

# Stabilizing the Ball on Beam System with Analog Feedback

by

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Submitted to the Department of Mechanical Engineering  
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Bachelor of Science in Mechanical Engineering

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May 7, 2004

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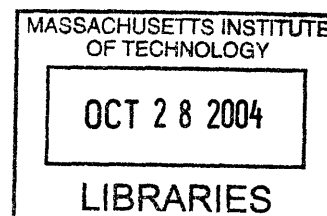
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## Abstract

A mechanical ball on beam system was stabilized to demonstrate the capabilities of control systems. This demonstration system is intended for use in control theory classes such as 6.011 Introduction to Communication, Control, and Signal Processing, 6.302 Feedback Systems, and 6.003 Signals and Systems. Control of this unstable system is achieved through classical control methods taught in 6.302. The compensators are implemented in analog circuitry. The system was successfully demonstrated in a 6.011 lecture (April 5, 2004). A lab kit system was designed for future 6.302 students.

Thesis Supervisor: Dr. Kent Lundberg  
Title: Lecturer



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# Chapter 1

## Introduction

Feedback control can be used to stabilize unstable systems. Demonstration of a working, well-tuned control system can be the highlight of an introductory control systems class. The ball on beam is intuitively unstable yet can be stabilized with an appropriate controller. Demonstration of a stabilized ball on beam system would be a valuable visual aid in teaching control theory.

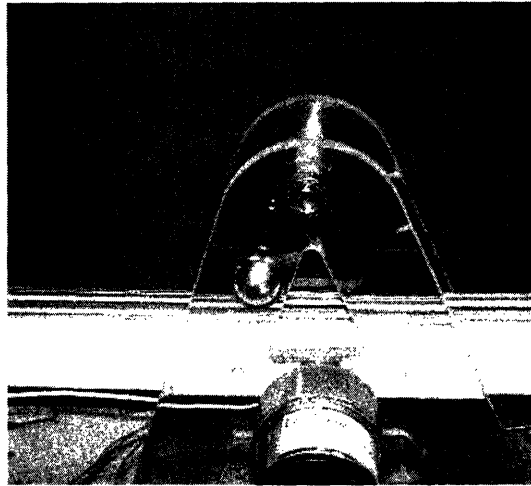


Figure 1-1: Ball Balancer Demonstration

This thesis report describes the design, manufacture, and implementation of a ball on beam control system for use as a demonstration system and laboratory project. The open loop ball on beam system is inherently unstable and presents a challenging control systems problem as the signal path from motor current to ball position includes

four poles at the origin.

The purpose of this report is to document the methods implemented in the design and construction of the ball balancer demonstration system shown in Figure 1-1. This system was built as a classroom demonstration for the Department of Electrical Engineering and Computer Science at MIT. This document emphasizes the design and construction of the sensors and control loop. The demonstration system constructed as a part of this thesis may be used to demonstrate control systems applications to future students.

This document begins with a brief description of the problem and background to the ball on beam system in Chapter 1. The discussion then shifts to a description of previously existing systems including the Ball Balance System for Undergraduates [1] and a project completed as a 6.302 Ball Balance Lab in the Fall of 2003.

Chapter 3 provides theoretical analysis of stabilizing the ball on beam system. The equations of motion and modeling methods are described as well as approximations and linearizations which were used in the design of the control system. This section includes a discussion of the control theory and topologies that could be used to stabilize the ball on beam system

Chapter 4 describes the construction of the demonstration system. The demonstration system includes significant improvements beyond the 6.302 Ball Balance Lab and performs much better. The system output is presented and compared to the 6.302 Ball Balance Lab.

Chapter 5 discusses the sensor design, possible improvements, as well as quantitative tests of various sensors which could be used in future iterations. Designing robust and accurate sensors has proven to be a large part of the design of the and improvements to the sensor systems could lead to increased performance in the balancer system.

The final section describes modifications to the existing lab kit [2] which may aid future students in designing a control system for an undergraduate laboratory project. This kit was designed to be simple, inexpensive, and easy to manufacture for use as a laboratory project in undergraduate controls classes.

# Chapter 2

## Existing Systems

Stabilization of the ball on beam system has been achieved in research settings. Most existing systems are too costly or complicated to be used as either a lab kit for students or a demonstration system for lectures.

### 2.1 Electronic Design Magazine

In a November 20, 1995 article in *Electronic Design Magazine*, Bob Pease describes his ball balance system [3]. Pease's system stabilizes the ball on beam system using only inexpensive analog components. He compares his solution to a system implemented with complicated and expensive computer controlled stabilization methods using "fuzzy logic".

The system built by Pease uses minor loop compensation and proportional, integral, and derivative (PID) compensators to stabilize the system. Using this approach, Pease successfully stopped the ball in the center of the beam with very little overshoot. This system provides a model for the control design and is a good benchmark for how well analog feedback can stabilize a ball on beam system.

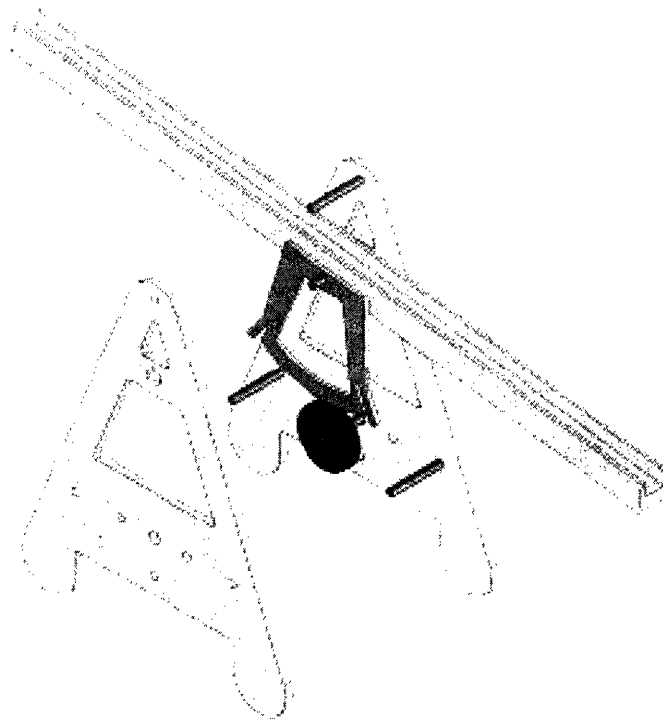


Figure 2-1: Existing Lab Kit

## 2.2 A Ball Balance System for Undergraduates

As part of a project at MIT, a ball balance lab kit was designed by Evencio Rosales under the supervision of Professor David Trumper and Dr. Kent Lundberg [2]. This kit provides a good foundation for students building a ball balance system.

The lab kit provides a sturdy basic structure of a ball on beam system. Figure 2-1 shows an exploded view of the kit. The acrylic “A” frames hold the beam which is free to pivot on a bearing. The plastic frame allows for easy alterations and modifications which may be necessary for a particular setup. This flexibility allows a choice of motors, sensors, and control topologies while providing much of the mechanical design and construction which would normally go into building a ball on beam system.

