tr>
<tr>
<td>V.17</td>
<td>NGS Closure Phase of 3C 345</td>
<td>73</td>
</tr>
</tbody>
</table>
LIST OF FIGURES (continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>V.18a-c</td>
<td>Brightness Distribution Map of 3C 345 from the Fourier Inversion Method</td>
<td>111</td>
</tr>
<tr>
<td>V.19</td>
<td>Position Angle and Component Separation of 3C 345 as a Function of Date</td>
<td>116</td>
</tr>
<tr>
<td>V.20</td>
<td>Total and Model Flux of 3C 345 as a Function of Date</td>
<td>121</td>
</tr>
<tr>
<td>VI.1</td>
<td>Residual Fringe Phase versus Time for the First 15 Observations of 3C 345 in 1972 July</td>
<td>129</td>
</tr>
<tr>
<td>VI.2</td>
<td>Residual Fringe Rate for 3C 345 in 1972 July</td>
<td>131</td>
</tr>
<tr>
<td>VI.3</td>
<td>&quot;New&quot; Residual Fringe Phase versus Time for the First 15 Observations of 3C 345 in 1972 July</td>
<td>132</td>
</tr>
<tr>
<td>VI.4</td>
<td>Residual Fringe Rates for Several Possible Phase Connections of the First 15 Observations of 3C 345 in 1972 July</td>
<td>134</td>
</tr>
<tr>
<td>VI.5</td>
<td>The Structure Phase of 3C 345 for 1974 May Calculated from the Standard and Fourier Series Models</td>
<td>139</td>
</tr>
<tr>
<td>VI.6</td>
<td>Coordinates of NRAO 512</td>
<td>146</td>
</tr>
<tr>
<td>VI.7</td>
<td>1974 May HG Post-Fit Residual Fringe Phases from the Differenced Phase Solution for the Coordinates of NRAO 512</td>
<td>148</td>
</tr>
<tr>
<td>VI.8a&amp;b</td>
<td>1971 October HG Post-Fit Residual Phases from the Differenced Phase Solution for the Coordinates of NRAO 512, With and Without a Phase-Connection Error</td>
<td>150</td>
</tr>
<tr>
<td>VII.1a&amp;b</td>
<td>Blue Magnitude of 3C 345 versus Date</td>
<td>157</td>
</tr>
<tr>
<td>VII.1c</td>
<td>Optical Linear Polarization of 3C 345 versus Date</td>
<td>157</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>VII.2</td>
<td>Radio Flux of 3C 345 versus Date and Wavelength</td>
<td>164</td>
</tr>
<tr>
<td>VII.3</td>
<td>Radio Spectra of 3C 345</td>
<td>168</td>
</tr>
<tr>
<td>VII.4</td>
<td>Radio Linear Polarization of 3C 345 versus Date and Wavelength</td>
<td>171</td>
</tr>
<tr>
<td>VII.5</td>
<td>Measured Position Angle of Linear Polarization of 3C 345 versus Wavelength Squared</td>
<td>174</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>List of Experiments and Equipment</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>Total Flux Measurements</td>
<td>47</td>
</tr>
<tr>
<td>3</td>
<td>Standard Model Parameters for 3C 345</td>
<td>92</td>
</tr>
<tr>
<td>4</td>
<td>List of Differenced Fringe Phase Experiments</td>
<td>136</td>
</tr>
<tr>
<td>5</td>
<td>Coordinates of NRAO 512</td>
<td>143</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

The earliest very-long-baseline interferometry (VLBI) experiments indicated that some extragalactic radio sources, particularly quasars, have very small "compact" components with sizes of the order of 1 to 10 milliarcseconds or smaller (for example, Clark et al., 1968 a and b; Kellermann et al., 1968; Clarke et al., 1969; Jauncey et al., 1970; Kellermann et al., 1970). The small sizes of these compact radio sources suggested that they would be excellent candidates for use in a test of general relativity based on the apparent change in the positions of sources due to the bending of light as it passes close to the sun. The quasars 3C 279, which is occulted by the sun in mid-October, and 3C 273 were chosen for a VLBI light-bending experiment in 1970 in which the difference in the coordinates of the two quasars was measured as a function of the distance between 3C 279 and the sun. Due to problems introduced by the solar corona, this first relativity test did not prove as accurate as had been hoped (Robertson et al., 1971; Margolis, 1973); however, the measurements did reveal that both quasars showed fine structure when observed with a long baseline (Knight et al., 1971). Follow-up experiments
in 1971 February, when the sources were far from the sun, yielded the exciting result that the fine structure of the quasars was time variable on the scale of a few months (Whitney et al., 1971; Cohen et al., 1971).

The findings from these observations sparked a series of on-going experiments to determine the radio fine-structure of quasars and related sources, such as Seyfert galaxies, and to monitor any time variation of that structure (for example, Wittels et al., 1975; Niell et al., 1975; Shaffer et al., 1975). The experiments in the early 1970's usually involved only a single baseline; however, although it quickly became apparent that many sources exhibited fine structure, the data from a single baseline posed more questions than they answered. By 1973, it became practice to employ three or four antennas in simultaneous observation; the data resulting from these multi-antenna experiments offered the possibility of using more sophisticated modelling techniques to distinguish among possible simple models for some sources as well as to describe those sources which were too complicated for simple models. In addition, the use of three or more antennas for the first time provided some phase information for VLBI source modelling (Rogers et al., 1974).

Concurrent with the observations of radio fine-structure, VLBI was being used to determine very accurate positions for the radio sources (Rogers et al., 1973). In addition,
the technique for measuring differential positions used in the relativity tests was applied to a second, very closely-spaced pair of sources, 3C 345 and NRAO 512, and their relative separation monitored to search for time variability.

This thesis focuses on the fine-structure of 3C 345 and the relative position of 3C 345 and NRAO 512. Chapter II gives a short summary of the VLBI observables and their information content with respect to source structure and source coordinates. Chapter III is a brief description of the experimental setup. The methods used to analyze the brightness distribution of the sources are described in Chapter IV and the results of this analysis, in Chapter V. Chapter VI gives the methods, results, and implications of the determination of the separation of 3C 345 and NRAO 512 as a function of time. Finally, Chapter VII includes a summary of the optical, X-ray, and other radio observations of 3C 345, and a brief discussion of questions that remain.
CHAPTER II

OBSERVABLES

Everything that can be learned about extragalactic radio sources from long baseline interferometry is contained in the observables, fringe amplitude and fringe phase, and their dependence on time and frequency. For an interferometer composed of antennas at sites j and k, the measured fringe amplitude is approximately equal to twice the fraction of correlated bits minus one, and when normalized gives $F_{jk}^C$, the amount of source flux* that is correlated at a given interferometer resolution. The measured fringe phase, $\phi_{jk}^m$, is the frequency times the measured time delay, which is defined as the time lag between receipt of the source signal at the two observing sites as measured by the clock at site j, designated the reference clock. The time derivative of fringe phase with respect to the reference time is called fringe rate, and the derivative with respect to frequency, group delay+.

*In this thesis, the term "flux" refers to the power incident per unit area per unit frequency interval; the modifier "density" is consistently omitted.

+For a more detailed description of the geometry, the resolution, and the observables, see Whitney (1974) and Robertson (1975). Only a brief summary will be given here.
In this thesis, two kinds of information about the sources are of interest: the brightness distribution of the source, and the separation of two closely spaced sources as a function of time. The brightness distribution is determined from the correlated flux data and the portion of the fringe phase due to the delays between receipt of signals from different parts of the source, called structure phase. The separation of two sources is determined very accurately by an analysis of the differences of the fringe phases of the two sources, and less accurately by use of the differences in group delays and delay rates.

II. 1. Source Structure

The brightness distribution, \( B(x,y) \), of a radio source is related to the correlated flux and structure phase through the Fourier transform (Kraus, 1966):

\[
V(u,v) = F_c(u,v)e^{i\phi^S(u,v)} = \int B(x,y)e^{-2\pi i(ux + vy)}dx\,dy
\]

(II.1)

\( V(u,v) \) is called the complex fringe visibility, and \( u \) and \( v \) are the spatial-frequency transform pair corresponding to the lengths \( x \) and \( y \) in coordinate space; \( F_c \), the correlated flux, and \( \phi^S \), the structure phase, are functions of \( u \) and \( v \). For measurements made with the \( jk \) interferometer, \( u_{jk} \) and \( v_{jk} \) are the resolutions of the interferometer in the \( \hat{x} \) and \( \hat{y} \) directions, respectively, where \( \hat{x} \) and \( \hat{y} \) are cartesian unit vectors;
$F_{jk}^C$ and $\phi_{jk}^S$ are the correlated flux and structure phase of the interferometer with $u_{jk}$ and $v_{jk}$ resolution*. The convention used here is that $\hat{y}$ points in the north direction and $\hat{x}$ points in the direction of increasing right ascension.

The resolution of the interferometer is just the length of the baseline vector, $\hat{b}_{jk}$, the vector from the remote to the reference site, projected into the plane perpendicular to the line of sight to the source, which is in the $\hat{e}$ direction. Therefore the components of the resolution in the $\hat{x}$ and $\hat{y}$ directions are

$$u_{jk} = \frac{[\hat{b}_{jk} - (\hat{b}_{jk} \cdot \hat{e})\hat{e}] \cdot \hat{x}}{\lambda}$$

$$v_{jk} = \frac{[\hat{b}_{jk} - (\hat{b}_{jk} \cdot \hat{e})\hat{e}] \cdot \hat{y}}{\lambda}$$

where $\lambda$ is the wavelength of the observations. Writing $\hat{b}_{jk}, \hat{x}, \hat{y}$, and $\hat{e}$ in an earth-centered, non-rotating, cartesian coordinate system with unit vectors $\hat{z}_1$ in the direction of the true equinox of date, $\hat{z}_3$ in the direction of the instantaneous spin axis of the earth, and $\hat{z}_2 = \hat{z}_3 \times \hat{z}_1$, we have

*Both the brightness distribution and the complex visibility are functions of frequency. Since all the measurements discussed in this thesis were made around 7.8 GHz, the frequency dependence will be suppressed in the equations and most of the discussion; however, it should be remembered.
\( \hat{b}_{jk} = b_{jk} \{ \cos D_{jk} \{ \cos (A_{jk}^0 + \Omega t_s) \hat{z}_1 + \sin (A_{jk}^0 + \Omega t_s) \hat{z}_2 + \sin D_{jk} \hat{z}_3 \} \} \)

\[
\begin{align*}
\hat{x} &= -\sin \delta \hat{z}_1 + \cos \delta \hat{z}_2 \\
\hat{y} &= -\sin \delta [\cos \alpha \hat{z}_1 + \sin \alpha \hat{z}_2] + \cos \delta \hat{z}_3 \\
\hat{e} &= \cos \delta [\cos \alpha \hat{z}_1 + \sin \alpha \hat{z}_2] + \sin \delta \hat{z}_3
\end{align*}
\]

(II.3)

and \( u_{jk} \) and \( v_{jk} \) are given by

\[
\begin{align*}
u_{jk} &= \frac{b_{jk}}{\lambda} \cos D_{jk} \sin (A_{jk}^0 + \Omega t_s - \alpha) \\
v_{jk} &= \frac{b_{jk}}{\lambda} [-\cos D_{jk} \sin \delta \cos (A_{jk}^0 + \Omega t_s - \alpha) + \sin D_{jk} \cos \delta]
\end{align*}
\]

(II.4)

In the above equations \( b_{jk} \) is the baseline length, \( D_{jk} \) is the baseline vector declination, \( A_{jk}^0 \) is the baseline vector right ascension when the sidereal time, \( t_s \), at the Greenwich meridian is zero, \( \Omega \) is the Earth's rotation rate, and \( \alpha \) and \( \delta \) are the right ascension and declination of the source, respectively.

Equations (II.4) are parameterized equations for an ellipse with the origin at \( u_{jk} = 0, v_{jk} = \frac{b_{jk}}{\lambda} \sin D_{jk} \cos \delta \) and semi-axes \( \frac{b_{jk}}{\lambda} \cos D_{jk} \) in the \( \hat{x} \) direction and \( \frac{b_{jk}}{\lambda} \cos D_{jk} \sin \delta \) in the \( \hat{y} \) direction. Thus, for a given baseline, the higher the declination of the source, the closer the u-v "track" is to a circle centered at \( u = 0, v = 0 \).

The source structure can be determined by measuring the correlated flux and structure phase and inverting the Fourier transform to obtain the brightness distribution.

Ideally, one would like to know the brightness as a function of frequency, and, for a polarized source, the brightness
for all four Stokes parameters; however, most experiments are conducted at a single frequency and are sensitive to only a single polarization. Even with these restrictions there are two major problems: the first is that the structure phase cannot be extracted from the phase measured on the jk baseline because we do not have sufficient precision to untangle it from all the other contributions to the measured fringe phase. Therefore for single baseline experiments, only correlated flux data are available for source structure determination. Second, observations along the u-v ellipse of a single, or even several baselines usually do not fulfill the criteria of sampling theory so that the transform cannot be uniquely inverted, even if both amplitude and phase data were available. As a result, more indirect techniques are used which can result in a model which fits the data well but is not the only model which would do so.

II. 2. Source Positions

For convenience in handling many types of observations, the theoretical model for the measured fringe phase of a VLBI experiment is expressed in a coordinate system with origin at the solar system barycenter, and general relativity is assumed valid. Then this fringe phase on a baseline from reference site j to remote site k is given by

$$\phi_{jk}^t(t) = \omega [\Delta_0^t + \tau_j^c + \tau_j^a - \tau_j^p] + \psi_j + \phi_j^s + 2n\pi$$

(II.5)
where $\Delta t_{jk}$, the geometric time delay, is

$$\Delta t_{jk}(t) \approx \frac{1}{c} [\overrightarrow{B}_{jk}(t) \cdot \hat{e}]$$  \hspace{1cm} (II.6)

and $\tau_{jk}^c$, the difference in deviations of the clocks at sites $j$ and $k$ from atomic time, is described by a polynomial series

$$\tau_{jk}^c = \sum_{q} [a_{jq}(t-t_o)q - a_{kq}(t-t_o-\Delta t_{jk})q]$$

$$= (a_{jo} - a_{ko}) + [a_{jl}(t-t_o) - a_{kl}(t-t_o-\Delta t_{jk})] + \text{higher order terms}$$  \hspace{1cm} (II.7)

In the above expressions, $t$ is coordinate time; $\omega$ is the (angular) radio frequency; $\tau_{jk}^a$ is the delay introduced by the neutral atmosphere; $\tau_{jk}^p$ is the delay due to the difference in the number of charged particles along the path to each site and is principally due to the ionosphere; $\psi_{jk}$ is the difference of the antenna-feed position angles; $\phi_{jk}^s$ is the structure phase; $n$ is an integer giving the ambiguity; $a_{jq}$ and $a_{kq}$ are the coefficients of the clock polynomial; $t_o$ is the reference time, usually taken as the midpoint of some set (one or a few days) of observations.

The fringe rate can then be written

$$\dot{\phi}_{jk}(t) \equiv \frac{d\phi_{jk}(t)}{dT_j} \approx \omega \left[ \frac{d\Delta t_{jk}}{dt} + \dot{\tau}_{jk}^c + \dot{\tau}_{jk}^a - \dot{\tau}_{jk}^p \right] + \psi_{jk} + \dot{\phi}_{jk}^s$$  \hspace{1cm} (II.8)

where a dot indicates the derivative with respect to coordinate time, and $T_j$, the time kept by the clock at site $j$,
can be written

\[ T_j = t + \text{UTC} - \text{Al} - 32.15 + \sum_{q=0}^{Q} \alpha_{jq}(t-t_0)^q \]  \hspace{1cm} (II.9)

Here UTC is coordinated universal time; Al is U.S. Naval Observatory (USNO) atomic time; and 32.15 is an epoch offset between Al and coordinate time chosen for convenience in other astronomical observations. Finally, the group delay is written

\[ \tau_{jk}(t) = \Delta t + \tau_{jk}^c + \tau_{jk}^a + \tau_{jk}^p + (\phi_{jk}^s)' + \psi'_{jk} \]  \hspace{1cm} (II.10)

where the prime denotes differentiation with respect to frequency.

For fringe phase, fringe rate, and group delay, the term proportional to \( \Delta t_{jk}(t) \) contains all the information about the source position. Using Equations (II.3), we write the geometric delay

\[ \Delta t_{jk}(t) = \frac{b}{c}[\cos D_{jk}\cos \delta \cos (\cot_{jk} + \delta t_s - \alpha) + \sin D_{jk} \sin \delta] \]  \hspace{1cm} (II.11)

The methods used to estimate source positions and baseline parameters from the fringe rate and group delay data are described in Robertson (1975).

By observing a closely spaced pair of sources, designated 1 and 2, and assuming the location of source 1, the difference of the fringe phases, rates, and group delays for the two
sources can be used to improve the accuracy of the coordinates of source 2. In order to use fringe phase in this procedure, it is necessary to determine the ambiguity, n, between successive measurements of phase. One method of "phase connection" was used with features in the water-vapor source W3(OH) (Reisz et al., 1973). In the experiments discussed here, phase connection was assured by rapidly switching between a pair of sources, with a cycle time of 4 to 8 minutes between successive observations of the same source. Then the differenced, phase-connected fringe phase is given by

\[
\Delta \phi_{jk}(t) = \omega \left[ -\frac{1}{c} B_{jk} \cdot (\hat{e}_1 - \hat{e}_2) + (\tau_{jk}^a)_{1} - (\tau_{jk}^a)_{2} - (\tau_{jk}^p)_{1} + (\tau_{jk}^p)_{2} \right] \\
+ (\phi_{jk}^s)_{1} - (\phi_{jk}^s)_{2} + \text{constant}
\]

(II.12)

For a sufficiently closely spaced pair, such as the quasars 3C 345 and NRAO 512, and for the cycle times used in our experiments, \( \tau_{jk}^c \) and \( \psi_{jk} \) tend to vanish. The atmosphere (both neutral and charged) can be modelled, although for this source pair, the solution for the location of source 2 is quite insensitive to the parameters of the atmosphere model (see Section VI.3). The structure phases, of course, do not cancel, nor do the noise terms. Similar differenced equations can also be written for the fringe rate and group delay and all three differenced quantities used to determine the location of the second source with respect to the first. The
phase-connected fringe phase data give the most accurate results.

