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IPA-based Tuning of Queue Admission Control under Imperfect Information [1]

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ABSTRACT

The Infinitesimal Perturbation Analysis (IPA) is applied to the queue admission control problem for a system modeled by tandem queues. The control takes the form of holding a customer in a controller buffer with a low holding cost until admitting that customer to the main queueing system with a high holding cost. An open-loop dynamic admission control policy simple to implement is proposed for a control of the queueing system. The basic idea of this policy is to ensure minimal interadmission time, θ, in order to prevent congestion in the main queueing system; θ is left as a design parameter. A simple gradient estimate of the total steady state holding cost with respect to θ is derived using Infinitesimal Perturbation Analysis techniques. A stochastic gradient-like algorithm based on this estimate is studied for optimizing θ, and convergence is proven under certain conditions.

Key words: perturbation analysis, stochastic gradient–like algorithm, open–loop admission control, minimal interadmission time

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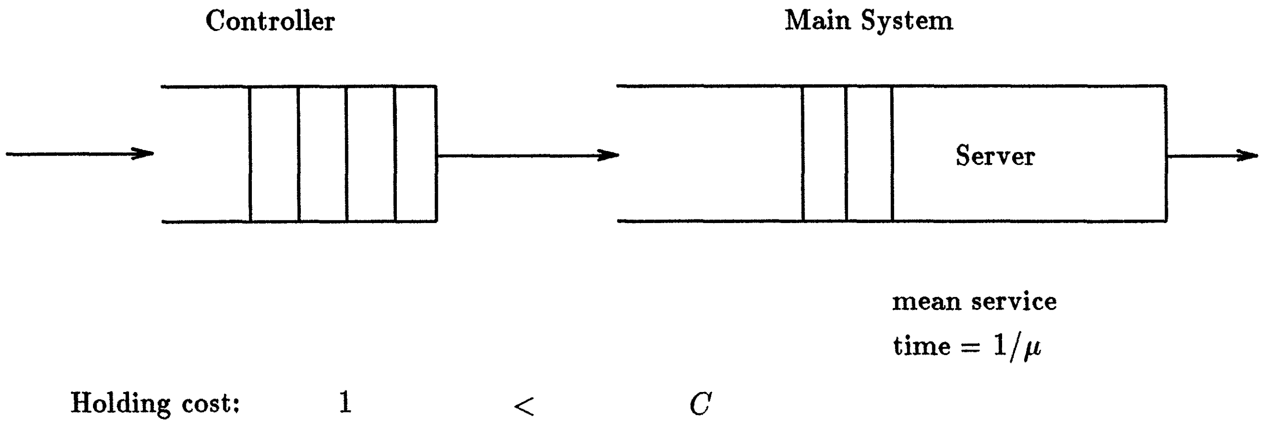

Figure 1: **System model**

1 Introduction

Perturbation analysis has been recently studied actively due to its efficiency in estimating performance sensitivity with respect to a parameter in discrete event simulation [1] – [15]. The control of a queueing system under imperfect information has recently received significant attention [16]–[20]. This paper illustrates the use of the infinitesimal perturbation analysis for the control of a queueing system with imperfect information.

The system model studied in this paper is depicted in Figure 1; it consists of a controller and a main system. Customers arrive and get queued initially at the controller buffer and are eventually admitted to the main system. The holding cost of the main system is more expensive than that of the controller buffer. The controller decides the time that these customers are admitted into the main system without knowledge of the state of the main system's queue. The primary motivation of this model is the flow control of high-speed communication networks [16] [18] [19].

In this paper, an admission control policy that is very simple to implement is proposed. The essential idea of this policy is to ensure minimal interadmission time, θ, in order to prevent congestion in the main queueing system with a high holding cost; θ is left as a design parameter. Most of this paper is devoted to the perturbation analysis technique for estimating the sensitivity of the holding cost with respect to θ and stochastic optimization based on this estimate. Due to special aspects of the system model and the proposed control policy, the gradient estimate based on the

Infinitesimal Perturbation Analysis has a simple and interesting form. In section 3, the IPA estimator will be derived, and its properties (such as strong consistency) will be proven. In section 4, a stochastic gradient-like algorithm for tuning θ based on this estimate is studied in order to minimize the holding cost, and a convergence of this algorithm is proven under certain conditions.