The kit provides a sector drive system for gearing down the DC motor sufficiently to drive the rotational inertia of the beam and ball. This allows the use of an inexpensive gearless DC motor without requiring expensive gears or belt driven systems.

A complete bill of materials for the lab kit and description of the kit components is included in Appendix A.

## **2.3 6.302 Ball Balancer Laboratory**

The lab kit described above was provided to a select number of students in 6.302 during the Fall 2003 semester. Most of the students were able to balance the ball for extended periods of time, although with varying performance. The author along with fellow student Brandon Kam were able to achieve a stable system.

As a part of the final project, the feedback sensors were chosen by the students to provide signals to the control system. The beam angle was measuring using a MEMS tilt sensor from Analog Devices (ADXL202). The tilt sensor measured the beam angle by measuring the gravitational component parallel to the beam. The ball position sensor was a linear potentiometer design built using nickel-chromium wire wrapped around a nylon rod.

The Ball Balancer successfully kept the ball on the track for more than four hours at a time. After some initial transient, the ball position settled into a steady oscillation around the center of the beam. The 6.302 Ball Balancer solution was highly sensitive to slight variations in input voltages, loop gains, and the initial motor position which made this solution impractical as a demonstration system.





# Chapter 3

## Theoretical Model

The first step in controlling an unstable system is to describe the system using appropriate mathematical models. These models are then used to design a control system which, when applied to the open loop system, produce the desired system behavior.

### 3.1 System Description

Figure 3-1 shows the geometry of the ball on beam system. The system consists of a ball which is free to roll along a channel in a beam. The beam is free to rotate about the pivot point. System input is achieved through a geared motor which is coupled to the beam.

The overall system can be broken down into two subsystems. The ball on beam dynamics describes the ball position response to an input of beam angle. The motor and beam system describes the beam angle response to the motor input current. These two subsystems are directly coupled to each other and cannot be completely separated. To simplify this analysis, some dynamic coupling is modeled as a system disturbance.

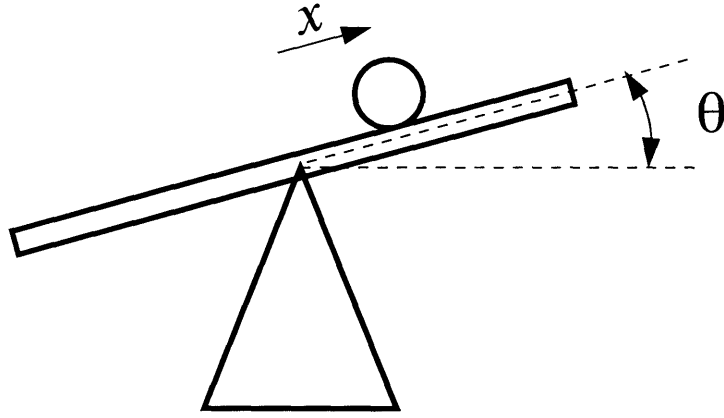


Figure 3-1: Geometry of Ball on Beam System

### 3.1.1 Dynamic Models

The ball motion is constrained to rolling on the track. The ball is modeled as rolling with no slip along a surface which is tilted to an angle  $\theta$ . The component of gravity which accelerates the ball is,

$$F_x = mg \sin \theta \quad (3.1)$$

where  $F_x$  is force parallel to the track. Invoking the no-slip condition, the ball accelerates as,

$$\left(m + \frac{J}{r^2}\right) \ddot{x} = F_x \quad (3.2)$$

where  $J$  is the moment of inertia about the center of the ball. The moment of inertia of a rolling ball is  $J = \frac{2}{5}mr^2$ . The rolling mass accelerates just as a sliding object with the mass  $m_{eq} = m + \frac{J}{r^2}$ . Combining equations 3.1 and 3.2 leads to the equation of motion relating the beam angle and the ball position,

$$\sin \theta = \frac{m_{eq}}{mg} \ddot{x} \quad (3.3)$$

for any rolling sphere on an incline plane. This equation assumes a static beam and no other forces on the ball. In the ball on beam system, the angular acceleration of the beam will generate some centrifugal force on the ball and will change the ball

dynamics. In this analysis, this effect is assumed to be small and is modeled as a disturbance to the system.

The dynamics of the motor and beam system are modeled as a simple DC motor and inertial load. This system is studied extensively in 6.302. The equation of motion relating the motor torque to the beam angle is,

$$\tau = i_{motor} k_t = (J_{motor} + J_{load}) \ddot{\theta} \quad (3.4)$$

where  $k_t$  is the motor constant and  $J_{load}$  is the effective load on the motor as seen through the gear box. This model assumes there is no ball on the track. Addition of the ball will increase the  $J_{load}$  as it adds an offset mass on the beam. This effect is less than if the ball was fixed to the track at a certain position ( $x \neq 0$ ), because the ball is free to roll on the track. An additional torque is exerted on the beam due to the force of gravity on the ball. This effect is modeled as a disturbance and is ignored in the controller design.

The overall system can be linearized using the small angle approximation ( $\sin \theta = \theta$ ). The transfer functions,

$$H_1(s) = \frac{\theta(s)}{I(s)} = \frac{k_t}{J_{tot} s^2} \quad (3.5)$$

$$H_2(s) = \frac{X(s)}{\theta(s)} = \frac{mg}{m_{eq} s^2} \quad (3.6)$$

$$H(s) = H_1(s)H_2(s) = \frac{X(s)}{I(s)} = \frac{mgk_t}{J_{tot} s^4} \quad (3.7)$$

express the (linearized) dynamic coupling between the motor current and the ball position. This transfer function has four poles at the origin, indicating a difficult control problem.

### 3.1.2 Open-Loop Response

Using the model above, the open-loop system has four poles at the origin. The time-domain representation of this is four integrators in the forward path which will

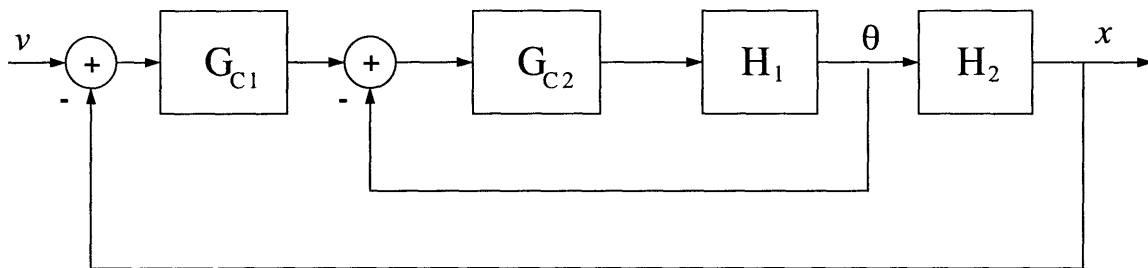


Figure 3-2: Minor Loop Compensation Block Diagram

magnify any input offset into the system. Physically, this can be seen as an input current into the motor will cause an angular acceleration of the beam. The increasing beam angle will lead to increased ball acceleration and eventually infinite ball position.