When phase connection can be accomplished for a pair of sources on all three baselines of a closed loop, the redundancy can be used to estimate two of the three coordinates of one of the observing sites as well as the location of source 2.
CHAPTER III
EXPERIMENTS

III. 1. General Description

The experiments discussed in this thesis were carried out at several-week to several-month intervals between 1971 October and 1974 October, and used five antennas whose locations and diameters are: Haystack, Massachusetts, 120-foot (36.6 m); Goldstone, California, 210-foot (64 m); NRAO, West Virginia, 140-foot (42.7 m); Onsala, Sweden, 84-foot (25.6 m); and Fairbanks, Alaska, 85-foot (26 m). They will hereafter be designated H, G, N, S, and F, respectively. In Table 1, Column 1 gives the dates of the experiments; Column 2 lists the antennas used; Column 3 gives the polarization; Column 4 indicates whether or not wide-band synthesis (see below) was used; Columns 5 through 9 indicate how the radiometry was recorded at each of the stations; columns 10 through 14 indicate the frequency standard used at each station; Column 15 lists any special problems that occurred.

Wide-band synthesis was achieved by rapid switching among 5 or 6 channels each 360 kHz wide, centered at frequencies 1-50 MHz from the central channel at 7850 MHz; in some cases the data from one or more of the separate channels were not used. This synthesis technique is described in Whitney (1974) and Hinteregger (1972). Hydrogen-maser frequency standards were used in all experiments at all stations except G, where
TABLE 1
List of Experiments and Equipment

<table>
<thead>
<tr>
<th>Date</th>
<th>Antennas</th>
<th>Pol.</th>
<th>Freq.</th>
<th>H</th>
<th>G</th>
<th>N</th>
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<tr>
<td>1 1971 October 10</td>
<td>H,G</td>
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<td>N</td>
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<tr>
<td>2 1972 April 14,15</td>
<td>H,G</td>
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<td>3 1972 May 9,10</td>
<td>H,G,F</td>
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<tr>
<td>4 1972 May 29,30</td>
<td>H,G,F</td>
<td>LCP</td>
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<td>C</td>
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<tr>
<td>5 1972 June 3</td>
<td>H,F</td>
<td>LCP</td>
<td>Y</td>
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<td>6 1972 June 6,7</td>
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<td>11 1973 February 4,5</td>
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<td>H,G,1,S</td>
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<td>U</td>
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<tr>
<td>14 1973 May 22,23</td>
<td>H,S</td>
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<tr>
<td>15 1973 August 10-14</td>
<td>H,G,1,N,S</td>
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<td>H,G,1,N,S</td>
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<td>NAR</td>
<td>C,B C</td>
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<td>19 1974 April 29-May 3</td>
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<td>NAR</td>
<td>C,B C</td>
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<tr>
<td>20 1974 July 8-12</td>
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<td>U</td>
<td>NAR</td>
<td>C,B C</td>
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<td>NAR</td>
<td>C,B</td>
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*See footnotes on following page.
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<th>Frequency Standard</th>
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<td>21</td>
<td></td>
</tr>
</tbody>
</table>
Footnotes for Table 1, page 25.

1. G participated only 26 hours.
2. No frequency switching when G was participating.
3. G participated only 12 hours.
4. During a short part of this experiment, the H and G antennas switched between LCP and RCP. During this period, frequency-switching was turned off. Three RCP measurements of 1633+38 are reported; all other measurements in this thesis are LCP.
5. U stands for the Univac 490 computer at Haystack; NAR stands for noise-adding radiometer; C stands for a chart recording; B stands for beam switching.
rubidium or cesium standards had to be used during several 1973 experiments. The effect of the phase noise added by these less stable frequency standards was reduced by processing coherently over segments of data shorter than the standard 180-second tape, and then averaging the segments incoherently.

The Mark I system was used to record the data on magnetic tape. This system and our data processing programs were checked (Rogers, 1973) by simultaneously recording tapes using two separate video converters. The input to each converter consisted of an independent noise source plus a fraction of the same noise source. This fraction of correlated signal was measured with square-law detectors that were attached to each converter output, and the results were compared with the output of the processing programs for a set of fringe rates in the range from 0.5 to 20,000 Hz, which encompassed the observed fringe rates. The fraction of correlated noise was also varied to range between 0.03 and 0.28. The comparison showed that the difference varied but was always within 6% -- about three times the uncertainty of the square-law detector measurements. The instrumentation and data processing programs through the stage where the fringe amplitude and the measured fringe phase, fringe rate, and group delay are calculated are described in Whitney (1974).
III. 2. Radiometry Data

Radiometry data, required for the normalization of the fringe amplitude, were recorded using a variety of techniques (see Table 1). These temperature measurements were generally wide-band; the only exceptions were measurements at G after the noise-adding radiometer was added, and at H, where, if there were substantial differences between the narrow and wide-band measurements, the narrow-band data were used. No attempt has been made to measure the variation of system gain over the individual frequency channels. At H, G, and N, system temperature (either $T_{ON}$ or $T_{OFF}$) and source (antenna) temperature ($T_A = T_{ON} - T_{OFF}$) were normally measured for each 3-minute observation. At H, a calibrated noise tube was used in conjunction with the U490 computer which continuously recorded system temperature averaged over 6-12 seconds. At G, until 1972 November, a chart recording, periodically calibrated by a noise tube, was used; this was later augmented by very accurate measurements with each observation made with a noise adding radiometer (NAR). At N, a chart recording calibrated by a noise tube was used for system temperature, however antenna temperatures were measured by beam switching with an off-axis receiver horn.

If heavy rain caused the system temperature to fluctuate wildly on a time scale less than three minutes, the fringe amplitude data were discarded as unusable because they could
not be normalized. If the antenna temperature could not be measured because the source was too weak to be seen above the noise, the antenna temperature was calculated using that measured at another site during the same experiment and the antenna effective areas, $A_e$. For example

$$ (T_A)_H = \frac{(A_e)_H}{(A_e)_G} (T_A)_G $$

(III.1)

and then

$$ (T_{ON})_H = (T_{OFF})_H + (T_A)_H $$

(III.2)

The values used for $(A_e)_G$ and $(T_A)_G$ were those for the elevation angle corresponding to the maximum effective area as determined by calibration with strong sources. The effective areas of all other antennas were assumed not to vary with elevation angle.

At O, the system temperature $(T_{OFF})_O$ of about 60°K, the antenna efficiency on the order of 10 to 15%, and the radiometer bandwidth of 2 MHz yielded a system sensitivity insufficient to determine source temperatures reliably so that all source temperatures were calculated from those measured at another site. At F, a system temperature of 300°K, previously measured, was used for all observations; no sources could be detected above the system temperature so that here, also, all source temperatures were calculated.
III. 3. Scheduling

A typical VLBI experiment is designed to obtain both source structure and position information. Generally 2 or 3 three-minute observations are taken on a source before the antennas move to the next source. Two or three sources are observed concurrently with an interval of 30 to 60 minutes between successive sets of observations of the same source. For an experiment 24 to 30 hours in length, usually 10 to 15 sources are observed.

If a primary goal of the experiment is the determination of source and baseline coordinates, generally 3 or 4 sources are observed concurrently and a larger total number of sources is observed. The result is that the time between successive observations of the same source is longer, and the total span of time during which a source is observed is usually shorter. For many sources, the fringe amplitude varies sufficiently slowly that such gaps between successive measurements of the amplitude do not hamper the analysis of the source structure. However, for sources such as the Seyfert galaxy 3C84, whose fringe amplitude shows rapid variation with baseline projection, only short intervals between observations can be tolerated without serious loss of information. It is particularly important for observations to be closely spaced when the fringe amplitude displays a deep minimum. Unfortunately it is not usually possible to predict in advance where these minima will
be located as the structure of a source is often time variable. Schedules for experiments designed primarily for the study of source structure usually have only 1 or 2 sources under observation at a time, and fewer sources in the overall span of the experiment. Large differences in the right ascension and declination of sources observed concurrently are avoided to reduce the time wasted in moving the antennas.

When one goal of the experiment is to determine the separation of a pair of closely spaced sources using differenced fringe phase data, the observing schedule is designed so that the antennas cycle between 2 sources, with a 4 to 8 minute cycle time, for a minimum of 4 to 5 hours. The shorter the cycle time, the more reliable the determination of the behavior of the phase in the gap between observations of the same source, and the more exact the cancellation of the phase terms due to clock drifts. In addition, such closely spaced observations of the source facilitate the study of source structure.
CHAPTER IV
SOURCE STRUCTURE - METHODS FOR ANALYSIS

For the sources that will be discussed in this thesis, each experiment between 1971 October and 1973 March produced usable fringe amplitude data on the HG baseline; the 1973 May experiment gave HS amplitudes. As was described in Chapter II.1, the structure phase information is lost because it cannot be extracted from the other constituents of the measured fringe phase. All of the later experiments (see Table 1) yielded fringe amplitudes on three or more baselines and, by combining the fringe phases measured on a triangle of baselines, they also yielded at least a few values of "closure" phase. The following sections will discuss the preparation of the amplitude data for modelling, the definition of closure phase, and the methods of source modelling including a standard model.

IV. 1. Amplitude Data Preparation

For the experiments through 1973 May, the normalization of the fringe amplitude data and an error analysis were described in detail in Wittels et al. (1975). The only amendment to that discussion is that the bias in the fringe amplitude -- the last entry in Table 2 of that paper -- has been removed from the data by subtraction of the theoretical estimate of the bias. The procedures described in Wittels et al. (1975) were also used to handle the data from ex-
periments after 1973 May; however, several new problems arose which warrant attention. All but the first of these are noted in the last column of Table 1 for the appropriate experiment(s).

(i) **Efficiency of the S Antenna**

When the 1973 May data were processed, the efficiency of the S antenna was believed to be about 0.12±0.01 (see Wittels et al., 1975). However, by March of 1974, the use of this value for efficiency in the calculation of S antenna temperatures began to produce a contradiction: the few HS and NS correlated fluxes calculated from the 1974 March observations of NRAO 512 came out slightly greater than the source's total flux (i.e. normalized amplitude slightly greater than 1). A somewhat higher efficiency would be required to lower the normalized amplitudes below 1.

Between the 1974 March and May experiments, the temperature of the noise tube used to calibrate system temperatures at S was changed from its earlier value of 85°2K to between 35 to 40°K. Unfortunately, careful measurements of the noise tube temperature were never made and are no longer possible to make. During the 1974 July experiment, observations of strong sources were used to determine the ratio of the noise tube temperature to the antenna efficiency by measuring the ratio of source total flux to the source temperature. The noise tube temperature and antenna efficiency could not be individually determined; however, the ratio was found to be
about 250°±50°K, so that a 40°K temperature for the noise tube was consistent with an efficiency of 0.16±0.03, or a 35°K noise tube temperature with an efficiency of 0.14±0.03. However, the value for the ratio is still not consistent with the 1974 May experiment: when the 1974 May NRAO 512 data from HS and NS are normalized using 38°K and 0.15, the normalized amplitudes are greater than 1. The result is that there is an unknown scaling error, which did not seem to have the same value for all experiments, in all correlated flux data from baselines using S (HS, NS, and GS). The data from 1973 May through 1974 March were processed using 8°2 and 12% while those from 1974 May and after were processed using 38° and 15%.

The two most straightforward methods for removing this uncertainty are: (a) Observe simultaneously with a baseline whose scaling is known and whose u-v track crosses the u-v tracks of one or more of the S baselines. Measurements made at the intersections of the tracks can be used to fix the S scaling. None of the other three baselines used in these experiments fit the criterion. (b) Observe one or several sources known to be unresolved; then the normalized amplitudes would be 1 and the scaling could be determined. Unfortunately there is no source which we are certain is unresolved when observed with our present S baselines. Two additional difficulties are that such proposed methods of determining the S scaling may not be valid for experiments
back in 1973 if, for example, the efficiency has changed with time. Second, the antenna which was used for these experiments is unavailable at present for VLBI observations and may continue to be so.

The effects on the model parameters of the uncertainty of the scaling of data from baselines using the S antenna are discussed with the results in Chapter V.

(ii) **Rb or Cs Standard**

For experiments when a hydrogen-maser frequency standard was unavailable at G, Cesium or Rubidium standards were used. The effect of the resulting increase in phase noise was reduced by processing coherently over a shorter segment of data (see Section III. 1).

(iii) **Parity Errors**

Tape-recording difficulties at N and S resulted in very large numbers of parity errors which significantly degraded the fringe amplitudes on baselines using these two antennas. For 1973 August and October, observations with more than 800 parity errors -- a full 3-minute observation has 900 records -- were discarded while those with fewer than 800 had the records containing errors dropped from the processing; however, the scatter of the data points is still much greater than the scatter for similar data from the 1974 May and July experiments. For the 1974 January, March, and
May experiments the recording problem was alleviated by a processing modification; the tape recorders were fixed prior to the 1974 July experiment.

(iv) Bandpass

At N, during the 1974 March experiment, interference severely distorted the apparent spectral response of the system, resulting in an approximately uniform underestimation of the fringe amplitudes on baselines using N. The largest part of this underestimation was removed by scaling the amplitude data by a factor of 1.6 based on the assumption that the ratio of the HN correlated flux to the total flux was about the same in 1974 March as it had been in 1973 August and October for the sources 3C 84, 3C 273, and PKS 2134+004. Nevertheless some unknown fraction of the scale error probably still remains.

(v) No S Radiometry

Because of the loss of the radiometry chart recording from S for the 1973 August experiment, all data using the S station were processed using a constant value of system temperature, $T_{OFF}$. The major errors in the correlated fluxes introduced by this procedure are: an unknown scaling error, an elevation-angle-dependent bias, and a systematic error depending on the temperature of the cooled receiver. These errors affect the correlated fluxes for this experiment for all baselines using S.
Finally, it is important to note that when the signal sinks down below the noise level, as is the case in the minima of 3C 345 measured using the HS and NS baselines, the amplitude estimated by the processing programs must be considered as an upper limit, an unknown fraction of which is due to noise.

IV. 2. Definition of Closure Phase

For experiments in which three or more antennas are used simultaneously, the fringe phases from three baselines forming a closed loop can be combined in such a way as to eliminate the fluctuations due to atmosphere and clocks; the resulting closure phase can be used for source modelling either independently or in conjunction with the fringe amplitude (Rogers et al., 1974).

The closure phase is calculated by writing the fringe phase, Equation (II.5), for each of the three baselines and summing them with the appropriate sign. Using the information that

\[ \Delta t_{jk}(t_j) + \Delta t_{k\ell}(t_k) - \Delta t_{j\ell}(t_j) = 0 \] (IV.1)

which follows from the definition

\[ \Delta t_{jk}(t_j) \equiv t_j - t_k \] (IV.2)

almost all terms cancel and we find the closure phase is given by
\[ \phi_{jkl}^C \equiv \phi_{jk}^m(t_j) + \phi_{kl}^m(t_k) - \phi_{jl}^m(t_j) \]

\[(IV.3)\]

where the \( \phi^S \)'s are structure phases. Note that because the \( \phi^m \)'s are such rapidly varying functions of time, it is important to calculate \( \phi^C \) using \( \phi^m \)'s taken at the appropriate epoch.

Thus \( \phi_{jkl}^C \) can be used to extract source structure information from the measured fringe phase, helping to eliminate the first problem of source structure determination mentioned in Chapter II.1. However, the second problem, the lack of sufficient data to invert the fringe visibility uniquely, has not been solved.

IV. 3. Methods of Source Modelling

IV. 3. a. The Standard Model

Two separate approaches have been tried to produce source models from the correlated fluxes and closure phases. The first is to proceed by choosing a brightness distribution for the source, calculating the transform, and least-squares fitting the measured correlated fluxes and closure phases to determine the parameters of the chosen brightness distribution.

For certain simple choices of the brightness \( B(x,y) \), the fringe visibility [Equation (II.1)] can be easily integrated. For example, for two circular Gaussian components (see
Figure IV.1 with fluxes $S_o$ and $S_1$, separation $r$, position angle $P$, and $e^{-1/2}$ widths* $\sigma_0$ and $\sigma_1$, the brightness distribution is given by:

$$B(x,y) = \frac{S_o}{2\pi\sigma_o^2} e^{-\frac{(x^2+y^2)}{2\sigma_o^2}}$$

$$+ \frac{S_1}{2\pi\sigma_1^2} e^{-\frac{[(x-x_1)^2+(y-y_1)^2]}{2\sigma_1^2}}$$

(IV.4)

In the coordinate system where the $\hat{y}$ points north, and $\hat{x}$ points in the direction of increasing right ascension, $S_o$ is centered at the origin and $S_1$ is at position $(x_1, y_1)$, where $r = \sqrt{x_1^2+y_1^2}$ and $\tan P = y_1/x_1$.

Alternatively, we can define $S = S_o + S_1$ and $\xi = (S_o - S_1)/S$ so that

$$B(x,y) = \frac{S}{2}[\frac{(1+\xi)}{e^{\frac{-(x^2+y^2)}{2\sigma_o^2}}} + \frac{(1-\xi)}{e^{\frac{-(x-x_1)^2+(y-y_1)^2}{2\sigma_1^2}}}]$$

(IV.5)

where $S$ is the total flux of the model and $\xi$ is the relative difference in strength. The fringe visibility+ is then

(Appendix A)

$$V(u,v) = \frac{S}{2}[\frac{\xi}{e^{\frac{-2\pi^2\sigma_0^2(u^2+v^2)}}} + \frac{(1-\xi)}{e^{\frac{-2\pi^2\sigma_1^2(u^2+v^2)}}} e^{-2\pi ir(\sin P + v\cos P)}]$$

(IV.6)

* Hereafter called HPHW for half-power-half-width.

