Use of Axiomatic Design Principles to Develop Vehicle Suspension Controls for Variable Stiffness and Ride Height

by

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ABSTRACT

Axiomatic Design principles are used to design a vehicle suspension system. The use of Axiomatic Design helps to guide the design of a decoupled system. The Design Matrix (DM) illustrates the independence among the Functional Requirements (FRs) and the Design Parameters (DPs). The ultimate goal is the design of a fully independent suspension system in which the FRs stiffness, ride height, and damping can be varied as needed. To achieve the three FRs, three DPs are chosen – the volume of an air spring for stiffness, the volume of fluid in a fluid chamber for ride height, and orifice control for damping.

This thesis investigates two DPs in depth, the air spring and fluid chamber. The nonlinearity of the air spring is studied and its effect on the system as a whole is simulated in Simulink. Two control systems are proposed in which stiffness and ride height are kept constant. The desired values for stiffness and ride height are predetermined by the user or by an optimization algorithm. The physical design for the control systems is also proposed in this thesis. The design for the air spring system uses an electropneumatic design, and the design for the fluid chamber system uses an electrohydraulic servovalve design.

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1.0 Introduction

The suspension system of a car is one of the car’s most integral parts. It prevents the car body from shaking and vibrating from road noise, helps to sustain wheel contact with the ground, interacts with the steering system to provide vehicle control, and also aids in maintaining a comfortable car ride. A basic suspension system includes springs and dampers. There are three parameters that characterize a suspension system – the stiffness, the damping, and the ride height. The levels and values of these parameters for a specific car are determined by the type of car and the type of user.

There are tradeoffs associated with each parameter. For example, a car with a stiff suspension handles well but does not give a comfortable ride. This would be suitable for a road enthusiast or race car driver, and a softer suspension would be more suitable for a family. With ride height, a low one gives the driver stability at high speeds, but a high one allows the driver to see better and also allows the car body to clear the ground in rough and bumpy road conditions. The high ride height, however, causes the center of gravity of the car to be higher, and therefore more unstable.

Ideally, the three parameters for any given car would be able to be set for specific drivers and road conditions. In order to achieve this, a fully adjustable suspension system must be constructed. Subsequent chapters of this thesis will review active suspension systems and axiomatic design, and will describe the design for a proposed suspension system that allows for the independent variation of the three parameters.
2.0 Active Vehicle Suspension Systems

Suspension systems can be categorized as the following – passive, self-leveling, semi-active, and full-active. In recent years, active components have been integrated into suspension systems in order to improve comfort of ride, handling of vehicle, etc. Passive suspension systems are comprised of conventional springs and shock absorbers with properties that do not depend on time. Self-leveling suspensions can adjust for variations in load. In semi-active suspensions, the properties of the spring and damping elements can be changed by external control. Semi-active suspensions can be further broken down into slow-active, low-bandwidth, and high-bandwidth systems. Slow-active systems, also called adaptive systems, allow for the spring and damping properties to be switched among a number of distinct levels. The changes in these properties are due to road and driving conditions. In low-bandwidth systems, the changes in spring and damper properties are made in response to the low-frequencies of the sprung mass. In high-bandwidth systems, changes are made due to both the high-frequency axle oscillations and low-frequency sprung mass oscillations. Full-active suspensions use actuators in place of springs and dampers in order to produce desired forces in the suspension system. [1]

Active suspension systems are a research topic of great interest as evidenced by comprehensive surveys compiled in [2, 3]. The value of these suspension systems has been recognized by [4] in which the authors have investigated varying the stiffness and damping coefficient. They have also recognized the disadvantages of fully active suspensions – “complexity, power requirements, and cost.” However, according to [4], the use of passive systems, in which measured external signals are used to actively change the parameters of the system, “can come close to optimal performance.”

Variable damping is already a recognized technology in high-end automobiles today [5, 6, 7, 8]. Ride height adjusters are also a reality in recent years’ luxury cars in additional to variable damping [1, 7, 8]. These adjusters are responsive to the car’s velocity, the roughness of the terrain, and the car’s load. However, with these systems, there is no independent control of the stiffness. The stiffness of these systems is the value that results after setting the damping coefficient and ride height. In [9], the need for variable stiffness and the need for variable ride
height are developed. Deo and Suh have demonstrated that softer suspensions provide more 
comfort and that stiffer suspensions provide better vehicle handling.