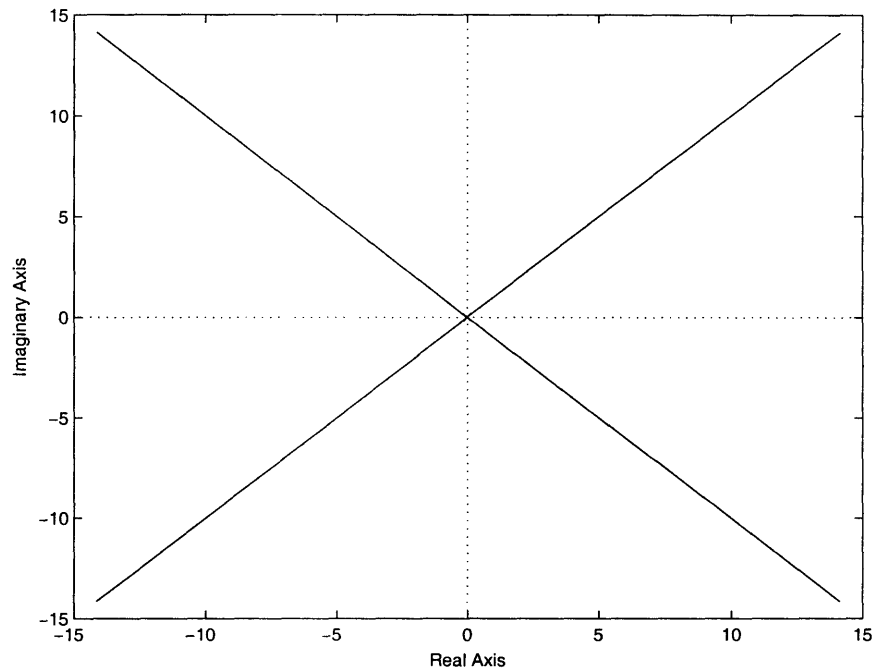
The goal of this control application will be to eliminate this instability and provide a controlled system which will accept an input voltage and will output a ball position proportional to the input. For this to be accomplished, all four poles must be brought into the left-half plane. A robust control system will be insensitive to initial ball position, initial beam angle, and possible disturbance sources.

## 3.2 Control Design

The control system implemented in this thesis was very similar to the topology used by Bob Pease in his analog ball on beam system [3] and in the 6.302 Ball Balance Lab project. Minor loop feedback is used to first stabilize the motor loop, creating a motor angular position servo system. That system is then used in the major loop to stabilize the ball position. Appropriate compensators are used in both the minor and outer loops. The block diagram of the system is shown in Figure 3-2.

The system dynamics are captured in the transfer functions  $H_1$  and  $H_2$  which are defined above in equations. The controller transfer functions, represented by  $G_{C1}$  and  $G_{C2}$  are designed to provide optimum bandwidth and phase margin for the system. Pease used PID compensators in his design. This project used lead compensators for stabilization. Both approaches were successful in stabilizing the system.

Figure 3-3: Open Loop Root-locus Diagram



Both PID and lead compensation are classical control methods, they rely on access to the error signal (beam angle or ball position) and use that signal to generate a control signal. In this control design, two of the states of the system are used in the feedback loops.

Root-locus analysis of this control approach shows that, for appropriate gain selections, a stable system can be achieved. Figure 3-3 shows the open-loop root locus of the system. Since there are four poles at the origin, two poles immediately move into the right-half plane, causing the system to be unstable.

Using minor loop compensation isolates two of the system poles and allows compensation on just those poles. In this system, the two poles at the origin from the motor are stabilized with a lead compensator. The root-locus of the minor loop system is shown in Figure 3-4. For appropriate gain selection, the system poles move into the left-half plane, stabilizing the minor loop system.

The complete system root-locus diagram is shown in Figure 3-5. The major loop lead compensator brings the remaining system poles into the left-half plane. This root-locus analysis shows that using minor loop compensation with appropriate com-

Figure 3-4: Minor Loop Root-locus Diagram

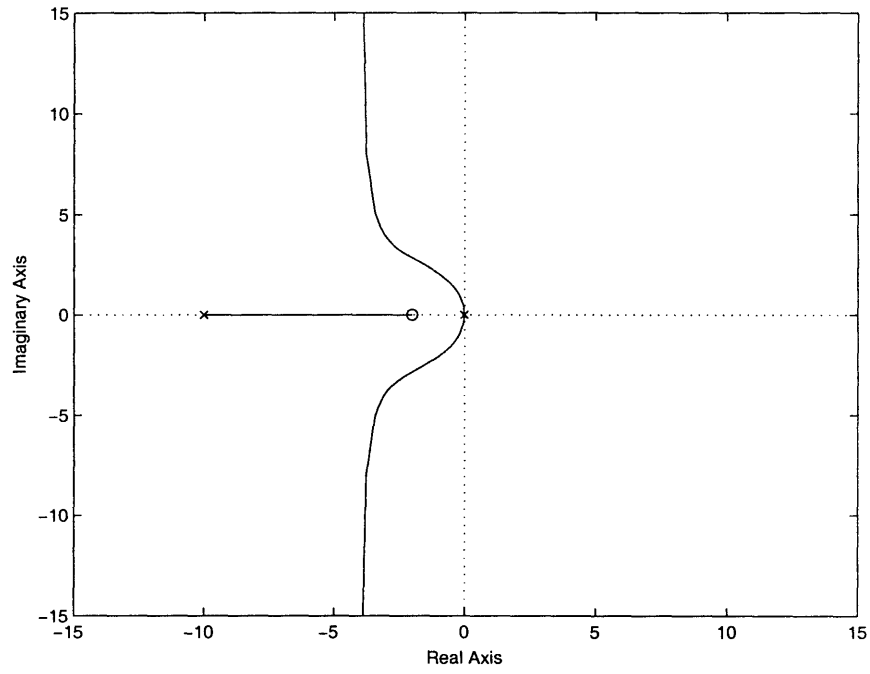
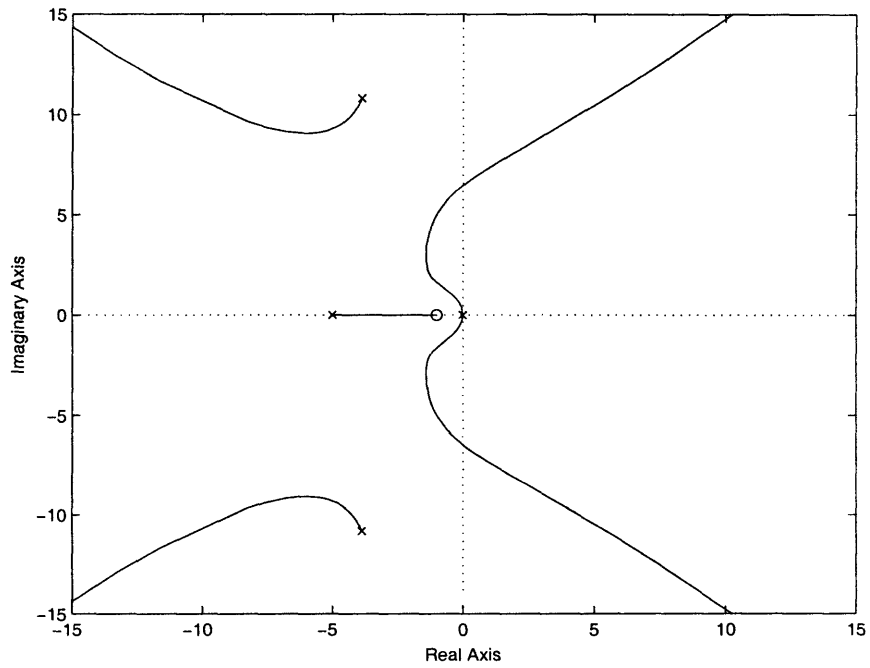


Figure 3-5: Complete System Root-locus Diagram



pensators and gain selection, the ball on beam system can be stabilized.