2 Ensured Minimal Interadmission Time Policy

The performance measure considered in this paper is a weighted sum of the steady-state queueing delays of a customer in the controller and in the main system. A more generalized version of this queue control problem has been studied [16] using the theory of dynamic programming. However, the results are very limited: they might specify some properties of an optimal policy but this is not necessarily of any use in designing an optimal or near optimal policy. The exact specification of the optimal policy seems very difficult. For this reason, we bypass dynamic programming techniques in this paper and concentrate instead on whatever progress is possible in approximating optimal policies.

Suppose the service time is deterministic, for example $1/\mu$. If the time since the last admission is more than $1/\mu$, one can be sure that the main system is empty; therefore, a customer, if there is any, in the controller must be admitted. If the time since the last admission is less than $1/\mu$, one can be sure that the main system is busy, so a customer in the controller must wait until $1/\mu$ time units have elapsed since the last admission. In other words, the controller admits customers as soon as possible, subject to the constraint that the time between consecutive admissions is no less than $1/\mu$. We suggest applying such a simple scheme to the general case of random service time. We designate this scheme as *Ensured Minimal Interadmission Time (EMIT)* . In the case of random service time, however, the average service time $1/\mu$ may not be the best minimal ensured time between admissions. This ensured minimal time between admissions is left as a design parameter that is to be tuned according to the the holding cost C and the statistics of the service time in the main system. This parameter is denoted by θ.

3 An IPA–based Estimator

In this section, the sensitivity of the performance measure to the parameter θ will be estimated through the technique of infinitesimal perturbation analysis (IPA) [8].

Properties such as unbiasedness and strong consistency of this IPA estimator will also be established. The motivation of perturbation analysis is that only one sample path is used to estimate the gradient instead of two; computation is thereby reduced by roughly a factor of two. Also, more significantly, the variance of the gradient estimate is reduced as a result of using only one sample path; therefore faster convergence is expected for the stochastic optimization algorithm based on this gradient estimate.

Denote by X_i the service time of the i-th customer. Denote by A_i the interarrival time between the i-th and the $(i+1)$-st customer. Both $\{X_i\}$ and $\{A_i\}$ are sequences of independent and identically distributed random variables, and these two sequences are independent. We assume that both X_i and A_i have an absolutely continuous probability distribution and that $E(X_i) < E(A_i)$. We will denote a particular realization of arrival and service processes by $\xi \in \Omega$, where Ω is the sample space. A sequence $X_1(\xi), A_1(\xi), X_2(\xi), A_2(\xi), \cdots$ completely specifies ξ. Denote the i-th customer's waiting time in the controller by $W_i(\theta, \xi)$, and the response time in the main system by $T_i(\theta, \xi)$. Denote by $W(\theta)$ the steady-state waiting time in the controller. Denote by $T(\theta)$ the steady-state average response time in the main system. The expected cost function, which is a typical performance measure in this paper, is denoted by

$$J(\theta) \equiv EW(\theta) + CET(\theta) \tag{1}$$

We assume that the system is initially empty, and that the first arrival is admitted to the main system immediately. For a finite number, L, of customers, the sample performance function can be naturally defined as

$$G_L(\theta, \xi) = \frac{1}{L} \sum_{i=1}^{L} [\ W_i(\theta, \xi) + CT_i(\theta, \xi)\ ] \tag{2}$$

From this sample function, we derive a sample derivative with respect to θ, and use it as an estimate of the derivative of our performance measure. The next lemma shows that the sample performance function is continuous and piecewise linear in θ.

Lemma 1 *Both functions $W_i(\theta, \xi)$ and $T_i(\theta, \xi)$ are continuous and piecewise linear functions of θ for any sample path ξ for a finite number of customers.*

Proof:

$$\begin{aligned} W_1(\theta, \xi) &= 0 \\ W_{i+1}(\theta, \xi) &= \max[\ 0,\ W_i(\theta, \xi) + \theta - A_i(\xi)\ ] \end{aligned}$$

By induction, $W_i(\theta, \xi)$ is a continuous and piecewise linear function of θ for any sample path ξ. Denote the difference between the admission time of the $(i+1)$-st customer and the i-th customer by $B_i(\theta, \xi)$. Then

$$\begin{aligned} B_i(\theta, \xi) &= \{ \sum_{j=1}^{i} A_j(\xi) + W_{i+1}(\theta, \xi) \} - \{ \sum_{j=1}^{i-1} A_j(\xi) + W_i(\theta, \xi) \} \\ &= A_i(\xi) + W_{i+1}(\theta, \xi) - W_i(\theta, \xi) \end{aligned}$$