Previous work has been done in [9] on a fully customizable suspension system with independent 
control of stiffness. Deo and Suh have proposed a design through the use of axiomatic design for 
such a system. An alternative design for an adaptive suspension system with independent 
control of stiffness, damping, and ride height responsive to road conditions and user input is 
proposed in this thesis. The alternative design uses an air spring to provide variable stiffness and 
a fluid chamber to generate an appropriate ride height. Novel component designs and control 
systems designs for these two elements are detailed in this thesis.
3.0 Axiomatic Design

Axiomatic Design is a method of design in which design is mapped out into four domains – customer, function, design, and process. The characteristic vectors of these domains are Customer Attributes \{\text{CAs}\}, Functional Requirements \{\text{FRs}\}, Design Parameters \{\text{DPs}\}, and Process Variables \{\text{PVs}\}. Product design requires mapping from the function to the physical domains, and process design requires mapping from the physical to the process domains.

FRs are defined as the least number of independent requirements needed to fully describe the overall intention of the product. The DPs are the physical attributes of the design that satisfy the FRs. Two axioms guide the design of the DPs – the Independence Axiom and the Information Axiom. The Independence Axiom states that independence should be maintained among the FRs. The Information Axiom states that information content of the design should be minimized.

There is a one-to-one correspondence between the FRs and DPs, and a Design Matrix, DM, is used to represent the effect of DPs on FRs:

\[
\begin{bmatrix}
FR_1 \\
FR_2
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\text{DP}_1 \\
\text{DP}_2
\end{bmatrix},
\]

where \(A_{11}\) shows the effect of \(\text{DP}_1\) on \(\text{FR}_1\) and \(A_{21}\) shows the effect of \(\text{DP}_1\) on \(\text{FR}_2\), etc. The Independence Axiom is fulfilled if the DM is diagonal or triangular. If the DM is diagonal, the design is uncoupled, and each FR can be fulfilled by modifying a single DP. If the DM is triangular, the design is decoupled, and the FRs can only be fulfilled if the DPs are set in the proper sequence. If in Equation 1, there were no effect of \(\text{DP}_2\) on \(\text{FR}_1\) (i.e. \(A_{12} = 0\)), then the DPs would have to be determined in the order \(\text{DP}_1\) followed by \(\text{DP}_2\). If the DM is completely filled out, the design is coupled, and the fulfillment of FRs is more difficult due to the couplings. Varying one DP would affect more than one FR.

According to the Information Axiom, DPs with the least information (i.e. the least complexity) are better solutions. Information is defined as:
\[ I = \log\left(\frac{1}{p}\right), \quad (2) \]

where \( I \) is information, and \( p \) is the probability of fulfilling the FR. For an uncoupled design, the total information content equals the sum of the information content for each DP. [10]
4.0 Proposed Design

In order to fully understand existing suspension systems and the improvements that could be made, the kinematics of linear systems were studied. It is very important that suspension kinematics not change with the modifications.

4.1 Linear System

The most common design for front-wheel suspension systems is the short long arm configuration shown in Figure 1. The schematic in Figure 1 can be modeled as the quarter-car, single degree of freedom system shown in Figure 2.

![Figure 1. Kinematic schematic of existing short long arm suspension system.](image)

![Figure 2. Quater-car, single degree of freedom model.](image)

In Figure 2, $M_s$ is the sprung mass, $K$ is the stiffness of the linear spring, $C$ is the damping coefficient, $X_s$ is the suspension travel, $X_r$ is the road disturbance, and $F$ is the force on the
suspension system due to the load in the car, inertial forces, forces generated during driving, etc. The equation of motion for this system is

\[ M_s \ddot{x}_s + C(\dot{x}_s - \dot{x}_c) + K(x_s - x_c) - F = 0. \]  \hspace{1cm} (3)