This same system is studied in 6.011 Introduction to Communication, Control, and Signal Processing. Control of the system in the 6.011 analysis is achieved through full state feedback where each of the four state variables are used to generate a control signal. Full state feedback can be more effective in stabilizing unstable systems, but is often impractical due to the number of sensors which are needed to sense all the state variables. The 6.011 analysis also covers feedback control using observers. This method only requires access to one output signal but would require more complex control circuitry. Further analysis of this problem can be found in the 6.011 lecture notes [4].





# Chapter 4

## Sensor Issues

One of the biggest challenges in designing the ball on beam system is the development of appropriate sensors for detecting the ball position and beam angle. Although the control design was the main focus of this project, a large part of the total effort was spent in developing appropriate sensors.

### 4.1 Beam Angle Sensing

Detection of the beam angle was the easier of the two sensor issues in the ball on beam system. Angular offsets are commonly measured with potentiometers which work well due to their inherent rotational motion.

The 6.302 Ball Balancer Lab used a different approach. An ADXL202 accelerometer was mounted on the beam shaft to measure the component of gravity parallel to the beam. The magnitude of the acceleration measured is proportional to the sine of the beam angle.

The beam angle sensor should be linear in its response around  $\theta = 0$  and should have little or no noise when the beam is stationary. Ease of mounting is also important in reducing the design and construction effort in building a ball balancer.

The accelerometer has the advantage of being very easy to mount as the alignment issues of connecting to both the shaft and the frame. The potentiometer angle sensor is much more common, has a more linear response, and has a much larger dynamic

range and lower noise.

Both of these sensors have been shown to work in the ball balance feedback loop. Future work should include more quantitative tests of advantages and disadvantages of either sensor.

## 4.2 Ball Position Sensing

Ball position detection proved to be one of the most difficult problems in the construction of the ball balance system. Accurate and speedy detection of the ball position is critical in controlling the position of the ball. Several different sensor designs have been tested with varied success.

Ball position sensors can be classified into two categories. The linear potentiometer sensors rely on the conductivity of the ball to short a voltage across two rails. Other sensors rely on reflection of other signals off the ball. The ball position sensor should produce an output linearly proportional to the ball position with a bandwidth of at least 100 Hz (to avoid sensor dynamics interfering with the control design) and very little noise.

### 4.2.1 Linear Potentiometer

A variety of linear potentiometer designs have been tested as ball position sensors. The main difficulty in constructing a sensor is producing a linear voltage distribution on one rail. The steel ball makes contact at some point along the powered rail and shorts that voltage to the wiper rail.

Figure 4-1 shows a functional diagram of the linear potentiometer ball position sensor. The ball rolls between two rails shown in the figure. The powered rail has a high resistance and is powered to create a linear voltage distribution along its length. The ball shorts the voltage at the position of the ball to the wiper rail, acting as the wiper of a potentiometer. The voltage on the wiper rail is proportional to the ball position.

The sensor in Pease's system was composed of a length of model train track which

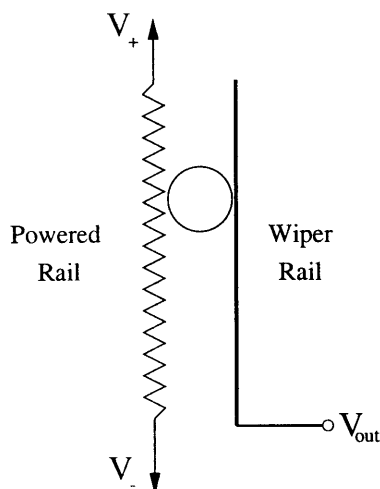


Figure 4-1: Linear Potentiometer Schematic

has a resistivity on the order of a few ohms per foot. A current source was connected through the track which produced a voltage distribution which followed  $V = IR$ . Due to the low track resistance, the currents needed to produce significant voltages were rather high. Pease used a 0.5 A current to produce a distribution of 200 mV across the 3 foot track.

Pease's sensor provided a linear response, good ball contact, low noise, and high bandwidth. Driving the large current, however, required a lot of power, which could result in heating of the rail or other undesired effects. This linear potentiometer design could be improved by replacing the model train track with a material with higher resistance.

Nickel-chromium (nichrome) wire has a high resistivity and has many of the same benefits of the model train track (low contact resistance, low noise, high bandwidth). Mounting the wire to allow the ball to roll and make good electrical contact with a wire is difficult. An improved version was built with a wire wrapped around an insulating rod. This configuration increased the resistance of the rail by increasing the length but suffered from some linearity issues as the windings were not always consistently spaced.

Other options for construction of a linear potentiometer have been explored. Rosales tested conductive plastic in his experimentation only to find that the pressure of

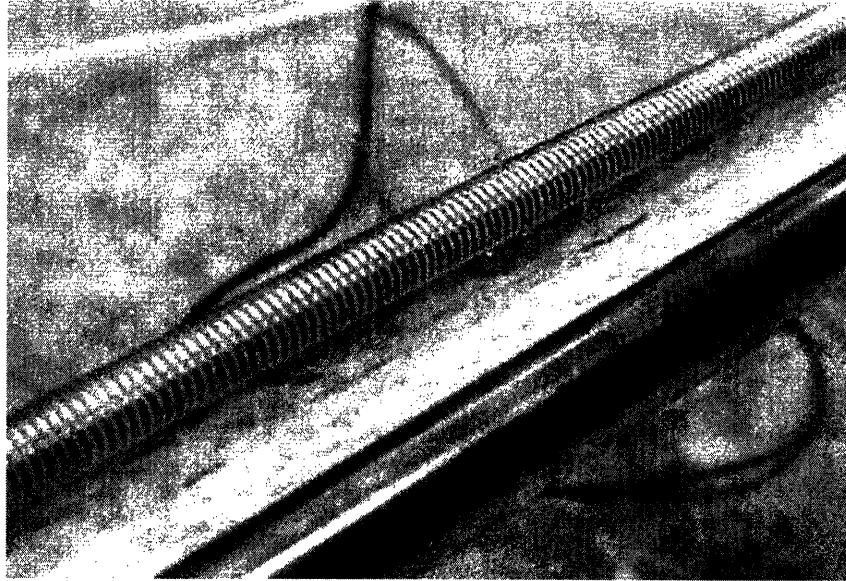


Figure 4-2: Linear Potentiometer Ball Position Sensor

a rolling ball was insufficient to make good contact with the plastic. Other materials include welding rod, or other types of train track. Further experimentation may lead to improved linear potentiometer ball sensors.