Therefore, $B_i(\theta, \xi)$ is also a continuous and piecewise linear function of θ for any sample path ξ. Note that

$$\begin{aligned} T_1(\theta, \xi) &= \theta \\ T_{i+1}(\theta, \xi) &= \max[\, 0,\ T_i(\theta, \xi) - B_i(\theta, \xi)\,] + X_{i+1}(\xi) \end{aligned}$$

Again by induction, $T_i(\theta, \xi)$ is continuous and piecewise linear function of θ for any sample path ξ. **Q.E.D.**

Since G_L is piecewise linear, its sample derivative is merely its slope, except at the point at which the slope changes. The derivative does not exist for those values of θ at which the slope changes. It will become clear later that for a fixed $\hat{\theta}$, the sample performance function is differentiable at $\hat{\theta}$ with probability 1. We now discuss a technique to calculate and simply represent the sample derivative $\frac{\partial G_L}{\partial \theta}(\theta, \xi)$.

1. The admissions into the main system can be partitioned according to the inter-admission times. Interadmission times strictly greater than θ are demarcations of this partition. Each segment is comprised of a train of admissions separated by length θ in time. The admissions in a segment of size m can be chronologically labeled as $0, 1, 2, \cdots, m-1$. Each admission belongs to a segment and can be labeled as above. We define a mapping $\tilde{l}$ from the set of admissions to the set of nonnegative integers representing such labels. We denote the admission label of the n-th admission, or the n-th customer, by $\tilde{l}_n(\theta, \xi)$.

2. For a finite number of customers, we can perturb θ with probability 1 by a sufficiently small $\Delta\theta$ without changing the admission label of any customer. If θ is increased by a sufficiently small $\Delta\theta$, the n-th admission is delayed by $\tilde{l}_n(\theta, \xi)\ \Delta\theta$. The reason is that a sufficiently small change of θ does not affect the admission time of the first customer in a segment and delays successive admissions within a segment by successive multiples of $\Delta\theta$. (See Figure 2.)

3. The queue size of the main system is governed by the service times of customers as well as the admission times. With probability 1, there is a sufficiently small