+ See Chapter II for definitions of $u$, $v$, and $V(u,v)$.
Figure IV.1  Geometry for the two-component model of radio brightness. The components have flux $S_0$ and $S_1$, Gaussian HPHW's $\sigma_0$ and $\sigma_1$; component 1 is located at Cartesian position $(x_1, y_1)$ or cylindric position $(r, \phi)$ with respect to component 0, which is centered at the origin.
This model has six parameters, which we define as $S$, $\xi$, $r$, $P$, $\sigma_1^2$, and $\Delta\sigma^2 = \sigma_o^2 - \sigma_1^2$. This particular set of parameters was chosen to simplify the equations, and to point out that the structure phase is only dependent on four of the six parameters. For the $j_k$ baseline, the correlated flux, which depends on all six parameters, is given by

$$
(P_{jk}^C)^{ts} = \frac{S}{2} e^{-4\pi^2 \sigma_0^2 (u_{jk}^2 + v_{jk}^2)} \left[ e^{-4\pi^2 \Delta\sigma^2 (u_{jk}^2 + v_{jk}^2)} + (1-\xi)^2 -2\pi^2 \Delta\sigma^2 (u_{jk}^2 + v_{jk}^2) \cos[2\pi r(u_{jk} \sin P + v_{jk} \cos P)] \right]^{IV.7}
$$

The structure phase, which depends only on $\xi$, $r$, $P$, and $\Delta\sigma^2$, is given by

$$
(\phi_{jk}^S)^{ts} = \tan^{-1} \left[ \frac{-(1-\xi) \sin[2\pi r(u_{jk} \sin P + v_{jk} \cos P)]}{(1+\xi) e^{-2\pi^2 \Delta\sigma^2 (u_{jk}^2 + v_{jk}^2)} + (1-\xi) \cos[2\pi r(u_{jk} \sin P + v_{jk} \cos P)]} \right]^{IV.8}
$$

The closure phase for the $j_k l$ triangle is then

$$
(\phi_{jkl}^C)^{ts} = (\phi_{jk}^S)^{ts} + (\phi_{kl}^S)^{ts} - (\phi_{jl}^S)^{ts}^{IV.9}
$$

Since the model phase is generally a slowly varying function of time, $(\phi_{jkl}^C)^{ts}$ can be calculated from values of $(\phi_{jkl}^S)^{ts}$ taken at the same epoch.

*The superscript ts stands for theoretical-standard model, whereas tf stands for theoretical-Fourier model.*
The standard model is defined as a slightly more complex version of the above; the components are allowed to be elliptical with HPHW's $\sigma_{ox}$, $\sigma_{oy}$, $\sigma_{lx}$ and $\sigma_{ly}$. The two variable parameters $\sigma_{l}^{2}$ and $\Delta \sigma^{2}$ become four -- $\sigma_{lx}^{2}$, $\Delta \sigma_{x}^{2} = \sigma_{ox}^{2}$ $- \sigma_{lx}^{2}$, $\sigma_{ly}^{2}$ and $\Delta \sigma_{y}^{2} = \sigma_{oy}^{2} - \sigma_{ly}^{2}$ -- so that the exponential expressions in Equations (IV.7) and (IV.8) now have the form

$\sigma_{lx}^{2} u_{jk}^{2} + \sigma_{ly}^{2} v_{jk}^{2}$ and $(\Delta \sigma_{x}^{2} u_{jk}^{2} + \Delta \sigma_{y}^{2} v_{jk}^{2})$, instead of $\sigma_{l}^{2} (u_{jk}^{2} + v_{jk}^{2})$ and $\Delta \sigma^{2} (u_{jk}^{2} + v_{jk}^{2})$, respectively. The model can describe anything between a single point component at the origin ($\xi = 1$, $\sigma_{lx}^{2} = \sigma_{ly}^{2} = \Delta \sigma_{x}^{2} = \Delta \sigma_{y}^{2} = 0$), and two elliptical Gaussians of unequal size and strength, where the ellipses are required to have principal axes in either the $\hat{x}$ or $\hat{y}$ direction. This model was chosen as a standard because it is sufficiently complex to provide reasonable descriptions of the three sources discussed in the following chapter.

IV. 3. b. Modelling Using Fourier Series and Transform Inversion

Rogers et al. (1974) describe a 1-dimensional method of source modelling in which the source is assumed to be extended along a line. The measured correlated fluxes and closure phases are then fit with a 1-dimensional Fourier cosine and sine series, respectively, where the series are functions of the resolution in the direction along the line source. The fringe visibility is then constructed from the series coef-
ficients and inverted to give a brightness map of the source.

This method has been extended to two dimensions for use in modelling 3C 345. The correlated flux, an even function, is represented by a series of the form

\[
(f^C_{jk}) = \sum_{\mu=1}^{N} \sum_{\nu=1}^{M} a_{\mu\nu} \cos[\pi(\mu-1) \frac{u_{jk}}{u_m} + \pi(\nu-1) \frac{v_{jk}}{v_m}] \quad (IV.10)
\]

and the single baseline phase, an odd function, by

\[
(\phi^S_{jk}) = \sum_{\mu=1}^{N'} \sum_{\nu=1}^{M'} c_{\mu\nu} \sin[\pi(\mu-1) \frac{u_{jk}}{u_m} + \pi(\nu-1) \frac{v_{jk}}{v_m}] \quad (IV.11)
\]

where \( c_{11} \) is not counted as a parameter since the sine of zero is zero. \( u_m \) and \( v_m \) are the maximum values of \( u_{jk} \) and \( v_{jk} \) encountered in the data to be used in the Fourier fit, and correspond to the highest resolution obtained with the interferometer. Finally, the closure phase is given by

\[
(\phi^C_{jk\ell}) = \sum_{\mu=1}^{N'} \sum_{\nu=1}^{M'} c_{\mu\nu} \{ \sin[\pi(\mu-1) \frac{u_{jk}}{u_m} + \pi(\nu-1) \frac{v_{jk}}{v_m}] \\
- \sin[\pi(\mu-1) \frac{u_{jk}}{u_m} + \pi(\nu-1) \frac{v_{jk}}{v_m}] \\
+ \sin[\pi(\mu-1) \frac{u_{jk}}{u_m} + \pi(\nu-1) \frac{v_{jk}}{v_m}] \}
\quad (IV.12)
\]

It is important to remember that the set of baselines forms a closed loop, so that \( u_{\ell k} = u_{j\ell} - u_{jk} \) and \( v_{\ell k} = v_{j\ell} - v_{jk} \) and \( \phi_{ijk}^C \) is a function only of \( u_{jk}, u_{j\ell}, v_{jk}, \) and \( v_{j\ell} \).
After estimating values of $a_{\mu\nu}$ and $c_{\mu\nu}$, we find the brightness distribution by inverting the fringe visibility using the fast-Fourier transform (FFT). The FFT is calculated in a square 32 x 32 grid with the origin at the center; the $u$ and $v$ spacings between the grid points are $u_m/16$ and $v_m/16$, respectively, giving $x$ and $y$ spacings of $1/2u_m$ and $1/2v_m$, respectively.

If there is a priori information that the source to be modelled is primarily one-dimensional, that is, extended mainly in one direction and compact in the perpendicular direction, the number of terms required to produce a good series representation of the data can be reduced by choosing to compute the series coefficients in a coordinate system that is aligned with the source. For example, consider the source pictured in Figure IV.1; a considerable savings in terms can be realized if the primed directions are used for the Fourier series fit since the structure will depend primarily on $u'$ and only the lowest order terms in $v'$ will be required, where $u'$ and $v'$ are the spatial frequencies in the directions corresponding to $x'$ and $y'$, respectively. Rotated axes were particularly useful in the modelling of 3C 345.
CHAPTER V
SOURCE STRUCTURE RESULTS

Three sources have been selected for discussion: NRAO 512, primarily because knowledge of its structure is needed for the differential phase measurements discussed in Chapter VI; 1633 + 38 because it is an excellent candidate for future three-source differential phase measurements in conjunction with NRAO 512 and 3C 345; and 3C 345 because it is an extremely interesting astronomical object. Table 2 lists the total fluxes of the sources as measured during the experiments as well as the fluxes of the calibration sources. The three sources have approximately the same coordinates so that the u-v tracks* along which they are observed are very similar. These tracks, marked off in (GST) hours, for the six baselines used for these observations are shown in Figure V.1; u and v are in units of $10^6$ wavelengths.

V. 1. NRAO 512

NRAO 512 is a quasar with a redshift, determined from only two spectral lines, of 1.67 (Lynds, 1975). The optical magnitude is about 18.5; however, the optical source has been observed to undergo rapid flaring of more than a magnitude over

*See Equation (II.4) for the time-dependent expressions for u and v.
Table 2*

Total Flux Measurements (Jy)\(^1\)

<table>
<thead>
<tr>
<th>Date</th>
<th>3C 274(^3)</th>
<th>PKS 2134+004(^2)</th>
<th>DR 21(^3)</th>
<th>NRAO 512</th>
<th>3C 345</th>
<th>1633 + 38</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971 Oct 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.0±0.5</td>
</tr>
<tr>
<td>1972 May 9,10</td>
<td>49.1±2.0</td>
<td></td>
<td>2.0±0.3</td>
<td></td>
<td>10.9±0.9</td>
<td></td>
</tr>
<tr>
<td>1972 May 29,30</td>
<td></td>
<td>12.3±0.3</td>
<td></td>
<td></td>
<td>9.6±1.0</td>
<td></td>
</tr>
<tr>
<td>1972 Jun 27,28</td>
<td></td>
<td>12.1±0.3</td>
<td></td>
<td></td>
<td>8.8±0.5</td>
<td></td>
</tr>
<tr>
<td>1972 Jul 3,4(^4)</td>
<td></td>
<td></td>
<td>1.5±0.2</td>
<td></td>
<td>8.3±1.1</td>
<td></td>
</tr>
<tr>
<td>1972 Aug 29,30</td>
<td></td>
<td>12.0±0.2</td>
<td></td>
<td></td>
<td>10.0±0.6</td>
<td></td>
</tr>
<tr>
<td>1973 Feb 4,5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.8±0.6</td>
</tr>
<tr>
<td>1973 Mar 30,31</td>
<td>49.1±2.0</td>
<td></td>
<td>21.7±0.7</td>
<td></td>
<td>11.4±0.6</td>
<td></td>
</tr>
<tr>
<td>1973 May 17-19</td>
<td>49.1±2.0</td>
<td></td>
<td>21.7±0.7</td>
<td></td>
<td>12.4±1.2</td>
<td></td>
</tr>
<tr>
<td>1973 May 22-23</td>
<td>49.1±2.0</td>
<td></td>
<td>21.7±0.7</td>
<td></td>
<td>12.4±1.2</td>
<td></td>
</tr>
<tr>
<td>1973 Aug 10-14</td>
<td>49.1±2.0</td>
<td></td>
<td>21.7±0.7</td>
<td></td>
<td>11.8±0.6</td>
<td></td>
</tr>
<tr>
<td>1973 Oct 12-16</td>
<td>49.1±2.0</td>
<td></td>
<td>21.7±0.7</td>
<td></td>
<td>12.0±0.6</td>
<td></td>
</tr>
<tr>
<td>1974 Jan 22,23</td>
<td>49.1±2.0</td>
<td></td>
<td>21.7±0.7</td>
<td>1.6±0.1</td>
<td>11.2±0.4</td>
<td></td>
</tr>
<tr>
<td>1974 Mar 3-7</td>
<td>49.1±2.0</td>
<td></td>
<td>21.7±0.7</td>
<td>1.9±0.5</td>
<td>11.6±0.8</td>
<td></td>
</tr>
<tr>
<td>1974 Apr 29- May 3</td>
<td>21.7±0.7</td>
<td>1.8±0.1</td>
<td>12.2±0.4</td>
<td>5.0±0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1974 Jul 8-12</td>
<td></td>
<td></td>
<td>21.7±0.7</td>
<td></td>
<td>11.9±0.5</td>
<td></td>
</tr>
<tr>
<td>1974 Oct 26-7</td>
<td></td>
<td></td>
<td>21.7±0.7</td>
<td></td>
<td>11.2±0.3</td>
<td></td>
</tr>
</tbody>
</table>

*Footnote references will be found on p.48 (following).
Footnotes for Table 2 on p.47

1. Each column gives the total flux and associated standard error, both in Janskys; 1 Jy = $10^{-26}$ W m$^{-2}$ Hz$^{-1}$.

2. The total fluxes for this calibrator were obtained from the data of Dent (private communication) by interpolation.

3. The fluxes of the calibration sources 3C 274 [Baars and Hartsuijker, 1972] and DR 21 [Dent, 1972a] were assumed time-invariant. Correction factors of 1.04 and 1.024, respectively, were applied for partial resolution by the 212 beam at G.

4. There were no good calibration sources observed on 1972 July 3,4; the scale was fixed by calibration on 1972 June 27, 28.
Figure V.1  The u-v tracks of the source 3C 345 observed with the HN, HG, NG, HS, NS, and GS interferometers. The units of u and v are millions of wavelengths; the tracks are labelled with tic marks on the (GST) hour. 1633+38 and NRAO 512, whose coordinates are very similar to those of 3C 345, have almost identical u-v tracks.
a time-scale of a few days (Folsom et al., 1970). The radio power has also exhibited outbursts (Locke et al., 1969), but on a time scale of months.

Our early observations of NRAO 512 were made in late 1971 and mid-1972. The 1971 October fringe amplitudes were not usable due to rain at the H site. The correlated flux results from 1972 May and July are shown in Figure V.2 and refer to the scale on the left, whereas the normalized amplitudes, N.A., defined as correlated flux divided by total flux, are referred to the right-hand scale. NRAO 512 was again observed in 1974, briefly in January and March, and in May. The correlated flux data from these experiments are shown in Figure V.3a-f. On all plots of $F_C$, the arrow on the time axis marks interferometer hour angle (≡ IHA) equal to zero.

Closure phases from 1974 May are plotted in Figure V.4. Although all data are shown for all four triangles, it must be remembered that 4 antennas define only 3 independent closure-phase loops. The error bars denote one standard deviation and include random but not systematic errors.

Taken alone, the first two sets of $F_C$ data indicate that in 1972 May and July, NRAO 512 had an unresolved compact component containing about 76% and 67% of the total power, respectively. The remaining power, in each case about 0.45 to 0.50 Jy, would comprise a halo of about constant flux, the compact component being principally responsible for the
Figure V.2  Correlated flux versus Greenwich sidereal time of NRAO 512 observed with the HG interferometer in 1972. The normalized amplitudes are given by the axes on the right with the symbol for the data of each experiment shown beneath the corresponding axis. The arrow on the time axis marks interferometer hour angle equal to zero. Typical error bars are shown. The lines through the data points show the mean level.

Figures V.3a-f  The same as Figure V.2 but for 1974 observations with the HN, HG, NG, HS, NS, and GS baselines, respectively. For the first three figures, the lines through the data points show the mean level.

Figure V.4  Closure-phase versus Greenwich sidereal time of NRAO 512 observed in 1974 May with all four loops formed by the H, N, G and S antennas. Typical error bars are shown. The line through the data points shows the mean level of the HNG data points.
Figure V.2
Figure V.3a

Figure V.3b

Figure V.3c
Figure V.4
decrease in total power from 2.0 to 1.4 Jy between the two experiments. However, two HG measurements of Kellermann et al. (1970) and two of Cohen et al. (1971), when the source was much weaker -- total power 0.7 and 0.8 Jy, respectively -- indicated that the compact component contained about 80 to 85% of the power, leaving only about 0.10 to 0.15 Jy for the halo. One possible explanation is that the compact component increased in flux after the 1971 observations and was partially resolved by the 1972 observations.

A few values of $F_C$ for NRAO 512 were measured in early 1974; however, in 1974 May, six baselines were used to observe NRAO 512 for several hours. The $F_{HC}^C$ data are essentially flat at about 1.65 Jy, or 89% of the flux. The $F_{HG}^C$ and $F_{NG}^C$ data are also flat except for a pronounced dip beginning about 0 hours GST which also appeared in the $F_{GS}^C$ data. This dip is due to systematic effects at G where the source is passing through zenith. The evidence for this conclusion is as follows: For a source, like NRAO 512, with a declination of about 40°, the elevation at G is above 80° during the passage through zenith, causing such a rapid change in azimuth that the antenna has difficulty tracking the source; the result is that the pointing during an observation is variable. Evidence for this effect can be seen in the radiometry chart recordings taken during the experiment. In addition, source temperatures were not measured with each observation due to lack of time
so that overall pointing errors are not normalized out. Finally, 3C 345, which was observed alternately with NRAO 512 and is at virtually the same declination, shows the same degradation of fringe amplitude during the passage through zenith in this experiment. A similar systematic degradation of fringe amplitude was not seen in 1972 because the G antenna bearings were in considerably better condition at that time, allowing somewhat faster and smoother slewing.

The means of the first $2\frac{1}{2}$ hours of the HG and NG correlated fluxes are 1.47 and 1.45 Jy, respectively, or about 80% of the total flux. Since this percentage is lower than that measured with the HN baseline, the compact component has been partially resolved. The combined HN, NG, and HG amplitudes indicate that the compact component has an HPHW less than about $0.3 \times 10^{-3}$ arc seconds. Unfortunately, the $F^C$'s obtained on baselines formed with S cannot be used to help with a better determination of the HPHW because of the scaling problem discussed in the previous chapter (see Section IV.1), although the $F^C_{HS}$ and $F^C_{NS}$ data still appear approximately flat. By contrast, however, $F^C_{GS}$ shows evidence of a slight maximum between 2230 and 2300 GST, not coincident with the maximum resolution of this baseline. This maximum is the first evidence of possible structure in the compact component of NRAO 512.

From the standard model (Section IV.3.a), it is evident that the model phase is zero for a baseline on which the source appears to be only a single Gaussian component. Thus, for
NRAO 512, $\phi^C_{\text{HNG}}$ should be zero; in fact, the mean value is $-1.5\pm0.5$. $\phi^C_{\text{HNS}}$ also should be zero; however, there are too few points to verify this deduction. $\phi^C_{\text{HGS}}$ and $\phi^C_{\text{NGS}}$ should possibly show some structure as indicated by the $F^C_{\text{GS}}$ amplitudes; unfortunately the phases show too much scatter to draw any conclusions along these lines.

Taken together, the observations of NRAO 512 indicate that the compact component

(i) has time variable brightness;

(ii) can contain at least 90% of the source flux, as indicated by the 1974 $F^C_{\text{HN}}$ data, leaving little or perhaps no halo;

(iii) is slightly resolved in 1974 May, with a size of $\leq 0.3 \times 10^{-3}$ as indicated by the correlated flux data from the HN baseline compared with those from the HG and NG baselines, and may also have been resolved in 1972;

(iv) may begin to show structure on the GS baseline where the resolution is over twice that of the HG baseline and about 25% greater than the NS baseline.