The current version of this nichrome wire based linear potentiometer design uses small gauge wire wrapped around a nylon threaded rod. The experimental version of this sensor is shown in Figure 4-2. The threaded rod insures consistent wire spacing but increases the space between windings, resulting in a more linear, but smaller resistance. The wiper rail is a brass tube which makes good electrical contact with the rolling ball.

### 4.2.2 Active Systems

Active detection of the ball involves a sensor which sends a signal then measures the returning signal which has been altered by the presence of the ball. These systems are more universal in their ability to detect the distance to any object and are used in a variety of other applications.

Infra-red range-finders are currently used in a variety of applications. These range-finders shine an infra-red light on the target and measure the amount of reflected

light on the sensor. Reflected light dissipates with a  $1/r^2$  dependence, leading to non-linear sensor response and effectively varied gain in different portions of the sensor. Additionally, the sensor has a limited operating range. Benefits of the IR range-finder include a fast response and reliability.

Ultrasonic range-finders use the speed of sound and accurate timing to measure the distance to an object. The range-finder produces a sound pulse which is reflected off the object. By measuring the time between sending and receiving the pulse, the distance to the object can be measured. Ultrasonic detectors have high bandwidth, high accuracy, and are generally reliable in detecting objects. The ultrasonic pulse has a large dispersion angle and thus will detect not only the ball, but also the frame and other objects near the beam. This effect and their prohibitive cost (\$30 ea. in bulk) reduces the feasibility of using ultrasonic range-finders as ball position sensors.

## 4.3 Experimental Systems

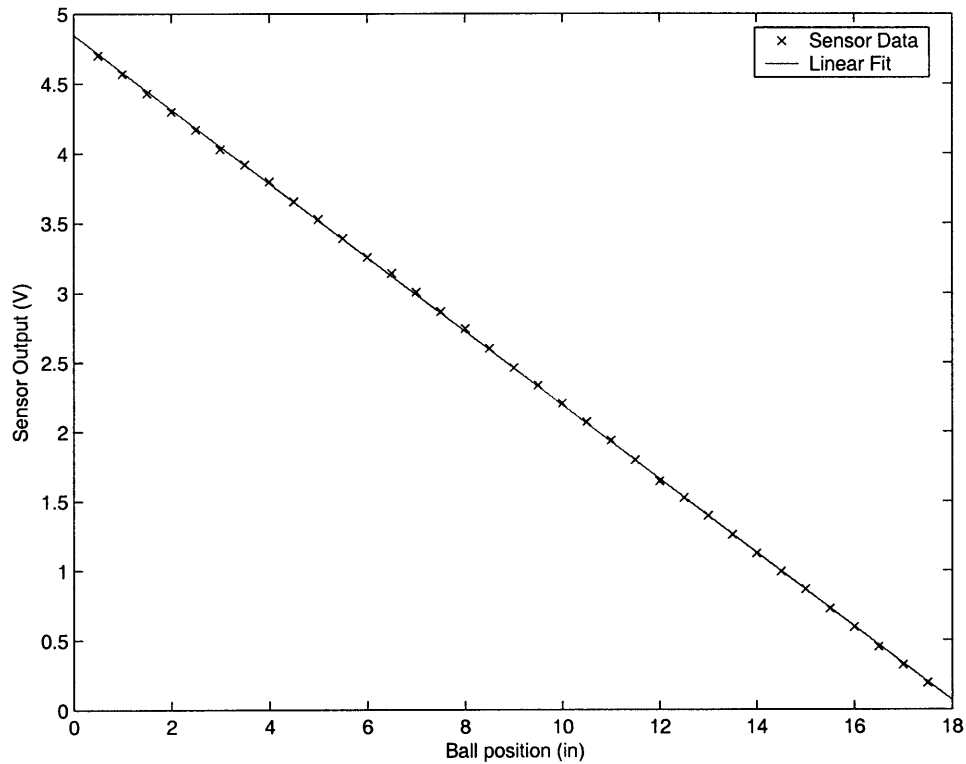
As part of this thesis project, two of the sensors described above were constructed and tested. One linear potentiometer design and one active sensor were tested. The data from these tests may be used to choose appropriate sensors and develop better sensors in the future.

### 4.3.1 Threaded Rod and Wire

The linear potentiometer design tested as a part of this project was constructed by wrapping 26 gauge nichrome wire around a #10-32 nylon threaded rod. A brass tube was used as a wiper rail. The threaded rod ensured even spacing between the wire coils and provided electrical insulation between coils. Small gauge wire was chosen to fit nearly completely within the threads of the nylon. The completed sensor allowed the ball to roll smoothly along the track.

Figure 4-3 shows the data taken at various ball positions on this sensor. The data clearly shows a linear output characteristic. The RMS error of the linear fit is 0.012 volts which corresponds to a linear error of 0.048 inches. This indicates a highly linear

Figure 4-3: Linear Potentiometer Data



response and a good curve fit.

The dynamic response of this sensor was measured by examining the sensor output while the ball was moving. When rolling, the ball would occasionally lose contact with the powered rail due to the spacing between the wire coils. This momentary loss of power produced high-frequency noise in the output of the sensor which needed to be filtered out in the control electronics.

### 4.3.2 IR Range-finder

Figure 4-4 shows the infrared range-finder as tested as a part of this experiment. The sensor was mounted to one end of a beam. Two brass tubes were mounted on the beam to create a track for the ball to roll in. This configuration was tested in a typical lab setting, near windows and under fluorescent lighting. Moving locations or changing lighting conditions had no significant impact on the sensor performance.

Figure 4-5 shows the response characteristic of the IR range-finder. The response