4.2 Development of Design

The same basic configuration can be used for an adaptive suspension system so that suspension kinematics are not changed. Since the goal is to design a fully customizable suspension system, the functional requirements are variable stiffness, variable ride height, and variable damping. In the redesigned system, variable stiffness can be accomplished with an air spring. If we model the air in the air spring as ideal, and the process as adiabatic, we know that

\[ PV^n = P_0V_0^\gamma = \text{constant}, \]  \hspace{1cm} (4)

where \( P \) is the pressure in the air spring, \( V \) is the volume of the air spring, \( P_0 \) is the initial pressure, \( V_0 \) is the initial volume, and \( \gamma \) is the specific heat ratio which is equal to \( c_p/c_v \) where \( c_p \) is the specific heat capacity of a gas at constant pressure and \( c_v \) is the specific heat capacity of a gas at constant volume. For air, this ratio is equal to \(~1.4\). From Equation 4, assuming air, we find that \( P = NV^n \) where \( N \) is a constant, and in Figure 3, the pressure vs. volume curve (\( N = 1 \)) is plotted to give us a general idea of the nonlinearity of the air spring.

![Figure 3. Nonlinear pressure vs. volume curve for the air spring.](image)
For a regular spring, we know that

\[ dF = -kdx, \]  \hspace{1cm} (5)

where \( F \) is the force on the spring, \( k \) is the stiffness, and \( x \) is the deflection of the spring. From basic principles and geometric relationships,

\[ dF = AdP \]  \hspace{1cm} (6)

and

\[ dx = \frac{dV}{A}, \]  \hspace{1cm} (7)

where \( A \) is the cross sectional area of the air spring. After differentiating Equation 4, combining Equations 5-7, and rearranging terms, we find that

\[ k = \frac{\gamma P_0 V_0^\gamma A^2}{V^{\gamma+1}}. \]  \hspace{1cm} (8)

We can simplify Equation 8 by substituting in Equation 3 so that

\[ k = \frac{\gamma PA^2}{V}. \]  \hspace{1cm} (9)

By controlling \( V \), the volume air spring, we can change the stiffness \( k \) of the air spring.

Variable ride height can be obtained with a fluid chamber that extends and contracts based on how much fluid is inside the chamber. A simple geometric relationship is used to determine the height of the fluid chamber:
where $U$ is the height of the fluid, $V_f$ is the volume of fluid inside the chamber, and $A$ is the cross sectional area of the chamber. Again, we control the volume in order to change ride height.

The existing technology of orifice control can be used to obtain variable damping. The system is configured as shown in Figure 4. The absence of the damper in the schematic is for the sake of simplicity. The damper would connect the lower arm to the vehicle frame so that it would be in parallel with the spring.

![Figure 4. Kinematic representation of redesigned suspension system.](image)

The design matrix is as follows:

$$
\begin{cases}
\text{Stiffness} & = \begin{bmatrix} X & O & O \\ X & X & O \end{bmatrix} \text{Volume of Air} \\
\text{Ride Height} & = \begin{bmatrix} X & X & O \\ O & O & X \end{bmatrix} \text{Volume of Fluid} \\
\text{Damping} & = \begin{bmatrix} X & O & O \\ O & O & X \end{bmatrix} \text{Orifice Control}
\end{cases}
$$

(11)

Because the DP air spring changes both FRs stiffness and ride height (i.e. the design is decoupled), the setting of the air spring must be determined before the setting of the fluid
chamber can be established. Also, for damping, since the FR is not affected by other DPs and orifice control does not affect other FRs, we can forgo the analysis of this FR/DP pair for reason of simplicity. And instead, rely on the technology that is well established.

4.3 Behavior of System

The fluid inside the fluid chamber is modeled as incompressible; thus, the fluid chamber acts as a rigid body, and we can model the system as shown in Figure 5 where $K$, the stiffness of the nonlinear air spring, is what was derived in Equation 8.

$$\text{Figure 5. Model of redesigned suspension system.}$$

The equation of motion of the quarter-car, single degree of freedom model in Figure 5 can still be modeled with the equation in Equation 3. However, we must remember that here, $K$ is nonlinearly dependent on the difference between the suspension travel and road disturbance. Substituting Equation 8 into Equation 3, we obtain

$$M_s \ddot{x}_s + C(x_s - \dot{x}_r) + \frac{\gamma P_0 V_0^y}{A^{y-1}} (x_s - x_r)^y - F = 0. \quad (12)$$