Because NRAO 512 appears to be a single compact component, perhaps with a halo, when viewed with the HN, HG, and NG interferometers, the structure phases on these baselines will be taken to be zero for the differential phase calculations of Chapter VI.
V. 2. **1633 + 38**

1633 + 38 is a time-variable radio source, associated with a blue stellar object of about 17th magnitude, with a radic spectrum that clearly indicates a compact component (Pauliny-Toth et al., 1973). The source was first observed with VLBI in 1974 May to check that it did, in fact, have a high normalized amplitude. In 1974 July, because of its proximity to 3C 345, it was used as the calibration source in a polarization test. Finally, in 1974 October it was observed more extensively to confirm the earlier indications that it did, indeed, have a very small compact component.

The correlated flux data for 1633 + 38 are shown in Figure V.5a-f and the closure phase data in Figures V.6a-c. The following qualifications should be noted before any conclusions can be drawn:

(i) The data using the H antenna from the July experiment have a somewhat larger scatter than those from October because only half the usual number of records were processed to produce each data point.

(ii) The 1st, 3rd, and 5th July HG data points (Figure V.5b) were based on right circular polarization observations. All other data in this thesis were based on left circular polarization observations.

(iii) The July data involving the S antenna are suspect because, in addition to the unknown scaling discussed in the previous chapter, all observations were made
Figures V.5a-f Correlated flux versus Greenwich sidereal time of 1633+38 observed with the HN, HG, NG, HS, NS, and GS interferometers, respectively. The normalized amplitudes are given by the axes on the right with the symbol for the data of each experiment shown beneath the corresponding axis. The arrows on the time axes mark interferometer hour angle equal to zero. Typical error bars are shown. For the first three figures, the lines through the data points show the mean level.

Figures V.6a-c Closure phase versus Greenwich sidereal time of 1633+38 observed with the HNG, HNS, and HGS and NGS triangles of antennas, respectively. Typical error bars are shown. For the HNG loop, the line through the points shows the mean level of the 1974 October data.
Figure V.5a

Figure V.5b
Figure V.5c

Figure V.5d
Figure V.6a

Figure V.5b

Figure V.6c
with the antenna elevation angle below 12° so that the pointing may have been quite poor and the source temperatures consequently were probably overestimated. These two effects may partially compensate for one another; however, the former is a systematic error whereas the latter is a random error, independent for each time the antenna is pointed.

The \( F_C^{HN} \) data from both October and July indicate that the compact component of 1633+38 contains between about 85 and 90% of the total power. The normalized amplitudes from the HG and NG interferometers are slightly lower, indicating that, were the source a Gaussian, it would have an HPHW less than about \( 0.2 \times 10^{-3} \) arcseconds, smaller than that for NRAO 512. Notice that the normalized amplitudes are about the same for the two experiments even though the total flux changed by 12%. The \( F_C \) data from the three baselines using S do not really add anything to these conclusions, nor do they contradict them.

The \( \phi_C^{HNG} \) data from the October experiment are approximately flat with an average value of \(-3.2 \pm 4.2\), in agreement with the conclusion that the compact component is essentially unresolved with these baselines. The remaining HNG closure phases are also all close to zero. For the closure loops including the S antenna, the data are much noisier. As with the amplitudes, these data do not really add anything, nor do they contradict anything. The few closure phases using the GS baselines include no indications that the source begins to show appreciable structure at this resolution.
V. 3. 3C 345

3C 345, a quasar with a redshift of 0.595, is an optical, radio, and polarization variable (see, for example, optical: Goldsmith and Kinman, 1965; Kinman et al., 1968; and Lü, 1972; radio: Dent, 1965, 1972b; and Kellermann and Pauliny-Toth, 1968; polarization: Seielstad and Berge, 1975; and Aller, 1970), as well as a possible X-ray source (Giacconi et al., 1974; and Bahcall, 1974). Early long baseline radio observations (for example, Clark et al., 1968b; Kellermann et al., 1970) suggested that 3C 345 would be a good candidate for more detailed study with VLBI. The first such observations were made by Cohen et al. (1971) on 1971 Feb 28 using the HG interferometer. These data could not distinguish among several simple models.

Single baseline HG observations of 3C 345 were made by us at intervals of a few weeks to a few months between 1971 October and 1973 March. The correlated fluxes from these experiments are shown in Figure V.7a and b. All the observations, including those of Cohen et al. in 1971, exhibit the same minimum at about 22+ hours (GST), at the maximum resolution of the interferometer, and both our 1972 May 9 data and Cohen et al.'s data have a maximum about 5 1/2 hours later; yet there are important differences among the observations. Even considering that the Cohen et al. data appear to be normalized by a factor of about 1.5 higher than our data (Wittels et al., 1975 -- see the discussion of PKS 2134+00), and that the 1971 Oc-
Figures V.7a & b Correlated flux versus Greenwich sidereal time of 3C 345 observed with the HG interferometer between 1971 October and 1973 March. The normalized amplitudes are given by the axes on the right with the symbol for the data of each experiment shown beneath the corresponding axis. The arrows on the time axes mark interferometer hour angle equal to zero. Typical error bars are shown. The curves through the data are labelled with the number corresponding to the appropriate entry in Table 3.
ober data show large scatter due to bad weather, it is clear that the minimum was deepening between early 1971 and 1972 May, more than can be accounted for by a simple scaling by source total flux.

The 1973 May experiment was the first in a series of three- and four-station experiments which resulted in a huge quantity of both amplitude and closure phase data for 3C 345. The resulting correlated fluxes are shown in Figure V.8a and b (HN), V.9a and b (HG), V.10a and b (NG), V.11a-d (HS), V.12a and b (NS), and V.13 (GS); the closure phases are shown in Figures V.14 a-d (HNS), V.15 a&b (HNG), V.16 (HGS), and V.17 (NGS). These simultaneous observations with several baselines, and the accompanying closure phase data, carried the possibility of distinguishing among simple models as well as the possibility of using more sophisticated methods of source mapping.

The data from most experiments were modelled by least-squares fitting using the standard model described in Chapter IV.3.a. Selected results for the model parameters are given in Table 3 and are discussed below. The columns in Table 3 give the following information: Column 1, the experiment date; Columns 2 and 3, the interferometers used for the correlated flux and closure-phase measurements on which the least-squares solution was based; Column 4, the flux of the two components in Jy; Column 5, the percent of the total
Figures V.8a&amp;b  The same as Figures V.7a&amp;b but for observations with the HN interferometer after 1973 March. The curve labelled "Fourier" corresponds to the Fourier series model for the 1974 May data.

Figures V.9a&amp;b  The same as Figures V.8a&amp;b but for observations with the HG interferometer.

Figures V.10a&amp;b  The same as Figures V.8a&amp;b but for observations with the NG interferometer.

Figures V.11a-d  The same as Figures V.8a&amp;b but for observations with the HS interferometer.

Figures V.12a&amp;b  The same as Figures V.8a&amp;b but for observations with the NS interferometer.

Figure V.13  The same as Figures V.8a&amp;b but for observations with the GS interferometer.

Figures V.14a-d  Closure phase versus Greenwich sidereal time of 3C 345 observed with the HNS triangle of antennas. Typical error bars are shown. The curves through the data are labelled with the number corresponding to the appropriate entry in Table 3. The curve labelled "Fourier" corresponds to the Fourier series model for the 1974 May data.

Figures V.15a&amp;b  The same as Figures V.14a-d but for observations with the HNG triangle of antennas.
Figure V.16 The same as Figures V.14a–d but for observations with the HGS triangle of antennas.

Figure V.17 The same as Figure V.14a–d but for observations with the NGS triangle of antennas.
Figure V.10a
Figure V.10b
Figure V.11b
Figure V.12a
Figure V.14
Figure V.15a

Figure V.15b
Figure V.17
### TABLE 3

Standard Model Parameters for 3C 345

<table>
<thead>
<tr>
<th>Experiment Date</th>
<th>Data Used in Fit</th>
<th>S(Jy)</th>
<th>% of $P_T$</th>
<th>$\xi$</th>
<th>$r(10^{-3}\text{a.s.})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 '71 Feb$^2$</td>
<td>1</td>
<td>10.2 (2)$^3$</td>
<td>86%</td>
<td>0.27 (6)</td>
<td>0.97 (5)</td>
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<tr>
<td>2</td>
<td>1</td>
<td>6.8 (2)</td>
<td>58%</td>
<td>0.29 (5)</td>
<td>0.99 (2)</td>
</tr>
<tr>
<td>3 '71 Oct</td>
<td>1</td>
<td>5.1 (3)</td>
<td>51%</td>
<td>0.26 (2)</td>
<td>0.95 (5)</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>5.1 (3)</td>
<td>51%</td>
<td>0.23 (2)</td>
<td>0.94 (5)</td>
</tr>
<tr>
<td>5'72 May</td>
<td>1</td>
<td>7.4 (1)</td>
<td>68%</td>
<td>0.13 (2)</td>
<td>1.08 (0)</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>7.3 (1)</td>
<td>67%</td>
<td>0.15 (2)</td>
<td>1.14 (0)</td>
</tr>
<tr>
<td>7 '72 June &amp; July</td>
<td>1</td>
<td>7.0 (2)</td>
<td>82%</td>
<td>0.10 (1)</td>
<td>1.02 (0)</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>7.2 (2)</td>
<td>84%</td>
<td>0.09 (1)</td>
<td>1.06 (0)</td>
</tr>
<tr>
<td>9 '73 Feb &amp; March</td>
<td>1</td>
<td>9.3 (3)</td>
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<td>0.12 (2)</td>
<td>1.07 (2)</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
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<td>4,5</td>
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<td>1.09 (2)</td>
</tr>
<tr>
<td>16</td>
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<td>0.24 (3)</td>
<td>1.22 (2)</td>
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</table>

*Footnotes are on page 96 following table.*
<table>
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<th>$\sigma_1 (x10^{-3}\text{as})$</th>
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<th>$\phi^C$ (°)</th>
<th>Number of Points</th>
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</tr>
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</tr>
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<tr>
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<td>% of F_T</td>
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</tr>
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<tr>
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<tr>
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<td>1.15 (1)</td>
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<td>( \sigma_1 \times 10^{-3} \text{as} )</td>
<td>( F^C \text{(Jy)} )</td>
<td>( \phi^C (^\circ) )</td>
<td>( F^C )</td>
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<td>---</td>
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<td>23</td>
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<td>( y: 0.06 ) (6)</td>
<td>( \equiv 0.25 ) (2)</td>
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<tr>
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<td>( y: 0.18 ) (9)</td>
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</tr>
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<td>F = 0</td>
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<td>( y: 0.25 ) (1)</td>
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<td>29</td>
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<td>0.24</td>
<td>-</td>
</tr>
<tr>
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<td>0.39</td>
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<td>( y: 0.20 ) (11)</td>
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<td>( \equiv 0.20 ) (11)</td>
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</table>

TABLE 3 (continued)
Footnotes for Table 3:

1. The baselines are designated 1 = HG, 2 = HN, 3 = NG,
   4 = HS, 5 = NS, 6 = GS, and the closure phase loops,
   1 = HNG, 2 = HNS, 3 = HGS, 4 = NGS.

2. Entry 2 is the published data of Cohen et al. (1971) divided by 1.5.

3. The number in parentheses is the error in the final
   significant digit(s) shown.

4. F means the parameter was fixed during the solution at the
   value shown.

5. The source sizes were constrained to be equal in the
   solution.
flux contained in the two components; Column 6, the relative
difference in strength of the components; Columns 7 and 8,
the separation in units of $10^{-3}$ arcseconds and position angle
in degrees, respectively, of the two components; Columns 9 and
10, the HPHW's of the components -- the labels $x$, $y$ mean that
the components are elliptical Gaussians with HPHW's in the $x$
and $y$ direction as given (see Figure IV.1); Columns 11 and 12,
the RMS of the correlated flux and closure-phase residuals, res-
pectively; Columns 13 and 14, the number of correlated flux and
closure-phase data points, respectively, used in the least-
squares solution.

In addition, the 1974 May experiment yielded sufficient
information in both amplitude and phase to allow Fourier fit-
ting and transform inversion.

V. 3. a. **Discussion of the Individual Experiments**

(i) **1971 Feb 28**

The data of Cohen et al. (1971) from their
1971 experiment were modelled with two point components
($\text{HPHW} = 0$) using the standard model so that comparison could
be readily made with later experiments. The resulting para-
meters for these data, both as published and scaled down by
a factor of 1.5, are shown in entries 1 and 2 of Table 3. The
standard errors are large because there are only 14 data
points*. Using a position angle of 103° as indicated by Cohen et al. results in slightly larger value for the separation, though less than the error bar, and a larger RMS of the residuals.

(ii) 1971 October through 1973 March

Like the data of Cohen et al., all of our early HG correlated fluxes are well modelled by two point-components. Least-squares fitting to the standard model with component HPHW’s fixed at zero was done separately for four groups of data, with the total flux nearly constant in each group: 1971 Oct, 1972 May, 1972 late June and early July, and 1973 February and March (see Table 1 for experiment list). For the latter three cases, S, ξ, r, and P were varied in the solution; however, for the 1971 October data, P was fixed at 103°, since P is very poorly determined by these data. The curves from these models are shown in Figure V.7a and b and the parameters are shown as entries 3, 5, 7, and 9 in Table 3. It should be noted that the only substantial effect of fixing the component sizes at some small non-zero value such as 0.1 to 0.2 \( \times 10^{-3} \) arcseconds, is to increase S and hence the percent of the total power contained in the two components.

Because the data from the later experiments tended to be more consistent with a P of 105° or higher, these four

*The point at GST=2.72 (IHA=4.2) was deleted from the fits both because later experiments indicate that it was spuriously low, and because, if included in the modelling, it severely biased the residuals [see Cohen et al. (1971) for the data].
groups of data were refit with $P$ fixed at $105^\circ$. -- entries 4, 6, 8, and 10 in Table 3. As expected, with one fewer free parameter, the solutions did not fit the data as well. The question of whether or not there has been a true shift in position angle will be discussed in Section V.3.c; however, it is important to notice that for 3 of the 4 data groups, the primary effect of an increase in $P$ is an increase in the component separation, $r$, of between $0.04$ to $0.07 \times 10^{-3}$ arc-seconds.

As with the Cohen et al. data, there are probably other simple models that would describe these early data sets quite well. It requires data on additional baselines to begin to gain confidence in a particular kind of simple model.

(iii) 1973 May

Although this was a three-station experiment, 3C 345 was only observed with the HS interferometer. The correlated fluxes are shown in Figure V.11a, where the solid curve corresponds to the model parameters from the solution using the 1973 February and March HG data (Entry 9 in Table 3). Clearly this model fits the locations of the maxima and minima of the HS data quite well; nevertheless, the overall level of the model curve is too high, especially since the total flux rose slightly between March and May. The fact that the model curve agrees better at low than high correlated flux indicates that the discrepancy may be at least partially due to an overall scale error, such as could result if the efficiency
of the S antenna had been lower than assumed, a distinct possibility in view of the scaling problems with S uncovered during the later experiments. However, these later data seemed to indicate that the S efficiency was actually slightly higher than assumed.

Another possible explanation is that the components are not points but have physical size. Entries 13 and 14 in Table 3 indicate that component HPHW's of just under 0.02 x 10^{-3} substantially reduce the RMS of the residuals, and the dashed curve in Figure V.11a, calculated from the Entry 14 parameters, shows that the systematic pattern of the residuals for this model also virtually disappears. As mentioned in (i) above, the only parameter which changes substantially when the component sizes are increased is S. If there remains a scale error in these HS data, its principal effects would be on the component sizes and S.

(iv) 1973 August

Because this experiment was plagued with difficulties, no modelling was done with this data set. The principal problems were the tape recorders at N and S that ruined most of the data and left the remainder with large random errors, and the loss of all radiometry data from S, causing additional unknown scaling and systematic errors in the normalization. Finally, there are only 9 observations with the HG baseline (Figures V.9a).
Although these data also had large random errors due to recording problems, model fitting was attempted. A least-squares solution based on HS and NS amplitudes (Figures V.11b and V.12a) for point components immediately pointed out the necessity for sources with non-zero size (Entry 15): the lower height of the principal maximum on the NS baseline than on the HS baseline indicates extension in the direction perpendicular to $P$, whereas the lower height of the first subsidiary maximum on the NS baseline than on the HS baseline indicates extension along the $P$ direction* (see the $u$–$v$ curves, Figure V.1).

A solution for equal size elliptical components based on the $F^c$ data from all three baselines is given in Entry 16 of Table 3. The principal changes from the comparable results for May are an increase in component separation and a change in the relative component strengths. Theoretical curves for this model are shown in Figures V.8a, V.11b and V.12a and are labelled with the number from the corresponding entry in Table 3. The fit is considerably improved over that for point components, although there are still some systematic differences between the observed and the calculated points; these differences are primarily due to systematic errors in the data

*S scaling problems do not affect these conclusions since both HS and NS would be scaled by the same factor.
and perhaps additional asymmetry in the source, the most likely form of this asymmetry being that the components should differ in size.

The \( \phi_{\text{HNS}} \) data for this experiment are shown in Figure V.14a. They were not used in the least-squares solution but the model curve for elliptical components is drawn through the data. The gaps which appear in \( \phi_{\text{HNS}} \) between about 20 to 22 hours GST and 01 to 02 hours GST, for all the experiments, are a confirmation of the minima in the fringe amplitude on the HS and NS baselines in these intervals; as mentioned at the end of Chapter IV.1, at these times, the signal sinks below the noise so that fringes are lost and the closure phase cannot be calculated.

(vi) 1974 January

Because of scheduling conflicts at the G site, this experiment yielded only 10 GS values for \( F^c \), 11 HG values, and only 4 values of \( \phi_{\text{HGS}}^c \). The remaining \( F^c \) data were from the HS baseline. With so little information, it was not possible to find parameters for a useful solution for the standard model. The problem is compounded by the scaling errors in the S data and the lack of HN data to help determine the flux contained in the model components. A solution based on only the HG and HS correlated
fluxes indicated a value of r of about 1".23(2) x 10^{-3} and a P of about 105^\circ(1); however, the HS data require a value of S too high for the HG data so that the resulting residuals are highly systematic. The data are presented in Figures V.9a, V.11c, V.13, and V.16 without theoretical curves.

(vii) 1974 March

The F_c data for 1974 March are shown in Figures V.8a, V.11c, and V.12a with theoretical curves calculated from the Entry 18 parameters in Table 3. It is instructive to compare the data from this experiment with those from 1973 October as both involve only the HN, HS, and NS baselines and cover about the same span of u-v track. Least-squares solutions to each set of data indicate a value of S of about 9.2, a value of \xi of between about 0.18 to 0.25, and elliptical components of comparable size; however, the best-fit position angle and component separation for the March data are both greater than the values from the analysis of the October data; it is important to note that forcing the P's to be equal eliminates the difference in the r's.