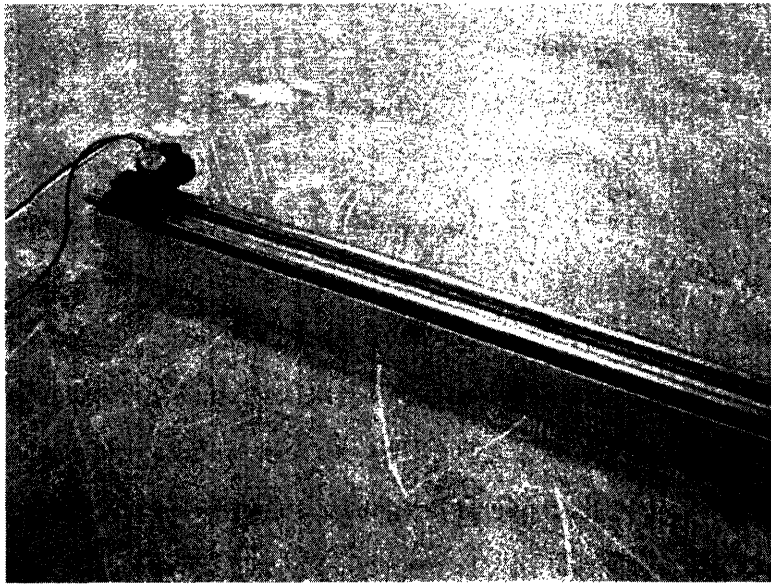
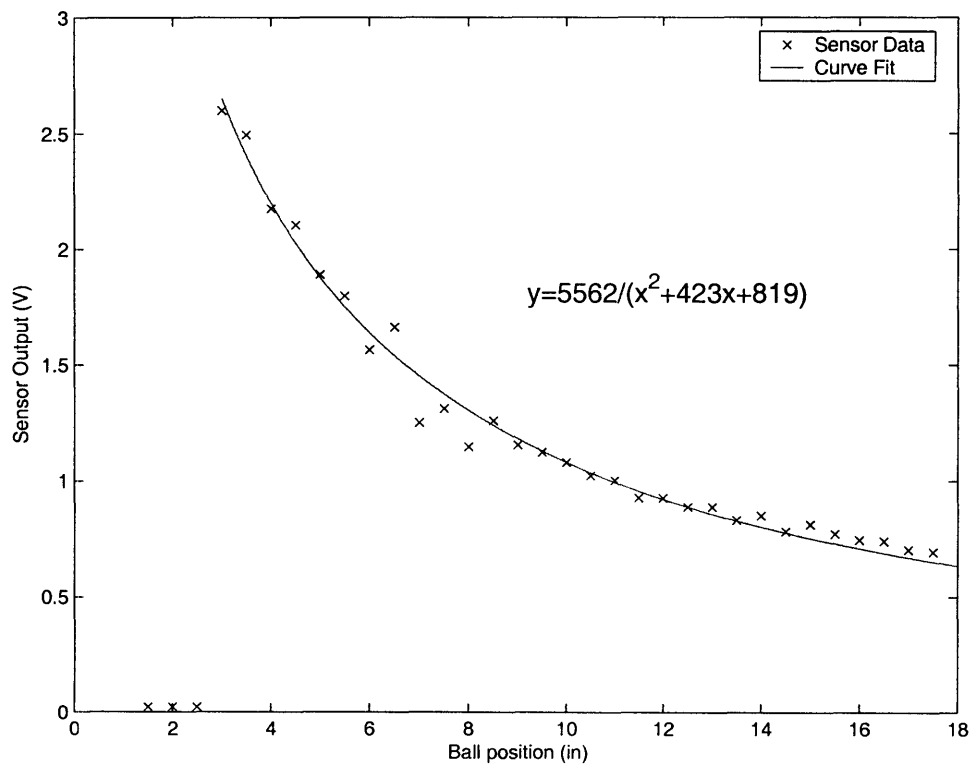


Figure 4-4: IR Range-finder

Figure 4-5: IR Range-finder Data



is clearly non-linear. The response is expected to have a  $1/r^2$  dependence as the light intensity dissipates over a distance.

The IR sensor also suffered from a limited range. The first three data points were too close to allow the signal to spread sufficiently to reach the detector while any points farther than 18 inches would be difficult to distinguish as the slope becomes very low.

The fitted curve is shown with the data in Figure 4-5. The fit has a RMS error of 0.068 volts which corresponds to different linear error at different ranges. At close range, the RMS linear error is 0.136 inches while at farther ranges, the RMS linear error can be as high as 0.816 inches. The fact that this curve is non-linear adds further complications. Since the slope of the response changes over position, the gain of the incremental response varies and the sensor gain changes. A system using this sensor would have to be designed to work over a range of gains and with varying uncertainty in ball position.



# Chapter 5

## Demonstration System

One of the main goals of this thesis is to provide a robust, working demonstration system for the Department of Electrical Engineering and Computer Science at MIT. This demonstration system may be used to demonstrate control systems in classes such as 2.003 Modeling Dynamics and Control, 6.003 Signals and Systems, 6.302 Feedback Systems, or 6.011 Introduction to Communication, Control, and Signal Processing. The demonstration system is based on previous work conducted for the 6.302 lab. Redesign of the system concentrated on improving robustness for ease of use.

### 5.1 Improvements

The demonstration system is heavily based on the solution to the 6.302 Ball Balancer final project described in Section 2.2. Modifications and improvements to both the actuation and sensing systems were completed to make the system more robust. The drive circuitry was redesigned for simplicity, modularity, and robustness.

The system built as a final project for 6.302 used a Winchester drive motor which severely limited the range in which the beam torque was linear with motor current. This motor was combined with sector drive gearing to drive the beam position. The angular limitations of the motor, combined with slipping of the sector drive gearing resulted in a very high sensitivity to relative motor and beam angles. This problem

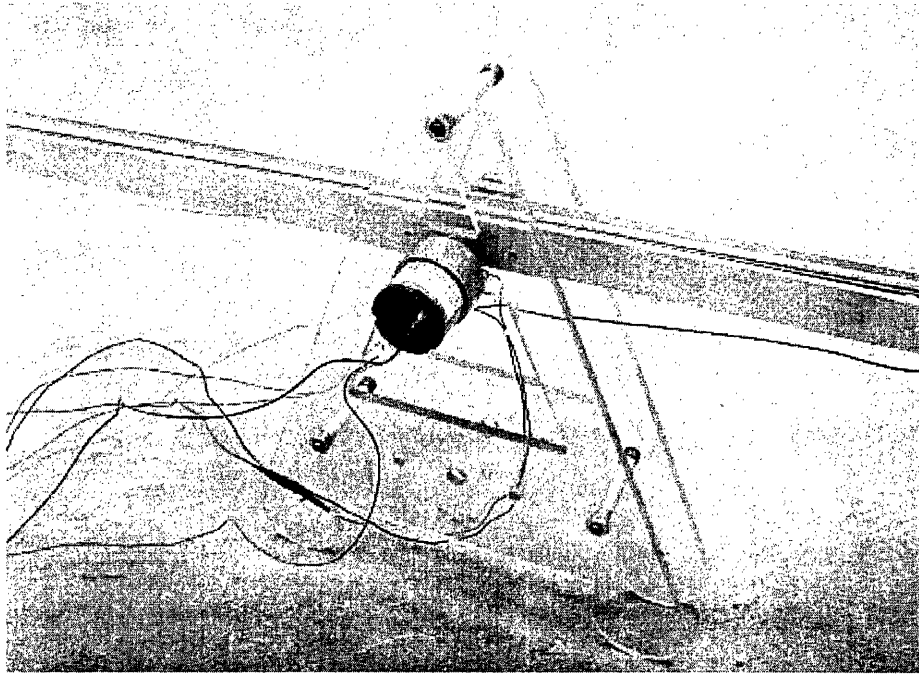


Figure 5-1: Completed Demonstration System

was avoided in the demonstration system by replacing both these components with a geared motor which directly drives the beam.

The circuitry for the 6.302 Ball Balancer lab was very sensitive to small adjustments in gain setting or voltages sources. The output stage was the only thing which survived nearly untouched in the redesign of the circuitry. The output stage remains a current controlled operational amplifier with a push-pull output stage to increase the current driving capability of the circuit. The redesigned circuitry in the demonstration system used a similar topology but was less sensitive to small adjustments.

## 5.2 Final Design and Construction

The final design of the demonstration system included a potentiometer and the geared motor directly coupled to the drive shaft. This eliminated the need for any power transfer or coupling to other sources and was accomplished with a simple shaft with set-screws on either end. The beam was pressed onto the shaft, completing the critical module of the design. The motor and potentiometer were then mounted to the frame.