We can model Equation 12 in Matlab’s Simulink environment as shown in Figure 6. Note that in this figure, $K$ is not the stiffness of the spring, but the constant in Equation 12 that is multiplied with $(x_s-x_r)^y$ and $B$ is used for the damping coefficient in place of $C$. For the Simulink simulation
that was performed in this thesis, to which Figure 6 belongs, the road input $x_r$ is modeled as a sine function with a frequency of 5 rad/s and amplitude of 1. All constants are set equal to 1, and $\gamma$ is set equal to 1.4 to represent air. A constant value of 100 is fed into the summing junction along with $x_s$ and $x_r$ in order to provide the function block $[A - Y]$ with a positive input value. A simulation was run from 0.1 seconds to 100 seconds with two inputs – the constant force $F$ and the sinusoidal road input $x_r$ shown in Figure 7. The result of the simulation is shown in Figure 8. It is interesting to note that the resulting suspension travel without any control is a sinusoid oscillating around a line with positive slope. The frequency of the oscillations is identical to the frequency of the sinusoidal road input.

![Block diagram of governing equation of motion used in Simulink simulation.](image)

**Figure 6.** Block diagram of governing equation of motion used in Simulink simulation.
Figure 7. Graph of sinusoidal road input $x_r$ from $t_i = 0.1s$ to $t_f = 100s$ with frequency 5 rad/s and amplitude 1.

Figure 8. Graph of the output, suspension travel, $x_s$ from $t_i = 0.1s$ to $t_f = 100s$. 
5.0 Design of Control Systems

In order to fulfill the functional requirements of our system, we have to be able to control the two design parameters – the volume of air in the air spring, and the volume of fluid in the fluid chamber. The ultimate goal is to keep both ride height and stiffness constant at their predetermined levels. There are several options available for the user when inputting his preferences. One option is to have predetermined modes like luxury, sport, etc. for the user to select. Each mode would have a specific stiffness, ride height, and damping associated with it. Another option is to have the parameters determined by an algorithm, based on road conditions, velocity of the car, the user’s driving behavior, etc.

5.1 Stiffness Control

The goal of the controller is to keep the stiffness in the spring at a constant level. Equation 9 is used to calculate the required volume of the air spring for a specified stiffness. Since the cross-sectional area of the air spring stays constant, we can use the extension of the spring to find the volume (volume = cross-sectional area * extension). As a result, Equation 8 can be rewritten as:

\[ k = \frac{\gamma P_0 V_0 r}{A r^{-1} x^r} \]  \hspace{1cm} (13)

From geometric relationships, the extension of the spring can be expressed as a function of the angles shown in Figure 9:

\[ x = x_0 + L_3 \sin \theta_2 - L_2 \sin \theta_1, \] \hspace{1cm} (14)

where \(x\) is the desired spring extension, \(x_0\) is the “equilibrium” spring height when \(\theta_1\) and \(\theta_2\) are equal to 0. Because the user is inputting a specific stiffness \(k\), calculations can be done internally to find the desired extension \(x\) of the spring unique to the \(k\).
The controller can then control the extension $x$ of the spring. A block diagram of the control system is shown in Figure 10. Two encoders on the upper and lower arms of the suspension give a measurement of $x_{act}$, the actual value of the spring extension. This value is compared with the desired value of the spring extension $x_{des}$, and the error is fed into the PID controller of the motor. Unity feedback is used.

\[ x_{act} = \frac{1}{K_1 s^2 + K_2 s} \]

**Figure 10.** Block diagram for controlling stiffness.

5.2 Ride Height Control

As with stiffness, ride height is to be kept constant. The required volume of fluid for a height $U$ of the fluid chamber is given by Equation 10. The ride height of the car is equal to the sum of the extension of the spring and the height of the fluid chamber, and it is also equal to the
difference between the suspension travel and road noise. It can be calculated if the angle \( \theta_i \) in Figure 9 is known:

\[
\text{ride height} = u_0 + x_0 + L_2 \sin \theta_i = x_s - x_r, \tag{15}
\]

where \( u_0 \) is the “equilibrium” fluid chamber height defined as the height of the fluid chamber when \( \theta_2 \) is equal to 0.

Figure 11 shows the block diagram for the control system for ride height. A low pass filter is necessary since we would like to filter out the high frequency road noise. The actual ride height \( (x_s - x_r)_{\text{act}} \) is measured by the encoder on the lower arm of the suspension system, and the error is fed through a low pass filter and a controller in order to obtain the desired fluid chamber height \( U_{\text{des}} \). The controller of the electrohydraulic servovalve, which is controlling the height of the fluid chamber, sets \( U_{\text{des}} \) and feeds the result into the plant.