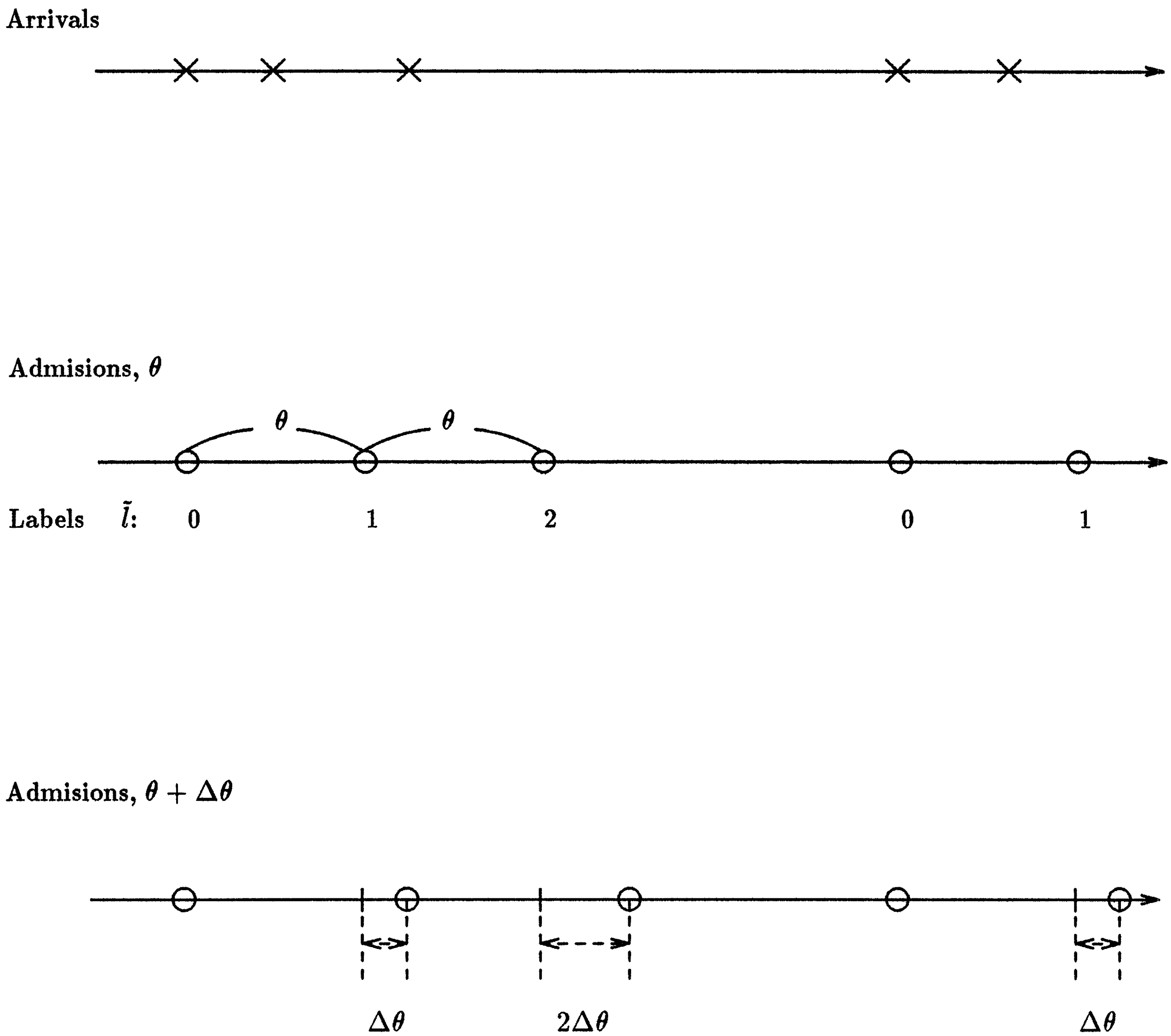

Figure 2: **Slope of $W(\theta, \xi)$ with respect to θ**

$\Delta\theta$ such that changing θ by $\Delta\theta$ neither merges any two busy periods into one, nor separates any single busy period into two.

4. The change of the response time of a customer is equal to the change of its service completion time minus the change of its admission time.

5. If the admission that initiates a busy period is delayed by a sufficiently small $\delta > 0$, all the service completions in that busy period are delayed by δ. If the busy period initiating admission is not delayed, (i.e. if this admission has label $\tilde{l} = 0$), the service completion times remain unchanged.

6. Denote by $b_n(\theta, \xi)$ the label of the admission that initiates the busy period in which customer n is served. The change of the response time of customer n is

$$(b_n(\theta,\xi) - \tilde{l}_n(\theta,\xi)\ )\Delta\theta \tag{3}$$

Note that for G_L to be nondifferentiable at θ, the service completions or admissions must coincide in time at least once among L customers in the sample path. This event has probability 0. The total change is linear in $\Delta\theta$ for a sufficiently small range of $\Delta\theta$. Therefore, the derivative of the sample performance function is

$$\begin{aligned} \frac{\partial G_L}{\partial\theta}(\theta,\xi) &= \frac{1}{L}\sum_{i=1}^{L}[\ \tilde{l}_i(\theta,\xi) + C\{\ b_i(\theta,\xi) - \tilde{l}_i(\theta,\xi)\ \}] \qquad a.s. \\ &\equiv \frac{1}{L}\sum_{i=1}^{L} l_i(\theta,\xi)\ , \end{aligned} \tag{4}$$

where the label $l_i(\theta,\xi)$ of customer i is defined $l_i(\theta,\xi) = \tilde{l}_i(\theta,\xi) + C(\ b_i(\theta,\xi) - \tilde{l}_i(\theta,\xi)\ )$. We use this as an estimate of the gradient of the performance measure.

3.1 Performance during the Transient Period

Consider the expected value of the average cost of L customers $EG_L(\theta,\xi)$ as a performance measure. We claim that the estimate of $\frac{\partial}{\partial\theta}E[G_L(\theta,\xi)]$ given by (4) is unbiased. The concept of dominant differentiability from [8, p64] is used for the proof.