For both sets of data, P and r are primarily determined from the locations of the maxima and minima of the HS and NS correlated fluxes. These locations cannot be pinpointed very well because near the minima, fringes are lost, and
near the maxima there are too few data points and there is a large scatter in those data that do exist. In spite of these problems, the consistency in the parameters for these two similar sets of data is encouraging.

(viii) 1974 May

The 1974 May 4-station experiment yielded more 3C 345 data, both amplitude and phase, than any of the other experiments. Because a reasonable number of observations was made on each of the 6 baselines, these data were particularly useful in helping to determine the sensitivity of the parameters to the level of complication of the model and to the subset of the data used for the least-squares solution. In addition, this was the first experiment that produced sufficient closure-phase data for closure phase to become an important addition to the modelling technique (also see Section V.3.b).

A great many combinations of baselines and number of variable parameters were tried in the model fitting of the 1974 May data; a selected set of parameter results are shown in entries 20 to 24 of Table 3. The following conclusions, not in order of importance, can be drawn*:

(1) If the components are more equal in size, either because the solution requires them to be equal or because the data are better fit by components that

*However, not all conclusions follow from the selection of solutions given in Table 3.
are the same size, they are more equal in flux, that is, they have a smaller value of $\xi$.

2) The use of closure phase data in the solution requires the components to be more unequal in both size and strength than the correlated flux data alone, and places the smaller, weaker component to the south and east (direction of increasing $\alpha$) of the larger component (see Figure IV.1).

3) Position angle and component separation are positively correlated so that increasing one increases the other. This positive correlation also holds for all previous data sets except 1971 February and October, where decreasing $P$ increases $r$.

4) The amplitude data are better fit by a slightly smaller position angle and separation than are the closure-phase data.

5) The only significant effects of reducing the scale of the HS, NS and GS correlated fluxes by as much as 45% is to dramatically increase the component sizes and to make the components more circular; the flux in the model, $S$, which is principally set by the data from the shortest (HN) baseline, is not affected.

6) Changing from circular to elliptical components in solutions based on all the data from this
experiment increases the position angle by several degrees. In general the position angle varies with component size and shape as well as with the choice of data.

The implications of these conclusions for the long-term behavior of the source will be discussed in Section V.3.c.

The parameters from the solution for a fit to the standard model for all the amplitude and phase data are shown as Entry 24 of Table 3; the curves calculated from these parameters are shown with the data in Figures V.8b, V.9b, V.10a, V.11d, V.12b, V.13 for $F^C$, and V.14c, V.15a, V.16, and V.17 for $\phi^C$. Although the shapes of the theoretical curves are reasonable, there are clear systematic discrepancies between the model and the data.

The most notable of these systematic errors is in the level of the correlated flux curves: because the data from the baselines involving S are probably scaled too high, the least-squares solution may have to compromise between the scales of the three shorter and three longer baselines. The result is that the theoretical curves are too low for the HS and NS data and too high for the HG and NG data. The least systematic scaling error is for the HN data, which primarily limit the total flux of the model, and the GS data, which limit the component sizes. Decreasing the model flux or increasing the component sizes
results in a better fit to the data obtained from the shorter baselines, but a worse fit to those from the longer ones. Increasing flux or decreasing sizes yields the opposite results.

If the model has, as an additional variable parameter (compared with Entry 24), a single scale factor for the three baselines using S, the HS, NS, and GS amplitudes are scaled down by a factor of $0.66 \pm 0.03$. The only significant change in the model parameters is that the component HPHW's become larger — $\xi, \Delta \sigma_x$ and $\Delta \sigma_y$, which are restricted by the phase, do not change. The root-mean-square of the residuals for correlated flux decreases to 0.61 and almost all of the systematic differences between the model and the correlated flux data disappears. The remaining discrepancies between the model and the data are due primarily to other small-

It is interesting to note one specific contribution made by the closure-phase data: the ambiguity in the orientation of the components is removed since the phase is an odd function of $u$ and $v$.

(ix) **1974 July**

Although this experiment should have yielded almost as much information as the May experiment, difficulties at G resulted in very few observations with the HG, NG, and GS baselines. The data are shown in Figures V.8b, V.9b, V.10b, V.11d, V.12b, V.13, V.14d, V.15b, and V.17 with
theoretical curves calculated from the parameters in Entry 28 of Table 3. Entry 28 was used because this solution was based on both correlated-flux and closure-phase data.

The parameters, Entry 28 of Table 3, for this experiment are very similar to those from the May experiment and the model curves and data have the same systematic discrepancies. The only striking difference between the results of the two experiments is found in the HGS and NGS closure-phase data between about 0210 and 0300 GST: the two May data points have a negative slope indicating that the closure phase "turns over", whereas the two July data points have a positive slope, indicating that the closure phase has wrapped around from +180° to -180°. A test of the parameter sensitivity to correlated flux versus closure phase has not been done.

(x) 1974 October

Although only the H, N, and G antennas were available for this experiment, striking changes from the previous two experiments were evident in both the correlated fluxes and closure phases. The data are shown in Figures V.8b, V.9b, V.10b, and V.15b along with model curves for unequal circular components, Entry 30 in Table 3, and equal-size elliptical components, Entry 31; the latter is a slightly better fit to the amplitude, and the former to the phase. Both
cases indicate that the primary change from the previous results is an increase in component separation, r.

V. 3. b. Results of Fourier Modelling

Only the 1974 May experiment yielded sufficient data to produce useful maps of 3C 345 by the Fourier fitting and inversion method (see Section IV.3.b). The Fourier coefficients were calculated along axes u' and v' at a position angle of 15°; because 3C 345 is elongated along a position angle of about 105°, the use of rotated axes required fewer terms to attain a good fit to the data. Curves for an N=10, M=2* fit to all the correlated flux data and an N'=30, M'=2 fit to the closure phase data are shown in Figures V.8b, V.9b, V.10a, V.11d, V.12b, V.13, V.14c, V.15a, V.16, and V.17.

The Fourier series for the correlated flux shows systematic deviations from the data that are similar to those of the standard model and, again, are primarily due to scaling errors. Increasing the number of terms in the series decreases the deviations, but only by adding high frequency oscillations. These oscillations are unrealistic as they would indicate that the amplitude was fluctuating on a time scale shorter than the interval between observations. If such short-period oscillations were present, the data should exhibit scatter significantly greater than the indicated errors, and they do not. However, these additional high frequency terms do not appre-

*See Section IV.3.b for definitions of notation.
ciably affect the map of the brightness distribution resulting from the inversion. The Fourier series for the phase required the inclusion of many high frequency terms in order to imitate the very sharp changes in the closure phase. Once again the small oscillations do not appreciably affect the source brightness distribution.

Figure V.18a shows the contour map obtained by inverting the complex visibility computed from the series coefficients using a 32 x 32 grid with a u' spacing of 10.6 x 10^6 wavelengths and a v' spacing of 9.9 x 10^6 wavelengths so that the size of the grid corresponded to the largest values of u' and v' in the observations. The resulting map has an (x', y') grid spacing of 0.63 x 10^{-3} and 0.65 x 10^{-3}, respectively. Only the closest sidelobes are shown, with negative flux indicated by dashed lines. All remaining sidelobes are at the 5 to 10% level.

Figure V.18b shows a more detailed map of the main lobe made by using a grid spacing of 19.4 x 10^6 wavelengths in both the u' and v' directions and by setting both the amplitude and phase to zero everywhere outside the region of actual observation, that is, when |u'| > 163 x 10^6 and/or |v'| > 159 x 10^6 wavelengths. This map has a square (x', y') grid spacing of 0.33 x 10^{-3}. The cuts labelled I, II, and III in Figure V.18b correspond to the three cross-sections shown in Figure V.18c; the solid line is a smooth sketch through the inversion grid points, marked by crosses, whereas the dashed line corresponds to the standard model calculated from the
Figures V.18a-c (a) and (b) show the brightness distribution map of 3C 345 resulting from Fourier transforming the series representations for the 1974 May correlated-flux and closure-phase data. For the map in (a), the grid spacing was 0.63 x 10^-3 along the line connecting the components and 0.65 x 10^-3 perpendicular to it, whereas for the map in (b), the grid spacing was 0.33 x 10^-3 in both directions. The dashed lines in (a) indicate negative sidelobes; sidelobes are not shown in (b). The cross-sections labelled Cuts I, II, and III in (b) are shown in (c): The solid line is a sketch through the fluxes at the inversion grid points, shown by crosses; the dashed line is calculated from the standard model using the parameters of Entry 24 in Table 3; the dotted line is a suggested decomposition of the solid curve into two components. The solid and dashed curves are scaled to have approximately the ratio of integrated flux (area under the curves) given by the models. The maps in (a) and (b) both have the orientation shown by the axes labelled NORTH and INCREASING α in (b).
a. CONTOURS at -5, -1, 1, 5, 10, 15, and 19 times the maximum brightness

b. CONTOURS at 1, 5, 10, 15, and 19 times the maximum brightness

Figure V.18
Figure V.18
parameters in Entry 24 of Table 3. The two models are scaled to have the appropriate strengths: the two-component model has an integrated brightness of 10.1 Jy, whereas the Fourier model contains 9.9 Jy (including both the main source and the sidelobes). The dotted line is a suggested decomposition of the two components of the brightness determined by the Fourier method. The origin of the inversion map has been moved with respect to that of the standard model to facilitate comparison of the two structures.

Clearly the standard model and the "Fourier" model agree quite well on the orientation and the separation of the two components. The main disagreement is that the Fourier model requires the second component to be closer both in strength and size to the central component. This discrepancy comes about primarily for two reasons: first, between 2300 and 2315 GST (e.g. Entry 24), the model phase "wraps around" from +180° to -180°, whereas the Fourier series model phase "turns over". Thus, for the Fourier series representation of the source, components will be more alike in size and strength than the components resulting from the standard model. Second, the standard model contains no information about the limiting resolution of the interferometers used for the observations, so that, for example, the smaller component can have zero size in the y' direction. The inversion technique has a resolution explicitly given by the grid size; it can never produce components of zero HFW. If the resolution of the brightness
distribution map is further increased, the smaller component does become narrower in the y' direction and the relative difference in strength becomes closer to the $\xi$ of the standard model.

V. 3. c. Secular Trends in the Structure of 3C 345

Figure V.19 shows the position angle and component separation of 3C 345 as a function of date. The number beside each point corresponds to the appropriate entry in Table 3, and an F beside a position angle indicates that $P$ was fixed at that value for the solution. The point labelled S on each plot was taken from Shaffer et al. (1975). For points without error bars, except those labelled f or S, the errors are too small to show.

A discussion of secular trends in the model parameters must take into account the effects of data and model selection. For the experiments through 1973 May, data were available from only one baseline and there was insufficient information to determine component sizes. For comparison, data from selected baselines from the later experiments were also modelled by point components (see entries 11, 15, 17, 20, 25, 26, 29 of Table 3). In general the point components tend to be closer together and at a smaller position angle than the Gaussian components.

The standard model indicates that the separation of the two components of 3C 345 is slowly increasing. The best-fit
Figure V.19 The position angle and component separation of 3C 345 as a function of date. The points are labelled with the number corresponding to the appropriate entry in Table 3; an f means the position angle was fixed at the value shown in the solution. The points labelled S are from Shaffer et al. (1975). Error bars are shown where they are not smaller than the symbol. The curves shown through the component separation data correspond to the best-fit lines through the points labelled 2, 3, 5, 7, 9, 14, 16, 18, 24, 28, and 31, and 2, 3, 5, 7, 9, 11, 15, 17, 20, 25, 26, and 29 (see Table 3).
Figure V.19
straight line through the separations given by Entries 2, 3, 5, 7, 9, 14, 16, 18, 24, 28, 31 in Table 3, has a slope of $1.6 \times 10^{-17}$ rad/sec, which, for $q = 1$, $H_0 = 60$ km/sec/Mpc, and $z = 0.595$, corresponds to an apparent separation velocity of about 3 times the speed of light. If the separations for point components are used (Entries 2, 3, 5, 7, 9, 11, 15, 17, 20, 25, 26, 29 in Table 3), the slope is $1.2 \times 10^{-17}$ rad/sec and the apparent separation velocity is slightly over $2c$. Assuming that two components are a good representation of the source, the component separation is seen to increase independent of the details of the model.

Knight et al. (1971) and Whitney et al. (1971) first presented data that seemed to imply an apparent expansion velocity greater than the speed of light for 3C 279. Later, similar behavior was observed for the Seyfert galaxy, 3C 120 (Shapiro et al., 1973). Dent (1972c) suggested that if the 3C 120 data were interpreted as being due to spatially independent outbursts, then the difference in component separations would be due to components at different locations being "on" or "off", and no expansion would be required. However, one of the many problems with such a model is that the component separation should be as likely to decrease as increase. The results for 3C 345 in Figure V.19 show a trend of continuous increase in separation, especially if only points corresponding to models with the same number of variable parameters are considered. Both 3C 279 and 3C 120 showed an
increase of component separation, and, in fact, no source has
been seen to exhibit an apparent decrease in component separa-
tion. Without such evidence, the spatially-independent-out-
burst model seems hard to accept as valid.

The position angle for the two components, at first
glance, also appears to show a secular trend, especially if
only entries 2, 3, 5, 7, 9, 14, 16, 18, 24, 28, and 31 of
Table 3 are considered. However, careful scrutiny of the later
experiments, where the effects of selection can be tested,
casts serious doubt on any conclusions involving systematic
changes of \( P \). For example, for the 1974 May experiment, the
Entry 24 set of parameters in Table 3 shows a value for \( P \) of
107°, but the best-fit \( P \) for the HG correlated fluxes modelled
by two point-components, Entry 20, is only 102°, more consistent
with the earlier single-baseline experiments. Furthermore, if,
for this same experiment, the correlated fluxes from the HN,
HG, NG, HS and NS baselines and the HNG and HNS closure phases
are modelled by two equal, circular, Gaussian components, the
best-fit \( P \) is 110°. For most of the experiments, a shift of
1° to 3° in the position angle does not greatly increase the
RMS of the residuals or introduce significant systematic trends
in the residuals. A far more serious effect is produced by
changing the component shapes. It is not possible to conclude
whether or not the position angle changes systematically because
data and model selection effects are of the same order as, if
not larger than, any systematic changes in $P$.

Figure V.20 shows the total flux and model flux for 3C 345 as a function of date, with curves sketched through the points. The model flux is shown for three separate cases -- circular or elliptical Gaussian components, point components from solutions using HG and/or NG data, and point components from solutions using HS and/or NS data. Point components are separated from Gaussians because the former always require less flux in the model than the latter. HG, NG solutions are separated from HS, NS solutions because of the scaling uncertainty in the baselines using $S$. The point labelled $S$ is from Shaffer et al. (1975); the other model fluxes are labelled with the number from the appropriate entry in Table 3.

In general, the model flux follows the trends of the measured total flux, although the difference between the total and model flux for the point components is highly variable. The best determinations of model flux are for those experiments after mid-1973 when HN data became available. For these later experiments, the model flux for Gaussian components remains at about 80 to 85% of the total flux, indicating that the total flux changes are primarily due to changes in the flux of the components.

V. 3. d. Other Simple Models

Although these observations of 3C 345 are, for the most part, well described by two components, the modelling
Figure V.20 The total and model flux of 3C 345 versus date. The model flux is shown separately for three cases: circular or elliptical Gaussian components (HPHW ≠ 0), point components (HPHW = 0) for solutions based on HG and/or NG data, and point components for solutions based on HS and/or NS data. The points are labelled with the number corresponding to the appropriate entry in Table 3. The point labelled S is from Shaffer et al. (1975) and is for components with HPHW ≠ 0. The total fluxes are from Table 2 and the references cited. The curves through the data points are sketches only and have no theoretical basis.
Figure V.20
techniques employed above do not result in a unique description of the source. It is quite possible that, with increased resolution, one or both of the components seen on the scale of present resolution will be decomposed into two or more smaller pieces. Perhaps the first question to ask is whether or not there exists a third component located between the two discussed above, since a gradual decrease in the strength of such a component can produce the type of long term changes in the data which result in the conclusion of source expansion if only two components are considered.

There are primarily two pieces of information which bear on this question. The first is the Fourier inversion map: if a third component is present between the two main components*, it must be very weak and narrow or the main components must be very asymmetric in the $x'$ direction (see Figure V.18c). Clearly a small component cannot be ruled out. The second type of information is the source total flux. The errors in the early measurements of total flux are sufficiently large to cast doubt on the validity of the variations, but after the beginning of 1973 the increase in total flux from about 11.2 to 12.4 Jy is significant. There is no indication of a systematic decrease in flux that would have to accompany the decay of a third component; however, if the third component had been almost gone by the end of 1974, the errors in the measurements are sufficiently large to have masked the decay. Subsequent measurements would be consistent with, but would not confirm this decay if they showed no further increase in the

* A third component outside seems even less likely (see Figure V.18b).
separation of the two components of the standard model.

At present, the compact source in 3C 345 appears to consist of two main components, with the weaker one to the east (direction of increasing $\alpha$) and slightly south of the stronger. A definitive decision on whether or not there are additional weak components or whether the two main components are composed of several smaller "knots" of flux will have to await higher resolution measurements. In any event, the interpretation of the time variability of the source must always be viewed in terms of the assumptions inherent in the models.
CHAPTER VI

RELATIVE SOURCE POSITIONS FROM DIFFERENCED PHASE MEASUREMENTS

Differenced phase experiments are designed to obtain very accurate estimates of the separation of two sources by unambiguously measuring the phase of each source over a reasonable fraction of 24 hours, and differencing the two phases to remove instrumental and propagation-medium effects. For an experiment having only one antenna at each end of the baseline, antennas are scheduled to switch between the sources with a cycle time of 4 to 8 minutes, so that noise common to the two sources, having a time scale longer than the cycle time, will be subtracted out by differencing the phases of the two sources. Also, if the sources are sufficiently close together, the radiation from the two sources passes along similar ray-paths so that effects of the propagation medium will, for the most part, cancel.