**Figure 11.** Block diagram for controlling ride height.
6.0 Physical Design of System

Actual designs for the air spring and fluid chamber systems are proposed in the following sections. An electropneumatic system was devised for the air spring, and an electrohydraulic system was developed for the fluid chamber.

6.1 Air Spring System

The component design for the air spring system is shown in Figure 12.

![Figure 12. Schematic of design for air spring system.](image)

Atmospheric air is used for reasons of simplicity since a compressed air tank would involve more components. However, because we are using atmospheric air, a filter must be implemented before the compressor to filter out dust particles in the air that can damage the compressor. DC brushless motors are common motors used to drive compressors and pumps [11]. We can use this type of motor to power the compressor. It takes its signal from the control system described in Section 5.1. An encoder on the lower arm of the suspension measures the angle of the arm relative to its “equilibrium” position at 0 degrees. Another encoder on the upper arm of the spring measures the angle of the upper arm. However, the upper arm can only be moved if fluid is taken out or added to the fluid chamber, therefore the angle of the upper arm is a known value. According to our DM, we must first determine the volume of the air spring before the height of the fluid chamber can change. Changing the volume of the air spring will only affect the angle in the lower arm. In this way, the volume of the spring can be measured.
When air is needed, a signal is sent to the motor to begin powering the compressor. The filter located after the compressor is included in part to allow the air coming out of the compressor to cool down a little, and in part to filter out water and oil particles that may have gotten into the air during compression. Valves 1, 2, and 3 are one-way directional valves so that air cannot flow backwards in the system. Valve 1 is a check valve with two positions – open and closed. It is operated by a solenoid that will take its signal from the motor. Once the motor is activated, the solenoid will trigger. When the motor stops (the encoder on the lower control arm sends a signal to the motor that the air spring no longer needs more air), the solenoid is de-triggered, and the valve is closed. Valves 2 and 3 are proportionally controlled by motors that receive a signal from the encoder on the lower arm of the suspension. If the desired volume change of the air spring is relatively large, the valves are opened more to allow for more air flow. If the desired volume change is relatively small, then the valves are only opened a small amount. Valve 2 leads to the inlet of the air spring, allowing volume into the air spring, and Valve 3 allows air out of the spring.

6.2 Fluid Chamber System

A schematic of the fluid chamber system is shown in Figure 13. This type of setup is characteristic of an electrohydraulic servovalve. These servovalves can be sorted into two broad categories – single-stage and two-stage. In a single-stage servovalve, the motor is directly connected to a spool valve, and it also positions the valve. The type of motor generally used is a permanent magnet torque motor [12]. In this design, a single-stage servovalve is used. The appropriate analysis and transfer functions of these servovalves can be found in [12].
The fluid chamber is expandable and contractible in the vertical direction. An “equilibrium” value is set for the volume of fluid inside the fluid chamber. Any added volume is added to this value, and any removed volume is subtracted from this value. The height of the fluid chamber is dependent on the angle of the lower arm shown in Figure 9. The relationship is expressed by Equation 15.

When the control system described in Section 5.2 signals that more volume of fluid is needed, the motor powering the pump is activated. Fluid is drawn from the reservoir into the pump and subsequently through the spool valve into the fluid chamber. The amount of the change in volume to the desired volume dictates how much the motor powering the spool valve will allow the pathway to open. When less volume is needed in the fluid chamber, the motor opens the other side of the spool valve to allow the fluid back into the oil reservoir.
7.0 Conclusion

This thesis examines a proposed design for a fully customizable vehicle suspension system with independent control of stiffness, ride height, and damping. More specifically, it discusses the control systems and their component designs for the control of stiffness and ride height. To obtain variable stiffness, an air spring is used in place of a conventional spring. By changing the volume of the air spring, the stiffness parameter of the air spring can be varied. A fluid chamber is used to compensate for the volume changes of the air spring and also to achieve a variable ride height. The advantages of a fully customizable suspension system are illustrated by the Design Matrix which is obtained through the use of Axiomatic Design principles. The DM shows us that the proposed design in this thesis is a decoupled design.

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