Theorem 2 *The transient performance measure $EG_L(\theta)$ is continuously differentiable, and the estimator in formula (4) is unbiased; that is,*

$$E\frac{1}{L}\sum_{i=1}^{L} l_i(\theta,\xi) = \frac{\partial}{\partial\theta}EG_L(\theta)$$

Proof: Define $r(\theta, \Delta\theta, \xi)$ such that

$$G_L(\theta + \Delta\theta, \xi) = G_L(\theta, \xi) + \frac{\partial G_L}{\partial \theta}(\theta, \xi)\Delta\theta + r(\theta, \Delta\theta, \xi)$$

Since $\frac{\partial G_L}{\partial \theta}(\theta, \xi)$ almost surely exists,

$$\lim_{\Delta\theta \to 0} \frac{r(\theta, \Delta\theta, \xi)}{\Delta\theta} = 0 \quad a.s.$$

Since both W_i and T_i are continuous and piecewise linear in θ for any ξ, and labels satisfy

$$\tilde{l}_i(\theta, \xi), b_i(\theta, \xi) \ \leq \ L \quad \forall \theta, \xi, \forall i \leq L, \tag{5}$$

we have $|W_i(\theta + \Delta\theta, \xi) - W_i(\theta, \xi)| \leq L\Delta\theta$ and $|T_i(\theta + \Delta\theta, \xi) - T_i(\theta, \xi)| \leq 2L\Delta\theta$ for all θ, $\Delta\theta$, ξ. Therefore, for all θ, $\Delta\theta$, ξ, we have

$$|G_L(\theta + \Delta\theta, \xi) - G_L(\theta, \xi)| \ \leq \ (L + 2CL)\Delta\theta \tag{6}$$

Combining (4), (5), (6), we have

$$\left| \frac{r(\theta, \Delta\theta, \xi)}{\Delta\theta} \right| = \left| \frac{G_L(\theta + \Delta\theta, \xi) - G_L(\theta, \xi)}{\Delta\theta} - \frac{\partial G_L}{\partial \theta}(\theta, \xi) \right| \leq 2(L + 2CL) \quad a.s.$$

Hence, using Lebesgue's dominated convergence theorem [21, p44], we establish

$$E\frac{\partial G_L}{\partial \theta}(\theta, \xi) = \frac{\partial}{\partial \theta} EG_L(\theta)$$

Also, from (5) we have $\left| \frac{1}{L} \sum_{i=1}^{L} l_i(\theta, \xi) \right| \leq L + 2CL$, so again by the dominated convergence theorem

$$\lim_{\theta \to \theta_0} \frac{dEG_L}{d\theta}(\theta) = \lim_{\theta \to \theta_0} E\frac{1}{L}\sum_{i=1}^{L} l_i(\theta, \xi) = \frac{dEG_L}{d\theta}(\theta_0)$$

Therefore, $\frac{dEG_L}{d\theta}(\theta)$ is continuous. **Q.E.D.**

3.2 Steady State Performance

In this subsection, we show that the estimator given by (4) almost surely converges to the gradient of the steady-state performance measure as the number of customers grows (strong consistency). Results of [10] cannot be used to establish the strong consistency because the sample performance function (2) is not convex or concave of θ. This paper applies and extends the techniques used in [5].

The state of the system is fully specified by the number of customers in the controller, the number of customers in the main system, the time since the last admission, and the time since the last service completion. We define a "regenerative point" as a time at which

- 1) Both the controller and the main system are empty.
- 2) At least time θ has elapsed since the latest admission.

(The system exits from the regenerative point when the next customer arrives.) Define $N_k(\theta)$ to be the number of customers served between the $(k-1)$-th and the k-th regenerative points. Then, $\{N_k(\theta)\}$ is an i.i.d. process. We refer to the interval between the $(k-1)$-th and the k-th regenerative points as the k-th period. Note that $N_k(\theta)$ is a stopping time with respect to the increasing sequence of $\sigma - algebras$ $\{\mathcal{F}_i\}$, where

$$\mathcal{F}_i \equiv \sigma(X_1, A_1, X_2, A_2, \ldots, X_i, A_i)$$

Define the accumulated number of customers, $R_k(\theta)$ by

$$R_k(\theta) = \sum_{j=1}^{k} N_j(\theta) \quad k \geq 1 \qquad R_0(\theta) = 0$$

Note that for each k and θ, $R_k(\theta)$ is again a stopping time with respect to $\{\mathcal{F}_i\}$. The following lemma is a key conceptual tool that allows us to remove the θ- dependency of certain functions from our analysis. This lemma states that for each fixed sample path, when a system with parameter θ_{max} reaches a regenerative point, a system with smaller parameter $\theta < \theta_{max}$ also reaches a regenerative point.