Consider the differenced phase $\Delta \phi_{jk}$, given in Equation (II.12), for two sources designated 1 and 2. As mentioned in Chapter II, the position information is given by the geometric delay term, which, for the differenced delay, is given by:

$$\delta \Delta t_{jk} \equiv \Delta t_{jk}^{(1)} - \Delta t_{jk}^{(2)} = \frac{1}{c} b_{jk} \cdot (\hat{e}_1 - \hat{e}_2)$$

$$= \frac{b_{jk}}{c} \left[ \cos D_{jk} \left( \cos A_{jk} \left( \cos \delta_1 \cos \alpha_1 - \cos \delta_2 \cos \alpha_2 \right) \right. \\
- \left. \cos \delta_2 \cos \alpha_2 \right) + \sin A_{jk} \left( \cos \delta_1 \sin \alpha_1 - \cos \delta_2 \sin \alpha_2 \right) \right]$$

+ $\sin D_{jk} \left( \sin \delta_1 - \sin \delta_2 \right)$
where \( A_{jk} = A^O_{jk} + \omega t_s \). If source 1 is designated the reference source, we can define offset coordinates \( \Delta \alpha \) and \( \Delta \delta \) by

\[
\begin{align*}
\alpha_2 & \equiv \alpha_1 + \Delta \alpha \equiv \alpha + \Delta \alpha \\
\delta_2 & \equiv \delta_1 + \Delta \delta \equiv \delta + \Delta \delta
\end{align*}
\]  
(VI.2)

so that

\[
\delta \Delta t_{jk} = -\frac{b_{jk}}{c} \{ \cos D_{jk} \{ \cos A_{jk} (\cos \delta \cos \alpha - \cos(\delta + \Delta \delta) \cos (\alpha + \Delta \alpha)) \\
+ \sin A_{jk} (\cos \delta \sin \alpha - \cos(\delta + \Delta \delta) \sin (\alpha + \Delta \alpha)) \} \\
+ \sin D_{jk} [\sin \delta - \sin(\delta + \Delta \delta) ] \}
\]  
(VI.3)

The procedure is then to measure the fringe phase, \( \phi^m_{jk} \), for both sources over a several-hour period, to remove 2\( \pi \) ambiguities between consecutive measurements of the phase of each source -- a procedure called "phase connection" -- to remove the structure phase term, \( \phi^S_{jk} \), for each source by subtracting out the phase calculated from the source models, and finally, to apply a least-squares-fitting procedure to the differenced phases to estimate \( \Delta \alpha \) and \( \Delta \delta \).

VI. 1. Phase Connection

Since the measured phase, \( \phi^m_{jk} \), is a rapidly varying function of time, it is much easier to remove 2\( \pi \) ambiguities between observations after the time dependence has been slowed by subtracting out an a priori model and working with the
residual phase. The model used to describe the terms in Equation (II.5) is as follows: For the geometric delay, Equation (II.6) is used with \textit{a priori} values assumed for \( \dot{\delta}, \Omega, \) and \( \delta. \) The constant clock, \((a_{j0} - a_{k0})\) is fixed at a value determined during the original correlation of the data tapes; the clock rates and all higher order terms in the clock polynomials are assumed to be zero. The two atmosphere terms are modelled by a semi-empirical formula developed by C. C. Chao (1970) and given by

\[
T(E) = \frac{Z}{\sin E + \frac{A}{\tan E + B}} \tag{VI.4}
\]

where \( T(E) \) is the phase thickness of the atmosphere, \( Z \) is the zenith phase thickness of the atmosphere, \( E \) is the source elevation angle at the station, a known function for a source of given \( \alpha \) and \( \delta, \) and \( A \) and \( B \) are empirically determined constants given by

\[
A = 0.00143 \text{ and } B = 0.0445
\]

The zenith thickness at all sites was assumed to be 7 nsec, which corresponds to \( \sim 350 \) radians of phase. The atmospheric contribution to the phase is the difference of two terms of the form of Equation (VI.4), one for each station. The phase term due to the rotation of the antenna feeds and to the structure phase are assumed to be zero as they are generally negligible for purposes of phase connection. The residual phase
is then defined by

$$\phi_{jk}^R(t) = \phi_{jk}^m(t) - \phi_{jk}^{ap}(t) \quad (VI.5)$$

where \(\phi_{jk}^{ap}(t)\) is the a priori model phase. The residual fringe rate, \(\dot{\phi}_{jk}^R(t)\), is also calculated by removing the time derivative of the above a priori model terms from the measured fringe rate \(\dot{\phi}_{jk}^m(t)\).

One final effort is made to slow the time variation of the residual phase. The residual fringe rate as a function of time is modelled by a polynomial series -- generally only 3 to 5 terms are required for a good fit -- and the resulting function is integrated and subtracted from the residual fringe phase. The new residual fringe phase is then

$$\phi_{jk}^{NR}(t) = \phi_{jk}^R(t) - \int_{t_0}^{t} \phi_{jk}^{PR}(t) \, dt = \phi_{jk}^R(t) - \phi_{jk}^{PR}(t) \quad (VI.6)$$

where \(\phi_{jk}^{PR}(t)\) is the polynomial fit to the residual fringe rate and \(\phi_{jk}^{PR}(t)\) is the fringe phase from the polynomial. The resulting residual fringe phase \(\phi_{jk}^{NR}(t)\), calculated record by record for each observation, generally has a very small slope.

Figure VI.1 shows the residual phase, \(\phi_{HG}^R(t)\), versus time before the polynomial is removed, for the first 15 observations of 3C 345 in July 1972. Each observation is about 50 seconds long and each point is an average over 7 records or 1.4 seconds. The interval between the end of one observation and the start of the next is 250 seconds.
Figure VI.1 The residual fringe phase versus time of 3C 345 for the first fifteen observations in 1972 July. Each observation is approximately 50 seconds in length whereas the interval between the end of each observation and the beginning of the next is 250 seconds, so that the time axis is discontinuous between observations.
so that the time scale has a discontinuity between observations. The residual fringe rates $\phi^R_{HG}(t)$ are shown in Figure VI.2 for the entire set of 3C 345 observations for this date; the curve through the data is a four-term polynomial fit to these rates. Figure VI.3 shows the "new" residual phase $\phi^{NR}_{HG}(t)$ (i.e. the residual phase to the a priori model plus the time integral of the polynomial) for the same 15 observations.

From $\phi^{NR}(t)$ it is usually possible to determine by eye how properly to connect successive observations. The connections suggested are indicated in Fig. VI.3 by solid lines connecting the end of each observation to the beginning of the next. These visual predictions are tested by computing the best-fit straight line through the residual phases for each pair of adjacent observations of the same source, using the record-by-record values of $\phi^{NR}_{jk}(t)$. If the choice of ambiguity corresponding to the best-fit line agrees with the visual choice, the phase connection between the two observations is generally correct. For the July data, the only discrepancy between the two methods is for the connection between observations 10 and 11: the computer chooses a phase change $2\pi$ less than that chosen by human judgment. The computer's choice is shown by a dashed line between observations 10 and 11 in Figure VI.3.
Figure VI.2 Residual fringe rate versus time for the observations of 3C 345 in 1972 July. The curve is a 4-term polynomial series fit to the data points.
Figure VI.3 The same as Figure VI.1 but for the "new" residual fringe phase, that is, the fringe phase less the integrated polynomial series representation of the residual fringe rate [Equation (VI.6)]. The solid lines connecting successive observations are the phase connections suggested both visually and by the computer, except for the connection between observations 10 and 11. Here the solid line is the visual choice for connection and the dashed line is the choice of the computer algorithm.
At this stage, any discrepancies or questions in the phase connection can usually be settled in one of two ways. The first is simply to difference the total phases of the two sources, where the total phases have been computed by adding the phase of the a priori model, the polynomial phase, and the residual phase as connected. The consequent reduction of noise often makes it possible to spot an incorrect choice that may have looked only suspicious in the phases of the individual sources.

The second method is to examine a plot of fringe rate displaying several possible ambiguities in connecting the "new" residual phases from one observation to the next. Figure VI.4 shows such a plot for the same 15 observations of 3C 345 in 1972 July. For each pair of adjacent observations, the possible choices of fringe rate which will connect the "new" residual phases (Figure VI.3) are indicated by the letters A, B, C, D, and E. The letter C indicates the computer's selection, whereas the letters A, B, D, and E indicate the fringe rates for the possible connections which are -2, -1, 1, and 2 ambiguities from the computer's selection. From this plot it is clear that the computer's connection between observations 10 and 11 is almost certainly wrong by $2\pi$; that is, the phase of observation 11 should be $2\pi$ higher than the value used by the computer. It should be noted that a human phase-connector can often do better than the computer, since the human has the benefit
Figure VI.4 The residual fringe rate corresponding to several possible ambiguities in connecting the "new" residual fringe phases of 3C 345 for the first 15 observations in 1972 July. For each pair of observations, the letter C designates the fringe rate for the computer's suggested phase connection; the letters A, B, D, and E designate the fringe rates for the connections with additional ambiguities of $-2\pi$, $-\pi$, $\pi$, and $2\pi$ around the computer's selection.
of examining many observations at once and noticing trends, while the machine is usually given, at least by this program, only 2 or 3 observations at a time, and is oblivious to what it has just done and ignorant of what it has not yet seen.

After all these methods of checking for errors in the phase connection have been tried, the total fringe phases are recomputed from the a priori model, polynomial fit, and residuals. Subsequent calculations are done using one value of total fringe phase for each observation; the value chosen is either that for the beginning time or mid-point time of the observation, with either one or the other time reference used throughout each experiment.

VI. 2. Removal of Source Structure

As was noted in Chapter II, differencing the phases of the two sources does not remove any contributions to the phase from the structure of the sources, which are presumed independent. However, if a reliable model for source structure can be determined from the correlated-flux and closure-phase data, the structure term can be subtracted from the measured phase of each source.

Four experiments (see Table 4) were performed to determine the differenced fringe phase and thus the separation of the quasars 3C 345 and NRAO 512. The correlated flux data for the first three, conducted on 10 October 1971, 9 May 1972, and 4 July 1972, and the correlated-flux and closure-phase data
Table 4
List of Differenced Fringe Phase Experiments

<table>
<thead>
<tr>
<th>Date</th>
<th>Start UT</th>
<th>End UT</th>
<th>Baseline</th>
<th>Cycle Time (min)</th>
<th>Wide Band Synthesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Oct '71</td>
<td>1633</td>
<td>2155</td>
<td>HG</td>
<td>5</td>
<td>N</td>
</tr>
<tr>
<td>9 May '72</td>
<td>0800</td>
<td>1345</td>
<td>HG</td>
<td>5</td>
<td>Y</td>
</tr>
<tr>
<td>4 July '72</td>
<td>0008</td>
<td>0405</td>
<td>HG</td>
<td>5</td>
<td>N</td>
</tr>
<tr>
<td>2 May '74</td>
<td>0708</td>
<td>1046</td>
<td>HG</td>
<td>4</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HN</td>
<td>8</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>NG</td>
<td>8</td>
<td>Y</td>
</tr>
</tbody>
</table>
for the last experiment, on 2 May 1974, were discussed in Chapter V. For this May 1974 experiment, the source separation was determined from the combined data of the HN, HG, and NG baselines, so that structure phase had to be calculated for all three baselines.

For the earlier three experiments, the conclusions of Chapter V indicate that NRAO 512 had an unresolved compact component when viewed with the HG interferometer, so that the structure phase for this baseline is zero [see Equation (IV.10) with $\xi=1$]. Thus $\phi^S_{HG}$ was taken as zero for all three experiments. For the 1974 May experiment, although the 1974 May data from the GS baseline did appear to show structure for NRAO 512, the data from the shorter baselines used for the source separations did not, and $\phi^S_{HG}$, $\phi^S_{HG}'$, and $\phi^S_{NG}$ were all taken to be zero.

In contrast, 3C 345 clearly had structure visible on the baselines used for all four experiments. In all cases, two-component models give reasonable representations of the data and were therefore used to calculate the structure phase. For the first three experiments the structure phase was calculated for two point components [Equation (IV.8) with $\Delta \sigma^2 = 0$] using the parameters listed in Table 3 (Entries 3, 5, and 7). It should be remembered that Equation (IV.8) is formulated with the origin at the center of the brighter component ($\xi > 0$) -- see Figure IV.1 -- so that the source separation calculated will be that from
the compact component of NRAO 512 to the center of the brighter component of 3C 345.

For the 1974 May experiment, the structure phase was again calculated for two components using Equation (IV.8) but for unequal elliptical components and using the parameter values $r = 1.26 \times 10^{-3}$, $P = 107^\circ$, $\xi = .322$, $\Delta \sigma_x^2 = 7.66 \times 10^{-8}$ arcsec$^2$ $\Delta \sigma_y^2 = 3.40 \times 10^{-8}$ arcsec$^2$. In addition, the 3C 345 structure phase was calculated from the Fourier series coefficients found in the two-dimensional 30x2 fit to the closure phases from this experiment. Figure VI.5 shows these structure phases for both the two-component model and the Fourier series model for all three baselines used in this experiment. The source separation was then estimated separately for the two independent models. It is important to remember that for the Fourier series model, the separation is that from the compact component of NRAO 512 to the origin of the brightness distribution map of 3C 345 obtained from the inversion of the series fit (see Figure V.18b), which origin is also approximately at the center of the brighter component.

VI. 3. Source Separation Results

Once the structure phases had been removed, the phases from the two sources were differenced and a least-squares-fitting procedure applied to determine the separation of the two sources. The computer program used for these least-squares solutions is described by Robertson (1975). As mentioned
Figure VI.5  Theoretical values for the 1974 May structure phase of 3C 345 calculated from the standard model(s) and the Fourier series model (f) for the HN, NG, and HG interferometers.
Figure VI.5
in Chapter II, calculations are done in a coordinate system using the solar-system barycenter as origin, coordinate time as the time variable, and general relativity is assumed valid. BIH values were used for UT1. All parameters in the models for polar motion, earth tides and the atmosphere above each site were fixed as were all higher order clock terms. Also fixed were the coordinates of the H and G sites and the right ascension and declination of 3C 345. The only variable parameters were the right ascension and the declination of NRAO 512, a clock offset*, and the longitude and radius of the N site in the 1974 May 2 solutions using data from the HG, HN, and NG baseline.

Table 5 gives results for the location of NRAO 512 when the fixed parameters and starting values for the variable parameters are as follows:

\[\text{\begin{tabular}{|c|c|c|c|}
\hline
Parameter & Value & Unit & \\
\hline
Right ascension & & & \\
Declination & & & \\
Longitude & & & \\
Radius & & & \\
\hline
\end{tabular}}\]

*For some solutions based on the 1971 Oct data, separate clock offset parameters were used for the first and second halves of the experiment. One parameter applied to the data before, and the second after, a 40-minute gap in the observations that was due to instrumental failures.

\[\text{\begin{tabular}{|c|c|}
\hline
BIH & Bureau International de l'Heure. \\
\hline
\end{tabular}}\]
Source Coordinates (1950.0)

\[ \alpha_{345}' \delta_{345} = 16^h 41^m 17.634758504, \quad 39^\circ 54' 10.9583766302 \]

\[ \alpha_{512}' \delta_{512} = 16 \ 38 \ 48.19999 \quad 39 \ 52 \ 30.22866 \]

Earth-Centered Cylindric Station Coordinates (radius, longitude, and polar component)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>4700.64332 km</td>
<td>71.488650 km</td>
</tr>
<tr>
<td>G</td>
<td>5204.14078</td>
<td>116.888563 km</td>
</tr>
<tr>
<td>N</td>
<td>5003.16504</td>
<td>79.835964 km</td>
</tr>
</tbody>
</table>

Atmosphere Zenith Thickness

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>7.99 x 10^{-9} sec</td>
</tr>
<tr>
<td>G</td>
<td>6.74 x 10^{-9}</td>
</tr>
<tr>
<td>N</td>
<td>6.74 x 10^{-9}</td>
</tr>
</tbody>
</table>

Clock Terms

All orders 0.0

Polar Motion, UT1, and Tidal Models

See Robertson (1975)

Source Models

As discussed in Section VI.2.
Table 5

Coordinates of NRAO 512 (1950.0)\textsuperscript{+}

<table>
<thead>
<tr>
<th></th>
<th>$\alpha (16^h \ 38^m \ 48.8^s)$</th>
<th>$\delta (39^\circ \ 52' \ 30'')$</th>
<th>RMS (10^-12 sec)</th>
<th>USNO-BIH (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. All models used</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Oct '71</td>
<td>0.19815 (4)</td>
<td>0.2308 (7)</td>
<td>12.2</td>
<td></td>
</tr>
<tr>
<td>9 May '72</td>
<td>0.19812 (3)</td>
<td>0.2328 (6)</td>
<td>19.4</td>
<td></td>
</tr>
<tr>
<td>4 July '72</td>
<td>0.19816 (6)</td>
<td>0.2319 (5)</td>
<td>10.6</td>
<td></td>
</tr>
<tr>
<td>2 May '74*</td>
<td>0.19805 (6)</td>
<td>0.2322 (4)</td>
<td>13.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.19816 (6)</td>
<td>0.2331 (4)</td>
<td>14.0</td>
<td></td>
</tr>
<tr>
<td>Av. of 1st four:</td>
<td>0.19812</td>
<td>0.2319</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. No structure model removed.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Oct '71</td>
<td>0.19814 (4)</td>
<td>0.2311 (7)</td>
<td>11.7</td>
<td></td>
</tr>
<tr>
<td>9 May '72</td>
<td>0.19806 (2)</td>
<td>0.2317 (6)</td>
<td>18.9</td>
<td></td>
</tr>
<tr>
<td>4 July '72</td>
<td>0.19814 (6)</td>
<td>0.2326 (5)</td>
<td>10.4</td>
<td></td>
</tr>
<tr>
<td>2 May '74</td>
<td>0.19815 (6)</td>
<td>0.2316 (4)</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>Av.:</td>
<td>0.19812</td>
<td>0.2317</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. No structure model removed and no model for earth tides.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Oct '71</td>
<td>0.19816 (4)</td>
<td>0.2309 (7)</td>
<td>11.7</td>
<td></td>
</tr>
<tr>
<td>9 May '72</td>
<td>0.19802 (2)</td>
<td>0.2316 (6)</td>
<td>18.7</td>
<td></td>
</tr>
<tr>
<td>4 July '72</td>
<td>0.19811 (6)</td>
<td>0.2333 (5)</td>
<td>9.8</td>
<td></td>
</tr>
<tr>
<td>2 May '74</td>
<td>0.19826 (7)</td>
<td>0.2325 (4)</td>
<td>13.4</td>
<td></td>
</tr>
<tr>
<td>D. No structure model removed and USNO rather than BIH values of Al-UT1 used.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Oct '71</td>
<td>0.19814 (4)</td>
<td>0.2274 (7)</td>
<td>11.7</td>
<td>-11.5</td>
</tr>
<tr>
<td>9 May '72</td>
<td>0.19805 (2)</td>
<td>0.2316 (6)</td>
<td>18.8</td>
<td>- 0.5</td>
</tr>
<tr>
<td>4 July '72</td>
<td>0.19815 (6)</td>
<td>0.2303 (5)</td>
<td>9.8</td>
<td>- 6.5</td>
</tr>
<tr>
<td>2 May '74</td>
<td>0.19816 (7)</td>
<td>0.2318 (4)</td>
<td>13.3</td>
<td>1.5</td>
</tr>
</tbody>
</table>

\textsuperscript{+}These source coordinates are free of the effects of elliptic aberration. See Rogers et al., 1973.