### 5.2.1 Circuit Design

The drive circuitry for the demonstration system was designed to run off of +/- 15 volt power supplies with a maximum current draw of 1.5 amps. This power supply can be found in most labs and is sufficient to power the motor, which draws the most power in the system. The motor is driven with an op-amp and a large push-pull stage. This provides fast response and a large current sourcing capability. The sensors are powered by +/- 5 volt power supplies which produce usable signals with large dynamic ranges on the output of the sensors.

The control circuitry was largely based on the circuitry in the 6.302 ball balancer. Lead compensators and adders with adjustable gain are used to generate the appropriate control signals. A modular approach was used in designing the control circuitry to ease debugging, and tuning.

The circuit schematic shown in Figure 5-2 shows the control circuitry used in the demonstration system. The circuitry is divided into five stages. The first and last stages are included in the lab kit to allow students to concentrate on control design and implementation

The first block shown is a pre-amplifier for the ball position feedback. The input of this amplifier is filtered to eliminate noise from the rolling contact. The amplifier provides some gain on the signal and serves as a buffer between the sensor and control circuitry.

The second stage is an adder which sums the ball position input and the signal from the ball position sensor. The feedback loop of this stage also provides compensation for the ball position loop ( $G_{C1}$  in Figure 3-2).

The third stage adds the compensated ball position error signal and the beam angle signal to generate the beam angle error signal used in the minor loop. This stage also includes variable gain for the minor loop.

The fourth block in the circuit diagram is a lead compensator for the minor loop system ( $G_{C2}$  in Figure 3-2). This compensator is designed to produce optimal motor response with minimal phase lag.

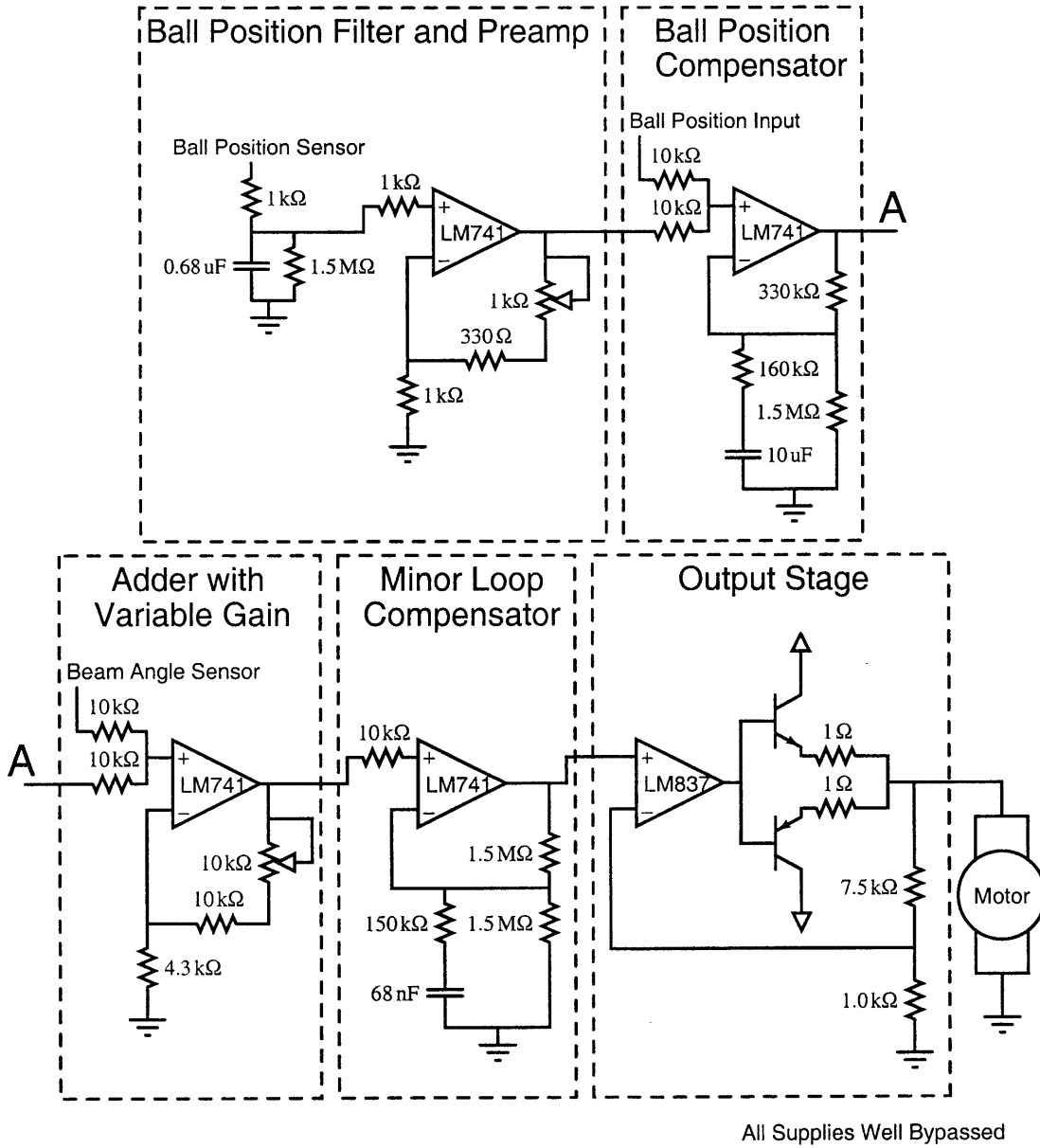


Figure 5-2: Demonstration System Circuit Schematic

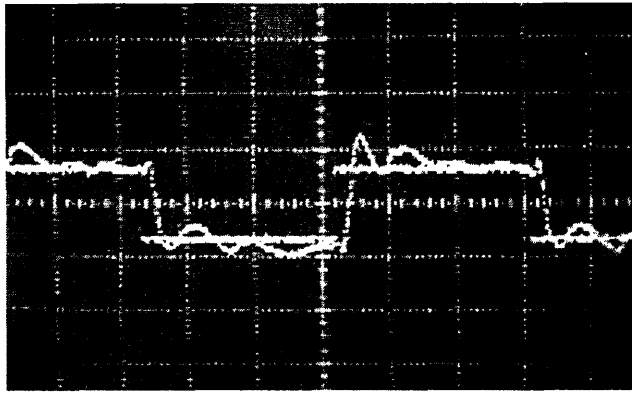


Figure 5-3: Demonstration System Ball Position Response

The final stage of the circuit diagram is the output stage. It provides both current and voltage gain to drive the large load of the motor. Feedback in this stage regulates the power into the motor and provides a linear motor response to an input signal.

### 5.3 Performance

After some fine-tuning of the loop gains, the demonstration system successfully stabilized the ball on the beam. The system was able to bring the ball to a complete stop and successfully held the ball on various points along the beam.

Figure 5-3 shows the ball position response to a square wave driving signal. The signals displayed are the driving signal from the function generator and the output of the ball position sensor. The x-axis divisions are 0.5 seconds per division and the y-axis divisions are 0.5 volts per division. The response is nearly second order despite unmodeled non-linearities.