Lemma 3 *For each sample path ξ, there exists a sequence $\{K_k\}$ such that*

$$R_k(\theta_{max}) = R_{K_k}(\theta)$$

That is, $R_k(\theta_{max})$ is a subsequence of $R_k(\theta)$.

Proof: For any realization of the random variables $X_1(\xi), A_1(\xi), X_2(\xi), A_2(\xi), \ldots$ each customer is admitted earlier in the case of the parameter θ than the case of θ_{max}, if $\theta < \theta_{max}$. As a result, each customer departs earlier for the parameter θ. It suffices to prove that for an arbitrary ξ, if a system with parameter θ_{max} reaches a regenerative point at some time, a system with any $\theta < \theta_{max}$, is also in a regenerative point. Suppose that at time t, a regenerative point is reached in the case of parameter θ_{max}. Then, at that time, both queues are empty, and the latest admission occurred before $t-\theta_{max}$. There is no arrival between $t-\theta_{max}$ and t. Consider $\theta < \theta_{max}$ for the identical realization ξ. At time t, both queues are empty because each customer departs earlier than the case of θ_{max}. The latest admission occurs before $t-\theta$ because each customer is admitted earlier than the case of θ_{max}. Therefore, at time t, the system with parameter θ is at a regenerative point. **Q.E.D.**

Throughout this paper we assume that the probability distributions of X and A are such that

$$E[N_k(\theta)^4] \;<\; \infty, \quad \forall \theta < E(X) \tag{7}$$

The following lemma concerns the relationship between the cost function given by (1) and the estimator. This relationship is useful in establishing strong consistency of our estimator. Later, this lemma will be used in establishing convergence of an IPA-based stochastic algorithm.

Lemma 4 *For an arbitrary $\theta_{max} < E(A)$, the cost function $J(\theta)$ is continuously differentiable in $[0, \theta_{max}]$, and*

$$\frac{dJ}{d\theta}(\theta) = \frac{E\sum_{i=1}^{N(\theta)} l_i(\theta,\xi)}{EN(\theta)}$$

Proof: Since $R_k(\theta_{max})$ is a number of customers served between certain regenerative points for the case of θ, from the theory of regenerative systems [22],

$$EW(\theta,\xi) = \frac{E\sum_{i=1}^{N(\theta_{max})} W_i(\theta)}{EN(\theta_{max})} \qquad ET(\theta,\xi) = \frac{E\sum_{i=1}^{N(\theta_{max})} T_i(\theta)}{EN(\theta_{max})}$$

With probability 1, we have

$$\left| \frac{d}{d\theta} \sum_{i=1}^{N(\theta_{max})} W_i(\theta,\xi) \right| \;=\; \left| \sum_{i=1}^{N(\theta_{max})} \tilde{l}_i(\theta,\xi) \right| \leq \sum_{i=1}^{N(\theta_{max})} \left| \tilde{l}_i(\theta,\xi) \right| \leq N(\theta_{max})^2 \tag{8}$$

$$\begin{aligned} \left| \frac{d}{d\theta} \sum_{i=1}^{N(\theta_{max})} T_i(\theta) \right| = \left| \sum_{i=1}^{N(\theta_{max})} \{b_i(\theta,\xi) - \tilde{l}_i(\theta,\xi)\} \right| \;&\leq\; \sum_{i=1}^{N(\theta_{max})} \left| b_i(\theta,\xi) - \tilde{l}_i(\theta,\xi)\} \right| \\ &\leq\; 2N(\theta_{max})^2 \end{aligned} \tag{9}$$

Assumption (7) gives $E\left[N(\theta_{max})^2\right] < \infty$. Also, both W_i and T_i are continuous and piecewise linear functions of θ for any sample path ξ. Therefore, from the dominated convergence theorem [21, p44], and by an argument identical to the one used in Theorem 2, we have

$$\frac{d}{d\theta} E \sum_{i=1}^{N(\theta_{max})} W_i(\theta,\xi) = E \sum_{i=1}^{N(\theta_{max})} \frac{d}{d\theta} W_i(\theta,\xi) = E \sum_{i=1}^{N(\theta_{max})} \tilde{l}_i(\theta,\xi)$$

$$\frac{d}{d\theta} E \sum_{i=1}^{N(\theta_{max})} T_i(\theta,\xi) = E \sum_{i=1}^{N(\theta_{max})} \frac{d}{d\theta} T_i(\theta,\xi) = E \sum_{i=1}^{N(\theta_{max})} \{b_i(\theta,\xi) - \tilde{l}_i(\theta,\xi)\}$$