*The first set of values for 2 May '74 is for a two-component model of 3C 345 while the second set is for the Fourier series model of 3C 345.
Table 5 (continued)

<table>
<thead>
<tr>
<th>E*</th>
<th>1971 October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Model and Parameters</td>
<td>( \alpha (16^h 38^m 48^s) )</td>
</tr>
<tr>
<td>Omitting the final two hours of observations</td>
<td>.19813 (3)</td>
</tr>
<tr>
<td>Allowing the zenith thicknesses of the atmosphere to be variable parameters</td>
<td>.19805 (5)</td>
</tr>
<tr>
<td>Changing the Z-component of the baseline by 2 meters</td>
<td>.19811 (1)</td>
</tr>
<tr>
<td>Changing the longitude of the G antenna by ( 1^\circ x10^{-5} )</td>
<td>.19814 (1)</td>
</tr>
<tr>
<td>Leaving a ( 2\pi ) error in the phase-connection (see Fig. VI.8)</td>
<td>.19778 (3)</td>
</tr>
</tbody>
</table>

F* Effects of the difference in clock drift rates (1971 October).

<table>
<thead>
<tr>
<th>Difference</th>
<th>( \alpha (16^h 38^m 48^s) )</th>
<th>( \delta (39^\circ 52' 30'') )</th>
<th>RMS ( (10^{-12} \text{ sec}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C 345 minus NRAO 512</td>
<td>.19811 (3)</td>
<td>.2311 (3)</td>
<td>11.9</td>
</tr>
<tr>
<td>3C 345 minus NRAO 512</td>
<td>.19814 (3)</td>
<td>.2321 (3)</td>
<td>12.0</td>
</tr>
<tr>
<td>Difference interpolated points</td>
<td>.19812 (2)</td>
<td>.2315 (2)</td>
<td>9.7</td>
</tr>
</tbody>
</table>

* These results form a self-consistent set but should not be compared to the results of A through D because the models used in the solutions were not identical.
Part A of Table 5 gives the position for a solution including all the models previously described while Part B gives the position when no structure phase was removed, which is equivalent to assuming that 3C 345 was unresolved. The one standard deviation error in the final significant figure, calculated for the weighted postfit RMS residual scaled to unity, is given to the right in parentheses, and the RMS of the residuals is given in picoseconds. Figure VI.6 is a plot of these position results for NRAO 512. Figure VI.7 shows a typical plot of residuals, those for the HG baseline in 1974 May 2 for the solution given in Part B of the table.

In order to estimate the accuracy of the results, the sensitivity of the coordinate solutions to small changes in the fixed parameters or in the models was tested. For example, Part C of Table 5 shows the effects of eliminating solid earth tides from the model; generally the change in coordinates is no worse than about 2 standard deviations. The solutions in Part D of Table 5 were calculated using USNO rather than B I H values for the difference between atomic and universal time (Al-UT1). Here, because $\Delta \delta \ll \Delta \alpha$, primarily the declination should be affected, and the biggest change in declination should occur for the experiment that coincided with the biggest difference between the USNO and B I H values; both of these suppositions are confirmed.
Figure VI.6  Position of NRAO 512 relative to the fixed position of 3C 345. The coordinates are shown for the solutions of both parts A and B of Table 5.
Figure VI.6
Figure VI.7 The 1974 May HG post-fit residual fringe phases from the differenced phase solution for the coordinates of NRAO 512.
Part E of Table 5 shows the results of a self-consistent set of tests performed on the 1971 Oct data. The only substantial effect on the resulting coordinates is for the solution based on a data set containing a $2\pi$ error in the phase connection. Figure VI.8a shows the residuals from this solution. It is evident that a $2\pi$ phase-connection error occurred in the 40-minute gap in the observations, due to instrumental difficulties. Removing the $2\pi$ error flattens the residuals and substantially reduces the RMS value (Figure VI.8b).

Part F of Table 5 tests the effect of having ignored the difference of the clock drift rates. For the first entry, the phase of each 3C 345 observation was differenced with that of the NRAO 512 observation which followed 2 minutes later. For the second entry, the phase of each NRAO 512 observation was differenced with that of the 3C 345 observation which followed 3 minutes later. Because the pairs of phases that were differenced were not measured simultaneously, the results are affected by the differences in the drift rates of the clocks in the interval between the measurement of the two phases to be subtracted. Since this interval was 2 minutes in one case and 3 minutes in the other, the resulting coordinates are not the same. It was not possible to determine useful values for the clock rates in a

*These tests were performed only to check the internal consistency of the data and the results should not be compared to the previous results because several small differences existed in the model used in the coordinate solution.*
Figure VI.8a&b  1971 October HG post-fit residual fringe phases from the differenced phase solution for the coordinates of NRAO 512. (a) shows the residuals when a $2\pi$ error was inserted in the phase connection across the gap in the observations, whereas (b) shows the residuals when this $2\pi$ error was not inserted.
Figure VI.8
simultaneous solution with the source coordinates. A method to circumvent this difficulty is to use each pair of observations of the same source to interpolate a value of the phase for that source for the intermediate time when the opposite source was observed. The solution based on differencing each observed phase with the interpolated phase of the other source for the same time is given in the third entry. As expected, the results lie between the results of the first and second solutions. The solutions in Parts A through D in the table were based on differencing each measured 3C 345 phase with the following measured NRAO 512 phase, or vice versa; however, in the solutions for a given experiment the order of the sources in the differencing was always the same. In Part E, the solutions were based on both measured and interpolated phases.

Finally, two sensitivity tests using the 1972 May data show first that a change of 0.1 in the declination of 3C 345 produces no significant change in the difference in the coordinates of 3C 345 and NRAO 512, whereas a change of 0.07 in the right ascension of 3C 345 changes the declination separation of the two sources by 0.0002 but does not affect their right ascension separation. The second test shows that an error of 2 meters in polar motion has no effect, to the accuracy quoted, on the NRAO 512 coordinates.

Because the solutions from 1974 May were based on data from three baselines, HG, HN, and NG, it was also possible to
solve for two of the three coordinates of one station. Since the N location was least well known, the radius and longitude of the N site were allowed to vary in the solutions. For the cases corresponding to Part A of Table 5, the radius solution was 5003.1675 (5) km (2-component model), 5003.1671 (5) km (Fourier series model), and for Part B, 5003.1670 (5) km. In all three solutions, the resulting longitude was 79°83599 (1). Robertson (1975) found 5003.1690 (2) km and 79°835960 (3) for the radius and longitude, respectively, of the N site based on VLBI observations with the HN interferometer. The differences between his values and the results based on the differenced phases are about four times the error in the latter, a discrepancy which remains unexplained.

VI. 4. Discussion

The estimated coordinates of NRAO 512 with respect to 3C 345 show no trend with time. The straight lines from a weighted least-squares fit using the four values of right ascension and declination given in Part A of the table have slopes of $-0.060\pm0.030 \times 10^{-3}$/year and $0.026\pm0.073 \times 10^{-3}$/year respectively. Using the values given in Part B, the slopes are $0.026\pm0.060 \times 10^{-3}$/year and $-0.020\pm0.062 \times 10^{-3}$/year, respectively. The one standard deviation errors in the slopes are based on the weighted postfit RMS of the residuals scaled to 1. Based on this lack of change in separation, the difference in the coordinates of 3C 345 and NRAO 512 is best given by the weighted mean of the values in Part A of Table 5:
Δα = 2^m 29^S 43663 (2)
Δδ = 1' 40" 7262 (8)

where the error represents the scatter in the four observations.

One possible source of confusion in the limit on the rate of change of the separation of the two sources is whether, for a source with structure such as 3C 345, the center of brightness corresponds to the center of mass. However, if we observe the source over a period of time long compared to the time between outbursts, the estimate of the limiting velocity will be an average over the outbursts and hence is likely to apply to the center of mass of the source. For the solutions given in Table 5, the separation determined was from NRAO 512 to the center of the brighter component of 3C 345. However, between 1971 October and 1974 March, this brighter component only moved about 0.00002 in right ascension and 0"0001 in declination with respect to the center of brightness, so that the conclusions concerning the separation of the two sources are not affected by the secular trends in the 3C 345 structure.

The distance to the sources can be estimated in the following two ways:
(i) If 3C 345 is assumed to be extragalactic, the 1971 October and 1972 May experiments combine to give a lower limit on the parallax distance to NRAO 512 of about 3 kpc, or as far as the edge of the disk of the Galaxy.

(ii) If each source has a transverse velocity of the order of 100 km/sec and the difference in the transverse velocity vectors is also of the order of 100 km/sec or greater, then both sources have a proper motion distance of at least 35 kpc, well outside the disk of the Galaxy, but still one and a half times closer than M31.

In order to cast doubt on the theory that quasars are local objects ejected from the center of the Galaxy, an independent estimate of quasar distances must be able, at a minimum, to require the quasars to be farther away than the nearest few galaxies. With an order of magnitude increase in resolution, an order of magnitude increase in the span of observation time, and a statistical sample of quasar pairs, VLBI observations should be able to provide the required minimum distance estimate.

*Clearly, the reverse assumption, that NRAO 512 is extragalactic, allows the conclusions to be drawn about the distance to 3C 345.*
CHAPTER VII

FURTHER DISCUSSION OF 3C 345

VII. 1. SUMMARY OF AVAILABLE INFORMATION

VII. 1. a. Optical Data

Figure VII.1a shows the blue magnitude light curve of 3C 345 from measurements of plates in the Harvard collection (Angione, 1973). More detailed measurements, beginning in 1965 are shown in Figure VII.1b (Kinman et al., 1968; Symth and Wolsstencroft, 1970; Tritton and Selmes, 1971; Lü, 1972; and Hackney, 1974). The most remarkable features of these data are the sharp outbursts, lasting only weeks, which can increase the brightness by almost 2 magnitudes, and which appear to have been occurring at least since about 1900.

Kinman et al. (1968) suggest that the optical radiation of 3C 345 consists of three components: a constant component of about 17.3 magnitudes, a slowly-varying component, which was turned off during 1965 and 1966, had turned on by the beginning of 1967 and began to fade toward the end of 1967, and a rapidly varying component which produces the flares. In addition, small, very short period fluctuations may be present, as indicated by the scatter in the observations being greater than that of the measurements of comparison stars. As supporting evidence for the two variable components they offer optical polarization data from 1967, shown in Figure VII.1c. The solid line through the
Historical light curve of blue magnitude versus date for 3C 345. Data are from Angione (1973).

Light curve of blue magnitude versus date for 3C 345. The data are from Kinman et al. (1968) (·), Smyth and Wolstencroft (1970)(v), Tritton and Selmes (1971)(o), Lü (1972) (+), and Hackney (1974)(x). The arrows mark the positions of what Kinman et al. call "periodic" flares.

Percent linear polarization and position angle of linear polarization for the optical radiation of 3C 345. Data are from Kinman et al. (1968). The arrows mark the positions of optical flares. The solid line is Kinman et al.'s suggestion for the position angle of the polarization of a slowly-varying component; the dashed lines mark the excursions from the solid line due to an outburst of the flaring component.
Figure VII.1a
Figure VII.1c
position angles gives the suggested contribution of the slowly-varying component, which is on at this time and is about 17% polarized. The excursions from this solid line, shown as dashed lines, are then due to the flaring component which produces radiation polarized at a different position angle and also has a high fractional polarization.

Kinman et al. also conclude that the flaring component shows periodicity with a period of 80.4 days, and a longer period of 321.5 days. Peaks corresponding to the 80.4-day period are marked in Figure VII.1b and Figure VII.1c with arrows. Many of the missing peaks may have escaped observation due to gaps in the data, but a few, such as the one expected about JD 2439300, simply did not occur. The observations immediately following the end of Kinman et al.'s observing program are generally too widely spaced to offer a thorough test of this periodicity, although Hackney concludes that her data and that of Lü are not compatible with the 80.4-day period. The major outburst of mid-1969 is particularly interesting because of its double peak, the first of which corresponds to the 80.4-day period; however, the second, and larger peak occurs about 45 days later, out of synchronization. Perhaps the phenomena responsible for the outbursts are transient, lasting only a few years, so that the periodicity observed at one epoch may not persist.

The later observations also indicate that after the disappearance of the slowly-varying component at the beginning of
1968, the constant component is only about 17.5 magnitudes, although this decrease may be due to a small difference in scaling between measurements of the different observers. By early 1971, the slowly-varying component has reappeared, although the level of its magnitude is uncertain due to the scatter in the data after mid-1970.

Finally, Burbidge and Burbidge (1966) and Wampler (1967) have observed changes in the frequency, strength, and structure of the MgII emission line of 3C 345, which appear to be associated with the outburst in the fall of 1965, and which occurred on a similar time scale.

VII. 1. b. X-Ray Data

Giacconi et al. (1974) report that UHURU object number 3U1639+40 may be identified with 3C 345. The X-ray object has right ascension between 16\text{h} 32\text{m} 0\text{s} and 16\text{h} 46\text{m} 24\text{s}, and declination between 39\text{o} 16' 48" and 41\text{o} 12' 0". 3C 345 falls well within this error box, with right ascension 16\text{h} 41\text{m} 17.6 and declination 39\text{o} 54' 11". Another possible identification for the source of the X-rays is the cluster Abell 2199, although this object is considered less likely than 3C 345 because it is near the edge of the error box. Clusters of galaxies are common X-ray sources whereas 3 C 273 is the only quasar definitely identified with an X-ray source.

The X-ray flux of this UHURU object is 4.0±0.6 in units of UHURU count rate in a 2 to 6 kev band. Using the conversion
factor suggested by Giacconi et al. (of 1.7 x 10^{-11±30%} erg cm^{-2} sec^{-1}) times the count rate, and the 4 kev bandwidth, we find an average X-ray flux of (7±2) x 10^{-32} Jy. This compares to 10^{-28} to 10^{-30} Jy for the optical flux and about 10^{-25} Jy for the radio flux.

VII. 1. c. Radio Data

Figure VII.2 shows the radio flux of 3C 345 as a function of time for wavelengths between 0.33 and 40 cm.

[Fogarty et al., 1971, 0.33 cm (+); Hobbs and Waak, 1970, 0.95 and 1.65 cm (o); Dent and Hobbs, 1973, 0.95 cm (x); Dent et al., 1974, 1.8 cm (+); Kellermann and Pauliny-Toth, 1968, 2, 11, 22, and 40 cm (x); Seaquist et al., 1974, 2.2 cm (o); Medd et al., 1968, 2.8 cm (+); Bignell and Seaquist, 1973, 2.8 and 4.5 cm (o); Kellermann and Pauliny-Toth, 1973, 2.8 cm (x); Dent, 1965, 3.8 (o); Aller and Haddock, 1967, 3.8 cm (+); Dent and Kajoian, 1972, 3.8 cm (x); Aller, 1970, 3.8 cm (o); this thesis, 3.8 cm (Θ); Pauliny-Toth and Kellermann, 1968, 6 cm (x); Kellermann et al., 1969, 6, 11, and 21 cm (v); Seielstad and Berge, 1975, 6 cm (o); Kellermann et al., 1970, 13 cm (o); Broderick et al., 1972, 13 cm (z); Berge and Seielstad, 1972, 18 cm (Θ); Kellermann et al., 1968, 18 cm (y); Pauliny-Toth and Kellermann, 1966, 22 cm (+); Bologna et al., 1969, 21.2 cm (z); and Haves et al., 1974, 31 cm (o)]. These data together with those from Pauliny-Toth and Kellermann (1966), Erickson and Cronyn (1965), and Pauliny-Toth et al. (1966) were used to plot.
Figure VII.2 The radio flux of 3C 345 versus date and wavelength of the observations. (See the text for references.)
Figure VII.2
Figure VII.2 (continued)
radio spectra at epochs 1966.0, 1967.0, 1969.5, 1971.0 and 1973.0, shown in Figure VII.3; the points at frequencies below 200 MHz are the same for all the epochs, whereas those above 200 MHz are interpolated from the data in Figure VII.2. The dashed lines show a suggested decomposition into outbursts for the spectra from 1967.0 and 1973.0.

One particularly interesting feature of the radio flux data is the large increase in flux, first evident in the 2-cm data in 1966, which can be followed through the 13-cm data as it progresses to longer wavelengths. Conventional synchrotron radiation theories predict that an outburst of relativistic electrons should be visible first at higher frequencies; as the cloud expands and the energy distribution of the electrons evolves, the maximum flux should progress to lower frequencies and decrease. The radio spectrum from 1966.0 in Figure VII.3 shows evidence of this outburst at the highest frequencies. By 1973.0 the maximum in the spectrum due to the outburst is at about 7.0 GHz and the peak of a new outburst is visible at higher frequencies; the dashed lines in Figure VII.3 are a suggested decomposition of the spectrum into outbursts. All of the 3C 345 long-baseline interferometry data discussed in Chapter V were taken after 1971.0, when it appears that two outbursts were visible at 7.8 GHz; however, one cannot reliably draw a one-to-one connection between the two physical components of the brightness distribution and the two outbursts
Figure VII.3 The radio spectrum of 3C 345 for epochs 1966.5, 1967.0, 1969.5, 1971.0 and 1973.0 (see the text for references.) The data for 1971.0 and 1973.0 have been raised one decade with respect to the data from the earlier epochs for easier viewing. The solid curves are sketches through the points; the dashed curves are suggested decompositions of the 1967.0 and 1973.0 spectra into components.
evident in the spectrum.