### 5.4 Lecture Demonstration

This system was successfully demonstrated in a 6.011 Introduction to Communication, Control and Signal Processing lecture on April 5, 2004. The demonstration system successfully balanced a 3/4" ball for the full hour. With the ball position input driven with a square wave, the system was able to track the input.



# Chapter 6

## Lab Kit Design

One of the goals of this project is to produce a lab kit version of the ball balancer which will be readily accessible to students studying 6.302 or other introductory feedback control classes. This lab kit has been designed and built, based heavily upon the work presented by Rosales.

### 6.1 Design Requirements and Approach

The demonstration system described above was built with emphasis on robustness for use as a lecture demonstration. Thus, it should withstand the hazards of transportation and possible jarring over time. The lab kit design concentrates on creating an easy-to-construct and inexpensive system which can be given out as a lab kit to undergraduate students.

One of the most important factors in keeping the cost of the system low is to design around an inexpensive motor. The Cannon CKT26-T5 motor is readily available to the 6.302 staff and relatively inexpensive. This motor has the added benefit of being used throughout much of the 6.302 lab work and thus is familiar and well characterized by the students.

To make the control problem easily accessible to the students both the sensors and motor driving circuitry must be readily available. The motor driver and the sensors were designed to be simple to build and easy to use.

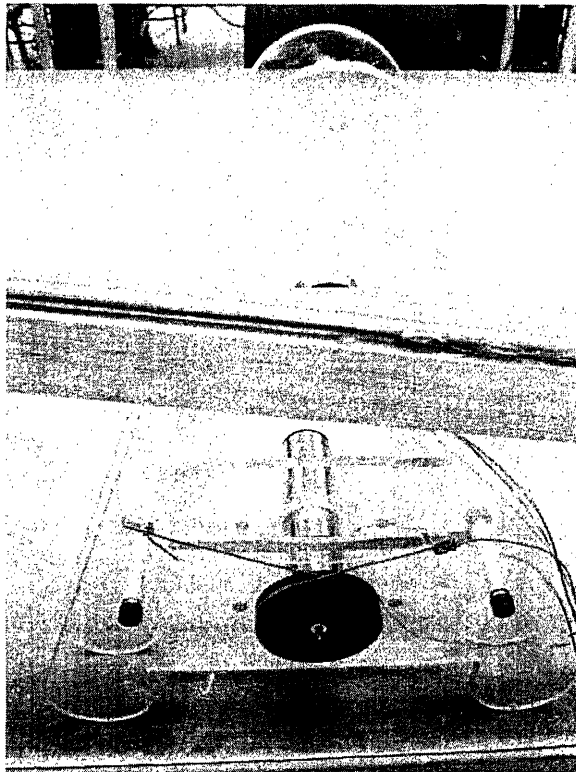


Figure 6-1: Assembled Lab Kit

## 6.2 Final Design

The final lab kit system used the lab kit designed by Rosales with modifications to suit the needs of a 6.302 lab kit. The motor mounting was modified to adapt to the available motors and appropriate sensors were designed to fit onto the existing kit. The circuitry provided with the kit was designed to isolate the control problem by making feedback and control inputs available. The final design is readily accessible to students with only an electrical engineering background.

Figure 6.2 shows a completed version of the final system. The linear potentiometer design is mounted to the beam. A CKT26-T5 motor is mounted within the frame and connected to the sector drive system to gear down the motor actuation. A rotary potentiometer serves as both the angle sensor and the beam pivot point.



## 6.2.1 Physical Modifications

The kit provided by Rosales' work is specifically designed for a particular motor and does not include any feedback sensors. For use in 6.302 it was appropriate to include sensors and to replace the motor with the motor used in other experiments in the class.

There were two key issues in mounting the motor. The 6.302 motor does not include sufficient screw mounting options to mount the face to the plastic frame. Instead, the entire motor was mounted within the frame by drilling a 1 inch diameter hole and press-fitting the motor in the hole. The pulley of the lab kit design needed to be mounted to the motor shaft. The original motor had a larger shaft and thus a fitting was designed to adapt the new motor to the pulley.

The control design described above requires feeding back both the beam angle and ball position. The lab kit incorporates potentiometers for feeding back both signals.

Angle sensing is accomplished by coupling a potentiometer to the shaft. The potentiometer is press fit into a pair of mounting holes in the frame. The shaft of the potentiometer is press fit into the beam, providing mechanical coupling and a pivot point for the beam.

The ball position sensor included in the lab kit is the threaded rod based linear potentiometer described in Section 4.3.1. The wire-wrapped threaded rod and wiper rail can be glued to the beam to provide a rolling surface for the ball.

## 6.2.2 Circuitry Provided

The purpose of the lab kit is to provide an interesting and challenging control systems problem to undergraduate students. To allow the students to concentrate on designing appropriate controllers, some circuit schematics will be provided. The sensor input filter and buffer and the motor output stage can be provided to the students to lessen the analog design for student who do not have strong backgrounds in circuit design.

The sensors are powered by +/- 5 volt reference supplies to produce large scale, zero-centered signals. These power supplies are easily generated by three terminal

five volt regulators. The LM7805 and LM7905 can be used as power supplies for the sensors.

## **6.3 Results**

Appendix A lists the full modifications to the existing lab kit system. These modifications are designed to make the mechanical and electrical design portions of this laboratory project easier so students may concentrate on the control design.

This kit and appropriate instructions will be provided to students in 6.302 in the Fall of 2004. Future improvements on the lab kit system may involve further integration of the electronics to reduce the number of parts.

# Appendix A

## Lab Kit Materials

The lab kit designed by Rosales as a part of his undergraduate thesis [2] includes many of the basic parts required to build a ball on beam balancer. Table A.1 lists the kit contents. Rosales estimates the total kit cost at \$20 plus manufacturing costs.

The redesigned lab kit is based on the materials included in the original kit with the addition of the Cannon motor and appropriate sensors. Table A.2 lists the modified and additional items in the new lab kit.

Table A.1: Bill of Materials - Existing Lab Kit

<b>Item</b>	<b>Quantity</b>	<b>Description</b>
"A" Frame	2	Custom Polycarbonate Frame
Sector Transmission	1	Custom Polycarbonate Sector Drive
Motor	1	Winchester Drive DC Motor
Motor Pulley	1	Delrin Pulley with Set Screw
Beam	1	2 Foot Basswood Beam
Ball	1	3/4 in Stainless Steel Ball
Frame Bracings	3	1/4" x 2 - 1/4" Bolts and 3/8" Al Tube
Beam Shaft	1	1/4" Al Tube
Shaft Bearings	2	1/4" Nylon Bearing

Table A.2: Bill of Materials - New Lab Kit

<b>Item</b>	<b>Quantity</b>	<b>Description</b>
"A" Frame	2	Modified Motor and Beam Pivot Mount
Motor	1	Cannon CKT26-T5 DC Motor
Motor Pulley	1	Modified Motor Attachment
Beam	1	1.5 Foot Basswood Beam
Potentiometer	1	1/4" Shaft Potentiometer
Linear Pot Rod	1	1.5 Foot #10-32 Nylon Threaded Rod
Nichrome Wire	125	Ft. 26 GA

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