Therefore,

$$\frac{d}{d\theta}J(\theta) = \frac{E\sum_{i=1}^{N(\theta_{max})} l_i(\theta,\xi)}{EN(\theta_{max})} \tag{10}$$

Lemma 3 states $N(\theta_{max}) = \sum_{k=1}^{K_1} N_k(\theta) = R_{K_1}(\theta)$ for some K_1, so the following equations can be obtained using Wald's equality [23, p.460] as in [5].

$$\begin{aligned} E[\ N(\theta_{max})\ ] &= E(K_1)E(N_1) && (11) \\ E[\ \sum_{i=1}^{N(\theta_{max})} l_i(\theta,\xi)\ ] &= E(K_1)E[\ \sum_{i=1}^{N(\theta)} l_i(\theta,\xi)\ ] && (12) \end{aligned}$$

(See [18] for details.) From (10), (11), and (12), we prove

$$\frac{d}{d\theta}J(\theta) = \frac{E\sum_{i=1}^{N(\theta)} l_i(\theta,\xi)}{EN(\theta)}$$

Also, from (8) and (9) we have $|\sum_{i=1}^{N(\theta)} l_i(\theta,\xi)| \leq (1+C)N(\theta_{max})^2$, so by the dominated convergence theorem $\frac{dJ}{d\theta}$ is continuous in $[0,\theta_{max}]$. **Q.E.D.**

Now we can prove strong consistency of our estimator.

Theorem 5

$$\lim_{L\to\infty} \frac{1}{L}\sum_{i=1}^{L} l_i(\theta,\xi) = \frac{dJ}{d\theta}(\theta) \qquad a.s.$$

Proof: Denote the number of occurrences of regenerative points until service completion of L customers by $M_L(\theta,\xi) = \max\{k = 1,2,\cdots | R_k(\theta) \leq L\}$, then we have $\lim_{L\to\infty} M_L(\theta,\xi) = \infty$ with probability 1. Using the strong law of large numbers,

$$\lim_{L\to\infty} \frac{L}{M_L} = \frac{\sum_{k=1}^{M_L} N_k + L - R_{M_L}}{M_L} = E(N) \qquad a.s. \tag{13}$$

Again, by the strong law of large numbers

$$\begin{aligned} \lim_{L\to\infty} \frac{\sum_{i=1}^{L} l_i(\theta,\xi)}{M_L} &= \lim_{L\to\infty} \frac{\sum_{k=1}^{M_L}\sum_{i=R_{k-1}+1}^{R_k} l_i(\theta,\xi) + \sum_{i=R_{M_L}+1}^{L} l_i(\theta,\xi)}{M_L} \\ &= E\sum_{i=1}^{N(\theta)} l_i(\theta,\xi) \qquad a.s. && (14) \end{aligned}$$

From equations (13) (14) and from Lemma 4, we have

$$\lim_{L\to\infty} \frac{1}{L}\sum_{i=1}^{L} l_i(\theta,\xi) = \frac{E\sum_{i=1}^{N(\theta)} l_i(\theta,\xi)}{EN(\theta)} = \frac{dJ}{d\theta}(\theta)$$

Q.E.D.

4 A Stochastic Gradient–like Algorithm

In this section, we discuss how to optimize the introduced parameter θ. The main difficulty in this optimization is that the cost function is not known in closed form, and can be evaluated only through simulation. As an optimization algorithm, this paper suggests a stochastic gradient-like algorithm [24][25], which resorts to simulation and the IPA at each iteration to estimate of the gradient of the cost function. IPA–based optimization algorithms have been previously studied [5] [26] [27] [28]. The contribution of this section is to provide an example that applies the IPA-based stochastic optimization algorithm to an open–loop admission control problem.

Let θ^* be a minimizer of $J(\theta)$ defined in (1). As θ approaches $E(A)$, $W(\theta)$ grows to infinity; therefore, we can pick some $\theta_{max} < E(A)$ such that we can be sure that $\theta^* \in [0, \theta_{max}]$ through crude analysis. We simulate customers until a regenerative point is reached and we denote the numerator of the IPA estimate in equation (4) by

$$\hat{h}(\theta) \equiv \sum_{i=1}^{N(\theta,\xi)} l_i(\theta, \xi)$$

We apply the stochastic optimization algorithm in [5] with this gradient estimate.