The linear polarization of 3C 345 at centimeter wavelengths integrated over the source is shown in Figure VII.4 as percent polarization as a function of date [Hobbs and Waak, 1972, 0.96 cm ($\circ$); Hobbs et al., 1968, 2.1 cm ($\circ$); Hobbs and Hollinger, 1968, 2.1 cm (+); Seaquist et al., 1974, 2.2 cm ($\times$); Bignell and Seaquist, 1973, 2.8 and 4.5 cm (+); Berge and Seielstad, 1969, 3.1 cm ($\circ$); Aller and Haddock, 1967, 3.8 cm ($\times$); Aller, 1970, 3.8 cm ($\circ$); Sastry et al., 1967, 6 cm ($\circ$); Berge and Seielstad, 1972, 6 and 18 cm ($\times$); Seielstad and Berge, 1975, 6 cm (+); Maltby and Seielstad, 1966, 10.6 cm ($\circ$); Haves et al., 1974, 31 cm ($\times$); and Conway et al., 1972, 49 and 73 cm (+)]. The fluctuations at the shorter wavelengths can be quite large, between zero and 7%, whereas the percent polarization at wavelengths longer than about 15 cm is about 5%, although there are only 1 or 2 measurements at each frequency. Pacholczyk and Gregory (1973) have shown that it is possible to produce the observed dependence of percent polarization and position angle on frequency for sources like 3C 345 (flat spectrum sources) by a superposition of spectral components whose polarization is typical of that for steep spectrum sources (monotonic decrease of polarization with wavelength). Faraday dispersion (differential Faraday rotation of polarized emission from different parts of the source) is also suggested as a mechanism responsible for the dependence of polarization on wavelength (see, for example, Kronberg et al., 1972). Unfortunately, it is not
Figure VII.4  The percent linear polarization of the radio
radiation of 3C 345 versus date and wave-
length. (See the text for references.)
possible to draw a good spectrum for percent polarization as a function of frequency due to the variability of the source and the lack of coincident measurements at many frequencies.

The position angle of linear polarization is plotted as a function of wavelength squared in Figure VII.5 (the data in Figure VII.4 and Bologna et al., 1969, 21.2 cm; Berge and Seielstad, 1967, 18 cm; Kronberg and Conway, 1970, 49 cm; and Morris and Berge, 1964, 10.3, 18, 21.2 and 21.6 cm). From the best fit straight line through the position angles, the rotation measure (RM) of the Faraday rotation due to charged particles and magnetic field along the path from 3C 345 to the observer, and the intrinsic polarization angle (IPA) of the source can be determined (Rose, 1973) since

\[ \chi(\lambda^2) = \lambda^2 \text{RM} + \text{IPA} \]

where \( \lambda \) is the wavelength and \( \chi(\lambda^2) \) is the measured position angle*. For the line in Figure VII.5, which is the result of a least-squares fit to the measurements at wavelengths above 10.3 cm, RM is 18.2 radians/m² and IPA is 47°, in good agreement with the values of 18.9±1.9 and 46.4±4.6, respectively, calculated by Bignell (1973). Measurements at shorter wavelengths are shown as a band between 0 and 100°; all but a few

*Note that since the measured position angle is ambiguous by \( n \pi \) the measurements of \( \chi \) at longer wavelengths have been increased by the number of \( n \pi \)'s necessary to give a straight line.
Figure VII.5  The measured position angle of the linear polarization of the radio radiation of 3C 345 versus wavelength squared. (See the text for references.) The data points at longer wavelengths have been increased by the number of $n\pi$'s necessary to give a straight line. The slope of the line gives the rotation measure of the medium along the line of slight and the intercept gives the intrinsic polarization angle of the radiation from the source. The shaded area at short wavelengths indicates the variability of the polarization of the source in this region.
RM = 18.2 rad/m²

PA = 47°

Figure VII.5
of the measured position angles below 6 cm fall in this band with typical errors of a few to about 25°. The Faraday rotation is thought to be due primarily to the interstellar medium since values of rotation measure for a large sample of sources show a dependence on both the galactic latitude and longitude of the sources (Berge and Seielstad, 1967).

Several attempts have been made to measure the integrated circular polarization of 3C 345. At 21 cm, Conway et al. (1971) measured 0.008±0.026*; at 18.7 cm, Berge and Seielstad (1972) measured 0.10±0.07; at 9.3 cm, Seaquist (1969) measured -0.18±0.34 and 0.07±0.32. Seielstad and Berge (1975) have 1 out 10 measurements at 6 cm where the percent circular polarization is just over 3 standard deviations: 0.31±0.10 percent right circularly polarized. Errors for the 9 remaining measurements are between ±0.06 and ±0.34 percent, and the measured values are all less than one standard deviation. One measurement of Berge and Seielstad (1967) at 3.1 cm was less than the one sigma error of 0.50, and four measurements of Seaquist et al. (1974) at 2.2 cm have the values: -0.28±0.19, 0.51±0.24, 1.58±1.16, and 0.51±0.19. Clearly the evidence for circular polarization is quite weak, although indications are that it is more likely to be present at shorter wavelengths.

*Positive means right circularly polarized.
VII. 2. TOPICS FOR FURTHER RESEARCH

Synchrotron radiation is generally accepted as the mechanism responsible for the radio radiation from quasars and related objects. Models based on the injection of a cloud of relativistic electrons with a power law spectrum into a region containing a magnetic field, with subsequent adiabatic expansion of the cloud (Shklovskii, 1960; Van de Laan, 1966; Kellermann, 1966; Pauliny-Toth and Kellermann, 1966; Ozernoi and Ulanovskii 1974; Aller, 1970; and others) give good qualitative and sometimes quantitative agreement with the observed radio spectra, polarization, and their time variations. The high optical polarization measured for 3C 345 suggests that the synchrotron mechanism may also be responsible for some fraction of the optical radiation. However, there are still many unanswered questions, some of which are:

(i) If 3C 345 has two radio components which are separating with time, how can the velocity of separation be explained? Several suggestions have been proposed (for example, Rees, 1966), but no one mechanism is preferred by the observations.

(ii) How comparable are the physical mechanisms responsible for the observed behavior of sources with different redshifts and hence different size scales for the same angular separations and velocities?

(iii) How are the radio structure components of 3C 345 related to the outbursts seen in the spectrum? Is there a one-
to-one correspondence? Are the three optical components suggested by Kinman et al. (1968) related to the radio components? Produced by the same mechanism?

(iv) Both Kinman et al. (1968) and Morrison (1969) suggest that the optical flares of 3C 345 are hot spots on the "surface" of a rotating object with a period of slightly less than a year. Is the optical periodicity real? Is there radio periodicity?

(v) What is the physical mechanism producing bursts of relativistic electrons seen as increases in the radio flux, first at shorter wavelengths, and then at longer ones? Two suggestions are energetic stellar collisions (Spitzer and Saslaw, 1966; Gold et al., 1965), or multiple supernovae (Colgate, 1967; Colgate et al., 1975). How can these theories be better tested experimentally?

(vi) If 3C 345 produces X-rays, how are they generated? What is the relation to the optical and radio behavior?

Long baseline interferometry can be particularly useful in helping to clarify some of the observational aspects of these questions. Experiments involving increased numbers of cooperating antennas will allow improved mapping of the structure of the brightness distribution as a function of time. In addition, VLBI polarization experiments could help to further restrict theoretical models for the magnetic fields responsible for the presumed synchrotron radiation.
APPENDIX A

CALCULATION OF THE COMPLEX FRINGE

VISIBILITY FROM THE BRIGHTNESS DISTRIBUTION

Using Equation (II.1), we can calculate the complex fringe visibility \( V(u,v) \) for the brightness distribution \( B(x,y) \). For the brightness distribution given in Equation (IV.6), the visibility is given by

\[
V(u,v) = \frac{S_0}{2\pi\sigma_0^2} \int_{-\infty}^{\infty} e^{-x^2/2\sigma_0^2} e^{-2\pi iux} \, dx \int_{-\infty}^{\infty} e^{-y^2/2\sigma_0^2} e^{-2\pi ivy} \, dy
\]

\[
+ \frac{S_1}{2\pi\sigma_1^2} \int_{-\infty}^{\infty} e^{-(x-x_1)^2/2\sigma_1^2} e^{-2\pi iux} \, dx \int_{-\infty}^{\infty} e^{-(y-y_1)^2/2\sigma_1^2} \, dy.
\]

If the integral \( I \) is defined

\[
I(q_1,\sigma,s) \equiv \int_{-\infty}^{\infty} e^{-\frac{(q-q_1)^2}{2\sigma^2}} e^{-2\pi iqs} \, dq
\]

then \( V(u,v) \) is simply

\[
V(u,v) = \frac{S_0}{2\pi\sigma_0^2} I(0,\sigma_0,u)I(0,\sigma_0,v) + \frac{S_1}{2\pi\sigma_1^2} I(x_1,\sigma_1,u)I(y_1,\sigma_1,v)
\]

The integral \( I(q_1,\sigma,s) \), can be calculated by the method of "completing the square". Writing the exponent of \( e \) as
\[-\frac{q^2}{2\sigma^2} + q\left(\frac{q_1}{\sigma} - 2\pi s\right) - \frac{q_1}{2\sigma^2} + 2\pi^2 s^2 \sigma^2 - 2\pi^2 \sigma^2 s^2 + 2\pi i q_1 s - 2\pi i q_1 s\]

we can write

\[I(q_1, \sigma, s) = e^{-2\pi^2 \sigma^2 s^2} e^{-2\pi i q_1 s} \int_\sigma^{\infty} e^{-\frac{(q_1}{\sigma^2 \sqrt{2}} - 2\pi i q_1 s} dq\]

Now defining

\[q' = \frac{q}{\sigma^2 \sqrt{2}} + \pi s \sqrt{2} \sigma - \frac{q_1}{\sigma^2 \sqrt{2}}\]

\[dq' = \frac{1}{\sigma^{2 \sqrt{2}}} dq\]

we have

\[I(q_1, \sigma, s) = \sqrt{2} \sigma e^{-2\pi^2 \sigma^2 s^2} e^{-\frac{q_1}{\sigma^2 \sqrt{2}} \sigma} \int_\sigma^{\infty} e^{-q'^2} dq'\]

Since

\[\int_{-\infty}^{\infty} e^{-q'^2} dq' = \sqrt{\pi}\]

the integral is given by

\[I(q_1, \sigma, s) = \sqrt{2} \sigma e^{-2\pi^2 \sigma^2 s^2} e^{-2\pi i q_1 s}\]

and \(V(u, v)\) is given by

\[V(u, v) = S_0 e^{-2\pi^2 \sigma^2 (u^2 + v^2)} + S_1 e^{-2\pi^2 \sigma^2 (u^2 + v^2)} - 2\pi i (x_1 u + y_1 v)\]

Using \(x_1\) and \(y_1\) as defined in Figure IV.1, and the alternate parameters \(S = S_0 + S_1\), and \(\xi = \frac{1}{2} (S_0 - S_1) / S\), we have Equation (IV.8):
\[ V(u,v) = \frac{S}{2} \left[ (1+\xi)e^{-2\pi^2 \sigma_0^2 (u^2+v^2)} + (1-\xi)e^{-2\pi^2 \sigma_1^2 (u^2+v^2)} \right] e^{-2\pi i r(u \sin \phi + v \cos \phi)} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>coefficient in the model for the delay due to atmosphere [Equation (VI.4)]</td>
</tr>
<tr>
<td>Al</td>
<td>U. S. Naval Observatory atomic time</td>
</tr>
<tr>
<td>$A_e$</td>
<td>antenna effective area</td>
</tr>
<tr>
<td>$A_{jk}$, $A^o_{jk}$</td>
<td>right ascension of the $jk$ baseline vector, as a function of time and when the sidereal time at the Greenwich meridian is zero, respectively</td>
</tr>
<tr>
<td>B</td>
<td>coefficient in the model for the delay due to the atmosphere [Equation (VI.4)]</td>
</tr>
<tr>
<td>$B(x,y)$</td>
<td>brightness distribution of the source as a function of spatial position</td>
</tr>
<tr>
<td>$D_{jk}$</td>
<td>declination of the $jk$ baseline</td>
</tr>
<tr>
<td>$E$</td>
<td>source elevation angle [Equation (VI.4)]</td>
</tr>
<tr>
<td>$F$</td>
<td>the Fairbanks, Alaska, antenna</td>
</tr>
<tr>
<td>$F^c$</td>
<td>correlated flux</td>
</tr>
<tr>
<td>$F^c_{jk}$</td>
<td>correlated flux measured by the $jk$ interferometer</td>
</tr>
<tr>
<td>$F_T$</td>
<td>total flux</td>
</tr>
<tr>
<td>$G$</td>
<td>the Goldstone, California, antenna</td>
</tr>
<tr>
<td>GST</td>
<td>Greenwich sidereal time</td>
</tr>
<tr>
<td>H</td>
<td>the Haystack, Massachusetts, antenna</td>
</tr>
<tr>
<td>HPHW</td>
<td>the $e^{-1/2}$ half-width of a Gaussian component</td>
</tr>
<tr>
<td>IPA</td>
<td>the intrinsic position angle of the linearly polarized radiation of a source</td>
</tr>
</tbody>
</table>
M, M'
the number of terms in the v direction in the
Fourier series representations for the correlated
flux and closure phase, respectively

N
the NRAO, West Virginia, antenna

N, N'
the number of terms in the u direction in the
Fourier series representations for the correlated
flux and closure phase, respectively

P
position angle of the smaller, weaker component
of the standard model with respect to the larger,
stronger component, measured counter-clockwise
from north (Figure IV.1)

RM
the rotation measure of the medium along the line
of sight to the source

S
the Onsala, Sweden, antenna

S
the total model flux

S_0, S_1
the model flux of the stronger and weaker components
of the standard model, respectively (Figure IV.1)

T_j
time kept by the clock at site j

T(E)
the phase thickness of the atmosphere

[T(Equation (VI.4))]

T_A
the antenna temperature of the source

T_{ON, OFF}
the on-source and off-source system temperatures,
respectively

UT
universal time
\( V(u,v) \) the complex fringe visibility

\( z \) the zenith phase thickness of the atmosphere

[Equation (VI.4)]

\( a_{\mu \nu} \) coefficients in the Fourier sine series representation of the correlated flux

\( \hat{b}_{jk}, b_{jk} \) the baseline vector for the \( jk \) interferometer and its magnitude, respectively

\( c \) the speed of light

\( c_{\mu \nu} \) coefficients in the Fourier cosine series representation of closure phase

\( \hat{e} \) unit vector in the direction of the source

\( j, k, \lambda \) subscripts designating a site or antenna

\( q \) index of summation in the clock polynomial

\( r \) radial separation of the two components of the standard model (Figure IV.1)

\( t \) coordinate time

\( t_s \) sidereal time

\( u, v \) spatial frequencies

\( u_{jk}, v_{jk} \) resolution of the \( jk \) interferometer in the direction of increasing \( \alpha \), and in the north direction, respectively

\( x, y \) spatial positions

\( x_1, y_1 \) coordinate position of the smaller, weaker component of the standard model (Figure IV.1)
\( \hat{x}, \hat{y} \) unit vectors in the direction of increasing \( \alpha \), and in the north direction, respectively

\( \hat{z}_1, \hat{z}_2, \hat{z}_3 \) unit vectors of an earth-centered, non-rotating coordinate system; \( \hat{z}_1 \) points in the direction of the true equinox of date, \( \hat{z}_3 \) in the direction of the instantaneous spin axis of the earth, and \( \hat{z}_2 = \hat{z}_3 \times \hat{z}_1 \)

\( \alpha \) source right ascension

\( \alpha_{jq} \) coefficients in the polynomial series representation of the clock at site \( j \)

\( \delta \) source declination

\( \delta \Delta t_{jk} \) the difference of the geometric time delays of two sources observed with the \( jk \) interferometer

\( \Delta t_{jk} \) the geometric time delay of a source observed with the \( jk \) interferometer

\( \Delta \alpha \) the difference of the right ascensions of two sources

\( \Delta \delta \) the difference of the declinations of two sources

\( \Delta \sigma \) the difference of the HPHW's of two circular, Gaussian components

\( \Delta \sigma_x, \Delta \sigma_y \) the difference of the HPHW's in the \( \hat{x} \) and \( \hat{y} \) directions, respectively, of two elliptical, Gaussian components

\( \Delta \phi_{jk} \) the difference of the fringe phases of two sources observed with the \( jk \) interferometer
\( \lambda \) wavelength of the observations

\( \mu, \nu \) indices of summation in the Fourier series representations of correlated flux and closure phase

\( \xi \) relative difference in strength of the two components of the standard model

\( \sigma_0, \sigma_1 \) circular HPHW's of the two Gaussian components of the standard model

\( \sigma_{ox}, \sigma_{lx} \) elliptical HPHW's in the x and y directions, respectively, of the two Gaussian components of the standard model

\( \sigma_{oy}, \sigma_{ly} \)

\( \tau_{jk} \) the group delay of a source observed with the jk interferometer

\( \tau^c_{jk} \) the delay due to the clocks for a source observed with the jk interferometer

\( \tau^a_{jk} \) the delay due to the atmosphere for a source observed with the jk interferometer

\( \tau^p_{jk} \) the delay due to the difference in the number of charged particles along the line of site to each antenna of the jk interferometer

\( \phi^m_{jk} \) the fringe phase measured by the jk interferometer

\( \phi^t_{jk} \) the theoretical fringe phase of the jk interferometer

\( \phi^s \) the structure phase

\( \phi^s_{jk} \) the structure phase for a source observed with the jk interferometer
the theoretical structure phase calculated from the standard model, and the Fourier series model, respectively

the residual fringe phase calculated by integrating the polynomial series for the residual fringe rate for a source observed with the jk interferometer

the residual fringe phase for a source observed with the jk interferometer, before and after, respectively, the integrated polynomial series for the residual fringe rate has been removed

the fringe rate for a source observed with the jk interferometer

the polynomial series representation of the residual fringe rate for a source observed with the jk interferometer

the closure phase for a source observed with the 3 interferometers formed by the j, k, and \( \ell \) antennas

the measured position angle of the linearly polarized radiation of a source

the phase due to the difference of the antenna-feed position angles for a source observed with the jk interferometer
\( \omega \) the radio (angular) frequency of the observations

\( \Omega \) the rate of rotation of the earth
BIBLIOGRAPHY