Algorithm 6

$$\theta_{n+1} = \pi_{n+1}[\ \theta_n + a_n \hat{h}(\theta_n)\ ]$$

where π is a projection operator defined by

$$\pi_{n+1}[x] = \begin{cases} x & \text{if } x \in [0, \theta_{max}] \\ \theta_n & \text{otherwise} \end{cases}$$

As for the step size a_n, the following restrictions are imposed:
Sequence $\{a_n\}$ is adapted to the sequence of σ – algebras

$$\mathcal{G}_n(\theta) \equiv \mathcal{F}_{R_n(\theta)} \equiv \{A \in \mathcal{F} \mid A \cap \{R_n(\theta) = l\} \in \mathcal{F}_l,\ \forall l\} \quad \text{where } \mathcal{F} = \cap_{i=1}^{\infty} \mathcal{F}_i$$

and

$$\sum_{n=1}^{\infty} a_n = \infty \quad a.s. \tag{15}$$

$$\sum_{n=1}^{\infty} a_n^2 < \infty \quad a.s. \tag{16}$$

(A typical example is $a_n = 1/n$.)

The conditions for a convergence of this algorithm is specified in the following lemma along with assumption (7).

Theorem 7 *If J has a unique minimum, θ in $[0, \theta_{max}]$, and*

$$\frac{dJ}{d\theta}(\theta) \neq 0 \quad \forall \theta \neq \theta^* \ ,$$

then, Algorithm 6 converges with probability 1 *to the minimum; that is,*

$$\theta_n \rightarrow \theta^* \quad a.s.$$

Proof: Algorithm 6 can be written as

$$\theta_{n+1} = \pi_{n+1}\left[\ \theta_n + a_n E\{N(\theta_n)\}\frac{\hat{h}(\theta_n)}{E\{N(\theta_n)\}}\ \right]$$

Define the error of the estimate by

$$\mathcal{E}_n \equiv \frac{\hat{h}(\theta_n)}{E\{N(\theta_n)\}} - \frac{dJ}{d\theta}(\theta_n)$$

Then, from Lemma 4

$$E[\mathcal{E}_{n+1}|\mathcal{G}_n] \ = \ 0 \tag{17}$$

Using Lemma 4 and a few lines of algebra, we can easily derive

$$E[\mathcal{E}^2_{n+1}|\mathcal{G}_n] \ \leq \ E[\ N(\theta_{max})^4\ ] < \infty \tag{18}$$

Also,

$$\sum_{n=1}^{\infty} a_n E\{N(\theta_n)\} \ \geq \ \sum_{n=1}^{\infty} a_n = \infty \ \ a.s. \tag{19}$$

$$\sum_{n=1}^{\infty} a_n^2 [E\{N(\theta_n)\}]^2 \ \leq \ \sum_{n=1}^{\infty} a_n^2 [E\{N(\theta_{max})\}]^2 < \infty \ \ a.s. \tag{20}$$

Lemma 4 ensures that $J(\theta)$ is continuously differentiable in $[0, \theta_{max}]$, and implies that the derivative of J is bounded in $[0, \theta_{max}]$. Therefore, relations (17), (18), (19), (20) guarantee almost sure convergence (see [3] for standard conditions for convergence). **Q.E.D.**

5 Discussion

The optimal admission scheme bases its admission decision on the number of customers in the controller buffer and the contemporary probability distribution of the main system's queue size. On the other hand, the $EMIT$ scheme introduced in this paper makes the admission decision without considering the number of customers in the controller buffer. The $EMIT$ scheme can be improved by also using information on the controller's queue size. We can keep track of the time elapsed since the latest admission as in the $EMIT$ scheme, but apply a different threshold depending on the controller's queue size. That is, at each moment, if the controller queue size is n, we admit a customer only if the time since the latest admission is more than a certain threshold $\theta(n)$ that depends on n. The set of $\theta(n)$ is to be tuned according to the cost C and the statistics of the service time. For example, we can truncate the set of $\theta(n)$ (if the controller queue size is N or more, apply a threshold $\theta(N)$), and optimize the vector,

$$(\theta(1), \theta(2), \cdots, \theta(N)).$$

The study of an N-dimensional stochastic algorithm that optimizes this vector parameter is left for future work.

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